CHAPTER

4 DETERMINANTS

Chapter Objectives

This chapter will help you understand :

- ***Determinants :*** *Introduction, Types of determinants, Properties of determinants, Area of triangles, Minors and co-factors, Adjoint and inverse of a matrix and Application of determinants and matrices.*

Quick Review

- The Wronskian of solutions of a linear ODE is a determinant. It plays a central role in spectral theory (Hill's equation with periodic co-efficients), and therefore in stability analysis of travelling waves in PDEs.
- Perron's eigen value of an irreducible non-negative matrix is a very nice use of the multi-linearity of the determinant.
- The nth root of the determinant is a concave function over the $n \times n$ Hermitian positive definite matrices. This is at the basis of many developments in modern analysis, via the Brunn-Minkowski inequality.
- In control theory, the Routh-Hurwitz algorithm, which checks whether a system is stable or not, is based on the calculation of determinants.
- As mentioned by J.M., Slater determinants are used in quantum chemistry.

TIPS...

- *Along with numerical, some theories and concepts are also important in this chapter.*
- *Properties of determinants are very important, especially properties number 6 mentioned in NCERT.*
- *In expansion of determinants, decide well before solving about one style expansion by row or column.*

TRICKS...

- *You could always do the calculation twice, once with the top row as a starting point and one (say) with the bottom row.*
- *If the matrix is structured so that a certain row or column has a lot of zeros in it then you must be sure to take advantage of this.*
- *You might consider Pivotal Condensation. Pivotal condensation can be extremely tedious; it may-or-may-not be time-effective in solving problems.*

Know the Links

- *www.sosmath.com/matrix/determ0/determ0.html*
- *https://www.math.drexel.edu/~jwd25/LM_SPRING_07/lectures/lecture4B.html*
- *https://www.ironsidegroup.com/.../determinants-the-answer-to-a-framework-manager-...*

Multiple Choice Questions

(1 mark each)

Q. 1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is

(a) 3 **(b)** ± 3

(c) ± 6 **(d)** 6

[NCERT Exemp. Ex. 4.3, Q. 24, Page 80]

Ans. Correct option : (c)

Explanation : Given that

$$\therefore \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix},$$

$$\Rightarrow 2x^2 - 40 = 18 + 14$$

$$\Rightarrow \quad 2x^2 = 32 + 40$$

$$\Rightarrow \quad x^2 = \frac{72}{2} = 36$$

$$\therefore \quad x = \pm 6$$

Q. 2. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

(a) $a^3 + b^3 + c^3$ **(b)** $3bc$

(c) $a^3 + b^3 + c^3 - 3abc$ **(d)** None of these

[NCERT Exemp. Ex. 4.3, Q. 25, Page 80]

Ans. Correct option : (d)

Explanation : We have

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} = \begin{vmatrix} a+c & b+c+a & a \\ b+c & c+a+b & b \\ c+b & a+b+c & c \end{vmatrix}$$

$[\because C_1 \to C_1 + C_2 \text{ and } C_2 \to C_2 + C_3]$

$$= (a+b+c)\begin{vmatrix} a+c & 1 & a \\ b+c & 1 & b \\ c+b & 1 & c \end{vmatrix}$$

[Taking $(a+b+c)$ common from C_2]

$$= (a+b+c)\begin{vmatrix} a-b & 0 & a-c \\ 0 & 0 & b-c \\ c+b & 1 & c \end{vmatrix}$$

$[\because R_2 \to R_2 - R_3 \text{ and } R_1 \to R_1 - R_3]$

$= (a+b+c) - [(b-c)(a-b)]$

[Expanding along R_2]

$= (a+b+c)(b-c)(a-b)$

Q. 3. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. Then, the value of k will be

(a) 9 **(b) 3**

(c) –9 **(d) 6**

[NCERT Exemp. Ex. 4.3, Q. 26, Page 80]

Ans. Correct option : (b)

***Explanation* :** We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$$

[Expanding along R_1]

$$9 = \frac{1}{2}[-3(-k) - 0 + 1(3k)]$$

$\Rightarrow 18 = 3k + 3k = 6k$

$\therefore \quad k = \frac{18}{6} = 3$

Q. 4. The determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ is equal to

(a) $abc(b-c)(c-a)(a-b)$

(b) $(b-c)(c-a)(a-b)$

(c) $(a+b+c)(b-c)(c-a)(a-b)$

(d) None of these

[NCERT Exemp. Ex. 4.3, Q. 27, Page 80]

Ans. Correct option : (d)

***Explanation* :** We have

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

$$= (b-a)^2\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

[on taking (b – a) common from C_1 and C_3 each]

$$= (b-a)^2\begin{vmatrix} b-c & b-c & c \\ a-b & a-b & b \\ c-a & c-a & a \end{vmatrix}$$

$[\because C_1 \to C_1 - C_3]$

$= 0$

[Since, two columns C_1 and C_2 are identical, so the value of determinant is zero.]

Q. 5. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 **(b) 2**

(c) 1 **(d) 3**

[NCERT Exemp. Ex. 4.3, Q. 28, Page 81]

Ans. Correct option : (c)

***Explanation* :** We have,

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 2\cos x + \sin x & \cos x & \cos x \\ 2\cos x + \cos x & \sin x & \cos x \\ 2\cos x + \cos x & \cos x & \sin x \end{vmatrix} = 0$$

On taking $(2\cos x + \sin x)$ common from the C_1, we get

$$\Rightarrow (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x) \end{vmatrix}$$

$[\because R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1]$

$$\Rightarrow (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & (\sin x - \cos x) \end{vmatrix} = 0$$

Expanding along C_1,

$(2\cos x + \sin x)[1.(\sin x - \cos x)^2] = 0$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

Either $\quad 2\cos x = -\sin x$

$\Rightarrow \quad \cos x = -\frac{1}{2}\sin x$

$\Rightarrow \quad \tan x = -2$

But here for, we get $-1 \le \tan x \le 1$. So, no solution is possible and for

$(\sin x - \cos x)^2 = 0, \quad \sin x = \cos x$

$\Rightarrow \quad \tan x = 1 = \tan\frac{\pi}{4}$ ...(i)

$\therefore \quad x = \frac{\pi}{4}$

Thus, one distinct real root exists.

Q. 6. If A, B and C are angles of a triangle, then the determinant $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to

(a) 0 **(b) –1**

(c) 1 **(d) None of these**

[NCERT Exemp. Ex. 4.3, Q. 29, Page 81]

Ans. **Correct option :** (a)

Explanation : We have, $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$

Applying $C_1 \to aC_1 + bC_2 + cC_3$,

$$\begin{vmatrix} -a + b\cos C + c\cos B & \cos C & \cos B \\ a\cos C - b + c\cos A & -1 & \cos A \\ a\cos B + b\cos A - c & \cos A & -1 \end{vmatrix}$$

Also, by projection rule in a triangle, we know that $a = b\cos C + c\cos B$, $b = c\cos A + a\cos C$ and $c = a\cos B + b\cos A$

Using above equation in column first, we get

$$\begin{vmatrix} -a + a & \cos C & \cos B \\ b - b & -1 & \cos A \\ c - c & \cos A & -1 \end{vmatrix} = \begin{vmatrix} 0 & \cos C & \cos B \\ 0 & -1 & \cos A \\ 0 & \cos A & -1 \end{vmatrix} = 0$$

[Since, determinant having all elements of any column or row gives value of determinant as zero]

Q. 7. **If** $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$**, then** $\lim_{t \to 0} \frac{f(t)}{t^2}$ **is equal to**

(a) 0 **(b)** –1
(c) 2 **(d)** 3

[NCERT Exemp. Ex. 4.3, Q. 30, Page 81]

Ans. **Correct option :** (a)

Explanation : We have,

$$f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix},$$

Expanding along C_1,

$$= \cos t\left(t^2 - 2t^2\right) - 2\sin t\left(t^2 - t\right) + \sin t\left(2t^2 - t\right)$$
$$= -t^2\cos t - \left(t^2 - t\right)2\sin t + \left(2t^2 - t\right)\sin t$$
$$= -t^2\cos t - t^2.2\sin t + t.2\sin t + 2t^2\sin t - t\sin t$$
$$= -t^2\cos t + t\sin t$$

$$\therefore \lim_{t\to 0}\frac{f(t)}{t^2} = \lim_{t\to 0}\frac{(-t^2\cos t)}{t^2} + \lim_{t\to 0}\frac{t\sin t}{t^2}$$
$$= -\lim_{t\to 0}\cos t + \lim_{t\to 0}\frac{\sin t}{t}$$
$$= -1 + 1$$
$$\left[\lim_{t\to 0}\frac{\sin t}{t} = 1 \text{ and } \cos 0 = 1\right]$$
$$= 0$$

Q. 8. **The maximum value of** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ **is (θ is real number)**

(a) $\frac{1}{2}$ **(b)** $\frac{\sqrt{3}}{2}$

(c) $\sqrt{2}$ **(d)** $\frac{2\sqrt{3}}{4}$

[NCERT Exemp. Ex. 4.3, Q. 31, Page 81]

Ans. **Correct option :** (a)

Explanation : Given that,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$$

[Applying $C_1 \to C_2 - C_3$ and $C_2 \to C_1 - C_3$]

$$= \begin{vmatrix} 0 & 0 & 1 \\ 0 & \sin\theta & 1 \\ \cos\theta & 0 & 1 \end{vmatrix}$$
$$= -\sin\theta.\cos\theta$$
$$= -\frac{1}{2}.2\sin\theta.\cos\theta$$
$$= -\frac{1}{2}\sin 2\theta$$

So, maximum value of Δ is $\frac{1}{2}$ when $\sin 2\theta = -1$

Q. 9. **If** $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$**, then**

(a) $f(a) = 0$ **(b)** $f(b) = 0$
(c) $f(0) = 0$ **(d)** $f(1) = 0$

[NCERT Exemp. Ex. 4.3, Q. 32, Page 82]

Ans. **Correct option :** (c)

Explanation : Given that,

$$f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$$

$$\Rightarrow f(a) = \begin{vmatrix} 0 & 0 & a-b \\ a+a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & a-b \\ 2a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix}$$
$$= [(a-b)\{2a.(a+c)\}] \neq 0$$

$$\therefore f(b) = \begin{vmatrix} 0 & b-a & b-b \\ b+a & 0 & b-c \\ b+b & b+c & 0 \end{vmatrix} = \begin{vmatrix} 0 & b-a & 0 \\ b+a & 0 & b-c \\ 2b & b+c & 0 \end{vmatrix}$$
$$= (b-a)[2b(b-c)]$$
$$= 2b(b-a)(b-c) \neq 0$$

$$\therefore f(0) = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$= a\,(bc) - b(ac)$$
$$= abc - abc$$
$$= 0$$

Q. 10. **If** $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$**. Then A^{-1} exist if**

(a) $\lambda = 2$ **(b)** $\lambda \neq 2$
(c) $\lambda \neq -2$ **(d)** None of these

[NCERT Exemp. Ex. 4.3, Q. 33, Page 82]

Ans. **Correct option :** (d)

Explanation : Given that,

$$A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$$

Expanding along R_1,

$|A| = 2(6-5) - \lambda(-5) - 3(-2)$

$= 2 + 5\lambda + 6$

We know that A^{-1} exists, if A is non-singular matrix, *i.e.*, $|A| \neq 0$

$\therefore 2 + 5\lambda + 6 \neq 0$

$\Rightarrow \quad 5\lambda \neq -8$

$\therefore \quad \lambda \neq \frac{-8}{5}$

So, A^{-1} exists if and only if $\lambda \neq \frac{-8}{5}$.

Q. 11. If A and B are invertible matrices, then which of the following is not correct?

(a) $adj.\, A = |A|.A^{-1}$

(b) $\det(A^{-1}) = [\det(A)]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

(d) $(A+B)^{-1} = B^{-1} + A^{-1}$

[NCERT Exemp. Ex. 4.3, Q. 34, Page 82]

Ans. Correct option : (d)

Explanation : Since, A and B are invertible matrices, so, we can say that

$$(AB)^{-1} = B^{-1}A^{-1} \quad ...(i)$$

Also, $A^{-1} = \frac{1}{|A|}(adj\ A)$

$\Rightarrow \quad adj\ A = A^{-1}.|A| \quad ...(ii)$

Also, $\det(A)^{-1} = [\det(A)]^{-1}$

$\Rightarrow \quad \det(A)^{-1} = \frac{1}{[\det(A)]}$

$\Rightarrow \det(A).\det(A)^{-1} = 1 \quad ...(iii)$

From equation (iii), we conclude that it is true.

Again, $(A+B)^{-1} = \frac{1}{|(A+B)|} adj\ (A+B)$

$\Rightarrow \quad (A+B)^{-1} \neq B^{-1} + A^{-1} \quad ...(iv)$

Q. 12. If x, y and z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then value of $x^{-1} + y^{-1} + z^{-1}$ is

(a) xyz **(b)** $x^{-1}y^{-1}z^{-1}$

(c) $-x-y-z$ **(d)** -1

[NCERT Exemp. Ex. 4.3, Q. 35, Page 82]

Ans. Correct option : (d)

Explanation : We have,

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0,$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} x & 1 & 1 \\ 0 & y & 1 \\ -z & -z & 1+z \end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow x[y(1+z)+z] - 0 + 1(yz) = 0$

$\Rightarrow \quad x(y+yz+z) + yz = 0$

$\Rightarrow \quad xy + xyz + xz + yz = 0$

$\Rightarrow \quad \frac{xy}{xyz} + \frac{xyz}{xyz} + \frac{xz}{xyz} + \frac{yz}{xyz} = 0$

[On dividing (xyz) from both sides]

$\Rightarrow \quad \frac{1}{x} + 1 + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -1$

$\therefore \quad x^{-1} + y^{-1} + z^{-1} = -1$

Q. 13. The value of the determinant $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is

(a) $9x^2(x+y)$ **(b)** $9y^2(x+y)$

(c) $3y^2(x+y)$ **(d)** $7x^2(x+y)$

[NCERT Exemp. Ex. 4.3, Q. 36, Page 82]

Ans. Correct option : (b)

Explanation : Given that,

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ and $C_3 \to C_3 - C_2$

$$\begin{vmatrix} 3(x+y) & x+y & y \\ 3(x+y) & x & y \\ 3(x+y) & x+2y & -2y \end{vmatrix}$$

$$= 3(x+y)\begin{vmatrix} 1 & (x+y) & y \\ 1 & x & y \\ 1 & (x+2y) & -2y \end{vmatrix}$$

[Taking $3(x+y)$ common from first column]

$$= 3(x+y)\begin{vmatrix} 0 & y & 0 \\ 1 & x & y \\ 1 & (x+2y) & -2y \end{vmatrix} \quad [\text{Applying } R_1 \to R_1 - R_2]$$

Expanding along R_1

$= 3(x+y)[-y(-2y-y)]$

$= 3y^2.3(x+y)$

$= 9y^2(x+y)$

Q. 14. There are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$. Then sum of these numbers is

(a) 4 **(b)** 5

(c) –4 **(d)** 9

[NCERT Exemp. Ex. 4.3, Q. 37, Page 83]

Ans. Correct option : (c)

Explanation : We have,

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

Expanding along first column

$$\Rightarrow 1(2a^2+4)-2(-4a-20)+0=86$$
$$\Rightarrow \quad 2a^2+4+8a+40=86$$
$$\Rightarrow \quad 2a^2+8a+44-86=0$$
$$\Rightarrow \quad a^2+4a-21=0$$
$$\Rightarrow \quad a^2+7a-3a-21=0$$
$$\Rightarrow \quad (a+7)(a-3)=0$$
$$\therefore a=-7,\ 3$$

Required sum $= -7+3=-4$

Q. 15. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to

(a) 6 **(b)** ± 6

(c) -6 **(d)** 0

[NCERT Ex. 4.1, Q. 8, Page 109]

Ans. Correct option : (b)

Explanation : Given that,

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$\Rightarrow x^2-36=36-36$$
$$\Rightarrow \quad x^2-36=0$$
$$\Rightarrow \quad x^2=36$$
$$\Rightarrow \quad x=\pm 6$$

Q. 16. Choose the correct answer

Let A be a square matrix of order 3×3, then $|kA|$ is equal to

(a) $k|A|$ **(b)** $k^2|A|$

(c) $k^3|A|$ **(d)** $3k|A|$

[NCERT Ex. 4.2, Q. 15, Page 121]

Ans. Correct option : (c)

Explanation **:** We know that, A be a square matrix of order 3×3

$$\text{Let } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$\text{Then, } kA = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{bmatrix}$$

$$\therefore \quad |kA| = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{bmatrix}$$

$$= k^3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

[Taking out common factor k from each row]

$$= k^3|A|$$
$$\therefore \quad |kA| = k^3|A|$$

Q. 17. Which of the following is correct?

(a) Determinant is a square matrix.

(b) Determinant is a number associated to a matrix.

(c) Determinant is a number associated to a square matrix.

(d) None of these. [NCERT Ex. 4.2, Q. 16, Page 121]

Ans. Correct option : (c)

Explanation **:** We know that every square matrix, $A = [a_{ij}]$ of order n. We can associate a number called the determinant of a square matrix A, where $a_{ij} = (i, j)^{th}$ element of A.

Thus, the determinant is a number associated to a square matrix.

Q. 18. If area of triangle is 35 sq. units with vertices (2, –6), (5, 4) and (k, 4). Then k is

(a) 12 **(b)** –2

(c) –12, –2 **(d)** 12, –2

[NCERT Ex. 4.3, Q. 5, Page 123]

Ans. Correct option : (d)

Explanation **:** The area of the triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(4-4)+6(5-k)+1(20-4k)]$$
$$= \frac{1}{2}[30-6k+20-4k]$$
$$= \frac{1}{2}[50-10k]$$
$$= 25-5k$$

It is given that the area of the triangle is ± 35.

Therefore, we have

$$\Rightarrow 25-5k=\pm 35$$
$$\Rightarrow 5(5-k)=\pm 35$$
$$\Rightarrow \quad 5-k=\pm 7$$

When $5-k=-7$, $k=5+7=12$.

When $5-k=7$, $k=5-7=-2$.

Hence, $k=12, -2$.

Q. 19. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactors of a_{ij}, then value of Δ is *given by*

(a) $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}$

(b) $a_{11}A_{11}+a_{12}A_{21}+a_{13}A_{31}$

(c) $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$

(d) $a_{11}A_{11}+a_{21}A_{21}+a_{31}A_{31}$

[NCERT Ex. 4.4, Q. 5, Page 126]

Ans. Correct option : (d)

Explanation **:** We know that :

Δ = Sum of the product of the elements of a column (or a row) with their corresponding co-factors

$\therefore \Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the value of Δ is given by the expression given in alternative D.

Q. 20. Let A be a non-singular square matrix of order 3 × 3. Then $|adj\ A|$ is equal to

(a) $|A|$ **(b)** $|A|^2$

(c) $|A|^3$ **(d)** $3|A|$

[NCERT Ex. 4.5, Q. 17, Page 132]

Ans. Correct option : (b)

***Explanation* :** We know that,

$$(adj\,A)A = |A|\,I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow \quad |(adj\,A)A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\Rightarrow |adj\,A|\,|A| - |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - |A|^3\,(I)$$

$$\therefore \quad |adj\,A| = |A|^2$$

Q. 21. If A is an invertible matrix of order 2, then $\det\left(A^{-1}\right)$ is equal to

(a) $\det(A)$ **(b)** $\frac{1}{\det(A)}$

(c) 1 **(d)** 0

[NCERT Ex. 4.5, Q. 18, Page 132]

Ans. Correct option : (b)

***Explanation* :** Given that A is an invertible matrix,

A^{-1} exists and $A^{-1} = \frac{1}{|A|} adj.A.$

As matrix A is order of 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then, $|A| = ad - bc$ and $adj.A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now,

$$A^{-1} = \frac{1}{|A|} adj.A = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore \quad |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$= \frac{1}{|A|^2}(ad - bc)$$

$$= \frac{1}{|A|^2}.|A|$$

$$= \frac{1}{|A|}$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)}$$

Q. 22. Choose the correct answer.

If a, b, c, are in A.P., then the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

(a) 0 **(b)** 1

(c) x **(d)** $2x$

[NCERT Misc. Ex. Q. 17, Page 143]

Ans. Correct option : (a)

Explanation : Given that,

$$\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+(a+c) \\ x+4 & x+5 & x+2c \end{vmatrix}$$

[$2b = a + c$ as a, b and c are in A.P.]

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$, we have

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+(a+c) \\ 1 & 1 & c-a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we have

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+(a+c) \\ 1 & 1 & c-a \end{vmatrix}$$

Here, all the elements of the first row (R_1) are zero. Hence, we have $\Delta = 0$.

Q. 23. Choose the correct answer.

If x, y, z are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **is**

(a) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ **(b)** $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(c) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **(d)** $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[NCERT Misc. Ex. Q. 18, Page 143]

Ans. Correct option : (a)

Explanation : As we know that from the given question,

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$\therefore \quad |A| = x(yz - 0) = xyz \neq 0$$

Now, $A_{11} = yz, A_{12} = 0, A_{13} = 0$

$A_{21} = 0, A_{22} = xz, A_{23} = 0$

$A_{31} = 0, A_{32} = 0, A_{33} = xy$

$$\therefore adj.A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj.A$$

$$= \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$= \begin{bmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

$$= \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

Q. 24. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ where 0 $0 \le \theta \le 2\pi$, then

(a) $\det(A) = 0$

(b) $\det(A) \in (2, \infty)$

(c) $\det(A) \in (2, 4)$

(d) $\det(A) \in [2, 4]$

[NCERT Misc. Ex. Q. 19, Page 143]

Ans. Correct option : (d)

Explanation : As we know that from the question,

$$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$$\therefore |A| = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$$

$$= 1 + \sin^2\theta + \sin^2\theta + 1$$

$$= 2 + 2\sin^2\theta$$

$$= 2(1+\sin^2\theta)$$

Now,

$0 \le \theta \le 2\pi$

$\Rightarrow -1 \le \sin\theta \le 1$

Very Short Answer Type Questions (1 or 2 mark each)

Q. 1. If A is a matrix of order 3 × 3, then $|3A|$ = _______.

[NCERT Exemp. Ex. 4.3, Q. 38, Page 83]

Ans. $|3A| = 3 \times 3 \times 3\ |A| = 27|A|$. **[1]**

Q. 2. If A is invertible matrix of order 3 × 3, then $|A^{-1}|$ is equal to _______.

[NCERT Exemp. Ex. 4.3, Q. 39, Page 83]

Ans. $|A^{-1}| = \frac{1}{|A|}$ [Since, $|A|.|A^{-1}| = 1$] **[1]**

Q. 3. If $x, y, z \in R$, then the value of determinant $\begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$ is equal to _______.

[NCERT Exemp. Ex. 4.3, Q. 40, Page 83]

Ans. We have,

$$\begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix} = \begin{vmatrix} (2.2^x)(2.2^{-x}) & (2^x-2^{-x})^2 & 1 \\ (2.3^x)(2.3^{-x}) & (3^x-3^{-x})^2 & 1 \\ (2.4^x)(2.4^{-x}) & (4^x-4^{-x})^2 & 1 \end{vmatrix}$$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

$[\because C_1 \to C_1 - C_2]$

$$= \begin{vmatrix} 4 & (2^x-2^{-x})^2 & 1 \\ 4 & (3^x-3^{-x})^2 & 1 \\ 4 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$$

$= 0$

[Since, C_1 and C_3 are proportional to each other.] **[2]**

Q. 4. If $\cos 2\theta = 0$, then $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$ = ________

[NCERT Exemp. Ex. 4.3, Q. 41, Page 83]

Ans. Given that, $\cos 2\theta = 0$

$$\Rightarrow \cos 2\theta = \cos\frac{\pi}{2}$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

$$\therefore \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ and } \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\therefore \begin{vmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{vmatrix}^2$$

Expanding along R_1,

$$= \left[-\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) + \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right)\right]^2$$

$$= \left[\frac{-2}{2\sqrt{2}}\right]^2$$

$$=\left(-\frac{1}{\sqrt{2}}\right)^2$$

$$=\frac{1}{2}$$

[2]

Q. 5. If A is a matrix of order 3 × 3, then $(A^2)^{-1}$ = ________.

[NCERT Exemp. Ex. 4.3, Q. 42, Page 83]

Ans. A is a matrix of order 3 × 3, then $(A^2)^{-1} = (A^{-1})^2$. **[1]**

Q. 6. If A is a matrix of order 3 × 3, then number of minors in determinant of A are ________.

[NCERT Exemp. Ex. 4.3, Q. 43, Page 83]

Ans. A are 9 [Since, in a 3 × 3 matrix, these are 9 elements.] **[1]**

Q. 7. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to ________.

[NCERT Exemp. Ex. 4.3, Q. 44, Page 83]

Ans.

Let $\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix}$

Expanding along R_1,

$$\Delta = a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$$ **[1]**

= Sum of products of elements of R_1 with their corresponding co-factors.

Q. 8. If $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x\end{vmatrix}=0$ then other two roots are ________.

[NCERT Exemp. Ex. 4.3, Q. 45, Page 83]

Ans. Since, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x\end{vmatrix}=0$

Expanding along R_1,

$$\Rightarrow x(x^2-12)-3(2x-14)+7(12-7x)=0$$

$$\Rightarrow \quad x^3-12x-6x+42+84-49x=0$$

$$\Rightarrow \quad x^3-67x+126=0 \quad \ldots(i)$$

Here, $126\times 1 = 9\times 2\times 7$

For $x=2$,

$$\Rightarrow \quad 2^3-67\times 2+126=134-134=0$$

Hence, $x=2$ is a root.

For $x=7$,

$$\Rightarrow \quad 7^3-67\times 7+126=469-469=0$$

Hence, $x=7$ is also a root.

[2]

Q. 9. Evaluate $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0\end{vmatrix}=$ ________

[NCERT Exemp. Ex. 4.3, Q. 46, Page 83]

Ans. Given that, $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0\end{vmatrix}$

Applying $C_1 \to C_1 - C_3$, we get

$$=\begin{vmatrix} z-x & xyz & x-z \\ z-x & 0 & y-z \\ z-x & z-y & 0\end{vmatrix}$$

[Taking $(z-x)$ common from column first]

$$=(z-x)\begin{vmatrix} 1 & xyz & x-z \\ 1 & 0 & y-z \\ 1 & z-y & 0\end{vmatrix}$$

Expanding along R_1,

$$=(z-x)[1.\{-(y-z)(z-y)\}-xyz(z-y)+(x-z)(z-y)]$$

$$=(z-x)(z-y)(-y+z-xyz+x-z)$$

$$=(z-x)(z-y)(x-y-xyz)$$

[2]

Q. 10. If $f(x)=\begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47}\end{vmatrix} = A + Bx + Cx^2 + \ldots$, then A = ______

[NCERT Exemp. Ex. 4.3, Q. 47, Page 84]

Ans. Given that, $f(x)=\begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47}\end{vmatrix}$

Now,

$$f(x)=(1+x)^{17}(1+x)^{23}(1+x)^{41}\begin{vmatrix} 1 & (1+x)^{2} & (1+x)^{6} \\ 1 & (1+x)^{6} & (1+x)^{11} \\ 1 & (1+x)^{2} & (1+x)^{6}\end{vmatrix}=0$$

[Since R_1 and R_3 are identical.]

$A=0$ **[1]**

Q. 11. State True or False for the statement :

$(A^3)^{-1}=(A^{-1})^3$ where A is a square matrix and $|A| \neq 0$. **[NCERT Exemp. Ex. 4.3, Q. 48, Page 84]**

Ans. **True,** since, $(A^n)^{-1}=(A^{-1})^n$ where $n \in N$. **[2]**

Q. 12. State True or False for the statement :

$(aA)^{-1}=\frac{1}{a}A^{-1}$, where a is any real number and A is a square matrix.

[NCERT Exemp. Ex. 4.3, Q. 49, Page 84]

Ans. **False,** since, we know that, if A is a non-singular square matrix, then for any scalar a (non-zero). aA is invertible such that

$$(aA)\left(\frac{1}{a}A^{-1}\right)=\left(a,\frac{1}{a}\right)(A.A^{-1})$$

i.e., (aA) is inverse of $\left(\frac{1}{a}A^{-1}\right)$ or $(aA)^{-1}=\frac{1}{a}A^{-1}$,

where 'a' is any non-zero scalar.

In the above statement, a is any real number. **[2]**

Q. 13. State True or False for the statement :

$|A^{-1}| \neq |A|^{-1}$, where A is non-singular matrix.

[NCERT Exemp. Ex. 4.3, Q. 50, Page 84]

Ans. **False,** $|A^{-1}| \neq |A|^{-1}$, where A is non-singular matrix. **[2]**

Q. 14. State True or False for the statement :

If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, then $|3AB| = 27 \times 5 \times 3 = 405$.

[NCERT Exemp. Ex. 4.3, Q. 51, Page 84]

Ans. **True**

We know that,

$|AB| = |A|.|B|$

$|3AB| = 27|AB|$

$= 27|A|.|B|$

$= 27 \times 5 \times 3$

$= 405$ **[2]**

Q. 15. State True or False for the statement :

If the value of a third-order determinant is 12, then the value of the determinant formed by replacing each element by its co-factor will be 144.

[NCERT Exemp. Ex. 4.3, Q. 52, Page 84]

Ans. **True,** let A is the determinant.

$\therefore |A| = 12$

Also, we know that, if A is a square matrix of order n, then $|adj\ A| = |A|^{n-1}$

For $n = 3, |adj\ A| = |A|^{3-1} = |A|^2$

$= (12)^2 = 144$ **[2]**

Q. 16. State True or False for the statement :

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

where a, b and c are in A.P.

[NCERT Exemp. Ex. 4.3, Q. 53, Page 84]

Ans. True,

Since, a, b and c are in AP, then $2b = a + c$

$$\therefore \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$[\because R_1 \to R_1 + R_3]$

$$\Rightarrow \begin{vmatrix} 2x+4 & 2x+6 & 2x+a+c \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$[\because 2b = a+c]$

$$\Rightarrow \begin{vmatrix} 2(x+2) & 2(x+3) & 2(x+b) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$\Rightarrow \quad 0 = 0$

[Since R_1 and R_2 are proportional to each other.] **[2]**

Q. 17. State True or False for the statement :

$|adj.A| = |A|^2$ **where A is a square matrix of order two.** **[NCERT Exemp. Ex. 4.3, Q. 54, Page 84]**

Ans. **False,**

If A is a square matrix of order n, then

$|adj.A| = |A|^{n-1}$

$\Rightarrow |adj.A| = |A|^{2-1} = |A| \quad [\because n = 2]$ **[2]**

Q. 18. State True or False for the statement :

The determinant $\begin{vmatrix} \sin A & \cos A & \sin A + \cos B \\ \sin B & \cos A & \sin B + \cos B \\ \sin C & \cos A & \sin C + \cos B \end{vmatrix}$ **is\ equal to zero.**

[NCERT Exemp. Ex. 4.3, Q. 55, Page 84]

Ans. **True,**

Since, $\begin{vmatrix} \sin A & \cos A & \sin A + \cos B \\ \sin B & \cos A & \sin B + \cos B \\ \sin C & \cos A & \sin C + \cos B \end{vmatrix}$

$$= \begin{vmatrix} \sin A & \cos A & \sin A \\ \sin B & \cos A & \sin B \\ \sin C & \cos A & \sin C \end{vmatrix} + \begin{vmatrix} \sin A & \cos A & \cos B \\ \sin B & \cos A & \cos B \\ \sin C & \cos A & \cos B \end{vmatrix}$$

$$= 0 + \begin{vmatrix} \sin A & \cos A & \cos B \\ \sin B & \cos A & \cos B \\ \sin C & \cos A & \cos B \end{vmatrix}$$

[Since, in the first determinant C_1 and C_3 are identicals.]

$$= \cos A.\cos B \begin{vmatrix} \sin A & 1 & 1 \\ \sin B & 1 & 1 \\ \sin C & 1 & 1 \end{vmatrix}$$

[Taking cos A common from C_2 and cos B common from C_3]

=0 [Since, C_2 and C_3 are identical] **[2]**

Q. 19. State True or False for the statement :

If the determinant $\begin{vmatrix} x+a & p+u & l+f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$ **splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K is 8.**

[NCERT Exemp. Ex. 4.3, Q.56, Page 84]

Ans. **True,**

Given that, $\begin{vmatrix} x+a & p+u & l+f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} x & p & l \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix} + \begin{vmatrix} a & u & f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$$

[Spliting first row]

$$\Rightarrow \begin{vmatrix} x & p & l \\ y & q & m \\ z+c & r+w & n+h \end{vmatrix} + \begin{vmatrix} x & p & l \\ b & v & g \\ z+c & r+w & n+h \end{vmatrix}$$

$$+ \begin{vmatrix} a & u & f \\ y & q & m \\ z+c & r+w & n+h \end{vmatrix} + \begin{vmatrix} a & u & f \\ b & v & g \\ z+c & r+w & n+h \end{vmatrix}$$

[Spliting second row]

Similarly, we can split these 4 determinants in 8 determinants by splitting each one in two determinants further. **[2]**

Q. 20. State True or False for the statement :

Let $\Delta = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = 16$**, then** $\Delta_1 = \begin{vmatrix} p+x & a+x & a+p \\ q+y & b+y & b+q \\ r+z & c+z & c+r \end{vmatrix} = 32$

[NCERT Exemp. Ex. 4.3, Q. 57, Page 85]

Ans. **True,**

We have $\Delta = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = 16$

and we have to prove, $\Delta_1 = \begin{vmatrix} p+x & a+x & a+p \\ q+y & b+y & b+q \\ r+z & c+z & c+r \end{vmatrix} = 32$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Delta_1 = \begin{vmatrix} 2p+2x+2a & a+x & a+p \\ 2q+2y+2b & b+y & b+q \\ 2r+2z+2c & c+z & c+r \end{vmatrix}$$

[Taking 2 common from C_1 and then $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$]

$$= 2\begin{vmatrix} p & x-p & a+p \\ q & y-q & b+q \\ r & z-r & c+r \end{vmatrix}$$

$$= 2\begin{vmatrix} p & x & a+p \\ q & y & b+q \\ r & z & c+r \end{vmatrix} - \begin{vmatrix} p & p & a+p \\ q & q & b+q \\ r & r & c+r \end{vmatrix}$$

[Since, two columns C_1 and C_2 are identicals]

$$= 2\begin{vmatrix} p & x & a+p \\ q & y & b+q \\ r & z & c+r \end{vmatrix} - 0$$

[Since, C_1 and C_3 are identical in second determinant and in first determinant, $C_1 \leftrightarrow C_2$ and then $C_1 \leftrightarrow C_3$]

$$= 2\begin{vmatrix} p & x & a \\ q & y & b \\ r & z & c \end{vmatrix} + 2\begin{vmatrix} p & x & p \\ q & y & q \\ r & z & r \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} + 0$$

= 2 16 [$\because \Delta = 16$]

= 32

Hence proved. **[2]**

Q. 21. State True or False for the statement :

The maximum value of $\begin{vmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1+\sin\theta} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1+\cos\theta} \end{vmatrix}$

is $\mathbf{\frac{1}{2}}$. **[NCERT Exemp. Ex. 4.3, Q. 58, Page 85]**

Ans. **True**,

Since,

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & 0 \\ 0 & 0 & \cos\theta \end{vmatrix} \quad [\because R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1]$$

On expanding along third row, we get the value of the determinant

$= \cos\theta . \sin\theta$

$= \frac{1}{2}\sin 2\theta$

$= \frac{1}{2}$

when θ is 45° which gives maximum value. **[2]**

Q. 22. Evaluate the determinants $\begin{vmatrix} \mathbf{2} & \mathbf{4} \\ \mathbf{-5} & \mathbf{-1} \end{vmatrix}$

[NCERT Ex. 4.1, Q. 1, Page 108]

Ans. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5)$ **[1]**

$= -2 + 20$

$= 18$

Q. 23. Evaluate the determinants

(i) $\begin{vmatrix} \mathbf{\cos\theta} & \mathbf{-\sin\theta} \\ \mathbf{\sin\theta} & \mathbf{\cos\theta} \end{vmatrix}$

(ii) $\begin{vmatrix} \mathbf{x^2 - x + 1} & \mathbf{x-1} \\ \mathbf{x+1} & \mathbf{x+1} \end{vmatrix}$ **[NCERT Ex. 4.1, Q. 2, Page 108]**

Ans. (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta)$

$= \cos^2\theta + \sin^2\theta$

$= 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

$= (x^2 - x + 1)(x+1) - (x-1)(x+1)$

$= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$ **[2]**

Q. 24. If $\mathbf{A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}}$ **then show that** $\mathbf{|2A| = 4|A|}$.

[NCERT Ex. 4.1, Q. 3, Page 108]

Ans. The given matrix is

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

$\therefore \quad 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

LHS $= |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$= 2 \times 4 - 4 \times 8$

$= 8 - 32$

$= -24$

Now, $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$

$= 1 \times 2 - 2 \times 4$

$= 2 - 8$

$= -6$

$\therefore$ RHS $= 4|A| = 4 \times (-6) = -24$

$\therefore$ LHS = RHS **[2]**

Q. 25. Evaluate the determinants

(i) $\begin{vmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{-2} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{3} & \mathbf{-5} & \mathbf{0} \end{vmatrix}$ **(ii)** $\begin{vmatrix} \mathbf{3} & \mathbf{-4} & \mathbf{5} \\ \mathbf{1} & \mathbf{1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \end{vmatrix}$

(iii) $\begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-3} \\ \mathbf{-2} & \mathbf{3} & \mathbf{0} \end{vmatrix}$ **(iv)** $\begin{vmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{-2} \\ \mathbf{0} & \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{-5} & \mathbf{0} \end{vmatrix}$

[NCERT Ex. 4.1, Q. 5, Page 108]

Ans. **(i)** Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}$$

$= (-15+3)$

$= -12$ **[2]**

(ii) Let $A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

By expanding along the first row, we have

$$|A| = 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$= 3(1+6) + 4(1+4) + 5(3-2)$

$= 3(7) + 4(5) + 5(1)$

$= 21 + 20 + 5$

$= 46$ **[2]**

(iii) Let $A = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

By expanding along the first row, we have

$$|A| = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

$= 0 - 1(0-6) + 2(-3-0)$

$= -1(-6) + 2(-3)$

$= 6 - 6$

$= 0$ **[2]**

(iv) Let $A = \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

By expanding along the first column, we have

$$|A| = 2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3\begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$$

$= 2(0-5) - 0 + 3(1+4)$

$= -10 + 15$

$= 5$ **[2]**

Q. 26. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ Find |A|.

[NCERT Ex. 4.1, Q. 6, Page 109]

Ans. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

By expanding along the first row, we have

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$= 1(-9+12) - 1(-18+15) - 2(8-5)$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 6 - 6$

$= 0$ **[2]**

Q. 27. Find values of *x*, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ **[NCERT Ex. 4.1, Q. 7, Page 109]**

Ans. **(i)** $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **[1]**

$\Rightarrow \quad 2\times1 - 5\times4 = 2x\times x - 6\times4$

$\Rightarrow \quad 2 - 20 = 2x^2 - 24$

$\Rightarrow \quad 2x^2 = 6$

$\Rightarrow \quad x^2 = 3$

$\Rightarrow \quad x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ **[1]**

$\Rightarrow \quad 2\times5 - 3\times4 = x\times5 - 3\times2x$

$\Rightarrow \quad 10 - 12 = 5x - 6x$

$\Rightarrow \quad -2 = -x$

$\Rightarrow \quad x = 2$

Q. 28. Using the property of determinants and without expanding, prove that $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

[NCERT Ex. 4.2, Q. 1, Page 119]

Ans. $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix} + \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} = 0 + 0 = 0$

Here, the two columns of the determinant are identical. **[2]**

Q. 29. Using the property of determinants and without expanding, prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

[NCERT Ex. 4.2, Q. 2, Page 119]

Ans. Let we assume that $\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2$, we have

$$\Delta = \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ -(a-c) & -(b-a) & -(c-b) \end{vmatrix}$$

$$= \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ (a-c) & (b-a) & (c-b) \end{vmatrix}$$

Here, the two rows R_1 and R_3 are identical. Therefore, $\Delta = 0$ **[2]**

Q. 30. Using the property of determinants and without expanding, prove that $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

[NCERT Ex. 4.2, Q. 3, Page 119]

Ans.

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63+2 \\ 3 & 8 & 72+3 \\ 5 & 9 & 81+5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} + \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 9(7) \\ 3 & 8 & 9(8) \\ 5 & 9 & 9(9) \end{vmatrix} + 0$$

[Two coulmns are identical.]

$$= 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$

$$= 0$$

[Two coulmns are identical.] **[2]**

Q. 31. Using the property of determinants and without expanding, prove that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

[NCERT Ex. 4.2, Q. 4, Page 119]

Ans. Let we assume that, $\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

By applying $C_3 \to C_3 + C_2$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix} = 0$$

$$\Rightarrow (ab+bc+ca)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} = 0$$

Here, two columns C_1 and C_3 are proportional.

$\Rightarrow \Delta = 0$ **[2]**

Q. 32. Find area of the triangle with vertices at the point given in each of the following

(i) (1, 0), (6, 0), (4, 3)

(ii) (2, 7), (1, 1), (10, 8)

(iii) (–2, –3), (3, 2), (–1, –8)

[NCERT Ex. 4.3, Q. 1, Page 122]

Ans. (i) The area of the triangle with vertices (1, 0), (6, 0) and (4, 3) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(0-3) - 0(6-4) + 1(18-0)]$$

$$= \frac{1}{2}[-3+18]$$

$$= \frac{15}{2} \text{ square units}$$

[2]

(ii) The area of the triangle with vertices (2, 7), (1, 1) and (10, 8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)]$$

$$= \frac{1}{2}[2(-7) - 7(-9) + 1(-2)]$$

$$= \frac{1}{2}[-14+63-2]$$

$$= \frac{1}{2}[-16+63]$$

$$= \frac{47}{2} \text{ square units}$$

[2]

(iii) The area of the triangle with vertices (−2, −3), (3, 2) and (−1, −8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-24+2)]$$

$$= \frac{1}{2}[-2(10) + 3(4) + 1(-22)]$$

$$= \frac{1}{2}[-20+12-22]$$

$$= -\frac{30}{2}$$

$$= -15 \text{ square units}$$

Hence, the area of the triangle is $|-15| = 15$ square units. **[2]**

Q. 33. Find adjoint of each of the matrices $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

[NCERT Ex. 4.5, Q. 1, Page 131]

Ans. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

We have,

$A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = 1$

$$\therefore adj.A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

[2]

Q. 34. Find the inverse of each of the matrices (if it exists) $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$.

[NCERT Ex. 4.5, Q. 5, Page 132]

Ans. Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$.

We have

$|A| = 6 + 8 = 14$

Now,

$A_{11} = 3,\ A_{12} = -4,\ A_{21} = 2,\ A_{22} = 2$

$$\therefore adj.A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

[2]

Q. 35. Find the inverse of each of the matrices (if it exists) $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$. [NCERT Ex. 4.5, Q. 6, Page 132]

Ans. Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

We have,

$$|A| = -2 + 15 = 13$$

Now,

$$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$$

$$\therefore adj.A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

[2]

Q. 36. Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$ [NCERT Ex. 4.6, Q. 1, Page 136]

Ans. The given system of equations is :

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Now,

$$|A| = 1(3) - 2(2)$$
$$= 3 - 4$$
$$= -1 \neq 0$$

Hence, A is non-singular.

Thus, A^{-1} exists.

Therefore, the given system of two equations will be consistent. **[2]**

Q. 37. Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$ [NCERT Ex. 4.6, Q. 2, Page 136]

Ans. The given system of equations is :

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now,

$$|A| = 2(1) - (-1)(1)$$
$$= 2 + 1$$
$$= 3 \neq 0$$

Hence, A is non-singular.

Thus, A^{-1} exists.

Therefore, the given system of two equations will be consistent. **[2]**

Q. 38. Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$ [NCERT Ex. 4.6, Q. 3, Page 136]

Ans. The given system of equations is

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Now,

$$|A| = 1(6) - 3(2)$$
$$= 6 - 6$$
$$= 0$$

$\therefore A$ is a singular matrix.

Now,

$$(adj.A) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(adj.A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent. **[2]**

Q. 39. Solve system of linear equations, using matrix method.

$5x + 2y = 3$

$3x + 2y = 5$ [NCERT Ex. 4.6, Q. 10, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Now,

$$|A| = 10 - 6 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists. **[2]**

Now $A_{11} = 2, A_{12} = -3, A_{21} = -2, A_{22} = 5$

$$\therefore \text{Adj. A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(adj A) = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{-3}{4} & \frac{5}{4} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-3}{4} & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$x = 1, y = 4$$

Q. 40. Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

[NCERT Misc. Ex. Q. 10, Page 142]

Ans. $\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$\Delta = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$

Expanding along C_1 , we have

$\Delta = 1(xy - 0) = xy$ **[2]**

Q. 41. Using the properties of determinants, evaluate $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

[NCERT Exemp. Ex. 4.3, Q. 1, Page 77]

Ans. We have, $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$

$\begin{vmatrix} x^2-2x+2 & x-1 \\ 0 & x+1 \end{vmatrix}$

On expanding, we get

$=(x^2-2x+2).(x+1)-(x-1).0$

$=x^3-2x^2+2x+x^2-2x+2$

$=x^3-x^2+2$ **[2]**

Q. 42. Using the properties of determinants, evaluate $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$.

[NCERT Exemp. Ex. 4.3, Q. 2, Page 77]

Ans. We have $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

On applying R_1 □ $R_1 - R_2$ and R_2 □ $R_2 - R_3$

$= \begin{vmatrix} a & -a & 0 \\ 0 & a & -a \\ x & y & a+z \end{vmatrix}$

Applying $C_2 \to C_2 + C_1$

$= \begin{vmatrix} a & 0 & 0 \\ 0 & a & -a \\ x & x+y & a+z \end{vmatrix}$

$=a(a^2+az+ax+ay)$

$=a^2(a+z+x+y)$ **[2]**

Q. 43. Using the properties of determinants, evaluate $\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$.

[NCERT Exemp. Ex. 4.3, Q. 3, Page 77]

Ans. We have, $\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$

Taking x^2, y^2 and z^2 common from C_1, C_2 and C_3, respectively.

$= x^2y^2z^2 \begin{vmatrix} 0 & x & x \\ y & 0 & y \\ z & z & 0 \end{vmatrix}$

Applying $C_2 \to C_2 - C_3$

$= x^2y^2z^2 \begin{vmatrix} 0 & 0 & x \\ y & -y & y \\ z & z & 0 \end{vmatrix}$

$= x^2y^2z^2[x(yz+yz)]$

$= x^2y^2z^2.2xyz$

$= 2x^3y^3z^3$ **[2]**

Q. 44. Using the properties of determinants, evaluate $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$.

[NCERT Exemp. Ex. 4.3, Q. 5, Page 77]

Ans. We have, $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} 3x+4 & 3x+4 & 3x+4 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$

$= (3x+4) \begin{vmatrix} 1 & 1 & 1 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$= (3x+4) \begin{vmatrix} 0 & 0 & 1 \\ -4 & 4 & x \\ 0 & -4 & x+4 \end{vmatrix}$

$= 16(3x+4)$ **[2]**

Q. 45. Using the properties of determinants, prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$.

[NCERT Exemp. Ex. 4.3, Q. 9, Page 78]

Ans. Given that,

$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

Taking $(a-1)$ common from R_1 and R_2

$$=(a-1)^2\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Expanding alone R_3

$$=(a-1)^2[1.(a+1)-2]$$

$$=(a-1)^3$$ **[2]**

Q. 46. If $a_1, a_2, a_3, ..., a_v$ ··· are in G.P., then prove that the determinant $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$ is independent of r.

[NCERT Exemp. Ex. 4.3, Q. 14, Page 78]

Ans. We know that, $a_{r+1} = AR^{(r+1)-1} = AR^r$

where r = rth term of a GP, A = First term of a GP and R = Common difference of GP

We have, $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$

$$=\begin{vmatrix} AR^r & AR^{r+4} & AR^{r+8} \\ AR^{r+6} & AR^{r+10} & AR^{r+14} \\ AR^{r+10} & AR^{r+16} & AR^{r+20} \end{vmatrix}$$

$$=AR^r.AR^{r+6}.AR^{r+10}\begin{vmatrix} 1 & R^4 & R^8 \\ 1 & R^4 & R^8 \\ 1 & R^6 & R^{10} \end{vmatrix}$$

[Taking AR^r, AR^{r+6} and AR^{r+10} common from the row R_1, R_2 and R_3, respectively]

$= 0$ [Since, R_1 and R_2 are identicals] **[2]**

Q. 47. Show that the points $(a+5, a-4)$, $(a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.

[NCERT Exemp. Ex. 4.3, Q. 15, Page 78]

Ans. Given, the points are

$a+5, a-4), (a-2, a+3)$ and (a, a)

$$\Delta=\frac{1}{2}\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$

$$=\frac{1}{2}\begin{vmatrix} 5 & -4 & 0 \\ -2 & 3 & 0 \\ a & a & 1 \end{vmatrix}$$

$$=\frac{1}{2}[1(15-8)]$$

$$=\frac{7}{2}\neq 0$$

Hence, given points form a triangle, *i.e.*, points do not lie in a straight line. **[2]**

Q. 48. Given $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ Compute A^{-1} and show that $2A^{-1} = 9I - A$. **[CBSE Board, Delhi Region, 2018]**

Ans.

Given, $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$$\Rightarrow A^{-1}=\frac{1}{14-12}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}=\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Now,

LHS $= 2A^{-1}$

$$=2.\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$ **[2]**

$$=\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

RHS $= 9I - A$

$$=9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 9-2 & 0-(-3) \\ 0-(-4) & 9-7 \end{bmatrix}$$

$$=\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Since, LHS = RHS

Hence proved. **[2]**

Q. 49. If $|A| = 3$ and $A^{-1}=\begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$ then write the *adj. A*.

[CBSE Board, Foreign Scheme, 2017]

Ans. $adj.A=3\begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}=\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ **[1]**

Q. 50. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.

[CBSE Board, Delhi Region, 2016]

Ans. Let we assume that,

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}=\sin\theta.\cos\theta$$ **[½]**

$$=\frac{1}{2}\sin 2\theta \qquad \left[\therefore \text{Maximum value}=\frac{1}{2}\right]$$ **[½]**

Q. 51. Write the value of x. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}=8$, write the value of x.

[CBSE Board, Foreign Scheme, 2016]

Ans. On expanding we get

$$\Rightarrow x[-x^2-1]-\sin\theta[-x\sin\theta-\cos\theta]+\cos\theta[-\sin\theta + x.\cos\theta]=8$$

$$\Rightarrow -x^3-x+x.\sin^2\theta+\sin\theta.\cos\theta-\sin\theta.\cos\theta + x.\cos^2\theta=8$$

But we know that $\cos^2\theta+\sin^2\theta=1$

So,

$$\Rightarrow -x^3-x+x=8$$

$$\Rightarrow x^3=-8$$

$$\Rightarrow x=-2$$ **[1]**

Short Answer Type Questions (3 or 4 marks each)

Q. 1. If $A=\begin{bmatrix}1&0&1\\0&1&2\\0&0&4\end{bmatrix}$ then show that $|3A|=27|A|$.

[NCERT Ex. 4.1, Q. 4, Page 108]

Ans. The given matrix is $A=\begin{bmatrix}1&0&1\\0&1&2\\0&0&4\end{bmatrix}$

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$$|A|=1\begin{vmatrix}1&2\\0&4\end{vmatrix}-0\begin{vmatrix}0&1\\0&4\end{vmatrix}+0\begin{vmatrix}0&1\\1&2\end{vmatrix}$$

$$=1(4-0)-0+0$$

$$=4$$

Therefore, $27|A|=27(4)=108$ ...(i)

Now, $3A=3\begin{bmatrix}1&0&1\\0&1&2\\0&0&4\end{bmatrix}=\begin{bmatrix}3&0&3\\0&3&6\\0&0&12\end{bmatrix}$

Therefore, $|3A|=3\begin{vmatrix}3&6\\0&12\end{vmatrix}+0\begin{vmatrix}0&3\\0&12\end{vmatrix}+0\begin{vmatrix}0&3\\3&6\end{vmatrix}$

$=3(36-0)=3(36)=108$ ...(ii)

From equations (i) and (ii), we get

$|3A|=27|A|$

Hence proved. **[3]**

Q. 2. Using the property of determinants and without expanding, prove that $\begin{vmatrix}0&a&-b\\-a&0&-c\\b&c&0\end{vmatrix}=0$

[NCERT Ex. 4.2, Q. 6, Page 120]

Ans. We have,

$$\Delta=\begin{vmatrix}0&a&-b\\-a&0&-c\\b&c&0\end{vmatrix}=0$$

Applying $R_1\to cR_1$, we get

$$\Delta=\frac{1}{c}\begin{vmatrix}0&ac&-bc\\-a&0&-c\\b&c&0\end{vmatrix}$$

Applying $R_1\to R_1-bR_2$, we get

$$\Delta=\frac{1}{c}\begin{vmatrix}ab&ac&0\\-a&0&-c\\b&c&0\end{vmatrix}$$

$$=\frac{a}{c}\begin{vmatrix}b&c&0\\-a&0&-c\\b&c&0\end{vmatrix}$$

Here, the two rows R_1 and R_3 are identical.

So, $\Delta=0$. **[3]**

Q. 3. Using the property of determinants and without expanding, prove that $\begin{vmatrix}-a^2&ab&ac\\ba&-b^2&bc\\ca&cb&-c^2\end{vmatrix}=4a^2b^2c^2$.

[NCERT Ex. 4.2, Q. 7, Page 120]

Ans. Taking LHS and we get

$$\Delta=\begin{vmatrix}-a^2&ab&ac\\ba&-b^2&bc\\ca&cb&-c^2\end{vmatrix}$$

$$=abc\begin{vmatrix}-a&b&c\\a&-b&c\\a&b&-c\end{vmatrix}$$

[Taking out factors a,b,c from R_1,R_2 and R_3]

$$=a^2b^2c^2\begin{vmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{vmatrix}$$

[Taking out factors a,b,c from C_1,C_2 and C_3]

Applying $R_2\to R_2+R_1$ and $R_3\to R_3+R_1$ we have

$$\Delta=\begin{vmatrix}-1&1&1\\0&0&2\\0&2&0\end{vmatrix}$$

$$=a^2b^2c^2(-1)\begin{vmatrix}0&2\\2&0\end{vmatrix}$$

$$=-a^2b^2c^2(0-4)$$

$$=4a^2b^2c^2$$

Hence proved. **[3]**

Q. 4. (i) Find equation of line joining (1, 2) and (3, 6) using determinants.

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants. [NCERT Ex. 4.3, Q. 4, Page 123]

Ans. **(i)** Let $P(x, y)$ be any point on the line joining points $A(1, 2)$ and $B(3, 6)$. Then, the points A, B and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \quad \frac{1}{2}\begin{vmatrix}1&2&1\\3&6&1\\x&y&1\end{vmatrix}=0$$

$$\Rightarrow \frac{1}{2}\left[1(6-y)-2(3-x)+1(3y-6x)\right]=0$$

$$\Rightarrow \quad 6-y-6+2x+3y-6x=0$$

$$\Rightarrow \quad 2y-4x=0$$

$$\Rightarrow \quad y=2x$$

Hence, the equation of the line joining the given points is $y = 2x$. **[3]**

(ii) Let $P(x, y)$ be any point on the line joining points $A(3, 1)$ and $B(9, 3)$. Then, the points A, B and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[3(3-y)-1(9-x)+1(9y-3x)] = 0$$

$$\Rightarrow \quad 9-3y-9+x+9y-3x = 0$$

$$\Rightarrow \quad 6y-2x = 0$$

$$\Rightarrow \quad x-3y = 0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$. **[3]**

Q. 5. Write Minors and Co-factors of the elements of following determinants :

(i) $\begin{vmatrix} \mathbf{2} & \mathbf{-4} \\ \mathbf{0} & \mathbf{3} \end{vmatrix}$ **(ii)** $\begin{vmatrix} \boldsymbol{a} & \boldsymbol{c} \\ \boldsymbol{b} & \boldsymbol{d} \end{vmatrix}$

[NCERT Ex. 4.4, Q. 1, Page 126]

Ans. **(i)** The given determinant is $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Minor of element a_{ij} is M_{ij}.

Therefore,

$M_{11} =$ Minor of element $a_{11} = 3$

$M_{12} =$ Minor of element $a_{12} = 0$

$M_{21} =$ Minor of element $a_{21} = -4$

$M_{22} =$ Minor of element $a_{22} = 2$

Co-factor of a_{ij} is $A_{ij} = (-1^{)\, i+j} M_{ij}$.

Therefore,

$$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$$

$$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$$

$$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (\ 2)\ 2$$

[3]

(ii) The given determinant is $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Minor of element a_{ij} is M_{ij} .

Therefore,

$M_{11} =$ Minor of element $a_{11} = d$.

$M_{12} =$ Minor of element $a_{12} = b$

$M_{21} =$ Minor of element $a_{21} = c$

$M_{22} =$ Minor of element $a_{22} = a$

Co-factor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

Therefore,

$$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$$

$$A_{21} = (-1)^{2+1} M_{21} = (1)^3 (c) = -c$$

$$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$$

[3]

Q. 6. Using Co-factors of elements of second row, evaluate $\boldsymbol{\Delta} = \begin{vmatrix} \mathbf{5} & \mathbf{3} & \mathbf{8} \\ \mathbf{2} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \end{vmatrix}$

[NCERT Ex. 4.4, Q. 3, Page 126]

Ans. The given determinant is $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

We have,

$$M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9-16 = -7$$

Therefore, $A_{21} =$ co-factor of $a_{21} = (-1)^{2+1} M_{21} = 7$

$$M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15-8 = 7$$

Therefore, $A_{22} =$ co-factor of $a_{22} = (-1)^{2+2} M_{22} = 7$

$$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10-3 = 7$$

Therefore, $A_{23} =$ co-factor of $a_{23} = (-1)^{2+3} M_{23} = -7$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding co-factors.

Therefore, $\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$$= 2\ (7) + 0(7) + 1(-7)$$

$$= 14 - 7$$

$$= 7$$

[3]

Q. 7. Find adjoint of each of the matrices $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{5} \\ \mathbf{-2} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

[NCERT Ex. 4.5, Q. 2, Page 131]

Ans. Let we assume that, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

We have,

$$A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3-0 = 3$$

$$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0+6 = 6$$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1+4 = 5$$

$$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$$

$$A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$$

$$A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

Hence, $adj.A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$$=\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$

[3]

Q. 8. Verify A(adj. A) = (adj A) A = |A|I.

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

[NCERT Ex. 4.5, Q. 3, Page 131]

Ans. Let we assume that,

$$A=\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

We have,

$$|A| = -12-(-12) = -12+12 = 0$$

$$\therefore |A|I = 0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A_{11} = -6, A_{12} = 4, A_{21} = -3, A_{22} = 2$$

$$\therefore adj.A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

Now,

$$A(adj.A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also, $(adj.A)A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A(adj.A) = (adj.A)A = |A|I$

[3]

Q. 9. Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}.$$

[NCERT Ex. 4.5, Q. 7, Page 132]

Ans. Let we assume that,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

We have,

$$|A| = 1(10-0)-2(0-0)+3(0-0) = 10$$

Now,

$$A_{11} = 10-0 = 10, \quad A_{12} = -(0-0) = 0, \quad A_{13} = 0-0 = 0$$
$$A_{21} = -(10-0) = -10, A_{22} = 5-0 = 5, \quad A_{23} = -(0-0) = 0$$
$$A_{31} = 8-6 = 2, \quad A_{32} = -(4-0) = -4, A_{33} = 2-0 = 2$$

$$\therefore adj.A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}adj.A = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

[3]

Q. 10. Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}.$$

[NCERT Ex. 4.5, Q. 8, Page 132]

Ans. Let we assume that,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

We have,

$$|A| = 1(-3-0)-0+0 = -3$$

Now,

$$A_{11} = -3-0 = -3, A_{12} = -(-3-0) = 3, A_{13} = 6-15 = -9$$
$$A_{21} = -(0-0) = 0, A_{22} = -1-0 = -1, \quad A_{23} = -(2-0) = -2$$
$$A_{31} = 0-0 = 0, \quad A_{32} = -(0-0) = 0, \quad A_{33} = 3-0 = 3$$

$$\therefore adj.A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}adj.A = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

[3]

Q. 11. Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}.$$

[NCERT Ex. 4.5, Q. 9, Page 132]

Ans. Let we assume that,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

We have,

$$|A| = 2(-1-0)-1(4-0)+3(8-7)$$
$$= 2(-1)-1(4)+3(1)$$
$$= -2-4+3$$
$$= -3$$

Now,

$$A_{11} = -1-0 = -1, A_{12} = -(4-0) = -4, A_{13} = 8-7 = 1$$
$$A_{21} = -(1-6) = 5, A_{22} = 2+21 = 23, \quad A_{23} = -(4+7) = -11$$
$$A_{31} = 0+3 = 3, \quad A_{32} = -(0-12) = 12, \quad A_{33} = -2-4 = -6$$

$$\therefore adj.A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}adj.A = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

[3]

Q. 12. Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}.$$

[NCERT Ex. 4.5, Q. 10, Page 132]

Ans. Let we assume that,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

By expanding along C_1, we get

$$|A| = 1(8-6) - 0(-4+4) + 3(3-4)$$
$$= 2-3$$
$$= -1$$

Now,

$$A_{11} = 8-6 = -2, A_{12} = -(0+9) = -9, A_{13} = 0-6 = -6$$
$$A_{21} = -(-4+4) = 0, A_{22} = (4-6) = -2, A_{23} = (-2+3) = 1$$
$$A_{31} = 3-4 = -1, A_{32} = -(-3-0) = 3, A_{33} = 2-0 = 2$$

$$\therefore adj.A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj.A = -1\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

[3]

Q. 13. Find the inverse of each of the matrices (if it exists) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$.

[NCERT Ex. 4.5, Q. 11, Page 132]

Ans. Let we assume that,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

We have,

$$|A| = 1(-\cos^2\alpha - \sin^2\alpha)$$
$$= -(\cos^2\alpha + \sin^2\alpha)$$
$$= -1$$

Now,

$$A_{11} = -\cos^2\alpha - \sin^2\alpha = -1, A_{12} = 0, A_{13} = 0$$
$$A_{21} = 0, A_{22} = -\cos\alpha, A_{23} = -\sin\alpha$$
$$A_{31} = 0, A_{32} = -\sin\alpha, A_{33} = \cos\alpha$$

$$\therefore adj.A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.adj.A = -1\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

[3]

Q. 14. Examine the consistency of the system of equations.

$$\boldsymbol{x + y + z = 1}$$
$$\boldsymbol{2x + 3y + 2z = 2}$$
$$\boldsymbol{ax + ay + 2az = 4}$$

[NCERT Ex. 4.6, Q. 4, Page 136]

Ans. The given system of equations is :

$$x + y + z = 1$$
$$2x + 3y + 2z = 2$$
$$ax + ay + 2az = 4$$

This system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Now,

$$|A| = 1(6a-2a) - 1(4a-2a) + 1(2a-3a)$$
$$= 4a - 2a - a = 4a - 3a = a \neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

[3]

Q. 15. Examine the consistency of the system of equations.

$$\boldsymbol{3x - y - 2z = 2}$$
$$\boldsymbol{2y - z = -1}$$
$$\boldsymbol{3x - 5y = 3}$$

[NCERT Ex. 4.6, Q. 5, Page 136]

Ans. The given system of equations is :

$$3x - y - 2z = 2$$
$$2y - z = -1$$
$$3x - 5y = 3$$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Now,

$$|A| = 3(0-5) - 0 + 3(1+4) = -15 + 15 = 0$$

$\therefore$ A is a singular matrix.

Now,

$$(adj.A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore (adj.A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent. **[3]**

Q. 16. Examine the consistency of the system of equations.

$$\boldsymbol{5x - y + 4z = 5}$$
$$\boldsymbol{2x + 3y + 5z = 2}$$
$$\boldsymbol{5x - 2y + 6z = -1}$$

[NCERT Ex. 4.6, Q. 6, Page 136]

Ans. The given system of equations is :

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

Now,

$|A| = 5(18+10)+1(12-25)+4(-4-15)$

$= 5(28)+1(-13)+4(-19)$

$= 140-13-76$

$= 51 \neq 0$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent. **[3]**

Q. 17. Solve system of linear equations, using matrix method.

$5x+2y=4$

$7x+3y=5$ [NCERT Ex. 4.6, Q. 7, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Now, $|A| = 15-14 = 1 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|}(adj.A)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$\therefore \quad X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Hence, $x = 2$ and $y = -3$ **[3]**

Q. 18. Solve system of linear equations, using matrix method.

$2x-y=-2$

$3x+4y=3$ [NCERT Ex. 4.6, Q. 8, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Now,

$|A| = 8+3 = 11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A} adj.A = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\therefore \quad X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} \frac{-5}{11} \\ \frac{12}{11} \end{bmatrix}$$

Hence, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$ **[3]**

Q. 19. Solve system of linear equations, using matrix method.

$4x-3y=3$

$3x-5y=7$ [NCERT Ex. 4.6, Q. 9, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Now,

$|A| = -20+9 = -11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|}(adj.A) = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 15-21 \\ 9-28 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix} = \begin{bmatrix} \frac{-6}{11} \\ \frac{-19}{11} \end{bmatrix}$$

Hence, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$. **[3]**

Q. 20. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

[NCERT Misc. Ex. Q. 1, Page 141]

Ans. We know that,

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= -x^3 - x + x$

$= -x^3$(Indenpendent of θ)

Hence, Δ is independent of θ. **[3]**

Q. 21. Without expanding the determinant, prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$.

[NCERT Misc. Ex. Q. 2, Page 141]

Ans. Taking LHS, we get

$$\text{LHS} = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$$

$[R_1 \to aR_1, R_2 \to bR_2, \text{ and } R_3 \to cR_3]$

$$= \frac{1}{abc}.abc\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

[Taking out factor abc from C_3]

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

[Applying $C_1 \leftrightarrow C_3$ and $C_2 \leftrightarrow C_3$]

Hence proved. **[3]**

Q. 22. Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

[NCERT Misc. Ex. Q. 3, Page 141]

Ans. Given that,

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have

$$\Delta = -\sin\alpha(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha)$$
$$+ \cos\alpha(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta)$$
$$= \sin^2\alpha(\sin^2\beta + \cos^2\beta)$$
$$+ \cos^2\alpha(\cos^2\beta + \sin^2\beta)$$
$$= \sin^2\alpha(1) + \cos^2\alpha(1)$$
$$= 1$$

[3]

Q. 23. Using properties of determinants, prove that $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma).$

[NCERT Misc. Ex. Q. 11, Page 142]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta-\alpha & \beta^2-\alpha^2 & \alpha-\beta \\ \gamma-\alpha & \gamma^2-\alpha^2 & \alpha-\gamma \end{vmatrix}$$

$$= (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ 1 & \beta+\alpha & -1 \\ 1 & \gamma+\alpha & -1 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we have

$$\Delta = (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ 1 & \beta+\alpha & -1 \\ 0 & \gamma-\beta & 0 \end{vmatrix}$$

Expanding along R_3, we have

$$\Delta = (\beta-\alpha)(\gamma-\alpha)[-(\gamma-\beta)(-\alpha-\beta-\gamma)]$$
$$= -(\beta-\alpha)(\gamma-\alpha)(\beta-\gamma)(\alpha+\beta+\gamma)]$$
$$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

Hence proved. **[3]**

Q. 24. Using properties of determinants, prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1.$

[NCERT Misc. Ex. Q. 14, Page 142]

Ans. Taking LHS,

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we have

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix}$$

Applying $R_3 \to R_3 - 3R_2$, we have

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1, we have

$$\Delta = 1\begin{vmatrix} 1 & 2+p \\ 0 & 1 \end{vmatrix} = 1(1-0) = 1$$

Hence proved. **[3]**

Q. 25. Using properties of determinants, prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

[NCERT Misc. Ex. Q. 15, Page 142]

Ans. We are taking LHS

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

$$= \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta\sin\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we have

$$\Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \cos\alpha\cos\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \cos\beta\cos\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \cos\gamma\cos\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

Here, two columns C_1 and C_2 are identical.

$\therefore \Delta = 0$

Hence proved. **[3]**

Q. 26. Using the properties of determinants, evaluate $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

[NCERT Exemp. Ex. 4.3, Q. 4, Page 77]

Ans. We have,

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$,

$$=\begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

$$=(x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$

[Taking $(x+y+z)$ common from column C_1]

$$=(x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & 2z+x \end{vmatrix}$$

[$\because R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

Now, expanding along first column, we get

$=(x+y+z).1[(2y+x)(2z+x)-(x-y)(x-z)]$

$=(x+y+z)(4yz+2yx+2xz+x^2-x^2+xz+yx-yz)$

$=(x+y+z)(3yz+3yx+3xz)$

$=3(x+y+z)(yz+yx+xz)$ **[3]**

Q. 27. Using the properties of determinants, evaluate

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}.$$

[NCERT Exemp. Ex. 4.3, Q. 6, Page 77]

Ans. We have,

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Taking $(a+b+c)$ common from the first row]

$$=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$]

$$=(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -(a+b+c) & 2b \\ a+b+c & a+b+c & c-b-a \end{vmatrix}$$

Expanding along R_1,

$=(a+b+c)[1\times 0+(a+b+c)^2]$

$=(a+b+c)^3$ **[3]**

Q. 28. Using the properties of determinants, prove that

$$\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}=0$$

[NCERT Exemp. Ex. 4.3, Q. 7, Page 77]

Ans.

$$\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}$$

[Multiplying R_1, R_2, R_3 by x, y, z respectively]

$$=\frac{1}{xyz}\begin{vmatrix} xy^2z^2 & xyz & xy+xz \\ x^2yz^2 & xyz & yz+xy \\ x^2y^2z & xyz & xz+yz \end{vmatrix}$$

[Taking (xyz) common from C_1 and C_2]

$$=\frac{1}{xyz}(xyz^2)\begin{vmatrix} yz & 1 & xy+xz \\ xz & 1 & yz+xy \\ xy & 1 & xz+yz \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$

$$=xyz\begin{vmatrix} yz & 1 & xy+xz+zx \\ xz & 1 & yz+xy+zx \\ xy & 1 & xz+yz+zx \end{vmatrix}$$

[Taking $(xy+yz+zx)$ common from C_3]

$$=xyz(xy+yz+zx)\begin{vmatrix} yz & 1 & 1 \\ xz & 1 & 1 \\ xy & 1 & 1 \end{vmatrix}$$

$=0$ [$\because C_2$ and C_3 are identical] **[3]**

Q. 29. Using the properties of determinants, prove that

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}=4xyz.$$

[NCERT Exemp. Ex. 4.3, Q. 8, Page 77]

Ans.

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$$=\begin{vmatrix} 2(y+z) & z & y \\ 2(z+x) & z+x & x \\ 2(y+x) & x & x+y \end{vmatrix}$$

$$=2\begin{vmatrix} y+z & z & y \\ z+x & z+x & x \\ y+x & x & x+y \end{vmatrix}$$

[Applying $C_1 \to C_1 - C_2$]

$$=2\begin{vmatrix} y & z & y \\ 0 & z+x & x \\ y & x & x+y \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_1$]

$$=2\begin{vmatrix} y & z & 0 \\ 0 & z+x & x \\ y & x & x \end{vmatrix}$$

[Applying $R_3 \to R_3 - R_1$]

$$=2\begin{vmatrix} y & z & 0 \\ 0 & z+x & x \\ 0 & x-z & x \end{vmatrix}$$

$=2y[(z+x)x-x(x-z)]$

$=2y(2xz)$

$=4xyz$ **[3]**

Q. 30. If $A+B+C=0$, then prove that

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}=0$$

[NCERT Exemp. Ex. 4.3, Q. 10, Page 78]

Ans. $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 1(1-\cos^2 A) - \cos C(\cos C$

$-\cos A.\cos B) + \cos B(\cos C.\cos A - \cos B)$

$= \sin^2 A - \cos^2 C + \cos A.\cos$

$B.\cos C + \cos A.\cos B.\cos C - \cos^2 B$

$= \sin^2 A - \cos^2 B + 2\cos A.\cos$

$B.\cos C - \cos^2 C$

$= -\cos(A+B).\cos(A-B) + 2\cos$

$A.\cos B.\cos C - \cos^2 C$

$[\because \cos^2 B - \sin^2 A = \cos(A+B).\cos(A-B)]$

$= -\cos(-C).\cos(A-B)$

$+ \cos C(2\cos A.\cos B - \cos C)$

$= -\cos C(\cos A.\cos B + \sin A.\sin B$

$- 2\cos A.\cos B + \cos C)$

$= \cos C(\cos A.\cos B - \sin A.\sin B - \cos C)$

$= \cos C[\cos(A+B) - \cos C]$

$= \cos C(\cos C - \cos C)$

[As $\cos C = \cos(A+B)$]

$= 0$ **[3]**

Q. 31. If the co-ordinates of the vertices of an equilateral triangle with sides of length '*a*' are (x_1, y_1), (x_2, y_2), (x_3, y_3), then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \dfrac{3a^4}{4}$.

[NCERT Exemp. Ex. 4.3, Q. 11, Page 78]

Ans. The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3), is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Also area of an equilateral triangle with side a is given by

$$\Delta = \frac{\sqrt{3}}{4}a^2$$

$$\therefore \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{\sqrt{3}}{4}a^2$$

Squaring both sides, we get

$$\Rightarrow \Delta^2 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3}{16}a^4$$

or $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \dfrac{3a^4}{4}$ **[3]**

Q. 32. Find the value of θ satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$

[NCERT Exemp. Ex. 4.3, Q. 12, Page 78]

Ans. We have,

$$\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$$

Expanding along C_3, we get

$\sin 3\theta \times (28-21) - \cos 2\theta \times (-7-7) - 2(3+4) = 0$

$\Rightarrow 7\sin 3\theta + 14\cos 2\theta - 14 = 0$

$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0$

$\Rightarrow (3\sin\theta - 4\sin^3\theta) + 2(1 - 2\sin^2\theta) - 2 = 0$

$\Rightarrow 4\sin^3\theta - 4\sin^2\theta + 3\sin\theta = 0$

$\Rightarrow \sin\theta(4\sin^2\theta - 4\sin\theta + 3) = 0$

$\Rightarrow \sin\theta(4\sin^2\theta - 6\sin\theta + 2\sin\theta + 3) = 0$

$\Rightarrow \sin\theta(2\sin\theta + 1)(2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = -\dfrac{1}{2}$ or $\sin\theta = \dfrac{3}{2}$

$\Rightarrow \theta = n\pi$ or $\theta = m\pi + (-1)^n\left(-\dfrac{\pi}{6}\right); m, n \in Z$

$\Rightarrow \sin\theta = -\dfrac{3}{2}$ is not possible.

[3]

Q. 33. Find the value of x if, $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$

[NCERT Exemp. Ex. 4.3, Q. 13, Page 78]

Ans. We have,

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$\Rightarrow \begin{vmatrix} 12+x & 12+x & 12+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Taking $(12+x)$ common from R_1]

$$\Rightarrow (12+x)\begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$]

$$\Rightarrow (12+x)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -2x & 4+x \\ 2x & 2x & 4-x \end{vmatrix} = 0$$

$\Rightarrow (12+x)[0 - (-2x)(2x)] = 0$

$\Rightarrow (12+x)(4x^2) = 0$

$\therefore x = -12, 0$ **[3]**

Q. 34. Using matrix method, solve the system of equations

$3x + 2y - 2z = 3,$

$x + 2y + 3z = 6$

$2x - y + z = 2.$

[NCERT Exemp. Ex. 4.3, Q. 19, Page 79]

Ans. Given that system of equations is

$3x+2y-2z=3,$

$x+2y+3z=6$

$2x-y+z=2.$

In the form of $AX=B$,

$$\begin{bmatrix}3 & 2 & -2\\1 & 2 & 3\\2 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\6\\2\end{bmatrix}$$

For A^{-1}

$$|A|=|3(5)-2(1-6)+(-2)(-5)|$$
$$=|15+10+10|$$
$$=|35|\neq 0$$

$\therefore A_{11}=5, A_{12}=5, A_{13}=-5, A_{21}=0, A_{22}=7, A_{23}$
$=7, A_{31}=10, A_{32}=-11$ and $A_{33}=4$

$$\therefore adj.A=\begin{vmatrix}5 & 5 & -5\\0 & 7 & 7\\10 & -11 & 4\end{vmatrix}=\begin{vmatrix}5 & 0 & 10\\5 & 7 & -11\\-5 & 7 & 4\end{vmatrix}$$

$$\text{Now, } A^{-1}=\frac{adj.A}{|A|}=\frac{1}{35}\begin{vmatrix}5 & 0 & 10\\5 & 7 & -11\\-5 & 7 & 4\end{vmatrix}$$

For $X=A^{-1}B$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{35}\begin{bmatrix}5 & 0 & 10\\5 & 7 & -11\\-5 & 7 & 4\end{bmatrix}\begin{bmatrix}3\\6\\2\end{bmatrix}$$

$$=\frac{1}{35}\begin{bmatrix}15+20\\15+42-22\\-15+42+8\end{bmatrix}$$

$$=\frac{1}{35}\begin{bmatrix}35\\35\\35\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\therefore x=1, y=1, z=1$ **[3]**

Q. 35. Using properties of determinates, prove that

$$\begin{vmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1+3x}\\\mathbf{1+3y} & \mathbf{1} & \mathbf{1}\\\mathbf{1} & \mathbf{1+3z} & \mathbf{1}\end{vmatrix}=\mathbf{9(3xyz+xy+yz+zx)}$$

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$\begin{vmatrix}1 & 1 & 1+3x\\1+3y & 1 & 1\\1 & 1+3z & 1\end{vmatrix}$$

Apply $R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$$\begin{vmatrix}1 & 1 & 1+3x\\1+3y & 1 & 1\\1 & 1+3z & 1\end{vmatrix}=\begin{vmatrix}-3y & 0 & 3x\\3y & -3z & 0\\1 & 1+3z & 1\end{vmatrix}$$

Taking 3 common from R_1 and R_2, we get

$$\begin{vmatrix}-3y & 0 & 3x\\3y & -3z & 0\\1 & 1+3z & 1\end{vmatrix}=(3)(3)\begin{vmatrix}-y & 0 & x\\y & -z & 0\\1 & 1+3z & 1\end{vmatrix}$$

$$=9[(-y)(-z)-0+x(y+3yz+z)]$$
$$=9[yz+xy+3xyz+xz]$$
$$=9(3xyz+xy+yz+zx)$$

Hence proved. **[4]**

Q. 36. Using properties of determinants, prove that

$$\begin{vmatrix}\mathbf{x} & \mathbf{x+y} & \mathbf{x+2y}\\\mathbf{x+2y} & \mathbf{x} & \mathbf{x+y}\\\mathbf{x+y} & \mathbf{x+2y} & \mathbf{x}\end{vmatrix}=\mathbf{9y^2(x+y)}$$

[CBSE Board, Delhi Region, 2017]

Ans.

$$\begin{vmatrix}x & x+y & x+2y\\x+2y & x & x+y\\x+y & x+2y & x\end{vmatrix}$$

$C_1\to C_1+C_2+C_3$

$$=3(x+y)\begin{vmatrix}1 & x+y & x+2y\\1 & x & x+y\\1 & x+2y & x\end{vmatrix}$$

$R_1\to R_1-R_2, R_3\to R_3-R_2$

$$=3(x+y)\begin{vmatrix}0 & y & y\\1 & x & x+y\\0 & 2y & -y\end{vmatrix}$$

$$=-3(x+y)(-y^2-2y^2)$$
$$=9y^2(x+y)$$

[4]

Q. 37. Using properties of determinants, prove that

$$\begin{vmatrix}\mathbf{a^2+2a} & \mathbf{2a+1} & \mathbf{1}\\\mathbf{2a+1} & \mathbf{a+2} & \mathbf{1}\\\mathbf{3} & \mathbf{3} & \mathbf{1}\end{vmatrix}=\mathbf{(a-1)^3}$$

[CBSE Board, All India Region, 2017].

Ans.

$$\Delta=\begin{vmatrix}a^2+2a & 2a+1 & 1\\2a+1 & a+2 & 1\\3 & 3 & 1\end{vmatrix}$$

$R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$$\Delta=\begin{vmatrix}a^2-1 & a-1 & 0\\2(a-1) & a-1 & 0\\3 & 3 & 1\end{vmatrix}$$

$$=(a-1)^2\begin{vmatrix}a+1 & 1 & 0\\2 & 1 & 0\\3 & 3 & 1\end{vmatrix}$$

On expanding, we get

$$=(a-1)^2.(a-1)$$
$$=(a-1)^3$$

[4]

Long Answer Type Questions

(5 or 6 marks each)

Q. 1. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

[NCERT Ex. 4.2, Q. 5, Page 119]

Ans.

$$\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix} + \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

$= \Delta_1 + \Delta_2$ (say) ...(i)

Now, $\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$, we have

$$\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta_1 = \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \leftrightarrow R_3$ and $R_2 \leftrightarrow R_3$ we have

$$\Delta_1 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad ...(ii)$$

$$\Delta_2 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, we have

$$\Delta_2 = \begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta_2 = \begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \leftrightarrow R_2$ and $R_2 \leftrightarrow R_3$ we have

$$\Delta_2 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad ...(iii)$$

From (i), (ii) and (iii), we have

$$\Delta = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Hence proved. **[5]**

Q. 2. By using properties of determinants, show that :

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

[NCERT Ex. 4.2, Q. 8, Page 120]

Ans.

(i) Let we assume that

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we have

$$\Delta = \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (b-c)(c-a)\begin{vmatrix} 0 & -1 & -a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2$ we have

$$\Delta = (b-c)(c-a)\begin{vmatrix} 0 & 0 & -a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along C_1, we have

$$\Delta = (a-b)(b-c)(c-a)\begin{vmatrix} 0 & -1 \\ 1 & b+c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

Hence proved. **[2½]**

(ii) Let we assume that

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we have

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ a^3-c^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 1\\ a-c & b-c & c\\ (a-c)(a^2+ac+c^2) & (b-c)(b^2+bc+c^2) & c^3\end{vmatrix}$$

$$=(c-a)(b-c)\begin{vmatrix}0 & 0 & 1\\ -1 & 1 & c\\ -(a^2+ac+c^2) & (b^2+bc+c^2) & c^3\end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$ we have

$$\Delta=(c-a)(b-c)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ (b^2-a^2)+(bc-ac) & (b^2+bc+c^2) & c^3\end{vmatrix}$$

$$=(b-c)(c-a)(a-b)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -(a+b+c) & (b^2+bc+c^2) & c^3\end{vmatrix}$$

$$=(a-b)(b-c)(c-a)(a+b+c)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -1 & (b^2+bc+c^2) & c^3\end{vmatrix}$$

Expanding along C_1, we have

$$\Delta=(a-b)(b-c)(c-a)(a+b+c)(-1)\begin{vmatrix}0 & 1\\ 1 & c\end{vmatrix}$$

$$=(a-b)(b-c)(c-a)(a+b+c)$$

Hence proved. **[2½]**

Q. 3. By using properties of determinants, show that :

$$\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$$

[NCERT Ex. 4.2, Q. 9, Page 120]

Ans. Let we assume that,

$$\Delta=\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we have

$$\Delta=\begin{vmatrix}x & x^2 & yz\\ y-x & y^2-x^2 & zx-yz\\ z-x & z^2-x^2 & xy-yz\end{vmatrix}$$

$$=\begin{vmatrix}x & x^2 & yz\\ -(x-y) & -(x-y)(x+y) & z(x-y)\\ (z-x) & (z-x)(z+x) & -y(z-x)\end{vmatrix}$$

$$=(x-y)(z-x)\begin{vmatrix}x & x^2 & yz\\ -1 & -x-y & z\\ 1 & z+x & -y\end{vmatrix}$$

Applying $R_3 \to R_3 + R_2$ we have

$$\Delta=(x-y)(z-x)\begin{vmatrix}x & x^2 & yz\\ -1 & -x-y & z\\ 0 & z-y & z-y\end{vmatrix}$$

$$=(x-y)(z-x)(z-y)\begin{vmatrix}x & x^2 & yz\\ -1 & -x-y & z\\ 0 & 1 & 1\end{vmatrix}$$

Expanding along R_3 we have

$$\Delta=[(x-y)(z-x)(z-y)]\left[(-1)\begin{vmatrix}x & yz\\ -1 & z\end{vmatrix}+1\begin{vmatrix}x & x^2\\ -1 & -x-y\end{vmatrix}\right]$$

$$=(x-y)(z-x)(z-y)\left[(-xz-yz)+(-x^2-xy+x^2)\right]$$

$$=-(x-y)(z-x)(z-y)(xy+yz+zx)$$

$$=(x-y)(y-z)(z-x)(xy+yz+zx)$$

Hence proved. **[5]**

Q. 4. By using properties of determinants, show that :

(i) $\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}=(5x+4)(4-x)^2$

(ii) $\begin{vmatrix}y+k & y & y\\ y & y+k & y\\ y & y & y+k\end{vmatrix}=k^2(3y+k)$

[NCERT Ex. 4.2, Q. 10, Page 120]

Ans.

(i) Let we assume that,

$$\Delta=\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ we have

$$\Delta=\begin{vmatrix}5x+4 & 5x+4 & 5x+4\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}$$

$$=(5x+4)\begin{vmatrix}1 & 1 & 1\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ we have

$$\Delta=(5x+4)\begin{vmatrix}1 & 0 & 0\\ 2x & -x+4 & 0\\ 2x & 0 & -x+4\end{vmatrix}$$

$$=(5x+4)(4-x)(4-x)\begin{vmatrix}1 & 0 & 0\\ 2x & 1 & 0\\ 2x & 0 & 1\end{vmatrix}$$

Expanding along C_3 we have

$$\Delta=(5x+4)(4-x)^2\begin{vmatrix}1 & 0\\ 2x & 1\end{vmatrix}$$

$$=(5x+4)(4-x)^2$$

Hence proved. **[2½]**

(ii) Let we assume that

$$\Delta=\begin{vmatrix}y+k & y & y\\ y & y+k & y\\ y & y & y+k\end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ we have

$$\Delta=\begin{vmatrix}3y+k & 3y+k & 3y+k\\ y & y+k & y\\ y & y & y+k\end{vmatrix}$$

$$=(3y+k)\begin{vmatrix}1 & 1 & 1\\ y & y+k & y\\ y & y & y+k\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ we have

$$\Delta = (3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}$$

$$= k^2(3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & 1 & 0 \\ y & 0 & 1 \end{vmatrix}$$

Expanding along C_3 we have

$$\Delta = k^2(3y+k)\begin{vmatrix} 1 & 0 \\ y & 1 \end{vmatrix}$$

$$= k^2(3y+k)$$

Hence proved. **[2½]**

Q. 5. By using properties of determinants, show that :

(i) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(ii) $$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

[NCERT Ex. 4.2, Q. 11, Page 120]

Ans. **(i)** Let we assume that,

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we have

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

Expanding along C_3, we have

$$\Delta = (a+b+c)^3(-1)(-1)$$

$$= (a+b+c)^3$$

Hence proved. **[2½]**

(ii) Let we assume that,

$$\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we have

$$\Delta = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Delta = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)^3\begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we have

$$\Delta = 2(x+y+z)^3(1)(1-0) = 2(x+y+z)^3$$

Hence proved. **[2½]**

Q. 6. By using properties of determinants, show that :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

[NCERT Ex. 4.2, Q. 12, Page 121]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have

$$\Delta = (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

$$= (1-x^3)(1-x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

Expanding along R_1, we have

$$\Delta = (1-x^3)(1-x)(1)\begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$$

$$= (1-x^3)(1-x)(1+x+x^2)$$

$$= (1-x^3)(1-x^3)$$

$$= (1-x^3)^2$$

Hence proved. **[5]**

Q. 7. By using properties of determinants, show that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

[NCERT Ex. 4.2, Q. 13, Page 121]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + bR_3$ and $R_2 \to R_2 - aR_3$, we have

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expanding along R_1, we have

$$\Delta = (1+a^2+b^2)^2 \left[(1)\begin{vmatrix} 1 & a \\ -2a & 1-a^2-b^2 \end{vmatrix} - b\begin{vmatrix} 0 & 1 \\ 2b & -2a \end{vmatrix} \right]$$

$$= (1+a^2+b^2)^2 \left[1-a^2-b^2+2a^2-b(-2b)\right]$$

$$= (1+a^2+b^2)^2(1+a^2+b^2)$$

$$= (1+a^2+b^2)^3$$

Hence proved. **[5]**

Q. 8. By using properties of determinants, show that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

[NCERT Ex. 4.2, Q. 14, Page 121]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a, b and c from R_1, R_2 and R_3 respectively, we have

$$\Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \to aC_1$, $C_2 \to bC_2$, and $C_3 \to cC_3$, we have

$$\Delta = abc \times \frac{1}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we get

$$\Delta = (-1)\begin{vmatrix} b^2 & c^2 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} a^2+1 & b^2 \\ -1 & 1 \end{vmatrix}$$

$$= -1(-c^2) + (a^2+1+b^2)$$

$$= 1+a^2+b^2+c^2$$

Hence proved. **[5]**

Q. 9. Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

[NCERT Ex. 4.3, Q. 2, Page 123]

Ans. Area of ΔABC is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

[Applying $R_3 \to R_3 + R_2$]

$= 0$ [All elements of R_3 are 0]

Thus, the area of the triangle formed by points *A*, *B* and *C* is zero. Hence, the points *A*, *B* and *C* are collinear.

Q. 10. Find values of *k* if area of triangle is 4 sq. units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$

(ii) $(-2, 0), (0, 4), (0, k)$

[NCERT Ex. 4.3, Q. 3, Page 123]

Ans. We know that the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is the absolute value of the determinant (Δ), where

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.

Therefore, $\Delta = \pm 4$. **[1]**

(i) The area of the triangle with vertices (k, 0), (4, 0) and (0, 2) is given by the relation

$$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

When $-k+4=-4$, $k=8$.

When $-k+4=4$, $k=0$.

Hence, $k = 0, 8$ **[2]**

(ii) The area of the triangle with vertices (−2, 0), (0, 4) and (0, k) is given by the relation

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(4-\text{k})]$$

$$= k-4$$

$\therefore k-4 = \pm 4$

When $k-4=-4$, $k=0$

When $k-4=4$, $k=8$

Hence, $k = 0, 8$ **[2]**

Q. 11. Write Minors and Co-factors of the elements of following determinants :

(i) $\begin{vmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{vmatrix}$

(ii) $\begin{vmatrix} \mathbf{1} & \mathbf{0} & \mathbf{4} \\ \mathbf{3} & \mathbf{5} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \end{vmatrix}$ **[NCERT Ex. 4.4, Q. 2, Page 126]**

Ans. (i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

By the definition of minors and co-factors, we have

$$M_{11} = \text{Minor of } a_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$M_{12} = \text{Minor of } a_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$M_{13} = \text{Minor of } a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{21} = \text{Minor of } a_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$M_{22} = \text{Minor of } a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$M_{23} = \text{Minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{31} = \text{Minor of } a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$M_{32} = \text{Minor of } a_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{33} = \text{Minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{11} = \text{Co-factor of } a_{11} = (-1)^{1+1} M_{11} = 1$$

$$A_{12} = \text{Co-factor of } a_{12} = (-1)^{1+2} M_{12} = 0$$

$$A_{13} = \text{Co-factor of } a_{13} = (1)^{1+3} M_{13} = 0$$

$$A_{21} = \text{Co-factor of } a_{21} = (-1)^{2+1} M_{21} = 0$$

$$A_{22} = \text{Co-factor of } a_{22} = (-1)^{2+2} M_{22} = 1$$

$$A_{23} = \text{Co-factor of } a_{23} = (-1)^{2+3} M_{23} = 0$$

$$A_{31} = \text{Co-factor of } a_{31} = (-1)^{3+1} M_{31} = 0$$

$$A_{32} = \text{Co-factor of } a_{32} = (-1)^{3+2} M_{32} = 0$$

$$A_{33} = \text{Co-factor of } a_{33} = (-1)^{3+3} M_{33} = 1$$

[5]

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

By definition of minors and co-factors, we have

$$M_{11} = \text{Minor of } a_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10+1 = 11$$

$$M_{12} = \text{Minor of } a_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6-0 = 6$$

$$M_{13} = \text{Minor of } a_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3-0 = 3$$

$$M_{21} = \text{Minor of } a_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0-4 = -4$$

$$M_{22} = \text{Minor of } a_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2-0 = 2$$

$$M_{23} = \text{Minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$$

$$M_{31} = \text{Minor of } a_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0-20 = -20$$

$$M_{32} = \text{Minor of } a_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1-12 = -13$$

$$M_{33} = \text{Minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5-0 = 5$$

A_{11} = Co-factor of a_{11} = $(-1)^{1+1} M_{11}$ **= 11**

A_{12} = Co-factor of a_{12} = $(-1)^{1+2} M_{12}$ **= –6**

A_{13} = Co-factor of a_{13} = $(-1)^{1+3} M_{13}$ **= 3**

A_{21} = Co-factor of a_{21} = $(-1)^{2+1} M_{21}$ **= 4**

A_{22} = Co-factor of a_{22} = $(-1)^{2+2} M_{22}$ **= 2**

A_{23} = Co-factor of a_{23} = $(-1)^{2+3} M_{23}$ **= –1**

A_{31} = Co-factor of a_{31} = $(-1)^{3+1} M_{31}$ **= –20**

A_{32} = Co-factor of a_{32} = $(-1)^{3+2} M_{32}$ **= 13**

A_{33} = Co-factor of a_{33} = $(-1)^{3+3} M_{33}$ **= 5** **[5]**

Q. 12. Using Co-factors of elements of third column, evaluate $\Delta = \begin{vmatrix} \mathbf{1} & x & yz \\ \mathbf{1} & y & zx \\ \mathbf{1} & z & xy \end{vmatrix}$

[NCERT Ex. 4.4, Q. 4, Page 126]

Ans. The given determinant is $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

We have

$$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z-y$$

$$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z-x$$

$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

Therefore,

A_{13} = Co-factor of $a_{13} = (-1)^{1+3} M_{13} = (z - y)$

A_{23} = Co-factor of $a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$

A_{33} = Co-factor of $a_{33} = (-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding co-factors.

Therefore, $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + x^2z + x^2z - xz^2 + xy^2 - x^2y$

$= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$

$= z(x^2 - y^2) + z^2(y - x) + xy(y - x)$

$= z(x-y)(x+y) + z^2(y-x) + xy(y-x)$

$= (x-y)[zx + zy - z^2 - xy]$

$= (x-y)[\,z(x-z) + y(z-x)]$

$= (x-y)(z-x)[-z+y]$

$= (x-y)(y-z)(z-x)$

Hence, $\Delta = (x-y)(y-z)(z-x)$ **[5]**

Q. 13. Verify $A(adj.A) = (adj.A)A = |A|I$.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

[NCERT Ex. 4.5, Q. 4, Page 131]

Ans. Given that,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$|A| = 1(0-0) + 1(9+2) + 2(0-0) = 11$

$$\therefore |A|I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now,

$A_{11} = 0, A_{12} = -(9+2) = -11, A_{13} = 0$

$A_{21} = -(-3-0) = 3, A_{22} = 3 - 2 = 1, A_{23} = -(0+1) = -1$

$A_{31} = 2 - 0 = 2, A_{32} = -(-2-6) = 8, A_{33} = 0 + 3 = 3$

$$\therefore adj.A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now,

$$A(adj.A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Also,

$$\therefore (adj.A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence, $A(adj.A) = (adj.A)A = |A|I$. **[5]**

Q. 14. **If** $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **Verify that** $A^3 - 6A^2 + 9A - 4I$ **=0 and hence find** A^{-1}.

[NCERT Ex. 4.5, Q. 16, Page 132]

Ans.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now, $A^3 - 6A^2 + 9A - 4I = 0$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$=\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}-\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Now, we will find the value of A^{-1}

$A^3-6A^2+9A-4I=0$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+9AA^{-1}-4IA^{-1}=0$

[Post-multiplying by A^{-1} as $|A|\neq 0$]

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+9AA^{-1}=4(IA^{-1})$

$\Rightarrow AAI-6AI+9I=4A^{-1}$

$\Rightarrow A^2-6A+9I=4A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{4}(A^2-6A+9I)$ ...(i)

$A^2-6A+9I$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-6\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}+9\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-\begin{bmatrix}12 & -6 & 6\\ -6 & 12 & -6\\ 6 & -6 & 12\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

From equation (i), we have

$$A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

[5]

Q. 15. Solve system of linear equations, using matrix method.

$\mathbf{2x+y+z=1}$

$\mathbf{x-2y-z=\frac{3}{2}}$

$\mathbf{3y-5z=9}$

[NCERT Ex. 4.6, Q. 11, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2 & 1 & 1\\ 1 & -2 & -1\\ 0 & 3 & -5\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$$

Now,

$|A|=2(10+3)-1(-5-3)+0$

$=2(13)-1(-8)$

$=26+8$

$=34\neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=13, A_{12}=5, A_{13}=3$

$A_{21}=8, A_{22}=-10, A_{23}=-6$

$A_{31}=1, A_{32}=3, A_{33}=-5$

$$\therefore A^{-1}=\frac{1}{|A|}(adj.A)=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{34}\begin{bmatrix}13+12+9\\ 5-15+27\\ 3-9-45\end{bmatrix}=\frac{1}{34}\begin{bmatrix}34\\ 17\\ -51\end{bmatrix}$$

$$=\begin{bmatrix}1\\ \frac{1}{2}\\ -\frac{3}{2}\end{bmatrix}$$

Hence, $x=1, y=\frac{1}{2}$ and $z=-\frac{3}{2}$

[5]

Q. 16. Solve system of linear equations, using matrix method.

$\mathbf{x-y+z=4}$

$\mathbf{2x+y-3z=0}$

$\mathbf{x+y+z=2}$

[NCERT Ex. 4.6, Q. 12, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

Now,

$|A|=1(1+3)+1(2+3)+1(2-1)=4+5+1=10\neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=4, A_{12}=-5, A_{13}=1$

$A_{21}=2, A_{22}=0, A_{23}=-2$

$A_{31}=2, A_{32}=5, A_{33}=3$

$$\therefore \quad A^{-1}=\frac{1}{|A|}(adj.A)=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

$$\therefore \quad X=A^{-1}B=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{bmatrix}=\frac{1}{10}\begin{bmatrix}20\\ -10\\ 10\end{bmatrix}$$

$$=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$$

Hence, $x=2, y=-1$ and $z=1$

[5]

Q. 17. Solve system of linear equations, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

[NCERT Ex. 4.6, Q. 13, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

Now,

$$|A| = 2(4+1)-3(-2-3)+3(-1+6)$$
$$= 2(5)-3(-5)+3(5)$$
$$= 10+15+15$$
$$= 40 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=5, A_{12}=5, A_{13}=5$

$A_{21}=3, A_{22}=-13, A_{23}=11$

$A_{31}=9, A_{32}=1, A_{33}=-7$

$$\therefore\quad A^{-1}=\frac{1}{|A|}(adj.A)=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}$$

$$\therefore\quad X=A^{-1}B=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}25-12+27\\25+52+3\\25-44-21\end{bmatrix}=\frac{1}{40}\begin{bmatrix}40\\80\\-40\end{bmatrix}$$

$$=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

Hence, $x=1, y=2$ and $z=-1$ **[5]**

Q. 18. Solve system of linear equations, using matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

[NCERT Ex. 4.6, Q. 14, Page 136]

Ans. The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}7\\-5\\12\end{bmatrix}$$

Now,

$$|A| = 1(12-5)+1(9+10)+2(-3-8)$$
$$= 7+19-22$$
$$= 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=7, A_{12}=-19, A_{13}=-11$

$A_{21}=1, A_{22}=-1, A_{23}=-1$

$A_{31}=-3, A_{32}=11, A_{33}=7$

$$\therefore\quad A^{-1}=\frac{1}{|A|}(adj.A)=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$$

$$\therefore\quad X=A^{-1}B=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}\begin{bmatrix}7\\-5\\12\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}49-5-36\\-133+5+132\\-77+5+84\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix}$$

$$=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

Hence, $x=2, y=1$ and $z=3$ **[5]**

Q. 19. If $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations.

$2x-3y+5z=11$

$3x+2y-4z=-5$

$x+y-2z=-3$

[NCERT Ex. 4.6, Q. 15, Page 137]

Ans. Given that, $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$

$$\therefore |A| = 2(-4+4)+3(-6+4)+5(3-2)$$
$$= 0-6+5=-1\neq 0$$

Now,

$A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$$\therefore A^{-1}=\frac{1}{|A|}(adj.A)=-\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}$$

...(i)

Now, the given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$$

The solution of the system of equations is given by $X = A^{-1}B$

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix} \quad \text{[By using Eq. (i)]}$$

$$=\begin{bmatrix}0-5+6\\-22-45+69\\-11-25+39\end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2$ and $z = 3$ **[5]**

Q. 20. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs. 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs. 70. Find cost of each item per kg by matrix method.

[NCERT Ex. 4.6, Q. 16, Page 138]

Ans. Let the cost of onions, wheat and rice per kg be ₹ x, ₹ y and ₹ z respectively. Then the given situation can be represented as :

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70$$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$|A| = 4(12-12) - 3(6-36) + 2(4-24)$$
$$= 0 + 90 - 40 = 50 \neq 0$$

Now, $A_{11} = 0, A_{12} = 30, A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adj. A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj. A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now,

$X = A^{-1}B$

$$\Rightarrow \quad X = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore$ Hence, $x = 5, y = 8$ and $z = 8$

Hence, the cost of onions is ₹ 5 per kg, the cost of wheat is ₹ 8 per kg, and the cost of rice is ₹ 8 per kg.

[5]

Q. 21. If a, b and c are real numbers, and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either $a + b + c = 0$ or $a = b = c$.

[NCERT Misc. Ex. Q. 4, Page 141]

Ans. Given that,

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$\Delta = 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = 2(a+b+c)(1)[(b-c)(c-b) - (b-a)(c-a)]$$
$$= 2(a+b+c)[-b^2 - c^2 + 2bc - bc + ba + ac - a^2]$$
$$= 2(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2]$$

It is given that $\Delta = 0$.

$$(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2] = 0$$

$\Rightarrow$ Either $a + b + c = 0, ab + bc + ca - a^2 - b^2 - c^2 = 0$.

Now,

$$ab + bc + ca - a^2 - b^2 - c^2 = 0$$
$$\Rightarrow -2ab - 2bc - 2ca + 2a^2 + 2b^2 + 2c^2 = 0$$
$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$
$$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0$$

$[(a-b)^2, (b-c)^2, (c-a)^2$ are non-negative$]$

$$\Rightarrow (a-b) = (b-c) = (c-a) = 0$$
$$\Rightarrow a = b = c$$

Hence, if $\Delta = 0$, then either $a + b + c = 0$ or $a = b = c$. [5]

Q. 22. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$

[NCERT Misc. Ex. Q. 5, Page 141]

Ans. Given that,

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$(3x+a)\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1, we have

$$(3x+a)\left[1 \times a^2\right] = 0$$

$$\Rightarrow \quad a^2(3x+a) = 0$$

But $a \neq 0$

Therefore, we have :

$$3x + a = 0$$

$$\Rightarrow \quad x = -\frac{a}{3}$$

[5]

Q. 23. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

[NCERT Misc. Ex. Q. 6, Page 141]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking out common factors a, b, c from C_1, C_2 and C_3, we have

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we have

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ b-a & b & -a \end{vmatrix}$$

Applying $R_2 \to R_2 + R_1$, we have

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b-a & b & -a \end{vmatrix}$$

Applying $R_3 \to R_3 + R_2$, we have

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 2b & 2b & 0 \end{vmatrix}$$

$$= 2ab^2c\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 1 & 1 & 0 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we have:

$$\Delta = 2ab^2c\begin{vmatrix} a & c-a & a+c \\ a+b & -a & a \\ 1 & 0 & 0 \end{vmatrix}$$

Expanding along R_3, we have

$$\Delta = 2ab^2c\left[a(c-a) + a(a+c)\right]$$

$$= 2ab^2c[ac - a^2 + a^2 + ac]$$

$$= 2ab^2c\left[2ac\right]$$

$$= 4a^2b^2c^2$$

Hence, the given result is proved. **[5]**

Q. 24. If

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}, \text{ find } (AB)^{-1}.$$

[NCERT Misc. Ex. Q. 7, Page 141]

Ans. We know that $(AB)^{-1} = B^{-1}A^{-1}$

$$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\therefore |B| = 1 \times (3-0) - 2(-1) - 2(2-0) = 3 + 2 - 4 = 5 - 4 = 1$$

Now, $A_{11} = 3, A_{12} = 1, A_{13} = 2$

$A_{21} = 2, A_{22} = 1, A_{23} = 2$

$A_{31} = 6, A_{32} = 2, A_{33} = 5$

$$\therefore adj.B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$B^{-1} = \frac{1}{|B|}.adj.B$$

$$\therefore \quad B^{-1} = 1.\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

[5]

Q. 25. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ **Verify that**

(i) $[adj.A]^{-1} = adj.(A^{-1})$

(ii) $(A^{-1})^{-1} = A$

[NCERT Misc. Ex. Q. 8, Page 142]

Ans. Given that,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\therefore |A| = 1(15-1) + 2(-10-1) + 1(-2-3)$$
$$= 14 - 22 - 5$$
$$= -13$$

Now,

$A_{11} = 14, A_{12} = 11, A_{13} = -5$
$A_{21} = 11, A_{22} = 4, A_{23} = -3$
$A_{31} = -5, A_{32} = -3, A_{13} = -1$

$$\therefore adj.A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(adj.A)$$

$$= -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

[1]

(i) $[adj.A]^{-1} = adj.(A^{-1})$

$$|adj.A| = 14(-4-9) - 11(-11-15) - 5(-33+20)$$
$$= 14(-13) - 11(-26) - (5)(-13)$$
$$= -182 + 286 + 65$$
$$= 169$$

We have,

$$adj.(adj.A) = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$\therefore \quad [adj.A]^{-1} = \frac{1}{|adj.A|}(adj.(adj.A))$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$\therefore adj.A^{-1} \begin{bmatrix} -\frac{4}{169} - \frac{9}{169} & -\left(-\frac{11}{169} - \frac{15}{169}\right) & -\frac{33}{169} + \frac{20}{169} \\ -\left(-\frac{11}{169} - \frac{15}{169}\right) & -\frac{14}{169} - \frac{25}{169} & -\left(-\frac{42}{169} + \frac{55}{169}\right) \\ -\frac{33}{169} + \frac{20}{169} & -\left(-\frac{42}{169} + \frac{55}{169}\right) & \frac{56}{169} - \frac{121}{169} \end{bmatrix}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

[2]

Hence, $[adj.A]^{-1} = adj.(A^{-1})$.

(ii) $(A^{-1})^{-1} = A$

We have shown that,

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

and, $adj.A^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$

Now,

$$|A^{-1}| = \left(\frac{1}{13}\right)^3 [-14(-13) + 11(-26) + 5(-13)]$$
$$= \left(\frac{1}{13}\right)^3 (-169)$$
$$= -\frac{1}{13}$$

$$\therefore \quad (A^{-1})^{-1} = \frac{adj.A^{-1}}{|A^{-1}|}$$

$$= \frac{1}{\left(-\frac{1}{13}\right)} \times \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$= A$$

Hence, $(A^{-1})^{-1} = A$

[2]

Q. 26. **Evaluate** $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

[NCERT Misc. Ex. Q. 9, Page 142]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta = \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have

$$\Delta = 2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix}$$

Expanding along R_1, we get

$$\Delta = 2(x+y)\left[-x^2 + y(x-y)\right]$$

$$= -2(x+y)(x^2+y^2-yx)$$

$$= -2(x^3+y^3)$$ **[5]**

Q. 27. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x),$$

where p is any scalar

[NCERT Misc. Ex. Q. 12, Page 142]

Ans. Let we assume that,

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y-x & y^2-x^2 & p(y^3-x^3) \\ z-x & z^2-x^2 & p(z^3-x^3) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^2 \\ 1 & y+x & p(y^2+x^2+xy) \\ 1 & z+x & p(z^2+x^2+xz) \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we have

$$\Delta = (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^2+x^2+xy) \\ 0 & z-y & p(z-y)(x+y+z) \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^2+x^2+xy) \\ 0 & 1 & p(x+y+z) \end{vmatrix}$$

Expanding along R_3, we have

$$\Delta = (x-y)(y-z)(z-x)$$
$$\left[(-1)(p)(xy^2+x^3+x^2y)+1+px^3+p(x+y+z)(xy)\right]$$
$$= (x-y)(y-z)(z-x)$$
$$\left[-pxy^2-px^3-px^2y+1+px^3+px^2y+pxy^2+pxyz\right]$$
$$= (x-y)(y-z)(z-x)[1+pxyz]$$

Hence proved. **[5]**

Q. 28. Using properties of determinants, prove that :

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

[NCERT Misc. Ex. Q. 13, Page 142]

Ans. Taking LHS, we get

$$\Delta = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we have

$$\Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Delta = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1, we have

$$\Delta = (a+b+c)[(2b+a)(2c+a)-(a-b)(a-c)]$$
$$= (a+b+c)[4bc+2ab+2ac+a^2-a^2+ac+ba-bc]$$
$$= (a+b+c)(3ab+3bc+3ca)$$
$$= 3(a+b+c)(ab+bc+ca)$$

Hence proved. **[5]**

Q. 29. Solve the system of the following equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

[NCERT Misc. Ex. Q. 16, Page 142]

Ans. Given that,

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Then the given system of equations is as follows

$$2p+3q+10r=4$$
$$4p-6q+5r=1$$
$$6p+9q-20r=2$$

This system can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Now, $|A| = 2(120-45)-3(-80-30)+10(36+36)$

$$= 150+330+720$$
$$= 1200$$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A_{11}=75, A_{12}=110, A_{13}=72$

$A_{21}=150, A_{22}=-100, A_{23}=0$

$A_{31}=75, A_{32}=30, A_{33}=-24$

$$\therefore \quad A^{-1}=\frac{1}{|A|}adj.A$$

$$=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

Now,

$$X=A^{-1}B$$

$$\Rightarrow \begin{bmatrix}p\\ q\\ r\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$=\frac{1}{1200}\begin{bmatrix}300+150+150\\ 440-100+60\\ 288+0-48\end{bmatrix}$$

$$=\frac{1}{1200}\begin{bmatrix}600\\ 400\\ 240\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{5}\end{bmatrix}$$

$$\therefore \quad p=\frac{1}{2}, q=\frac{1}{3} \text{ and } r=\frac{1}{5}$$

Hence, $x=2, y=3$ and $z=5$

[5]

Q. 30. Show that the ΔABC is an isosceles triangle if the determinant

$$\Delta=\begin{vmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\\ \mathbf{1+\cos A} & \mathbf{1+\cos B} & \mathbf{1+\cos C}\\ \mathbf{\cos^2 A+\cos A} & \mathbf{\cos^2 B+\cos B} & \mathbf{\cos^2 C+\cos C}\end{vmatrix}=\mathbf{0}$$

[NCERT Exemp. Ex. 4.3, Q.16, Page 78, CBSE Board, All India Region, 2016]

Ans. We have,

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1+\cos A & 1+\cos B & 1+\cos C\\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C\end{vmatrix}=0$$

[Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$]

$$\Rightarrow \begin{vmatrix}0 & 0 & 1\\ \cos A-\cos C & \cos B-\cos C & 1+\cos C\\ \cos^2 A+\cos A-\cos^2 C-\cos C & \cos^2 B+\cos B-\cos^2 C-\cos C & \cos^2 C+\cos C\end{vmatrix}=0$$

[Taking (cos$A$–cosC) common from C_1 and (cos$B$–cosC) common from C_2]

$$\Rightarrow (\cos A-\cos C)(\cos B-\cos C)$$

$$\begin{vmatrix}0 & 0 & 1\\ 1 & 1 & 1+\cos C\\ \cos A+\cos C+1 & \cos B+\cos C+1 & \cos^2 C+\cos C\end{vmatrix}=0$$

[Applying $C_1 \to C_1 - C_2$]

$$\Rightarrow (\cos A-\cos C)(\cos B-\cos C)$$

$$\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & 1+\cos C\\ \cos A-\cos B & \cos B+\cos C+1 & \cos^2 C+\cos C\end{vmatrix}=0$$

Expanding along C_1, we get

$$\Rightarrow (\cos A-\cos C)(\cos B-\cos C)(\cos B-\cos A)=0$$

$$\Rightarrow \cos A=\cos C \text{ or } \cos B=\cos C \text{ or } \cos B=\cos A$$

$$\Rightarrow A=C \text{ or } B=C \text{ or } B=A$$

Hence, ΔABC is an isosceles triangle.

[5]

Q. 31. Find A^{-1} if $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ and show that $A^{-1}=\frac{A^2-3I}{2}$

[NCERT Exemp. Ex. 4.3, Q.17, Page 79]

Ans. Given that, $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$

Co-factors are:

$A_{11}=-1, A_{12}=1, A_{13}=1$

$A_{21}=1, A_{22}=-1, A_{23}=1$

$A_{31}=1, A_{32}=1, A_{33}=-1$

$$\therefore \quad adj.A=\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}^T=\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}$$

$$|A|=0-1(-1)+1.1=2$$

$$\therefore \quad A^{-1}=\frac{adj.A}{|A|}=\frac{1}{2}\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}$$

Now,

$$A^2=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$$

$$\therefore \frac{A^2-3I}{2}=\frac{1}{2}\left\{\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}=A^{-1}$$

Hence proved.

[5]

Q. 32. If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$,

Using A^{-1}**, solve the system of linear equations** $x-2y=10$, $2x-y-z=8$ **and** $-2y+z=7$.

[NCERT Exemp. Ex. 4.3, Q.18, Page 79]

Ans. We have,

$$A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix} \quad ...(i)$$

$$\therefore |A|=1(-3)-2(-2)+0=1\neq 0$$

Now,

$A_{11}=-3, A_{12}=2, A_{13}=2$

$A_{21}=-2, A_{22}=1, A_{23}=1$

$A_{31}=-4, A_{32}=2, A_{33}=3$

$$\therefore adj.A=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}^T=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$$

$$\therefore A^{-1}=\frac{adj.A}{|A|}=\frac{1}{1}.\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$$

$$\Rightarrow A^{-1}=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix} \quad .....(ii)$$

Also, we have the system of linear equation as

$x-2y=10,$

$2x-y-z=8$

and $-2y+z=7$

In the form of $CX=D$,

$$\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

where, $C=\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $D=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$

We know that,

$(A^T)^{-1}=(A^{-1})^T$

$$\therefore C^T=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}=A \quad \text{[By using Eq. (i)]}$$

$\therefore X=C^{-1}D$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

$$=\begin{bmatrix}-30+16+14\\ -20+8+7\\ -40+16+21\end{bmatrix}$$

$$=\begin{bmatrix}0\\ -5\\ -3\end{bmatrix}$$

Hence, $x=0, y=-5$ and $z=-3$ **[5]**

Q. 33. Given $A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ **find** BA **and use this to solve the system of equations** $y+2z=7, x-y=3$ **and** $2x+3y+4z=17$.

[NCERT Exemp. Ex. 4.3, Q.20, Page 79]

Ans. We have,

$$A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$$

$$\therefore BA=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}=6I$$

$$\therefore B^{-1}=\frac{A}{6}=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix} \quad ....(i)$$

Given system of equations is

$y+2z=7,$

$x-y=3$

and $2x+3y+4z=17$

or

$\therefore AX=B$

$$\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}$$

$\therefore X=A^{-1}B$

$$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}^{-1}\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}6+34-28\\ -12+34-28\\ 6-17+35\end{bmatrix}=\frac{1}{6}\begin{bmatrix}12\\ -6\\ 24\end{bmatrix}$$

$$=\begin{bmatrix}2\\ -1\\ 4\end{bmatrix}$$

Hence, $x=2, y=-1$ and $z=4$

[5]

Q. 34. If $a+b+c\neq 0$ **and** $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=0$, **then prove that** $a=b=c$.

[NCERT Exemp. Ex. 4.3, Q. 21, Page 79]

Ans. Given that, $a+b+c\neq 0$ and $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=0$,

Let $A=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$

$$=\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ b & c & a\\ c & a & b\end{vmatrix} \quad [\because R_1\to R_1+R_2+R_3]$$

$$=(a+b+c)\begin{vmatrix}1 & 1 & 1\\ b & c & a\\ c & a & b\end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-a & c-a & a \\ c-b & a-b & b \end{vmatrix} \quad [\because C_1 \to C_1 - C_3 \text{ and } C_2 \to C_2 - C_3]$$

Expanding along R_1,

$= (a+b+c)[1(b-a)(a-b)-(c-a)(c-b)]$

$= (a+b+c)(ba-b^2-a^2+ab-c^2+cb+ac-ab)$

$= -\frac{1}{2}(a+b+c)\times(-2)(-a^2-b^2-c^2+ab+bc+ca)$

$= -\frac{1}{2}(a+b+c)[a^2+b^2+c^2-2ab-2bc-2ca+a^2+b^2+c^2]$

$= -\frac{1}{2}(a+b+c)[a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ca]$

$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

Also, $A = 0$

$\Rightarrow -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 0$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2 = 0 \quad [\because a+b+c \neq 0, \text{given}]$

$\Rightarrow a-b = b-c = c-a = 0$

$\Rightarrow a = b = c$

Hence proved. **[5]**

Q. 35. Prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ **is divisible by $(a+b+c)$ and find the quotient.**

[NCERT Exemp. Ex. 4.3, Q. 22, Page 79]

Ans. Given that, $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$

Let we assume that,

$$\Delta = \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} bc-a^2-ca+b^2 & ca-b^2-ab+c^2 & ab-c^2 \\ ca-b^2-ab+c^2 & ab-c^2-bc+a^2 & bc-a^2 \\ ab-c^2-bc+a^2 & bc-a^2-ca+b^2 & ca-b^2 \end{vmatrix}$$

$[\because C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3]$

$$= \begin{vmatrix} (b-a)(a+b+c) & (c-b)(a+b+c) & ab-c^2 \\ (c-a)(a+b+c) & (a-c)(a+b+c) & bc-a^2 \\ (a-c)(a+b+c) & (b-a)(a+b+c) & ca-b^2 \end{vmatrix}$$

$$= (a+b+c)^2\begin{vmatrix} b-a & c-b & ab-c^2 \\ c-a & a-c & bc-a^2 \\ a-c & b-a & ca-b^2 \end{vmatrix}$$

[Taking $(a+b+c)$ common from C_1 and C_2 each]

$$= (a+b+c)^2\begin{vmatrix} 0 & 0 & ab+bc+ca-(a^2+b^2+c^2) \\ c-b & a-c & bc-a^2 \\ a-c & b-a & ca-b^2 \end{vmatrix}$$

$[\because R_1 \to R_1 + R_2 + R_3]$

$\Rightarrow -\frac{1}{b} = -1$

$\Rightarrow b = 1$

and

$\Rightarrow \frac{-3-a}{b} = 1$

$\Rightarrow -3-a = 1$

$\Rightarrow a = -4$

Hence, given determinant is divisible by $(a+b+c)$ and quotient is

$(a^3+b^3+c^3-3abc)[(a-b)^2+(b-c^2)+(c-a)^2]$ **[5]**

Q. 36. If $x+y+z=0$, then prove that

$$\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

[NCERT Exemp. Ex. 4.3, Q. 23, Page 80]

Ans. Given that, $x+y+z=0$

Now, $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} \quad xyz\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Taking LHS, we get

$$\text{LHS} = \begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix}$$

$= xa(za.ya - xb.xc) - yb(yc.ya - xb.zb) + zc(yc.xc - za.zb)$

$= xa(a^2yz - x^2bc) - yb(y^2ac - b^2xz) + zc(c^2xy - z^2ab)$

$= xyza^3 - x^3abc - y^3abc + b^3xyz + c^3xyz - z^3abc$

$= xyz(a^3+b^3+c^3) - abc(x^3+y^3+z^3)$

$= xyz(a^3+b^3+c^3) - abc(3xyz)$

$[\because x+y+z=0 \Rightarrow x^3+y^3+z^3-3xz]$

$= xyz(a^3+b^3+c^3-3abc)$ .....(i)

Now, $\text{RHS} = xyz\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

$$= xyz\begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix} \quad [\because C_1 \to C_1 + C_2 + C_3]$$

$$= xyz(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

[Taking $(a+b+c)$ common from C_1]

$$= xyz(a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & a-c & b-a \\ 1 & c & a \end{vmatrix}$$

$[\because R_1 \to R_1 - R_3 \text{ and } R_2 \to R_2 - R_3]$

Expanding along C_1,

$= xyz(a+b+c)[1(b-c)(b-a)-(a-c)(c-a)]$

$= xyz(a+b+c)(b^2-ab-bc+ca+a^2+c^2-2ac)$

$= xyz(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= xyz(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= xyz(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= xyz(a^3+b^3+c^3-3abc)$ ...(ii)

LHS=RHS

$$\Rightarrow \begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Hence proved. **[5]**

Q. 37. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

[NCERT Ex. 4.5, Q. 12, Page 132]

Ans. We have, $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$|A| = 15 - 14 = 1$

Now,

$A_{11} = 5, A_2 = -2$

$A_{21} = -7, A_{22} = 3$

$$\therefore adj.A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.adj.A = 1.\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Now, let $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$|B| = 54 - 56 = -2$

Now,

$B_{11} = 9, B_{12} = -8$

$B_{21} = -7, B_{22} = 6$

$$\therefore adj.B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|A|}.adj.B = -\frac{1}{2}.\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$$

Now,

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad ...(i)$$

Then,

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

Therefore, we have $|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2$

Also,

$$adj.(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \frac{1}{|AB|}.adj.(AB) = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad ....(ii)$$

From (i) and (ii), we have

$(AB)^{-1} = B^{-1}A^{-1}$.

Hence, the given result is proved. **[5]**

Q. 38. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

[NCERT Ex. 4.5, Q. 13, Page 132]

Ans. Given that,

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$\therefore A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A^2 - 5A + 7I = 0$

$\therefore A.A - 5A = -7I$

$\Rightarrow A.A(A^{-1}) - 5A.A^{-1} = -7IA^{-1}$

[Post-multiplying by A^{-1} as $|A| \neq 0$]

$\Rightarrow A(AA^{-1}) - 5I = -7A^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$

$$\Rightarrow A^{-1} = -\frac{1}{7}(A - 5I)$$

$$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$$

$$\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

[5]

Q. 39. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the numbers a and b such that $A^2 + aA + bI = 0$.

[NCERT Ex. 4.5, Q. 14, Page 132]

Ans.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Now,

$$A^2 + aA + bI = 0$$

$$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = 0$$

[Post-multiplying by A^{-1} as $|A| \neq 0$]

$$\Rightarrow A(AA^{-1}) + aI + b(IA^{-1}) = 0$$

$$\Rightarrow AI + aI + bA^{-1} = 0$$

$$\Rightarrow A + aI = -bA^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{b}(A + aI)$$

Now,

$$A^{-1} = \frac{1}{|A|}adj.A = \frac{1}{1}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

We have,

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{b}\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}\right)$$

$$= -\frac{1}{b}\begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3-a}{b} & -\frac{2}{b} \\ -\frac{1}{b} & \frac{-1-a}{b} \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have

$$\Rightarrow \quad -\frac{1}{b} = -1$$

$$\Rightarrow \quad b = 1$$

and

$$\Rightarrow \quad \frac{-3-a}{b} = 1$$

$$\Rightarrow \quad -3-a = 1$$

$$\Rightarrow \quad a = -4$$

Hence, −4 and 1 are the required values of a and b respectively. **[5]**

Q. 40. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence, find A^{-1}.

[NCERT Ex. 4.5, Q. 15, Page 132]

Ans.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & -5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Thus, $A^3 - 6A^2 + 5A + 11I = 0$

Now,

$A^3 - 6A^2 + 5A + 11I = 0$

$$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 5AA^{-1} + 11IA^{-1} = 0$$

[Post-multiplying by A^{-1} as $|A| \neq 0$]

$$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = -11(IA^{-1})$$

$$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I) \qquad \text{...(i)}$$

Now,

$A^2 - 6A + 5I$

$$=\begin{bmatrix}4 & 2 & 1\\-3 & 8 & -14\\7 & -3 & 14\end{bmatrix}-6\begin{bmatrix}1 & 1 & 1\\1 & 2 & -3\\2 & -1 & 3\end{bmatrix}+5\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 1\\-3 & 8 & -14\\7 & -3 & 14\end{bmatrix}-\begin{bmatrix}6 & 6 & 6\\6 & 12 & -18\\12 & -6 & 18\end{bmatrix}+\begin{bmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}9 & 2 & 1\\-3 & 13 & -14\\7 & -3 & 19\end{bmatrix}-\begin{bmatrix}6 & 6 & 6\\6 & 12 & -18\\12 & -6 & 18\end{bmatrix}+\begin{bmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}9 & 2 & 1\\-3 & 13 & -14\\7 & -3 & 19\end{bmatrix}-\begin{bmatrix}6 & 6 & 6\\6 & 12 & -18\\12 & -6 & 18\end{bmatrix}$$

$$=\begin{bmatrix}3 & -4 & -5\\-9 & 1 & 4\\-5 & 3 & 1\end{bmatrix}$$

From equation (i), we have

$$A^{-1}=-\frac{1}{11}\begin{bmatrix}3 & -4 & -5\\-9 & 1 & 4\\-5 & 3 & 1\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-3 & 4 & 5\\9 & -1 & -4\\5 & -3 & -1\end{bmatrix}$$

[5]

Q. 41. If $A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$ find A^{-1} Use it to solve the system of equations.

$$2x-3y+5z=11$$
$$3x+2y-4z=-5$$
$$x+y-2z=-3$$

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$$

$$\Rightarrow |A|=2(-4+4)+3(-6+4)+5(3-2)$$
$$=-6+5$$
$$=-1$$

Now,

$$adj.A=\begin{bmatrix}(-1)^{1+1}\begin{vmatrix}2 & -4\\1 & -2\end{vmatrix} & (-1)^{1+2}\begin{vmatrix}3 & -4\\1 & -2\end{vmatrix} & (-1)^{1+2}\begin{vmatrix}3 & 2\\1 & 1\end{vmatrix}\\(-1)^{2+1}\begin{vmatrix}1 & 5\\-3 & -2\end{vmatrix} & (-1)^{2+2}\begin{vmatrix}2 & 5\\1 & -2\end{vmatrix} & (-1)^{2+3}\begin{vmatrix}2 & -3\\1 & 1\end{vmatrix}\\(-1)^{3+1}\begin{vmatrix}-3 & 5\\2 & -4\end{vmatrix} & (-1)^{3+2}\begin{vmatrix}2 & 5\\3 & -4\end{vmatrix} & (-1)^{3+3}\begin{vmatrix}2 & -3\\3 & 2\end{vmatrix}\end{bmatrix}^T$$

$$=\begin{bmatrix}0 & 2 & 1\\-1 & -9 & -5\\2 & 23 & 13\end{bmatrix}^T=\begin{bmatrix}0 & -1 & 2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}$$

$$\therefore A^{-1}=-1\begin{bmatrix}0 & -1 & 2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}=\begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}$$

We can write the given equation as $AX = B$

$$\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$\therefore x=1, y=2$ and $z=3$

[6]

Q. 42. If $A=\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations $x - y = 3$; $2x + 3y + 4z = 17$ and $y + 2z = 7$.

[CBSE Board, Foreign Scheme, 2017]

Ans.

$$\text{Getting } AB=\begin{bmatrix}6 & 0 & 0\\0 & 6 & 0\\0 & 0 & 6\end{bmatrix}=6I$$

Given system of equations can be written as

$$\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}13\\17\\7\end{bmatrix}$$

i.e., $AX = C \Rightarrow X=A^{-1}C=\frac{1}{6}BC\left(\because AB=6I\Rightarrow A^{-1}=\frac{1}{6}B\right)$

$$=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix}=\begin{bmatrix}2\\-1\\4\end{bmatrix}$$

$\Rightarrow x=2,\ y=-1,\ z=4$

[6]

Q. 43. Prove that $\begin{vmatrix}yz-x^2 & zx-y^2 & xy-z^2\\zx-y^2 & xy-z^2 & yz-x^2\\xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.

Ans. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

Taking $(x+y+z)$ common from C_1 and C_2

$$\Delta=\begin{vmatrix}y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2\\z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2\\x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2\end{vmatrix}$$

Taking $(x + y + z)$ common from C_1 and C_2

$$\Rightarrow \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = (x+y+z) \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$

Expanding to get

$\Delta = (x + y + z)^2 (xy + yz + zx - x^2 - y^2 - z^2)^2$

Hence Δ is divisible by $(x + y + z)$ and the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$ **[6]**

Q. 44. Using elementary transformations, find the inverse of the matrix $A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use it to solve the following system of linear equations :

$8x + 4y + 3z = 19$

$2x + y + z = 5$

$x + 2y + 2z = 7$

[CBSE Board, Delhi Region, 2016]

Ans. Given that, $A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_3$
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 - 4R_2$
$$\begin{bmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{bmatrix} A$$

$R_1 \to \frac{1}{3}R_1$ and $R_3 \to -R_3$
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 2/3 \\ 1 & 4 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - R_3$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 2, z = 1$ **[6]**

Q. 45. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs. 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs. 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs. 70. Using matrix method, finfd the cost of each variety of pen.

[CBSE Board, All India Region, 2016]

Ans. Let the cost of one pen of variety 'A', 'B' and 'C' be ₹ x, ₹ y and ₹ z respectively then the system of equations is :

$$x + y + z = 21$$
$$4x + 3y + 2z = 60$$
$$6x + 2y + +3z = 70$$

Matrix form of the system is :

$$A.X = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$|A| = (5) - 1(0) + 1(-10) = -5$

Co-factors of the matrix A are :

$C_{11} = 5;$ $C_{21} = -1;$ $C_{31} = -1$

$C_{12} = 0;$ $C_{22} = -3;$ $C_{32} = -1$

$C_{13} = -10$ $C_{23} = 4;$ $C_{33} = -1$

$$\therefore A^{-1} = \frac{1}{|A|} Adj.A = \frac{1}{-5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 & -1 \\ -10 & 4 & \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5, y = 8, z = 8$$

[6]

Q. 46. If a, b and c are all non-zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$ then prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

[CBSE Board, Foreign Scheme, 2016]

Ans. Given that, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

$$abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow abc \begin{bmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{bmatrix} = 0$$

$$\Rightarrow abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{bmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 1 & \frac{1}{b}+1 & \frac{1}{c}\\ 1 & \frac{1}{b} & \frac{1}{c}+1\end{bmatrix}=0$$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\Rightarrow abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=0$$

$\because$ a, b, c, $\neq 0$

$$\therefore\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

[6]

Q. 47. Find *adj.A* and verify that $A(adj.A) = (adj.A)A = |A|I^3$. If

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

[CBSE Board, Foreign, 2016]

Ans.

$$|A|=1$$

$$adj.A=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$A(adj.A)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$$|A|I_3=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

[6]

Some Commonly Made Errors

- Students do not write the formula when doing numerical.
- Students confuse in Inverse and Transpose of the matrix.
- Students do not put right sign convention when finding adjoint.
- Students get confuse in finding co-factor and minor of the matrix.

EXPERT ADVICE

- *For the sums of Matrix Elementary Operation, do not change rows and columns together in the same sum.*
- *In the Determinant sums, to get full marks, students must use the properties of determinant.*
- *For row transformation, change only row, and for column transformation, change only column.*
- *Always Try to Use Direct Methods for the Solution of Linear Algebraic Equations.*