CHAPTER

3 MATRICES

Chapter Objectives

This chapter will help you understand :

- ***Matrices*** *: Order of matrix; Types of matrices; Operations on matrices; Properties of multiplication of matrices; Transpose of matrix; Symmetric and skew symmetric matrices; Elementary operations and Invertible matrix.*

Quick Review

- Matrix is a concept of linear algebra, and it has wide applications in many fields, including economics, statistics, computer programming, operations research, industrial organisation and engineering.
- Multiplication of two matrices can be illustrated from input-output models which are used to illustrate producer theory in economics.
- The concept of matrices originated in the work of the two mathematicians Arthur Cayley and James Sylvester while solving a system of linear equations.
- In 1857, Cayley wrote Memoir on the Theory of Matrices.
- Information content of a tree is given by the negative log of the sum of the probabilities of the trees it permits (Thorley et al., 1998).

Know the Links

- *https://brilliant.org/wiki/matrices/*
- *www.maths.surrey.ac.uk/explore/emmaspages/option1.html*
- *https://www.britannica.com/science/matrix-mathematics*

TIPS...

- *Setting up the Matrix for Solving.*
- *Write your equations in standard form.*
- *Transfer the numbers from the system of equations into a matrix.*
- *Draw a large square bracket around full matrix.*

TRICKS...

- *Recognise the form of the solution matrix.*
- *Use scalar multiplication.*
- *Use row-addition or row-subtraction.*
- *Combine row-addition and scalar multiplication in a single step.*
- *Work from top to bottom first.*

Multiple Choice Questions

(1 mark each)

Q. 1. $A=[a_{ij}]_{m\times n}$ **is a square matrix, if**

(a) $m < n$ (b) $m > n$

(c) $m = n$ (d) None of these

[NCERT Ex. 3.1, Q. 8, Page 65]

Ans. Correct option : (c)

Explanation : It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore,

$A=[a_{ij}]_{m\times n}$ is a square matrix, if $m = n$.

Q. 2. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix}, \begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$$

(a) $x=\frac{-1}{3}, y=7$

(b) Not possible to find

(c) $y=7, x=\frac{-2}{3}$

(d) $x=\frac{-1}{3}, y=\frac{-2}{3}$

[NCERT Ex. 3.1, Q. 9, Page 65]

Ans. Correct option : (b)

Explanation : It is given that

$$\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix} = \begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$$

Equating the corresponding elements, we get:

$3x+7=0$

$\Rightarrow \quad x=-\frac{7}{3}$

$5=y-2$

$\Rightarrow \quad y=7$

$$y+1=8$$
$$\Rightarrow \quad y=7$$
$$2-3x=4$$
$$\Rightarrow \quad x=-\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Q. 3. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(a) 27 **(b) 18**
(c) 81 **(d) 512**

[NCERT Ex. 3.1, Q. 10, Page 65]

Ans. Correct option : (d)

Explanation : The given matrix of the order 3 × 3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$.

Q. 4. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. The restriction on n, k and p so that $PY + WY$ will be defined are:

(a) $k = 3, p = n$ **(b) k is arbitrary, $p = 2$**
(c) p is arbitrary, $k = 3$ **(d) $k = 2, p = 3$**

[NCERT Ex. 3.2, Q. 21, Page 83]

Ans. Correct option : (a)

Explanation : Matrices P and Y are of the orders $p \times k$ and $3 \times k$, respectively. Therefore, matrix PY will be defined if $k = 3$. Consequently, PY will be of the order $p \times k$. Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order $n \times k$. Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$. Thus, $k = 3$ and $p = n$ are the restrictions on n, k, and p so that $PY + WY$ will be defined.

Q. 5. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(a) $p \times 2$ **(b) $2 \times n$**
(c) $n \times 3$ **(d) $p \times n$**

[NCERT Ex. 3.2, Q. 22, Page 83]

Ans. Correct option : (b)

Explanation : Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$, *i.e.*, $2 \times n$ [Since $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Q. 6. If A and B are symmetric matrices of same order, $AB - BA$ is a:

(a) Skew-symmetric matrix **(b) Symmetric matrix**
(c) Zero matrix **(d) Identity matrix**

[NCERT Ex. 3.3, Q. 11, Page 90]

Ans. Correct option : (a)

Explanation : Given that,

A and B are symmetric matrices.

$$\Rightarrow \quad A = A' \text{ and } B = B'$$

Now, $(AB - BA)' = (AB)' - (BA)'$ (i)

$\Rightarrow \quad (AB - BA)' = B'A' - A'B'$ **[By reversal law]**

$\Rightarrow \quad (AB - BA)' = BA - AB$ **[From Eq. (i)]**

$\Rightarrow \quad (AB - BA)' = -(AB - BA)$

$\therefore$ $(AB - BA)$ is a skew matrix.

Q. 7. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is :

(a) $\frac{\pi}{6}$ **(b) $\frac{\pi}{3}$**
(c) π **(d) $\frac{3\pi}{2}$**

[NCERT Ex. 3.3, Q. 12, Page 90]

Ans. Correct option : (b)

Explanation:

Given that,

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Also $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

$$\Rightarrow 2\cos\alpha = 1$$
$$\Rightarrow \quad \cos\alpha = \frac{1}{2}$$
$$\Rightarrow \quad \cos\alpha = \cos\frac{\pi}{3}$$
$$\therefore \quad \alpha = \frac{\pi}{3}$$

Q. 8. Matrices A and B will be inverse of each other only if

(a) $AB = BA$ **(b) $AB = BA = 0$**
(c) $AB = 0, BA = I$ **(d) $AB = BA = I$**

[NCERT Ex. 3.4, Q. 18, Page 97]

Ans. Correct option : (d)

Explanation : We know that if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A.

In this case, it is clear that A is the inverse of B. Thus, matrices A and B will be the inverse of each other only if $AB = BA = I$.

Q. 9. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(a) $1 + \alpha^2 + \beta\gamma = 0$

(b) $1 - \alpha^2 + \beta\gamma = 0$
(c) $1 - \alpha^2 - \beta\gamma = 0$
(d) $1 + \alpha^2 - \beta\gamma = 0$

[NCERT Misc. Ex. Q. 13, Page 101]

Ans. Correct option : (c)

Explanation :

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\therefore A^2 = A \cdot A$$

$$= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$$

Now,

$$A^2 = I$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have :

$$\alpha^2 + \beta\gamma = 1$$

$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Q. 10. If the matrix A is both symmetric and skew symmetric, then

(a) A is a diagonal matrix
(b) A is a zero matrix
(c) A is a square matrix
(d) None of these **[NCERT Misc. Ex. Q. 14, Page 101]**

Ans. Correct option : (b)

Explanation : If A is both symmetric and skew-symmetric matrices, then we should have,

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = 0$$

$$\Rightarrow 2A = 0$$

$$\Rightarrow A = 0$$

Therefore, A is a zero matrix.

Q. 11. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A **(b)** $I - A$
(c) I **(d)** $3A$

[NCERT Misc. Ex. Q. 15, Page 101]

Ans. Correct option : (c)

Explanation:

$$(I+A)^3 - 7A = I^3 + A^3 + 3I^2A + 3A^2I - 7A$$

$$= I + A^3 + 3A + 3A^2 - 7A$$

$$= I + A^2 \cdot A + 3A + 3A - 7A \quad [\because A^2 = A]$$

$$= I + A \cdot A - A$$

$$= I + A^2 - A$$

$$= I + A - A$$

$$= I$$

$$\therefore (I+A)^3 - 7A = I$$

Q. 12. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

(a) square matrix
(b) diagonal matrix
(c) unit matrix
(d) None of these

[NCERT Exemp. Ex. 3.3, Q. 53, Page 59]

Ans. Correct option : (a)

Explanation : We know that, in a square matrix number of rows is equal to the number of columns.

So, the matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a square matrix.

Q. 13. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 **(b)** 27
(c) 81 **(d)** 512

[NCERT Exemp. Ex. 3.3, Q. 54, Page 59]

Ans. Correct option : (d)

Explanation : Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is 2^9 *i.e.*, 512.

Q. 14. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

(a) $x = 3, y = 1$ **(b)** $x = 2, y = 3$
(c) $x = 2, y = 4$ **(d)** $x = 3, y = 3$

[NCERT Exemp. Ex. 3.3, Q. 55, Page 59]

Ans. Correct option : (b)

Explanation : We have,

$$4x = x + 6$$

$$\Rightarrow x = 2$$

And

$$4x = 7y - 13$$

$$\Rightarrow 8 = 7y - 13$$

$$\Rightarrow 7y = 21$$

$$\Rightarrow y = 3$$

$$\therefore x + y = 2 + 3 = 5$$

Q. 15. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ and

$B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$ then $A - B$ is equal to

(a) I **(b)** O
(c) $2I$ **(d)** $\frac{1}{2}I$

[NCERT Exemp. Ex. 3.3, Q. 56, Page 60]

Ans. Correct option : (d)

Explanation : We have,

$$A=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}$$

and

$$B=\frac{1}{\pi}\begin{bmatrix}-\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$$

$$A-B=\begin{bmatrix}\frac{1}{\pi}\left(\sin^{-1}x\pi+\cos^{-1}x\pi\right) & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi}-\tan^{-1}\frac{x}{\pi}\right)\\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi}-\sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\left(\cot^{-1}\pi x+\tan^{-1}x\right)\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{\pi}\left(\frac{\pi}{2}\right) & 0\\ 0 & \frac{\pi}{2}\end{bmatrix}\quad\left[\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} \text{ and } \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$=\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\frac{1}{2}I$$

Q. 16. If A and B are two matrices of the order $3\times m$ and $3\times n$, respectively, and $m = n$, then the order of matrix $(5A - 2B)$ is

(a) $m\times 3$
(b) 3×3
(c) $m\times n$
(d) $3\times n$ **[NCERT Exemp. Ex. 3.3, Q. 57, Page 60]**

Ans. Correct option : (d)

Explanation : $A_{3\times m}$ and $B_{3\times n}$ are two matrices. If $m = n$, then A and B have same orders $3\times n$ as each, so the order of $(5A - 2B)$ should be same as $3\times n$.

Q. 17. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ then A^2 is equal to

(a) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ **(b)** $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}$ **(d)** $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

[NCERT Exemp. Ex. 3.3, Q. 58, Page 60]

Ans. Correct option : (d)

Explanation:

$$A^2 = A\cdot A$$

$$=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Q. 18. If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij}= 1$ if $i \neq j$, and 0 if $i = j$ then A^2 is equal to

(a) I **(b)** A
(c) 0 **(d) None of these**

[NCERT Exemp. Ex. 3.3, Q. 59, Page 60]

Ans. Correct option : (a)

Explanation : We have,

$A = [a_{ij}]_{2\times 2}$, where $a_{ij}= 1$ if $i \neq j$ and 0 if $i = j$.

$$\therefore A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

And

$$A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=I$$

Q. 19. The matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$ is a

(a) identity matrix
(b) symmetric matrix
(c) skew-symmetric matrix
(d) None of these

[NCERT Exemp. Ex. 3.3, Q. 60, Page 60]

Ans. Correct option : (b)

Explanation : $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}=A$$

So, the given matrix is a symmetric matrix.
[Since, in a square matrix A, if $A' = A$, then A is called symmetric matrix.]

Q. 20. The matrix $\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$ is a

(a) diagonal matrix
(b) symmetric matrix
(c) skew symmetric matrix
(d) scalar matrix

[NCERT Exemp. Ex. 3.3, Q. 61, Page 61]

Ans. Correct option : (c)

Explanation : We know that, in a square matrix, if b_{ij}, when $i \neq j$ then it is said to be a diagonal matrix. Here, b_{12}, b_{13}.... $\neq 0$ so the given matrix is not a diagonal matrix.

Now, $B=\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$

$$B'=\begin{bmatrix}0 & 5 & -8\\ -5 & 0 & -12\\ -8 & 12 & 0\end{bmatrix}$$

$$=-\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$$

$$=-B$$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if $B' = -B$, then it is called skew-symmetric matrix.

Q. 21. If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is

(a) $m \times m$ **(b)** $n \times n$

(c) $n \times m$ **(d)** $m \times n$

[NCERT Exemp. Ex. 3.3, Q. 62, Page 61]

Ans. Correct option : (d)

Explanation : Let, $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$

$B' = [b_{ji}]_{q\times p}$

Now, AB′ is defined, so $n = q$ and $B'A$ is also defined, so $p = m$

$\therefore$ Order of $B' = [b_{ji}]_{n\times m}$

And order of $B = [b_{ij}]_{m\times n}$

Q. 22. If A and B are matrices of same order, then ($AB' - BA'$) is a

(a) skew symmetric matrix

(b) null matrix

(c) symmetric matrix

(d) unit matrix

[NCERT Exemp. Ex. 3.3, Q. 63, Page 61]

Ans. Correct option : (a)

Explanation : We have matrices A and B of same order.

Let, $P(AB' - BA')$

Then,

$$P' = (AB' - BA')'$$
$$= (AB')' - (BA')'$$
$$= (B')'(A)' - (A')'B'$$
$$= BA' - AB'$$
$$= -(AB' - BA')$$
$$= -P$$

Hence, $(AB' - BA')$ is a skew symmetric matrix.

Q. 23. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A **(b)** $I - A$

(c) $I + A$ **(d)** $3A$

[NCERT Exemp. Ex. 3.3, Q. 64, Page 61]

Ans. Correct option : (a)

Explanation : We have, $A^2 = I$

$$\therefore (A-I)^3 + (A+I)^3 - 7A$$
$$= \left[(A-I)+(A+I)\left\{(A-I)^2+(A+I)^2-(A-I)(A+I)\right\}\right] - 7A \quad \left[\because a^3+b^3=(a+b)(a^2+b^2-ab)\right]$$
$$= \left[(2A)\left\{A^2+I^2-2AI+A^2+I^2+2AI-(A^2-I^2)\right\}\right]-7A$$
$$= 2A\left[I+I^2+I+I^2-A^2+I^2\right]-7A \quad \left[\because A^2 = AI\right]$$
$$= 2A[5I - I] - 7A$$
$$= 8AI - 7AI \quad [\because A = AI]$$
$$= AI = A$$

Q. 24. For any two matrices A and B, we have

(a) $AB = BA$

(b) $AB \neq BA$

(c) $AB = 0$

(d) None of the above

[NCERT Exemp. Ex. 3.3, Q. 65, Page 61]

Ans. Correct option : (d)

Explanation : For any two matrices A and B, we may have $AB = BA = I$, $AB \neq BA$ and $AB = 0$ but it is not always true.

Q. 25. On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation

$$\begin{bmatrix}1 & -3\\2 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 4\end{bmatrix}$$ **we have**

(a) $\begin{bmatrix}1 & -5\\0 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\-2 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\2 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & -5\\0 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\-0 & 2\end{bmatrix}$

(c) $\begin{bmatrix}1 & -5\\2 & 0\end{bmatrix} = \begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\-2 & 4\end{bmatrix}$

(d) $\begin{bmatrix}1 & -5\\2 & 0\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\2 & 0\end{bmatrix}$

[NCERT Exemp. Ex. 3.3, Q. 66, Page 61]

Ans. Correct option : (d)

Explanation : As we know that

$$\begin{bmatrix}1 & -3\\2 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 4\end{bmatrix}$$

By using $C_2 \to C_2 - 2C_1$,

$$\begin{bmatrix}1 & -5\\2 & 0\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\2 & 0\end{bmatrix}$$

Since, on using elementary column operation on $X = AB$, we apply these operations simultaneously on X and on the second matrix B of the product AB on RHS.

Q. 26. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation

$$\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$ **we have**

(a) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(b) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}-1 & -3\\1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\1 & -7\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(d) $\begin{bmatrix}4 & 2\\-5 & -7\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

[NCERT Exemp. Ex. 3.3, Q. 67, Page 62]

Ans. Correct option : (a)

Explanation : We have, $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

Using elementary row operation, we have

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$

Since, on using elementary row operation on $X = AB$, we apply these operations simultaneously on X and on the first matrix A of the product AB on RHS.

Q. 27. ________ matrix is both symmetric and skew symmetric matrix.
[NCERT Exemp. Ex. 3.3, Q. 68, Page 62]

Ans. Null

Q. 28. The product of any matrix by the scalar ________ is the null matrix.
[NCERT Exemp. Ex. 3.3, Q. 71, Page 62]

Ans. 0 (zero)

Q. 29. If A is skew-symmetric, then kA is a ________. (where, k is any scalar).
[NCERT Exemp. Ex. 3.3, Q. 77, Page 63]

Ans. Skew-symmetric matrix.

Q. 30. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeroes in one or more, then A^{-1} ________.
[NCERT Exemp. Ex. 3.3, Q. 81, Page 63]

Ans. Does not exist.

Very Short Answer Type Questions (1 or 2 marks each)

Q. 1. Sum of two skew symmetric matrices is always ________ matrix.
[NCERT Exemp. Ex. 3.3, Q. 69, Page 62]

Ans. Let A is a given matrix, then $(-A)$ is a skew-symmetric matrix. Similarly, for a given matrix B is a skew-symmetric matrix. **[½]**

Hence, $-A - B = -(A + B)$; sum of two skew-symmetric matrices is always skew symmetric matrix. **[½]**

Q. 2. The negative of a matrix is obtained by multiplying it by ________.
[NCERT Exemp. Ex. 3.3, Q. 70, Page 62]

Ans. Let A is a given matrix.

$\therefore -A = -1[A]$

So, the negative of a matrix is obtained by multiplying it by -1. **[1]**

Q. 3. A matrix which is not a square matrix is called a ________ matrix.
[NCERT Exemp. Ex. 3.3, Q. 72, Page 62]

Ans. A matrix which is not a square matrix is called a rectangular matrix. For example, a rectangular matrix is $A = [a_{ij}]_{m \times n}$, where $m \neq n$. **[1]**

Q. 4. Matrix multiplication is ________ over addition.
[NCERT Exemp. Ex. 3.3, Q. 73, Page 62]

Ans. Matrix multiplication is distributive over addition.

e.g., for three matrices A, B and C, we have

(i) $A(B + C) = AB + AC$

(ii) $(A + B)C = AC + BC$ **[2]**

Q. 5. If A is a symmetric matrix, then A^3 is a ________ matrix. [NCERT Exemp. Ex. 3.3, Q. 74, Page 62]

Ans. If A is a symmetric matrix, then A^3 is a symmetric matrix.

$\because \quad A' = A$

$\therefore (A^3)' = A'^3$

$= A^3 \left[\because (A')^n = (A^n)'\right]$ **[1]**

Q. 6. If A is a skew-symmetric matrix, then A^2 is a ________. [NCERT Exemp. Ex. 3.3, Q. 75, Page 62]

Ans. If A is a skew-symmetric matrix, then A^2 is a symmetric matrix.

$\because \quad A' = -A$

$\therefore (A^2)' = A'^2$

$= (-A)^2 \quad [\because A' = -A]$

$= A^2$ **[1]**

Q. 7. If A and B are square matrices of the same order, then

(i) $(AB)' =$ ________.

(ii) $(kA)' =$ ________. (Where, k is any scalar)

(iii) $[k(A - B)]' =$ ________.
[NCERT Exemp. Ex. 3.3, Q. 76, Page 63]

Ans. (i) $(AB)' = B'A'$

(ii) $(kA)' = kA'$

(iii) $[k(A - B)]' = k(A' - B')$ **[2]**

Q. 8. If A and B are symmetric matrices, then

(i) $AB - BA$ is a ________.

(ii) $BA - 2AB$ is a ________.
[NCERT Exemp. Ex. 3.3, Q. 78, Page 63]

Ans. (i) $AB - BA$ is a skew-symmetric matrix.

Since, $[AB - BA]' = (AB') - (BA)'$

$= B'A' - A'B' \quad \left[\because (AB)' = B'A'\right]$

$= BA - AB \quad [\because A' = A \text{ and } B' = B]$

$= -[AB - BA]$

So, $[AB - BA]$ is a skew-symmetric matrix. **[1]**

(ii) $[BA - 2AB]$ is neither a symmetric, nor a skew-symmetric matrix.

$\therefore (BA - 2AB)' = (BA)' - 2(AB)'$

$= A'B' - 2B'A'$

$= AB - 2BA$

$= -(2BA - AB)$

So, $[BA - 2AB]$ is neither a symmetric, nor a skew-symmetric matrix. **[1]**

Q. 9. If A is symmetric matrix, then $B'AB$ is ________.
[NCERT Exemp. Ex. 3.3, Q. 79, Page 63]

Ans. If A is a symmetric matrix, then $B'AB$ is a symmetric matrix. Then, $B'AB$ is a symmetric matrix.

$\because [B'AB]' = [B'(AB)]'$

$= (AB)'(B')' \quad \left[\because (AB')' = B'A'\right]$

$= B'A'B$

$= [B'A'B]$

So, $B'AB$ is a symmetric matrix. **[2]**

Q. 10. If A and B are symmetric matrices of same order, then AB is symmetric if and only if ________.
[NCERT Exemp. Ex. 3.3, Q. 80, Page 63]

Ans. If A and B are symmetric matrices of same order, then AB is symmetric if and only if $AB = BA$.

$\therefore (AB)' = B'A' \quad [\because AB = BA]$

$= BA$

$= AB$ **[2]**

Q. 11. State true or false in the given statement : *A* matrix denotes a number.

[NCERT Exemp. Ex. 3.3, Q. 82, Page 63]

Ans. False, a matrix is an ordered rectangular array of numbers of functions. **[1]**

Q. 12. State true or false in the given statement : Matrices of any order can be added.

[NCERT Exemp. Ex. 3.3, Q. 83, Page 63]

Ans. False, two matrices are added, if they are of the same order. **[1]**

Q. 13. State true or false in the given statement : Two matrices are equal if they have same number of rows and same number of columns.

[NCERT Exemp. Ex. 3.3, Q. 84, Page 63]

Ans. False, if two matrices have same number of rows and same number of columns, then they are said to be square matrix and if two square matrices have same elements in both the matrices, only then they are called equal. **[2]**

Q. 14. State true or false in the given statement : Matrices of different order cannot be subtracted.

[NCERT Exemp. Ex. 3.3, Q. 85, Page 63]

Ans. True, two matrices of same order can be subtracted. **[1]**

Q. 15. State true or false in the given statement : Matrix addition is associative as well as commutative.

[NCERT Exemp. Ex. 3.3, Q. 86, Page 63]

Ans. True, matrix addition is associative as well as commutative, *i.e.,*

$(A+B)+C = A+(B+C)$ and $A+B = B+A$

where A, B and C are matrices of same order. **[2]**

Q. 16. State true or false in the given statement : Matrix multiplication is commutative.

[NCERT Exemp. Ex. 3.3, Q. 87, Page 63]

Ans. False, since it is possible when AB and BA are both defined. **[1]**

Q. 17. State true or false in the given statement : *A* square matrix where every element is unity is called an identity matrix.

[NCERT Exemp. Ex. 3.3, Q. 88, Page 63]

Ans. False, since $AB \neq BA$ in an identity matrix, the diagonal elements are all one and rest are all zero. **[1]**

Q. 18. State true or false in the given statement : If *A* and *B* are two square matrices of the same order, then $A + B = B + A$.

[NCERT Exemp. Ex. 3.3, Q. 89, Page 63]

Ans. True, since matrix addition is commutative, *i.e.,* $A + B = B + A$, where A and B are two square matrices. **[1]**

Q. 19. State true or false in the given statement : If *A* and *B* are two matrices of the same order, then $A - B = B - A$. [NCERT Exemp. Ex. 3.3, Q. 90, Page 63]

Ans. False, since the addition of two matrices of same order are commutative.

$\therefore A+(-B) = A-B = -[B-A] \neq B-A$ **[2]**

Q. 20. State true or false in the given statement : If matrix $AB = 0$, then $A = 0$ or $B = 0$ or both *A* and *B* are null matrices.

[NCERT Exemp. Ex. 3.3, Q. 91, Page 63]

Ans. False, since for two non-zero matrices A and B of same order, it can be possible that $A.B = 0$ = null matrix. **[1]**

Q. 21. State true or false in the given statement : Transpose of a column matrix is a column matrix.

[NCERT Exemp. Ex. 3.3, Q. 92, Page 63]

Ans. False, transpose of a column matrix is a row matrix. **[1]**

Q. 22. State true or false in the given statement : If *A* and *B* are two square matrices of the same order, then $AB = BA$. [NCERT Exemp. Ex. 3.3, Q. 93, Page 63]

Ans. False, for two square matrices of same order it is not always true that $AB = BA$. **[1]**

Q. 23. State true or false in the given statement : If each of the three matrices of the same order are symmetric, then their sum is a symmetric matrix.

[NCERT Exemp. Ex. 3.3, Q. 94, Page 63]

Ans. True, let A, B and C are three matrices of same order.

$\therefore A' = A,\ B' = B$ and $C' = C$

$\therefore (A+B+C)' = A' + B' + C'$

$= (A+B+C)$ **[2]**

Q. 24. State true or false in the given statement : If *A* and *B* are any two matrices of the same order, then $(AB)' = A'B'$.

[NCERT Exemp. Ex. 3.3, Q. 95, Page 64]

Ans. False, $\because (AB)' = B'A'$ **[1]**

Q. 25. State true or false in the given statement : If $(AB)' = B'A'$, where *A* and *B* are not square matrices, then number of rows in *A* is equal to number of columns in *B* and number of columns in *A* is equal to number of rows in *B*.

[NCERT Exemp. Ex. 3.3, Q. 96, Page 64]

Ans. True,

Let, A is of order $m \times n$ and B is of order $p \times q$

Since, $(AB)' = B'A'$

$\therefore A_{(m\times n)}B_{p\times q}$ is defined.

$\Rightarrow n = p$ (i)

And AB is of order $m \times q$.

$\Rightarrow (AB)'$ is of order $q \times m$. (ii)

Also, B' is of order $q \times p$ and A' is of order $n \times m$.

$\therefore B'A'$ is of order.

$\Rightarrow p = n$

And $B'A'$ is of order $q \times m$. (iii)

Also, equality of matrix $(AB)' = B'A'$, we get the given statement as true.

e.g., if A is order (3×1) and B is order of (1×3), we get Order of $(AB)'$ = Order of $(B'A') = 3 \times 3$ **[2]**

Q. 26. State true or false in the given statement : If *A*, *B* and *C* are square matrices of same order, then $AB = AC$ always implies that $B = C$.

[NCERT Exemp. Ex. 3.3, Q. 97, Page 64]

Ans. False, if $AB = AC = 0$, then it can be possible that B and C are two non-zero matrices such that $B \neq C$.

$\therefore A \cdot B = 0 = A \cdot C$

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ And

$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow AB = AC$ but $B \neq C$ **[2]**

Q. 27. State true or false in the given statement : AA' is always a symmetric matrix for any matrix A.
[NCERT Exemp. Ex. 3.3, Q. 98, Page 64]

Ans. True,

$\because [AA'] = (A')'A' = [AA']$ **[1]**

Q. 28. If $A = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{vmatrix}$ then AB and BA are defined and equal.
[NCERT Exemp. Ex. 3.3, Q. 99, Page 64]

Ans. False, since AB is defined,

$\therefore AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 14 & 20 \\ 22 & 25 \end{bmatrix}$

Also, BA is defined.

$\therefore \quad BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 18 & 4 \\ 13 & 32 & 6 \\ 5 & 10 & 0 \end{bmatrix}$

$\therefore \quad AB \neq BA$ **[2]**

Q. 29. State true or false in the given statement : If A is skew-symmetric matrix, then A^2 is a symmetric matrix. [NCERT Exemp. Ex. 3.3, Q. 100, Page 64]

Ans. True,

$\because \quad [A^2]' = [A']^2$

$= [-A]^2 \qquad [\because A' = -A]$

$= A^2$

Hence, A^2 is symmetric matrix. **[2]**

Q. 30. $(AB)^{-1} = A^{-1}.\ B^{-1}$, where A and B are invertible matrices satisfying commutative property with respect to multiplication.
[NCERT Exemp. Ex. 3.3, Q. 101, Page 64]

Ans. True, we know that, if A and B are invertible matrices of the same order, then

$(AB)^{-1} = (BA)^{-1}$

$[\because AB = BA]$

Here, $(AB)^{-1} = (AB)^{-1}$

$\Rightarrow \qquad B^{-1}A^{-1} = A^{-1}B^{-1}$

[Since A and B are satisfying commutative property with respect to multiplications.] **[2]**

Q. 31. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ write:

(i) The order of the matrix,

(ii) The number of elements,

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.
[NCERT Ex. 3.1, Q. 1, Page 64]

Ans. (i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is 3×4. **[1]**

(ii) Since the order of the matrix is 3×4, there are $3 \times 4 = 12$ elements in it. **[½]**

(iii) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$ **[½]**

Q. 32. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?
[NCERT Ex. 3.1, Q. 2, Page 64]

Ans. We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are : (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6) and (6, 4).

Hence, the possible orders of a matrix having 24 elements are:

$(1 \times 24), (24 \times 1), (2 \times 12), (12 \times 2), (3 \times 8), (8 \times 3), (4 \times 6)$ and (6×4)

Thus (1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1×13 and 13×1. **[2]**

Q. 33. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?
[NCERT Ex. 3.1, Q. 3, Page 64]

Ans. We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18. The ordered pairs are : (1, 18), (18, 1), (2, 9), (9, 2), (3, 6) and (6, 3).

Hence, the possible orders of a matrix having 18 elements are:

$1 \times 18, 18 \times 1, 2 \times 9, 9 \times 2, 3 \times 6$, and 6×3

Thus (1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1. **[2]**

Q. 34. Find the value of a, b, c and d from the equation:

$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

[NCERT Ex. 3.1, Q. 7, Page 64]

Ans. $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$a - b = -1 \quad \ldots (a_{11})$

From (a_{21}), we have:

$b = 2a$

Then, from (a_{11}), we have:

$a - 2a = -1$

$\Rightarrow \quad a = 1$

$\Rightarrow \quad b = 2$

Now, from (a_{12}), we have:

$2 \times 1 + c = 5 \Rightarrow c = 3$

From (a_{22}) we have:

$3 \times 3 + d = 13$

$\Rightarrow \; 9 + d = 13$

$\Rightarrow \quad d = 4$

Therefore, $a = 1$, $b = 2$, $c = 3$ and $d = 4$. **[2]**

Q. 35. Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

[NCERT Ex. 3.2, Q. 2, Page 80]

Ans.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix}$

$= \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$ **[2]**

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$

$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$ **[2]**

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$ **[2]**

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad [\because \sin^2 x + \cos^2 x = 1]$ **[2]**

Q. 36. Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} \\ 2 \\ \\ \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$ **[NCERT Ex. 3.2, Q. 3, Page 80]**

Ans.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & -b(-b)+a(a) \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$ **[2]**

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$ **[2]**

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$ [2]

(iv) $$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$ [2]

(v) $$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$ [2]

(vi) $$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$ [2]

Q. 37. If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ **and** $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ **then compute 3A – 5B. [NCERT Ex. 3.2, Q. 5, Page 81]**

Ans.

$$3A-5B = 3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ [2]

Q. 38. Simplify

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

[NCERT Ex. 3.2, Q. 6, Page 81]

Ans. $$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta \\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\because \cos^2\theta+\sin^2\theta = 1)$$ [2]

Q. 39. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X+Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

[NCERT Ex. 3.2, Q. 8, Page 81]

Ans.

$$2X+Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$2X+\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$ [2]

Q. 40. Find the value of x and y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

[NCERT Ex. 3.2, Q. 9, Page 81]

Ans. $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ **[1]**

Comparing the corresponding elements of these two matrices, we have

$2+y=5$

$\Rightarrow \quad y=3$

$2x+2=8$

$\Rightarrow \quad x=3$

Therefore, $x = 3$ and $y = 3$. **[1]**

Q. 41. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix}5\\ \frac{1}{2}\\ -1\end{bmatrix}$ **(ii)** $\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

(iii) $\begin{bmatrix}-1 & 5 & 6\\ \sqrt{3} & 5 & 6\\ 2 & 3 & -1\end{bmatrix}$ **[NCERT Ex. 3.3, Q. 1, Page 88]**

Ans.

(i) Let, $A=\begin{bmatrix}5\\ \frac{1}{2}\\ -1\end{bmatrix}$

Transpose of $A = A'$ or $A^T=\begin{bmatrix}5 & \frac{1}{2} & -1\end{bmatrix}$ **[2]**

(ii) $\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

Transpose of $A = A'$ or $A^T=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$ **[2]**

(iii) $\begin{bmatrix}-1 & 5 & 6\\ \sqrt{3} & 5 & 6\\ 2 & 3 & -1\end{bmatrix}$

Transpose of $A = A'$ or $A^T=\begin{bmatrix}-1 & \sqrt{3} & 2\\5 & 5 & 3\\6 & 6 & -1\end{bmatrix}$ **[2]**

Q. 42. If $A'=\begin{bmatrix}-2 & 3\\1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$ then find $(A + 2B)'$.

[NCERT Ex. 3.3, Q. 4, Page 88]

Ans. Given that,

$A'=\begin{bmatrix}-2 & 3\\1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$, then $(A')'=A=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}$

$A+2B=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$

$=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\2 & 4\end{bmatrix}$

$=\begin{bmatrix}-2-2 & 1+0\\3+2 & 2+4\end{bmatrix}$

$=\begin{bmatrix}-4 & 1\\5 & 6\end{bmatrix}$

Therefore,

$(A+2B)'=\begin{bmatrix}-4 & 1\\5 & 6\end{bmatrix}'$

$=\begin{bmatrix}-4 & 5\\1 & 6\end{bmatrix}$ **[2]**

Q. 43. For the matrices A and B, verify that $(AB)' = B'A'$, where:

(i) $A=\begin{bmatrix}1\\-4\\3\end{bmatrix}$, $B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}0\\1\\2\end{bmatrix}$, $B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

[NCERT Ex. 3.3, Q. 5, Page 88]

Ans.

(i) $AB=\begin{bmatrix}1\\-4\\3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

$=\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$

$\text{LHS}=(AB)'$

$=\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}'$

$=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$

$\text{RHS}=B'A'$

$=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}'\begin{bmatrix}1\\-4\\3\end{bmatrix}'$

$=\begin{bmatrix}-1\\2\\1\end{bmatrix}\begin{bmatrix}1 & -4 & 3\end{bmatrix}$

$=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$

$\therefore$ LHS=RHS

Hence proved. **[2]**

(ii) $AB=\begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

$=\begin{bmatrix}0 & 0 & 0\\1 & 5 & 7\\2 & 10 & 14\end{bmatrix}$

LHS $(AB)'$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$\text{RHS} = B'A'$

$$= \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}' \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$\therefore$ LHS=RHS

Hence proved. **[2]**

Q. 44. (i) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then verify that $A'A = I$.

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$ then verify that $A'A = I$.

[NCERT Ex. 3.3, Q. 6, Page 89]

Ans.

$\text{LHS} = A'A$

(i)

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}' \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

=I=RHS **[2]**

(ii) LHS=$A'A$

$$= \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}' \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

=I=RHS **[2]**

Q. 45. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

[NCERT Ex. 3.3, Q. 7, Page 89]

Ans. (i) Given that,

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \qquad \ldots(\text{i})$$

Changing rows of matrix, A as the columns of new matrix $A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$

$\therefore A' = A$

Therefore, by definitions of symmetric matrix, A is a symmetric matrix. **[2]**

(ii) Given that,

$$= \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \ldots(\text{i})$$

$$A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}'$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Taking (-1) common, $A' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$

[From Eq. (i)]

Therefore, by definition matrix A is a skew-symmetric matrix **[2]**

Q. 46. For a matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ verify that:

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew symmetric matrix.

[NCERT Ex. 3.3, Q. 8, Page 89]

Ans. (i) Given that,

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

Let,

$B = A + A'$

$$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 5+6\\6+5 & 7+7\end{bmatrix}$$

$$=\begin{bmatrix}2 & 11\\11 & 14\end{bmatrix}$$

$$\therefore B'=\begin{bmatrix}2 & 11\\11 & 14\end{bmatrix}'=\begin{bmatrix}2 & 11\\11 & 14\end{bmatrix}=B$$

$\therefore B=A+A'$ is a symmetric matrix. **[2]**

(ii) Given that,

$$A=\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}$$

Let,

$$B=A-A'$$

$$=\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}-\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}'$$

$$=\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}-\begin{bmatrix}1 & 6\\5 & 7\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 5-6\\6-5 & 7-7\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

$$\therefore B'=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}'=\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$

Taking (-1) common, $-\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}=-B$

$\therefore B=A-A'$ is a skew-symmetric matrix. **[2]**

Q. 47. Using elementary transformations, find the inverse of each of the matrices, if it exists in given matrices. $\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$. **[NCERT Ex. 3.4, Q. 1, Page 97]**

Ans.

Let, $A=\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

We know that $A=IA$

$$\therefore \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix}1 & -1\\0 & 5\end{bmatrix}=\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}A \quad (R_2 \to R_2-2R_1)$$

$$\Rightarrow \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\-\frac{2}{5} & \frac{1}{5}\end{bmatrix}A \quad \left(R_2 \to \frac{1}{5}R_2\right)$$

$$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}\frac{3}{2} & \frac{1}{5}\\-\frac{2}{5} & \frac{1}{5}\end{bmatrix} \quad (R_1 \to R_1+R_2)$$

$$\therefore A^{-1}=\begin{bmatrix}\frac{3}{5} & \frac{1}{5}\\-\frac{2}{5} & \frac{1}{5}\end{bmatrix}$$

[2]

Q. 48. Find the inverse of each of the matrices, if it exists in given matrices.

$$\begin{bmatrix}\mathbf{2} & \mathbf{1}\\\mathbf{1} & \mathbf{1}\end{bmatrix}$$

[NCERT Ex. 3.4, Q. 2, Page 97]

Ans.

Let, $A=\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}A \quad (R_1 \to R_1-R_2)$$

$$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}A \quad (R_2 \to R_2-R_1)$$

$$\therefore \quad A^{-1}=\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}$$

[2]

Q. 49. Find the inverse of each of the matrices, if it exists in given matrices.

$$\begin{bmatrix}\mathbf{1} & \mathbf{3}\\\mathbf{2} & \mathbf{7}\end{bmatrix}$$

[NCERT Ex. 3.4, Q. 3, Page 97]

Ans.

$$A=\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}$$

We know that $A=IA$

$$\therefore \begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}A \quad (R_2 \to R_2-2R_1)$$

$$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix}A \quad (R_1 \to R_1-3R_2)$$

$$\therefore \quad A^{-1}=\begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix}$$

[2]

Q. 50. Find the inverse of each of the matrices, if it exists in given matritces.

$$\begin{bmatrix}\mathbf{2} & \mathbf{1}\\\mathbf{4} & \mathbf{2}\end{bmatrix}$$

[NCERT Ex. 3.4, Q. 14, Page 97]

Ans.

Let, $A=\begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}$

We know that $A=IA$

$$\therefore \begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1-\frac{1}{2}R_2$, we have

$$\begin{bmatrix}0 & 0\\4 & 2\end{bmatrix}=\begin{bmatrix}1 & -\frac{1}{2}\\0 & 1\end{bmatrix}A$$

Now, in the above equation, we can see that all the zeros in the first row of the matrix is on the LHS. Therefore, A^{-1} does not exist. **[2]**

Q. 51. If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

[NCERT Misc. Ex. Q. 4, Page 100]

Ans. It is given that A and B are symmetric matrices. Therefore, we have

$A' = A$ and $B' = B$ ...(i)

Now, $(AB-BA)'=(AB)'-(BA)'\left[(A-B)'=A'-B'\right]$

$= B'A' - A'B' \left[(AB)' = B'A'\right]$

$= BA - AB \left[\text{Using Eq. (i)}\right]$

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Thus, $(AB – BA)$ is a skew-symmetric matrix. **[2]**

Q. 52. For what values of x : $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$

[NCERT Misc. Ex. Q. 7, Page 100]

Ans. We have,

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$\Rightarrow \left[6(0)+2(2)+4(x)\right]=0$$

$$\Rightarrow [4+4x]=[0]$$

$$\Rightarrow 4+4x=0$$

Thus, the required value of x is –1. **[2]**

Q. 53. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

[NCERT Exemp. Ex. 3.3, Q. 1, Page 52]

Ans. We know that, if a matrix is of order $m \times n$ it has mn elements, where m and n are natural numbers. We have,

$m \times n = 28$

$\Rightarrow (m,n)=\{(1,28),(2,14),(4,7),(7,4),(14,2),(28,1)\}$

So, the possible orders are :

$1\times28,\ 2\times14,\ 4\times7,\ 7\times4,\ 14\times2,\ 28\times1$

Also, if it has 13 elements, then $m\times n=13$

$(m,n)=\{(1,13),(13,1)\}$

Hence, the possible orders are $1\times13,\ 13\times1$ **[2]**

Q. 54. In the matrix $A=\begin{bmatrix}a & 1 & x\\2 & \sqrt{3} & x^2-y\\0 & 5 & \frac{-2}{5}\end{bmatrix}$ write:

(i) the order of the matrix A

(ii) the number of elements

(iii) elements a_{23}, a_{31} and a_{12}

[NCERT Exemp. Ex. 3.3, Q. 2, Page 52]

Ans. We have, $A=\begin{bmatrix}a & 1 & x\\2 & \sqrt{3} & x^2-y\\0 & 5 & \frac{-2}{5}\end{bmatrix}$

(i) The order of matrix $A = 3 \times 3$ **[½]**

(ii) The number of elements $= 3 \times 3 = 9$

[Since, the number of elements in an m x n matrix will be equal to $m \times n = mn$] **[½]**

(iii) $a_{23} = x^2 - y, a_{31} = 0, a_{12} = 1$

[Since, we know that a_{ij} is a representation of element tying in the i_{th} row and j_{th} column.] **[1]**

Q. 55. Construct $a_{2\times2}$ matrix where:

(i) $a_{ij}=\dfrac{(i-2j)^2}{2}$

(ii) $a_{ij}=|-2i+3j|$

[NCERT Exemp. Ex. 3.3, Q. 3, Page 53]

Ans. We know that, the notation $A[a_{ij}]_{m\times n}$, namely it indicates that A is a matrix of order $m \times n$.

(i) Here, $A=\left[a_{ij}\right]_{2\times2}$

$$\Rightarrow A=\frac{(i-2j)^2}{2},\ 1\le i\le 2;\ 1\le j\le 2$$

$$\therefore\ a_{11}=\frac{(1-2)^2}{2}=\frac{1}{2}$$

$$a_{12}=\frac{(1-2\times2)^2}{2}=\frac{9}{2}$$

$$a_{21}=\frac{(2-2\times1)^2}{2}=0$$

$$a_{22}=\frac{(2-2\times2)^2}{2}=2 \quad ...(i)$$

Thus, $A=\begin{bmatrix}\frac{1}{2} & \frac{9}{2}\\0 & 2\end{bmatrix}_{2\times2}$ **[2]**

(ii) Here, $A=\left[a_{ij}\right]_{2\times2}=|-2i+3j|,\ 1\le i\le 2;\ 1\le j\le 2$

$$\therefore\ a_{11}=|-2\times1+3\times1|=1$$

$$a_{12}=|-2\times1+3\times2|=4 \quad [\because |-1|=1]$$

$$a_{21}=|-2\times2+3\times1|=1$$

$$a_{22}=|-2\times2+3\times2|=2$$

$$\therefore\ A=\begin{vmatrix}1 & 4\\1 & 2\end{vmatrix}_{2\times2}$$

[2]

Q. 56. Construct a matrix 3×2 whose elements are given by $a_{ij}=e^{ix}\sin jx$.

[NCERT Exemp. Ex. 3.3, Q. 4, Page 53]

Ans. Since, $A=\left[a_{ij}\right]_{m\times n}\ 1\le i\le m$ and $1\le j\le n,\ i,\ j\in N$

$$\therefore A=\left[e^{i\cdot x}\sin jx\right]_{3\times2};\ 1\le i\le 3;\ 1\le j\le 2$$

$$\Rightarrow a_{11}=e^{1\cdot x}\cdot\sin1\cdot x=e^x\sin x$$

$$a_{12}=e^{1\cdot x}\cdot\sin2\cdot x=e^x\sin2x$$

$$a_{21} = e^{2\cdot x}\cdot \sin 1\cdot x = e^{2x}\sin x$$
$$a_{22} = e^{2\cdot x}\cdot \sin 2\cdot x = e^{2x}\sin 2x$$
$$a_{31} = e^{3\cdot x}\cdot \sin 1\cdot x = e^{3x}\sin x$$
$$a_{32} = e^{3\cdot x}\cdot \sin 2\cdot x = e^{3x}\sin 2x$$
$$A = \begin{bmatrix} e^{x}\sin x & e^{x}\sin 2x \\ e^{2x}\sin x & e^{2x}\sin 2x \\ e^{3x}\sin x & e^{3x}\sin 2x \end{bmatrix}_{3\times 2}$$ **[2]**

Q. 57. Find values of *a* and *b* if $A = B$, where $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 5, Page 53]

Ans. By equality of matrices we know that each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

$\therefore \quad a_{11} = b_{11}$

$\Rightarrow \quad a+4 = 2a+2$

$\Rightarrow \quad a = 2$

$a_{12} = b_{12}$

$\Rightarrow \quad 3b = b^2+2$

$\Rightarrow \quad b^2 = 3b-2$

and $a_{22} = b_{22}$

$\Rightarrow \quad -6 = b^2-5b$

$\Rightarrow \quad -6 = 3b-2-5b \quad [\because b^2 = 3b-2]$

$\Rightarrow \quad 2b = 4$

$\Rightarrow \quad b = 2$

$\therefore \quad a = 2$ and $b = 2$ **[2]**

Q. 58. If possible, find the sum of the matrices A and B, where $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 6, Page 53]

Ans. We have,

$$A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}_{2\times 2} \text{ and } B = \begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}_{2\times 3}$$

Here, A and B are of different orders. Also, we know that the addition of two matrices A and B is possible only if order of both the matrices A and B should be same.

Hence, the sum of matrices A and b is not possible. **[2]**

Q. 59. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, then find:

(i) $X + Y$

(ii) $2X - 3Y$

(iii) a matrix Z such that $X + Y + Z$ is zero matrix.

[NCERT Exemp. Ex. 3.3, Q. 7, Page 53]

Ans. Given that,

(i) $$X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}_{2\times 3} \text{ and } Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}_{2\times 3}$$

$$X+Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

(ii) $$2X = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$ [2]
$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix}$$

and $$3Y = 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$2X-3Y = \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$ **[2]**

(iii) $$X+Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & +1 \end{bmatrix}$$

Also, $$X+Y+Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We see that Z is the additive inverse of $(X+Y)$ or negative of $(X+Y)$.

$$Z = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix} \quad [\because Z = -(X+Y)]$$ **[2]**

Q. 60. Find non-zero values of a : satisfying the matrix equation $\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (\quad +8) & 24 \\ (10) & 6 \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 8, Page 53]

Ans. Given that,

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ (10) & 6x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$\Rightarrow \quad 2x+10x = 48$

$\Rightarrow \quad 12x = 48$

$\therefore \quad x = \dfrac{48}{12} = 4$ **[2]**

Q. 61. Show by an example that for $A \neq 0, B \neq 0$, AB = 0.

[NCERT Exemp. Ex. 3.3, Q. 16, Page 54]

Ans. Let $A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \neq 0$

$$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence proved. **[1]**

Q. 62. Given

$$A=\begin{bmatrix}2&4&0\\3&9&6\end{bmatrix} \text{ and } B=\begin{bmatrix}1&4\\2&8\\1&3\end{bmatrix} \text{ is } (AB)'=B'A'?$$

[NCERT Exemp. Ex. 3.3, Q. 17, Page. 54]

Ans. We have $A=\begin{bmatrix}2&4&0\\3&9&6\end{bmatrix}_{2\times3}$ and $B=\begin{bmatrix}1&4\\2&8\\1&3\end{bmatrix}_{3\times2}$

$$\therefore \quad AB=\begin{bmatrix}2+8+0 & 8+32+0\\3+18+6 & 12+72+18\end{bmatrix}=\begin{bmatrix}10&40\\27&102\end{bmatrix}$$

and $(AB)'=\begin{bmatrix}10&27\\40&102\end{bmatrix}$ ...(i)

Also, $B'=\begin{bmatrix}1&2&1\\4&8&3\end{bmatrix}_{2\times3}$ and $A'=\begin{bmatrix}2&3\\4&9\\0&6\end{bmatrix}_{3\times2}$

$$\therefore B'A'=\begin{bmatrix}2+8+0 & 3+18+6\\8+32+0 & 12+72+18\end{bmatrix}=\begin{bmatrix}10&27\\40&102\end{bmatrix} \quad ...(ii)$$

Thus, we see that, $(AB)'=B'A'$
[By using Eqs. (i) and (ii)] **[2]**

Q. 63. Give an example of matrices A, B and C, such that $AB=AC$, where A is non-zero matrix but $B\neq C$.
[NCERT Exemp. Ex. 3.3, Q. 21, Page 55]

Ans. Let $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}, B=\begin{bmatrix}2&3\\4&0\end{bmatrix}$ and $C=\begin{bmatrix}2&3\\4&4\end{bmatrix}$ $[\because B\neq C]$

$$\therefore AB=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}2&3\\4&0\end{bmatrix}=\begin{bmatrix}2&3\\0&0\end{bmatrix} \quad ...(i)$$

and $AC=\begin{bmatrix}1&0\\0&0\end{bmatrix}\cdot\begin{bmatrix}2&3\\4&4\end{bmatrix}=\begin{bmatrix}2&3\\0&0\end{bmatrix}$ ...(ii)

Thus, we see that $AB=AC$ [By using Eqs. (i) and (ii)]
Where, A is non-zero matrix, but $B\neq C$. **[2]**

Q. 64. If $P=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}$ and $Q=\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}$, then prove that:

$$PQ=\begin{bmatrix}xa&0&0\\0&yb&0\\0&0&zc\end{bmatrix}=QP.$$

[NCERT Exemp. Ex. 3.3, Q. 23, Page. 55]

Ans. $PQ=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}=\begin{bmatrix}xa&0&0\\0&yb&0\\0&0&zc\end{bmatrix}$ ...(i)

and $QP=\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}=\begin{bmatrix}ax&0&0\\0&by&0\\0&0&cz\end{bmatrix}$ ...(ii)

Thus, we see that, $PQ=QP$ [By using Eqs. (i) and (ii)]
Hence proved. **[2]**

Q. 65. If $\begin{bmatrix}2&1&3\end{bmatrix}\begin{bmatrix}-1&0&-1\\-1&1&0\\0&1&1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}=$ A, Find A

[NCERT Exemp. Ex. 3.3, Q. 24, Page 55]

Ans.

We have $\begin{bmatrix}2&1&3\end{bmatrix}\begin{bmatrix}-1&0&-1\\-1&1&0\\0&1&1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}=A$

$$\therefore A=\begin{bmatrix}2&1&3\end{bmatrix}\begin{bmatrix}-1&0&-1\\-1&1&0\\0&1&1\end{bmatrix}$$

$$=\begin{bmatrix}-2-1+0 & 0+1+3 & -2+0+3\end{bmatrix}$$

$$=\begin{bmatrix}-3&4&1\end{bmatrix}$$

Now, $A=\begin{bmatrix}-3&4&1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$$=\begin{bmatrix}-3+0-1\end{bmatrix}$$

$$=\begin{bmatrix}-4\end{bmatrix}$$

[2]

Q. 66. If $A=\begin{bmatrix}0&-1&2\\4&3&-4\end{bmatrix}$ and $B=\begin{bmatrix}4&0\\1&3\\2&6\end{bmatrix}$, then verify that:

(i) $(A')'=A$

(ii) $(AB)'=B'A'$

(iii) $(kA)'=(kA')$

[NCERT Exemp. Ex. 3.3, Q. 27, Page 56]

Ans. We have, $A=\begin{bmatrix}0&-1&2\\4&3&-4\end{bmatrix}$ and $B=\begin{bmatrix}4&0\\1&3\\2&6\end{bmatrix}$

(i) We have to verify that, $(A')'=A$

$$\therefore \quad A'=\begin{bmatrix}0&4\\-1&3\\2&-4\end{bmatrix}$$

and $(A')'=\begin{bmatrix}0&-1&2\\4&3&-4\end{bmatrix}=A$

Hence proved.

[2]

(ii) We have to verify that, $AB'=B'A'$

$$\therefore \quad AB=\begin{bmatrix}3&9\\11&-15\end{bmatrix}$$

$$\Rightarrow \quad (AB)'=\begin{bmatrix}3&11\\9&-15\end{bmatrix}$$

and $B'A'=\begin{bmatrix}4&1&2\\0&3&6\end{bmatrix}\begin{bmatrix}0&4\\-1&3\\2&-4\end{bmatrix}$

$$=\begin{bmatrix}3&11\\9&-15\end{bmatrix}$$

$$=(AB)'$$

Hence proved.

[2]

(iii) We have to verify that $(kA)'=(kA')$

Now, $(kA)=\begin{bmatrix}0&-k&2k\\4k&3k&-4k\end{bmatrix}$

and $(kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

Also, $kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

$= (kA)'$

Hence Proved. **[2]**

Q. 67. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$ then verify that:

(i) $(2A + B)' = 2A' + B'$

(ii) $(A - B)' = A' - B'$

[NCERT Exemp. Ex. 3.3, Q. 28, Page 56]

Ans. We have,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$$

(i) $(2A+B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$

and $(2A+B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$

Also, $2A' + B' = 2\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$

$= (2A+B)'$

Hence proved. **[2]**

(ii) $(A-B) = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$

and $(A-B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

Also, $A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

$= (A-B)'$

Hence proved. **[2]**

Q. 68. Show that $A'A$ and AA' are both symmetric matrices for any matrix A.

[NCERT Exemp. Ex. 3.3, Q. 29, Page 56]

Ans. Let $P = A'A$

$\therefore \quad P' = (A'A)'$

$= A'(A')' \quad [\because (AB')' = B'A']$

$= A'A = P$

So, $A'A$ is symmetric matrix for any matrix A.

Similarly,

Let $Q = AA'$

$Q' = (AA')'$

$= (A')'A'$

$= AA'$

$= Q$

So, AA' is symmetric matrix for any matrix A. **[2]**

Q. 69. Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons.

[NCERT Exemp. Ex. 3.3, Q. 30, Page 56]

Ans. Since, A and B are square matrices of order 3×3

$AB^2 = AB \cdot AB$

$= ABAB$

$= AABB \quad [\because AB = BA]$

$= A^2B^2$

So, $AB^2 = A^2B^2$ is true when $AB = BA$. **[1]**

Q. 70. Show that if A and B are square matrices such that AB = BA, then $(A + B)^2 = A^2 + B^2 + 2AB$.

[NCERT Exemp. Ex. 3.3, Q. 31, Page 56]

Ans. Since, A and B are square matrices such that AB = BA

$\therefore (A + B)^2 = (A + B)\cdot(A + B)$

$= A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2 \quad [\because AB = BA]$

$= A^2 + 2AB + B^2$

Hence proved.

Q. 71. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$.

If $a = 4$, $b = -2$, then show that:

(a) $A + (B + C) = (A + B) + C$

(b) $A(BC) = (AB)C$

(c) $(a + b)B = aB + bB$

(d) $a(C - A) = aC - aA$

(e) $(A^T)^T = A$

(f) $(bA)^T = bA^T$

(g) $(AB)^T = B^TA^T$

(h) $(A - B)C = AC - BC$

(i) $(A - B)^T = A^T - B^T$

[NCERT Exemp. Ex. 3.3, Q. 32, Page 56]

Ans. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$a = 4,\ b = -2$

(a) $A + (B+C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

and $(A+B)+C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

$= A+(B+C)$

Hence proved. **[2]**

(b) $(BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ **[2]**

$= \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$

and $A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$

$= \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix}$

$= \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$

Also, $(AB) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$

$= A(BC)$

Hence proved.

(c)

$(a+b)B = (4-2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ $[\because a = 4,\ b = -2]$

$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

$\because \quad aB + bB = 4B - 2B$

$= \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

$= (a+b)B$

Hence proved. **[2]**

(d) $(C-A) = \begin{bmatrix} 2-1 & 0-2 \\ 1+1 & -2-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$

and $a(C-A) = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$ $[\because a = 4]$

Also, $aC - aA = \begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$

$= a(C-A)$

Hence proved. **[2]**

(e) $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Now,

$(A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A$

Hence proved. **[2]**

(f) $(bA)^T = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T$ $[\because b = -2]$

$= \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

and $A^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore \quad bA^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix} = (bA)^T$

Hence proved. **[2]**

(g) $AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

Now,

$B^T A^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix} = (AB)^T$

Hence proved. **[2]**

(h) $(A-B) = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}$

$(A-B)C = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$ ...(i)

Now, $AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$ ...(ii)

and $BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$ ...(iii)

$\therefore AC - BC = \begin{bmatrix} 4-8 & -4-0 \\ 1-7 & -6+10 \end{bmatrix}$ [By using Eqs. (ii) and (iii)]

$= \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

$= (A-B)C$ [By using Eq. (i)]

Hence proved. **[2]**

(i) $(A-B)^T = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$

$$A^T - B^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$$

$$= (A-B)^T$$

Hence proved. **[2]**

Q. 72. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then show that $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 33, Page 57]

Ans. We have,

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \cos\theta.\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos 2\theta \end{bmatrix} \quad \begin{bmatrix} \because \cos^2\theta - \sin^2\theta \\ = \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \quad [\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$$

Hence proved. **[2]**

Q. 73. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$, then show that $(A+B)^2 = A^2 + B^2$.

[NCERT Exemp. Ex. 3.3, Q. 34, Page 57]

Ans. We have,

$$A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } x^2 = -1$$

$$\therefore (A+B) = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

and

$$(A+B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \quad \ldots(i)$$

Also, $A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix}$$

and $B^2 = B \cdot B$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, $A^2 + B^2 = \begin{bmatrix} -x^2+1 & 0 \\ 0 & -x^2+1 \end{bmatrix}$

$$= \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

$$= (A+B)^2 \quad \text{[By using Eq. (i)]}$$

Hence proved. **[2]**

Q. 74. Verify that $A^2 = I$, when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 35, Page 57]

Ans. We have,

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \quad [\because A^2 = A \cdot A]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Hence proved. **[2]**

Q. 75. Find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

[NCERT Exemp. Ex. 3.3, Q. 41, Page 58]

Ans. We have,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ c+d-1 & 3+2d \end{bmatrix}$$

$\Rightarrow \quad 3a = a+4$

$\therefore \quad a = 2$

$\Rightarrow \quad 3b = 6+a+b$

$\Rightarrow \quad 3b - b = 8 \quad [\because a = 2]$

$\therefore \quad b = 4$

$3d = 3+2d$

$\therefore \quad d = 3$

$\Rightarrow \quad 3c = c+d-1$

$\Rightarrow \quad 2c = 3-1$

$\therefore \quad c = 1$

Thus, we have

$a = 2,\ b = 4,\ c = 1$ and $d = 3$ **[2]**

Q. 76. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$ then find $A^2 + 2A + 7I$.

[NCERT Exemp. Ex. 3.3, Q. 43, Page 58]

Ans. We have,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \quad [\because A^2 = A \cdot A]$$

$$= \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

[2]

Q. 77. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A^{-1} = A'$, then find the value of α.

[NCERT Exemp. Ex. 3.3, Q. 44, Page 58]

Ans. We have,

$$A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}$$

$$\text{and } A' = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

Also, $A^{-1} = A'$

$\Rightarrow \quad AA^{-1} = AA'$

$$\Rightarrow \quad I = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2 a + \sin^2 a & 0 \\ 0 & \sin^2 a + \cos^2 a \end{bmatrix}$$

By using equality of matrices, we get

$\cos^2 a + \sin^2 a = 1$

Which is true for all real values of α. **[2]**

Q. 78. If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of *a*, *b* and *c*.

[NCERT Exemp. Ex. 3.3, Q. 45, Page 58]

Ans. Let,

$$A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

Since, A is skew-symmetric matrix.

$\therefore \quad A' = -A$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & +1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get

$a = -2,\ c = -3$ and $b = -b$

$\Rightarrow b = 0$

$\therefore\ a = -2,\ b = 0$ and $c = -3$ **[2]**

Q. 79. If A is square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

[NCERT Exemp. Ex. 3.3, Q. 47, Page 58]

Ans. Since, $A^2 = A$

and $(I + A)\cdot(I + A) = I^2 + IA + AI + A^2$

$= I^2 + 2AI + A^2$

$= I + 2A + A = I + 3A$

and $(I + A)\cdot(I + A)(I + A) = (I + A)(I + 3A)$

$= I^2 + 3AI + AI + 3A^2$

$= I + 4AI + 3A$

$= I + 7A = 7A + I$

Hence proved. **[2]**

Q. 80. If *A*, *B* are square matrices of same order and *B* is a skew-symmetric matrix, show that *A′BA* is skew symmetric.

[NCERT Exemp. Ex. 3.3, Q. 48, Page 58]

Ans. Since, A and B are square matrices of same order and B is a skew-symmetric matrix, *i.e.*, $B' = -B$.

Now, we have to prove that $A'BA$ is a skew-symmetric matrix.

$(A'BA)' = A'BA'$

$= BA'A' \quad [\because AB' = B'A']$

$= A'B'A$

$= A'(-BA)$

$= -A'BA$

Hence, $A'BA$ is a skew-symmetric matrix. **[2]**

Q. 81. If matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the value of '*a*' and '*b*'.

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

We know that if A is a skew symmetric matrix then, $A^T = -A$

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

$\therefore -a = 2$ and $-b = -3$

$\Rightarrow\ a = -2$ and $b = 3$ **[1]**

Q. 82. Given, $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$. Compute A^{-1} and show that $2A^{-1} = 9I - A$.

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{14-12}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Now,

$$\text{LHS} = 2A^{-1}$$

$$= 2\left(\frac{1}{2}\right)\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\text{RHS} = 9I - A$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9-2 & 0-(-3) \\ 0-(-4) & 9-7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Since, LHS = RHS

Hence proved. **[2]**

Q. 83. If A is a 3 × 3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

[CBSE Board, Delhi Region, 2017]

Ans.
$$A^{-1} = \frac{AdjA}{|A|}$$

$$\therefore |A^{-1}| = \frac{|AdjA|}{|A|} = \frac{|A|^{3-1}}{|A|}$$

[∵ If A is a non-singular matrix of order n, then $|adj(A)| = |A|^{n-1}$]

$$= \frac{|A|^2}{|A|}$$

As we are given that $|A^{-1}| = |A|^k$

$\therefore \quad k = -1$ **[1]**

Q. 84. Show that all the diagonal elements of a skew symmetric matrix are zero.

[CBSE Board, Delhi Region, 2017]

Ans. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix.

A is skew symmetric matrix.

$\therefore \quad A = -A$

$\Rightarrow \; a_{ij} = a_{ji} \forall i, j$

For diagonal elements $i = j$,

$\Rightarrow \; 2a_{ii} = 0$

$\Rightarrow \; 2_{ii} = 0$

Thus, diagonal elements are zero. **[2]**

Q. 85. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$.

[CBSE Board, All India Region, 2017]

Ans. Method 1 : Any skew symmetric matrix of order 3 is:

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$|A| = -a(bc) + a(bc)$$
$$= 0$$

Method 2 : Since A is a skew-symmetric matrix

$\therefore \quad A^T = -A$

$\therefore \quad |A^T| = |-A|$

$\quad = (-1)^3 \cdot |A|$

$\Rightarrow \quad |A| = -|A|$

$\Rightarrow \quad 2|A| = 0$ or $|A| = 0$. **[2]**

Q. 86. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then find the value of $|2AB|$.

[CBSE Board, Foreign Scheme, 2017]

Ans. We know that,

$|A| = -1$ and $|B| = 3$

$|2AB| = 2^3\,|AB| = 8 \times |A| \times |B|$

So,

$|2AB| = 8\,|A| \times |B| = 8 \times (-1) \times 3 = -24$ **[2]**

Q. 87. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

[CBSE Board, Delhi Region, 2016]

Ans. Given that,

$$(A-I)^3 + (A+I)^3 - 7A$$
$$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$$
$$= 2A^3 + 6AI^2 - 7A$$
$$= 2A \cdot A^2 + 6AI^2 - 7A$$
$$= 8A - 7A$$
$$= A$$

[1]

Q. 88. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

[CBSE Board, Delhi Region, 2016]

Ans. We have,

$$A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

It is given that the matrix is symmetric.

$\therefore A = A'$

$$\Rightarrow \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, by equality of matrices, we get:

$$2b = 3$$

$$\Rightarrow \quad b = \frac{3}{2}$$

and $3a = -2$

$$\Rightarrow \quad a = \frac{-2}{3}$$

Therefore, $a = \frac{-2}{3}$ and $b = \frac{3}{2}$

[1]

Q. 89. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

[CBSE Board, All India Region, 2016]

Ans. Given that,

Order is 2×2.

Number of elements = 4 in which 1, 2 or 3 (Three) entry can be filled and can repeat.

Number of possible matrices = $3^4 = 81$ **[1]**

Q. 90. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation: $\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}$.

[CBSE Board, All India Region, 2016]

Ans. By using this operation, $C_2 \to C_2 + 2C_1$ on given matrices, we get

$$\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$$ **[1]**

Q. 91. If $(2 \quad 1 \quad 3)\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = A$ then write the order of matrix A.

[CBSE Board, Foreign Scheme, 2016]

Ans.

$$[2 \quad 1 \quad 3]_{1\times3}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{3\times3}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

$$[2 \quad 1 \quad 3]_{1\times3}\begin{bmatrix} (-1)\times1 + 0\times0 + (-1)\times(-1) \\ (-1)\times1 + 1\times0 + 0\times(-1) \\ 0\times1 + 1\times0 + 1\times(-1) \end{bmatrix} = A$$

$$[2 \quad 1 \quad 3]_{1\times3}\begin{bmatrix} -1+0+1 \\ -1+0+0 \\ 0+0-1 \end{bmatrix} = A$$

$$[2 \quad 1 \quad 3]_{1\times3}\begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = A$$

$$[2\times0 + 1\times(-1) + 3\times(-1)] = A$$

$$[0-1-3] = A$$

$$[-4]_{1\times1} = A$$

The order of the matrix will be 1×1. **[1]**

Q. 92. If $A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ is written as $A = P + Q$, where P is a symmetric matrix and Q is skew symmetric matrix, then write the matrix P.

[CBSE board, Foreign Scheme, 2016]

Ans. We know that,

$$P = \frac{1}{2}(A + A')$$

Given that,

$$A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$$

$$A' = \begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$$

Now,

$$P = \frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 5+7 \\ 7+5 & 9+9 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 12 \\ 12 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$$

Hence proved. **[1]**

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **(ii) $a_{ij} = \frac{i}{j}$**

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$ **[NCERT Ex. 3.1, Q. 4, Page 64]**

Ans. In general, a 2×2 matrix is given by

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(i) $a_{ij} = \frac{(i+j)^2}{2}; i, j = 1, 2$

Therefore,

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2 \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2} \quad a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Therefore, the required matrix is $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ **[1]**

(ii) $a_{ij} = \frac{i}{j}, i, j = 1, 2$

$$\therefore a_{11} = \frac{1}{1} = 1 \quad a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2 \quad a_{22} = \frac{2}{2} = 1$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$ **[1]**

(iii) $a_{ij} = \frac{(i+2j)^2}{2}, i, j = 1, 2$

$$\therefore a_{11} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} \quad a_{12} = \frac{(1+4)^2}{2} = \frac{5^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = 8 \quad a_{22} = \frac{(2+4)^2}{2} = \frac{6^2}{2} = 18$$

Therefore, the required matrix is $A=\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$ **[1]**

Q. 2. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **(ii)** $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ **[NCERT Ex. 3.1, Q. 6, Page 64]**

Ans.

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

x = 1, y = 4 and z = 3 **[1]**

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y = 6, xy = 8, 5 + z = 5$

$\Rightarrow x - y = \pm 2$

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$\therefore x = 4, y = 2$, and $z = 0$

or

$x = 2, y = 4$, and $z = 0$ **[1]**

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y + z = 9$ … (i)

$x + z = 5$ … (ii)

$y + z = 7$ … (iii)

From equations (i) and (ii), we have:

$y + 5 = 9$

$\Rightarrow y = 4$

Then, from equation (iii), we have:

$4 + z = 7$

$\Rightarrow z = 3$

$\therefore x + z = 5$

$\Rightarrow x = 2$

$\therefore x = 2, y = 4$, and $z = 3$ **[1]**

Q. 3. Solve the equation for x, y, z and t if

$2\begin{bmatrix} x & z \\ y & t \end{bmatrix}+3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}=3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

[NCERT Ex. 3.2, Q. 10, Page 81]

Ans. $2\begin{bmatrix} x & z \\ y & t \end{bmatrix}+3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}=3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix}+\begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix}=\begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix}=\begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$ **[1]**

The corresponding elements of these two matrices, we get

$2x + 3 = 9$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

$2y = 12$

$\Rightarrow y = 6$

$2z - 3 = 15$

$\Rightarrow 2z = 18$

$\Rightarrow z = 9$

$2t + 6 = 18$

$\Rightarrow 2t = 12$

$\Rightarrow t = 6$

$\therefore x = 3, y = 6, z = 9$ and $t = 6$ **[2]**

Q. 4. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$. Find values of x and y.

[NCERT Ex. 3.2, Q. 11, Page 81]

Ans. $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix}+\begin{bmatrix} -y \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$ **[1]**

Comparing the corresponding elements of these two matrices, we get:

$2x - y = 10$ and $3x + y = 5$

Adding these two equations, we have:

$5x = 15 \Rightarrow x = 3$

Now, $3x + y = 5$

$\Rightarrow y = 5 - 3x$

$\Rightarrow y = 5 - 9 = -4$

$\therefore x = 3$ and $y = -4$. **[2]**

Q. 5. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$. Find the values of x, y, z and w.

[NCERT Ex. 3.2, Q. 12, Page 81]

Ans. $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}=\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$ **[1]**

Comparing the corresponding elements of these two matrices, we get:

$3x = x + 4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

$3y = 6 + x + y$

$\Rightarrow 2y = 6 + x = 6 + 2 = 8$

$\Rightarrow y = 4$

$3w = 2w + 3$

$\Rightarrow W = 3$

$3z = -1 + z + w$

$\Rightarrow 2z = -1 + W = -1 + 3 = 2$

$\Rightarrow z = 1$

$\therefore x = 2, y = 4, z = 1, \text{ and } W = 3$ **[2]**

Q. 6. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$. Show that .

$F(x)F(y) = F(x+y)$

[NCERT Ex. 3.2, Q. 13, Page 82]

Ans.
$$F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x)F(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & \begin{matrix}-\cos x\sin y\\ -\sin x\cos y+0\end{matrix} & 0\\ \sin x\cos y+\cos x\sin y+0 & \begin{matrix}-\sin x\sin y\\ +\cos x\cos y+0\end{matrix} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$ **[3]**

$= F(x+y)$

$\therefore F(x)F(y) = F(x+y)$

Q. 7. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. Find k so that $A^2 = kA - 2I$. **[NCERT Ex. 3.2, Q. 17, Page 82]**

Ans.
$$A^2 = A\cdot A$$
$$=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$$
$$=\begin{bmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{bmatrix}$$
$$=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=k\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k & -2k\\ 4k & -2k\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k-2 & -2k\\ 4k & -2k-2\end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1. **[3]**

Q. 8. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

[NCERT Ex. 3.2, Q. 20, Page 83]

Ans. The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books. The selling prices of a chemistry book, a physics book, and an economics book are respectively given as ₹ 80, ₹60, and ₹40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\ 60\\ 40\end{bmatrix}$$

$= 12[10 \times 80 + 8 \times 60 + 10 \times 40]$

$= 12\,(800 + 480 + 400)$

$= 12(1680)$

$= 20160$ **[3]**

Q. 9. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$ when

$A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$.

[NCERT Ex. 3.3, Q. 9, Page 89]

Ans. Given that,

$$A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

Now,

$$A+A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}+\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$=\begin{bmatrix}0+0 & a-a & b-b\\ -a+a & 0+0 & c-c\\ -b+b & -c+c & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Therefore,

$$\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Now,

$$A-A'=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}-\begin{bmatrix}0&-a&-b\\a&0&-c\\b&c&0\end{bmatrix}$$

$$=\begin{bmatrix}0-0&a+a&b+b\\-a-a&0-0&c+c\\-b-b&-c-c&0-0\end{bmatrix}$$

$$=\begin{bmatrix}0&2a&2b\\-2a&0&2c\\-2b&-2c&0\end{bmatrix}$$

Thus,

$$\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&2a&2b\\-2a&0&2c\\-2b&-2c&0\end{bmatrix}$$

$$=\begin{bmatrix}0&-a&b\\-a&0&c\\-b&-c&0\end{bmatrix}$$

[3]

Q. 10. Express the following matrices as the sum of a symmetric and skew symmetric matrix :

(i) $\begin{bmatrix}3&5\\1&-1\end{bmatrix}$ **(ii)** $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

(iii) $\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&3\end{bmatrix}$ **(iv)** $\begin{bmatrix}1&5\\-1&2\end{bmatrix}$

[NCERT Ex. 3.3, Q. 10, Page 89]

Ans. (i) Given that,

$$A=\begin{bmatrix}3&5\\1&-1\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}3&1\\5&-1\end{bmatrix}$$

$$\text{Symmetric matrix}=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\left(\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}6&6\\6&-2\end{bmatrix}$$

$$=\begin{bmatrix}3&3\\3&-1\end{bmatrix}$$

$$\text{Skew symmetric matrix}=\frac{1}{2}(A-A')$$

$$=\frac{1}{2}\left(\begin{bmatrix}3&5\\1&-1\end{bmatrix}-\begin{bmatrix}3&1\\5&-1\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}0&4\\-4&0\end{bmatrix}$$

$$=\begin{bmatrix}0&2\\-2&0\end{bmatrix}$$

Therefore, given matrix A is sum of symmetric matrix $\begin{bmatrix}3&3\\3&-1\end{bmatrix}$ and skew symmetric matrix $\begin{bmatrix}0&2\\-2&0\end{bmatrix}$. **[3]**

(ii) Given that,

$$A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$$

$$\text{Symmetric matrix}=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\left(\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}+\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}12&-4&4\\-4&6&-2\\4&-2&6\end{bmatrix}$$

$$=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$$

$$\text{Skew symmetric matrix}=\frac{1}{2}(A-A')$$

$$=\frac{1}{2}\left(\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}-\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Therefore, given matrix A is sum of symmetric matrix $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ and skew symmetric matrix $\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$. **[3]**

(iii) Given that

$$A=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}$$

$$A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Symmetric matrix $= \frac{1}{2}(A + A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

Skew symmetric matrix $= \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Therefore, given matrix A is sum of symmetric matrix $\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$ and skew symmetric matrix $\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$. **[3]**

(iv) Given that,

$$A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

$$\therefore\ A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

Symmetric matrix $= \frac{1}{2}(A + A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

Skew symmetric matrix $= \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Therefore, given matrix A is sum of symmetric matrix $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and skew symmetric matrix $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$. **[3]**

Q. 11. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

[NCERT Ex. 3.4, Q. 4, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

We know that $A = IA$

$$\therefore\ \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_2 \to R_2 - 5R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_1 \to R_1 + 3R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad (R_2 \to -2R_1)$$

$$\therefore\quad A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

[3]

Q. 12. Find the inverse of each of the matrices, if it exists in given matrices $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

[NCERT Ex. 3.4, Q. 5, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_2 \to R_2 - 7R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad (R_2 \to 2R_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

[3]

Q. 13. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 6, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_2 \to R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_1 \to R_1 - 5R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \to 2R_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

[3]

Q. 14. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 7, Page 97]

Ans. Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad (C_1 \to C_1 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad (C_2 \to C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \quad (C_1 \to C_1 - C_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

[3]

Q. 15. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

[NCERT Ex. 3.4, Q. 8, Page 97]

Ans. Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad (R_2 \to R_2 - 3R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad (R_1 \to R_1 - R_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

[3]

Q. 16. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 9, Page 97]

Ans. Let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad (R_1 \to R_1 - 3R_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

[3]

Q. 17. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 10, Page 97]

Ans. Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \quad (C_1 \to C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad (C_2 \to C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \quad \left(C_2 \to \frac{1}{2}C_2\right)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

[3]

Q. 18. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 11, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \quad (C_2 \to C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \quad (C_1 \to C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \quad \left(C_1 \to \frac{1}{2}C_1\right)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

[3]

Q. 19. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 12, Page 97]

Ans. Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix}A \quad \left(R_1 \to \frac{1}{6}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix}A \quad (R_2 \to R_2 + 2R_1)$$

Now, in the above equation, we can see that all the zeros in the second row of the matrix on the LHS. Therefore, A^{-1} does not exist. **[3]**

Q. 20. Find the inverse of each of the matrices, if it exists in given matrices is $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 13, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}A \quad (R_1 \to R_1 + R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}A \quad (R_2 \to R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}A \quad (R_1 \to R_1 + R_2)$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

[3]

Q. 21. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric. [NCERT Misc. Ex. Q. 5 Page 100]

Ans. We suppose that A is a symmetric matrix, then

$A' = A$ … (i)

Consider,

$$(B'AB)' = \{B'(AB)\}'$$
$$= (AB)'(B')' \quad [(AB)' = B'A']$$
$$= B'A'(B) \quad [(B')' = B]$$
$$= B'(A'B)$$
$$= B'(AB) \quad [\text{By using Eq. (i)}]$$
$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then,

$A' = -A$

Consider,

$$(B'AB)' = [B'(AB)]'$$
$$= (AB)'(B')'$$
$$= (B'A')B$$
$$= B'(-A)B$$
$$= -B'AB$$

Therefore,

$$(B'AB)' = -B'AB$$

Thus, if A is a skew-symmetric matrix, then $B'AB$ is a skew-symmetric matrix. Hence, if A is a symmetric or skew-symmetric matrix, then $B'AB$ is a symmetric or skew symmetric matrix accordingly. **[3]**

Q. 22. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$. Satisfy the equation $A'A = I$.

[NCERT Misc. Ex. Q. 6, Page 100]

Ans. It is given that,

$$A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}$$

Now, $A'A = I$

$$\Rightarrow \begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0+x^2+x^2 & 0+xy-xy & 0-xz+xz\\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz\\ 0-xz+zx & 2yz-yz-yz & z^2+z^2+z^2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3z^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2=1$$

$$\therefore x=\pm\frac{1}{\sqrt{2}}$$

$$6y^2=1$$

$$\therefore y=\pm\frac{1}{\sqrt{6}}$$

$$3z^2=1$$

$$\therefore z=\pm\frac{1}{\sqrt{3}}$$

[3]

Q. 23. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$. Show that $A^2-5A+7I=0$.
[NCERT Misc. Ex. Q. 8, Page 100]

Ans. It is given that,

$$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

Therefore,

$$A^2=A\cdot A$$

$$=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$$LHS = A^2 - 5A + 7I$$

$$\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$= 0 = \text{RHS}$$

$$\therefore A^2-5A+7I=0$$

[3]

Q. 24. Find x, if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$.
[NCERT Misc. Ex. Q. 9, Page 100]

Ans. We have,

$$\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}x+0-2 & 0-10+0 & 2x-5-3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}x-2 & -10 & 2x-8\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}x(x-2)-40+2x-8\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}x^2-2x-40+2x-8\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x^2-48\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$$\therefore \quad x^2-48=0$$

$$\Rightarrow \quad x^2=48$$

$$\Rightarrow \quad x=\pm 4\sqrt{3}$$

[3]

Q. 25. Find the value of x, if $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$.
[NCERT Exemp. Ex. 3.3, Q. 10, Page 54]

Ans. We have,

$$\begin{bmatrix}1 & x & 1\end{bmatrix}_{1\times 3}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}_{3\times 3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times 1}=0$$

$$\Rightarrow \begin{bmatrix}1+2x+15 & 3+5x+3 & 2+x+2\end{bmatrix}_{1\times 3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times 1}=0$$

$$\Rightarrow \begin{bmatrix}16+2x & 5x+6 & x+4\end{bmatrix}_{1\times 3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times 1}=0$$

$$\Rightarrow \begin{bmatrix}16+2x+(5x+6)\cdot 2+(x+4)\cdot x\end{bmatrix}_{1\times 1}=0$$

$$\Rightarrow \begin{bmatrix}16+2x+10x+12+x^2+4x\end{bmatrix}=0$$

$$\Rightarrow \quad [x^2+16x+28]=0$$
$$\Rightarrow [x^2+2x+14x+28]=0$$
$$\Rightarrow \quad (x+2)(x+14)=0$$
$$\therefore \quad x=-2,-14$$
[3]

Q. 26. Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.

[NCERT Exemp. Ex. 3.3, Q. 11, Page 54]

Ans. We have,

$$A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$$

$$A^2 = A\cdot A$$
$$=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$$
$$=\begin{bmatrix}25-3 & 15-6\\ -5+2 & -3+4\end{bmatrix}$$
$$=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}$$

$$3A=3\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$$
$$=\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}$$
$$7I=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

Therefore,

$$A^2-3A-7I=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}-\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$
$$=\begin{bmatrix}22-15-7 & 9-9-0\\ -3+3-0 & 1+6-7\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$=0$$

Hence proved.

Since, $A^2-3A-7I=0$

$$A^{-1}[(A^2)-3A-7I]=A^{-1}0$$
$$A^{-1}A\cdot A-3A^{-1}A-7A^{-1}I=0 \quad [\because A^{-1}0=0]$$
$$IA-3I-7A^{-1}=0 \quad [\because A^{-1}A=I]$$
$$A-3I-7A^{-1}=0 \quad [\because A^{-1}I=A^{-1}]$$
$$-7A^{-1}=-A+3I$$
$$=\begin{bmatrix}-5 & -3\\ 1 & 2\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$
$$=\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix}$$
$$A^{-1}=\frac{-1}{7}\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix}$$
[3]

Q. 27. Find A, if $\begin{bmatrix}4\\1\\3\end{bmatrix}A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 13, Page 54]

Ans. We have,

$$\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1} A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

Let $A = [x\ y\ z]$

$$\therefore \begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1}[x\ y\ z]_{1\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$
$$\Rightarrow \begin{bmatrix}4x & 4y & 4z\\ x & y & z\\ 3x & 3y & 3z\end{bmatrix}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$$

$\Rightarrow$ $4x = -4$

$\Rightarrow$ $x = -1$

$\Rightarrow$ $4y = 8$

$\Rightarrow$ $y = 2$

and $4z = 4$

$\Rightarrow$ $z = 1$

$\therefore$ $A = [-1\ 2\ 1]$ **[3]**

Q. 28. If $A=\begin{bmatrix}3 & -4\\ 1 & 1\\ 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}$, then verify $(BA)^2 \neq B^2A^2$.

[NCERT Exemp. Ex. 3.3, Q. 14, Page 54]

Ans. We have,

$$A=\begin{bmatrix}3 & -4\\ 1 & 1\\ 2 & 0\end{bmatrix}_{3\times2} \text{ and } B=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}_{2\times3}$$
$$BA=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}_{2\times3}\begin{bmatrix}3 & -4\\ 1 & 1\\ 2 & 0\end{bmatrix}_{3\times2}$$
$$=\begin{bmatrix}6+1+4 & -8+1+0\\ 3+2+8 & -4+2+0\end{bmatrix}$$
$$=\begin{bmatrix}11 & -7\\ 13 & -2\end{bmatrix}$$
$$\text{and } (BA)\cdot(BA)=\begin{bmatrix}11 & -7\\ 13 & -2\end{bmatrix}\begin{bmatrix}11 & -7\\ 13 & -2\end{bmatrix}$$
$$(BA)^2=\begin{bmatrix}121-91 & -77+14\\ 143-26 & -91+4\end{bmatrix}$$
$$=\begin{bmatrix}30 & -63\\ 117 & -87\end{bmatrix} \quad \ldots\textbf{(i)}$$
$$\text{Also, } B^2=B\cdot B=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}_{2\times3}\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}_{2\times3}$$

So B^2 is not possible, since the B is not a square matrix.

Hence, $(BA)^2 \neq B^2A^2$. **[3]**

Q. 29. If possible, find the value of *BA* and AB, where

$$A=\begin{bmatrix}2&1&2\\1&2&4\end{bmatrix} \text{ and } B=\begin{bmatrix}4&1\\2&3\\1&2\end{bmatrix}.$$

[NCERT Exemp. Ex. 3.3, Q. 15, Page 54]

Ans. We have,

$$A=\begin{bmatrix}2&1&2\\1&2&4\end{bmatrix} \text{ and } B=\begin{bmatrix}4&1\\2&3\\1&2\end{bmatrix}$$

So, *AB* and *BA* both are possible.

[Since, in both *AB* and *BA*, the number of columns of first is equal to the number of rows of second.]

$$AB=\begin{bmatrix}2&1&2\\1&2&4\end{bmatrix}_{2\times3}\begin{bmatrix}4&1\\2&3\\1&2\end{bmatrix}_{3\times2}$$

$$=\begin{bmatrix}8+2+2&2+3+4\\4+4+4&1+6+8\end{bmatrix}$$

$$=\begin{bmatrix}12&9\\12&15\end{bmatrix}$$

$$BA=\begin{bmatrix}4&1\\2&3\\1&2\end{bmatrix}_{3\times2}\begin{bmatrix}2&1&2\\1&2&4\end{bmatrix}_{2\times3}$$

$$=\begin{bmatrix}8+1&4+2&8+4\\4+3&2+6&4+12\\2+2&1+4&2+8\end{bmatrix}$$

$$=\begin{bmatrix}9&6&12\\7&8&16\\4&5&10\end{bmatrix}$$

[3]

Q. 30. Solve for x and y, $x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=0$.

[NCERT Exemp. Ex. 3.3, Q. 18, Page 54]

Ans. We have,

$$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}2x\\x\end{bmatrix}+\begin{bmatrix}3\cdot y\\5\cdot y\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}2x+3y-8\\x+5y+11\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\therefore \quad 2x+3y-8=0$$

$$\Rightarrow \quad 4x+6y=16 \quad \ldots(i)$$

$$\text{and } x+5y-11=0$$

$$\Rightarrow \quad 4x+20y=44 \quad \ldots\textbf{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get:

$$14y=28$$

$$\Rightarrow \quad y=2$$

$$\therefore 2x+3\times2-8=0$$

$$\Rightarrow \quad 2x=2$$

$$\Rightarrow \quad x=1$$

$$\therefore \quad x=1 \text{ and } y=2$$

[3]

Q. 31. If *X* and *y* are 2 × 2 matrices, then solve the following matrix equations for *X* and *Y*

$$2X+2Y=\begin{bmatrix}2&3\\4&0\end{bmatrix},\ 3X+2Y=\begin{bmatrix}-2&2\\1&-5\end{bmatrix}.$$

[NCERT Exemp. Ex. 3.3, Q. 19, Page 55]

Ans. We have,

$$2X+2Y=\begin{bmatrix}2&3\\4&0\end{bmatrix} \quad \ldots(i)$$

$$\text{and } 3X+2Y=\begin{bmatrix}-2&2\\1&-5\end{bmatrix} \quad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get:

$$\therefore (3X+2Y)-(2X+3Y)=\begin{bmatrix}-2-2&2-3\\1-4&-5-0\end{bmatrix}$$

$$(X-Y)=\begin{bmatrix}-4&-1\\-3&-5\end{bmatrix} \quad \ldots(iii)$$

On adding Eqs. (i) and (ii), we get:

$$(5X+5Y)=\begin{bmatrix}0&5\\5&-5\end{bmatrix}$$

$$(X+Y)=\frac{1}{5}\begin{bmatrix}0&5\\5&-5\end{bmatrix}$$

$$=\begin{bmatrix}0&1\\1&-1\end{bmatrix} \quad \ldots(iv)$$

On adding Eqs. (iii) and (iv), we get:

$$(X-Y)+(X+Y)=\begin{bmatrix}-4&0\\-2&-6\end{bmatrix}$$

$$2X=2\begin{bmatrix}-2&0\\-1&-3\end{bmatrix}$$

$$\therefore \quad X=\begin{bmatrix}-2&0\\-1&-3\end{bmatrix}$$

From Eq. (iv), we have

$$\begin{bmatrix}-2&0\\-1&-3\end{bmatrix}+Y=\begin{bmatrix}0&1\\1&-1\end{bmatrix}$$

$$Y=\begin{bmatrix}2&1\\2&2\end{bmatrix} \text{ and } X=\begin{bmatrix}-2&0\\-1&-3\end{bmatrix}$$

[3]

Q. 32. If *A* = [3 5] and *B* = [7 3] then find a non-zero matrix *C* such that *AC* = *BC*.

[NCERT Exemp. Ex. 3.3, Q. 20, Page 55]

Ans. We have,

$$A=[3\quad 5]_{1\times2} \text{ and } B=[7\quad 3]_{1\times2}$$

Let $C=\begin{bmatrix}x\\y\end{bmatrix}_{2\times1}$ is a non-zero matrix of order 2 × 1.

$$AC=[3\quad 5]\begin{bmatrix}x\\y\end{bmatrix}$$

$$=[3x+5y]$$

$$\text{and } BC=[7\quad 3]\begin{bmatrix}x\\y\end{bmatrix}$$

$$=[7x+3y]$$

For *AC* = *BC*,

$$[3x+5y]=[7x+3y]$$

On using equality of matrix, we get

$3x + 5y = 7x + 3y$

$\Rightarrow \quad 4x = 2y$

$\Rightarrow \quad x = \frac{1}{2}y$

$\Rightarrow \quad y = 2x$

$\therefore \quad C = \begin{bmatrix} x \\ 2x \end{bmatrix}$

We see that on taking Column of order 2 × 1, 2 × 2, 2 × 3, …, we get

$\therefore C = \begin{bmatrix} x \\ 2x \end{bmatrix}, \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix}, \begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix} \ldots$

In general,

$\therefore C = \begin{bmatrix} k \\ 2k \end{bmatrix}, \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix}, \text{etc.}$

[3]

Q. 33. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, verify:

(i) $(AB)C = A(BC)$.

(ii) $A(B + C) = AB + AC$.

[NCERT Exemp. Ex. 3.3, Q. 22, Page 55]

Ans. We have,

$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

(i) $(AB) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix}$

$= \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$

and $(AB)C = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 8+5 & 0 \\ -1+10 & 0 \end{bmatrix}$

$= \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$ …(i)

Again, $(BC) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2-3 & 0 \\ 3+4 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$

and $A(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1+14 & 0 \\ +2+7 & 0 \end{bmatrix}$

$= \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$ …(ii)

$\therefore (AB)C = A(BC)$ [By using Eqs. (i) and (ii)] **[3]**

(ii) $(B+C) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

and $A \cdot (B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix}$

$= \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$ …(iii)

Also, $AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix}$

$= \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$

And

$AC = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & 0 \\ -2-1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$

$\therefore \quad AB + AC = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$

$\Rightarrow \quad AB + AC = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$ …(iv)

From Eqs. (iii) and (iv), we have

$A(B + C) = AB + AC$ **[3]**

Q. 34. If $A = [2 \quad 1], B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that $A(B + C) = (AB + AC)$.

[NCERT Exemp. Ex. 3.3, Q. 25, Page 55]

Ans. We have to verify that, $A(B + C) = AB + AC$

We have,

$A = [2 \quad 1], B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$\therefore A(B+C) = [2 \quad 1]\begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix}$

$= [2 \quad 1]\begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix}$

$= [8+9 \quad 10+7 \quad 10+8]$

$= [17 \quad 17 \quad 18]$ …(i)

Also, $AB = [2 \quad 1]\begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$

$= [10+8 \quad 6+7 \quad 8+6]$

$= [18 \quad 13 \quad 14]$

and $AC = \begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2+1 & 4+0 & 2+2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & 4 \end{bmatrix}$

$\therefore AB + AC = \begin{bmatrix} 18 & 13 & 14 \end{bmatrix} + \begin{bmatrix} -1 & 4 & 4 \end{bmatrix}$

$= \begin{bmatrix} 17 & 17 & 18 \end{bmatrix}$ ...(ii)

$\therefore A(B+C) = (AB + AC)$ [By using Eqs. (i) and (ii)]

Hence proved. **[3]**

Q. 35. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$ where I is 3×3 unit matrix.

[NCERT Exemp. Ex. 3.3, Q. 26, Page 56]

Ans. We have,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\therefore\ A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \text{...(i)}$$

Now,

$$A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

and

$$A(A+I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \text{...(ii)}$$

Thus, we see that $A^2 + A = A(A + I)$ [By using Eqs. (i) and (ii)] **[3]**

Q. 36. Prove by Mathematical Induction that $(A')^n = (A^n)'$ where $n \in N$ for any square matrix A.

[NCERT Exemp. Ex. 3.3, Q. 36, Page 57]

Ans. Let $P(n) : (A')^n = (A^n)'$

$\therefore \quad P(1) : (A)^1 = (A)'$

$\Rightarrow \quad A' = A' \Rightarrow P(1)$ is true.

Now, $P(k) : (A')^k = (A^k)'$

where $k \in N$

and $P(k+1) : (A')^{k+1} = (A^{k+1})'$

where $P(k + 1)$ is true whenever $P(k)$ is true.

$\therefore P(k+1) : (A')^k . (A')^1 = [A^{k+1}]'$

$(A^k)'. (A)' = [A^{k+1}]'$

$(A \cdot A^k)' = [A^{k+1}]'$ [$\because (A')^k = (A^k)'$ and $(AB) = B'A'$]

$(A^{k+1})' = [A^{k+1}]'$

Hence proved. **[3]**

Q. 37. Find inverse, by elementary row operations (if possible), of the following matrices:

(i) $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

[NCERT Exemp. Ex. 3.3, Q. 37, Page 57]

Ans. Let $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

In order to use elementary row operations we may write $A = IA$.

$\therefore \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A$ [$\because R_2 \to R_2 + 5R_1$]

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A$ [$\because R_2 \to \frac{1}{22} R_2$]

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A$ [$\because R_1 \to R_1 - 3R_2$]

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix} A$

$\Rightarrow I = BA$, where B is the inverse of A.

$B = \frac{1}{22}\begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix}$

[2]

(ii) Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

In order to use elementary row operations, we write $A = IA$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A$ [$\because R_2 \to R_2 + 2R_1$]

Since, we obtain all zeroes in a raw of the matrix A on LHS, so A^{-1} does not exist. **[2]**

Q. 38. Find the values of x, y, z and w.

If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & W \\ 0 & 6 \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 38, Page 57]

Ans. We have,

$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & W \\ 0 & 6 \end{bmatrix}$

By equality of matrix, $x + y = 6$ and $xy = 8$

$\Rightarrow x = 6 - y$ and $(6 - y)y = 8$

$\Rightarrow \quad y^2 - 4y - 2y + 8 = 0$

$\Rightarrow \quad (y - 2)(y - 4) = 0$

$\Rightarrow \quad y = 2$ or $y = 4$

$\therefore \quad x = 6 - 2 = 4$

or $x = 6 - 4 = 2$ [Because $x = 6 - y$]

Also, $z + 6 = 0$

$\Rightarrow \quad z = -6$ and $W = 4$

$\therefore \quad x = 2, y = 4$ or $x = 4, y = 2, z = -6$ and $W = 4$ **[3]**

Q. 39. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that $3A + 5B + 2C$ is a null matrix. **[NCERT Exemp. Ex. 3.3, Q. 39, Page 57]**

Ans. We have,

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$$

$$\text{Let } C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore \quad 3A + 5B + 2C = 0$$

$$\Rightarrow \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 2a + 48 = 0 \Rightarrow a = -24$

Also, $\quad 20 + 2b = 0 \Rightarrow b = -10$

$\quad 56 + 2c = 0 \Rightarrow c = -28$

and $\quad 76 + 2d = 0 \Rightarrow d = -38$

$$\therefore \quad c = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

[3]

Q. 40. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ then find $A^2 - 5A - 14I$. Hence, obtain A^3. **[NCERT Exemp. Ex. 3.3, Q. 40, Page 58]**

Ans. We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \quad \ldots\text{(i)}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \quad \ldots\text{(ii)}$$

$$\therefore A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $\quad A^2 - 5A - 14I = 0$

$\Rightarrow \quad A \cdot \times A^2 - 5A \cdot A - 14AI = 0$

$\Rightarrow A^3 - 5A^2 - 14A = 0$ [$\because AI = A$]

$\Rightarrow \quad A^3 = 5A^2 + 14A$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

[By using Eqs. (i) and (iii)]

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

[3]

Q. 41. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$. **[NCERT Exemp. Ex. 3.3, Q. 42, Page 58]**

Ans. We have,

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3\times3}$$

From the given equation, it is clear that order of A should be 2×3.

$$\text{Let} A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0d & b+0\cdot e & c+0\cdot f \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

By equality of matrices, we get

$a = 1, b = -2, c = -5$ and $2a - d = -1$

$\Rightarrow \quad d = 2a + 1 = 3;$

$\Rightarrow 2b - e = -8$

$\Rightarrow \quad e = 2(-2) + 8 = 4$

$2c - f = -10$

$\Rightarrow \quad f = 2c + 10 = 0$

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

[3]

Q. 42. If $AB = BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n = A^nB^n$. **[NCERT Exemp. Ex. 3.3, Q. 49, Page 58]**

Ans. Let $P(n) : (AB)^n = A^nB^n$

$\therefore \quad P(1) : (AB)^1 = A^1B^1$

$\Rightarrow \quad AB = AB$

So, $P(1)$ is true.

Now, $P(k) : (AB)^k = A^kB^k, k \in N$

So, $P(K)$ is true, whenever $P(k + 1)$ is true.

$\therefore P(K + 1 : (AB)^{k+1} = A^{k+1} B^{k+1}$

$\Rightarrow AB^k AB^1$ …(i)

$\Rightarrow A^k B^k BA \Rightarrow A^kB^{k+1} A$ [$\because AB = BA$]

$Ak \cdot A \cdot B^{k+1} \Rightarrow A^{k+1} B^{k+1}$

$(AB)^{k+1} = A^{k+1} B^{k+1}$

So, $P(k + 1)$ is true for all $n \in N$, whenever $P(k)$ is true.

By mathematical induction, $(AB) = A^nB^n$ is true for all $n \in N$. **[3]**

Q. 43. Find x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$

[NCERT Exemp. Ex. 3.3, Q. 50, Page 59]

Ans. We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \text{and } A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Also, $A' = A^{-1}$

$\Rightarrow AA' = AA^{-1}$ $[\because AA^{-1} = I]$

$\Rightarrow AA' = I$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2y^2 - z^2 = 0$$
$$2y^2 = z^2$$
$$4y^2 + z^2 = 1$$
$$2 \cdot z^2 + z^2 = 1$$
$$z = \pm\frac{1}{\sqrt{3}}$$
$$y^2 = \frac{z^2}{2}$$
$$y = \pm\frac{1}{\sqrt{6}}$$

Also, $x^2 + y^2 + z^2 = 1$

$$x^2 = 1 - y^2 - z^2$$
$$= 1 - \frac{1}{6} - \frac{1}{3}$$
$$= 1 - \frac{3}{6}$$
$$= \frac{1}{2}$$
$$x = \pm\frac{1}{\sqrt{2}}$$

Therefore, $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$ **[3]**

Q. 44. Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

[NCERT Exemp. Ex. 3.3, Q. 52, Page 59]

Ans. We have,

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, $\frac{A+A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

and $\frac{A-A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

$$\frac{A+A'}{2} + \frac{A-A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

This is the required expression. **[3]**

Q. 45. Find a matrix D such that $CD - AB = 0$. Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$.

[CBSE Board, Delhi Region, 2017]

Ans. Let,

$$D = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$ **[½]**

$$CD = AB$$

$$\Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$ **[1+1]**

$2x + 5z = 3, 3x + 8z = 43; 2y + 5w = 0, 3y + 8w = 22$

Solving we get $x = -191, y = -110, z = 77, W = 44$ **[1]**

$$\therefore D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$$ **[½]**

Q. 46. Find matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$.

[CBSE Board, All India Region, 2017]

Ans. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$ **[1]**

$$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$ **[1]**

$\Rightarrow \quad 2a - c = -1, 2b - d = -8$

$a = 1, b = -2$ **[1]**

$-3a + 4c = 9, -3b + 4d = 22$

Solving to get $a = 1, b = -2, c = 3$ and $d = 4$

$$\therefore A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$ **[1]**

Q. 47. Find matrix X so that $X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$.

[CBSE Board, Foreign Scheme, 2017]

Ans. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **[1]**

then, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ **[1]**

Equating and solving to get a = 1, $b = -2, c = 2$ and $d = 0$ **[1½]**

$$X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$ **[½]**

Q. 48. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received `2,800 as interest. However, if trust had interchanged money in bonds, they would have got `100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question? [CBSE board, All India Region, 2016]

Ans. Let `$x$ be invested in first bond and `y be invested in second bond then the system of equations is:

$$\left.\begin{aligned} \frac{10x}{100} + \frac{12y}{100} = 2800 \\ \frac{12x}{100} + \frac{10y}{100} = 2700 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 5x + 6y = 140000 \\ 6x + 5y = 135000 \end{aligned}\right\}$$ **[1]**

Let $A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$

$\therefore A \cdot X = B$

$$|A| = -11; A^{-1} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$$ **[1]**

$\therefore$ Solution is

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$$

$$= \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

$\therefore \quad x = 10{,}000$ and $y = 15{,}000$

$\therefore$ Amount invested = $Rs.\ 25{,}000$ **[½+½]**

Value : Caring elders **[1]**

Q. 49. A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is `9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is `26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society?

[CBSE Board, Foreign Scheme, 2016]

Ans. Let each poor child Pay `$x$ per month and each rich child pay `y per month.

$\therefore 20x + 5y = 9000$ **[½]**

$5x + 25y = 26000$ **[½]**

In matrix form, we have

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$ **[1]**

$AX = B$

$\Rightarrow \quad X = A^{-1}B$

$$A^{-1} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

$$= \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 1000$ **[1]**

Value : Compassion or any relevant value **[1]**

Long Answer Type Questions (5 or 6 marks each)

Q. 1. Construct a 3 × 4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

[NCERT Ex. 3.1, Q. 5, Page 64]

Ans. In general, a 3 × 4 matrix is given by

(i) $a_{ij} = \frac{1}{2}|-3i + j|, i = 1,2,3$ and $j = 1,2,3,4$

$$\therefore a_{11} = \frac{1}{2}|-3\times1+1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2}|-3\times2+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3\times3+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3\times1+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3\times2+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2}|-3\times3+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3\times1+3| = \frac{1}{2}|-3+3| = 0$$

$$a_{23} = \frac{1}{2}|-3\times2+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3\times3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3\times1+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}|-3\times2+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2}|-3\times3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$$

Therefore, the required matrix is:

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

[2½]

(ii) $a_{ij} = 2i - j, i = 1,2,3$ and $j = 1,2,3,4$

$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$
$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$
$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$
$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$
$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$
$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$
$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$
$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$
$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$
$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$
$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$
$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$

Therefore, the required matrix is:

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

[2½]

Q. 2. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following:

(i) $A + B$ **(ii) $A - B$**
(iii) $3A - C$ **(iv) AB**
(v) BA **[NCERT Ex. 3.2, Q. 1, Page 80]**

Ans.

(i) $$A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$ **[1]**

$$= \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii) $$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$ **[1]**

$$= \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii) $$3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$ **[1]**

$$= \begin{bmatrix} 3\times2 & 3\times4 \\ 3\times3 & 3\times2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B. Therefore, AB is defined as:

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 23 \\ -1 & 19 \end{bmatrix}$$

[1]

(v) Matrix B has 2 columns. This number is equal to the number of rows in matrix A. Therefore, BA is defined as:

$$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

[1]

Q. 3. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute $(A + B)$ and $(B - C)$. Also, verify that $A + (B - C) = (A + B) - C$.

[NCERT Ex. 3.2, Q. 4, Page 81]

Ans.

$$A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$ **[2]**

$$B-C=\begin{bmatrix}3&-1&2\\4&2&5\\2&0&3\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

$$=\begin{bmatrix}3-4&-1-1&2-2\\4-0&2-3&5-2\\2-1&0-(-2)&3-3\end{bmatrix}$$

$$=\begin{bmatrix}-1&-2&0\\4&-1&3\\1&2&0\end{bmatrix}$$

[2]

$$A+(B-C)=\begin{bmatrix}1&2&3\\5&0&2\\1&1&1\end{bmatrix}+\begin{bmatrix}-1&-2&0\\4&-1&3\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}1+(-1)&2+(-2)&-3+0\\5+4&0+(-1)&2+3\\1+1&-1+2&1+0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&3\\9&-1&5\\2&1&1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4&1&1\\9&2&7\\3&-1&4\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

$$=\begin{bmatrix}4-4&1-1&-1-2\\9-0&2-3&7-2\\3-1&-1--2&4-3\end{bmatrix}$$

$$=\begin{bmatrix}0&0&3\\9&-1&5\\2&1&1\end{bmatrix}$$

Hence, we have verified that $A+(B-C)=(A+B)-C$. **[3]**

Q. 4. Find X and Y, if

(i) $X+Y=\begin{bmatrix}7&0\\2&5\end{bmatrix}$ **and** $X-Y=\begin{bmatrix}3&0\\0&3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$ **and** $3X+2Y=\begin{bmatrix}2&-2\\-1&5\end{bmatrix}$

[NCERT Ex. 3.2, Q. 7, Page 81]

Ans.

(i) $X+Y=\begin{bmatrix}7&0\\2&5\end{bmatrix}$ …(i)

$X-Y=\begin{bmatrix}3&0\\0&3\end{bmatrix}$ …(ii)

Adding equations (i) and (ii), we get:

$$2X=\begin{bmatrix}7&0\\2&5\end{bmatrix}+\begin{bmatrix}3&0\\0&3\end{bmatrix}$$

$$=\begin{bmatrix}7+3&0+0\\2+0&5+3\end{bmatrix}$$

$$=\begin{bmatrix}10&0\\2&8\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}10&0\\2&8\end{bmatrix}$$

$$=\begin{bmatrix}5&0\\1&4\end{bmatrix}$$

Now,

$$X+Y=\begin{bmatrix}7&0\\2&5\end{bmatrix}$$

$$\begin{bmatrix}5&0\\1&4\end{bmatrix}+Y=\begin{bmatrix}7&0\\2&5\end{bmatrix}$$

$$Y=\begin{bmatrix}7&0\\2&5\end{bmatrix}-\begin{bmatrix}5&0\\1&4\end{bmatrix}$$

$$=\begin{bmatrix}7-5&0-0\\2-1&5-4\end{bmatrix}$$

$$=\begin{bmatrix}2&0\\1&1\end{bmatrix}$$

(ii)

$2X+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$ **…(iii)**

$3X+2Y=\begin{bmatrix}2&-2\\-1&5\end{bmatrix}$ …(iv)

Multiplying equations (iii) with (2), we get:

$$2(2X+3Y)=2\begin{bmatrix}2&3\\4&0\end{bmatrix}$$

$\Rightarrow 4X+6Y=\begin{bmatrix}4&6\\8&0\end{bmatrix}$ …(v)

Multiplying equations (iv) with (3), we get:

$$3(3X+2Y)=3\begin{bmatrix}2&-2\\-1&5\end{bmatrix}$$

$\Rightarrow 9X+6Y=\begin{bmatrix}6&-6\\-3&15\end{bmatrix}$ …(vi)

From equations (v) and (vi), we have:

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix}4&6\\8&0\end{bmatrix}-\begin{bmatrix}6&-6\\-3&15\end{bmatrix}$$

$$-5X=\begin{bmatrix}4-6&6-(-6)\\8-(-3)&0-15\end{bmatrix}$$

$$=\begin{bmatrix}-2&12\\11&-15\end{bmatrix}$$

$$X=-\frac{1}{5}\begin{bmatrix}-2&12\\11&-15\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{5}&-\frac{12}{5}\\-\frac{11}{5}&3\end{bmatrix}$$

[2½]

$$\because\ 2X+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$$

$$2\begin{bmatrix}\frac{2}{5}&-\frac{12}{5}\\-\frac{11}{5}&3\end{bmatrix}+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$$

$$\begin{bmatrix}\frac{4}{5}&-\frac{24}{5}\\-\frac{22}{5}&6\end{bmatrix}+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$$

$$3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}-\begin{bmatrix}\frac{4}{5}&-\frac{24}{5}\\-\frac{22}{5}&6\end{bmatrix}$$

$$3Y = \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

[2½]

Q. 5. Show that

(i) $\begin{bmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{6} & \mathbf{7} \end{bmatrix}\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} \end{bmatrix} \neq \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}\begin{bmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{6} & \mathbf{7} \end{bmatrix}$

(ii) $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix} \neq \begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$

[NCERT Ex. 3.2, Q. 14, Page 82]

Ans.

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

[2½]

(ii)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) \\ 1(0)+2(1)+3(4) & \\ 0(0)+1(1)+0(4) & \\ 1(0)+1(1)+0(4) & \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) \\ -1(3)+1(0)+0(0) & \\ 0(3)+(-1)(0)+1(0) & \\ 2(3)+3(0)+4(0) & \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

[2½]

Q. 6. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{-1} & \mathbf{0} \end{bmatrix}$

[NCERT Ex. 3.2, Q. 15, Page 82]

Ans. We have $A^2 = A \times A$

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) \\ 2(1)+0(3)+1(0) & \\ 2(1)+1(3)+3(0) & \\ 1(1)+(-1)(3)+0(0) & \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Therefore,

$$A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

[5]

Q. 7. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.

[NCERT Ex. 3.2, Q. 16, Page 82]

Ans. $A^2 = AA$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, $A^3 = A^2 A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

$\therefore A^3 - 6A^2 + 7A + 2I = 0$ **[5]**

Q. 8. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

[NCERT Ex. 3.2, Q. 18, Page 82]

Ans. On the LHS

$I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \quad \ldots\text{(i)}$$

On the RHS

$(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix} \quad \ldots\text{(ii)}$$

$$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from equations (i) and (ii), we get LHS = RHS **[5]**

Q. 9. A trust fund has ₹30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) ₹1800 **(b) ₹2000**

[NCERT Ex. 3.2, Q. 19, Page 82]

Ans. (a) Let ₹x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹$(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year. Therefore, in order to obtain an annual total interest of ₹1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \quad \left[\because \text{SI for 1 year} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}\right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of ₹ 1800, the trust fund should invest ₹ 15000 in the first bond and the remaining ₹ 15000 in the second bond. **[2½]**

(b) Let ₹ x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹ (30000 – x). Therefore, in order to obtain an annual total interest of ₹ 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of ₹ 2000, the trust fund should invest ₹ 5000 in the first bond and the remaining ₹ 25000 in the second bond. **[2½]**

Q. 10. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ **then verify that:**

(i) $(A + B)' = A' + B'$ **(ii)** $(A - B)' = A' - B'$

[NCERT Ex. 3.3, Q. 2, Page 88]

Ans. (i) $$A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\text{LHS} = (A+B)'$$

$$= \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}'$$

$$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$\text{RHS} = A' + B'$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

∴ LHS = RHS

Hence proved. **[2½]**

(ii) $$A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\text{LHS} = (A-B)'$$

$$= \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$\text{RHS} = A' - B'$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ 3 & 9 & 1 \end{bmatrix}' - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

∴ LHS = RHS

Hence proved. **[2½]**

Q. 11. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that:

(i) $(A + B)' = A' + B'$

(ii) $(A - B)' = A' - B'$ [NCERT Ex. 3.3, Q. 3, Page 80]

Ans. Given that,

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \text{ then}$$

$$(A')' = A$$

$$= \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

(i) $$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & -1+2 & 0+1 \\ 4+1 & 2+2 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\text{LHS} = (A+B)'$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$\text{RHS} = A' + B'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & 4+1 \\ -1+2 & 2+2 \\ 0+1 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$\therefore$ LHS = RHS

Hence proved. **[2½]**

(ii) $$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & -1-2 & 0-1 \\ 4-1 & 2-2 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

LHS = $(A - B)'$

$$= \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}'$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$\text{RHS} = A' - B'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$\therefore$ LHS = RHS

Hence proved. **[2½]**

Q. 12. Find the inverse of each of the matrices, if it exists in given matrices $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 15, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ (R_2 \to R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ \left(R_2 \to \frac{1}{5}\right)$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ (R_1 \to R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ -\frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A \ \begin{pmatrix} R_1 \to R_1 + R_2 \text{ and} \\ R_3 \to R_3 + 2R_2 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A \quad (R_3 \to R_3 + 3R_1)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left(R_3 \to \frac{1}{5} R_3\right)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & 0 & -\frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (R_1 \to R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (R_1 \to (-1) R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

[5]

Q. 13. Find the inverse of each of the matrices, if it exists in given matrices $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 16, Page 97]

Ans. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_3$ and $R_2 \to R_2 + 8R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{25} R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 - 10R_3$, and $R_2 \to R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

[5]

Q. 14. Find the inverse of each of the matrices, if it exists in given matrices $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

[NCERT Ex. 3.4, Q. 17, Page 97]

Ans. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2} R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$ and $R_2 \to R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

[5]

Q. 15. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in N$. **[NCERT Misc. Ex. Q. 1, Page 100]**

Ans. It is given that,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

To show : $P(n) : (aI + bA)^n = a^nI + na^{n-1} bA, n \in N$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$P(1) : (aI + bA) = aI + ba^0 A = aI + bA$

Therefore, the result is true.

For $n = 1$.

Let the result be true for $n = k$., *i.e.*,

$P(k) : (aI + bA)^k = a^kI + ka^{k-1} bA$

Now, we prove that the result is true for $n = k + 1$.

Consider,

$$\begin{aligned}(aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^k I + ka^{k-1} bA)(aI + bA) \\ &= a^{k+1}I + ka^kbAI + a^kbIA + ka^{k-1}b^2A^2 \\ &= a^{k+1}I + (k+1)a^kbA + ka^{k-1}b^2A^2 \quad \text{...(i)}\end{aligned}$$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

From equation (i), we have:

$$\begin{aligned}(aI + bA)^{k+1} &= a^{k+1}I + (k+1)a^kbA + 0 \\ &= a^{k+1}I + (k+1)\, a^kbA\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$(aI + bA)^n = a^nI + na^{n-1}bA$ where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, n \in N$.

[5]

Q. 16. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Prove that

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$$

[NCERT Misc. Ex. Q. 2, Page 100]

Ans. It is given that,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To show: $P(n) : A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1) : \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k) : A^k \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Now, $A^{k+1} = A\,A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus by the principle of mathematical induction, we have:

$$A = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$$

[5]

Q. 17. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ Then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer.

[NCERT Misc. Ex. Q. 3, Page 100]

Ans. It is given that,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

To prove : $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in N$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$., *i.e.*,

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in N$$

Now, we prove that the result is true for $n = k + 1$.

Consider,

$A^{k+1} = A^k A$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1-2k & -4k-1(1-2k) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$$ **[5]**

Q. 18. A manufacturer produces three products *x*, *y* and *z* which he sells in two markets. Annual sales are indicated below:

Market	Products	Products	Products
1	10,000	2,000	18,000
2	6,000	20,000	8,000

(a) If unit sale prices of *x*, *y* and *z* are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit. [NCERT Misc. Ex. Q. 10, Page 101]

Ans. **(a)** The unit sale prices of *x*, *y* and *z* are respectively given as `2.50, `1.50 and `1.00. Consequently, the total revenue in market 1 can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$

$= 25000 + 3000 + 18000$

$= 46000$

The total revenue in market 2 can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$

$= 15000 + 30000 + 8000$

$= 53000$

Therefore, the total revenue in market 1 is `46000 and the same in market 2 is `53000. **[2½]**

(b) The unit cost prices of *x*, *y* and *z* are respectively given as `2.00, `1.00 and 50 paise. Consequently, the total cost prices of all the products in market 1 can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$= 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50$

$= 20000 + 2000 + 9000$

$= 31000$

Since the total revenue in market 1 is `46000, the gross profit in this market

= (`46000 – `31000) = `15000.

The total cost prices of all the products in market 2 can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50$

$= 12000 + 20000 + 4000$

$= \text{Rs } 36000$

Since the total revenue in market 2 is `53000, the gross profit in this market

= (`53000 – `36000) = `17000. **[2½]**

Q. 19. Find the matrix *X* so that

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$

[NCERT Misc. Ex. Q. 11, Page 101]

Ans. It is given that,

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the RHS of the equation is a 2×3 matrix and the one given on the LHS of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let

$$X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$ **[2½]**

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7, \quad 2a+5c=-8 \quad 3a+6c=-9$

$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$

Now, $a + 4c = -7$

$\Rightarrow \quad a = -7 - 4c$

$\therefore \quad 2a + 5c = -8$

$\Rightarrow -14 - 8c + 5c = -8$

$\Rightarrow \quad -3c = 6$

$\Rightarrow \quad c = -2$

$\therefore \quad a = -7 - 4(-2) = -7 + 8 = 1$

Now, $\quad b + 4d = 2$

$\Rightarrow \quad b = 2 - 4d$

$\therefore \quad 2b + 5d = 4$

$\Rightarrow \quad 4 - 8d + 5d = 4$

$\Rightarrow \quad -3d = 0$

$\Rightarrow \quad d = 0$

$\therefore \quad b = 2 - 4(0) = 2$

Thus, a = 1, $b = 2$, $c = -2$ and $d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$. **[2½]**

Q. 20. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in$ N. [NCERT Misc. Ex. Q. 12, Page 101]

Ans. A and B are square matrices of the same order such that $AB = BA$.

To prove : $P(n) : AB^n = B^nA, n \in N$

For $n = 1$, we have:

$P(1) : AB = BA$ [Given]

$\Rightarrow \quad AB^1 = B^1A$

Therefore, the result is true for $n = 1$.

Let the result be true for n = k.

$P(k) : AB^k = B^kA \quad \ldots(i)$

Now, we prove that the result is true for $n = k + 1$.

$AB^{k+1} = AB^kB$

$= (B^kA)B$ [By using Eq. (i)]

$= B^k(AB)$ [By associative law]

$= B^k(BA)$ [$AB = BA$ (Given)]

$= (B^kB)A$ [By associative law]

$= B^{k+1}A$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have

$AB^n = B^nA, n \in$ N.

Now, we prove that

$(AB)^n = A^nB^n$

For all $n \in$ N

For $n = 1$, we have : $(AB)^1 = A^1B^1 = AB$

Therefore, the result is true for $n = 1$.

Let the result be true for n = k.

$(AB)^k = A^kB^k \quad \ldots(ii)$

Now, we prove that the result is true for $n = k + 1$.

$(AB)^{k+1} = (AB)^k(AB)$

$= (A^kB^k)(AB)$ [By using Eq. (ii)]

$= A^k (B^k A)B$ [By associative law]

$= A^k (AB^k)B$ [$AB^n = B^n A$ for all $n \in$ N)]

$= (A^k A)(B^kB)$ [By associative law]

$= A^{k+1}B^{k+1}$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^nB^n$, for all natural numbers. **[5]**

Q. 21. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that $(A + B)(A - B) \neq A^2 - B^2$.

[NCERT Exemp. Ex. 3.3, Q. 9, Page 53]

Ans. We have,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}_{2\times2}$$

$$\text{and } (A-B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

Since, $(A + B)(A - B)$ is defined, if the number of columns of $(A + B)$ is equal to the number of rows of $(A - B)$, so here multiplication of matrices $(A + B).(A - B)$ is possible.

Now,

$$(A+B)_{2\times2} \cdot (A-B)_{2\times2} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad \ldots(i)$$

Also, $\quad A^2 = AA$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

And

$B^2 = B \cdot B$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \ldots(ii)$$

Thus, we see that

$(A + B)(A - B) \neq A^2 - B^2$ [By using Eqs. (i) and (ii)]

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

Hence proved. **[5]**

Q. 22. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

[NCERT Exemp. Ex. 3.3, Q. 12, Page 54]

Ans. We have,

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times2} A \cdot \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a-6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad -6a-3c+10b+5d = 1$ ...(i)

$\Rightarrow \quad 4a + 2c - 6b - 3d = 0$ ...(ii)

$\Rightarrow -9a - 6c + 15b + 10d = 0$ ...(iii)

$6a + 4c - 9b - 6d = 1$ ...(iv)

On adding Eqs. (i) and (iv), we get:

$c + b - d = 2$

$\Rightarrow \quad d = c + b - 2$ ...(v)

On adding Eqs. (ii) and (iii), we get:

$-5a - 4c + 9b + 7d = 0$ ...(vi)

On adding Eqs. (vi) and (iv), we get:

$a + 0 + 0 + d = 1$

$\Rightarrow \quad d = 1 - a$ ...(vii)

From Eqs. (v) and (vii),

$c + b - 2 = 1 - a$

$\Rightarrow \quad a + b + c = 3$ ...(viii)

$a = 3 - b - c$

Now, using the values of a and d in Eq. (iii), we get:

$-9(3 - b - c) - 6c + 15b + 10(-2 + b + c) = 0$

$\Rightarrow -27 + 9b + 9c - 6c + 15b - 20 + 10b + 10c = 0$

$\Rightarrow 34b + 13c = 47$ ...(ix)

Now, using the values of a and d in Eqs. (ii), we get:

$4(3 - b - c) + 2c - 6b - 3(b + c - 2) = 0$

$\Rightarrow 12 - 4b - 4c + 2c - 6b - 3b - 3c + 6 = 0$

$\Rightarrow \quad -13b - 5c = -18$ ...(x)

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get:

$-169b - 65c = -234$

$\underline{170b + 65c = 235}$

$b = 1$

$\Rightarrow \quad -13 \times 1 - 5c = -18$ [From Eq. (x)]

$\Rightarrow \quad -5c = -18 + 13 = -5$

$\Rightarrow \quad c = 1$

$\therefore \quad a = 3 - 1 - 1 = 1$ and $d = 1 - 1 = 0$

$$\therefore \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

[5]

Q. 23. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x)\, P(y) = P(x + y) = P(y)\cdot P(x)$.

[NCERT Exemp. Ex. 3.3, Q. 46, Page 58]

Ans. We have,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

Now, $P(x)\cdot P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

$$= \begin{bmatrix} \cos x\cdot\cos y - \sin x\cdot\sin y & \cos x\cdot\sin y + \sin x\cdot\cos y \\ -\sin x\cdot\cos y - \cos x\cdot\sin y & -\sin x\cdot\sin y + \cos x\cdot\cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \quad \text{...(i)}$$

$$\begin{bmatrix} \because \cos(x+y) = \cos x\cdot\cos y - \sin x\cdot\sin y \text{ and} \\ \sin(x+y) = \sin x\cdot\cos y + \cos x\cdot\sin y \end{bmatrix}$$

and $P(x+y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$ ...(ii)

Also, $P(y)\cdot P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$$= \begin{bmatrix} \cos y\cdot\cos x - \sin y\cdot\sin x & \cos y\cdot\sin x + \sin y\cdot\cos x \\ -\sin y\cdot\cos x - \sin x\cdot\cos y & -\sin y\cdot\sin x + \cos y\cdot\cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \quad \text{...(iii)}$$

Thus, we see from the Eqs. (i), (ii) and (iii) that,

$P(x)\cdot P(y) = P(x + y) = P(y)\cdot P(x)$

Hence proved. **[5]**

Q. 24. If possible, using elementary row transformations, find the inverse of the following matrices:

(i) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

[NCERT Exemp. Ex. 3.3, Q. 51, Page 59]

Ans. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as $A = IA$

(i) $\because \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_1 \to R_1 + R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \; [\because R_2 \to R_2 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because R_1 \to R_1 + R_2 \\ \text{and } R_3 \to -1 \cdot R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because R_1 \to R_1 + 10R_3 \\ \text{and } R_2 \to R_2 + 17R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because R_1 \to -1R_1 \text{ and} \\ R_2 \to -1R_2 \end{bmatrix}$$

So, the inverse of A is $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$

(ii) $\therefore \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because R_2 \to R_2 + R_3 \\ \text{and } R_1 \to R_1 - 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + R_1]$$

Since, second row of the matrix A on LHS containing all zeroes, so we can say that inverse of matrix of matrix A does not exist.

(iii) $\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \; [\because R_2 \to R_2 - R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because R_2 \to R_2 - R_1 \text{ and} \\ R_3 \to R_3 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because R_3 \to R_3 + R_1 \text{ and} \\ R_2 \to R_2 - \frac{1}{2}R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A \begin{bmatrix} \because R_1 \to \frac{1}{2}R_1 \text{ and} \\ R_3 \to 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \begin{bmatrix} \because R_1 \to R_1 + \frac{1}{2}R_3 \text{ and} \\ R_2 \to R_2 - \frac{5}{2}R_3 \end{bmatrix}$$

Hence, $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ is the inverse of given matrix A. **[5]**

Q. 25. Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

We know that, $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$ **[6]**

Q. 26. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 9$; $-x + 2y - 2z = 4$; $2x - 3y + 4z = -3$.

[CBSE Board, Delhi Region, 2017]

Ans.

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AB = I$ **[1]**

$\Rightarrow \; A^{-1} = B$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$ **[1]**

Given equations in matrix form are:

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$ **[1]**

$AX = C$ **[½]**

$X = (A)^{-1}C = (A^{-1})C$ **[1]**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$ **[1]**

$x = 0, y = 5$ and $z = 3$ **[1]**

Q. 27. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ **and use it to solve the system of equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$.**

[CBSE Board, All India Region, 2017]

Ans. Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(i) **[1½]**

Given equations can be written as:

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$ **[1]**

$\Rightarrow AX = B$

From equation (i), we have

$$A^{-1} = \frac{1}{8} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$$ **[1]**

$X = A^{-1}B$

$$= \frac{1}{8} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$ **[1]**

$$\frac{1}{8} \begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$ **[1]**

$\Rightarrow x = 3, y = -2$ and $z = -1$ **[½]**

Some Commonly Made Errors

- Generally, students confuse in general formula of skew symmetric or symmetric matrices.
- Students confuse in transpose of a matrix or inverse of a matrix.
- Students does not use right sign convention of matrix element.
- Students forget the identity of the matrices.

EXPERT ADVICE

- *Features of judgment matrix aggregation.*
- *Matrix aggregation method is based on improved GSO Algorithm.*
- *Experimental Analysis of a matrix.*
- *Comparing and combining dimensions of decision matrices.*

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