CHAPTER

7 INTEGRALS

Chapter Objectives

This chapter will help you understand :

- ***Integrals :*** *Introduction; Integration as an inverse process of differentiation; Methods of integration; Integrals of some particular functions; Integration by partial fractions; Integration by parts; Definite integral; Evaluation of definite integrals by substitution; and Some properties of definite integrals.*

Quick Review

- Integration is the act of bringing together smaller components into a single system that functions as one.
- Integration is the inverse process to differentiation. So instead of multiplying by the index and reducing the index by one, we increase the index by one and divide by the new index.
- The first documented systematic technique capable of determining integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus (ca. 370 BC), who sought to find areas and volumes by breaking them up into an infinite number of divisions for which the area or volume was known.
- This method was further developed and employed by Archimedes in the third century BC and used to calculate areas for parabolas and an approximation to the area of a circle.
- The major advance in integration came in the seventeenth century with the independent discovery of the fundamental theorem of calculus by Newton and Leibniz. The theorem demonstrates a connection between integration and differentiation.

TIPS...

- *Understand the concept of integration of rational-irrational, trigonometric, exponential and logarithmic functions.*
- *Study the proper method to solve integration-based concepts.*
- *Keep a separate list of important concepts and formulae to revise as often as possible.*
- *Try to practice a variety of questions from different topics to gain speed.*

TRICKS...

- *Manipulations of definite integrals may rely upon specific limits for the integral, like with odd and even functions, or they may require directly changing the integrand itself, through some types of substitution.*
- *However, most integrals require a combination of techniques, and many of the more complicated approaches, like interpretation as a double integral, require multiple steps to reduce the expression.*

Know the Links

- *https://brilliant.org/wiki/integration-tricks/*
- *https://www.whitman.edu/mathematics/calculus_online/chapter09.html*
- *https://www.intmath.com/integration/integration-mini-lecture-indefinite-definite.php*
- *https://www.mathsisfun.com/calculus/integration-introduction.html*

Multiple Choice Questions

(1 mark each)

Q. 1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ **is equal to**

(a) $2(\sin x + x\cos\theta) + C$

(b) $2(\sin x - x\cos\theta) + C$

(c) $2(\sin x + 2x\cos\theta) + C$

(d) $2(\sin x - 2x\cos\theta) + C$

[NCERT Exemp. Ex. 7.3, Q. 48, Page 166]

Ans. Correct option : (a)

Explanation : Let,

$$I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$$

$$= \int \frac{(2\cos^2 x - 1 - 2\cos^2\theta + 1)}{\cos x - \cos\theta} dx$$

$$= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{(\cos x - \cos\theta)} dx$$

$$= 2\int (\cos x + \cos\theta) dx$$

$$= 2\sin x + 2\cos\theta + C$$

Q. 2. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x + 1)\,dx$ is

(a) 0 **(b)** 2

(c) π **(d)** 1

[NCERT Ex. 7.11, Q. 20, Page 347]

Ans. Correct option : (c)

Explanation : Let,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right) dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\cdot dx$$

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ and if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$$I = 0 + 0 + 0 + 2\int_0^{\frac{\pi}{2}} 1\cdot dx$$

$$= 2[x]_0^{\frac{\pi}{2}} = \frac{2\pi}{2} = \pi$$

Q. 3. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is

(a) 2 **(b)** $\frac{3}{4}$

(c) 0 **(d)** –2

[NCERT Ex. 7.11, Q. 21, Page 347]

Ans. Correct option : (c)

Explanation :

Let $$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx \quad \ldots(i)$$

$$I = \int_0^{\frac{\pi}{2}} \log\left[\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)}\right] dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) dx \quad \text{...(ii)} \quad \ldots\text{(ii)}$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{\log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) + \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right)\right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x} \times \frac{4 + 3\cos x}{4 + 3\sin x}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0 dx$$

$$\Rightarrow I = 0$$

Q. 4. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(a) $\tan^{-1}(e^x) + C$

(b) $\tan^{-1}(e^{-x}) + C$

(c) $\log(e^x - e^{-x}) + C$

(d) $\log(e^x + e^{-x}) + C$

[NCERT Misc. Ex. Q. 41, Page 353]

Ans. Correct option : (a)

Explanation :

Let $I = \int \frac{dx}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx$

Also, let $e^x = t$

$\Rightarrow \quad e^x\,dx = dt$

$$\therefore \quad I = \int \frac{dt}{1 + t^2}$$

$$= \tan^{-1} t + C$$

$$= \tan^{-1}\left(e^x\right) + C$$

Q. 5. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

(a) $\frac{-1}{\sin x + \cos x} + C$

(b) $\log|\sin x + \cos x| + C$

(c) $\log|\sin x - \cos x| + C$

(d) $\frac{1}{(\sin x + \cos x)^2}$

[NCERT Misc. Ex. Q. 42, Page 354]

Ans. Correct option : (b)

Explanation :

Let $I = \frac{\cos 2x}{(\cos x + \sin x)^2}$

$$I = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{\cos x - \sin x}{\cos + \sin x} dx$$

Let $\cos x + \sin x = t$

$\Rightarrow (\cos x - \sin x)dx = dt$

$$\therefore \quad I = \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|\cos x + \sin x| + C$$

Q. 6. If $f(a + b - x) = f(x)$, then $\int_a^b x\,f(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b - x)dx$ **(b)** $\frac{a+b}{2}\int_a^b f(b + x)dx$

(c) $\frac{b-a}{2}\int_a^b f(x)dx$ **(d)** $\frac{a+b}{2}\int_a^b f(x)dx$

[NCERT Misc. Ex. Q. 43, Page 353]

Ans. Correct option : (d)

Explanation :

Let $I = \int_a^b x\,f(x)dx \quad \ldots(i)$

$$I=\int_a^b (a+b-x)f(a+b-x)dx$$

$$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$$\Rightarrow \quad I=\int_a^b (a+b-x)f(x)dx$$

$$\Rightarrow \quad I=(a+b)\int_a^b f(x)dx - I \qquad [\text{Using Eq. (i)}]$$

$$\Rightarrow \quad I+I=(a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad 2I=(a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad I=\left(\frac{a+b}{2}\right)\int_a^b f(x)dx$$

Q. 7. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(a) 1 **(b) 0**

(c) –1 **(d) $\frac{\pi}{4}$**

[NCERT Misc. Ex. Q. 44, Page 353]

Ans. Correct option : (b)

Explanation :

Let $I=\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$$\Rightarrow I=\int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$$

$$\Rightarrow I=\int_0^1 \left[\tan^{-1}x-\tan^{-1}(1-x)\right]dx \qquad \text{...(i)}$$

$$\Rightarrow I=\int_0^1 \left[\tan^{-1}(1-x)-\tan^{-1}(1-1+x)\right]dx$$

$$\Rightarrow I=\int_0^1 \left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx$$

$$\Rightarrow I=\int_0^1 \left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx \qquad \text{...(ii)}$$

Adding equations (i) and (ii), we obtain

$$2I=\int_0^1 \left(\tan^{-1}x+\tan^{-1}(1-x)-\tan^{-1}(1-x)-\tan^{-1}x\right)dx$$

$$\Rightarrow 2I=0$$

$$\Rightarrow \quad I=0$$

Q. 8. $\dfrac{dx}{\sin(x-a)\ \sin(x-b)}$ is equal to

(a) $\sin\ (b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(b) $\text{cosec}\ (b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

(c) $\text{cosec}\ (b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(d) $\sin\ (b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

[NCERT Exemp. Ex. 7.3, Q. 49, Page. 167]

Ans. Correct option : (c)

Explanation : Let,

$$I=\int \frac{dx}{\sin(x-a)\sin(x-b)}$$

$$=\frac{1}{\sin(b-a)}\int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int \frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int \frac{\sin(x-a)\cos(x-b)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int [\cot(x-b)-\cot(x-a)]dx$$

$$=\frac{1}{\sin(b-a)}[\log|\sin(x-b)|-\log|\sin(x-a)|]+C$$

$$=\text{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$$

Q. 9. $\int \sqrt{1+x^2}dx$ is equal to

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

(b) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+C$

(c) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+C$

(d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)+C$

[NCERT Ex. 7.7, Q. 10, Page 330]

Ans. Correct option : (a)

Explanation : It is known that,

$$\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\therefore \int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

Q. 10. $\int\sqrt{x^2-8x+7}dx$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

[NCERT Ex. 7.7, Q. 11, Page 330]

Ans. Correct option : (d)

Explanation :

Let $I=\int\sqrt{x^2-8x+7}dx$

$$=\int\sqrt{(x^2-8x+16)-9}dx$$

$$=\int\sqrt{(x-4)^2-(3)^2}dx$$

It is known that,

$$\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$$

Q. 11. $\int \frac{x\,dx}{(x-1)(x-2)}$ **equals**

(a) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$

(b) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$

(d) $\log|(x-1)(x-2)|+C$

[NCERT Ex. 7.5, Q. 22, Page 323]

Ans. Correct option : (b)

Explanation :

Let, $\frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$

$x=A(x-2)+B(x-1)$ ...( ) ...(i)

Substituting x = 1 and 2 in Eq. (1), we obtain

A = –1 and B = 2

$\therefore \frac{x}{(x-1)(x-2)}=-\frac{1}{(x-1)}+\frac{2}{(x-2)}$

$\Rightarrow \int\frac{x}{(x-1)(x-2)}dx=\int\left\{\frac{-1}{(x-1)}+\frac{2}{(x-2)}\right\}dx$

$=-\log|x-1|+2\log|x-2|+C$

$=\log\left|\frac{(x-2)^2}{x-1}\right|+C$

Q. 12. The value of integral $\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$ **is**

(a) 6 **(b)** 0

(c) 3 **(d)** 4

[NCERT Ex. 7.10, Q. 9, Page 340]

Ans. Correct option : (a)

Explanation :

Let $I=\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$

Also, let x = sin $\theta \Rightarrow dx$ = cos $\theta d\theta$

When $x=\frac{1}{3}$, $\theta=\sin^{-1}\left(\frac{1}{3}\right)$ and when x = 1, $\theta=\frac{\pi}{2}$

$\Rightarrow I=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta-\sin^3\theta)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\,d\theta$

$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(1-\sin^2\theta)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\,d\theta$

$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta}\cos\theta\,d\theta$

$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta\sin^2\theta}\cos\theta\,d\theta$

$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}}\text{cosec}^2\theta\,d\theta$

$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}(\cot\theta)^{\frac{5}{3}}\text{cosec}^2\theta\,d\theta$

Let,

$\cot\theta=t$

$\Rightarrow -\text{cosec }2\theta d\theta=dt$

When $\theta=\sin^{-1}\left(\frac{1}{3}\right)$, $t=2\sqrt{2}$ and when $\theta=\frac{\pi}{2}$, $t = \theta$

$\therefore I=-\int_{2\sqrt{2}}^{0}(t)^{\frac{5}{3}}dt$

$=-\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$

$=-\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$

$=-\frac{3}{8}\left[-(2\sqrt{2})^{\frac{8}{3}}\right]$

$=\frac{3}{8}\left[(\sqrt{8})^{\frac{8}{3}}\right]$

$=\frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$

$=\frac{3}{8}[16]$

$=3\times 2$

$=6$

Q. 13. If $f(x)=\int_0^x t\sin t\,dt$ **, then** $f'(x)$ **is**

(a) cos x + x sin x

(b) x sin x

(c) x cos x

(d) sin x + x cos x **[NCERT Ex. 7.10, Q. 10, Page 340]**

Ans. Correct option : (b)

Explanation :

$f(x)=\int_0^x t\sin t\,dt$

Integrating by parts, we obtain

$f(x)=t\int_0^x \sin t\,dt-\int_0^x\left\{\left(\frac{d}{dt}t\right)\int\sin t\,dt\right\}dt$

$=\left[t(-\cos t)\right]_0^x-\int_0^x(-\cos t)dt$

$=\left[-t\cos t+\sin t\right]_0^x$

$=-x\cos x+\sin x$

$\Rightarrow f'(x)=-\left[\{x(-\sin x)\}+\cos x\right]+\cos x$

$=x\sin x-\cos x+\cos x$

$=x\sin x$

Q. 14. $\int\frac{dx}{x(x^2+1)}$ **equals**

(a) $\log|x|-\frac{1}{2}\log(x^2+1)+C$

(b) $\log|x|+\frac{1}{2}\log(x^2+1)+C$

(c) $-\log|x|+\frac{1}{2}\log(x^2+1)+C$

(d) $\frac{1}{2}\log|x|+\log(x^2+1)+C$

[NCERT Ex. 7.5, Q. 23, Page 323]

Ans. Correct option : (a)

Explanation :

Let $\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$1=A(x^2+1)+(Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

$A+B=0$

$C=0$

$A=1$

On solving these equations, we obtain

$A = 1, B = -1$, and $C = 0$

$\therefore \quad \frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-x}{x^2+1}$

$\Rightarrow \int\frac{1}{x(x^2+1)}dx=\int\left\{\frac{1}{x}-\frac{x}{x^2+1}\right\}dx$

$=\log|x|-\frac{1}{2}\log|x^2+1|+C$

Q. 15. $\int \tan^{-1}\sqrt{x}\,dx$ **is equal to**

(a) $(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

(d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

[NCERT Exemp. Ex. 7.3, Q. 50, Page 167]

Ans. Correct option : (a)

Explanation : Let,

$I=\int 1\cdot\tan^{-1}\sqrt{x}\;dx$

$=\tan^{-1}\sqrt{x}\cdot x-\frac{1}{2}\int\frac{1}{(1+x)}\cdot\frac{2}{\sqrt{x}}dx$

$=x\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{2}{\sqrt{x}(1+x)}dx$

Put $x = t^2 \Rightarrow dx = 2t\,dt$

$\therefore \quad I=x\tan^{-1}\sqrt{x}-\int\frac{t}{t(1+t^2)}dt$

$=x\tan^{-1}\sqrt{x}-\int\frac{t^2}{1+t^2}dt$

$=x\tan^{-1}\sqrt{x}-\int\left(1-\frac{1}{1+t^2}\right)dt$

$=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}t+C$

$=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}\sqrt{x}+C$

$=(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

Q. 16. $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx$ **is equal to**

(a) $\frac{e^x}{1+x^2}+C$ **(b)** $\frac{-e^x}{1+x^2}+C$

(c) $\frac{e^x}{(1+x^2)^2}+C$ **(d)** $\frac{-e^x}{(1+x^2)^2}+C$

[NCERT Exemp. Ex. 7.3, Q. 51, Page 167]

Ans. Correct option : (a)

Explanation :

$\int e^x\left(\frac{1-x}{1+x^2}\right)^2dx=\int e^x\frac{1+x^2-2x}{(1+x^2)^2}dx$

$=\int e^x\left[\frac{1}{(1+x^2)}-\frac{2x}{(1+x^2)^2}\right]dx$

$=\int e^x[f(x)+f'(x)]dx$, where $f(x)=\frac{1}{1+x^2}$

$=e^xf(x)+C=\frac{e^x}{1+x^2}+C$

Q. 17. $\int x^2e^{x^3}dx$ **is equal to**

(a) $\frac{1}{3}e^{x^3}+C$ **(b)** $\frac{1}{3}e^{x^2}+C$

(c) $\frac{1}{2}e^{x^3}+C$ **(d)** $\frac{1}{2}e^{x^2}+C$

[NCERT Ex. 7.6, Q. 23, Page 328]

Ans. Correct option : (a)

Explanation :

Let $I=\int x^2e^{x^3}dx$

Also, let $x^3 = t \Rightarrow 3x^2\,dx = dt$

$\Rightarrow I=\frac{1}{3}\int e^t dt$

$=\frac{1}{3}(e^t)+C$

$=\frac{1}{3}e^{x^3}+C$

Q. 18. $\int e^x\sec x\,(1+\tan x)\,dx$ **is equal to**

(a) $e^x\cos x + C$ **(b)** $e^x\sec x + C$

(c) $e^x\sin x + C$ **(d)** $e^x\tan x + C$

[NCERT Ex. 7.6, Q. 24, Page 328]

Ans. Correct option : (b)

Explanation : $\int e^x\sec x(1+\tan x)dx$

Let $I=\int e^x\sec x(1+\tan x)dx=\int e^x(\sec x+\sec x\tan x)dx$

Also, let $\sec x=f(x)\Rightarrow \sec x\tan x=f'(x)$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

$\therefore I = e^x\sec x + C$

Q. 19. $\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$ **is equal to**

(a) $\tan x + \cot x + C$

(b) $\tan x + \operatorname{cosec} x + C$

(c) $-\tan x + \cot x + C$

(d) $\tan x + \sec x + C$ **[NCERT Ex. 7.3, Q. 23, Page 307]**

Ans. Correct option : (a)

Explanation :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int (\sec^2 x - \text{cosec}^2 x) dx$$

$$= \tan x + \cot x + C$$

Q. 20. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ **is equal to**

(a) $-\cot(ex^x) + C$ **(b)** $\tan(xe^x) + C$

(c) $\tan(e^x) + C$ **(d)** $\cot(e^x) + C$

[NCERT Ex. 7.3, Q. 24, Page 307]

Ans. Correct option : (b)

Explanation : $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$

Let $e^x x = t$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1) dx = dt$$

$$e^x(x+1) dx = dt$$

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t \, dt$$

$$= \tan t + C$$

$$= \tan(e^x \cdot x) + C$$

Q. 21. $\int \frac{10x^9 + 10^x \log_e 10 \, dx}{x^{10} + 10^x}$ **is equal to**

(a) $10^x - x^{10} + C$

(b) $10^x + x^{10} + C$

(c) $(10^x - x^{10})^{-1} + C$

(d) $\log(10^x + x^{10}) + C$

[NCERT Ex. 7.2, Q. 38, Page 305]

Ans. Correct option : (d)

Explanation :

Let $x^{10} + 10^x = t$

$$(10x^9 + 10^x \log_e 10) dx = dt$$

$$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log(10^x + x^{10}) + C$$

Q. 22. $\int \frac{dx}{\sin^2 x \cos^2 x}$ **is equal to**

(a) $\tan x + \cot x + C$ **(b)** $\tan x - \cot x + C$

(c) $\tan x \cot x + C$ **(d)** $\tan x - \cot 2x + C$

[NCERT Ex. 7.2, Q. 39, Page 305]

Ans. Correct option : (b)

Explanation :

Let $I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \text{cosec}^2 \, x dx$$

$$= \tan x - \cot x + C$$

Q. 23. $\int \frac{x^9}{(4x^2+1)^6} dx$ **is equal to**

(a) $\frac{1}{5x}\left(4 + \frac{1}{x^2}\right)^{-5} + C$ **(b)** $\frac{1}{5}\left(4 + \frac{1}{x^2}\right)^{-5} + C$

(c) $\frac{1}{10x}(1+4)^{-5} + C$ **(d)** $\frac{1}{10}\left(\frac{1}{x^2} + 4\right)^{-5} + C$

[NCERT Exemp. Ex. 7.3, Q. 52, Page 167]

Ans. Correct option : (d)

Explanation : Let

$$I = \int \frac{x^9}{(4x^2+1)^6} dx$$

$$= \int \frac{x^9}{x^{12}\left(4 + \frac{1}{x^2}\right)^6} dx$$

$$= \int \frac{dx}{x^3\left(4 + \frac{1}{x^2}\right)^6}$$

Put $4 + \frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3} dx = dt$

$$\Rightarrow \frac{1}{x^3} dx = -\frac{1}{2} dt$$

$$\therefore \quad I = -\frac{1}{2}\int \frac{dt}{t^6} = -\frac{1}{2}\left[\frac{t^{-6+1}}{-6+1}\right] + C$$

$$= \frac{1}{10}\left[\frac{1}{t^5}\right] + C$$

$$= \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C$$

Q. 24. **If**

$$\int \left[\begin{array}{l} \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| \\ +b\tan^{-1} x + \frac{1}{5}\log|x+2| + C \end{array}\right]$$ **, then**

(a) $a = \frac{-1}{10}, b = \frac{-2}{5}$ **(b)** $a = \frac{1}{10}, b = -\frac{2}{5}$

(c) $a = \frac{-1}{10}, b = \frac{2}{5}$ **(d)** $a = \frac{1}{10}, b = \frac{2}{5}$

[NCERT Exemp. Ex. 7.3, Q. 53, Page 168]

Ans. Correct option : (c)

Explanation : Given that,

$$\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1} x + \frac{1}{5}\log|x+2| + C$$

Now, $I = \int \frac{dx}{(x+2)(x^2+1)}$

$$\frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \ 1 = A(x^2+1) + (Bx+C)(x+2)$$

$$\Rightarrow \ 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$\Rightarrow \ 1 = (A+B)x^2 + (2B+C)x + A + 2C$$

$$\Rightarrow \ A + B = 0, \ A + 2C = 1, \ 2B + C = 0$$

We have, $A = \frac{1}{5}$, $B = -\frac{1}{5}$ and $C = \frac{2}{5}$

$$\therefore \int \frac{dx}{(x+2)(x^2+1)} = \frac{1}{5}\int \frac{1}{x+2}dx + \int \frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}dx$$

$$= \frac{1}{5}\int \frac{1}{x+2}dx - \frac{1}{5}\int \frac{x}{1+x^2}dx + \frac{1}{5}\int \frac{2}{1+x^2}dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{10}\log|1+x^2| + \frac{2}{5}\tan^{-1}x + C$$

$$\therefore b = \frac{2}{5} \text{ and } a = \frac{-1}{10}$$

Q. 25. $\int \frac{x^3}{x+1}$ **is equal to**

(a) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

[NCERT Exemp. Ex. 7.3, Q. 54, Page 168]

Ans. Correct option : (d)

Explanation : Let,

$$I = \int \frac{x^3}{x+1}dx$$

$$= \int\left((x^2 - x + 1) - \frac{1}{(x+1)}\right)dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$$

Q. 26. $\int \frac{x+\sin x}{1+\cos x}dx$ **is equal to**

(a) $\log|1+\cos x| + C$ **(b)** $\log|x+\sin x| + C$

(c) $x - \tan\frac{x}{2} + C$ **(d)** $x \cdot \tan\frac{x}{2} + C$

[NCERT Exemp. Ex. 7.3, Q. 55, Page 168]

Ans. Correct option : (d)

Explanation : Let,

$$I = \int \frac{x+\sin x}{1+\cos x}dx$$

$$= \int \frac{x}{1+\cos x}dx + \int \frac{\sin x}{1+\cos x}dx$$

$$= \int \frac{x}{2\cos^2 x/2}dx + \int \frac{2\sin x/2\cos x/2}{2\cos^2 x/2}dx$$

$$= \frac{1}{2}\int x\sec^2 x/2dx + \int \tan x/2dx$$

$$= \frac{1}{2}\left[x\cdot\tan x/2\cdot 2 - \int \tan\frac{x}{2}\cdot 2dx\right] + \int \tan\frac{x}{2}dx$$

$$= x\cdot\tan\frac{x}{2} + C$$

Q. 27. If $\frac{x^3dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$**, then**

(a) $a = \frac{1}{3}$, $b = 1$ **(b)** $a = \frac{-1}{3}$, $b = 1$

(c) $a = \frac{-1}{3}$, $b = -1$ **(d)** $a = \frac{1}{3}$, $b = -1$

[NCERT Exemp. Ex. 7.3, Q. 56, Page 168]

Ans. Correct option : (d)

Explanation : Let,

$$I = \int \frac{x^3}{\sqrt{1+x^2}}dx = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$$

$$\therefore I = \int \frac{x^3}{\sqrt{1+x^2}}dx = \int \frac{x^2\cdot x}{\sqrt{1+x^2}}dx$$

Put $1+x^2 = t^2$

$\Rightarrow 2x\,dx = 2t\,dt$

$$\therefore I = \int \frac{t(t^2-1)}{t}dt = \frac{t^3}{3} - t + C$$

$$= \frac{1}{3}(1+x^2)^{3/2} - \sqrt{1+x^2} + C$$

$$\therefore a = \frac{1}{3} \text{ and } b = -1$$

Q. 28. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$ **is equal to**

(a) 1 **(b)** 2

(c) 3 **(d)** 4

[NCERT Exemp. Ex. 7.3, Q. 57, Page 168]

Ans. Correct option : (a)

Explanation : Let

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$$

$$= \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x}$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$$= \int_0^{\pi/4} \sec^2 x dx$$

$$= [\tan x]_0^{\pi/4}$$

$$= 1$$

Q. 29. $\int \frac{dx}{x^2+2x+2}$ **equals**

(a) $x\tan^{-1}(x+1) + C$ **(b)** $\tan^{-1}(x+1) + C$

(c) $(x+1)x\tan^{-1} + C$ **(d)** $\tan^{-1} + C$

[NCERT Ex. 7.4, Q. 24, Page 316]

Ans. Correct option : (b)

Explanation :

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$$

$$= \int \frac{1}{(x+1)^2+(1)^2}dx$$

$$= \left[\tan^{-1}(x+1)\right] + C$$

Q. 30. $\int \frac{dx}{\sqrt{9x-4x^2}}$ **is equal to**

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right) + C$ **(b)** $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$ **(d)** $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

[NCERT Ex. 7.4, Q. 25, Page 316]

Ans. Correct option : (b)

Explanation :

$$\int\frac{dx}{\sqrt{9x-4x^2}}=\int\frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$=\int\frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}}dx$$

$$=\int\frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$=\frac{1}{2}\int\frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$=\frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C$$

$$\left(\because\int\frac{dy}{\sqrt{a^2-y^2}}=\sin^{-1}\frac{y}{a}+C\right)$$

$$=\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Q. 31. $\int_0^{\frac{\pi}{2}}\sqrt{1-\sin 2x dx}$ **is equal to**

(a) $2\sqrt{2}$ **(b)** $2\left(\sqrt{2}+1\right)$

(c) 2 **(d)** $2\left(\sqrt{2}-1\right)$

[NCERT Exemp. Ex. 7.3, Q. 58, Page 169]

Ans. Correct option : (d)

Explanation : Let

$$I=\int_0^{\pi/2}\sqrt{1-\sin 2x}\ dx$$

$$=\int_0^{\pi/4}\sqrt{(\cos x-\sin x)^2}\ dx+\int_{\pi/4}^{\pi/2}\sqrt{(\sin x-\cos x)^2}\ dx$$

$$=[\sin x+\cos x]_0^{\pi/4}+[-\cos x-\sin x]_{\pi/4}^{\pi/2}$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-0-1+\left(-0-1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

$$=2\sqrt{2}-2=2(\sqrt{2}-1)$$

Q. 32. The anti-derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ **equals**

(a) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$ **(b)** $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^2+C$

(c) $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$ **(d)** $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

[NCERT Ex. 7.1, Q. 21, Page 299]

Ans. Correct option : (c)

Explanation :

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Q. 33. $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$ **equals**

(a) $\frac{\pi}{3}$ **(b)** $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ **(d)** $\frac{\ }{12}$

[NCERT Ex. 7.9, Q. 21, Page 338]

Ans. Correct option : (d)

Explanation :

$$\int\frac{dx}{1+x^2}=\tan^{-1}x=F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}}\frac{dx}{1+x^2}=F\left(\sqrt{3}\right)-F(1)$$

$$=\tan^{-1}\sqrt{3}-\tan^{-1}1$$

$$=\frac{\pi}{3}-\frac{\pi}{4}$$

$$=\frac{\pi}{12}$$

Q. 34. $\int_0^{\frac{2}{3}}\frac{dx}{4+9x^2}$ **equals**

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$ **(d)** $\frac{\pi}{4}$

[NCERT Ex. 7.9, Q. 22, Page 338]

Ans. Correct option : (c)

Explanation :

$$\int\frac{dx}{4+9x^2}=\int\frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore\int\frac{dx}{(2)^2+(3x)^2}=\frac{1}{3}\int\frac{dt}{(2)^2+(t)^2}$$

$$=\frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$

$$=\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$

$$=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}}\frac{dx}{4+9x^2}=F\left(\frac{2}{3}\right)-F(0)$$

Q. 35. If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ **such that** $f(2) = 0$**. Then** $f(x)$ **is**

(a) $x^4+\frac{1}{x^3}-\frac{129}{8}$ **(b)** $x^3+\frac{1}{x^4}+\frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

[NCERT Ex. 7.1, Q. 22, Page 299]

Ans. Correct option : (a)

Explanation : It is given that,

$$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$ Anti-derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}dx$$

$$f(x) = 4\int x^3 dx - 3\int \left(x^{-4}\right)dx$$

$$\therefore f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore \quad f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow \quad C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow \quad C = \frac{-129}{8}$$

$$\therefore \quad f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Very Short Answer Type Questions

(1 or 2 marks each)

Q. 1. Integrate the function $\frac{2x}{1+x^2}$.

[NCERT Ex. 7.2, Q. 1, Page 304]

Ans. Let,

$$1 + x^2 = t$$

$$\therefore \quad 2x\,dx = dt$$

$$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

$$= \log\left|1+x^2\right| + C$$

$$= \log\left(1+x^2\right) + C$$

[2]

Q. 2. Integrate the function

$\frac{(\log x)^2}{x}$. **[NCERT Ex. 7.2, Q. 2, Page 304]**

Ans. Let,

$$\log|x| = t$$

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{\left(\log|x|\right)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{\left(\log|x|\right)^3}{3} + C$$

[2]

Q. 3. Integrate the function

$\frac{(1+\log x)^2}{x}$. **[NCERT Ex. 7.2, Q. 35, Page 305]**

Ans. Let,

$$1 + \log x = t$$

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(1+\log x)^3}{3} + C$$

[2]

Q. 4. Integrate the function

$\frac{}{x \quad x\log x}$. **[NCERT Ex. 7.2, Q. 3, Page 304]**

Ans. $\frac{1}{x + x\log x} = \frac{1}{x(1+\log x)}$

Let $1 + \log x = t$

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{1}{x(1+\log x)}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

$$= \log|1 + \log x| + C$$

[2]

Q. 5. Integrate the function sin x sin (cos x).

[NCERT Ex. 7.2, Q. 4, Page 304]

Ans. $\sin x \cdot \sin\,(\cos x)$

Let $\cos x = t$

$\therefore$ **–sin** $x\,dx = dt$

$$\Rightarrow \int \sin x \cdot \sin(\cos x)dx = -\int \sin t\; dt$$

$$= -[-\cos t] + C$$

$$= \cos t + C$$

$$= \cos(\cos x) + C$$

[2]

Q. 6. Find the integrals of the function sin $4x$ sin $8x$.

[NCERT Ex. 7.3, Q. 7, Page 307]

Ans. It is known that,

$$\sin A \cdot \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\therefore \int \sin 4x \;\sin\; 8x\; dx$$

$$= \int \tfrac{1}{2}\{\cos(4x-8x) - \cos(4x+8x)\}dx$$

$$= \tfrac{1}{2}\int \{\cos(-4x) - \cos(12x)\}dx$$

[2]

$= \frac{1}{2}\int(\cos 4x - \cos 12x)dx$

$= \frac{1}{2}\left(\frac{\sin 4x}{4} - \frac{\sin 12x}{12} + C\right)$

Q. 7. Find an anti-derivative (or integral) of the function sin 2*x* by the method of inspection.
[NCERT Ex. 7.1, Q. 1, Page 299]

Ans. The anti-derivative of sin 2*x* is a function of *x* whose derivative is sin 2*x*.

It is known that,

$\frac{d}{dx}(\cos 2x) = -2\sin 2x$

$\Rightarrow \quad \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$

$\therefore \quad \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$

Therefore, the anti-derivative of sin 2*x* is $-\frac{1}{2}\cos 2x$ **[2]**

Q. 8. Find an anti-derivative (or integral) of the function cos 3*x* by the method of inspection.
[NCERT Ex. 7.1, Q. 2, Page 299]

Ans. The anti-derivative of cos 3*x* is a function of *x* whose derivative is cos 3*x*.

It is known that,

$\frac{d}{dx}(\sin 3x) = 3\cos 3x$

$\Rightarrow \quad \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$

$\therefore \quad \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$

Therefore, the anti-derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x.$ **[2]**

Q. 9. Evaluate the definite integral of $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$.
[NCERT Ex. 7.9, Q. 5, Page 338]

Ans. Let $I = \int_0^{\frac{\pi}{2}} \cos 2x \, dx$

$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{2}\right) - F(0)$

$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$

$= \frac{1}{2}[\sin \pi - \sin 0]$

$= \frac{1}{2}[0 - 0] = 0$ **[2]**

Q. 10. Evaluate the definite integral of $\int_4^5 e^x dx$.
[NCERT Ex. 7.9, Q. 6, Page 338]

Ans. Let $I = \int_4^5 e^x dx$

$\int e^x dx = e^x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(5) - F(4)$

$= e^5 - e^4$

$= e^4(e-1)$ **[2]**

Q. 11. Evaluate the definite integral of $\int_0^{\frac{\pi}{4}} \tan x \, dx$.
[NCERT Ex. 7.9, Q. 7, Page 338]

Ans. Let,

$I = \int_0^{\frac{\pi}{4}} \tan x \, dx$

$\int \tan x \, dx = -\log|\cos x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$

$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$

$= -\log(2)^{-\frac{1}{2}}$

$= \frac{1}{2}\log 2$ **[2]**

Q. 12. Evaluate the definite integral of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}\, x \, dx$.
[NCERT Ex. 7.9, Q. 8, Page 338]

Ans. Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}\, x \, dx$

$\int \text{cosec}\, x \, dx = -\log|\text{cosec}\, x - \cot x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$

$= -\log\left|\text{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| + \log\left|\text{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$

$= -\log\left|\sqrt{2} - 1\right| + \log\left|2 - \sqrt{3}\right|$

$= -\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$ **[2]**

Q. 13. Evaluate the definite integral of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.
[NCERT Ex. 7.9, Q. 9, Page 338]

Ans. Let $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$ **[2]**

Q. 14. Evaluate the definite integral of $\int_0^1 \frac{dx}{1+x^2}$.
[NCERT Ex. 7.9, Q. 10, Page 338]

Ans. Let $I=\int_0^1 \frac{dx}{\sqrt{1+x^2}}$

$$\int \frac{dx}{\sqrt{1+x^2}}=\tan^{-1}x=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=F(1)-F(0)$$
$$=\tan^{-1}(1)-\tan^{-1}(0)$$
$$=\frac{\pi}{4}$$

[2]

Q. 15. Evaluate the definite integral of $\int_2^3 \frac{dx}{x^2-1}$.

[NCERT Ex. 7.9, Q. 11, Page 338]

Ans. Let $I=\int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1}=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|$$
$$=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=F(3)-F(2)$$
$$=\frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right|-\log\left|\frac{2-1}{2+1}\right|\right]$$
$$=\frac{1}{2}\left[\log\left|\frac{2}{4}\right|-\log\left|\frac{1}{3}\right|\right]$$
$$=\frac{1}{2}\left[\log\left|\frac{2}{4}\right|-\log\left|\frac{1}{3}\right|\right]$$
$$=\frac{1}{2}\left[\log\frac{3}{2}\right]$$

[2]

Q. 16. Evaluate the definite integral of $\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$.

[NCERT Ex. 7.9, Q. 12, Page 338]

Ans. Let $I=\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$

$$\int \cos^2 x\,dx=\int\left(\frac{1+\cos 2x}{2}\right)dx=\frac{x}{2}+\frac{\sin 2x}{4}$$
$$=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=\left[F\left(\frac{\pi}{2}\right)-F(0)\right]$$
$$=\frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$
$$=\frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$
$$=\frac{\pi}{4}$$

[2]

Q. 17. Evaluate the definite integral of $\int_2^3 \frac{x\,dx}{x^2+1}$.

[NCERT Ex. 7.9, Q. 13, Page 338]

Ans. Let $I=\int_2^3 \frac{x}{x^2+1}dx$

$$\int \frac{x}{x^2+1}dx=\frac{1}{2}\int\frac{2x}{x^2+1}dx=\frac{1}{2}\log\left(1+x^2\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=F(3)-F(2)$$
$$=\frac{1}{2}\left[\log\left(1+(3)^2\right)-\log\left(1+(2)^2\right)\right]$$
$$=\frac{1}{2}[\log(10)-\log(5)]$$
$$=\frac{1}{2}\log\left(\frac{10}{5}\right)=\frac{1}{2}\log 2$$

[2]

Q. 18. Find an anti-derivative (or integral) of the function e^{2x} by the method of inspection.

[NCERT Ex. 7.1, Q. 3, Page 299]

Ans. The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$$
$$\Rightarrow\quad e^{2x}=\frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$
$$\therefore\quad e^{2x}=\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti-derivative of e^{2x} is $\frac{1}{2}e^{2x}$. **[2]**

Q. 19. Integrate the function $x\sqrt{1+2x^2}$.

[NCERT Ex. 7.2, Q. 8, Page 304]

Ans. Let $1+2x^2=t$

$$\therefore 4x\,dx=dt$$

$$\int x\sqrt{1+2x^2}\,dx=\int\frac{\sqrt{t}\,dt}{4}$$
$$=\frac{1}{4}\int t^{\frac{1}{2}}dt$$
$$=\frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$
$$=\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+C$$

[2]

Q. 20. Integrate the function $(4x+2)\sqrt{x^2+x+1}$.

[NCERT Ex. 7.2, Q. 9, Page 304]

Ans. Let $x^2+x+1=t$

$$\therefore (2x+1)dx=dt$$

$$\int(4x+2)\sqrt{x^2+x+1}\,dx=\int 2\sqrt{t}\,dt$$
$$=2\int\sqrt{t}\,dt$$
$$=2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$
$$=\frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

[2]

Q. 21. Integrate the function $\frac{1}{x-\sqrt{x}}$.

[NCERT Ex. 7.2, Q. 10, Page 304]

Ans. $$\frac{1}{x-\sqrt{x}}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right)=t$

$$\therefore \quad \frac{1}{2\sqrt{x}}dx=dt$$

$$\Rightarrow \int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx=\int\frac{2}{t}dt$$

$$=2\log|t|+C$$

$$=2\log\left|\sqrt{x}-1\right|+C$$ **[2]**

Q. 22. Integrate the function $\frac{x}{\sqrt{x+4}}, x>0$.

[NCERT Ex. 7.2, Q. 11, Page 304]

Ans. Let $I=\int\frac{x}{\sqrt{x+4}}dx$

Put $x+4=t$

$\Rightarrow \quad dx=dt$

Now, $I=\int\frac{(t-4)}{\sqrt{t}}dt$

$$=\int\left(\sqrt{t}-4t^{-1/2}\right)+C$$

$$=\frac{2}{3}t^{3/2}-4\left(2t^{1/2}\right)+C$$

$$=\frac{2}{3}\cdot t\cdot t^{1/2}-8t^{1/2}+C$$

$$=\frac{2}{3}(x+4)\sqrt{x+4}-8\sqrt{x+4}+C$$

$$=\frac{2}{3}x\sqrt{x+4}+\frac{8}{3}\sqrt{x+4}-8\sqrt{x+4}+C$$

$$=\frac{2}{3}x\sqrt{x+4}-\frac{16}{3}\sqrt{x+4}+C$$

$$=\frac{2}{3}(\sqrt{x+4})(x-8)+C$$ **[2]**

Q. 23. Integrate the function $(x^3-1)^{\frac{1}{3}}x^5$.

[NCERT Ex. 7.2, Q. 12, Page 304]

Ans. Let $x^3-1=t$

$$\therefore 3x^2dx=dt$$

$$\int\left(x^3-1\right)^{\frac{1}{3}}x^5dx=\int\left(x^3-1\right)^{\frac{1}{3}}x^3\cdot x^2\ dx$$

$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$

$$=\frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$

$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$$

$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$

$$=\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+C$$ **[2]**

Q. 24. Integrate the function $\frac{x^2}{(2+3x^3)^3}$.

[NCERT Ex. 7.2, Q. 13, Page 304]

Ans. Let $2+3x^3=t$

$$\therefore 9x^2\ dx=dt$$

$$\Rightarrow \int\frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$

$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$

$$=\frac{-1}{18\left(2+3x^3\right)^2}+C$$ **[2]**

Q. 25. Integrate the function $\frac{1}{x(\log x)^m}, x>0, m\neq 1$.

[NCERT Ex. 7.2, Q. 14, Page 304]

Ans. Let $\log x=t$

$$\therefore \frac{1}{x}dx=dt$$

$$\int\frac{1}{x(\log x)^m}dx=\int\frac{dt}{(t)^m}$$

$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$

$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$ **[2]**

Q. 26. Integrate the function $\frac{x}{9-4x^2}$.

[NCERT Ex. 7.2, Q. 15, Page 304]

Ans. Let $9-4x^2=t$

$$\therefore -8x\ dx=dt$$

$$\Rightarrow \int\frac{x}{9-4x^2}dx=\frac{-1}{8}\int\frac{1}{t}dt$$

$$=\frac{-1}{8}\log|t|+C$$

$$=\frac{-1}{8}\log\left|9-4x^2\right|+C$$ **[2]**

Q. 27. Integrate the function e^{2x+3}.

[NCERT Ex. 7.2, Q. 16, Page 304]

Ans. Let $2x+3=t$

$$\therefore 2dx=dt$$

$$\Rightarrow \int e^{2x+3}dx=\frac{1}{2}\int e^t\ dt$$

$$=\frac{1}{2}\left(e^t\right)+C$$

$$=\frac{1}{2}e^{(2x+3)}+C$$ **[2]**

Q. 28. Integrate the function $\frac{x}{e^{x^2}}$.

[NCERT Ex. 7.2, Q. 17, Page 304]

Ans. Let $x^2=t$

$\therefore 2xdx = dt$

$$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^t}dt$$

$$= \frac{1}{2}\int e^{-t}dt$$

$$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$$

$$= -\frac{1}{2}e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

[2]

Q. 29. Integrate the function $\frac{e^{\tan^{-1} x}}{1+x^2}$.

[NCERT Ex. 7.2, Q. 18, Page 305]

Ans. Let $\tan^{-1} x = t$

$$\therefore \frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2}dx = \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1} x} + C$$

[2]

Q. 30. Find an anti-derivative (or integral) of the function $(ax + b)^2$ by the method of inspection.

[NCERT Ex. 7.1, Q. 4, Page 299]

Ans. The anti-derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$

$$\Rightarrow \quad (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore \quad (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti-derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

[2]

Q. 31. Find an anti-derivative (or integral) of the function $\sin 2x - 4e^{3x}$ by the method of inspection.

[NCERT Ex. 7.1, Q. 5, Page 299]

Ans. The anti-derivative of $\left(\sin 2x - 4e^{3x}\right)$ is the function of x whose derivative is $\left(\sin 2x - 4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti-derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

[2]

Q. 32. Find the integral of $\int (4e^{3x} + 1)dx$.

[NCERT Ex. 7.1, Q. 6, Page 299]

Ans. $\int (4e^{3x}+1)dx = 4\int e^{3x}dx + \int 1dx$

$$= 4\left(\frac{e^{3x}}{3}\right) + x + C$$

$$= \frac{4}{3}e^{3x} + x + C$$

[2]

Q. 33. Find the integral of $\int x^2\left(1 - \frac{1}{x^2}\right)dx$.

[NCERT Ex. 7.1, Q. 7, Page 299]

Ans. $\int x^2\left(1-\frac{1}{x^2}\right)dx = \int \left(x^2 - 1\right)dx$

$$= \int x^2 dx - \int 1dx$$

$$= \frac{x^3}{3} - x + C$$

[2]

Q. 34. Find the integral of $\int (ax^2 + bx + c)dx$.

[NCERT Ex. 7.1, Q. 8, Page 299]

Ans. $\int (ax^2+bx+c)dx = a\int x^2 dx + b\int xdx + c\int 1\cdot dx$

$$= a\left(\frac{x^3}{3}\right) + b\left(\frac{x^2}{2}\right) + cx + C$$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

[2]

Q. 35. Find the integral of $\int (2x^2 + e^x)dx$.

[NCERT Ex. 7.1, Q. 9, Page 299]

Ans. $\int (2x^2+e^x)dx = 2\int x^2 dx + \int e^x dx$

$$= 2\left(\frac{x^3}{3}\right) + e^x + C$$

$$= \frac{2}{3}x^3 + e^x + C$$

[2]

Q. 36. Find the integral of $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$.

[NCERT Ex. 7.1, Q. 10, Page 299]

Ans. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + \frac{1}{x} - 2\right)dx$

$$= \int xdx + \int \frac{1}{x}dx - 2\int 1\cdot dx$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

[2]

Q. 37. Find the integral of $\int \frac{x^3 + 5x^2 - 4}{x^2}dx$.

[NCERT Ex. 7.1, Q. 11, Page 299]

Ans. $\int \frac{x^3+5x^2-4}{x^2}dx = \int \left(x + 5 - 4x^{-2}\right)dx$

$$= \int xdx + 5\int 1\cdot dx - 4\int x^{-2}dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

[2]

Q. 38. Find the integral of $\int \frac{x^3 + 3x + 4}{\sqrt{x}}dx$.

[NCERT Ex. 7.1, Q. 12, Page 299]

Ans. $\int \frac{x^3+3x+4}{\sqrt{x}}dx = \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right)dx$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$

[2]

$$=\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$$

$$=\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

Q. 39. Find the integral of $\int\frac{x^3-x^2+x-1}{x-1}dx$.

[NCERT Ex. 7.1, Q. 13, Page 299]

Ans. Given that,

$$\int\frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$=\int(x^2+1)dx$$

$$=\int x^2dx+\int 1dx$$

$$=\frac{x^3}{3}+x+C$$ **[2]**

Q. 40. Integrate the function sec^2 (7 – 4x).

[NCERT Ex. 7.2, Q. 22, Page 305]

Ans. Let $7-4x=t$

$$\therefore -4dx=dt$$

$$\therefore\int\sec^2(7-4x)dx=\frac{-1}{4}\int\sec^2 t\,dt$$

$$=\frac{-1}{4}(\tan t)+C$$

$$=\frac{-1}{4}\tan(7-4x)+C$$ **[2]**

Q. 41. Integrate the function $\frac{\cos\sqrt{x}}{\sqrt{x}}$.

[NCERT Ex. 7.2, Q. 26, Page 305]

Ans. Let $\sqrt{x}=t$

$$\therefore\frac{1}{2\sqrt{x}}dx=dt$$

$$\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx=2\int\cos t\,dt$$ **[2]**

$$=2\sin t+C$$

$$=2\sin\sqrt{x}+C$$

Q. 42. Integrate the function e^x (sin$x$ + cos$x$).

[NCERT Ex. 7.6, Q. 16, Page 328]

Ans. Let $I=\int e^x(\sin x+\cos x)dx$

Let $f(x)=\sin x$

$$f'(x)=\cos x$$

$$\therefore I=\int e^x\{f(x)+f'(x)\}dx$$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

$$\therefore I=e^x\sin x+C$$ **[2]**

Q. 43. Integrate the function $\frac{xe^x}{(1+x)^2}$.

[NCERT Ex. 7.6, Q. 17, Page 328]

Ans. Let,

$$I=\int\frac{xe^x}{(1+x)^2}dx$$

$$=\int e^x\left\{\frac{x}{(1+x)^2}\right\}dx$$

$$=\int e^x\left\{\frac{1+x-1}{(1+x)^2}\right\}dx$$

$$=\int e^x\left\{\frac{1}{1+x}-\frac{1}{(1+x)^2}\right\}dx$$

Let $f(x)=\frac{1}{1+x}\Rightarrow f'(x)=\frac{-1}{(1+x)^2}$

$$\Rightarrow\int\frac{xe^x}{(1+x)^2}dx=\int e^x\{f(x)+f'(x)\}dx$$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

$$\therefore\int\frac{xe^x}{(1+x)^2}dx=\frac{e^x}{1+x}+C$$ **[2]**

Q. 44. Integrate the function $\sqrt{\sin 2x}\,\cos 2x$.

[NCERT Ex. 7.2, Q. 27, Page 305]

Ans. Let $\sin 2x=t$

$$\therefore 2\cos 2x\,dx=dt$$

$$\Rightarrow\int\sqrt{\sin 2x}\cos 2x\,dx=\frac{1}{2}\int\sqrt{t}\,dt$$

$$=\frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{1}{3}t^{\frac{3}{2}}+C$$

$$=\frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C$$ **[2]**

Q. 45. By using the properties of definite integrals, evaluate the integral of $\int_0^a\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$.

[NCERT Ex. 7.11, Q. 17, Page 347]

Ans. Let $I=\int_0^a\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(i)

It is known that, $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$I=\int_0^a\frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$$ ...(ii)

Adding equations (i) and (ii), we obtain

$$2I=\int_0^a\frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I=\int_0^a 1\cdot dx$$

$$\Rightarrow 2I=[x]_0^a$$

$$\Rightarrow 2I=a$$

$$\Rightarrow I=\frac{a}{2}$$ **[2]**

Q. 46. Integrate the function cos^3 x $e^{\log\sin x}$.

[NCERT Misc. Ex. Q. 15, Page 352]

Ans. $\cos^3 xe^{\log\sin x}=\cos^3 x\times\sin x$

Let $\cos x=t\Rightarrow -\sin x\,dx=dt$

$$\Rightarrow\int\cos^3 xe^{\log\sin}\,dx=\int\cos^3 x\sin x dx$$

$$=-\int t^3\cdot dt$$

$$=-\frac{t^4}{4}+C=\frac{-\cos^4 x}{4}+C$$ **[2]**

Q. 47. Integrate the function $e^{3\log x}(x^4+1)^{-1}$.
[NCERT Misc. Ex. Q. 16, Page 352]

Ans. $e^{3\log x}(x^4+1)^{-1} = e^{\log x^3}\left(x^4+1\right)^{-1}$

$$= \frac{x^3}{\left(x^4+1\right)}$$

Let $x^4 + 1 = t \Rightarrow 4x^3\,dx = dt$

$$\int e^{3\log x}\left(x^4+1\right)^{-1}dx = \int \frac{x^3}{\left(x^4+1\right)}dx$$

$$= \frac{1}{4}\int \frac{dt}{t}$$

$$= \frac{1}{4}\log|t| + C \quad \textbf{[2]}$$

$$= \frac{1}{4}\log\left|x^4+1\right| + C$$

$$= \frac{1}{4}\log\left(x^4+1\right) + C$$

Q. 48. Integrate the function $f'(ax+b)[f(ax+b)]^n$.
[NCERT Misc. Ex. Q. 17, Page 352]

Ans. $f'(ax+b)\left[f(ax+b)\right]^n$

Let $f(ax+b) = t \Rightarrow af'(ax+b)dx = dt$

$$\int f'(ax+b)\left[f(ax+b)\right]^n dx = \frac{1}{a}\int t^n dt$$

$$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$$

$$= \frac{1}{a(n+1)}\left(f(ax+b)\right)^{n+1} + C$$

[2]

Q. 49. By using the properties of definite integrals, evaluate the integral of $\int_0^4 |x-1|\,dx$.
[NCERT Ex. 7.11, Q. 18, Page 347]

Ans. $I = \int_0^4 |x-1|\,dx$

It can be seen that, $(x-1) \le 0$ when $0 \le x \le 1$ and $(x-1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1|\,dx + \int_1^4 |x-1|\,dx \quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1)\,dx + \int_0^4 (x-1)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4 \quad \textbf{[2]}$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Q. 50. Show that $\int_0^a f(x)g(x)dx = 2\int_a^a f(x)dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$.
[NCERT Ex. 7.11, Q. 19, Page 347]

Ans. Let $I = \int_0^a f(x)g(x)dx \quad \text{...(i)}$

$$\Rightarrow I = \int_0^a f(a-x)g(a-x)dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^a f(x)g(a-x)dx \quad \text{...(ii)}$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^a \left\{f(x)g(x) + f(x)g(a-x)\right\}dx$$

$$\Rightarrow 2I = \int_0^a f(x)\left\{g(x) + g(a-x)\right\}dx$$

$$\Rightarrow 2I = \int_0^a f(x) \times 4dx \quad \left[g(x) + g(a-x) = 4\right]$$

$$\Rightarrow \quad I = 2\int_0^a f(x)dx$$

So that,

$$\Rightarrow \int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx \quad \textbf{[2]}$$

Q. 51. Integrate the function $\dfrac{\cos x}{\sqrt{1+\sin x}}$.
[NCERT Ex. 7.2, Q. 28, Page 305]

Ans. Let $1 + \sin x = t$

$\therefore \cos x\; dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}}dx = \int \frac{dt}{\sqrt{t}}$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C \quad \textbf{[2]}$$

$$= 2\sqrt{1+\sin x} + C$$

Q. 52. Integrate the function $\dfrac{x^3}{\sqrt{1-x^8}}$.
[NCERT Misc. Ex. Q. 12, Page 352]

Ans. Given that,

$$\frac{x^3}{\sqrt{1-x^8}}$$

Let $x^4 = t \Rightarrow 4x^3\,dx = dt$

$$\int \frac{x^3}{\sqrt{1-x^8}}dx = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{4}\sin^{-1} t + C$$

$$= \frac{1}{4}\sin^{-1}\left(x^4\right) + C \quad \textbf{[2]}$$

Q. 53. Integrate the function $\dfrac{e^x}{(1+e^x)(2+e^x)}$.
[NCERT Misc. Ex. Q. 13, Page 352]

Ans. $\dfrac{e^x}{(1+e^x)\,(2+e^x)}$

Let $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \frac{e^x}{(1+e^x)\,(2+e^x)}dx = \int \frac{dt}{(t+1)\,(t+2)}$$

$$= \int\left[\frac{1}{t+1} - \frac{1}{t+2}\right]dt$$

$$= \log|t+1| - \log|t+2| + C$$

$$= \log\left|\frac{t+1}{t+2}\right| + C$$

$$= \log\left|\frac{e^x+1}{e^x+2}\right| + C$$

Q. 54. Integrate the function cot x log sin x.
[NCERT Ex. 7.2, Q. 29, Page 305]

Ans. Let log sin $x = t$

$$\Rightarrow \frac{1}{\sin x}\cdot\cos x\ dx = dt$$

$$\therefore \cot x\ dx = dt$$

$$\Rightarrow \int \cot x \log\sin x\ dx = \int t\ dt$$

$$= \frac{t^2}{2} + C$$

$$= \frac{1}{2}(\log\sin x)^2 + C$$ **[2]**

Q. 55. Integrate the function $\frac{\sin x}{1+\cos x}$.
[NCERT Ex. 7.2, Q. 30, Page 305]

Ans. Let $1 + \cos x = t$

Differentiating the above term, we have

$-\sin x\ dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x}dx = \int -\frac{dt}{t}$$

$$= -\log|t| + C$$

$$= -\log|1+\cos x| + C$$ **[2]**

Q. 56. Integrate the function $\frac{\sin x}{(1+\cos x)^2}$.
[NCERT Ex. 7.2, Q. 31, Page 305]

Ans. Let $1 + \cos x = t$

Differentiating the above term, we have

$$\therefore -\sin x\ dx = dt$$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}dx = \int -\frac{dt}{t^2}$$

$$= -\int t^{-2}dt$$

$$= \frac{1}{t} + C$$

$$= \frac{1}{1+\cos x} + C$$ **[2]**

Q. 57. Integrate the function $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$.
[NCERT Ex. 7.2, Q. 23, Page 305]

Ans. Let $\sin^{-1} x = t$

$$\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\Rightarrow \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\ dt$$ **[2]**

$$= \frac{t^2}{2} + C$$

$$= \frac{(\sin^{-1}x)^2}{2} + C$$

Q. 58. Integrate the function $\sqrt{4-x^2}$.
[NCERT Ex. 7.7, Q. 1, Page 330]

Ans. Let $I = \int\sqrt{4-x^2}dx = \int\sqrt{(2)^2-(x)^2}dx$

It is known that,

$$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\therefore I = \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$$

$$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$$ **[2]**

Q. 59. By using the properties of definite integrals, evaluate the integral of $\int_{-5}^{5}|x+2|dx$.
[NCERT Ex. 7.11, Q. 5, Page 347]

Ans. Let $I = \int_{-5}^{5}|x+2|dx$

It can be seen that $(x + 2) \le 0$ on $[-5, -2]$ and $(x + 2) \ge 0$ on $[-2, 5]$.

$$\therefore I = \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5}(x+2)dx$$

$$\left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = -\left[\frac{x^2}{2}+2x\right]_{-5}^{-2} + \left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right]$$

$$+ \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$ **[2]**

$$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$= 29$$

Q. 60. By using the properties of definite integrals, evaluate the integral of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\ dx$.
[NCERT Ex. 7.11, Q. 13, Page 347]

Ans. Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x dx$

As $\sin^7(-x) = (\sin(-x))^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\ dx = 0$$ **[2]**

Q. 61. By using the properties of definite integrals, evaluate the integral of $\int_0^{2\pi}\cos^5 x\ dx$.
[NCERT Ex. 7.11, Q. 14, Page 347]

Ans. Let $I = \int_0^{2\pi}\cos^5 x\ dx$

$\cos^5(2\pi - x) = \cos^5 x$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2x-x) = f(x)$$

$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi}\cos^5 x dx$$ **[2]**

$$\Rightarrow I = 2(0) = 0 \qquad [\because \cos^5(\pi - x) = -\cos^5 x]$$

Q. 62. By using the properties of definite integrals, evaluate the integral of $\int_0^{\frac{\pi}{2}}\frac{\sin x - \cos x}{1+\sin x\cos x}dx$.
[NCERT Ex. 7.11, Q. 15, Page 347]

Ans. Let $I=\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}dx$ ...(i)

$$\Rightarrow \quad I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$\left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\sin x\cos x}dx$ ...(ii)

Adding equations (i) and (ii), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{0}{1+\sin x\cos x}dx$$

$\Rightarrow I=0$ **[2]**

Q. 63. By using the properties of definite integrals, evaluate the integral of $\int_0^8|x-5|dx$.

[NCERT Ex. 7.11, Q. 6, Page 347]

Ans. Let $I=\int_0^8|x-5|dx$

It can be seen that $(x-5)\le 0$ on [2, 5] and $(x-5)\ge 0$ on [5, 8].

$$I=\int_2^5-(x-5)dx+\int_5^8(x-5)dx$$

$$\left(\int_a^b d(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$=-\left[\frac{x^2}{2}-5x\right]_2^5+\left[\frac{x^2}{2}-5x\right]_5^8$$ **[2]**

$$=-\left[\frac{25}{2}-25-2+10\right]+\left[32-40-\frac{25}{2}+25\right]$$

$$=9$$

Q. 64. By using the properties of definite integrals, evaluate the integral of $\int_0^1 x(1-x)^n dx$.

[NCERT Ex. 7.11, Q. 7, Page 347]

Ans. Let $I=\int_0^1 x(1-x)^n dx$

$$\therefore \quad I=\int_0^1(1-x)(1-(1-x))^n dx$$

$$=\int_0^1(1-x)(x)^n dx$$

$$=\int_0^1\left(x^n-x^{n+1}\right)dx$$

$$=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1 \quad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$=\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$ **[2]**

$$=\frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$=\frac{1}{(n+1)(n+2)}$$

Q. 65. Integrate the function $\sqrt{1-4x^2}$.

[NCERT Ex. 7.7, Q. 2, Page 330]

Ans. Let $I=\int\sqrt{1-4x^2}dx=\int\sqrt{(1)^2-(2x)^2}dx$

Let $2x=t\Rightarrow 2dx=dt$

$$\therefore \quad I=\frac{1}{2}\int\sqrt{(1)^2-(t)^2}dt$$

It is known that,

$$\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

$$\Rightarrow \quad I=\frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$$

$$=\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$$

$$=\frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$ **[2]**

$$=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

Q. 66. Integrate the function $\sqrt{x^2+4x+6}$.

[NCERT Ex. 7.7, Q. 3, Page 330]

Ans. Let $I=\int\sqrt{x^2+4x+6}\ dx$

$$=\int\sqrt{x^2+4x+4+2}\ dx$$

$$=\int\sqrt{(x^2+4x+4)+2}\ dx$$

$$=\int\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\ dx$$

It is known that,

$$\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\therefore \quad I=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

$$=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$ **[2]**

Q. 67. Integrate the function $\sqrt{x^2+4x+1}$.

[NCERT Ex. 7.7, Q. 4, Page 330]

Ans. Let $I=\int\sqrt{x^2+4x+1}\ dx$

$$=\int\sqrt{(x^2+4x+4)-3}\ dx$$

$$=\int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\ dx$$

It is known that,

$$\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$\therefore \quad I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$$ **[2]**

Q. 68. Integrate the function $\sqrt{1-4x-x^2}$.

[NCERT Ex. 7.7, Q. 5, Page 330]

Ans. Let $I=\int\sqrt{1-4x-x^2}\ dx$

$$= \int \sqrt{1-(x^2+4x+4-4)}\,dx$$
$$= \int \sqrt{1+4-(x+2)^2}\,dx$$
$$= \int \sqrt{(\sqrt{5})^2-(x+2)^2}\,dx$$

It is known that,

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$
$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C \quad [2]$$

Q. 69. Integrate the function $\sqrt{x^2+4x-5}$.

[NCERT Ex. 7.7, Q. 6, Page 330]

Ans. Let $I = \int \sqrt{x^2+4x-5}\,dx$

$$= \int \sqrt{(x^2+4x+4)-9}\,dx$$
$$= \int \sqrt{(x^2+2)^2-(3)^2}\,dx$$

It is known that,

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$
$$\therefore \quad I = \frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C \quad [2]$$

Q. 70. Find the integral of $\int (1-x)\sqrt{x}\,dx$.

[NCERT Ex. 7.1, Q. 14, Page 299]

Ans. $\int (1-x)\sqrt{x}dx = \int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$

$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$
$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C \quad [2]$$
$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}+C$$

Q. 71. Find the integral of $\int \sqrt{x}(3x^2+2x+3)dx$.

[NCERT Ex. 7.1, Q. 15, Page 299]

Ans. $\int \sqrt{x}(3x^2+2x+3)dx = \int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$
$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right)+2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)+3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+C \quad [2]$$
$$= \frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+C$$

Q. 72. Find the integral of $\int (2x-3\cos x+e^x)\,dx$.

[NCERT Ex. 7.1, Q. 16, Page 299]

Ans. $\int (2x-3\cos x+e^x)\,dx$

$$= 2\int xdx - 3\int \cos xdx + \int e^x dx$$
$$= \frac{2x^2}{2} - 3(\sin x)+e^x+C \quad [2]$$
$$= x^2 - 3\sin x + e^x + C$$

Q. 73. Find the integral of $\int (2x^2-3\sin x+5\sqrt{x})dx$.

[NCERT Ex. 7.1, Q. 17, Page 299]

Ans.

$$\int (2x^2-3\sin x+5\sqrt{x})dx = 2\int x^2dx - 3\int \sin xdx + 5\int x^{\frac{1}{2}}dx$$
$$= \frac{2x^3}{3} - 3(-\cos x)+5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$
$$= \frac{2}{3}x^3+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C \quad [2]$$

Q. 74. Find the integral of $\int \sec x(\sec x+\tan x)\,dx$.

[NCERT Ex. 7.1, Q. 18, Page 299]

Ans. $\int \sec x(\sec x+\tan x)\,dx = \int(\sec^2 x+\sec x\tan x)\,dx$

$$= \int \sec^2 xdx + \int \sec x\tan xdx \quad [2]$$
$$= \tan x + \sec x + C$$

Q. 75. Integrate the function $\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$.

[NCERT Misc. Ex. Q. 8, Page 352]

Ans.

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}} = \frac{e^{4\log x}(e^{\log x}-1)}{e^{2\log x}(e^{\log x}-1)}$$
$$= e^{2\log x}$$
$$= e^{\log x^2}$$
$$= x^2$$
$$\therefore \int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx = \int x^2dx = \frac{x^3}{3}+C \quad [2]$$

Q. 76. Integrate the function $\frac{\cos x}{\sqrt{4-\sin^2 x}}$.

[NCERT Misc. Ex. Q. 9, Page 352]

Ans. $\frac{\cos x}{\sqrt{4-\sin^2 x}}$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{4-\sin^2 x}}dx = \int \frac{dt}{(2)^2-(t)^2}$$
$$= \sin^{-1}\left(\frac{t}{2}\right)+C$$
$$= \sin^{-1}\left(\frac{\sin x}{2}\right)+C \quad [2]$$

Q. 77. Integrate the function $\frac{3x^2}{x^6+1}$.

[NCERT Ex. 7.4, Q. 1, Page 315]

Ans. Let $x^3 = t$

$\therefore \quad 3x^2\,dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1}$$
$$= \tan^{1} t + C$$
$$= \tan^{-1}(x^3)+C \quad [2]$$

Q. 78. Integrate the function $\frac{1}{\sqrt{1+4x^2}}$.

[NCERT Ex. 7.4, Q. 1, Page 315]

Ans. Let $2x = t$

$\therefore \quad 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C$$

$$\left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + C$$

[2]

Q. 79. Integrate the function $\frac{1}{\sqrt{(2-x)^2+1}}$.

[NCERT Ex. 7.4, Q. 3, Page 315]

Ans. Let $2 - x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$

$$= -\log\left|t+\sqrt{t^2+1}+C\right|$$

$$\left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C \quad \textbf{[2]}$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

Q. 80. Integrate the function $\frac{1}{\sqrt{9-25x^2}}$.

[NCERT Ex. 7.4, Q. 4, Page 315]

Ans. Let $5x = t$

$\therefore \quad 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}dx = \frac{1}{5}\int \frac{1}{\sqrt{9-t^2}}dt$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}dt$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C \quad \textbf{[2]}$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

Q. 81. Integrate the function $\frac{3x}{1+2x^4}$.

[NCERT Ex. 7.4, Q. 5, Page 315]

Ans. Let $\sqrt{2}x^2 = t$

$\therefore \quad 2\sqrt{2}x\ dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$$

$$= \frac{3}{2\sqrt{2}}\left[\tan^{-1}t\right] + C$$

$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right) + C$$

[2]

Q. 82. Evaluate the definite integral of $\int_{-1}^{1}(x+1)\,dx$.

[NCERT Ex. 7.9, Q. 1, Page 338]

Ans. **Let** $I = \int_{-1}^{1}(x+1)dx$

$$\int (x+1)dx = \frac{x^2}{2} + x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(-1)$$

$$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)$$

$$= \frac{1}{2}+1-\frac{1}{2}+1$$

$$= 2$$

[2]

Q. 83. Evaluate the definite integral of $\int_{2}^{3}\frac{1}{x}dx$.

[NCERT Ex. 7.9, Q. 2, Page 338]

Ans. **Let** $I = \int_{2}^{3}\frac{1}{x}dx$

$$\int \frac{1}{x}dx = \log|x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \log|3| - \log|2| = \log\frac{3}{2}$$

[2]

Q. 84. Integrate the function $\frac{x^2}{1-x^6}$.

[NCERT Ex. 7.4, Q. 6, Page 315]

Ans. Let $x^3 = t$

$\therefore \quad 3x^2dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$$

$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

[2]

Q. 85. Find the integral of $\int \frac{\sec^2 x}{\mathrm{cosec}^2 x}dx$.

[NCERT Ex. 7.1, Q. 19, Page 299]

Ans. Given that,

$$\int \frac{\sec^2 x}{\mathrm{cosec}^2 x}dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x}dx$$

$$= \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1)dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

[2]

Q. 86. Find the integral of $\int \frac{2-3\sin x}{\cos^2 x}dx$.

[NCERT Ex. 7.1, Q. 20, Page 299]

Ans. $\int \frac{2-3\sin x}{\cos^2 x}dx = \int\left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right)dx$

$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$ **[2]**

$= 2\tan x - 3\sec x + C$

Q. 87. Integrate the function $\frac{x^2}{\sqrt{x^6+a^6}}$.

[NCERT Ex. 7.4, Q. 8, Page 315]

Ans. Let $x^3 = t$

$\Rightarrow \quad 3x^2\, dx = dt$

$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}}$

$= \frac{1}{3}\log\left|t + \sqrt{t^2+a^6}\right| + C$

$= \frac{1}{3}\log\left|x^3 + \sqrt{x^6+a^6}\right| + C$ **[2]**

Q. 88. Integrate the function $\frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$.

[NCERT Ex. 7.4, Q. 9, Page 315]

Ans. Let $\tan x = t$

$\therefore \quad \sec^2 x\, dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx = \int \frac{dt}{\sqrt{t^2+2^2}}$

$= \log\left|t + \sqrt{t^2+4}\right| + C$

$= \log\left|\tan x + \sqrt{\tan^2 x+4}\right| + C$ **[2]**

Q. 89. Find : $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}dx$.

[CBSE Board, All India Region, 2017]

Ans. $-\log|\sin 2x| + c$

$\Rightarrow \log|\sec x| - \log|\sin x| + c.$ **[1]**

Q. 90. Find : $\int \frac{dx}{5-8x-x^2}$.

[CBSE Board, All India Region, 2017]

Ans. $\int \frac{dx}{5-8x-x^2} = \int \frac{dx}{(\sqrt{21})^2-(x+4)^2}$

$= \frac{1}{2\sqrt{21}}\log\left|\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right| + c$ **[2]**

Q. 91. Evaluate : $\int_2^3 3^x dx$.

[CBSE Board, Delhi Region, 2017]

Ans. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3}\right]_2^3 = \frac{18}{\log 3}$ **[1]**

Q. 92. Find : $\int \frac{dx}{x^2+4x+8}$.

[CBSE Board, Delhi Region, 2017]

Ans. $\int \frac{dx}{x^2+4x+8} = \int \frac{dx}{(x+2)^2+(2)^2}$

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$ **[2]**

Q. 93. Evaluate : $\int_0^{2\pi} \cos^5 x\, dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $\int_0^{2\pi}\cos^5 x\, dx = 2\int_0^{\pi}\cos^5 x\, dx$

and $2\int_0^{\pi}\cos^5 x\, dx = 0 \Rightarrow \int_0^{2\pi}\cos^5 x\, dx = 0$ **[1]**

Q. 94. Find : $\int \frac{dx}{\sqrt{3-2x-x^2}}$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $I = \int \frac{dx}{\sqrt{(2)^2-(x+1)^2}}$

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c$ **[2]**

Q. 95. Verify : $\int \frac{2x+3}{x^2+3x}dx = \log|x^2+3x| + C$.

[NCERT Exemp. Ex. 7.3, Q. 2, Page 163]

Ans. Let $I = \int \frac{2x+3}{x^2+3x}dx$

Put $x^2 + 3x = t$

$\Rightarrow (2x+3)dx = dt$

$\therefore \quad I = \int \frac{1}{t}dt = \log|t| + C$

$= \log|(x^2+3x)| + C$ **[2]**

Q. 96. Evaluate : $\int \frac{(x^2+2)dx}{x+1}$.

[NCERT Exemp. Ex. 7.3, Q. 3, Page 163]

Ans. Let $I = \int \frac{x^2+2}{x+1}dx$

$= \int\left(x - 1 + \frac{3}{x+1}\right)dx$

$= \int (x-1)dx + 3\int \frac{1}{x+1}dx$

$= \frac{x^2}{2} - x + 3\log|(x+1)| + C$ **[2]**

Q. 97. Evaluate : $\int \frac{(1+\cos x)}{x+\sin x}dx$.

[NCERT Exemp. Ex. 7.3, Q. 5, Page 164]

Ans. Consider that,

$I = \int \frac{(1+\cos x)}{x+\sin x}dx$

Let $x + \sin x = t \Rightarrow (1+\cos x)dx = dt$

$\therefore \quad I = \int \frac{1}{t}dt = \log|t| + C$

$= \log|(x+\sin x)| + C$ **[2]**

Q. 98. Evaluate : $\int \frac{dx}{1+\cos x}$.

[NCERT Exemp. Ex. 7.3, Q. 6, Page 164]

Ans. Let

$$I = \int \frac{dx}{1+\cos x} = \int \frac{dx}{1+2\cos^2\frac{x}{2}-1}$$

$$= \frac{1}{2}\int \frac{1}{\cos^2\frac{x}{2}}dx = \frac{1}{2}\int \sec^2\frac{x}{2}dx \quad \textbf{[2]}$$

$$= \frac{1}{2}\tan\frac{x}{2}\cdot 2 + C = \tan\frac{x}{2} + C \quad [\because \sec^2 x\, dx = \tan x]$$

Q. 99. Evaluate : $\int \tan^2 x \sec^4 x\, dx$.

[NCERT Exemp. Ex. 7.3, Q. 7, Page 164]

Ans. Let $I = \int \tan^2 x \sec^4 x\, dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore \quad I = \int t^2(1+t^2)dt = \int (t^2+t^4)dt$$

$$= \frac{t^3}{3} + \frac{t^5}{5} + C = \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C \quad \textbf{[2]}$$

Q. 100. Evaluate : $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx$.

[NCERT Exemp. Ex. 7.3, Q. 8, Page 164]

Ans. Let

$$I = \int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx = \int \frac{(\sin x + \cos x)}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}}dx$$

$$= \int \frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}}dx = \int 1 dx = x + C \quad \textbf{[2]}$$

Q. 101. Evaluate : $\int \sqrt{1+\sin x}dx$.

[NCERT Exemp. Ex. 7.3, Q. 9, Page 164]

Ans. Let $I = \int \sqrt{1+\sin x}dx$

$$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$\left[\because \sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1\right]$$

$$= \int \sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}dx = \int \left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx \quad \textbf{[2]}$$

$$= -\cos\frac{x}{2}\cdot 2 + \sin\frac{x}{2}\cdot 2 + C = -2\cos\frac{x}{2} + 2\sin\frac{x}{2} + C$$

Q. 102. Evaluate : $\int \frac{\sqrt{1+x^2}}{x^4}dx$.

[NCERT Exemp. Ex. 7.3, Q. 13, Page 164]

Ans. Let $I = \int \frac{\sqrt{1+x^2}}{x^4}dx = \int \frac{\sqrt{1+x^2}}{x}\cdot\frac{1}{x^3}dx$

$$= \int \sqrt{\frac{1+x^2}{x^2}}\cdot\frac{1}{x^3}dx = \int \sqrt{\frac{1}{x^2}+1}\cdot\frac{1}{x^3}dx$$

Put $1 + \frac{1}{x^2} = t^2 \Rightarrow \frac{-2}{x^3}dx = 2t\, dt$

$$\Rightarrow \quad -\frac{1}{x^3} = t\, dt$$

$$\therefore I = -\int t^2 dt = -\frac{t^3}{3} + C = -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + C \quad \textbf{[2]}$$

Q. 103. Evaluate : $\int \frac{dx}{\sqrt{16-9x^2}}$.

[NCERT Exemp. Ex. 7.3, Q. 14, Page 164]

Ans. Let

$$I = \int \frac{dx}{\sqrt{16-9x^2}} = \int \frac{dx}{(4)^2-(3x)^2}dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C \quad \textbf{[2]}$$

Q. 104. Evaluate : $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$.

[NCERT Exemp. Ex. 7.3, Q. 24, Page 165]

Ans. Let $I = \int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx = \int \frac{\sqrt{x}}{\sqrt{(a^{3/2})^2-(x^{3/2})^2}}$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}x^{1/2}dx = dt$

$$\therefore \quad I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} = \frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}} + C$$

$$= \frac{2}{3}\sin^{-1}\frac{x^{3/2}}{a^{3/2}} + C = \frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}} + C \quad \textbf{[2]}$$

Q. 105. Evaluate : $\int \frac{dx}{x\sqrt{x^4-1}}$. (Hint : Put $x^2 = \sec\theta$)

[NCERT Exemp. Ex. 7.3, Q. 26, Page 165]

Ans. Let $I = \int \frac{dx}{x\sqrt{x^4-1}}$

Put $x^2 = \sec\theta \Rightarrow \theta = \sec^{-1} x^2$

$2x\, dx = \sec\theta \ \tan\theta\ d\theta$

$$\therefore \quad I = \frac{1}{2}\ \frac{\sec\theta\ \tan\theta}{\sec\theta\tan\theta}d\theta = \frac{1}{2}\ \ d\theta = \frac{1}{2}\theta + C$$

$$= \frac{1}{2}\sec^{-1}(x^2) + C \quad \textbf{[2]}$$

Q. 106. Evaluate : $\int_0^1 \frac{dx}{e^x+e^{-x}}$.

[NCERT Exemp. Ex. 7.3, Q. 29, Page 165]

Ans. Let $I = \int_0^1 \frac{dx}{e^x+e^{-x}} = \int_0^1 \frac{e^x}{1+e^{2x}}dx$

Put $e^x = t$

$\Rightarrow e^x dx = dt$

$$\therefore \quad I = \int_1^e \frac{dt}{1+t^2} = [\tan^{-1} t]_1^e$$

$$= \tan^{-1} e - \tan^{-1} 1$$

$$= \tan^{-1} e - \frac{\pi}{4} \quad \textbf{[2]}$$

Q. 107. Evaluate : $\int_0^1 \frac{xdx}{\sqrt{1+x^2}}$.

[NCERT Exemp. Ex. 7.3, Q. 32, Page 165]

Ans. Let $I = \int_0^1 \frac{xdx}{\sqrt{1+x^2}}dx$

Put $1 + x^2 = t^2$

$\Rightarrow \quad 2x\, dx = 2tdt$

$\Rightarrow \quad x\, dx = tdt$

$$\therefore \quad I = \int_1^{\sqrt{2}} \frac{tdt}{t}$$

$$= [t]_1^{\sqrt{2}} = \sqrt{2} - 1 \quad \textbf{[2]}$$

Q. 108. Evaluate : $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}}$.

[NCERT Exemp. Ex. 7.3, Q. 41, Page 166]

Ans. Let $I = \int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}} dx$

$$= \int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^2\sqrt{1+\cos x}} dx$$

$$= \int_{\pi/3}^{\pi/2} \frac{1}{(1-\cos^2 x)} dx = \int_{\pi/3}^{\pi/2} \frac{1}{\sin^2 x} dx \quad \textbf{[2]}$$

$$= \int_{\pi/3}^{\pi 2/} \text{cosec}^2 x\ dx = [-\cot x]_{\pi/3}^{\pi/2}$$

$$= -\left[\cot\frac{\pi}{2} - \cot\frac{\pi}{3}\right] = -\left[0 - \frac{1}{\sqrt{3}}\right] = +\frac{1}{\sqrt{3}}$$

Q. 109. Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \frac{\cos^2 x - \sin^2 x + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx \quad \textbf{[2]}$$

$$= \int \sec^2 x dx$$

$$= \tan x + C$$

Q. 110. Evaluate the integral using substitution : $\int_0^1 \frac{x}{x^2+1} dx$. **[NCERT Ex. 7.10, Q. 1, Page 340]**

Ans. $\int_0^1 \frac{x}{x^2+1} dx$

Let $x^2 + 1 = t \Rightarrow 2xdx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$

$$= \frac{1}{2}\left[\log|t|\right]_1^2$$

$$= \frac{1}{2}[\log 2 - \log 1] \quad \textbf{[2]}$$

$$= \frac{1}{2}\log 2$$

Q. 111. Evaluate the integral using substitution : $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi d\phi$.**[NCERT Ex. 7.10, Q. 2, Page 340]**

Ans. Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4 - \cos\phi\ d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} + \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 \quad \textbf{[2]}$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231} = \frac{64}{231}$$

Q. 112. Evaluate : $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$ **is equal to _______.**

[NCERT Exemp. Ex. 7.3, Q. 59, Page 168]

Ans. Let $I = \int_0^{\pi/2} \cos x e^{\sin x} dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

As $x \to 0$, then $t \to 0$

and $x \to \pi/2$, then $t \to 1$

$$\therefore I = \int_0^1 e^t dt = [e^t]_0^1$$

$$= e^1 - e^0 = e - 1 \quad \textbf{[2]}$$

Q. 113. Evaluate : $\int \frac{x+3}{(x+4)^2} e^x dx =$ **__________.**

[NCERT Exemp. Ex. 7.3, Q. 60, Page 168]

Ans. Let $I = \int \frac{x+3}{(x+4)^2} e^x dx$

$$= \int \frac{e^x}{x+4} - \int \frac{e^x}{(x+4)^2} dx$$

$$= \int e^x \left(\frac{1}{(x+4)} - \frac{1}{(x+4)^2}\right) dx \quad \textbf{[2]}$$

$$= e^x\left(\frac{1}{x+4}\right) + C \quad \left[\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$$

Q. 114. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$**, then** $a =$ **__________.**

[NCERT Exemp. Ex. 7.3, Q. 61, Page 168]

Ans. Let $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$

Now, $\int_0^a \frac{1}{4\left(\frac{1}{4}+x^2\right)} dx = \frac{2}{4}[\tan^{-1} 2x]_0^a$

$$= \frac{1}{2}\tan^{-1} 2a - 0 = \pi/8$$

$$\frac{1}{2}\tan^{-1} 2a = \frac{\pi}{8}$$

$$\tan^{-1} 2a = \pi/4$$

$$2a = 1$$

$$\therefore \quad a = \frac{1}{2} \quad \textbf{[2]}$$

Q. 115. Evaluate : $\int \frac{\sin x}{3+4\cos^2 x}dx =$ __________.

[NCERT Exemp. Ex. 7.3, Q. 62, Page 168]

Ans. Let $I = \int \frac{\sin x}{3+4\cos^2 x}dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore \quad I = -\int \frac{dt}{3+4t^2} = -\frac{1}{4}\int \frac{dt}{\left(\frac{\sqrt{3}}{2}\right)^2 + t^2}$$

$$= -\frac{1}{4}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}} + C \quad \textbf{[2]}$$

$$= -\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$$

Q. 116. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x\, dx$ **is** __________.

[NCERT Exemp. Ex. 7.3, Q. 63, Page 168]

Ans. We have, $f(x) = \int_{-\pi}^{\pi} \sin^3 x \cos^2 x\, dx$

$$f(-x) = \int_{-\pi}^{\pi} \sin^3(-x)\cos^2(-x)dx$$

$$= -f(x)$$

Since $f(x)$ is an odd function

So that, $\int_{-\pi}^{\pi} \sin^3 x \cos^2(x)dx = 0$ **[2]**

Q. 117. Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5-\cos^2\theta - 4\sin\theta}d\theta.$

[CBSE Board, Delhi Region, 2016]

Ans. $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}d\theta$

$\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$ **[1]**

$$\therefore \quad I = \int \frac{3t-2}{t^2-4t+4}dt = \int \frac{3t-2}{(t-2)^2}dt$$

$$= \int \frac{3(t-2)}{(t-2)^2}dt + 4\int \frac{1}{(t-2)^2}dt$$

$$= 3\log|t-2| - \frac{4}{(t-2)} + C \quad \textbf{[1]}$$

$$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$$

Q. 118. Evaluate : $\int_0^{\pi} e^{2x}\cdot\sin\left(\frac{\pi}{4}+x\right)dx$ **.**

[CBSE Board, Delhi Region, 2016]

Ans. Let $I = \int_0^{\pi}\sin\left(\frac{\pi}{4}+x\right)e^{2x}dx$

$$= \sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\Bigg]_0^{\pi} - \int_0^{\pi}\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}dx$$

$$I = \left[\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\right\}\right]_0^{\pi}$$

$$+\frac{1}{2}\int_0^{\pi} -\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}dx$$

$$\frac{5}{4}I = \left\{\frac{1}{4}\left[2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right]e^{2x}\right\}_0^{\pi}$$

$$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\right\}\right]$$

$$= \frac{-1}{5\sqrt{2}}\left(e^{2\pi}+1\right) \quad \textbf{[2]}$$

Q. 119. Evaluate : $\int_0^{\pi}\frac{x}{1+\sin\alpha\sin x}dx$ **.**

[CBSE Board, Foreign Scheme, 2017]

Ans. $I = \int_0^{\pi}\frac{x}{1+\sin\alpha\sin x}dx$ ...(i)

$$I = \int_0^{\pi}\frac{\pi - x}{1+\sin\alpha\sin(\pi - x)}dx$$

$$I = \int_0^{\pi}\frac{\pi - x}{1+\sin\alpha\sin x} \quad \text{...(ii)}$$

on adding (i) and (ii)

$$2I = \pi\int_0^{\pi}\frac{dx}{1+\sin\alpha\sin x}$$

$$= 2\pi\int_0^{\pi/2}\frac{dx}{1+\sin\alpha\sin x} \quad \textbf{[1]}$$

$$I = \pi\int_0^{\pi/2}\frac{dx}{1+\sin\alpha\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$I = \pi\int_0^1\frac{2dt}{1+t^2+2t\sin\alpha} \quad \left[\text{Put}\tan\frac{x}{2} = t\right]$$

$$\Rightarrow I = 2\pi\int_0^1\frac{dt}{(t+\sin\alpha)^2+\cos^2\alpha}$$

$$= \frac{2\pi}{\cos\alpha}\left[\tan^{-1}\left(\frac{t+\sin\alpha}{\cos\alpha}\right)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos\alpha}\left(\frac{\pi}{2}-\alpha\right) \quad \textbf{[1]}$$

Short Answer Type Questions (3 and 4 marks each)

Q. 1. Integrate the function sin(ax + b) cos(ax + b).

[NCERT Ex. 7.2, Q. 5, Page 304]

Ans. Given that,

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2}$$

$$= \frac{\sin 2(ax+b)}{2}$$

Let $2(ax + b) = t$

$$\therefore \quad 2a\,dx = dt$$

$$\int\frac{\sin 2(ax+b)}{2}dx = \frac{1}{2}\int\frac{\sin t\, dt}{2a}$$

$$= \frac{1}{4a}[-\cos t] + C$$

$$= \frac{-1}{4a}\cos 2(ax+b) + C \quad \textbf{[3]}$$

Q. 2. Integrate the function $\dfrac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$.
[NCERT Ex. 7.2, Q. 24, Page 305]

Ans. Given that,

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$\therefore$ $(-3\sin x + 2\cos x)\,dx = dt$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\int\frac{dt}{2t}=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|2\sin x+3\cos x|+C$$ **[3]**

Q. 3. Integrate the function $\dfrac{1}{\cos^2 x(1-\tan x)^2}$.
[NCERT Ex. 7.2, Q. 25, Page 305]

Ans. Given that,

$$\frac{1}{\cos^2 x(1-\tan x)^2}=\frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1 - \tan x) = t$

$\therefore$ $-\sec^2 x\,dx = dt$

$$\Rightarrow \int\frac{\sec^2 x}{(1-\tan x)^2}dx=\int\frac{-dt}{t^2}=-\int t^{-2}dt$$

$$=+\frac{1}{t}+C$$ **[3]**

$$=\frac{1}{(1-\tan x)}+C$$

Q. 4. Integrate the function $\sqrt{ax+b}$.
[NCERT Ex. 7.2, Q. 6, Page 304]

Ans. Let $ax + b = t$

$\Rightarrow$ $a\,dx = dt$

$\therefore$ $dx=\frac{1}{a}dt$

$$\int(ax+b)^{\frac{1}{2}}dx=\frac{1}{a}\int t^{\frac{1}{2}}dt$$ **[3]**

$$=\frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$$

Q. 5. Integrate the function $x\sqrt{x+2}$.
[NCERT Ex. 7.2, Q. 7, Page 304]

Ans. Let $(x+2) = t$

$\therefore$ $dx = dt$

$$\int x\sqrt{x+2}\,dx=\int(t-2)\sqrt{t}\,dt=\int\left(t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right)dt$$

$$=\int t^{\frac{3}{2}}dt-2\int t^{\frac{1}{2}}dt$$

$$=\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$$ **[3]**

$$=\frac{t^{\frac{5}{2}}}{\frac{5}{2}}-2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C=\frac{2}{5}t^{\frac{5}{2}}-\frac{4}{3}t^{\frac{3}{2}}+C$$

$$=\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$$

Q. 6. Find : $\displaystyle\int\frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx$.
[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$\int\frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx$$

Let $\sin x = t$

$\Rightarrow \cos x\,dx = dt$

$$\therefore \int\frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx=\int\frac{2}{(1-t)(1+t^2)}dt$$

Using partial fraction, we get

$$\frac{2}{(1-t)(1+t^2)}=\frac{A}{1-t}+\frac{Bt+C}{1+t^2}$$

$\Rightarrow 2 = A + At^2 + Bt + C - Bt^2 - Ct$

$\Rightarrow 2 = (A - B)t^2 + (B - C)t + A + C$

$\therefore$ $A - B = \mathbf{0}$, $B - C = 0$ and $A + C = \mathbf{2}$

Solving all these equations, we get

$A = B = C = 1$

Substituting the values, we get

$$2\int\frac{1}{(1-t)(1+t^2)}dt=\int\frac{1}{1-t}dt+\int\frac{t+1}{1+t^2}dt$$

$$=\int\frac{1}{1-t}dt+\int\frac{t}{1+t^2}dt+\int\frac{1}{1+t^2}dt$$

$$=\int\frac{1}{1-t}dt+\frac{1}{2}\int\frac{2t}{1+t^2}dt+\int\frac{1}{1+t^2}dt$$

$$=-\log|1-t|+\frac{1}{2}\log|1+t^2|+\tan^{-1}t+C$$

$$=-\log|1-t|+\log\left|\sqrt{1+t^2}\right|+\tan^{-1}t+C$$

$$=\log\frac{\left|\sqrt{1+t^2}\right|}{|1-t|}+\tan^{-1}t+C$$

Substitute back the value of t in

$$\log\frac{\left|\sqrt{1+t^2}\right|}{|1-t|}+\tan^{-1}t+C$$.

$$\log\frac{\left|\sqrt{1+\sin^2 x}\right|}{|1-\sin x|}+\tan^{-1}(\sin x)+C$$ **[4]**

Q. 7. Evaluate : $\displaystyle\int_0^{\pi/4}\frac{\sin x+\cos x}{16+9\sin 2x}dx$.
[CBSE Board, Delhi Region, 2018]

Ans. $$I=\int_0^{\pi/4}\frac{\sin x+\cos x}{16+9\sin x}dx$$

Put $(-\cos x + \sin x) = t$

or $(\sin x + \cos x)\,dx = dt$

$x = 0$, $t = -1$

$x = \pi/4$, $t = 0$

$$I=\int_{-1}^{0}\frac{dt}{16+9(1-t^2)}$$

$$=\int_{-1}^{0}\frac{dt}{25-9t^2}$$

$$=\frac{1}{9}\int_{-1}^{0}\frac{dt}{\left(\frac{5}{3}\right)^2-t^2}$$

$$= \frac{1}{9}\left[\frac{1}{\frac{10}{3}}\log\left|\frac{\frac{5}{3}+t}{\frac{5}{3}-t}\right|\right]_{-1}^{0}$$

$$= \frac{1}{30}\log 4$$ **[4]**

Q. 8. Evaluate : $\int_1^3 \left(x^2+3x+e^x\right)dx$, as the limit of the sum. [CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$\int_1^3 \left(x^2+3x+e^x\right)dx,$$

Here, $a = 1$, $b = 3f(x) = x^2 + 3x + e^x$

$$\therefore h = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$\int_1^3 \left(x^2+3x+e^x\right)dx = \lim_{h\to 0}\left[\begin{array}{l} f(1)+f(1+h)+f(1+2h) \\ +f(1+3h)......+f(1+(n-1)h)\end{array}\right]$$

$$= \lim_{h\to 0}\left[\begin{array}{l} 4+e+(1+h)^2+3(1+h)+e^{1+h}+(1+2h)^2+3(1+2h) \\ +e^{1+2h}+(1+3h)^2+3(1+3h)+e^{1+3h}+..... \\ .....+(1+(n-1)h)^2+3(1+(n-1)h)+e^{1+(n-1)h}\end{array}\right]$$

$$= \lim_{h\to 0}\left[\begin{array}{l} 4n+e+h^2\dfrac{(n-1)(n-2)(2n-3)}{6}+2h\left(\dfrac{n^2-n}{2}\right) \\ +3h\left(\dfrac{n^2-n}{2}\right)+e\dfrac{e^h\left[e^{h(n-1)}-1\right]}{e^h-1}\end{array}\right]$$

$$= \lim_{h\to 0}\left[\begin{array}{l} 4n+e+h^2\dfrac{(n-1)(n-2)(2n-3)}{6}+2h\left(\dfrac{n^2-n}{2}\right) \\ +3h\left(\dfrac{n^2-n}{2}\right)+e\dfrac{e^h\left[e^{h(n-1)}-1\right]}{e^h-1}\end{array}\right]$$

$$= \frac{62}{3}+e^3-e$$

[4]

Q. 9. Evaluate : $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$.

[NCERT Exemp. Ex. 7.3, Q. 38, Page 165]

Ans. Let,

$$I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)}dx$$

Now, $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x-3)}$

$\Rightarrow 2x-1 = A(x+2)(x-3) + B(x-3)(x-1) + C(x-1)(x+2)$

Put $x = 3$, then

$6 - 1 = C(3-1)(3+2)$

$\Rightarrow \quad 5 = 10C$

$\therefore \quad C = \frac{1}{2}$

Put $x = 1$, then

$(2-1) = A(1+2)(1-3)$

$1 = -6A$

$A = -1/6$

Now, put $x = -2$, then

$-4-1 = B(-2-1)(-2-3)$

$\Rightarrow -5 = 15B$

$\Rightarrow B = -\frac{1}{3}$

$$\therefore \quad I = -\frac{1}{6}\int\frac{1}{x-1}dx - \frac{1}{3}\int\frac{1}{x+2}dx + \frac{1}{2}\int\frac{1}{x-3}dx$$

$$= -\frac{1}{6}\log|(x-1)| - \frac{1}{3}\log|(x+2)| + \frac{1}{2}\log|(x-3)| + C$$

$$= -\log|(x-1)|^{1/6} - \log|(x+2)|^{1/3} + \log|(x-3)|^{1/2} + C$$

$$= \log\left|\frac{\sqrt{x-3}}{(x-1)^{1/6}(x+2)^{1/3}}\right| + C$$ **[3]**

Q. 10. Integrate the function $\frac{(x+1)(x+\log x)^2}{x}$.

[NCERT Ex. 7.2, Q. 36, Page 305]

Ans. Given that,

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2$$

$$= \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\left(1+\frac{1}{x}\right)dx = dt$$ **[3]**

$$\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{1}{3}(x+\log x)^3 + C$$

Q. 11. Evaluate definite integral as limit of sums : $\int_{-1}^{1} e^x dx$.

[NCERT Ex. 7.8, Q. 5, Page 334]

Ans. Let $I = \int_{-1}^{1} e^x dx$ It is known that,

$$\int_a^b \left[\begin{array}{l} f(x)dx = (b-a)\lim\limits_{n\to\infty}\dfrac{1}{n}\left[f(a)+f(a+h)...f(a+(n-1)h)\right], \\ \text{where } h = \dfrac{b-a}{n}\end{array}\right]$$

Here, $a = -1$, $b = 1$ and $f(x) = e^x$

$$\therefore \quad = \frac{1 \quad 1}{n} = \frac{2}{n}$$

$$\therefore \quad = (1+1)\lim_{\to\infty}-\left[\begin{array}{l} f(-1)+f\left(-1+\dfrac{2}{n}\right)+f\left(-1+2\cdot\dfrac{2}{n}\right) \\ +...+\ \ \left(- \ \ +\dfrac{(\quad 1)2}{}\right)\end{array}\right]$$

$$= \lim_{\to\infty}-\left[e \quad + e^{\left(-1+\frac{2}{n}\right)} + e^{\left(-1+2\cdot\frac{2}{n}\right)} + ...e^{\left(-1+(\ \ -1)\frac{2}{n}\right)}\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$= 2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}-1}}{e^{\frac{2}{n}-1}}\right]$$ **[1½]**

$$= e^{-1} \times 2 \lim_{n\to\infty} \frac{1}{n} \left[\frac{e^2 - 1}{e^{\frac{2}{n}-1}} \right]$$

$$= \frac{e^{-1} \times 2(e^2 - 1)}{\lim_{\frac{2}{n}\to 0} \left(\frac{e^{\frac{2}{n}} - 1}{\frac{2}{n}} \right) \times 2}$$

$$= e^{-1} \left[\frac{2(e^2 - 1)}{2} \right] \quad \left[\because \lim_{h\to 0} \left(\frac{e^h - 1}{h} \right) = 1 \right]$$ **[1½]**

$$= \frac{e^2 - 1}{e}$$

$$= \left(e - \frac{1}{e} \right)$$

Q. 12. Integrate the function $\frac{\sin x}{\sin(x-a)}$.

[NCERT Misc. Ex. Q. 7, Page 352]

Ans. Given that,

$$\frac{\sin x}{\sin(x-a)}$$

Let $x - a = t$

$\Rightarrow \quad dx = dt$

$$\int \frac{\sin x}{\sin(x-a)} dx = \int \frac{\sin(t+a)}{\sin t} dt$$

$$= \int \frac{\sin t \cos a + \cos t \sin a}{\sin t} dt$$

$$= \int (\cos a + \cot t \sin a) dt$$

$$= t \cos a + \sin a \log|\sin t| + C_1$$

$$= (x-a)\cos a + \sin a \log|\sin(x-a)| + C_1$$

$$= x\cos a + \sin a \log|\sin(x-a)| - a\cos a + C_1$$

$$= \sin a \log|\sin(x-a)| + x\cos a + C$$ **[3]**

Q. 13. Evaluate the definite integral $\int_1^2 (4x^3 - 5x^2 + 6x + 9)dx$.

[NCERT Ex. 7.9, Q. 3, Page 338]

Ans. Let,

$$I = \int_1^2 (4x^3 - 5x^2 + 6x + 9)dx$$

$$\int (4x^3 - 5x^2 + 6x + 9)dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(2) - F(1)$

$$I = \left\{ 2^4 - \frac{5\cdot(2)^3}{3} + 3(2)^2 + 9(2) \right\} - \left\{ (1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1) \right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18 \right) - \left(1 - \frac{5}{3} + 3 + 9 \right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3} = \frac{64}{3}$$ **[3]**

Q. 14. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \sin 2x\, dx$.

[NCERT Ex. 7.9, Q. 4, Page 338]

Ans. Let,

$$I = \int_0^{\frac{\pi}{4}} \sin 2x\, dx$$

$$\int \sin 2x\, dx = \left(\frac{-\cos 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0 \right]$$ **[3]**

$$= -\frac{1}{2}[0 - 1]$$

$$= \frac{1}{2}$$

Q. 15. Evaluate the definite integral as limit of sums : $\int_0^4 (x + e^{2x})dx$. **[NCERT Ex. 7.8, Q. 6, Page 334]**

Ans. It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} f(a) + f(a+h) + ... \\ + f(a + (n-1)h) \end{array} \right], \text{where } h = \frac{b-a}{n}$$

Here, $a = 0, b = 4$ and $f(x) = x + e^{2x}$

$$\therefore h = \frac{4-0}{n} = \frac{4}{n}$$

$$\Rightarrow \int_0^4 (x + e^{2x})dx = (4-0)\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} f(0) + f(h) + f(2h) \\ +... + f((n-1)h) \end{array} \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} (0 + e^0) + (h + e^{2h}) + (2h + e^{2\cdot 2h}) \\ +.... + \{(n-1)h + e^{2(n-1)h}\} \end{array} \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} 1 + (h + e^{2h}) + (2h + e^{4h}) \\ +... + \{(n-1)h + e^{2(n-1)h}\} \end{array} \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} \{h + 2h + 3h + ... + (n-1)h\} \\ +(1 + e^{2h} + e^{4h} + ... + e^{2(n-1)h}) \end{array} \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\begin{array}{l} h\{1 + 2 + ...(n-1)\} \\ + \left(\frac{e^{2hn} - 1}{e^{2h} - 1} \right) \end{array} \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\frac{(h(n-1)n)}{2} + \left(\frac{e^{2hn} - 1}{e^{2h} - 1} \right) \right]$$

$$= 4\lim_{n\to\infty} \frac{1}{n} \left[\frac{4}{n} \cdot \frac{(n-1)n}{2} + \left(\frac{e^8 - 1}{e^{\frac{8}{n}} - 1} \right) \right]$$

[1½]

$$= 4(2) + 4\lim_{n\to\infty} \frac{(e^8 - 1)}{\left(\frac{e^{\frac{8}{n}} - 1}{\frac{8}{n}}\right) 8}$$

$$= 8 + \frac{4\cdot(e^8 - 1)}{8} \qquad \left(\because \lim_{x\to\infty}\frac{e^x - 1}{x} = 1\right)$$

$$= 8 + \frac{e^8 - 1}{2}$$

$$= \frac{15 + e^8}{2}$$ [1½]

Q. 16. Integrate the function $\frac{x^3 \sin(\tan^{-1} x^4)}{1 + x^8}$.

[NCERT Ex. 7.2, Q. 37, Page 305]

Ans. Let $x^4 = t$

$\therefore$ $4x^3\,dx = dt$

$$\Rightarrow \int \frac{x^3 \sin(\tan^{-1} x^4)}{1 + x^8} dx = \frac{1}{4}\int \frac{\sin(\tan^{-1} t)}{1 + t^2} dt \qquad ...(i)$$

Let $\tan^{-1} t = u$

$\therefore$ $\frac{1}{1+t^2} dt = du$

From equation (i), we obtain

$$\int \frac{x^3 \sin(\tan^{-1} x^4) dx}{1 + x^8} = \frac{1}{4}\int \sin u\, du$$

$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos(\tan^{-1} t) + C$$

$$= \frac{-1}{4}\cos(\tan^{-1} x^4) + C$$ [3]

Q. 17. Evaluate : $\int e^{\tan^{-1}x}\left(\frac{1 + x + x^2}{1 + x^2}\right)dx$.

[NCERT Exemp. Ex. 7.3, Q. 39, Page 166]

Ans. Let,

$$I = \int e^{\tan^{-1} x}\left(\frac{1 + x + x^2}{1 + x^2}\right)dx$$

$$= \int e^{\tan^{-1} x}\left(\frac{1 + x^2}{1 + x^2} + \frac{x}{1 + x^2}\right)dx$$

$$= \int e^{\tan^{-1} x} dx + \int \frac{x e^{\tan^{-1} x}}{1 + x^2} dx$$

$I = I_1 + I_2$ ...(i)

Now, $I_2 = \int \frac{x e^{\tan^{-1} x}}{1 + x^2} dx$

Put $\tan^{-1} x = t \Rightarrow x = \tan t$

$$\Rightarrow \frac{1}{1 + x^2} dx = dt$$

$\therefore$ $I = \int \underset{\text{I}}{\tan t} \cdot \underset{\text{II}}{e^t} dt$

$$= \tan t \cdot e^t - \int \sec^2 t \cdot e^t dt + C$$

$$= \tan t \cdot e^t - \int (1 + \tan^2 t) e^t dt + C$$

$$[\because \sec^2\theta = 1 + \tan^2\theta]$$

$$I_2 = \tan t \cdot e^t - \int (1 + x^2)\frac{e^{\tan^{-1} x}}{1 + x^2} dx + C$$

$$I_2 = \tan t \cdot e^t - \int e^{\tan^{-1} x} dx + C$$

$$I = \int e^{\tan^{-1} x} dx + \tan t \cdot e^t - \int e^{\tan^{-1}} dx + C$$

$$= \tan t \cdot e^t + C$$

$$= x e^{\tan^{-1} x} + C$$ [3]

Q. 18. Evaluate : $\int \sin^{-1}\sqrt{\frac{x}{a + x}}dx$. (Hint : Put $x = a\tan^2\theta$)

[NCERT Exemp. Ex. 7.3, Q. 40, Page 166]

Ans. Let,

$$I = \int \sin^{-1}\sqrt{\frac{x}{a + x}} dx$$

Put $x = a\tan^2\theta$

$\Rightarrow dx = 2a\tan\theta\sec^2\theta d\theta$

$$\therefore I = \int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a + a\tan^2\theta}}(2a\tan\theta\cdot\sec^2\theta)d\theta$$

$$= 2a\int \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)(\tan\theta\cdot\sec^2\theta)d\theta$$

$$= 2a\int \sin^{-1}(\sin\theta)\tan\theta\cdot\sec^2\theta\, d\theta$$ [1½]

$$= 2a\int \underset{\text{I}}{\theta}\cdot \underset{\text{II}}{\tan\theta\sec^2\theta}\, d\theta$$

$$= 2a\left[\theta\cdot\int \tan\theta\cdot\sec^2\theta d\theta - \int\left(\frac{d}{d\theta}\theta\cdot\int\tan\theta\cdot\sec^2\theta\, d\theta\right)d\theta\right]$$

$$\left[\begin{array}{l}\text{Put} \quad \tan\theta = t \\ \Rightarrow \sec\theta\cdot\tan\theta\cdot d\theta = dt \\ \Rightarrow \int \tan\theta\sec^2\theta d\theta = \int t\, dt\end{array}\right]$$

$$= 2a\left[\theta\cdot\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right]$$

$$= a\theta\tan^2\theta - a\int(\sec^2\theta - 1)d\theta$$ [1½]

$$= a\theta\cdot\tan^2\theta - a\tan\theta + a\theta + C$$

$$= a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + C$$

Q. 19. Integrate the function $\frac{e^{2x} - 1}{e^{2x} + 1}$.

[NCERT Ex. 7.2, Q. 19, Page 305]

Ans. Given that,

$$\frac{e^{2x} - 1}{e^{2x} + 1}$$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{(e^{2x} - 1)}{e^x}}{\frac{(e^{2x} + 1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$\therefore (e^x - e^{-x})dx = dt$

$$\Rightarrow \int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{dt}{t}$$
$$= \log|t| + C$$
$$= \log\left|e^x + e^{-x}\right| + C \quad \textbf{[3]}$$

Q. 20. Integrate the function $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$.

[NCERT Ex. 7.2, Q. 20, Page 305]

Ans. Let $e^{2x} + e^{-2x} = t$

$$\therefore \left(2e^{2x} - 2e^{-2x}\right)dx = dt$$
$$\Rightarrow 2\left(e^{2x} - e^{-2x}\right)dx = dt$$
$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx = \int\frac{dt}{2t} = \frac{1}{2}\int\frac{1}{t}dt \quad \textbf{[3]}$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log\left|e^{2x}+e^{-2x}\right| + C$$

Q. 21. Integrate the function $\tan^2(2x-3)$.

[NCERT Ex. 7.2, Q. 21, Page 305]

Ans. Given that,

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let $2x - 3 = t$

$\therefore \quad 2dx = dt$

$$\int \tan^2(2x-3)dx = \int\left[\sec^2(2x-3) - 1\right]dx$$
$$= \int\left[\sec^2 t - 1\right]\frac{dt}{2}$$
$$= \frac{1}{2}\left[\int \sec^2 t\,dt - \int 1dt\right] \quad \textbf{[3]}$$
$$= \frac{1}{2}\left[\tan t - t + C\right]$$
$$= \frac{1}{2}\left[\tan(2x-3) - (2x-3) + C\right]$$

Q. 22. Integrate the function $\dfrac{5x+3}{\sqrt{x^2+4x+10}}$.

[NCERT Ex. 7.4, Q. 23, Page 316]

Ans. Let,

$$5x + 3 = A\frac{d}{dx}\left(x^2+4x+10\right) + B$$
$$\Rightarrow 5x + 3 = A(2x+4) + B$$

Equation the coefficients of x and constant term, we obtain

$$2A = 5 \Rightarrow A = \frac{5}{2}$$
$$4A + B = 3 \Rightarrow B = -7$$
$$\therefore \quad 5x + 3 = \frac{5}{2}(2x+4) - 7$$
$$\Rightarrow \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int\frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$
$$= \frac{5}{2}\int\frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int\frac{1}{\sqrt{x^2+4x+10}}dx$$

Let,

$$I_1 = \int\frac{2x+4}{\sqrt{x^2+4x+10}}dx \text{ and } I_2 = \int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\therefore \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \quad \ldots\text{(i)} \textbf{ [1½]}$$

Then, $I_1 = \int\frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Let $x^2 + 4x + 10 = t$

$\therefore \quad (2x+4)dx = dt$

$$\Rightarrow I_1 = \int\frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \quad \ldots\text{(ii)}$$
$$I_2 = \int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$= \int\frac{1}{\sqrt{\left(x^2+4x+4\right)+6}}dx$$
$$= \int\frac{1}{\sqrt{(x+2)^2+\left(\sqrt{6}\right)^2}}dx$$
$$= \log\left|(x+2)+\sqrt{x^2+4x+10}\right| \quad \ldots\text{(iii)}$$

Using equations (ii) and (iii) in equation (i), we obtain

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\left[2\sqrt{x^2+4x+10}\right] - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$
$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C \quad \textbf{[1½]}$$

Q. 23. Integrate the function $e^x\left(\dfrac{1+\sin x}{1+\cos x}\right)$.

[NCERT Ex. 7.6, Q. 18, Page 328]

Ans.

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right) = e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$
$$= \frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$
$$= \frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$
$$= \frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2 \quad \textbf{[1½]}$$
$$= \frac{1}{2}e^x\left(1+\tan\frac{x}{2}\right)^2$$
$$= \frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$
$$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)}{(1+\cos x)} = e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] \quad \ldots(i)$$

Let $\tan\frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

From equation (i), we obtain

$$\int \frac{e^x(1+\sin x)}{(1+\cos x)}dx = e^x\tan\frac{x}{2}+C$$ [1½]

Q. 24. Integrate the function $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$.

[NCERT Ex. 7.6, Q. 19, Page 328]

Ans. Let,

$$I = \int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$$

Also, let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

$$\therefore I = \frac{e^x}{x}+C$$ [3]

Q. 25. Integrate the function $\frac{(x-3)e^x}{(x-1)^3}$.

[NCERT Ex. 7.6, Q. 20, Page 328]

Ans.

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx = \int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$= \int e^x\left\{\frac{1}{(x-1)^2}-\frac{1}{(x-1)^3}\right\}dx$$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

$$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^3}\right\}dx = \frac{e^x}{(x-1)^2}+C$$ [3]

Q. 26. Integrate the function $e^{2x}\sin x$.

[NCERT Ex. 7.6, Q. 21, Page 328]

Ans. Let,

$$I = \int e^{2x}\sin x\, dx \quad \ldots(i)$$

Integrating by parts, we obtain

$$I = \sin x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$= \sin x\cdot\frac{e^{2x}}{2} - \int\cos x\cdot\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x\, dx$$

Again, integrating by parts, we obtain

$$I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$= \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x dx\right]$$ [1½]

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I \quad \text{[From Eq. (i)]}$$

$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right]+C$$

$$\Rightarrow I = \frac{e^{2x}}{5}[2\sin x - \cos x]+C$$

[1½]

Q. 27. Integrate the function $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

[NCERT Ex. 7.6, Q. 22, Page 328]

Ans. Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta\, d\theta$$

$$= 2\int\theta\cdot\sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$$

$$= 2\left[\theta\cdot\tan\theta - \int\tan\theta d\theta\right]$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right]+C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right]+C$$ [3]

$$= 2x\tan^{-1}x + 2\log(1+x^2)^{-\frac{1}{2}}+C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right]+C$$

$$= 2x\tan^{-1}x - \log(1+x^2)+C$$

Q. 28. Find the integrals of the function $\sin^2(2x+5)$.

[NCERT Ex. 7.3, Q. 1, Page 307]

Ans.

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2}$$

$$= \frac{1-\cos(4x+10)}{2}$$

$$\int\sin^2(2x+5)dx = \int\frac{1-\cos(4x+10)}{2}dx$$ [3]

$$= \frac{1}{2}\int 1\, dx - \frac{1}{2}\int\cos(4x+10)dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10)+C$$

Q. 29. Find the integrals of the function $\sin 3x\cos 4x$.

[NCERT Ex. 7.3, Q. 2, Page 307]

Ans. It is known that,

$$\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$$

$$\int\sin 3x\cos 4x\,dx=\frac{1}{2}\int\{\sin(3x+4x)+\sin(3x-4x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x-\sin x\}dx$$

$$=\frac{1}{2}\int\sin 7x\,dx-\frac{1}{2}\int\sin x\,dx$$

$$=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C$$

$$=\frac{-\cos 7x}{14}+\frac{\cos x}{2}+C$$ **[3]**

Q. 30. Find the integrals of the function $\cos 2x\cos 4x\cos 6x$.

[NCERT Ex. 7.3, Q. 3, Page 307]

Ans. It is known that,

$$\cos A\cos B=\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$$

$$\therefore\int\cos 2x(\cos 4x\cdot\cos 6x)dx$$

$$=\int\cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\}dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos^2 2x\}dx$$

$$=\frac{1}{2}\int\left[\begin{matrix}\frac{1}{2}\{\cos(2x+10x)+\cos(2x-10x)\}\\+\left(\frac{1+\cos 4x}{2}\right)\end{matrix}\right]dx$$ **[3]**

$$=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)dx$$

$$=\frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

Q. 31. Find the integrals of the function $\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}$.

[NCERT Ex. 7.3, Q. 17, Page 307]

Ans. $$\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}=\frac{\sin^3 x}{\sin^2 x\cos^2 x}+\frac{\cos^3 x}{\sin^2 x\cos^2 x}$$

$$=\frac{\sin x}{\cos^2 x}+\frac{\cos x}{\sin^2 x}$$

$$=\tan x\sec x+\cot x\,\text{cosec}\,x$$

$$\therefore\int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx=\int(\tan x\sec x+\cot x\,\text{cosec}\,x)dx$$

$$=\sec x-\text{cosec}\,x+C$$ **[3]**

Q. 32. Find the integrals of the function $\frac{\cos 2x+2\sin^2 x}{\cos^2 x}$.

[NCERT Ex. 7.3, Q. 18, Page 307]

Ans.

$$\frac{\cos 2x+2\sin^2 x}{\cos^2 x}=\frac{\cos 2x+(1-\cos 2x)}{\cos^2 x}$$

$$[\because\cos 2x=1-2\sin^2 x]$$

$$=\frac{1}{\cos^2 x}=\sec^2 x$$

$$\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx=\int\sec^2 x\,dx=\tan x+C$$ **[3]**

Q. 33. Find the integrals of the function $\frac{1}{\sin x\cos^3 x}$.

[NCERT Ex. 7.3, Q. 19, Page 307]

Ans. $$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$

$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$

$$\frac{1}{\sin x\cos^3 x}=\tan x\sec^2 x+\frac{\frac{1}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}}$$ **[3]**

$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

Q. 34. Find the integrals of the function $\frac{\cos 2x}{(\cos x+\sin x)^2}$.

[NCERT Ex. 7.3, Q. 20, Page 307]

Ans. $$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}$$

$$=\frac{\cos 2x}{1+\sin 2x}$$

$$\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int\frac{\cos 2x}{(1+\sin 2x)}dx$$

Let $1+\sin 2x=t$

$$2\cos 2x\,dx=dt$$

$$\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int\frac{1}{t}dt=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$ **[3]**

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log|\sin x+\cos x|+C$$

Q. 35. Find the integrals of the function $\sin^3(2x+1)$.

[NCERT Ex. 7.3, Q. 4, Page 307]

Ans. Let $$I=\int\sin^3(2x+1)$$

$$\Rightarrow\int\sin^3(2x+1)dx=\int\sin^2(2x+1)\cdot\sin(2x+1)dx$$

$$=\int(1-\cos^2(2x+1))\sin(2x+1)dx$$

Let $\cos(2x+1)=t$

$$\Rightarrow -2\sin(2x+1)dx=dt$$

$$\Rightarrow\quad\sin(2x+1)dx=\frac{-dt}{2}$$

$$\Rightarrow\quad I=\frac{-1}{2}\int(1-t^2)dt$$

$$=\frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{6}\right\}$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{12} + C$$

[3]

Q. 36. Find the integrals of the function $\sin^3 x \cos^3 x$.
[NCERT Ex. 7.3, Q. 5, Page 307]

Ans. Let $I = \int \sin^3 x \cos^3 x \cdot dx$

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$

$$= \int \cos^3 x (1 - \cos^2 x) \sin x \cdot dx$$

Let $\cos x = t$

$-\sin x \cdot dx = dt$

$$I = -\int t^3 (1 - t^2) dt$$

$$= -\int (t^3 - t^5) dt$$

$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$

[3]

$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

Q. 37. Integrate the function $\frac{1}{1+\cot x}$.
[NCERT Ex. 7.2, Q. 32, Page 305]

Ans. Let,

$$I = \int \frac{1}{1+\cot x} dx$$

$$= \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{2\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let $\sin x + \cos x = t$

$\Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore \quad I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

[3]

$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

Q. 38. Integrate the function $\frac{1}{1-\tan x}$.
[NCERT Ex. 7.2, Q. 33, Page 305]

Ans. Let,

$$I = \int \frac{1}{1-\tan x} dx$$

$$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{2\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put $\cos x - \sin x = t$

$\Rightarrow (-\sin x - \cos x)\, dx = dt$

$$\therefore \quad I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

[3]

$$= \frac{x}{2} - \frac{1}{2} \log|\cos x - \sin x| + C$$

Q. 39. Integrate the functions $\frac{1}{\sqrt{x^2+2x+2}}$.
[NCERT Ex. 7.4, Q. 10, Page 316]

Ans. Given that,

$$\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}} dx$$

Let $x + 1 = t$

$\therefore \quad dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+1}} dx = \int \frac{1}{\sqrt{t^2+1}} dt$$

$$= \log\left|t + \sqrt{t^2+1}\right| + C$$

$$= \log\left|(x+1) + \sqrt{(x+1)^2 + 1}\right| + C$$

[3]

$$= \log\left|(x+1) + \sqrt{x^2+2x+2}\right| + C$$

Q. 40. Integrate the function $\frac{4x+1}{\sqrt{2x^2+x-3}}$.
[NCERT Ex. 7.4, Q. 16, Page 316]

Ans. Let, $4x + 1 = A\frac{d}{dx}(2x^2 + x - 3) + B$

$$\Rightarrow 4x + 1 = A(4x+1) + B$$

$$\Rightarrow 4x + 1 = 4Ax + A + B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$4A = 4 \Rightarrow A = 1$$

$$A + B = 1 \Rightarrow B = 0$$

Let $2x^2 + x - 3 = t$

$\therefore \quad (4x+1)dx = dt$

$$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx = \int \frac{1}{\sqrt{t}} dt$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{2x^2+x-3} + C$$

[3]

Q. 41. Integrate the function $\frac{x+2}{\sqrt{x^2-1}}$.
[NCERT Ex. 7.4, Q. 17, Page 316]

Ans. Let $x+2 = A\frac{d}{dx}(x^2-1)+B$ ...(i)

$\Rightarrow x+2 = A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \frac{1}{2}$

$B = 2$

From equation (i), we obtain

$(x+2) = \frac{1}{2}(2x)+2$

Then, $\int\frac{x+2}{\sqrt{x^2-1}}dx = \int\frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$

$= \frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx + \int\frac{2}{\sqrt{x^2-1}}dx$ ...(ii) [1½]

In $= \frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t \Rightarrow 2xdx = dt$

$= \frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int\frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\left[2\sqrt{t}\right]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

Then, $\int\frac{2}{\sqrt{x^2-1}}dx = 2\int\frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$

From equation (ii), we obtain

$\int\frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$ [1½]

Q. 42. Integrate the function $\sqrt{1+3x-x^2}$.
[NCERT Ex. 7.7, Q. 7, Page 330]

Ans. Let,

$I = \int\sqrt{1+3x-x^2}dx$

$= \int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}dx$

$= \int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}dx$

$= \int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}dx$

It is known that,

$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$ [3]

$= \frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$

Q. 43. By using the properties of definite integrals, evaluate the integral $\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$.
[NCERT Ex. 7.11, Q. 8, Page 347]

Ans. Let,

$I = \int_0^{\frac{\pi}{4}}\log(1+\tan x)dx$ ...(i)

$\therefore \quad I = \int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}}\log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}}\log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}}\log\frac{2}{(1+\tan x)}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}}\log 2dx - \int_0^{\frac{\pi}{4}}\log(1+\tan x)dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}}\log 2dx - 1 \quad$ [From Eq. (i)]

$\Rightarrow \quad 2I = [x\log 2]_0^{\frac{\pi}{4}}$

$\Rightarrow \quad 2I = \frac{\pi}{4}\log 2$

$\Rightarrow \quad I = \frac{\pi}{8}\log 2$ [3]

Q. 44. By using the properties of definite integrals, evaluate the integral $\int_0^2 x\sqrt{2-x}\,dx$.
[NCERT Ex. 7.11, Q. 9, Page 347]

Ans. Let,

$I = \int_0^2 x\sqrt{2-x}dx$

$I = \int_0^2(2-x)\sqrt{x}dx$

$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$= \int_0^2\left\{2x^{\frac{1}{2}}-x^{\frac{3}{2}}\right\}dx$

$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}}-\frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4\times 2\sqrt{2}}{3}-\frac{2}{5}\times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3}-\frac{8\sqrt{2}}{5}$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + C$$ **[3]**

Q. 45. Integrate the function $\frac{2 + \sin 2x}{1 + \cos 2x} e^x$.

[NCERT Misc. Ex. Q. 21, Page 353]

Ans.

$$I = \int \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) e^x$$

$$= \int \left(\frac{2 + 2\sin x \cos x}{2\cos^2 x} \right) e^x$$

$$= \int \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) e^x$$

$$= \int \left(\sec^2 x + \tan x \right) e^x$$

Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$\therefore \quad I = \int \left[f(x) + f'(x) \right] e^x dx$

$$= e^x f(x) + C$$

$$= e^x \tan x + C$$ **[3]**

Q. 46. Integrate the function $\frac{x^2 + x + 1}{(x+1)^2 (x+2)}$.

[NCERT Misc. Ex. Q. 22, Page 353]

Ans. Let,

$$\frac{x^2 + x + 1}{(x+1)^2 (x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)} \quad \ldots\text{(i)}$$

$$\Rightarrow x^2 + x + 1 = A(x+1)(x+2) + B(x+2) + C(x^2 + 2x + 1)$$

$$\Rightarrow x^2 + x + 1 = A(x^2 + 3x + 2) + B(x+2) + C(x^2 + 2x + 1)$$

$$\Rightarrow x^2 + x + 1 = (A + C)x^2 + (3A + B + 2C)x + (2A + 2B + C)$$

Equating the coefficients of x^2, x and constant term, we obtain

$$A + C = 1$$
$$3A + B + 2C = 1$$
$$2A + 2B + C = 1$$

On solving these equations, we obtain $A = -2$, $B = 1$ and $C = 3$

From equation (i), we obtain

$$\frac{x^2 + x + 1}{(x+1)^2 (x+2)} = \frac{-2}{(x+1)} + \frac{3}{(x+2)} + \frac{1}{(x+1)^2}$$

$$\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx$$

$$= -2\int \frac{1}{x+1} dx + 3\int \frac{1}{(x+2)} dx + \int \frac{1}{(x+1)^2} dx$$

$$= -2\log|x+1| + 3\log|x+2| - \frac{1}{(x+1)} + C$$ **[3]**

Q. 47. Integrate the function $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$.

[NCERT Misc. Ex. Q. 23, Page 353]

Ans. Given that,

$$I = \tan^{-1}\sqrt{\frac{1-x}{1+x}} dx$$

Let $x = \cos\theta$

$\Rightarrow dx = -\sin\theta \, d\theta$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} (-\sin\theta \, d\theta)$$

$$= -\int \tan^{-1}\sqrt{\frac{2\sin^2 \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2}}} \sin\theta \, d\theta$$

$$= -\int \tan^{-1} \tan\frac{\theta}{2} \cdot \sin\theta \, d\theta$$

$$= -\frac{1}{2}\int \theta \cdot \sin\theta \, d\theta$$

$$= -\frac{1}{2}\left[\theta \cdot (-\cos\theta) - \int 1 \cdot (-\cos\theta) d\theta \right]$$

$$= -\frac{1}{2}\left[-\theta\cos\theta + \sin\theta \right]$$

$$= +\frac{1}{2}\theta\cos\theta - \frac{1}{2}\sin\theta$$

$$= \frac{1}{2}\cos^{-1} x \cdot x - \frac{1}{2}\sqrt{1-x^2} + C$$

$$= \frac{x}{2}\cos^{-1} x - \frac{1}{2}\sqrt{1-x^2} + C$$

$$= \frac{1}{2}\left(x\cos^{-1} x - \sqrt{1-x^2} \right) + C$$ **[3]**

Q. 48. Evaluate the definite integral $\int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$.

[NCERT Misc. Ex. Q. 25, Page 353]

Ans. Given that,

$$I = \int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2 \frac{x}{2}} \right) dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{\operatorname{cosec}^2 \frac{x}{2}}{2} - \cot\frac{x}{2} \right) dx$$

Let, $f(x) = -\cot\frac{x}{2}$

$$f'(x) = -\left(-\frac{1}{2}\operatorname{cosec}^2 \frac{x}{2} \right)$$

$$= \frac{1}{2}\operatorname{cosec}^2 \frac{x}{2}$$

$$I = \int_{\frac{\pi}{2}}^{\pi} e^x \left[f(x) + f'(x) \right] dx$$

$$= \left[e^x \cdot f(x) dx \right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\left[e^x \cdot \cot\frac{x}{2} \right]_{\frac{\pi}{2}}^{\pi}$$ **[1½]**

$$= -\left[e^{x}\cdot\cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\left[e^{\pi}\times\cot\frac{\pi}{2} - e^{\frac{\pi}{2}}\times\cot\frac{\pi}{4}\right]$$

$$= -\left[e^{\pi}\times 0 - e^{\frac{\pi}{2}}\times 1\right]$$

$$= e^{\frac{\pi}{2}}$$ **[1½]**

Q. 49. By using the properties of definite integrals, evaluate the integral $\int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)dx$.

[NCERT Ex. 7.11, Q. 10, Page 347]

Ans. Let,

$$I = \int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\{2\log\sin x - \log(2\sin x\cos x)\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\{2\log\sin x - \log\sin x - \log\cos x - \log 2\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\{\log\sin x - \log\cos x - \log 2\}dx \quad \ldots(i)$$

It is known that,

$$\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\{\log\cos x - \log\sin x - \log 2\}dx \quad \ldots(ii)$$

Adding (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}}(-\log 2 - \log 2)dx$$

$$\Rightarrow 2I = -2\log 2\int_0^{\frac{\pi}{2}}1\times dx$$

$$\Rightarrow I = -\log 2\left[\frac{\pi}{2}\right]$$

$$\Rightarrow I = \frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I = \frac{\pi}{2}\left[\log\frac{1}{2}\right]$$

$$\Rightarrow I = \frac{\pi}{2}\log\frac{1}{2}$$

[3]

Q. 50. By using the properties of definite integrals, evaluate the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\,dx$.

[NCERT Ex. 7.11, Q. 11, Page 347]

Ans. Let,

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\,dx$$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$$

$$I = 2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx$$

$$= 2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}dx$$

$$= \int_0^{\frac{\pi}{2}}(1-\cos 2x)dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

[3]

Q. 51. Evaluate the definite integral $\int_0^1 \frac{2x+3}{5x^2+1}dx$.

[NCERT Ex. 7.9, Q. 14, Page 338]

Ans. Let, $I = \int_0^1 \frac{2x+3}{5x^2+1}dx$

$$\int\frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int\frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int\frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int\frac{10x}{5x^2+1}dx + 3\int\frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int\frac{10x}{5x^2+1}dx + 3\int\frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log(5x^2+1) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log(5x^2+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5}x)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})\right\}$$

$$-\left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\} = \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

[3]

Q. 52. Evaluate the definite integral $\int_0^1 xe^{x^2}dx$.

[NCERT Ex. 7.9, Q. 15, Page 338]

Ans. Let $I = \int_0^1 xe^{x^2}dx$

Put $x^2 = t$

$\Rightarrow 2x\,dx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1$,

$$\therefore \quad I = \frac{1}{2}\int_0^1 e^t dt$$

$$\frac{1}{2}\int_0^1 e^t dt = \frac{1}{2}e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$$= \frac{1}{2}e - \frac{1}{2}e^0$$

$$= \frac{1}{2}(e-1)$$

[3]

Q. 53. Evaluate the definite integral $\int_1^2 \frac{5x^2}{x^2+4x+3}$.

[NCERT Ex. 7.9, Q. 16, Page 338]

Ans. Let,

$$I=\int_1^2 \frac{5x^2}{x^2+4x+3}dx$$

Dividing $5x^2$ by $x^2 + 4x + 3$, we obtain

$$I=\int_1^2 \left\{5-\frac{20x+15}{x^2+4x+3}\right\}dx$$

$$=\int_1^2 5dx-\int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$=[5x]_1^2-\int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$$I=5-I_1 \text{ where } I=\int_1^2 \frac{20x+15}{x^2+4x+3}dx \quad ...(i)$$

Consider $\quad I_1=\int_1^2 \frac{20x+15}{x^2+4x+3}dx$

Let $\quad 20x+15=A\frac{d}{dx}\left(x^2+4x+3\right)+B$

$$=2Ax+(4A+B)$$ **[1½]**

Equating the coefficients of x and constant term, we obtain

$A = 10$ and $B = -25$

$$\Rightarrow I_1=10\int_1^2 \frac{2x+4}{x^2+4x+3}dx-25\int_1^2 \frac{dx}{x^2+4x+3}$$

Let $x^2 + 4x + 3 = t$

$\Rightarrow \quad (2x + 4)dx = dt$

$$I_1=10\int\frac{dt}{t}-25\int\frac{dx}{(x+2)^2-1^2}$$

$$=10\log t-25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$

$$=\left[10\log\left(x^2+4x+3\right)\right]_1^2-25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$

$$=[10\log 15-10\log 8]-25\left[\frac{1}{2}\log\frac{3}{5}-\frac{1}{2}\log\frac{2}{4}\right]$$

$$=[10\log(5\times 3)-10\log(4\times 2)]-\frac{25}{2}$$
$$[\log 3-\log 5-\log 2+\log 4]$$

$$=[10\log 5+10\log 3-10\log 4-10\log 2]-\frac{25}{2}$$
$$[\log 3-\log 5-\log 2+\log 4]$$

$$=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]$$
$$\log 3+\left[-10+\frac{25}{2}\right]\log 2$$

$$=\frac{45}{2}\log 5-\frac{45}{2}\log 4-\frac{5}{2}\log 3+\frac{5}{2}\log 2$$

$$=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in equation (i), we obtain

$$I=5-\left[\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}\right]$$

$$=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]$$ **[1½]**

Q. 54. Evaluate the definite integral $\int_0^{\frac{\pi}{4}}(2\sec^2 x+x^3+2)dx$.

[NCERT Ex. 7.9, Q. 17, Page 338]

Ans. Let, $\quad I=\int_0^{\frac{\pi}{4}}\left(2\sec^2 x+x^3+2\right)dx$

$$\int\left(2\sec^2 x+x^3+2\right)dx=2\tan x+\frac{x^4}{4}+2x$$
$$=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=F\left(\frac{\pi}{4}\right)-F(0)$$

$$=\left\{\left(2\tan\frac{\pi}{4}+\frac{1}{4}\left(\frac{\pi}{4}\right)^4+2\left(\frac{\pi}{4}\right)\right)-(2\tan 0+0+0)\right\}$$

$$=2\tan\frac{\pi}{4}+\frac{\pi^4}{4^5}+\frac{\pi}{2}$$

$$=2+\frac{\pi}{2}+\frac{\pi^4}{1024}$$ **[3]**

Q. 55. Evaluate the definite integral

$$\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx.$$

[NCERT Ex. 7.9, Q. 18, Page 338]

Ans. Let,

$$I=\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$$

$$=-\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)dx$$

$$=-\int_0^{\pi}\cos x\,dx$$

$$\int\cos x dx=\sin x=F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(\pi) - F(0)$

$= \sin\pi - \sin 0$

$= 0$ **[3]**

Q. 56. Evaluate the definite integral $\int_0^2 \frac{6x+3}{x^2+4}dx$.

[NCERT Ex. 7.9, Q. 19, Page 338]

Ans. Let,

$$\int_0^2 \frac{6x+3}{x^2+4}dx$$

$$\int\frac{6x+3}{x^2+4}dx=3\int\frac{2x+1}{x^2+4}dx$$

$$=3\int\frac{2x}{x^2+4}dx+3\int\frac{1}{x^2+4}dx$$

$$=3\log\left(x^2+4\right)+\frac{3}{2}\tan^{-1}\frac{x}{2}=F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(2) - F(0)$

$$=\left\{3\log\left(2^2+4\right)+\frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\}$$
$$-\left\{3\log(0+4)+\frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$ **[3]**

Q. 57. Evaluate the definite integral $\int_0^1\left(xe^x + \sin\frac{\pi x}{4}\right)dx$.

[NCERT Ex. 7.9, Q. 20, Page 338]

Ans. Let,

$$I = \int_0^1\left(xe^x + \sin\frac{\pi x}{4}\right)dx$$

$$I = \int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx + \left\{\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right\}$$

$$= xe^x - \int e^x dx - \frac{4}{\pi}\cos\frac{\pi x}{4}$$

$$= xe^x - e^x - \frac{4}{\pi}\cos\frac{\pi x}{4}$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$$= \left(1\cdot e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4}\right) - \left(0\cdot e^0 - e^0 - \frac{4}{\pi}\cos 0\right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$ **[3]**

Q. 58. Prove : $\int_0^1 xe^x dx = 1$.

[NCERT Misc. Ex. Q. 35, Page 353]

Ans. Let,

$$I = \int_0^1 xe^x dx$$

Integrating by parts, we obtain

$$I = x\int_0^1 e^x dx - \int_0^1\left\{\left(\frac{d}{dx}(x)\right)\int e^x dx\right\}dx$$

$$= \left[xe^x\right]_0^1 - \left[e^x\right]_0^1$$

$$= e - e + 1$$

$$= 1$$

Hence, the given result is proved. **[3]**

Q. 59. Prove : $\int_{-1}^1 x^{17}\cos^4 x\, dx = 0$

[NCERT Misc. Ex. Q. 36, Page 353]

Ans. Let,

$$I = \int_{-1}^1 x^{17}\cos^4 x\, dx$$

Also, let $f(x) = x^{17}\cos^4 x$

$\Rightarrow \quad f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

Therefore, $f(x)$ is an odd function.

It is known that if $f(x)$ is an odd function, then $\int_{-a}^a f(x)dx = 0$

$\therefore \quad I = \int_{-1}^1 x^{17}\cos^4 x\, dx = 0$

Hence, the given result is proved. **[3]**

Q. 60. Prove : $\int_0^{\frac{\pi}{2}}\sin^3 x\, dx = \frac{2}{3}$.

[NCERT Misc. Ex. Q. 37, Page 353]

Ans. Let,

$$I = \int_0^{\frac{\pi}{2}}\sin^3 x\, dx$$

$$I = \int_0^{\frac{\pi}{2}}\sin^2 x\cdot\sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}}(1-\cos^2 x)\sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}}\sin x\, dx - \int_0^{\frac{\pi}{2}}\cos^2 x\cdot\sin x\, dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved. **[3]**

Q. 61. Prove : $\int_0^{\frac{\pi}{4}} 2\tan^3 x\, dx = 1 - \log 2$.

[NCERT Misc. Ex. Q. 38, Page 353]

Ans. Let,

$$I = \int_0^{\frac{\pi}{4}} 2\tan^3 x\, dx$$

$$I = 2\int_0^{\frac{\pi}{4}}\tan^2 x\tan x\, dx = 2\int_0^{\frac{\pi}{4}}(\sec^2 x - 1)\tan x\, dx$$

$$= 2\int_0^{\frac{\pi}{4}}\sec^2 x\tan x\, dx - 2\int_0^{\frac{\pi}{4}}\tan x\, dx$$

$$= 2\left[\frac{\tan^2 x}{2}\right]_0^{\frac{\pi}{4}} + 2[\log\cos x]_0^{\frac{\pi}{4}}$$

$$= 1 + 2\left[\log\cos\frac{\pi}{4} - \log\cos 0\right]$$

$$= 1 + 2\left[\log\frac{1}{\sqrt{2}} - \log 1\right]$$

$$= 1 - \log 2 - \log 1 = 1 - \log 2$$

Hence, the given result is proved. **[3]**

Q. 62. Prove : $\int_0^1 \sin^{-1}x\, dx = \frac{\pi}{2} - 1$.

[NCERT Misc. Ex. Q. 39, Page 353]

Ans. Let,

$$I = \int_0^1 \sin^{-1}x\, dx$$

$$\therefore I = \int_0^1 \sin^{-1}x\cdot 1\cdot dx$$

Integrating by parts, we obtain

$$I = \left[\sin^{-1}x\cdot x\right]_0^1 - \int_0^1\frac{1}{\sqrt{1-x^2}}\cdot x\, dx$$

$$= \left[x\sin^{-1}x\right]_0^1 + \frac{1}{2}\int_0^1\frac{(-2x)}{\sqrt{1-x^2}}dx$$

Let $1 - x^2 = t$

$\Rightarrow -2x\, dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 0$

$$I=\left[x\sin^{-1}x\right]_0^1+\frac{1}{2}\int_1^0\frac{dt}{\sqrt{t}}$$

$$=\left[x\sin^{-1}x\right]_0^1+\frac{1}{2}\left[2\sqrt{t}\right]_1^0$$

$$=\sin^{-1}(1)+\left[-\sqrt{1}\right]$$

$$=\frac{\pi}{2}-1$$

Hence, the given result is proved. **[3]**

Q. 63. Integrate the function $\sqrt{x^2+3x}$.

[NCERT Ex. 7.7, Q. 8, Page 330]

Ans. Let,

$$I=\int\sqrt{x^2+3x}\,dx$$

$$=\int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$

$$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is know that,

$$\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$\therefore I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

[3]

Q. 64. Integrate the function $\sqrt{1+\frac{x^2}{9}}$.

[NCERT Ex. 7.7, Q. 9, Page 330]

Ans. Let,

$$I=\int\sqrt{1+\frac{x^2}{9}}dx$$

$$=\frac{1}{3}\int\sqrt{9+x^2}dx$$

$$=\frac{1}{3}\int\sqrt{(3)^2+x^2}dx$$

It is known that,

$$\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

[3]

Q. 65. Integrate the function x sin x

[NCERT Ex. 7.6, Q. 1, Page 327]

Ans. Let,

$$I=\int x\sin x\,dx$$

Taking x as first function and sin x as second function and integrating by parts, we obtain

$$I=x\int\sin x\,dx-\int\left\{\left(\frac{d}{dx}x\right)\int\sin x\,dx\right\}dx$$

$$=x(-\cos x)-\int 1\cdot(-\cos x)dx$$

$$=-x\cos x+\sin x+C$$

[3]

Q. 66. Integrate the function x sin $3x$

[NCERT Ex. 7.6, Q. 2, Page 327]

Ans. Let,

$$I=\int x\sin 3x\,dx$$

Taking x as first function and sin $3x$ as second function and integrating by parts, we obtain

$$I=x\int\sin 3x\,dx-\int\left\{\left(\frac{d}{dx}x\right)\int\sin 3x\,dx\right\}dx$$

$$=x\left(\frac{-\cos 3x}{3}\right)-\int 1\cdot\left(\frac{-\cos 3x}{3}\right)dx$$

$$=\frac{-x\cos 3x}{3}+\frac{1}{3}\int\cos 3x\,dx$$

$$=\frac{-x\cos 3x}{3}+\frac{1}{9}\sin 3x+C$$

[3]

Q. 67. Integrate the function x^2e^x.

[NCERT Ex. 7.6, Q. 3, Page 327]

Ans. Let,

$$I=\int x^2e^xdx$$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I=x^2\int e^xdx-\int\left\{\left(\frac{d}{dx}x^2\right)\int e^xdx\right\}dx$$

$$=x^2e^x-\int 2x\cdot e^xdx$$

$$=x^2e^x-2\int x\cdot e^xdx$$

Again integrating by parts, we obtain

$$=x^2e^x-2\left[x.\int e^xdx-\int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^xdx\right\}dx\right]$$

$$=x^2e^x-2\left[xe^x-\int e^xdx\right]$$

$$=x^2e^x-2\left[xe^x-e^x\right]$$

$$=x^2e^x-2xe^x+2e^x+C$$

$$=e^x\left(x^2-2x+2\right)+C$$

[3]

Q. 68. Integrate the function x log x.

[NCERT Ex. 7.6, Q. 4, Page 327]

Ans. Let,

$$I=\int x\log x dx$$

Taking log x as first function and x as second function and integrating by parts, we obtain

$$I=\log x\int x\,dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x\,dx\right\}dx$$

$$=\log x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\log x}{2}-\int\frac{x}{2}dx$$

$$=\frac{x^2\log x}{2}-\int\frac{x}{2}dx$$

$$=\frac{x^2\log x}{2}-\frac{x^2}{4}+C$$

[3]

Q. 69. Integrate the function x log $2x$.

[NCERT Ex. 7.6, Q. 5, Page 327]

Ans. Let,

$$I=\int x\log 2x\,dx$$

Taking log $2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\,dx - \int\left\{\left(\frac{d}{dx}\log 2x\right)\int x\,dx\right\}dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2}dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2}dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$ **[3]**

Q. 70. Integrate the function $x^2 \log x$.
[NCERT Ex. 7.6, Q. 6, Page 327]

Ans. Let,

$$I = \int x^2 \log x\,dx$$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\,dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x^2\,dx\right\}dx$$

$$= \log x\left(\frac{x^3}{2}\right) - \int \frac{1}{x}\cdot\frac{x^3}{3}dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3}dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$ **[3]**

Q. 71. Integrate the function $x \sin^{-1} x$.
[NCERT Ex. 7.6, Q. 7, Page 327]

Ans. Let,

$$I = \int x \sin^{-1} x\,dx$$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\sin^{-1} x\right)\int x\,dx\right\}dx$$

$$= \sin^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int\left\{\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int\left\{\sqrt{1-x^2}dx - \int \frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x - \sin^{-1} x\right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1} x - \frac{1}{2}\sin^{-1} x + C$$

$$= \frac{1}{4}\left(2x^2-1\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + C$$ **[3]**

Q. 72. Integrate the function $x \tan^{-1} x$.
[NCERT Ex. 7.6, Q. 8, Page 327]

Ans. Let,

$$I = \int x \tan^{-1} x dx$$

Taking $\tan^{-1}$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\tan^{-1} x\right)\int x\,dx\right\}dx$$

$$= \tan^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + C$$ **[3]**

Q. 73. Integrate the function $x \cos^{-1}$ x.
[NCERT Ex. 7.6, Q. 9, Page 327]

Ans. Let,

$$I = \int x \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \cos^{-1} x \int x\,dx - \int\left\{\left(\frac{d}{dx}\cos^{-1} x\right)\int x\,dx\right\}dx$$

$$= \cos^{-1} x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \sqrt{1-x^2}dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1} x \quad \text{...(i)}$$ **[1½]**

where, $I_1 = \int \sqrt{1-x^2}dx$

$$\Rightarrow I_1 = \sqrt{1-x^2}\int 1dx - \int \frac{d}{dx}\sqrt{1-x^2}\int 1dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}}\cdot x\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{\int \sqrt{1-x^2}dx + \int \frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{I_1 + \cos^{-1} x\right\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1} x$$

$$\therefore \quad I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1} x$$

Substituting in equation (i), we obtain

$$I = \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x\right) - \frac{1}{2}\cos^{-1}x$$

$$= \frac{(2x^2-1)}{4}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + C$$ **[1½]**

Q. 74. Integrate the function $(\sin^{-1}x)^2$.

[NCERT Ex. 7.6, Q. 10, Page 327]

Ans. Let,

$$I = \int(\sin^{-1}x)^2 \cdot 1\,dx$$

Taking $(\sin^{-1}x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\sin^{-1}x)^2\int 1dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)^2 \cdot \int 1\cdot dx\right\}dx$$

$$= (\sin^{-1}x)^2 \cdot x - \int\frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot x\,dx$$

$$= x(\sin^{-1}x)^2 + \int\sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$

$$= x(\sin^{-1}x)^2 + \left[\begin{array}{l}\sin^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx \\ -\int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\end{array}\right]$$

$$= x(\sin^{-1}x)^2 + \left[\begin{array}{l}\sin^{-1}x \cdot 2\sqrt{1-x^2} \\ -\int\frac{1}{\sqrt{1-x}} \cdot 2\sqrt{1-x^2}dx\end{array}\right]$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - \int 2dx$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - 2x + C$$ **[3]**

Q. 75. Integrate the function $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$.

[NCERT Ex. 7.6, Q. 11, Page 327]

Ans. Let,

$$I = \int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$$

$$= \frac{-1}{2}\int\frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}xdx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\begin{array}{l}\left(\frac{d}{dx}\cos^{-1}x\right) \\ \int\frac{-2x}{\sqrt{1-x^2}}dx\end{array}\right\}dx\right]$$

$$= \frac{-1}{2}\left[\cos^{-1}x \cdot 2\sqrt{1-x^2} - \int\frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + \int 2dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + 2x\right] + C$$

$$= -\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + C$$ **[3]**

Q. 76. Integrate the function $x\sec^2 x$.

[NCERT Ex. 7.6, Q. 12, Page 327]

Ans. Let,

$$I = \int\sec^2 x$$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x\int\sec^2 x\,dx - \int\left\{\left\{\frac{d}{dx}x\right\}\int\sec^2 x\,dx\right\}dx$$

$$= x\tan x - \int 1\cdot\tan xdx$$

$$= x\tan x + \log|\cos x| + C$$ **[3]**

Q. 77. Integrate the function $\tan^{-1}x$.

[NCERT Ex. 7.6, Q. 13, Page 327]

Ans. Let $I = \int 1\cdot\tan^1 x\,dx$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1}x\int 1dx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$$

$$= \tan^{-1}x \cdot x - \int\frac{1}{1+x^2} \cdot x\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$= x\tan^{-1}x - \frac{1}{2}\log|1+x^2| + C$$

$$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$$ **[3]**

Q. 78. Integrate the function $x(\log x)^2$.

[NCERT Ex. 7.6, Q. 14, Page 327]

Ans. $I = \int x(\log x)^2 dx$

Taking $(\log x)^2$ as first function and x as second function and integrating by parts, we obtain

$$I = (\log x)^2\int xdx - \int\left[\left\{\frac{d}{dx}(\log x)^2\right\}\int xdx\right]dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\int 2\log x \cdot \frac{1}{x} \cdot \frac{x^2}{2}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \int x\log x\,dx$$

Again integrating by parts, we obtain

$$I = \frac{x^2}{2}(\log x)^2 - \left[\log x\int x\,dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x\,dx\right\}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2}\log x - \int\frac{1}{x} \cdot \frac{x^2}{2}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$ **[3]**

Q. 79. Integrate the function $(x^2 + 1)\log x$.

[NCERT Ex. 7.6, Q. 15, Page 327]

Ans. Let,

$$I = \int(x^2+1)\log x\,dx$$

$$= \int x^2\log x\,dx + \int\log x\,dx$$

Let $I = I_1 + I_2$ ...(i)

Where, $I_1 = \int x^2\log x\,dx$ and $I_2 = \int\log x\,dx$

$$I_1 = \int x^2\log x\,dx$$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$I_1 = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right) \qquad \ldots\text{(ii) [1½]}$$

$$I_2 = \int \log x \, dx$$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2 \qquad \ldots\text{(iii)}$$

Using equations (ii) and (iii) in equation (i), we obtain

$$I_2 = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

[1½]

Q. 80. Evaluate the integral using substitution :

$$\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx.$$

[NCERT Ex. 7.10, Q. 3, Page 340]

Ans. Let,

$$I = \int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2 \theta d\theta$

When $x = 0, \theta = 0$ and when $x = 1, \theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1} \left(\frac{2\tan\theta}{1+\tan^2\theta} \right) \sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta) \sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta \, d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta \, d\theta$$

Taking θ as first function and $\sec^2 \theta$ as second function and integrating by pats, we obtain

$$I = 2\left[\theta \int \sec^2\theta \, d\theta - \int \left\{ \left(\frac{d}{dx}\theta \right) \int \sec^2\theta \, d\theta \right\} d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta - \int \tan\theta \, d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan\theta + \log|\cos\theta| \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} \tan\frac{\pi}{4} + \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0| \right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}} \right) - \log 1 \right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2 \right]$$

$$= \frac{\pi}{2} - \log 2$$

[3]

Q. 81. Evaluate the integral using substitution :

$$\int_0^2 x\sqrt{x+2} \qquad \textbf{(Put } x+2 = t^2\textbf{).}$$

[NCERT Ex. 7.10, Q. 4, Page 340]

Ans. Let,

$$x + 2 = t^2$$

$$\Rightarrow \quad dx = 2t dt$$

When $x = 0, t = \sqrt{2}$ and when $x = 2, t = 2$

$$\therefore \int_0^2 x\sqrt{x+2} dx = \int_{\sqrt{2}}^2 (t^2 - 2)\sqrt{t^2} \, 2t dt$$

$$= 2\int_{\sqrt{2}}^2 (t^2 - 2) t^2 dt$$

$$= 2\int_{\sqrt{2}}^2 (t^4 - 2t^2) dt$$

$$= 2\left[\frac{t^5}{5} - \frac{2t^3}{3} \right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3} \right]$$

$$= 2\left[\frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15} \right]$$

$$= 2\left[\frac{16 + 8\sqrt{2}}{15} \right]$$

$$= \frac{16(2+\sqrt{2})}{15}$$

$$= \frac{16\sqrt{2}(\sqrt{2}+1)}{15}$$

[3]

Q. 82. Integrate the function $\frac{1}{\sqrt{x+a}+\sqrt{x+b}}$**.**

[NCERT Misc. Ex. Q. 2, Page 352]

Ans.

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}} = \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$= \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}$$

$$= \frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)}{a-b}$$

$$\Rightarrow \int \frac{1}{\sqrt{x+a}-\sqrt{x+b}} dx = \frac{1}{a-b} \int \left(\sqrt{x+a} - \sqrt{x+b} \right) dx$$

$$= \frac{1}{(a-b)} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

$$= \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + C$$

[3]

Q. 83. Integrate the functions

$$\frac{1}{x\sqrt{ax-x^2}} \qquad \left[\textbf{Hint : Put } x=\frac{a}{t}\right].$$

[NCERT Misc. Ex. Q. 3, Page 352]

Ans. Let,

$$x=\frac{a}{t}$$

$$\Rightarrow dx=-\frac{a}{t^2}dt$$

$$\int\frac{1}{x\sqrt{ax-x^2}}dx=\int\frac{1}{\frac{a}{t}\sqrt{a\cdot\frac{a}{t}-\left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}dt\right)$$

$$=-\int\frac{1}{at}\cdot\frac{1}{\sqrt{\frac{1}{t}-\frac{1}{t^2}}}dt$$

$$=-\frac{1}{a}\int\frac{1}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}dt$$

$$=-\frac{1}{a}\int\frac{1}{\sqrt{t-1}}dt$$

$$=-\frac{1}{a}\left[2\sqrt{t-1}\right]+C$$

$$=-\frac{1}{a}\left[2\sqrt{\frac{a}{x}-1}\right]+C$$

$$=-\frac{2}{a}\left[\frac{\sqrt{a-x}}{\sqrt{x}}\right]+C$$

$$=-\frac{2}{a}\left[\sqrt{\frac{a-x}{x}}\right]+C$$

[3]

Q. 84. Integrate the function $\dfrac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$.

[NCERT Misc. Ex. Q. 4, Page 352]

Ans. Given that,

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{\frac{-3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{\frac{-3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{\frac{-3}{4}}$$

Let, $\quad \dfrac{1}{x^4}=t$

$$\Rightarrow \quad -\frac{4}{x^5}dx=dt$$

$$\Rightarrow \quad \frac{1}{x^5}dx=-\frac{dt}{4}$$

[1½]

$$\therefore\int\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=\frac{-1}{4}\int(1+t)^{\frac{-3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

[1½]

Q. 85. Integrate the function $\dfrac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$.

[NCERT Misc. Ex. Q. 5, Page 352]

Ans. Given that,

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$$

Let $\quad x=t^6$

$\Rightarrow \quad dx=6t^5\,dt$

$$\therefore\int\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx=\int\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}dx$$

$$=\int\frac{6t^5}{t^2(1+t)}dt$$

$$=6\int\frac{t^3}{(1+t)}dt$$

On dividing, we obtain

$$\int\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx=6\int\left\{\left(t^2-t+1\right)-\frac{1}{1+t}\right\}dt$$

$$=6\left[\left(\frac{t^3}{3}\right)-\left(\frac{t^2}{2}\right)+t-\log|1+t|\right]$$

$$=2x^{\frac{1}{2}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

$$=2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

[3]

Q. 86. Evaluate the integral using substitution :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx.$$ **[NCERT Ex. 7.10, Q. 5, Page 340]**

Ans. Given that,

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Let $\cos x = t$

$\Rightarrow -\sin x\, dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right] = \frac{\pi}{4}$$ **[3]**

Q. 87. By using the properties of definite integrals, evaluate the integral of $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$.

[NCERT Ex. 7.11, Q. 1, Page 347]

Ans. Let,

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \quad \ldots(i)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \quad \left(\int_0^0 f(x) dx = \int_0^0 f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x\, dx \quad \ldots(ii)$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)$$

$$\Rightarrow 2I \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$ **[3]**

Q. 88. By using the properties of definite integrals, evaluate the integral of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$

[NCERT Ex. 7.11, Q. 2, Page 347]

Ans. Given that,

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \ldots(i)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$$

$$\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}} dx \quad \ldots(ii)$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\Rightarrow 2t = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$ **[3]**

Q. 89. By using the properties of definite integrals, evaluate the integral of $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x\, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$.

[NCERT Ex. 7.11, Q. 3, Page 347]

Ans. Let,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(i)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx$$

$$\left(\int_0^a (x) dx = \int_0^a f(a-x) dx\right)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots(ii)$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$ **[3]**

Q. 90. Integrate the function $\frac{\sin^8 - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$.

[NCERT Misc. Ex. Q. 10, Page 352]

Ans.

$$\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$$

$$= \frac{\left(\sin^4 x + \cos^4 x\right)\left(\sin^4 x - \cos^4 x\right)}{\sin^2 x + \cos^2 x - \sin^2 x \cos^2 x - \sin^2 x \cos^2 x}$$

$$= \frac{\left(\sin^4 x + \cos^4 x\right)\left(\sin^2 x + \cos^2 x\right)\left(\sin^2 x - \cos^2 x\right)}{\left(\sin^2 x - \sin^2 x \cos^2 x\right) + \left(\cos^2 x - \sin^2 x \cos^2 x\right)}$$

$$= \frac{\left(\sin^4 x + \cos^4 x\right)\left(\sin^2 x - \cos^2 x\right)}{\sin^2 x\left(1 - \cos^2 x\right) + \cos^2 x\left(1 - \sin^2 x\right)}$$

$$= \frac{-\left(\sin^4 x + \cos^4 x\right)\left(\cos^2 x - \sin^2 x\right)}{\left(\sin^4 x + \cos^4 x\right)}$$

$= -\cos 2x$

$$\therefore \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx = \int -\cos 2x dx = -\frac{\sin 2x}{2} + C$$

[3]

Q. 91. Integrate the function $\frac{1}{\cos(x+a)\cos(x+b)}$.

[NCERT Misc. Ex. Q. 11, Page 352]

Ans. Given that,

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by sin (*a* – *b*), we obtain

$$\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cdot\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)} - \frac{\sin(x+b)}{\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x+a) - \tan(x+b)\right]$$ **[1½]**

$$\int \frac{1}{\cos(x+a)\cos(x+b)}dx = \frac{1}{\sin(a-b)}\int\left[\tan(x+a) - \tan(x+b)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log|\cos(x+a)| + \log|\cos(x+b)|\right] + C$$

$$= \frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C$$

[1½]

Q. 92. By using the properties of definite integrals, evaluate the integral of $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x\, dx}{\sin^5 x + \cos^5 x}$.

[NCERT Ex. 7.11, Q. 4, Page 347]

Ans. Let,

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}dx \qquad \ldots(i)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)}dx$$

$$\left(\int_0^a f(x)dx = \int_0^a (a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}dx \qquad \ldots(ii)$$

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

[3]

Q. 93. Evaluate the integral using substitution : $\int_0^2 \frac{dx}{x + 4 - x^2}$. **[NCERT Ex. 7.10, Q. 6, Page 340]**

Ans.

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-(x^2 - x - 4)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2 - x + \frac{1}{4} - \frac{1}{4} - 4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x - \frac{1}{2}\right)^2 - \frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}$$

Let $x - \frac{1}{2} = t \Rightarrow dx = dt$

When $x = 0, t = -\frac{1}{2}$ and when $x = 2, t = -\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

[1½]

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)}\log\frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t}\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log\frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \frac{\log\frac{\sqrt{17}}{2} - \frac{1}{2}}{\log\frac{\sqrt{17}}{2} + \frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log\frac{\sqrt{17}+3}{\sqrt{17}-3} - \log\frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}}\log\frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}+1}$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{7}}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{(5+\sqrt{17})(5+\sqrt{17})}{25-17}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{42+10\sqrt{17}}{8}\right)$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{21+5\sqrt{17}}{4}\right)$$ **[1½]**

Q. 94. Evaluate the integral using substitution : $\int_{-1}^{1}\frac{dx}{x^2+2x+5}$. [NCERT Ex. 7.10, Q. 7, Page 340]

Ans. $$\int_{-1}^{1}\frac{dx}{x^2+2x+5}=\int_{-1}^{1}\frac{dx}{(x^2+2x+1)+4}$$

$$=\int_{-1}^{1}\frac{dx}{(x+1)^2+(2)^2}$$

Let $x+1=t \Rightarrow dx=dt$

When $x=-1$, $t=0$ and when $x=1$, $t=2$

$$\therefore \int_{-1}^{1}\frac{dx}{(x+1)^2+(2)^2}=\int_{0}^{2}\frac{dt}{t^2+2^2}$$

$$=\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$ **[3]**

$$=\frac{1}{2}\tan^{-1}1-\frac{1}{2}\tan^{-1}0$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{8}$$

Q. 95. Evaluate the integral using substitution : $\int_{1}^{2}\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$. [NCERT Ex. 7.10, Q. 8, Page 340]

Ans. Given that,

$$\int_{1}^{2}\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x=t \Rightarrow 2dx=dt$

When $x=1$, $t=2$ and when $x=2$, $t=4$

$$\therefore \int_{1}^{2}\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx=\frac{1}{2}\int_{2}^{4}\left(\frac{2}{t}-\frac{2}{t^2}\right)e^{t}dt$$

$$=\int_{2}^{4}\left(\frac{1}{t}-\frac{1}{t^2}\right)e^{t}dt$$

Let $$\frac{1}{t}=f(t)$$

Then, $$f'(t)=-\frac{1}{t^2}$$

$$\int_{2}^{4}\left(\frac{1}{t}-\frac{1}{t^2}\right)e^{t}dt=\int_{2}^{4}e^{t}\left[f(t)+f'(t)\right]dt$$

$$=\left[e^{t}f(t)\right]_2^4$$

$$=\left[e^{t}\cdot\frac{1}{t}\right]_2^4=\frac{e^2(e^2-2)}{4}$$ **[3]**

Q. 96. Integrate the function $\frac{1}{9x^2+6x+5}$.

[NCERT Ex. 7.4, Q. 11, Page 316]

Ans. $$\int\frac{1}{9x^2+6x+5}dx=\int\frac{1}{(3x+1)^2+2^2}dx$$

Let $(3x+1)=t$

$\therefore$ $3\,dx=dt$

$$\int\frac{1}{(3x+1)^2+2^2}dx=\frac{1}{3}\int\frac{1}{t^2+2^2}dt$$

$$=\frac{1}{3\times2}\tan^{-1}\frac{t}{2}+C$$

$$=\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+C$$

[3]

Q. 97. Integrate the function $\frac{1}{\sqrt{7-6x-x^2}}$.

[NCERT Ex. 7.4, Q. 12, Page 316]

Ans. Function $7-6x-x^2$ can be written as $7-(x^2+6x+9-9)$.

Therefore,

$7-(x^2+6x+9-9)$

$16-(x^2+6x+9)$

$16-(x+3)^2$

$(4)^2-(x+3)^2$

$$\therefore \int\frac{1}{\sqrt{7-6x-x^2}}dx=\int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow$ $dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\int\frac{1}{\sqrt{(4)^2-(t)^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$

$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

[3]

Q. 98. Find the integrals of the function $\frac{1-\cos x}{1+\cos x}$.

[NCERT Ex. 7.3, Q. 8, Page 307]

Ans. $$\left[\begin{array}{l}\because 2\sin^2\frac{x}{2}=1-\cos x\\ \text{and } 2\cos^2\frac{x}{2}=1+\cos x\end{array}\right]$$

$$\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}=\left(\sec^2\frac{x}{2}-1\right)$$

$$\int\frac{1-\cos x}{1+\cos x}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$=\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C=2\tan\frac{x}{2}-x+C$$

[3]

Q. 99. Find the integrals of the function $\frac{\cos x}{1+\cos x}$.

[NCERT Ex. 7.3, Q. 9, Page 307]

Ans.

$$\left[\begin{array}{l}\because \cos x = \cos^2\dfrac{x}{2}\sin^2\dfrac{x}{2}\\ \text{and } \cos x = 2\cos^2\dfrac{x}{2}-1\end{array}\right]$$

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\int\frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx = \frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C = x - \tan\frac{x}{2} + C$$

[3]

Q. 100. Integrate the function $\dfrac{\sqrt{\tan x}}{\sin x\cos x}$.

[NCERT Ex. 7.2, Q. 34, Page 305]

Ans. Let,

$$I = \int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$$

$$= \int\frac{\sqrt{\tan x}\times\cos x}{\sin x\cos x\times\cos x}dx$$

$$= \int\frac{\sqrt{\tan x}}{\tan x\cos^2 x}dx$$

$$= \int\frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \int\frac{dt}{\sqrt{t}} = 2\sqrt{t}+C = 2\sqrt{\tan x}+C$$

[3]

Q. 101. Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\tan x\, dx}{1+m^2\tan^2 x}$.

[NCERT Exemp. Ex. 7.3, Q. 30, Page 165]

Ans. Let,

$$I = \int_0^{\pi/2}\frac{\tan x\, dx}{1+m^2\tan^2 x}dx$$

$$= \int_0^{\pi/2}\frac{\frac{\sin x}{\cos x}}{1+m^2\cdot\frac{\sin^2 x}{\cos^2 x}}dx$$

$$= \int_0^{\pi/2}\frac{\frac{\sin x}{\cos x}}{\frac{\cos^2 x+m^2\sin^2 x}{\cos^2 x}}dx$$

$$= \int_0^{\pi/2}\frac{\sin x\cos x\, dx}{1-\sin^2 x+m^2\sin^2 x}dx$$

$$= \int_0^{\pi/2}\frac{\sin x\cos x}{1-\sin^2 x(1-m^2)}dx$$

[2]

Put $\sin^2 x = t$, we have

$\Rightarrow 2\sin x.\cos x\, dx = dt$

$$I = \frac{1}{2}\int_0^1\frac{dt}{1-t(1-m^2)}$$

$$= \frac{1}{2}\left[-\log\left|1-t(1-m^2)\right|\cdot\frac{1}{1-m^2}\right]_0^1$$

$$= \frac{1}{2}\left[-\log\left|m^2\right|\cdot\frac{1}{1-m^2}\right] = \frac{2}{2}\cdot\frac{\log m}{(m^2-1)}$$

$$= \log\frac{m}{m^2-1}$$

[1]

Q. 102. Evaluate : $\displaystyle\int_0^{\pi} x\sin x\cos^2 x dx$.

[NCERT Exemp. Ex. 7.3, Q. 33, Page 165]

Ans. Let $I = \int_0^{\pi} x\sin x\cos^2 x dx$ ...(i)

and $I = \int_0^{\pi}(\pi - x)\sin(\pi - x)\cos^2(\pi - x)dx$

$\Rightarrow I = \int_0^{\pi}(\pi - x)\sin x\cos^2 x\, dx$ ...(ii)

On adding equations (i) and (ii), we get

$$2I = \int_0^{\pi}\pi\sin x\cos^2 x\, dx$$

Put $\cos x = t$

$\Rightarrow -\sin x dx = dt$

As $x \to 0$, then $t \to 1$

and $x \to \pi$, then $t \to -1$

$$\therefore\quad I = -\pi\int_1^{-1}t^2 dt \Rightarrow I = -\pi\left[\frac{t^3}{3}\right]_1^{-1}$$

$$\Rightarrow 2I = -\frac{\pi}{3}[-1-1]$$

$$\Rightarrow 2I = \frac{2\pi}{3}$$

$$\therefore\quad I = \frac{\pi}{3}$$

[3]

Q. 103. Evaluate : $\displaystyle\int_1^2\frac{dx}{\sqrt{(x-1)(2-x)}}$.

[NCERT Exemp. Ex. 7.3, Q. 31, Page 165]

Ans. Let,

$$I = \int_1^2\frac{dx}{\sqrt{(x-1)(2-x)}} = \int_1^2\frac{dx}{\sqrt{2x-x^2-2+x}}$$

$$= \int_1^2\frac{dx}{\sqrt{-(x^2-3x+2)}}$$

[1]

$$= \int_1^2\frac{dx}{\sqrt{-\left[x^2-2\cdot\frac{3}{2}x+\left(\frac{3}{2}\right)^2+2-\frac{9}{4}\right]}}$$

$$= \int_1^2\frac{dx}{\sqrt{-\left\{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2\right\}}}$$

$$= \int_1^2\frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{1}{2}}\right)\right]_1^2$$

$$= \left[\sin^{-1}(2x-3)\right]_1^2$$
$$= \sin^{-1} - \sin^{-1}(-1)$$
$$= \frac{\pi}{2} + \frac{\pi}{2} \qquad \left[\because \sin\frac{\pi}{2} = 1 \text{ and } \sin(-\theta) = -\sin\theta\right]$$
$$= \pi$$

[2]

Q. 104. Verify : $\int \frac{2x-1}{2x+3}dx = x - \log\left|(2x+3)^2\right| + C$.

[NCERT Exemp. Ex. 7.3, Q. 1, Page 163]

Ans. Let,

$$I = \int \frac{2x-1}{2x+3}dx$$
$$= \int \frac{2x+3-3-1}{2x+3}dx$$
$$= \int 1dx - 4\int \frac{1}{2x+3}dx = x - \int \frac{4}{2\left(x+\frac{3}{2}\right)}dx$$
$$= x - 2\log\left|\left(x+\frac{3}{2}\right)\right|C' = x - 2\log\left|\left(\frac{2x+3}{2}\right)\right| + C'$$
$$= x - 2\log\left|(2x+3)\right| + 2\log 2 + C'$$
$$\left[\because \log\frac{m}{n} = \log m - \log n\right]$$
$$= x - \log\left|(2x+3)^2\right| + C'$$
$$[\because C = 2\log 2 + C']$$

[3]

Q. 105. Evaluate : $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}dx$.

[NCERT Exemp. Ex. 7.3, Q. 4, Page 163]

Ans. Let,

$$I = \int\left(\frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\right)dx$$
$$= \int\left(\frac{e^{\log x^6} - e^{\log x^5}}{e^{\log x^4} - e^{\log x^3}}\right)dx \qquad \left[\because a\log b = \log b^a\right]$$
$$= \int\left(\frac{x^6 - x^5}{x^4 - x^3}\right)dx \qquad \left[\because e^{\log x} = x\right]$$
$$= \int\left(\frac{x^3 - x^2}{x-1}\right)dx = \int \frac{x^2(x-1)}{x-1}dx$$
$$= \int x^2 dx = \frac{x^3}{3} + C$$

[3]

Q. 106. Evaluate : $\int \frac{x}{\sqrt{x}+1}dx$ $\left(\textbf{Hint : Put } \sqrt{x} = z\right)$.

[NCERT Exemp. Ex. 7.3, Q. 10, Page 164]

Ans. Let,

$$I = \int \frac{x}{\sqrt{x}+1}dx$$

Put $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}dx = dt$$
$$dx = 2\sqrt{x}dt = 2tdt$$
$$I = 2\int \frac{t^2 \cdot t}{t+1}dt = 2\int \frac{t^3}{t+1}dt$$
$$= 2\int \frac{t^3+1-1}{t+1}dt = 2\int \frac{(t+1)(t^2-t+1)}{t+1}dt - 2\int \frac{1}{t+1}dt$$
$$= 2\int (t^2-t+1)dt - 2\int \frac{1}{t+1}dt$$
$$= 2\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + C$$
$$= 2\left[\frac{x\sqrt{x}}{3} - \frac{x}{2} + \sqrt{x} - \log\left|\sqrt{x}+1\right|\right] + C$$

[3]

Q. 107. Evaluate : $\int \sqrt{\frac{a+x}{a-x}}$.

[NCERT Exemp. Ex. 7.3, Q. 11, Page 164]

Ans. Let,

$$I = \int \sqrt{\frac{a+x}{a-x}}dx$$

Put $x = a\cos 2\theta$

$\Rightarrow dx = -a\sin 2\theta\, 2d\theta$

$$\therefore I = -2\int \sqrt{\frac{a+a\cos 2\theta}{a-a\cos 2\theta}} \cdot a\sin 2\theta d\theta$$
$$\left[\because \cos 2\theta = \frac{x}{a} \Rightarrow 2\theta = \cos^{-1}\frac{x}{a} \Rightarrow \theta = \frac{1}{2}\cos^{-1}\frac{x}{a}\right]$$
$$= -2a\int \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\sin 2\theta d\theta = -2a\int \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}\sin 2\theta d\theta$$
$$= -2a\int \cot\theta \cdot \sin 2\theta d\theta = -2a\int \frac{\cos\theta}{\sin\theta} \cdot 2\sin\theta \cdot \cos\theta d\theta$$
$$= -4a\int \cos^2\theta d\theta = -2a\int (1+\cos 2\theta)d\theta$$
$$= -2a\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$
$$= -2a\left[\frac{1}{2}\cos^{-1}\frac{x}{a} + \frac{1}{2}\sqrt{1-\frac{x^2}{a^2}}\right] + C$$
$$= -a\left[\cos^{-1}\left(\frac{x}{a}\right) + \sqrt{1-\frac{x^2}{a^2}}\right] + C$$

[3]

Q. 108. Evaluate : $\int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}}dx$ $\left(\textbf{Hint : Put } x = z^4\right)$.

[NCERT Exemp. Ex. 7.3, Q. 12, Page 164]

Ans. Let,

$$I = \int \frac{x^{1/2}}{1+x^{3/4}}dx$$

Put $x = t^4$

$\Rightarrow dx = 4t^3$dt

$$I = 4\int \frac{t^2(t^3)}{1+t^3}dt = 4\int\left(t^2 - \frac{t^2}{1+t^3}\right)dt$$
$$I = 4\int t^2 dt - 4\int \frac{t^2}{1+t^3}dt$$
$$I = I_1 - I_2$$
$$I_1 = 4\int t^2 dt = 4\cdot\frac{t^3}{3} + C_1 = \frac{4}{3}x^{3/4} + C_1$$

Now, $I_2 = 4\int \frac{t^2}{1+t^3}dt$

Again, put $1+t^3 = z$

$$3t^2dt = dz$$

$$t^2dt = \frac{1}{3}dz = \frac{4}{3}\int \frac{1}{z}dz$$

$$= \frac{4}{3}\log|z| + C_2 = \frac{4}{3}\log\left|1+t^3\right| + C_2$$

$$= \frac{4}{3}\log\left|1+x^{3/4}\right| + C_2$$

$$I = \frac{4}{3}x^{3/4} + C_1 - \frac{4}{3}\log\left|(1+x^{3/4})\right| - C_2$$

$$= \frac{4}{3}x^{3/4} - \log(1+x^{3/4}) + C$$

$$[\because C = C_1 - C_2]$$

[3]

Q. 109. Evaluate : $\int \frac{dt}{\sqrt{3t-2t^2}}$.

[NCERT Exemp. Ex. 7.3, Q. 15, Page 164]

Ans.

$$I = \int \frac{dt}{\sqrt{3t-2t^2}} = \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{-\left(t^2 - \frac{3}{2}t\right)}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{-\left[\left(t^2 - 2\cdot\frac{1}{2}\cdot\frac{3}{2}t\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{-\left[\left(t-\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]}} = \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2 - \left(t-\frac{3}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t-\frac{3}{4}}{\frac{3}{4}}\right) + C = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4t-3}{3}\right) + C$$

[3]

Q. 110. Evaluate : $\int \frac{3x-1}{\sqrt{x^2+9}}dx$.

[NCERT Exemp. Ex. 7.3, Q. 16, Page 164]

Ans. Let,

$$I = \int \frac{3x-1}{\sqrt{x^2+9}}dx$$

$$= \int \frac{3x}{\sqrt{x^2+9}}dx - \int \frac{1}{\sqrt{x^2+9}}dx$$

$$I = I_1 - I_2$$

Now, $I_1 = \int \frac{3x}{\sqrt{x^2+9}}$

Put $x^2+9 = t^2$

$$2xdx = 2tdt$$

$$xdx = tdt$$

$$I_1 = 3\int \frac{t}{t}dt$$

$$= 3\int dt = 3t + C_1 = 3\sqrt{x^2+9} + C_1$$

and $I_2 = \int \frac{1}{\sqrt{x^2+9}}dx = \int \frac{1}{\sqrt{x^2+(3)^2}}dx$

$$= \log\left|x+\sqrt{x^2+9}\right| + C_2$$

$$I = 3\sqrt{x^2+9} + C_1 - \log\left|x+\sqrt{x^2+9}\right| - C_2$$

$$= 3\sqrt{x^2+9} - \log\left|x+\sqrt{x^2+9}\right| + C$$

$$[\because C = C_1 - C_2]$$

[3]

Q. 111. Integrate the rational function $\frac{x}{(x+1)(x+2)}$.

[NCERT Ex. 7.5, Q. 1, Page 322]

Ans. Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow$ $x = A(x+2) + B(x+1)$

Equation the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C$$

$$= \log\frac{(x+2)^2}{(x+1)} + C$$

[3]

Q. 112. Integrate the rational function $\frac{1}{x^2-9}$.

[NCERT Ex. 7.5, Q. 2, Page 322]

Ans. Let,

$$\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$$

$$1 = A(x-3) + B(x+3)$$

Equation the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$$A = \frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)}dx = \int\left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)}\right)dx$$ **[3]**

$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + C$$

$$= \frac{1}{6}\log\left|\frac{(x-3)}{(x+3)} + C\right|$$

Q. 113. Integrate the rational function $\frac{3x-1}{(x-1)(x-2)(x-3)}$.

[NCERT Ex. 7.5, Q. 3, Page 322]

Ans. Let,

$$\frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$$

$3x - 1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ...(i)

Substituting $x = 1, 2$ and respectively in equation (i), we obtain

$A = 1, B = -5$ and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}$$

$$\Rightarrow \int\frac{3x-1}{(x-1)(x-2)(x-3)}dx$$

$$=\int\left\{\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}\right\}dx$$

$$=\log|x-1|-5\log|x-2|+4\log|x-3|+C$$ **[3]**

Q. 114. Integrate the rational function $\frac{x}{(x-1)(x-2)(x-3)}$.

[NCERT Ex. 7.5, Q. 4, Page 322]

Ans. Let,

$$\frac{x}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$$

$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ...(i)

Substituting $x = 1, 2$ and 3 respectively in equation (i), we obtain

$A=\frac{1}{2}, B=-2$ and $C=\frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}$$

$$\Rightarrow \int\frac{x}{(x-1)(x-2)(x-3)}dx$$

$$=\int\left\{\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}\right\}dx$$

$$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$$ **[3]**

Q. 115. Integrate the rational function $\frac{2x}{x^2+3x+2}$.

[NCERT Ex. 7.5, Q. 5, Page 322]

Ans. Let,

$$\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$$

$2x = A(x + 2) + B(x + 1)$ ...(i)

Substituting $x = -1$ and -2 in equation (i), we obtain

$A = -2$ and $B = 4$

$$\therefore \quad \frac{2x}{(x+1)(x+2)}=\frac{-2}{(x+1)}+\frac{4}{(x+2)}$$

$$\Rightarrow \int\frac{2x}{(x+1)(x+2)}dx=\int\left\{\frac{4}{(x+2)}-\frac{2}{(x+1)}\right\}dx$$

$$=4\log|x+2|-2\log|x+1|+C$$ **[3]**

Q. 116. Integrate the rational function $\frac{1-x^2}{x(1-2x)}$.

[NCERT Ex. 7.5, Q. 6, Page 322]

Ans. It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let,

$$\frac{2-x}{x(1-2x)}=\frac{A}{x}+\frac{B}{(1-2x)}$$

$$\Rightarrow (2-x)=A(1-2x)+Bx \text{ ...(i)}$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (i), we obtain

$A = 2$ and $B = 3$

$$\therefore \frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{1-2x}$$ **[1½]**

Substituting in equation (i), we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left\{\frac{2}{x}+\frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{1-2x}\right)\right\}dx$$

$$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$$ **[1½]**

Q. 117. Integrate the rational function $\frac{x}{(x^2+1)(x-1)}$.

[NCERT Ex. 7.5, Q. 7, Page 322]

Ans. Let,

$$\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-1)} \quad \text{...(i)}$$

$x = (Ax + B)(x - 1) + C(x^2 + 1)$

$x = Ax^2 - Ax + Bx - B + Cx^2 + C$

Equation the coefficients of x^2, x and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A=-\frac{1}{2}, B=\frac{1}{2}$ and $C=\frac{1}{2}$

From equation (i), we obtain

$$\frac{x}{(x^2+1)(x-1)}=\frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1}+\frac{\frac{1}{2}}{(x-1)}$$

$$\int\frac{x}{(x^2+1)(x-1)}=-\frac{1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=-\frac{1}{4}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$ **[1½]**

Consider $\int\frac{2x}{x^2+1}dx$, let $(x^2+1)=t \rightarrow 2xdx=dt$

$$\Rightarrow \int\frac{2x}{x^2+1}dx=\int\frac{dt}{t}=\log|t|=\log|x^2+1|$$

$$\therefore \int\left[\frac{x}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C\right]$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C \quad \textbf{[1½]}$$

Q. 118. Integrate the rational function $\dfrac{x}{(x-1)^2(x+2)}$.

[NCERT Ex. 7.5, Q. 8, Page 322]

Ans. Let,

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B = \frac{1}{3}$$

Equation the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int\frac{x}{(x-1)^2(x+2)}dx$$

$$= \frac{2}{9}\int\frac{1}{(x-1)}dx + \frac{1}{3}\int\frac{1}{(x-1)^2}dx - \frac{2}{9}\int\frac{1}{(x+2)}dx \quad \textbf{[1½]}$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C \quad \textbf{[½]}$$

Q. 119. Integrate the rational function $\dfrac{3x+5}{x^3-x^2-x+1}$.

[NCERT Ex. 7.5, Q 9 , Page 322]

Ans. Given that,

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

Let,

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$3x + 5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$3x + 5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$ ...(i)

Substituting $x = 1$ in equation (i), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$ **[1½]**

On solving, we obtain

$A = -\frac{1}{2}$ and $C = \frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\Rightarrow \int\frac{3x+5}{(x-1)^2(x+1)}dx$$

$$= -\frac{1}{2}\int\frac{1}{x-1}dx + 4\int\frac{1}{(x-1)^2}dx + \frac{1}{2}\int\frac{1}{(x+1)}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + C$$

[1½]

Q. 120. Integrate the rational function $\dfrac{2x-3}{(x^2-1)(2x+3)}$.

[NCERT Ex. 7.5, Q. 10, Page 322]

Ans. Given that,

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let,

$$\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$$

$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$

$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$

$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$

Equating the coefficients of x^2 and x, we obtain

$B = -\frac{1}{10}$, $A = \frac{5}{2}$ and $C = -\frac{24}{5}$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$$

$$\Rightarrow \int\frac{2x-3}{(x^2-1)(2x+3)}dx = \frac{5}{2}\int\frac{1}{(x+1)}dx - \frac{1}{10}\int\frac{1}{x-1}dx - \frac{24}{5}\int\frac{1}{(2x+3)}dx$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3|$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

[3]

Q. 121. Integrate the rational function $\dfrac{5x}{(x+1)(x^2-4)}$.

[NCERT Ex. 7.5, Q. 11, Page 322]

Ans. Given that,

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let,

$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$$

$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2)$ ...(i)

Substituting $x = -1, -2$ and 2 respectively in equation (i), we obtain

$A = -\frac{5}{3}, B = -\frac{5}{2}$ and $C = -\frac{5}{6}$

$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} - \frac{5}{6(x-2)}$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)}dx = \frac{5}{3}\int \frac{1}{(x+1)}dx - \frac{5}{2}\int \frac{1}{(x+2)}dx + \frac{5}{6}\int \frac{1}{(x-2)}dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

[3]

Q. 122. Integrate the rational function $\frac{x^3+x+1}{x^2-1}$.

[NCERT Ex. 7.5, Q. 12, Page 322]

Ans. It can be seen that given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$

Let,

$\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$2x + 1 = A(x-1) + B(x+1)$ ...(i)

Substituting $x = 1$ and -1 in equation (i), we obtain

$A = \frac{1}{2}$ and $B = \frac{3}{2}$

$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx = \int x dx + \frac{1}{2}\int \frac{1}{(x+1)}dx + \frac{3}{2}\int \frac{1}{(x-1)}dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

[3]

Q. 123. Integrate the rational function $\frac{2}{(1-x)(1+x^2)}$.

[NCERT Ex. 7.5, Q. 13, Page 322]

Ans. Let,

$\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

$2 = A(1 + x^2) + (Bx + C)(1 - x)$

$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

Equating the coefficients of x^2, x and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1, B = 1$ and $C = 1$

$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$$

$$= -\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{2x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$$

[3]

Q. 124. Integrate the rational function $\frac{3x-1}{(x+2)^2}$.

[NCERT Ex. 7.5, Q. 14, Page 322]

Ans. Let,

$\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$

$\Rightarrow 3x - 1 = A(x + 2) + B$

Equation the coefficients of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}dx = 3\int \frac{1}{(x+2)}dx - 7\int \frac{1}{(x+2)^2}dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{7}{(x+2)} + C$$

[3]

Q. 125. Integrate the rational function $\frac{1}{x^4-1}$.

[NCERT Ex. 7.5, Q. 15, Page 322]

Ans. Given that,

$\frac{1}{(x^4-1)} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x+1)(x-1)(1+x^2)}$

Let,

$\frac{1}{(x+1)(x-1)(1+x^2)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)}$

$1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1)$

$1 = A(x^3 + x - x^2 - 1) + B(x^3 + x + x^2 + 1) + Cx^3 + Dx^2 - Cx - D$

$1 = (A + B + C)x^3 + (-A + B + D)x^2 + (A + B - C)x + (-A + B - D)$

Equating the coefficients of x^3, x^2, x and constant term, we obtain

$A + B + C = 0$

$-A + B + D = 0$

$A + B - C = 0$

$-A + B - D = 1$

On solving these equations, we obtain

$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0$ and $D = -\frac{1}{2}$

$\therefore \frac{1}{x^4-1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}$

$$\Rightarrow \int \frac{1}{x^4-1}dx = -\frac{1}{4}\log|x-1| + \frac{1}{4}\log|x-1| - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$$

[3]

Q. 126. Integrate the rational function : $\frac{1}{x(x^n+1)}$.

[Hint : Multiply numerator and denominator by x^{n-1} and put $x^n = t$]

[NCERT Ex. 7.5, Q. 16, Page 322]

Ans. Given that,

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)}=\frac{x^{n-1}}{x^{n-1}x(x+1)}=\frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n = t$

$\Rightarrow$ $x^{n-1}\,dx = 1/n\,dt$

$$\therefore \int\frac{1}{x(x^n+1)}dx=\int\frac{x^{n-1}}{x^n(x^n+1)}dx=\frac{1}{n}\int\frac{1}{t(t+1)}dt$$

Let $\frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{(t+1)}$

$1 = A(1+t) + Bt$ ...(i)

Substituting $t = 0, -1$ in equation (i), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(1+t)}$$ **[1½]**

$$\Rightarrow \int\frac{1}{x(x^n+1)}dx=\frac{1}{n}\int\left\{\frac{1}{t}-\frac{1}{(t+1)}\right\}dx$$

$$=\frac{1}{n}\left[\log|t|-\log|t+1|\right]+C$$

$$=-\frac{1}{n}\left[\log|x^n|-\log|x^n+1|\right]+C$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$$ **[1½]**

Q. 127. Integrate the rational function $\frac{\cos x}{(1-\sin x)(2-\sin x)}$.

[Hint : Put $\sin x = t$]

[NCERT Ex. 7.5, Q. 17, Page 323]

Ans. Given that,

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t$

$\Rightarrow$ $\cos x\,dx = dt$

$$\therefore \int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)}=\frac{A}{(1-t)}+\frac{B}{(2-t)}$

$1 = A(2-t) + B(1-t)$ ...(i)

Substituting $t = 2$ and then $t = 1$ in equation (i), we obtain $A = 1$ and $B = -1$

$$\therefore \frac{1}{(1-t)(2-t)}=\frac{1}{(1-t)}-\frac{1}{(2-t)}$$

$$\Rightarrow \int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\left\{\frac{1}{1-t}-\frac{1}{(2-t)}\right\}dt$$

$$=-\log|1-t|+\log|2-t|+C$$

[3]

Q. 128. Integrate the rational function $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$.

[NCERT Ex. 7.5, Q. 18, Page 323]

Ans. Given that,

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}=1\frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

Let,

$$\frac{4x^2+10}{(x^2+3)(x^2+4)}=\frac{Ax+B}{(x^2+3)}+\frac{Cx+D}{(x^2+4)}$$

$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$

$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$

$4x^2 + 10 = (A + C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$

Equating the coefficients of x^3, x^2, x and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0, B = -2, C = 0$ and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)}=\frac{-2}{(x^2+3)}+\frac{6}{(x^2+4)}$$ **[1½]**

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}=1-\left(\frac{-2}{(x^2+3)}+\frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx=\int\left\{1+\frac{2}{(x^2+3)}-\frac{6}{(x^2+4)}\right\}dx$$

$$=\int\left\{1+\frac{2}{x^2+(\sqrt{3})^2}-\frac{6}{x^2+2^2}\right\}$$

$$=x+2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)-6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right)+C$$

$$=x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+C$$

[1½]

Q. 129. Integrate the rational function $\frac{2x}{(x^2+1)(x^2+3)}$.

[NCERT Ex. 7.5, Q. 19, Page 323]

Ans. Given that,

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Let $x^2 = t$

$\Rightarrow$ $2x\,dx = dt$

$$\therefore \int\frac{2x}{(x^2+1)(x^2+3)}dx=\int\frac{dt}{(t+1)(t+3)}$$ ...(i)

Let $\frac{1}{(t+1)(t+3)}=\frac{A}{(t+1)}+\frac{B}{(t+3)}$

$1 = A(t + 3) + B(t + 1)$ ...(ii)

Substituting $t = -3$ and $t = -1$ in equation (i), we obtain

$$A=\frac{1}{2} \text{ and } B=-\frac{1}{2}$$

$$\therefore \quad \frac{1}{(t+1)(t+3)}=\frac{1}{2(t+1)}-\frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)}dx=\int\left\{\frac{1}{2(t+1)}-\frac{1}{2(t+3)}\right\}dt$$

[1½]

$$=\frac{1}{2}\log|t+1|-\frac{1}{2}\log|t+3|+C$$

$$=\frac{1}{2}\log\left|\frac{t+1}{t+3}\right|+C$$

$$=\frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right|+C$$ **[1½]**

Q. 130. Integrate the rational function $\frac{1}{x(x^4-1)}$.

[NCERT Ex. 7.5, Q. 20, Page 323]

Ans. Given that,

$$\frac{1}{x(x^4-1)}$$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x(x^4-1)}=\frac{x^3}{x^4(x^4-1)}$$

$$\therefore \int\frac{1}{x(x^4-1)}dx=\int\frac{x^3}{x^4(x^4-1)}dx$$

Let $x^4 = t$

$\Rightarrow \quad 4x^3\,dx = dt$

$$\therefore \int\frac{1}{x(x^4-1)}dx=\frac{1}{4}\int\frac{dt}{t(t-1)}$$

Let,

$$\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{(t-1)}$$

$$1 = A(t-1) + Bt \quad \ldots\text{(i)}$$ **[1½]**

Substituting $t = 0$ and 1 in equation (i), we obtain

$$A = -1 \text{ and } B = 1$$

$$\frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\int\frac{1}{x(x^4-1)}dx=\frac{1}{4}\int\left\{\frac{-1}{t}+\frac{1}{t-1}\right\}dt$$

$$=\frac{1}{4}\left[-\log|t|+\log|t-1|\right]+C$$

$$=\frac{1}{4}\log\left|\frac{t-1}{t}\right|+C$$

$$=\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$$ **[1½]**

Q. 131. Integrate the rational function $\frac{1}{(e^x-1)}$ [Hint : Put $e^x = t$].

[NCERT Ex. 7.5, Q. 21, Page 323]

Ans. Given that,

$$\frac{1}{(e^x-1)}$$

Let $e^x = t$

$\Rightarrow e^x\,dx = dt$

$$\Rightarrow \int\frac{1}{e^x-1}dx=\int\frac{1}{t-1}\times\frac{dt}{t}=\int\frac{1}{t(t-1)}dt$$

Let $\quad \frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}$

$$1 = A(t-1) + Bt \quad \ldots\text{(i)}$$

Substituting $t = 1$ and $t = 0$ in equation (i), we obtain

$$A = -1 \text{ and } B = 1$$

$$\therefore \quad \frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int\frac{1}{t(t-1)}dt=\log\left|\frac{t-1}{t}\right|+C$$

$$=\log\left|\frac{e^x-1}{e^x}\right|+C$$

[3]

Q. 132. Evaluate : $\int\sqrt{5-2x+x^2dx}$.

[NCERT Exemp. Ex. 7.3, Q. 17, Page 164]

Ans. Let,

$$I=\int\sqrt{5-2x+x^2}dx=\int\sqrt{x^2-2x+1+4}dx$$

$$=\int\sqrt{(x-1)^2+(2)^2}dx=\int\sqrt{(2)^2+(x-1)^2}dx$$

$$=\frac{x-1}{2}\sqrt{2^2+(x-1)^2}+2\log\left|x-1+\sqrt{2^2+(x-1)^2}\right|+C$$

$$=\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|x-1+\sqrt{5-2x+x^2}\right|+C$$

[3]

Q. 133. Evaluate : $\int\frac{x}{x^4-1}dx$.

[NCERT Exemp. Ex. 7.3, Q. 18, Page. 164]

Ans. Let,

$$I=\int\frac{x}{x^4-1}dx$$

Put $\quad x^2 = t$

$\Rightarrow \quad 2xdx = dt$

$\Rightarrow \quad xdx=\frac{1}{2}dt$

$$\therefore \quad I=\frac{1}{2}\int\frac{dt}{t^2-1}=\frac{1}{2}\cdot\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+C$$

$$\left[\because \int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C\right]$$

$$=\frac{1}{4}\left[\log|x^2-1|-\log|x^2+1|\right]+C$$

[3]

Q. 134. Integrate the functions $\frac{x-1}{\sqrt{x^2-1}}$.

[NCERT Ex. 7.4, Q. 7, Page 315]

Ans. $$\int\frac{x-1}{\sqrt{x^2-1}}dx=\int\frac{x}{\sqrt{x^2-1}}dx-\int\frac{1}{\sqrt{x^2-1}}dx \quad \ldots\text{(i)}$$

For $\int\frac{x}{\sqrt{x^2-1}}dx$, let $x^2-1=t \Rightarrow 2x\,dx=dt$

$$\therefore \int\frac{x}{\sqrt{x^2-1}}dx=\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\frac{1}{2}\int t^{-\frac{1}{2}}dt$$

$$=\frac{1}{2}\left[2t^{\frac{1}{2}}\right]=\sqrt{t}=\sqrt{x^2-1}$$

From equation (i), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx$$

$$\left[\because \int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$

$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C \quad [3]$$

Q. 135. Evaluate : $\int \frac{x^2}{1-x^4}dx$ [Put $x^2 = t$].

[NCERT Exemp. Ex. 7.3, Q. 19, Page 164]

Ans. Given that,

$$I = \int \frac{x^2}{1-x^4}dx$$

$$= \int \frac{\left(\frac{1}{2}+\frac{x^2}{2}-\frac{1}{2}+\frac{x^2}{2}\right)}{(1-x^2)(1+x^2)}dx \quad [\because a^2-b^2=(a+b)(a-b)]$$

$$= \int \frac{\frac{1}{2}(1+x^2)-\frac{1}{2}(1-x^2)}{(1-x^2)(1+x^2)}dx$$

$$= \int \frac{\frac{1}{2}(1+x^2)}{(1-x^2)(1+x^2)}dx - \frac{1}{2}\int \frac{(1-x^2)}{(1-x^2)(1+x^2)}dx$$

$$= \frac{1}{2}\int \frac{1}{1-x^2}dx - \frac{1}{2}\int \frac{1}{1+x^2}dx$$

$$= \frac{1}{2}\cdot\frac{1}{2}\log\left|\frac{1+x}{1-x}\right| + C_1 - \frac{1}{2}\tan^{-1}x + C_2$$

$$= \frac{1}{4}\log\left|\frac{1+x}{1-x}\right| - \frac{1}{2}\tan^{-1}x + C \quad [\because C = C_1 + C_2] \quad [3]$$

Q. 136. Evaluate : $\int \sqrt{2ax - x^2 dx}$.

[NCERT Exemp. Ex. 7.3, Q. 20, Page 164]

Ans. $I = \int \sqrt{2ax-x^2}dx = \int \sqrt{-(x^2-2ax)}dx$

$$= \int \sqrt{-(x^2-2ax+a^2-a^2)}dx = \int \sqrt{-(x-a)^2+a^2}dx$$

$$= \int \sqrt{a^2-(x-a)^2}dx$$

$$= \frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$= \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C \quad [3]$$

Q. 137. Evaluate : $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$.

[NCERT Exemp. Ex. 7.3, Q. 21, Page 164]

Ans. Let,

$$I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx = \int \frac{\sin^{-1}x}{(1-x^2)\sqrt{1-x^2}}dx$$

Put $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$

and $x = \sin t \Rightarrow 1-x^2 = \cos^2 t$

$\Rightarrow \cos t = \sqrt{1-x^2}$

$\therefore \quad I = \int \frac{t}{\cos^2 t}dt = \int t\cdot\sec^2 t\,dt$

$$= t\cdot\int \sec^2 t\,dt - \int\left(\frac{d}{dt}t\cdot\int\sec^2 t\,dt\right)dt$$

$$= t\cdot\tan t - \int 1\cdot\tan t\,dt$$

$$= t\tan t + \log|\cos t| + C \quad [\because \int \tan x dx = -\log|\cos x| + C]$$

$$= \sin^{-1}x\cdot\frac{x}{\sqrt{1-x^2}} + \log\left|\sqrt{1-x^2}\right| + C \quad [3]$$

Q. 138. Evaluate : $\int \frac{(\cos 5x + \cos 4x)}{1-2\cos 3x}dx$.

[NCERT Exemp. Ex. 7.3, Q. 22, Page 164]

Ans. $I = \int \frac{\cos 5x + \cos 4x}{1-2\cos 3x}dx = \int \frac{2\cos\frac{9x}{2}\cdot\cos\frac{x}{2}}{1-2\left(2\cos^2\frac{3x}{2}-1\right)}dx$

$$\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\times\cos\frac{C-D}{2} \text{ and } \cos 2x = 2\cos^2 x - 1\right]$$

$$I = \int \frac{2\cos\frac{9x}{2}\cdot\cos\frac{x}{2}}{3-4\cos^2\frac{3x}{2}}dx = -\int \frac{2\cos\frac{9x}{2}\cdot\cos\frac{x}{2}}{4\cos^2\frac{3x}{2}-3}dx$$

$$= -\int \frac{2\cos\frac{9x}{2}\cdot\cos\frac{x}{2}\cdot\cos\frac{3x}{2}}{4\cos^3\frac{3x}{2}-3\cos\frac{3x}{2}}dx$$

$$\left[\text{Multiply and divide by} \cos\frac{3x}{2}\right]$$

$$= -\int \frac{2\cos\frac{9x}{2}\cdot\cos\frac{x}{2}\cdot\cos\frac{3x}{2}}{\cos 3\cdot\frac{3x}{2}}dx = -\int 2\cos\frac{3x}{2}\cdot\cos\frac{x}{2}dx$$

$$= -\int\left\{\cos\left(\frac{3x}{2}+\frac{x}{2}\right)+\cos\left(\frac{3x}{2}-\frac{x}{2}\right)\right\}dx$$

$$= -\int(\cos 2x + \cos x)dx$$

$$= -\left[\frac{\sin 2x}{2} + \sin x\right] + C = -\frac{1}{2}\sin 2x - \sin x + C$$

[3]

Q. 139. Evaluate : $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}dx$.

[NCERT Exemp. Ex. 7.3, Q. 23, Page 164]

Ans. $I = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x\cos^2 x}dx$

$$= \int \frac{(\sin^2 x)^3 + (\cos^2 x)^3}{\sin^2 x\cdot\cos^2 x}dx$$

$$= \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x)}{\sin^2\cdot\cos^2 x}dx$$

$$= \int \frac{\sin^4 x}{\sin^2 x\cos^2 x}dx + \int \frac{\cos^4 x}{\sin^2 x\cdot\cos^2 x}dx - \int \frac{\sin^2 x\cos^2 x}{\sin^2 x\cdot\cos^2 x}dx$$

$$= \int \tan^2 x dx + \int \cot^2 x dx - \int 1 dx$$

$$= \int(\sec^2 x - 1)dx + \int(\text{cosec}^2 x - 1)dx - \int 1 dx$$

$$= \int \sec^2 x dx + \int \text{cosec}^2 x dx - 3\int dx$$

$$= \tan x - \cot x - 3x + C$$ **[3]**

Q. 140. Evaluate : $\int \frac{\cos x - \cos 2x}{1 - \cos x} dx$.

[NCERT Exemp. Ex. 7.3, Q. 25, Page 165]

Ans. Let,

$$I = \int \frac{\cos x - \cos 2x}{1 - \cos x} dx = \int \frac{2\sin\frac{3x}{2} \cdot \sin\frac{x}{2}}{1 - 1 + 2\sin^2\frac{x}{2}} dx$$

$$= 2\int \frac{\sin\frac{3x}{2} . \sin\frac{x}{2}}{2\sin^2\frac{x}{2}} dx = \int \frac{\sin\frac{3x}{2}}{\sin\frac{x}{2}} dx$$ **[1]**

$$= \int \frac{3\sin\frac{x}{2} - 4\sin^3\frac{x}{2}}{\sin\frac{x}{2}} dx \quad \left[\because \sin 3x = 3\sin x - 4\sin^3 x\right]$$

$$= 3\int dx - 4\int \sin^2\frac{x}{2} dx = 3\int dx - 4\int \frac{1 - \cos x}{2} dx$$

$$= 3\int dx - 2\int dx + 2\int \cos x \, dx$$

$$= \int dx + 2\int \cos x \, dx = x + 2\sin x + C = 2\sin x + x + C$$ **[2]**

Q. 141. Evaluate $\int_0^2 (x^2 + 3) dx$ as limit of sum.

[NCERT Exemp. Ex. 7.3, Q. 27, Page 165]

Ans. Let $I = \int_0^2 (x^2 + 3) dx$

Here, $a = 0, b = 2$ and $h = \frac{b-a}{n} = \frac{2-0}{n}$

$\Rightarrow \quad h = \frac{2}{n}$

$\Rightarrow \quad nh = 2$

$\Rightarrow f(x) = (x^2 + 3)$

Now $\int_0^2 (x^2 + 3) dx = \lim_{h \to 0} h\,[f(0) +$

$f(0+h) + f(0+2h) + ... + f\{0 + (n-1)h\}]$ ...(i)

$\because \quad f(0) = 3$

$\Rightarrow f(0+h) = h^2 + 3, f(0+2h) = 4h^2 + 3 = 2^2h^2 + 3$

$f[0 + (n-1)h] = (n^2 - 2n + 1)h + 3 = (n-1)^2 h + 3$

From equation (i), we have

$$\int_0^2 (x^2 + 3) dx = \lim_{h \to 0} h[3 + h^2 + 3 + 2^2h^2 + 3 + 3^2h^2 + 3 + ... + (n-1)^2 h^2 + 3]$$

$$= \lim_{h \to 0} h[3n + h^2\{1^2 + 2^2 + ... + (n-1)^2\}]$$

$$= \lim_{h \to 0} h\left[3n + h^2\left(\frac{(n-1)(2n-2+1)(n-1+1)}{6}\right)\right]$$ **[1]**

$$\left[\because \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{h \to 0} h\left[3n + h^2\left(\frac{(n^2 - n)(2n-1)}{6}\right)\right]$$

$$= \lim_{h \to 0} h\left[3n + \frac{h^2}{6}\left(2n^3 - n^2 - 2n^2 + n\right)\right]$$

$$= \lim_{h \to 0}\left[3nh + \frac{2n^3h^3 - 3n^2h^2 \cdot h + nh \cdot h^2}{6}\right]$$

$$= \lim_{h \to 0}\left[3 \cdot 2 + \frac{2 \cdot 8 - 3 \cdot 2^2 \cdot h + 2 \cdot h^2}{6}\right]$$

$$= \lim_{h \to 0}\left[6 + \frac{16 - 12h + 2h^2}{6}\right]$$

$$= 6 + \frac{16}{6} = 6 + \frac{8}{3} = \frac{26}{3}$$ **[2]**

Q. 142. Evaluate $\int_0^2 e^x dx$ as limit of sum.

[NCERT Exemp. Ex. 7.3, Q. 28, Page. 165]

Ans. Let,

$$I = \int_0^2 e^x dx$$

Here, $\quad a = 0$ and $b = 2$

$\therefore \quad h = \frac{b-a}{n}$

$\Rightarrow \quad nh = 2$ and $f(x) = e^x$

Now, $\int_0^2 e^x dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + ... + f\{0 + (n-1)h\}]$

$$\therefore \quad I = \lim_{h \to 0} h[1 + e^h + e^{2h} + ... + e^{(n-1)h}]$$

$$= \lim_{h \to 0} h\left[\frac{1 \cdot (e^h)^n - 1}{e^h - 1}\right] = \lim_{h \to 0} h\left(\frac{e^{nh} - 1}{e^h - 1}\right)$$

$$= \lim_{h \to 0} h\left(\frac{e^2 - 1}{e^h - 1}\right)$$

$$= e^2 \lim_{h \to 0} \frac{h}{e^h - 1} - \lim_{h \to 0} \frac{h}{e^h - 1}$$

$$\left[\because \lim_{h \to 0} \frac{h}{e^h - 1} = 1\right]$$

$$= e^2 - 1$$ **[3]**

Q. 143. Evaluate : $\int_0^1 x \log(1 + 2x) dx$.

[NCERT Exemp. Ex. 7.3, Q. 45, Page 166]

Ans. Let,

$$I = \int_0^1 x \log(1 + 2x) dx$$

$$= \left[\log(1+2x)\frac{x^2}{2}\right]_0^1 - \int \frac{1}{1+2x} \cdot 2 \cdot \frac{x^2}{2} dx$$

$$= \frac{1}{2}[x^2 \log(1+2x)]_0^1 - \int \frac{x^2}{1+2x} dx$$

$$= \frac{1}{2}[1\log 3 - 0] - \left[\int_0^1 \left(\frac{x}{2} - \frac{\frac{x}{2}}{1+2x}\right) dx\right]$$

$$= \frac{1}{2}\log 3 - \frac{1}{2}\int_0^1 x \, dx + \frac{1}{2}\int_0^1 \frac{x}{1+2x} dx$$

$$= \frac{1}{2}\log 3 - \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\int_0^1 \frac{\frac{1}{2}(2x+1-1)}{(2x+1)} dx$$

$= \frac{1}{2}\log 3 - \frac{1}{2}\int_0^1 x\,dx + \frac{1}{2}\int_0^1 \frac{x}{1+2x}dx$

$= \frac{1}{2}\log 3 - \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\int_0^1 \frac{\frac{1}{2}(2x+1-1)}{(2x+1)}dx$

$= \frac{1}{2}\log 3 - \frac{1}{2}\left[\frac{1}{2} - 0\right] + \frac{1}{4}\int_0^1 dx - \frac{1}{4}\int_0^1 \frac{1}{1+2x}dx$

$= \frac{1}{2}\log 3 - \frac{1}{4} + \frac{1}{4}[x]_0^1 - \frac{1}{8}[\log|(1+2x)|]_0^1$

$= \frac{1}{2}\log 3 - \frac{1}{4} + \frac{1}{4} - \frac{1}{8}[\log 3 - \log 1]$

$= \frac{1}{2}\log 3 - \frac{1}{8}\log 3$

$= \frac{3}{8}\log 3$ **[3]**

Q. 144. Find : $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$.

[CBSE Board, All India Region, 2017]

Ans. $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta = \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}d\theta$

$= \int \frac{dt}{(4+t^2)(1+4t^2)}$, where $\sin\theta = t$

$= \int \frac{-\frac{1}{15}}{4+t^2}dt + \int \frac{\frac{4}{15}}{1+4t^2}dt$

$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + \frac{4}{30}\tan^{-1}(2t) + c$

$= -\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + \frac{2}{15}\tan^{-1}(2\sin\theta) + c$ **[4]**

Q. 145. Evaluate : $\int_0^\pi \frac{x\tan x}{\sec x + \tan x}dx$.

[CBSE Board, All India Region, 2017]

Ans. $I = \int_0^\pi \frac{x\tan x}{\sec x + \tan x}dx = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x + \tan x}dx$

$\Rightarrow 2I = \pi\int_0^\pi \frac{\tan x}{\sec x + \tan x}dx$

$= \pi\int_0^\pi \tan x(\sec x - \tan x)dx$

$I = \frac{\pi}{2}\int_0^\pi [\sec x - \tan x + x]_0^\pi$

$= \frac{\pi(\pi-2)}{2}$ **[4]**

Q. 146. Evaluate : $\int_1^4 \{|x-1| + |x-2| + |x-4|\}dx$.

[CBSE Board, All India Region, 2017]

Ans.

$I = \int_1^4 \{|x-1| + |x-2| + |x-4|\}dx$

$= \int_1^4 (x-1)dx - \int_1^2 (x-2)dx + \int_1^4 (x-2)dx - \int_1^4 (x-4)dx$

$= \left.\frac{(x-1)^2}{2}\right]_1^4 - \left.\frac{(x-2)^2}{2}\right]_1^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 - \left.\frac{(x-4)^2}{2}\right]_1^4$

$= \frac{9}{2} + \frac{1}{2} + 2 + \frac{9}{2}$

$= 11\frac{1}{2}$ or $\frac{23}{2}$ **[4]**

Q. 147. Find : $\int \frac{e^x}{(2+e^x)(4+e^{2x})}dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $I = \int \frac{dt}{(2+t)(4+t^2)}$ where $e^x = t$

Now, $\frac{1}{(2+t)(4+t^2)} = \frac{1}{8(2+t)} - \frac{1}{8}\left(\frac{t-2}{4+t^2}\right)$

$\Rightarrow \int \frac{dt}{(2+t)(4+t^2)} = \frac{1}{8}\log|2+t| - \frac{1}{16}\log|4+t^2| + \frac{1}{8}\tan^{-1}\left(\frac{t}{2}\right) + c$

[4]

$\Rightarrow \int \frac{e^x dx}{(2+e^x)(4+e^{2x})} = \frac{1}{8}\log|2+e^x| - \frac{1}{16}\log|4+e^{2x}| + \frac{1}{8}\tan^{-1}\left(\frac{e^x}{2}\right) + c$

Q. 148. Evaluate : $\int_{-2}^{1} |x^3 - x|dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $\int_{-2}^1 |x^3 - x|dx$

$= \int_{-2}^{-1} -(x^3-x)dx + \int_{-1}^0 (x^3-x)dx - \int_0^1 -(x^3-x)dx$

$= \left|\frac{x^2}{2} - \frac{x^4}{4}\right|_{-2}^{-1} + \left|\frac{x^4}{4} - \frac{x^2}{2}\right|_{-1}^{0} - \left|\frac{x^4}{4} - \frac{x^2}{2}\right|_0^1$

$= \frac{11}{4}$ **[4]**

Q. 149. Find : $\int e^{2x}\sin(3x+1)dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $I = \int e^{2x}\sin(3x+1)dx$

$= \sin(3x+1)\cdot\frac{e^{2x}}{2} - \int 3\cos(3x+1)\cdot\frac{e^{2x}}{2}dx$

$= \frac{e^{2x}}{2}\cdot\sin(3x+1) - \frac{3}{4}\left[\cos(3x+1)\cdot\frac{e^{2x}}{2} - \int -3\sin(3x+1)\cdot\frac{e^{2x}}{2}dx\right]$

$= \frac{e^{2x}}{2}\sin(3x+1) - \frac{3}{4}\cos(3x+1)\cdot e^{2x} - \frac{9}{4}I + c$

$\Rightarrow \frac{13}{4}I = \frac{e^{2x}}{4}[2\sin(3x+1) - 3\cos(3x+1)] + c$

$\Rightarrow I = \frac{e^{2x}}{13}[2\sin(3x+1) - 3\cos(3x+1)] + c$ **[4]**

Q. 150. Find : $\int \frac{2x}{(x^2+1)(x^2+2)^2}dx$.

[CBSE Board, Delhi Region, 2017]

Ans. $\int \frac{2x}{(x^2+1)(x^2+2)^2} = \int \frac{dy}{(y+1)(y+2)^2}$

[By substituting $x^2 = y$]

$= \int \frac{dy}{y+1} - \int \frac{dy}{y+2} - \int \frac{dy}{(y+2)^2}$

(By using partial fraction)

$= \log(y+1) - \log(y+2) + \frac{1}{y+2} + C$

$= \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + C$ **[4]**

Q. 151. Evaluate : $\int_0^{3/2} |x \sin \pi x| dx$.

[CBSE Board, Delhi Region, 2017]

Ans. $I = \int_0^{3/2} |x \sin \pi x| dx$

$= \int_0^1 x \sin \pi x \cdot dx - \int_1^{3/2} x \sin \pi x \, dx$

$= \left[-x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_0^1 - \left[-\frac{x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_1^{3/2}$

$= \frac{2}{\pi} + \frac{1}{\pi^2}$ **[4]**

Q. 152. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

[CBSE Board, Delhi Region, 2017]

Ans. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$

Put $\cos x = t$ and $-\sin x \, dx = dt$

$= -\pi \int_1^{-1} \frac{dt}{1+t^2}$

$= \pi [\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{2}$

$\Rightarrow \quad I = \frac{\pi^2}{4}$ **[4]**

Q. 153. Find : $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$.

[CBSE Board, Delhi Region, 2016]

Ans. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2} \cdot x^{1/2} dx = dt$ or $\sqrt{x} \, dx = \frac{2}{3} dt$

$I = \frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$

$= \frac{2}{3} \cdot \sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$

$= \frac{2}{3} \sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$ **[4]**

Q. 154. Evaluate : $\int_{-1}^{2} |x^2 - x| dx$.

[CBSE Board, Delhi Region, 2016]

Ans. $I = \int_{-1}^{2} |x^2 - x| dx$

$= \int_{-1}^{0} (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$

$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$ **[4]**

Q. 155. Find : $\int \frac{x^2}{x^4 + x^2 - 2} dx$.

[CBSE Board, All India Region, 2016]

Ans. Let,

$x^2 = t$

$\frac{x^2}{x^4 + x^2 - 2} = \frac{x^2}{(x^2-1)(x^2+2)}$

$= \frac{t}{(t-1)(t+2)}$

$= \frac{A}{t-1} + \frac{B}{t+2}$

Solving for A and B to get,

$A = \frac{1}{3}, B = \frac{2}{3}$

$\int \frac{x^2}{x^4 + x^2 - 2} dx = \frac{1}{3} \int \frac{1}{x^2 - 1} dx + \frac{2}{3} \int \frac{1}{x^2 + 2} dx$

$= \frac{1}{6} \log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + C$ **[4]**

Q. 156. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$.

[CBSE Board, All India Region, 2016]

Ans. Let,

$I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$, Also I

$= \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx$

Adding to get,

$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$

$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$

$= \frac{1}{\sqrt{2}} \log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\pi/2}$

$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right\}$

$\Rightarrow I = \frac{1}{2\sqrt{2}} \begin{Bmatrix} \log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right| \\ \text{or } \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| \end{Bmatrix}$ **[4]**

Q. 157. Evaluate : $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$.

[CBSE Board, All India Region, 2016]

Ans. $\int_0^{3/2} |x \cos \pi x| dx = \int_0^{1/2} x \cos \pi x \, dx - \int_{1/2}^{3/2} x \cos \pi x \, dx$

$= \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_{1/2}^{3/2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2}$ **[4]**

Q. 158. Find : $\int (3x+1)\sqrt{4-3x-2x^2}\,dx$.

[CBSE Board, All India Region, 2016]

Ans. $\int (3x+1)\sqrt{4-3x-2x^2}\,dx$

$$= -\frac{3}{4}\int(-4x-3)\sqrt{4-3x-2x^2}\,dx - \frac{5}{4}\int\sqrt{4-3x-2x^2}\,dx$$

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x+\frac{3}{4}\right)^2}$$

$$= -\frac{1}{2}(4-3-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\left\{\begin{array}{l}\frac{4x+3}{8}\sqrt{\frac{41}{16}-\left(x+\frac{3}{4}\right)^2} \\ +\frac{41}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\end{array}\right\}+C$$

$$= -\frac{1}{2}(4-3-2x^2)^{3/2} - \frac{5}{4}\left\{\begin{array}{l}\frac{4x+3}{8}\sqrt{4-3x-2x^2} \\ +\frac{41\sqrt{2}}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\end{array}\right\}+C$$ [4]

Q. 159. Find : $\int (2x+5)\sqrt{10-4x-3x^2}\,dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $I = \int(2x+5)\sqrt{10-4x-3x^2}\,dx$

$$= -\frac{1}{3}\int(-4-6x)\sqrt{10-4x-3x^2}\,dx + \frac{11}{3}\sqrt{10-4x-3x^2}\,dx$$

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\int\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2}\,dx$$

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}$$

$$\left[\frac{\left(x-\frac{2}{3}\right)-\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2-\left(x-\frac{2}{3}\right)^2}}{2}+\frac{17}{9}\sin^{-1}\frac{3x-2}{\sqrt{34}}\right]+C$$ **[4]**

Q. 160. Find : $\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. $x^2 = y$ (say)

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5}$$

By using partial fraction, we get

$$A = \frac{1}{4}, B = \frac{27}{4}$$

$$\int\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}dx$$

$$= \int 1.dx + \frac{1}{4}\int\frac{dx}{x^2+3} + \frac{27}{4}\int\frac{dx}{x^2-5}$$

$$= x + \frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + C$$ **[4]**

Q. 161. Find : $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$.

[CBSE Board, Foreign Scheme, 2017]

Ans. Given that,

$$I = \int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$$

Put $\sin^{-1}x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

$$\int t\cdot\sin t\,dt$$

$$= -t\cos t + \sin t + c$$

$$= -\sqrt{1-x^2}\sin^{-1}x + x + c$$ **[4]**

Long Answer Type Questions

(5 or 6 marks each)

Q. 1. Evaluate : $\int_0^{\pi} x\log\sin x\,dx$.

[NCERT Exemp. Ex. 7.3, Q. 46, Page 166]

Ans. Let,

$$I = \int_0^{\pi} x\log\sin x\,dx \quad \ldots\text{(i)}$$

$$I = \int_0^{\pi}(\pi-x)\log\sin(\pi-x)\,dx$$

$$= \int_0^{\pi}(\pi-x)\log\sin x\,dx \quad \ldots\text{(ii)}$$

$$2I = \pi\int_0^{\pi}\log\sin x\,dx \quad \ldots\text{(iii)}$$

$$2I = 2\pi\int_0^{\pi/2}\log\sin x\,dx \quad \left[\because \int_0^{2a}f(x)dx = 2\int_0^a f(x)dx\right]$$

$$I = \pi\int_0^{\pi/2}\log\sin x\,dx \quad \ldots\text{(iv)}$$

Now, $I = \pi\int_0^{\pi/2}\log\sin(\pi/2-x)\,dx$ …(v)

On adding equations (iv) and (v), we get

$$2I = \pi\int_0^{\pi/2}(\log\sin x + \log\cos x)dx$$

$$2I = \pi\int_0^{\pi/2}\log\sin x\cos x\,dx$$

$$= \pi\int_0^{\pi/2}\log\frac{2\sin x\cos x}{2}dx$$

$$2I = \pi\int_0^{\pi/2}(\log\sin 2x - \log 2)dx$$

$$2I = \pi\int_0^{\pi/2}\log\sin 2x\,dx - \pi\int_0^{\pi/2}\log 2\,dx$$ **[2½]**

Put $2x = t \Rightarrow dx = \frac{1}{2}dt$

As $x \to 0$, then $t \to 0$ and $x \to \frac{\pi}{2}$, then $t \to \pi$

$$\therefore\ 2I = \frac{\pi}{2}\int_0^{\pi}\log\sin t\,dt - \frac{\pi^2}{2}\log 2$$

$$\Rightarrow 2I = \frac{\pi}{2}\int_0^{\pi}\log\sin x\,dx - \frac{\pi^2}{2}\log 2$$

$$\Rightarrow 2I = I - \frac{\pi^2}{2}\log 2 \quad \text{[From Eq. (iii)]}$$

$$\therefore \quad I = -\frac{\pi^2}{2}\log 2 = \frac{\pi^2}{2}\log\left(\frac{1}{2}\right)$$

[2½]

Q. 2. Evaluate the definite integral as limit of sum : $\int_a^b x dx$. **[NCERT Ex. 7.8, Q. 1, Page 334]**

Ans. It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\begin{bmatrix} f(a)+f(a+h)+\ldots \\ +f(a+(n-1)h)\end{bmatrix},$$

where $h = \dfrac{b-a}{n}$

Here, $a = a, b = b,$ and $f(x) = x$

$$\int_a^b xdx = (b-a)\lim_{n\to\infty}\frac{1}{n}\begin{bmatrix} a+(a+h)\ldots \\ (a+2h)\ldots a+(n-1)h\end{bmatrix}$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[\begin{pmatrix} a+a+a+\ldots \\ +n-\text{times}\end{pmatrix}\begin{pmatrix} h+2h+3h+\ldots \\ +(n-1)h\end{pmatrix}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h(1+2+3+\ldots+(n-1))\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + \frac{n(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{n}{n}\left[a + \frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)h}{2}\right]$$

[2½]

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)(b-a)}{2n}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$= (b-a)\left[a + \frac{(b-a)}{2}\right]$$

$$= (b-a)\left[\frac{2a+b-a}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2}$$

$$= \frac{1}{2}(b^2-a^2)$$

[2½]

Q. 3. Evaluate the definite integral as limit of sum : $\int_0^5 (x+1)dx$. **[NCERT Ex. 7.8, Q. 2, Page 334]**

Ans. Let $I = \int_0^5 (x+1)dx$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right],$$

Where $h = \dfrac{b-a}{n}$

Here, $a = 0, b = 5$ and $f(x) = (x+1)$

$$h = \frac{5-0}{n} = \frac{5}{n}$$

$$\int_0^5 (x+1)dx = (5-0)\lim_{n\to\infty}\frac{1}{n}\begin{bmatrix} f(0)+f\left(\frac{5}{n}\right)+\ldots \\ +f\left((n-1)\frac{5}{n}\right)\end{bmatrix}$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\begin{bmatrix}\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n} \\ +\ldots(n-1)\frac{5}{n}\end{bmatrix}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$= 5\lim_{n\to\infty}\frac{n}{n}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 5\left[1+\frac{5}{2}\right] = 5\left[\frac{7}{2}\right] = \frac{35}{2}$$

[5]

Q. 4. Evaluate the definite integral as limit of sum : $\int_2^3 x^2 dx$. **[NCERT Ex. 7.8, Q. 3, Page 334]**

Ans. It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\begin{bmatrix} f(a)+f(a+h) \\ +f(a+2h)\ldots f\{a+(n-1)h\}\end{bmatrix},$$

where $h = \dfrac{b-a}{n}$

Here, $a = 2, b = 3,$ and $f(x) = x^2$

$$h = \frac{3-2}{n} = \frac{1}{n}$$

$$\int_2^3 x^2 dx = (3-2)\lim_{n\to\infty}\frac{1}{n}\begin{bmatrix} f(2)+f\left(2+\frac{1}{n}\right) \\ +f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\end{bmatrix}$$

$$= \lim_{n\to\infty}\frac{1}{n}\begin{bmatrix}(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2 \\ +\ldots\left(2+\frac{(n-1)}{n}\right)^2\end{bmatrix}$$

$$= \lim_{n\to\infty}\frac{1}{n}\begin{bmatrix}2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\} \\ +\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\end{bmatrix}$$

$$= \lim_{n\to\infty}\frac{1}{n}\begin{bmatrix}\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\} \\ +2\cdot2\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\end{bmatrix}$$

$$= \lim_{n\to\infty}\frac{1}{n}\begin{bmatrix}4n+\frac{1}{n^2}\left\{1^2+2^2+3^3\ldots+(n-1)^2\right\} \\ +\frac{4}{n}\{1+2+\ldots+(n-1)\}\end{bmatrix}$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}\\ +\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\end{array}\right]$$ **[2½]**

$$= \lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$= \lim_{n\to\infty}\frac{n}{n}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$= 4+\frac{2}{6}+2=\frac{19}{3}$$ **[2½]**

Q. 5. Evaluate the definite integral as limit of sum : $\int_1^4 (x^2-x)dx$. [NCERT Ex. 7.8, Q. 4, Page 334]

Ans. Let $= \int_1^4 (x^2-x)dx$

$$= \int_1^4 x^2dx - \int_1^4 xdx$$

Let $I = I_1 - I_2$, where $I_1 = \int_1^4 x^2dx$ and $I_2 = \int_1^4 xdx$ …(i)

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a)+f(a+h)+f(a+(n-1)h)],$$

where $h = \frac{b-a}{n}$

For $I_1 = \int_1^4 x^2dx$,

$a = 1, b = 4$, and $f(x) = x^2$

$\therefore h = \frac{4-1}{n} = \frac{3}{n}$

$$I_1 = \int_1^4 x^2dx = (4-1)\lim_{n\to\infty}[f(1)+f(1+h)+\ldots+f(1+(n-1)h)]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}1^2+\left(1+\frac{3}{n}\right)^2+\left(1+2\cdot\frac{3}{n}\right)^2\\ +\ldots\left(1+\frac{(n-1)3}{n}\right)^2\end{array}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}1^2+\left\{1^2+\left(\frac{3}{n}\right)^2+2\cdot\frac{3}{n}\right\}\\ +\ldots+\left\{\begin{array}{l}1^2+\left(\frac{(n-1)3}{n}\right)^2\\ +\frac{2\cdot(n-1)\cdot 3}{n}\end{array}\right\}\end{array}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}\left(\underbrace{1^2+\ldots+1^2}_{n\text{ times}}\right)+\left(\frac{3}{n}\right)^2\{1^2+2^2+\ldots+(n-1)^2\}\\ +2\cdot\frac{3}{n}\{1+2+\ldots+(n-1)\}\end{array}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\}+\frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{6n-6}{2}\right]$$

$$= 3\lim_{n\to\infty}\left[1+\frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+3-\frac{3}{n}\right]$$

$$= 3[1+3+3] = 3[7]$$ **[2½]**

$I_1 = 21$ …(ii)

For $I_2 = \int_1^4 xdx$,

$a = 1, b = 4$, and $f(x) = x$

$$h = \frac{4-1}{n} = \frac{3}{n}$$

$$I_2 = (4-1)\lim_{n\to\infty}\frac{1}{n}[f(1)+f(1+h)+\ldots f(a+(n-1)h)]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}[1+(1+h)+\ldots+(1+(n-1)h)]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\ldots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+\ldots+1}_{n\text{ times}}\right)+\frac{3}{n}(1+2+\ldots+(n-1))\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 3\left[1+\frac{3}{2}\right] = 3\left[\frac{5}{2}\right]$$

$I_2 = \frac{15}{2}$ …(iii)

From equations (ii) and (iii), we obtain

$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$ **[2½]**

Q. 6. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log(\sin x + \cos x)dx$.

[NCERT Exemp. Ex. 7.3, Q. 47, Page 166]

Ans. $I = \int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x)dx$ …(i)

$$I = \int_{-\pi/4}^{\pi/4} \log\left\{\sin\left(\frac{\pi}{4}-\frac{\pi}{4}-x\right)+\cos\left(\frac{\pi}{4}-\frac{\pi}{4}-x\right)\right\}dx$$

$$= \int_{-\pi/4}^{\pi/4} \log\{\sin(-x)+\cos(-x)\}dx$$

and $I = \int_{-\pi/4}^{\pi 4} \log(\cos x - \sin x)dx$ …(ii)

From equations (i) and (ii),

$$2I = \int_{-\pi/4}^{\pi 4} \log\cos 2x dx$$

$2I = \int_0^{\pi/4} \log\cos 2x dx$ …(iii)

$[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x), if\ f(-x) = f(x)]$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

As $x \to 0$, then $t \to 0$

and $x \to \frac{\pi}{4}$, then $t \to \frac{\pi}{2}$

$2I = \frac{1}{2}\int_0^{\pi/2} \log\cos t dt$ …(iv)

$2I = \frac{1}{2}\int_0^{\pi/2} \log\cos\left(\frac{\pi}{2}-t\right)dt \quad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow 2I = \frac{1}{2}\int_0^{\pi/2} \log\sin t dx$ …(v)

[2½]

On adding equations (iv) and (v), we get

$$4I=\frac{1}{2}\int_0^{\pi/2}\log\sin t\cos t dt$$

$$\Rightarrow \quad 4I=\frac{1}{2}\int_0^{\pi/2}\log\frac{\sin 2t}{2}dt$$

$$\Rightarrow \quad 4I=\frac{1}{2}\int_0^{\pi/2}\log\sin 2dx-\frac{1}{2}\int_0^{\pi/2}\log 2dx$$

$$\Rightarrow \quad 4I=\frac{1}{2}\int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-2x\right)dx-\log 2\cdot\frac{\pi}{4}$$

$$\Rightarrow \quad 4I=\frac{1}{2}\int_0^{\pi/2}\log\cos 2x dx-\frac{\pi}{4}\log 2$$

$$\Rightarrow \quad 4I=\frac{1}{2}\int_0^{\pi/4}\log\cos 2x dx-\frac{\pi}{4}\log 2$$

$$\left[\because \int_0^{2a}f(x)=2\int_0^{a}f(x)dx\right]$$

$$\Rightarrow \quad 4I=2I-\frac{\pi}{4}\log 2 \quad \text{[From Eq. (iii)]}$$

$$\therefore \quad I=-\frac{\pi}{8}\log 2=\frac{\pi}{8}\log\left(\frac{1}{2}\right)$$

[2½]

Q. 7. Evaluate : $\int e^{-3x}\cos^3 x dx$.

[NCERT Exemp. Ex. 7.3, Q. 42, Page 166]

Ans. Let,

$$I=\int e^{-3x}\cos^3 x dx$$

$$=\cos^3 x\int e^{-3x}dx-\int\left(\frac{d}{dx}\cos^3 x\int e^{-3x}dx\right)dx$$

$$=\cos^3 x\cdot\frac{e^{-3x}}{-3}-\int(-3\cos^2 x)\sin x\cdot\frac{e^{-3x}}{-3}dx$$

$$=-\frac{1}{3}\cos^3 xe^{-3x}-\int\cos^2 x\sin xe^{-3x}dx$$

$$=-\frac{1}{3}\cos^3 xe^{-3x}-\int(1-\sin^2 x)\sin xe^{-3x}dx$$

$$=-\frac{1}{3}\cos^3 xe^{-3x}-\int\sin xe^{-3x}dx+\int\sin^3 xe^{-3x}dx$$

$$=-\frac{1}{3}\cos^3 xe^{-3x}-\int\sin xe^{-3x}dx+\sin^3 x\cdot\frac{e^{-3x}}{-3}$$

$$-\int 3\sin^2 x\cos x\cdot\frac{e^{-3x}}{-3}dx$$

$$=-\frac{1}{3}\cos^3 xe^{-3x}-\int\sin xe^{-3x}dx-\frac{1}{3}\sin^3 xe^{-3x}$$

$$+\int(1-\cos^2 x)\cos xe^{-3x}dx$$

$$I=-\frac{1}{3}\cos^3 xe^{-3x}-\int\sin xe^{-3x}-\frac{1}{3}\sin^3 xe^{-3x}$$

$$+\int\cos xe^{-3x}dx-\int\cos^3 xe^{-3x}dx$$

$$2I=\frac{e^{-3x}}{3}\left[\cos^3 x+\sin^3 x\right]-\left[\sin x\cdot\frac{e^{-3x}}{-3}-\int\cos x\cdot\frac{e^{-3x}}{-3}dx\right]$$

$$+\int\cos xe^{-3x}dx$$

$$2I=\frac{e^{-3x}}{-3}\left[\cos^3 x+\sin^3 x\right]+\frac{1}{3}\sin x.e^{-3x}$$

$$-\frac{1}{3}\int\cos x\cdot e^{-3x}dx+\int\cos xe^{-3x}dx$$

$$2I=\frac{e^{-3x}}{-3}\left[\cos^3 x+\sin^3 x\right]+\frac{1}{3}\sin xe^{-3x}+\frac{2}{3}\int\cos xe^{-3x}dx$$

[2½]

Now, let $I_1=\int\cos x e^{-3x}dx$

$$I_1=\cos x\cdot\frac{e^{-3x}}{-3}-\int(-\sin x)\cdot\frac{e^{-3x}}{-3}dx$$

$$I_1=\frac{-1}{3}\cos x\cdot e^{-3x}-\frac{1}{3}\int\sin x.e^{-3x}dx$$

$$=-\frac{1}{3}\cos x.e^{-3x}-\frac{1}{3}\left[\sin x.\frac{e^{-3x}}{-3}-\int\cos x\cdot\frac{e^{-3x}}{-3}dx\right]$$

$$=-\frac{1}{3}\cos x\cdot e^{-3x}+\frac{1}{9}\sin x\cdot e^{-3x}-\frac{1}{9}\int\cos x\cdot e^{-3x}dx$$

$$I_1+\frac{1}{9}I_1=-\frac{1}{3}e^{-3x}\cdot\cos x+\frac{1}{9}\sin x\cdot e^{-3x}$$

$$\left(\frac{10}{9}\right)I_1=-\frac{1}{3}e^{-3x}\cdot\cos x+\frac{1}{9}\sin\cdot e^{-3x}$$

$$I_1=\frac{-3}{10}e^{-3x}\cdot\cos x+\frac{1}{10}e^{-3x}\sin x$$

$$2I=-\frac{1}{3}e^{-3x}\left[\sin^3 x+\cos^3 x\right]+\frac{1}{3}\sin x\cdot e^{-3x}$$

$$-\frac{3}{10}e^{-3x}\cdot\cos x+\frac{1}{10}e^{-3x}\cdot\sin x+C$$

$$\therefore I=-\frac{1}{6}e^{-3x}\left[\sin^3 x+\cos^3 x\right]+\frac{13}{30}e^{-3x}\cdot\sin x$$

$$-\frac{3}{10}e^{-3x}\cdot\cos x+C$$

$$\left[\begin{array}{l}\because \sin 3x=3\sin x-4\sin^3 x \text{ and } \cos 3x\\ =4\cos^3 x-3\cos x\end{array}\right]$$

$$=\frac{e^{-3x}}{24}\left[\sin 3x-\cos 3x\right]+\frac{3e^{-3x}}{40}\left[\sin x-3\cos x\right]+C$$

[2½]

Q. 8. Evaluate : $\int\sqrt{\tan x}dx$ (Hint : Put $\tan x=t^2$) .

[NCERT Exemp. Ex. 7.3, Q. 43, Page 166]

Ans. Let,

$$I=\int\sqrt{\tan x}dx$$

Put $\tan x=t^2$

$\sec^2 x dx=2t\ dt$

$$I=\int t\cdot\frac{2t}{\sec^2 x}dt$$

$$=2\int\frac{t^2}{1+t^4}dt$$

$$=\int\frac{(t^2+1)+(t^2-1)}{(1+t^4)}dt$$

$$=\int\frac{t^2+1}{1+t^4}dt+\int\frac{t^2-1}{1+t^4}dt$$

$$=\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt+\int\frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}$$

$$=\int\frac{1-\left(-\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2}+\int\frac{1+\left(-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2}dt$$

[2½]

Put $u = t - \frac{1}{t}$

$$du = \left(1 + \frac{1}{t^2}\right)dt$$

and $v = t + \frac{1}{t}$

$$dv = \left(1 - \frac{1}{t^2}\right)dt$$

$$I = \int \frac{du}{u^2 + (\sqrt{2})^2} + \int \frac{dv}{v^2 - (\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1}\right| + C$$

[2½]

Q. 9. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$.

(Hint : Divide numerator and denominator by $\cos^4 x$) [NCERT Exemp. Ex. 7.3, Q. 44, Page 166]

Ans. Let,

$$I = \int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$$

Divide numerator and denominator by $\cos^4 x$, we get

$$I = \int_0^{\pi/2} \frac{\sec^4 x dx}{(a^2 + b^2\tan^2 x)^2}$$

$$= \int_0^{\pi/2} \frac{(1+\tan^2 x)\sec^2 x dx}{(a^2 + b^2\tan^2 x)^2}$$

Put $\tan x = t$

$\sec^2 x dx = dt$

As $x \to 0$, then $t \to 0$

and $x \to \frac{\pi}{2}$, then $t \to \infty$

$$I = \int_0^{\infty} \frac{(1+t^2)}{\left(a^2 + b^2t^2\right)^2}$$

Let $t^2 = u$ Then,

$$\frac{1+u}{(a^2+b^2u)^2} = \frac{A}{(a^2+b^2u)} + \frac{B}{(a^2+b^2u)^2}$$

$$1 + u = A(a^2 + b^2u) + B$$

On comparing the coefficient of x and constant term on both sides, we get

$a^2A + B = 1$ and $b^2A = 1$

$\therefore A = \frac{1}{b^2}$ **[2½]**

Now, $\frac{a^2}{b^2} + B = 1$

$$B = 1 - \frac{a^2}{b^2} = \frac{b^2 - a^2}{b^2}$$

$$I = \int_0^{\infty} \frac{(1+t^2)}{(a^2+b^2t^2)^2}$$

$$= \frac{1}{b^2}\int_0^{\infty} \frac{dt}{a^2+b^2t^2} + \frac{b^2-a^2}{b^2}\int_0^{\infty} \frac{dt}{(a^2+b^2t^2)^2}$$

$$= \frac{1}{b^2}\int_0^{\infty} \frac{dt}{b^2\left(\frac{a^2}{b^2}+t^2\right)} + \frac{b^2-a^2}{b^2}\int_0^{\infty} \frac{dt}{(a^2+b^2t^2)^2}$$

$$= \frac{1}{ab^3}\left[\tan^{-1}\left(\frac{tb}{a}\right)\right]_0^{\infty} + \frac{b^2-a^2}{b^2}\left(\frac{\pi}{4}\cdot\frac{1}{a^3b}\right)$$

$$= \frac{1}{ab^3}\left[\tan^{-1}\infty - \tan^{-1}0\right] + \frac{\pi}{4}\cdot\frac{b^2-a^2}{(a^3b^3)}$$

$$= \frac{\pi}{2ab^3} + \frac{\pi}{4}\cdot\frac{b^2-a^2}{(a^3b^3)}$$

$$= \pi\left(\frac{2a^2+b^2-a^2}{4a^3b^3}\right) = \frac{\pi}{4}\left(\frac{a^2+b^2}{a^3b^3}\right)$$

[2½]

Q. 10. Integrate the function $\frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^4}$.

[NCERT Misc. Ex. Q. 24, Page 353]

Ans.

$$\frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^4} = \frac{\sqrt{x^2+1}}{x^4}\left[\log(x^2+1) - \log x^2\right]$$

$$= \frac{\sqrt{x^2+1}}{x^4}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$

$$= \frac{\sqrt{x^2+1}}{x^4}\log\left(1+\frac{1}{x^2}\right)$$

$$= \frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

$$= \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

Let,

$$1 + \frac{1}{x^2} = t$$

$$\frac{-2}{x^3}dx = dt$$

$$I = \int \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)dx$$

$$= -\frac{1}{2}\int \sqrt{t}\log t\, dt$$

$$= -\frac{1}{2}\int t^{\frac{1}{2}}\cdot\log t\, dt$$

[2½]

Integrating by parts, we obtain

$$I = -\frac{1}{2}\left[\log t\cdot\int t^{\frac{1}{2}}dt - \left\{\left(\frac{d}{dt}\log t\right)\int t^{\frac{1}{2}}dt\right\}dt\right]$$

$$= -\frac{1}{2}\left[\log t\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int\frac{1}{t}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t - \frac{2}{3}\int t^{\frac{1}{2}}dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t - \frac{4}{9}t^{\frac{3}{2}}\right] + C$$

$$= -\frac{1}{3}t^{\frac{3}{2}}\log t + \frac{2}{9}t^{\frac{3}{2}} + C$$

$$= -\frac{1}{3}t^{\frac{3}{2}}\left[\log t - \frac{2}{3}\right] + C$$

$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$$

[2½]

Q. 11. Integrate the function $\frac{1}{\sqrt{(x-1)(x-2)}}$.

[NCERT Ex. 7.4, Q. 13, Page 316]

Ans. Functions $(x-1)(x-2)$ can be written as x^2-3x+2. Therefore,

$$x^2-3x+2 = x^2-3x+\frac{9}{4}-\frac{9}{4}+2$$

$$= \left(x-\frac{3}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\int\frac{1}{\sqrt{(x-1)(x-2)}}dx = \int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$dx = dt$

$$\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx = \int\frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}dt$$

$$= \log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right| + C$$

$$= \log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right| + C$$

[5]

Q. 12. Integrate the function $\frac{1}{\sqrt{8+3x-x^2}}$.

[NCERT Ex. 7.4, Q. 14, Page 316]

Ans. Function $8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right) = \frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\int\frac{1}{\sqrt{8+3x-x^2}}dx = \int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$dx = dt$

$$\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx = \int\frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right) + C$$

$$= \sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C$$

$$= \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + C$$

[5]

Q. 13. Integrate the function $\frac{1}{\sqrt{(x-a)(x-b)}}$.

[NCERT Ex. 7.4, Q. 15, Page 316]

Ans. Function $(x-a)(x-b)$ can be written as $x^2-(a+b)x+ab$.

Therefore,

$$x^2-(a+b)x+ab = x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$

$$= \left[x-\left(\frac{a+b}{2}\right)\right]^2 - \frac{(a-b)^2}{4}$$

$$\int\frac{1}{\sqrt{(x-a)(x-b)}}dx = \int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t$

$dx = dt$

$$\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx = \int\frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$

$$= \log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right| + C$$

$$= \log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right| + C$$

[5]

Q. 14. By using the properties of definite integrals, evaluate the integral of $\int_0^{\pi}\frac{x\,dx}{1+\sin x}$.

[NCERT Ex. 7.11, Q. 12, Page 347]

Ans. Let $I = \int_0^{\pi}\frac{x\,dx}{1+\sin x}$ ...(i)

$$I = \int_0^{\pi}\frac{(\pi-x)}{1+\sin(\pi-x)}dx \quad \left[\because \int_0^a f(x)dx = \int_0^a (a-x)dx\right]$$

$$= \int_0^{\pi}\frac{(\pi-x)}{1+\sin x}dx \quad \text{...(ii)}$$

Adding (i) and (ii), we obtain

$$2I = \int_0^{\pi}\frac{\pi}{1+\sin x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} dx$$

$$\Rightarrow 2I = \pi \left[\tan x - \sec x\right]_0^{\pi}$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$ **[5]**

Q. 15. Integrate the function $\dfrac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}$.

[NCERT Misc. Ex. Q. 18, Page 352]

Ans. $$\frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} = \frac{1}{\sqrt{\sin^3 x(\sin x \cos\alpha + \cos x \sin\alpha)}}$$

$$= \frac{1}{\sqrt{\sin^4 x \cos\alpha + \sin^3 x \cos x \sin\alpha}}$$

$$= \frac{1}{\sin^2 x\sqrt{\cos\alpha + \cot x \sin\alpha}}$$

$$= \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x \sin\alpha}}$$

Let cos α + cot x sin α = t – cosec² x sin α $dx = t$

$$\int \frac{1}{\sin^3 x \sin(x+\alpha)} dx = \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x \sin\alpha}} dx$$

$$= \frac{-1}{\sin\alpha} \int \frac{dt}{\sqrt{t}}$$

$$= \frac{-1}{\sin\alpha}\left[2\sqrt{t}\right] + C$$

$$= \frac{-1}{\sin\alpha}\left[2\sqrt{\cos\alpha + \cot x \sin\alpha}\right] + C$$

$$= \frac{-2}{\sin\alpha}\sqrt{\cos\alpha + \frac{\cos x \sin\alpha}{\sin x}} + C$$

$$= \frac{-2}{\sin\alpha}\sqrt{\frac{\sin x \cos\alpha + \cos x \sin\alpha}{\sin x}} + C$$

$$= -\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$$ **[5]**

Q. 16. Integrate the function $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}, x \in [0,1]$.

[NCERT Misc. Ex. Q. 19, Page 352]

Ans. Let $I = \int \dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

It is known that,

$$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$$

$$I = \int \frac{\left(\frac{\pi}{2} - \cos^{-1}\sqrt{x}\right) - \cos^{-1}\sqrt{x}}{\frac{\pi}{2}} dx$$

$$= \frac{2}{\pi}\int\left(\frac{\pi}{2} - 2\cos^{-1}\sqrt{x}\right)dx$$

$$= \frac{2}{\pi}\cdot\frac{\pi}{2}\int 1.dx - \frac{4}{\pi}\int \cos^{-1}\sqrt{x}dx$$

$$= x - \frac{4}{\pi}\int \cos^{-1}\sqrt{x}dx \quad \ldots(i)$$

Let $I_1 = \int \cos^{-1}\sqrt{x}dx$

Also, let $\sqrt{x} = t \Rightarrow dx = 2t\ dt$

$$\Rightarrow I_1 = 2\int \cos^{-1} t.t\ dt$$

$$= 2\left[\cos^{-1} t \cdot \frac{t^2}{2} - \int \frac{-1}{\sqrt{1-t^2}} \cdot \frac{t^2}{2} dt\right]$$

$$= t^2 \cos^{-1} t + \int \frac{t^2}{\sqrt{1-t^2}} dt$$

$$= t^2 \cos^{-1} t - \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt$$

$$= t^2 \cos^{-1} t - \int \sqrt{1-t^2} dt + \int \frac{1}{\sqrt{1-t^2}} dt$$

$$= t^2 \cos^{-1} t - \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}\sin^{-1} t + \sin^{-1} t$$

$$= t^2 \cos^{-1} t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t$$ **[2½]**

From equation (i), we obtain

$$I = x - \frac{4}{\pi}\left[t^2 \cos^{-1} t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t\right]$$

$$= x - \frac{\pi}{4}\left[x\cos^{-1}\sqrt{x} - \frac{\sqrt{x}}{2}\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{x}\right]$$

$$= x - \frac{\pi}{4}\left[x\left(\frac{\pi}{2} - \sin^{-1}(\sqrt{x})\right) - \frac{\sqrt{x-x^2}}{2} + \frac{1}{2}\sin^{-1}(\sqrt{x})\right]$$

$$= x - \ x + \frac{4}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - \frac{2}{\pi}\sin^{-1}\sqrt{x}$$

$$= -x + \frac{2}{x}\left[(2x-1)\sin^{-1}\sqrt{x}\right] + \frac{2}{\pi}\sqrt{x-x^2} + C$$

$$= \frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - x + C$$ **[2½]**

Q. 17. Integrate the function $\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$.

[NCERT Misc. Ex. Q. 20, Page 353]

Ans. $$I = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

Let $x = \cos^2\theta$

$dx = -2\sin\theta \cos\theta\ d\theta$

$$I = \int \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-2\sin\theta \cos\theta) d\theta$$

$$= -\int \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}} \sin 2\theta\, d\theta$$

$$= -\int \tan\frac{\theta}{2} \cdot 2\sin\theta \cos\theta\, d\theta$$

$$= -2\int \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta\, d\theta$$ **[2½]**

$$= -4\int \sin^2\frac{\theta}{2}\cos\theta\, d\theta$$

$$= -4\int \sin^2\frac{\theta}{2}\cdot\left(2\cos^2\frac{\theta}{2}-1\right)d\theta$$
$$= -4\int\left(2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)d\theta$$
$$= -8\int\sin^2\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}d\theta+4\int\sin^2\frac{\theta}{2}d\theta$$
$$= -2\int\sin^2\theta\, d\theta+4\int\sin^2\frac{\theta}{2}d\theta$$
$$= -2\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta+4\int\frac{1-\cos\theta}{2}d\theta$$
$$= -2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]+4\left[\frac{\theta}{2}-\frac{\sin\theta}{2}\right]+C$$
$$= -\theta+\frac{\sin 2\theta}{2}+2\theta-2\sin\theta+C$$
$$= \theta+\frac{\sin 2\theta}{2}-2\sin\theta+C$$
$$= \theta+\frac{2\sin\theta\cos\theta}{2}-2\sin\theta+C$$
$$= \theta+\sqrt{1-\cos^2\theta}\cdot\cos\theta-2\sqrt{1-\cos^2\theta}+C$$
$$= \cos^{-1}\sqrt{x}+\sqrt{1-x}\cdot\sqrt{x}-2\sqrt{1-x}+C$$
$$= -2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}+C$$
$$= -2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+C$$

[2½]

Q. 18. Evaluate : $\int_0^{\frac{1}{2}}\frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (Hint : Let $x = \sin\theta$).

[NCERT Exemp. Ex. 7.3, Q. 34, Page 165]

Ans. Let $\int_0^{1/2}\frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\Rightarrow x \to 0$, then $\theta \to 0$

and $x \to \frac{1}{2}$, then $\theta \to \frac{\pi}{6}$

$$I = \int_0^{\pi/6}\frac{\cos\theta}{(1+\sin^2\theta)\cos\theta}d\theta$$
$$= \int_0^{\pi/6}\frac{1}{1+\sin^2\theta}d\theta$$
$$= \int_0^{\pi/6}\frac{1}{\cos^2\theta(\sec^2\theta+\tan^2\theta)}d\theta$$
$$= \int_0^{\pi/6}\frac{\sec^2\theta}{\sec^2\theta+\tan^2\theta}d\theta$$
$$= \int_0^{\pi/6}\frac{\sec^2\theta}{1+\tan^2\theta+\tan^2\theta}d\theta$$
$$= \int_0^{\pi/6}\frac{\sec^2\theta}{1+2\tan^2\theta}d\theta$$

[2½]

Again, put $\tan\theta = t$

$\Rightarrow \sec^2\theta\ d\theta = dt$

As $\theta \to 0$, then $t \to 0$

and $\theta \to \frac{\pi}{6}$, then $t \to \frac{1}{\sqrt{3}}$

$$I = \int_0^{1/\sqrt{3}}\frac{dt}{1+2t^2}$$
$$= \frac{1}{2}\int_0^{1/\sqrt{3}}\frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2+t^2}$$
$$= \frac{1}{2}\cdot\frac{1}{1/\sqrt{2}}\left[\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}}\right]_0^{1/\sqrt{3}}$$
$$= \frac{1}{\sqrt{2}}[\tan^{-1}(\sqrt{2}t)]_0^{1/\sqrt{3}}$$
$$= \frac{1}{\sqrt{2}}\left[\tan^{-1}\sqrt{\frac{2}{3}}-0\right]$$
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

[2½]

Q. 19. Find the integrals of the function $\sin^{-1}(\cos x)$.

[NCERT Ex. 7.3, Q. 21, Page 307]

Ans. $\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1-t^2}$

$(-\sin x)dx = dt$

$$dx = \frac{-dt}{\sin x} = \frac{-dt}{\sqrt{1-t^2}}$$
$$\int\sin^{-1}(\cos x)dx = \int\sin^{-1}t\left(\frac{-dt}{\sqrt{1-t^2}}\right) = -\int\frac{\sin^{-1}t}{\sqrt{1-t^2}}dt$$

[2½]

Let $\sin^{-1}t = u$

$$\frac{1}{\sqrt{1-t^2}}dt = du$$
$$\int\sin^{-1}(\cos x)dx = -\int u\,du$$
$$= -\frac{u^2}{2}+C$$
$$= \frac{-(\sin^{-1}t)^2}{2}+C$$
$$= \frac{-[\sin^{-1}(\cos x)]^2}{2}+C \quad \ldots\text{(i)}$$

It is known that,

$$\sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}$$
$$\sin^{-1}(\cos x) = \frac{\pi}{2}-\cos^{-1}(\cos x)$$
$$= \left(\frac{\pi}{2}-x\right)$$

Substituting in equation (i), we obtain

$$\int\sin^{-1}(\cos x)dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2}+C$$
$$= -\frac{1}{2}\left(\frac{\pi^2}{4}+x^2-\pi x\right)+C$$

$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$ **[2½]**

Q. 20. Find the integrals of the function $\frac{1}{\cos(x-a)\cos(x-b)}$.

[NCERT Ex. 7.3, Q. 22, Page 307]

Ans. $\frac{1}{\cos(x-a)\cos(x-b)}$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\therefore \int \frac{1}{\cos(x-a)\cos(x-b)}dx$$

$$= \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)| + \log|\cos(x-a)|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right] + C$$ **[5]**

Q. 21. Find the integrals of the function $\sin x \sin 2x \sin 3x$.

[NCERT Ex. 7.3, Q. 6, Page 307]

Ans. It is known that,

$$\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$\int \sin x \ \sin 2x \ \sin 3x \ dx$$

$$= \int\left[\sin x \cdot \frac{1}{2}\{\cos(2x-3x) - \cos(2x+3x)\}\right]dx$$

$$= \frac{1}{2}\int(\sin x\cos(-x) - \sin x\cos 5x)dx$$

$$= \frac{1}{2}\int(\sin x\cos x - \sin x\cos 5x)dx$$

$$= \frac{1}{2}\int\frac{\sin 2x}{2}dx - \frac{1}{2}\int\sin x\cos 5x \ dx$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x) + \sin(x-5x)\right\}dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int(\sin 6x + \sin(-4x))dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{6} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$ **[5]**

Q. 22. Evaluate the definite integral of $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x}dx$.

[NCERT Misc. Ex. Q. 26, Page 353]

Ans. Let,

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x}dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\frac{(\sin x \cos x)}{\cos^4 x}}{\frac{(\cos^4 x + \sin^4 x)}{\cos^4 x}}dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{1+\tan^4 x}dx$$

Let $\tan^2 x = t$

$\Rightarrow 2\tan x \sec^2 x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{4}, t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}1 - \tan^{-1}0\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}\right] = \frac{\pi}{8}$$

[5]

Q. 23. Evaluate the definite integral of $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4\sin^2 x}$.

[NCERT Misc. Ex. Q. 27, Page 353]

Ans.

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1-\cos^2 x)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1-\cos^2 x)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 - 4\cos^2 x}dx$$

$$\Rightarrow I = \frac{-1}{3}\int_0^{\frac{\pi}{2}} \frac{4-3\cos^2 x - 4}{4-3\cos^2 x}dx$$

$$\Rightarrow I = \frac{-1}{3}\int_0^{\frac{\pi}{2}} \frac{4-3\cos^2 x}{4-3\cos^2 x}dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4}{4-3\cos^2 x}dx$$

$$\Rightarrow I = \frac{-1}{3}\int_0^{\frac{\pi}{2}} 1dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4\sec^2 x}{4\sec^2 x - 3}dx$$

$$\Rightarrow I = \frac{-1}{3}[x]_0^{\frac{\pi}{2}} + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4\sec^2 x}{4(1+\tan^2 x)-3}dx$$

$$\Rightarrow I = -\frac{\pi}{6} + \frac{2}{3}\int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x}dx \quad \ldots(i)$$ **[2½]**

Consider, $\int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x}dx$

Let $2\tan x = t$

$2\sec^2 x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \infty$

$$\int_0^{\frac{\pi}{2}} \frac{2\sec^2 x}{1+4\tan^2 x}dx = \int_0^{\infty} \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_0^{\infty}$$

$$= \left[\tan^{-1}(\infty) - \tan^{-1}(0)\right] = \frac{\pi}{2}$$

Therefore, from equation (i), we obtain

$$I=-\frac{\pi}{6}+\frac{2}{3}\left[\frac{\pi}{2}\right]=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$ **[2½]**

Q. 24. Evaluate the definite integral of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$.

[NCERT Misc. Ex. Q. 28, Page 353]

Ans. Let

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{-(-\sin 2x)}}dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{-(-1+1-2\sin x\cos x)}}dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{1-(\sin^2 x+\cos^2 x-2\sin x\cos x)}}dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)dx}{\sqrt{1-(\sin x-\cos x)^2}}$$

Let $(\sin x-\cos x)=t$

$(\sin x+\cos x)dx=dt$

When $x=\frac{\pi}{6}, t=\left(\frac{1-\sqrt{3}}{2}\right)$ and when $x=\frac{\pi}{3}, t=\left(\frac{\sqrt{3}-1}{2}\right)$

$$I=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}=\int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$ **[2½]**

As $\frac{1}{\sqrt{1-(-t)^2}}=\frac{1}{\sqrt{1-t^2}}$, therefore, $\frac{1}{\sqrt{1-t^2}}$ is an even function.

It is known that if $f(x)$ is an even function then,

$$\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)dx$$

$$I=2\int_0^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

$$=\left[2\sin^{-1}t\right]_0^{\frac{\sqrt{3}-1}{2}}$$

$$=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$ **[2½]**

Q. 25. Evaluate the definite integral of $\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$.

[NCERT Misc. Ex. Q. 29, Page 353]

Ans. Let,

$$I=\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$=\int_0^1\frac{1}{(\sqrt{1+x}-\sqrt{x})}\times\frac{(\sqrt{1+x}+\sqrt{x})}{(\sqrt{1+x}+\sqrt{x})}dx$$

$$=\int_0^1\frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$$

$$=\int_0^1\sqrt{1+x}dx+\int_0^1\sqrt{x}dx$$

$$=\left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1+\left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$=\frac{2}{3}\left[(2)^{\frac{3}{2}}-1\right]+\frac{2}{3}[1]$$

$$=\frac{2}{3}(2)^{\frac{3}{2}}=\frac{2\cdot 2\sqrt{2}}{3}=\frac{4\sqrt{2}}{3}$$ **[5]**

Q. 26. Evaluate the definite integral $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

[NCERT Misc. Ex. Q. 30, Page 353]

Ans. Let,

$$I=\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$$

Also, let $\sin x-\cos x=t$

$\Rightarrow\quad(\cos x+\sin x)dx=dt$

When $x=0, t=-1$ and when $x=\frac{\pi}{4}, t=0$

$\Rightarrow\quad(\sin x-\cos x)^2=t^2$

$\Rightarrow\sin^2 x+\cos^2 x-2\sin x\cos x=t^2$

$\Rightarrow\quad 1-\sin 2x=t^2$

$\Rightarrow\quad \sin 2x=1-t^2$ **[2½]**

$$I=\int_{-1}^{0}\frac{dt}{9+16(1-t^2)}=\int_{-1}^{0}\frac{dt}{9+16-16t^2}$$

$$=\int_{-1}^{0}\frac{dt}{25-16t^2}=\int_{-1}^{0}\frac{dt}{(5)^2-(4t)^2}$$

$$=\frac{1}{4}\left[\frac{1}{2(5)}\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$$

$$=\frac{1}{40}\left[\log(1)-\log\left|\frac{1}{9}\right|\right]$$

$$=\frac{1}{40}\log 9$$ **[2½]**

Q. 27. Evaluate the definite integral of $\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx$.

[NCERT Misc. Ex. Q. 31, Page 353]

Ans. Let,

$$I=\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin x\cos x\tan^{-1}(\sin x)dx$$

Also, let $\sin x=t$

$\cos x dx=dt$

When $x=0, t=0$ and when $x=\frac{\pi}{2}, t=1$

$$\Rightarrow I=2\int_0^1 t\tan^{-1}(t)dt \quad \ldots(i)$$

Consider $\int t\cdot\tan^{-1}t\,dt$

$$=\tan^{-1}t\cdot\int t\,dt-\int\left\{\frac{d}{dt}\left(\tan^{-1}t\right)\int t\,dt\right\}dt$$

$$=\tan^{-1}t\cdot\frac{t^2}{2}-\int\frac{1}{1+t^2}\cdot\frac{t^2}{2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int\frac{t^2+1-1}{1+t^2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int 1dt+\frac{1}{2}\int\frac{1}{1+t^2}dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \cdot t + \frac{1}{2} \tan^{-1} t$$ **[2½]**

$$\int_0^1 t \cdot \tan^{-1} t \, dt = \left[\frac{t^2 \cdot \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 1\right] = \frac{\pi}{4} - \frac{1}{2}$$

From equation (i), we obtain

$$I = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$$ **[2½]**

Q. 28. Evaluate the definite integral of $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$.

[NCERT Misc. Ex. Q. 32, Page 353]

Ans. Let $I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$ …(i)

$$I = \int_0^\pi \left\{ \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} \right\} dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a (a - x)dx \right]$$

$$= \int_0^\pi \left\{ \frac{-(\pi - x)\tan x}{-(\sec x + \tan x)} \right\} dx$$

$$\Rightarrow \quad I = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x + \tan x} dx$$ **…(ii)** **[2½]**

$$2I = \int_0^\pi \frac{\pi \tan x}{\sec x + \tan x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\sin x + 1 - 1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi 1.dx - \pi \int_0^\pi \frac{1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi [x]_0^\pi - \pi \int_0^\pi \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi^2 - \pi \int_0^\pi (\sec^2 x - \tan x \sec x) dx$$

$$\Rightarrow 2I = \pi^2 - \pi [\tan x - \sec x]_0^\pi$$

$$\Rightarrow 2I = \pi^2 - \pi [\tan \pi - \sec \pi - \tan 0 + \sec 0]$$

$$\Rightarrow 2I = \pi^2 - \pi [0 - (-1) - 0 + 1]$$

$$\Rightarrow 2I = \pi^2 - 2\pi$$

$$\Rightarrow 2I = \pi(\pi - 2)$$

$$\Rightarrow I = \frac{\pi}{2}(\pi - 2)$$ **[2½]**

Q. 29. Evaluate the definite integral of $\int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$.

[NCERT Misc. Ex. Q. 33, Page 353]

Ans. Let $I = \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$

$$\Rightarrow \quad I = \int_1^4 |x - 1| dx + \int_1^4 |x - 2| dx + \int_1^4 |x - 3| dx$$

$$I = I_1 + I_2 + I_3$$ …(i)

where, $I_1 = \int_1^4 |x - 1| dx, I_2 = \int_1^4 |x - 2| dx$, and $I_3 = \int_1^4 |x - 3| dx$

$$I_1 = \int_1^4 |x - 1| dx$$

$(x - 1) \geq 0$ for $1 \leq x \leq 4$

$$\therefore I_1 = \int_1^4 (x - 1) dx$$

$$\Rightarrow I_1 = \left[\frac{x^2}{2} - x \right]_1^4$$

$$\Rightarrow \quad I_1 = \left[8 - 4 - \frac{1}{2} + 1 \right] = \frac{9}{2}$$ …(ii) **[2½]**

$$I_2 = \int_1^4 |x - 2| dx$$

$x - 2 \geq 0$ for $2 \leq x \leq 4$ and $x - 2 \leq 0$ for $1 \leq x \leq 2$

$$\therefore \quad I_2 = \int_1^2 (2 - x) dx + \int_2^4 (x - 2) dx$$

$$\Rightarrow \quad I_2 = \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^4$$

$$\Rightarrow \quad I_2 = \left[4 - 2 - 2 + \frac{1}{2} \right] + [8 - 8 - 2 + 4]$$

$$\Rightarrow \quad I_2 = \frac{1}{2} + 2 = \frac{5}{2}$$ …(iii)

$$I_3 = \int_1^4 |x - 3| dx$$

$x - 3 \geq 0$ for $3 \leq x \leq 4$ and $x - 3 \leq 0$ for $1 \leq x \leq 3$

$$I_3 = \int_1^3 (3 - x) dx + \int_3^4 (x - 3) dx$$

$$= \left[3x - \frac{x^2}{2} \right]_1^3 + \left[\frac{x^2}{2} - 3x \right]_3^4$$

$$= \left[9 - \frac{9}{2} - 3 + \frac{1}{2} \right] + \left[8 - 12 - \frac{9}{2} + 9 \right]$$

$$\therefore \quad I_3 = [6 - 4] + \left[\frac{1}{2} \right] = \frac{5}{2}$$ …(iv)

From equations (i) to (iv), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$ **[2½]**

Q. 30. Prove : $\int_1^3 \frac{dx}{x^2(x + 1)} = \frac{2}{3} + \log \frac{2}{3}$.

[NCERT Misc. Ex. Q. 34, Page 353]

Ans. Let,

$$I = \int_1^3 \frac{dx}{x^2(x + 1)}$$

Also, let $\frac{1}{x^2(x + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1}$

$$\Rightarrow 1 = Ax(x + 1) + B(x + 1) + C(x^2)$$

$$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$A + B = 0$

$B = 1$

On solving these equations, we obtain

$A = -1, C = 1$, and $B = 1$

$$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$$

$$\Rightarrow \quad I = \int_1^3 \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$$ **[2½]**

$$= \left[-\log x - \frac{1}{x} + \log(x+1) \right]_1^3$$

$$= \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x} \right]_1^3$$

$$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$$

$$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$$

$$= \log 2 - \log 3 + \frac{2}{3} = \log\left(\frac{2}{3}\right) + \frac{2}{3}$$

Hence, the given result is proved. **[2½]**

Q. 31. Evaluate : $\int \frac{x^2 dx}{x^4 - x^2 - 12}$.

[NCERT Exemp. Ex. 7.3, Q. 35, Page 165]

Ans. Let, $$I = \int \frac{x^2}{x^4 - x^2 - 12} dx$$

$$= \int \frac{x^2}{x^4 - 4x^2 + 3x^2 - 12} dx$$

$$= \int \frac{x^2 dx}{x^2(x^2-4) + 3(x^2-4)}$$

$$= \int \frac{x^2 dx}{(x^2-4)(x^2+3)}$$

Now, $\frac{x^2}{(x^2-4)(x^2+3)}$ $\quad [\text{Let } x^2 = t]$

$$\frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3}$$

$$t = A(t+3) + B(t-4)$$

On comparing the coefficients of t on both sides, we get

$$A + B = 1$$

$$\Rightarrow \quad 3A - 4B = 0$$

$$\Rightarrow 3(1-B) - 4B = 0$$

$$\Rightarrow \quad 3 - 3B - 4B = 0$$

$$\Rightarrow \quad 7B = 3$$

$$\Rightarrow \quad B = \frac{3}{7}$$

If $B = \frac{3}{7}$, then $A + \frac{3}{7} = 1$ **[2½]**

$$A = 1 - \frac{3}{7} = \frac{4}{7}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7(x^2-4)} + \frac{3}{7(x^2+3)}$$

$$I = \frac{4}{7} \int \frac{1}{x^2 - (2)^2} dx + \frac{3}{7} \int \frac{1}{x^2 + (\sqrt{3})^2} dx$$

$$= \frac{4}{7} \cdot \frac{1}{2 \cdot 2} \log\left|\frac{x-2}{x+2}\right| + \frac{3}{7} \cdot \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C$$

$$= \frac{1}{7} \log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7} \tan^{-1} \frac{x}{\sqrt{3}} + C$$ **[2½]**

Q. 32. Integrate the function $\frac{5x}{(x+1)(x^2+9)}$.

[NCERT Misc. Ex. Q. 6, Page 352]

Ans. Let $$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+9)} \quad \ldots(i)$$

$$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$$

$$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}$, and $C = \frac{9}{2}$

From equation (i), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)} + \frac{\frac{x}{2} + \frac{9}{2}}{(x^2+9)}$$ **[2½]**

$$\int \frac{5x}{(x+1)(x^2+9)} dx$$

$$= \int \left\{ \frac{-1}{2(x+1)} + \frac{(x+9)}{2(x^2+9)} \right\} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{2} \int \frac{x}{x^2+9} dx + \frac{9}{2} \int \frac{1}{x^2+9} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \int \frac{2x}{x^2+9} dx + \frac{9}{2} \int \frac{1}{x^2+9} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \log|x^2+9| + \frac{9}{2} \cdot \frac{1}{3} \tan^{-1} \frac{x}{3}$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \log(x^2+9) + \frac{3}{2} \tan^{-1} \frac{x}{3} + C$$ **[2½]**

Q. 33. Integrate the function $\frac{5x-2}{1+2x+3x^2}$.

[NCERT Ex. 7.4, Q. 18, Page 316]

Ans. Let $5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$

$$\Rightarrow \quad 5x - 2 = A(2 + 6x) + B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$5 = 6A \Rightarrow A = \frac{5}{6}$$

$$2A + B = -2 \Rightarrow B = -\frac{11}{3}$$

$$\therefore 5x - 2 = \frac{5}{6}(2 + 6x) + \left(-\frac{11}{3}\right)$$

$$\Rightarrow \int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} dx$$

$$= \frac{5}{6} \int \frac{2+6x}{1+2x+3x^2} dx - \frac{11}{3} \int \frac{1}{1+2x+3x^2} dx$$

Let $I_1 = \int \frac{2+6x}{1+2x+3x^2} dx$ and $I_2 = \int \frac{1}{1+2x+3x^2} dx$

$$\therefore \int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \ldots(i)$$

$$I_1 = \int \frac{2+6x}{1+2x+3x^2}dx$$

Let $1+2x+3x^2 = t$

$\Rightarrow \quad (2+6x)dx = dt$

$\therefore \quad I_1 = \int \frac{dt}{t}$

$I_1 = \log|t|$

$I_1 = \log|1+2x+3x^2| \quad \ldots\text{(ii)}$

$$I_2 = \int \frac{1}{1+2x+3x^3}dx$$

$1+2x+3x^2$ can be written as

$1+3\left(x^2+\frac{2}{3}x\right)$.

Therefore,

$$1+3\left(x^2+\frac{2}{3}x\right) = 1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$$

$$= 1+3\left(x+\frac{1}{3}\right)^2 - \frac{1}{3}$$

$$= \frac{2}{3}+3\left(x+\frac{1}{3}\right)^2$$

$$= 3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]$$

$$= 3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$$

$$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]}dx$$

[2½]

$$= \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]$$

$$= \frac{1}{3}\left[\frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) \quad \ldots\text{(iii)}$$

Substituting equations (ii) and (iii) in equation (i), we obtain

$$\int \frac{5x-2}{1+2x+3x^2}dx = \frac{5}{6}\left[\log|1+2x+3x^2|\right] - \frac{11}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]+C$$

$$= \frac{5}{6}\log|1+2x+3x^2|$$

$$= \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$$

[2½]

Q. 34. Integrate the function $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$.

[NCERT Ex. 7.4, Q. 19, Page 316]

Ans. $\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$

Let $\quad 6x+7 = A\frac{d}{dx}(x^2-9x+20)+B$

$\Rightarrow \quad 6x+7 = A(2x-9)+B$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

$-9A+B = 7 \Rightarrow B = 34$

$\therefore 6x+7 = 3(2x-9)+34$

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$

$$= 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int \frac{1}{\sqrt{x^2-9x+20}}dx$$

Let $I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$$\therefore \int \frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1+34I_2$$

…(i) **[2½]**

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$$

Let $x^2-9x+20 = t$

$\Rightarrow \quad (2x-9)dx = dt$

$\Rightarrow \quad I_1 = \frac{dt}{\sqrt{t}}$

$I_1 = 2\sqrt{t}$

$I_1 = 2\sqrt{x^2-9x+20} \quad \ldots\text{(ii)}$

and $\quad I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$x^2-9x+20$ can be written as $x^2-9x+20+\frac{81}{4}-\frac{81}{4}$.

Therefore,

$$x^2-9x+20+\frac{81}{4}-\frac{81}{4}$$

$$= \left(x-\frac{9}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2 = \int \frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

$$I_2 = \log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right| \quad \ldots\text{(iii)}$$

Substituting equations (ii) and (iii) in (i), we obtain

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3\left[2\sqrt{x^2-9x+20}\right] + 34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$

$$= 6\sqrt{x^2 - 9x + 20} + 34\log \left[\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\right] + C$$ **[2½]**

Q. 35. Integrate the function $\frac{x+2}{\sqrt{4x - x^2}}$.

[NCERT Ex. 7.4, Q. 20, Page 316]

Ans. Let $x + 2 = A\frac{d}{dx}(4x - x^2) + B$

$\Rightarrow x + 2 = A(4 - 2x) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

$$4A + B = 2 \Rightarrow B = 4$$

$$\Rightarrow \quad (x+2) = -\frac{1}{2}(4 - 2x) + 4$$

$$\therefore \int \frac{x+2}{\sqrt{4x - x^2}}dx = \int \frac{\frac{-1}{2}(4-2x) + 4}{\sqrt{4x - x^2}}dx$$

$$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int \frac{1}{\sqrt{4x-x^2}}dx$$

Let $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2 \int \frac{1}{\sqrt{4x-x^2}}dx$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}I_1 + 4I_2 \quad \ldots(i)$$

Then, $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x - x^2 = t$

$\Rightarrow (4 - 2x)dx = dt$

$$\Rightarrow \quad I_1 = \int \frac{1}{\sqrt{t}}dt = 2\sqrt{t} = 2\sqrt{4x - x^2} \quad \ldots(ii)$$

$$I_2 = \int \frac{1}{\sqrt{4x-x^2}}dx$$

$$\Rightarrow \quad 4x - x^2 = -(-4x + x^2)$$

$$= (-4x + x^2 + 4 - 4)$$

$$= 4 - (x-2)^2$$

$$= (2)^2 - (x-2)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}}dx = \sin^{-1}\left(\frac{x-2}{2}\right) \quad \ldots(iii)$$

Using equations (ii) and (iii) in (i), we obtain **[2½]**

$$\int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}\left(2\sqrt{4x-x^2}\right) + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

$$= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$ **[2½]**

Q. 36. Integrate the function $\frac{x+2}{\sqrt{x^2 + 2x + 3}}$.

[NCERT Ex. 7.4, Q. 21, Page 316]

Ans. $$\int \frac{(x+2)}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2 \quad \ldots(i)$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x+2)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad \ldots(ii)$$ **[2½]**

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$\Rightarrow x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x+1)^2 + (\sqrt{2})^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})^2}}dx = \log\left|(x+1) + \sqrt{x^2+2x+3}\right| \quad \ldots(iii)$$

Using equations (ii) and (iii) in (i), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$$

$$= \sqrt{x^2+2x+3} + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$$ **[2½]**

Q. 37. Integrate the function $\frac{x+3}{x^2 - 2x - 5}$.

[NCERT Ex. 7.4, Q. 22, Page 316]

Ans. Let $(x+3) = A\frac{d}{dx}(x^2 - 2x - 5) + B$

$(x+3) = A(2x - 2) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1$$

$$A = \frac{1}{2}$$

$$-2A + B = 3$$

$$B = 4$$

$$(x+3) = \frac{1}{2}(2x-2) + 4$$

$$\int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2) + 4}{x^2-2x-5}dx$$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

Let $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

and $I_2 = \int \frac{1}{x^2-2x-5}dx$

$\therefore \int \frac{x+3}{(x^2-2x-5)}dx = \frac{1}{2}I_1 + 4I_2$ ...(i) **[2½]**

Then, $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x-2)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5|$ ...(ii)

$I_2 = \int \frac{1}{x^2-2x-5}dx$

$= \int \frac{1}{(x^2-2x+1)-6}dx$

$= \int \frac{1}{(x-1)^2+(\sqrt{6})^2}dx$

$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right)$ ...(iii)

Substituting (ii) and (iii) in (i), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log|x^2-2x-5| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

$$= \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$ **[2½]**

Q. 38. Evaluate : $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$.

[NCERT Exemp. Ex. 7.3, Q. 36, Page 165]

Ans. Let $I = \int \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$

Now, $\frac{x^2}{(x^2+a^2)(x^2+b^2)}$ $[\text{Let } x^2 = t]$

$= \frac{t}{(t+a^2)(t+b^2)} = \frac{A}{(t+a^2)} + \frac{B}{(t+b^2)}$

$t = A(t+b^2) + B(t+a^2)$

On comparing the coefficients of t, we get

$A + B = 1$ ...(i)

$b^2A + a^2B = 0$ ...(ii)

$b^2(1-B) + a^2B = 0$

$b^2 - b^2B + a^2B = 0$

$b^2 + (a^2-b^2)B = 0$

$B = \frac{-b^2}{a^2-b^2}$

$= \frac{b^2}{b^2-a^2}$

From equation (i), we have

$A + \frac{b^2}{b^2-a^2} = 1$

$A = \frac{b^2-a^2-b^2}{b^2-a^2}$

$= \frac{-a^2}{b^2-a^2}$

[2½]

$$I = \int \frac{-a^2}{(b^2-a^2)(x^2+a^2)}dx + \int \frac{b^2}{b^2-a^2}\cdot\frac{1}{x^2+b^2}dx$$

$$= \frac{-a^2}{(b^2-a^2)}\int \frac{1}{x^2+a^2}dx + \frac{b^2}{b^2-a^2}\int \frac{1}{x^2+b^2}dx$$

$$= \frac{-a^2}{b^2-a^2}\cdot\frac{1}{a}\tan^{-1}\frac{x}{a} + \frac{b^2}{b^2-a^2}\cdot\frac{1}{b}\tan^{-1}\frac{x}{b}$$

$$= \frac{1}{b^2-a^2}\left[-a\tan^{-1}\frac{x}{a} + b\tan^{-1}\frac{x}{b}\right]$$

$$= \frac{1}{a^2-b^2}\left[a\tan^{-1}\frac{x}{a} - b\tan^{-1}\frac{x}{b}\right]$$ **[2½]**

Q. 39. Find the integrals of the function $\sin^4 x$.

[NCERT Ex. 7.3, Q. 10, Page 307]

Ans. $\sin^4 x = \sin^2 x \sin^2 x$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{4}(1-\cos 2x)^2$

$= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right]$

$= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right]$

$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]$

$= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]$

$\int \sin^4 x\,dx = \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]dx$

$= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \left(\frac{2\sin 2x}{2}\right)\right] + C$

$= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} - 2\sin 2x\right] + C$

$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

[5]

Q. 40. Find the integrals of the function $\cos^4 2x$.

[NCERT Ex. 7.3, Q. 11, Page 307]

Ans. $\cos^4 2x = \left(\cos^2 2x\right)^2$

$= \left(\frac{1+\cos 4x}{2}\right)^2$

$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$

$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right) + 2\cos 4x\right]$

$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2} + 2\cos 4x\right]$

$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2} + 2\cos 4x\right]$

$$\int \cos^4 2x dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$
$$=\frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$ **[5]**

Q. 41. Find the integrals of the function $\frac{\sin^2 x}{1+\cos x}$.

[NCERT Ex. 7.3, Q. 12, Page 307]

Ans. $$\frac{\sin^2 x}{1+\cos x}=\frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$\left[\because \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$
$$=2\sin^2\frac{x}{2}$$
$$=1-\cos x$$

$$\int\frac{\sin^2 x}{1+\cos x}dx=\int(1-\cos x)dx$$
$$=x-\sin x+C$$ **[5]**

Q. 42. Find the integrals of the function $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$.

[NCERT Ex. 7.3, Q. 13, Page 307]

Ans. $$\left[\because \cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2[\cos(x)+\cos\alpha]$$
$$=2\cos x+2\cos\alpha$$

$$\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int[2\cos x+2\cos\alpha]$$
$$=2[\sin x+x\cos\alpha]+C$$ **[5]**

Q. 43. Integrate the function $\frac{1}{x-x^3}$.

[NCERT Misc. Ex. Q. 1, Page 352]

Ans. $$\frac{1}{x-x^3}=\frac{1}{x(1-x^2)}$$
$$=\frac{1}{x(1-x)(1+x)}$$

Let,

$$\frac{1}{x(1-x)(1+x)}=\frac{A}{x}+\frac{B}{(1-x)}+\frac{C}{1+x} \quad \ldots(i)$$
$$\Rightarrow 1=A(1-x^2)+Bx(1+x)+Cx(1-x)$$
$$\Rightarrow 1=A-Ax^2+Bx+Bx^2+Cx-Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$$-A+B-C=0$$
$$B+C=0$$
$$A=1$$

On solving these equations, we obtain **[2½]**

$$A=1, B=\frac{1}{2}, \text{ and } C=-\frac{1}{2}$$

From equation (i), we obtain

$$\frac{1}{x(1-x)(1+x)}=\frac{1}{x}+\frac{1}{2(1-x)}-\frac{1}{2(1+x)}$$
$$\int\frac{1}{x(1-x)(1+x)}dx=\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{1-x}dx-\frac{1}{2}\int\frac{1}{1+x}dx$$
$$=\log|x|-\frac{1}{2}\log|(1-x)|-\frac{1}{2}\log|(1+x)|$$
$$=\log|x|-\log\left|(1-x)^{\frac{1}{2}}\right|-\log\left|(1+x)^{\frac{1}{2}}\right|$$
$$=\log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right|+C$$
$$=\log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right|+C$$
$$=\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$$

[2½]

Q. 44. Find the integrals of the function $\frac{\cos x-\sin x}{1+\sin 2x}$.

[NCERT Ex. 7.3, Q. 14, Page 307]

Ans. $$\left[\because \sin^2 x+\cos^2 x=1 \text{ and } \sin 2x = 2\sin x\cos x\right]$$

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{(\sin^2 x+\cos^2)+2\sin x\cos x}$$
$$=\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$$

Let $\sin x+\cos x=t$

$(\cos x-\sin x)dx=dt$

$$\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$$

$$= \int \frac{dt}{t^2} = \int t^{-2} dt$$

$$= -t^{-1} + C = -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$ **[5]**

Q. 45. Find the integrals of the function $\tan^3 2x \sec 2x$.
[NCERT Ex. 7.3, Q. 15, Page 307]

Ans. $\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$

$$= (\sec^2 2x - 1)\tan 2x \sec 2x$$

$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$

$$\int \tan^3 2x \sec 2x\, dx = \int \sec^2 2x \tan 2x \sec 2x\, dx - \int \tan 2x \sec 2x\, dx$$

$$= \int \sec^2 2x \tan 2x \sec 2x\, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$2 \sec 2x \tan 2x\, dx = dt$

$$\int \tan^3 2x \sec 2x\, dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$

$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$

$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$ **[5]**

Q. 46. Find the integrals of the function $\tan^4 x$.
[NCERT Ex. 7.3, Q. 16, Page 307]

Ans. $\tan^4 x = \tan^2 x \tan^2 x$

$$= (\sec^2 x - 1)\tan^2 x$$

$$= \sec^2 x \tan^2 x - \tan^2 x$$

$$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$$

$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

$$\int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx + \int 1 \cdot dx$$

$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C \qquad \text{...(i)}$$

Consider $\int \sec^2 x \tan^2 x\, dx$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\int \sec^2 x \tan^2 x dx = \int t^2 dt$$

$$= \frac{t^2}{3} = \frac{\tan^3 x}{3}$$

From equation (i), we obtain

$$\therefore \int \tan^4 x\, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$ **[5]**

Q. 47. Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x}$.
[NCERT Exemp. Ex. 7.3, Q. 37, Page 165]

Ans. Let $I = \int_0^{\pi} \frac{x}{1 + \sin x} dx$ ...(i)

And $I = \int_0^{\pi} \frac{\pi - x}{1 + \sin(\pi - x)} dx = \int_0^{\pi} \frac{\pi - x}{1 + \sin x} dx$ ...(ii)

On adding equations (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \sin x} dx$$

$$= \pi \int_0^{\pi} \frac{(1 - \sin x)dx}{(1 + \sin x)(1 - \sin x)}$$

$$= \pi \int_0^{\pi} \frac{(1 - \sin x)dx}{\cos^2 x}$$

$$= \pi \int_0^{\pi} (\sec^2 x - \tan x \cdot \sec x) dx$$

$$= \pi \int_0^{\pi} \sec^2 x\, dx - \pi \int_0^{\pi} \sec x \cdot \tan x\, dx$$

$$= \pi [\tan x]_0^{\pi} - \pi [\sec x]_0^{\pi}$$

$$= \pi [\tan x - \sec x]_0^{\pi}$$

$$= \pi [\tan \pi - \sec \pi - \tan 0 + \sec 0]$$

$$2I = \pi [0 + 1 - 0 + 1]$$

$$2I = 2\pi$$

$$I = \pi$$ **[5]**

Q. 48. By using the properties of definite integrals, evaluate the integral $\int_0^{\pi} \log(1 + \cos x)dx$.
[NCERT Ex. 7.11, Q. 16, Page 347]

Ans. Let $I = \int_0^{\pi} \log(1 + \cos x)dx$ ...(i)

$$I = \int_0^{\pi} \log(1 + \cos(\pi - x))dx \quad \left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x)dx$ ...(ii)

Adding equations (i) and (ii), we obtain

$$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\}dx$$

$$= \int_0^{\pi} \log(1 - \cos^2 x)dx$$

$$= \int_0^{\pi} \log \sin^2 x\, dx$$

$$= 2\int_0^{\pi} \log \sin x\, dx$$

$\therefore I = \int_0^{\pi} \log \sin x\, dx$ ...(iii)

$\sin(\pi - x) = \sin x$

$\therefore I = 2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right)dx = 2\int_0^{\frac{\pi}{2}} \log \cos x\, dx$ ...(iv)**[2½]**

Adding equations (iii) and (iv), we obtain

$$2I = 2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x)dx$$

$$I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2)dx$$

$$= \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2)dx$$

$$= \int_0^{\frac{\pi}{2}} \log \sin 2x\, dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 0, t = 0$

and when $x = \frac{\pi}{2}, t = \pi$

$\therefore \quad I = \frac{1}{2}\int_0^{\pi} \log \sin t\, dt - \frac{\pi}{2}\log 2$

$\Rightarrow \ I = \frac{I}{2} - \frac{\pi}{2}\log 2 \quad$ [From Eq. (iii)]

$\Rightarrow \frac{I}{2} = -\frac{\pi}{2}\log 2$

$\Rightarrow \ I = -\pi \log 2$ **[2½]**

Q. 49. Integrate the function $\frac{1}{(x^2+1)(x^2+4)}$.

[NCERT Misc. Ex. Q. 14, Page 352]

Ans. As we know that,

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)} \quad \ldots\text{(i)}$$

$\Rightarrow 1 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 1)$

$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$

Equation the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$A = 0, B = \frac{1}{3}, C = 0,$ and $D = -\frac{1}{3}$ **[2½]**

From equation (i), we obtain

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}$$

$$\int \frac{1}{(x^2+1)(x^2+4)}dx = \frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$$

$$= \frac{1}{3}\tan^{-1} x - \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$\int \frac{1}{(x^2+1)(x^2+4)}dx = \frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$$

$$= \frac{1}{3}\tan^{-1} x - \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= \frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$$ **[2½]**

Q. 50. Evaluate $\int_0^1 e^{2-3x}dx$ as a limit of a sum.

[NCERT Misc. Ex. Q. 40, Page 353]

Ans. Let,

$I = \int_0^1 e^{2-3x}dx$

It is known that,

$$\int_0^1 f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a)+f(a+h)+\ldots+f(a+(n-1)h)]$$

where, $h = \frac{b-a}{n}$

Here, $a = 0$, $b = 1$, and $f(x) = e^{2-3x}$

$$h = \frac{1-0}{n} = \frac{1}{n}$$

$$\int_0^1 e^{2-3x}dx = (1-0)\lim_{n\to\infty}\frac{1}{n}\left[\begin{matrix} f(0)+f(0+h) \\ +\ldots+f(0+(n-1)h)\end{matrix}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2 + e^{2-3h} + \ldots e^{2-3(n-1)h}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{1 + e^{-3h} + e^{-6h} + e^{-9h} + \ldots e^{-3(n-1)h}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-3h}\right)^n}{1-\left(e^{-3h}\right)}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-\frac{3}{n}\times n}\right)}{1-e^{-\frac{3}{n}}}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2\left(1-e^{-3}\right)}{1-e^{-\frac{3}{n}}}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{e^{-\frac{3}{n}}-1}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\left(-\frac{1}{3}\right)\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}\lim_{n\to\infty}\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}(1) \qquad \left[\because \lim_{n\to\infty}\frac{x}{e^x-1}\right]$$

$$= \frac{-e^{-1}+e^2}{3}$$

$$= \frac{1}{3}\left(e^2 - \frac{1}{e}\right)$$ **[5]**

Some Commonly Made Errors

- Properly use the formula.
- Many challenging integration problems can be solved surprisingly quickly by simply knowing the right technique to apply. While finding the right technique can be a matter of ingenuity, there are a dozen or so many techniques that permit a more comprehensive approach to solve the definite integrals.

EXPERT ADVICE

- *Always remember standard Formulae to solve integration-based problems.*
- *Knowing the nature of the functions before starting to solve the problem.*
- *Don't be in a rush to solve problems. In Board papers, both speed and strike-rate matter. You need to be quick as well as accurate to achieve high scores. High speed with low accuracy can actually ruin your results.*
- *Use graphs whenever possible.*
- *Practice questions from previous year papers, sample papers and model papers within the time frame you will have at the final exam.*
- *Focus on solving as many problems as you can, rather than just reading theories, formulae and solutions.*

OSWAAL LEARNING TOOLS

***For* Suggested Online Videos**

Visit : *https://goo.gl/UwnRHr* **Or Scan the Code**	**Visit :** *https://youtu.be/YH8nd6lCzEY* **Or Scan the Code**
Visit : *https://goo.gl/aCn6st* **Or Scan the Code**	**Visit :** *https://goo.gl/dWLZee* **Or Scan the Code**