# Real Numbers

## Euclid's

**Division Lemma**

Given positive integers $a$ and $b$, there exist unique integers $q$ and $r$ satisfying $a = bq + r$ ; $0 \le r < b$

**Division Algorithm**

Steps to obtain the HCF of two positive integers, say $c$ and $d$, with $c > d$

- Step 1: Apply Euclid's Division Lemma, to $c$ & $d$. $c = dq + r$
- Step 2: If $r$ = zero, $d$ is the HCF of $c$ and $d$. If $r \neq 0$, apply Euclid's Division to $d$ and $r$
- Step 3: Continue the process till the remainder is zero

## Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur

Composite Number $x = P_1 \times P_2 \times P_3 \ldots \times P_n$, where $P_1, P_2, \ldots, P_n$ are prime numbers

## Prime Factorization Method

For any two positive integers, $a$ and $b$

HCF $(a, b)$ × LCM $(a, b) = a \times b$

For Example

$f(x) = 3x^2y$

$g(x) = 6xy^2$

HCF = $3xy$

LCM = $6x^2y^2$

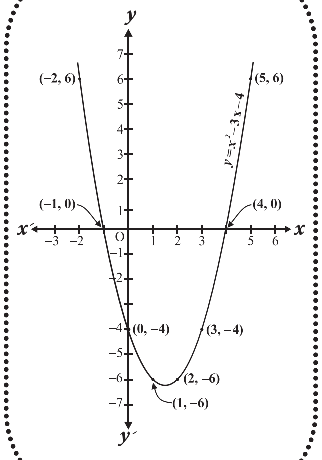
## Theorems

1. Let $p$ be a prime number. If $p$ divides $a^2$, then $p$ divides $a$, where $a$ is a positive integer
2. $\sqrt{2}, \sqrt{3}$ are irrational
3. Let $x$ be a rational number whose decimal expansion terminates. Then $x$ can be expressed in the form, $\frac{p}{q}$ where $p$ & $q$ are coprime, the prime factorisation of $q$ is of the form $2^n 5^m$ where $n$, $m$ are non-negative integers
4. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of $q$ is of the form $2^n 5^m$ where $n$, $m$ are non-negative integers. Then, $x$ has a decimal expansion which terminates.
5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of $q$ is not of the form of $2^n 5^m$ where $n$, $m$ are non-negative integers. Then, $x$ has a decimal expansion which is non-terminating repeating and recurring

# Polynomials

## Graphical Representation Quadratic Polynomial

**Parabola**




$y = x^2 - 3x - 4$

## Degree of Polynomial

Highest power of $x$ in Polynomial, $p(x)$

**Types**

| Polynomial | Degree | General Form |
|---|---|---|
| Linear | 1 | $ax+b$ |
| Quadratic | 2 | $ax^2+bx+c$, $a \neq 0$ |
| Cubic | 3 | $ax^3+bx^2+cx+d$, $a \neq 0$ |

## Division Algorithm

If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then – $p(x) = g(x) \times q(x) + r(x)$ where, $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

## Relationship-Zeroes and Coefficient of Polynomials

**Quadratic**

$\alpha$ and $\beta$ are zeroes of Quadratic Polynomial

$ax^2 + bx + c$

Then, Sum of zeroes,

$\alpha + \beta = -\frac{b}{a}$

Product of zeroes

$\alpha\beta = \frac{c}{a}$

**Cubic**

$\alpha$, $\beta$ and $\gamma$ are zeroes of Cubic Polynomial

$ax^3 + bx^2 + cx + d$

Sum of zeroes,

$\alpha + \beta + \gamma = -\frac{b}{a}$

Sum of products of the zeroes taken two at a time

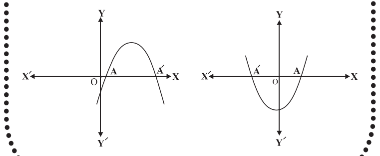
$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

Product of zeroes

$\alpha\beta\gamma = -\frac{d}{a}$

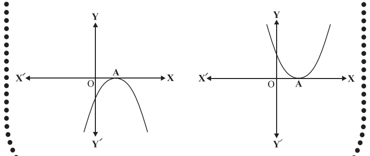
## Zeroes of Polynomial Graphically

**Case1- Graph cuts $x$-axis at 2 points**



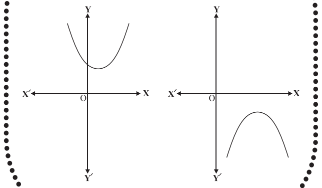

Number of Zeroes 2

**Case2- Graph cuts $x$-axis at exactly one point**




Number of Zeroes 1

**Case3- Graph does not cut $x$-axis**




Number of Zeroes 0

# Pair of Linear Equations in Two Variables

## General Form

$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
$a_1, b_1, c_1, a_2, b_2, c_2$, – Real numbers

## Solution Graphically

Each solution $(x, y)$, corresponds to a point on the line representing the equation and vice-versa

## Graphical Representation

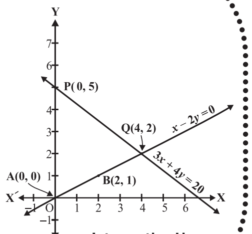
Pair of Lines: = $x-2y = 0$
$3x+4y-20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios: $= \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Algebraic Interpretation : Exactly one solution – consistent (unique)

Graphical Representation



Intersecting Lines

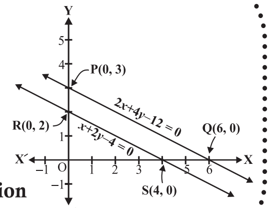
Pair of Lines: = $x+2y-4 = 0$
$2x+4y-12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios $= \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Algebraic Interpretation : No solution – Inconsistent

Graphical Representation



Parallel Lines

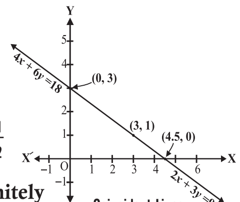
Pair of Lines: = $2x+3y-9 = 0$
$4x+6y-18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-9}{-18}$

Compare the Ratios $= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation = Infinitely many solutions – Dependent

Graphical Representation



Coincident Lines

## Algebraic Methods

### By Substitution

Solve: $7x-15y = 2$ –(i)
$x+2y = 3$ –(ii)

Solution: From equation (ii), $x = 3-2y$
substitute value of $x$ in eq. (i)
$7(3-2y)-15y = 2$

$-29y = -19 \Leftrightarrow y = \frac{19}{29}$

Now, from $x = 3 - 2y$

$x = 3 - 2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

### By Elimination

Solve: $x+3y = 6$ – (i)
$2x+3y = 12$ – (ii)
Now, Adding equation (i) and (ii)
$3x = 18$ or $x = 6$
Again, from (i) ×2– (ii)
$3y = 0$ or, $y = 0$
Hence, $x = 6, y = 0$

### By Cross-Multiplication

Solve: $2x+3y-46 = 0$ –(i)
$3x+5y-74 = 0$ –(ii)

Solution: By cross-multiplication method

$$\begin{array}{cccccc} & x & & y & & 1 \\ 3 & -46 & & 2 & & 3 \\ 5 & -74 & & 3 & & 5 \end{array}$$

Then, $\frac{x}{3(-74)-5(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{2(5)-3(3)}$

$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$

$\frac{x}{8} = \frac{y}{10} = \frac{1}{1} \Leftrightarrow \frac{x}{8} = \frac{1}{1}$ and $\frac{y}{10} = \frac{1}{1}$

*i.e.* $x = 8$ and $y = 10$

# Quadratic Equations

## Meaning

Equation of degree 2, in one variable

## General Form

$ax^2+bx+c=0$
$a, b, c$ – real numbers
$a \neq 0$

## Quadratic Formula

Roots of $ax^2+bx+c=0$ are given by

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

## Nature of Roots

For quadratic equation $ax^2+bx+c=0$, $b^2-4ac$ is Discriminant (D)

1. $D > 0$ Two distinct real root
2. $D = 0$ Two equal real roots
3. $D < 0$ No real roots (imaginary)

## Solution of a Quadratic Equation

### By Completing the Square

Solve: $2x^2-5x+3=0$

Solution: $2x^2-5x+3=0$

$$x^2-\frac{5}{2}x+\frac{3}{2}=0$$

$$\left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2+\frac{3}{2}=0 \Leftrightarrow \left(x-\frac{5}{4}\right)^2-\frac{1}{16}=0$$

$$\left(x-\frac{5}{4}\right)^2=\frac{1}{16} \Leftrightarrow x-\frac{5}{4}=\pm\frac{1}{4}$$

$$x=\frac{5}{4}+\frac{1}{4} \text{ or } x=\frac{5}{4}-\frac{1}{4}$$

$$x=\frac{3}{2} \text{ or } x=1$$

### By Factorization

Find roots of $6x^2-x-2=0$

Solution: $6x^2+3x-4x-2=0$

$3x(2x+1)-2(2x+1)=0$

$(3x-2)(2x+1)=0$

The roots of $6x^2-x-2=0$

$(3x-2)=0$ or $(2x+1)=0$

$x=\frac{2}{3}$ or $x=\frac{-1}{2}$

Roots are $\frac{2}{3}, \frac{-1}{2}$

# Arithmetic Progressions

## Definition

List of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. Fixed number is called common difference.

## General form

$a, a+d, a+2d, a+3d, \ldots a+(n-1)d$

## Common Difference

- Fixed number in arithmetic progression which provides the to and fro terms by adding/subtracting from the present number.
- Can be positive or negative.

## $n^{th}$ term

From beginning

$a_n = a+(n-1)d$

Here

$a \rightarrow$ first term

$d \rightarrow$ common difference

$a_n \rightarrow n^{th}$ term

From the end

$a_n = l-(n-1)d$

Here

$l \rightarrow$ last term

$d \rightarrow$ common difference

$a_n \rightarrow n^{th}$ term

## Sum (S)

When first & last terms are given :

$S_n = \frac{n}{2}(a+a_n)$ or $S_n = \frac{n}{2}(a+l)$

$a \rightarrow$ first term

$n \rightarrow$ total terms

$a_n \rightarrow n^{th}$ term

$l \rightarrow$ last term

When first term and common differnce are given :

$S_n = \frac{n}{2}(2a+(n-1)d)$

$a \rightarrow$ first term

$d \rightarrow$ common difference

$n \rightarrow$ total terms

## Arithmetic mean

If $a$, $b$, $c$, are in AP,

$b = \frac{a+c}{2}$

$b$ is arithmetic mean

## Examples

Sum of first n positive integers

Let $s_n = 1 + 2 + 3 + \ldots n$

$a = 1$, last term $l = n$

$s_n = \frac{n(a+l)}{2} = \frac{n(1+n)}{2}$

How many 2-digit numbers are divisible by 3?

2-digit numbers divisible by 3 are 12, 15, 18, ... 99

$a = 12$, $d = 3$, $a_n = 99$

$a_n = a + (n-1)d$

$99 = 12 + (n-1)3$

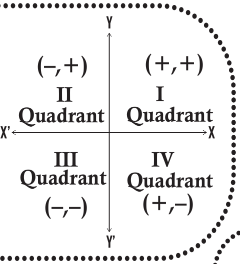
*i.e.,* $n-1 = \frac{87}{3} = 29$

$n = 30$

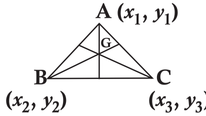
# Lines (In Two-Dimensions)

## Meaning

Study of algebraic equations on graphs

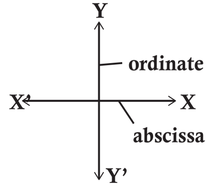



## Centroid




$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

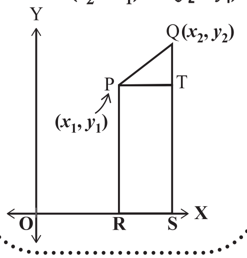
## Coordinate axis




Horizontal = $x$–axis (Abscissa)
Vertical = $y$–axis (Ordinate)

## Distance formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$




### Example

Are the following points vertices of a square : (1, 7), (4, 2), (–1, –1), (–4, 4)?
A (1, 7); B = (4, 2); C = (–1, –1); D = (–4, 4)

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{34}$

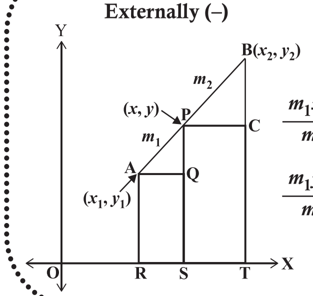
$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD.
All four sides and diagonals are equal Hence, ABCD is a square

## Section formula

Internally (+)
Externally (–)

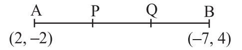



$$\frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}$$

$$\frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}$$

### Example

Find point of Trisection of line segment AB, A(2, –2) and B(–7, 4)




$$\text{Coordinate of } P = \left(\frac{1(-7) + 2(2)}{1+2}, \frac{1(4) + 2(-2)}{1+2}\right)$$

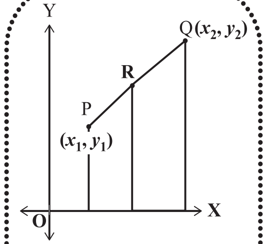
*i.e.,* (–1, 0)

Coordinate of Q

$$Q = \left(\frac{2(-7) + 1(2)}{2+1}, \frac{2(4) + 1(-2)}{2+1}\right)$$

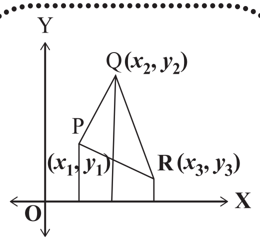
*i.e.,* (–4, 2)

## Mid-point Line Segment




$$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

## Area of Triangle




$$\text{Area} = \frac{1}{2}\,|\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\,|$$

# Triangles

## Theorems

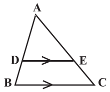
**Summary**

In ΔABC, let DE ∥ BC. Then,

(i) $\frac{AD}{DB} = \frac{AE}{EC}$

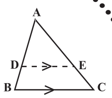
(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

(iii) $\frac{AD}{AB} = \frac{AE}{AC}$



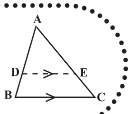
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If, DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$

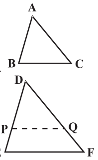


2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If $\frac{AD}{DB} = \frac{AE}{EC}$ then, DE || BC



3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)

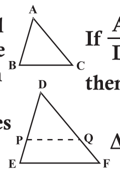


If ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

ΔABC~ΔDEF

4. If in two triangles, sides of one triangle are proportional to (*i.e.*, in the same ratio of ) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)

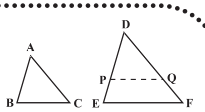


If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

then, ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

ΔABC~ΔDEF

5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)

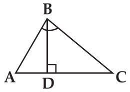


If $\frac{AB}{DE} = \frac{AC}{DF}$ & ∠A = ∠D

then, ΔABC ~ ΔDEF

## Right angled triangle theorem Pythagoras

1. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.



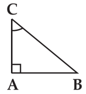
In right ΔABC, BD⊥AC,

then, ΔADB ~ ΔABC

ΔBDC ~ ΔABC

ΔADB ~ ΔBDC

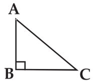
2. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



In right ΔABC,

$BC^2 = AB^2 + AC^2$

3. In a triangle, if square of one side is equal to the sum of the squares of other two sides, then the angle opposite the first side is a right angle.



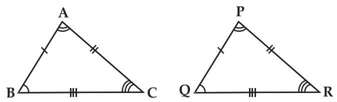
If $AC^2 = AB^2 + BC^2$

then, ∠ B = 90°

## Similarity

(i) Corresponding angles are equal

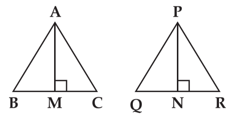
(ii) Corresponding sides are in the same ratio



ΔABC ~ ΔPQR

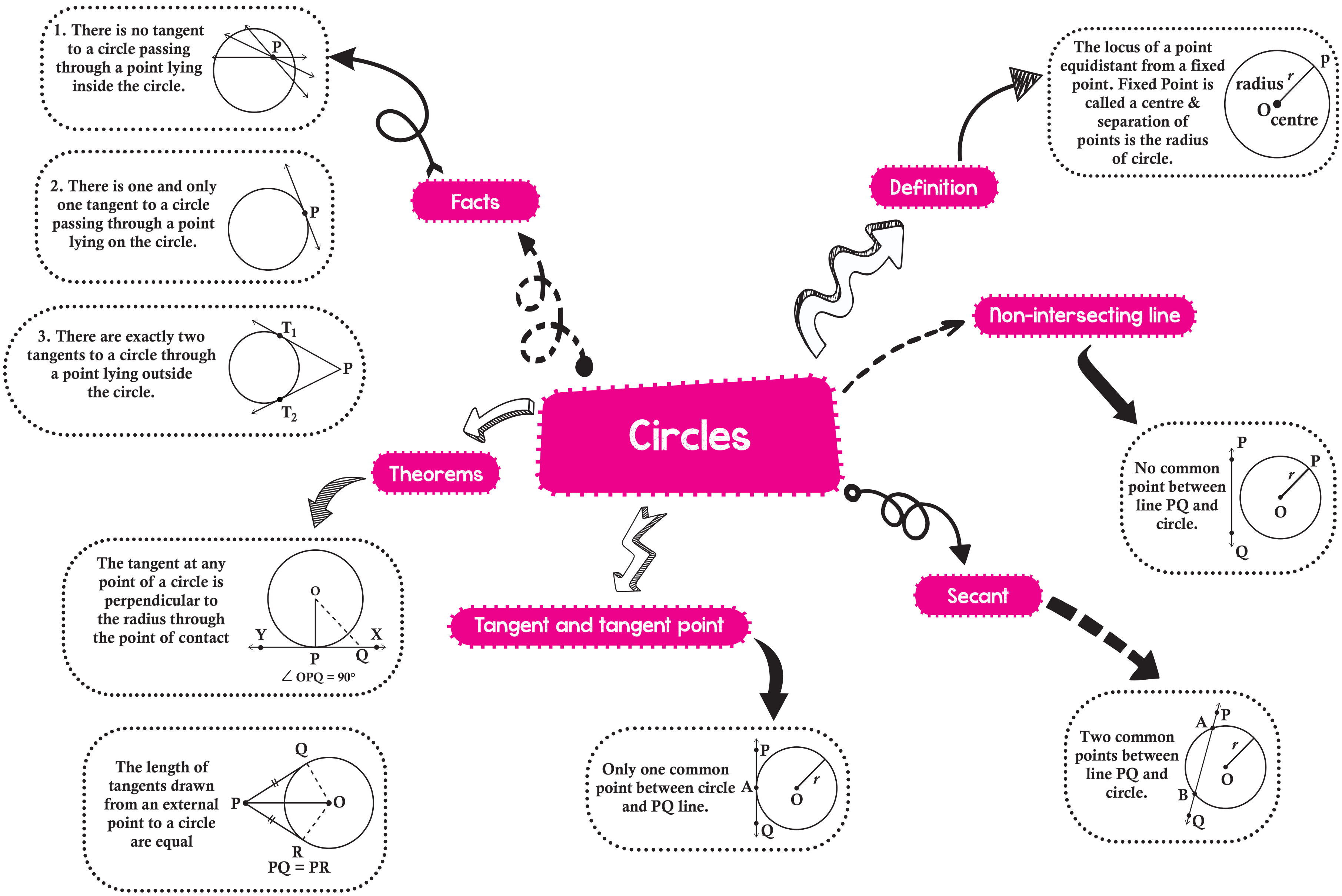
## Area of Similar Triangles

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides



Here ΔABC ~ ΔPQR

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$



Circles
Definition
The locus of a point equidistant from a fixed point. Fixed Point is called a centre & separation of points is the radius of circle.
radius
r
O
centre
P
Non-intersecting line
No common point between line PQ and circle.
P
Q
P
r
O
Secant
Two common points between line PQ and circle.
A
P
B
Q
r
O
Tangent and tangent point
Only one common point between circle and PQ line.
P
A
Q
r
O
Theorems
The tangent at any point of a circle is perpendicular to the radius through the point of contact
O
Y
X
P
Q
∠ OPQ = 90°
The length of tangents drawn from an external point to a circle are equal
Q
P
O
R
PQ = PR
Facts
1. There is no tangent to a circle passing through a point lying inside the circle.
P
2. There is one and only one tangent to a circle passing through a point lying on the circle.
P
3. There are exactly two tangents to a circle through a point lying outside the circle.
T1
T2
P

# Constructions

## Definition

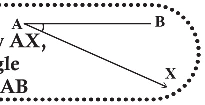
To draw geometrical shapes using compasses, ruler etc

## Line Segment Division of line in ratios

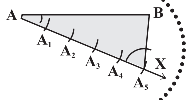
### Method 1

Given: Line segment, ratio (3 : 2)

Step 1. Draw any ray AX, Making acute angle with line segment AB

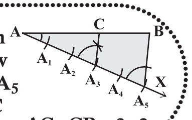



Step 2. Locate 5 points $A_1, A_2, A_3, A_4, A_5$ at equal distances ($A_1 = A_2 = A_3 = A_4 = A_5$). Join $BA_5$




Step 3. Through $A_3$ ($m$ = 3), draw line parallel to $BA_5$ cutting AB at C

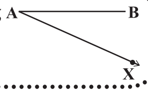
AC : CB = 3 : 2



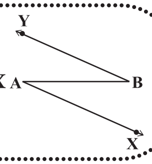

### Method 2

Given: Line segment, ratio (3 : 2)

1. Draw any ray AX making an acute angle with line segment AB

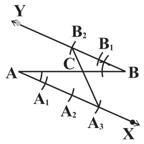



2. Draw ray BY || AX




3. Locate $A_1, A_2, A_3$ ($m$ = 3) on AX and $B_1, B_2$ ($n$ = 2) on BY Join $A_3 B_2$, intersecting AB at C

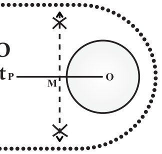
AC : CB = 3 : 2



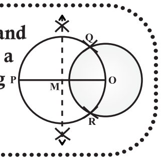

## Tangent to circle

Given: Circle with centre O and point P outside it.

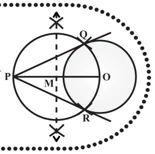
Step 1. Join PO and bisect it at mid-point M




Step 2. M as centre and radius = MO draw a circle, intersecting given circle at Q and R



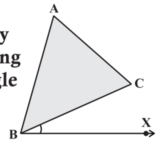

Step 3. Join PQ and PR, required tangents to the circle



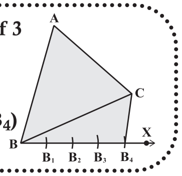

## Triangle similar to given triangle

Construct a triangle similar to a given $\Delta ABC$ with sides $\frac{3}{4}$ of the corresponding sides of $\Delta ABC$

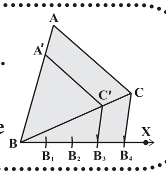
1. Draw any ray BX making an acute angle with BC




2. Locate 4 points (greater of 3 and 4 ) on BX at equal distance from each other ($BB_1 = B_1 B_2 = B_2 B_3 = B_3 B_4$) Join $B_4$ C




3. Draw line parallel to $B_4$ C from $B_3$ intersecting BC at C'. Draw line || to AC from C' intersecting AB at A' A'BC' is the required triangle



# Introduction to Trigonometry and Trigonometric Identities

## Trigonometry

Study of relationships between the sides & angles of a right triangle.

## Trigonometry Ratio

Sine of ∠A $= \frac{BC}{AC}$

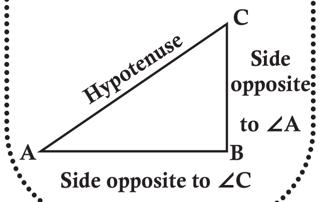
Cosine of ∠A $= \frac{AB}{AC}$

Tangent of ∠A $= \frac{BC}{AB}$

Cosecant of ∠A $= \frac{AC}{BC}$

Secant of ∠A $= \frac{AC}{AB}$

Cotangent of ∠A $= \frac{AB}{BC}$




## Values

| ∠A | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| sin A | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos A | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan A | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not (∞) defined |
| cot A | Not (∞) defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |
| sec A | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not (∞) defined |
| cosec A | Not (∞) defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |

## Example

Express tan A, cos A in terms of sin A

Solution : Since, $\cos^2 A + \sin^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$ *i.e.* $\cos A = \sqrt{1 - \sin^2 A}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

## Trigonometric Identities

$\cos^2 A + \sin^2 A = 1$

$1 + \tan^2 A = \sec^2 A; 0 \le A \le 90°$

$\cot^2 A + 1 = \text{cosec}^2 A; 0 \le A \le 90°$

## Complementary Angles

$\sin(90° - A) = \cos A$

$\cos(90° - A) = \sin A$

$\tan(90° - A) = \cot A$

$\cot(90° - A) = \tan A$

$\sec(90° - A) = \text{cosec } A$

$\text{cosec}(90° - A) = \sec A$

Note: How to learn the relation

"Some people have" $\sin\theta = \frac{P}{H}$

"Curly Brown Hair" $\cos\theta = \frac{B}{H}$

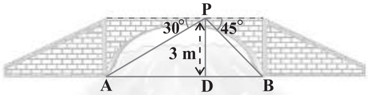
"through proper Brushing" $\tan\theta = \frac{P}{B}$

# Heights and Distances

## Examples

**Distance**

Determine width AB




From figure, AB = AD + DB

In right ΔAPD, ∠A = 30°, ∠D = 90°

$$\tan 30° = \frac{PD}{AD} \text{ i.e., } AD = 3\sqrt{3} \text{ m}$$

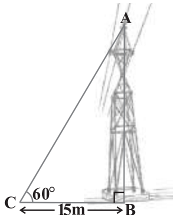
In right ΔBPD, ∠B = 45°, ∠D = 90°

$$\tan 45° = \frac{PD}{BD} \text{ i.e., } BD = 3$$

$\therefore$ AB = $(3\sqrt{3} + 3)$m = $3(\sqrt{3} + 1)$m

**Object Height**

Determine height of object AB




In ABC, ∠B = 90°, ∠C = 60°

Here, $\tan 60° = \frac{AB}{BC}$

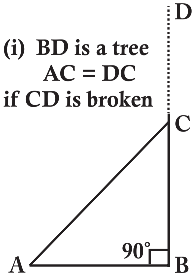
$$\sqrt{3} = \frac{AB}{15}$$

*i.e.,* AB = $15\sqrt{3}$m

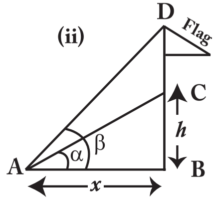
## Application-Trigonometric Ratios (To determine)

### height / length of an object

(i) BD is a tree
AC = DC
if CD is broken




(ii)

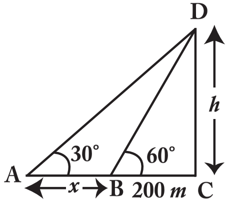



Find flag length

$$\tan \alpha = \frac{h}{x} \quad ...(i)$$

$$\tan \beta = \frac{h+DC}{x} \quad ...(ii)$$

### Distance between two objects



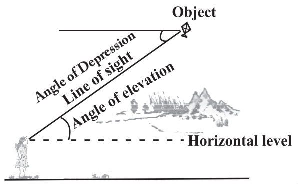

Find $x$ and $h$

$$\tan 60° = \frac{h}{200} \quad ...(i)$$

$$\tan 60° = \frac{h}{x+200} \quad ...(ii)$$

## Measuring Angles

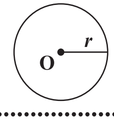
### Angle of Elevation is equal to Angle of Depression



# Areas Related to Circles

## Circle

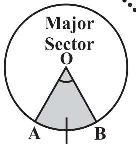
- Circumference = π × diameter = $2\pi r$
- Area = $\pi r^2$




## Sector

### Meaning

Portion of the circular region enclosed by two radius and the corresponding arc

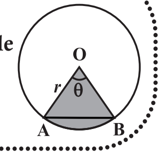



### Formula

**Area**

$A = \frac{\theta}{360^\circ} \times \text{area of circle}$

$A = \frac{\theta}{360^\circ} \times \pi r^2$




**Length of arc**

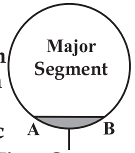
$L = \frac{\theta}{360^\circ} \times \text{circumference}$

$L = \frac{\theta}{360^\circ} \times 2\pi r$

## Segment

### Meaning

Portion of the circular region enclosed between a chord and the corresponding arc

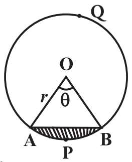



### Formula

Area = Area of the corresponding sector– Area of the corresponding triangle

$= \frac{\theta}{360^\circ} \times \pi r^2 - \text{area of } \Delta OAB$

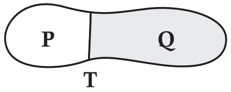
$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$




Area of Sector $= \frac{1}{2} \times L \times r$

## Area - Combination of figures

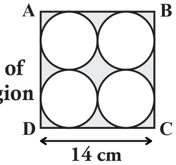
### Meaning




Area of T = Area of P + Area of Q

### Example

Find Area of shaded region




Area of square ABCD = $14 \times 14$ cm$^2$ = 196 cm$^2$

Diameter of each circle, $D = \frac{14}{2} = 7$ cm

For each circle, radius $(r) = \frac{7}{2}$ cm

Area of 1 circle = $\pi r^2$

Area of 4 circles $= 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$ cm$^2$

$= \frac{154}{4} \times 4$ cm$^2$

$= 154$ cm$^2$

Area of shaded region = Area of ABCD– Area of 4 circles

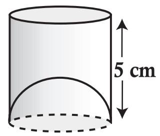
$= (196 - 154) = 42$ cm$^2$

# Surface Areas and Volumes

## Surface Area
Sum of surface areas of the faces of solid.

## Volume
Quantity of 3-D space enclosed by a hollow/closed solid.

## Combination of Solids
Given – Inner diameter of the Cylindrical glass = 5 cm
Height = 5 cm




Find – Actual capacity of Cylindrical glass

Solution :– Apparent capacity of the glass = $\pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 5$ cm$^3$

$= 98.125$ cm$^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$, if $r = 2.5$cm

$= \frac{2}{3} \times 3.14 \times (2.5)^3$ cm$^3$ $= 32.71$ cm$^3$

Actual capacity = Apparent capacity – Volume of hemisphere

$= 98.125 - 32.71$

$= 65.42$ cm$^3$

## Conversion of Solids
A copper rod – Diameter 1cm, length 8cm converted into a wire of length 18m
Find the thickness of the wire.

Solution : Volume of the rod $= \pi\left(\frac{1}{2}\right)^2 \times 8$cm$^3$

$= 2\pi$ cm$^3$

Let $r$ be the radius of cross-section of the wire, volume $= \pi \times r^2 \times 1800$ cm$^3$

$\therefore \quad \pi \times r^2 \times 1800 = 2\pi$

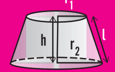
$r^2 = \frac{1}{900}$

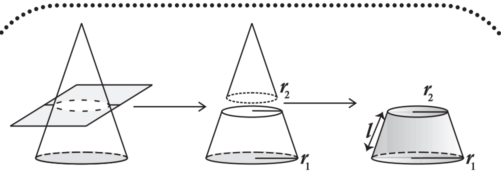
$r = \frac{1}{30}$ cm

Thickness = Diameter of the cross-section

$= \frac{1}{15}$ cm $= 0.07$ cm

## Frustum of Cone




A cone sliced by a plane parallel to base → The two parts separated → Frustum of a cone

**Curved Surface Area**

CSA $= \pi l\,(r_1 + r_2)$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

**Total Surface Area**

TSA $= \pi l\,(r_1 + r_2) + \pi(r_1^2 + r_2^2)$

**Volume**

$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

# Statistics

## Definition
A collection, analysis, interpretation of quantitative data

## Cumulative Frequency
Frequency obtained by adding the frequencies of all the classes preceding the giving class

## Class Mark
$$\left[\frac{\text{Upper class limit} + \text{Lower class limit}}{2}\right]$$

## Grouped Data

### Mean
- **Direct Method (Long cut)** $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
- **Assumed Mean (Short cut)** $\bar{x} = a + \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

  Here, $d_i = x_i - a$
- **Step Deviation** $\bar{x} = a + \dfrac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

  Here, $u_i = \dfrac{x_i - a}{h}$

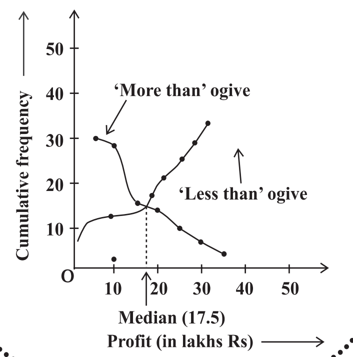
### Median
**Median** $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$

### Mode
**Mode** $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

## Emperical Relationship
3 Median = Mode + 2 Mean

## Ogive

### Meaning
Representation of cumulative frequencies with respect to given class intervals

### A Cumulative Frequency Graph


# Probability

## Theoretical Probability

- $0 \leq P(E) \leq 1$
- What we expect to happen in an experiment
- $P(E) = \dfrac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$

## Experimental Probability

- What actually happens in an experiment
- $P(E) = \dfrac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$

## Value

Sum of probabilities of all elementary events is 1.
For events A, B, C; $P(A) + P(B) + P(C) = 1$

## Definitions

**Elementary Event**
Event having only one outcome of the experiment

**Complementary Event**
For event E, complement event, $P(\bar{E}) = 1 - P(E)$

**Sure or Certain Event**
When event having probability to occur as 1

## Examples

**Dice**
Two dice are rolled, what is probability of getting 12 as a sum?

Solution : Number of possible outcome $= 6^2 = 36$

Number of favourable outcomes = 1

$P(E) = \dfrac{1}{36}$

**Coin**
When a coin is tossed, what would be the probability of appearing head ?

Solution : Total outcomes = 2
Favourable outcomes = 1

Required Prob. $P(E) = \dfrac{1}{2}$

**Card**
What is the probability of getting an ace from a pack of 52 cards?

Solution : Number of favourable outcomes = 4

Number of possible outcomes = 52

$P(E) = \dfrac{4}{52} = \dfrac{1}{13}$