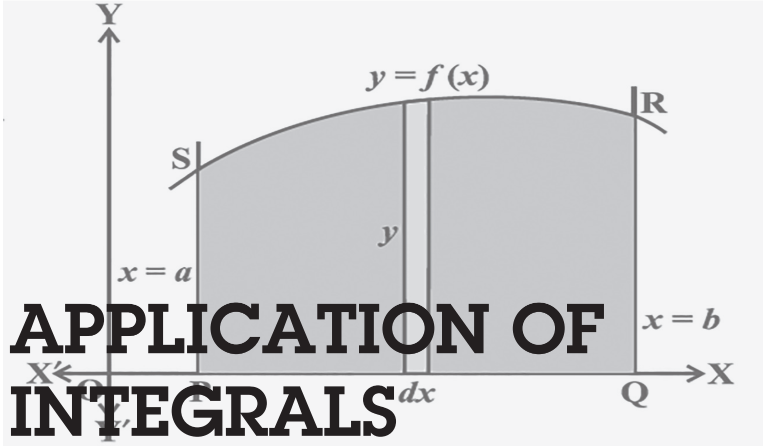
CHAPTER

8 APPLICATION OF INTEGRALS



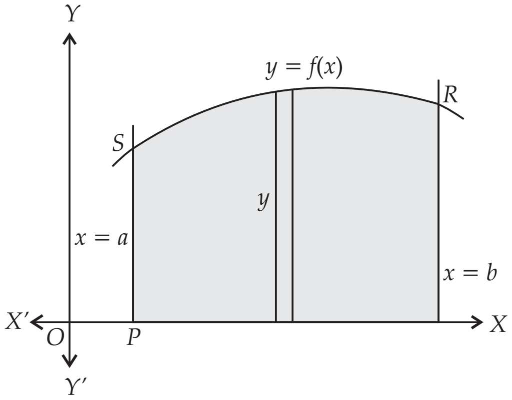
Chapter Objectives

This chapter will help you understand :

- *Area of the region bounded by a curve and a line.*
- *Areas between two curves.*

Quick Review

- Consider the easy way of finding the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$. From Figure, we can find area under the curve (AUC) as composed of large number of very thin vertical strips. Consider an arbitrary strip of height y and width dx, then dA (area of the elementary strip) $= ydx$, where, $y = f(x)$.



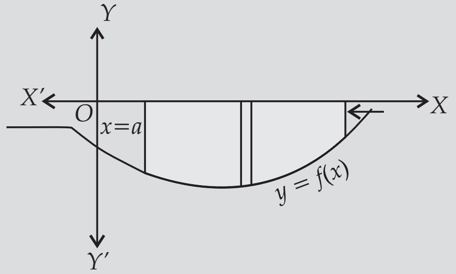
- This area is called the elementary area, which is located at an arbitrary position within the region, which is specified by some value of x between a and b. We can think of the total area A of the region between x-axis, ordinates $x = a$, $x = b$ and the curve $y = f(x)$ as the result of adding up the elementary areas of thin strips across the region PQRSP. Symbolically, we express :

$$A = \int_a^b dA = \int_a^b ydx = \int_a^b f(x)dx$$

- The area A of the region bounded by the curve $x = g(y)$, y-axis and the lines $y = c$, $y = d$ is given by :

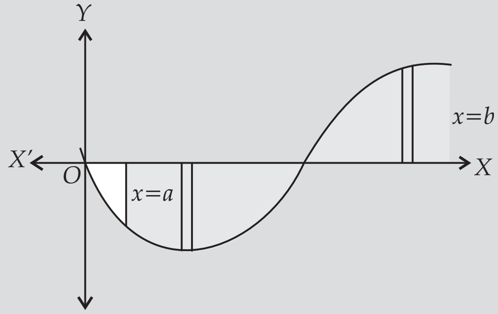
TIPS...

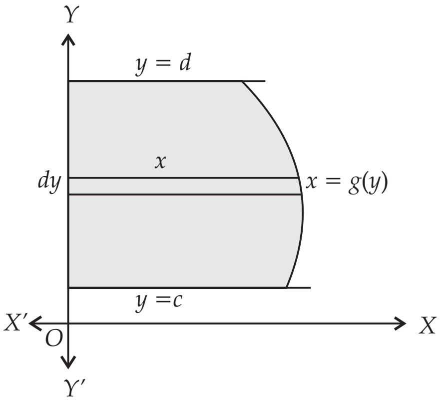
- *If the position of the curve under consideration is below the x-axis, then since $f(x) < 0$ from $x = a$ to $x = b$, as shown in Figure, the area bounded by the curve, x-axis and the ordinates $x = a$, $x = b$ comes out to be negative. But, it is only the numerical value of the area which is taken into consideration. Thus, if the area is negative, we take its absolute value,*



$$A = \left| \int_a^b f(x)dx \right|$$

- *It may happen that some portion of the curve is above x-axis and some is below the x-axis as shown in the given figure. Here, $A_1 < 0$ and $A_2 > 0$. Therefore, the area A bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is given by*





$$A = \int_c^d x\,dy = \int_c^d g(y)\,dy$$

> **TRICKS...**
> - *Learn all integral formulae from chapter 'Integration'.*
> - *Convert equations of curves in standard form before finding area between two curves.*

- **Similarly the area of the region enclosed between two curves** $y = f(x)$, $y = g(x)$ **and the lines** $x = a$, $x = b$ **is given by the formula,**

  $A = \int_b^a [f(x) - g(x)]dx$ [where, $f(x) \geq g(x)$ in (a, b)]

- **If** $f(x) \geq g(x)$ **in** $[a, c]$ **and** $f(x) \leq g(x)$ **in** $[c, b]$, $a < c < b$, **then**

  $$A = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$$

Know the Links

- *www.teacherschoice.com.au/maths_library/calculus/area_under_a_curve.htm*
- *https://www.intmath.com/applications-integration/2-area-under-curve.php*
- *https://revisionmaths.com/advanced-level-maths-revision/pure-maths/calculus/area-under-curve*

Multiple Choice Questions **(1 mark each)**

Q. 1. The area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \leq x \leq \pi/2$ is

(a) $\sqrt{2}$ sq. units **(b) $(\sqrt{2}+1)$ sq. units**

(c) $(\sqrt{2}-1)$ sq. units **(d) $(2\sqrt{2}-1)$ sq. units**

[NCERT Exemp. Ex. 8.3, Q. 24, Page 177, NCERT Exemp. Ex. Q. 19, Page 376]

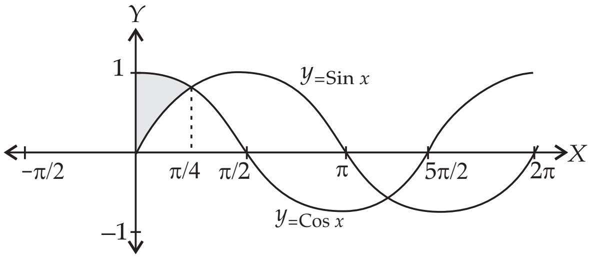
Ans. Correct option : (c)

Explanation : We have $y = \cos x$ and $y = \sin x$, where $0 \leq x \leq \frac{\pi}{2}$.

We get $\cos x = \sin x$

$x = \frac{\pi}{4}$

From the figure, area of the shaded region,



$$A = \int (\cos x + \sin x)\,dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}}$$

$$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = (\sqrt{2}-1) \text{ sq. units}$$

Q. 2. The area of the region bounded by the curve $x^2 = 4y$ and the straight-line $x = 4y - 2$ is

(a) $\frac{3}{8}$ sq. units **(b) $\frac{5}{8}$ sq. units**

(c) $\frac{7}{8}$ sq. units **(d) $\frac{9}{8}$ sq. units**

[NCERT Exemp. Ex. 8.3, Q. 25, Page 177]

Ans. Correct option : (d)

Explanation :

$x^2 = x + 2$

$x^2 - x - 2 = 0$

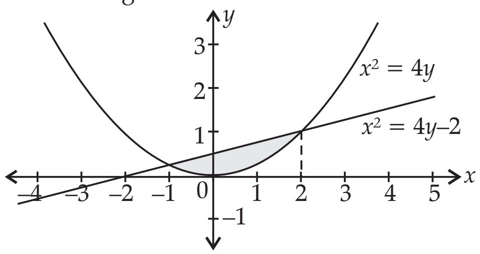
$(x-2)(x+1) = 0$

$x = -1,\ 2$

For $x = -1,\ y = \frac{1}{4}$ and for $x = 2, y = 1$

Points of intersection are $(-1, \frac{1}{4})$ and (2, 1).

Graphs of parabola $x^2 = 4y$ and $x = 4y - 2$ are shown in figure :



$$A = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[8 - \frac{1}{2} - 3\right]$$

$$= \frac{1}{4}\left[8 - \frac{1}{2} - 3\right]$$

$$= \frac{9}{8} \text{ sq. units}$$

Q. 3. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

(a) 8π sq. units **(b) 20π sq. units**

(c) 16π sq. units **(d) 256π sq. units**

[NCERT Exemp. Ex. 8.3, Q. 26, Page 177]

Ans. Correct option : (a)

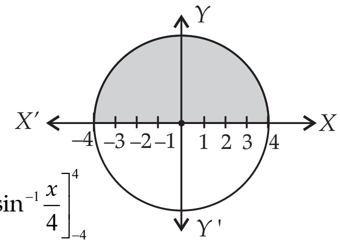
Explanation : We have $y = \sqrt{16 - x^2}$

$$y^2 = 16 - x^2,\ y \geq 0$$

$$y^2 + x^2 = 16,\ y \geq 0$$

Graph of above function is semi-circle lying above the graph as shown in the adjacent figure.

From the figure, area of the shaded region,



$$A = \int_{-4}^{4}(\sqrt{16 - x^2})dx$$

$$= \int_{-4}^{4}(\sqrt{4^2 - x^2})dx$$

$$= \left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$$

$$= \left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}\frac{4}{4}\right] - \left[-\frac{4}{2}\sqrt{4^2 - (-4)^2} + 8\sin^{-1}\left(\frac{-4}{4}\right)\right]$$

$$= 0 + 8\sin^{-1}1 - 0 - 8\sin^{-1}(-1)$$

$$= 4\pi + 4\pi$$

$$= 8\pi \text{ sq. units}$$

Q. 4. Area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is

(a) 16π sq. units **(b) 4π sq. units**

(c) 32π sq. units **(d) 24π sq. units**

[NCERT Exemp. Ex. 8.3, Q. 27, Page 178]

Ans. Correct option : (b)

Explanation : We have $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in the first quadrant.

Solving $y = x$ with the circle

$$x^2 + x^2 = 32$$

$$x^2 = 16$$

$$x = 4 \quad \text{(In the first quadrant)}$$

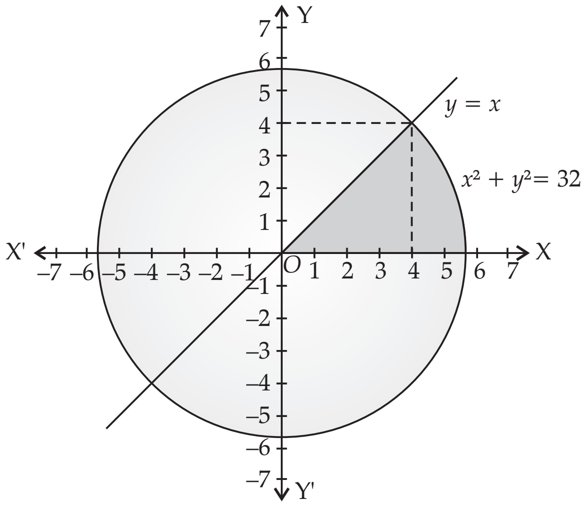
When $x = 4$, $y = 4$ for the point of intersection of the circle with the x-axis.

Put $y = 0$

$$x^2 + 0 = 32$$

$$x = \pm 4\sqrt{2}$$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2}, 0)$.



From the above figure, area of the shaded region,

$$A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \left[\frac{16}{2}\right] + \left[\begin{matrix} 0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16^2} \\ -16\sin^{-1}\frac{4}{4\sqrt{2}} \end{matrix}\right]$$

$$= 8 + \left[16\ \pi/2 - 2\sqrt{16} - 16\ \frac{\pi}{4}\right] = 8 + [8\ \pi - 8 - 4\ \pi]$$

$$= 4\pi \text{ sq. units}$$

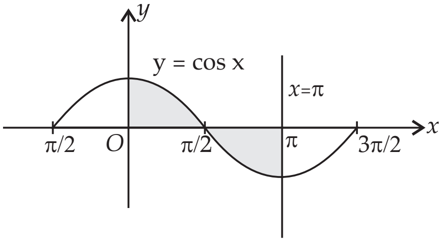
Q. 5. Area of the region bounded by the curve y = cos x between $x = 0$ and $x = \pi$ is

(a) 2 sq. units **(b) 4 sq. units**
(c) 3 sq. units **(d) 1 sq. unit**

[NCERT Exemp. Ex. 8.3, Q. 28, Page 178]

Ans. Correct option : (a)

Explanation : We have $y = \cos x,\ x = 0,\ x = \pi$



From the figure, area of the shaded region,

$$A = \int_0^{\pi} |\cos x| dx + \int_0^{\pi/2} \cos x dx$$

$$= 2[\sin x]_0^{\frac{\pi}{2}} = 2 \text{ sq. units}$$

Q. 6. The area of the region bounded by parabola $y^2 = x$ and the straight line 2y = x is

(a) $\frac{4}{3}$ sq. units **(b) 1 sq. units**

(c) $\frac{2}{3}$ sq. units **(d) $\frac{1}{3}$ sq. units**

[NCERT Exemp. Ex. 8.3, Q. 29, Page 178]

Ans. Correct option : (a)

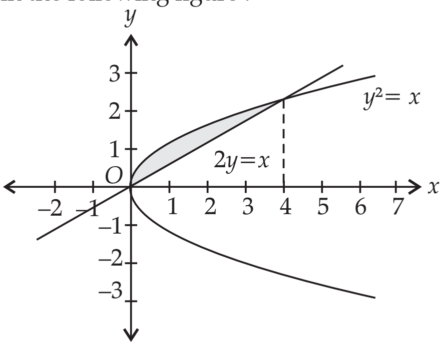
Explanation : When $y^2 = x$ and $2y = x$

Solving we get $y^2 = 2y$

$\Rightarrow y = 0,\ 2$ and when $y = 2,\ x = 4$

So, points of intersection are (0, 0) and (4, 2).

Graphs of parabola $y^2 = x$ and $2y = x$ are as shown in the following figure :



From the figure, area of the shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}.\frac{x^2}{2}\right]_0^4$$

$$= \frac{2}{3}.(4)^{3/2} - \frac{16}{4} - 0$$

$$= \frac{16}{3} - 4$$

$$= \frac{4}{3} \text{ sq. units}$$

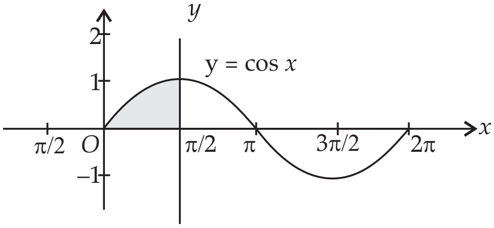
Q. 7. The area of the region bounded by the curve y = sin$x$ between the ordinates $x = 0,\ x = \frac{\pi}{2}$ and the axis is

(a) 2 sq. units **(b) 4 sq. units**
(c) 3 sq. units **(d) 1 sq. units**

[NCERT Exemp. Ex. 8.3, Q. 30, Page 178]

Ans. Correct option : (d)

Explanation : We have y = **sin**x, $0 \le x \le \frac{\pi}{2}$.



From the figure, area of the shaded region,

$$A = \int_0^{\pi/2} \sin x dx$$

$$= [-\cos x]_0^{\pi/2}$$

$$= \left[-\cos\frac{\pi}{2} + \cos 0\right]$$

$$= 0 + 1$$

= 1 sq. unit

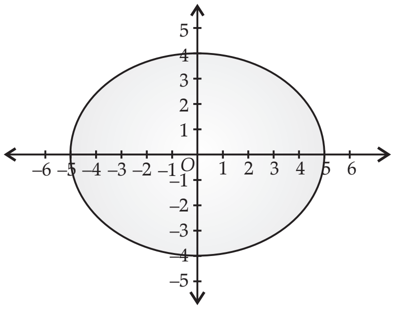
Q. 8. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(a) 20π sq. units **(b) $20\pi^2$ sq. units**
(c) $16\pi^2$ sq. units **(d) 25π sq. units**

[NCERT Exemp. Ex. 8.3, Q. 31, Page 178]

Ans. Correct option : (a)

Explanation : We have $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$, which is ellipse with its axes as coordinate axes.



$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

$$y^2 = 16\left(1 - \frac{x^2}{25}\right)$$

$y=\frac{4}{5}\sqrt{5^2-x^2}$

From the figure, area of the shaded region,

$$A=4\int_0^5 \frac{4}{5}\sqrt{5^2-x^2}\,dx$$

$$=\frac{16}{5}\left[\frac{x}{2}\sqrt{5^2-x^2}-\frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$=\frac{16}{5}\left[0+\frac{5^2}{2}\sin^{-1}1-0-0\right]$$

$$=\frac{16}{5}.\frac{25}{2}.\frac{\pi}{2}$$

$= 20\pi$ sq. units

Q. 9. The area of the region bounded by the circle $x^2+y^2=1$ is

(a) 2π **sq. units** **(b)** π **sq. units**
(c) 3π **sq. units** **(d)** 4π **sq. units**

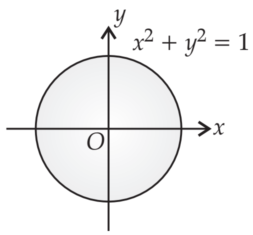
[NCERT Exemp. Ex. 8.3, Q. 32, Page 178]

Ans. Correct option : (b)

Explanation : We have, $x^2+y^2=1$, which is a circle having centre at (0, 0) and radius '1' unit.

$\Rightarrow y^2=1-x^2$

$y=\sqrt{1-x^2}$



From the figure, area of the shaded region,

$$A=4\int_0^1 \sqrt{1^2-x^2}\,dx$$

$$=4\left[\frac{x}{2}\sqrt{1^2-x^2}-\frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$=4\left[0+\frac{1^2}{2}\times\frac{\pi}{2}-0-0\right]$$

$= \pi$ sq. units

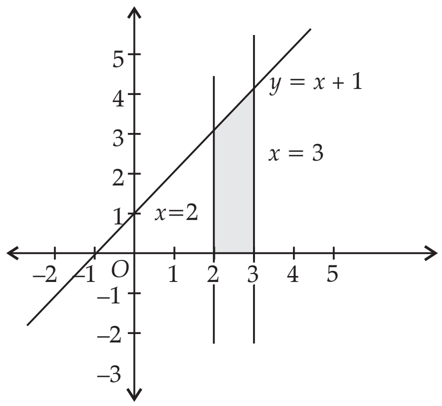
Q. 10. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is

(a) $\frac{7}{2}$ **sq. units** **(b)** $\frac{9}{2}$ **sq. units**
(c) $\frac{11}{2}$ **sq. units** **(d)** $\frac{13}{2}$ **sq. units**

[NCERT Exemp. Ex. 8.3, Q. 33, Page 178]

Ans. Correct option : (a)

Explanation :



From the figure, area of the shaded region,

$$A=\int_2^3 (x+1)\,dx$$

$$=\left[\frac{x^2}{2}+x\right]_2^3$$

$$=\left[\frac{9}{2}+3-\frac{4}{2}-2\right]$$

$=\frac{7}{2}$ sq. units

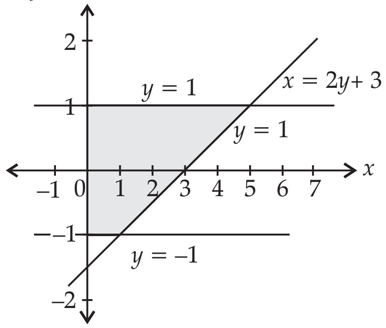
Q. 11. The area of the region bounded by the curve $x = 2y + 3$ and the y lines y = 1 and $y = -1$ is,

(a) **4 sq. units** **(b)** $\frac{3}{2}$ **sq. units**
(c) **6 sq. units** **(d)** **8 sq. units**

[NCERT Exemp. Ex. 8.3, Q. 34, Page 178]

Ans. Correct option : (c)

Explanation :



From the figure, area of the shaded region,

$$A=\int_{-1}^{1}(2y+3)\,dy$$

$$=\left[y^2+3y\right]_{-1}^{1}$$

$$=\left[1+3-1+3\right]$$

$= 6$ sq. units

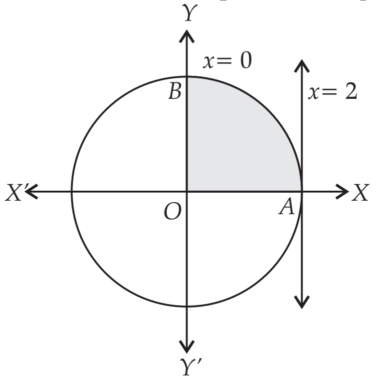
Q. 12. Area lying in the first quadrant and bounded by circle $x^2+y^2=4$ and the lines $x = 0$ and $x = 2$ is

(a) π **(b)** $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{4}$

[NCERT Ex. 8.1, Q. 12, Page 366]

Ans. Correct option : (a)

Explanation : The area bounded by the circle and the lines in the first quadrant is represented as :



$$A = \int_0^2 y\,dx$$

$$= \int_0^2 \sqrt{4 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$= \pi$ sq. units

Q. 13. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

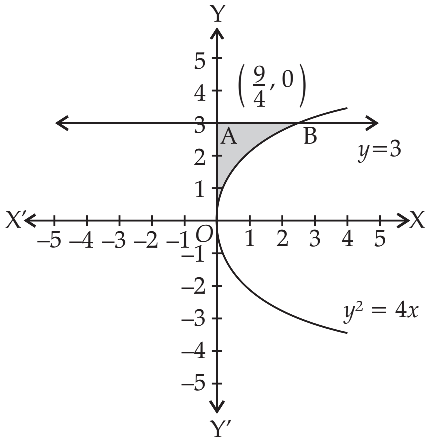
(a) **2** **(b)** $\mathbf{\frac{9}{4}}$

(c) $\mathbf{\frac{9}{3}}$ **(d)** $\mathbf{\frac{9}{2}}$

[NCERT Ex. 8.1, Q. 13, Page 366]

Ans. Correct option : (b)

Explanation : The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as :



Area of OAB $= A = \int_0^3 x\,dy$

$$= \int_0^3 \frac{y^2}{4}dy$$

$$= \frac{1}{4}\left[\frac{y^3}{4}\right]_0^3$$

$$= \frac{1}{12} \times 27$$

$$= \frac{9}{4} \text{ sq. units}$$

Q. 14. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$

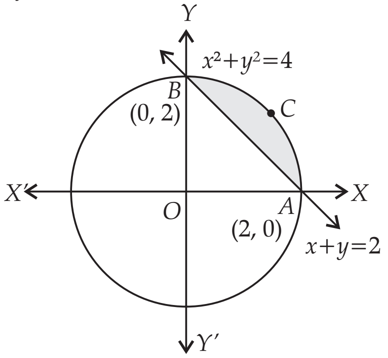
(a) $\mathbf{2(\pi - 2)}$ **(b)** $\mathbf{\pi - 2}$

(c) $\mathbf{2\pi - 1}$ **(d)** $\mathbf{2(\pi + 2)}$

[NCERT Ex. 8.2, Q. 6, Page 372]

Ans. Correct option : (b)

Explanation : The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line, $x + y = 2$ is represented by the shaded area ACBA as :



It can be observed that

Area of ACBA = Area of OACBO − Area of ΔAOB

$$A = \int_0^2 \sqrt{4 - x^2}\,dx - \int_0^2 (2 - x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x}{2}\right]_0^2$$

$$= \left[2 \times \frac{\pi}{2}\right] - [4 - 2]$$

$= \pi - 2$ sq. units

Q. 15. Area lying between the curve $y^2 = 4x$ and $y = 2x$

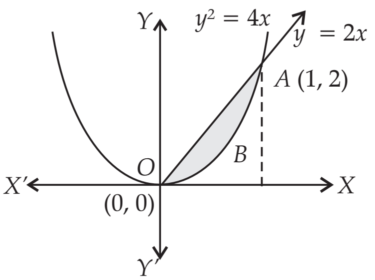
(a) $\mathbf{\frac{2}{3}}$ **(b)** $\mathbf{\frac{1}{3}}$

(c) $\mathbf{\frac{1}{4}}$ **(d)** $\mathbf{\frac{3}{4}}$

[NCERT Ex. 8.2, Q. 7, Page 372]

Ans. Correct option : (b)

Explanation : The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area OBAO as



The points of intersection of the curves are O(0, 0) and A(1, 2).

We draw AC perpendicular to x-axis such that coordinate of C is (1, 0).

Area of OBAO = Area of ΔOCA – Area of OCABO

$$A = \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^1$$

$$= \left|\left[1 - \frac{4}{3}\right]\right|$$

$$= \left|-\frac{1}{3}\right|$$

$$= \frac{1}{3} \text{ sq. units}$$

Q. 16. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(a) -9 **(b)** $-\frac{15}{4}$

(c) $\frac{15}{4}$ **(d)** $\frac{17}{4}$

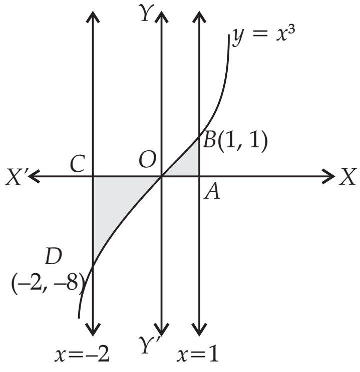
[NCERT Misc. Ex. Q. 16, Page 376]

Ans. Correct option : (b)

Explanation : Required area,

$$A = \int_{-2}^{1} y\,dx$$

$$= \int_{-2}^{1} x^3\,dx$$



$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left(\frac{1}{4} - 4\right)$$

$$= -\frac{15}{4}$$

$$\therefore \text{Area} = \left|-\frac{15}{4}\right| = \frac{15}{4} \text{ sq. units}$$

Q. 17. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

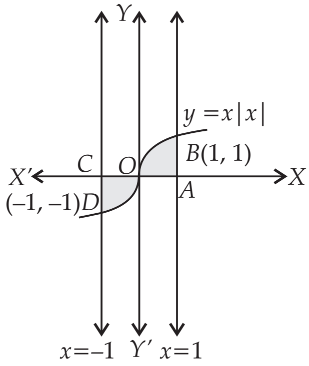
(a) 0 **(b)** 1/3

(c) 2/3 **(d)** 4/3

[NCERT Misc. Ex. Q. 17, Page 376]

Ans. Correct option : (c)

Explanation :



Required, area

$$A = \int_{-1}^{1} y\,dx$$

$$= \int_{-1}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= \frac{2}{3} \text{ sq. units}$$

Q. 18. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

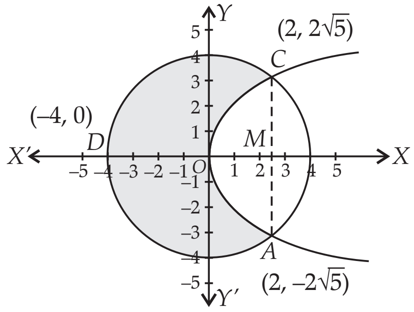
(a) $\frac{4}{3}(4\pi - \sqrt{3})$ **(b)** $\frac{4}{3}(4\pi + \sqrt{3})$

(c) $\frac{4}{3}(8\pi - \sqrt{3})$ **(d)** $\frac{4}{3}(8\pi + \sqrt{3})$

[NCERT Misc. Ex. Q. 18, Page 376]

Ans. Correct option : (c)

Explanation :



Area bounded by the circle and parabola

$= 2[\text{area } (OAMO) + \text{area } (AMBA)]$

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

$$= 2\int_0^2 \sqrt{6x}\,dx + 2\int_2^4 \sqrt{16 - x^2}\,dx$$

$$= 2\sqrt{6}\int_0^2 \sqrt{x}\,dx + 2\int_2^4 \sqrt{16 - x^2}\,dx$$

$$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4$$

$$= \frac{4\sqrt{6}}{2}\left(2\sqrt{2} - 0\right) + 2\left[\left\{0 + 8\sin^{-1}(1)\right\} - \left\{2\sqrt{3} + 8\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$$

$$= \frac{16\sqrt{3}}{3} + 2\left[8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right]$$

$=\frac{16\sqrt{3}}{3}+2\left(4\pi-2\sqrt{3}-\frac{4\pi}{3}\right)$

$=\frac{16\sqrt{3}}{3}+8\pi-4\sqrt{3}-\frac{8\pi}{3}$

$=\frac{16\sqrt{3}+24\pi-4\sqrt{3}-8\pi}{3}$

$=\frac{16\pi+12\sqrt{3}}{3}$

$=\frac{4}{3}\left[4\pi+\sqrt{3}\right]$ sq. units

Area of circle $=\pi(r)^2$

$=\pi(4)^2$

$=16$ sq. units

$\therefore$ Required area $=16\pi-\frac{4}{3}\left(4\pi+\sqrt{3}\right)$

$=16\pi-\frac{16\pi}{3}-\frac{4\sqrt{3}}{3}$

$=\frac{32\pi}{3}-\frac{4\sqrt{3}}{3}$

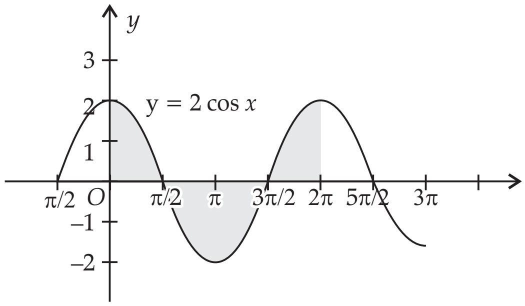
$=\frac{4}{3}\left[8\pi-\sqrt{3}\right]$ sq. units

Very Short Answer Type Questions (2 marks each)

Q. 1. Find the area bounded by the curve $y = 2\cos x$ and the x-axis from $x = 0$ to $x = 2\pi$.
[NCERT Exemp. Ex. 8.3, Q. 22, Page 177]

Ans. We have $y=2\cos x, 0\le x\le 2\pi$

Graph of the functions is as shown in the following figure.



From the figure, area of the shaded region

$A=\int_0^{2\pi}|2\cos x|dx=4\int_0^{\pi/2}(2\cos x)dx=8[\sin x]_0^{\pi/2}$

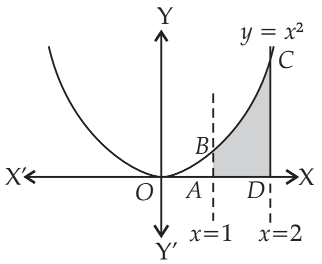
$=8$ sq. units **[2]**

Q. 2. Find the area under the given curves and given lines :

(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x-axis

[NCERT Misc. Ex. Q. 1, Page 375]



Ans. **(i)** Required area = Area $ABCD$

$=\int_1^2 ydx$

Here, $y=x^2$

$=\int_1^2 x^2dx$

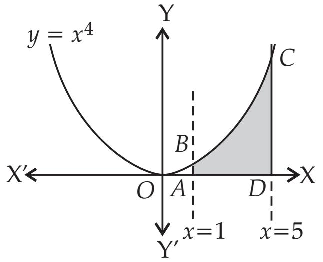
$=\left[\frac{x^3}{3}\right]_1^2$

$=\frac{1}{3}\left[2^3-1^3\right]$

$=\frac{1}{3}[8-1]$

$=\frac{7}{3}$ sq. units **[1]**

(ii)



Required area = Area ABCD

$=\int_1^5 ydx$

Here, $y = x^4$

$=\int_1^5 x^4dx$

$=\left[\frac{x^5}{5}\right]_1^5$

$=\frac{5^5}{5}-\frac{1^5}{5}$ **[1]**

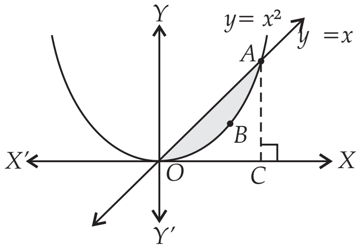
$=5^4-\frac{1}{5}$

$=625-0.2$

$=624.8$ sq. units

Q. 3. Find the area between the curves $y = x$ and $y = x^2$.
[NCERT Misc. Ex. Q. 2, Page 375]

Ans. The required area is represented by the shaded area $OBAO$ as



The points of intersection of the curves, $y = x$ and $y = x^2$, is A (1, 1).

We draw AC perpendicular to x-axis.

$\therefore$ Area ($OBAO$) = Area (ΔOCA) – Area ($OCABO$) …(1)

$$= \int_0^1 x dx - \int_0^1 x^2 dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6} \text{ sq.units}$$

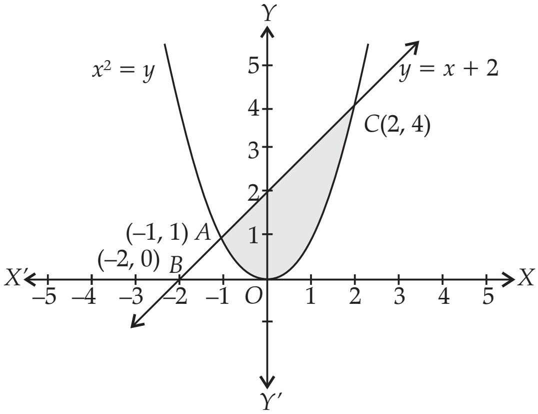
[2]

Q. 4. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis.

[NCERT Misc. Ex. Q. 10, Page 375]

[NCERT Exemp. Ex.8.3 Q. 6, Page 176]

Ans. The area of the region enclosed by the parabola, $x^2 = y$, the line $y = x + 2$, and x-axis is represented by the shaded region $OACO$ as



The point of intersection of the parabola, $x^2 = y$, and the line $y = x + 2$, is A (–1, 1) and C (2, 4).

$$\text{Area of } OACO = \int_{-1}^{2} (x+2)\,dx - \int_{-1}^{2} x^2 dx$$

$$\Rightarrow \text{Area of } OACO = \left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{3}\left[x^3\right]_{-1}^{2}$$

$$\Rightarrow \text{Area of } OACO = \left[\left\{\frac{(2)^2}{2} + 2(2)\right\} - \left\{\frac{(-1)^2}{2} + 2(-1)\right\}\right] - \frac{1}{3}\left[(2)^3 - (-1)^3\right]$$

$$\Rightarrow \text{Area of } OACO = \left[2 + 4 - \left(\frac{1}{2} - 2\right)\right] - \frac{1}{3}(8+1)$$

$$\Rightarrow \text{Area of } OACO = 6 + \frac{3}{2} - 3$$

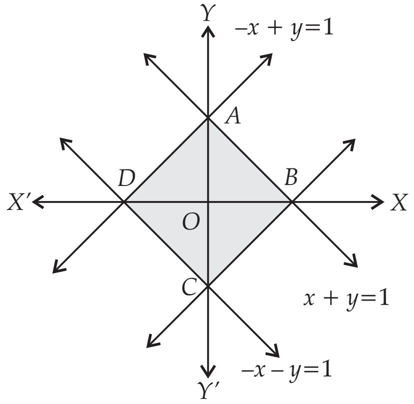
$$\Rightarrow \text{Area of } OACO = 3 + \frac{3}{2} = \frac{9}{2} \text{ sq. units}$$

[2]

Q. 5. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$.

[NCERT Misc. Ex. Q. 11, Page 375]

Ans. The area bounded by the curve, $|x| + |y| = 1$, is represented by the shaded region $ADCB$ as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, –1), and D (–1, 0).

It can be observed that the given curve is symmetrical about x-axis and y-axis

$\therefore$ Area $ADCB$ = 4 × Area $OBAO$

$$= 4\int_0^4 (1-x)dx$$

$$= 4\left(x - \frac{x^2}{2}\right)_0^1$$

$$= 4\left[1 - \frac{1}{2}\right]$$

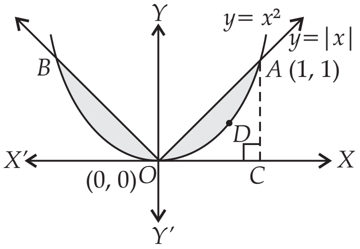
$$= 4\left(\frac{1}{2}\right)$$

$$= 2 \text{ sq. units}$$

[2]

Q. 6. Find the area bounded by curves $\{(x, y) : y \geq x^2$ and $y = |x|\}$. [NCERT Misc. Ex. Q. 12, Page 376]

Ans. The area bounded by the curves, $\{(x,y) : y \geq x^2 \text{ and } y = |x|\}$, is represented by the shaded region as



It can be observed that the required area is symmetrical about y-axis

$$\text{Required area} = 2[\text{Area}(OCAO) - \text{Area}(OCADO)]$$

$$= \left[\int_0^1 x dx - \int_0^1 x^2 dx\right]$$

$$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$$

[2]

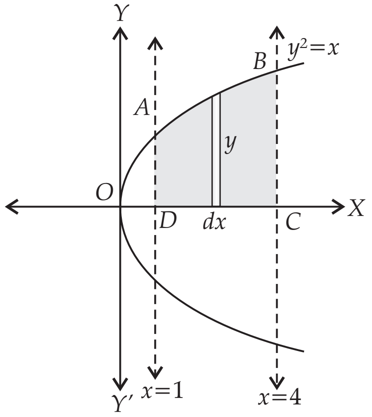
$$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2\left[\frac{1}{6}\right] = \frac{1}{3} \text{ sq. units}$$

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis in the first quadrant. [NCERT Ex. 8.1, Q. 1, Page 365]

Ans. The area of the region bounded by the curve, $y^2 = x$ the lines, $x = 1$ and $x = 4$, and the x-axis is the area of $ABCD$.



[1]

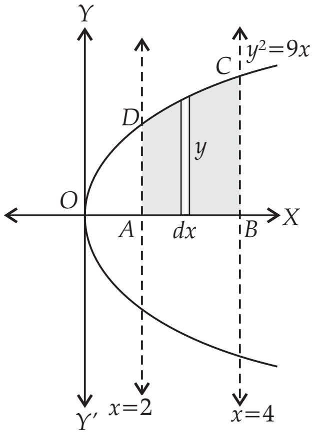
$$\text{Area of } ABCD = \int_1^4 y\,dx = \int_1^4 \sqrt{x}\,dx$$

$$= \left[\frac{x^{3/2}}{\frac{3}{2}}\right]_1^4 = \frac{2}{3}\left[(4)^{3/2} - (1)^{3/2}\right] \quad [2]$$

$$= \frac{2}{3}[8-1] = \frac{14}{3} \text{ sq. units}$$

Q. 2. Find the area of the region bounded by $y^2 = 9x$, $x = 2, x = 4$ and the x-axis in the first quadrant. [NCERT Ex. 8.1, Q. 2, Page 365]

Ans. The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$, and $x = 4$, and the x-axis is the area of $ABCD$.



$$\text{Area of } ABCD = \int_2^4 y\,dx = \int_1^4 3\sqrt{x}\,dx$$

$$= 3\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_2^4 = 2\left[(4)^{3/2} - (2)^{3/2}\right]$$

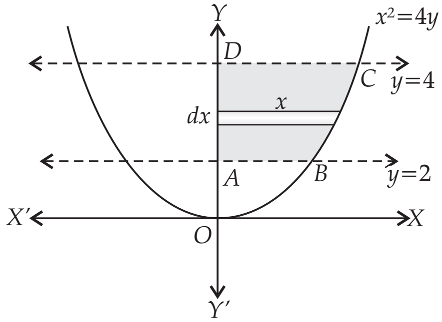
$$= 2[8 - 2\sqrt{2}].$$

$$= [16 - 4\sqrt{2}] \text{ sq. units}$$

[1]

Q. 3. Find the area of the region bounded by $x^2 = 4$, $y = 2$, $y = 4$ and the y-axis in the first quadrant. [NCERT Ex. 8.1, Q. 3, Page 366]

Ans.



[1]

The area of the region bounded by the curve $x^2 = 4y$, $y = 2$, and $y = 4$, and the y-axis is the area of $ABCD$.

$$\text{Area of } ABCD = \int_2^4 x\,dy = 2\int_1^4 \sqrt{y}\,dy$$

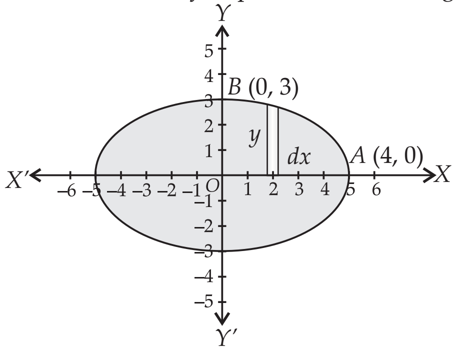
$$= 2\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_2^4 = \frac{4}{3}\left[(4)^{3/2} - (1)^{3/2}\right] \quad [2]$$

$$= \frac{4}{3}[8 - 2\sqrt{2}] = \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ sq. units}$$

Q. 4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. [NCERT Ex. 8.1, Q. 4, Page 366]

Ans. It can be observed that the ellipse is symmetrical about x-axis and y-axis.

$\therefore$ Area bounded by ellipse = 4 × Area of region OAB



[½]

$$\text{Area of region } OAB = \int_0^4 y\,dx$$

$$= \int_0^4 3\sqrt{1 - \frac{x^2}{16}}\,dx$$

$$= \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\,dx$$

$$= \frac{3}{4}\left[2\sqrt{16-16} + 8\sin^{-1}(1) - 8\sin^{-1}(0)\right]$$

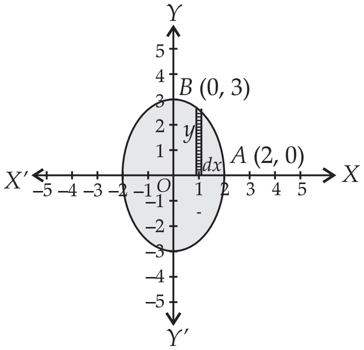
$=\frac{3}{4}\left[\frac{8\pi}{2}\right]$

$=3\pi$ sq. units **[2]**

Area bounded by ellipse = 4 × Area of region *OAB*

$=4\times 3\pi=12\pi$ sq. units **[½]**

Q. 5. Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. [NCERT Ex. 8.1, Q. 5, Page 366]



[½]

It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis. Thus,

Area bounded by ellipse = 4 × Area of region *OAB*

Area of region $OAB=\int_0^2 y\,dx$

$$=\int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx$$

$$=\frac{3}{2}\int_0^2\sqrt{4-x^2}\,dx$$

$$=\frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

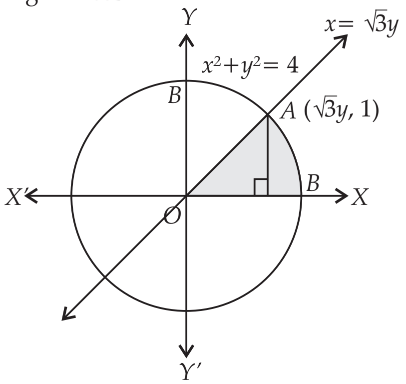
$$=\frac{3}{4}\left[\frac{2\pi}{2}\right]=\frac{3\pi}{2}\text{ sq. units}$$ **[2]**

Area bounded by ellipse = 4 × Area of region *OAB*

$$=4\times\frac{3\pi}{2}=6\pi\text{ sq. units}$$ **[½]**

Q. 6. Find the area of the region in the first quadrant enclosed by *x*-axis, line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$. [NCERT Ex. 8.1, Q. 6, Page 366]

Ans. The area of the region bounded by the circle, $x^2+y^2=4$, $x=\sqrt{3}y$, and the *x*-axis is the area of region *OAB*.



The point of intersection of the line and circle in the first quadrant is $(\sqrt{3}, 1)$.

Area of ΔOAB = Area of ΔOCB + Area of ΔABC

Area of ΔOCB

$=\frac{1}{2}\times OC\times AC=\frac{1}{2}\times\sqrt{3}\times 1=\frac{\sqrt{3}}{2}$ sq. units **[1]**

Area of $\Delta ABC=\int_{\sqrt{3}}^{2} y\,dx$

$$=\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$=\left[2\times\frac{\pi}{2}-\frac{\sqrt{3}}{2}\sqrt{4-3}-\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$=\left[\pi-\frac{\sqrt{3}\pi}{2}-\frac{2\pi}{3}\right]$$

$$=\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]\text{sq. units}$$ **[2]**

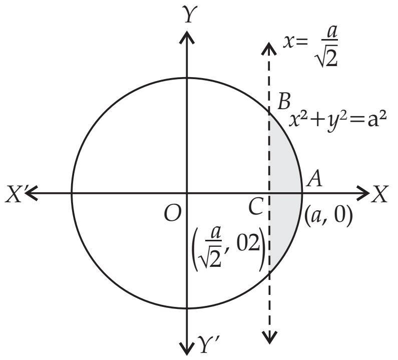
Area enclosed by *x*-axis the line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$ in the first quadrant

$$=\frac{\sqrt{3}}{2}+\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]=\frac{\pi}{3}\text{ units.}$$

Q. 7. Find the area of the smaller part of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{\sqrt{2}}$.

[NCERT Ex. 8.1, Q. 7, Page 366]

Ans. The area of the smaller part of the circle, $x^2+y^2=a^2$ cut-off by the line $x=\frac{a}{\sqrt{2}}$ is the area of ABCD.



[1]

It can be observed that the area of *ABCD* is symmetrical about *x*-axis.

Area of *ABCD* = 2 × Area of region *ABC*

Area of region $ABC=\int_{a/\sqrt{2}}^{a} y\,dx$

$$=\int_{a/\sqrt{2}}^{a}\sqrt{a^2-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{a/\sqrt{2}}^{a}$$

$$=\left[\frac{a^2}{2}\times\frac{\pi}{2}-\frac{a}{2\sqrt{2}}\sqrt{a^2-\frac{a^2}{2}}-\frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$=\frac{a^2\pi}{4}-\frac{a^2}{4}-\frac{a^2\pi}{8}$

$=\frac{a^2}{4}\left[\pi-1-\frac{\pi}{2}\right]$

$=\frac{a^2}{4}\left(\frac{\pi}{2}-1\right)$sq. units

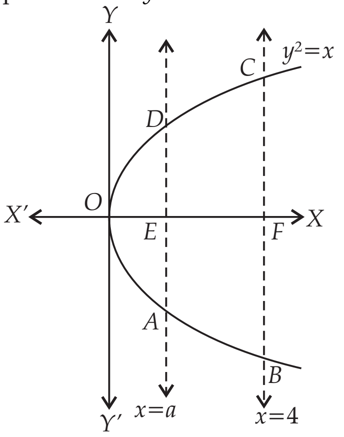
$$=2\left[\frac{a^2}{4}\left(\frac{\pi}{2}-1\right)\right]$$

Area of ABCD $=\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ sq. units **[2]**

Therefore, the area of smaller part of the circle $x^2+y^2=a^2$ cut-off by the line $x=\frac{a}{\sqrt{2}}$ is $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ sq.units.

Q. 8. The area between $x=y^2$ and $x=4$ is divided into two equal parts by the line $x=a$, find the value of a. [NCERT Ex. 8.1, Q. 8, Page 366]

Ans. The line $x=a$, divides the area bounded by the parabola $x=y^2$ and $x=4$ into two equal parts.



[1]

It can be observed that the given area is symmetrical about x-axis. Therefore,

Area of OED = Area of $EFCD$

Area of $OED=\int_0^a y\,dx$

$=\int_0^a \sqrt{x}\,dx$

$=\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$

$=\frac{2}{3}(a)^{3/2}$ **(i)**

Area of $EFCD=\int_0^4 \sqrt{x}\,dx$

$=\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$=\frac{2}{3}(8-a)^{3/2}$ **(ii)**

From equations (1) and (2), we obtain

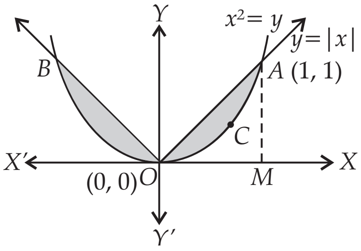
$\frac{2}{3}(a)^{3/2}=\frac{2}{3}(8-a)^{\frac{3}{2}}$

$2(a)^{3/2}=8$

$(a)^{3/2}=4$

$a=(4)^{3/2}$ **[2]**

Q. 9. Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$. [NCERT Ex. 8.1, Q. 9, Page 366]



[1]

The area bounded by the parabola $x^2=y$ and the line $y=|x|$, can be represented as:

The given area is symmetrical about y-axis.

Area of $OACO$ = Area of $ODBO$

The point of intersection of parabola $x^2=y$ and line $y=x$ is A(1, 1).

Area of $OACO$ = Area of ΔOAB – Area of $OBACO$

Area $\Delta OAB=\frac{1}{2}\times OB\times AB=\frac{1}{2}\times1\times1=\frac{1}{2}$ sq. units

Area of $OBACO=\int_0^1 y\,dx$

$=\int_0^1 x^2\,dx$

$=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$ sq. units

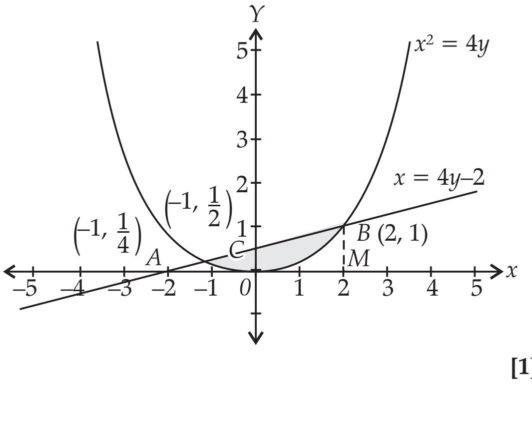
Area of $OACO$= Area of ΔOAB – Area of $OBACO$

$=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ sq. units

Required area $=2\left[\frac{1}{6}\right]=\frac{1}{3}$ sq. units. **[2]**

Q. 10. Find the area bounded by the curve $x^2=4y$ and the line $x=4y-2$. [NCERT Ex. 8.1, Q. 10, Page 366]

Ans.



[1]

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y - 2$, is represented by the shaded area *OBAO*.

Let *A* and *B* be the points of intersection of the line and parabola.

Coordinates of point *A* are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point *B* are (2, 1).

We draw *AL* and *BM* perpendicular to *x*-axis.

It can be observed that

Area of *OBAO*=Area of *OMBC* – Area of *OMBO*

$$= \int_0^2 \frac{x+2}{4}dx - \int_0^2 \frac{x^2}{4}dx$$

$$= \frac{1}{4}\left[\frac{x^2+2}{4}dx\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}dx\right]_0^2$$

$$= \frac{1}{4}[2+4] - \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{5}{6}$$

Similarly, Area of *OBAO* = Area of *OMBC* – Area of *OMBO*

$$= \int_{-1}^0 \frac{x+2}{4}dx - \int_{-1}^0 \frac{x^2}{4}dx$$

$$= \frac{1}{4}\left[\frac{x^2+2}{4}dx\right]_{-1}^0 - \frac{1}{4}\left[\frac{x^3}{3}dx\right]_{-1}^0$$

$$= -\frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \left[-\frac{1}{4}\left(\frac{(-1)^3}{3}\right)\right]$$

$$= \frac{1}{2} - \frac{1}{8} - \frac{1}{12}$$

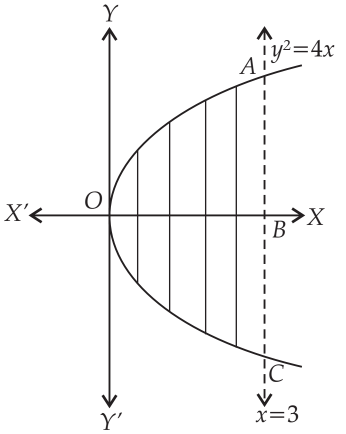
$$= \frac{7}{24}$$

Required area $= \left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$ sq. units **[2]**

Q. 11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

[NCERT Ex. 8.1, Q. 11, Page 366]

Ans.



[1]

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area of OACO.

The area of OACO is symmetrical about *x*-axis.

Area of OACO = 2 (Area of OABO)

Area of OACO $= 2\int_0^3 y dx$

$$= \int_0^3 2\sqrt{x}dx$$

$$= 4\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^3$$

$$= \frac{8}{3}\left[(3)^{3/2}\right]$$

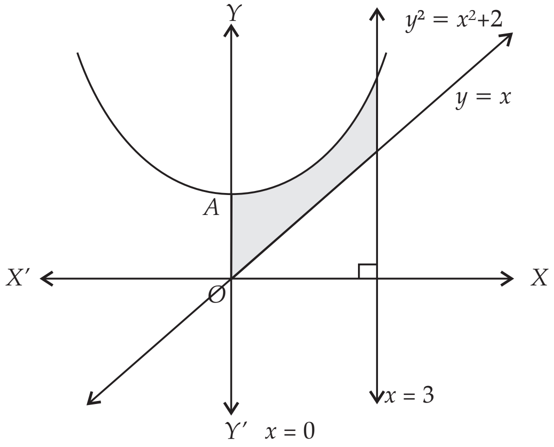
$$= 8\sqrt{3}$$

Required area $= 8\sqrt{3}$ sq. units. **[2]**

Q. 12. Find the area of the region bounded by the curves, $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

[NCERT Ex. 8.2, Q. 3, Page 371]

Ans. The area bounded by the curves, $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$, is represented by the shaded area *OCBAO* as :



[1]

Then,

Area of *OCBAO* = Area of *ODBAO* – Area of *ODCO*

$$A = \int_0^3 (x^2+2)dx - \int_0^1 xdx$$

$$= \left[\frac{x^3}{3} + 2x\right]_0^3 - \left[\frac{x^2}{3}\right]_0^3$$

$$= 15 - \frac{9}{2} = \frac{21}{2} \text{ sq. units}$$

[2]

Q. 13. Find the area of the region bounded by the curves $y^2 = 9x$ and $y = 3x$.

[NCERT Exemp. Ex. 8.3, Q. 1, Page 176]

Ans. We have, $y^2 = 9x$, $y = 3x$

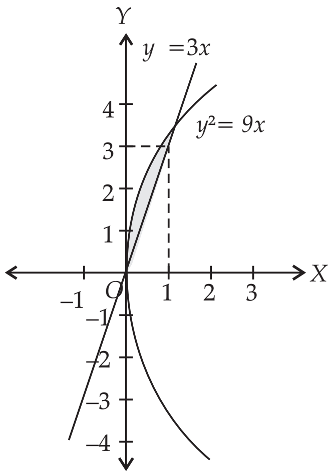
Solving $y^2 = 3(3x) = 3y$

$y = 0, 3$

When $y = 0$, $x = 0$ and when $y = 3$, $x = 1$

So points of intersection are $(0, 3)$ and $(1, 3)$.

Graph of parabola $y^2 = 9x$ and $y = 3x$ is as shown in the following figure :



[1]

From the figure,

Area of the shaded region $= \int_1^2 (\sqrt{9x} - 3x)\, dx$

$$= 3\int_1^2 x^{\frac{1}{2}}\, dx - 3\int_0^1 x\, dx$$

$$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$$

$$= 3\left(\frac{2}{3} - 0\right) - 3\left(\frac{1}{2} - 0\right)$$

$$= \frac{1}{2} \text{ sq. units}$$ **[2]**

Q. 14. Find the area of the region bounded by the parabola, $y^2 = 2px$ and $x^2 = 2py$.

[NCERT Exemp. Ex. 8.3, Q. 2, Page 176]

Ans. We have $y^2 = 2px$ and $x^2 = 2py$

Solving curves, we get

$$x^4 = 4p^2y^2$$

$$x^4 = 4p^2(2px)$$

$$x^4 = 8p^3x$$

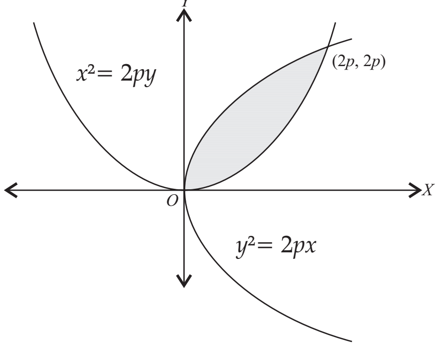
$$x(x^3 - 8p^3) = 0$$

$$x = 0,\ 2p$$

When, $x = 0,\ y = 0$ and when, $x = 2p,\ y = 2p$

So, points of intersection are $(0, 0)$ and $(2p, 2p)$.

Graph of both the parabolas is as shown in the following figure :



[1]

From the figure,

Area of the shaded region,

$$A = \int_0^{2p}\left[\sqrt{2px} - \frac{x^2}{2p}\right] dx$$

$$= \sqrt{2p}\int_0^{2p} x^{\frac{1}{2}} dx - \frac{1}{2p}\int_0^{2p} x^2 dx$$

$$= \sqrt{2p}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^{2p} - \frac{1}{2p}\left[\frac{x^3}{3}\right]_0^{2p}$$

$$= \sqrt{2p}\left(\frac{2}{3}.2\sqrt{2}p^{\frac{3}{2}}\right) - \frac{1}{2p}\left(\frac{1}{3}8p^3\right)$$

$$= \frac{8}{3}p^2 - \frac{4}{3}p^2 = \frac{4p^2}{3} \text{ sq. units}$$

[2]

Q. 15. Find the area of the region bounded by the curve $y = x^3$, $y = x + 6$ and $x = 0$.

[NCERT Exemp. Ex. 8.3,Q. 3, Page 176]

Ans. We have, $y = x^3$, $y = x + 6$, $x = 0$

Graph of function is as shown in the following figure.

Solving, $y = x^3$ and $y = x + 6$ we get

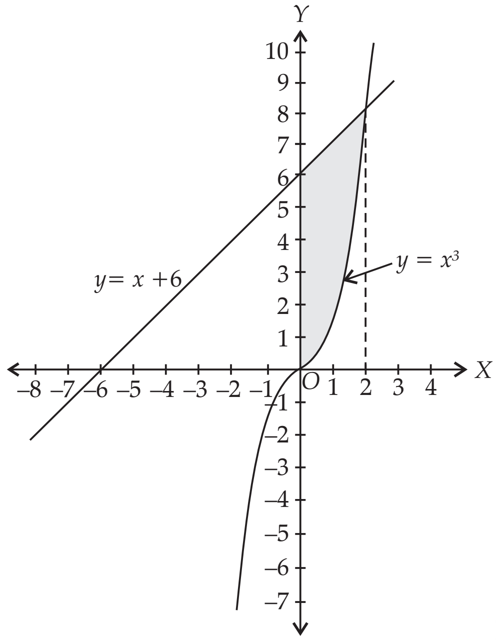
$$x^3 = x + 6$$

$$x^3 - x - 6 = 0$$

$$x^2(x-2) + 2x(x-2) + 3(x-2) = 0$$

$$(x-2)(x^2 + 2x + 3) = 0$$

$x = 2$, With 2 imaginary points.



[1½]

Clearly $x = 2$ satisfies the above equation.

Also from the figure it is clear that there is only one point of intersection.

From the figure, area of the shaded region,

$$A = \int_0^2 (x + 6 - x^3)dx$$

$$=\left[\frac{x^2}{2}+6x-\frac{x^4}{4}\right]_0^2=\frac{4}{2}+12=10 \text{ sq. units.}$$ **[1½]**

Q. 16. Find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$.
[NCERT Exemp. Ex. 8.3, Q. 4, Page 176]

Ans. We have $y^2 = 4x,\ x^2 = 4y$

Solving curves, we get

$$x^4 = 16y^2$$

$$x^4 = 16(4x)$$

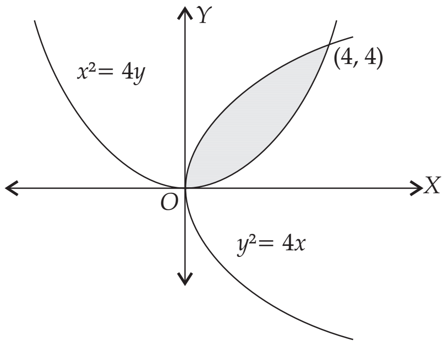
$$x^4 = 64x$$

$$x(x^3 - 4^3) = 0$$

$$x = 0,\ 4$$

When, $x = 0,\ y = 0$ and when $x = 4,\ y = 4$

So, points of intersection are $(0, 0)$ and $(4, 4)$.



[1]

From the figure, area of the shaded region,

$$A=\int_0^4\left[\sqrt{4x}-\frac{x^2}{4}\right]dx$$

$$=\sqrt{4}\int_0^4 x^{\frac{1}{2}}dx-\frac{1}{4}\int_0^4 x^2dx$$

$$=\sqrt{4}\left[\frac{2}{3}x^{3/2}\right]_0^4-\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$=\sqrt{4}\left(\frac{2}{3}.4^{3/2}\right)-\frac{1}{4}\left(\frac{1}{3}4^3\right)$$

$$=\frac{32}{3}-\frac{16}{3}=\frac{16}{3} \text{ sq. units}$$ **[2]**

Q. 17. Find the area of the region included between $y^2 = 9x$ and $y = x$.
[NCERT Exemp. Ex. 8.3, Q. 5, Page 176]

Ans. We have, $y^2 = 9x$ and $y = x$

Solving above equations :

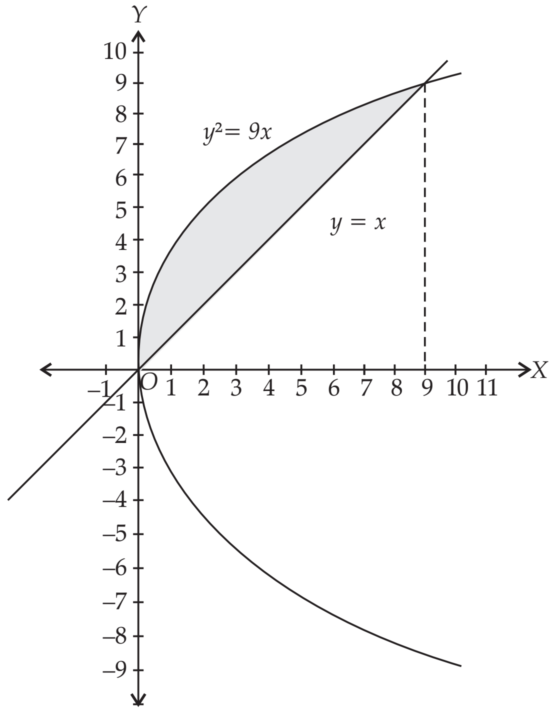
$$y^2 = 9y$$

$$\Rightarrow y = 0 \text{ or } 9$$

When $y = 0,\ x = 0$ and $y = 9,\ x = 9$

So points of intersection are $(0, 0)$ and $(9, 9)$.

Graphs of parabola are as shown in the following figure :



[1]

From the figure, area of the shaded region,

$$A=\int_0^9(\sqrt{9x}-x)dx$$

$$=\int_0^9(3x^{1/2}-x)dx$$

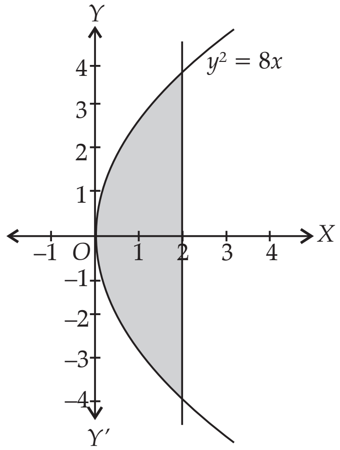
$$=\left[3\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_0^9$$

$$=54-\frac{81}{2}=\frac{27}{2} \text{ sq. units.}$$ **[2]**

Q. 18. Find the area of the region bounded by the line $x = 2$ and the parabola $y^2 = 8x$.
[NCERT Exemp. Ex. 8.3, Q. 7, Page 176]

Ans. We have, $y^2 = 8x$ and $x = 2$

Graphs of parabola and line are as shown in the following figure :



[1]

From the figure, area of the shaded region

$$A = 2\int_0^2 (\sqrt{8x})dx$$

$$= 4\sqrt{2}\int_0^2 (x^{1/2})dx$$

$$= 4\sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2$$

$$= 4\sqrt{2}\left[\frac{2}{3}\times 2\sqrt{2} - 0\right]_0^2$$

$$= \frac{32}{3} \text{ sq. units.}$$

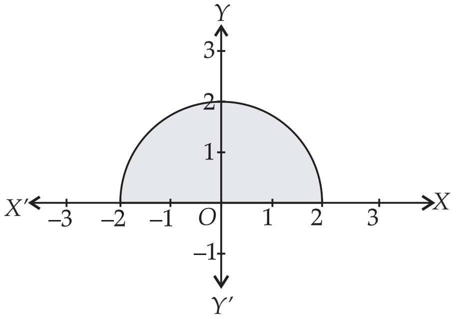
[2]

Q. 19. Sketch the region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and, find the area of the region using integration.
[NCERT Exemp. Ex. 8.3, Q. 8, Page 176]

Ans. We have $y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2,\ y \ge 0$

$y^2 + x^2 = 4,\ y \ge 0$



[1]

Graph of the above function is semi-circle lying above the graph is as shown in the above figure.

From the figure, area of the shaded region,

$$A = \int_{-2}^{2} (\sqrt{4 - x^2})dx$$

$$= \int_{-2}^{2} (\sqrt{2^2 - x^2})dx$$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= 0 + 2\sin^{-1}1 - 0 - 2\sin^{-1}(-1)$$

$$= 2\times\frac{\pi}{2} + 2\times\frac{\pi}{2}$$

$$= 2\pi \text{ sq. units.}$$

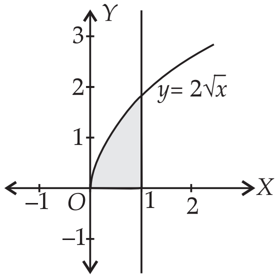
[2]

Q. 20. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.
[NCERT Exemp. Ex. 8.3, Q. 9, Page 176]

Ans. We have, $y = 2\sqrt{x}$

Or $y^2 = 4x,\ x \ge 0$

The graph of the above function is a part of parabola lying above,



[1]

The graph is as shown in the above figure.

From the figure, area of the shaded region,

$$A = \int_0^1 2\sqrt{x}dx$$

$$= 2\left[\frac{2}{3}x^{3/2}\right]_0^1$$

$$= 2\left(\frac{2}{3}\right)$$

$$= \frac{4}{3} \text{ sq. units}$$

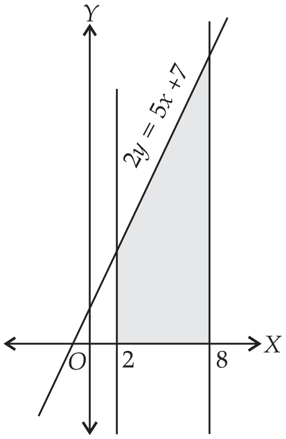
[2]

Q. 21. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x– axis and the lines $x = 2$ and $x = 8$.
[NCERT Exemp. Ex. 8.3, Q. 10, Page 176]

Ans. We have, $2y = 5x + 7$

Or $y = \frac{5x}{2} + \frac{7}{2}$

The graph is as shown in the following figure :



[1]

From the figure, area of the shaded region,

$$A = \int_2^8 \frac{5x + 7}{2}dx$$

$$= \frac{1}{2}\int_2^8 (5x + 7)dx$$

$$= \frac{1}{2}\left[\frac{5x^2}{2} + 7x\right]_2^8$$

$$= \frac{1}{2}\left[5.32 + 7.8 - 10 - 14\right]$$

$$= 96 \text{ sq. units}$$

[2]

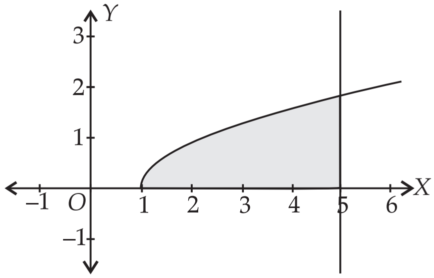
Q. 22. Draw a rough sketch of the curve $y=\sqrt{x-1}$ in the interval [1, 5]. Find the area under the curve and between the lines $x = 1$ and $x = 5$.

[NCERT Exemp. Ex. 8.3, Q. 11, Page 177]

Ans. We have $y=\sqrt{x-1}$

$$y^2 = x-1$$

The graph of above function is parabola with vertex (1, 0) and lying above x-axis and for $x \in [1, 5]$, graph is as shown in the following figure :



[1]

From the figure, area of the shaded region,

$$A=\int_1^5 (x-1)^{\frac{1}{2}}dx$$

$$=\left[\frac{2}{3}(x-1)^{3/2}\right]_1^5$$

$$=\frac{16}{3} \text{ sq. units}$$

[2]

Q. 23. Determine the area under the curve $y=\sqrt{a^2-x^2}$ included between the lines $x = 0$ and $x = a$.

[NCERT Exemp. Ex. 8.3, Q. 12, Page 177]

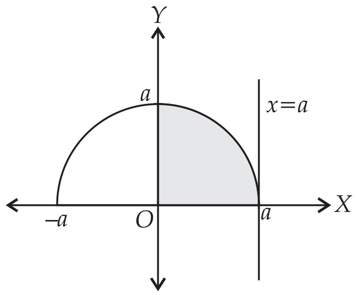
Ans. We have $y=\sqrt{a^2-x^2}$

$$y^2 = a^2 - x^2$$

$$y^2 + x^2 = a^2$$

Graph of above function is semi-circle lying above x-axis.

The graph is as shown in the following figure .



[1]

From the figure, area of the shaded region,

$$A=\int_0^a (\sqrt{a^2-x^2})dx$$

$$=\left[\frac{x}{2}(\sqrt{a^2-x^2})^{\frac{3}{2}}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=\left[0+\frac{a^2}{2}\sin^{-1}1-0-\frac{a^2}{2}\sin^{-1}0\right]$$

$$=\frac{a^2}{2}\times\frac{\pi}{2}$$

$$=\frac{\pi a^2}{4} \text{ sq. units}$$

[2]

Q. 24. Find the area of the region bounded by $y=\sqrt{x}$ and $y = x$.

[NCERT Exemp. Ex. 8.3, Q. 13, Page 177]

Ans. We have $y=\sqrt{x}$ and $y = x$

On solving, we get

$$x=\sqrt{x}$$

$$x^2 = x$$

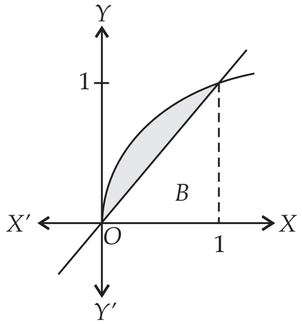
$$x^2 - x = 0$$

$$x = 0, 1$$

At $x = 0$, $y = 0$ and at $x = 1$, $y = 1$

Thus, curves intersect at (0, 0) and (1, 1).

Graph of $y=\sqrt{x}$ is part of parabola lying above x-axis.



[1]

The graph is as shown in the above figure.

From the figure, area of the shaded region,

$$A=\int_0^1 (\sqrt{x}-x)dx$$

$$=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1$$

$$=\frac{2}{3}-\frac{1}{2}=\frac{1}{6} \text{ sq. units}$$

[2]

Q. 25. Find the area enclosed by the curve $y = -x^2$ and the straight line $x + y + 2 = 0$.

[NCERT Exemp. Ex. 8.3, Q. 14, Page 177]

Ans. We have, $y = -x^2$ and $x + y + 2 = 0$

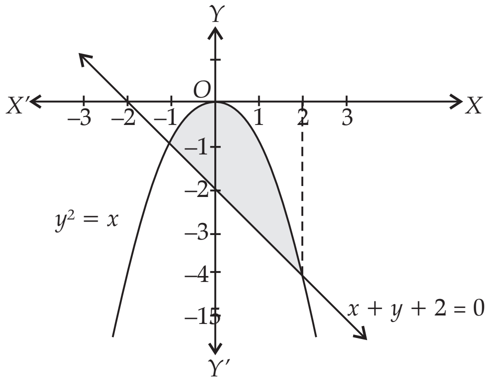
On solving we get,

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, \ -1$$



[2]

The graph of the above function is downward parabola.

$$A = \int_{-1}^{2} (-x^2 - (-x-2))dx$$

$$= \int_{-1}^{2} (x + 2 - x^2)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left[\frac{8}{3} - \frac{4}{2} - 4 + \frac{1}{3} + \frac{1}{2} - 2\right]$$

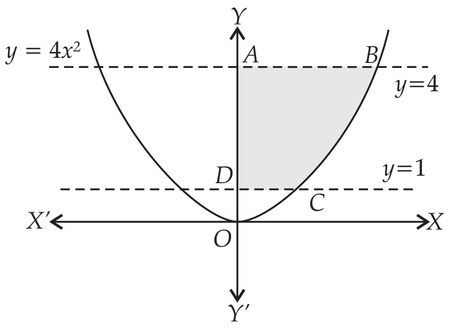
$$= \left|\frac{16 - 12 - 24 + 2 + 3 - 12}{6}\right|$$

$$= \left|\frac{-27}{6}\right| = \frac{9}{2} \text{ sq. units}$$

[1]

Q. 26 Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$. [NCERT Misc. Ex. Q. 3, Page 375]

Ans.



Required area = Area *ABCD*

$$= \int_1^4 x\, dy$$

Here,

$$y = 4x^2$$

$$4x^2 = y$$

$$x^2 = \frac{y}{4}$$

$$x = \sqrt{\frac{y}{4}}$$

$$x = \frac{\sqrt{y}}{2}$$

Thus,

$$\text{Area required} = \int_1^4 x\, dy$$

$$= \int_1^4 \frac{\sqrt{y}}{2} dy$$

$$= \frac{1}{2}\int_1^4 y^{1/2} dy$$

$$= \frac{1}{2}\left[\frac{y^{1/2+1}}{\frac{1}{2}+1}\right]_1^4$$

$$= \frac{1}{2}\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{1}{2} \times \frac{2}{3}\left[y^{3/2}\right]_1^4$$

$$= \frac{1}{3}\left[4^{3/2} - 1^{3/2}\right]$$

$$= \frac{1}{3}\left[(2^2)^{3/2} - 1\right]$$

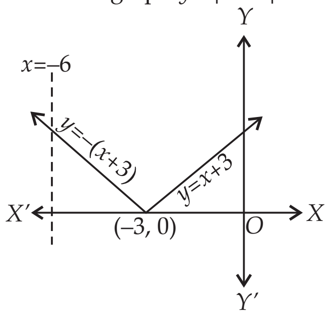
$$= \frac{1}{3}\left[2^3 - 1\right]$$

$$= \frac{7}{3} \text{ sq. units}$$

[3]

Q. 27. Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3| dx$. [NCERT Misc. Ex. Q. 4, Page 375]

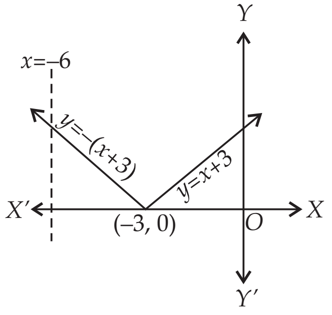
Ans. Draw the graph $y = |x+3|$



$$y = |x+3| = \begin{cases} x+3 \text{ for } x+3 \geq 0 \\ -(x+3) \text{ for } x+3 < 0 \end{cases}$$

$$= \begin{cases} x+3 \text{ for } x \geq -3 \\ -(x+3) \text{ for } x < -3 \end{cases}$$

Finding $\int_{-6}^{0} |x+3|\, dx$



$$\int_{-6}^{0} |x+3|\, dx = \int_{-6}^{-3} |x+3|\, dx - \int_{-3}^{0} |x+3|\, dx$$

$$= \int_{-6}^{-3} -(x+3)dx + \int_{-3}^{0} (x+3)dx$$

$$= \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= \frac{-(-3)^2}{2} - 3\times(-3) - \left[\frac{-(-6)^2}{2} - 3(-6)\right]$$

$$+ \left[-\frac{0^2}{2} + 3\times 0\right] - \left[\frac{(-3)^2}{2} + 3\times(-3)\right]$$

$$= \frac{-9}{2} - (-9) - \left[\frac{-36}{2} - (-18)\right] - \left[\frac{9}{2} - 9\right]$$

$= \frac{-9}{2} + 9 + 0 - \frac{9}{2} + 9$

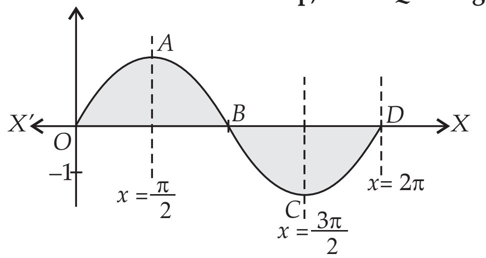
$= -9 + 18$

$= 9$ sq. units

[3]

Q. 28. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$. [NCERT Misc. Ex. Q. 5, Page 375, NCERT Exemp, Ex. 8.3 Q. 17 Page 177]

Ans.



Area Required = Area OAB + Area BCD

Area OAB $= \int_0^{\pi} y\,dx$

Here, $y = \sin x$

$= \int_0^{\pi} \sin x\,dx$

$= -\left[-\cos x\right]_0^{\pi}$

$= -[\cos\pi - \cos 0]$

$= -[-1-1]$

$= -[-2]$

$= 2$

Area BCD $= \int_{\pi}^{2\pi} y\,dx$

Here, $y = \sin x$

$= \int_{\pi}^{2\pi} \sin x\,dx$

$= \left[-\cos x\right]_{\pi}^{2\pi}$

$= -[\cos 2\pi - \cos\pi]$

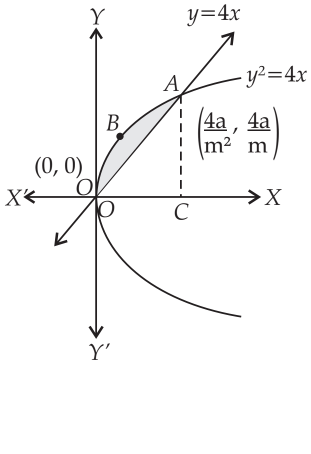
$= -[1-(-1)]$

$= -2$

Since area cannot be negative,
Area BCD=2
Hence,
Required area = Area OAB + Area BCD **[3]**
=2+2
=4 sq.units

Q. 29. Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$. [NCERT Misc. Ex. Q. 6, Page 375]

Ans. The area enclosed between the parabola, $y^2 = 4ax$, and the line $y = mx$, is represented by the shaded area $OABO$ as



The points of intersection of both the curves (0, 0) and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$

We draw AC perpendicular to x-axis.

$\therefore$ Area $OABO$ = Area $OCABO$ – Area (ΔOCA)

$= \int_0^{4a/m^2} 2\sqrt{ax}\,dx - \int_0^{4a/m^2} mx dx$

$= 2\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^{4a/m^2} - m\left[\frac{x^2}{2}\right]_0^{4a/m^2}$

$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{3/2} - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2\right]$

$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$

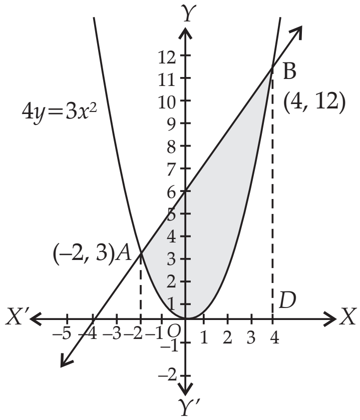
$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$

$= \frac{8a^2}{3m^3}$ sq. units

[3]

Q. 30. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$. [NCERT Misc. Ex. Q. 7, Page 375]

Ans. The area enclosed between the parabola, $4y = 3x^2$, and line, $2y = 3x + 12$, is represented by the shaded area $OBAO$ as



The points of intersection of the given curves are A (–2, 3) and (4, 12).

We draw AC and BD perpendicular to x-axis.

$\therefore$ Area $OBAO$ = Area $CDBA$ – (Area $ODBO$ +Area $OACO$)

$= \int_{-2}^{4} \frac{1}{2}(3x+12)dx - \int_{-2}^{4} \frac{3x^2}{4}dx$

$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{2}[24 + 48 - 6 + 24] - \frac{1}{2}[64 + 8]$

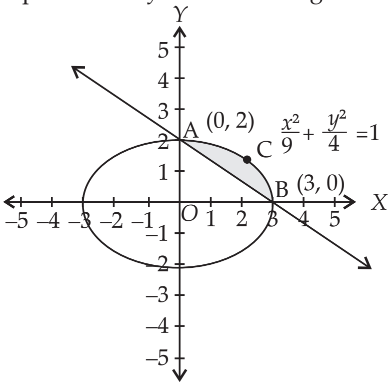
$= \frac{1}{2}[90] - \frac{1}{4}[72]$

$= 45 - 18$

$= 27$ sq. units **[3]**

Q. 31. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. [NCERT Misc. Ex. Q. 8, Page 375]

Ans. The area of the smaller region bounded by the ellipse, $\frac{x^2}{9}+\frac{y^2}{4}=1$, and the line $\frac{x}{3}+\frac{y}{2}=1$, is represented by the shaded region $BCAB$ as :



$\therefore$ Area $BCAB$ = Area ($OBCAO$) – Area ($OBAO$)

$$=\int_0^3 2\sqrt{1-\frac{x^2}{9}}dx-\int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$=\frac{2}{3}\left[\int_0^3\sqrt{9-x^2}dx\right]-\frac{2}{3}\int_0^3(3-x)dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right]-\frac{2}{3}\left[9-\frac{9}{2}\right]$$

$$=\frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$$

$$=\frac{2}{3}\times\frac{9}{4}(\pi-2)$$

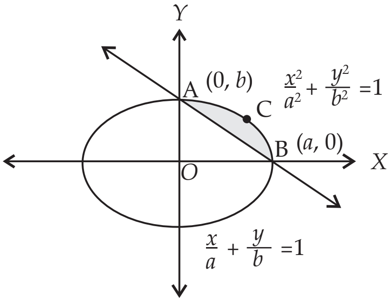
$$=\frac{3}{2}(\pi-2)\text{ sq.units}$$

[3]

Q. 32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

[NCERT Misc. Ex. Q. 9, Page 375]

Ans. The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and the line, $\frac{x}{a}+\frac{y}{b}=1$, is represented by the shaded region $BCAB$ as



$\therefore$ Area $BCAB$ = Area ($OBCAO$) – Area ($OBAO$)

$$=\int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx-\int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$=\frac{b}{a}\int_0^a\sqrt{a^2-x^2}dx-\frac{b}{a}\int_0^a(a-x)dx$$

$$=\frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a-\left\{ax-\frac{x^2}{2}\right\}_0^a\right]$$

$$=\frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\}-\left\{a^2-\frac{a^2}{2}\right\}\right]$$

$$=\frac{b}{a}\left[\frac{a^2\pi}{4}-\frac{a^2}{2}\right]$$

$$=\frac{ba^2}{2a}\left[\frac{\pi}{2}-1\right]$$

$$=\frac{ab}{2}\left[\frac{\pi}{2}-1\right]$$

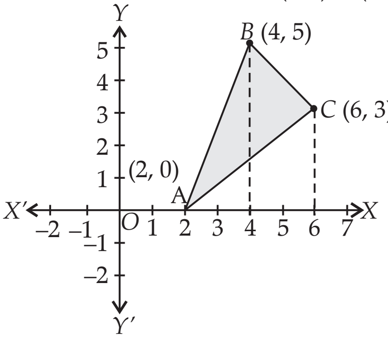
$$=\frac{ab}{4}(\pi-2)\text{ sq. units}$$

[3]

Q. 33. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(2, 0)$, B (4, 5) and C (6, 3).

[NCERT Misc. Ex. Q. 13, Page 376]

Ans. The vertices of ΔABC are A (2, 0), B (4, 5), and C (6, 3)



Equation of line segment AB is

$$y-0=\frac{5-0}{4-2}(x-2)$$

$$2y=5x-10$$

$$y=\frac{5}{2}(x-2) \quad ...(i)$$

Equation of line segment BC is

$$-5=\frac{3-5}{6-4}(x-4)$$

$$2y-10=-2x+8$$

$$2y=-2x+18$$

$$y=-x+9 \quad ...(ii)$$

Equation of line segment CA is

$$y-3=\frac{0-3}{2-6}(x-6)$$

$$-4y+12=-3x+18$$

$$4y=3x-6$$

$$y=\frac{3}{4}(x-2) \quad ...(iii)$$

Area (ΔABC) = Area ($ABLA$) + Area ($BLMCB$) – Area ($ACMA$)

$$=\int_2^4\frac{5}{2}(x-2)dx+\int_4^6(-x+9)dx-\int_2^6\frac{3}{4}(x-2)dx$$

$$=\frac{5}{2}\left[\frac{x^2}{2}-2x\right]_2^4+\left[\frac{-x^2}{2}+9x\right]_4^6-\frac{3}{4}\left[\frac{x^2}{2}-2x\right]_2^6$$

$$=\frac{5}{2}[8-8-2+4]+[-18+54+8-36]-\frac{3}{4}[18-12-2+4]$$

$$=5+8-\frac{3}{4}(8)$$

$= 13 - 6$

$= 7$ sq.units **[3]**

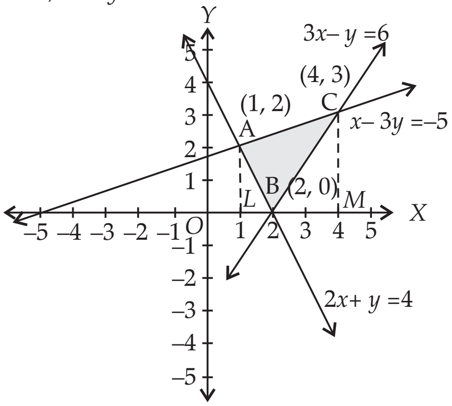
Q. 34. Using the method of integration find the area of the region bounded by lines : $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$ [NCERT Misc. Ex. Q. 14, Page 376]

Ans. The given equations of lines are

$2x + y = 4$ … (i)

$2x - 2y = 6$ …(ii)

And, $x - 3y + 5 = 0$ … (iii)



The area of the region bounded by the lines is the area of ΔABC. AL and CM are the perpendiculars on x-axis.

Area (ΔABC) = Area ($ALMCA$) – Area (ALB) – Area (CMB)

$$= \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x-6}{2}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x - x^2\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$= \frac{1}{3}\left[8+30-\frac{1}{2}-5\right] - [8-4-4+1] - \frac{1}{2}[24-24-6+12]$$

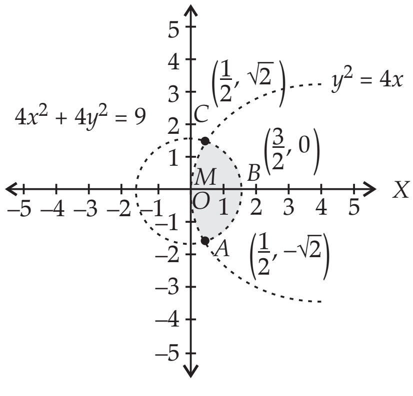
$$= \left(\frac{1}{3}\times\frac{45}{2}\right) - [1] - \frac{1}{2}[6]$$

$$= \frac{15}{2} - 1 - 3$$

$$= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ sq.units}$$ **[3]**

Q. 35. Find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$ [NCERT Misc. Ex. Q. 15, Page 376]

Ans. The area bounded by the curves, $\{(x,y): y^2 \le 4x, 4x^2+4y^2 \le 9\}$, is represented as



The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given $OABCO$.

It can be observed that are $OABCO$ is symmetrical about x-axis.

$\therefore$ Area $OABCO$ = 2 × Area OBC

Area $OBCO$ = Area OMC + Area MBC

$$= \int_0^{1/2} 2\sqrt{x}\, dx + \int_{1/2}^{3/2} \frac{1}{2}\sqrt{9-4x^2}\, dx$$

$$= \int_0^{1/2} 2\sqrt{x}\, dx + \int_{1/2}^{3/2} \frac{1}{2}\sqrt{(3)^2-(2x)^2}\, dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

When $x = \frac{3}{2}$, $t = 3$ and when $x = \frac{1}{2}$, $t = 1$

$$= \int_0^{1/2} 2\sqrt{x}\, dx + \frac{1}{4}\int_1^3 \sqrt{(3)^2-(t)^2}\, dt$$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{1/2} + \frac{1}{4}\left[\frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{t}{3}\right)\right]_1^3$$

$$= 2\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}}\right] + \frac{1}{4}\left[\left\{\frac{3}{2}\sqrt{9-(3)^2} + \frac{9}{2}\sin^{-1}\left(\frac{3}{3}\right)\right\} - \left\{\frac{1}{2}\sqrt{9-(1)^2} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$$

$$= \frac{2}{3\sqrt{2}} + \frac{1}{4}\left[\left\{0 + \frac{9}{2}\sin^{-1}(1)\right\} - \left\{\frac{1}{2}\sqrt{8} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$$

$$= \frac{2}{3\sqrt{2}} + \frac{1}{4}\left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$$

$$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$$

$$= \frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}$$

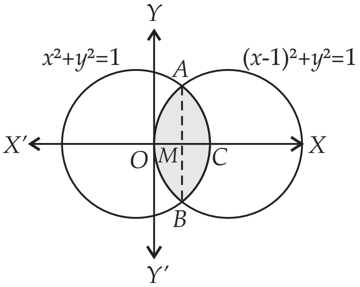
Therefore, the required area is

$$\left[2x\left(\frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}\right)\right] = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}} \text{ sq. units.}$$

[3]

Q. 36. Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ [NCERT Ex. 8.2, Q. 2, Page 371]

Ans. The areas bounded by the curves, $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, is represented by the shaded area as



On solving the equation, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, we obtain the point of intersection as $A\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

It can be observed that the required area is symmetrical about x-axis

$\therefore$ Area $OBCAO = 2 \times$ Area $OCAO$

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are $\left(\frac{1}{2},0\right)$.

$\Rightarrow$ *Area OCAO=Area OMAO + Area MCAM*

$$=\left[\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx+\int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$=\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{1/2}^{1}$$

$$=\left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)-\frac{1}{2}\sin^{-1}(-1)\right]$$

$$+\left[\frac{1}{2}\sin^{-1}(1)-\frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2}-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{8}+\frac{1}{2}\left(-\frac{\pi}{6}\right)-\frac{1}{2}\left(-\frac{\pi}{2}\right)\right]+\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{\sqrt{3}}{8}-\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{12}+\frac{\pi}{4}+\frac{\pi}{4}-\frac{\pi}{12}\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{6}+\frac{\pi}{2}\right]$$

$$=\left[\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right]$$

Therefore, required area $OBCAO$

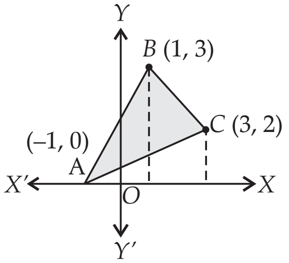
$2\times\left(\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right)=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ sq. units **[3]**

Q. 37. Using integration find the area of region bounded by the triangle whose vertices are (– 1, 0), (1, 3) and (3, 2). [NCERT Ex. 8.2, Q. 4, Page 371]

Ans. BL and CM are drawn perpendicular to x-axis

It can be observed in the following figure that,

Area (ΔACB) – Area ($ALBA$) +Area ($BLMCB$) – Area ($AMCA$) … (i)



Equation of line segment AB is

$$y-0=\frac{3-0}{1+1}(x+1)$$

$$y=\frac{3}{2}(x+1)$$

$$\therefore \text{Area }(ALBA)=\int_{-1}^{1}\frac{3}{2}(x+1)dx=\frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1}$$

$$=\frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right]=3\text{ sq.units}$$

Equation of line segment BC is

$$y-3=\frac{2-3}{3-1}(x-1)$$

$$y=\frac{1}{2}(-x+7)$$

$$\therefore \text{Area}(BLMCB)=\int_1^3\frac{1}{2}(-x+7)dx=\frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_1^3$$

$$=\frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right]=5\text{ sq.units}$$

Equation of line segment AC is

$$y-0=\frac{2-0}{3+1}(x+1)$$

$$y=\frac{1}{2}(x+1)$$

$$\therefore \text{Area}(AMCA)=\frac{1}{2}\int_{-1}^{3}(x+1)dx=\frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$=\frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right]=4\text{ sq.units}$$

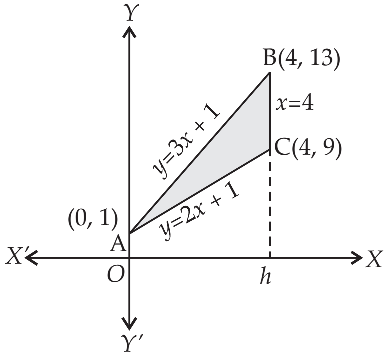
Therefore, from equation (i), we obtain

Area (ΔABC) = (3 + 5 – 4) = 4 sq.units **[3]**

Q. 38. Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$. [NCERT Ex. 8.2, Q. 5, Page 371]

Ans. The equations of sides of the triangle are $y = 2x+1$, $y = 3x + 1$, and $x = 4$.

On solving these equations, we obtain the vertices of triangle as A (0, 1), B (4, 13), and C (4, 9).



It can be observed that,

Area (ΔACB) = Area ($OLBAO$) – Area ($OLCAO$)

$$=\int_0^4(3x+1)dx-\int_0^4(2x+1)dx$$

$$=\left[\frac{3x^2}{2}+x\right]_0^4-\left[\frac{2x^2}{2}+x\right]_0^4$$

$$=(24+4)-(16+4)$$

$$=28-20$$

$$=8\text{ sq.units}$$

[3]

Q. 39. Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$ [CBSE Board, Foreign Scheme, 2016]

Ans. Equation of given curves

$y^2 = 4ax$ and $x^2 = 4by$

Their point of intersections are (0, 0) and $\left(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3}\right)$

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y},\ \text{slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \text{...(i)}$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b},\ \text{slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \text{...(ii)}$$

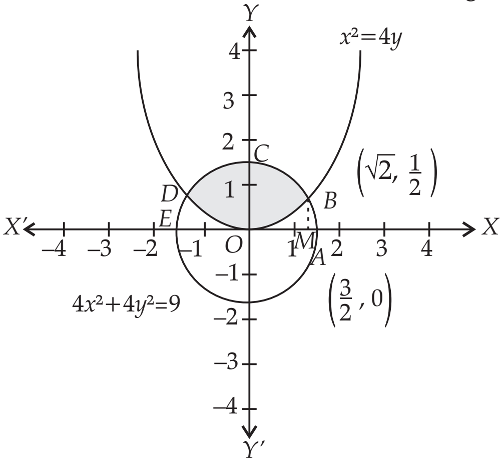
At (0, 0), angle between two curves is 90° **[4]**

Long Answer Type Questions

(5 or 6 marks each)

Q. 1. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

[NCERT Ex. 8.2, Q. 1, Page 371]



Ans. The required area is represented by the shaded region *OBCDO*. Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$

It can be observed that the required area is symmetrical about *y*-axis.

$\therefore$ Area of *OBCDO* = 2 × Area of *OBCO*

We draw *BM* perpendicular to *OA*.

Therefore, the coordinates of *M* are $\left(\sqrt{2}, 0\right)$.

Therefore,

Area of *OBCO* = Area of *OMBCO* – Area of *OMBO*

$$= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}}dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}}dx$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9-4x^2}\,dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2 dx$$

$$= \frac{1}{4}\left[x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^2}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9-8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}\left(\sqrt{2}\right)^3$$

$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin\ \frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area *OBCDO*

$$= \left(2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right)$$

$$= \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$$

[5]

Q. 2. Find the area bounded by the curve $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and x-axis.

[NCERT Exemp. Ex. 8.3, Q. 15, Page 177]

Ans. We have $y = \sqrt{x}$ and $x = 2y + 3$

On solving we get,

$$y = \sqrt{2y+3},\ y \ge 0$$

$$y^2 = 2y + 3,\ y \ge 0$$

$$y^2 - 2y - 3 = 0,\ y \ge 0$$

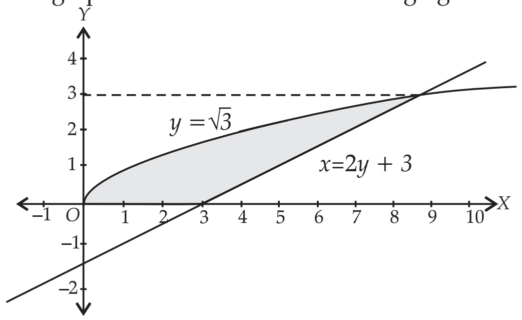
$$(y-3)(y+1) = 0,\ y \ge 0$$

$$y = 3, -1$$

[2]

The graph of function $y = \sqrt{x}$ is part of parabola $y^2 = x$ lying above *x*-axis.

The graph is as shown in the following figure :



[1½]

From the figure, area of the shaded region,

$$A = \int_0^3 (2y + 3 - y^2)dy$$

$$= \left[\frac{2y^2}{2} + 3y - \frac{y^3}{3}\right]_0^3$$

$$= \left[\frac{18}{2} + 9 - 9 - 0\right]$$

=9 sq.units **[1½]**

Q. 3. Find the region bounded by the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$.

[NCERT Exemp. Ex. 8.3, Q. 16, Page 177]

Ans. We have, $y^2=2x$ and $x^2+y^2=4x$

$y^2=2x$ is parabola opening to the right of positive direction x-axis :

$x^2+y^2=4x$

$\Rightarrow (x-2)^2+y^2=4$ is a circle having centre at (2, 0) and radius 2.

Solving curves

$\Rightarrow x^2+2x=4x$

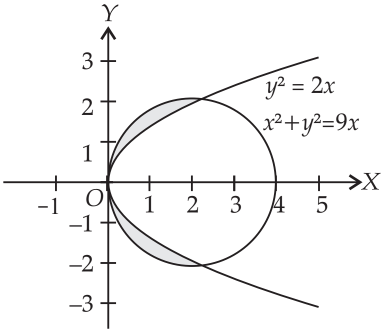
$x^2-2x=0$

$\Rightarrow x = 0,\ 2$

When $x=0, y=0$ and when $x=2,\ y=\pm 2$

Point of intersection are (0, 0), (2, 2) and (2, –2) **[2]**

Graph is as shown below.



[1]

From the figure, area of the shaded region,

$$A = 2\int_0^2 \sqrt{2^2-(x-2)^2} - 2\int_0^2 \sqrt{2x}dx$$

$$=2\left[\left[\frac{x-2}{2}\sqrt{2^2-(x-2)^2}+\frac{2^2}{2}\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^2-\left[\sqrt{2}\frac{x^{3/2}}{3/2}\right]_0^2\right]$$

$$=2\left[\left(0+0-0+2.\frac{\pi}{2}\right)-\frac{2\sqrt{2}}{3}(2^{\frac{3}{2}}-0)\right]$$

$$=2\pi-\frac{16}{3}$$

$$=2\left(\pi-\frac{8}{3}\right) \text{ sq. units}$$

[2]

Q. 4. Find the area of region bounded by the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration.

[NCERT Exemp. Ex. 8.3, Q. 18, Page 177]

Ans. We have vertices of the ΔABC as A (–1, 1), B (0, 5) and C (3, 2).

Equation of AB is $y-1=\left(\frac{5-1}{0+1}\right)(x+1)$

$$y-1=4x+4$$

$$y=4x+5 \quad \textbf{(i)}$$

Equation of BC is $y-5=\left(\frac{2-5}{3-0}\right)(x-0)$

$$y-5=-x$$

$$y=5-x \quad \textbf{(ii)}$$

Equation of line AC is $y-1=\left(\frac{2-1}{3+1}\right)(x+1)$

$$y-1=\frac{1}{4}(x+1).$$

$$4y=x+5 \quad \textbf{(iii)}$$

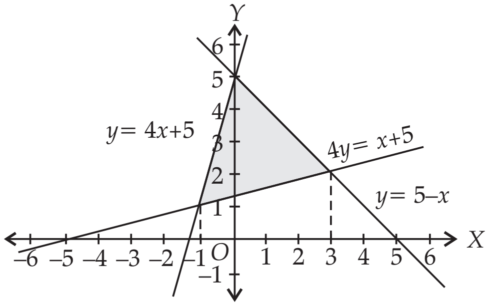
[2]

Solving equations (i) and (ii) we get point of intersection (0, 5)

Solving equations (ii) and (iii) we get point of intersection (3, 2)

Solving equations (i) and (iii) we get point of intersection (–1, 1)

These lines are plotted on coordinate plane as shown in following figure :



[1]

From the figure, area of the shaded region,

$$A=\int_{-1}^{0}(4x+5)\,dx+\int_{0}^{3}(5-x)\,dx-\int_{-1}^{3}\left(\frac{x+5}{4}\right)dx$$

$$=\left[\frac{4x^2}{2}+5x\right]_{-1}^{0}+\left[5x-\frac{x^2}{2}\right]_{0}^{3}-\frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$$

$$=\left[0-2+5\right]+\left[15-\frac{9}{2}-0\right]-\frac{1}{4}\left[\frac{9}{2}+15-\frac{1}{2}\right]$$

$$=\frac{15}{2} \text{ sq.units}$$

[2]

Q. 5. Draw a rough sketch of the region $(x, y): y^2 \le 6ax$ and $x^2+y^2 \le 16a^2$. Also find the area of the region sketched using method of integration.

[NCERT Exemp. Ex. 8.3, Q. 19, Page 177]

Ans. We have $y^2 \le 6ax$, which represents the region interior of parabola $y^2=6ax$ towards focus and $x^2+y^2 \le 16a^2$ represents the interior to circle $x^2+y^2 \le 16a^2$

On solving circle and parabola we get,

$$x^2+6ax=16a^2$$

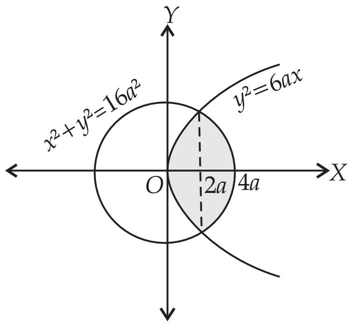
$$x^2+6ax-16a^2=0$$

$$(x-2)(x+8a)=0$$

$$x=2a, -8a$$

[1]

Putting value $x = 2a$ in parabola we get, the graph of functions are as shown in the figure :



[2]

From the figure area of the shaded portion is,

$$A = 2\left[\int_0^{2a} \sqrt{6ax}\,dx + \int_{2a}^{-4a} \sqrt{(4a)^2 - x^2}\,dx\right]$$

$$= 2\left[\left[\sqrt{6a} \times \frac{2x^{\frac{3}{2}}}{3}\right]_0^{2a} + \left[\frac{x}{2}\sqrt{(4a)^2 - x^2} + \frac{(4a)^2}{2}\sin^{-1}\frac{x}{4a}\right]_{2a}^{4a}\right]$$

$$= 2\left[\sqrt{6a}\,\frac{2}{3}(2a)^{\frac{3}{2}} + 8a^2.\frac{\pi}{2} - \frac{2a}{2}\sqrt{16a^2 - 4a^2} - 8a^2.\frac{\pi}{6}\right]$$

On solving the above equation

$$= 2\left[\frac{2}{3}\sqrt{3}a^2 + \frac{8a^2\pi}{3}\right]$$

$$= \frac{4}{3}a^2(\sqrt{3} + 4\pi) \text{ sq.units}$$

[2]

Q. 6. Compute the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

[NCERT Exemp. Ex. 8.3, Q. 21, Page 177]

Ans. $y = 4x + 5$ **(i)**

$y = 5 - x$ **(ii)**

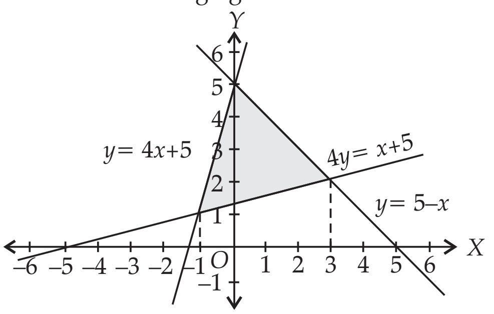
$4y = x + 5$ **(iii)**

On solving the equations (i) and (ii) we get point of intersection (0, 5)

On solving the equations (ii) and (iii) we get point of intersection (3, 2)

On solving the equations (i) and (iii) we get point of intersection (−1, 1) **[2]**

These lines are plotted on coordinate plane as shown in following figure



[1]

From the figure, area of shaded region,

$$A = \int_{-1}^{0} (4x+5)\,dx + \int_0^3 (5-x)\,dx - \int_{-1}^{3} \left(\frac{x+5}{4}\right)dx$$

$$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$$

$$= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2}\right]$$

$$= \frac{15}{2} \text{ sq.units}$$

[2]

Q. 7. Draw a rough sketch of the given curve, $y = 1 + |x + 1|$, $x = -3$, $x = 3$ and $y = 0$. Find the area of region bounded by them, using integration.

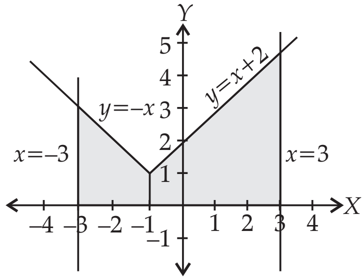
[NCERT Exemp. Ex. 8.3, Q. 23, Page 177]

Ans. We have $y = 1 + |x + 1|$, $x = -3$ $x = 3$ and $y = 0$,

Now $|x+1| = \begin{Bmatrix} -x-1, & x < -1 \\ +x+1, & x \geq -1 \end{Bmatrix}$

$$y = 1 + |x+1| = \begin{Bmatrix} -x, & x < -1 \\ +x+2, & x \geq -1 \end{Bmatrix}$$

Graph of the above function with $x = -3$, $x = 3$ as is shown in the following figure :



[3]

From the figure, area of the shaded region,

$$A = 2\int_{-3}^{-1} -x\,dx + \int_{-1}^{3} (x+2)\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\left[\frac{1}{2} - \frac{9}{2}\right] + \left[\frac{9}{2} + 6 - \frac{1}{2} + 2\right]$$

$= 16$ sq. units **[2]**

Q. 8. Use the method of integration find the area of ΔABC, coordinates of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$.

[CBSE Board, All India Region, 2017]

Ans. We have vertices of a ΔABC as $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$.

Equation of AB is $y - 1 = \left(\frac{6-1}{6-4}\right)(x+4)$

$$2y - 2 = 5x - 20$$

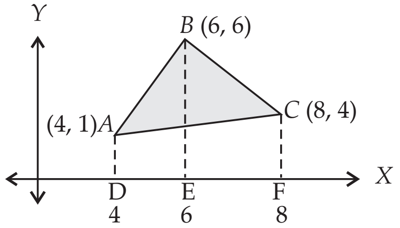
$$y = \frac{5x}{2} - 9$$ **(i)**

Equation of BC is $y - 6 = \left(\frac{4-6}{8-6}\right)(x-6)$

$$y = 12 - x$$ **(ii)**

Equation of line AC is $y - 1 = \left(\frac{4-1}{8+4}\right)(x-4)$

$$y=\frac{3x}{4}-2 \qquad \textbf{(iii)}$$



[3]

These lines are plotted on coordinate plane as shown in following figure :

From the figure, area of the shaded region,

$$A=\int_4^6\left(\frac{5x}{2}-9\right)dx+\int_6^8(12-x)dx-\int_4^6\left(\frac{3x}{4}-2\right)dx$$

$$=\left[\frac{5x^2}{4}-9x\right]_4^6+\left[12x-\frac{x^2}{2}\right]_6^8-\left[\frac{3x^2}{8}-2x\right]_4^6$$

$=7$ sq. units

[3]

Q. 9. Find the area enclosed between the parabola $4y=3x^2$ and straight line $3x-2y+12=0$.

[CBSE Board, All India Region, 2017]

Ans. Given that the curve $4y=3x^2$ (i)

The line $3x-2y+12=0$ (ii)

From equation (ii)

$$y=\frac{3x+12}{2}$$

Putting value of y in equation (i)

$6x+24=3x^2$

$x=4,-2$

When $x=4$ then $y=12$

When $x=-2$ then $y=3$

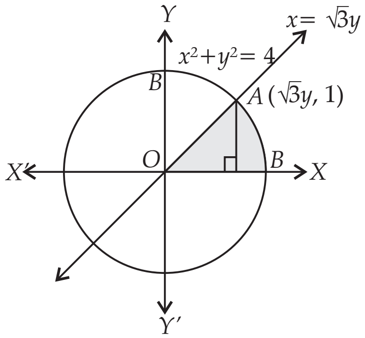
Required area

$$A=\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3x^2}{4}\right)dx$$

$=27$ sq. units. **[6]**

Q. 10. Find the area of the region in the first quadrant enclosed by x-axis, line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$. **[CBSE Board, Delhi Region, 2017]**

Ans. The area of the region bounded by the circle $x^2+y^2=4$, $x=\sqrt{3}\ y$, and the x-axis is the area of OAB.



The point of intersection of the line and circle lies in the first quadrant $(\sqrt{3},1)$.

Area of OAB = Area of ΔACB + Area of ΔABC

$$\text{Area of }\Delta ACB=\frac{1}{2}\times OC\times AC=\frac{1}{2}\times\sqrt{3}\times 1=\frac{\sqrt{3}}{2}$$

$$\text{Area of }\Delta ABC=\int_{\sqrt{3}}^{2}y\,dx$$

$$=\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$=\left[2\times\frac{\pi}{2}-\frac{\sqrt{3}}{2}\sqrt{4-3}-\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$=\left[\pi-\frac{\sqrt{3}\pi}{2}-\frac{2\pi}{3}\right]$$

$$=\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]\text{ sq. units.}$$

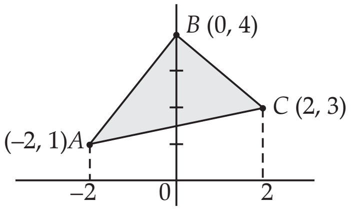
Area enclosed by x axis the line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$ in the first quadrant $=\frac{\sqrt{3}}{2}+\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right]=\frac{\pi}{3}$ units. **[6]**

Q. 11. Using integration, find the area of region bounded by the triangle whose vertices are (–2, 1), (0, 4) and (2, 3). **[CBSE Board, Delhi Region, 2017]**

Ans. Equation of AB : $y=\frac{3}{2}x+4$

Equation of BC : $y=4-\frac{x}{2}$

Equation of AC : $y=\frac{1}{2}x+2$



[3]

Required area

$$=\int_{-2}^{0}\left(\frac{3}{2}x+4\right)dx+\int_{0}^{2}\left(4-\frac{x}{2}\right)dx-\int_{-2}^{2}\left(\frac{1}{2}x+2\right)dx$$

$$=\left[\frac{3x^2}{4}+4x\right]_{-2}^{0}+\left[4x-\frac{x^2}{2}\right]_{0}^{2}-\left[\frac{x^2}{4}+2x\right]_{-2}^{2}$$

$=5+7-8$

$=4$ sq. units. **[3]**

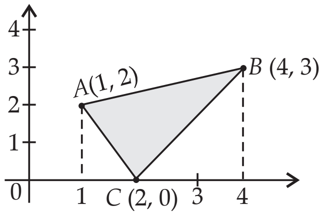
Q. 12. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are $A(1, 2)$, $B(2, 0)$ and $C(4, 3)$.

[CBSE Board, Foreign Scheme, 2017]

Ans. Equation of AB : $y=\frac{x+5}{3}$

Equation of BC : $y=\frac{3x}{2}-2$

Equation of AC : $y=4-2x$



Required area,

$$A = \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)\,dx - \int_2^4 \left(\frac{3x}{2}-2\right)dx$$

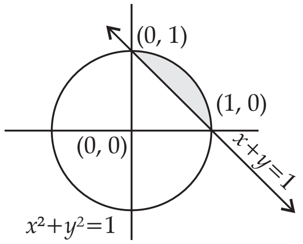
$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x - x^2\right]_1^2 - \left[\frac{3x^2}{4}-3x\right]_2^4$$

$$= \frac{15}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units.}$$ **[6]**

Q. 13. Using integration, find the area of the region $(x^2 + y^2 \le 1 \le x + y)$

[CBSE Board, Foreign Scheme, 2017]

Ans.



Required area,

$$A = \left(\int_0^1 \sqrt{1-x^2} - (1-x)\right)dx$$

$$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}\sin^{-1}(1) - \frac{1}{2}$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ sq. units}$$ **[6]**

Q. 14. Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

[NCERT Exemp. Ex. 8.3, Q. 20, Page 177]

Ans. We have lines

$x + 2y = 2$ …(i)

$y - x = 1$ …(ii)

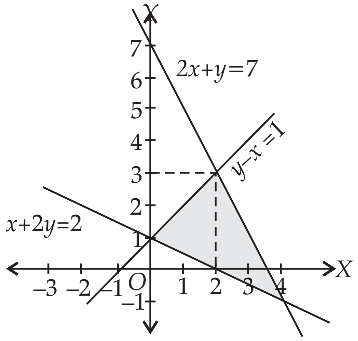
and $2x + y = 7$ …(iii)

Solving (i) and (ii), we get point of intersection (0, 1)

Solving (ii) and (iii), we get point of intersection (2, 3)

Solving (i) and (iii), we get point of intersection (4, –1)

These lines are plotted on coordinate plane as shown in the following figure :



∴ From the figure, area of the shaded region

$$A = \int_0^2 \left(x + 1 - \frac{2-x}{2}\right)dx + \int_2^4 \left(7 - 2x - \frac{2-x}{2}\right)dx$$

$$= \int_0^2 \frac{3x}{2}dx + \int_2^4 \left(6 - \frac{3}{2}x\right)dx$$

$$= \left[\frac{3x^2}{4}\right]_0^2 + \left[6x - \frac{3x^2}{4}\right]_2^4$$

$$= 3 + (24-12) - (12-3) = 6 \text{ sq. units.}$$ **[5]**

Q. 15. Using integration find the area of the region $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}$.

[CBSE Board, Delhi Region, 2016]

Ans. We have

$x^2 + y^2 \le 2ax$

$\Rightarrow x^2 - 2ax + y^2 \le 0$

$\Rightarrow x^2 - 2ax + a^2 - a^2 + y^2 \le 0$

$\Rightarrow (x-a)^2 + y^2 \le a^2.$

Also,

$y^2 \ge ax$

To find the point of intersection of $(x-a)^2 + y^2 = a^2$ and $y^2 = ax$

Substituting, $y^2 = ax$ in $(x-a)^2 + y^2 = a^2$,

$\Rightarrow (x - a)^2 + ax = a^2$

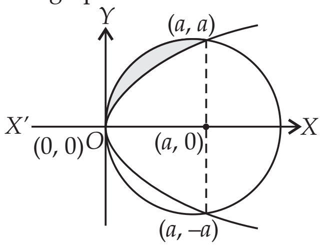
$\Rightarrow x^2 + a^2 - 2ax + ax = a^2$

$\Rightarrow x^2 - ax = 0$

$\Rightarrow x(x-a) = 0$

$\Rightarrow x = 0, a$

The graph is as follows :



Area of the shaded portion is :

$$\text{Area} = \int_0^a \left(\sqrt{a^2 - (x-a)^2} - \sqrt{ax}\right)dx$$

$$\text{Area} = \left(\frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \frac{2\sqrt{a}}{3}(x)^{3/2}\right)_0^a$$

$$\text{Area} = \left(0+0-0-\frac{a^2}{2}\sin^{-1}(-1)-\frac{2\sqrt{a}}{3}(a)^{3/2}\right)_0^a$$

$$\text{Area} = \frac{a^2}{2}\frac{\pi}{2}-\frac{2}{3}a^2$$

$$\text{Area} = a^2\left(\frac{\pi}{4}-\frac{2}{3}\right) \text{ sq. units.}$$

[6]

Q. 16. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$

[CBSE Board, Delhi Region, 2018]

Ans. Given : $y = x$ and $x^2 + y^2 = 32$

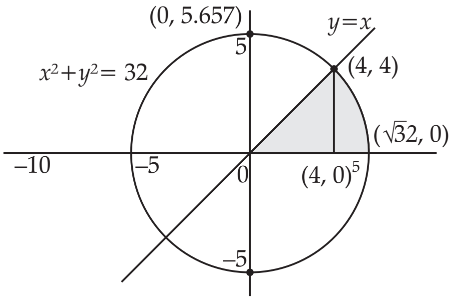
Substitute $y = x$ in $x^2 + y^2 = 32$, we get

$x^2 + x^2 = 32$

$\Rightarrow 2x^2 = 32$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$



From the above figure, area of the shaded region,

$$A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$

$$A = \left|\frac{x^2}{2}\right|_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$A = \frac{16}{2} + \left[0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{32-16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$$

$$= 8 + \left[16\frac{\pi}{2} - 2\times 4 - 16\times\frac{\pi}{4}\right]$$

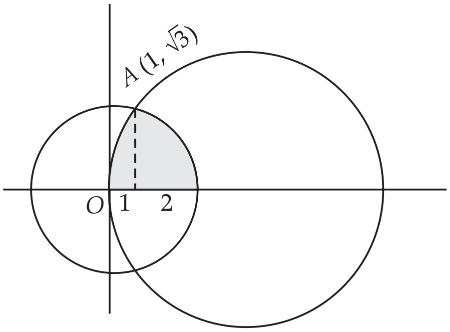
$$= 8 + [8\pi - 8 - 4\pi]$$

$$= 4\pi \text{ sq. units}$$

[6]

Q. 17. Using integration find the area of the region bounded by the curves $y = \sqrt{4-x^2}$, $x^2 + y^2 - 4x = 0$ and the x-axis. **[CBSE Board, Foreign Scheme, 2016]**

Ans. Their point of intersection is $(1, \sqrt{3})$



Required Area,

$$A = \int_0^1 \sqrt{(2)^2-(x-2)^2}\,dx + \int_1^2 \sqrt{2^2-x^2}\,dx$$

$$= \left[\frac{(x-2)\sqrt{4x-x^2}}{2} + 2\sin^{-1}\frac{x-2}{2}\right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$$

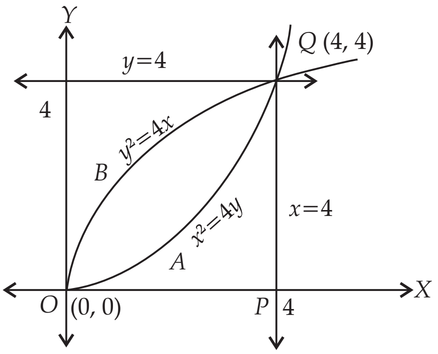
$$= \left(\frac{5\pi}{3} - \sqrt{3}\right) \text{sq. units}$$

[6]

Q. 18. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x=0$, $x=4$ and $y=0$ into three equal parts.

[CBSE Board, All India Region, 2016]

Ans. Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



$$\text{area }(OAQBO) = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left[\left\{\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right\}\right]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

$$\text{area }(OPQAO) = \int_0^4 \frac{x^2}{4}dx = \left.\frac{1}{12}x^3\right]_0^4 = \frac{16}{3}$$

$$\text{area }(OBQRO) = \int_0^4 \frac{y^2}{4}dy = \left.\frac{1}{12}y^3\right]_0^4 = \frac{16}{3}$$

[6]

Hence, produced the areas of the three regions are equal.

Some Commonly Made Errors

➢ Equations for all curves are different and have different standard form of representation.

EXPERT ADVICE

- *Learn all the integral formulae from the chapter.*
- *Learn all the graphs of parabola, circle, lines, etc.*
- *Try to solve conditions and make graphs based on the given questions.*

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