CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

Chapter Objectives

- *Introduction.*
- *Basic concepts.*
- *Properties of inverse trigonometric functions.*

Quick Review

- The inverse trigonometric functions are also known as arcus functions, ant-trigonometric functions or cyclometric functions.
- The most common convention is to name inverse trigonometric functions using an arc- prefix, e.g., arc sin (x), arc cos (x), arc tan (x), etc.
- The notations $\sin^{-1}$ (x), $\cos^{-1}$ (x), $\tan^{-1}$ (x), etc., as introduced by John Herschel in 1813, are often used as well in British sources, and this convention compiles with the notation of an inverse function.
- The inverse trigonometric functions logically conflict with the common semantics for expressions like $\sin^2$ (x), which refer to numeric power rather than function composition, and therefore, may result in confusion between multiplicative inverse and compositional inverse.
- Since 2009, the ISO 80000-2 standard has removed the ambiguity by solely specifying the "arc" prefix for the inverse functions.

Know the Links

- *www.themathpage.com/aTrig/inverseTrig.htm*
- *https://www.varsitytutors.com/hotmath/hotmath.../inverse-trigonometric-functions*

TIPS...

- *Convert all sec, cosec, cot, and tan to sin and cos. Most of these can be done by using the quotient and reciprocal identities.*
- *Check all the angles for sums and differences and use the appropriate identities to remove them.*
- *Replace cos powers greater than 2 with sin powers using the Pythagorean Identities.*
- *Expand any equations you can, combine like terms, and simplify the equations.*

TRICKS...

- *Consider the below equation, which has to be reduced to its simplest form.*

  $$arc\ \tan\frac{\sqrt{1+x^2}-1}{x},\ x \neq 0$$

- *You must be aware of the Domain and Range of the function.*
- *One important fact to remember is that the inverse may not be a "function" itself. If it has two or more points in a vertical line it is just a "relation".*
- *You should just realise that the inverse of any function is its REFLECTION in the line $y = x$.*

Multiple Choice Questions (1 mark each)

Q. 1. Which of the following is the principal value branch of $\cos^{-1}x$?

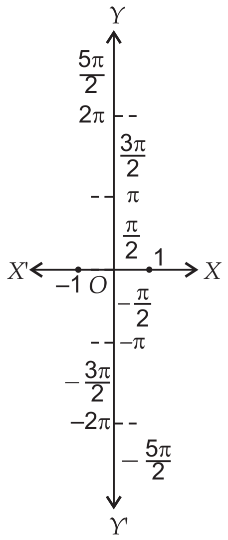
(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **(b)** $[0,\pi]$

(c) $[0,\pi]$ **(d)** $(0,\pi)-\left\{\frac{\pi}{2}\right\}$

[NCERT Exemp. Ex. 2.3, Q. 20, Page 37]

Ans. Correct option : (c)

Explanation : As we know that the principal value of $\cos^{-1}x$ is $[0,\pi]$.



$y = \cos^{-1}x$

Q. 2. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

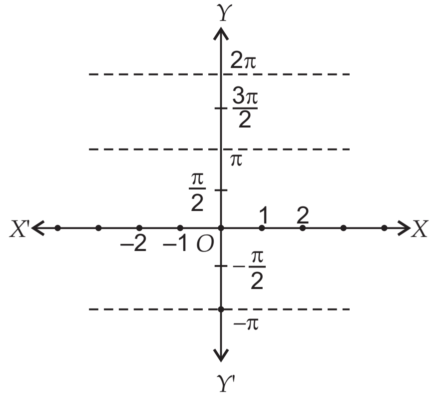
(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

[NCERT Exemp. Ex. 2.3, Q. 21, Page 37]

Ans. Correct option : (d)

Explanation : As we know that the principal value of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.



$y = \text{cosec}^{-1}x$

Q. 3. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x equals

(a) 0 (b) 1

(c) –1 (d) $\frac{1}{2}$

[NCERT Exemp. Ex. 2.3, Q. 22, Page 37]

Ans. Correct option : (b)

Explanation : Given that,

$$3\tan^{-1}x + \cot^{-1}x = \pi$$

Now, we have,

$$3\tan^{-1}x + \cot^{-1}x = \pi$$

$\Rightarrow 2\tan^{-1}x + (\tan^{-1}x + \cot^{-1}x) = \pi$

Using this property, we get

$\Rightarrow 2\tan^{-1}x + \left(\frac{\pi}{2}\right) = \pi \qquad \left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \quad 2\tan^{-1}x = \pi - \frac{\pi}{2}$

$\Rightarrow \quad 2\tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow \quad x = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad x = 1$

Q. 4. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$

(c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

[NCERT Exemp. Ex. 2.3, Q. 23, Page 37]

Ans. Correct option : (d)

Explanation : Let,

$$= \sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$[\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2} + x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$[\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10}$$

$$\left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

Q. 5. The domain of function $\cos^{-1}(2x - 1)$ is

(a) [0,1] (b) [–1, 1]

(c) (–1,1) (d) $[0, \pi]$

[NCERT Exemp. Ex. 2.3, Q. 24, Page 38]

Ans. Correct option : (a)

Explanation :

We have $\cos^{-1}(2x - 1)$

$\Rightarrow -1 \le 2x - 1 \le 1 \qquad [\because x \in [-1, 1]]$

$\Rightarrow \quad 0 \le 2x \le 2$

$\Rightarrow \quad 0 \le x \le 1$

$\Rightarrow \quad \in [0, 1]$

Q. 6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x - 1}$ is

(a) [1,2] (b) [–1,1]

(c) [0,1] (d) None of these

[NCERT Exemp. Ex. 2.3, Q. 25, Page 38]

Ans. Correct option : (a)

Explanation : $f(x)=\sin^{-1}\sqrt{x-1}$

$\Rightarrow 0\le x-1\le 1$ $[\because \sqrt{x-1}\ge 0 \text{ and } -1\le\sqrt{x-1}\le 1]$

$\Rightarrow 1\le x\le 2$

$\therefore x\in[1,2]$

Q. 7. If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then x is equal to

(a) $\frac{1}{5}$ **(b)** $\frac{2}{5}$

(c) 0 **(d)** 1

[NCERT Exemp. Ex. 2.3, Q. 26, Page 38]

Ans. Correct option : (b)

Explanation : Given that,

$$\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\cos^{-1}0$$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\cos^{-1}\cos\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\frac{2}{5}$$

$$\left[\because \sin^{-1}\frac{2}{5}+\cos^{-1}x=\frac{\pi}{2}\right]$$

$$\Rightarrow \cos^{-1}x=\cos^{-1}\frac{2}{5}$$

$$\Rightarrow x=\frac{2}{5}$$

Q. 8. The value of $\sin[2\tan^{-1}(0.75)]$ is equal to

(a) 0.75 **(b)** 1.5

(c) 0.96 **(d)** sin1.5

[NCERT Exemp. Ex. 2.3, Q. 27, Page 38]

Ans. Correct option : (c)

Explanation : We have,

$$\sin[2\tan^{-1}(0.75)]=\sin\left(2\tan^{-1}\frac{3}{4}\right)\quad\left[\because 0.75=\frac{75}{100}=\frac{3}{4}\right]$$

$$=\sin\left(\sin^{-1}\frac{2\cdot\frac{3}{4}}{1+\frac{9}{16}}\right)$$

$$=\sin\left[\sin^{-1}\frac{\frac{3}{2}}{\frac{25}{16}}\right]$$

$$=\sin\left[\sin^{-1}\left(\frac{48}{50}\right)\right]$$

$$=\sin\left[\sin^{-1}\left(\frac{24}{25}\right)\right]$$

$$=\frac{24}{25}$$

$$=0.96$$

Q. 9. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{3\pi}{2}$

(c) $\frac{5\pi}{2}$ **(d)** $\frac{7\pi}{2}$

[NCERT Exemp. Ex. 2.3, Q. 28, Page 38]

Ans. Correct option : (a)

Explanation : We have, $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

$$=\cos^{-1}\left[\cos\left(2\pi-\frac{\pi}{2}\right)\right]$$

$$\left[\because \cos\left(2\pi-\frac{\pi}{2}\right)=\cos\frac{\pi}{2}\right]$$

$$=\cos^{-1}\cos\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$$

$$[(\therefore \cos^{-1})(\cos x)=x,\ x\in[0,\pi]]$$

$$=\cos^{-1}\left(\cos\frac{\pi}{2}\right)$$

$$=\frac{\pi}{2}\ [\because \cos^{-1}(\cos x)=x \text{ if } x\in[0,\pi]]$$

$$\Rightarrow \cos^{-1}\left(\cos\frac{3\pi}{2}\right)=\frac{\pi}{2}$$

Q. 10. The value of expression $2\sec^{-1}2+\sin^{-1}\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{6}$ **(b)** $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$ **(d)** 1

[NCERT Exemp. Ex. 2.3, Q. 29, Page 38]

Ans. Correct option : (b)

Explanation : We have, $2\sec^{-1}2+\sin^{-1}\left(\frac{1}{2}\right)$

$$=2\sec^{-1}\sec\frac{\pi}{3}+\sin^{-1}\sin\frac{\pi}{6}$$

$$=2\times\frac{\pi}{3}+\frac{\pi}{6}\quad[\because \sec^{-1}(\sec x)=x \text{ and } \sin^{-1}(\sin x)=x]$$

$$=\frac{4\pi+\pi}{6}$$

$$=\frac{5\pi}{6}$$

Q. 11. If $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$, then $\cot^{-1}x+\cot^{-1}y$ equals to

(a) $\frac{\pi}{5}$ **(b)** $\frac{2\pi}{5}$

(c) $\frac{3\pi}{5}$ **(d)** π

[NCERT Exemp. Ex. 2.3, Q. 30, Page 38]

Ans. Correct option : (a)

Explanation : We have,

$$\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$$

$$\Rightarrow \frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y=\frac{4\pi}{5}\left[\because \tan^{-1}+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$\Rightarrow \quad -\left(\cot^{-1}x+\cot^{-1}y\right)=\frac{4\pi}{5}-\pi$$

$$\Rightarrow \quad \cot^{-1}x+\cot^{-1}y=-\left(-\frac{\pi}{5}\right)$$

$$\Rightarrow \quad \cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$$

Q. 12. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in]0,\ 1[$, then the value of x is

(a) 0 **(b)** $\frac{a}{2}$

(c) a **(d)** $\frac{2a}{1-a^2}$

[NCERT Exemp. Ex. 2.3, Q. 31, Page 38]

Ans. Correct option : (d)

Explanation : We have,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Let we assume that $a = \tan\theta \Rightarrow \theta = \tan^{-1}a$

$$\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow \sin^{-1}\sin 2\theta+\cos^{-1}\cos 2\theta=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow 2\theta+2\theta=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow 4\tan^{-1}a=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow (2\times 2)\tan^{-1}a=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Using this property, we have

$$\left[\because 2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$2\times\tan^{-1}\frac{2a}{1-a^2}=\tan^{-1}\frac{2x}{1-x^2}$$

$$\Rightarrow \tan^{-1}\frac{2\left(\frac{2a}{1-a^2}\right)}{1-\left(\frac{2a}{1-a^2}\right)^2}=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow \quad \left(\frac{2a}{1-a^2}\right)=x$$

Q. 13. The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is

(a) $\frac{25}{24}$ **(b)** $\frac{25}{7}$

(c) $\frac{24}{25}$ **(d)** $\frac{7}{24}$

[NCERT Exemp. Ex. 2.3, Q. 32, Page 39]

Ans. Correct option : (d)

Explanation :

Let, $\cos^{-1}\left(\frac{7}{25}\right)=x$

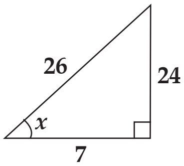
$$\Rightarrow \quad \cos x=\frac{7}{25}$$

$$\therefore \quad \sin x=\sqrt{1-\cos^2 x}$$

$$=\sqrt{1-\left(\frac{7}{25}\right)^2}$$

$$=\sqrt{\frac{625-49}{625}}$$

$$=\frac{24}{25}$$



$$\cot x=\frac{\cos x}{\sin x}$$

$$=\frac{\frac{7}{25}}{\frac{24}{25}}$$

$$=\frac{7}{24}$$

$$\Rightarrow x=\cot^{-1}\left(\frac{7}{24}\right)$$

$$=\cos^{-1}\left(\frac{7}{25}\right)$$

$$\therefore \cot\left(\cos^{-1}\frac{7}{25}\right)$$

$$=\cot\left(\cot^{-1}\right. \qquad \left[\because \cos^{-1}\frac{7}{25}=\cot^{-1}\frac{7}{24}\right]$$

$$=\frac{7}{24}$$

Q. 14. The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

(a) $2+\sqrt{5}$ **(b)** $\sqrt{5}-2$

(c) $\frac{\sqrt{5}+2}{2}$ **(d)** $5+\sqrt{2}$

[NCERT Exemp. Ex. 2.3, Q. 33, Page 39]

Ans. Correct option : (b)

Explanation :

Let, $\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}=\theta$

$$\Rightarrow \quad \cos^{-1}\frac{2}{\sqrt{5}}=2\theta$$

$$\Rightarrow \quad \cos 2\theta=\frac{2}{\sqrt{5}}$$

$$\therefore (1-2\sin^2\theta)=\frac{2}{\sqrt{5}}$$

$$\sin^2\theta=\frac{1}{2}-\frac{1}{\sqrt{5}}$$

$$\sin\theta=\sqrt{\frac{1}{2}-\frac{1}{\sqrt{5}}}$$

We know that $\cos^2\theta=1-\sin^2\theta$

$$\cos\theta=\sqrt{\frac{1}{2}+\frac{1}{\sqrt{5}}}$$

We also know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

On solving, we get the final term given as :

$\Rightarrow \tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$

$= \tan\ \tan^{-1}\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}}$

$= \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}}$

$= \sqrt{\frac{(\sqrt{5}-2)^2}{5-4}}$

$= \sqrt{5}-2$

$\Rightarrow \tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \sqrt{5}-2$

Q. 15. If $|x| \le 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $4\tan^{-1}x$ **(b)** 0

(c) $\frac{\pi}{2}$ **(d)** π

[NCERT Exemp. Ex. 2.3, Q. 34, Page 39]

Ans. Correct option : (a)

Explanation : We have,

$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Let $x = \tan\theta$

$\therefore 2\tan^{-1}\tan\theta + \sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}$

$= 2\theta + \sin^{-1}\sin 2\theta \qquad \left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$

$= 2\theta + 2\theta$

$= 4\theta$

$= 4\tan^{-1}x \qquad [\because \theta = \tan^{-1}x]$

Q. 16. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$ equals

(a) 0 **(b)** 1

(c) 6 **(d)** 12

[NCERT Exemp. Ex. 2.3, Q. 35, Page 39]

Ans. Correct option : (c)

Explanation : We have,

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$,

We know that, $0 \le \cos^{-1}x \le \pi$

$\Rightarrow \cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

If and only if, $\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$

$\Rightarrow \cos\pi = \alpha = \beta = \gamma$

$\Rightarrow -1 = \alpha = \beta = \gamma$

$\Rightarrow \alpha = \beta = \gamma = -1$

$\therefore \alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$

$= -1(-1-1)-1(1-1)-1(-1-1)$

$= 2+2+2 = 6$

Q. 17. The number of real solutions of the equation : $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is

(a) 0 **(b)** 1

(c) 2 **(d)** ∞

[NCERT Exemp. Ex. 2.3, Q. 36, Page 39]

Ans. Correct option : (a)

Explanation : We have,

$\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x), \left[\frac{\pi}{2}, \pi\right]$

$\Rightarrow \sqrt{1+2\cos^2 x - 1} = \sqrt{2}\cos^{-1}(\cos x)$

$\Rightarrow \sqrt{2}\cos x = \sqrt{2}\cos^{-1}(\cos x)$

$\Rightarrow \cos x = x \qquad [\because \cos^{-1}(\cos x) = x]$

Which is not true for any real value of x.

Hence, there is no solution possible for the given equation.

Q. 18. If $\cos^{-1}x > \sin^{-1}x$, then

(a) $\frac{1}{\sqrt{2}} < x \le 1$ **(b)** $0 \le x < \frac{1}{\sqrt{2}}$

(c) $-1 \le x < \frac{1}{\sqrt{2}}$ **(d)** $x > 0$

[NCERT Exemp. Ex. 2.3, Q. 37, Page 39]

Ans. Correct option : (c)

Explanation : Given that, $\cos^{-1}x > \sin^{-1}x$ where $x \in [-1, 1]$

Now, we have

$\cos^{-1}x \quad \sin^{-1}x$

$\Rightarrow \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x$

$\Rightarrow \frac{\pi}{2} > 2\sin^{-1}x$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4}$ ...(i)

But $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ ...(ii)

From (i) and (ii) $\sin^{-1}x < \frac{\pi}{4}$

$\Rightarrow \sin\left(-\frac{\pi}{2}\right) \le x < \sin\frac{\pi}{4}$

$\Rightarrow -1 \le x < \frac{1}{\sqrt{2}}$

Q. 19. Solve $\sin(\tan^{-1}x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ **(b)** $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ **(d)** $\frac{x}{\sqrt{1+x^2}}$

[NCERT Misc. Ex. Q. 15, Page 52]

Ans. Correct option : (d)

Explanation : Let us, $\tan^{-1}x = y$.

$\Rightarrow \tan y = x$

$\sin y = \frac{x}{\sqrt{1+x^2}}$

$$\therefore y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Therefore, we have

$$\sin\left(\tan^{-1} x\right) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

$$= \frac{x}{\sqrt{1+x^2}}$$

Q. 20. Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$

(c) 0 (d) $\frac{1}{2}$

[NCERT Misc. Ex. Q. 16, Page 52]

Ans. Correct option : (c)

Explanation : Given that,

$$\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$$

Let, $x = \sin y$, then $y = \sin^{-1}x$

$$\therefore \sin^{-1}(1-\sin y) - 2y = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}(1-\sin y) = \frac{\pi}{2} + 2y$$

$$\Rightarrow \quad 1 - \sin y = \sin\left(\frac{\pi}{2} + 2y\right)$$

$$\Rightarrow \quad 1 - \sin y = \cos 2y$$

$$\Rightarrow \quad 1 - \sin y = 1 - 2\sin^2 y \quad [\because \cos 2y = 1 - 2\sin^2 y]$$

$$\Rightarrow \quad 2\sin^2 y - \sin y = 0$$

$$\Rightarrow \quad 2x^2 - x = 0 \quad [\because x = \sin y]$$

$$\Rightarrow \quad x(2x-1) = 0$$

$$\Rightarrow \quad x = 0 \text{ or, } x = \frac{1}{2}$$

But $x = \frac{1}{2}$ does not satisfy the given equation.

$\therefore x = 0$ is the solution of the given equation.

Q. 21. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(a) π (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

[NCERT Ex. 2.1, Q. 14, Page 42]

Ans. Correct option : (b)

Explanation:

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\frac{\pi}{3}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

$$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Let $\sec^{-1}(-2) = y$. Then,

$$\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}.$$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.

$$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$$

Hence,

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3}$$

$$= -\frac{\pi}{3}$$

Q. 22. Solve that : $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{-3\pi}{4}$

[NCERT Misc. Ex. Q. 17, Page 52]

Ans. Correct option : (c)

Explanation :

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$$

$$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right]$$

$$\left[\therefore \tan^{-1} a - \tan^{-1} b = \tan^{-1}\frac{a-b}{1+ab}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}}\right]$$

$$= \tan^{-1}\left(\frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy}\right)$$

$$= \tan^{-1}\left(\frac{x^2 + y^2}{x^2 + y^2}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

Q. 23. If $\sin^{-1}x = y$, then

(a) $0 \le y \le \pi$ (b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

[NCERT Ex. 2.1, Q. 13, Page 42]

Ans. Correct option : (b)

Explanation :

Given, $\sin^{-1}x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Hence, $\left[-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right]$.

Q. 24. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ **is equal to**

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

[NCERT Ex. 2.2, Q. 19, Page 48]

Ans. Correct option : (b)

Explanation :

$$\text{Given, } \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$
$$= \cos^{-1}\left[\cos\left(2\pi - \frac{5\pi}{6}\right)\right]$$

Since, $\frac{5\pi}{6} \in [0,\pi]$

Thus, $\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

Q. 25. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ **is equal to**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1

[NCERT Ex. 2.2, Q. 20, Page 48]

Ans. Correct option : (d)

Explanation :

$$\text{Given, } \sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$
$$= \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$$
$$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$$
$$= \sin\left[\frac{3\pi}{6}\right]$$
$$= \sin\left[\frac{\pi}{2}\right]$$
$$= 1$$

Q. 26. $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ **is equal to**

(a) π (b) $-\frac{\pi}{2}$

(c) 0 (d) $2\sqrt{3}$

[NCERT Ex. 2.2, Q. 21, Page 48; CBSE Board, Delhi Region, 2018]

Ans. Correct option : (b)

Explanation :

$$\text{Here, } \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$
$$= \tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3}) \quad \because [\cot^{-1}(-x) = \pi - \cot^{-1}x]$$
$$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$$
$$= \tan^{-1}\sqrt{3} + \tan^{-1}\frac{1}{\sqrt{3}} - \pi$$
$$= \left[\tan^{-1}\frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{1-\sqrt{3}\times\frac{1}{\sqrt{3}}}\right] - \pi$$
$$= \left[\tan^{-1}\frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{0}\right] - \pi$$
$$= \tan^{-1}\infty - \pi$$
$$= \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$
$$= -\frac{\pi}{2}$$

Very Short Answer Type Questions (1 or 2 marks each)

Q. 1. The principle value of $\cos^{-1}\left(-\frac{1}{2}\right)$ **is__________________.**

[NCERT Exemp. Ex. 2.3, Q. 38, Page 40]

Ans. Given that,

$$\cos^{-1}\left(-\frac{1}{2}\right)$$
$$= \cos^{-1}\left(-\cos\frac{\pi}{3}\right)$$
$$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right] \quad [\because \cos(\pi - \theta) = \cos\theta]$$
$$= \cos^{-1}\left(\cos\frac{2\pi}{3}\right)$$
$$= \frac{2\pi}{3} \quad [\because \cos^{-1}(\cos x) = x \text{ if } x \in [0,\pi]]$$

Hence, the principle value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$. **[2]**

Q. 2. The value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ **is ________________.**

[NCERT Exemp. Ex. 2.3, Q. 39, Page 40]

Ans. We know that $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$\therefore \sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

$= \sin^{-1}\sin\left(\pi - \frac{3\pi}{5}\right)$

$= \sin^{-1}\left(\sin\frac{2\pi}{5}\right)$

$= \frac{2\pi}{5}$ **[2]**

Q. 3. If $\cos\left(\tan^{-1}x + \cot^{-1}\sqrt{3}\right) = 0$, then the value of x is ______________.

[NCERT Exemp. Ex. 2.3, Q. 40, Page 40]

Ans. We have, $\cos\left(\tan^{-1}x + \cot^{-1}\sqrt{3}\right) = 0$

$\Rightarrow \tan^{-1}x + \cot^{-1}\sqrt{3} = \cos^{-1}0$

$\Rightarrow \tan^{-1}x + \cot^{-1}\sqrt{3} = \cos^{-1}\cos\frac{\pi}{2}$

$\Rightarrow \tan^{-1}x + \cot^{-1}\sqrt{3} = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{2} - \cot^{-1}\sqrt{3}$

$\Rightarrow \tan^{-1}x = \tan^{-1}\sqrt{3}$

$\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \therefore x = \sqrt{3}$ **[2]**

Q. 4. The set of values of $\sec^{-1}\frac{1}{2}$ is ______________.

[NCERT Exemp. Ex. 2.3, Q. 41, Page 40]

Ans. Since, domain of $\sec^{-1}\frac{1}{2}$ is $R-(-1,1)$

$\Rightarrow (-\infty,-1) \cup [1,\infty)$

So, there is no set of values that exist for $\sec^{-1}\frac{1}{2}$.

So, ϕ is the answer. **[2]**

Q. 5. The principle value of $\tan^{-1}\sqrt{3}$ is ____________.

[NCERT Exemp. Ex. 2.3, Q. 42, Page 40]

Ans. Given that,

$\because \tan^{-1}\sqrt{3} = \tan^{-1}\tan\left(\frac{\pi}{3}\right)$

$\left[\because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$= \left(\frac{\pi}{3}\right)$ **[2]**

Q. 6. The value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$ is______________.

[NCERT Exemp. Ex. 2.3, Q. 43, Page 40]

Ans. We have $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$

$= \cos^{-1}\cos\left(4\pi + \frac{2\pi}{3}\right)$

cos cos— $\quad [\because \cos(2n\pi + \theta) = \cos\theta]$

$= \frac{2\pi}{3} \quad \{\because \cos^{-1}(\cos x) = x, x \in [0,\pi]\}$ **[1]**

Note to remember that,

$\cos^{-1}\left(\cos\frac{14\pi}{3}\right) \ne \frac{14\pi}{3}$

Since, $\frac{14\pi}{3} \notin [0,\pi]$ **[1]**

Q. 7. The value of $\cos\left(\sin^{-1}x + \cos^{-1}x\right)$, where $|x| \le 1$ is _________. [NCERT Exemp. Ex. 2.3, Q. 44, Page 40]

Ans. $\cos\left(\sin^{-1}x + \cos^{-1}x\right)$

$\Rightarrow \cos\frac{\pi}{2} = 0 \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$ **[2]**

Q. 8. The value of expression $\tan\left(\frac{\sin^{-1}x + \cos^{-1}x}{2}\right)$, when $x = \frac{\sqrt{3}}{2}$, is ____________.

[NCERT Exemp. Ex. 2.3, Q. 45, Page 40]

Ans.

$\tan\left(\frac{\sin^{-1}x + \cos^{-1}x}{2}\right)$

$= \tan\left(\frac{\frac{\pi}{2}}{2}\right) \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \tan\frac{\pi}{4} = 1$ **[2]**

Q. 9. If $y = 2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$, for all x then _________ $<y<$ ________.

[NCERT Exemp. Ex. 2.3, Q. 46, Page 40]

Ans. Given that,

$y = 2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$

$\Rightarrow y = 2\tan^{-1}x + 2\tan^{-1}x$

$\left[\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$

$\Rightarrow y = 4\tan^{-1}x$ **[1]**

Now, according to principle value of $\tan^{-1}x$, we know that,

$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

$\Rightarrow -\frac{4\pi}{2} < 4\tan^{-1}x < \frac{4\pi}{2}$

$\Rightarrow -2\pi < 4\tan^{-1}x < 2\pi$

$\Rightarrow -2\pi < y < 2\pi \quad [\because y = 4\tan^{-1}x]$ **[1]**

Q. 10. The result $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ is true when value of xy is ______________.

[NCERT Exemp. Ex. 2.3, Q. 47, Page 40]

Ans. We know that given equation $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ is true because xy is greater than -1, *i.e.*, $xy > -1$. **[2]**

Q. 11. The value of $\cot^{-1}(-x)$ for all $x \in R$ in terms of $\cot^{-1}x$ is __________.

[NCERT Exemp. Ex. 2.3, Q. 48, Page 40]

Ans. We know that $=\pi - \cot^{-1}x, x\in R$.
So, the value of $\cot^{-1}(-x)$ is $\pi - \cot^{-1}x$. **[2]**

Q. 12. State True or False for the following statement: All trigonometric functions have inverse over their respective domains.
[NCERT Exemp. Ex. 2.3, Q. 49, Page 40]

Ans. False, we know that all trigonometric functions have inverse over their respective domains. **[2]**

Q. 13. The value of the expression $(\cos^{-1}x)^2$ is equal to $\sec^2 x$. **[NCERT Exemp. Ex. 2.3, Q. 50, Page 40]**

Ans. False, $\because (\cos^{-1}x)^2 = \left(\sec^{-1}\frac{1}{x}\right)^2 \neq \sec^2 x$ **[2]**

Q. 14. State True or False for the following statement: The domain of trigonometric functions can be restricted to any one of their branch (not necessarily principal value) in order to obtain their inverse functions.
[NCERT Exemp. Ex. 2.3, Q. 51, Page 40]

Ans. True, we know that the domain of trigonometric functions is restricted in their domain to obtain their inverse functions. **[2]**

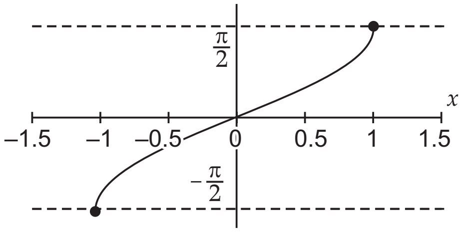
Q. 15. State True or False for the following statement: The least numerical value, either positive or negative of angle θ is called principal value of the inverse trigonometric function.
[NCERT Exemp. Ex. 2.3, Q. 52, Page 40]

Ans. True, we know that, the smallest numerical value, either positive or negative of θ is called the principal value of the function. **[2]**

Q. 16. State True or False for the following statement: The graph of inverse trigonometric function can be obtained from the graph of their corresponding trigonometric function by interchanging *x*- and y-axes. **[NCERT Exemp. Ex. 2.3, Q. 53, Page 40]**

Ans. True, we know that, the graph of an inverse function can be obtained from the corresponding graph of original function as a mirror image (*i.e.*, reflection) along the line $y = x$. **[1]**

Arc sin x



[1]

Q. 17. State True or False for the following statement: The minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$, $n\in N$, is valid for 5.
[NCERT Exemp. Ex. 2.3, Q. 54, Page 41]

Ans. False,
Given that $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$

$\Rightarrow \frac{n}{\pi} > \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \frac{n}{\pi} > 1$

$\Rightarrow n > \pi$

$\Rightarrow n = 4$

$[\because n\in N \text{ and } \pi = 3.14]$ **[2]**

Q. 18. State True or False for the following statement: The principal value of $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{1}{2}\right)\right]$ is $\frac{\pi}{3}$.
[NCERT Exemp. Ex. 2.3, Q. 55, Page 41]

Ans. True,
Given that,

$\sin^{-1}\left[\cos\left(\sin^{-1}\frac{1}{2}\right)\right]$

$\Rightarrow \sin^{-1}\left[\cos\left(\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$= \sin^{-1}\left[\cos\frac{\pi}{6}\right] \quad \left[\because \sin^{-1}(\sin x) = x \text{ if } x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3} \quad \left[\because \sin^{-1}(\sin x) = x \text{ if } x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$ **[2]**

Q. 19. Find the principal values of $\sin^{-1}\left(-\frac{1}{2}\right)$.
[NCERT Ex. 2.1, Q. 1, Page 41]

Ans. Let $y = \sin^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \sin y = \frac{-1}{2}$

$\Rightarrow \sin y = \sin\left(\frac{-\pi}{6}\right)$

Range of principle value of $\sin^{-1}$ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Hence, the principal value $\sin^{-1}\left(-\frac{1}{2}\right)$ is $\frac{-\pi}{6}$. **[2]**

Q. 20. Find the principal values of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
[NCERT Ex. 2.1, Q. 2, Page 41]

Ans. Let, $y = \cos^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \cos\frac{\pi}{6}$

We know that principal values of $\cos^{-1}$ is $[0,\pi]$.

Hence, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$. **[2]**

Q. 21. Find the principal values of $\text{cosec}^{-1}(2)$.
[NCERT Ex. 2.1, Q. 3, Page 41]

Ans. Let, $y = \text{cosec}^{-1}\, 2$

$\Rightarrow \text{cosec}\, y = 2$

$\Rightarrow \text{cosec}\, y = \text{cosec}\left(\frac{\pi}{6}\right)$

Range of principal value of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right] - \{0\}$.

So the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$. **[2]**

Q. 22. Find the principal value of $\tan^{-1}(-\sqrt{3})$.

[NCERT Ex. 2.1, Q. 4, Page 41]

Ans. Let $y = \tan^{-1}(-\sqrt{3})$

$\Rightarrow \tan y = -\sqrt{3}$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{3}\right)$

Range of the principal value of $\tan^{-1}$ is between $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Therefore, the principal value $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$. **[2]**

Q. 23. Find the principal values of $\cos^{-1}\left(-\frac{1}{2}\right)$.

[NCERT Ex. 2.1, Q. 5, Page 41]

Ans. Let, $= \cos\left(--\right)$

$\Rightarrow \cos y = -\frac{1}{2}$

$\Rightarrow \cos y = \cos\left(\frac{2\pi}{3}\right)$

Range of principal value of $\cos^{-1}$ is $[0, \pi]$.

Hence, the principal $\cos^{-1}\left(\frac{-1}{2}\right)$ value is $\frac{2\pi}{3}$. **[2]**

Q. 24. Find the principal values of $\tan^{-1}(-1)$.

[NCERT Ex. 2.1, Q. 6, Page 41]

Ans. Let, $y = \tan^{-1}(-1)$

$\Rightarrow \tan y = -1$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{4}\right)$

Range of principal value of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Therefore, the principal value of $\tan^{-1}(-1)$ is $\left(-\frac{\pi}{4}\right)$. **[2]**

Q. 25. Find the principal values of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

[NCERT Ex. 2.1, Q. 7, Page 42]

Ans. Let, $y = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \sec y = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec y = \sec\left(\frac{\pi}{6}\right)$

Range of principal value of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

Therefore, $\frac{\pi}{6}$ is the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$. **[2]**

Q. 26. Find the principal values of $\cot^{-1}(\sqrt{3})$.

[NCERT Ex. 2.1, Q. 8, Page 42]

Ans. Let, $y = \cot^{-1}(\sqrt{3})$

$\Rightarrow \cot y = \sqrt{3}$

$\Rightarrow \cot y = \cot\frac{\pi}{6}$

Range of principal value of $\cot^{-1}$ is $[0, \pi]$.

Hence, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$. **[2]**

Q. 27. Find the principal values of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

[NCERT Ex. 2.1, Q. 9, Page 42]

Ans. Let, $y = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \cos y = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos y = \cos\left(\frac{3\pi}{4}\right)$

Range of principal value of $\cos^{-1}$ is between $[0, \pi]$.

Hence, the principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$. **[2]**

Q. 28. Find the principal values of $\text{cosec}^{-1}(-\sqrt{2})$.

[NCERT Ex. 2.1, Q. 10, Page 42]

Ans. Let, $y = \text{cosec}^{-1}(-\sqrt{2})$

$\Rightarrow \text{cosec } y = -\sqrt{2}$

$\Rightarrow \text{cosec } y = \text{cosec}\left(\frac{-\pi}{4}\right)$

Range of principal value of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

The principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$. **[2]**

Q. 29. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

[NCERT Ex. 2.1, Q. 12, Page 42]

Ans. Let, $\cos^{-1}\left(\frac{1}{2}\right) = x$.

Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos\left(\frac{}{2}\right) = \frac{}{3}$

Let, $\sin^{-1}\left(\frac{1}{2}\right) = y$.

Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6}$$

$$= \frac{\pi}{3} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

[2]

Q. 30. Prove that $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

[NCERT Ex. 2.2, Q. 1, Page 47; CBSE Board, Delhi Region, 2018]

Ans. In this question, we are using concept of $\sin 3\theta$.

To prove : $3\sin^{-1}x = \sin^{-1}(3x-4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Let, $x = \sin\theta$.

Then, $\sin^{-1}x = \theta$.

We have,

$$\text{RHS} = \sin^{-1}(3x-4x^3)$$
$$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$$
$$= \sin^{-1}(\sin 3\theta) \quad [\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta]$$
$$= 3\theta$$
$$= 3\sin^{-1}x$$
$$= \text{LHS}$$

$\therefore$ LHS = RHS. Hence proved. **[2]**

Q. 31. Prove that $3\cos^{-1}x = \cos^{-1}(4x^3-3x),\ x \in \left[\frac{1}{2}, 1\right]$.

[NCERT Ex. 2.2, Q. 2, Page 47]

Ans. Let, $x = \cos\theta$.

Then, $\cos^{-1}x = \theta$.

We have,

$$\text{RHS} = \cos^{-1}(4x^3-3x)$$
$$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$$
$$= \cos^{-1}(\cos 3\theta)$$
$$= 3\theta$$
$$= 3\cos^{-1}x$$

$\therefore$ LHS = RHS. Hence proved. **[2]**

Q. 32. Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.

[NCERT Ex. 2.2, Q. 3, Page 47]

Ans. **To prove :** $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

We have,

$$\text{LHS} = \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$$
$$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right) \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

[1]

$$= \tan^{-1}\left(\frac{\frac{48+77}{11\times 24}}{\frac{11\times 24-14}{11\times 24}}\right)$$
$$= \tan^{-1}\left(\frac{48+77}{264-14}\right)$$
$$= \tan^{-1}\left(\frac{125}{250}\right)$$
$$= \tan^{-1}\left(\frac{1}{2}\right)$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS. Hence proved. **[1]**

Q. 33. Write the function in the simplest form : $\tan \frac{\sqrt{1+\quad} \quad 1}{\quad}, \quad \neq 0$.

[NCERT Ex. 2.2, Q. 5, Page 47]

Ans. Given function $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$\because 1+\tan^2\theta = \sec^2\theta$, where x tends to $\tan\theta$.

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$
$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$
$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$
$$= \frac{\theta}{2}$$
$$= \frac{1}{2}\tan^{-1}x$$

[2]

Q. 34. Write the function in the simplest form : $\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$ [NCERT Ex. 2.2, Q. 6, Page 47]

Ans. Given function

$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$

Put $x = \text{cosec}\,\theta$

$\Rightarrow \theta = \text{cosec}^{-1}x$

$$\therefore \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\text{cosec}^2\theta - 1}}$$

$$= \tan^{-1}\left(\frac{1}{\cot\theta}\right) \quad [\because \text{cosec}^2\theta - \cot^2\theta = 1]$$

$$= \tan^{-1}(\tan\theta)$$

Using this formula : $\left[\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}\right]$

$$= \theta = \text{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$$

[2]

Q. 35. Write the function in simplest form : $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$.

[NCERT Ex. 2.2, Q. 7, Page 47]

Ans. Given that,

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ 0 < x < \pi$$

$$= \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2}$$ **[2]**

Q. 36. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

[NCERT Ex. 2.2, Q. 11, Page 48]

Ans. $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$$

$$= \tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$ **[2]**

Q. 37. Find the value of $\cot\left(\tan^{-1}a + \cot^{-1}a\right)$.

[NCERT Ex. 2.2, Q. 12, Page 48]

Ans. Given that,

$\cot\left(\tan^{-1}a + \cot^{-1}a\right)$

$$= \cot\left(\frac{\pi}{2}\right) \qquad \left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$$

$$= 0$$ **[2]**

Q. 38. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.

[NCERT Ex. 2.2, Q. 14, Page 48]

Ans. $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$$\therefore \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1$$

$$\sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1}x$$

$$\sin^{-1}\frac{1}{5} = \sin^{-1}x \quad \because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}.$$

We use the identity : $\sin^{-1}1 = \frac{\pi}{2}$ and

$$\therefore x = \frac{1}{5}$$ **[2]**

Q. 39. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

[NCERT Misc. Ex. Q. 1, Page 51]

Ans. We know that $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$, which is the principle value of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$$

where,

$$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$$

$$= \frac{\pi}{6} \qquad \frac{\pi}{6} \in [0, \pi]$$ **[2]**

Q. 40. Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

[NCERT Misc. Ex. Q. 2, Page 51]

Ans. We know that $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\therefore \tan^{-1}\tan\left(\frac{7\pi}{6}\right) = \tan^{-1}\tan\left(\pi + \frac{\pi}{6}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$$

$$\because \tan(\pi + x) = \tan x$$

$$= \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$ **[2]**

Q. 41. Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.

[NCERT Misc. Ex. Q. 5, Page 51]

Ans. Let, $\cos^{-1}\frac{4}{5} = A$ and $\cos^{-1}\frac{12}{13} = B$

So, $\cos A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$

Hence, $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$

We know that,

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= \frac{4}{5}\times\frac{12}{13} - \frac{3}{5}\times\frac{5}{13}$$

$$= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$$

$$\Rightarrow \cos(A+B) = \frac{33}{65}$$

$$A + B = \cos^{-1}\frac{33}{65}$$

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{13}{13} = \cos^{-1}\frac{33}{65}$$

Hence proved. **[2]**

Q. 42. Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$.

[NCERT Misc. Ex. Q. 6, Page 51]

Ans. Given that,

$$\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$$

Let $\sin^{-1}\frac{3}{5}=A$ and $\cos^{-1}\frac{12}{13}=B$

So, $\sin A=\frac{3}{5}$ and $\cos B=\frac{12}{13}$

Hence, $\cos A=\frac{4}{5}$ and $\sin B=\frac{5}{13}$

We know that,

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$=\frac{3}{5}\times\frac{12}{13}+\frac{4}{5}\times\frac{5}{13}$$

$$=\frac{36}{65}+\frac{20}{65}$$

$$\Rightarrow \sin(A+B)=\frac{56}{65}$$

$$\Rightarrow A+B=\sin^{-1}\frac{56}{65}$$

Thus, $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Hence proved. **[2]**

Q. 43. Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$.

[NCERT Misc. Ex. Q. 9, Page 52]

Ans. Let $x=\tan^2\theta$.

Then, $\sqrt{x}=\tan\theta \Rightarrow \theta=\tan^{-1}\sqrt{x}$.

$$\therefore \frac{1-x}{1+x}=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$$

$$\left[\because \cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

$$\text{RHS}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\cos^{-1}(\cos 2\theta)$$

$$=\frac{1}{2}\times 2\theta$$

$$=\theta$$

$$=\tan^{-1}\sqrt{x}$$

$$=\text{LHS}$$

$\therefore$ LHS = RHS Hence proved. **[2]**

Q. 44. Solve that $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$.

[NCERT Misc. Ex. Q. 13, Page 52]

[NCERT Exemp. Ex. 2.3, Q. 9, Page 36]

[CBSE Board, Delhi Region, 2016]

Ans. Given : $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\operatorname{cosec} x)$$

$$\left[\because 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\Rightarrow \frac{2\cos x}{1-\cos^2 x}=2\operatorname{cosec} x$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x}=\frac{2}{\sin x}$$

$$\Rightarrow \tan x=1$$

$$\therefore x=\frac{\pi}{4}$$ **[2]**

Q. 45. Solve that $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x, (x>0)$.

[NCERT Misc. Ex. Q. 14, Page 52]

Ans. Given : $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x\ (x>0)$

$$\Rightarrow \tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x$$

$$\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$\Rightarrow \frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow x=\tan\frac{\pi}{6}$$

$$\therefore x=\frac{1}{\sqrt{3}}$$ **[2]**

Q. 46. Find the values of the expression $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

[NCERT Ex. 2.2, Q. 16, Page 48]

Ans.

Given, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$$

$$=\sin^{-1}\left(\sin\frac{\pi}{3}\right) \quad [\because \sin(\pi-\theta)=\sin\theta]$$

$$=\frac{\pi}{3}$$

Since, $\frac{\pi}{3}\in\left[-\frac{\pi}{3},\frac{\pi}{2}\right]$

$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\frac{\pi}{3}$$ [2]

Q. 47. Find the values of each of the expression $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ **[NCERT Ex. 2.2, Q. 17, Page 48]**

Ans.

Given, $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$=\tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right]$$

$$\Rightarrow -\frac{\pi}{4}$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right)=-\frac{\pi}{4}$$ **[2]**

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Find the value of:

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

[NCERT Ex. 2.1, Q. 11, Page 42]

Ans. Let, $\tan^{-1}(1) = x$.

$\tan x = 1$

Then, $= \tan\frac{\pi}{4}\cdot$

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let, $\cos^{-1}\left(-\frac{1}{2}\right) = y$.

Then, $\cos y = -\frac{1}{2}$

$= -\cos\left(\frac{\pi}{3}\right)$

$= \cos\left(\pi - \frac{\pi}{3}\right)$ $[\because \cos(\pi-\theta) = -\cos\theta]$

$= \cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let, $\sin^{-1}\left(-\frac{1}{2}\right) = z$. Then,

$\sin z = -\frac{1}{2}$

$= -\sin\left(\frac{\pi}{6}\right)$

$= \sin\left(-\frac{\pi}{6}\right)$.

$\therefore \quad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now, we have,

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$

$$= \frac{3\pi+8\pi-2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

[3]

Q. 2. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

[NCERT Ex. 2.2, Q. 4, Page 47]

Ans. To prove : $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$\text{LHS} = 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\frac{1}{7}\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\frac{1}{\left(\frac{3}{4}\right)}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\left(\frac{4}{3}+\frac{1}{7}\right)}{\left(1-\frac{4}{3}\cdot\frac{1}{7}\right)}\left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{\left(\frac{28+3}{21}\right)}{\left(\frac{21-4}{21}\right)}$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

$= \text{RHS}$

$\therefore$ LMS = RMS Hence proved. [3]

Q. 3. Write the function in the simplest form:

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right),\ x<\pi$$

[NCERT Ex. 2.2, Q. 8, Page 47]

Ans. Given that,

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$$

$$= \tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \tan^{-1}(1)-\tan^{-1}(\tan x)\left[\because \tan^{-1}\frac{x-y}{1+xy} = \tan^{-1}x-\tan^{-1}y\right]$$

$$= \frac{\pi}{4}-x$$

Note that : $\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x-\tan^{-1}y$ **[3]**

Q. 4. Write the function in the simplest form:

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}},\ |x|<a$$

[NCERT Ex. 2.2, Q. 9, Page 48]

Ans. Given functions, $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

Put $x = a\sin\theta$

$\Rightarrow \frac{x}{a} = \sin\theta$

$\Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$

$\therefore \tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

$$= \sin^{-1}\frac{x}{a}$$ **[3]**

Q. 5. Write the function in the simplest form :

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right),\ a > 0;\ \frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}.$$

[NCERT Ex. 2.2, Q. 10, Page 48]

Ans. Hence Proved.

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$$

Put $x = a\tan\theta$

$$\Rightarrow \frac{x}{a} = \tan\theta$$

$$\Rightarrow \theta = \tan^{-1}\frac{x}{a}$$

$$\therefore \tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = \tan^{-1}\left(\frac{3a^2\cdot a\tan\theta - a^3\tan^3\theta}{a^3 - 3a\cdot a^2\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$\left[\because \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right]$$ **[3]**

$$= 3\theta$$

$$= 3\tan^{-1}\frac{x}{a}$$

Q. 6. Find the value of

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right],\ |x| < 1,\ y > 0 \text{ and } xy < 1.$$

[NCERT Ex. 2.2, Q. 13, Page 48]

Ans.

Let, $x = \tan\theta$.

Then, $\theta = \tan^{-1}x$.

$$\therefore \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)\left[\because \sin 2A = \left(\frac{2\tan A}{1+\tan^2\theta}\right)\right]$$

$$= 2\theta$$

$$= 2\tan^{-1}x$$

Let, $y = \tan\phi$.

Then, $\phi = \tan^{-1}y$.

$$\therefore \cos^{-1}\frac{1-y^2}{1+y^2} = \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)$$

$$= \cos^{-1}(\cos 2\phi)\left[\because \cos 2A = \left(\frac{1-\tan^2 A}{1+\tan^2 A}\right)\right]$$

$$= 2\phi$$

$$= 2\tan^{-1}y$$

$$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$= \tan\frac{1}{2}[2\tan^{-1}x + 2\tan^{-1}y]$$

$$= \tan[\tan^{-1}x + \tan^{-1}y]$$

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\because \tan^{-1}A + \tan^{-1}B = \tan^{-1} = \frac{A+B}{1-AB}$$

$$= \frac{x+y}{1-xy}$$

Q. 7. Prove that $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$ **[3]**

[NCERT Misc. Ex. Q. 3, Page 51]

Ans. Let $\sin^{-1}\frac{3}{5} = x$.

Then, $\sin x = \frac{3}{5}$.

$$\cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \frac{4}{5}$$

$$\therefore \tan x = \frac{3}{4}$$

$$\therefore x = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$$ **[1]**

Now, we have :

$$\text{LHS} = 2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right)$$

$$= \tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right)\quad\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\frac{24}{7}$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS. Hence proved. **[2]**

Q. 8. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

[NCERT Misc. Ex. Q. 4, Page 51]

Ans.

$$\text{LHS}=\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$$

$$=\tan^{-1}\frac{8}{\sqrt{17^2-8^2}}+\tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$$

$$\left[\because \sin^{-1}\frac{a}{b}=\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$$

$$=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left[\frac{\left(\frac{8}{15}+\frac{3}{4}\right)}{\left(1-\frac{8}{15}\times\frac{3}{4}\right)}\right]$$

$$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\tan^{-1}\left[\frac{\left(\frac{32+45}{15\times 4}\right)}{\left(\frac{15\times 4-8\times 3}{15\times 4}\right)}\right]$$

$$=\tan^{-1}\left[\frac{\left(\frac{77}{60}\right)}{\left(\frac{36}{60}\right)}\right]$$

$$=\tan^{-1}\left(\frac{77}{36}\right)$$

$= \text{RHS}$

$\therefore$ LHS = RHS. Hence proved, [3]

Q. 9. Prove that

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\quad x\in\left(0,\frac{\pi}{4}\right)$$

[NCERT Misc. Ex. Q. 10, Page 52]

Ans. Consider,

$$\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$$

$$=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}\quad\text{(By rationalising)}$$

$$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$$

$$=\frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x}$$

$$=\frac{1+\cos x}{\sin x}$$

$$=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\qquad\left[\frac{\because 1+\cos A=2\cos^2\frac{A}{2}}{\&\ 2\sin A=2\sin\frac{A}{2}\cos\frac{A}{2}}\right]$$

$$=\cot\frac{x}{2}$$

$$\text{LHS}=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$=\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

$= \text{RHS}$

$\therefore$ LHS = RHS. Hence proved, [3]

Q. 10. Prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\quad -\frac{1}{\sqrt{2}}\le x\le 1$$

[Hint : put $x = \cos 2\theta$]

[NCERT Misc. Ex. Q. 11, Page 52]

Ans.

Put $x=\cos 2\theta$ so that $\theta=\frac{1}{2}\cos^{-1}x$. Then, we have :

$$\text{LHS}=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$\left[\frac{\because 1+\cos 2A=2\cos^2 A}{\&\ 1-\cos 2A=2\sin^2 A}\right]$$

$$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$=\tan^{-1}1-\tan^{-1}\tan\theta$$

$$\left[\because \tan^{-1}\left(\frac{x-}{1+x}\right)=\tan^{-1}x-\tan^{-1}\right]$$

$$=\frac{\pi}{4}-\theta$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

$= \text{RHS}$

$\therefore$ LHS = RHS. Hence proved. [3]

Q. 11. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.

[NCERT Misc. Ex. Q. 12, Page 52]

Ans.

$$\text{LHS}=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$=\frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)$$

$$=\frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)\qquad\left[\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right]\quad\text{...(i)}$$

Now, let $\cos^{-1}\frac{1}{3}=x$.

Then, $\cos x=\frac{1}{3}$

$$\sin x=\sqrt{1-\left(\frac{1}{3}\right)^2}=\frac{2\sqrt{2}}{3}$$

$$\therefore \quad x=\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$\cos^{-1}\frac{1}{3}=\sin^{-1}\frac{2\sqrt{2}}{3}$$

$\therefore$ LHS $=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}=$ RHS Hence proved.

[3]

Q. 12. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

[NCERT Exemp. Ex. 2.3, Q. 1, Page 35]

Ans. We have,

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

$$=\tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$$=\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]+\cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$$=-\frac{\pi}{6}+\frac{\pi}{6}$$

$$=0$$

[As we know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\cos^{-1}$ is $[0,\pi]$.]

Hence, the value of

$\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)=0.$ **[3]**

Q. 13. Evaluate : $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$.

[NCERT Exemp. Ex. 2.3, Q. 2, Page 35]

Ans. We have,

$$\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$$

$$=\cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right)+\frac{\pi}{6}\right] \quad \left[\because \cos\frac{5\pi}{6}=\frac{-\sqrt{3}}{2}\right]$$

$$=\cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right) \quad \{\because \cos^{-1}\cos x=x,\ x\in[0,\pi]\}$$

$$=\cos\left(\frac{6\pi}{6}\right)$$

$$=\cos(\pi)=-1$$

[3]

Q. 14. Prove that $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=7$.

[NCERT Exemp. Ex. 2.3, Q. 3, Page 35]

Ans.

$$\text{LHS}=\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$$

$$=\cot\left(\frac{\pi}{4}-2\tan^{-1}\frac{1}{3}\right)$$

$$=\cot\left[\frac{\pi}{4}-\tan^{-1}\left\{\frac{\left(2\times\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right\}\right]$$

$$\left[\because\ 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$=\cot\left(\frac{\pi}{4}-\tan^{-1}\frac{3}{4}\right)$$

$$=\cot\left(\frac{\pi}{4}-\cot^{-1}\frac{4}{3}\right)$$

$$=\frac{\cot\frac{\pi}{4}\cot\left(\cot^{-1}\frac{4}{3}\right)+1}{\cot\left(\cot^{-1}\frac{4}{3}\right)-\cot\frac{\pi}{4}}$$

$$\left[\because\ \cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}\right]$$

$$=\frac{\frac{4}{3}+1}{\frac{4}{3}-1}=\frac{\frac{7}{3}}{\frac{1}{3}}=7=\text{RHS}$$

[3]

$\therefore$ LHS = RHS. Hence proved.

Q. 15. Find the value of

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

[NCERT Exemp. Ex. 2.3, Q. 4, Page 35]

Ans. We have,

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$$

$$=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sqrt{3}\right)+\tan^{-1}\left[-\sin\left(\frac{\pi}{2}\right)\right]$$

$$=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sqrt{3}\right)+\tan^{-1}[-1]$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\tan^{-1}\left(\tan\frac{\pi}{3}\right)+\tan^{-1}\left[-\tan\frac{\pi}{4}\right]$$

$$=\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]+\tan^{-1}\left(\tan\frac{\pi}{3}\right)+\tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$$

[As we know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.]

$$=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}$$

$$=\frac{-2\pi+4\pi-3\pi}{12}$$

$$=-\frac{\pi}{12}$$

[3]

Q. 16. Find the value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.

[NCERT Exemp. Ex. 2.3, Q. 5, Page 35]

Ans. We have,

$$\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$
$$= \tan^{-1}\tan\left(\pi - \frac{\pi}{3}\right)$$
$$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right) \quad [\because \tan^{-1}(-x) = -\tan^{-1}x]$$
$$= \tan^{-1}\tan\left(\frac{-\pi}{3}\right) \quad \left[\because \tan^{-1}(\tan x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$
$$= -\frac{\pi}{3}$$

[3]

Q. 17. Show that $2\tan^{-1}(-3) = -\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$.

[NCERT Exemp. Ex. 2.3, Q. 6, Page 35]

Ans. LHS = $2\tan^{-1}(-3)$

$= -2\tan^{-1}(3)$ [*As* $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$$= -\tan^{-1}\left[\frac{2\times 3}{1-(3)^2}\right] \quad \left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$
$$= -\tan^{-1}\left[\frac{6}{-8}\right]$$
$$= -\tan^{-1}\left(-\frac{3}{4}\right)$$
$$= -\cot^{-1}\left(-\frac{4}{3}\right) \quad \left[\because \tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)\right]$$
$$= -\left[\frac{\pi}{2} - \tan^{-1}\left(-\frac{4}{3}\right)\right] \quad \left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$$
$$= -\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$$

= RHS **[3]**

$\therefore$ LHS = RHS. Hence proved.

Q. 18. Find the real solution of the equation : $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$.

[NCERT Exemp. Ex. 2.3, Q. 7, Page 36]

Ans. Given that,

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$
$$\Rightarrow \quad \cot^{-1}\sqrt{x(x+1)} = \sin^{-1}\sqrt{x^2+x+1}$$
$$\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$$
$$\Rightarrow \quad \sin^{-1}\frac{1}{\sqrt{x^2+x+1}} = \sin^{-1}\sqrt{x^2+x+1} \quad [2]$$
$$\left[As\ \cot^{-1}\frac{a}{b} = \sin^{-1}\frac{b}{\sqrt{a^2+b^2}}\right]$$
$$\Rightarrow \quad \frac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$$
$$\Rightarrow \quad x^2 + x + 1 = 1$$
$$\Rightarrow \quad x^2 + x = 0$$
$$\Rightarrow \quad x(x+1) = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x = -1$$

[1]

Q. 19. Find the value of the expression : $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2})$.

[NCERT Exemp. Ex. 2.3, Q. 8, Page 36]

Ans. Given that,

$$\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2})$$
$$= \sin\left[\tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right)\right] + \cos\left(\tan^{-1}2\sqrt{2}\right) \quad [1]$$
$$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right] \quad [½]$$
$$= \sin\left[\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)\right] + \cos\left(\tan^{-1}2\sqrt{2}\right)$$
$$= \sin\left[\tan^{-1}\frac{3}{4}\right] + \cos\left(\tan^{-1}2\sqrt{2}\right)$$
$$= \sin\left[\sin^{-1}\frac{3}{\sqrt{9+16}}\right] + \cos\left(\cos^{-1}\frac{1}{\sqrt{8+1}}\right)$$
$$\left[\because \left(\tan^{-1}\frac{a}{b} = \sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right) \text{ and } \left(\tan^{-1}\frac{a}{b} = \cos^{-1}\frac{b}{\sqrt{a^2+b^2}}\right)\right]$$
$$= \sin\left[\sin^{-1}\frac{3}{5}\right] + \cos\left(\cos^{-1}\frac{1}{3}\right) \quad [1]$$
$$= \frac{3}{5} + \frac{1}{3}$$
$$= \frac{9+5}{15}$$
$$= \frac{14}{15}$$

[½]

Q. 20. Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$.

[NCERT Exemp. Ex. 2.3, Q. 10, Page 36]

Ans. We have,

$$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$$
$$\Rightarrow \cos\left[\cos^{-1}\left(\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}\right)\right] = \sin\left(2.2\tan^{-1}\frac{1}{3}\right)$$
$$\left[\because 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{\frac{48}{49}}{\frac{50}{49}}\right)\right] = \sin\left[2.\left(\tan^{-1}\frac{\frac{2}{3}}{1-\left(\frac{1}{3}\right)^2}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{48\times 49}{50\times 49}\right)\right] = \sin\left[2\tan^{-1}\left(\frac{18}{24}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{24}{25}\right)\right] = \sin\left(2\tan^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{24}{25}\right)\right] = \sin\left(\sin^{-1}\frac{2\times\frac{3}{4}}{1+\frac{9}{16}}\right)$$

$$\left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right]$$

$$\Rightarrow \quad \frac{24}{25} = \sin\left(\sin^{-1}\frac{\frac{3}{2}}{\frac{25}{16}}\right)$$

$$\Rightarrow \quad \frac{24}{25} = \frac{48}{50}$$

$$\therefore \quad \frac{24}{25} = \frac{24}{25}$$

$\therefore$ LHS = RHS. Hence proved. **[3]**

Q. 21. Solve the following equation :

$$\mathbf{\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right).}$$

[NCERT Exemp. Ex. 2.3, Q. 11, Page 36]

Ans. Given that,

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1^2}}\right) = \sin\left(\sin^{-1}\frac{4}{\sqrt{4^2+3^2}}\right)$$

$$\left[\because \tan^{-1}\frac{a}{b} = \cos^{-1}\frac{b}{\sqrt{b^2+a^2}} \text{ and } \cot^{-1}\frac{a}{b} = \sin^{-1}\frac{b}{\sqrt{b^2+a^2}}\right]$$

$$\Rightarrow \frac{1}{\sqrt{x^2+1^2}} = \frac{4}{\sqrt{4^2+3^2}}$$

$$\Rightarrow \frac{1}{\sqrt{x^2+1}} = \frac{4}{5}$$

Squaring both sides, we have

$$\frac{1}{x^2+1} = \frac{16}{25}$$

$\Rightarrow \quad 25 = 16x^2 + 16$

$\Rightarrow \quad 16x^2 = 9$

$$\Rightarrow \quad x^2 = \frac{9}{16}$$

$$\Rightarrow \quad x = \pm\frac{3}{4}$$

Hence, $x = \frac{3}{4}$ or $x = -\frac{3}{4}$ **[3]**

Q. 22. Find the simplified form of $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$, where $x \in \left[\frac{-3\pi}{4}, \frac{\pi}{4}\right]$.

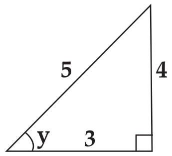
[NCERT Exemp. Ex. 2.3, Q. 13, Page 36]

Ans. Given :

Let, $\cos y = \frac{3}{5}$

$\Rightarrow \sin y = \frac{4}{5}$

$$\therefore y = \cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5} = \tan^{-1}\left(\frac{4}{3}\right)$$ **[2]**



$\therefore \cos^{-1}[\cos y.\cos x + \sin y.\sin x]$

$= \cos^{-1}[\cos(y - x)]$

$[\therefore \cos(A - B) = \cos A.\cos B + \sin A.\sin B]$

$= y - x \qquad \left[\because y = \tan^{-1}\frac{4}{3}\right]$ **[1]**

$= \tan^{-1}\frac{4}{3} - x$

Q. 23. Show that $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$.

[NCERT Exemp. Ex. 2.3, Q. 15, Page 36]
[NCERT Misc. Ex. Q. 7, Page 51]

Ans. LHS $= \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

$$= \tan^{-1}\frac{5}{\sqrt{13^2-5^2}} + \tan^{-1}\frac{\sqrt{5^2-3^2}}{3}$$

$$\left[\because \sin^{-1}\frac{a}{b} = \tan^{-1}\frac{a}{\sqrt{b^2-a^2}} \text{ and } \cos^{-1}\frac{a}{b} = \tan^{-1}\frac{\sqrt{b^2-a^2}}{a}\right]$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$

$$= \tan^{-1}\left[\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right]$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{\left(\frac{15+48}{12\times 3}\right)}{\left(\frac{12\times 3-5\times 4}{12\times 3}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\left(\frac{63}{36}\right)}{\left(\frac{16}{36}\right)}\right]$$

$= \tan^{-1}\left(\frac{63}{16}\right)$

= RHS **[3]**

$\therefore$ LHS = RHS. Hence proved.

Q. 24. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$.

[NCERT Exemp. Ex. 2.3, Q. 16, Page 36]

Ans. $\text{LHS} = \tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}$

$$= \tan^{-1}\left[\frac{\left(\frac{1}{4}+\frac{2}{9}\right)}{1-\left(\frac{1}{4}\times\frac{2}{9}\right)}\right]$$

$$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{\left(\frac{9+8}{4\times 9}\right)}{\left(\frac{4\times 9-1\times 2}{4\times 9}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\left(\frac{17}{36}\right)}{\left(\frac{34}{36}\right)}\right]$$

$$= \tan^{-1}\left(\frac{17}{34}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \sin^{-1}\frac{1}{\sqrt{1^2+2^2}} \qquad \left[\because \tan^{-1}\frac{a}{b}=\sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right]$$

$$= \sin^{-1}\frac{1}{\sqrt{5}}$$

= RHS **[3]**

$\therefore$ LHS = RHS. Hence proved.

Q. 25. If $a_1, a_2, a_3, \ldots\ldots$ an is an arithmetic progression with common difference d, then evaluate the following expression :

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

[NCERT Exemp. Ex. 2.3, Q. 19, Page 37]

Ans. We have, $a_1 = a, a_2 = a+d, a_3 = a+2d\ldots$

And $d = a_2-a_1 = a_3-a_2 = a_4-a_3 = \ldots =$ an – an$_{-1}$

Given that,

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$$= \tan\left[\tan^{-1}\frac{a_2-a_1}{1+a_2\cdot a_1}+\tan^{-1}\frac{a_3-a_2}{1+a_3\cdot a_2}+\ldots+\tan^{-1}\frac{a_n-a_{n-1}}{1+a_n\cdot a_{n-1}}\right]$$

$$= \tan\left[\left(\tan^{-1}a_2-\tan^{-1}a_1\right)+\left(\tan^{-1}a_3-\tan^{-1}a_2\right)+\ldots+\left(\tan^{-1}a_n-\tan^{-1}a_{n-1}\right)\right]$$

$$= \tan\left[\tan^{-1}a_n-\tan^{-1}a_1\right]$$

$$= \tan\left[\tan^{-1}\frac{a_n-a_1}{1+a_n\cdot a_1}\right]$$

$$\left[\therefore \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \frac{a_n-a_1}{1+a_n\cdot a_1} \qquad \left[\because \tan\left(\tan^{-1}x\right)=x\right]$$ **[3]**

Q. 26. Prove that

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$$

[CBSE Board, Delhi Region, 2017]

Ans. Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=x$ **[½]**

$$\text{LHS} = \tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)$$

$$= \frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$$ **[1½]**

$$= \frac{2(1+\tan^2 x)}{1-\tan^2 x}=\frac{2}{\cos 2x}$$ **[1]**

$$= \frac{2b}{a} = \text{RHS}$$ **[1]**

$\therefore$ LHS = RHS. Hence proved.

Q. 27. Find the value of

$$\cot\frac{1}{2}\left[\cos^{-1}\frac{2x}{1+x^2}+\sin^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0$$

and $xy < 1$. [CBSE Board, Foreign Scheme, 2017]

Ans. $\cot\frac{1}{2}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\sin^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$

$$= \cot\frac{1}{2}\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{\pi}{2}-\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$ **[1]**

$$= \cot\frac{1}{2}[\pi-2\tan^{-1}x-2\tan^{-1}y]$$ **[1]**

$$= \cot\left[\frac{\pi}{2}-(\tan^{-1}x+\tan^{-1}y)\right]$$ **[1]**

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]=\frac{x+y}{1-xy}$$ **[1]**

Q. 28. If $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x. [CBSE Board, All India Region, 2017]

Ans. $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\frac{x-3}{x-4}\cdot\frac{x+3}{x+4}}\right)=\frac{\pi}{4}$$ **[1½]**

$$\Rightarrow \frac{2x^2-24}{-7}$$

$$=1\Rightarrow x^2=\frac{17}{2}$$ **[1½]**

$$\Rightarrow x=\pm\sqrt{\frac{17}{2}}$$ **[1]**

Q. 29. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

[CBSE Board, Delhi Region, 2016]
[NCERT Misc, Ex. Q. 8 Page 51]

Ans. $\text{LHS}=\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\cdot\frac{1}{8}}\right)$ **[1]**

$$=\tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)$$ **[1]**

$$=\tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\cdot\frac{11}{23}}\right)$$

$$=\tan^{-1}\left(\frac{325}{325}\right)$$ **[1]**

$$=\tan^{-1}(1)=\frac{\pi}{4}$$ **[1]**

Q. 30. Solve the equation for x:
$\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$.

[CBSE Board, All India Region, 2016]

Ans. $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow \sin^{-1}(1-x)$

$$=\frac{\pi}{2}-2\sin^{-1}x$$ **[1]**

$$\Rightarrow 1-x=\sin\left(\frac{\pi}{2}-2\sin^{-1}x\right)$$

$$\Rightarrow 1-x=\cos(2\sin^{-1}x)$$

$$\Rightarrow 1-x=1-2\sin^2(\sin^{-1}x)$$ **[1]**

$$1-x=1-2x^2$$ **[1]**

Solving we get, $x=0$ or $x=\frac{1}{2}$ **[1]**

Q. 31. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that

$$\frac{x^2}{a^2}-2\frac{xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$

[CBSE Board, All India Region, 2016]

Ans. From the given equation :

$$\Rightarrow \cos^{-1}\frac{x}{a}=\alpha-\cos^{-1}\frac{y}{b}$$

$$\Rightarrow \frac{x}{a}=\cos\left(\alpha-\cos^{-1}\frac{y}{b}\right)$$

$$\Rightarrow \frac{x}{a}=\cos\alpha\cdot\cos\left(\cos^{-1}\frac{y}{b}\right)$$

$$+\sin\alpha\sin\left(\cos^{-1}\frac{y}{b}\right)$$ [2]

$$\Rightarrow \frac{x}{a}=\frac{y\cdot\cos\alpha}{b}+\sin\alpha\sqrt{1-\frac{y^2}{b^2}}$$

$$\Rightarrow \frac{x}{a}-\frac{y}{b}\cos\alpha=\sin\alpha\sqrt{1-\frac{y^2}{b^2}}$$ **[1]**

Squaring both sides, we have

$$\Rightarrow \left(\frac{x}{a}-y\frac{\cos\alpha}{b}\right)^2=\left(\sin\alpha\sqrt{1-\frac{y^2}{b^2}}\right)^2$$ **[½]**

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy}{ab}\cdot\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$ **[½]**

Q. 32. Solve for x :

$$\tan^{-1}\left(\frac{x-2}{x-1}\right)+\tan^{-1}\left(\frac{x+2}{x+1}\right)=\frac{\pi}{4}$$

[CBSE Board, Foreign Scheme, 2016]

Ans.

$$\tan^{-1}\left[\frac{\frac{x-2}{x-1}+\frac{x+2}{x+1}}{1-\frac{x-2}{x-1}\cdot\frac{x+2}{x+1}}\right]=\frac{\pi}{4}$$ **[1½]**

$$\Rightarrow \frac{2x^2-4}{3}=\tan\frac{\pi}{4}$$ **[1½]**

$$\Rightarrow x=\pm\sqrt{\frac{7}{2}}$$ **[1]**

Long Answer Type Questions (5 or 6 marks)

Q. 1. Prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

[NCERT Exemp. Ex. 2.3, Q. 12, Page 36]

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

[NCERT Exemp. Ex. 2.3, Q. 12, Page 36]

Ans.

$$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

[Let, $x^2 = \cos y$]

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos y}+\sqrt{1-\cos y}}{\sqrt{1+\cos y}-\sqrt{1-\cos y}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{y}{2}}+\sqrt{2\sin^2\frac{y}{2}}}{\sqrt{2\cos^2\frac{y}{2}}-\sqrt{2\sin^2\frac{y}{2}}}\right)$$

$$\left[\because\ 1+\cos y = 2\cos^2\frac{y}{2} \text{ and } 1-\cos y = 2\sin^2\frac{y}{2}\right]$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\frac{y}{2}+\sqrt{2}\sin\frac{y}{2}}{\sqrt{2}\cos\frac{y}{2}-\sqrt{2}\sin\frac{y}{2}}\right)$$

$$\left[\text{Dividing each term by } \sqrt{2}\cos\frac{y}{2}\right]$$

$$= \tan^{-1}\left(\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}}\right)$$ **[3]**

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\frac{y}{2}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{y}{2}}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)\right]$$

$$= \frac{\pi}{4}+\frac{y}{2}$$

$$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2 \qquad [\because x^2 = \cos y]$$

= RHS **[2]**

$\therefore$ LHS = RHS. Hence proved.

Q. 2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{77}{85}$.

[NCERT Exemp. Ex. 2.3, Q. 14, Page 36]

Ans. $\text{LHS} = \sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$

$$= \tan^{-1}\frac{8}{\sqrt{17^2-8^2}}+\tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$$

$$\left[\because \sin^{-1}\frac{a}{b} = \tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$$

$$= \tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}$$ **[1]**

$$= \tan^{-1}\left[\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right]$$

$$\left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\tan\left[\frac{\frac{32\quad 45}{15\times 4}}{\frac{15\times 4-8\times 3}{15\times 4}}\right]$$

$$\tan\left[\frac{\frac{77}{60}}{\frac{36}{60}}\right]$$

$$\tan\ \frac{77}{36}$$ **[1]**

$$= \sin^{-1}\frac{77}{\sqrt{77^2+36^2}} \qquad \left[\because \tan^{-1}\frac{a}{b} = \cos^{-1}\frac{b}{\sqrt{a^2+b^2}}\right]$$

$$= \sin^{-1}\frac{77}{\sqrt{5929+1296}}$$

$$= \sin^{-1}\frac{77}{\sqrt{7225}}$$

$$= \sin^{-1}\frac{77}{85}$$

= R.H.S. **[3]**

$\therefore$ LHS = RHS. Hence proved.

Q. 3. Find the value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$.

[NCERT Exemp. Ex. 2.3, Q. 17, Page 36]

Ans. We have,

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$$

$$= 2.2\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$$

$$= 2\cdot\left[\tan^{-1}\frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2}\right]-\tan^{-1}\frac{1}{239}$$

$$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= 2\cdot\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)\right]-\tan^{-1}\frac{1}{239}$$

$$= 2\cdot\left[\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)\right]-\tan^{-1}\frac{1}{239}$$ [2]

$$= 2\tan^{-1}\frac{5}{12}-\tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\frac{2\cdot\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} - \tan^{-1}\frac{1}{239}$$

$$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{25}{144}}\right) - \tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{144\times 5}{119\times 6}\right) - \tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\cdot\frac{1}{239}}\right)$$

$$\left[\because \tan^{-1}x - \tan^{-1} = \left(\frac{x-y}{1+xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{120\times 239 - 119}{119\times 239 + 120}\right)$$

$$= \tan^{-1}\left[\frac{28680-119}{28441+120}\right]$$

$$= \tan^{-1}\frac{28561}{28561}$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$ **[3]**

Q. 4. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?

[NCERT Exemp. Ex. 2.3, Q. 18, Page 37]

Ans. We have,

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$

LHS $= \tan\left[\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)\right]$

Let,

$$\frac{1}{2}\sin^{-1}\frac{3}{4} = \theta$$

$$\Rightarrow \quad \sin^{-1}\frac{3}{4} = 2\theta$$

$$\Rightarrow \quad \sin 2\theta = \frac{3}{4}$$

$$\Rightarrow \quad \frac{2\tan\theta}{1+\tan^2\theta} = \frac{3}{4}$$

$$\Rightarrow \quad 3 + 3\tan^2\theta = 8\tan\theta$$

$$\Rightarrow 3\tan^2\theta - 8\tan\theta + 3 = 0$$

Let,

$$\tan\theta = y$$

$$\therefore \quad 3y^2 - 8y + 3 = 0$$

$$\Rightarrow \quad y = \frac{8\pm\sqrt{64-4\times 3\times 3}}{2\times 3}$$

$$= \frac{8\pm\sqrt{28}}{6}$$

$$= \frac{2\left[4\pm\sqrt{7}\right]}{2\cdot 3}$$

$$\Rightarrow \quad \tan\theta = \frac{4\pm\sqrt{7}}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{4\pm\sqrt{7}}{3}\right]$$

$$\left[\text{But}\frac{4+\sqrt{7}}{3} > \frac{1}{2}\cdot\frac{\pi}{2}, \text{Since } \max\left\{\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)\right\} = 1\right]$$

[3]

$$\therefore \text{LHS} = \tan\left[\tan^{-1}\left(\frac{4-\sqrt{7}}{3}\right)\right] = \frac{4-\sqrt{7}}{3} = \text{RHS}$$

Note : Since, $-\frac{\pi}{2} \le \sin^{-1}\frac{3}{4} \le \frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{4} \le \frac{1}{2}\sin^{-1}\frac{3}{4} \le \frac{\pi}{4}$$

$$\therefore \tan\left(\frac{-\pi}{4}\right) \le \tan\frac{1}{2}\left(\sin^{-1}\frac{3}{4}\right) \le \tan\frac{\pi}{4}$$

$$\Rightarrow -1 \le \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) \le 1$$

[2]

Q. 5. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4'}$ then find the value of x.

[NCERT Ex. 2.2, Q. 15, Page 48]

Ans. Given, $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)}}{\frac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)(x+2)}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{(x^2+2x-x-2)+(x^2-2x+x-2)}{(x^2-4)-(x^2-1)}\right] = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-x^2+1}\right] = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{2x^2-4}{-3}\right] = \frac{\pi}{4}$$

$$\Rightarrow \quad \frac{2x^2-4}{-3} = \tan^{-1}$$

$$\Rightarrow \quad \frac{2x^2-4}{-3} = 1$$

$$\Rightarrow \quad 2x^2 - 4 = -3$$

$$\Rightarrow \quad 2x^2 = -3 + 4 = 1$$

$$\Rightarrow \quad x^2 = \frac{1}{2}$$

$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$ **[5]**

Q. 6. Find the values of the expression

$$\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

[NCERT Ex. 2.2, Q. 18, Page 48]

Ans. Given, $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Since, $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ and $\cot^{-1}x = \tan^{-1}\frac{1}{x}$

Therefore, above given expression can be written as

$$\tan\left(\tan^{-1}\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}+\tan^{-1}\frac{1}{\frac{3}{2}}\right)$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{5}}{\sqrt{\frac{16}{25}}}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{5}\times\frac{5}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right]$$

$$=\tan\left[\tan^{-1}\frac{\frac{17}{12}}{1-\frac{6}{12}}\right]$$

$$=\tan\left[\tan^{-1}\frac{17}{12}\times\frac{12}{6}\right]$$

$$=\tan\left[\tan^{-1}\frac{17}{6}\right]$$

$$=\frac{17}{6}$$

Thus, $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{17}{6}$

[5]

Some Commonly Made Errors

- Cosine is a function and cos is used to denote that we are dealing with the cosine function.
- Errors due to limited applicability of models.
- Errors due to applying properties of a familiar model like sin, cos, tan, etc. in a less familiar situation.
- Errors due to quickly associating sin identity.

EXPERT ADVICE

- *Student needs more practice with the law of sine so that you have got enough brain power available to pay attention to all the moving parts while you are trying to solve the problem.*
- *Students should have remembered the powers of trigonometric functions.*
- *Students should have aware about the inverse trigonometric notation.*
- *Students should have learned all the trigonometric functions identities to make easy solution in less time.*

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