Sample Question Paper, 2021-22

(Issued by CBSE Board on 14th January, 2022)

MATHEMATICS STANDARD(Term- II)

SOLVED

Time: 2 Hours **Max. Marks: 40**

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

Section - A

[2 Marks Each]

1. Find the value of $a_{25} - a_{15}$ for the AP: 6, 9, 12, 15, ...

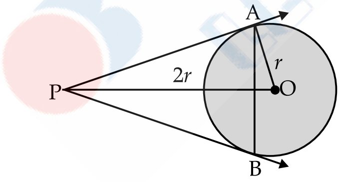
OR

If 7 times the seventh term of the *AP* is equal to 5 times the fifth term, then find the value of its 12th term.

2. Find the value of m so that the quadratic equation $mx(5x-6)+q=0$ has two equal roots.

3. From a point P, two tangents PA and PB are drawn to a circle C(0, r).

If $OP = 2r$, then find $\angle APB$. What type of triangle is APB?



4. The curved surface area of a right circular cone is 12320 cm². If the radius of its base is 56 cm, then find its height.

5. Mrs. Garg recorded the marks obtained by her students in the following table. She calculated the modal marks of the students of the class as 45. While printing the data, a blank was left. Find the missing frequency in the table given below.

Marks Obtained	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of Students	5	10	–	6	3

6. If Ritu were younger by 5 years than what she really is, then the square of her age would have been 11 more than five times her present age. What is her present age?

OR

Solve for x : $9x^2 - 6px + (p^2 - q^2) = 0$

Section - B

[3 Marks Each]

7. Following is the distribution of the long jump competition in which 250 students participated. Find the median distance jumped by the students. Interpret the median

Distance (in m)	0 – 1	1 – 2	2 – 3	3 – 4	4 – 5
Number of Students	40	80	62	38	30

8. Construct a pair of tangents to a circle of radius 4 cm, which are inclined to each other at an angle of 60°.

9. The distribution given below shows the runs scored by batsmen in one-day cricket matches. Find the mean number of runs.

Runs scored	0 – 40	40 – 80	80 – 120	1200 – 160	160 – 200
Number of batsmen	12	20	35	30	23

10. Two vertical poles of different heights are standing 20 m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. (Take $\sqrt{3}$ = 1.73)

OR

A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building. (Take $\sqrt{3}$ = 1.73)

Section - C

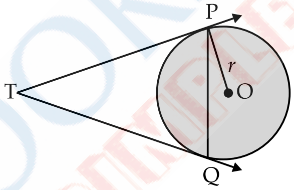
[4 Marks Each]

11. The internal and external radii of a spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder. Also find the total surface area of the cylinder. (Take $\pi = \frac{22}{7}$)

OR

Two tangents *TP* and *TQ* are drawn to a circle with centre *O* from an external point *T*.

Prove that $\angle PTQ = 2\angle OPQ$



12. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre.

Case Study-1

13. Trigonometry in the form of triangulation forms the basis of navigation, whether it is by land, sea or air. GPS a radio navigation system helps to locate our position on earth with the help of satellites.

A guard, stationed at the top of a 240 m tower, observed an unidentified boat coming towards it. A clinometer or inclinometer is an instrument used for measuring angles or slopes(tilt). The guard used the clinometer to measure the angle of depression of the boat coming towards the lighthouse and found it to be 30°.

(Lighthouse of Mumbai Harbour. Picture credits - Times of India Travel)

(i) Make a labelled figure on the basis of the given information and calculate the distance of the boat from the foot of the observation tower. **[2]**

(ii) After 10 minutes, the guard observed that the boat was approaching the tower and its distance from tower is reduced by $240(\sqrt{3} - 1)$ m. He immediately raised the alarm. What was the new angle of depression of the boat from the top of the observation tower? **[2]**

Case Study-2

14. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.

Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour.

But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind answer the following questions:

(i) Form an A.P representing the number of push-ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished? **[2]**

(ii) Find the total number of push-ups performed by Nitesh up to the day his goal is achieved. **[2]**

■■■

CBSE Marking Scheme 2021-22
(Issued by Board)

Section - A

1.

$$a = 6, d = 3;$$
$$a_{25} = 6 + 24(3) = 78$$
$$a_{15} = 6 + 14(3) = 48$$
$$a_{25} - a_{15} = 78 - 48$$
$$a_{25} - a_{15} = 30$$

OR

$$7(a + 6d) = 5(a + 4d)$$
$$\Rightarrow \quad 2a + 22d = 0$$
$$\Rightarrow \quad a + 11d = 0$$
$$\Rightarrow \quad t_{12} = 0$$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

6, 9, 12, 15, ... is the given A.P.

First term $= a = 6$

Common Difference $= d = 9 - 6 = 3$

n^{th} term $= a_n = a + (n-1)d$

25^{th} term $= a_{25}$

$$= 6 + (25-1) \times 3$$
$$= 6 + (24) \times 3$$
$$= 6 + 72$$
$$= 78$$

15^{th} term $= a_{15}$

$$= 6 + (15-1) \times 3$$
$$= 6 + (14) \times 3$$
$$= 6 + 42$$
$$= 48$$

$$a_{25} - a_{15} = 78 - 48 = 30$$

OR

Let 'a' and 'd' be the first term and common difference of AP

n^{th} term $= a_n = a + (n-1)d$

7^{th} term $= a_7 = a + (7-1)d = a + 6d$

5^{th} term $= a_5 = a + (5-1)d = a + 4d$

According to the Questions,

$$7a_7 = 5a_5$$
$$7(a + 6d) = 5(a + 4d)$$
$$7a + 42d = 5a + 20d$$
$$7a - 5a + 42d - 20d = 0$$
$$2a + 22d = 0$$
$$2(a + 11d) = 0$$
$$a + 11d = 0 \quad ...(i)$$

12^{th} term $= a_{12}$

$$= a + 11d$$
$$= 0 \quad \text{(From (i))}$$

Hence, 12^{th} term is zero.

2. Given $5mx^2 - 6mx + 9 = 0$

Since, $b^2 - 4ac = 0$

for equal roots

$$(-6m)^2 - 4(5m)(9) = 0$$
$$\Rightarrow \quad 36m(m-5) = 0$$
$$\Rightarrow \quad m = 0, 5; \text{ rejecting } m = 0,$$

we get $m = 5$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Since, $mx(5x-6) + 9 = 0$ has equal roots

$\therefore$ Discriminant $= 0$

$$\Rightarrow \quad b^2 - 4ac = 0 \quad ...(i)$$

Quadratic equation can be written as

$$5mx^2 - 6mx + 9 = 0$$

Here, $a = 5m, b = -6m, c = 9$

Put in (i),

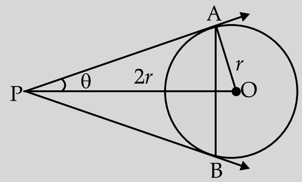
$$(6m)^2 - 4(5m)(9) = 0 \qquad m = 0$$
$$= 36m^2 - 180m = 0 \qquad \text{or } m = 5$$
$$= 36m(m-5) = 0$$

Either $m = 0$ or $m = 5$

Since, $m = 0$ is not possible

Hence, $m = 5$

3.



Let $\angle APO = \theta$

$$\sin\theta = \frac{OA}{OP} = \frac{1}{2} \Rightarrow \theta = 30°$$
$$\Rightarrow \quad \angle APB = 2\theta = 60°$$

Also $\angle PAB = \angle PBA = 60°$

$(\because PA = PB)$

$\Rightarrow \Delta APB$ is equilateral

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Radius of given circle $OA = r$ units

Let, $\angle APO = \theta$

Radius is always perpendicular to tangent

$= \quad OA \perp AP$

So, $\angle OAP = 90°$

$\therefore \Delta OAP$ is a right angled triangle

In ΔOAP,

$$\sin\theta = \frac{OA}{OP}$$

$$\sin\theta = \frac{r}{2r} = \frac{1}{2}$$

$$\sin\theta = \sin 30°$$

$\Rightarrow \quad \theta = 30°$

$\angle APO = 30°$

$\angle APB = 2 \times \angle APO = 2 \times 30° = 60°$

$PA = PB$ [Length of tangents from an external point are equal in length]

Let, $\angle ABP = \angle BAP = x$

So, from ΔAPB,

$\angle APB + \angle ABP + \angle BAP = 180°$ [Angle sum property of triangle]

$60° + x + x = 180°$

or $x = 60°$

$\therefore$ ΔAPB is equilateral triangle.

4. CSA (cone) $= \pi rl = 12320\ \text{cm}^2$

$$\frac{22}{7} \times 56 \times l = 12320$$

$$l = 70\ \text{cm}$$

$$h = \sqrt{70^2 - 56^2} = 42\ \text{cm}$$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Let Height of cone $= h$

Radius of cone $= 56$ cm

C.S.A of cone $= 12320\ \text{cm}^2$

Let slant height of cone $= l$ cm

Curved surface area of cone $= \pi rl$

$\Rightarrow \quad 12320 = \frac{22}{7} \times 56 \times l$

$\Rightarrow \quad \frac{12320 \times 7}{22 \times 56} = l$

$\Rightarrow \quad l = 70$ cm

Now, $l^2 = h^2 + r^2$

$\Rightarrow \quad (70)^2 = h^2 + (56)^2$

$\Rightarrow \quad h^2 = 4900 - 3136$

$\Rightarrow \quad h^2 = 1764$

$\Rightarrow \quad h^2 = 42^2$

$\Rightarrow \quad h = 42$ cm

5. Modal class is 40 – 60, $l = 40$,

$h = 20, f_1 = = ?,$

$f_0 = 10, f_2 = 6$

$$45 = 40 + 20 \times \left[\frac{f_1 - 10}{2f_1 - 10 - 6}\right]$$

$$\Rightarrow \quad \frac{1}{4} = \frac{f_1 - 10}{2f_1 - 16}$$

$\Rightarrow \quad 2f_1 - 16 = 4f_1 - 40$

$\Rightarrow \quad f_1 = 12$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Marks obtained	Number of students
0 - 20	5
20 - 40	10
40 - 60	x (Say)
60 - 80	6
80 - 100	3

Mode $= 45$

$\Rightarrow$ Modal class $= 40 - 60$

Lower limit of modal class $(l) = 40$

Size of modal class $(h) = 20$

Frequency corresponding to modal class

$(F_1) = x$

Frequency preceding to modal class

$(F_0) = 10$

Frequency preceding to modal class $(F_2) = 6$

$$\text{Mode} = l + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} \times h$$

$$\Rightarrow \quad 45 = 40 + \frac{x - 10}{2x - 10 - 6} \times 20$$

$$\Rightarrow \quad 5 = \frac{x - 10}{2x - 16} \times 20$$

$$\Rightarrow \quad \frac{5}{20} = \frac{x - 10}{2x - 16}$$

$$\frac{1}{4} = \frac{x - 10}{2x - 16}$$

$\Rightarrow \quad 2F_1 - 16 = 4F_1 - 40$

$\Rightarrow \quad 2F_1 - 4F_1 = -40 + 16$

$\Rightarrow \quad -2F_1 = -24$

$\Rightarrow \quad F_1 = 12$

6. Let the present age of Ritu be x years

$$(x-5)^2 = 5x + 11$$
$$x^2 - 15x + 14 = 0$$
$$(x-14)(x-1) = 0$$
$$\Rightarrow \quad x = 1 \text{ or } 14$$

$x = 14$ years (rejecting $x = 1$ as in that case Ritu's age 5 years ago will be – ve)

OR

$$9x^2 - 6px + (p^2 - q^2) = 0$$
$$a = 9, b = -6p, c = p^2 - q^2$$
$$D = b^2 - 4ac$$
$$= (-6p)^2 - 4(9)(p^2 - q^2)$$
$$= 36q^2$$
$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6p \pm 6q}{18}$$
$$= \frac{p+q}{3} \text{ or } \frac{p-q}{3}$$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Let present age of Ritu = x years

As per question given the following equation can be formed:

$$\Rightarrow \quad (x-5)^2 = 11 + 5x$$
$$\Rightarrow \quad x^2 - 10x + 25 = 11 + 5x$$
$$\Rightarrow \quad x^2 - 10x - 5x + 25 - 11 = 0$$
$$\Rightarrow \quad x^2 - 15x + 14 = 0$$
$$\Rightarrow \quad x^2 - 14x - 1x + 14 = 0$$
$$\Rightarrow \quad x(x-14) - 1(x-14) = 0$$
$$\Rightarrow \quad (x-14)(x-1) = 0$$
$$\Rightarrow \quad x - 14 = 0, \; x - 1 = 0$$
$$x = 14, \; x = 1$$

Since, her age can't be 1, so her present age will be 14 years.

OR

$$9x^2 - 6px + (p^2 - q^2) = 0$$

In comparing with

$$ax^2 + bx + c = 0$$
$$a = 9, b = -6p, c = p^2 - q^2$$

Solve by quadratic formula,

$$D = b^2 - 4ac$$
$$= (-6p)^2 - 4(9)(p^2 - q^2)$$
$$= 36p^2 - 36p^2 + 36q^2$$
$$= 36q^2$$

Now, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6p) \pm \sqrt{36q^2}}{2(9)}$$
$$= \frac{6p \pm 6q}{18}$$
$$\therefore \quad x = \frac{p+q}{3} \text{ or } \frac{p-q}{3}$$

Section - B

7.

Distance (in m)	0 – 1	1 – 2	2 – 3	3 – 4	4 – 5
Number of Students	40	80	62	38	30
cf	40	120	182	220	250

$$\frac{n}{2} = \frac{250}{2}$$
$$= 125 \Rightarrow \text{median class is } 2 - 3,$$
$$l = 2, h = 1, cf = 120, f = 62$$
$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$
$$= 2 + \frac{5}{62}$$
$$= \frac{129}{62} = 2\frac{5}{62} \text{ m or } 2.08 \text{ m}$$

50% of students jumped below $2\frac{5}{62}$ m and 50% above it. **[CBSE Marking Scheme, 2021-22]**

Detailed Solution:

Distance (in m)	No. of students (Frequency)	Cumulative frequency
0 – 1	40	40
1 – 2	80	120
2 – 3	62	182
3 – 4	38	220
4 – 5	30	250
	$N = 250$	

Take $\frac{N}{2} = \frac{250}{2} = 125$

$\therefore$ Median class $= 2 - 3$

Lower limit of median class (l) = 2

Size of median class (h) = 1

Frequency corresponding to median class (f) = 62

Total number of observations Frequency (N) = 250

Cumulative frequency preceding Median class ($c.f.$) = 120

$$\text{Median} = l + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 2 + \frac{125-120}{62} \times 1$$

$$= 2 + \frac{5}{62}$$

$$= \frac{129}{62}$$

$$= 2\frac{5}{62} \text{ m or } 2.08 \text{ m}$$

$\therefore$ 50% of students jumped below 2.08 m and 50% of students jumped above 2.08 m.

8. Draw circle of radius 4cm

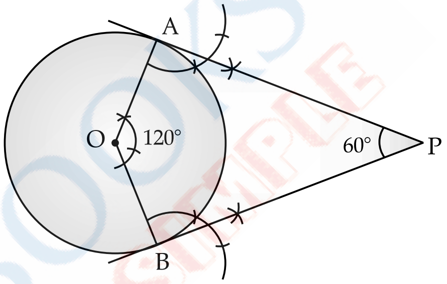
Draw OA and construct $\angle AOB = 120°$

Draw $\angle OBP = \angle OBP = 90°$

PA and PB are required tangents

[CBSE Marking Scheme, 2021-22]

Detailed Solution:



Steps of construction:

1. Draw a circle of radius 4 cm.
2. Draw two radii having an angle of 120°.
3. Let the radii intersect circle at A and B.
4. Draw angle of 90° on both A and B.
5. The point where both rays of 90° intersect is P.
6. PA and PB are the required tangents.

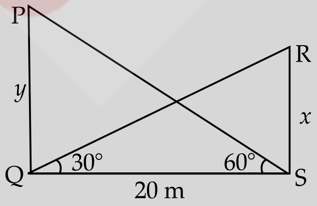
9.

Runs Scored	0 – 40	40 – 80	80 – 120	120 – 160	160 – 200	Total
Number of Batsmen (f_i)	12	20	35	30	23	$\Sigma f_i = 120$
x_i	20	60	100	140	180	
$f_i x_i$	240	1200	3500	4200	4140	$\Sigma f_i x_i = 13280$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{13280}{120} = 110.67 \text{ runs}$$

[CBSE Marking Scheme, 2021-22]

10.



In ΔPQS, $\tan 60° = \frac{y}{20}$

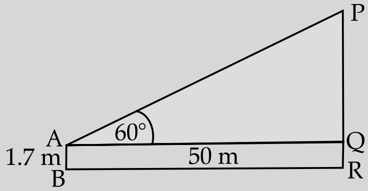
$\Rightarrow$ $y = 20\sqrt{3}\text{m}$

In ΔRSQ, $\tan 30° = \frac{x}{20}$

$\Rightarrow$ $x = \frac{20}{\sqrt{3}}\text{m}$

$$y - x = 20\sqrt{3} - \frac{20}{\sqrt{3}} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.06 \text{ m}$$

OR



Let PR be the building and AB be the boy

In ΔPQR, $\tan 60° = \frac{PQ}{50} \Rightarrow PQ = 50\sqrt{3}\text{m}$

Height of the building $= (50\sqrt{3} + 1.7)\text{m}$

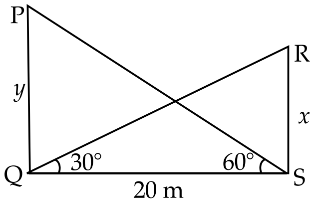
$= 88.2$ m

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Let the heights of two pole be y and x.

Distance between the poles is $QS = 20$ m.



In ΔPQS, $\angle Q = 90°$

$$\tan 60° = \frac{PQ}{QS}$$

$$\sqrt{3} = \frac{y}{20}$$

$$y = 20\sqrt{3} \text{ m}$$

In ΔRSQ, $\angle S = 90°$

$$\tan 30° = \frac{RS}{QS}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{20}$$

$$\frac{20}{\sqrt{3}} = x$$

$$x = \frac{20}{3}\sqrt{3} \text{ m}$$

Difference between their heights

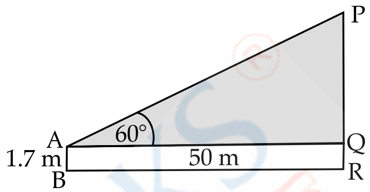
$$y - x = \left(20\sqrt{3} - \frac{20}{3}\sqrt{3}\right)\text{m}$$

$$= \frac{20}{3}\sqrt{3} \times 2$$

$$= \frac{40}{3}\sqrt{3}.$$

$$= 23.07\text{m}$$

OR



Height of Boy $= AB = 1.7$ m

$= QR = 1.7$ m

Distance between Boy and building

$BR = 50$ m

$\Rightarrow$ $AQ = 50$ m

In ΔPQA, $\angle Q = 90°$

$$\tan 60° = \frac{PQ}{AQ}$$

$$\sqrt{3} = \frac{PQ}{50}$$

$$50\sqrt{3} = PQ$$

Total height of the building $= PQ + QR$

$= 86.5 + 1.7$

$= 88.2$ m.

Section - C

11. Volume of shell = Volume of cylinder

$$\Rightarrow \frac{4\pi}{3}[5^3 - 3^3] = \pi(7)^2 h$$

$$\Rightarrow h = \frac{8}{3} = 2\frac{2}{3}\text{cm}$$

TSA of cylinder is $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7 \times \left(7 + \frac{8}{3}\right)$$

$$= 44 \times \frac{29}{3}$$

$$= \frac{1276}{3}\text{ cm}^2 \text{ or } 425.33 \text{ cm}^2$$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

Internal radius (r) = 3 cm

External radius (R) = 5 cm

Radius of cylinder (R_1) $= \frac{14}{2} = 7$ cm

Let height of cylinder $= h$ cm

According to the question,

[When one shape is reshaped into another shape the volumes are same of both shapes]

$$\frac{4}{3}\pi(R^3 - r^3) = \pi(R_1)^2 h$$

$$\frac{4}{3}(5^3 - 3^3) = (7)^2 h$$

$$\frac{4}{3} \times (125 - 27) = 49h$$

$$\frac{4}{3} \times \frac{98}{49} = h$$

$$h = \frac{8}{3}\text{m or } 2\frac{2}{3}\text{ cm}$$

$$\text{TSA of cylinder} = 2\pi r\,(h + R_1)$$

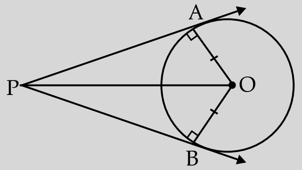
$$= 2 \times \frac{22}{7} \times 7\left(\frac{8}{3}+7\right)$$

$$= 44 \times \frac{29}{3}$$

$$= \frac{1276}{3}\text{ cm}^2$$

or 425.33 cm^2

12.



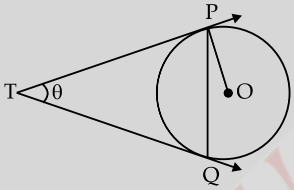
$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$

$\Rightarrow 90° + 90° + \angle APB + \angle AOB = 360°$

($\because$ Tangent $\perp$ radius)

$\Rightarrow \angle APB + \angle AOB = 180°$

OR



Let $\angle PTQ = \theta$

TPQ is an isosceles triangle

$$\angle TPQ = \angle TQP = \frac{1}{2}(180° - \theta)$$

$$= 90° - \frac{\theta}{2}$$

$$\angle OPT = 90°$$

$$\angle OPQ = \angle OPT - \angle TPQ$$

$$= 90° - \left(90° - \frac{\theta}{2}\right)$$

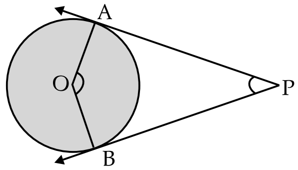
$$= \frac{\theta}{2}$$

$$\angle OPQ = \frac{1}{2}\angle PTQ$$

$$2\angle OPQ = \angle PTQ$$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:



Given: PA and PB are two tangents to a circle with centre O.

To Prove:

$$\angle APB + \angle AOB = 180°$$

Proof: $\square OAPB$ is a quadrilateral, OA is radius and PA & PB are tangents.

$\therefore \quad \angle OAP = \angle OBP = 90°$

[Radius is always perpendicular to tangent]

By angle sum property of quadrilateral.

$\therefore \angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$

$\angle AOB + 90° + \angle APB + 90° = 360°$

$\therefore \quad \angle APB + \angle AOB = 180°$

Hence Proved.

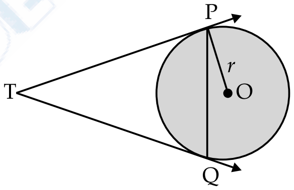
OR

Given: A circle with centre O two tangents TP and TQ to the circle where P and Q are the point of contact.

To Prove: $\angle PTQ = 2\angle OPQ$

Proof: $TP = TQ$

[length of tangents drawn from same external point to a circle are equal]



$\therefore \quad \angle TQP = \angle TPQ \quad$ ...(i)

[Angles opposite to equal sides are equal]

Now, PT is a tangent & OP is radius

$\therefore \quad OP \perp TP$[Radius and Tangent are perpendicular to each other]

$\Rightarrow \quad \angle OPT = 90°$

$\angle OPQ + \angle TPQ = 90°$

$\angle TPQ = 90° - \angle OPQ \quad$ ...(ii)

In ΔPTQ,

By angle sum property of triangle,

$\angle TPQ + \angle TQP + \angle PTQ = 180°$ [From (i)]

$\angle TPQ + \angle TPQ + \angle PTQ = 180°$

$\Rightarrow \quad 2[TPQ + \angle PTQ = 180°$

$\Rightarrow \quad 2(90° - \angle OPQ) + \angle PTQ = 180°$ [From (ii)]

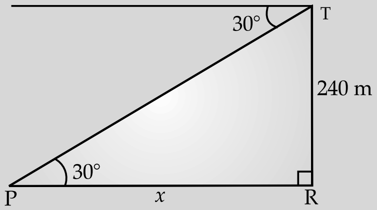
$\Rightarrow \quad 180 - 2\angle OPQ + \angle PTQ = 180°$

$\angle PTQ = 2\angle OPQ$

Hence Proved.

Case Study-1

13. (i)



In ΔPTR, $\tan 30° = \frac{240}{x} \Rightarrow x = 240\sqrt{3}$ m

(ii) Distance of boat from tower

$$= 240\sqrt{3} - 240(\sqrt{3}-1)$$

$$= 240 \text{ m}$$

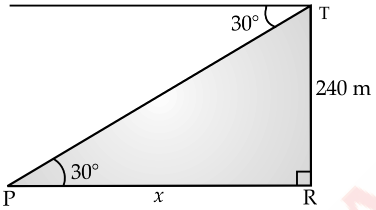
Let the angle of depression = θ

$$\tan\theta = \frac{240}{240} = 1$$

$\Rightarrow$ $\theta = 45°$

[CBSE Marking Scheme, 2021-22]

Detailed Solution:



(i) Height of light house = 240 m

In ΔPRT, $\angle R = 90°$

$$\tan 30° = \frac{240}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{240}{x}$$

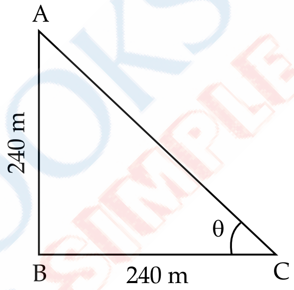
$$x = 240\sqrt{3} \text{ m.}$$

Distance of foot from the foot of observation tower $= 240\sqrt{3}$ m.

(ii) After 10 minutes,

Distance between boat and light house is reduced by

$$= 24\left(\sqrt{3}-1\right) \text{m}$$



$\therefore$ Distance between boat and light house

$$= 240\sqrt{3} - 240\left(\sqrt{3}-1\right)$$

$$= 240\sqrt{3} - 240\sqrt{3} + 240$$

$$= 240 \text{ m}$$

Let the point C is the new position of boat,

$$\tan\theta = \frac{AB}{BC} = \frac{240}{240}$$

$\Rightarrow$ $\theta = 45°$

Case Study-2

14. (i) 3000, 3005, 3010, ..., 3900

$$a_n = a + (n-1)d$$

$$3900 = 3000 + (n-1)5$$

$\Rightarrow$ $900 = 5n - 5 \Rightarrow 5n = 905$

$\Rightarrow$ $n = 181$

Minimum number of days of practice = n – 1

= 180 days

(ii) $S_n = \frac{n}{2}(a + l)$

$$= \frac{181}{2} \times (3000 + 3900)$$

= 624450 push-ups

[CBSE Marking Scheme, 2021-22]

Detailed Solution:

(i) By the given situation the A.P. to be formed is:

3000, 3005, 3010, ... 3900

First term = a = 3000

Common Difference = d = 3005 – 3000

d = 5

According to the problem

n^{th} term (a_n) = 3900

To Find n

$$n^{\text{th}} \text{ term} = a_n$$

$$= a + (n-1)d$$

$$3900 = 3000 + (n-1)5$$

$$3900 - 3000 = (n-1)5$$
$$900 = (n-1)5$$
$$\frac{900}{5} = n - 1$$
$$180 + 1 = n$$
or $$n = 181$$

Minimum number of days he needs to practice before his goal is accomplished

$$= 181 - 1$$
[Excluding the last day]
$$= 180$$

(ii) Total Number of push-ups performed means sum of all push-ups he did in 181 days.

$$\therefore \quad S_n = \frac{n}{2}[a + l] \quad \text{...(i)}$$

S_n = Sum of all push-ups in 181 days.

n = number of days = 181

a = first term of the A.P.

= 3000

l = last term = 3900

Put all values in (i)

$$\Rightarrow \quad S_n = \frac{181}{2}[3000 + 3900]$$
$$= \frac{181}{2}[6900]$$
$$= 624450$$

Total number of Push-ups performed in 181 days

= 624450.

●●●

Solved Paper, 2021-22

MATHEMATICS (STANDARD)

Term-I, Set-4

Series : JSK/2	**Question Paper Code No. 030/2/4**

Time allowed : 90 Minutes **Max. Marks : 40**

General Instructions :

*(i) The question paper contains **50** questions out of which **40** questions are to be attenpted. All questions carry equal marks.*

*(ii) The question paper consists of **three** sections – Section A, B and C.*

*(iii) **Section–A** contains of **20** questions. Attempt any **16** questions from **Q. No. 1 to 20**.*

*(iv) **Section–B** also contains **20** questions. Attempt any **16** questions from **Q. No. 21 to 40**.*

*(v) **Section–C** contains of two Case Studies containing **5** questions in each case. Attempt any **4** questions from **Q. No. 41 to 45** and another **4** from **Q. No. 46 to 50**.*

(vi) There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.

(vii) There is no negative marking.

SECTION-A

Q. No. 1 to 20 are of 1 mark each. Attempt any 16 from Q. No. 1 to 20.

1. The exponent of 5 in the prime factorisation of 3750 is

(a) 3 **(b)** 4

(c) 5 **(d)** 6

2. The graph of a polynomial $P(x)$ cuts the x-axis at 3 points and touches it at 2 other points. The number of zeroes of $P(x)$ is

(a) 1 **(b)** 2

(c) 3 **(d)** 5

3. The values of x and y satisfying the two equations $32x + 33y = 34$, $33x + 32y = 31$ respectively are

(a) –1, 2 **(b)** –1, 4

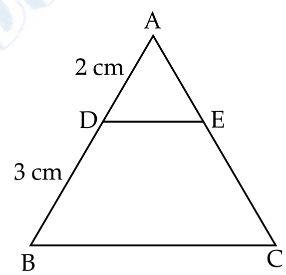
(c) 1, –2 **(d)** –1, –4

4. If $A(3, \sqrt{3})$, $B(0, 0)$ and $C(3, k)$ are the three vertices of an equilateral triangle ABC, then the value of k is

(a) 2 **(b)** –3

(c) $-\sqrt{3}$ **(d)** $-\sqrt{2}$

5. In figure, $DE \parallel BC$, $AD = 2$ cm and $BD = 3$ cm, then ar(ΔABC) : ar(ΔADE) is equal to



(a) 4 : 25 **(b)** 2 : 3

(c) 9 : 4 **(d)** 25 : 4

6. If $\cot\theta = \frac{1}{\sqrt{3}}$, the value of $\sec^2\theta + \operatorname{cosec}^2\theta$ is

(a) 1 **(b)** $\frac{40}{9}$

(c) $\frac{38}{9}$ **(d)** $5\frac{1}{3}$

7. The area of a quadrant of a circle where the circumference of circle is 176 m, is

(a) 2464 m^2 **(b)** 1232 m^2

(c) 616 m^2 **(d)** 308 m^2

8. For an event E, $P(E) + P(\bar{E}) = x$, then the value of $x^3 - 3$ is

(a) –2 (b) 2

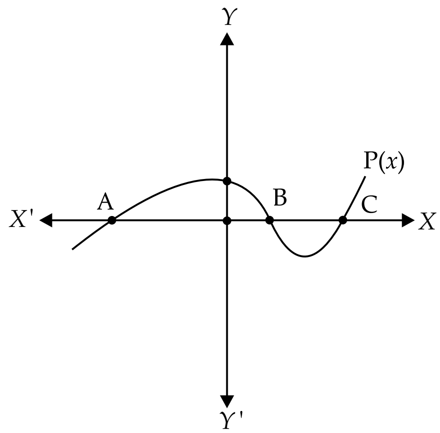
(c) 1 (d) –1

9. What is the greatest possible speed at which a girl can walk 95 m and 171 m in an exact number of minutes?

(a) 17 m/min. **(b)** 19 m/min.

(c) 23 m/min. **(d)** 13 m/min.

10. In figure, the graph of a polynomial $P(x)$ is shown. The number of zeroes of $P(x)$ is



(a) 1 **(b)** 2
(c) 3 **(d)** 4

11. Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be
(a) $9x + 8y = 7$ **(b)** $-12x - 8y = 7$
(c) $-12x + 8y = 7$ **(d)** $12x + 8y = 7$

12. Three vertices of a parallelogram $ABCD$ are $A(1, 4)$, $B(-2, 3)$ and $C(5, 8)$. The ordinate of the fourth vertex D is
(a) 8 **(b)** 9
(c) 7 **(d)** 6

13. In ΔABC and ΔDEF, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \frac{1}{2}DE$. Then the two triangles are
(a) Congruent, but not similar
(b) Similar, but not congruent
(c) Neither congruent nor similar
(d) Congruent as well as similar

14. In ΔABC right angled at B, $\sin A = \frac{7}{25}$, then the value of $\cos C$ is
(a) $\frac{7}{25}$ **(b)** $\frac{24}{25}$
(c) $\frac{7}{24}$ **(d)** $\frac{24}{7}$

15. The minute hand of a clock is 84 cm long. The distance covered by the tip of minute hand from 10:10 am to 10:25 am is
(a) 44 cm **(b)** 88 cm
(c) 132 cm **(d)** 176 cm

16. The probability that the drawn card from a pack of 52 cards is neither an ace nor a spade is
(a) $\frac{9}{13}$ **(b)** $\frac{35}{52}$
(c) $\frac{10}{13}$ **(d)** $\frac{19}{26}$

17. Three alarm clocks ring their alarms at regular intervals of 20 min., 25 min. and 30 min. respectively. If they first beep together at 12 noon, at what time will they beep again for the first time?
(a) 4 : 00 pm **(b)** 4 : 30 pm
(c) 5 : 00 pm **(d)** 5 : 30 pm

18. A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively is
(a) $k[x^2 - 8x + 5]$ **(b)** $k[x^2 + 8x + 5]$
(c) $k[x^2 - 5x + 8]$ **(d)** $k[x^2 + 5x + 8]$

19. Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$. The values of y are
(a) 1, –7 **(b)** –1, 7
(c) 2, 7 **(d)** –2, –7

20. Given that $\sec\theta = \sqrt{2}$, the value of $\frac{1+\tan\theta}{\sin\theta}$ is
(a) $2\sqrt{2}$ (b) $\sqrt{2}$
(c) $3\sqrt{2}$ **(d)** 2

SECTION-B

Q. No. 21 to 40 are of 1 mark each. Attempt any 16 from Q. 21 to 40.

21. The greatest number which when divides 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively is
(a) 575 **(b)** 450
(c) 750 **(d)** 625

22. Which of the following cannot be the probability of an event?
(a) 0.01 **(b)** 3%
(c) $\frac{16}{17}$ **(d)** $\frac{17}{16}$

23. The diameter of a car wheel is 42 cm. The number of complete revolutions it will make in moving 132 km is
(a) 10^4 **(b)** 10^5
(c) 10^6 **(d)** 10^3

24. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is
(a) 1 **(b)** $\frac{1}{2}$
(c) $\frac{\sqrt{2}}{2}$ **(d)** $\sqrt{2}$

25. The ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7) is
(a) 3 : 2 **(b)** 2 : 3
(c) 3 : 4 **(d)** 4 : 3

26. If $x - 1$ is a factor of the polynomial $p(x) = x^3 + ax^2 + 2b$ and $a + b = 4$, then
(a) $a = 5, b = -1$ **(b)** $a = 9, b = -5$
(c) $a = 7, b = -3$ **(d)** $a = 3, b = 1$

27. If a and b are two coprime numbers, then a^3 and b^3 are
(a) Coprime **(b)** Not coprime
(c) Even **(d)** Odd

28. The area of a square that can be inscribed in a circle of area $\frac{1408}{7}$ cm^2 is
(a) 321 cm^2 **(b)** 642 cm^2
(c) 128 cm^2 **(d)** 256 cm^2

29. If $A(4, -2)$, $B(7, -2)$ and $C(7, 9)$ are the vertices of a ΔABC, then ΔABC is

(a) equilateral triangle
(b) isosceles triangle
(c) right angled triangle
(d) isosceles right angled triangle

30. If α, β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k+6)x + 2(2k-1)$, then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$, is

(a) -7 **(b)** 7
(c) -3 **(d)** 3

31. If n is a natural number, then $2(5^n + 6^n)$ always ends with

(a) 1 **(b)** 4
(c) 3 **(d)** 2

32. The line segment joining the points $P(-3, 2)$ and $Q(5, 7)$ is divided by the y-axis in the ratio

(a) 3 : 1 **(b)** 3 : 4
(c) 3 : 2 **(d)** 3 : 5

33. If $a \cot\theta + b \operatorname{cosec}\theta = p$ and $b \cot\theta + a \operatorname{cosec}\theta = q$, then $p^2 - q^2 =$

(a) $a^2 - b^2$ **(b)** $b^2 - a^2$
(c) $a^2 + b^2$ **(d)** $b - a$

34. If the perimeter of a circle is half to that of a square, then the ratio of the area of the circle to the area of the square is

(a) 22 : 7 **(b)** 11 : 7
(c) 7 : 11 **(d)** 7 : 22

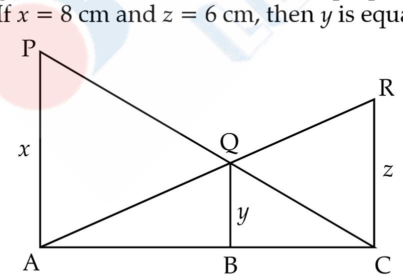
35. A dice is rolled twice. The probability that 5 will not come up either time is

(a) $\frac{11}{36}$ **(b)** $\frac{1}{3}$
(c) $\frac{13}{36}$ **(d)** $\frac{25}{36}$

36. The LCM of two numbers is 2400. Which of the following CAN NOT be their HCF?

(a) 300 **(b)** 400
(c) 500 **(d)** 600

37. In figure, PQ, QB and RC are each perpendicular to AC. If $x = 8$ cm and $z = 6$ cm, then y is equal to



(a) $\frac{56}{7}$ cm **(b)** $\frac{7}{56}$ cm
(c) $\frac{25}{7}$ cm **(d)** $\frac{24}{7}$ cm

38. In a ΔABC, $\angle A = x^{\circ}$, $\angle B = (3x-2)^{\circ}$, $\angle C = y^{\circ}$. Also $\angle C - \angle B = 9^{\circ}$. The sum of the greatest and the smallest angles of this triangle is

(a) 107° **(b)** 135°
(c) 155° **(d)** 145°

39. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is

(a) $\frac{p^2+1}{2p}$ **(b)** $\frac{p^2-1}{2p}$
(c) $\frac{p^2-1}{p^2+1}$ **(d)** $\frac{p^2+1}{p^2-1}$

40. The base BC of an equilateral ΔABC lies on the y-axis. The co-ordinates of C are $(0, -3)$. If the origin is the mid-point of the base BC, what are the co-ordinates of A and B?

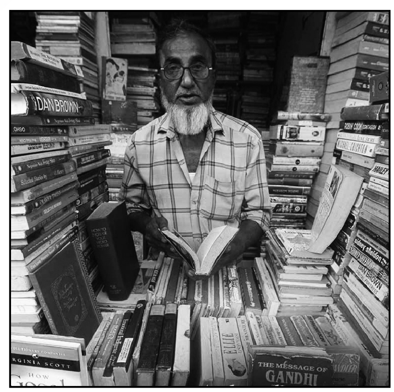
(a) $A(\sqrt{3}, 0)$, $B(0, 3)$ **(b)** $A(\pm 3\sqrt{3}, 0)$, $B(3, 0)$
(c) $A(\pm 3\sqrt{3}, 0)$, $B(0, 3)$ **(d)** $A(-\sqrt{3}, 0)$, $B(3, 0)$

SECTION-C

Q. No. 41-45 are based on Case Study-I, you have to answer any (4) four questions. Q. No. 46-50 are based on Case Study-II, you have to answer any (4) four questions.

Case Study-I

A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc. He takes a fixed charge for the first two days and an additional charge for subsequent day. Amruta paid ₹22 for a book and kept for 6 days; while Radhika paid ₹16 for keeping the book for 4 days.



Assume that the fixed charge be ₹x and additional charge (per day) be ₹y.

Based on the above information, answer any four of the following questions:

41. The situation of amount paid by Radhika, is algebraically represented by

(a) $x - 4y = 16$ **(b)** $x + 4y = 16$
(c) $x - 2y = 16$ **(d)** $x + 2y = 16$

42. The situation of amount paid by Amruta, is algebraically represented by

(a) $x - 2y = 11$ **(b)** $x - 2y = 22$
(c) $x + 4y = 22$ **(d)** $x - 4y = 11$

43. What are the fixed charges for a book?

(a) ₹9 **(b)** ₹13
(c) ₹10 **(d)** ₹15

44. What are the additional charges for each subsequent day for a book?

(a) ₹6 **(b)** ₹5
(c) ₹4 **(d)** ₹3

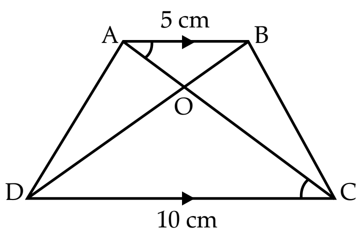
45. Which is the total amount paid by both, if both of them have kept the book for 2 more days?

(a) ₹35 **(b)** ₹52
(c) ₹50 **(d)** ₹58

Case Study-II

A farmer has a field in the shape of trapezium, whose map with scale 1 cm = 20 m, is given below:

The field is divided into four parts by joining the opposite vertices.



Based on the above information, answer any four of the following questions:

46. The two triangular regions *AOB* and *COD*

(a) Similar by AA criterion
(b) Similar by SAS criterion
(c) Similar by RHS criterion
(d) Not similar

47. The ratio of the area of the ΔAOB to the area of ΔCOD, is

(a) 4 : 1 **(b)** 1 : 4
(c) 1 : 2 **(d)** 2 : 1

48. If the ratio of the perimeter of ΔAOB to the perimeter of ΔCOD of would have been 1 : 4, then

(a) $AB = 2CD$ **(b)** $AB = 4CD$
(c) $CD = 2AB$ **(d)** $CD = 4AB$

49. If in Δs *AOD* and *BOC*, $\frac{AO}{BC} = \frac{AD}{BO} = \frac{OD}{OC}$, then

(a) $\Delta AOD \sim \Delta BOC$ **(b)** $\Delta AOD \sim \Delta BCO$
(c) $\Delta ADO \sim \Delta BCO$ **(d)** $\Delta ODA \sim \Delta OBC$

50. If the ratio of areas of two similar triangles *AOB* and *COD* is 1 : 4, then which of the following statements is true?

(a) The ratio of their perimeters is 3 : 4
(b) The corresponding altitudes have a ratio 1 : 2
(c) The medians have a ratio 1 : 4
(d) The angle bisectors have a ratio 1 : 16

❑❑❑

Solutions

SECTION-A

1. **(b)** 4

Explanation: According to the prime factorisation, 3750 can be written as

$3750 = 5 \times 5 \times 5 \times 5 \times 3 \times 2 = 5^4 \times 3^1 \times 2^1$

It is clear from above, that exponent of 5 in the prime factorisation of 3750 is 4.

2. **(d)** 5

Explanation: According to the property of the polynomials,

Number of zeroes = Number of points at which graph intersects the *x*-axis.

It is mentioned in the question that, the graph intersects *x*-axis at 3 points and it touches it at 2 further points.

This means that the graph intersects the *x*-axis at 5 different points.

Therefore, number of zeroes = 5.

3. **(a)** –1, 2

Explanation: The given equations are,

$$32x + 33y = 34 \quad ...(i)$$

$$\& \quad 33x + 32y = 31 \quad ...(ii)$$

Subtract eq.(ii) from eq.(i)

$$-x + y = 3$$

or $y = 3 + x$

Put this value of *y* in (i), we get

$$32x + 33(3+x) = 34$$

$\Rightarrow$ $32x + 99 + 33x = 34$

$\Rightarrow$ $65x = 34 - 99$

$\Rightarrow$ $65x = -65$

or $x = -1$

Also, $y = 3 + x$

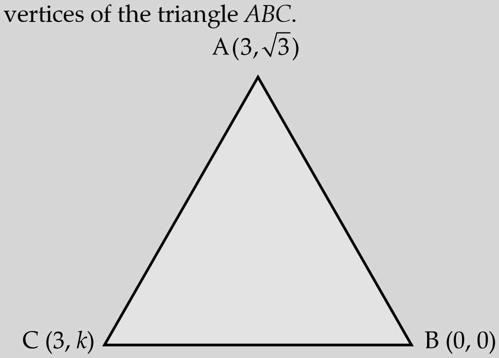
$\Rightarrow$ $y = 3 + (-1)$

$= 3 - 1 = 2$

Hence, the correct solution is $x = -1$ and $y = 2$.

4. **(c)** $\sqrt{3}$

Explanation: $A(3, \sqrt{3})$, $B(0, 0)$ and $C(3, k)$ are the vertices of the triangle *ABC*.



As in the equilateral triangle ABC all sides are equal.

Then, apply distance formula for sides AB and BC.

According to the distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(3-0)^2 + (\sqrt{3}-0)^2}$$

$$= \sqrt{9+3} = \sqrt{12} \text{ units}$$

$$BC = \sqrt{(3-0)^2 + (k-0)^2}$$

$$= \sqrt{9+k^2} \text{ units}$$

Now, $AB = BC$

$$\sqrt{12} = \sqrt{9+k^2}$$

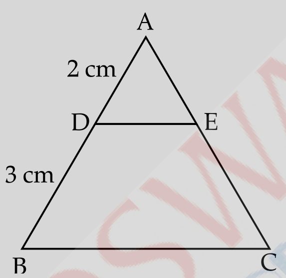
or $12 = 9 + k^2$

or $k^2 = 3$

or $k = \pm\sqrt{3}$

5. **(d)** 25 : 4

Explanation:



First we need to prove that ΔABC & ΔADE are similar.

$\angle ADE = \angle ABC$ [Corresponding angles]

$\angle A = \angle A$ [Common]

By, AA similarity rule $\Delta ABC \sim \Delta ADE$

According to the theorem, "Ratio of area of similar triangles is equal to the square of the ratio of corresponding sides."

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(ADE)} = \frac{AB^2}{AD^2}$$

$$= \frac{(2+3)^2}{(2)^2} = \frac{5^2}{2^2} = \frac{25}{4}$$

6. **(d)** $5\frac{1}{3}$

Explanation: It is given that

$$\cot\theta = \frac{1}{\sqrt{3}} = \cot 60^o$$

$\Rightarrow \quad \theta = 60^o$

Substituting the value of θ

$$\sec^2\theta + \text{cosec}^2\theta = \sec^2 60^o + \text{cosec}^2 60^o$$

$$= (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 4 + \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

7. **(c)** 616 m^2

Explanation: It is given that circumference of the circle is 176 cm^2

$\Rightarrow \quad 2\pi r = 176$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 176$

$\Rightarrow \quad r = \frac{176 \times 7}{2 \times 22} = 28$ cm

Also, in a quadrant $\theta = 90^o$

$$\text{Area of quadrant} = \frac{\theta}{360^o} \times \pi r^2$$

$$= \frac{90^o}{360^o} \times \frac{22}{7} \times 28 \times 28$$

$$= 616 \text{ cm}^2$$

8. **(a)** –2

Explanation: Given

$P(E) + P(\bar{E}) = x$ ...(i)

Also, according to the law of probability,

$P(E) + P(\bar{E}) = 1$ ...(ii)

From (i) and (ii), we get

$x = 1$

Put value of x in $x^3 - 3$, we get

$$x^3 - 3 = (1)^3 - 3 = 1 - 3 = -2$$

9. **(b)** 19 m/min.

Explanation: As the girl needs to walk 95 m and 171 m at the exact number of minutes.

So, we have to find HCF of 95 and 171.

According to prime factorisation of 95 and 171

$95 = 5 \times 19$

$171 = 3 \times 3 \times 19$

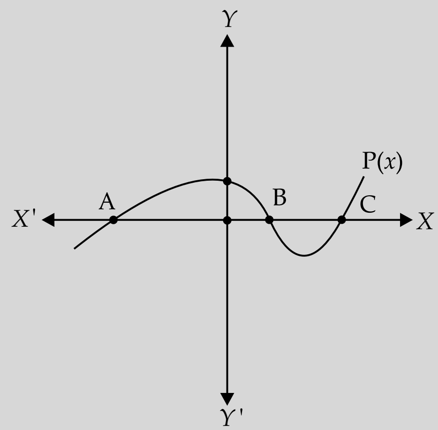
HCF(95, 171) = 19

Hence, greatest possible speed is 19 m/min.

10. **(c)** 3

Explanation: According to the property of the polynomials,

Number of zeroes = Number of points at which graph intersects the x-axis.



From the figure it is clear that the graph intersects X-axis at three different points. Therefore, the polynomial has 3 zeroes.

11. **(c)** $-12x + 8y = 7$

Explanation: The given equation is

$$3x - 2y = 5$$

According to the condition that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel

then $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Taking option (C) and applying the above condition on it and in the given equation.

or $$\frac{3}{-12} = \frac{-2}{8} \neq \frac{5}{7}$$

12. **(b)** 9

Explanation: Let $A(1, 4)$ $B(-2, 3)$ $C(5, 8)$ and $D(a, b)$ are the vertices of a parallelogram.

Midpoint of diagonal AC

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{1+5}{2}, \frac{4+8}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{12}{2}\right) = (3, 6)$$

Midpoint of diagonal BD

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2+a}{2}, \frac{3+b}{2}\right)$$

The diagonals of the parallelogram bisect each other. The diagonals share same mid-point.

$\therefore$ $$(3, 6) = \left(\frac{-2+a}{2}, \frac{3+b}{2}\right)$$

On comparing both sides, we get

$$3 = \frac{-2+a}{2} \text{ and } 6 = \frac{3+b}{2}$$

In the question value of ordinate is asked,

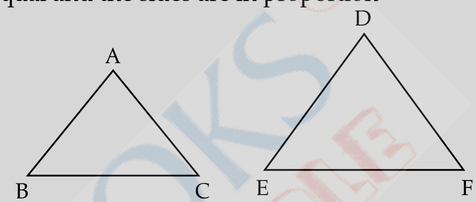
$$6 = \frac{3+b}{2}$$

$$12 = 3 + b$$

or $$b = 9$$

13. **(b)** Similar, but not congruent

Explanation: According to the definition of similarity of two triangles, "Two triangles are similar when their corresponding angles are equal and the sides are in proportion"



According to the question,

$$\angle F = \angle C \text{ and } \angle B = \angle E$$

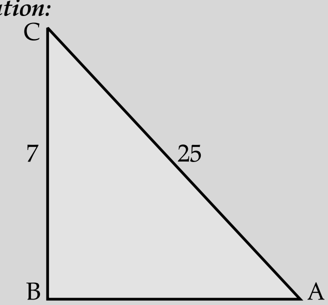
Since, $$AB = \frac{1}{2}DE \quad \text{[Given]}$$

Also $$\frac{AB}{DE} = \frac{1}{2}$$

Which means the triangles are similar but not congruent.

14. **(a)** $\frac{7}{25}$

Explanation:



$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

and $$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\sin A = \frac{BC}{AC} = \frac{7}{25}$$

$\therefore$ $$\cos C = \frac{\text{BC}}{\text{AC}} = \frac{7}{25}$$

15. **(c)** 132 cm

Explanation: Length of minute hand = Radius of the quadrant/sector so formed = 84 cm.

In 1 minute, minute hand makes an angle of 6°.

Therefore, in 15 minutes it makes an angle of $15 \times 6^{\circ} = 90^{\circ}$

Distance covered by the tip of the minute hand

$$= \text{Length of arc}$$

$$= \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 84$$

$$= 132 \text{ cm.}$$

16. **(a)** $\frac{9}{13}$

Explanation: Total ace cards = 4 and total spade cards = 13 – 1 = 12 (One card among aces is also a spade)

Cards which are neither ace or spade

$$= 52 - 16 = 36$$

$$\text{Required probability} = \frac{36}{52} = \frac{9}{13}$$

17. **(c)** 5 : 00 pm

Explanation: Time when they ring together

= LCM (20, 25, 30)

According to prime factorisation,

$$20 = 2 \times 2 \times 5$$
$$25 = 5 \times 5$$
$$30 = 2 \times 3 \times 5$$
$$\text{LCM}(20, 25, 30) = 2 \times 2 \times 3 \times 5 \times 5 = 300$$

Thus, 3 bells ring together after 300 minutes or 5 hours.

Since, they rang together first at 12 noon, then they ring together again at 5 pm

18. **(a)** $k[x^2 - 8x + 5]$

Explanation: For any quadratic polynomial,

$$ax^2 + bx + c$$

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$8 = \frac{-b}{a}$$

$$\frac{8}{1} = \frac{-b}{a}$$

or $b = -8k, a = 1k$

$$\text{Also, product of zeroes} = \frac{c}{a}$$

$$5 = \frac{c}{a}$$

$$\frac{5}{1} = \frac{c}{a}$$

or $c = 5k, a = 1k$

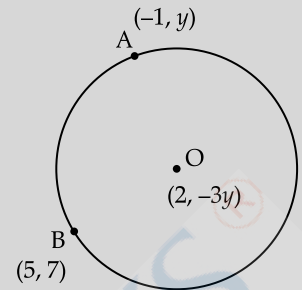
Polynomial whose sum of zeroes or product of zeroes are given,

$$\text{Required Polynomial} = ax^2 + bx + c$$
$$= kx^2 - 8kx + 5k$$
$$= k(x^2 - 8x + 5)$$

19. **(b)** –1, 7

Explanation: As points A and B lie on the circle and O is the centre.

AO and BO will be the radii of the circle.



So, $AO = BO$

$$\Rightarrow \sqrt{(2-(-1))^2 + (-3y - y)^2} = \sqrt{(2-5)^2 + (-3y-7)^2}$$

(Applying distance formula on both AO and BO)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow (3)^2 + (-4y)^2 = (-3)^2 + (-3y-7)^2$$
$$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$$
$$\Rightarrow 16y^2 - 9y^2 - 42y - 49 = 0$$
$$\Rightarrow 7y^2 - 42y - 49 = 0$$
$$\Rightarrow 7(y^2 - 6y - 49) = 0$$
$$\Rightarrow y^2 - 7y + 1y - 49 = 0$$
$$\Rightarrow y(y-7) + 1(y-7) = 0$$
$$\Rightarrow (y-7)(y+1) = 0$$
$$\Rightarrow y = 7, -1$$

20. **(a)** $2\sqrt{2}$

Explanation: It is given that

$$\sec\theta = \sqrt{2} \quad \text{...(i)}$$

Also, $\sec 45^\circ = \sqrt{2}$ ...(ii)

From (i) and (ii), we get

$$\theta = 45^\circ$$

Put value of θ in $\frac{1+\tan\theta}{\sin\theta}$,

$$\Rightarrow \frac{1+\tan\theta}{\sin\theta} = \frac{1+\tan 45^\circ}{\sin 45^\circ}$$

$$\Rightarrow \frac{1+1}{\frac{1}{\sqrt{2}}}$$

or $2\sqrt{2}$

SECTION-B

21. **(d)** 625

Explanation: First subtract the remainders from their respective numbers,

$$1251 - 1 = 1250$$
$$9377 - 2 = 9375$$
$$15628 - 3 = 15625$$

According to the prime factorisation,

$$1250 = 2 \times 5 \times 5 \times 5 \times 5$$
$$9375 = 3 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

HCF(1250, 9375, 15625) $= 5 \times 5 \times 5 \times 5 = 625$

22. **(d)** $\frac{17}{16}$

Explanation: Probability of an event is always a proper fraction.

Also, $0 \le P(E) \le 1$

But $\frac{17}{16} > 1$

Therefore, $\frac{17}{16}$ can never be probability of any event.

23. **(b)** 10^5

Explanation: Diameter of wheel = 42 cm

Radius of the wheel $= \frac{42}{2} = 21$ cm

Distance in 1 revolution

= Circumference of the wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 3$

= 132 cm

Total distance covered by the wheel

= 132 km = 132 × 100000 cm = 13200000 cm

Number of revolutions

$$= \frac{\text{Total distance covered by wheel}}{\text{Distance covered in 1 revolution}}$$

$$= \frac{13200000}{132} = 100000 = 10^5$$

24. **(c)** $\frac{\sqrt{2}}{2}$

Explanation:

$$\tan\theta + \cot\theta = 2$$

or $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

or $\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = 2$

or $\sin^2\theta + \cos^2\theta = 2\sin\theta\cos\theta$

or $1 = 2\sin\theta\cos\theta$

or $\sin\theta\cos\theta = \frac{1}{2}$ ...(i)

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$$
$$= 1 + 2 \times \frac{1}{2}$$
$$= 1 + 1 = 2$$

Therefore,

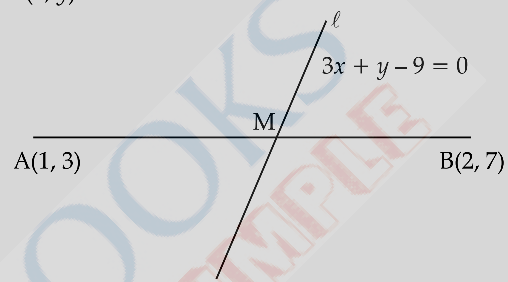
$\sin\theta + \cos\theta = \sqrt{2}$ ...(ii)

Now taking,

$$\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)^3 - 3\sin\theta\cos\theta(\sin\theta + \cos\theta)$$
$$= (\sqrt{2})^3 - 3 \times \frac{1}{2} \times \sqrt{2}$$
$$= 2\sqrt{2} - \frac{3}{2}\sqrt{2} = \frac{\sqrt{2}}{2}$$

25. **(c)** 3 : 4

Explanation: Let the the point of intersection be $M(x, y)$.



Let the line λ divides the line AB in the ratio $k : 1$.

According to the section formula,

$$M(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$
$$= \left(\frac{k(2) + 1(1)}{k+1}, \frac{k(7) + 1(3)}{k+1}\right)$$
$$= \left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$$

This point M lies on the line λ.

Therefore, $3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$

or $6k + 3 + 7k + 3 - 9(k+1) = 0$

or $4k - 3 = 0$

or $k = \frac{3}{4}$

The ratio is $k : 1$ or 3 : 4.

26. **(b)** $a = 9, b = -5$

Explanation: Given,

$p(x) = x^3 + ax^2 + 2b$

$a + b = 4$ ...(i)

$x - 1$ is a factor of the polynomial $P(x)$, which means $x = 1$ is a zero of the polynomial $p(x)$.

∴ $p(1) = 0$

or $(1)^3 + a(1)^2 + 2b = 0$

or $1 + a + 2b = 0$

or $a + 2b = -1$ ...(ii)

Subtracting (i) from (ii), we get

$b = -5$

Substituting the value of b in (i), we get $a = 9$

∴ $a = 9$ & $b = -5$

27. (a) Coprime

Explanation: As a and b are co-prime then a^3 and b^3 are also co-prime.

We can understand above situation with the help of an example.

Let $a = 3$ and $b = 4$

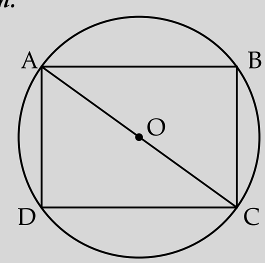
$$a^3 = 3^3 = 27 \text{ and } b^3 = 4^3 = 64$$

Clearly, HCF(a, b) = HCF(3, 4) = 1

Then, HCF(a^3, b^3) = HCF(27, 64) = 1

28. (c) 128 cm^2

Explanation:



$$\text{Area of circle} = \frac{1408}{7}\text{ cm}^2 \quad \text{[Given]}$$

or $$\pi r^2 = \frac{1408}{7}$$

or $$\frac{22}{7} \times r^2 = \frac{1408}{7}$$

or $$r^2 = \frac{1408}{7} \times \frac{7}{22}$$

or $$r = \sqrt{64}$$
$$= 8 \text{ cm}$$

Diameter of circle = $2r$ = 16 cm

As square is inscribed in the circle, diameter of circle = diagonal of square = 16 cm

$$\text{Area of square} = \frac{(\text{diagonal of square})^2}{2}$$
$$= \frac{16^2}{2} = \frac{256}{2} = 128 \text{ cm}^2$$

29. (c) right angled triangle

Explanation: $A(4, -2)$, $B(7, -2)$ and $C(7, 9)$ are the vertices of a triangle.

Using distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{[7-4]^2 + [-2-(-2)]^2}$$
$$= \sqrt{3^2 + 0} = 3$$
$$BC = \sqrt{[7-7]^2 + [9-(-2)]^2}$$
$$= \sqrt{0 + 11^2} = 11$$
$$AC = \sqrt{[7-4]^2 + [9-(-2)]^2}$$
$$= \sqrt{3^2 + 11^2}$$
$$= \sqrt{9 + 121} = \sqrt{129}$$

Clearly, they are not equilateral or isosceles.

Also, $AC^2 = AB^2 + BC^2$

Which mean it is following Pythagoras theorem.

$\therefore$ ΔABC is a right angled triangle.

30. (b) 7

Explanation: $p(x) = x^2 - (k + 6)x + 2(2k - 1)$ is the given polynomial

Here, $a = 1$, $b = -(k + 6)$ & C = $2(2k - 1)$

$$\text{Sum of zeroes} = \alpha + \beta = \frac{-b}{a} = k + 6$$

$$\text{Product of zeroes} = \alpha\beta = \frac{c}{a} = \frac{2(2k-1)}{1} = 2(2k-1)$$

It is given that,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$\Rightarrow$ $$k + 6 = \frac{1}{2}2(2k-1)$$

$\Rightarrow$ $$k + 6 = 2k - 1$$

$\Rightarrow$ $$-k = -7$$

or $$k = 7$$

31. (d) 2

Explanation: Let us take an example of different powers of 5.

As, $5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$ …….

It is clear from above example that 5^n will always end with 5.

Similarly, 6^n will always end with 6.

So, $5^n + 6^n$ will always end with 5 + 6 = 11

Also, $2(5^n + 6^n)$ always ends with 2 × 11 = 22 i.e., it will always end with 2.

32. (d) 3 : 5

Explanation: Let the point on y-axis which divides the line PQ is $M(0, y)$ and the ratio be $k : 1$.

According to the section formula,

$$M(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$M(0, y) = \left(\frac{5k + (-3)}{k+1}, \frac{k(7) + 1(2)}{k+1}\right)$$

On comparing, we get

$$0 = \frac{5k-3}{k+1}$$

or $5k - 3 = 0$

or $k = \frac{3}{5}$

33. (b) $b^2 - a^2$

Explanation: $a \cot\theta + b \operatorname{cosec}\theta = p$ and $b \cot\theta + a \operatorname{cosec}\theta = q$ are the given equations.

Taking, $p^2 - q^2$

$= (a \cot\theta + b \operatorname{cosec}\theta)^2 - (b \cot\theta + a \operatorname{cosec}\theta)^2$

$= a^2\cot^2\theta + b^2\operatorname{cosec}^2\theta + 2ab.\cot\theta.\operatorname{cosec}\theta$
$\quad - b^2\cot^2\theta - a^2\operatorname{cosec}^2\theta - 2ab.\cot\theta.\operatorname{cosec}\theta$

$= a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\cot^2\theta - \operatorname{cosec}^2\theta)$

$= a^2(-1) + b^2(-1)$

$= b^2 - a^2$ [using, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$]

34. (d) 7 : 22

Explanation: Let radius of the circle be r cm and side of the square is a cm.

According to the question, perimeter of the circle is half of perimeter of the square.

$$\Rightarrow \quad 2\pi r = \frac{1}{2}(4a)$$

$$\Rightarrow \quad r = \frac{2a}{2\neq}$$

or $\frac{r}{a} = \frac{1}{\neq}$

$$\frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\neq r^2}{a^2}$$

$$= \pi \times \frac{1}{\pi^2} = \frac{1}{\pi} \text{ or } \frac{7}{22}$$

35. (d) $\frac{25}{36}$

Explanation: All possible events are written below:

(1 1)	(1 2)	(1 3)	(1 4)	(1 5)	(1 6)
(2 1)	(2 2)	(2 3)	(2 4)	(2 5)	(2 6)
(3 1)	(3 2)	(3 3)	(3 4)	(3 5)	(3 6)
(4 1)	(4 2)	(4 3)	(4 4)	(4 5)	(4 6)
(5 1)	(5 2)	(5 3)	(5 4)	(5 5)	(5 6)
(6 1)	(6 2)	(6 3)	(6 4)	(6 5)	(6 6)

Total events = 36

Out of the events in which 5 will not come up either time are (1, 1) (1, 2) (1, 3) (1, 4) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 6).

No. of required events in = 25

$$\text{Required probability} = \frac{25}{36}$$

36. (c) 500

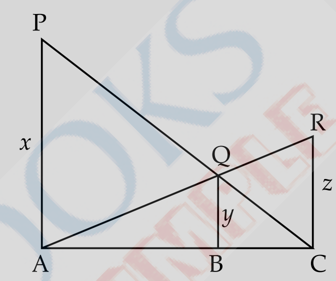
Explanation: According to the property, HCF of two numbers is also a factor of LCM of same two numbers.

Out of all the options, only (C) 500 is not a factor of 2400.

Therefore, 500 cannot be the HCF.

37. (d) $\frac{24}{7}$ cm

Explanation:



$$\Rightarrow \quad \frac{BQ}{AP} = \frac{CB}{CA}$$

[Since, $\Delta CBQ \sim \Delta CAP$]

$$\Rightarrow \quad \frac{y}{x} = \frac{BQ}{CR} \quad \ldots\text{(i)}$$

In ΔACR, we have $BQ \parallel CR$

$$\Rightarrow \quad \frac{BQ}{AP} = \frac{AB}{AC}$$

[Since, $\Delta ABQ \sim \Delta ACR$]

$$\Rightarrow \quad \frac{y}{z} = \frac{AB}{AC}$$

Adding (i) and (ii), we get

$$\frac{y}{x} + \frac{y}{z} = \frac{CB}{AC} + \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{y}{x} + \frac{y}{z} = \frac{AB+BC}{AC}$$

$$\Rightarrow \quad \frac{y}{x} + \frac{y}{z} = \frac{AC}{AC}$$

$$\Rightarrow \quad \frac{y}{x} + \frac{y}{z} = 1$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

Put $x = 8$ and $z = 6$

$$\frac{1}{y} = \frac{1}{8} + \frac{1}{6}$$

$$= \frac{14}{48} = \frac{7}{24}$$

$$\Rightarrow \quad y = \frac{24}{7}$$

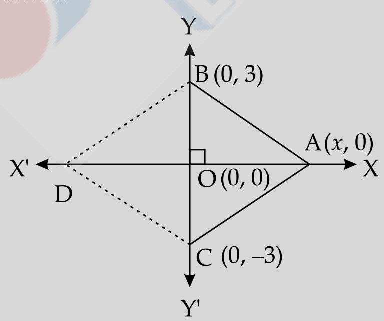
38. **(a)** 107^o

> ***Explanation:*** $\angle A = x^o$, $\angle B = 3x - 2^o$ and $\angle C = y^o$
> Sum of angles in a triangle is 180^o.
> Therefore, $x + 3x - 2 + y = 180^o$
> or $4x + y = 182$ ...(i)
> Also, $\angle C - \angle B = 90^o$
> or $y - (3x - 2) = 90^o$
> or $y - 3x = 70^o$ ...(ii)
> Subtracting (ii) from (i), we get
> $7x = 175$
> or $x = 250$
> Put $x = 25^o$ in (ii), we get $y = 82^o$
> Therefore,
> $\angle A = 25^o$, $\angle B = 3x - 2 = 3(25) - 2 = 73^o$
> And $\angle C = y^o$
> Sum of greatest and smallest angle
> $= 82^o + 25^o = 107^o$

39. **(b)** $\dfrac{p^2-1}{2p}$

> ***Explanation:*** $\sec\theta + \tan\theta = p$ ...(i)
> is the given equation.
> Since, $1 + \tan^2\theta = \sec^2\theta$
> or $\sec\theta = \sqrt{1+\tan^2\theta}$
> Put this value in (i), we get
> $\sqrt{1+\tan^2\theta} + \tan\theta = p$
> or $\sqrt{1+\tan^2\theta} = p - \tan\theta$
> Squaring both sides, we get
> $1 + \tan^2\theta = p^2 + \tan^2\theta - 2p\tan\theta$
> or $1 = p^2 - 2p(\tan\theta)$
> or $1 - p^2 = -2p\tan\theta$
> or $\tan\theta = \dfrac{p^2-1}{2p}$

40. **(c)** $A(\pm 3\sqrt{3}, 0)$, $B(0, 3)$

> ***Explanation:***
>
> 
>
>
> O is the midpoint of the base BC
> i.e., O is the midpoint of B and $C(0, -3)$
> Therefore, coordinates of point B is $(0, 3)$
> So, $BC = 6$ units.
> Let the coordinates of point A be $(x, 0)$.
> Using distance formula,
> $AB = \sqrt{(0-x)^2+(3-0)^2}$
> $= \sqrt{x^2+9}$
> $BC = \sqrt{(0-0)^2+(-3-3)^2}$
> $= \sqrt{36}$
> Also, $BC = AB$
> $\sqrt{x^2+9} = \sqrt{36}$
> $x^2 = 27$
> or $x = \pm 3\sqrt{3}$
> Coordinates of A and B are $(\pm 3\sqrt{3}, 0)$ and $(0, 3)$ respectively.

SECTION-C

Case Study-I

41. **(d)** $x + 2y = 16$

> ***Explanation:*** Let the fixed charge for two days be ₹x and additional charge be ₹y per day.
> As Radhika has taken book for 4 days.
> It means that Radhika will pay fixed charge for first two days and pays additional charges for next two days.
> $x + 2y = 16$

42. **(c)** $x + 4y = 22$

> ***Explanation:*** As the fixed charge for two days be ₹x and additional charge be ₹y per day
> It means that Amruta will pay fixed charge for first two days and pays additional charges for next four days.
> $x + 4y = 22$

43. **(c)** ₹10

> ***Explanation:***
> $x + 2y = 16$ ...(i)
> $x + 4y = 22$ ...(ii)
> Subtracting (ii) from (i), we get
> $y = 3$ and put this value of x in (i), we get $x = 10$.
> Therefore, fixed charge is x = ₹10.

44. **(d)** ₹3

> ***Explanation:*** From solution of Q.43, we get $y = 3$.
> Therefore, additional charges is y = ₹3.

45. **(c)** ₹50

> ***Explanation:*** For two more days price charged will be
> $2y = 2 \times 3 = 6$
> Total money paid by Amruta and Radhika is $22 + 16 + 6 + 6$ = ₹50

Case Study-II

46. **(b)** Similar by SAS criterion

47. **(b)** 1 : 4

Explanation: According to the theorem, "Ratio of area of similar triangles is equal to the square of the ratio of corresponding sides".

$$\frac{ar(\Delta AOB)}{ar(COD)} = \frac{(AB)^2}{(CD)^2}$$

$$= \frac{25}{100} = 1:4$$

48. **(d)** $CD = 4AB$

Explanation:

$$\frac{\text{Perimeter of } \Delta AOB}{\text{Perimeter of } \Delta COD} = \frac{1}{4}$$

Also, $$\frac{\text{Perimeter of } \Delta AOB}{\text{Perimeter of } \Delta COD} = \frac{AB}{CD}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{AB}{CD}$$

$$\Rightarrow \quad CD = 4AB$$

49. **(b)** $\Delta AOD \sim \Delta BCO$

50. **(c)** The medians have a ratio 1 : 4

❑❑❑

Term – I

OMR SHEET

Booklet Series
A

Use English Numbers / Letters only. Use Blue / Black Ball Point Pen to write in box.

Booklet Series

A B C D

Subject

Roll Number

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

Name

Test Date

Invigilator's Signature

Student's Signature

Certified that all the entries In this section have been properly filled by the student

Proper Marking

The OMR Sheet will be computer checked Fill the circles completely and dark enough for proper detection, Use ballpen (black or blue) for marking.

A B ● D

Avoid Improper Marking

Partially Filled

Lightly Filled

Test Center Code

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

IMPORTANT

The candidate should check that the Test Book Series printed on the OMR Sheet is the same as printed on the Test Booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.

Darken the circle for each question.

Q.No.	Response	Q.No.	Response	Q.No.	Response	Q.No.	Response
01	A B C D	16	A B C D	31	A B C D	46	A B C D
02	A B C D	17	A B C D	32	A B C D	47	A B C D
03	A B C D	18	A B C D	33	A B C D	48	A B C D
04	A B C D	19	A B C D	34	A B C D	49	A B C D
05	A B C D	20	A B C D	35	A B C D	50	A B C D
06	A B C D	21	A B C D	36	A B C D	51	A B C D
07	A B C D	22	A B C D	37	A B C D	52	A B C D
08	A B C D	23	A B C D	38	A B C D	53	A B C D
09	A B C D	24	A B C D	39	A B C D	54	A B C D
10	A B C D	25	A B C D	40	A B C D	55	A B C D
11	A B C D	26	A B C D	41	A B C D	56	A B C D
12	A B C D	27	A B C D	42	A B C D	57	A B C D
13	A B C D	28	A B C D	43	A B C D	58	A B C D
14	A B C D	29	A B C D	44	A B C D	59	A B C D
15	A B C D	30	A B C D	45	A B C D	60	A B C D