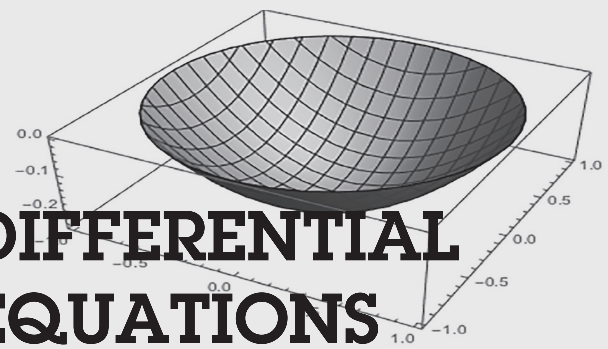
CHAPTER

9 DIFFERENTIAL EQUATIONS



Chapter Objectives

This chapter will help you understand :

- ***Differential equations : Introduction to differential equations; Basic concepts; General and Particular solutions of differential equations; Methods to solve First-order and First-degree differential equations.***

Quick Review

- Differential equation is an equation involving derivatives of an unknown function and possibly the function itself as well as the independent variables.
- Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from biology, economics, physics, chemistry, and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over the time.
- Some other uses of differential equations include :
  - In medicine, for modelling cancer growth or the spread of disease.
  - In engineering, for describing the movement of electricity.
  - In chemistry, for modelling chemical reactions and to computer radio-active half-life.
  - In economics, to find optimum investment strategies.
  - In physics, to describe the motion of waves, pendulums or chaotic systems. It is also used in physics with Newton's Second Law of Motion and the Law of Cooling.
  - In Hooke's Law, for modelling the motion of a spring or in representing models for population growth and money flow/circulation.

TIPS...

- *Differential equation is a linear differential equation if the degree of function and its derivatives are all 1, otherwise, the equation is said to be a non-linear differential equation.*
- *Linear differential equations are notable because they have solutions that can be added together in linear combinations to form further solutions.*

TRICKS...

- *Try to identify the order of differential equation as the order of the highest derivative taken in the equation.*
- *The general solutions to ordinary differential equations are not unique but introduce arbitrary constants. The number of constants is equal to the order of equation in most instances.*
- *In applications, these constants are subject to be evaluated given the initial conditions : the function and its derivatives at $x = 0$. The number of initial conditions required to find a particular solution of a differential equation is also equal to the order of equation in most cases.*

Know the Links

- *http ://www.analyzemath.com/calculus/Differential_Equations/applications.html*
- *http ://tutorial.math.lamar.edu/Classes/DE/IntroBasic.aspx*

Multiple Choice Questions

(1 mark each)

Q. 1. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right) \text{ is}$$

(a) 1 **(b) 2**

(c) 3 **(d) not defined**

[NCERT Exemp. Ex. 9.3, Q. 34, Page 195]

Ans. Correct option : (d)

Explanation : The degree of above differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

Q. 2. The degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2} \text{ is}$$

(a) 4 **(b) $\frac{3}{2}$**

(c) not defined **(d) 2**

[NCERT Exemp. Ex. 9.3, Q. 35, Page 195]

Ans. Correct option : (d)

Explanation : Given that,

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$$

On squaring both sides, we get

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3}=\left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

Q. 3. The order and degree of the differential equation

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}+x^{1/5}=0 \text{ respectively, are}$$

(a) 2 and 4 **(b) 2 and 2**

(c) 2 and 3 **(d) 3 and 3**

[NCERT Exemp. Ex. 9.3, Q. 36, Page 195]

Ans. Correct option : (a)

Explanation :

Given that, $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}=-x^{1/5}$

$$\Rightarrow \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}=-x^{1/5}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^{1/4}=-\left(x^{1/5}+\frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^{1/2}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we have

$$\frac{dy}{dx}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^4$$

Order = 2, degree = 4

Q. 4. If $y = e^{-x}$ $(A \cos x + B \sin x)$, then y is a solution of

(a) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$ **(b) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$**

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ **(d) $\frac{d^2y}{dx^2}+2y=0$**

[NCERT Exemp. Ex. 9.3, Q. 37, Page 195]

Ans. Correct option : (c)

Explanation :

Given that, $y = e^{-x}$ $(A \cos x + B \sin x)$

On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx}=-e^{-x}(A\cos x+B\sin x)+e^{-x}(-A\sin x+B\cos x)$$

$$\frac{dy}{dx}=-y+e^{-x}(-A\sin x+B\cos x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{-dy}{dx}+e^{-x}(-A\cos x-B\sin x)-e^{-x}(-A\sin x+B\cos x)$$

$$\Rightarrow \frac{d\ y}{dx}=-\frac{dy}{dx}-y\left[\frac{dy}{dx}+y\right]$$

$$\Rightarrow \frac{d\ y}{dx}=-\frac{dy}{dx}-y-\frac{dy}{dx}-y$$

$$\Rightarrow \frac{d\ y}{dx}=-2\frac{dy}{dx}-2$$

$$\Rightarrow \frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$$

Q. 5. The differential equation for $y = A \cos\alpha x + B \sin\alpha x$, where A and B are arbitrary constants, is

(a) $\frac{d^2y}{dx^2}-\alpha^2y=0$ **(b) $\frac{d^2y}{dx^2}+\alpha^2y=0$**

(c) $\frac{d^2y}{dx^2}+\alpha y=0$ **(d) $\frac{d^2y}{dx^2}-\alpha y=0$**

[NCERT Exemp. Ex. 9.3, Q. 38, Page 196]

Ans. Correct option : (b)

Explanation :

Given, $y = A \cos \alpha x + B \sin \alpha x$

$$\Rightarrow \frac{dy}{dx}=-\alpha A\sin\alpha x+\alpha B\cos\alpha x$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-A\alpha^2\cos\alpha x-\alpha^2B\sin\alpha x$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\alpha^2(A\cos\alpha x+B\sin\alpha x)$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\alpha^2y$$

$$\Rightarrow \frac{d^2y}{dx^2}+\alpha^2y=0$$

Q. 6. The solution of differential equation $xdy - ydx$ =0 represents

(a) a rectangular hyperbola

(b) parabola whose vertex is at origin

(c) straight line passing through origin

(d) a circle whose centre is at origin

[NCERT Exemp. Ex. 9.3, Q. 39, Page 196]

Ans. Correct option : (c)

Explanation :

Given that, $xdy - ydx = 0$

$\Rightarrow xdy = ydx$

$$\Rightarrow \frac{dy}{y}=\frac{dx}{x}$$

On integrating both sides, we get

$$\log y=\log x+\log C$$

$$\Rightarrow \log y=\log Cx$$

$$\Rightarrow y=Cx$$

which is a straight line passing through the origin.

Q. 7. The integrating factor of differential equation

$$\cos x\frac{dy}{dx}+y\sin x=1 \text{ is}$$

(a) $\cos x$ **(b) $\tan x$**

(c) $\sec x$ **(d) $\sin x$**

[NCERT Exemp. Ex. 9.3, Q. 40, Page 196]

Ans. Correct option : (c)

Explanation :

Given that, $\cos x\frac{dy}{dx}+y\sin x=1$

$\Rightarrow \quad \frac{dy}{dx} + y\tan x = \sec x$

Here, $P = \tan x$ and $Q = \sec x$

$IF = e^{\int P dx} = e^{\int \tan x dx} = e^{\ln \sec x}$

$\therefore IF = \sec x$

Q. 8. Solution of the differential equation $\tan y \sec^2 x\, dx + \tan x \sec^2 y dy = 0$ is :

(a) $\tan x + \tan y = k$ **(b)** $\tan x - \tan y = k$

(c) $\frac{\tan x}{\tan y} = k$ **(d)** $\tan x \cdot \tan y = k$

[NCERT Exemp. Ex. 9.3, Q. 41, Page 196]

Ans. Correct option : (d)

Explanation :

Given that, $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$

$\Rightarrow \tan y \sec^2 x dx = -\tan x \sec^2 y dy$

$\Rightarrow \quad \frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy \quad$ ...(i)

On integrating both sides, we have

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

Put $\tan x = t$ in LHS integral, we get

$\sec^2 x\, dx = dt \Rightarrow \sec^2 x\, dx = dt$

and $\tan y = u$ in RHS integral, we get

$\sec^2 y dy = du$

On substituting these values in EQ. (i), we get

$\int \frac{dt}{t} = -\int \frac{du}{u}$

$\log t = -\log u + \log k$

$\Rightarrow \quad \log(t \cdot u) = \log k$

$\Rightarrow \quad \log(\tan x \tan y) = \log k$

$\Rightarrow \quad \tan x \tan y = k$

Q. 9. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree :

(a) 1 **(b)** 2

(c) 3 **(d)** 4

[NCERT Exemp. Ex. 9.3, Q. 42, Page 196]

Ans. Correct option : (a)

Explanation :

Given that, $y = Ax + A^3$

$\Rightarrow \frac{dy}{dx} = A$

[We can differential above equation only once because it has only one arbitrary constant.]

$\therefore$ Degree = 1

Q. 10. The integrating factor of

$\frac{xdy}{dx} - y = x^4 - 3x$ **is**

(a) x **(b)** $\log x$

(c) $\frac{1}{x}$ **(d)** $-x$

[NCERT Exemp. Ex. 9.3, Q. 43, Page 196]

Ans. Correct option : (c)

Explanation :

Given that, $x\frac{dy}{dx} - y = x^4 - 3x$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Here, $P = -\frac{1}{x}$, $Q = x^3 - 3$

$\therefore \; IF = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$

Q. 11. The solution of

$\frac{dy}{dx} - y = 1$, $y(0) = 1$ **is given by**

(a) $xy = -e^x$ **(b)** $xy = -e^{-x}$

(c) $xy = -1$ **(d)** $y = 2e^x - 1$

[NCERT Exemp. Ex. 9.3, Q. 44, Page 196]

Ans. Correct option : (d)

Explanation : Given that,

$\frac{dy}{dx} - y = 1$

$\Rightarrow \quad \frac{dy}{dx} = 1 + y$

$\Rightarrow \quad \frac{dy}{1+y} = dx$

On integrating both sides, we get

$\log(1+y) = x + C$

When $x = 0$ and $y = 1$, then

$\log 2 = 0 + C$

$\Rightarrow C = \log 2$

The required solution is

$\log(1+y) = x + \log 2$

$\Rightarrow \log\left(\frac{1+y}{2}\right) = x$

$\Rightarrow \quad \frac{1+y}{2} = e^x$

$\Rightarrow \quad 1 + y = 2e^x$

$\Rightarrow \quad y = 2e^x - 1$

Q. 12. The number of solutions of

$\frac{dy}{dx} = \frac{y+1}{x-1}$, **when $y(1) = 2$ is**

(a) none **(b)** one

(c) two **(d)** infinite

[NCERT Exemp. Ex. 9.3, Q. 45, Page 197]

Ans. Correct option : (b)

Explanation :

Given that, $\frac{dy}{dx} = \frac{y+1}{x-1}$

$\Rightarrow \frac{dy}{y+1} = \frac{dx}{x-1}$

On integrating both sides, we get

$\log(y+1) = \log(x-1) - \log C$

$C(y+1) = (x-1)$

$\Rightarrow \quad C = \frac{x-1}{y+1}$

When $x = 1$ and $y = 2$, then $C = 0$

So, the required solution is $x - 1 = 0$

Hence, only one solution can exist.

Q. 13. Which of the following is a second-order differential equation?

(a) $(y')^2 + x = y^2$ (b) $y'y'' + y = \sin x$

(c) $y''' + (y'')^2 + y = 0$ (d) $y' = y^2$

[NCERT Exemp. Ex. 9.3, Q. 46, Page 197]

Ans. Correct option : (b)

Explanation :

The second-order differential equation is $y'y''+y = \sin x$.

Q. 14. The integrating factor of differential equation $(1-x^2)\dfrac{dy}{dx} - xy = 1$ is

(a) $-x$ (b) $\dfrac{x}{1+x^2}$

(c) $\sqrt{1-x^2}$ (d) $\dfrac{1}{2}\log(1-x^2)$

[NCERT Exemp. Ex. 9.3, Q. 47, Page 197]

Ans. Correct option : (c)

Explanation :

Given that, $(1-x^2)\dfrac{dy}{dx} - xy = 1$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{x}{1-x^2}y = \dfrac{1}{1-x^2}$

which is a linear differential equation.

IF $= e^{-\int \frac{x}{1-x^2}dx}$

Put $1-x^2 = t \Rightarrow -2xdx = dt \Rightarrow xdx = -\dfrac{dt}{2}$

Now, IF $= e^{\frac{1}{2}\int \frac{dt}{t}} = e^{\frac{1}{2}\log t} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1-x^2}$

Q. 15. $\tan^{-1}x + \tan^{-1}y = C$ is general solution of the differential equation

(a) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$ (b) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(c) $(1+x^2)dy + (1+y^2)dx = 0$

(d) $(1+x^2)dx + (1+y^2)dy = 0$

[NCERT Exemp. Ex. 9.3, Q. 48, Page 197]

Ans. Correct option : (c)

Explanation :

Given that, $\tan^{-1}x + \tan^{-1}y = C$

On differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{1+y^2}\cdot\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$\Rightarrow (1+x^2)dy + (1+y^2)dx = 0$

Q. 16. The differential equation $y\dfrac{dy}{dx} + x = C$ represents

(a) family of hyperbolas (b) family of parabolas

(c) family of ellipses (d) family of circles

[NCERT Exemp. Ex. 9.3, Q. 49, Page 197]

Ans. Correct option : (d)

Explanation :

Given that, $y\dfrac{dy}{dx} + x = C$

$\Rightarrow y\dfrac{dy}{dx} = C - x$

$\Rightarrow yd\,y = (C-x)dx$

On integrating both sides, we get

$$\frac{y^2}{2} = Cx - \frac{x^2}{2} + K$$

$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{2} = Cx + K$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} - Cx = K$$

which represent family of circles.

Q. 17. The general solution of $e^x \cos y dx - e^x \sin y dy = 0$ is

(a) $e^x \cos y = k$ (b) $e^x \sin y = k$

(c) $e^x = k \cos y$ (d) $e^x = k \sin y$

[NCERT Exemp. Ex. 9.3, Q. 50, Page 197]

Ans. Correct option : (a)

Explanation :

Given that, $e^x \cos ydx - e^x \sin ydy = 0$

$\Rightarrow e^x \cos ydx = e^x \sin ydy$

$\Rightarrow \quad \dfrac{dx}{dy} = \tan y$

$\Rightarrow \quad dx = \tan ydy$

On integrating both sides, we get

$x = \log\sec y + C$

$\Rightarrow \quad x - C = \log\sec y$

$\Rightarrow \quad \sec y = e^{x-C}$

$\Rightarrow \quad \sec y = e^x e^{-C}$

$\Rightarrow \quad \dfrac{1}{\cos y} = \dfrac{e^x}{e^C}$

$\Rightarrow e^x \cos y = e^C$

$\Rightarrow e^x \cos y = K \quad$ [where, $K = e^C$]

Q. 18. The degree of differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0 \text{ is}$$

(a) 1 (b) 2

(c) 3 (d) 5

[NCERT Exemp. Ex. 9.3, Q. 51, Page 197]

Ans. Correct option : (a)

Explanation :

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$$

We know that, the degree of a differential equation is exponent of highest order derivative.

$\therefore$ Degree = 1

Q. 19. The solution of

$\dfrac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is

(a) $y = e^x(x-1)$ (b) $y = xe^{-x}$

(c) $y = xe^{-x} + 1$ (d) $y = (x+1)e^{-x}$

[NCERT Exemp. Ex. 9.3, Q. 52, Page 197]

Ans. Correct option : (b)

Explanation :

Given that, $\dfrac{dy}{dx} + y = e^{-x}$

Here, $P = 1$, $Q = e^{-x}$

IF $= e^{\int Pdx} = e^{\int dx} = e^x$

The general solution is

$y \cdot e^x = \int e^{-x} e^x dx + C$

$\Rightarrow y \cdot e^x = \int dx + C$

$\Rightarrow y \cdot e^x = x + C$ ...(i)

When $x = 0$ and $y = 0$, then

$0 = 0 + C \Rightarrow C = 0$

EQ. (i) becomes $y \cdot e^x = x$

$\Rightarrow y = xe^{-x}$

Q. 20. The integrating factor of differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

(a) $\cos x$ **(b)** $\sec x$

(c) $e^{\cos x}$ **(d)** $e^{\sec x}$

[NCERT Exemp. Ex. 9.3, Q. 53, Page 198]

Ans. Correct option : (b)

Explanation :

Given that, $\frac{dy}{dx} + y\tan x - \sec x = 0$

Here, $P = \tan x$, $Q = \sec x$

$IF = e^{\int P dx} = e^{\int \tan x dx}$

$= e^{(\log \sec x)}$

$= \sec x$

Q. 21. The solution of differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1} x$ **(b)** $y - x = k(1 + xy)$

(c) $x = \tan^{-1} y$ **(d)** $\tan (xy) = k$

[NCERT Exemp. Ex. 9.3, Q. 54, Page 198]

Ans. Correct option : (b)

Explanation :

Given that, $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

On integrating both sides, we get

$\tan^{-1} y = \tan^{-1} x + C$

$\Rightarrow \tan^{-1} y - \tan^{-1} x = C$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = C$

$\Rightarrow \frac{y-x}{1+xy} = \tan C$

$\Rightarrow y - x = \tan C(1+xy)$

$\Rightarrow y - x = k(1+xy)$

where, $k = \tan C$

Q. 22. The integrating factor of differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{x}{e^x}$ **(b)** $\frac{e^x}{x}$

(c) xe^x **(d)** e^x

[NCERT Exemp. Ex. 9.3, Q. 55, Page 198]

Ans. Correct option : (b)

Explanation :

Given that, $\frac{dy}{dx} + y = \frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1+y}{x} - y$

$\Rightarrow \frac{dy}{dx} = \frac{1+y-xy}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} + \frac{y(1-x)}{x}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = \frac{1}{x}$

Here, $P = \frac{-(1-x)}{x}$, $Q = \frac{1}{x}$

$IF = e^{\int P dx} = e^{-\int \frac{1-x}{x} dx} = e^{\int \frac{x-1}{x} dx}$

$= e^{\int \left(1 - \frac{1}{x}\right) dx}$

$= e^{\int x - \log x}$

$= e^x \cdot e^{\log\left(\frac{1}{x}\right)}$

$= e^x \cdot \frac{1}{x}$

Q. 23. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation

(a) $\frac{dy}{dx} + my = 0$ **(b)** $\frac{dy}{dx} - my = 0$

(c) $\frac{d^2y}{dx^2} - m^2y = 0$ **(d)** $\frac{d^2y}{dx^2} + m^2y = 0$

[NCERT Exemp. Ex. 9.3, Q. 56, Page 198]

Ans. Correct option : (c)

Explanation :

Given that, $y = ae^{mx} + be^{-mx}$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = mae^{mx} - bme^{-mx}$

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = m^2ae^{mx} + bm^2e^{-mx}$

$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mn} + be^{-mn})$

$\Rightarrow \frac{d^2y}{dx^2} = m^2y$

$\Rightarrow \frac{d^2y}{dx^2} - m^2y = 0$

Q. 24. The solution of differential equation $\cos x \sin y dx + \sin x \cos y dy = 0$ is

(a) $\frac{\sin x}{\sin y} = C$ **(b)** $\sin x \sin y = C$

(c) $\sin x + \sin y = C$ **(d)** $\cos x \cos y = C$

[NCERT Exemp. Ex. 9.3, Q. 57, Page 198]

Ans. Correct option : (b)

Explanation :

Given differential equation is

$\cos x \sin y dx + \sin x \cos y dy = 0$

$\Rightarrow \cos x \sin y dx = -\sin x \cos y dy$

$\Rightarrow \frac{\cos x}{\sin x} dx = -\frac{\cos y}{\sin y} dy$

$\Rightarrow \quad \cot x\, dx = -\cot y dy$

On integrating both sides, we get

log sin x = –log sin y + log C

$\Rightarrow \log \sin x \sin y = \log C$

$\Rightarrow \quad \sin x \cdot \sin y = C$

Q. 25. The solution of $x\frac{dy}{dx} + y = e^x$ is

(a) $y = \frac{e^x}{x} + \frac{k}{x}$ **(b)** $y = xe^x + Cx$

(c) $y = xe^x + k$ **(d)** $x = \frac{e^y}{y} + \frac{k}{y}$

[NCERT Exemp. Ex. 9.3, Q. 58, Page 198]

Ans. Correct option : (a)

Explanation :

Given that, $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

which is a linear differential equation.

$\therefore \quad \text{IF} = e^{\int \frac{1}{x} dx} = e^{(\log x)} = x$

The general solution is $y \cdot x = \int \left(\frac{e^x}{x} \cdot x\right) dx$

$\Rightarrow y \cdot x = \int e^x dx$

$\Rightarrow y \cdot x = e^x + k$

$\Rightarrow \quad y = \frac{e^x}{x} + \frac{k}{x}$

Q. 26. The differential equation of the family of curves $x^2 + y^2 - 2ay = 0$, where a is arbitrary constant, is

(a) $(x^2 - y^2)\frac{dy}{dx} = 2xy$ **(b)** $2(x^2 + y^2)\frac{dy}{dx} = xy$

(c) $2(x^2 - y^2)\frac{dy}{dx} = xy$ **(d)** $(x^2 + y^2)\frac{dy}{dx} = 2xy$

[NCERT Exemp. Ex. 9.3, Q. 59, Page 199]

Ans. Correct option : (a)

Explanation :

Given equation of curve is

$x^2 + y^2 - 2ay = 0$

$\Rightarrow \quad \frac{x^2 + y^2}{y} = 2a$

On differentiating both sides w.r.t. x, we get

$$\frac{y\left(2x + 2y\frac{dy}{dx}\right) - (x^2 + y^2)\frac{dy}{dx}}{y^2} = 0$$

$\Rightarrow 2xy + 2y^2\frac{dy}{dx} - (x^2 + y^2)\frac{dy}{dx} = 0$

$\Rightarrow \quad (2y^2 - x^2 - y^2)\frac{dy}{dx} = -2xy$

$\Rightarrow \quad (y^2 - x^2)\frac{dy}{dx} = -2xy$

$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} = 2xy$

Q. 27. The family $y = Ax + A^3$ of curves will correspond to a differential equation of order

(a) 3 **(b)** 2

(c) 1 **(d)** not defined

[NCERT Exemp. Ex. 9.3, Q. 60, Page 199]

Ans. Correct option : (c)

Explanation :

Given family of curves is $y = Ax + A^3$

$\Rightarrow \frac{dy}{dx} = A$ ...(i)

Replacing A by $\frac{dy}{dx}$ in EQ. (i) we get

$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$

$\therefore$ Order = 1

Q. 28. The general solution of $\frac{dy}{dx} = 2x\, e^{x^2 - y}$ is

(a) $e^{x^2 - y} = C$ **(b)** $e^{-y} + e^{x^2} = C$

(c) $e^y = e^{x^2} + C$ **(d)** $e^{x^2 + y} = C$

[NCERT Exemp. Ex. 9.3, Q. 61, Page 199]

Ans. Correct option : (c)

Explanation :

Given that, $\frac{dy}{dx} = 2x\, e^{x^2 - y} = 2x\, e^{x^2} \cdot e^{-y}$

$\Rightarrow e^y \frac{dy}{dx} = 2x\, e^{x^2}$

$\Rightarrow e^y\, dy = 2x\, e^{x^2} dx$

On integrating both sides, we get

$\int e^y\, dy = 2\int xe^{x^2} dx$

Put $x^2 = t$ in RHS integral, we get

$2x\, dx = dt$

$\int e^y\, dy = \int e^t\, dt$

$\Rightarrow \quad e^y = e^t + C$

$\Rightarrow \quad e^y = e^{x^2} + C$

Q. 29. The curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point is

(a) an ellipse

(b) parabola

(c) circle

(d) rectangular hyperbola

[NCERT Exemp. Ex. 9.3, Q. 62, Page 199]

Ans. Correct option : (d)

Explanation :

Slope of tangent to the curve $= \frac{dy}{dx}$

and ratio of abscissa to the ordinate $= \frac{x}{y}$

According to the question, $\frac{dy}{dx} = \frac{x}{y}$

$yd\,y = xd\,x$

On integrating both sides, we get

$$\frac{y^2}{2}=\frac{x^2}{2}+C$$

$\Rightarrow \frac{y^2}{2}-\frac{x^2}{2}=C$

$\Rightarrow y^2-x^2=2C$

which is an equation of rectangular hyperbola.

Q. 30. The general solution of differential equation

$$\frac{dy}{dx}=e^{\frac{x^2}{2}}+xy \text{ is}$$

(a) $y=Ce^{-x^2/2}$ **(b)** $y=Ce^{x^2/2}$

(c) $y=(x+C)e^{x^2/2}$ **(d)** $y=(C-x)e^{x^2/2}$

[NCERT Exemp. Ex. 9.3, Q. 63, Page 199]

Ans. Correct option : (c)

Explanation :

Given that, $\frac{dy}{dx}=e^{x^2/2}+xy$

$\Rightarrow \frac{dy}{dx}-xy=e^{x^2/2}$

Here, $P=-x,\ Q=e^{x^2/2}$

$\therefore \text{IF}=e^{\int -x\,dx}=e^{-x^2/2}$

The general solution is

$$y\cdot e^{-x^2/2}=\int e^{-x^2/2}.e^{x^2/2}dx+C$$

$\Rightarrow ye^{-x^2/2}=\int 1dx+C$

$\Rightarrow y\cdot e^{-x^2/2}=x+C$

$\Rightarrow \quad y=xe^{x^2/2}+Ce^{+x^2/2}$

$\Rightarrow \quad y=(x+C)e^{x^2/2}$

Q. 31. The solution of equation $(2y-1)\,dx-(2x+3)\,dy=0$ is

(a) $\frac{2x-1}{2y+3}=k$ **(b)** $\frac{2y+1}{2x-3}=k$

(c) $\frac{2x+3}{2y-1}=k$ **(d)** $\frac{2x-1}{2y-1}=k$

[NCERT Exemp. Ex. 9.3, Q. 64, Page 199]

Ans. Correct option : (c)

Explanation :

Given that, $(2y-1)dx-(2x+3)dy=0$

$\Rightarrow (2y-1)dx=(2x+3)dy$

$\Rightarrow \quad \frac{dx}{2x+3}=\frac{dy}{2y-1}$

On integrating both sides, we get

$$\frac{1}{2}\log(2x+3)=\frac{1}{2}\log(2y-1)+\log C$$

$\Rightarrow \frac{1}{2}[\log\cdot(2x+3)-\log(2y-1)]=\log C$

$\Rightarrow \quad \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right)=\log C$

$\Rightarrow \quad \left(\frac{2x+3}{2y-1}\right)^{1/2}=C$

$\Rightarrow \quad \frac{2x+3}{2y-1}=C^2$

$\Rightarrow \quad \frac{2x+3}{2y-1}=k,$ where $k=C^2$

Q. 32. The differential equation for which $y=a\cos x+b\sin x$ a solution, is

(a) $\frac{d^2y}{dx^2}+y=0$ **(b)** $\frac{d^2y}{dx^2}-y=0$

(c) $\frac{d^2y}{dx^2}+(a+b)y=0$ **(d)** $\frac{d^2y}{dx^2}+(a-b)y=0$

[NCERT Exemp. Ex. 9.3, Q. 65, Page 200]

Ans. Correct option : (a)

Explanation :

Given that, $y=a\cos x+b\sin x$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=-a\sin x+b\cos x$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-a\cos x-b\sin x$$

$\Rightarrow \quad \frac{d^2y}{dx^2}=-y$

$\Rightarrow \frac{d^2y}{dx^2}+y=0$

Q. 33. The solution of

$$\frac{dy}{dx}+y=e^{-x},\ y(0)=0 \text{ is}$$

(a) $y=e^{-x}(x-1)$ **(b)** $y=xe^{x}$

(c) $y=xe^{-x}+1$ **(d)** $y=xe^{-x}$

[NCERT Exemp. Ex. 9.3, Q. 66, Page 200]

Ans. Correct option : (d)

Explanation :

Given that, $\frac{dy}{dx}+y=e^{-x}$

which is a linear differential equation.

Here, $P=1$ and $Q=e^{-x}$

$\text{IF}=e^{\int dx}=e^{x}$

The general solution is

$y\cdot e^{x}=\int e^{-x}\cdot e^{x}dx+C$

$\Rightarrow ye^{x}=\int dx+C$

$\Rightarrow ye^{x}=x+C$ ...(i)

When $x=0$ and $y=0$ then, $0=0+C\Rightarrow C=0$

EQ. (i) becomes $y\cdot e^{x}=x\Rightarrow y=xe^{-x}$

Q. 34. The order and the degree of differential equations

$$\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4 \text{ are}$$

(a) 1, 4 **(b)** 3, 4

(c) 2, 4 **(d)** 3, 2

[NCERT Exemp. Ex. 9.3, Q. 67, Page 200]

Ans. Correct option : (d)

Explanation :

Given that, $\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4$

$\therefore$ Order = 3

and degree = 2

Q. 35. The order and the degree of differential equations $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$ are

(a) $2, \frac{3}{2}$ **(b)** 2, 3

(c) 2, 1 **(d)** 3, 4

[NCERT Exemp. Ex. 9.3, Q. 67, Page 200]

Ans. Correct option : (c)

Explanation :

Given that, $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$

$\therefore$ Order = 2 and degree = 1

Q. 36. The differential equation of family of curves $y^2 = 4a(x + a)$ is

(a) $y^2 = 4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$

(b) $2y\frac{dy}{dx} = 4a$

(c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

(d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2-y=0$

[NCERT Exemp. Ex. 9.3, Q. 69, Page 200]

Ans. Correct option : (d)

Explanation :

Given that, $y^2 = 4a(x + a)$ …(i)

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a \Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow y\frac{dy}{dx} = 2a \Rightarrow a = \frac{1}{2}y\frac{dy}{dx} \quad ...(ii)$$

On putting the value of a from EQ. (ii) in EQ. (i), we get

$$y^2 = 2y\frac{dy}{dx}\left(x+\frac{1}{2}y\frac{dy}{dx}\right)$$

$$\Rightarrow y^2 = 2xy\frac{dy}{dx}+y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2-y=0$$

Q. 37. Which of the following is the general solution of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$?

(a) $y = (Ax+ B)e^x$

(b) $y = (Ax + B)e^{-x}$

(c) $y = Ae^x + Be^{-x}$

(d) $y = A\cos x + B\sin x$

[NCERT Exemp. Ex. 9.3, Q. 70, Page 200]

Ans. Correct option : (a)

Explanation :

Given that, $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

$$D^2y - 2Dy + y = 0,$$

where $D = \frac{d}{dx}$

$$(D^2 - 2D + 1)y = 0$$

The auxiliary equation is $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0 \Rightarrow m = 1, 1$

Since, the roots are real and equal.

$\therefore$ CF $= (Ax+B)e^x \Rightarrow y = (Ax+B)e^x$

[Since, its roots of auxiliary equations are real and equal say (m), then [CF $= (C_1x + C_2)e^{mx}$]

Q. 38. The general solution of $\frac{dy}{dx}+y\tan x=\sec x$ is

(a) $y\sec x = \tan x + C$

(b) $y\tan x = \sec x + C$

(c) $\tan x = y\tan x + C$

(d) $x\sec x = \tan y + C$

[NCERT Exemp. Ex. 9.3, Q. 71, Page 201]

Ans. Correct option : (a)

Explanation :

Given differential equation is

$$\frac{dy}{dx}+y\tan x=\sec x$$

which is a linear differential equation

Here, $P = \tan x$, $Q = \sec x$,

$$\therefore \text{IF} = e^{\int \tan x d x} = e^{\log|\sec x|} = \sec x$$

The general solution is

$$y\cdot\sec x = \int \sec x\cdot\sec x + C$$

$$\Rightarrow y\cdot\sec x = \int \sec^2 x\, dx + C$$

$$\Rightarrow y\cdot\sec x = \tan x + C$$

Q. 39. The solution of differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$ is

(a) $x(y + \cos x) = \sin x + C$

(b) $x(y - \cos x) = \sin x + C$

(c) $xy\cos x = \sin x + C$

(d) $x(y + \cos x) = \cos x + C$

[NCERT Exemp. Ex. 9.3, Q. 72, Page 201]

Ans. Correct option : (a)

Explanation :

Given differential equation is

$$\frac{dy}{dx}+y\frac{1}{x} = \sin x$$

Which is linear differential equation.

$$P = \frac{1}{x} \text{ and } Q = \sin x$$

Here;

$$\therefore \text{IF} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

The general solution is,

$$y\cdot x = \int x\cdot\sin x dx + C \quad ...(i)$$

$$I = \int x \cdot \sin x dx$$

Take. $-x\cos x - \int -\cos x dx$

$= -x\cos x + \sin x$

Put the value of I in EQ. (1), we get

$xy = -x\cos x + \sin x + C$

$\Rightarrow \quad x(y + \cos x) = \sin x + C$

Q. 40. The general solution of differential equation $(e^x + 1)ydy = (y+1)e^x dx$ is

(a) $(y + 1) = k(e^x + 1)$

(b) $y + 1 = e^x + 1 + k$

(c) $y = \log\{k(y+1)(e^x+1)\}$

(d) $y = \log\{(e^x+1)/(y+1)\} + k$

[NCERT Exemp. Ex. 9.3, Q. 73, Page 201]

Ans. Correct option : (c)

Explanation :

Given differential equation

$$(e^x + 1)\,ydy = (y+1)e^x\,dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x(1+y)}{(e^x+1)y} \Rightarrow \frac{dx}{dy} = \frac{(e^x+1)y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{e^x y}{e^x(1+y)} + \frac{y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(1 + \frac{1}{e^x}\right)$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(\frac{e^x+1}{e^x}\right)$$

$$\Rightarrow \quad \left(\frac{y}{1+y}\right)dy = \left(\frac{e^x}{e^x+1}\right)dx$$

On integrating both sides, we get

$$\int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \quad \int \frac{1+y-1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \int 1dy - \int \frac{1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \quad y - \log|(1+y) = \log|(1+e^x) + \log k$$

$$\Rightarrow \quad y = \log(1+y) + \log(1+e^x) + \log(k)$$

$$\Rightarrow \quad y = \log\{k(1+y)(1+e^x)\}$$

Q. 41. The solution of differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $y = e^{x-y} - x^2e^{-y} + C$

(b) $e^v - e^x = \frac{x^3}{3} + C$

(c) $e^x + e^v = \frac{x^3}{3} + C$

(d) $e^x - e^v = \frac{x^3}{3} + C$

[NCERT Exemp. Ex. 9.3, Q. 74, Page 201]

Ans. Correct option : (b)

Explanation :

Given that, $\quad \frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$$\Rightarrow \quad \frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

$$\Rightarrow \quad e^y dy = (e^x + x^2)dx$$

On integrating both sides, we get

$$\int e^y dy = \int (e^x + x^2)dx$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + C$$

$$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + C$$

Q. 42. The solution of differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

(a) $y(1+x^2) = C + \tan^{-1}x$

(b) $\frac{y}{1+x^2} = C + \tan^{-1}x$

(c) $y\log(1+x^2) = C + \tan^{-1}x$

(d) $y(1+x^2) = C + \sin^{-1}x$

[NCERT Exemp. Ex. 9.3, Q. 75, Page 201]

Ans. Correct option : (a)

Explanation :

Given that, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$

Here, $\quad P = \frac{2x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

which is a linear differential equation.

$\therefore \quad IF = e^{\int \frac{2x}{1+x^2}dx}$

Put $\quad 1 + x^2 = t \Rightarrow 2xdx = dt$

$\therefore \quad IF = e^{\int \frac{dt}{1}} = e^{\log t} = e^{\log(1+x^2)} = 1 + x^2$

The general solution is

$$y \cdot (1+x^2) = \int (1+x^2)\frac{1}{(1+x^2)^2} + C$$

$$\Rightarrow y(1+x^2) = \int \frac{1}{1+x^2}dx + C$$

$$\Rightarrow y(1+x^2) = \tan^{-1}x + C$$

Q. 43. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is

(a) 3 **(b)** 2

(c) 1 **(d)** not defined

[NCERT Ex. 9.1, Q. 11, Page 383]

Ans. Correct option : (d)

Explanation :

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

The given differential equation is not a polynomial equation in its derivatives. Therefore, its degree is not defined.

Q. 44. The order of the differential equation

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0 \text{ is}$$

(a) 2 **(b) 1**

(c) 0 **(d) not defined**

[NCERT Ex. 9.1, Q. 12, Page 383]

Ans. Correct option : (a)

Explanation :

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

Q. 45. The numbers of arbitrary constants in the general solution of a differential equation of fourth order are :

(a) 0 **(b) 2**

(c) 3 **(d) 4**

[NCERT Ex. 9.2, Q. 11, Page 385]

Ans. Correct option : (d)

Explanation :

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth-order differential equation is four.

Q. 46. The numbers of arbitrary constants in the particular solution of a differential equation of third order are :

(a) 3 **(b) 2**

(c) 1 **(d) 0**

[NCERT Ex. 9.2, Q. 12, Page 385]

Ans. Correct option : (d)

Explanation :

In a particular solution of a differential equation, there are no arbitrary constants.

Q. 47. Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2}+y=0$ **(b)** $\frac{d^2y}{dx^2}-y=0$

(c) $\frac{d^2y}{dx^2}+1=0$ **(d)** $\frac{d^2y}{dx^2}-1=0$

[NCERT Ex. 9.3, Q. 11, Page 391]

Ans. Correct option : (b)

Explanation :

The given equation is :

$$y=c_1e^x+c_2e^{-x}$$

Differentiating with respect of x, we get :

$$\frac{dy}{dx}=c_1e^x-c_2e^{-x}$$

Again, differentiating with respect of x, we get :

$$\frac{d\ y}{dx}=c_1e^x+c_2e^{\ x}$$

$$\Rightarrow \frac{d^2y}{dx^2}=y$$

$$\Rightarrow \frac{d^2y}{dx^2}-y=0$$

This is the required differential equation of the given equation of curve.

Q. 48. Which of the following differential equations has $y = x$ as one of its particular solution?

(a) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$ **(b)** $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$

(c) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$ **(d)** $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

[NCERT Ex. 9.3, Q. 12, Page 391]

Ans. Correct option : (c)

Explanation :

The given equation of curve is $y=x$

Differentiating with respect to x, we get :

$$\frac{dy}{dx}=1 \quad ...(1)$$

Again, differentiating with respect to x, we get :

$$\frac{d^2y}{dx^2}=0 \quad ...(2)$$

Now, on substituting the values of $y, \frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0-x^2\cdot 1+x\cdot x$$

$$=-x^2+x^2$$

$$=0$$

Q. 49. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is

(a) $e^x+e^{-y}=C$ **(b)** $e^x+e^y=C$

(c) $e^{-x}+e^y=C$ **(d)** $e^{-x}+e^{-y}=C$

[NCERT Ex. 9.4, Q. 22, Page 397]

Ans. Correct option : (a)

Explanation :

$$\frac{dy}{dx}=e^{x+y}=e^x\cdot e^y$$

$$\Rightarrow \frac{dy}{e^y}=e^x dx$$

$$\Rightarrow e^{-y}dy=e^x dx$$

Integrating both sides, we get :

$$\int e^{-y}dy=\int e^x dx$$

$$\Rightarrow -e^{-y}=e^x+k$$

$$\Rightarrow e^x+e^{-y}=-k$$

$$\Rightarrow e^x+e^{-y}=C \qquad (C=-k)$$

Q. 50. A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution.

(a) $y = vx$ (b) $v = yx$

(c) $x = vy$ (d) $x = v$

[NCERT Ex. 9.5, Q. 16, Page 406]

Ans. Correct option : (c)

Explanation :

For solving the homogeneous equation of the form : $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$, we need to make the substitution as $x = vy$.

Q. 51. Which of the following is a homogeneous differential equation?

(a) $(4x + 6y + 5)\,dy - (3y + 2x + 4)\,dx = 0$

(b) $(xy)\,dx - (x^3 + y^3)\,dy = 0$

(c) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$

(d) $y^2dx + (x^2 - xy - y^2)\,dy = 0$

[NCERT Ex. 9.5, Q. 17, Page 407]

Ans. Correct option : (d)

Explanation :

Function $F(x, y)$ is said to be the homogenous function of degree n, if

$F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for any non-zero constant (λ).

Consider the equation given in alternative D :

$$y^2dx + (x^2 - xy - y^2)\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2 - xy - y^2} = \frac{y^2}{y^2 + xy - x^2}$$

$$\text{Let } F(x, y) = \frac{y^2}{y^2 + xy - x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{(\lambda y)^2}{(\lambda y)^2 + (\lambda x)(\lambda y) - (\lambda x)^2}$$

$$= \frac{\lambda^2 y^2}{\lambda^2(y^2 + xy - x^2)}$$

$$= \lambda^0\left(\frac{y^2}{y^2 + xy - x^2}\right)$$

$$= \lambda^0 . F(x, y)$$

Q. 52. The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(a) e^{-x} (b) e^{-y}

(c) $\frac{1}{x}$ (d) x

[NCERT Ex. 9.6, Q. 18, Page 414]

Ans. Correct option : (c)

Explanation :

The given differential equation is :

$$x\frac{dy}{dx} - y = 2x^2$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$$

This is a linear differential equation of the form :

$$\frac{dy}{dx} + py = Q \text{ (where } p = -\frac{1}{x} \text{ and } Q = 2x\text{)}$$

The integrating factor (IF) is given by the relation, $e^{\int p\,dx}$

$$\therefore \text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$$

Q. 53. The Integrating Factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay(-1 < y < 1)$ is

(a) $\frac{1}{y^2 - 1}$ (b) $\frac{1}{\sqrt{y^2 - 1}}$

(c) $\frac{1}{1 - y^2}$ (d) $\frac{1}{\sqrt{1 - y^2}}$

[NCERT Ex. 9.6, Q. 19, Page 414]

Ans. Correct option : (d)

Explanation :

The given differential equation is :

$$(1 - y^2)\frac{dx}{dy} + yx = ay$$

$$\Rightarrow \frac{dx}{dy} + \frac{yx}{1 - y^2} = \frac{ay}{1 - y^2}$$

This is a linear differential equation of the form :

$$\frac{dx}{dy} + py = Q \text{ (where } p = \frac{y}{1 - y^2} \text{ and } Q = \frac{ay}{1 - y^2}\text{)}$$

The integrating factor (IF) is given by relation, $e^{\int p\,dy}$

$$\therefore \text{IF} = e^{\int p\,dy} = e^{\int \frac{y}{1 - y^2}dy} = e^{-\frac{1}{2}\log(1 - y^2)} = e^{\log\left[\frac{1}{\sqrt{1 - y^2}}\right]} = \frac{1}{\sqrt{1 - y^2}}$$

Q. 54. The general solution of the differential equation $\frac{y\,dx - x\,dy}{y} = 0$ is

(a) $xy = C$ (b) $x = Cy^2$

(c) $y = Cx$ (d) $y = Cx^2$

[NCERT Misc. Ex. Q. 16, Page 421]

Ans. Correct option : (c)

Explanation :

The given differential equation is :

$$\frac{ydx - xdy}{y} = 0$$

$$\Rightarrow \frac{ydx - xdy}{xy} = 0$$

$$\Rightarrow \frac{1}{x}dx - \frac{1}{y}dy = 0$$

Integrating both sides, we get :

$\log|x| - \log|y| = \log k$

$\Rightarrow \quad \log\left|\frac{x}{y}\right| = \log k$

$\Rightarrow \quad \frac{x}{y} = k$

$\Rightarrow \quad y = \frac{1}{k}x$

$\Rightarrow \quad y = Cx$ where $C = \frac{1}{k}$

Q. 55. The general solution of a differential equation of the type

$\frac{dx}{dy} + P_1 x = Q_1$ **is**

(a) $y\,e^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)\int\left(Q_1 e^{\int P_1 dy}\right)dy + C$

(b) $y \cdot e^{\int P_1 dx} = \int\left(Q_1 e^{\int P_1 dx}\right)dx + C$

(c) $x \cdot e^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$

(d) $xe^{\int P_1 dx} = \int\left(Q_1 e^{\int P_1 dx}\right)dx + C$

[NCERT Misc. Ex. Q. 17, Page 421]

Ans. Correct option : (c)

Explanation :

The integrating factor of the given differential equation $\frac{dx}{dy} + P_1 x = Q_1$ is $e^{\int P_1 dy}$

The general solution of the differential equation is given by,

$x(\text{IF}) = \int(Q \times \text{IF})dy + C$

$\Rightarrow x \cdot e^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$

Q. 56. The general solution of the differential equation $e^x\,dy + (y\,e^x + 2x)\,dx = 0$ is

(a) $x\,e^y + x^2 = C$ (b) $x\,e^y + y^2 = C$

(c) $y\,e^x + x^2 = C$ (d) $y\,e^y + x^2 = C$

[NCERT Misc. Ex. Q. 18, Page 421]

Ans. Correct option : (c)

Explanation :

The given differential equation is :

$e^x dy + \left(ye^x + 2x\right)dx = 0$

$\Rightarrow \quad e^x \frac{dy}{dx} + ye^x + 2x = 0$

$\Rightarrow \quad \frac{dy}{dx} + y = -2xe^{-x}$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = 1$ and $Q = -2xe^{-x}$

Now, IF $= e^{\int P dx} = e^{\int dx} = e^x$

The general solution of the given differential equation is given by,

$y(\text{IF}) = \int(Q \times \text{IF})dx + C$

$\Rightarrow \quad ye^x = \int(-2xe^{-x} \cdot e^x)dx + C$

$\Rightarrow \quad ye^x = -\int 2x dx + C$

$\Rightarrow \quad ye^x = -x^2 + C$

$\Rightarrow ye^x + x^2 = C$

Very Short Answer Type Questions **(1 or 2 marks each)**

Q. 1. Find the solution of

$\frac{dy}{dx} = 2^{y-x}$ **[NCERT Exemp. Ex. 9.3, Q. 1, Page 193]**

Ans.

Given that, $\quad \frac{dy}{dx} = 2^{y-x}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2^y}{2^x} \quad \left[\because a^{m-n} = \frac{a^m}{a^n}\right]$

$\Rightarrow \quad \frac{dy}{2^y} = \frac{dx}{2^x}$

On integrating both sides, we get

$\int 2^{-y}dy = \int 2^{-x}dx$

$\Rightarrow \quad \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$

$\Rightarrow -2^{-y} + 2^{-x} = +C\log 2$

$\Rightarrow \quad 2^{-x} - 2^{-y} = -C\log 2$

$\Rightarrow \quad 2^{-x} - 2^{-y} = k \quad$ [where, $k = +C\log 2$] **[2]**

Q. 2. Find the differential equation of all non-vertical lines in a plane.

[NCERT Exemp. Ex. 9.3, Q. 2, Page 193]

Ans. Since, the family of all non-vertical line is $y = mx$ $+C$, where $m \neq \tan\frac{\pi}{2}$.

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = m$

Again, differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = 0$

[2]

Q. 3. Determine order and degree (if defined) of differential equation

$\frac{d^4y}{dx^4} + \sin(y''') = 0$ **[NCERT Ex. 9.1, Q. 1, Page 382]**

Ans. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

$\Rightarrow y'''' + \sin(y''') = 0$

The highest order derivative present in the differential equation is y''''. Therefore, its order is four.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined. **[2]**

Q. 4. Determine order and degree (if defined) of differential equation

$y' + 5y = 0$ **[NCERT Ex. 9.1, Q. 2, Page 382]**

Ans. The given differential equation is :

$y' + 5y = 0$

The highest-order derivative present in the differential equation is y'. Therefore, its degree is one. **[2]**

Q. 5. Determine order and degree (if defined) of differential equation

$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ **[NCERT Ex. 9.1, Q. 3, Page 382]**

Ans. $\left(\frac{ds}{dt}\right)^4 + 3\frac{d^2s}{dt^2} = 0$

The highest order derivative present in the given differential equation is $\frac{d^2s}{dt^2}$. Therefore, its order is two.

It is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$. The power raised to $\frac{d^2s}{dt^2}$ is one.

Hence, its degree is one. **[2]**

Q. 6. Determine order and degree (if defined) of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

[NCERT Ex. 9.1, Q. 4, Page 382]

Ans. $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

The highest-order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

The given differential equation is not polynomial equation in its derivatives. Hence, its degree is not defined. **[2]**

Q. 7. Determine order and degree (if defined) of differential equation

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

[NCERT Ex. 9.1, Q. 5, Page 382]

Ans.
$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} - \cos 3x - \sin 3x = 0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

It is a polynomial equation in $\frac{d^2y}{dx^2}$ and the power raised to $\frac{d^2y}{dx^2}$ is one.

Hence, its degree is one. **[2]**

Q. 8. Determine order and degree (if defined) of differential equation

$$(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$$

[NCERT Ex. 9.1, Q. 6, Page 382]

Ans. $(y''')^2 + (y'')^3 + (y') + y^5 = 0$

The highest-order derivative present in the differential equation is y'''. Therefore, its order is three.

The given differential equation is a polynomial equation in y''', y'', and y'.

The highest power raised to y''' is two. Hence, its degree is two. **[2]**

Q. 9. Determine order and degree (if defined) of differential equation $y''' + 2y'' + y' = 0$

[NCERT Ex. 9.1, Q. 7, Page 382]

Ans. $y''' + 2y'' + y' = 0$

The highest-order derivative present in the differential equation is y'''. Therefore, its order is three.

It is polynomial equation in y''', y'', and y'. The highest power raised to y''' is one. Hence, its degree is one. **[2]**

Q. 10. Determine order and degree (if defined) of differential equation $y' + y = e^x$

[NCERT Ex. 9.1, Q. 8, Page 383]

Ans. $y' + y = e^x$

$\Rightarrow y' + y - e^x = 0$

The highest order derivative present in the differential equation in y'. Therefore, its order is one.

The given differential equation is a polynomial equation in y' and the highest power is raised to one. He y' is one. Hence, its degree is one. **[2]**

Q. 11. Determine order and degree (if defined) of differential equation $y'' + (y')^2 + 2y = 0$

[NCERT Ex. 9.1, Q. 9, Page 383]

Ans. $y'' + (y')^2 + 2y = 0$

The highest-order derivative present in the differential equation is y''. Therefore, its order is two.

The given differential equation is a polynomial equation in y'' and y', and the highest power raised to y'' is one.

Hence, its degree is one. **[2]**

Q. 12. Determine order and degree (if defined) of differential equation $y'' + 2y' + \sin y = 0$

[NCERT Ex. 9.1, Q. 10, Page 383]

Ans. $y'' + 2y' + \sin y = 0$

The highest order derivative present in the differential equation is y''. Therefore, its order is two.

This is a polynomial equation in y'' and y'', and the highest power raised to y'' is one. Hence, its degree is one. **[2]**

Q. 13. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$\boldsymbol{y = e^x + 1}$: $\boldsymbol{y'' - y' = 0}$

[NCERT Ex. 9.2, Q. 1, Page 385]

Ans. $y = e^x + 1$

Differentiating both sides of this equation with respect to x, we get :

$$\frac{dy}{dx} = \frac{d}{dx}(e^x + 1)$$

$$\Rightarrow y' = e^x \quad \ldots(1)$$

Now, differentiating EQ. (1) with respect to x, we get :

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$

$$\Rightarrow \quad y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the LHS as :

$$y'' - y' = e^x - e^x = 0 = \text{RHS}$$

Thus, the given function is the solution of the corresponding differential equation. **[2]**

Q. 14. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$\boldsymbol{y = x^2 + 2x + C}$: $\boldsymbol{y' - 2x - 2 = 0}$

[NCERT Ex. 9.2, Q. 2, Page 385]

Ans. $y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to x, we get :

$$y' = \frac{d}{dx}(x^2 + 2x + C)$$

$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get :

$$\text{LHS} = y' - 2x - 2 = 2x + 2 - 2x - 2 = 0 = \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[2]**

Q. 15. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$\boldsymbol{y = \cos x + C}$: $\boldsymbol{y' + \sin x = 0}$

[NCERT Ex. 9.2, Q. 3, Page 385]

Ans. $y = \cos x + C$

Differentiating both sides of this equation with respect to x, we get :

$$y' = \frac{d}{dx}(\cos x + C)$$

$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get :

$$\text{LHS} = y' + \sin x = -\sin x + \sin x = 0 = \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[2]**

Q. 16. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$\boldsymbol{y = \sqrt{1 + x^2}}$: $\boldsymbol{y' = \dfrac{xy}{1 + x^2}}$

[NCERT Ex. 9.2, Q. 4, Page 385]

Ans. $y = \sqrt{1 + x^2}$

Differentiating both sides of the equation with respect to x, we get :

$$y' = \frac{d}{dx}\left(\sqrt{1 + x^2}\right)$$

$$y' = \frac{1}{2\sqrt{1 + x^2}} \cdot \frac{d}{dx}(1 + x^2)$$

$$y' = \frac{2x}{2\sqrt{1 + x^2}}$$

$$y' = \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow y' = \frac{x}{1 + x^2} \times \sqrt{1 + x^2}$$

$$\Rightarrow y' = \frac{x}{1 + x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1 + x^2}$$

$\therefore$ LHS = RHS

Hence, the given function is the solution of the corresponding differential equation. **[2]**

Q. 17. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$\boldsymbol{y = Ax}$: $\boldsymbol{xy' = y (x \neq 0)}$

[NCERT Ex. 9.2, Q. 5, Page 385]

Ans. $y = Ax$

Differentiating both sides with respect to x, we get :

$$y' = \frac{d}{dx}(Ax)$$

$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get :

$$\text{LHS} = xy' = x \cdot A = Ax = y = \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[2]**

Q. 18. Find the general solution :

$$\boldsymbol{\frac{dy}{dx} = (1 + x^2)(1 + y^2)}$$

[NCERT Ex. 9.4, Q. 6, Page 396]

Ans. The given differential equation is :

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both sides of this equation, we get :

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx$$

$$\Rightarrow \tan^{-1} y = \int dx + \int x^2 dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

This is the required general solution of the given differential equation. **[2]**

Q. 19. Find the general solution of the differential equation $\boldsymbol{\dfrac{dy}{dx} + \sqrt{\dfrac{1 - y^2}{1 - x^2}} = 0}$

[NCERT Misc. Ex. Q. 6, Page 420]

Ans.

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow \quad \frac{dy}{\sqrt{1-y^2}}=\frac{-dx}{\sqrt{1-x^2}}$$

Integrating both sides, we get :

$$\sin^{-1}y=\sin^{-1}x+C$$

$$\Rightarrow \sin^{-1}x+\sin^{-1}y=C$$

[2]

Q. 20. For each of the differential equations given below, indicate its order and degree (if defined).

(i) $\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$

(ii) $\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$

(iii) $\frac{d^4y}{dx^4}-\sin\left(\frac{d^3y}{dx^3}\right)=0$

[NCERT Misc. Ex. Q. 1, Page 419]

Ans. **(i)** The differential equation is given as :

$$\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$$

$$\Rightarrow \frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y-\log x=0$$

The highest-order derivative present in the differential equation is $\frac{d^2y}{dx^2}$. Thus, its order is two.

The highest power raised to $\frac{d^2y}{dx^2}$ is one. Hence, its degree is one. **[1]**

(ii) The differential equation is given as :

$$\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y-\sin x=0$$

The highest-order derivative present in the differential equation is $\frac{dy}{dx}$. Thus, its order is one.

The highest power raised to $\frac{dy}{dx}$ is three. Hence, its degree is three. **[1]**

(iii) The differential equation is given as :

$$\frac{d^4y}{dx^4}-\sin\left(\frac{d^3y}{dx^3}\right)=0$$

The highest-order derivative present in the differential equation is $\frac{d^4y}{dx^4}$. Thus, its order is four.

However, the given differential equation is not a polynomial equation. Hence, its degree is not defined. **[1]**

Q. 21. Solve the differential equation

$$\frac{dy}{dx}+1=e^{x+y}$$

[NCERT Exemp. Ex. 9.3, Q. 7, Page 193]

Ans. Given differential equation is $\frac{dy}{dx}+1=e^{x+y}$ ...(i)

On substituting $x + y = t$, we get

$$1+\frac{dy}{dx}=\frac{dt}{dx}$$

EQ. (i) becomes $\frac{dt}{dx}=e^t$

$$\Rightarrow \quad e^{-t}dt=dx$$

$$\Rightarrow \quad -e^{-t}=x+C$$

$$\Rightarrow \quad \frac{-1}{e^{x+y}}=x+C$$

$$\Rightarrow \quad -1=(x+C)e^{x+y}$$

$$\Rightarrow \quad (x+C)e^{x+y}+1=0$$

[2]

Q. 22. Find the differential equation of system of concentric circles with centre (1, 2).

[NCERT Exemp. Ex. 9.3, Q. 24, Page 194]

Ans. The family of concentric circles with centre (1, 2) and radius a is given by,

$$(x-1)^2+(y-2)^2=a^2$$

$$\Rightarrow \quad x^2+1-2x+y^2+4-4y=a^2$$

$$\Rightarrow \quad x^2+y^2-2x-4y+5=a^2 \quad ...(i)$$

On differentiating EQ. (i) w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$$

$$\Rightarrow \quad (2y-4)\frac{dy}{dx}+2x-2=0$$

$$\Rightarrow \quad (y-2)\frac{dy}{dx}+(x-1)=0$$

[2]

Q. 23. (i) The degree of the differential equation $\frac{d^2y}{dx^2}+e^{\frac{dy}{dx}}=0$ is ....

(ii) The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=x$ is ....

(iii) The number of arbitrary constants in the general solution of a differential equation of order three is ……

(iv) $\frac{dx}{dx}+\frac{y}{x\log x}=\frac{1}{x}$ is an equation of the type …..

(v) General solution of the differential equation of the type $\frac{dx}{dy}+P_1x=Q_1$ is given by ……

(vi) The solution of the differential equation $\frac{xdy}{dx}+2y=x^2$ is ……

(vii) The solution of $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$ is ……..

(viii) The solution of the differential equation $ydx+(x+xy)dy=0$ is …….

(ix) General solution of $\frac{dy}{dx} + y = \sin x$ is ……

(x) The solution of differential equation cot $y\ dx = xdy$ is ……

(xi) The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is …….

[NCERT Exemp. Ex. 9.3, Q. 76, Page 201]

Ans. **(i)** Given differential equation is $\frac{d^2y}{dx^2} + e^{\frac{dy}{dx}} = 0$

Degree of this equation is not defined.

(ii) Given differential equation is $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x$

So, degree of this equation is two.

(iii) There are three arbitrary constants.

(iv) Given differential equation is $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{1}{x}$

The equation is of the type $\frac{dy}{dx} + Py = Q$

(v) Given differential equation is

$$\frac{dx}{dy} + P_1x = Q_1$$

The general solution is

$x \cdot \text{IF} = \int Q(\text{IF})dy + \text{C}$, That is, $xe^{\int pdy} = \int Q\{e^{\int pdy}\}dy + \text{C}$

(vi) Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$$

This equation of the form $\frac{dy}{dx} + Py = Q$.

$\therefore$ IF $= e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$

The general solution is

$$yx^2 = \int x \cdot x^2 dx + \text{C}$$

$$\Rightarrow \quad yx^2 = \frac{x^4}{4} + \text{C}$$

$$\Rightarrow \quad y = \frac{x^2}{4} + \text{C}x^{-2}$$

(vii) Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1+x^2} - \frac{4x^2}{1+x^2} = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

$\therefore$ IF $= e^{\int \frac{2x}{1+x^2}dx}$

Put $\quad 1+x^2 = t \Rightarrow 2xdx = dt$

$\therefore$ IF $= e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)} = 1+x^2$

The general solution is

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

$\therefore$ IF $= e^{\int \frac{2x}{1+x^2}dx}$

Put $\quad 1+x^2 = t \Rightarrow 2xdx = dt$

$\therefore$ IF $= e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)} = 1+x^2$

The general solution is

$$y \cdot (1+x^2) = \int (1+x^2)\frac{4x^2}{(1+x^2)}dx + \text{C}$$

$$\Rightarrow \quad (1+x^2)y = \int 4x^2 dx + \text{C}$$

$$\Rightarrow \quad (1+x^2)y = 4\frac{x^3}{3} + \text{C}$$

$$\Rightarrow \quad y = \frac{4x^3}{3(1+x^2)} + \text{C}(1+x^2)^{-1}$$

(viii) Given differential equation is

$\Rightarrow ydx + (x + xy)dy = 0$

$\Rightarrow ydx + x(1+y)dy = 0$

$$\Rightarrow \quad \frac{dx}{-x} = \left(\frac{1+y}{y}\right)dy$$

$$\Rightarrow \quad \int \frac{1}{2}dx = -\int\left(\frac{1}{y}+1\right)dy \quad \text{[On integrating]}$$

$$\Rightarrow \quad \log(x) = -\log(y) - y + \log A$$

$$\log(x) + \log(y) + y = \log A$$

$$\log(xy) + y = \log A$$

$$\Rightarrow \quad \log xy + \log e^y = \log A$$

$$\Rightarrow \quad xye^y = A$$

$$\Rightarrow \quad xy = Ae^{-y}$$

(ix) Given differential equation is

$$\frac{dy}{dx} + y = \sin x$$

$$\text{IF} = e^{\int 1.dx} = e^x$$

The general solution is

$$y \cdot e^x = \int e^x \sin x\ dx + \text{C} \quad ...(i)$$

Let $\quad I = \int e^x \sin x\ dx$

$$I = \sin xe^x - \int \cos xe^x dx$$

$$= \sin xe^x - \cos xe^x + \int(-\sin x)e^x dx$$

$$2I = e^x(\sin x - \cos x)$$

$$I = \frac{1}{2}e^x(\sin x - \cos x)$$

Form Eq. (i).

$$y \cdot e^x = \frac{e^x}{2}(\sin x - \cos x) + \text{C}$$

$$\Rightarrow \quad y = \frac{1}{2}(\sin x - \cos x) + \text{C} \cdot e^{-x}$$

(x) Given differential equation is

$$\cot ydx = xdy$$

$$\Rightarrow \quad \frac{1}{x}dx = \tan ydy$$

On integrating both sides, we get

$$\Rightarrow \quad \int \frac{1}{x}dx = \int \tan ydy$$

$$\Rightarrow \quad \log(x) = \log(\sec y) + \log \text{C}$$

$$\Rightarrow \log\left(\frac{x}{\sec y}\right) = \log C$$

$$\Rightarrow \frac{x}{\sec y} = C$$

$$\Rightarrow x = C\sec y$$

(xi) Given differential equation is

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} + y\left(1 - \frac{1}{x}\right) = \frac{1}{x}$$

$$\therefore \quad \text{IF} = e^{\int\left(1-\frac{1}{x}\right)dx}$$

$$= e^{x-\log x}$$

$$= e^{x}\cdot e^{-\log x} = \frac{e^x}{x}$$

Q. 24. State True or False for the following :

(i) **Integrating factor of the differential of the form $\frac{dx}{dy} + P_1x = Q_1$ is given by $e^{\int P_1 dy}$.**

(ii) **Solution of the differential equation of the type $\frac{dx}{dy} + P_1x = Q_1$ is given by $x\cdot \text{IF} = \int(\text{IF})\times Q_1\,dy$.**

(iii) **Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = f(x,y)$, where $f(x,y)$ is a homogeneous function of zero degree is $y = vx$.**

(iv) **Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = g(x,y)$, where $g(x,y)$ is a homogeneous function of the degree zero is $x = vy$.**

(v) **Number of arbitrary constants in the particular solution of a differential equation of order two is two.**

(vi) **The differential equation representing the family of circles $x^2 + (y-a)^2 = a^2$ will be of order two.**

(vii) **The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is $y^{2/3} - x^{2/3} = C$**

(viii) **Differential equation representing the family of curves $y = e^x(A\cos x + B\sin x)$ is $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$**

(ix) **The Solution of the differential equation $\frac{dy}{dx} = \frac{x+2y}{x}$ is $x + y = kx^2$.**

(x) **Solution of $\frac{xdy}{dx} = y + x\tan\frac{y}{x}$ is $\sin\left(\frac{y}{x}\right) = Cx$**

(xi) **The differential equation of all non-horizontal lines in a plane is $\frac{d^2x}{dy^2} = 0$.**

Ans. **(i)** True

Given differential equation,

$$\frac{dx}{dy} + P_1x = Q_1$$

$$\text{IF} = e^{\int p_1 dy}$$

(ii) True

(iii) True

(iv) True

(v) False

There is no arbitrary constant in the particular solution of a differential equation.

(vi) False

We know that, order of the differential equation = Number of arbitrary constant

Here, number of arbitrary constant = 1.

So, order is one.

(vii) True

Given differential equation,

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}}$$

$$\Rightarrow y^{-1/3}dy = x^{-1/3}dx$$

On integrating both sides, we get.

$$\int y^{-1/3}dy = \int x^{-1/3}dx$$

$$\Rightarrow \frac{y^{-1/3+1}}{\frac{-1}{3}+1} = \frac{x^{-1+3+1}}{\frac{-1}{3}+1} + C'$$

$$\Rightarrow \frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + C'$$

$$\Rightarrow y^{2/3} - x^{2/3} = C' \qquad \left[\text{Where, } \frac{2}{3}C' = C\right]$$

(viii) True

Given that, $y = e^x(A\cos x + B\sin x)$

On differentiating w.r.t x, we get

$$\frac{dy}{dx} = e^x(-A\sin x + B\cos x) + e^x(A\cos x + B\sin x)$$

$$\Rightarrow \frac{dy}{dx} - y = e^x(-A\sin x + B\cos x)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = \frac{dy}{dx} - y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

(ix) True

Given that,

$$\frac{dy}{dx} = \frac{x+2y}{x} \Rightarrow \frac{dy}{dx} = 1 + \frac{2}{x}\cdot y$$

$\Rightarrow \frac{dy}{dx} - \frac{2}{x}y = 1$

$$\text{IF} = e^{\int \frac{-2}{x}dx} = e^{-2\log x} = x^{-2}$$

The differential solution,

$$y \cdot x^{-2} = \int x^{-2} \cdot 1 dx + k$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{x^{-2+1}}{-2+1} + k$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{-1}{x} + k$$

$$\Rightarrow \quad y = -x + kx^2$$

$$\Rightarrow x + y = kx^2$$

(x) True

Given differential equation

$$\frac{xdy}{dx} = y + x\tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

Put $\frac{y}{x} = v$, That is, $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + \frac{sdv}{dx} \quad , \quad ...(i)$$

On substituting these values in equation (i), we get

$$\frac{xdv}{dx} + v = v + \tan v$$

$$\Rightarrow \quad \frac{dx}{x} = \frac{dv}{\tan v}$$

On integrating both sides, we get

$$\int \frac{1}{x}dx = \int \frac{1}{\tan v}dx$$

$$\Rightarrow \quad \log(x) = \log(\sin v) + \log C'$$

$$\Rightarrow \quad \log\left(\frac{x}{\sin v}\right) = \log C'$$

$$\Rightarrow \quad \frac{x}{\sin v} = C'$$

$$\Rightarrow \quad \sin v = Cx \qquad \left[\text{Where, } C = \frac{1}{C'}\right]$$

$$\Rightarrow \quad \sin\frac{y}{x} = Cx$$

(xi) True

Let any non-horizontal line in a plane is given by $y = mx + C$

On differentiating w.r.t. x, we get $\frac{dy}{dx}$

Again, differentiating w.r.t. x, we get $\frac{d^2y}{dx^2} = 0$

Q. 25. Find the differential equation representing the family of curves $y = ae^{bx+5}$ where a and b are arbitrary constants.

[CBSE Board, Delhi Region, 2018]

Ans. Given $y = ae^{bx+5}$

$$\Rightarrow \quad \frac{y}{a} = e^{bx+5}$$

Differentiate $y = ae^{bx+5}$ with respect of x

$$\frac{dy}{dx} = ae^{bx+5}(b)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{aby}{a} \qquad \left[\because \frac{y}{a} = eb^{x+5}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = by$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 \qquad \left[\therefore b = \frac{1}{y}\frac{dy}{dx}\right]$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

[2]

Short Answer Type Questions

(3 and 4 marks each)

Q. 1. Find the particular solution of the differential equation $e^x \tan y dx + (2 - e^x)\sec^2 y dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$

[CBSE Board, Delhi Region, 2018]

Ans. Given that : $e^x \tan y dx + (2 - e^x)\sec^2 y dy = 0$

$$\Rightarrow \quad e^x \tan y dx = (e^x - 2)\sec^2 y dy$$

$$\Rightarrow \quad \frac{e^x}{e^x - 2}dx = \frac{\sec^2 y}{\tan y}dy$$

Integrating both sides, we get

$$\int \frac{e^x}{e^x - 2}dx = \int \frac{\sec^2 y}{\tan y}dy$$

$$\Rightarrow \quad \int \frac{dt}{t} = \int \frac{ds}{s}$$

$$\left[\begin{array}{l}\text{Let } e^x - 2 = t \text{ and } y = s \\ \Rightarrow e^x dx = dt \text{ and } \sec^2 y dy = dx\end{array}\right]$$

$$\Rightarrow \quad \text{In } t = \text{In} s + \text{InC}$$

$$\Rightarrow \quad t = sC$$

Substituting $t = e^x - 2$ and $s = \tan y$, we get

$$e^x - 2 = C\tan y$$

Now, $y = \frac{\pi}{4}$ when $x = 0$

$$e^0 - 2 = C\left(\tan\frac{\pi}{4}\right)$$

$$\Rightarrow \quad -1 = C$$

$$\Rightarrow \quad C = -1$$

$$\therefore e^x - 2 = -\tan y$$

$$\Rightarrow \quad \tan y = 2 - e^x$$

$$\Rightarrow \quad y = \tan^{-1}(2 - e^x)$$

[4]

Q. 2. Find the particular solution of the differential equation $\frac{dy}{dx}+2y\tan x=\sin x$ given that $y=0$ when $x=\frac{x}{3}$

[CBSE Board, Delhi Region, 2018]

Ans.

Given that : $\frac{dy}{dx}+2y\tan x=\sin x$

Comparing $\frac{dy}{dx}+2\tan x=\sin x$ with general linear equation $\frac{dy}{dx}+Py=Q$, we get

$P=2\tan x$ and $Q=\sin x$

Now, integrating factor

$e^{2\int \tan x dx}=e^{2\log\sec x}=e^{\log\sec^2 x}=\sec^2 x$

The differential equation is given by

$$y\sec^2 x=\int \sin x\sec^2 x dx+C$$

$$\Rightarrow \quad y\sec^2 x=\int \frac{\sin x}{\cos x}\sec x dx+C$$

$$\Rightarrow \quad y\sec^2=\int \tan x\sec x dx+C$$

$$\Rightarrow \quad y\sec^2 x=\sec x+C$$

Now, $y=0$ when $x=\frac{\pi}{3}$

$$(0)\sec^2\left(\frac{\pi}{3}\right)=\sec\left(\frac{\pi}{3}\right)+C$$

$$\Rightarrow \quad C=-2$$

$\therefore y\sec^2 x=\sec x-2\Rightarrow y=\sec x-2\cos x$

Q. 3. Solve the differential equation :

$$y+x\frac{dy}{dx}=x-y\frac{dy}{dx}$$

[CBSE Board, All India Region, 2016]

Ans. The differential equation can be re-written as :

$\frac{dy}{dx}=\frac{x-y}{x+y}$, put $y=vx, \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow v+x\frac{dv}{dx}=\frac{1-v}{1+v}\Rightarrow \frac{1+v}{1-2v-v^2}dv=\frac{1}{x}dx$

Integrating we get

$$\Rightarrow \frac{1}{2}\int\frac{2V+2}{V^2+2V-1}dv=-\int\frac{1}{x}dx=\frac{1}{2}\log|V^2+2V-1|$$
$$-\log x+\log C$$

$\therefore$ Solution of the differential equation is :

$$\frac{1}{2}\log\left|\frac{y^2}{x^2}+\frac{2y}{x}-1\right|=\log C-\log x \text{ or, } y^2+2xy-x^2=C^2$$

[4]

Q. 4. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

[CBSE Board, All India Region, 2016]

Ans. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is $(-a, a)$.

$\therefore$ Equation of the family of circles is :

$$(x+a)^2+(y-a)^2=a^2,\ a\in R$$

$$\Rightarrow x^2+y^2+2ax-2ay+a^2=0$$

Differentiate w.r.t.

$$2x+2yy'+2a-2ay'=0\Rightarrow a=\frac{x+yy'}{y'-1}$$

$\therefore$ The differential equation is :

$$\left(x+\frac{x+yy'}{y'-1}\right)^2+\left(y-\frac{x+yy'}{y'-1}\right)=\left(\frac{x+yy'}{y'-1}\right)^2$$

$$\Rightarrow\left(\frac{xy'+yy'}{y'-1}\right)^2+\left(\frac{x+y}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2$$

[4]

Q. 5. Find the particular solution of the differential equation $(1-y^2)(1+\log x)dx+2xy\,dy=0$ given that $y=0$ when $x=1$

[CBSE Board, Delhi Region, 2016]

Ans. Given differential equation can be written as

$$\frac{(1+\log x)}{x}dx+\frac{2y}{1-y^2}dy=0$$

Integrating to get, $\frac{1}{2}(1+\log x)^2-\log|1-y^2|=C$

$x=1, y=0\Rightarrow C=\frac{1}{2}$

$\Rightarrow(1+\log x)^2-2\log|1-y^2|=1$

Q. 6. Find the general solution of the following differential equation :

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$$

[CBSE Board, Delhi Region, 2016]

Ans. Given differential equation can be written as

$$\frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{e^{\tan^{-1}y}}{1+y^2}$$

Integrating factor is $e^{\tan^{-1}y}$

$\therefore$ Solution is $x.\ e^{\tan^{-1}y}=\int e^{2\tan^{-1}y}\frac{1}{1+y^2}dy$

$\therefore\ x\,e^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+C$

[4]

Q. 7. Solve the following differential equation :

$$y^2dx+(x^2-xy+y^2)dy=0$$

[CBSE Board, Foreign Scheme, 2016]

Ans.

$$y^2dx+(x^2-xy+y^2)dy=0$$

$$\Rightarrow\frac{dx}{dy}=-\frac{(x^2-xy+y^2)}{y^2}$$

Put $\quad x=vy\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$

$$v+y\frac{dv}{dy}=\frac{-(v^2y^2-y^2v+y^2)}{y^2}$$

$$\Rightarrow\frac{dv}{v^2+1}=-\frac{dy}{y}$$

Integrating both sides

$\tan^{-1} v = -\log y + C$

$\Rightarrow \tan^{-1}\frac{x}{y} = -\log y + C$ **[4]**

Q. 8. Solve the following differential equation :
$(\cot^{-1} y + x)dy = (1+y^2)dx$

[CBSE Board, Foreign Scheme, 2016]

Ans.

$$\frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2}$$

$$\text{IF} = e^{-\int \frac{1}{1+y^2}dy} = e^{\cot^{-1} y}$$

Integrating, we get

$$x \cdot e^{\cot^{-1} y} = \int \frac{\cot^{-1} y \, e^{\cot^{-1} y}}{1+y^2} dy$$

Put $\cot^{-1} y = t$

$$= -\int te^t dt$$

$$= (1-t)e^1 + C$$

$\Rightarrow \quad x = (1 - \cot^{-1} y) + Ce^{-\cot^{-1} y}$ **[4]**

Q. 9. Solve the differential equation $(\tan^{-1}x - y)\, dx = (1 + x^2)\, dy$.

[CBSE Board, All India Region, 2017]

Ans. $(\tan^{-1} x - y)\, dx = (1+x^2)\, dy$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1} x}{1+x^2}$$

On comparing it with $\frac{dy}{dx} + Py = Q$

$$P = \frac{1}{1+x^2}, \; Q = \frac{\tan^{-1} x}{1+x^2}$$

$$\text{I.F} = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1} x}$$

The General Soln. in

$$y.e^{\tan^{-1} x} = \int e^{\tan^{-1} x} . \frac{\tan^{-1} x}{1+x^2} dx + C$$

Put $\tan^{-1} x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$y.e^{tan^{-1}x} = \int te^t \, dt + C$$

$$= (t-1)\, e^t + C$$

$$= (tan^{-1} x - 1)\, e^{\tan^{-1} x} + C$$

$\therefore \quad y = (tan^{-1} x - 1) + Ce^{-\tan^{-1} x}$

[4]

Q. 10. Solve $ydx - xdy = x^2 ydx$

[NCERT Exemp. Ex. 9.3, Q. 8, Page 193]

Ans.

Given that, $\quad ydx - xdy = x^2 ydx$

$$\Rightarrow \quad \frac{1}{x^2} - \frac{1}{xy}\cdot\frac{dy}{dx} = 1$$

[Dividing throughout by $x^2 ydx$]

$$\Rightarrow \quad -\frac{1}{xy}\cdot\frac{dy}{dx} + \frac{1}{x^2} - 1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{xy}{x^2} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(x - \frac{1}{x}\right)y = 0$$

Which is a linear differential equation. **[1½]**

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \left(x - \frac{1}{x}\right), Q = 0$$

$$\text{IF} = e^{\int pdx}$$

$$= e^{\int\left(x - \frac{1}{x}\right)dx}$$

$$= e^{\frac{x^2}{2} - \log x}$$

$$= \frac{1}{x} e^{\frac{x^2}{2}}$$ **[1½]**

The general solution is

$$y \cdot \frac{1}{x} e^{x^2/2} = \int 0 \cdot \frac{1}{x} e^{\frac{x^2}{2}} dx + C$$

$$y \cdot \frac{1}{x} e^{x^2/2} = C$$

$$y = Cxe^{-x^2/2}$$

Q. 11. Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$ and $x = 0$.

[NCERT Exemp. Ex. 9.3, Q. 9, Page 193]

Ans.

Given that, $\quad \frac{dy}{dx} = 1 + x + y^2 + xy^2$

$$\Rightarrow \quad \frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\Rightarrow \quad \frac{dy}{dx} = (1+y^2)(1+x)$$

$$\Rightarrow \quad \frac{dy}{1+y^2} = (1+x)dx$$

On integrating both sides, we get

$$\tan^{-1} y = x + \frac{x^2}{2} + K \qquad \ldots\text{(i)}$$

[1½]

When $y = 0$ and $x = 0$, then substituting these values in EQ. (i), we get

$$\tan^{-1}(0) = 0 + 0 + k$$

$$\Rightarrow \quad k = 0$$

$$\Rightarrow \quad \tan^{-1} y = x + \frac{x^2}{2}$$

$$\Rightarrow \quad y = \tan\left(x + \frac{x^2}{2}\right)$$

[1½]

Q. 12. Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y$

[NCERT Exemp. Ex. 9.3, Q. 10, Page 193]

Ans. Given that, $(x+2y^3)\dfrac{dy}{dx}=y$

$\Rightarrow$ $y\cdot\dfrac{dx}{dy}=x+2y^3$

$\Rightarrow$ $\dfrac{dx}{dy}=\dfrac{x}{y}+2y^2$

[Dividing throughout by y)

$\Rightarrow$ $\dfrac{dx}{dy}-\dfrac{x}{y}=2y^2$

Which is a linear differential equation. **[1½]**

On comparing it with $\dfrac{dx}{dy}+Px=Q,$ we get

$$P=-\frac{1}{y},\ Q=2y^2$$

$$\text{IF}=e^{\int -\frac{1}{y}dy}=e^{-\int \frac{1}{y}dy}$$

$\therefore$ $=e^{-\log y}=\dfrac{1}{y}$

The general solution is $x\cdot\dfrac{1}{y}=\int 2y^2\cdot\dfrac{1}{y}dx+\text{C}$

$\Rightarrow$ $\dfrac{x}{y}=\dfrac{2y^2}{2}+\text{C}$

$\Rightarrow$ $\dfrac{x}{y}=y^2+\text{C}$

$\Rightarrow$ $x=y^3+\text{C}y$ **[1½]**

Q. 13. If $y(x)$ is a solution of $\left(\dfrac{2+\sin x}{1+y}\right)\dfrac{dy}{dx}=-\cos x$ and $y\ (0)=1,$ then find the value of $y\left(\dfrac{\pi}{2}\right).$

[NCERT Exemp. Ex. 9.3, Q. 11, Page 193]

Ans.

Given that, $\left(\dfrac{2+\sin x}{1+y}\right)\dfrac{dy}{dx}=-\cos x$

$\Rightarrow$ $\dfrac{dy}{1+y}=-\dfrac{\cos x}{2+\sin x}dx$

On integrating both sides, we get

$$\int\frac{1}{1+y}dy=-\int\frac{\cos x}{2+\sin x}dx$$

$\Rightarrow$ $\log(1+y)=-\log(2+\sin x)+\log\text{C}$ **[1½]**

$\Rightarrow \log(1+y)+\log(2+\sin x)=\log\text{C}$

$\Rightarrow$ $\log(1+y)(2+\sin x)=\log\text{C}$

$\Rightarrow$ $(1+y)(2+\sin x)=\text{C}$

$\Rightarrow$ $1+y=\dfrac{\text{C}}{2+\sin x}$

$\Rightarrow$ $y=\dfrac{\text{C}}{2+\sin x}-1$ …(i)

When $x=0$ and $y=1,$ then

$$1=\frac{\text{C}}{2}-1$$

$\Rightarrow$ $\text{C}=4$

On putting C = 4 in Eq. (i), we get

$$y=\frac{4}{2+\sin x}-1$$

$\therefore$ $y\left(\dfrac{\pi}{2}\right)=\dfrac{4}{2+\sin\frac{\pi}{2}}-1=\dfrac{4}{2+1}-1$

$$=\frac{4}{3}-1=\frac{1}{3}$$ **[1½]**

Q. 14. If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt}-ty=1$ and $y\ (0)\ =-1,$ then show that $y(1)=-\dfrac{1}{2}.$

[NCERT Exemp. Ex. 9.3, Q. 12, Page 193]

Ans. Given that, $(1+t)\dfrac{dy}{dt}-ty=1$

$\Rightarrow$ $\dfrac{dy}{dt}-\left(\dfrac{t}{1+t}\right)y=\dfrac{1}{1+t}$

Which is a linear differential equation.

On comparing it with $\dfrac{dy}{dt}+Py=Q,$ we get

$$P=-\left(\frac{t}{1+t}\right), Q=\frac{1}{1+t}$$

$$\text{IF}=e^{-\int\frac{t}{1+t}dt}=e^{-\int\left(1-\frac{1}{1+t}\right)dt}=e^{-[t+\log(1+t)]}$$

$$=e^{-t}\cdot e^{\log(1+t)}$$

$$=e^{-t}(1+t)$$ **[1½]**

The general solution is

$$y(t)\cdot\frac{(1+t)}{e^t}=\int\frac{(1+t)\cdot e^{-t}}{(1+t)}dt+\text{C}$$

$\Rightarrow$ $y(t)=\dfrac{e^{-t}}{(-1)}\cdot\dfrac{e^t}{1+t}+\text{C}',$

$\left[\text{where C}'=\dfrac{\text{C}e^t}{1+t}\right]$

$\Rightarrow$ $y(t)=-\dfrac{1}{1+t}+\text{C}'$

When $t=0$ and $y=-1,$ then

$$-1=-1+\text{C}'\Rightarrow\text{C}'=0$$

$$y(t)=-\frac{1}{1+t}\Rightarrow y(1)=-\frac{1}{2}$$

[1½]

Q. 15. Form the differential equation having $y=(\sin^{-1}x)^2+A\cos^{-1}x+B,$ where A and B are arbitrary constants, as its general solution.

[NCERT Exemp. Ex. 9.3, Q. 13, Page 194]

Ans. Given that, $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$

On differentiating w.r.t. $x,$ we get

$$\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}+\frac{(-A)}{\sqrt{1-x^2}}$$

$\Rightarrow$ $\sqrt{1-x^2}\dfrac{dy}{dx}=2\sin^{-1}x-A$

Again, differentiating w.r.t. x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{-2x}{2\sqrt{1+x^2}}=\frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-\frac{x}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}\frac{dy}{dx}=2$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$$

which is the required differential equation. **[3]**

Q. 16. Form the differential equation of all circles which pass through origin and whose centres lie on *y*-axis. [NCERT Exemp. Ex. 9.3, Q. 14, Page 194]

Ans. It is given that, circles pass through origin and their centres lie on *V*-axis. Let $(0, k)$ be the centre of the circle and radius is k.

So, the equation of circle is

$$(x-0)^2+(y-k)^2=k^2$$

$$\Rightarrow \quad x^2+(y-k)^2=k^2$$

$$\Rightarrow \quad x^2+y^2-2ky=0$$

$$\Rightarrow \quad \frac{x^2+y^2}{2y}=k \quad \ldots(i)$$

On differentiating EQ. (i) w.r.t. x, we get

$$\frac{2y\left(2x+2y\frac{dy}{dx}\right)-(x^2+y^2)\frac{2dy}{dx}}{4y^2}=0$$

$$\Rightarrow \quad 4y\left(x+y\frac{dy}{dx}\right)-2(x^2+y^2)\frac{dy}{dx}=0$$

$$\Rightarrow \quad 4xy+4y^2\frac{dy}{dx}-2(x^2+y^2)\frac{dy}{dx}=0$$

$$\Rightarrow \quad [4y^2-2(x^2+y^2)]\frac{dy}{dx}+4xy=0$$

[1½]

$$(4y^2-2x^2-2y^2)\frac{dy}{dx}+4xy=0$$

$$(2y^2-2x^2)\frac{dy}{dx}+4xy=0$$

$$(y^2-x^2)\frac{dy}{dx}+2xy=0$$

$$(x^2-y^2)\frac{dy}{dx}-2xy=0$$

[1½]

Q. 17. Find the equation of a curve passing through origin and satisfying the differential equation

$$\boldsymbol{(1+x^2)\frac{dy}{dx}+2xy=4x^2}$$

[NCERT Exemp. Ex. 9.3, Q. 15, Page 194]

Ans.

Given that, $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{2x}{1+x^2}, Q=\frac{4x^2}{1+x^2}$$

$$\therefore \quad \text{IF}=e^{\int Pdx}=e^{\int\frac{2x}{1+x^2}dx}$$

Put $1+x^2=t \Rightarrow 2xdx=dt$

$$\text{IF}=e^{\int\frac{dt}{t}}=e^{\log t}=e^{\log(1+x^2)}=1+x^2$$

[1½]

$$y\cdot(1+x^2)=\int\frac{4x^2}{1+x^2}(1+x^2)dx+C$$

$$\Rightarrow \quad y\cdot(1+x^2)=\int 4x^2dx+C$$

$$\Rightarrow \quad y\cdot(1+x^2)=4\frac{x^3}{3}+C \quad \ldots(i)$$

Since, the curve passes through origin, then substituting

$x=0$ and $y=0$ in Eq. (i), we get

$C=0$

The required equation of curve is

$$y(1+x^2)=\frac{4x^3}{3}$$

$$\Rightarrow \quad y=\frac{4x^3}{3(1+x^2)}$$

[1½]

Q. 18. Solve

$$\boldsymbol{x^2\frac{dy}{dx}=x^2+xy+y^2}$$

[NCERT Exemp. Ex. 9.3, Q. 16, Page 194]

Ans.

Given that, $x^2\frac{dy}{dx}=x^2+xy+y^2$

$$\Rightarrow \quad \frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2} \quad \ldots\textbf{(i)}$$

Let $f(x,y)=1+\frac{y}{x}+\frac{y^2}{x^2}$

$$f(\lambda x,\lambda y)=1+\frac{\lambda y}{\lambda x}+\frac{\lambda^2y^2}{\lambda^2x^2}$$

$$f(\lambda x,\lambda y)=\lambda^0\left(1+\frac{y}{x}+\frac{y^2}{x^2}\right)$$

$$=\lambda^0 f(x,y)$$

[1½]

Which is homogenous expression of degree zero.

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

On substituting these values in EQ. (i), we get

$$\left(v+x\frac{dv}{dx}\right)=1+v+v^2$$

$$\Rightarrow \quad x\frac{dv}{dx}=1+v+v^2-v$$

$$\Rightarrow \quad x\frac{dv}{dx}=1+v^2$$

$$\Rightarrow \quad \frac{dv}{1+v^2}=\frac{dx}{x}$$

On integrating both sides, we get.

$$\tan^{-1}v = \log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C$$ **[1½]**

Q. 19. Find the general solution of the differential equation

$$(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$$

[NCERT Exemp. Ex. 9.3, Q. 17, Page 194]

Ans. Given, differential equation is

$$(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1+y^2) = -(x - e^{\tan^{-1}y})\frac{dy}{dx}$$

$$(1+y^2)\frac{dx}{dy} = -x + e^{\tan^{-1}y}$$

$$\Rightarrow \quad (1+y^2)\frac{dx}{dy} + x = e^{\tan^{-1}y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$$

[Dividing throughout by $(1+y^2)$]

Which is a linear differential equation. **[1½]**

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{1}{1+y^2}, Q = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{IF} = e^{\int pdy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

The general solution is $x \cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \cdot e^{\tan^{-1}y} dy + C$

$$\Rightarrow \quad x \cdot e^{\tan^{-1}y} = \int \frac{(e^{\tan^{-1}y})^2}{1+y^2} \cdot dy + C$$

Put $\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2}dy = dt$

$$\therefore \quad x \cdot e^{\tan^{-1}y} = \int e^{2t}dt + C$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + C$$

$$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + 2C$$

$$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k \quad [\because k = 2C]$$ **[1½]**

Q. 20. Find the general solution of

$$y^2dx + (x^2 - xy + y^2)dy = 0$$

[NCERT Exemp. Ex. 9.3, Q. 18, Page 194]

Ans. Given, differential equation is

$$y^2dx + (x^2 - xy + y^2)dy = 0$$

$$\Rightarrow \quad y^2dx = -(x^2 - xy + y^2)dy$$

$$\Rightarrow \quad y^2\frac{dx}{dy} = -(x^2 - xy + y^2)$$

$$\Rightarrow \quad \frac{dx}{dy} = -\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right) \quad \ldots\text{(i)}$$

Which is a homogeneous differential equation.

Put $\frac{x}{y} = v$ Or $x = vy$

$$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$$ **[1½]**

On substituting these values in EQ. (i), we get

$$v + y\frac{dv}{dy} = -[v^2 - v + 1]$$

$$\Rightarrow \quad y\frac{dv}{dy} = -v^2 + v - 1 - v$$

$$\Rightarrow \quad y\frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$$

On integrating both sides, we get

$$\tan^{-1}(v) = -\log y + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x}{y}\right) + \log y = C \quad \left[\because v = \frac{x}{y}\right]$$ **[1½]**

Q. 21. Solve $(x+y)(dx - dy) = dx + dy$

[NCERT Exemp. Ex. 9.3, Q. 19, Page 194]

Ans. Given differential equation is

$$(x+y)(dx - dy) = dx + dy$$

$$\Rightarrow \quad (x+y)\left(1 - \frac{dy}{dx}\right) = 1 + \frac{dy}{dx} \quad \ldots\text{(i)}$$

Put $x + y = z$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

On substituting these values in EQ. (i), we get

$$z\left(1 - \frac{dz}{dx} + 1\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad z\left(2 - \frac{dz}{dx}\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad 2z - z\frac{dz}{dx} - \frac{dz}{dx} = 0$$ **[1½]**

$$\Rightarrow \quad 2z - (z+1)\frac{dz}{dx} = 0$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{2z}{z+1}$$

$$\Rightarrow \quad \left(\frac{z+1}{z}\right)dz = 2dx$$

On integrating both sides, we get

$$\int\left(1 + \frac{1}{z}\right)dz = 2\int dx$$

$$\Rightarrow \quad z + \log z = 2x - \log C$$

$$\Rightarrow \quad (x+y) + \log(x+y) = 2x - \log C$$

$[\because z = x + y]$

$$\Rightarrow \quad 2x - x - y = \log C + \log(x+y)$$

$$\Rightarrow \quad x - y = \log|C(x+y)|$$

$$\Rightarrow \quad e^{x-y} = C(x+y)$$

$$\Rightarrow \quad (x+y) = \frac{1}{C}e^{x-y}$$

$$\Rightarrow \quad x + y = ke^{x-y} \quad \left[\because k = \frac{1}{C}\right]$$ **[1½]**

Q. 22. Solve :

$$2(y+3)-xy\frac{dy}{dx}=0, \text{ given that } y(1)=-2$$

[NCERT Exemp. Ex. 9.3, Q. 20, Page 194]

Ans. Given that, $2(y+3)-xy\frac{dy}{dx}=0$

$$\Rightarrow \quad 2(y+3)=xy\frac{dy}{dx}$$

$$\Rightarrow \quad 2\frac{dx}{x}=\left(\frac{y}{y+3}\right)dy$$

$$\Rightarrow \quad 2\cdot\frac{dx}{x}=\left(\frac{y+3-3}{y+3}\right)dy$$

$$\Rightarrow \quad 2\cdot\frac{dx}{x}=\left(1-\frac{3}{y+3}\right)dy$$

On integrating both sides, we get

$$2\log x=y-3\log(y+3)+C \qquad \text{...(i)}$$

When $x=1$ and $y=-2$, then

$$2\log 1=-2-3\log(-2+3)+C$$

$$\Rightarrow \quad 2\cdot 0=-2-3\cdot 0+C$$

$$\Rightarrow \quad C=2$$ **[1½]**

On substituting the value of C in EQ. (i), we get

$$2\log x=y-3\log(y+3)+2$$

$$\Rightarrow \quad 2\log x+3\log(y+3)=y+2$$

$$\Rightarrow \quad \log x^2+\log(y+3)^3=(y+2)$$

$$\Rightarrow \quad \log x^2(y+3)^3=y+2$$

$$\Rightarrow \quad x^2(y+3)^3=e^{y+2}$$ **[1½]**

Q. 23. Solve the differential equation $dy=\cos x(2-y\ \text{cosec}\ x)dx$ given that $y=2$, when $x=\frac{\pi}{2}$.

[NCERT Exemp. Ex. 9.3, Q. 21, Page 194]

Ans. Given differential equation,

$$dy=\cos x(2-y\ \text{cosec}\ x)dx$$

$$\Rightarrow \quad \frac{dy}{dx}=\cos x(2-y\ \text{cosec}\ x)$$

$$\Rightarrow \quad \frac{dy}{dx}=2\cos x-y\ \text{cosec}\ x\cdot\cos x$$

$$\Rightarrow \quad \frac{dy}{dx}=2\cos x-y\cot x$$

$$\Rightarrow \quad \frac{dy}{dx}+y\cot x=2\cos x$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q$, we get

$$P=\cot x, Q=2\cos x$$

$$\text{IF}=e^{\int pdx}=e^{\int\cot xdx}=e^{\log\sin x}=\sin x$$

The general solution is

$$y\cdot\sin x=\int 2\cos x\cdot\sin xdx+C$$

$$\Rightarrow \quad y\cdot\sin x=\int\sin 2xdx+C$$

$$[\because \sin 2x=2\sin x\cos x]$$

$$\Rightarrow \quad y\cdot\sin x=-\frac{\cos 2x}{2}+C \qquad \text{...(i)}$$

[1½]

When $x=\frac{\pi}{2}$ and $y=2$, then

$$2\cdot\sin\frac{\pi}{2}=-\frac{\cos\left(2\times\frac{\pi}{2}\right)}{2}+C$$

$$\Rightarrow \quad 2\cdot 1=+\frac{1}{2}+C$$

$$\Rightarrow \quad 2-\frac{1}{2}=C\Rightarrow\frac{4-1}{2}=C$$

$$\Rightarrow \quad C=\frac{3}{2}$$

On substituting the value of C in EQ. (i), we get

$$y\sin x=-\frac{1}{2}\cos 2x+\frac{3}{2}$$ **[1½]**

Q. 24. Form the differential equation by eliminating A and B in $Ax^2+By^2=1$.

[NCERT Exemp. Ex. 9.3, Q. 22, Page 194]

Ans. Given differential equation is $Ax^2+By^2=1$

On differentiating both sides w.r.t. x, we get

$$2Ax+2By\frac{dy}{dx}=0$$

$$\Rightarrow \quad 2By\frac{dy}{dx}=-2Ax$$

$$\Rightarrow \quad By\frac{dy}{dx}=-Ax\Rightarrow\frac{y}{x}\cdot\frac{dy}{dx}=-\frac{A}{B}$$ **[1½]**

Again, differentiating w.r.t. x, we get,

$$\frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=0$$

$$\frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)}{x^2}=0$$

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)=0$$

$$xy\,y''+x(y')^2-y\,y'=0$$ **[1½]**

Q. 25. Solve the differential equation $(1+y^2)\tan^{-1}x\,dx+2y(1+x^2)dy=0$

[NCERT Exemp. Ex. 9.3, Q. 23, Page 194]

Ans. Given differential equation is

$$(1+y^2)\tan^{-1}xdx+2y(1+x^2)dy=0$$

$$\Rightarrow \quad (1+y^2)\tan^{-1}xdx=-2y(1+x^2)dy$$

$$\Rightarrow \quad \frac{\tan^{-1}xdx}{1+x^2}=-\frac{2y}{1+y^2}dy$$

On integrating both sides, we get

$$\int\frac{\tan^{-1}x}{1+x^2}dx=-\int\frac{2y}{1+y^2}dy$$

and put $1+y^2=u$ in RHS, we get

$$2y\,dy=du$$

$$\Rightarrow \quad \int t\,dt=-\int\frac{1}{u}du\Rightarrow\frac{t^2}{2}=-\log u+C$$

$$\Rightarrow \quad \frac{1}{2}(\tan^{-1}x)^2=-\log(1+y^2)+C$$

$$\Rightarrow \frac{1}{2}(\tan^{-1}x)^2 + \log(1+y^2) = C$$ **[3]**

Q. 26. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, then find the value of x when $y = 3$.

[NCERT Exemp. Ex. 9.3, Q. 3, Page 193]

Ans. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$

$$\Rightarrow \int e^{2y}dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C \quad \text{...(i)}$$

When $x = 5$ and $y = 0$, then substituting these values in EQ. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$$

Eq. (i) becomes $e^{2y} = 2x - 9$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$$\therefore \quad x = \frac{(e^6+9)}{2}$$

[3]

Q. 27. Solve the differential equation

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$$

[NCERT Exemp. Ex. 9.3, Q. 4, Page 193]

Ans. Given differential equation is

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{x^2-1}\right)y = \frac{1}{(x^2-1)^2}$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2x}{x^2-1}, Q = \frac{1}{(x^2-1)^2}$$

$$\text{IF} = e^{\int Pdx} = e^{\int\left(\frac{2x}{x^2-1}\right)dx}$$

Put $x^2 - 1 = t \Rightarrow 2xdx = dt$ **[1½]**

$$\therefore \quad \text{IF} = e^{\int\frac{dt}{t}} = e^{\log t} = t = (x^2-1)$$

The complete solution is

$$y \cdot \text{IF} = \int Q \cdot \text{IF} + K$$

$$\Rightarrow y\cdot(x^2-1) = \int \frac{1}{(x^2-1)^2}\cdot(x^2-1)dx + K$$

$$\Rightarrow y\cdot(x^2-1) = \int \frac{dx}{(x^2-1)} + K$$

$$\Rightarrow y\cdot(x^2-1) = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + K$$ **[1½]**

Q. 28. Solve the differential equation $\frac{dy}{dx} + 2xy = y$.

[NCERT Exemp. Ex. 9.3, Q. 5, Page 193]

Ans. Given that, $\frac{dy}{dx} + 2xy = y$

Given that, $\frac{dy}{dx} + 2xy = y$

$$\Rightarrow \frac{dy}{dx} + 2xy - y = 0$$

$$\Rightarrow \frac{dy}{dx} + (2x-1)y = 0$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = (2x-1), Q = 0$$

$$\text{IF} = e^{\int Pdx} = e^{\int(2x-1)dx}$$

$$= e^{\left(\frac{2x^2}{2}-x\right)} = e^{x^2-x}$$

The complete solution is

$$y\cdot e^{x^2-x} = \int 0\cdot e^{x^2-x}dx + C$$

$$\Rightarrow y\cdot e^{x^2-x} = 0 + C$$

$$\Rightarrow y = Ce^{x-x^2}$$ **[3]**

Q. 29. Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$

[NCERT Exemp. Ex. 9.3, Q. 6, Page 193]

Ans. Given differential equation

$$\frac{dy}{dx} + ay = e^{mx}$$

Which is a liner differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = a, Q = e^{mx}$$

$$\text{IF} = e^{\int Pdx} = e^{\int adx} = e^{ax}$$

The general solution is

$$y\cdot e^{ax} = \int e^{mx}\cdot e^{ax}dx + C$$

$$\Rightarrow y.\cdot e^{ax} = \int e^{(m+a)x}dx + C$$

$$\Rightarrow y\cdot e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + C$$

$$\Rightarrow (m+a)y = \frac{e^{(m+a)x}}{e^{ax}} + \frac{(m+a)C}{e^{ax}}$$

$$\Rightarrow (m+a)y = e^{mx} + ke^{-ax}$$

$$[\because k = (m+a)C]$$ **[3]**

Q. 30. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$y = x\sin x : xy' = y + x\sqrt{x^2-y^2}$ ($x \neq 0$ and $x > y$ or $x < -y$)

[NCERT Ex. 9.2, Q. 6, Page 385]

Ans. $y = x\sin x$

Differentiating both sides of this equation with respect to x, we get :

$$y' = \frac{d}{dx}(x\sin x)$$

$$\Rightarrow y' = \sin x\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x\cos x$$

Substituting the value of y' in the given differential equation, we get :

$$\text{LHS} = xy' = x(\sin x + x\cos x)$$
$$= x\sin x + x^2\cos x$$
$$= y + x^2\cdot\sqrt{1-\sin^2 x}$$
$$= y + x^2\sqrt{1-\left(\frac{y}{x}\right)^2}$$
$$= y + x\sqrt{y^2 - x^2}$$
$$= \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[3]**

Q. 31. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$$\boldsymbol{xy = \log y + C : y' = \frac{y^2}{1-xy} (xy \neq 1)}$$

[NCERT Ex. 9.2, Q. 7, Page 385]

Ans. $xy = \log y + C$

Differentiating both sides of this equation with respect to x, we get :

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$
$$\Rightarrow \quad y\cdot\frac{d}{dx}(x) + x\cdot\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$
$$\Rightarrow \quad y + xy' = \frac{1}{y}y'$$
$$\Rightarrow \quad y^2 + xy\,y' = y'$$
$$\Rightarrow \quad (xy-1)y' = -y^2$$
$$\Rightarrow \quad y' = \frac{y^2}{1-xy}$$

$\therefore$ LHS = RHS

Hence, the given function is the solution of the corresponding differential equation. **[3]**

Q. 32. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$$\boldsymbol{y - \cos y = x : (y\sin y + \cos y + x)y' = y}$$

[NCERT Ex. 9.2, Q. 8, Page 385]

Ans. $y - \cos y = x$ **...(i)**

Differentiating both sides of the equation with respect to x, we get :

$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$
$$\Rightarrow \quad y' + \sin y\cdot y' = 1$$
$$\Rightarrow \quad y'(1+\sin y) = 1$$
$$\Rightarrow \quad y' = \frac{1}{1+\sin y}$$

Substituting the value of y′ in equation (i), we get :

$$\text{LHS} = (y\sin y + \cos y + x)y'$$
$$= (y\sin y + \cos y + y - \cos y)\times\frac{1}{1+\sin y}$$
$$= y(1+\sin y)\cdot\frac{1}{1+\sin y}$$
$$= y(1+\sin y)\cdot\frac{1}{1+\sin y}$$
$$= y$$
$$= \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[3]**

Q. 33. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$$\boldsymbol{x + y = \tan^{-1} y : y^2 y' + y^2 + 1 = 0}$$

[NCERT Ex. 9.2, Q. 9, Page 385]

Ans. $x + y = \tan^{-1} y$

Differentiating both sides of this equation with respect to x, we get :

$$\frac{d}{dx}(x+y) = \frac{d}{dx}(\tan^{-1} y)$$
$$\Rightarrow \quad 1 + y' = \left[\frac{1}{1+y^2}\right]y'$$
$$\Rightarrow \quad y'\left[\frac{1}{1+y^2} - 1\right] = 1$$
$$\Rightarrow \quad y'\left[\frac{1-(1+y^2)}{1+y^2}\right] = 1$$
$$\Rightarrow \quad y'\left[\frac{-y^2}{1+y^2}\right] = 1$$
$$\Rightarrow \quad y' = \frac{-(1+y^2)}{y^2}$$

Substituting the value of y' in the given differential equation, we get :

$$\text{LHS} = y^2y' + y^2 + 1 = y^2\left[\frac{-(1+y^2)}{y^2}\right] + y^2 + 1$$
$$= -1 - y^2 + y^2 + 1$$
$$= 0$$
$$= \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[3]**

Q. 34. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$$\boldsymbol{y = \sqrt{a^2 - x^2}\; x \in (-a, a) : x + y\frac{dy}{dx} = 0\ (y \neq 0)}$$

[NCERT Ex. 9.2, Q. 10, Page 385]

Ans. $y = \sqrt{a^2 - x^2}$

Differentiating both sides of this equation with respect for x, we get :

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2-x^2}\right)$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2-x^2}}\cdot\frac{d}{dx}(a^2-x^2)$$
$$= \frac{1}{2\sqrt{a^2-x^2}}(-2x)$$
$$= \frac{-x}{\sqrt{a^2-x^2}}$$

Substituting the value of $\frac{dy}{dx}$ in the given differential equation, we get :

$$\text{LHS} = x + y\frac{dy}{dx} = x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}}$$
$$= x - x$$
$$= 0$$
$$= \text{RHS}$$

Hence, the given function is the solution of the corresponding differential equation. **[3]**

Q. 35. Form a differential equation representing the given curve by eliminating arbitrary constants *a* and *b*.

$$\frac{x}{a} + \frac{y}{b} = 1$$ **[NCERT Ex. 9.3, Q. 1, Page 391]**

Ans. $\frac{x}{a} + \frac{y}{b} = 1$

Differentiating both sides of the given equation with respect to x, we get :

$$\frac{1}{a} + \frac{1}{b}\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{1}{a} + \frac{1}{b}y' = 0$$

Again, differentiating both sides with respect to x, we get :

$$0 + \frac{1}{b}y'' = 0$$
$$\Rightarrow \quad \frac{1}{b}y'' = 0$$
$$\Rightarrow \quad y'' = 0$$

Hence, the required differential equation of the given curve is $y''=0$. **[3]**

Q. 36. Form a differential equation representing the given curve by eliminating arbitrary constants *a* and *b*.

$$y^2 = a(b^2 - x^2)$$ **[NCERT Ex. 9.3, Q. 2, Page 391]**

Ans. $y^2 = a(b^2 - x^2)$

Differentiating both sides with respect to x, we get :

$$2y\frac{dy}{dx} = a(-2x)$$
$$\Rightarrow 2yy' = -2ax$$
$$\Rightarrow yy' = -ax \quad \text{...(1)}$$

Again, differentiating both sides with respect to x, we get :

$$y' \cdot y' + yy'' = -a$$
$$\Rightarrow (y')^2 + yy'' = -a \quad (2)$$

Dividing equation (2) by equation (1), we get :

$$\frac{(y')^2 + yy''}{yy'} = \frac{-a}{-ax}$$
$$\Rightarrow \quad xyy'' + x(y')^2 - yy'' = 0$$

This is the required differential equation of the given curve. **[3]**

Q. 37. Form a differential equation representing the given curve by eliminating arbitrary constants *a* and *b*.

$$y = a\,e^{3x} + b\,e^{-2x}$$ **[NCERT Ex. 9.3, Q. 3, Page 391]**

Ans. $y = ae^{3x} + be^{-2x}$ **...(1)**

Differentiating both sides with respect to x, we get :

$$y' = 3ae^{3x} - 2be^{-2x} \quad \text{...(2)}$$

Again, differentiating both sides with respect to x, we get :

$$y'' = 9ae^{3x} + 4be^{-2x} \quad \text{...(3)}$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get :

$$(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2bc^{-2x}) = 2y + y'$$
$$\Rightarrow \quad 5ae^{3x} = 2y + y'$$
$$\Rightarrow \quad ae^{3x} = \frac{2y + y'}{5}$$

Now, multiplying equation (1) with 3 and subtracting equation (2) from it, we get :

$$(3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) = 3y - y'$$
$$\Rightarrow \quad 5be^{-2x} = 3y - y'$$
$$\Rightarrow \quad be^{-2x} = \frac{3y - y'}{5}$$ **[1½]**

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get :

$$y'' = 9 \cdot \frac{(2y + y')}{5} + 4\frac{(3y - y')}{5}$$
$$\Rightarrow \quad y'' = \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5}$$
$$\Rightarrow \quad y'' = \frac{30y + 5y'}{5}$$
$$\Rightarrow \quad y'' = 6y + y'$$
$$\Rightarrow \quad y'' - y' - 6y = 0$$

This is the required differential equation of the given curve. **[1½]**

Q. 38. Form a differential equation representing the given curve by eliminating arbitrary constants *a* and *b*.

$$y = e^{2x}(a + bx)$$ **[NCERT Ex. 9.3, Q. 4, Page 391]**

Ans. $y = e^{2x}(a + bx)$ **...(1)**

Differentiating both sides with respect to x, we get :

$$y' = 2e^{2x}(a + bx) + e^{2x} \cdot b$$
$$\Rightarrow \quad y' = e^{2x}(2a + 2bx + b) \quad \text{...(2)}$$

Multiplying equation (1) with 2 and then subtracting it from equation (2), we get :

$$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx)$$
$$\Rightarrow \quad y' - 2y = be^{2x} \quad \text{...(3)}$$

Differentiating both sides with respect to x, we get :

$$y'' - 2y' = 2be^{2x} \quad \text{....(4)}$$

Dividing equation (4) by equation (3), we get :

$$\frac{y'' - 2y'}{y' - 2y} = 2$$
$$\Rightarrow \quad y'' - 2y' = 2y' - 4y$$
$$\Rightarrow \quad y'' - 4y' + 4y = 0$$

This is the required differential equation of the given curve. **[3]**

Q. 39. Form a differential equation representing the given curve by eliminating arbitrary constants *a* and *b*.

$$y = e^x(a\cos x + b\sin x)$$

[NCERT Ex. 9.3, Q. 5, Page 391]

Ans. $y = e^x(a\cos x + b\sin x)$ ...(1)

Differentiating both sides with respect to x, we get :

$$y' = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$$

$$\Rightarrow y' = e^x[(a+b)\cos x - (a-b)\sin x] \quad ...(2)$$

Again, differentiating with respect to x, we get :

$$y'' = e^x[(a+b)\cos x - (a-b)\sin x] + e^x[-(a+b)\sin x - (a-b)\cos x]$$

$$y'' = e^x[2b\cos x - 2a\sin x]$$

$$y'' = 2e^x(b\cos x - a\sin x)$$

$$\Rightarrow \frac{y''}{2} = e^x(b\cos x - a\sin x) \quad ...(3)$$

Adding equations (1) and (3), we get :

$$y + \frac{y''}{2} = e^x[(a+b)\cos x - (a-b)\sin x]$$

$$\Rightarrow y + \frac{y''}{2} = y'$$

$$\Rightarrow 2y + y'' = 2y'$$

$$\Rightarrow y'' - 2y' + 2y = 0$$

This is the required differential equation of the given curve. **[3]**

Q. 40. Form the differential equation of the family of circles touching the *y*-axis at origin.

[NCERT Ex. 9.3, Q. 6, Page 391]

Ans. The centre of the circle touching the y-axis at origin lies on the x-axis.

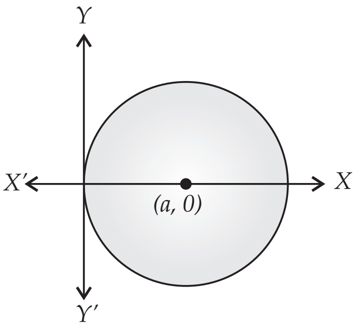
Let $(a, 0)$ be the centre of the circle.

Since it touches the y-axis at origin, its radius is a.

Now, the equation of the circle with centre $(a, 0)$ and radius (a) is

$$(x-a)^2 + y^2 = a^2$$

$$\Rightarrow x^2 + y^2 = 2ax \quad ...(1)$$



[1½]

Differentiating equation (1) with respect to x, we get :

$$2x + 2yy' = 2a$$

$$\Rightarrow x + yy' = a$$

Now, on substituting the value of a in equation (1), we get :

$$x^2 + y^2 = 2(x + yy')x$$

$$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$$

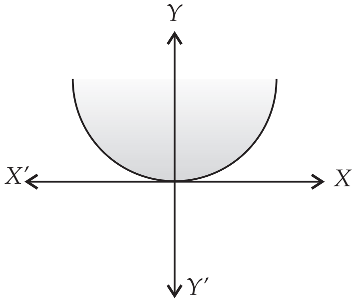
$$\Rightarrow 2xyy' + x^2 = y^2$$

This is the required differential equation. **[1½]**

Q. 41. Form the differential equation of the family of parabolas having vertex at origin and axis along positive *y*-axis.

[NCERT Ex. 9.3, Q. 7, Page 391]

Ans. The equation of the parabola having the vertex at origin and the axis along the positive y-axis is :



[1½]

Differentiating equation (1) with respect to x, we get :

$$2x = 4ay' \quad ...(2)$$

Dividing equation (2) by equation (1), we get :

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

$$\Rightarrow xy' = 2y$$

$$\Rightarrow xy' - 2y = 0$$

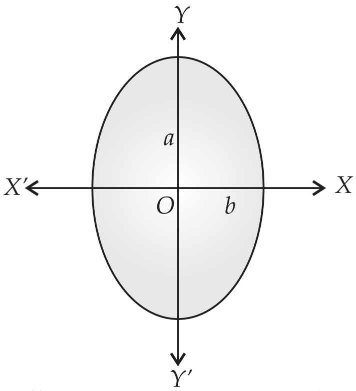
This is the required differential equation. **[1½]**

Q. 42. Form the differential equation of the family of ellipses having foci on *y*-axis and centre at origin.

[NCERT Ex. 9.3, Q. 8, Page 391]

Ans. The equation of the family of ellipses having foci on the y-axis and the centre at origin is as follows :

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad ...(1)$$



Differentiating equation (1) with respect to x, we get :

$$\frac{2x}{b^2} + \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad ...(2)$$

[1½]

Again, differentiating with respect to x, we get :

$$\frac{1}{b^2}+\frac{y'\cdot y'+y\cdot y''}{a^2}=0$$

$$\Rightarrow \frac{1}{b^2}+\frac{1}{a^2}(y'^2+yy'')=0$$

$$\Rightarrow \quad \frac{1}{b^2}=-\frac{1}{a^2}(y'^2+yy'')$$

Substituting this value in equation (2), we get :

$$x\left[-\frac{1}{a^2}\left((y')^2+yy''\right)\right]+\frac{yy'}{a^2}=0$$

$$\Rightarrow \quad -x(y')^2-xyy''+yy'=0$$

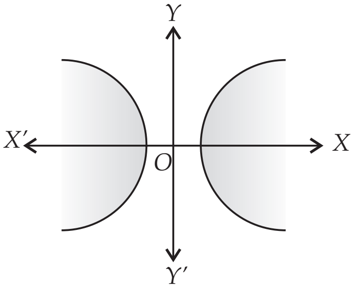
$$\Rightarrow \quad xyy''+x(y')^2-yy'=0$$

This is the required differential equation. **[1½]**

Q. 43. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin. [NCERT Ex. 9.3, Q. 9, Page 391]

Ans. The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is :

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad ...(1)$$



Differentiating both sides of equation (1) with respect to x, we get :

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0$$

$$\Rightarrow \frac{x}{a^2}-\frac{yy'}{b^2}=0 \qquad ...(2)$$

Again, differentiating both sides with respect to x, we get.

$$\frac{1}{a^2}-\frac{y'.y'+yy''}{b^2}=0$$

$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left((y')^2+yy''\right) \qquad \textbf{[1½]}$$

Substituting the values of $\frac{1}{a^2}$ in equation (2), we get.

$$\frac{x}{b^2}\left((y')^2+yy''\right)-\frac{yy'}{b^2}=0$$

$$\Rightarrow x(y')^2+xyy''-yy'=0$$

$$\Rightarrow xyy''+x(y')^2-yy'=0 \qquad \textbf{[1½]}$$

This is the required differential equation.

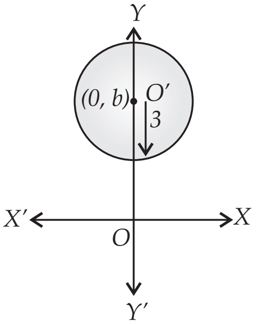
Q. 44. Form the differential equation of the family of circles having centre on y-axis and radius 3 units. [NCERT Ex. 9.3, Q. 10, Page 391]

Ans. Let the centre of the circle on y-axis be $(0,b)$.

The differential equation of the family of circles with centre at $(0,b)$ and radius 3 is as follows :

$$x^2+(y-b)^2=3^2$$

$$\Rightarrow x^2+(y-b)^2=9 \qquad ...(1)$$



Differentiating equation (1) with respect to x, we get.

$$2x+2(y-b)\cdot y'=0$$

$$\Rightarrow \quad (y-b)\cdot y'=-x$$

$$\Rightarrow \quad y-b=\frac{-x}{y'}$$

Substituting the value of $(y-b)$ in equation (1), we get.

$$x^2+\left(\frac{-x}{y'}\right)^2=9$$

$$\Rightarrow \quad x^2\left[1+\frac{1}{(y')^2}\right]=9$$

$$\Rightarrow \quad x^2\left((y')^2+1\right)=9(y')^2$$

$$\Rightarrow (x^2-9)(y')^2+x^2=0 \qquad \textbf{[1½]}$$

This is the required differential equation.

Q. 45. Find the general solution :

$$\boldsymbol{\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}}$$ **[NCERT Ex. 9.4, Q. 1, Page 395]**

Ans. The given differential equation is :

$$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx}=\left(\sec^2\frac{x}{2}-1\right)$$

Separating the variables, we get

$$dy=\left(\sec^2\frac{x}{2}-1\right)dx$$

Now, integrating both sides of this equation, we get :

$$\int dy=\int\left(\sec^2\frac{x}{2}-1\right)dx=\int\sec^2\frac{x}{2}dx-\int dx$$

$$\Rightarrow y=2\tan\frac{x}{2}-x+C \qquad \textbf{[3]}$$

This is the required general solution of the given differential equation

Q. 46. Find the general solution :

$$\frac{dy}{dx} = \sqrt{4-y^2}\ (-2 < y < 2)$$

[NCERT Ex. 9.4, Q. 2, Page 395]

Ans. The given differential equation is :

$$\frac{dy}{dx} = \sqrt{4-y^2}$$

Separating the variables, we get :

$$\Rightarrow \frac{dy}{\sqrt{4-y^2}} = dx$$

Now, integrating both sides of the equation, we get.

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$\Rightarrow \sin^{-1}\frac{y}{2} = x + C$$

$$\Rightarrow \frac{y}{2} = \sin(x+C)$$

$$\Rightarrow y = 2\sin(x+C)$$

[3]

This is the required general solution of the given differential equation.

Q. 47. Find the general solution :

$$\frac{dy}{dx} + y = 1 (y \neq 1)$$ **[NCERT Ex. 9.4, Q. 3, Page 396]**

Ans. The given differential equation is :

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow dy + ydx = dx$$

$$\Rightarrow dy = (1-y)dx$$

Separating the variables, we get :

$$\Rightarrow \frac{dy}{1-y} = dx$$

Now, integrating both sides, we get :

$$\Rightarrow \int \frac{dy}{(1-y)} = \int dx$$

$$\Rightarrow \log(1-y) = x + \log C$$

$$\Rightarrow -\log C - \log(1-y) = x$$

$$\Rightarrow \log C(1-y) = -x$$

$$\Rightarrow C(1-y) = e^{-x}$$

$$\Rightarrow 1-y = \frac{1}{C}e^{-x}$$

$$\Rightarrow y = 1 - \frac{1}{C}e^{-x}$$

$$\Rightarrow y = 1 + Ae^{-x} \qquad \left[\text{Where } A = -\frac{1}{C}\right]$$

[3]

This is the required general solution of the given differential equation.

Q. 48. Find the general solution :

$y\log y dx - xdy = 0$ **[NCERT Ex. 9.4, Q. 7, Page 396]**

Ans. The given differential equation is :

$$y\log y dx - xdy = 0$$

$$\Rightarrow y\log y dx = xdy$$

$$\Rightarrow y\log y dx = xdy$$

$$\Rightarrow \frac{dy}{y\log y} = \frac{dx}{x}$$

Integrating both sides, we get :

$$\int \frac{dy}{y\log y} = \int \frac{dx}{x}$$

Let log y=t.

$$\therefore \frac{d}{dy}(\log y) = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} = \frac{dt}{dy} \qquad ...(1)$$

$$\Rightarrow \frac{1}{y}dy = dt$$

[1½]

Substituting this value in equation (1), we get :

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \log t = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

[1½]

This is the required general solution of the given differential equation.

Q. 49. Find the general solution :

$$x^5\frac{dy}{dx} = -y^5$$

[NCERT Ex. 9.4, Q. 8, Page 396]

Ans. The given differential equation is :

$$x^5\frac{dy}{dx} = -y^5$$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

$$\Rightarrow \frac{dx}{x^5} + \frac{dy}{y^5} = 0$$

Integrating both sides, we get :

$$\int \frac{dx}{x^5} + \int \frac{dy}{y^5} = k \text{ (Where } k \text{ is any constant)}$$

$$\Rightarrow \int x^{-5}dx + \int y^{-5}dy = k$$

$$\Rightarrow \frac{x^{-4}}{-4} + \frac{y^{-4}}{-4} = k$$

$$\Rightarrow x^{-4} + y^{-4} = -4k$$

$$\Rightarrow x^{-4} + y^{-4} = C \qquad (C = -4k)$$

[3]

This is the required solution of the given differential equation.

Q. 50. Find a particular solution satisfying the given condition :

$$\cos\left(\frac{dy}{dx}\right) = a\ (a \in R);\ y = 2 \text{ when } x = 0$$

[NCERT Ex. 9.4, Q. 13, Page 396]

Ans.

$$\cos\left(\frac{dy}{dx}\right) = a$$

$\Rightarrow \quad \frac{dy}{dx} = \cos^{-1} a$

$\Rightarrow \quad dy = \cos^{-1} a dx$

Integrating both sides, we get :

$\int dy = \cos^{-1} a \int dx$

$\Rightarrow y = \cos^{-1} a \cdot x + C$

$\Rightarrow y = x\cos^{-1} a + C$

Now, y=2 when $x = 0$.

$\Rightarrow 2 = 0.\cos^{-1} a + C$

$\Rightarrow C = 2$ ...(2)

Substituting C=1 in equation (1), we get :

$$y = x\cos^{-1} a + 2$$

$\Rightarrow \quad \frac{y-2}{x} = \cos^{-1} a$

$\Rightarrow \cos\left(\frac{y-2}{x}\right) = a$

[3]

Q. 51. Find a particular solution satisfying the given condition :

$\boldsymbol{\frac{dy}{dx} = y\tan x; y = 1 \text{ when } x = 0}$

[NCERT Ex. 9.4, Q. 14, Page 396]

Ans. $\frac{dy}{dx} = y\tan x$

$\Rightarrow \frac{dy}{y} = \tan x dx$

Integrating both sides, we get :

$\int \frac{dy}{y} = -\int \tan x dx$

$\Rightarrow \log y = \log(\sec x) + \log C$

$\Rightarrow \log y = \log(C\sec x)$

$\Rightarrow \quad y = C\sec x$

Now, y=1 when $x = 0$

$\Rightarrow \quad 1 = C \times \sec 0$

$\Rightarrow \quad 1 = C \times 1$

$\Rightarrow \quad C = 1$ ...(1)

Substituting C = 1 in equation (1), we get

$y = \sec x$ **[3]**

Q. 52. For the differential equation

$\boldsymbol{xy\frac{dy}{dx} = (x+2)(y+2)}$

Find the solution of curve passing through the point (1, –1).

[NCERT Ex. 9.4, Q. 16, Page 396]

Ans. The differential equation of the given curve is :

$$xy\frac{dy}{dx} = (x+2)(y+2)$$

$\Rightarrow \quad \left(\frac{y}{y+2}\right)dy = \left(\frac{x+2}{x}\right)dx$

$\Rightarrow \left(1 - \frac{2}{y+2}\right)dy = \left(1 + \frac{2}{x}\right)dx$

Integrating on both sides, we get :

$\int\left(1 - \frac{2}{y+2}\right)dy = \int\left(1 + \frac{2}{x}\right)dx$

$\Rightarrow \int dy - 2\int \frac{1}{y+2} = \int dx + 2\int \frac{1}{x}dx$

$\Rightarrow y - 2\log(y+2) = x + 2\log x + C$

$\Rightarrow \quad y - x - C = \log x^2 + \log(y+2)^2$

$\Rightarrow \quad y - x - C = \log\left[x^2(y+2)^2\right]$ ...(1)

Now, the curve passes through point (1, −1)

$\Rightarrow -1 - 1 - C = \log\left[(1)^2(-1+2)^2\right]$

$\Rightarrow \quad -2 - C = \log 1 = 0$

$\Rightarrow \quad C = -2$

Substituting C = −2 in equation (1), we get :

$y - x + 2 = \log\left[x^2(y+2)^2\right]$

This is the required solution of the given curve.

Q. 53. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs. 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5}$ = 1.648).

[NCERT Ex. 9.4, Q. 21, Page 397]

Ans. Let p and t be the principal and time respectively. It is given that the principal increases continuously at the rate of 5% per year.

$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$

$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$

$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$

Integrating both sides, we get :

$\int \frac{dp}{p} = \frac{1}{20}\int dt$

$\Rightarrow \log p = \frac{t}{20} + C$

$\Rightarrow \quad p = e^{\frac{t}{20} + C}$ ...(1)

[3]

Now, when t=0, p=1000.

Hence, after 10 years the amount will worth Rs 1648.

Q. 54. For the differential equation find the general solution :

$\boldsymbol{\frac{dy}{dx} + 3y = e^{-2x}}$ **[NCERT Ex. 9.6, Q. 2, Page 413]**

Ans. The given differential equation is

$\frac{dy}{dx} + py = Q$ (where $p = 3$ and $Q = e^{-2x}$) Now,

IF $= e^{\int p dx} = e^{\int 3dx} = e^{3x}$.

The solution of the given differential equation is given by the relation,

$y(\text{IF}) = e^{\int p dx} = e^{\int 3dx} = e^{3x}$.

The solution of the given differential equation is given by the relation,

$$y(\text{IF})=\int(Q \times \text{IF})dx+C$$

$$\Rightarrow ye^{3x}=\int(e^{-2x}\times e^{3x})+C$$

$$\Rightarrow ye^{3x}=\int e^{x}dx+C$$

$$\Rightarrow ye^{3x}=e^{x}+C$$

$$\Rightarrow y=e^{-2x}+Ce^{-3x}$$

[3]

This is required general solution of the given differential equation.

Q. 55. For the differential equation find the general solution :

$$\frac{dy}{dx}+\frac{y}{x}=x^2$$

[NCERT Ex. 9.6, Q. 3, Page 413]

Ans. The given differential equation is :

$$\frac{dy}{dx}+py=Q(\text{where } p=\frac{1}{x} \text{ and } Q=x^2)$$

Now, $\text{IF}=e^{\int pdx}=e^{\int \frac{1}{x}dx}=e^{\log x}=x.$

The solution of the given differential equation is given by the relation.

$$y(\text{IF})=\int(Q\times \text{IF})dx+C$$

$$\Rightarrow y(x)=\int(x^2.x)dx+C$$

$$\Rightarrow \quad xy=\int x^3dx+C$$

$$\Rightarrow \quad xy=\frac{x^4}{4}+C$$

This is the required general solution of the given differential equation.

$$y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)\,dx+C$$

$$=\int(\sec x\tan x+\tan^2 x)\,dx+C$$

$$=\int \sec x\tan x\,dx+\int(\sec^2 x-1)dx+C$$

$$y(\sec x+\tan x)=\sec x+\tan x-x+C$$

[3]

Q. 56. For the differential equation find the general solution :

$$\frac{dy}{dx}+(\sec x)y=\tan x\left(0\le x<\frac{\pi}{2}\right)$$

[NCERT Ex. 9.6, Q. 4, Page 413]

Ans. The given differential equation is :

$$\frac{dy}{dx}+py=Q(\text{where } p=\sec x \text{ and } Q=\tan x)$$

Now, $\text{IF}=e^{\int pdx}=e^{\int \sec x dx}=e^{\log(\sec x+\tan x)}=\sec x+\tan x.$

The general solution of the given differential equation is given by the relation,

$$y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx+C$$

$$=\int(\sec x\tan x+\tan^2 x)dx+C$$

$$=\int \sec x\,.\,\tan x\,dx+\int(\sec^2 x-1)dx+C$$

$$y(\sec x+\tan x)=\sec x+\tan x-x+C$$

[3]

Q. 57. For the differential equation find the general solution :

$$y\,dx+(x-y^2)dy=0$$

[NCERT Ex. 9.6, Q. 11, Page 414]

Ans. $y\,dx+(x-y^2)dy=0$

$$\Rightarrow \quad ydx=(y^2-x)dy$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{y^2-x}{y}=y-\frac{x}{y}$$

$$\Rightarrow \frac{dx}{dy}+\frac{x}{y}=y$$

This is a linear differential equation of the form :

$$\frac{dy}{dx}+px=Q(\text{where } p=\frac{1}{2} \text{ and } Q=y)$$

Now, $\text{IF}=e^{\int pdy}=e^{\int \frac{1}{y}dy}=e^{\log y}=y.$

The general solution of the given differential equation is given by the relation.

$$x(\text{IF})=\int(Q\times \text{IF})dy+C$$

$$\Rightarrow xy=\int(y.y)dy+C$$

$$\Rightarrow xy=\int y^2dy+C$$

$$\Rightarrow xy=\frac{y^3}{3}+C$$

$$\Rightarrow \quad x=\frac{y^2}{3}+\frac{C}{y}$$

[3]

Q. 58. For the differential equation find the general solution :

$$(x+3y^2)\frac{dy}{dx}=y(y>0)$$

[NCERT Ex. 9.6, Q. 12, Page 414]

Ans. Given that,

$$(x+3y^2)\frac{dy}{dx}=y(y>0)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x+3y^2}$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{x+3y^2}{y}=\frac{x}{y}+3y$$

$$\Rightarrow \quad \frac{dx}{dy}-\frac{x}{y}=3y$$

This is a linear differential equation of the form :

$$\frac{dx}{dy}+px=Q \qquad \left(\text{Where } p=-\frac{1}{y} \text{ and } Q=3y\right)$$

Now, $\text{IF}=e^{\int pdy}=e^{-\int\frac{dy}{y}}=e^{-\log y}=e^{\log\left(\frac{1}{y}\right)}=\frac{1}{y}.$

The general solution of the given differential equation is given by the relation,

$$x(\text{IF})=\int(Q\times \text{IF})dy+C$$

$$\Rightarrow x\times\frac{1}{y}=\int\left(3y\times\frac{1}{y}\right)dy+C$$

$$\Rightarrow \quad \frac{x}{y}=3y+C$$

$$\Rightarrow \quad x=3y^2+Cy$$

[3]

Q. 59. For each of the exercises given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

(i) $y = a\,e^x + b\,e^{-x} + x^2 \;:\; x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy + x^2 - 2 = 0$

(ii) $y = e^x\left(a\cos x + b\sin x\right) : \dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

(iii) $y = x\sin 3x \;:\; \dfrac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$

(iv) $x^2 = 2y^2\log y \;:\; \left(x^2 + y^2\right)\dfrac{dy}{dx} - xy = 0$

[NCERT Misc. Ex. Q. 2, Page 420]

Ans. (i) $y = ae^x + be^{-x} + x^2$

Differentiating both sides with respect to x, we get :

$$\frac{dy}{dx} = a\frac{d}{dx}\left(e^x\right) + b\frac{d}{dx}\left(e^{-x}\right) + \frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow \frac{dy}{dx} = ae^x - be^{-x} + 2x$$

Again, differentiating both sides with respect to x, we get :

$$\frac{d^2y}{dx^2} = ae^x + be^{-x} + 2$$

Now, on substituting the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the differential equation, we get :

LHS

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2$$

$$= x\left(ae^x + be^{-x} + 2\right) + 2\left(ae^x - be^{-x} + 2x\right) - x\left(ae^x + be^{-x} + x^2\right) + x^2 - 2$$

$$= \left(axe^x + bxe^{-x} + 2x\right) + \left(2ae^x - 2be^{-x} + 4x\right) - \left(axe^x + bxe^{-x} + x^3\right) + x^2 - 2$$

$$= 2ae^x - 2be^{-x} + x^2 + 6x - 2$$

$$\neq 0$$

$\Rightarrow$ LHS $\neq$ RHS

Hence, the given function is not a solution of the corresponding differential equation. **[3]**

(ii) $y = e^x\left(a\cos x + b\sin x\right) = ae^x\cos x + be^x\sin x$

Differentiating both sides with respect to x, we get :

$$\frac{dy}{dx} = a\cdot\frac{d}{dx}\left(e^x\cos x\right) + b\cdot\frac{d}{dx}\left(e^x\sin x\right)$$

$$\Rightarrow \frac{dy}{dx} = a\left(e^x\cos x - e^x\sin x\right) + b\cdot\left(e^x\sin x + e^x\cos x\right)$$

$$\Rightarrow \frac{dy}{dx} = \left(a + b\right)e^x\cos x + \left(b - a\right)e^x\sin x$$

Again, differentiating both sides with respect to x, we get :

$$\frac{d^2y}{dx^2} = \left(a + b\right)\cdot\frac{d}{dx}\left(e^x\cos x\right) + \left(b - a\right)\frac{d}{dx}\left(e^x\sin x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(a + b\right)\cdot\left[e^x\cos x - e^x\sin x\right] + \left(b - a\right)\left[e^x\sin x + e^x\cos x\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^x\left[\left(a + b\right)\left(\cos x - \sin x\right) + \left(b - a\right)\left(\sin x + \cos x\right)\right]$$

$$\Rightarrow \frac{d^2y}{dx} = e^x\left[\begin{array}{l} a\cos x - a\sin x + b\cos x - b\sin x + \\ \qquad b\sin x - a\sin x - a\cos x\end{array}\right]$$

$$\Rightarrow \frac{d^2y}{dx} = \left[2e^x\left(b\cos x - a\sin x\right)\right]$$

[3]

Now, on substituting the values of $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in the LHS of the given differential equation, We get :

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y$$

$$= 2e^x\left(b\cos x - a\sin x\right) - 2e^x\left[\left(a + b\right)\cos x + \left(b - a\right)\sin x\right] + 2e^x\left(a\cos x + b\sin x\right)$$

$$= e^x\left[\begin{array}{l}\left(2b\cos x - 2a\sin x\right) - \left(2a\cos x + 2b\cos x\right) \\ -\left(2b\sin x - 2a\sin x\right) + \left(2a\cos x + 2b\sin x\right)\end{array}\right]$$

$$= e^x\left[\left(2b - 2a - 2b + 2a\right)\cos x\right] + e^x\left[\left(-2a - 2b + 2a + 2b\right)\sin x\right]$$

$$= 0$$

Hence, the given function is a solution of the corresponding differential equation. **[3]**

(iii) $y = x\sin 3x$

Differentiating both sides with respect to x, we get :

$$\frac{dy}{dx} = \frac{d}{dx}\left(x\sin 3x\right) = \sin 3x + x\cdot\cos 3x\cdot 3$$

$$\Rightarrow \frac{dy}{dx} = \sin 3x + 3x\cos 3x$$

Again, differentiating both sides with respect to x, we get :

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\sin 3x\right) + 3\frac{d}{dx}\left(x\cos 3x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 3\cos 3x + 3\left[\cos 3x + x\left(-\sin 3x\right)\cdot 3\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = 6\cos 3x - 9x\sin 3x$$

Substituting the value of $\frac{d^2y}{dx^2}$ in the LHS of the given differential equation, we get :

$$\frac{d^2y}{dx^2} + 9y - 6\cos 3x$$

$$= \left(6\cdot\cos 3x - 9x\sin 3x\right) + 9x\sin 3x - 6\cos x$$

$$= 0$$

Hence, the given function is a solution of the corresponding differential equation. **[3]**

(iv) $x^2 = 2y^2\log y$

Differentiating both sides with respect to x, we get :

$$2x\cdot = 2\frac{d}{dx}\left[y^2\log y\right]$$

$$\Rightarrow\; x = \left[2y\cdot\log y\cdot\frac{dy}{dx} + y^2\cdot\frac{1}{y}\cdot\frac{dy}{dx}\right]$$

$$\Rightarrow\; x = \frac{dy}{dx}\left(2y\log y + y\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y\left(1 + 2\log y\right)}$$

Substituting the value of $\frac{dy}{dx}$ in the LHS of the given differential equation, we get :

$$\left(x^2+y^2\right)\frac{dy}{dx}-xy$$
$$=\left(2y^2\log y+y^2\right)\cdot\frac{x}{y\left(1+2\log y\right)}-xy$$
$$=y^2\left(1+2\log y\right)\cdot\frac{x}{y\left(1+2\log y\right)}-xy$$
$$=xy-xy$$
$$=0$$

Hence, the given function is a solution of the corresponding differential equation. **[3]**

Q. 60. Find the general solution of the differential equation
***y dx* – (*x* + 2*y*²) *dy* = 0.**

[CBSE Board, All India Region, 2017]

Ans. Given differential equation can be written as

$$y\frac{dx}{dy}-x=2y^2 \text{ or } \frac{dx}{dy}-\frac{1}{y}\cdot x=2y$$

Integrating factor is $e^{-\log y}=\frac{1}{y}$

$\therefore$ Solution is $x\cdot\frac{1}{y}=\int 2dy=2y+C$

or $x=2y^2+Cy$. **[4]**

Q. 61. Find the general solution of the differential equation $dy/dx - y = \sin x$.

[CBSE Board, All India Region, 2017]

Ans. Given differential equation is $\frac{dy}{dx}-y=\sin x$

$\Rightarrow$ Integrating factor $= e^{-x}$

$\therefore$ Solution is $ye^{-x}=\frac{1}{2}\left(-\sin x-\cos x\right)e^{-x}+C$

or $y=-\frac{1}{2}\left(\sin x+\cos x\right)+Ce^{x}$ **[4]**

Q. 62. Solve the differential equation $x\, dy/dx + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$

[CBSE Board, Delhi Region, 2017]

Ans. The given equation can be written as

$$\frac{dy}{dx}+\frac{y}{x}=\cos x+\frac{\sin x}{x}$$

$$IF \quad =e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

$\therefore$ Solution is

$$y\cdot x=\int\left(x\cos x+\sin x\right)dx+C$$
$$\Rightarrow y\cdot x=x\sin x+C$$
$$\text{or } y=\sin x+\frac{C}{x}$$

when $x=\frac{\pi}{2}, y=1,$ we get $C=0$

Required solution is $y = \sin x$ **[4]**

Q. 63. Form the differential equation representing the family of curves given by $(x – a)^2 + 2y^2 = a^2$, where a is an arbitrary constant.

[NCERT Misc. Ex. Q. 3, Page 420]

Ans.
$$\left(x-a\right)^2+2y^2=a^2$$
$$\Rightarrow x^2+a^2-2ax+2y^2=a^2$$
$$\Rightarrow \quad 2y^2=2ax-x^2 \quad ...(1)$$

Differentiating with respect to x, we get :

$$2y\frac{dy}{dx}=\frac{2a-2x}{2}$$
$$\Rightarrow\frac{dy}{dx}=\frac{a-x}{2y}$$
$$\Rightarrow\frac{dy}{dx}=\frac{2ax-2x^2}{4xy} \quad ...(2)$$

From equation (1), we get :

$$2ax=2y^2+x^2$$

On substituting this value in equation (3), we get :

$$\frac{dy}{dx}=\frac{2y^2+x^2-2x^2}{4xy}$$
$$\Rightarrow\frac{dy}{dx}=\frac{2y^2-x^2}{4xy}$$

Hence, the differential equation of the family of curves is given as $\frac{dy}{dx}=\frac{2y^2-x^2}{4xy}$. **[3]**

Q. 64. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ is given by $\left(x+y+1\right)=A\left(1-x-y-2xy\right)$, where A is parameter.

[NCERT Misc. Ex. Q. 7, Page 420]

Ans.
$$\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$$
$$\Rightarrow \quad \frac{dy}{dx}=-\frac{\left(y^2+y+1\right)}{x^2+x+1}$$
$$\Rightarrow \quad \frac{dy}{y^2+y+1}=\frac{-dx}{x^2+x+1}$$
$$\Rightarrow\frac{dy}{y^2+y+1}+\frac{dx}{x^2+x+1}=0$$

Integrating both sides, we get :

$$\int\frac{dy}{y^2+y+1}+\int\frac{dx}{x^2+x+1}=C$$
$$\Rightarrow\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=C$$
$$\Rightarrow \quad \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]+\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]=C$$
$$\Rightarrow \quad \tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right]+\tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right]=\frac{\sqrt{3}C}{2}$$ **[1½]**
$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1-\frac{\left(2y+1\right)}{\sqrt{3}}\cdot\frac{\left(2x+1\right)}{\sqrt{3}}}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{2x+2y+2}{\sqrt{3}}}{1-\left(\frac{4xy+2x+2y+1}{3}\right)}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{2\sqrt{3}(x+y+1)}{3-4xy-2x-2y-1}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \frac{\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}=\tan\left(\frac{\sqrt{3}C}{2}\right)=B,$$

where $B=\tan\left(\frac{\sqrt{3}C}{2}\right)$

$$\Rightarrow x+y+1=\frac{2B}{\sqrt{3}}(1-xy-2xy)$$

$$\Rightarrow x+y+1=A(1-x-y-2xy), \text{ where } A=\frac{2B}{\sqrt{3}}$$

Hence, the given result is proved. **[1½]**

Q. 65. Find the equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ whose differential equation is $\sin x\cos y dx+\cos x\sin y dy=0$.

[NCERT Misc. Ex. Q. 8, Page 420]

Ans. The differential equation of the given curve is :

$$\sin x\cos y dx+\cos x\sin y dy=0$$

$$\Rightarrow \frac{\sin x\cos y dx+\cos x\sin y dy}{\cos x\cos y}=0$$

$$\Rightarrow \quad \tan x dx+\tan y dy=0$$

Integrating both sides, we get :

$$\log(\sec x)+\log(\sec y)=\log C$$

$$\log(\sec x\cdot\sec y)=\log C$$

$$\Rightarrow \sec x\cdot\sec y=C \qquad ...(1)$$

The curve passes through point $\left(0,\frac{\pi}{4}\right)$.

$$\therefore 1\times\sqrt{2}=C$$

$$\Rightarrow \quad C=\sqrt{2}$$

On substituting $C=\sqrt{2}$ in equation (1), we get :

$$\sec x\cdot\sec y=\sqrt{2}$$

$$\Rightarrow \sec x\cdot\frac{1}{\cos y}=\sqrt{2}$$

$$\Rightarrow \cos y=\frac{\sec x}{\sqrt{2}}$$

Hence, the required equation of the curve is $\cos y=\frac{\sec x}{\sqrt{2}}$. **[3]**

Q. 66. Find the particular solution of the differential equation $(1+e^{2x})dy+(1+y^2)e^x dx=0$, given that $y = 1$ when $x = 0$.[NCERT Misc. Ex. Q. 9, Page 420]

Ans. $(1+e^{2x})dy+(1+y^2)e^x dx=0$

$$\Rightarrow \quad \frac{dy}{1+y^2}+\frac{e^x dx}{1+e^{2x}}=0$$

Integrating both sides, we get :

$$\tan^{-1}y+\int\frac{e^x dx}{1+e^{2x}}=C \qquad ...(1)$$

Let $e^x=t\Rightarrow e^{2x}=t^2$.

$$\Rightarrow \frac{d}{dx}(e^x)=\frac{dt}{dx}$$

$$\Rightarrow \quad e^x=\frac{dt}{dx}$$

$$\Rightarrow \quad e^x dx=dt$$

Substituting these values in equation (1), we get :

$$\tan^{-1}y+\int\frac{dt}{1+t^2}=C$$

$$\Rightarrow \quad \tan^{-1}y+\tan^{-1}t=C$$

$$\Rightarrow \tan^{-1}y+\tan^{-1}(e^x)=C \qquad ...(2)$$

Now, $y = 1$ at $x = 0$.

Therefore, equation (2) becomes : **[1½]**

$$\tan^{-1}1+\tan^{-1}1=C$$

$$\Rightarrow \quad \frac{\pi}{4}+\frac{\pi}{4}=C$$

$$\Rightarrow \quad C=\frac{\pi}{2}$$

Substituting $C=\frac{\pi}{2}$ in equation (2), we get :

$$\tan^{-1}y+\tan^{-1}(e^x)=\frac{\pi}{2}$$

This is the required particular solution of the given differential equation. **[1½]**

Q. 67. Solve the differential equation

$$ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy\,(y\neq 0)$$

[NCERT Misc. Ex. Q. 10, Page 420]

Ans.

$$ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy$$

$$\Rightarrow \quad ye^{\frac{x}{y}}\frac{dx}{dy}=xe^{\frac{x}{y}}+y^2$$

$$\Rightarrow e^{\frac{x}{y}}\left[y\cdot\frac{dx}{dy}-x\right]=y^2$$

$$\Rightarrow e^{\frac{x}{y}}\left[\frac{y\cdot\frac{dx}{dy}-x}{y^2}\right]=1 \qquad ...(1)$$

Let $e^{\frac{x}{y}}=z$.

Differentiating it with respect to y, we get :

$$\frac{d}{dy}\left(e^{\frac{x}{y}}\right)=\frac{dz}{dy}$$

$$\Rightarrow \quad e^{\frac{x}{y}}\cdot\frac{d}{dy}\left(\frac{x}{y}\right)=\frac{dz}{dy}$$

$$\Rightarrow e^{\frac{x}{y}}\cdot\left[\frac{y\cdot\frac{dx}{dy}-x}{y^2}\right]=\frac{dz}{dy} \quad ...(2)$$

From equations (1) and (2), we get :

$$\frac{dz}{dy}$$

$$\Rightarrow dz=dy$$

Integrating both sides, we get :

$$z=y+C$$

$$\Rightarrow e^{\frac{x}{y}}=y+C$$

[3]

Q. 68. Find the particular solution of the differential equation $2ye^{x/y}dx+\left(y-2xe^{x/y}\right)dy=0$, given that $x = 0$ when $y = 1$.

[CBSE Board, Delhi Region/Foreign scheme, 2017]

Ans. Given differential equation can be written as

$$\frac{dx}{dy}=\frac{x}{y}-\frac{1}{2e^{x/y}}$$

Put $x=vy\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$

$$\therefore v+y\frac{dv}{dy}=v-\frac{1}{2e^v}$$

$$\Rightarrow \quad \int\frac{dy}{y}=-2\int e^v dv$$

$$\Rightarrow \quad \log|y|=-2e^v+C=-2e^{x/y}+C$$

when $x=0, y=1\Rightarrow C=2$

$$\therefore \log|y|=2\left(1-e^{x/y}\right)$$

[4]

Q. 69. Solve the differential equation

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1(x\neq 0)$$

[NCERT Misc. Ex. Q. 12, Page 421]

Ans.

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}$$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{\sqrt{x}}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

This equation is a liner differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{\sqrt{x}}$ and $Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

Now, $\text{IF}=e^{\int pdx}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

The general solution of the given differential equation is given by,

$$y(\text{IF})=\int(Q\times\text{IF})dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=\int\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}\times e^{2\sqrt{x}}\right)dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=\int\frac{1}{\sqrt{x}}dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=2\sqrt{x}+C$$

[3]

Q. 70. Find a particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\text{cosec } x$ $(x\neq 0)$, given that $y=0$ when $x=\frac{\pi}{2}$

[NCERT Misc. Ex. Q. 13, Page 421]

Ans. The given differential equation is :

$$\frac{dy}{dx}+y\cot x=4x\text{cosec } x$$

This equation is a liner differential equation of the form

$\frac{dy}{dx}+py=Q$, when $p=\cot x$ and $Q=4x\text{cosec } x$.

Now, $IF=e^{\int pdx}=e^{\int\cot xdx}=e^{\log|\sin x|}=\sin x$

The general solution of the given differential equation is given by,

$$y(\text{IF})=\int(Q\times\text{IF})dx+C$$

$$\Rightarrow y\sin x=\int(4x\text{cosec } x\cdot\sin x)dx+C$$

$$\Rightarrow y\sin x=4\int xdx+C$$

$$\Rightarrow y\sin x=4\cdot\frac{x^2}{2}+C$$

$$\Rightarrow y\sin x=2x^2+C$$

...(1)

[1½]

Now, $y=0$ at $x=\frac{\pi}{2}$.

Therefore, equation (1) becomes :

$$0=2\times\frac{\pi^2}{4}+C$$

$$\Rightarrow C=-\frac{\pi^2}{2}$$

Substituting $C=-\frac{\pi^2}{2}$ in equation (1), we get :

$$y\sin x=2x^2-\frac{\pi^2}{2}$$

This is the required particular solution of the given differential equation. **[1½]**

Q. 71. Find a particular solution of the differential equation $(x+1)dy/dx=2e^{-y}-1$, given that $y=0$ when $x=0$.

[NCERT Misc. Ex. Q. 14, Page 421]

Ans.

$$(x+1)\frac{dy}{dx}=2e^{-y}-1$$

$$\Rightarrow \frac{dy}{2e^{-y}-1}=\frac{dx}{x+1}$$

$$\Rightarrow \frac{e^y dy}{2-e^y}=\frac{dx}{x+1}$$

Integrating both sides, we get :

$$\int \frac{e^y dy}{2-e^y} = \log|x+1| + \log C \quad ...(1)$$

Let $2 - e^y = t$.

$$\therefore \frac{d}{dy}(2-e^y) = \frac{dt}{dy}$$

$$\Rightarrow \quad -e^y = \frac{dt}{dy}$$

$$\Rightarrow \quad e^y dt = -dt$$

Substituting this value in equation (1), we get : [1½]

$$\int \frac{-dt}{t} = \log|x+1| + \log C$$

$$\Rightarrow \quad -\log|t| = \log|C(x+1)|$$

$$\Rightarrow -\log|2-e^y| = \log|C(x+1)|$$

$$\Rightarrow \quad \frac{1}{2-e^y} = C(x+1)$$

$$\Rightarrow 2 - e^y = \frac{1}{C(x+1)} \quad ...(2)$$

Now, at $x = 0$ and $y = 0$, equation (2) becomes :

$$\Rightarrow 2 - 1 = \frac{1}{C}$$

$$\Rightarrow \quad C = 1$$

Substituting C = 1 in equation (2), we get :

$$2 - e^y = \frac{1}{x+1}$$

$$\Rightarrow e^y = 2 - \frac{1}{x+1}$$

$$\Rightarrow e^y = \frac{2x+2-1}{x+1}$$

$$\Rightarrow e^y = \frac{2x+1}{x+1}$$

$$\Rightarrow y = \log\left|\frac{2x+1}{x+1}\right|, (x \neq -1)$$

This is the required particular solution of the given differential equation. **[1½]**

Q. 72. Solve $x\frac{dy}{dx} = y(\log y - \log x + 1)$

[NCERT Exemp. Ex. 9.3, Q. 33, Page 195]

Ans. Given, $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$$\Rightarrow \quad x\frac{dy}{dx} = y\log\left(\frac{y}{x}+1\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x}+1\right) \quad ...(i)$$

Which is a homogeneous equation.

Put $\frac{y}{x} = v$ or $y = vx$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \textbf{[1½]}$$

On substituting these values in equation (i), we get

$$v + x\frac{dv}{dx} = v(\log v + 1)$$

$$\Rightarrow \quad x\frac{dv}{dx} = v(\log v + 1 - 1)$$

$$\Rightarrow \quad x\frac{dv}{dx} = v(\log v)$$

$$\Rightarrow \quad \frac{dv}{v\log v} = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

On putting $\log v = u$ in LHS integral, we get

$$\frac{1}{v}\cdot dv = du$$

$$\int \frac{du}{u} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log u = \log x + \log C$$

$$\Rightarrow \quad \log u = \log Cx$$

$$\Rightarrow \quad u = Cx$$

$$\Rightarrow \quad \log v = Cx$$

$$\Rightarrow \quad \log\left(\frac{y}{x}\right) = Cx$$

[1½]

Q. 73. Find the general solution of the differential equation

$$\boldsymbol{x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x.}$$

[CBSE Board, Foreign Scheme, 2017]

Ans. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = v + \frac{1}{\cos v}$$

$$\Rightarrow \quad \int \cos v \, dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \sin v = \log|x| + C$$

$$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = \log|x| + C$$

[4]

Long Answer Type Questions

(5 and 6 marks each)

Q. 1. Find the particular solution of the differential equation $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$, given that $y = 0$ when $x = 1$. **[CBSE Board, Foreign Scheme, 2017]**

Ans. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}}y}{1+y^2}$$

$$IF = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$$

Solution is given by

$$xe^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \times e^{\tan^{-1}y} dy = \int \frac{e^{2\tan^{-1}y}}{1+y^2} dy$$

$$\Rightarrow xe^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + C$$

when $x = 1, y = 0 \Rightarrow C = \frac{1}{2}$

$\therefore$ Solution is given by

$$xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}} + \frac{1}{2} \text{ or } x = \frac{1}{2}\left(e^{\tan^{-1}y} + e^{-\tan^{-1}y}\right)$$ **[6]**

Q. 2. Find the particular solution of the differential equation $(x - y)\, dy/dx = (x + 2y)$, given that $y = 0$ when $x = 1$. [CBSE Board, All India Region, 2017]

Ans.

$$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}}$$

$$\frac{y}{x} = v$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1}$$

$$\Rightarrow \int \frac{v-1}{v^2+v+1} dv = -\frac{dx}{x}$$

$$\Rightarrow \int \frac{2v+1-3}{v^2+v+1} dv = \int -\frac{2}{x} dx \Rightarrow \int \frac{2v+1}{v^2+v+1} dv - 3$$

$$\int \frac{1}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dv = -\int \frac{2}{x} dx$$

$$\Rightarrow \log\left|v^2+v+1\right| - 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + C$$

$$\Rightarrow \log\left|y^2+xy+x^2\right| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = C$$

$$x = 1, y = 0 \Rightarrow C = -2\sqrt{3}\cdot\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi$$

$$\therefore \log\left|y^2+xy+x^2\right| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$$

[6]

Q. 3. Solve $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.

[NCERT Exemp. Ex. 9.3, Q. 25, Page 194]

Ans. Given differential equation is

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$\Rightarrow y + x\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow x\frac{dy}{dx} + 2y = x(\sin x + \log x)$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$$ **[2½]**

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2}{x}, Q = \sin x + \log x$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$$

The general solution is

$$y \cdot x^2 = \int (\sin x + \log x)x^2 dx + C$$

$$\Rightarrow y \cdot x^2 = \int \left(x^2\sin x + x^2\log x\right)dx + C$$

$$\Rightarrow y \cdot x^2 = \int x^2 \sin x dx + \int x^2 \log x dx + C$$

$$\Rightarrow y \cdot x^2 = I_1 + I_2 + C \quad \text{...(i)}$$

Now, $I_1 = \int x^2 \sin x\, dx$

$$= x^2(-\cos x) + \int 2x\cos x dx$$

$$= -x^2\cos x + \left[2x(\sin x) - \int 2\sin x dx\right]$$

$$I_1 = -x^2\cos x + 2x\sin x + 2\cos x \quad \text{...(ii)}$$

and $I_2 = \int x^2 \log x dx$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{x^3}{3} dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2 dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^3}{3} \quad \text{...(iii)}$$

On substituting the value of I_1 and I_2 in equation (i), we get

$$y \cdot x^2 = -x^2\cos x + 2x\sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3 + C$$

$$\therefore y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x}{3}\log x - \frac{x}{9} + Cx^{-2}$$ **[2½]**

Q. 4. Find the general solution of $(1 + \tan y)(dx - dy) + 2x dy = 0$.

[NCERT Exemp. Ex. 9.3, Q. 26, Page 194]

Ans. Given differential equation is

$$(1 + \tan y)(dx - dy) + 2x\, dy = 0$$

On dividing throughout by dy, we get

$$(1+\tan y)\left(\frac{dx}{dy} - 1\right) + 2x = 0$$

$$\Rightarrow (1+\tan y)\frac{dx}{dy} - (1+\tan y) + 2x = 0$$

$$\Rightarrow (1+\tan y)\frac{dx}{dy} + 2x = (1+\tan y)$$

$$\Rightarrow \frac{dx}{dy} + \frac{2x}{1+\tan y} = 1$$

Which is a linear differential equation [2½]

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{2}{1+\tan y}, Q = 1$$

$$IF = e^{\int \frac{2}{1+\tan y}dy} = e^{\int \frac{2\cos y}{\cos y + \sin y}dy}$$

$$= e^{\int \frac{\cos y + \sin y + \cos y - \sin y}{\cos y + \sin y} dy}$$

$$= e^{\int \left(1 + \frac{\cos y - \sin y}{\cos y + \sin y}\right) dy} = e^{y + \log(\cos y + \sin y)}$$

$$= e^y.(\cos y + \sin y) \qquad \left[\because e^{\log x} = x\right]$$

The general solution is

$$x.e^y(\cos y + \sin y) = \int 1.e^y(\cos y + \sin y)dy + C$$

$$\Rightarrow \quad x.e^y(\cos y + \sin y) = \int e^y(\sin y + \cos y)dy + C$$

$$\Rightarrow \quad x.e^y(\cos y + \sin y) = e^y \sin y + C$$

$$\left[\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x)\right]$$

$$\Rightarrow \quad x(\sin y + \cos y) = \sin y + Ce^{-y}$$

[2½]

Q. 5. Solve $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$

[NCERT Exemp. Ex. 9.3, Q. 27, Page 194]

Ans. Given, $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$. **...(i)**

$x + y = z$

Put $\Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

On substituting these values in Eq. (i), we get

$$\left(\frac{dz}{dx} - 1\right) = \cos z + \sin z$$

$$\Rightarrow \quad \frac{dz}{dx} = (\cos z + \sin z + 1)$$

$$\Rightarrow \frac{dz}{\cos z + \sin z + 1} = dx$$

[2½]

On integrating both sides, we get

$$\int \frac{dz}{\cos z + \sin z + 1} = \int 1 dx$$

$$\Rightarrow \quad \int \frac{dz}{\frac{1 - \tan^2 z/2}{1 + \tan^2 z/2} + \frac{2\tan z/2}{1 + \tan^2 z/2} + 1} = \int dx$$

$$\Rightarrow \quad \int \frac{dz}{\frac{1 - \tan^2 z/2 + 2\tan z/2 + 1 + \tan^2 z/2}{(1 + \tan^2 z/2)}} = \int dx$$

$$\Rightarrow \quad \int \frac{(1 + \tan^2 z/2)dz}{2 + 2\tan\frac{z}{2}} = \int dx$$

$$\Rightarrow \quad \int \frac{\sec^2 z/2 dz}{2(1 + \tan z/2)} = \int dx$$

$Put 1 + \tan z/2 = t \Rightarrow \quad \left(\frac{1}{2}\sec^2 z/2\right)dz = dt$

$$\int \frac{dt}{t} = \int dx$$

$$\Rightarrow \quad \log|t| = x + c$$

$$\Rightarrow \quad \log|1 + \tan z/2| = x + c$$

$$\Rightarrow \quad \log\left|1 + \tan\frac{(x+y)}{2}\right| = x + c$$

[2½]

Q. 6. Find the particular solution of the differential equation $dx/dy - 3y \cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$.[CBSE Board, Foreign Scheme, 2017]

Ans. Here, IF= $e^{\int -3\cot x\, dx} = \frac{1}{\sin^3 x}$

Solution is given by,

$$y\left(\frac{1}{\sin^3 x}\right) = \int \frac{\sin 2x}{\sin^3 x}dx = 2\int \frac{\cos x}{\sin^2 x}dx$$

$$\Rightarrow \quad \frac{y}{\sin^3 x} = \frac{-2}{\sin x} + C$$

When $x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4$

$$\therefore \frac{y}{\sin^3 x} = \frac{-2}{\sin x} + 4 \text{ or } y = -2\sin^2 x + 4\sin^3 x$$

[6]

Q. 7. Find the particular solution of the differential equation tan x dy/dx= $2x$ tan x + $x^2 - y$; ($\tan x \neq 0$) given that $y = 0$ when $x = \Pi/2$.

[CBSE Board, Delhi Region, 2017]

Ans. Given equation can be written as

$$\Rightarrow \frac{dy}{dx} + (\cot x)y = 2x + x^2 \cot x$$

$$IF = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$$

Solution is, $y\sin x = \int (2x\sin x + x^2 \cos x)dx$

$$\Rightarrow \quad y\sin x = x^2 \sin x + C$$

when we get $C = \frac{-\pi^2}{4}$

$\therefore$ Required solution is, $4y \sin x = 4x^2 \sin x - \pi^2$

or, $y = x^2 - \pi^{2/4} \operatorname{cosec} x$ **[6]**

Q. 8. Find the particular solution of the differential equation
$dy = \cos x\,(2 - y \operatorname{cosec} x)\,dx$, given that $y = 2$ when $x = \frac{\pi}{2}$. [CBSE Board, Foreign Scheme, 2017]

Ans. Given differential equation can be written as

$$\frac{dy}{dx} + y \cot x = 2 \cos x$$

$$IF = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$$

Solution is given by

$$y \sin x = \int 2\sin x \cos x\, dx = \int \sin 2x\, dx$$

$$= \frac{-\cos 2x}{2} + C$$

When

$$x = \frac{\pi}{2}, y = 2$$

$$\Rightarrow C = \frac{3}{2}$$

Solution is given by

$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$ or $y = \operatorname{cosec} x + \sin x$ **[6]**

Q. 9. Find the general solution of $\frac{dy}{dx} - 3y = \sin 2x$.

[NCERT Exemp. Ex. 9.3, Q. 28, Page 194]

Ans. Given, $\frac{dy}{dx} - 3y = \sin 2x$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$P=-3, Q=\sin 2x$

$IF=e^{-3\int dx}=e^{-3x}$

The general solution is

$y.e^{-3x}=\int \underset{I}{\sin 2x}\ \underset{II}{e^{-3x}}\,dx$

Let $y.e^{-3x}=I$

$\therefore \quad I=\int \underset{II}{e^{-3x}}\ \underset{I}{\sin 2x}$

$\Rightarrow \quad I=\sin 2x\left(\frac{e^{-3x}}{-3}\right)-\int 2\cos 2x\left(\frac{e^{-3x}}{-3}\right)dx+c_1$

$\Rightarrow \quad I=-\frac{1}{3}e^{-3x}\sin 2x+\frac{2}{3}\int \underset{II}{e^{-3x}}\ \underset{I}{\cos 2x}\,dx+c_1$

$\Rightarrow \quad I=-\frac{1}{3}e^{-3x}\sin 2x+\frac{2}{3}$

$\left(\cos 2x\frac{e^{-3x}}{-3}-\int(-2\sin 2x)\frac{e^{-3x}}{-3x}dx\right)+c_1+c_2$

$\Rightarrow \quad I=-\frac{1}{3}e^{-3x}\sin 2x-\frac{2}{9}\cos 2xe^{-3x}-\frac{4}{9}I+c'$

[Where, $c'=c_1+c_2$]

$\Rightarrow I+\frac{4l}{9}2=+e^{-3x}\left(-\frac{1}{3}\sin 2x-\frac{2}{9}\cos 2x\right)+c'$

[2½]

$\Rightarrow \quad \frac{13}{9}I=e^{-3x}\left(-\frac{1}{3}\sin 2x-\frac{2}{9}\cos 2x\right)+C'$

$\Rightarrow \quad I=\frac{9}{13}e^{-3x}\left(-\frac{1}{3}\sin 2x-\frac{2}{9}\cos 2x\right)+C$

$\left[\text{Where } C=\frac{9c'}{13}\right]$

$\Rightarrow \quad I=\frac{3}{13}e^{-3x}\left(-\sin 2x-\frac{2}{3}\cos 2x\right)+C$

$\Rightarrow \quad =\frac{3}{13}e^{-3x}\frac{(-3\sin 2x-\cos 2x)}{3}+C$

$\Rightarrow \quad =\frac{e^{-3x}}{13}(-3\sin 2x-2\cos 2x)+C$

$\Rightarrow \quad I=-\frac{e^{-3x}}{13}(2\cos 2x+3\sin 2x)+C$

On substituting the value of I in Eq. (i), we get

$y.e^{-3x}=-\frac{e^{-3x}}{13}(2\cos 2x+3\sin 2x)+C$

$\Rightarrow \quad y=-\frac{1}{13}(2\cos 2x+3\sin 2x)+Ce^{3x}$ **[2½]**

Q. 10. Find the equation of a curve passing through (2, 1), if the slope of the tangent to the curve at any point

(x,y) is $\frac{x^2+y^2}{2xy}$.

[NCERT Exemp. Ex. 9.3, Q. 29, Page 194]

Ans. It is given that, the slope of tangent to the curve at point (x, y) is $\frac{x^2+y^2}{2xy}$ i.e.

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$$

Which is homogeneous differential equation.

Put $\quad y=vx$

$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

On substituting these values in Eq. (i), we get

$v+x\frac{dv}{dx}=\frac{1}{2}\left(\frac{1}{v}+v\right)$

$v+x\frac{dv}{dx}=\frac{1}{2}\left(\frac{1+v^2}{v}\right)$

$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$

$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2-2v^2}{2v}$

$\Rightarrow \quad x\frac{dv}{dx}=\frac{1-v^2}{2v}$

$\Rightarrow \frac{2v}{1-v^2}dv=\frac{dx}{x}$ **[2½]**

On integrating both sides, we get

$\int\frac{2v}{1-v^2}dv=\int\frac{dx}{x}$

Put $1-v^2=t$ in LHS, we get

$-2vdv=dt$

$\Rightarrow \quad -\int\frac{dt}{t}=\int\frac{dx}{x}$

$\Rightarrow \quad -\log t=\log x+\log C$

$\Rightarrow \quad -\log(1-v^2)=\log x+\log C$

$\Rightarrow \quad -\log\left(1-\frac{y^2}{x^2}\right)=\log x+\log C$

$\Rightarrow \log\left(\frac{x^2-y^2}{x^2}\right)=\log x+\log C$

$\Rightarrow \quad \log\left(\frac{x^2}{x^2-y^2}\right)=\log x+\log C$

$\Rightarrow \quad \frac{x^2}{x^2-y^2}=Cx \quad$ ...(ii)

Since, the curve passes through the point (2, 1).

$\therefore \quad \frac{(2)^2}{(2)^2-(1)^2}=C(2)$

$\Rightarrow C=\frac{2}{3}$

So, the required solution is $2(x^2-y^2)=3x$. **[2½]**

Q. 11. Find the equation of the curve through the point (1, 0), if the slope of the tangent to the curve at any point (x, y) is

$\frac{y-1}{x^2+x}$ [NCERT Exemp. Ex. 9.3, Q. 30, Page 195]

Ans. It is given that, slope of tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$

$$\therefore \left(\frac{dy}{dx}\right)_{(x,y)} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

On integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\Rightarrow \int \frac{dy}{y-1} = \int \frac{dx}{x(x+1)}$$

$$\Rightarrow \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \log(y-1) = \log x - \log(x+1) + \log C$$

$$\Rightarrow \log(y-1) = \log\left(\frac{xC}{x+1}\right)$$

[2½]

Since, the given curve passes through point (1, 0).

$$\therefore \quad 0 - 1 = \frac{1 \cdot C}{1+1}$$

$$\Rightarrow C = -2$$

The particular solution is $y - 1 = \frac{-2x}{x+1}$

$$\Rightarrow (y-1)(x+1) = -2x$$

$$\Rightarrow (y-1)(x+1) + 2x = 0$$

[2½]

Q. 12. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

[NCERT Exemp. Ex. 9.3, Q. 31, Page 195]

Ans. Slope of tangent to the curve $= \frac{dy}{dx}$ and difference of abscissa and ordinate $= x - y$

According to the question, $\frac{dy}{dx} = (x-y)^2$ ...(i)

Put $x - y = z$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 1 - \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$1 - \frac{dz}{dx} = z^2$$

$$\Rightarrow 1 - z^2 = \frac{dz}{dx}$$

$$\Rightarrow dx = \frac{dz}{1-z^2}$$

On integrating both sides, we get

$$\int dx = \int \frac{dz}{1-z^2}$$

$$\Rightarrow x = \frac{1}{2}\log\left|\frac{1+z}{1-z}\right| + C$$

$$\Rightarrow x = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| + C \quad \text{...(ii)}$$

Since, the curve passes through the origin.

$$\therefore \quad 0 = \frac{1}{2}\log\left|\frac{1+0-0}{1-0+0}\right| + C$$

$$\Rightarrow C = 0$$

[2½]

On substituting the value of C in EQ. (ii), we get

$$x = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right|$$

$$\Rightarrow 2x = \log\left|\frac{1+x-y}{1-x+y}\right|$$

$$\Rightarrow e^{2x} = \left|\frac{1+x-y}{1-x+y}\right|$$

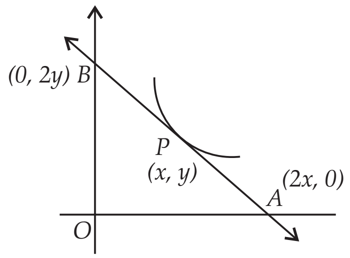
$$\Rightarrow (1-x+y)e^{2x} = 1+x-y$$

[2½]

Q. 13. Find the equation of a curve passing through the point (1, 1), if the tangent drawn at any point $P(x, y)$ on the curve meets the coordinate axes at A and B such that P is the mid-point of AB.

[NCERT Exemp. Ex. 9.3, Q. 32, Page 195]

Ans. The below figure obtained by the given information



Let the coordinate of the point P is (x,y). It is given that, P is mid-point of AB

So, the coordinates of points A and B are $(2x, 0)$ and $(0, 2y)$, respectively.

$$\therefore \text{Slope of } AB = \frac{0-2y}{2x-0} = -\frac{y}{x}$$

Since, the segment AB is a tangent to the curve at P.

$$\therefore \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

On integrating both sides, we get

$$\log y = -\log x + \log C$$

$$\log y = \log\frac{C}{x}$$

Since, the given curve passes through (1, 1).

$$\therefore \quad \log 1 = \log\frac{C}{1}$$

$$\Rightarrow \quad 0 = \log C$$

$$\Rightarrow \quad c = 1$$

$$\therefore \log y = \log\frac{1}{x}$$

$$\Rightarrow \quad y = \frac{1}{x}$$

$$\Rightarrow \quad xy = 1$$

[5]

Q. 14. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25,000 in the year 2004, what will be the population of the village in 2009? [NCERT Misc. Ex. Q. 15, Page 421]

Ans. Let the population at any instant (t) be y.

It is given that the rate of increase of population is proportional to the number of inhabitants at any instant.

$$\therefore \frac{dy}{dt} \alpha\, y$$

$$\Rightarrow \frac{dy}{dt} = ky \quad (k \text{ is a constant})$$

$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get :

$\log y = kt + C$ ... (1)

In the year 1999, $t=0$ and $y=20{,}000$

Therefore, we get :

$\log 20{,}000 = C$ ...(2)

In the year 2004, $t = 5$ and $y = 25{,}000$.

Therefore, we get :

$$\log 25{,}000 = k.5 + C \Rightarrow 5k = \log\left(\frac{25{,}000}{20{,}000}\right) = \log\left(\frac{5}{4}\right)$$ **[2½]**

$$\Rightarrow k = \frac{1}{5}\log\left(\frac{5}{4}\right)$$ **...(3)**

In the year 2009, $t = 10$ years.

Now, on substituting the values of t, k, and C in equation (1), we get :

$$\Rightarrow \log y = \log\left[20{,}000 \times \left(\frac{5}{4}\right)^2\right]$$

$$\Rightarrow \quad y = 20{,}000 \times \frac{5}{4} \times \frac{5}{4}$$

$$\Rightarrow \quad y = 31{,}250$$

Hence, the population of the village in 2009 will be 31,250. **[2½]**

Q. 15. Find a particular solution of the differential equation $(x - y)(dx + dy) = dx - dy$, given that $y = -1$, when $x = 0$. (Hint : put $x - y = t$) [NCERT Misc. Ex. Q. 11, Page 420]

Ans.

$$(x-y)(dx+dy) = dx - dy$$

$$\Rightarrow (x-y+1)dy = (1-x+y)dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-(x-y)}{1+(x-y)} \quad ...(1)$$

Let $x - y = t$.

$$\Rightarrow \quad \frac{d}{dx}(x-y) = \frac{dt}{dx}$$

$$\Rightarrow \quad 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad 1 - \frac{dt}{dx} = \frac{dy}{dx}$$

Substituting the values of $x - y$ and $\frac{dy}{dx}$ in equation (1), we get :

$$1 - \frac{dt}{dx} = \frac{1-t}{1+t}$$

$$\Rightarrow \quad \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t}\right)$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{2t}{1+t}$$

$$\Rightarrow \left(\frac{1+t}{t}\right)dt = 2dx$$

$$\Rightarrow \left(1+\frac{1}{t}\right)dt = 2dx \quad ...(2)$$

[2½]

Integrating both sides, we get :

$$t + \log|t| = 2x + C$$

$$\Rightarrow (x-y) + \log|x-y| = 2x + C$$

$$\Rightarrow \quad \log|x-y| = x + y + C$$

Now, $y = -1$ at $x = 0$. **...(3)**

Therefore, equation (3) becomes :

$$\log 1 = 0 - 1 + C$$

$$\Rightarrow \ C = 1$$

Substituting C = 1 in equation (3) we get :

$$\log|x-y| = x + y + 1$$

This is the required particular solution of the given differential equation. **[2½]**

Q. 16. Find the general solution :

$$\mathbf{\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0}$$

[NCERT Ex. 9.4, Q. 4, Page 396]

Ans. The given differential equation is :

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$$

$$\Rightarrow \frac{\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy}{\tan x \tan y} = 0$$

$$\Rightarrow \quad \frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

$$\Rightarrow \quad \frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$$

Integrating both sides of this equation, we get :

$$\int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$$

Let $\tan x = t$.

$$\therefore \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \quad \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \ \sec^2 x dx = dt$$

Now, $\int \frac{\sec^2 x}{\tan x}dx = \int \frac{1}{t}dt.$

$$= \log t$$

$$= \log(\tan x)$$

Similarly, $\int \frac{\sec^2 y}{\tan y} dy = \log(\tan y).$ **[2½]**

Substituting these values in equation (1), we get :

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\Rightarrow \log(\tan x) = \log\left(\frac{C}{\tan y}\right)$$

$$\Rightarrow \tan x = \frac{C}{\tan y}$$

$$\Rightarrow \tan x \tan y = C$$ **[2½]**

This is the required general solution of the given differential equation.

Q. 17. Find the general solution :
$\boldsymbol{(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0}$.

[NCERT Ex. 9.4, Q. 5, Page 396]

Ans. The given differential equation is :

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$$

$$\Rightarrow (e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

$$\Rightarrow dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx$$

Integrating both sides of this equation, we get :

$$\int dy = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx + C$$

$$\Rightarrow y = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx + C$$

Let $(e^x + e^{-x}) = t.$ **[2½]**

Differentiating both sides with respect to x, we get :

$$\frac{d}{dx}(e^x + e^{-x}) = \frac{dt}{dx}$$

$$\Rightarrow e^x - e^{-x} = \frac{dt}{dt}$$

$$\Rightarrow (e^x - e^{-x}) dx = dt$$

Substituting this value in equation (1), we get :

$$y = \int \frac{1}{t} dt + C$$

$$\Rightarrow y = \log(t) + C$$

$$\Rightarrow y = \log(e^x + e^{-x}) + C$$ **[2½]**

This is the required general solution of the given differential equation.

Q. 18. Find the general solution :

$\boldsymbol{\frac{dy}{dx} = \sin^{-1} x}$ **[NCERT Ex. 9.4, Q. 9, Page 396]**

Ans. The given differential equation

$$\frac{dy}{dx} = \sin^{-1} x$$

$$\Rightarrow dy = \sin^{-1} x\, dx$$

Integrating both sides, we get :

$$\int dy = \int \sin^{-1} x\, dx$$

$$\Rightarrow y = \int (\sin^{-1} x.1) dx$$

$$\Rightarrow y = \sin^{-1} x. \int (1) dx - \int \left[\left(\frac{d}{dx}(\sin^{-1} x). \int (1) dx\right)\right] dx$$

$$\Rightarrow y = \sin^{-1} x.x - \int \left(\frac{1}{\sqrt{1-x^2}}.x\right) dx$$

$$\Rightarrow y = x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \quad ...(1)$$

Let

$$1 - x^2 = t$$

$$\Rightarrow \frac{d}{dx}(1 - x^2) = \frac{dt}{dx}$$

$$\Rightarrow -2x = \frac{dt}{dx}$$

$$\Rightarrow x dx = -\frac{1}{2} dt$$ **[2½]**

Substituting this value in equation (1), we get :

$$y = x \sin^{-1} x + \int \frac{1}{2\sqrt{t}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2}. \int (t)^{-\frac{1}{2}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2}. \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{t} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$ **[2½]**

This is the required general solution of the given differential equation.

Q. 19. Find the general solution :

$\boldsymbol{e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0}$

[NCERT Ex. 9.4, Q. 10, Page 396]

Ans. The given differential equation is :

$$e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$$

$$(1 - e^x) \sec^2 y\, dy = -e^x \tan y\, dx$$

Separating the variables, we get :

$$\frac{\sec^2 y}{\tan y} dy = \frac{-e^x}{1 - e^x} dx$$

Integrating both sides, we get :

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{-e^x}{1 - e^x} dx \quad ...(1)$$

Let

$$\tan y = u.$$

$$\Rightarrow \frac{d}{dy}(\tan y) = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y dy = du$$

$$\therefore \int \frac{\sec^2 y}{\tan y} dy = \int \frac{du}{u} = \log u = \log(\tan y)$$ **[2½]**

Now, Let

$$1-e^x=t.$$

$$\therefore \frac{d}{dx}(1-e^x)=\frac{dt}{dx}$$

$$\Rightarrow \quad -e^x=\frac{dt}{dx}$$

$$\Rightarrow \quad -e^x dx=dt$$

$$\Rightarrow \int\frac{-e^x}{1-e^x}dx=\int\frac{dt}{t}=\log t=\log(1-e^x)$$

Substituting the values of $\int\frac{\sec^2 y}{\tan y}dy$ and $\int\frac{-e^x}{1-e^e}dx$ in equation (1), we get :

$$\Rightarrow \log(\tan\ y)=\log(1-e^x)+\log C$$

$$\Rightarrow \ \log(\tan y)=\log\left[C(1-e^x)\right]$$

$$\Rightarrow \quad \tan y=C(1-e^x)$$

[2½]

This is the required general solution of the given differential equation.

Q. 20. Find a particular solution satisfying the given condition :

$$\boldsymbol{(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x;\ y=1\text{ when } x=0}$$

[NCERT Ex. 9.4, Q. 11, Page 396]

Ans. The given differential equation is :

$$(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2x^2+x}{(x^3+x^2+x+1)}$$

$$\Rightarrow \quad dy=\frac{2x^2+x}{(x+1)(x^2+1)}dx$$

Integrating both sides, we get :

$$\int dy=\int\frac{2x^2+x}{(x+1)(x^2+1)}dx \quad \ldots(1)$$

$$\text{Let } \frac{2x^2+x}{(x+1)(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}. \quad \ldots(2)$$

$$\Rightarrow \quad \frac{2x^2+x}{(x+1)(x^2+1)}=\frac{Ax^2+A+(Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$\Rightarrow \quad 2x^2+x=Ax^2+A+Bx^2+Bx+Cx+C$$

$$\Rightarrow \quad 2x^2+x=(A+B)x^2+(B+C)x+(A+C)$$

[2½]

Comparing the coefficients of x^2 and x, we get :

$$A+B=2$$

$$B+C=1$$

$$A+C=0$$

Solving these equations, we get :

$$A=\frac{1}{2}, B=\frac{3}{2}, \text{ and } C=\frac{-1}{2}$$

Substituting the values of A, B, and C in equation (2), we get :

$$\frac{2x^2+x}{(x+1)(x^2+1)}=\frac{1}{2}.\frac{1}{(x+1)}+\frac{1}{2}\frac{(3x-1)}{(x^2+1)}$$

Therefore, equation (1) becomes :

$$\int dy=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{3x-1}{x^2+1}dx$$

$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}.\int\frac{2x}{x^2+1}dx-\frac{1}{2}\tan^{-1}x+C$$

$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+C$$

$$\Rightarrow y=\frac{1}{4}\left[2\log(x+1)+3\log(x^2+1)\right]-\frac{1}{2}\tan^{-1}x+C$$

$$\Rightarrow y=\frac{1}{4}\left[(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+C \quad \ldots(3)$$

Now, $y=1$ when $x=0$.

$$\Rightarrow I=\frac{1}{4}\log(1)-\frac{1}{2}\tan^{-1}0+C$$

$$\Rightarrow I=\frac{1}{4}\times\ 0-\frac{1}{2}\times 0+C$$

$$\Rightarrow C=1$$

[2½]

Substituting C = 1 in equation (3), we get :

$$y=\frac{1}{4}\left[\log(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Q. 21. Find a particular solution satisfying the given condition :

$$\boldsymbol{x(x^2-1)\frac{dy}{dx}=1;\ y=0\text{ when } x=2}$$

[NCERT Ex. 9.4, Q. 12, Page 396]

Ans.

$$x(x^2-1)\frac{dy}{dx}=1$$

$$\Rightarrow \quad dy=\frac{dx}{x(x^2-1)}$$

$$\Rightarrow \quad dy=\frac{1}{x(x-1)(x+1)}dx$$

Integrating both sides, we get :

$$\int dy=\int\frac{1}{x(x-1)(x+1)}dx \quad \ldots(1)$$

$$\text{Let } \frac{1}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}. \quad \ldots(2)$$

$$\Rightarrow \quad \frac{1}{x(x-1)(x+1)}=\frac{A(x-1)(x+1)+Bx(x+1)+Cx(x-1)}{x(x-1)(x+1)}$$

$$=\frac{(A+B+C)x^2+(B-C)x-A}{x(x-1)(x+1)}$$

Comparing the coefficients of x^2, x, and constant, we get :

$$A=-1$$

$$B-C=0$$

$$A+B+C=0$$

[2½]

Solving these equations, we get $B=\frac{1}{2}$ and $C=\frac{1}{2}$

Substituting the values of A, B, and C in equation (2), we get :

$$\frac{1}{x(x-1)(x+1)}=\frac{-1}{x}+\frac{1}{2(x-1)}+\frac{1}{2(x+1)}$$

Therefore, equation (1) becomes :

$$\int dy = -\int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{x+1}dx$$

$\Rightarrow \quad y = -\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \log k$

$\Rightarrow \quad y = \frac{1}{2}\log\left[\frac{k^2(x-1)(x+1)}{x^2}\right]$

Now, $\quad y = 0$ when $x = 2$.

$\Rightarrow \quad 0 = \frac{1}{2}\log\left[\frac{k^2(2-1)(2+1)}{4}\right]$

$\Rightarrow \log\left(\frac{3k^2}{4}\right) = 0$

$\Rightarrow \quad \frac{3k^2}{4} = 1$

$\Rightarrow \quad 3k^2 = 4$

$\Rightarrow \quad k^2 = \frac{4}{3}$

Substituting the value of k^2 in equation (3), we get :

$y = \frac{1}{2}\log\left[\frac{4(x-1)(x+1)}{3x^2}\right]$

$y = \frac{1}{2}\log\left[\frac{4(x^2-1)}{3x^2}\right]$

[2½]

Q. 22. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$. [NCERT Ex. 9.4, Q. 15, Page 396]

Ans. The differential equation of the curve is :

$y' = e^x \sin x$

$\Rightarrow \frac{dy}{dx} = e^x \sin x$

$\Rightarrow dy = e^x \sin x\, dx$

Integrating both sides, we get :

$\int dy = \int e^x \sin x dx$

Let

$I = \int e^x \sin x dx.$

$\Rightarrow \ I = \sin x \int e^x dx - \int\left(\frac{d}{dx}(\sin x).\int e^x dx\right)dx$

$\Rightarrow \ I = \sin x.e^x - \int \cos x.e^x dx$

$\Rightarrow \ I = \sin x.e^x - \left[\cos x.\int e^x dx - \int\left(\frac{d}{dx}(\cos x).\int e^x dx\right)dx\right]$

$\Rightarrow \ I = \sin x.e^x - \left[\cos x.e^x - \int(-\sin x).e^x dx\right]$

$\Rightarrow \ I = e^x \sin x - e^x \cos x - I$

$\Rightarrow 2I = e^x(\sin x - \cos x)$

$\Rightarrow \ I = \frac{e^x(\sin x - \cos x)}{2}$

[2½]

Substituting this value in equation (1), we get :

$y = \frac{e^x(\sin x - \cos x)}{2} + C \qquad \ldots(2)$

Now, the curve passes through point (0, 0)

$\therefore \ 0 = \frac{e^0(\sin 0 - \cos 0)}{2} + C$

$\Rightarrow \ 0 = \frac{1(0-1)}{2} + C$

$\Rightarrow C = \frac{1}{2}$

Substituting $C = \frac{1}{2}$ in equation (2), we get :

$y = \frac{e^x(\sin x - \cos x)}{2} + \frac{1}{2}$

$\Rightarrow \quad 2y = e^x(\sin x - \cos x) + 1$

$\Rightarrow \ 2y - 1 = e^x(\sin x - \cos x)$

Hence, the required equation of the curve is $2y - 1 = e^x(\sin x - \cos x)$

[2½]

Q. 23. Find the equation of a curve passing through the point (0, –2) given that at any point (*x*, *y*) on the curve, the product of the slope of its tangent and *y* coordinate of the point is equal to the *x* coordinate of the point. [NCERT Ex. 9.4, Q. 17, Page 396]

Ans. Let *x* and *y* be the *x*-coordinate and *y*- coordinate of the curve respectively.

We know that the slope of a tangent to the curve in the coordinate axis is given by the relation, $\frac{dy}{dx}$

According to the given information, we get :

$y.\frac{dy}{dx} = x$

$\Rightarrow \quad ydy = xdx$

Integrating both sides, we get :

$\int y\, dy = \int xdx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + C$

$\Rightarrow \ y^2 - x^2 = 2C$

Now, the curve passes through point $(0, -2)$.

$\therefore \ (-2)^2 - 0^2 = 2C$

$\Rightarrow \quad 2C = 4$

Substituting 2C = 4 in equation (1), we get :

$y^2 - x^2 = 4$

This is the required equation of the curve. **[5]**

Q. 24. At any point (*x*, *y*) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the curve given that it passes through (–2, 1). [NCERT Ex. 9.4, Q. 18, Page 396]

Ans. It is given that (*x*, *y*) is the point of contact of the curve and its tangent.

The slope (m_1) of the line segment joining (*x*, *y*) and $(-4, -3)$ is $\frac{y+3}{x+4}$.

We know that the slope of the tangent to the curve is given by the relation, $\frac{dy}{dx}$

$\therefore$ Slope (m_2) of the tangent $= \frac{dy}{dx}$

According to the given information,

$m_2 = 2m_1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2(y+3)}{x+4}$

$\Rightarrow \dfrac{dy}{y+3} = \dfrac{2dx}{x+4}$ **[2½]**

Integrating both sides, we get :

$\int \dfrac{dy}{y+3} = 2\int \dfrac{dx}{x+4}$

$\Rightarrow \log(y+3) = 2\log(x+4) + \log C$

$\Rightarrow \log(y+3) \log C(x+4)^2$

$\Rightarrow y+3 = C(x+4)^2$ …(1)

This is the general equation of the curve.

It is given that it passes through point (–2, 1)

$\Rightarrow 1+3 = C(-2+4)^2$

$\Rightarrow 4 = 4C$

$\Rightarrow C = 1$

Substituting C=1 in equation (1), we get :

$y+3 = (x+4)^2$ **[2½]**

This is the required equation of the curve.

Q. 25. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after *t* seconds.

[NCERT Ex. 9.4, Q. 19, Page 396]

Ans. Let the rate of change of the volume of the balloon be k (where k is a constant).

$\Rightarrow \dfrac{dv}{dt} = k$

$\Rightarrow \dfrac{d}{dt}\left(\dfrac{4}{3}\pi r^3\right) = k \quad \left[\text{Volume of sphere} = \dfrac{4}{3}\pi r^3\right]$

$\Rightarrow \dfrac{4}{3}\pi . 3r^2 . \dfrac{dr}{dt} = k$

$\Rightarrow 4\pi r^2 dr = k dt$

Integrating both sides, we get :

$4\pi \int r^2 dr = k \int dt$

$\Rightarrow 4\pi . \dfrac{r^3}{3} = kt + C$

$\Rightarrow 4\pi r^3 = 3(kt + C)$ …(1)

Now, at $t = 0, r = 3$:

$\Rightarrow 4\pi \times 3^3 = 3(k \times 0 + C)$

$\Rightarrow 108\pi = 3C$

$\Rightarrow C = 36\pi$

At $t = 3, r = 6$:

$\Rightarrow 4\pi \times 6^3 = 3(k \times 3 + C)$

$\Rightarrow 864\pi = 3(3k + 36\pi)$

$\Rightarrow 3k = -288\pi - 36\pi = 252\pi$

$\Rightarrow k = 84\pi$ **[2½]**

Substituting the values of k and C in equation (1), we get :

$4\pi r^3 = 3[84\pi t + 36\pi]$

$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$

$\Rightarrow r^3 = 63t + 27$

$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$ **[2½]**

Thus, the radius of the balloon after t seconds is $(63t+27)^{\frac{1}{3}}$

Q. 26. In a bank, principal increases continuously at the rate of *r*% per year. Find the value of *r* if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

[NCERT Ex. 9.4, Q. 20, Page 397]

Ans. Let p, t, and r represent the principal, time, and rate of interest, respectively.

It is given that the principal increases continuously at the rate of r% per year.

$\Rightarrow \dfrac{dp}{dt} = \left(\dfrac{r}{100}\right)p$

$\Rightarrow \dfrac{dp}{p} = \left(\dfrac{r}{100}\right)dt$

Integrating both sides, we get :

$\int \dfrac{dp}{p} = \dfrac{r}{100}\int dt$

$\Rightarrow \log p = \dfrac{rt}{100} + k$

$\Rightarrow p = e^{\frac{rt}{100}+k}$ …(1)

[2½]

It is given that when $t=0$, $p=100$.

$\Rightarrow 100 = e^k$ …(2)

Now, if $t=10$, then $p = 2 \times 100 = 200$

Therefore, equation (1) becomes :

$200 = e^{\frac{r}{10}+k}$

$\Rightarrow 200 = e^{\frac{r}{10}}.e^k$

$\Rightarrow 200 = e^{\frac{r}{10}}.100$

$\Rightarrow e^{\frac{r}{10}} = 2$

$\Rightarrow \dfrac{r}{10} = \log_e 2$

$\Rightarrow \dfrac{r}{10} = 0.6931$

$\Rightarrow r = 6.931$

Hence, the value of r is 6.39% **[2½]**

Q. 27. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present? [NCERT Ex. 9.4, Q. 22, Page 397]

Ans. Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$\therefore \frac{dy}{dt} \propto y$

$\Rightarrow \frac{dy}{dt} = ky$ (Where k is a constant)

$\Rightarrow \frac{dy}{y} = kdt$

Integrating both sides, we get :

$$\int \frac{dy}{y} = k\int dt$$

$\Rightarrow \log y = kt + C$ ...(1)

Let y_0 be the number of bacteria at $t=0$

$\Rightarrow \log y_0 = C$

Substituting the value of C in equation (1), we get :

$$\log y = kt + \log y_0$$

$\Rightarrow \log y - \log y_0 = kt$

$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$

$\Rightarrow kt = \log\left(\frac{y}{y_0}\right)$ ...(2)

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$\Rightarrow y = \frac{110}{100}y_0$

$\Rightarrow \frac{y}{y_0} = \frac{11}{10}$

Substituting this value in equation (2), we get :

$k.2 = \log\left(\frac{11}{10}\right)$

$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$

Therefore, equation (2) becomes :

$\frac{1}{2}\log\left(\frac{11}{10}\right).t = \log\left(\frac{y}{y_0}\right)$

$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)}$ **[2½]**

Now, let the time when the number of bacteria increases from 1,00,000 to 2,00,000 be t_1.

$\Rightarrow y = 2y_0$ at $t = t_1$

From equation (4), we get :

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 1,00,000 to 2,00,000. **[2½]**

Q. 28. Show that the given differential equation is homogeneous and solve

$$(x^2 + xy)dy = (x^2 + y^2)dx$$

[NCERT Ex. 9.5, Q. 1, Page 406]

Ans. The given differential equation, *i.e.*,

$(x^2 + xy)dy = (x^2 + y^2)dx$ can be w∝ritten as :

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Let $F(x, y) = \frac{x^2 + y^2}{x^2 + xy}$.

Now, $F(\lambda x, \lambda y) = \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)}$

$$= \frac{x^2 + y^2}{x^2 + xy} = \lambda^0.F(x, y) \quad ...(1)$$

[2½]

This shows that equation (1) is a homogeneous equation.

To solve it, we make the substitution as :

$y = vx$

Differentiating both sides with respect to x, we get :

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting the values of V and $\frac{dy}{dx}$ in equation in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{x^2 + (vx)^2}{x^2 + x(vx)}$$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + v^2}{1 + v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{1 + v} - v$

$= \frac{(1 + v^2) - v(1 + v)}{1 + v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - v}{1 + v}$

$\Rightarrow \left(\frac{1 + v}{1 - v}\right) = dv = \frac{dx}{x}$

$\Rightarrow \left(\frac{2 - 1 + v}{1 - v}\right)dv = \frac{dx}{x}$

$\Rightarrow \left(\frac{2}{1 - v} - 1\right)dv = \frac{dx}{x}$

Integrating both sides, we get :

$-2\log(1 - v) - v = \log x - \log k$

$\Rightarrow v = -2\log(1 - v) - \log x + \log k$

$\Rightarrow v = \log\left[\frac{k}{x(1 - v)^2}\right]$

$\Rightarrow \frac{y}{x} = \log\left[\frac{k}{x\left(1 - \frac{y}{x}\right)^2}\right]$

$\Rightarrow \frac{y}{x} = \log\left[\frac{kx}{(x - y)^2}\right]$

$$\Rightarrow \quad \frac{kx}{(x-y)^2} = e^{\frac{y}{x}}$$

$$\Rightarrow \quad (x-y)^2 = kxe^{-\frac{y}{x}}$$

[2½]

This is the required solution of the given differential equation.

Q. 29. Show that the given differential equation is homogeneous and solve $y' = \frac{x+y}{x}$.

[NCERT Ex. 9.5, Q. 2, Page 406]

Ans. The given differential equation is :

$$y' = \frac{x+y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x}$$

Let $F(x,y) = \frac{x+y}{x}$. ...**(1)**

$$\text{Now, } F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x}$$

$$= \frac{x+y}{x}$$

$$= \lambda^0 F(x,y)$$

The given differential equation is :

$$y' = \frac{x+y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x} \quad \text{...(1)}$$

Let $F(x,y) = \frac{x+y}{x}$.

$$\text{Now, } F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x}$$

$$= \frac{x+y}{x} = \lambda^0 F(x,y)$$

[2½]

Thus, the given equation is a homogeneous equation.

To solve it, we make the substitution as :

$y = vx$

Differentiating both sides with respect to x, we get :

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{x+vx}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = 1+v$$

$$x\frac{dv}{dx} = 1$$

$$\Rightarrow \quad dv = \frac{dx}{x}$$

Integrating both sides, we get :

$$v = \log\ x + C$$

$$\Rightarrow \quad \frac{y}{x} = \log x + C$$

$$\Rightarrow \quad y = x\log x + Cx$$

This is the required solution of the given differential equation. **[2½]**

Q. 30. Show that the given differential equation is homogeneous and solve

$(x-y)dy - (x+y)dx = 0$

[NCERT Ex. 9.5, Q. 3, Page 406]

Ans. The given differential equation is :

$$(x-y)dy - (x+y)dx = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x-y}$$

Let $F(x,y) = \frac{x+y}{x-y}$.

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y}$$

$$= \frac{x+y}{x-y} \quad \text{...(1)}$$

$$= \lambda^0 . F(x,y)$$

Thus, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{x+vx}{x-vx} = \frac{1+v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{1+v-v(1-v)}{1-v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \quad \frac{1-v}{(1+v^2)}dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{1+v^2} - \frac{v}{1-v^2}\right)dv = \frac{dx}{x}$$

[2½]

Integrating both sides, we get :

$$\tan^{-1}v - \frac{1}{2}\log(1+v^2) = \log x + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left[1+\left(\frac{y}{x}\right)^2\right] = \log x + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(\frac{x^2+y^2}{x^2}\right) = \log x + C$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\left[\log(x^2+y^2) - \log x^2\right] = \log x + C$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log(x^2+y^2) + C$$

This is the required solution of the given differential equation. **[2½]**

Q. 31. Show that the given differential equation is homogeneous and solve $(x^2-y^2)dx+2xy\,dy=0$

[NCERT Ex. 9.5, Q. 4, Page 406]

Ans. The given differential equation is

$$(x^2-y^2)dx+2xy\,dy=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-(x^2-y^2)}{2xy} \quad \dots\textbf{(1)}$$

$$\text{Let } F(x,y)=\frac{-(x^2-y^2)}{2xy}.$$

$$\therefore\ F(\lambda x,\lambda y)=\left[\frac{(\lambda x)^2-(\lambda y)^2}{2(\lambda x)(\lambda y)}\right]$$

$$=\frac{-(x^2-y^2)}{2xy}=\lambda^0.F(x,y)$$

[2½]

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitutions as :

$$y=vx$$

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v+x\frac{dv}{dx}=-\left[\frac{x^2-(vx)^2}{2x.(vx)}\right]$$

$$v+x\frac{dv}{dx}=\frac{v^2-1}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{v^2-1}{2v}-v$$

$$=\frac{v^2-1-2v^2}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=-\frac{(1+v^2)}{2v}$$

$$\Rightarrow \frac{2v}{1+v^2}dv=-\frac{dx}{x}$$

Integrating both sides, we get :

$$\log\ (1+v^2)=-\log x+\log C=\log\frac{C}{x}$$

$$\Rightarrow \quad 1+v^2=\frac{C}{x}$$

$$\Rightarrow \left[1+\frac{y^2}{x^2}\right]=\frac{C}{x}$$

$$\Rightarrow \quad x^2+y^2=Cx$$

This is the required solution of the differential equation. **[2½]**

Q. 32. Show that the given differential equation is homogeneous and solve

$$x^2\frac{dy}{dx}=x^2-2y^2+xy\cdot$$

[NCERT Ex. 9.5, Q. 5, Page 406]

Ans. The given differential equation is

$$x^2\frac{dy}{dx}=x^2-2y^2+xy$$

$$x^2\frac{dy}{dx}=x^2-2y^2+xy$$

$$\frac{dy}{dx}=\frac{x^2-2y+xy}{x^2} \quad \dots\textbf{(1)}$$

$$\text{Let } F(x,y)=\frac{x^2-2y^2+xy}{x^2}$$

$$\therefore F(\lambda x,\lambda y)=\frac{(\lambda x)^2-2(\lambda y)^2+(\lambda x)(\lambda y)}{(\lambda x)^2}$$

$$=\frac{x^2-2y^2+xy}{x^2}=\lambda^0.F(x,y)$$

[2½]

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v+x\frac{dv}{dx}=\frac{x^2-2(vx)^2+x.(vx)}{x^2}$$

$$\Rightarrow \quad v+x\frac{dv}{dx}=1-2v^2+v$$

$$\Rightarrow \quad x\frac{dv}{dx}=1-2v^2$$

$$\Rightarrow \quad \frac{dv}{1-2v^2}=\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}.\frac{dv}{\frac{1}{2}-v^2}=\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}.\left[\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}\right]=\frac{dx}{x}$$

Integrating both sides, we get :

$$\frac{1}{2}.\frac{1}{2x\frac{1}{\sqrt{2}}}\log\left|\frac{\frac{1}{\sqrt{2}}+v}{\frac{1}{\sqrt{2}}-v}\right|=\log|x|+C$$

$$\Rightarrow \quad \frac{1}{2\sqrt{2}}\log\left|\frac{\frac{1}{\sqrt{2}}+\frac{y}{x}}{\frac{1}{\sqrt{2}}-\frac{y}{x}}\right|=\log|x|+C$$

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{x+\sqrt{2}\,y}{x-\sqrt{2}\,y}\right|=\log|x|+C$$

This is the required solution for the given differential equation. **[2½]**

Q. 33. Show that the given differential equation is homogeneous and solve $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

[NCERT Ex. 9.5, Q. 6, Page 406]

Ans. $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

$$\Rightarrow \quad xdy = \left[y + \sqrt{x^2 + y^2}\right]dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \quad \ldots\textbf{(1)}$$

Let $F(x, y) = \dfrac{y + \sqrt{x^2 + y^2}}{x}$

$$\therefore \quad F(\lambda x, \lambda y) = \frac{\lambda x + \sqrt{(\lambda x)^2 + (\lambda y)^2}}{\lambda x}$$

$$= \frac{y + \sqrt{x^2 + y^2}}{x} = \lambda^0 . F(x, y)$$

[2½]

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of v and $\dfrac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + (vx)^2}}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get :

$$\log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log C$$

$$\Rightarrow \quad \log\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right| = \log|Cx|$$

$$\Rightarrow \log\left|\frac{y + \sqrt{x^2 + y^2}}{x}\right| = \log|Cx|$$

$$\Rightarrow \quad y + \sqrt{x^2 + y^2} = Cx^2$$

This is the required solution of the given differential equation. **[2½]**

Q. 34. Show that the given differential equation is homogeneous and solve

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$$

[NCERT Ex. 9.5, Q. 7, Page 406]

Ans. The given differential equation is

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$$

$$\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}x}$$

Let $F(x, y) = \dfrac{\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}x}$.

$$\therefore F(\lambda x, \lambda y) = \frac{\left\{\lambda x\cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y\sin\left(\frac{\lambda y}{\lambda x}\right)\right\}\lambda y}{\left\{\lambda y\sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x\sin\left(\frac{\lambda y}{\lambda x}\right)\right\}\lambda x}.$$

$$= \frac{\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}x}$$

$$= \lambda^0 . F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\dfrac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{(x\cos v + vx\sin v).vx}{(vx\sin v - x\cos v).x}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v - v^2\sin v + v\cos v}{v\sin v - \cos v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v - \cos v}$$

$$\Rightarrow \left[\frac{v\sin v - \cos v}{v\cos v}\right]dv = \frac{2dx}{x}$$

$$\Rightarrow \quad \left(\tan v - \frac{1}{v}\right)dv = \frac{2dx}{x}$$

Integrating both sides, we get :

$$\log(\sec v) - \log v = 2\log x + \log C$$

$$\Rightarrow \quad \log\left(\frac{\sec v}{v}\right) = \log\left(Cx^2\right)$$

$$\Rightarrow \quad \left(\frac{\sec v}{v}\right) = Cx^2$$

$$\Rightarrow \quad \sec v = Cx^2 v$$

$$\Rightarrow \quad \sec\left(\frac{y}{x}\right) = C.x^2.\frac{y}{x}$$

$$\Rightarrow \quad \sec\left(\frac{y}{x}\right) = Cxy$$

$$\Rightarrow \qquad \cos\left(\frac{y}{x}\right) = \frac{1}{Cxy} = \frac{1}{C}.\frac{1}{xy}$$

$$\Rightarrow \qquad xy\cos\left(\frac{y}{x}\right) = k \qquad \left(k = \frac{1}{C}\right)$$

[2½]

This is the required solution of the given differential equation. **[2½]**

Q. 35. Show that the given differential equation is homogeneous and solve

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

[NCERT Ex. 9.5, Q. 8, Page 406]

Ans.

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \qquad x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

Let ...**(1)**

$$F(x,y) = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}.$$

$$\therefore \qquad F(\lambda x, \lambda y) = \frac{\lambda y - \lambda x\sin\left(\frac{\lambda y}{\lambda x}\right)}{\lambda x}$$

$$= \frac{y - x\sin\left(\frac{y}{x}\right)}{x} = \lambda^0.F(x,y)$$

[2½]

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1) we get;

$$v + x\frac{dv}{dx} = \frac{vx - x\sin v}{x}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v - \sin v$$

$$\Rightarrow \quad -\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\Rightarrow \text{cosec}\, v dv = -\frac{dx}{x}$$

Integrating both sides, we get :

$$\log|\text{cosec}\, v - \cot v| = -\log x + \log C = \log\frac{C}{x}$$

$$\Rightarrow \text{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{C}{x}$$

$$\Rightarrow \quad \frac{1}{\sin\left(\frac{y}{x}\right)} - \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{C}{x}$$

$$\Rightarrow \quad x\left[1 - \cos\left(\frac{y}{x}\right)\right] = C\sin\left(\frac{y}{x}\right)$$

[2½]

This is the required solution of the given differential equation.

Q. 36. Show that the given differential equation is homogeneous and solve

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0\,.$$

[NCERT Ex. 9.5, Q. 9, Page 406]

Ans. $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$

$$\Rightarrow \qquad ydx = \left[2x - x\log\left(\frac{y}{x}\right)\right]dy$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

Let $F(x,y) = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}.$ ...**(1)**

$$\therefore \; F(\lambda x, \lambda y) = \frac{\lambda y}{2(\lambda x) - (\lambda x)\log\left(\frac{\lambda y}{\lambda x}\right)}$$

$$= \frac{y}{2x - \log\left(\frac{y}{x}\right)}\lambda^0.F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = \frac{}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{vx}{2x - x\log v}$$

$$\Rightarrow \qquad v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow \qquad x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$\Rightarrow \qquad x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$$

$$\Rightarrow \qquad x\frac{dv}{dx} = \frac{v\log v - v}{2 - \log v}$$

$$\Rightarrow \qquad \frac{2 - \log v}{v(\log v - 1)}dv = \frac{dx}{x}$$

$$\Rightarrow \qquad \left[\frac{1 + (1 - \log v)}{v(\log v - 1)}\right]dv = \frac{dx}{x}$$

$$\Rightarrow \left[\frac{1}{v(\log v-1)}-\frac{1}{v}\right]dv=\frac{dx}{x}$$

Integrating both sides, we get :

$$\int\frac{1}{v(\log v-1)}dv-\int\frac{1}{v}dv=\int\frac{1}{x}dx$$

$$\Rightarrow \int\frac{dv}{v(\log v-1)}-\log v=\log x+\log C \qquad \ldots\textbf{(2)}$$

Let

$$\log v-1=t$$

$$\Rightarrow \frac{d}{dv}(\log v-1)=\frac{dt}{dv}$$

$$\Rightarrow \frac{1}{v}=\frac{dt}{dv}$$

$$\Rightarrow \frac{dv}{v}=dt$$

[2½]

Therefore, equation (1) becomes :

$$\Rightarrow \int\frac{dt}{t}-\log v=\log x+\log C$$

$$\Rightarrow \log t-\log\left(\frac{y}{x}\right)=\log(Cx)$$

$$\Rightarrow \log\left[\log\left(\frac{y}{x}\right)-1\right]-\log\left(\frac{y}{x}\right)=\log(Cx)$$

$$\Rightarrow \log\left[\frac{\log\left(\frac{y}{x}\right)-1}{\frac{y}{x}}\right]=\log(Cx)$$

$$\Rightarrow \frac{x}{y}\left[\log\left(\frac{y}{x}\right)-1\right]=Cx$$

$$\Rightarrow \log\left(\frac{y}{x}\right)-1=Cy$$

This is the required solution of the given differential equation. **[2½]**

Q. 37. Show that the given differential equation is homogeneous and solve

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

[NCERT Ex. 9.5, Q. 10, Page 406]

Ans.

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

$$\Rightarrow \left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

$$\Rightarrow \left(1+e^{\frac{x}{y}}\right)dx=-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy$$

$$\Rightarrow \frac{dx}{dy}=\frac{-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}{1+e^{\frac{x}{y}}} \qquad \ldots\textbf{(1)}$$

Let $F(x,y)=\dfrac{-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}{1+e^{\frac{x}{y}}}$.

$$F(\lambda x,\lambda y)=\frac{-e^{\frac{\lambda x}{\lambda y}}\left(1-\frac{\lambda x}{\lambda y}\right)}{1+e^{\frac{\lambda x}{\lambda y}}}=\frac{-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}{1+e^{\frac{x}{y}}}=\lambda^0.F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$x=vy$$

$$\Rightarrow \frac{d}{dy}(x)=\frac{d}{dy}(vy)$$

$$\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$$

Substituting the values of x and $\frac{dx}{dy}$ in equation (1), we get :

$$v+y\frac{dv}{dy}=\frac{-e^{v}(1-v)}{1+e^{v}}$$

$$\Rightarrow y\frac{dv}{dy}=\frac{-e^{v}+ve^{v}}{1+e^{v}}-v$$

$$\Rightarrow y\frac{dv}{dy}=\frac{-e^{v}+ve^{v}-v-ve^{v}}{1+e^{v}}$$

$$\Rightarrow y\frac{dv}{dy}=-\left[\frac{v+e^{v}}{1+e^{v}}\right]$$

$$\Rightarrow \left[\frac{1+e^{v}}{v+e^{v}}\right]dv=-\frac{dy}{y}$$

[2½]

Integrating both sides, we get :

$$\Rightarrow \log(v+e^{v})=-\log y+\log C=\log\left(\frac{C}{y}\right)$$

$$\Rightarrow \left[\frac{x}{y}+e^{\frac{x}{y}}\right]=\frac{C}{y}$$

$$\Rightarrow x+ye^{\frac{x}{y}}=C$$

This is the required solution of the given differential equation. **[2½]**

Q. 38. Find the particular solution satisfying the given condition :

$$\boldsymbol{(x+y)dy+(x-y)dx=0;\ y=1 \text{ when } x=1}$$

[NCERT Ex. 9.5, Q. 11, Page 406]

Ans. $(x+y)dy+(x-y)dx=0$

$$\Rightarrow (x+y)dy=-(x-y)dx$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(x-y)}{x+y} \qquad \ldots\textbf{(1)}$$

Let $F(x,y)=\dfrac{-(x-y)}{x+y}$

$$\therefore F(\lambda x,\lambda y)=\frac{-(\lambda x-\lambda y)}{\lambda x-\lambda y}$$

$$=\frac{-(x-y)}{x+y}=\lambda^0\cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{d}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{-(x - vx)}{x + vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v = \frac{v-1-v(v+1)}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1-v^2-v}{v+1} = \frac{-(1+v^2)}{v+1}$$

$$\Rightarrow \frac{(v+1)}{1+v^2}dv = -\frac{dx}{x}$$

$$\Rightarrow \left[\frac{v}{1+v^2} + \frac{1}{1+v^2}\right]dv = -\frac{dx}{x}$$

Integrating both sides, we get :

$$\frac{1}{2}\log(1+v^2) + \tan^{-1}v = -\log x + k$$

$$\Rightarrow \log(1+v^2) + 2\tan^{-1}v = -2\log x + 2k$$

$$\Rightarrow \log\left[(1+v^2).x^2\right] + 2\tan^{-1}v = 2k$$

$$\Rightarrow \log\left[\left(1+\frac{y^2}{x^2}\right).x^2\right] + 2\tan^{-1}\frac{y}{x} = 2k$$

$$\Rightarrow \log(x^2+y^2) + 2\tan^{-1}\frac{y}{x} = 2k \quad \ldots(2)$$

[2½]

Now $y=1$ at $x=1$

$$\Rightarrow \log 2 + 2\tan^{-1}1 = 2k$$

$$\Rightarrow \log 2 + 2\times\frac{\pi}{4} = 2k$$

$$\Rightarrow \frac{\pi}{2} + \log 2 = 2k$$

Substituting the value of $2k$ in equation (2), we get :

$$\log(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$$

This is the required solution of the given differential equation. **[2½]**

Q. 39. Find the particular solution satisfying the given condition :

$$\boldsymbol{x^2dy + (xy+y^2)dx = 0; y = 1 \text{ when } x = 1}$$

[NCERT Ex. 9.5, Q. 12, Page 406]

Ans.

$$x^2dy + (xy+y^2)dx = 0$$

$$\Rightarrow x^2dy = -(xy+y^2)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(xy+y^2)}{x^2} \quad \ldots(1)$$

Let $F(x,y) = \frac{-(xy+y^2)}{x^2}$

$$\therefore F(\lambda x, \lambda y) = \frac{\left[\lambda x.\lambda y + (\lambda y)^2\right]}{(\lambda x)^2}$$

$$= \frac{-(xy+y^2)}{x^2} = \lambda^0.F(x,y)$$

Therefore, the given differential equation is a homogeneous equation

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v + x\frac{dv}{dx} = \frac{-\left[x.vx + (vx)^2\right]}{x^2} = -v - v^2$$

$$\Rightarrow x\frac{dv}{dx} = -v^2 - 2v = -v(v+2)$$

$$\Rightarrow \frac{dv}{v(v+2)} = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\left[\frac{(v+2)-v}{v(v+2)}\right]dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{v} - \frac{1}{v+2}\right]dv = -\frac{dx}{x}$$

[2½]

Integrating both sides, we get :

$$\frac{1}{2}[\log v - \log(v+2)] = -\log x + \log C$$

$$\Rightarrow \frac{1}{2}\log\left(\frac{v}{v+2}\right) = \log\frac{C}{x}$$

$$\Rightarrow \frac{v}{v+2} = \left(\frac{C}{x}\right)^2$$

$$\Rightarrow \frac{\frac{y}{x}}{\frac{y}{x}+2} = \left(\frac{C}{x}\right)^2$$

$$\Rightarrow \frac{y}{y+2x} = \frac{C^2}{x^2}$$

$$\Rightarrow \frac{x^2y}{y+2x} = C^2 \quad \ldots(2)$$

Now, $y=1$ at $x=1$.

$$\Rightarrow \frac{1}{1+2} = C^2$$

$$\Rightarrow C^2 = \frac{1}{3}$$

Substituting $C^2 = \frac{1}{3}$ in equation (2), we get :

$$\frac{x^2y}{y+2x} = \frac{1}{3}$$

$$\Rightarrow y + 2x = 3x^2y$$

This is the required solution of the given differential equation. **[2½]**

Q. 40. Find the particular solution satisfying the given condition :

$$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4} \text{ when } x = 1$$

[NCERT Ex. 9.5, Q. 13, Page 406]

Ans.

$$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-\left[x\sin^2\left(\frac{y}{x}\right)-y\right]}{x}$$

$$\text{Let } F(x,y)=\frac{-\left[x\sin^2\left(\frac{y}{x}\right)-y\right]}{x}$$

$$\therefore F(\lambda x,\lambda y)=\frac{-\left[\lambda x.\sin^2\left(\frac{\lambda x}{\lambda y}\right)-\lambda y\right]}{\lambda x}$$

$$=\frac{-\left[x\sin^2\left(\frac{y}{x}\right)-y\right]}{x}=\lambda^0.F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve this differential equation, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx}=v+x=\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v+x\frac{dv}{dx}=\frac{-\left[x\sin^2 v-vx\right]}{x}$$

$$\Rightarrow \quad v+x\frac{dv}{dx}=-\left[\sin^2 v-v\right]=v-\sin^2 v$$

$$\Rightarrow \quad x\frac{dv}{dx}=-\sin^2 v$$

$$\Rightarrow \quad \frac{dv}{\sin^2 v}=-\frac{dx}{dx}$$

$$\Rightarrow \cos ec^2 v dv=-\frac{dx}{x}$$ **[2½]**

Integrating both sides, we get :

$$-\cot v=-\log|x|-C$$

$$\Rightarrow \quad \cot v=\log|x|+C$$

$$\Rightarrow \cot\left(\frac{y}{x}\right)=\log|x|+\log C$$

$$\Rightarrow \cot\left(\frac{y}{x}\right)=\log|Cx| \quad \ldots\textbf{(2)}$$

Now, $y=\frac{\pi}{4}$ at $x=1$.

$$\Rightarrow \cot\left(\frac{\pi}{4}\right)=\log|C|$$

$$\Rightarrow \quad 1=\log C$$

$$\Rightarrow \quad C=e^1=e$$

Substituting C=e in equation (2), we get :

$$\cot\left(\frac{y}{x}\right)=\log|ex|$$

This is the required solution of the given differential equation. **[2½]**

Q. 41. Find the particular solution satisfying the given condition :

$$\frac{dy}{dx}-\frac{y}{x}+\cos ec\left(\frac{y}{x}\right)=0; y=0 \text{ when } x=1$$

[NCERT Ex. 9.5, Q. 14, Page 406]

Ans.

$$\frac{dy}{dx}-\frac{y}{x}+\text{cosec}\left(\frac{y}{x}\right)=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x}-\text{cosec}\left(\frac{y}{x}\right) \quad \ldots\textbf{(1)}$$

$$\text{Let } F(x,y)=\frac{y}{x}-\text{cosec}\left(\frac{y}{x}\right).$$

$$\therefore \quad F(\lambda x,\lambda y)=\frac{\lambda y}{\lambda x}-\text{cosec}\left(\frac{\lambda y}{\lambda x}\right)$$

$$\Rightarrow \quad F(\lambda x,\lambda y)=\frac{y}{x}-\text{cosec}\left(\frac{y}{x}\right)$$

$$=F(x,y)=\lambda^0.F(x,y)$$ **[2½]**

Therefore, the given differential equation is a homogeneous equation

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v+x\frac{dv}{dx}=v-\text{cosec}\, v$$

$$\Rightarrow -\frac{dv}{\text{cosec}\, v}=-\frac{dx}{x}$$

$$\Rightarrow -\sin v dv=\frac{dx}{x}$$

Integrating both sides, we get :

$$\cos v=\log x+\log C=\log|Cx|$$

$$\Rightarrow \cos\left(\frac{y}{x}\right)=\log|Cx| \quad \ldots\textbf{(2)}$$

This is the required solution of the given differential equation.

Now, y=0 at x=1.

$$\Rightarrow \cos(0)=\log C$$

$$\Rightarrow \quad 1=\log C$$

$$\Rightarrow \quad C=e^1=e$$

Substituting C=e in equation (2), we get :

$$\cos\left(\frac{y}{x}\right)=\log|(ex)|$$

This is the required solution of the given differential equation. **[2½]**

Q. 42. Find the particular solution satisfying the given condition :

$$2xy+y^2-2x^2\frac{dy}{dx}=0; y=2 \, when \, x=1$$

[NCERT Ex. 9.5, Q. 15, Page 406]

Ans.

$$2xy+y^2-2x^2\frac{dy}{dx}=0$$

$$\Rightarrow \quad 2x^2\frac{dy}{dx}=2xy+y^2$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2xy+y^2}{2x^2} \quad \ldots\textbf{(1)}$$

$$\text{Let } F(x,y)=\frac{2xy+y^2}{2x^2}$$

$$\therefore \quad F(\lambda x,\lambda y)=\frac{2(\lambda x)(\lambda y)+(\lambda y)^2}{2(\lambda x)^2}$$

$$=\frac{2xy+y^2}{2x^2}=\lambda^0.F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as :

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$$

$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get :

$$v+x\frac{dv}{dx}=\frac{2x(vx)+(vx)^2}{2x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{2v+v^2}{2}$$

$$\Rightarrow v+x\frac{dv}{dx}=v+\frac{v^2}{2}$$

$$\Rightarrow \quad \frac{2}{v^2}dv=\frac{dx}{x}$$ **[2½]**

Integrating both sides, we get :

$$2\cdot\frac{v^{-2+1}}{-2+1}=\log|x|+C$$

$$\Rightarrow \quad -\frac{2}{v}=\log|x|+C$$

$$\Rightarrow \quad -\frac{2}{\frac{y}{x}}=\log|x|+C$$

$$\Rightarrow \quad -\frac{2x}{y}=\log|x|+C$$

Now $y = 2$ and $x = 1$

$$\Rightarrow \quad -1=\log(1)+C$$

$$\Rightarrow \quad C=-1$$

Substituting C$=-1$ in equation (2) we get :

$$-\frac{2x}{y}=\log|x|-1$$

$$\Rightarrow \frac{2x}{y}=1-\log|x|$$

$$\Rightarrow \quad y=\frac{2x}{1-\log|x|},(x\neq 0, x\neq e)$$

This is the required solution of the given differential equation. **[2½]**

Q. 43. For the differential equation find the general solution :

$$\boldsymbol{\frac{dy}{dx}+2y=\sin x}$$

[NCERT Ex. 9.6, Q. 1, Page 413]

Ans. The given differential equation is $\frac{dy}{dx}+2y=\sin x$.

This is in the form of

$$\frac{dy}{dx}+py=Q \text{ (where } p=2 \text{ and } Q=\sin\ x)$$

Now, IF$=e^{\int pdx}=e^{\int 2dx}=e^{2x}$.

The solution of the given differential equation is given by the relation.

$$y(\text{IF})=\int(Q\times\text{ IF})dx+C$$

$$\Rightarrow ye^{2x}=\int\sin x.e^{2x}dx+C$$

$$\text{Let } I=\int\sin x.e^{2x}$$

$$\Rightarrow \quad I=\sin x.\int e^{2x}dx-\int\left(\frac{d}{dx}(\sin x).\int e^{2x}dx\right)dx$$

$$\Rightarrow \quad I=\sin x.\frac{e^{2x}}{2}-\int\left(\cos x.\frac{e^{2x}}{2}\right)dx$$ ...(1)

$$\Rightarrow \quad I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x.\int e^{2x}-\int\left(\frac{d}{dx}(\cos x).\int e^{2x}dx\right)dx\right]$$

$$\Rightarrow \quad I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x.\frac{e^{2x}}{2}-\int\left[(-\sin x).\frac{e^{2x}}{2}\right]dx\right]$$

$$\Rightarrow \quad I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}\int(\sin x.e^{2x})dx$$

$$\Rightarrow \quad I=\frac{e^{2x}}{4}(2\sin x-\cos x)-\frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}}{4}(2\sin x-\cos x)$$

$$\Rightarrow \quad I=\frac{e^{2x}}{5}(2\sin x-\cos x)$$ **[3]**

Therefore, equation (1) becomes :

$$ye^{2x}=\frac{e^{2x}}{5}(2\sin x-\cos x)+C$$

$$\Rightarrow y=\frac{1}{5}(2\sin x-\cos x)+Ce^{-2x}$$

This is the required general solution of the given differential equation. **[2]**

Q. 44. For the differential equation find the general solution :

$$\cos^2 x\frac{dy}{dx}+y=\tan x\left(0\leq x<\frac{\pi}{2}\right)$$

[NCERT Ex. 9.6, Q. 5, Page 413]

Ans. The given differential equation is :

$$\cos^2 x\frac{dy}{dx}+y=\tan x$$

$$\Rightarrow \frac{dy}{dx}+\sec^2 x.y=\sec^2 x\tan x$$

This equation is in the form of :

$$\frac{dy}{dx}+py=Q \text{ (where } p=\sec^2 x \text{ and } Q=\sec^2 x\tan x)$$

Now, IF$=e^{\int pdx}=e^{\int \sec^2 xdx}=e^{\tan x.}$

The general solution of the given differential equation is given by the relation.

$$y(\text{IF}) = \int (Q \times \text{IF})dx + C$$

$$\Rightarrow \quad y.e^{\tan x} = \int e^{\tan x}.\sec^2 x \tan x dx + C$$

Let $\tan x = t$

$$\Rightarrow \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \quad \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \quad \sec^2 x dx = dt$$ **[2½]**

Therefore, equation (1) becomes :

$$y.e^{\tan x} = \int (e^t.t)dt + C$$

$$\Rightarrow y.e^{\tan x} = \int (t.e^t)dt + C$$

$$\Rightarrow y.e^{\tan x} = t.\int e^t dt - \int \left(\frac{d}{dt}(t).\int e^t dt\right)dt + C$$

$$\Rightarrow y.e^{\tan x} = t.e^t - \int e^t dt + C$$

$$\Rightarrow \quad ye^{\tan x} = (t-1)e^t + C$$

$$\Rightarrow \quad ye^{\tan x} = (\tan x - 1)e^{\tan x} + C$$

$$\Rightarrow \quad y = (\tan\ x - 1) + Ce^{-\tan x}$$ **[2½]**

Q. 45. For the differential equation find the general solution :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

[NCERT Ex. 9.6, Q. 6, Page 413]

Ans. The given differential equation is :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x \log x$$

This equation is in the form of a linear differential equation as :

$$\frac{dy}{dx} + py = Q \ \ (\text{where } p = \frac{2}{x} \ and \ Q = x\log x)$$

Now, $\text{IF} = e^{\int pdx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$ **[2½]**

The general solution of the given differential equation is given by the relation.

$$y(\text{IF}) = \int (Q \times \text{IF})dx + C$$

$$\Rightarrow y.x^2 = \int (x \log x.x^2)dx + C$$

$$\Rightarrow x^2 y = \int (x^3 \log x)dx + C$$

$$\Rightarrow x^2 y = \log x.\int x^3 dx - \int \left[\frac{d}{dx}(\log x).\int x^3 dx\right]dx + C$$

$$\Rightarrow x^2 y = \log x.\frac{x^4}{4} - \int \left(\frac{1}{x}.\frac{x^4}{4}\right)dx + C$$

$$\Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4}\int x^3 dx + C$$

$$\Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4}.\frac{x^4}{4} + C$$

$$\Rightarrow x^2 y = \frac{1}{16}x^4(4\log x - 1) + C$$

$$\Rightarrow \quad y = \frac{1}{16}x^2(4\log x - 1) + Cx^{-2}$$ **[2½]**

Q. 46. For the differential equation find the general solution :

$$x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$$

[NCERT Ex. 9.6, Q. 7, Page 413]

Ans. The given differential equation is :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x}\log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x^2}$$

This equation is the form of a linear differential equation as :

$$\frac{dy}{dx} + py = Q \ \ (\text{where } p = \frac{1}{x \log x} \ and \ Q = \frac{2}{x^2})$$

Now, $\text{IF} = e^{\int pdx} = e^{\int \frac{1}{x\log}dx} = e^{\log(\log x)} = \log\ x$

The general solution of the given differential equation is given by the relation.

$$y(\text{IF}) = \int (Q \times \text{IF})dx + C$$

$$\Rightarrow y \log x = \int \left(\frac{2}{x^2}\log x\right)dx + C$$

Now,

$$\int \left(\frac{2}{x^2}\log x\right)dx = 2\int \left(\log x.\frac{1}{x^2}\right)dx$$

$$= 2\left[\log x.\int \frac{1}{x^2}dx - \int \left\{\frac{d}{dx}(\log x).\int \frac{1}{x^2}dx\right\}dx\right]$$

$$= 2\left[\log x\left(-\frac{1}{x}\right) - \int \left(\frac{1}{x}.\left(-\frac{1}{x}\right)\right)dx\right]$$

$$= 2\left[-\frac{\log x}{x} + \int \frac{1}{x^2}dx\right]$$

$$= 2\left[-\frac{\log x}{x} - \frac{1}{x}\right]$$

$$= -\frac{2}{x}(1 + \log x)$$

[2½]

Substituting the value of $\int \left(\frac{2}{x^2}\log x\right)\ dx$ in equation, we get :

$$y \log x = -\frac{2}{x}(1 + \log x) + C$$

This is the required general solution of the given differential equation. **[2½]**

Q. 47. For the differential equation find the general solution :

$$(1 + x^2)dy + 2xy\,dx = \cot x\,dx (x \neq 0)$$

[NCERT Ex. 9.6, Q. 8, Page 413]

Ans.

$$(1 + x^2)dy + 2xy\,dx = \cot x\,dx$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{\cot x}{1 + x^2}$$

This equation is a linear differential equation of the form :

$\frac{dy}{dx} + py = Q$ (where $p = \frac{2x}{1+x^2}$ and $Q = \frac{\cot x}{1+x^2}$)

Now, IF $= e^{\int pdx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$

The general solution of the given differential equation is given by the relation.

$$y(\text{IF}) = \int (Q\times \text{ IF})dx + C$$

$$\Rightarrow y(1+x^2) = \int \left[\frac{\cot x}{1+x^2}\text{x}(1+x^2)\right]dx + C$$

$$\Rightarrow y(1+x^2) = \int \cot x dx + C$$

$$\Rightarrow y(1+x^2) = \log|\sin x| + C$$

[5]

Q. 48. For the differential equation find the general solution :

$$x\frac{dy}{dx} + y - x + xy\cot x = 0 (x \neq 0)$$

[NCERT Ex. 9.6, Q. 9, Page 414]

Ans.

$$x\frac{dy}{dx} + y - x + xy\cot x = 0$$

$$\Rightarrow x\frac{dy}{dx} + y(1 + x\cot x) = x$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

This equation is a linear differential equation of the form :

$\frac{dy}{dx} + py = Q$ (where $p = \frac{1}{x} + \cot x$ *and* $Q = 1$)

Now, IF $= e^{\int pdx} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log x + \log(\sin x)}$

$= e^{\log(x\sin x)} = x\sin x.$

The general solution of the given differential equation is given by the relation.

$$y(\text{IF}) = \int (Q\times \text{ IF})dx + C$$

$$\Rightarrow y(x\sin x) = \int (1\times x\sin x)dx + C$$

$$\Rightarrow y(x\sin x) = \int (x\sin x)dx + C$$

$$\Rightarrow y(x\sin x) = x\int \sin x dx - \int\left[\frac{d}{dx}(x).\int \sin x dx\right] + C$$

$$\Rightarrow y(x\sin x) = x(-\cos x) - \int 1.(-\cos x)dx + C$$

$$\Rightarrow y(x\sin x) = -x\cos x + \sin x + C$$

$$\Rightarrow \quad y = \frac{-x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{C}{x\sin x}$$

$$\Rightarrow \quad y = -\cot .x + \frac{1}{x} + \frac{C}{x\sin x}$$

[5]

Q. 49. For the differential equation find the general solution :

$$(x+y)\frac{dy}{dx} = 1$$

[NCERT Ex. 9.6, Q. 10, Page 414]

Ans.

$$(x+y)\frac{dy}{dx} = 1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x+y}$$

$$\Rightarrow \quad \frac{dx}{dy} = x + y$$

$$\Rightarrow \frac{dx}{dy} - x = y$$

This is a linear differential equation of the form :

$\frac{dy}{dx} + px = Q$ (Where $p = -1$ and $Q = y$)

Now, IF $= e^{\int pdy} = e^{\int -dy} = e^{-y}$ **[2½]**

The general solution of the given differential equation is given by the relation.

$$x(\text{IF}) = \int (\text{Q}\times \text{IF})dy + C$$

$$\Rightarrow \quad xe^{-y} = \int (y.e^{-y})dy + C$$

$$\Rightarrow \quad xe^{-y} = y.\int e^{-y}dy - \int\left[\frac{d}{dy}(y)\int e^{-y}dy\right]dy + C$$

$$\Rightarrow \quad xe^{-y} = y(-e^{-y}) - \int(-e^{-y})dy + C$$

$$\Rightarrow \quad xe^{-y} = -ye^{-y} - e^{-y} + C$$

$$\Rightarrow \quad x = -y - 1 + Ce^{y}$$

$$\Rightarrow x + y + 1 = Ce^{y}$$

[2½]

Q. 50. For the differential equation, find a particular solution satisfying the given condition :

$$\frac{dy}{dx} + 2y\tan x = \sin x; y = 0 \text{ when } x = \frac{\pi}{3}$$

[NCERT Ex. 9.6, Q. 13, Page 414]

Ans. The given differential equation is

$$\frac{dy}{dx} + 2y\tan x = \sin x.$$

This is a linear equation of the form :

$\frac{dy}{dx} + py = Q$(Where $p = 2\tan x$ *and* $Q = \sin x$)

Now, IF$= e^{\int pdx} = e^{\int 2\tan x dx} = e^{2\log|\sec x|} = e^{\log(\sec^2 x)} = \sec^2 x$

The general solution of the given differential equation is given by the relation

$$y(\text{IF}) = \int (\text{Q}\times \text{IF})dx + C$$

$$\Rightarrow y(\sec^2 x) = \int (\sin x.\sec^2 x)dx + C$$

$$\Rightarrow \quad y\sec^2 x = \int (\sec x.\tan x)dx + C$$

$$\Rightarrow \quad y\sec^2 x = \sec x + C \quad \ldots(1)$$

Now, $y = 0$ at $x = \frac{\pi}{3}$

Therefore,

$$0\times\sec^2\frac{\pi}{3} = \sec\frac{\pi}{3} + C$$

$$\Rightarrow \quad 0 = 2 + C$$

$$\Rightarrow \quad C = -2$$

Substituting C=– 2 in equation (1), we get:

$$y\sec^2 x = \sec x - 2$$

$$\Rightarrow \quad y = \cos x - 2\cos^2 x$$

[2½]

Hence, the required solution of the given differential equation is $y = \cos\ x - 2\cos^2 x$ **[2½]**

Q. 51. For the differential equation, find a particular solution satisfying the given condition :

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0 \text{ when } x=1$$

[NCERT Ex. 9.6, Q. 14, Page 414]

Ans.

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$$

This is a linear differential equation of the form :

$$\frac{dy}{dx}+py=Q \quad \left(\text{Where } p=\frac{2x}{1+x^2} \text{ and } Q=\frac{1}{(1+x^2)^2}\right)$$

Now, IF$=e^{\int pdx}=e^{\int\frac{2xdx}{1+x^2}}=e^{\log(1+x^2)}=1+x^2$ **[2½]**

The general solution of the given differential equation is given by the relation.

$$y(\text{IF})=\int(Q\times\text{IF})dx+\text{C}$$

$$\Rightarrow y(1+x^2)=\int\left[\frac{1}{(1+x^2)^2}.(1+x^2)\right]dx+\text{C}$$

$$\Rightarrow y\left(1+x^2\right)=\int\frac{1}{1+x^2}dx+\text{C}$$

$$\Rightarrow y(1+x^2)=\tan^{-1}x+\text{C}$$

Now, $y=0$ at $x=1$.

Therefore,

$$0=\tan^{-1}1+\text{C}$$

$$\Rightarrow \quad \text{C}=-\frac{\pi}{4}$$

Substituting $\text{C}=-\frac{\pi}{4}$ in equation (1), we get :

$$y(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$$

This is the required solution of the given differential equation. **[2½]**

Q. 52. For the differential equation, find a particular solution satisfying the given condition :

$$\frac{dy}{dx}-3y\cot x=\sin 2x; y=2 \text{ when } x=\frac{\pi}{2}$$

[NCERT Ex. 9.6, Q. 15, Page 414]

Ans. The given differential equation is

$$\frac{dy}{dx}-3y\cot x=\sin 2x.$$

This is a linear differential equation of the form :

$$\frac{dy}{dx}+py=Q \text{ (where } p=-3\cot x \text{ and } Q=\sin 2x)$$

Now, IF$=e^{\int pdx}=e^{-3\int\cot xdx}=e^{-3\log|\sin x|}$

$$=e^{\log\left|\frac{1}{\sin^3 x}\right|}=\frac{1}{\sin^3 x}.$$

The general solution of the given differential equation is given by the relation.

$$y(\text{IF})=\int(Q\times IF)dx+C$$

$$\Rightarrow \quad y.\frac{1}{\sin^3 x}=\int\left[\sin 2x.\frac{1}{\sin^3 x}\right]dx+C$$

$$\Rightarrow \quad y\text{cosec}^3x=2\int(\cot x\text{cosec}x)dx+C \quad \ldots\textbf{(1)}$$

$$\Rightarrow \quad y\text{cosec}^3x=2\text{cosec}\,x+C$$

$$\Rightarrow \quad y=-\frac{2}{\text{cosec}^2x}+\frac{3}{\text{cosec}^3x}$$

$$\Rightarrow \quad y=-2\sin^2x+C\sin^3x$$

Now, $y=2$ at $x=\frac{\pi}{2}$.

Therefore, we get:

$$2=-2+\text{C}$$

$$\Rightarrow \quad \text{C}=4$$ **[2½]**

Substituting C = 4 in equation (1), we get :

$$y=-2\sin^2x+4\sin^3x$$

$$\Rightarrow y=4\sin^3x-2\sin^2x$$

This is the required particular solution of the given differential equation. **[2½]**

Q. 53. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (*x*, *y*) is equal to the sum of the coordinates of the point.

[NCERT Ex. 9.6, Q. 16, Page 414]

Ans.

Let $F(x, y)$ be the curve passing through the origin.

At point (x, y), the slope of the curve will be $\frac{dy}{dx}$.

According to the given information,

$$\frac{dy}{dx}=x+y$$

$$\Rightarrow \frac{dy}{dx}-y=x$$

This is a linear differential equation of the form :

$$\frac{dy}{dx}+py=Q \text{ (where } p=-1 \text{ and } Q=x)$$

Now, IF$=e^{\int pdx}=e^{\int(-1)dx}=e^{-x}$.

The general solution of the given differential equation is given by the relation.

$$y(\text{IF})=\int(\text{Q}\times\text{IF})dx+C$$

$$\Rightarrow \quad ye^{-x}=\int xe^{-x}dx+C \quad \ldots\textbf{(1)}$$

Now,

$$\int xe^{-x}dx=x\int e^{-x}dx-\int\left[\frac{d}{dx}(x).\int e^{-x}dx\right]dx.$$

$$=-xe^{-x}-\int -e^{-x}dx$$

$$=-xe^{-x}+\left(-e^{-x}\right)$$

$$=-e^{-x}(x+1)$$ **[2½]**

Substituting in equation (1), we get :

$$ye^{-x}=-e^{-x}(x+1)+C$$

$$\Rightarrow \quad y=-\left(x+1\right)+Ce^x$$

$$\Rightarrow x+y+1=Ce^x \quad \ldots\textbf{(2)}$$

The curve passes through the origin.

Therefore, equation (2) becomes :

$1=C$

$\Rightarrow C=1$

Substituting C = 1 in equation (2), we get :

$x+y+1=e^x$

Hence, the required equation of curve passing through the origin is $x+y+1=e^x$ **[2½]**

Q. 54. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

[NCERT Ex. 9.6, Q. 17, Page 414]

Ans. Let F(x, y) be the curve and let (x, y) be a point on the curve. The slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given information,

$$\frac{dy}{dx}+5=x+y$$

$$\Rightarrow \frac{dy}{dx}-y=x-5$$

This is a linear differential equation of the form :

$\frac{dy}{dx}+py=Q$ (where $p=-1$ and $Q=x-5$)

Now, IF $=e^{\int pdx}=e^{\int(-1)dx}=e^{-x}$

The general equation of the curve is given by the relation,

$$y(\text{IF})=\int(Q\times\text{IF})dx+C$$

$$\Rightarrow y.e^{-x}=\int(x-5)e^{-x}dx+C \quad \text{...(1)}$$

Now,

$$\int(x-5)e^{-x}dx=(x-5)\int e^{-x}dx-\int\left[\frac{d}{dx}(x-5).\int e^{-x}dx\right]dx.$$

$$=(x-5)(-e^{-x})-\int(-e^{-x})dx$$

$$=(5-x)e^{-x}+(-e^{-x})$$

$$=(4-x)e^{-x}$$

Therefore, equation (1) becomes:

$$ye^{-x}=(4-x)e^{-x}+C$$

$$\Rightarrow y=4-x+Ce^x$$

$$\Rightarrow x+y-4=Ce^x \quad \text{...(2)}$$

[2½]

The curve passes through point (0,2)

Therefore, equation (2) becomes :

$$0+2-4=Ce^0$$

$$\Rightarrow -2=C$$

$$\Rightarrow C=-2$$

Substituting C = –2 in equation (2), we get :

$$x+y-4=-2e^x$$

$$\Rightarrow y=4-x-2e^x$$

This is the required equation of the curve. **[2½]**

Q. 55. Prove that $x^2 - y^2 = C(x^2 + y^2)^2$ is the general solution of differential equation $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$, where C is a parameter.

[NCERT Misc. Ex. Q. 4, Page 420]

Ans.

$$\left(x^3-3xy^2\right)dx=\left(y^3-3x^2y\right)dy$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^3-3xy^2}{y^3-3x^2y} \quad \text{...(1)}$$

This is a homogeneous equation. To simplify it, we need to make the substitution as :

$$y=vx$$

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dv}{dx}$ in equation (1), we get:

$$v+x\frac{dv}{dx}=\frac{x^3-3x(vx)^2}{(vx)^3-3x^2(vx)}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{1-3v^2}{v^3-3v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1-3v^2}{v^3-3v}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1-3v^2-v(v^3-3v)}{v^3-3v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1-v^4}{v^3-3v}$$

$$\Rightarrow \left(\frac{v^3-3v}{1-v^4}\right)dv=\frac{dx}{x}$$

Integrating both sides, we get:

$$\int\left(\frac{v^3-3v}{1-v^4}\right)dv=\log x+\log C' \quad \text{...(2)}$$

Now, $\int\left(\frac{v^3-3v}{1-v^4}\right)dv=\int\frac{v^3dv}{1-v^4}-3\int\frac{vdv}{1-v^4}$

Let

$$1-v^4=t.$$

$$\therefore \frac{d}{dv}(1-v^4)=\frac{dt}{dv}$$

$$\Rightarrow -4v^3=\frac{dt}{dv}$$

$$\Rightarrow v^3dv=-\frac{dt}{4}$$

Now, $\quad I_1=\int\frac{-dt}{4t}=-\frac{1}{4}\log t=-\frac{1}{4}\log(1-v^4)$

And, $\quad I_2=\int\frac{vdv}{1-v^4}=\int\frac{vdv}{1-(v^2)^2}$

Let

$$v^2=p.$$

$$\therefore \frac{d}{dv}(v^2)=\frac{dp}{dv}$$

$$\Rightarrow 2v=\frac{dp}{dv}$$

$$\Rightarrow vdv=\frac{dp}{2}$$

$$\Rightarrow I_2=\frac{1}{2}\int\frac{dp}{1-p^2}=\frac{1}{2\times2}\log\left|\frac{1+p}{1-p}\right|$$

$$=\frac{1}{4}\log\left|\frac{1+v^2}{1-v^2}\right|$$

Substituting the values of I_1 and I_2 in equation (3), we get:

$$\int\left(\frac{v^3-3v}{1-v^4}\right)dv=-\frac{1}{4}\log(1-v^4)-\frac{3}{4}\log\left|\frac{1-v^2}{1+v^2}\right|$$

Therefore, equation (2) becomes:

$$\frac{1}{4}\log(1-v^4)-\frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right|=\log x+\log C'$$

[2½]

$$\Rightarrow -\frac{1}{4}\left[(1-v^4)\left(\frac{1+v^2}{1-v^2}\right)^3\right]=\log C'x$$

$$\Rightarrow \frac{(1+v^2)^4}{(1-v^2)^2}=(C'x)^{-4}$$

$$\Rightarrow \frac{\left(1+\frac{y^2}{x^2}\right)^4}{\left(1-\frac{y^2}{x^2}\right)^2}=\frac{1}{C'^4x^4}$$

$$\Rightarrow \frac{(x^2+y^2)^4}{x^4(x^2-y^2)^2}=\frac{1}{C'^4x^4}$$

$$\Rightarrow (x^2-y^2)^2=C'^4(x^2+y^2)^4$$

$$\Rightarrow (x^2-y^2)=C'^2(x^2+y^2)^2$$

$$\Rightarrow x^2-y^2=C(x^2+y^2)^2,$$

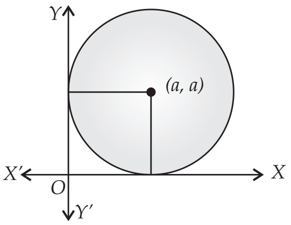
where $C=C'^2$

Hence, the given result is proved. **[2½]**

Q. 56. Form the differential equation of the family of circles in the first quadrant which touch the coordinate axes. [NCERT Misc. Ex. Q. 5, Page 420]

Ans. The equation of a circle in the first quadrant with centre (*a*, *a*) and radius (*a*) which touches the coordinate axes is :

$$(x-a)^2+(y-a)^2=a^2 \quad \text{...(1)}$$



Differentiating equation (1) with respect to x, we get :

$$2(x-a)+2(y-a)\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)+(y-a)y'=0$$

$$\Rightarrow x-a+yy'-ay'=0$$

$$\Rightarrow x+yy'-a(1+y')=0$$

$$\Rightarrow a=\frac{x+yy'}{1+y'}$$

[2½]

Substituting the value of a in equation (1), we get :

$$\left[x-\left(\frac{x+yy'}{1+y'}\right)\right]^2+\left[y-\left(\frac{x+yy'}{1+y'}\right)\right]^2=\left(\frac{x+yy'}{1+y'}\right)^2$$

$$\Rightarrow \left[\frac{(x-y)y'}{(1+y')}\right]^2+\left[\frac{y-x}{1+y'}\right]^2=\left[\frac{x+yy'}{1+y'}\right]^2$$

$$\Rightarrow (x-y)^2.y'^2+(x-y)^2=(x+yy')^2$$

$$\Rightarrow (x-y)^2\left[1+(y')^2\right]=(x+yy')^2$$

Hence, the required differential equation of the family of circles is $(x-y)^2\left[1+(y')^2\right]=(x+yy')^2$ **[2½]**

Some Commonly Made Errors

- Arbitrary error is that error in which the subject behaved arbitrarily and failed to take into account the constraints laid down in what was given.
- Usually student fails to carry out manipulations, though the principles involved may have been understood.
- They make computational and algebraic errors.
- Errors in transformation and process skill in solving the differential problems.

EXPERT ADVICE

- *Try to learn the procedures involved in solving the differential equations.*
- *Learn formulae through practicing.*
- *A differential equation is an equation that relates a function with one or more of its derivatives.*

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