CHAPTER

13 PROBABILITY

Chapter Objectives

This chapter will help you understand :

➢ ***Probability :*** *Conditional probability and its properties; Multiplication theorem on probability; Independent events, Bayes' theorem; Partition of a sample space, Theorem of total probability; Random variables and its Probability distributions; Mean of a random variable, Variance of a random variable; Bernoulli trials and Binomial distribution.*

Quick Review

❖ Properties of conditional probability : Let E and F be events associated with the sample space S of an experiment. Then

- $P(S|F) = P(F|F) = 1$

  $P[(A \cup B)|F] = P(A|F) + P(B|F) - P[(A \cap B|F)]$, where A and B are any two events associated with S.

- $P(E' \mid F) = 1 - P(E \mid F)$

❖ Let E and F be two events associated with a sample space of an experiment. Then

- $P(E \cap F) = P(E).P(F|E), P(E) \neq 0$
- $P(E \cap F) = P(F).P(E|F), P(F) \neq 0$
- If E, F and G are three events associated with a sample space, then
- $P(E \cap F \cap G) = P(E)\, P(F|E)\, P(G|E \cap F)$

❖ Binomial distribution : A random variable X taking values **0**, 1, 2, ..., n **is** said to have a binomial distribution with parameters n and p, if its probability distribution is distribution is given by $P(X = r) = {}^nc_r\, p^r\, q^{n-r}$, where $q = 1 - p$ and $r = 0, 1, 2, ..., n$.

❖ Probability is a fraction formula where you compare how many times something can occur to the potential number of times it can occur. The real life connection with Probability is to chance and most of the time winning something

TIPS...

- *Study the difference between random variable and mean of random variable.*
- *While dealing with more than one event, there are certain rules that must be followed. These rules depend greatly on : whether the events are dependent or independent.*

TRICKS...

- *Conceptual clarity is important in this chapter, so better try to understand the concept.*
- *Learn well about the terms related to Probability and its implications in the situation through various examples.*

Know the Links

☞ *https://whatis.techtarget.com/definition/probability*

☞ *https://www.teachoo.com/subjects/cbse-maths/class-12th/ch13-12th-probability/*

☞ *https://www.vedantu.com/ncert.../ncert-solutions-class-12-maths-chapter-13-probability...*

Multiple Choice Questions

(1 mark each)

Q. 1. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(a) $A \subset B$ **(b)** $B \subset A$

(c) $B = \phi$ **(d)** $A = \phi$

[NCERT Misc Ex. Q. 17, Page 584]

Ans. Correct option : (a)

Explanation :

$$P(A) \neq 0 \text{ and } P(B|A)=1$$

$$P(B|A)=\frac{P(B\cap A)}{P(A)}$$

$$1=\frac{P(B\cap A)}{P(A)}$$

$$P(A)=P(B\cap A)$$

$$\therefore A\subset B$$

Q. 2. If $P(A|B) > P(A)$, then which of the following is correct :

(a) $P(B|A) < P(B)$

(b) $P(A\cap B) < P(A) . P(B)$

(c) $P(B|A) > P(B)$

(d) $P(B|A) = P(B)$ **[NCERT Misc Ex. Q. 18, Page 584]**

Ans. Correct option : (c)

Explanation :

$$P(A|B)>P(A)$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)}>P(A)$$

$$\Rightarrow P(A\cap B)>P(A).P(B)$$

$$\Rightarrow \frac{P(A\cap B)}{P(A)}>P(B)$$

$$\Rightarrow P(B|A)>P(B)$$

Q. 3. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(a) $P(B|A) = 1$ **(b)** $P(A|B) = 1$

(c) $P(B|A) = 0$ **(d)** $P(B|A) = 0$

[NCERT Misc Ex. Q. 19, Page 584]

Ans. Correct option : (b)

Explanation :

$$P(A)+P(B)-P(A \text{ and } B)=P(A)$$

$$\Rightarrow P(A)+P(B)-P(A\cap B)=P(A)$$

$$\Rightarrow \quad P(B)-P(A\cap B)=0$$

$$\Rightarrow \quad P(A\cap B)=P(B)$$

$$\therefore \quad P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$$

Q. 4. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(a) 10^{-1} **(b)** $(1/2)^5$

(c) $(9/10)^5$ **(d)** $9/10$

[NCERT Ex. 13.5, Q. 14, Page 578]

Ans. Correct option : (c)

Explanation : The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denotes the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb,

$$p=\frac{10}{100}=\frac{1}{10}$$

$$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p=\frac{1}{10}$.

$$\therefore P(X=x)= {}^nC_xq^{n-x}p^x = {}^5C_x\left(\frac{9}{10}\right)^{5-x}\left(\frac{1}{10}\right)^x$$

P (none of the bulbs is defective) $= P(X=0)$

$$={}^5C_0.\left(\frac{9}{10}\right)^5=1.\left(\frac{9}{10}\right)^5=\left(\frac{9}{10}\right)^5$$

Q. 5. The probability that a student is not a swimmer is 1/5. Then the probability that out of five students, four are swimmers is

(a) ${}^5C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ **(b)** $\left(\frac{4}{5}\right)^4\frac{1}{5}$

(c) ${}^5C_1\frac{1}{5}\left(\frac{4}{5}\right)^4$ **(d) None of these**

[NCERT Ex. 13.5, Q. 15, Page 578]

Ans. Correct option : (a)

Explanation : The repeated selection of students who are swimmers are Bernoulli trial. Let X denotes the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q=\frac{1}{5}$

$$\therefore p=1-q=1-\frac{1}{5}=\frac{4}{5}$$

Clearly, X has a binomial distribution with $n = 5$ and $p=\frac{4}{5}$.

$$P(X=x)= {}^nC_xq^{n-x}p^x = {}^5C_x\left(\frac{1}{5}\right)^{5-x}.\left(\frac{4}{5}\right)^x$$

$$P(\text{4 students are swimmers})=P(X=4)= {}^5C_4\left(\frac{1}{5}\right).\left(\frac{4}{5}\right)^4$$

Q. 6. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(a) 1 **(b)** 2

(c) 5 **(d)** 8/3

[NCERT Ex. 13.4, Q. 16, Page 571]

Ans. Correct option : (b)

Explanation :

Let X be the random variable representing a number on the die.

The total number of observations is 6. Therefore,

$$P(X=1)=\frac{3}{6}=\frac{1}{2}$$
$$P(X=2)=\frac{2}{6}=\frac{1}{3}$$
$$P(X=5)=\frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	1/2	1/3	1/6

Mean = $E(X)$

$$=\sum p_i x_i$$
$$=\frac{1}{2}\times 1+\frac{1}{3}\times 2+\frac{1}{6}\times 5$$
$$=\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$$
$$=\frac{3+4+5}{6}=\frac{12}{6}=2$$

Q. 7. Suppose that two cards are drawn at random from a deck of cards. Let *X* be the number of aces obtained. Then the value of *E(X)* is

(a) 37/221 **(b) 5/13**

(c) 1/13 **(d) 2/13**

[NCERT Ex. 13.4, Q. 17, Page 571]

Ans. Correct option : (d)

Explanation : Let X denotes the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

$\therefore P\,(X = 0) = P$ (0 ace and 2 non-ace cards)

$$=\frac{{}^4C_0\times{}^{48}C_2}{{}^{52}C_2}=\frac{1128}{1326}$$

$P\,(X = 1) = P$ (1 ace and 1 non-ace cards)

$$=\frac{{}^4C_1\times{}^{48}C_1}{{}^{52}C_2}=\frac{192}{1326}$$

$P\,(X = 2) = P$ (2 ace and 0 non-ace cards)

$$=\frac{{}^4C_2\times{}^{48}C_0}{{}^{52}C_2}=\frac{6}{1326}$$

Thus, the probability distribution is a follows :

X	1	2	3
P(X)	1128/1326	192/1326	6/1326

Then,

$$E(X)=\sum p_i x_i$$
$$=0\times\frac{1128}{1326}+1\times\frac{192}{1326}+2\times\frac{6}{1326}$$
$$=\frac{204}{1326}=\frac{2}{13}$$

Q. 8. Probability that *A* speaks truth is 4/5. *A* coin is tossed. *A* reports that a head appears. The probability that actually there was head is

(a) 4/5 **(b) 1/2**

(c) 1/5 **(d) 2/5**

[NCERT Ex. 13.3, Q. 13, Page 557]

Ans. Correct option : (a)

Explanation : Let E_1 and E_2 be the events such that,

E_1 : A speaks truth

E_2 : A speaks false

Let X be the event that a head appears.

$$P(E_1)=\frac{4}{5}$$

Therefore,

$$P(E_2)=1-P(E_1)=1-\frac{4}{5}=\frac{1}{5}$$

If a coin is tossed, then it may result in either head (H) or tail (T).

The probability of getting a head is 1/2, whether A speaks truth or not.

$$\therefore P(X|E_1)=P(X|E_2)=\frac{1}{2}$$

The probability that there is actually a head is given by,

$$P(E_1|X)=\frac{P(E_1).P(X|E_1)}{P(E_1).P(X|E_1)+P(E_2).P(X|E_2)}$$
$$=\frac{\frac{4}{5}.\frac{1}{2}}{\frac{4}{5}.\frac{1}{2}+\frac{1}{5}.\frac{1}{2}}=\frac{\frac{1}{2}.\frac{4}{5}}{\frac{1}{2}\left(\frac{4}{5}+\frac{1}{5}\right)}$$
$$=\frac{\frac{4}{5}}{1}=\frac{4}{5}$$

Q. 9. If *A* and *B* are two events such that $A\subset B$ and *P(B)* ≠ 0, then which of the following is correct?

(a) $P(A|B) = P(B)/P(A)$

(b) $P(A|B) < P(A)$

(c) $P(A|B) \geq P(A)$

(d) $P(A|B) \geq P(A)$ **[NCERT Ex. 13.3, Q. 14, Page 557]**

Ans. Correct option : (c)

Explanation :

If $A\subset B$, then $A\cap B=A$

$\Rightarrow P(A\cap B)=P(A)$

Also, $P(A)<P(B)$ ...(i)

Consider,

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)} \quad ...(ii)$$

It is known that, $P(B)\leq 1$

$$\Rightarrow \frac{1}{P(B)}\geq 1$$
$$\Rightarrow \frac{P(A)}{P(B)}\geq P(A)$$

From equation (ii), we obtain

$\Rightarrow P(A|B)\geq P(A)$

$\therefore P(A|B)$ is not less than $P(A)$.

Q. 10. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(a) 0 **(b) 1/3**

(c) 1/12 **(d) 1/36**

[NCERT Ex. 13.2, Q. 17, Page 548]

Ans. Correct option : (d)

Explanation : When two dices are rolled, the number of outcomes is 36. The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

$\therefore \quad E = \{(2, 2)\}$

$\Rightarrow P(E) = \frac{1}{36}$

Q. 11. Two events A and B will be independent, if

(a) A and B are mutually exclusive

(b) $P(A'B') = [1 - P(A)]\,[1 - P(B)]$

(c) $P(A) = P(B)$

(d) $P(A) + P(B) = 1$ [NCERT Ex. 13.2, Q. 18, Page 548]

Ans. Correct option : (b)

Explanation : Two events A and B are said to be independent, if $P(AB) = P(A) \times P(B)$.

Consider the result given in alternative B.

$$P(A'B') = [1 - P(A)][1 - P(B)]$$
$$P(A' \cap B') = 1 - P(A) - P(B) + P(A).P(B)$$
$$1 - P(A \cap B) = 1 - P(A) - P(B) + P(A).P(B)$$
$$P(A \cup B) = P(A) + P(B) - P(A).P(B)$$
$$P(A) + P(B) - P(AB) = P(A) + P(B) - P(A).P(B)$$
$$P(AB) = P(A).P(B)$$

This implies that A and B are independent, if $P(A'B') = [1 - P(A)]\,[1 - P(B)]$

Distractor Rationale :

A. Let $P(A) = m$, $P(B) = n$, $0 < m, n < 1$

A and B are mutually exclusive.

$\therefore \; A \cap B = \phi$

$\Rightarrow P(AB) = 0$

However, $P(A).P(B) = mn \neq 0$

$\therefore P(A).P(B) \neq P(AB)$

C. Let A : Event of getting an odd number on throw of a die $= \{1, 3, 5\}$

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B. Event of getting an even number on throw of a die $= \{2, 4, 6\}$

$\Rightarrow \quad P(B) = \frac{3}{6} = \frac{1}{2}$

Here, $A \cap B = \phi$

$\therefore \quad P(AB) = 0$

$P(A).P(B) = \frac{1}{4} \neq 0$

$\Rightarrow P(A).P(B) \neq P(AB)$

D. From the above examples, it can be seen that

$\Rightarrow P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$

However, it cannot be inferred that A and B are independent.

Q. 12. If $P(A) = 1/2$, $P(B) = 0$, then $P(A|B)$ is

(a) 0 **(b) 1/2**

(c) not defined **(d) 1**

[NCERT Ex. 13.1, Q. 16, Page 539]

Ans. Correct option : (c)

Explanation : It is given that,

$P(A) = \frac{1}{2}$ and $P(B) = 0$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$

Therefore, $P(A|B)$ is not defined.

Q. 13. If A and B are events such that $P(A|B) = P(B|A)$, then

(a) $A \subset B$ but $A \neq B$ **(b) $A = B$**

(c) $A \cap B = \phi$ **(d) $P(A) = P(B)$**

[NCERT Ex. 13.1, Q. 17, Page 540]

Ans. Correct option : (d)

Explanation : It is given that,

$P(A|B) = P(B|A)$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad P(A) = P(B)$

Q. 14. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(a) 0.24 **(b) 0.3**

(c) 0.48 **(d) 0.96**

[NCERT Exemp. Ex. 13.3, Q. 61, Page 280]

Ans. Correct option : (d)

Explanation :

Here,

$P(A) = 0.4,\ P(B) = 0.8$ and $P(A/B) = 0.6$

$\because \quad P(B/A) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow \quad P(B \cap A) = P(B/A).P(A)$

$= 0.6 \times 0.4 = 0.24$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24$

$= 1.2 - 0.24 = 0.96$

Q. 15. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(a) $\left(\frac{9}{10}\right)^5$ **(b) $\frac{1}{2}\left(\frac{9}{10}\right)^4$**

(c) $\frac{1}{2}\left(\frac{9}{10}\right)^5 s$ **(d) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$**

[NCERT Exemp. Ex. 13.3, Q. 93, Page 285]

Ans. Correct option : (d)

Explanation : Here,

$n = 5,\ p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$

$r \leq 1$

$\Rightarrow r = 0, 1$

Also,

$P(X = r) = {}^nC_r p^r q^{n-r}$

$P(X = r) = P(r = 0) + P(r = 1)$

$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4$

$$=\left(\frac{9}{10}\right)^5+5.\frac{1}{10}.\left(\frac{9}{10}\right)^4$$

$$=\left(\frac{9}{10}\right)^5+\frac{1}{2}\left(\frac{9}{10}\right)^4$$

Q. 16. A and B are two students. Their chances of solving a problem correctly are 1/3 and 1/4, respectively. If the probability of their making a common error is, 1/20 and they obtain the same answer, then the probability of their answer to be correct is

(a) 1/12 **(b)** 1/40

(c) 13/120 **(d)** 10/13

[NCERT Exemp. Ex. 13.3, Q. 92, Page 285]

Ans. Correct option : (d)

Explanation : Let E_1 = Even that both A and B solve the problem

$$\therefore P(E_1)=\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$$

Let E_2 = Event that both A and B got incorrect solution of the problem

$$\therefore P(E_2)=\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$$

Let E = Event that they got same answer

Here,

$$P(E/E_1)=1,\ P(E/E_2)=\frac{1}{20}$$

$$P(E_1/E)=\frac{P(E_1\cap E)}{P(E)}$$

$$=\frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{12}\times 1}{\frac{1}{12}\times 1+\frac{1}{2}\times\frac{1}{20}}$$

$$=\frac{1/12}{\frac{10+3}{120}}=\frac{120}{12\times 13}=\frac{10}{13}$$

Q. 17. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in physics if she has failed in mathematics is

(a) 1/10 **(b)** 2/5

(c) 9/20 **(d)** 1/3

[NCERT Exemp. Ex. 13.3, Q. 91, Page 285]

Ans. Correct option : (b)

Explanation : Here,

$$P_{(Ph)}=\frac{30}{100}=\frac{3}{10}, P_{(M)}=\frac{25}{100}=\frac{1}{4} \text{ and } P_{(M\cap Ph)}=\frac{10}{100}=\frac{1}{10}$$

$$\therefore P\left(\frac{Ph}{M}\right)=\frac{P(Ph\cap M)}{P(M)}=\frac{\frac{1}{10}}{\frac{1}{4}}=\frac{2}{5}$$

Q. 18. Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If $P(x = r)/P(x = n - r)$ is independent of n and r, then p equals

(a) 1/2 **(b)** 1/3

(c) 1/5 **(d)** 1/7

[NCERT Exemp. Ex. 13.3, Q. 90, Page 284]

Ans. Correct option : (a)

Explanation :

$$\therefore P(X=r)={}^nC_r(p)^r(q)^{n-r}=\frac{n!}{(n-r)!r!}(p)^r(1-p)^{n-r}$$

$[\because q = 1 - p]$ ...(i)

$P(X=0)=(1-p)^n$ and

$$P(X=n-r)={}^nC_{n-r}(p)^{n-r}(q)^{n-(n-r)}$$

$$=\frac{n!}{(n-r)!r!}(p)^{n-r}(1-p)^r$$

$[\because q = 1 - p]$ $[Q\ {}^nC_r = {}^nC_{n-r}]$ ...(ii)

Now,

$$\frac{P(x=r)}{P(x=n-r)}=\frac{\frac{n!}{(n-r)!r!}p^r(1-p)^{n-r}}{\frac{n!}{(n-r)!r!}p^{n-r}(1-p)^{+r}}$$

$$=\left(\frac{1-p}{p}\right)^{n-r}\times\frac{1}{\left(\frac{1-p}{p}\right)^r}$$

[By using equations (i) and (ii)]

Above expression is independent of n and r, if

$$\frac{1-p}{p}=1$$

$$\Rightarrow \frac{1}{p}=2$$

$$\therefore\ p=\frac{1}{2}$$

Q. 19. For the following probability distribution :

X	1	2	3	4
$P(X)$	1/10	1/5	3/10	2/5

$E(X^2)$ is equal to

(a) 3 **(b)** 5

(c) 7 **(d)** 10

[NCERT Exemp. Ex. 13.3, Q. 89, Page 284]

Ans. Correct option : (d)

Explanation :

$$E(X^2)=\sum X^2\ P(X)$$

$$=1.\frac{1}{10}+4.\frac{1}{5}+9.\frac{3}{10}+16.\frac{2}{5}$$

$$=\frac{1}{10}+\frac{4}{5}+\frac{27}{10}+\frac{32}{5}$$

$$=\frac{1+8+27+64}{10}=10$$

Q. 20. For the following probability distribution :

X	−4	−3	−2	−1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to

(a) 0 **(b)** −1

(c) −2 **(d)** −1.8

[NCERT Exemp. Ex. 13.3, Q. 88, Page 284]

Ans. Correct option : (d)

Explanation :

$(X)=\sum X\,P(X)$

$=-4\times(0.1)+(-3\times0.2)+(-2\times0.3)+(-1\times0.2)+(0\times0.2)$

$=-0.4-0.6-0.6-0.2+0=-1.8$

Q. 21. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
$P(X)$	$5/k$	$7/k$	$9/k$	$11/k$

The value of k is

(a) 8 **(b) 16**

(c) 32 **(d) 48**

[NCERT Exemp. Ex. 13.3, Q. 87, Page 284]

Ans. Correct option : (c)

Explanation : We know that,

$$\sum P(X)=1$$

$$\Rightarrow \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1$$

$$\Rightarrow \frac{32}{k}=1$$

$\Rightarrow k=32$

Q. 22. The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is

(a) $^5C_4\,(0.7)^4\,(0.3)$ **(b) $^5C_1\,(0.7)\,(0.3)^4$**

(c) $^5C_4\,(0.7)^4\,(0.3)^4$ **(d) $(0.7)^4\,(0.3)$**

[NCERT Exemp. Ex. 13.3, Q. 86, Page 284]

Ans. Correct option : (a)

Explanation : Here,

$\bar{p}=0.3\Rightarrow p=0.7$ and $r=4$

$\therefore$ Required probability $=\,^5C_4\,(0.7)^4\,(0.3)$

Q. 23. The probability of guessing correctly at least 8 out of 10 answers on a true-false type examination is

(a) 7/64 **(b) 7/128**

(c) 45/1024 **(d) 7/41**

[NCERT Exemp. Ex. 13.3, Q. 85, Page 283]

Ans. Correct option : (b)

Explanation :

We know that,

$P(X=r)=\,^nC_r\,(p)^r\,(q)^{n-r}$

Here,

$n=10, p=\frac{1}{2}, q=\frac{1}{2}$ and $r\geq 8$ i.e., $r=8, 9, 10$

$\Rightarrow P(X=r)=P(r=8)+P(r=9)+P(r=10)$

$$={}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10-8}+{}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9}+{}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$$

$$=\left(\frac{10!}{8!2!}+\frac{10!}{9!1!}+1\right)\left(\frac{1}{2}\right)^{10}$$

$$=[45+10+1]\left(\frac{1}{2}\right)^{10}$$

$$=56\left(\frac{1}{2}\right)^{10}=\frac{7}{128}$$

Q. 24. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens, is

(a) $\frac{1}{13}\times\frac{1}{13}$ **(b) $\frac{1}{13}+\frac{1}{13}$**

(c) $\frac{1}{13}\times\frac{1}{17}$ **(d) $\frac{1}{13}\times\frac{4}{51}$**

[NCERT Exemp. Ex. 13.3, Q. 84, Page 283]

Ans. Correct option : (a)

Explanation :

Required probability $=\frac{4}{52}\cdot\frac{4}{52}=\frac{1}{13}\times\frac{1}{13}$

Q. 25. Which one is not a requirement of a binomial distribution?

(a) There are 2 outcomes for each trial.

(b) There is a fixed number of trials.

(c) The outcomes must be dependent on each other.

(d) The probability of success must be the same for all the trials.

[NCERT Exemp. Ex. 13.3, Q. 83, Page 283]

Ans. Correct option : (c)

Explanation : We know that, in a binomial distribution,

(i) There are two outcomes for each trial.

(ii) There is a fixed number of trials.

(iii) The probability of success must be the same for all the trials.

Q. 26. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(a) 1/18 **(b) 5/18**

(c) 1/5 **(d) 2/5**

[NCERT Exemp. Ex. 13.3, Q. 82, Page 283]

Ans. Correct option : (c)

Explanation : Let,

E_1 = Event that the sum of numbers on the dice was less than 6 and

E_2 = Event that the sum of numbers on the dice is 3.

$\therefore$ E_1 = {(1, 1), (1,2), (1,3), (1, 4), (2, 1), (2, 2), (2,3), (3,1), (3, 2), (4,1)}

$\Rightarrow n(E_1)=10$

and E_2={(1, 2), (2, 1)}

$\Rightarrow n(E_2)=2$

$\therefore$ Required probability $=\frac{2}{10}=\frac{1}{5}$

Q. 27. Eight coins are tossed together. The probability of getting exactly 3 heads is

(a) 1/256 **(b) 7/32**

(c) 5/32 **(d) 3/32**

[NCERT Exemp. Ex. 13.3, Q. 81, Page 283]

Ans. Correct option : (b)

Explanation :

We know that, probability distribution

$P(X=r)=\,^nC_r\,(p)^r\,q^{n-r}$

Here, $n=8,\ r=3,\ p=\frac{1}{2}$ and $q=\frac{1}{2}$

$$\therefore \text{Required probability}={}^8C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3}=\frac{8!}{5!3!}\left(\frac{1}{2}\right)^8$$

$$=\frac{8\cdot7\cdot6}{3\cdot2}\cdot\frac{1}{2^8}=\frac{7}{32}$$

Q. 28. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

(a) $\frac{33}{56}$ **(b)** $\frac{9}{64}$

(c) $\frac{1}{14}$ **(d)** $\frac{3}{28}$

[NCERT Exemp. Ex. 13.3, Q. 80, Page 283]

Ans. Correct option : (d)

Explanation :

Required probability $= \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$

Q. 29. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(a) $\frac{3}{28}$ **(b)** $\frac{2}{21}$

(c) $\frac{1}{28}$ **(d)** $\frac{167}{168}$

[NCERT Exemp. Ex. 13.3, Q. 79, Page 283]

Ans. Correct option : (a)

Explanation :

Probability of drawing 2 green balls and one blue ball

$$= P(G) \cdot P(G) \cdot P(B) + P(B) \cdot P(G) \cdot P(G) + P(G) \cdot P(B) \cdot P(G)$$

$$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} + \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} = \frac{3}{28}$$

Q. 30. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(a) 1/2 **(b)** 1/4

(c) 1/8 **(d)** 3/4

[NCERT Exemp. Ex. 13.3, Q. 78, Page 282]

Ans. Correct option : (c)

Explanation : Let,

E_1 = Event for getting an event number on die and

E_2 = Even that a spade card is selected

$\therefore P(E_1) = \frac{3}{6} = \frac{1}{2}$ and $P(E_2) = \frac{13}{52} = \frac{1}{4}$

Then, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

Q. 31. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$

(c) $\frac{2}{3}$ **(d)** $\frac{4}{7}$

[NCERT Exemp. Ex. 13.3, Q. 77, Page 282]

Ans. Correct option : (d)

Explanation : We have,

$S = \{B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)\}$

E_1 = Event that a family has at least one girl, then

$E_1 = \{(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)\}$

E_2 = Event that the eldest child is a girl, then

$E_2 = \{(G, B, B), (G, G, B), (G, B, G), (G, G, G)\}$

$\therefore E_1 \cap E_2 = \{(G,B,B),(G,G,B),(G,B,G),(G,G,G)\}$

$$\therefore \quad P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{4}{8}}{\frac{7}{8}} = \frac{4}{7}$$

Q. 32. Three persons, *A*, *B* and *C*, fire at a target in turn, starting with *A*. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(a) 0.024 **(b)** 0.188

(c) 0.336 **(d)** 0.452

[NCERT Exemp. Ex. 13.3, Q. 76, Page 282]

Ans. Correct option : (b)

Explanation : We have,

$P(A) = 0.4$, $P(B) = 0.3$, and $P(C) = 0.2$

$\therefore$ Probability of two hits

$$= P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A') \cdot P(B) \cdot P(C)$$

$$= 0.4 \times 0.3 \times 0.8 + 0.4 \times 0.7 \times 0.2 + 0.6 \times 0.3 \times 0.2$$

$$= 0.096 + 0.056 + 0.036 = 0.188$$

Q. 33. Refer to Question 34. The probability that exactly two of the three balls were red, the first ball being red, is

(a) $\frac{1}{3}$ **(b)** $\frac{4}{7}$

(c) $\frac{15}{28}$ **(d)** $\frac{5}{28}$

[NCERT Exemp. Ex. 13.3, Q. 75, Page 282]

Ans. Correct option : (b)

Explanation : Required probability

$$= P(R) \cdot P(B) + P(B) \cdot P(R)$$

$$= \frac{4}{7} \cdot \frac{3}{6} + \frac{3}{6} \cdot \frac{4}{7} \quad \text{(As first ball is Red)}$$

$$= \frac{4}{7}$$

Q. 34. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is

(a) $\frac{45}{196}$ **(b)** $\frac{135}{392}$

(c) $\frac{15}{56}$ **(d)** $\frac{15}{29}$

[NCERT Exemp. Ex. 13.3, Q. 74, Page 282]

Ans. Correct option : (c)

Explanation : Probability of getting exactly one red ball

$$= P(R) \cdot P(B) \cdot P(B) + P(B) \cdot P(R) \cdot P(B) + P(B) \cdot P(B) \cdot P(R)$$

$$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} = \frac{15}{56}$$

Q. 35. Two events E and F are independent. If $P(E) = 0.3$, $P(E \cup F) = 0.5$, then $P(E|F) - P(F|E)$ equals

(a) $\frac{2}{7}$ **(b)** $\frac{3}{35}$

(c) $\frac{1}{70}$ (d) $\frac{1}{7}$

[NCERT Exemp. Ex. 13.3, Q. 73, Page 282]

Ans. Correct option : (c)

Explanation : We have,

$P(E)=0.3$ and $P(E\cup F)=0.5$

Also, E and F are independent.

Now,

$$P(E\cup F)=P(E)+P(F)-P(E\cap F)$$

$$\Rightarrow \quad 0.5=0.3+P(F)-0.3P(F)$$

$$\Rightarrow \quad P(F)=\frac{0.5-0.3}{0.7}=\frac{2}{7}$$

$$\therefore \quad P(E/F)-P(F/E)$$

$= P(E)-P(F)$ (as E and F are independent)

$$=\frac{3}{10}-\frac{2}{7}=\frac{1}{70}$$

Q. 36. If the events *A* and *B* are independent, then *P*(*A*∩*B*) is equal to

(a) *P(A) + P(B)* **(b) *P(A) – P(B)***

(c) *P(A).P(B)* **(d) *P(A)|P(B)***

[NCERT Exemp. Ex. 13.3, Q. 72, Page 281]

Ans. Correct option : (c)

Explanation : If *A* and *B* are independent, then

$$P(A\cap B)=P(A)\cdot P(B)$$

Q. 37. Let *A* and *B* be two events such that $P(A)=\frac{3}{8}, P(B)=\frac{5}{8}$ and $P(A\cup B)=\frac{3}{4}$. Then $P(A|B).P(A|B)$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{3}{8}$

(c) $\frac{3}{20}$ (d) $\frac{6}{25}$

[NCERT Exemp. Ex. 13.3, Q. 71, Page 281]

Ans. Correct option : (d)

Explanation : We have,

$$P(A)=\frac{3}{8}, P(B)=\frac{5}{8} \text{ and } P(A\cup B)=\frac{3}{4}$$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$P(A\cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{1}{4}$$

$$\because \quad P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{5}{8}}=\frac{2}{5}$$

and $P(A'/B)=\dfrac{P(A'\cap B)}{P(B)}=\dfrac{P(B)-P(A\cap B)}{P(B)}$

$$=\frac{\frac{5}{8}-\frac{1}{4}}{\frac{5}{8}}=\frac{3}{5}$$

$$\therefore P(A/B)\cdot P(A'/B)=\frac{2}{5}\cdot\frac{3}{5}=\frac{6}{25}$$

Q. 38. If two events are independent, then

(a) they must be mutually exclusive.

(b) the sum of their probabilities must be equal to 1.

(c) (a) and (b) both are correct.

(d) None of the above is correct.

[NCERT Exemp. Ex. 13.3, Q. 70, Page 281]

Ans. Correct option : (d)

Explanation : If two events *A* and *B* are independent, then we know that

$$P(A\cap B)=P(A).P(B), P(A)\neq 0, P(B)\neq 0$$

Since, *A* and *B* have a common outcome.

Q. 39. If *A* and *B* are two independent events with $P(A)=\frac{3}{5}$ and $P(B)=\frac{4}{9}$, then $P(A\cap B)$ equals

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

[NCERT Exemp. Ex. 13.3, Q. 69, Page 281]

Ans. Correct option : (d)

Explanation : Since *A* and *B* are independent events, *A′* and *B′* are also independent. Therefore,

$$P(A'\cap B')=P(A')\cdot(B')$$

$$=(1-P(A)(1-P(B))$$

$$=\left(1-\frac{3}{5}\right)\left(1-\frac{4}{9}\right)$$

$$=\frac{2}{5}\cdot\frac{5}{9}=\frac{2}{9}$$

Q. 40. If *A* and *B* are such events that *P(A)* > 0 and *P(B)* ≠ 1, then *P* (*A* |*B*) equals

(a) 1 – *P*(*A*|*B*) **(b) 1 – *P*(*A* |*B*)**

(c) $\frac{1-P(A\cup B)}{P(B')}$ **(d) *P*(*A*)|*P*(*B*)**

[NCERT Exemp. Ex. 13.3, Q. 68, Page 281]

Ans. Correct option : (c)

Explanation : We have,

$P(A)>0$ and $P(B)\neq 1$

$$P(A'/B')=\frac{P(A'\cap B')}{P(B')}=\frac{1-P(A\cup B)}{P(B')}$$

Q. 41. Let $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$. Then *P*(*A*|*B*) is equal to

(a) $\frac{6}{13}$ (b) $\frac{4}{13}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

[NCERT Exemp. Ex. 13.3, Q. 67, Page 281]

Ans. Correct option : (d)

Explanation : Here,

$$P(A)=\frac{7}{13}, P(B)=\frac{9}{13} \text{ and } P(A\cap B)=\frac{4}{13}$$

$$P(A'/B)=\frac{P(A'\cap B)}{P(B)}$$

$$=\frac{P(B)-P(A\cap B)}{P(B)}$$

$$=\frac{\frac{9}{13}-\frac{4}{13}}{\frac{9}{13}}=\frac{5}{9}$$

Q. 42. If $P(B)=\frac{3}{5}$, $P(A|B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$, then $P(A\cup B)' + P(A'\cup B)=$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{2}$ (d) 1

[NCERT Exemp. Ex. 13.3, Q. 66, Page 280]

Ans. Correct option : (d)

Explanation : We have,

$$P(B)=\frac{3}{5}, P(A/B)=\frac{1}{2}$$

Therefore,

$$P(A\cap B)=P(A/B)\cdot P(B)$$
$$=\frac{1}{2}\cdot\frac{3}{5}=\frac{3}{10}$$

Now,

$$P(A\cap B)=P(A)+P(B)-P(A\cap B)$$
$$\therefore \quad P(A)=\frac{4}{5}-\frac{3}{5}+\frac{3}{10}=\frac{1}{2}$$
$$P(A\cup B)'=1-P(A\cup B)$$
$$=1-\frac{4}{5}=\frac{1}{5}$$

and $P(A'\cup B)=1-P(A-B)$

$$=1-[P(A)-P(A\cap B)]$$
$$=1-\left(\frac{1}{2}-\frac{3}{10}\right)=\frac{4}{5}$$
$$\therefore P(A\cup B)'+P(A'\cup B)=\frac{1}{5}+\frac{4}{5}=1$$

Q. 43. In Question 44, $P(B|A')$ is equal to

(a) $\frac{1}{5}$ (b) $\frac{3}{10}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

[NCERT Exemp. Ex. 13.3, Q. 65, Page 280]

Ans. Correct option : (d)

Explanation :

$$P(B/A')=\frac{P(B\cap A')}{P(A')}$$
$$=\frac{P(B)-P(B\cap A)}{1-P(A)}$$
$$=\frac{\frac{3}{5}-\frac{3}{10}}{1-\frac{1}{2}}=\frac{\frac{6-3}{10}}{\frac{1}{2}}=\frac{3}{5}$$

Q. 44. You are given that A and B are two events such that $P(B)=\frac{3}{5}$, $P(A|B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$, then $P(A)$ equals

(a) $\frac{3}{10}$ (b) $\frac{1}{5}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

[NCERT Exemp. Ex. 13.3, Q. 64, Page 280]

Ans. Correct option : (c)

Explanation : We have,

$$P(B)= =\frac{3}{5}, P(A/B)=\frac{1}{2} \text{ and } P(A\cup B)=\frac{4}{5}$$
$$\therefore P(A\cap B)=P(A/B)\cdot P(B)=\frac{1}{2}\cdot\frac{3}{5}=\frac{3}{10}$$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \quad \frac{4}{5}=P(A)+\frac{3}{5}-\frac{3}{10}$$
$$\therefore \quad P(A)=\frac{4}{5}-\frac{3}{5}+\frac{3}{10}=\frac{1}{2}$$

Q. 45. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A\cup B) = 0.5$. Then $P(B'\cap A)$ equals

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{10}$ (d) $\frac{1}{5}$

[NCERT Exemp. Ex. 13.3, Q. 63, Page 280]

Ans. Correct option : (d)

Explanation : We have,

$P(A) = 0.4$, $P(B) = 0.3$ and $P(A\cup B)=0.5$

Now,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$\Rightarrow \quad P(A\cap B)=0.4+0.3-0.5=0.2$$
$$\therefore \quad P(B'\cap A)=P(A)-P(A\cap B)$$
$$=0.4-0.2=0.2=\frac{1}{5}$$

Q. 46. If A and B are two events and $A \neq \phi$, $B \neq \phi$, then

(a) $P(A|B)=P(A).P(B)$ (b) $P(A|B)=\frac{P(A\cap B)}{P(B)}$

(c) $P(A|B).P(B|A)=1$ (d) $P(A|B)=P(A|P(B)$

[NCERT Exemp. Ex. 13.3, Q. 62, Page 280]

Ans. Correct option : (b)

Explanation :

If $A\neq\phi$ and $B\neq\phi$, then $P(A/B)=\frac{P(A\cap B)}{P(B)}$

Q. 47. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A\cup B)$ is equal to

(a) 0.24 (b) 0.3

(c) 0.48 (d) 0.96

[NCERT Exemp. Ex. 13.3, Q. 61, Page 280]

Ans. Correct option : (d)

Explanation : We have,

$P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$

$$\Rightarrow \quad P(B\cap A)=P(B/A)\cdot P(A)=0.6\times 0.4=0.24$$
$$\therefore \quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=0.4+0.8-0.24=0.96$$

Q. 48. If A and B are two events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A/B)=\frac{1}{4}$, then $P(A'\cap B')$ equals

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{16}$

[NCERT Exemp. Ex. 13.3, Q. 60, Page 279]

Ans. Correct option : (c)

Explanation : We have,

$$P(A)\ \frac{1}{2},\ P(B)=\frac{1}{3} \text{ and } P(A/B)=\frac{1}{4}$$

$$\Rightarrow \quad P(A \cap B) = P(A/B) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

Now,

$$P(A' \cap B') = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B]$$
$$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right] = 1 - \frac{9}{12} = \frac{3}{12} = \frac{1}{4}$$

Q. 49. If $P(A) = \frac{2}{5}$ $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A|B).P(B'|A')$ is equal to

(a) $\frac{5}{6}$ **(b) $\frac{5}{7}$**

(c) $\frac{25}{42}$ **(d) 1**

[NCERT Exemp. Ex. 13.3, Q. 59, Page 279]

Ans. Correct option : (c)

Explanation : We have,

$$P(A) = \frac{2}{5},\ P(B) = \frac{3}{10} \text{ and } P(A \cap B) = \frac{1}{5}$$

$$P(A'/B') \cdot P(B'/A') = \frac{P(A' \cap B')}{P(B')} \cdot \frac{P(A' \cap B')}{P(A')} = \frac{(P((A \cup B)'))^2}{P(A')P(B')}$$
$$= \frac{(1 - P(A \cup B))^2}{(1 - P(A))(1 - P(B))}$$
$$= \frac{(1 - P(A) + P(B) - P(A \cap B))^2}{(1 - P(A))(1 - P(B))}$$
$$= \frac{\left[1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)\right]^2}{\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{10}\right)} = \frac{\left(1 - \frac{1}{2}\right)^2}{\frac{3}{5} \cdot \frac{7}{10}} = \frac{25}{42}$$

Q. 50. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B|A) + P(A|B)$ equals

(a) $\frac{1}{4}$ **(b) $\frac{1}{3}$**

(c) $\frac{5}{12}$ **(d) $\frac{7}{12}$**

[NCERT Exemp. Ex. 13.3, Q. 58, Page 279]

Ans. Correct option : (d)

Explanation : We have,

$$P(A) = \frac{3}{10},\ P(B) = \frac{2}{5} \text{ and } P(A \cup B) = \frac{3}{5}$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\therefore \quad \frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B)$$
$$\therefore \quad P(A \cap B) = \frac{1}{10}$$
$$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Q. 51. If $P(A \cap B) = \frac{7}{10}$ and $P(B) = \frac{17}{20}$, then $(P(A|B)$ equals

(a) $\frac{14}{17}$ **(b) $\frac{17}{20}$**

(c) $\frac{7}{8}$ **(d) $\frac{1}{8}$**

[NCERT Exemp. Ex. 13.3, Q. 57, Page 279]

Ans. Correct option : (a)

Explanation : Here,

$$P(A \cap B) = \frac{7}{10} \text{ and } P(B) = \frac{17}{20}$$

Therefore,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{7/10}{17/20} = \frac{14}{17}$$

Q. 52. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $(P(B|A)$ is equal to

(a) $\frac{1}{10}$ **(b) $\frac{1}{8}$**

(c) $\frac{7}{8}$ **(d) $\frac{17}{20}$**

[NCERT Exemp. Ex. 13.3, Q. 56, Page 279]

Ans. Correct option : (c)

Explanation :

$$\because \quad P(A) = \frac{4}{5}, P(A \cap B) = \frac{7}{10}$$
$$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{7/10}{4/5} = \frac{7}{8}$$

Q. 53. If A and B are independent events, then A and B are also independent.

[NCERT Exemp. Ex. 13.3, Q. 95, Page 285]

Ans. True

Q. 54. If A and B are independent, then P (exactly one of A, B occurs) = $P(A)P(B') + P(B)P(A')$

[NCERT Exemp. Ex. 13.3, Q. 101, Page 286]

Ans. True

Q. 55. If A and B are mutually exclusive events, then they will be independent also.

[NCERT Exemp. Ex. 13.3, Q. 96, Page 285]

Ans. False

Q. 56. Two independent events are always mutually exclusive.

[NCERT Exemp. Ex. 13.3, Q. 97, Page 285]

Ans. False

Q. 57. If A and B are two independent events then $P(A$ and $B) = P(A).P(B)$.

[NCERT Exemp. Ex. 13.3, Q. 98, Page 286]

Ans. True

Q. 58. Another name for the mean of a probability distribution is expected value.

[NCERT Exemp. Ex. 13.3, Q. 99, Page 286]

Ans. True

$$E(X) = \sum X\, P(X) = \mu$$

Very Short Answer Type Questions (1 or 2 marks each)

Q. 1. Given that E and F are events such that $P(E)$ = 0.6, $P(F)$ = 0.3 and $P(E \cap F)$ = 0.2, find $P(E|F)$ and $P(F|E)$. [NCERT Ex. 13.1, Q. 1, Page 538]

Ans. It is given that,

$P(E) = 0.6, P(F) = 0.3, \text{ and } P(E \cap F) = 0.2$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

[2]

Q. 2. Compute $P(A|B)$, if $P(B)$ = 0.5 and $P(A \cap B)$ = 0.32. [NCERT Ex. 13.1, Q. 2, Page 538]

Ans. It is given that,

$P(B) = 0.5$ and $P(A \cap B) = 0.32$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$$

[2]

Q. 3. If $P(A)$ = 3/5 and $P(B)$ = 1/5, find $P(A \cap B)$ if A and B are independent events. [NCERT Ex. 13.2, Q. 1, Page 546]

Ans. It is given that,

$P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

A and B are independent events. Therefore,

$$P(A \cap B) = P(A) \cdot P(B) = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$$

[2]

Q. 4. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. [NCERT Ex. 13.2, Q. 2, Page 546]

Ans. There are 26 black cards in a deck of 52 cards.

Let $P(A)$ be the probability of getting a black card in the first draw.

$$\therefore P(A) = \frac{26}{52} = \frac{1}{2}$$

Let $P(B)$ be the probability of getting a black card on the second draw.

Since the card is not replaced.

$$\therefore P(B) = \frac{25}{51}$$

Thus, probability of getting both the cards black

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

[2]

Q. 5. Let E and F be events with $P(E)$ = 3/5, $P(F)$ = 3/10 and $P(E \cap F)$ = 1/5. Are E and F independent? [NCERT Ex. 13.2, Q. 6, Page 546]

Ans. It is given that,

$$P(E) = \frac{3}{5},\ P(F) = \frac{3}{10},\ \text{and } P(EF) = P(E \cap F) = \frac{1}{5}$$

$$P(E) \cdot P(F) = \frac{3}{5} \cdot \frac{3}{10} = \frac{9}{50} \neq \frac{1}{5}$$

$\Rightarrow P(E) \cdot P(F) \neq P(EF)$

Therefore, E and F are not independent. **[2]**

Q. 6. Let A and B be independent events with $P(A)$ = 0.3 and $P(B)$ = 0.4. Find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$

(iii) $P(A|B)$ (iv) $P(B|A)$

[NCERT Ex. 13.2, Q. 8, Page 547]

Ans. It is given that,

$P(A) = 0.3$ and $P(B) = 0.4$

(i) If A and B are independent events, then

$P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 12$

(ii) It is known that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$

(iii) It is known that,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A|B) = \frac{0.12}{0.4} = 0.3$$

(iv) It is known that,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{0.12}{0.3} = 0.4$$

[2]

Q. 7. Given two independent events A and B such that $P(A)$ = 0.3, $P(B)$ = 0.6. Find

(i) $P(A$ and $B)$ (ii) $P(A$ and not $B)$

(iii) $P(A$ or $B)$ (iv) $P($neither A nor $B)$

[NCERT Ex. 13.2, Q. 11, Page 546]

Ans. It is given that,

$P(A) = 0.3$ and $P(B) = 0.6$

Also, A and B are independent events.

(i) $\therefore P(A \text{ and } B) = P(A) \cdot P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$

(ii) $P(A \text{ and not } B) = P(A \cap B')$

$= P(A) - P(A \cap B)$

$= 0.3 - 0.18$

$= 0.12$

(iii) $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + (B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18$

$= 0.72$

(iv) $P \text{ (neither } A \text{ nor } B) = P(A' \cap B')$

$= P((A \cup B)')$

$= 1 - P(A \cup B)$

$= 1 - 0.72$

$= 0.28$ **[2]**

Q. 8. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
$P(X)$	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
$P(X)$	0.1	0.5	0.2	–0.1	0.3

(iii)

Y	–1	0	1
$P(Y)$	0.6	0.1	0.2

(iv)

Z	3	2	1	0	–1
$P(Z)$	0.3	0.2	0.4	0.1	0.03

[NCERT Ex. 13.4, Q. 1, Page 569]

Ans. It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = 0.4 + 0.4 + 0.2 = 1

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for $X = 3$, $P(X) = -0.1$

It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = $0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities = $0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$

Therefore, the given table is not a probability distribution of random variables. **[2]**

Q. 9. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

[NCERT Ex. 13.4, Q. 2, Page 570]

Ans. The two balls selected can be represented as *BB*, *BR*, *RB*, and *RR*, where *B* represents a black ball and R represents a red ball. X represents the number of black balls.

$\therefore X(BB) = 2$

$X(BR) = 1$

$X(RB) = 1$

$X(RR) = 0$

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable. **[2]**

Q. 10. Find the probability distribution of

(i) Number of heads in two tosses of a coin.

(ii) Number of tails in the simultaneous tosses of three coins.

(iii) Number of heads in four tosses of a coin.

[NCERT Ex. 13.4, Q. 4, Page 570]

Ans. (i) When one coin is tossed twice, the sample space is *(HH, HT, TH, TT)*

Let X represents the number of heads.

$\therefore X(HH) = 2, X(HT) = 1, X(TH) = 1, X(TT) = 0$

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$

$P(X = 0) = P(TT) = \frac{1}{4}$

$P(X = 1) = P(HT) + P(TH) = \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

$P(X = 2) = P(HH) = \frac{1}{4}$

Thus, the required probability distribution is as follows :

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

[2]

(ii) When three coins are tossed simultaneously, the sample space is {*HHH, HHT, HTH, HTT, THH, THT, TTH, TTT*}

Let X represents the number of tails.

It can be seen that X can take the value of 0, 1, 2 or 3.

$P(X = 0) = P(HHH) = \frac{1}{8}$

$P(X = 1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

$P(X = 2) = P(HTT) + P(THT) + P(TTH) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

$P(X = 3) = P(TTT) = \frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

[2]

(iii) When a coin is tossed four times, the sample space is

$$S = \begin{Bmatrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, \\ HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, \\ TTTH, TTTT \end{Bmatrix}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take value of 0, 1, 2, 3, or 4.

$P(X = 0) = P(TTTT) = \frac{1}{16}$

$P(X = 1) = P(TTTH) + P(TTHT) + P(THTT) + P(HTTT)$

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

$P(X = 2) = P(HHTT) + P(THHT) + P(TTHH) + P(HTTH) + P(HTHT) + P(THTH)$

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$

$P(X = 4) = P(HHHH) = \frac{1}{16}$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

[2]

Q. 11. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

(i) k **(ii)** $P(X < 3)$

(iii) $P(X > 6)$ **(iv)** $P(0 < X < 3)$

[NCERT Ex. 13.4, Q. 8, Page 570]

Ans. (i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow (10k - 1)(k + 1) = 0$

$$\Rightarrow \qquad k = -1, \frac{1}{10}$$

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore \qquad k = \frac{1}{10}$$

[2]

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$ **[2]**

(iii) $P(X > 6) = P(X = 7)$

$7k^2 + k$

$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$

$= \frac{7}{100} + \frac{1}{10}$

$= \frac{17}{100}$ **[2]**

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$ **[2]**

Q. 12. If X follows binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9$, $P(X = 3)$, then p = ___________

[NCERT Exemp. Ex. 13.3, Q. 106, Page 286]

Ans.

$\therefore P(X = 2) = 9 \cdot P(X = 3) \qquad$ (where, $n = 5$ and $q = 1 - p$)

$\Rightarrow \quad {}^5C_2 p^2 (1-p)^3 = 9 \cdot {}^5C_3 p^3 (1-p)^2$

$\Rightarrow \frac{5!}{2!3!} p^2 (1-p)^3 = 9 \cdot \frac{5!}{3!2!} p^3 (1-p)^2$

$\Rightarrow \qquad \frac{p^2(1-p)^3}{p^3(1-p)^2} = 9$

$\Rightarrow \qquad \frac{(1-p)}{p} = 9 \Rightarrow 9p + p = 1$

$\therefore \qquad p = \frac{1}{10}$

[2]

Q. 13. Let X be a random variable taking values $x_1, x_2, ..., x_n$ with probabilities $p_1, p_2, ..., p_n$, respectively. Then var.(X) = ________

[NCERT Exemp. Ex. 13.3, Q. 107, Page 286]

Ans. Var.(X) = $E(X)^2 - [E(X)]^2$

$$= \sum_{i=1}^{n} X^2 P(X) - \left[\sum_{i=1}^{n} X P(X)\right]^2$$

$$= \sum P_i x_i^2 - (\sum P_i x_i)^2$$ **[2]**

Q. 14. Let A and B be two events. If $P(A|B) = P(A)$, then A is __________ of B.

[NCERT Exemp. Ex. 13.3, Q. 108, Page 286]

Ans.

$\because \qquad P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \qquad P(A) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad P(A) \cdot P(B) = P(A \cap B)$

So, A is independent of B. **[2]**

Q. 15. If A and B are two events such that $P(A|B) = p$, $P(A) = p$, $P(B) = 1/3$ and $P(A \cup B) = 5/9$, then p = _____.

[NCERT Exemp. Ex. 13.3, Q. 104, Page 286]

Ans. Here,

$$P(A) = p, P(B) = \frac{1}{3} \text{ and } P(A \cup B) = \frac{5}{9}$$

$\because \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = p$

$\Rightarrow \quad P(A \cap B) = \frac{p}{3}$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \qquad \frac{5}{9} = p + \frac{1}{3} - \frac{p}{3}$

$\Rightarrow \qquad \frac{5}{9} - \frac{1}{3} = \frac{2p}{3}$

$\Rightarrow \qquad \frac{5-3}{9} = \frac{2p}{3}$

$\Rightarrow \qquad p = \frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$ **[2]**

Q. 16. If A and B are such that $P(A' \cup B') = 2/3$ and $P(A \cup B) = 5/9$, then $P(A') + P(B')$ = ...

[NCERT Exemp. Ex. 13.3, Q. 105, Page 286]

Ans. Here,

$$P(A' \cup B') = \frac{2}{3} \text{ and } P(A \cup B) = \frac{5}{9}$$

$$P(A' \cup B') = 1 - P(A \cap B)$$

$$\Rightarrow \quad \frac{2}{3} = 1 - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{9} = [1 - P(A') + (1 - P(B')] - \frac{1}{3}$$

$$\frac{5}{9} = \frac{2-1}{3} - [P(A') + P(B')]$$

$$P(A') + P(B') = 2 - \frac{1}{3} - \frac{5}{9}$$

$$= \frac{18-3-5}{9}$$

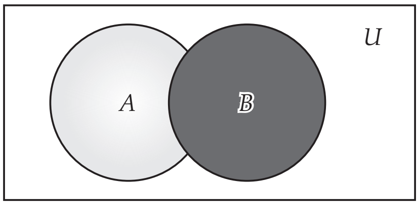
$$= \frac{10}{9}$$ **[2]**

Q. 17. If A and B' are independent events, then $P(A' \cup B) = 1 - P(A)\,P(B')$.

[NCERT Exemp. Ex. 13.3, Q. 100, Page 286]

Ans. True

$$P(A' \cup B) = 1 - P(A \cap B') = 1 - P(A)P(B')$$



$(A' \cup B)$ **[2]**

Q. 18. If A and B are two events such that $P(A) > 0$ and $P(A) + P(B) > 1$, then $P(B|A) \geq 1 - \frac{P(B')}{P(A)}$.

[NCERT Exemp. Ex. 13.3, Q. 102, Page 286]

Ans. False

$$\because P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{P(A) + P(B) - P(A \cup B)}{P(A)} > \frac{1 - P(A \cup B)}{P(A)}$$

[2]

Q. 19. If A, B and C are three independent events such that $P(A) = P(B) = P(C) = p$, then P (At least two of A, B, C occur) $= 3p^2 - 2p^3$

[NCERT Exemp. Ex. 13.3, Q. 103, Page 286]

Ans. True

P(at least two of A, B and C occur)

$p \times p \times (1 - p) + (1 - p)\,p\,p + p(1 - p)\,p + p\,p\,p$

$= p^2\,[1 - p + 1 - p + 1 - p + p]$

$= p^2\,(3 - 3p) + p^3$

$= 3p^2 - 3p^3 + p^3 = 3p^2 - 2p^3$ **[2]**

Q. 20. The probability that at least one of the two events A and B occurss is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\overline{A}) + P(\overline{B})$.

[NCERT Exemp. Ex. 13.3, Q. 3, Page 271]

Ans. We know that, $A \cup B$ denotes the occurrence of at least one of A and B and $A \cap B$ denotes the occurrence of both A and B, simultaneously.

Thus, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad 0.6 = P(A) + P(B) - 0.3$$

$$\Rightarrow \quad P(A) + P(B) = 0.9$$

$$\Rightarrow \quad [1 - P(\overline{A})] + [1 - P(\overline{B})] = 0.9$$

$$[\because P(A) = 1 - P(\overline{A}) \text{ and } P(B) = 1 - P(\overline{B})]$$

$$\Rightarrow \quad P(\overline{A}) + P(\overline{B}) = 2 - 0.9 = 1.1$$ **[2]**

Q. 21. Three events A, B and C have probabilities 2/5, 1/3 and 1/2, respectively. Given that $P(A \cap C) = 1/5$ and $P(B \cap C) = 1/4$, find the values of $P(C|B)$ and $P(A' \cap C')$.

[NCERT Exemp. Ex. 13.3, Q. 8, Page 272]

Ans. Here,

$$P(A) = \frac{2}{5}, P(B) = \frac{1}{3}, P(C) = \frac{1}{2}, P(A \cap C) = \frac{1}{5} \text{ and } P(B \cap C) = \frac{1}{4}$$

$$\therefore \quad P(C/B) = \frac{P(B \cap C)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

and $P(A' \cap C') = 1 - P(A \cup C)$

$$= 1 - [P(A) + P(C) - P(A \cap C)]$$

$$= 1 - \left[\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right]$$

$$= 1 - \left[\frac{4+5-2}{10}\right]$$

$$= 1 - \frac{7}{10} = \frac{3}{10}$$

[2]

Q. 22. Suppose 10,000 tickets are sold in a lottery each for Re 1. First prize is of Rs 3000 and the second prize is of Rs. 2000. There are three third prizes of Rs. 500 each. If you buy one ticket, what is your expectation.

[NCERT Exemp. Ex. 13.3, Q. 15, Page 273]

Ans. Let X is the random variable for the prize.

X	0	500	2000	3000
$P(X)$	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

Since, $E(X) = \sum XP(X)$

$$\therefore \quad E(X) = 0 \times \frac{9995}{10000} + \frac{1500}{10000} + \frac{2000}{10000} + \frac{3000}{10000}$$

$$= \frac{1500 + 2000 + 3000}{10000}$$

$$= \frac{6500}{10000} = \frac{13}{20} = \text{Rs. } 0.65$$ **[2]**

Q. 23. Bag I contains 3 black and 2 white balls, Bag II contains 2 black and 4 white balls. A bag and a ball is selected at random. Determine the probability of selecting a black ball.

[NCERT Exemp. Ex. 13.3, Q. 17, Page 273]

Ans. Bag I = $\{3B, 2W\}$, Bag II = $\{2B, 4W\}$

Let E_1 = Event that bag I is selected

E_2 = Event that bag II is selected

and E= Event that a black ball is selected

$$\Rightarrow\ P(E_1)=1/2, P(E_2)=\frac{1}{2}, P(E/E_1)=\frac{3}{5}, P(E/E_2)=\frac{2}{6}=\frac{1}{3}$$

$$\therefore\ P(E)=P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)$$

$$=\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{2}{6}=\frac{3}{10}+\frac{2}{12}$$

$$=\frac{18+10}{60}=\frac{28}{60}=\frac{7}{15}$$

[2]

Q. 24. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue?

[NCERT Exemp. Ex. 13.3, Q. 18, Page 273]

Ans. A box = {5 blue, 4 red}

Let E_1 is the event that first ball drawn is blue, E_2 is the event that first ball drawn is red and E is the event that second ball drawn is blue.

$\therefore P(E) = P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)$

$$=\frac{5}{9}\cdot\frac{4}{8}+\frac{4}{9}\cdot\frac{5}{8}=\frac{20}{72}+\frac{20}{72}=\frac{40}{72}=\frac{5}{9}$$ **[2]**

Q. 25. Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all the four cards are kings?

[NCERT Exemp. Ex. 13.3, Q. 19, Page 273]

Ans. Let E_1, E_2, E_3 and E_4 are the events that the first, second, third and fourth card is king, respectively.

$$\therefore\ P(E_1\cap E_2\cap E_3\cap E_4)=P(E_1)\cdot P(E_2/E_1)$$
$$\cdot P(E_3/E_1\cap E_2)\cdot P[E_4/(E_1\cap E_2\cap E_3\cap E_4)]$$

$$=\frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{1}{49}=\frac{24}{52\cdot 51\cdot 50\cdot 49}$$

$$=\frac{1}{13\cdot 17\cdot 25\cdot 49}=\frac{1}{270725}$$

[2]

Q. 26. *A* die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

[NCERT Exemp. Ex. 13.3, Q. 20, Page 273]

Ans. Here,

$$n=5, p=\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right)=\frac{1}{2}\text{ and } q=1-p=1-\frac{1}{2}=\frac{1}{2}$$

Also, r = 3

$$\therefore\ P(X=r)={}^nC_r,(p)^r(q)^{n-r}={}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$$

$$=\frac{5!}{3!2!}\cdot\frac{1}{8}\cdot\frac{1}{4}=\frac{10}{32}=\frac{5}{16}$$ **[2]**

Q. 27. A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?

[NCERT Exemp. Ex. 13.3, Q. 23, Page 273]

Ans. Probability of defective watch from a lot 100 watches $=\frac{10}{100}=\frac{1}{10}$

$$\therefore\ p=1/10, q=\frac{9}{10}, n=8\text{ and } r\ge 1$$

$$\therefore\ P(r\ge 1)=1-P(r=0)=1-{}^8C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{8-0}$$

$$=1-\frac{8!}{0!8!}\cdot\left(\frac{9}{10}\right)^8=1-\left(\frac{9}{10}\right)^8$$

[2]

Q. 28. Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

[NCERT Exemp. Ex. 13.3, Q. 32, Page 275]

Ans. Let,

E_1 = Event that fair coin is drawn

E_2 = Event that 2 headed coin is drawn

E = Event that tossed coin get a head

$\therefore P(E_1) = 1/2$, $P(E_2) = 1/2$, $P(E / E_1) = 1/2$ and $P(E / E_2) = 1$

Now, using Baye's theorem :

$$P(E_1/E)=\frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot 1}=\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$$ **[2]**

Q. 29. Two natural numbers *r*, *s* are drawn one at a time, without replacement from the set *S*= 1, 2, 3, ..., *n*. Find *P* [$r \le p | s \le p$], where $p \in S$.

[NCERT Exemp. Ex. 13.3, Q. 34, Page 275]

Ans. $\because$ Set $S = \{1, 2, 3, \ldots, n\}$

$$\therefore P(r\le p/s\le p)=\frac{P(p\cap S)}{P(S)}$$

$$=\frac{p-1}{n}\times\frac{n}{n-1}=\frac{p-1}{n-1}$$ **[2]**

Q. 30. *A* die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let *A* be the event "number obtained is even" and *B* be the event "number obtained is red". Find if *A* and *B* are independent events. [CBSE Board, All India Region, 2017]

Ans. Event,

A : Number obtained is even

B : Number obtained is red.

$$P(A)=\frac{3}{6}=\frac{1}{2}, P(B)=\frac{3}{6}=\frac{1}{2}$$

$$P(A\cap B)=P(\text{getting an even red number})=\frac{1}{6}$$

Since $P(A)\cdot P(B)=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}\ne P(P\cap B)$ which is $\frac{1}{6}$

$\therefore$ A and B are not independent events. **[2]**

Q. 31. Prove that if *E* and *F* are independent events, then the events *E* and *F′* are also independent.

[CBSE Board, Delhi Region, 2017]

Ans. $P(E\cap F') = P(E) - P(E\cap F)$

$= P(E) - P(E)\cdot P(F)$

$= P(E)[1 - P(F)]$

$= P(E)P(F')$

$\Rightarrow E$ and F' are independent events. **[2]**

Q. 32. If $P(A) = 0{\cdot}4$, $P(B) = p$, $P(A \cup B) = 0{\cdot}6$ and A and B are given to be independent events, find the value of 'p'. [CBSE Board, Foreign Scheme, 2017]

Ans. Given that,

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= P(A) + P(B) - P(A)\, P(B)$

[As A and B are independent events]

$\therefore\quad 0.6 = 0.4 + p - (0.4)p$

$\Rightarrow p = \dfrac{1}{3}$ **[2]**

Q. 33. In a game, a man wins ₹ 5 for getting a number than 4and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses. [CBSE Board, All India Region, 2016]

Ans. let X = Amount he wins then x = ₹ 5, 4, 3, –3

P= Probability of getting a no. $> 4 = \dfrac{1}{3}, q = 1 - p = \dfrac{2}{3}$

X	5	4	3	–3
$P(X)$	$\dfrac{1}{3}$	$\dfrac{2}{3}\cdot\dfrac{1}{3}=\dfrac{2}{9}$	$\left(\dfrac{2}{3}\right)^2\cdot\dfrac{1}{3}=\dfrac{4}{27}$	$\left(\dfrac{2}{3}\right)^3=\dfrac{8}{27}$

Expected amount he wins $= \Sigma XP(X) = \dfrac{5}{3} + \dfrac{8}{9} + \dfrac{12}{27} - \dfrac{24}{27}$

$= ₹\ \dfrac{19}{9}$ or ₹ $2\dfrac{1}{9}$ **[2]**

Q. 34. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white? [CBSE Board, All India Region, 2016]

Ans. Let,

E_1 = Event that all balls are white

E_2 = Event that 3 balls are white and 1 ball is non white

E_3 = Event that 2 balls are white and 2 balls are non-white

A = Event that 2 balls drawn without replacement are white

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = 1, P(A/E_2) = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2}, P(A/E_3) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$$

$$P(E_1/A) = \frac{1\cdot \frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{6}} = \frac{3}{5}$$

[2]

Q. 35. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die in number less than 4. [CBSE Board, Delhi Region, 2018]

Ans. Let,

A be the event when the sum of the observation is 8 = {(2, 6), (3, 5), (5, 3), (4, 4), (6, 2)}

$\therefore n(E) = 5$

Let B be the event when the observation on red die is less than 4

= {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (5, 3), (6, 2)……… (6, 3)}

$\therefore n(B) = 18$

$$\Rightarrow \quad P(B) = \frac{18}{36} = \frac{1}{2}$$

Now, $n(A\cap B) = 2$

$$\Rightarrow \quad n(A\cap B) = \frac{2}{36} = \frac{1}{18}$$

$$\therefore \quad P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{1}{2}} = \frac{1}{9}$$

[2]

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denotes the larger of the two numbers obtained. Find the mean and variance of X. [CBSE Board, Delhi Region, 2018]

Ans. The first positive integers are 1, 2, 3, 4 and 5.

We can select two numbers from 5 numbers in ${}^5P_2 = 5 \times 4 = 20$

Now, let X denotes the larger of two numbers, then X can take values from 2, 3, 4 and 5.

For X = 2, the possible observations are (1, 2) and (2, 1)

$\therefore P(X = 2) = \dfrac{2}{20}$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1) and (3, 2)

$\therefore P(X = 3) = \dfrac{4}{20}$

For X = 4, the possible are (1, 4), (2, 4), (3, 4), (4, 1), (4, 2) and (4, 3)

$\therefore P(X = 4) = \dfrac{6}{20}$

For X = 5, the possible observations are (1, 5), (2, 5), (3,5) (4, 5), (5, 1), (5, 2), (5, 3) and (5, 4)

$\therefore P(X = 8) = \dfrac{8}{20}$

X	2	3	4	5
$P(X)$	$\frac{2}{20}$	$\frac{4}{20}$	$\frac{6}{20}$	$\frac{8}{20}$

Therefore, Mean $[E(X)] = 2\times\frac{2}{20}+3\times\frac{4}{20}+4\times\frac{6}{20}+5\times\frac{8}{20}$

$=\frac{4}{20}+\frac{12}{20}+\frac{24}{20}+\frac{40}{20}$

$=\frac{80}{20}$

$=4$

$E(X^2)=2^2\times\frac{2}{20}+3^2\times\frac{4}{20}+4^2\times\frac{6}{20}+5^2\times\frac{8}{20}$

$=\frac{8}{20}+\frac{36}{20}+\frac{64}{20}+\frac{200}{20}$

$=\frac{340}{20}$

$=17$

Variance = $E(X^2) - [E(X)^2]$

$= 17 - (4)^2$

$= 1$

Therefore, mean and variance are 4 and 1 respectively. **[4]**

Q. 2. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail' what is the probability that she threw 3, 4, 5 or 6 with the die? [CBSE Board, Delhi Region, 2018]

Ans. Let A_1 be the event when she get 1 or 2 and A_1 be the event when she get 3, 4, 5 or 6.

$\therefore P(A_1)=\frac{2}{6}=\frac{1}{3}$ and $P(A_2)=\frac{4}{6}=\frac{2}{3}$

Le B be the event of getting exactly one tail.

$\therefore P(B/A_1)=\frac{3}{8}$ and $P(B/A_2)=\frac{1}{2}$

Required probability = $P(A_2/B)$

$=\frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)}$

$=\frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$

$=\frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{11}{24}}=\frac{8}{11}$ **[4]**

Q. 3. A box has 20 pens of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement, at most 2 are defective. [CBSE Board, Foreign Scheme, 2016]

Ans. Let selection of defective pen be considered success

$$\text{p}=\frac{2}{20}=\frac{1}{10}\cdot\text{q}=\frac{9}{10}$$

Required probability = $P(x = 0) + P(x = 1) + P(x = 2)$

$={}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5+{}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4+{}^5C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^3$

$=\left(\frac{9}{10}\right)^5+\frac{1}{2}\left(\frac{9}{10}\right)^4+\frac{1}{10}\times\left(\frac{9}{10}\right)^3$

$=\left(\frac{9}{10}\right)^3\times\frac{34}{25}$

[4]

Q. 4. Let, X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)\begin{cases}kx & , \text{ if } x=0 \text{ or } 1\\ 2kx & , \text{ if } x=2\\ k(5-x) & , \text{ if } x=3 \text{ or } 4'\\ 0 & , \text{ if } x>4\end{cases}$$

Where k is a positive constant. Find the value of k. Also find the probability that you will get admission in (i) exactly one college (ii) at most 2 colleges (iii) at least 2 colleges.

[CBSE Board, Foreign Scheme, 2016]

Ans. $\sum_{i=0}^{4}P(x_i)=1$

$\Rightarrow \quad 8k=1\Rightarrow k=\frac{1}{8}$

(i) $P(x=1)=\frac{1}{8}$

(ii) P(at most 2 colleges) = $P(0) + P(1) + P(2)$ $=\frac{5}{8}$

(iii) P(at least 2 colleges) = $1 - [P(x = 0) + P(x = 1)]$ **[4]**

$= -\frac{1}{8}=\frac{7}{8}$

Q. 5. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on the two drawn cards. Find the mean and variance of X.

[CBSE Board, All India Region, 2017]

Ans. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore$ X:	4	6	8	10	12
$P(X)$:	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$xP(X)$:	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{16}{6}$	$\frac{10}{6}$	$\frac{12}{6}$
$x^2P(X)$:	$\frac{16}{6}$	$\frac{36}{6}$	$\frac{128}{6}$	$\frac{100}{6}$	$\frac{144}{6}$

$\Sigma xP(x) = \frac{48}{6} = 8 \therefore \text{Mean} = 8$

Variance $= \Sigma x^2P(x) - [\Sigma xP(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$ **[4]**

Q. 6. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $Sp_i x_i^2 i = 2Sp_i x_i$, find the value of p. [CBSE Board, Delhi Region, 2017]

Ans.

x	$P(x)$
0	p
1	p
2	K
3	k

$\Sigma P(x) = 1$

$\Rightarrow 2p + 2k = 1$

$\therefore \quad k = \frac{1}{2} - p$

x_i	p_i	p_ix_i	$p_ix_i^2$
0	p	0	0
1	p	p	P
2	$\frac{1}{2} - p$	$1 - 2p$	$2 - 4p$
3	$\frac{1}{2} - p$	$\frac{3}{2} - 3p$	$\frac{9}{2} - 9p$
		$\frac{5}{2} - 4p$	$\frac{13}{2} - 12p$

As per problem, $\Sigma p_ix_i^2 = 2\Sigma p_ix_i$

$\Rightarrow \quad p = \frac{3}{8}$ **[4]**

Q. 7. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six.
Find the probability that it is actually a six.
Do you also agree that the value of truthfulness leads to more respect in the society?
[CBSE Board, Delhi Region, 2017]

Ans. Let H_1 be the event that 6 appears on throwing a die
H_2 be the event that 6 does not appear on throwing a die
E be the event that he reports it is six

$$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$$

$$P(H_1/E) = \frac{P(H_1)\cdot P(E/H_1)}{P(H_1)\cdot P(E/H_1) + P(H_2)P(E/H_2)}$$

$$= \frac{4}{9}$$

Relevant value : Yes, Truthfulness leads to more respect in society. **[4]**

Q. 8. There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on the two drawn cards. Find the mean and variance of X.
[CBSE Board, Foreign Scheme, 2017]

Ans. X can take the values 3, 4, 5, 6 and 7.

$\therefore\ X:$	3	4	5	6	7
$P(X)$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$
$=$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
$X.P(X):$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{3}$	$\frac{6}{6}$	$\frac{7}{6}$
$X^2P(X):$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{3}$	$\frac{36}{6}$	$\frac{49}{6}$

$\therefore$ Mean $= \Sigma X\cdot P(X) = \frac{30}{6} = 5$

Variance $= \Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2 = \frac{160}{6} - 25 = \frac{5}{3}$

[4]

Q. 9. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.
[CBSE Board, All India Region, 2017]

Ans. Let,

E_1 : Selecting a student with 100% attendance

E_2 : Selecting a student who is not regular

A : Selected student attains A grade.

$$P(E_1) = \frac{30}{100} \text{ and } P(E_2) = \frac{70}{100}$$

$$P(A/E_1) = \frac{70}{100} \text{ and } P(A/E_2) = \frac{10}{100}$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100} + \frac{70}{100}\times\frac{10}{100}}$$

$$= \frac{3}{4}$$

Regularity is required everywhere or any relevant value. **[4]**

Q. 10. In a shop X, 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are kept for sale while in shop Y, similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. Find the probability that it is

purchased from shop *Y*. What measures should be taken to stop adulteration?
[CBSE Board, Foreign Scheme, 2017]

Ans. Let,

E_1 = Ghee purshased from shop X

E_2 = Ghee purshased from shop Y

A = Getting adultered ghee

$$P(E_1) = P(E_2) = \frac{1}{2},$$

$$P(A/E_1) = \frac{4}{7},$$

$$P(A/E_2) = \frac{6}{11}$$

$$P(E_2/A) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{6}{11}+\frac{1}{2}\times\frac{4}{7}} = \frac{21}{43}$$

Second part : Stringent punishment for the adulterators or any suitable measure. **[4]**

Q. 11. Three persons *A, B* and *C* apply for a job of Manager in a Private Company. Chances of their selection (*A, B* and *C*) are in the ratio 1 :2 :4. The probabilities that *A, B* and *C* can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of *C*.
[CBSE Board, Delhi Region, 2016]

Ans. Let events are :

$E_1 : A$ is selected

$E_2 : B$ is selected

$E_3 : C$ is selected

A: Change is not introduced

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$$

$$P(A/E_1) = 0.2, P(A/E_2) = 0.5, P(A/E_3) = 0.7$$

$$\therefore P(E_3/A) = \frac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$$

$$= \frac{28}{40} = \frac{7}{10}$$

[4]

Q. 12. *A* and *B* throw a pair of dice, alternately. *A* wins the game if he gets a total of 7 and *B* wins the game if he gets a total of 10. If *A* starts the game, then find the probability that *B* wins.
[CBSE Board, Delhi Region, 2016]

Ans.

Prob. of success for $A = \frac{1}{6}$

Prob. of failure for $A = \frac{5}{6}$

Prob. of success for $B = \frac{1}{12}$

Prob. of failure for $B = \frac{11}{12}$

B can win in 2nd or 4th or 6th or....throw

$$\therefore P(B) = \left(\frac{5}{6}\cdot\frac{1}{12}\right)+\left(\frac{5}{6}\cdot\frac{11}{12}\cdot\frac{5}{6}\cdot\frac{1}{12}\right)+\left(\frac{5}{6}\cdot\frac{11}{12}\cdot\frac{5}{6}\cdot\frac{11}{12}\cdot\frac{5}{6}\cdot\frac{1}{12}\right)+\ldots$$

$$= \frac{5}{72}\left(1+\frac{55}{72}+\left(\frac{55}{72}\right)^2+\ldots.\right)$$

$$= \frac{5}{72}\times\frac{1}{1-\frac{55}{72}} = \frac{5}{72}\times\frac{72}{17} = \frac{5}{17}$$

[4]

Q. 13. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find
(i) $P(A\cap B)$ (ii) $P(A|B)$
(iii) $P(A\cup B)$ [NCERT Ex. 13.1, Q. 3, Page 538]

Ans. It is given that,

$P(A) = 0.8$, $P(B) = 0.5$, and $P(B|A) = 0.4$

(i) $P(B|A) = 0.4$

$$\therefore \frac{P(A\cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A\cap B)}{0.8} = 0.4$$

$$\Rightarrow P(A\cap B) = 0.32$$

(ii) $$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

(iii) $$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow P(A\cup B) = 0.8+0.5-0.32 = 0.98$$

[3]

Q. 14. Evaluate $P(A\cup B)$, if $2P(A) = P(B) = 5/13$ and $P(A|B) = 2/5$ [NCERT Ex. 13.1, Q. 4, Page 538]

Ans. It is given that,

$$2P(A) = P(B) = \frac{5}{13}$$

$$\Rightarrow P(A) = \frac{5}{26} \text{ and } P(B) = \frac{5}{13}$$

$$P(A|B) = \frac{2}{5}$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{2}{5}$$

$$\Rightarrow P(A\cap B) = \frac{2}{5}\times P(B) = \frac{2}{5}\times\frac{5}{13} = \frac{2}{13}$$

It is known that,

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow P(A\cup B) = \frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$

$$\Rightarrow P(A\cup B) = \frac{5+10-4}{26}$$

$$\Rightarrow P(A\cup B) = \frac{11}{26}$$

[3]

Q. 15. If $P(A) = 6/11$, $P(B) = 5/11$ and $P(A\cup B) = 7/11$, find
(i) $P(A\cap B)$ (ii) $P(A|B)$
(iii) $P(B|A)$ [NCERT Ex. 13.1, Q. 5, Page 538]

Ans. It is given that,

$$P(A) = \frac{6}{11}, P(B) = \frac{5}{11}, \text{ and } P(A\cup B) = \frac{7}{11}$$

(i) $P(A \cup B) = \frac{7}{11}$

$\therefore P(A) + P(B) - P(A \cap B) = \frac{7}{11}$

$\Rightarrow \quad \frac{6}{11} + \frac{5}{11} - P(A \cap B) = \frac{7}{11}$

$\Rightarrow \quad P(A \cap B) = \frac{11}{11} - \frac{7}{11} = \frac{4}{11}$

(ii) It is known that,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$$

(iii) It is known that,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6} = \frac{2}{3}$$

[3]

Q. 16. *A* coin is tossed three times, where

(i) *E* : head on third toss, *F* : heads on first two tosses

(ii) *E* : at least two heads, *F* : at most two heads

(iii) *E* : at most two tails, *F* : at least one tail

Determine *P(E|F)*.

[NCERT Ex. 13.1, Q. 6, Page 538]

Ans. If a coin is tossed three, then the sample space *S* is

S = {*HHH, HHT, HTH, HTT, THH, THT, TTH, TTT*}

It can be seen that the sample space has 8 elements.

(i) *E* = {*HHH, HTH, THH, TTH*}

F = {*HHH, HHT*}

$\therefore$ *E*∩*F* = {*HHH*}

$$P(F) = \frac{2}{8} = \frac{1}{4} \text{ and } P(E \cap F) = \frac{1}{8}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{4}{8} = \frac{1}{2}$$

(ii) *E* = {*HHH, HHT, HTH, THH*}

F = {*HHT, HTH, HTT, THH, THT, TTH, TTT*}

$\therefore$ *E*∩*F* = {*HHT, HTH, THH*}

Clearly,

$$P(E \cap F) = \frac{3}{8} \text{ and } P(F) = \frac{7}{8}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}$$

(iii) *E* = {*HHH, HHT, HTT, HTH, THH, THT, TTH*}

F = {*HHT, HTT, HTH, THH, THT, TTH, TTT*}

$\therefore$ *E*∩*F* = {*HHT, HTT, HTH, THH, THT, TTH*}

$$P(F) = \frac{7}{8} \text{ and } P(E \cap F) = \frac{6}{8}$$

Therefore,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$$

[3]

Q. 17. Two coins are tossed once, where

(i) *E* : tail appears on one coin, *F* : one coin shows head

(ii) *E* : no tail appears, F : no head appears Determine *P(E|F)*. **[NCERT Ex. 13.1, Q. 7, Page 539]**

Ans. If two coins are tossed once, then the sample space *S* is

S = {*HH, HT, TH, TT*}

(i) *E* = {*HT, TH*}

F = {*HT, TH*}

$\therefore \quad E \cap F = \{HT, TH\}$

$$P(F) = \frac{2}{4} = \frac{1}{2}$$

$$P(E \cap F) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{2} = 1$$

(ii) *E* = {*HH*}

F = {*TT*}

$\therefore \quad E \cap F = \Phi$

$$P(F) = \frac{1}{4} \text{ and } P(E \cap F) = 0$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{1}{4}} = 0$$

[3]

Q. 18. *A* die is thrown three times, *E* : 4 appears on the third toss, *F* : 6 and 5 appears respectively on first two tosses

Determine *P(E|F)*.

[NCERT Ex. 13.1, Q. 8, Page 539]

Ans. If a die is thrown three times, then the number of elements in the sample space = 6 × 6 × 6 = 216

$$E = \begin{bmatrix} (1,1,4),(1,2,4),...(1,6,4) \\ (2,1,4),(2,2,4),...(2,6,4) \\ (3,1,4),(3,2,4),...(3,6,4) \\ (4,1,4),(4,2,4),...(4,6,4) \\ (5,1,4),(5,2,4),...(5,6,4) \\ (6,1,4),(6,2,4),...(6,6,4) \end{bmatrix}$$

$$F = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$\therefore \quad E \cap F = \{(6,5,4)\}$

$$P(F) = \frac{6}{216} \text{ and } P(E \cap F) = \frac{1}{216}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

[3]

Q. 19. Mother, father and son line up at random for a family picture

***E* : son on one end, *F* : father in middle**

Determine *P(E|F)*.[NCERT Ex. 13.1, Q. 9, Page 539]

Ans. If mother (M), father (F) and son (S) line up for the family picture, then the sample space will be

$S = \{MFS, MSF, FMS, FSM, SMF, SFM\}$

$\Rightarrow \quad E = \{MFS, FMS, SMF, SFM\}$

$F = \{MFS, SFM\}$

$\therefore \quad E \cap F = \{MFS, SFM\}$

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

[3]

Q. 20. *A* box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15oranges out of which 12 are good and 3 are bad ones will be approved for sale.

[NCERT Ex. 13.2, Q. 3, Page 546]

Ans. Let A, B and C be the respective events that the first, second and third drawn orange is good.

Therefore, probability that first drawn orange is good, $P(A) = \frac{12}{15}$

The oranges are not replaced.

Therefore, probability of getting second orange good, $P(B) = \frac{11}{14}$

Similarly, probability of getting third orange is good, $P(C) = \frac{10}{13}$

The box is approved for sale, it all the three oranges are good.

Thus, probability of getting all the oranges good

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$$

Therefore, the probability that the box is approved for sale is $\frac{44}{91}$. **[3]**

Q. 21. *A* fair coin and an unbiased die are tossed. Let *A* be the event 'head appears on the coin' and *B* be the event '3 on the die'. Check whether *A* and *B* are independent events or not.

[NCERT Ex. 13.2, Q. 4, Page 546]

Ans. If a fair coin and an unbiased die are tossed, then the sample space S is given by,

$$S = \begin{Bmatrix} (H,1),(H,2),(H,3),(H,4),(H,5),(H,6), \\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \end{Bmatrix}$$

Let A : Head appears on the coin

$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),\}$

$$\Rightarrow P(A) = \frac{6}{12} = \frac{1}{2}$$

B : 3 on die $= \{(H,3), (T,3)\}$

$$P(B) = \frac{2}{12} = \frac{1}{6}$$

$$\therefore \quad A \cap B = \{(H,3)\}$$

$$P(A \cap B) = \frac{1}{12}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{6} = P(A \cap B)$$

Therefore, A and B are independent events. **[3]**

Q. 22. *A* die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let *A* be the event, 'the number is even,' and *B* be the event, 'the number is red'. Are *A* and *B* independent?

[NCERT Ex. 13.2, Q. 5, Page 546]

Ans. When a die is thrown, the sample space (S) is

$S = \{1, 2, 3, 4, 5, 6\}$

Let A : the number is even $= \{2, 4, 6\}$

$$\Rightarrow \quad P(A) = \frac{3}{6} = \frac{1}{2}$$

B : the number is red $= \{1, 2, 3\}$

$$\Rightarrow \quad P(B) = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \quad A \cap B = \{2\}$$

$$P(AB) = P(A \cap B) = \frac{1}{6}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6}$$

$$\Rightarrow P(A) \cdot P(B) \neq P(AB)$$

Therefore, A and B are not independent. **[3]**

Q. 23. Given that the events *A* and *B* are such that *P(A)* = 1/2, *P(A∪ B)* = 3/5 and *P(B)* = *p*. Find *p* if they are (i) mutually exclusive (ii) independent.

[NCERT Ex. 13.2, Q. 7, Page 547]

Ans. It is given that,

$$P(A) = \frac{1}{2},\ P(A \cap B) = \frac{3}{5},\ \text{and } P(B) = p$$

(i) When A and B are mutually exclusive, $A \cap B = \Phi$

$\therefore P(A \cap B) = 0$

Itisknownthat, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - 0$$

$$\Rightarrow \quad p = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

(ii) When A and B are independent,

$$P(A \cap B) = P(A) \cdot P(B) = -\frac{1}{2}p$$

It is known that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + \frac{p}{2}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow \quad p=\frac{2}{10}=\frac{1}{5}$$

[3]

Q. 24. If A and B are two events such that $P(A) = 1/4$, $P(B) = 1/2$ and $P(A \cap B) = 1/8$, find P (not A and not B). [NCERT Ex. 13.2, Q. 9, Page 547]

Ans. It is given that,

$$P(A)=\frac{1}{2} \text{ and } P(A \cap B)=\frac{1}{8}$$

$$P(\text{not on } A \text{ and not on } B) = P(A' \cap B')$$

$$= P\left((A \cup B)'\right)\left[A' \cap B' = (A \cup B)'\right]$$

$$= 1 - P(A \cup B)$$

$$= 1 - \left[P(A)+P(B)-P(A \cap B)\right]$$

$$= 1 - \left[\frac{1}{4}+\frac{1}{2}-\frac{1}{8}\right]$$

$$= 1 - \frac{5}{8} = \frac{3}{8}$$

[3]

**Q. 25. Events A and B are such that $P(A) = 1/2$, $P(B) = 7/12$ and P(not A or not B) = 1/4.
State whether A and B are independent? [NCERT Ex. 13.2, Q. 10, Page 547]**

Ans. It is given that,

$$P(A)=\frac{1}{2},\ P(B)=\frac{7}{12}, \text{ and } P(\text{not } A \text{ or not } B) = \frac{1}{4}$$

$$\Rightarrow \quad P(A' \cup B') = \frac{1}{4}$$

$$\Rightarrow P\left((A \cap B)'\right) = \frac{1}{4} \qquad \left[\because A' \cup B' = (A \cap B)'\right]$$

$$\Rightarrow 1 - P(A \cap B) = \frac{1}{4}$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{4}$$

However,

$$P(A)\cdot P(B) = \frac{1}{2}\cdot\frac{7}{12} = \frac{7}{24}$$

Here, $\frac{3}{4} \neq \frac{7}{24}$

$$\therefore P(A \cap B) \neq P(A)\cdot P(B)$$

Therefore, A and B are not independent events.

[3]

Q. 26. A die is tossed thrice. Find the probability of getting an odd number at least once. [NCERT Ex. 13.2, Q. 12, Page 547]

Ans. Probability of getting an odd number in a single throw of a die $=\frac{3}{6}=\frac{1}{2}$

Similarly, probability of getting an even number $=\frac{3}{6}=\frac{1}{2}$

Probability of getting an even number three times $=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

Therefore, probability of getting an odd number at least once

= 1 – Probability of getting an odd number in none of the throws

= 1 – Probability of getting an even number thrice

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

[3]

Q. 27. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red.
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.
[NCERT Ex. 13.2, Q. 13, Page 547]

Ans. Total number of balls = 18
Number of red balls = 8
Number of black balls = 10

(i) Probability of getting a red ball in the first draw $=\frac{8}{18}=\frac{4}{9}$

The ball is replaced after the first draw.

$\therefore$ Probability of getting a red ball in the second draw $=\frac{8}{18}=\frac{4}{9}$

Therefore, probability of getting both the balls red $=\frac{4}{9}\times\frac{4}{9}=\frac{16}{81}$

(ii) Probability of getting first ball black $=\frac{10}{18}=\frac{5}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as red $=\frac{8}{18}=\frac{4}{9}$

Therefore, probability of getting first ball as black and second ball as red $=\frac{5}{9}\times\frac{4}{9}=\frac{20}{81}$

(iii) Probability of getting first ball as red $=\frac{8}{18}=\frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as black $=\frac{10}{18}=\frac{5}{9}$

Therefore, probability of getting first ball as black and second ball as red $=\frac{4}{9}\times\frac{5}{9}=\frac{20}{81}$

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red + Probability of getting first ball red and second ball black

$$= \frac{20}{81}+\frac{20}{81}=\frac{40}{81}$$

[3]

Q. 28. Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved (ii) exactly one of them solves the problem. [NCERT Ex. 13.2, Q. 14, Page 547]

Ans. Probability of solving the problem by A, $P(A)=\frac{1}{2}$

Probability of solving the problem by B, $P(B)=\frac{1}{3}$

Since the problem is solved independently by A and B,

$$\therefore P(AB)=P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

$$P(A')=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$$

$$P(B')=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$$

(i) Probability that the problem is solved $=P(A\cup B)$

$=P(A)+P(B)-P(AB)$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

(ii) Probability that exactly one of them solves the problem is given by,

$P(A)\cdot P(B')+P(B)\cdot P(A')$

$$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}$$

$$=\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$$

[3]

Q. 29. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events *E* and *F* independent?

(i) *E* : 'the card drawn is a spade'
**   *F* : 'the card drawn is an ace'**

(ii) *E* : 'the card drawn is black'
**   *F* : 'the card drawn is a king'**

(iii) *E* : 'the card drawn is a king or queen'
**   *F* : 'the card drawn is a queen or jack'.**

[NCERT Ex. 13.2, Q. 15, Page 547]

Ans. **(i)** In a deck of 52 cards, 13 cards are spades and 4 cards are aces.

$\therefore P(E) = P(\text{card drawn is a spade}) = \frac{13}{52}=\frac{1}{4}$

$\therefore P(F) = P(\text{card drawn is an ace}) = \frac{4}{52}=\frac{1}{13}$

In the deck of cards, only 1 card is an ace of spades.

$P(EF) = P(\text{card drawn is spade and an ace}) = \frac{1}{52}$

$$P(E)\times P(F)=\frac{1}{4}\cdot\frac{1}{13}=\frac{1}{52}=P(EF)$$

$$P(E)\times P(F)=P(EF)$$

Therefore, the events ***E*** and F are independent.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are kings.

$\therefore P(E) = P(\text{card drawn is black}) = \frac{26}{52}=\frac{1}{2}$

$\therefore P(F) = P(\text{card drawn is a king}) = \frac{4}{52}=\frac{1}{13}$

In the pack of 52 cards, 2 cards are black as well as kings.

$\therefore P(EF) = P(\text{card drawn is a black king}) = \frac{2}{52}=\frac{1}{26}$

$$P(E)\times P(F)=\frac{1}{2}\cdot\frac{1}{13}=\frac{1}{26}=P(EF)$$

Therefore, the given events E and F are independent.

(iii) In a deck of 52 cards, 4 cards are kings 4 cards are queens, and 4 cards are jacks.

$\therefore P(E) = P(\text{card drawn is a king or a queen})$

$$=\frac{8}{52}=\frac{2}{13}$$

$\therefore P(F) = P(\text{card drawn is a queen or a jack})$

$$=\frac{8}{52}=\frac{2}{13}$$

There are 4 cards which are king or queen and queen or jack.

$\therefore P(EF) = P(\text{card drawn is a king or a queen, or queen or a jack})$

$$=\frac{4}{52}=\frac{1}{13}$$

$$P(E)\times P(F)=\frac{2}{13}\cdot\frac{2}{13}=\frac{4}{169}\neq\frac{1}{13}$$

$$\Rightarrow P(E)\cdot P(F)\neq P(EF)$$

Therefore, the given events E and F are not independent. **[3]**

Q. 30. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. *A* student is selected at random.

(i) Find the probability that she reads neither Hindi nor English newspapers.

(ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(iii) If she reads English newspaper, find the probability that she reads Hindi newspaper.

[NCERT Ex. 13.2, Q. 16, Page 547]

Ans. Let H denote the students who read Hindi newspaper and E denote the students who read English newspaper.

It is given that,

$$P(H)=60\%=\frac{6}{10}=\frac{3}{5}$$

$$P(E)=40\%=\frac{40}{100}=\frac{2}{5}$$

$$P(H\cap E)=20\%=\frac{20}{100}=\frac{1}{5}$$

(i) Probability that a student neither reads Hindi nor English newspaper is,

$$(H\cup E)'=1-P(H\cup E)$$

$$=1-\{P(H)+P(E)-P(H\cap E)\}$$

$$=1-\left(\frac{3}{5}+\frac{2}{5}-\frac{1}{5}\right)=1-\frac{4}{5}=\frac{1}{5}$$

(ii) Probability that a randomly chosen student reads English newspaper, if she reads Hindi newspaper, is given by $P(E|H)$.

$$P(E|H)=\frac{P(E\cap H)}{P(H)}=\frac{\frac{1}{5}}{\frac{3}{5}}=\frac{1}{3}$$

(iii) Probability that a randomly chosen student reads Hindi newspaper, if she reads English newspaper, is given by $P(H|E)$.

$$P(H|E) = \frac{P(H \cap E)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$$

[3]

Q. 31. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

[NCERT Ex. 13.3, Q. 1, Page 555]

Ans. The urn contains 5 red and 5 black balls.

Let a red ball be drawn in the first attempt.

$\therefore P$ (drawing a red ball) $= \frac{5}{10} = \frac{1}{2}$

It two red balls are added to the urn, then the urn contains 7 red and 5 black balls.

P (drawing a red ball) $= \frac{7}{12}$

Let a black ball be drawn in the first attempt.

$\therefore$ P (drawing a black ball in the first attempt) $= \frac{5}{10} = \frac{1}{2}$

It two black balls are added to the urn, then the urn contains 5 red and 7 black balls.

P (drawing a red ball) $= \frac{5}{12}$

Therefore, probability of drawing second ball as red is

$$\frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{1}{2}\left(\frac{7}{12} + \frac{5}{12}\right) = \frac{1}{2} \times 1 = \frac{1}{2}$$

[3]

Q. 32. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostler?

[NCERT Ex. 13.3, Q. 3, Page 556]

Ans. Let E_1 and E_2 be the events that the student is a hostler and a day scholar respectively and A be the event that the chosen student gets grade A.

$$\therefore P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 40\% = \frac{40}{100} = 0.4$$

$P(A|E_1) = P$(student getting an A grade is a hostler) $= 30\% = 0.3$

$P(A|E_2) = P$(student getting an A grade is a day scholar) $= 20\% = 0.2$

The probability that a randomly chosen student is a hostler, given that he has an A grade, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2}$$

$$= \frac{0.18}{0.26} = \frac{18}{26} = \frac{9}{13}$$

[3]

Q. 33. Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X? [NCERT Ex. 13.4, Q. 3, Page 570]

Ans. A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

$$\therefore X(6\,H,\ 0\,T) = |6 - 0| = 6$$

$$X(5\,H,\ 1\,T) = |5 - 1| = 4$$

$$X(4\,H,\ 2\,T) = |4 - 2| = 2$$

$$X(3\,H,\ 3\,T) = |3 - 3| = 0$$

$$X(2\,H,\ 4\,T) = |2 - 4| = 2$$

$$X(1\,H,\ 5\,T) = |1 - 5| = 4$$

$$X(0\,H,\ 6\,T) = |0 - 6| = 6$$

Thus, the possible values of X are 6, 4, 2, and 0.

[3]

Q. 34. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die.

[NCERT Ex. 13.4, Q. 5, Page 570]

Ans. When a die is tossed two times, we obtain $(6 \times 6) =$ 36 number of observations.

Let X be the random variable, which represents the number of successes.

(i) Here, success refers to the number greater than 4.

$P(X = 0) = P$(number less than or equal to 4 on both the tosses) $= \frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$

$P(X = 1) = P$(number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{4}{9}$$

$P(X = 2) = P$(number greater than 4 on both the tosses)

$$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Thus, the probability distribution is as follows.

X	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

$P(Y = 0) = P$(six appears on one of the dice)

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(Y = 1) = P$(six appears on at least one of the dice)

$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{11}{36}$ **[3]**

Thus, the required probability distribution is as follows.

Y	0	1
P (Y)	$\frac{25}{36}$	$\frac{11}{36}$

Q. 35. The random variable *X* has a probability distribution *P(X)* of the following form, where *k* is some number :

$$P(X) = \begin{cases} k, \text{ if } x = 0 \\ 2k, \text{ if } x = 1 \\ 3k, \text{ if } x = 2 \\ 0, \text{ otherwise} \end{cases}$$

(a) Determine the value of *k*.

(b) Find *P* ($X < 2$), *P* ($X \leq 2$), *P*($X \geq 2$).

[NCERT Ex. 13.4, Q. 9, Page 570]

Ans. **(a)** It is known that the sum of probabilities of a probability distribution of random variables is one.

$\therefore k + 2k + 3k + 0 = 1$

$\Rightarrow \quad 6k = 1$

$\therefore \quad k = \frac{1}{6}$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$= k + 2k$

$= 3k = \frac{3}{6}$

$\therefore k = \frac{1}{2}$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k$

$= 6k$

$\therefore \quad k = \frac{6}{6} = 1$

$P(X \geq 2) = P(X = 2) + P(X > 2)$

$= 3k + 0$

$= 3k$

$\therefore \quad k = \frac{3}{6} = \frac{1}{2}$ **[3]**

Q. 36. In a meeting, 70% of the members favour and 30% oppose a certain proposal. *A* member is selected at random and we take $X = 0$ if he opposed, and $X = 1$ if he is in favour. Find *E(X)* and Var.(*X*).

[NCERT Ex. 13.4, Q. 15, Page 570]

Ans. It is given that,

$P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$P(X = 1) = 70\% = \frac{70}{100} = 0.7$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, $E(X) = \Sigma X_i P(X_i)$

$= 0 \times 0.3 + 1 \times 0.7$

$= 0.7$

$E(X^2) = \Sigma X_i^2 P(X_i)$

$= 0^2 \times 0.3 + (1)^2 \times 0.7$

$= 0.7$

It is known that,

$\text{Var.}(X) = E(X^2) - [E(X)]^2$

$= 0.7 - (0.7)^2$

$= 0.7 - 0.49$

$= 0.21$ **[3]**

Q. 37. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is a spade? [NCERT Ex. 13.5, Q. 4, Page 577]

Ans. Let *X* represents the number of spade cards among the five cards drawn. Since drawing of card is with replacement, the trials are Bernoulli trials.

In a well shuffled deck of 52 cards, there are 13 spade cards.

$\Rightarrow p = \frac{13}{51} = \frac{1}{4}$

$\therefore \quad q = 1 - \frac{1}{4} = \frac{3}{4}$

X has a binomial distribution with $n = 5$ and $p = \frac{1}{4}$

$P(X = x) = {}^nC_x q^{n-x} p^x$ where $x = 0, 1, \ldots n$

$= {}^5C_x \left(\frac{3}{4}\right)^{5-x} \left(\frac{1}{4}\right)^x$

(i) *P* (all five cards are spades) $= P(X = 5)$

$= {}^5C_5 \left(\frac{3}{4}\right)^0 \cdot \left(\frac{1}{4}\right)^5$

$= 1 \cdot \frac{1}{1024} = \frac{1}{1024}$

(ii) *P* (only 3 cards are spades) $= P(X = 3)$

$= {}^5C_3 \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^3$

$= 10 \cdot \frac{9}{16} \cdot \frac{1}{64} = \frac{45}{512}$

(iii) *P* (none is a spade) $= P(X = 0)$

$= {}^5C_0 \left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^0$

$= 1 \cdot \frac{243}{1024} = \frac{243}{1024}$ **[3]**

Q. 38. *A* bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0? [NCERT Ex. 13.5, Q. 6, Page 577]

Ans. Let X denotes the number of balls marked with the digit 0 amongst the 4 balls drawn. Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binomial distribution with $n = 4$ and $p = \frac{1}{10}$

$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore P(X = x) = {}^nC_x q^{n-x} \cdot p^x$, $x = 1, 2, \ldots n$

$= {}^4C_x \left(\frac{9}{10}\right)^{4-x} \cdot \left(\frac{1}{10}\right)^x$

P (none marked with 0) $= P\ (X = 0)$

$= {}^4C_0 \left(\frac{9}{10}\right)^4 \cdot \left(\frac{1}{10}\right)^0$

$= 1 \cdot \left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^4$

[3]

Q. 39. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing? [NCERT Ex. 13.5, Q. 9, Page 577]

Ans. The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represents the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x$

$= {}^5C_x \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^x$

P (guessing more than 4 correct answers) $= P(X \geq 4)$

$= P(X = 4) + P(X = 5)$

$= {}^5C_4 \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^4 + {}^5C_5 \left(\frac{1}{3}\right)^5$

$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$

$= \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$

[3]

Q. 40. Find the probability of getting 5 exactly twice in 7 throws of a die. [NCERT Ex. 13.5, Q. 11, Page 577]

Ans. The repeated tossing of a die are Bernoulli trials. Let X represents the number of times of getting 5 in 7 throws of the die.

Probability of getting 5 in a single throw of the die, $p = \frac{1}{6}$

$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Clearly, X has the probability distribution with $n =$ 7 and $p = \frac{1}{6}$

$\therefore \mathrm{P}(\mathrm{X} = x) = {}^n\mathrm{C}_x q^{n-x} p^x = {}^7\mathrm{C}_x \left(\frac{5}{6}\right)^{7-x} \cdot \left(\frac{1}{6}\right)^x$

P (getting 5 exactly twice) $= P(X = 2)$

$= {}^7\mathrm{C}_2 \left(\frac{5}{6}\right)^5 \cdot \left(\frac{1}{6}\right)^2$

$= 21 \cdot \left(\frac{5}{6}\right)^5 \cdot \frac{1}{36} = \left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^5$

[3]

Q. 41. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective? [NCERT Ex. 13.5, Q. 13, Page 577]

Ans. The repeated selections of articles in a random sample space are Bernoulli trails. Let X denotes the number of times of selecting defective articles in random sample space of 12 articles.

Clearly, X has a binomial distribution with $x = 12$ and $p = 10\% = \frac{10}{100} = \frac{1}{10}$

$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x = {}^{12}C_x \left(\frac{9}{10}\right)^{12-x} \cdot \left(\frac{1}{10}\right)^x$

P (selecting 9 defective articles) $= {}^{12}C_9 \left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right)^9$

$= 220 \cdot \frac{9^3}{10^3} \cdot \frac{1}{10^9} = \frac{22 \times 9^3}{10^{11}}$

[3]

Q. 42. For a loaded die, the probabilities of outcomes are given as under : $P(1) = P(2) = 0.2$, $P(3) = P(5) = P(6) = 0.1$ and $P(4) = 0.3$. The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent. [NCERT Exemp. Ex. 13.3, Q. 1, Page 271]

Ans. For a loaded die, it is given that

$P(1) = P(2) = 0.2$

$P(3) = P(5) = P(6) = 0.1$

and $P(4) = 0.3$

Die is thrown two times.

Here, A = Same number each time

and B = Total score is 10 or more

$\therefore A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

So,

$P(A) = [P(1,\ 1) + P(2,\ 2) + P(3,\ 3) + P(4,\ 4) + P(5,\ 5)$
$+ P(6,\ 6)]$

$= [P(1)\cdot P(1) + P(2)\cdot P(2) + P(3)\cdot P(3) + P(4)\cdot P(4) + P(5)$
$\cdot P(5) + P(6)\cdot P(6)]$
$= [0.2\times 0.2 + 0.2\times 0.2 + 0.1\times 0.1 + 0.3\times 0.3 + 0.1\times 0.1$
$+0.1\times 0.1]$
$= 0.20$

and B = {(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)}
$\therefore$ $P(B)$ = P(4, 6)+ (6, 4) + P(5, 5) + P(5, 6) + P(6, 5) + P(6, 6)
$= P(4)\cdot P(6) + P(6)\cdot P(4) + P(5)\cdot P(5) + P(5)\cdot P(6) + P(6)$
$\cdot P(5) + P(6)\cdot P(6)$
$= 0.3\times 0.1 + 0.1\times 0.3 + 0.1\times 0.1 + 0.1\times 0.1 + 0.1\times 0.1$
$+0.1\times 0.1$
$= 0.10$

Also, $A\cap B = \{(5,\ 5)\ (6,\ 6)\}$
$\therefore P(A\cap B) = P(5,\ 5) + P(6,\ 6)$
$= P(5)\cdot P(5) + P(6)\cdot P(6)$
$= 0.1\times 0.1 + 0.1\times 0.1 = 0.01 + 0.01 = 0.02$

Here, $P(A\cap B) = 0.02$ and
$P(A)\cdot P(B) = 0.20\times 0.10 = 0.02$
Thus, $P(A\cap B) = P(A)\cdot P(B) = 0.02$
Hence, A and B are independent events. **[3]**

Q. 43. Refer to Question 42 above. If the die were fair, determine whether or not the events *A* and *B* are independent.
[NCERT Exemp. Ex. 13.3, Q. 2, Page 271]

Ans. We have,
A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}
$\therefore$ $n(A) = 6$ and $n(S) = 6^2 = 36$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

and B = {(4, 6), (6, 4), (5, 5), (6, 5), (5, 6) (6, 6)}
$\Rightarrow n(B) = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Also, $A\cap B = \{(5,\ 5),\ (6,\ 6)\}$

$$\therefore P(A\cap B) = \frac{2}{36} = \frac{1}{18}$$

Also, $P(A)\cdot P(B) = \frac{1}{36}$

Thus, $P(A\cap B) \neq P(A)\cdot P(B)$
So, A and B are not independent events. **[3]**

Q. 44. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of the three marbles drawn be black, if the first marble is red?
[NCERT Exemp. Ex. 13.3, Q. 4, Page 271]

Ans. Bag contains 5 red marbles and 3 black marbles.
For at least one of the three marbles drawn be black, if the first marble is red, then the following situations are possible.
Event E_1 = Second marble is black and third is red
Event E_2 = Second marble is black and third is also black (E_2)
Event E_3 = Second marble is red and third is black (E_3)
Let event R_i = drawing red marble in ith draw
Event B_i = drawing black marble in ith draw
Therefore,

$$P(E_1) = P(R_1)\cdot P(B_1/R_1)\cdot P(R_2/R_1B_1)$$
$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6}\cdot = \cdot\frac{5}{28}$$

$$P(E_2) = P(R_1)\cdot P(B_1/R_1)\cdot P(B_2/R_1B_1)$$
$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}\cdot = \cdot\frac{5}{56}$$

and

$$P(E_3) = P(R_1)\cdot P(R_2/R_1)\cdot P(B_1/R_1R_2)$$
$$= \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} = \frac{5}{28}$$

$\therefore$ Required probability = $P(E)$

$$= P(E_1) + P(E_2) + P(E_3) = \frac{5}{28} + \frac{5}{56} + \frac{5}{28} = \frac{25}{56}$$ **[3]**

Q. 45. Two dice are thrown together and the total score is noted. The events *E, F* and *G* are 'a total of 4', 'a total of 9 or more', and 'a total divisible by 5', respectively. Calculate *P(E)*, *P(F)* and *P(G)* and decide which pairs of events, if any, are independent.
[NCERT Exemp. Ex. 13.3, Q. 5, Page 272]

Ans. Two dice are thrown together *i.e.*,
$\therefore$ $n(S)$ = 36, where S is the sample space.
Event 'E' is 'a total of 4'.
$\therefore E$ = {(2, 2), (3, 1), (1, 3)}
Event 'F' is 'a total of 9 or more'.
$\therefore F$ = {(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)}
Event 'G' is 'a total divisible by 5'
$\therefore G$ = {(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)}
Here, $(E\cap F) = \phi$ and $(E\cap G) = \phi$

Also, $(F\cap G) = \{(4,\ 6),\ (6,\ 4),\ (5,\ 5)\}$

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(F) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

$$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

$$P(F\cap G) = \frac{3}{36} = \frac{1}{12}$$

and $P(F)\cdot P(G) = \frac{5}{18}\cdot\frac{7}{36} = \frac{35}{648}$

So, $P(F\cap G) \neq P(F)\cdot P(G)$
Hence, there is no pair which is independent. **[3]**

Q. 46. Explain why the experiment of tossing a coin three times is said to have binomial distribution.
[NCERT Exemp. Ex. 13.3, Q. 6, Page 272]

Ans. We know that, a random variable X taking values 0, 1, 2, …, n is said to have a binomial distribution with parameters n and P, it its probability distribution is given by

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

where, $q = 1 - p$
and $r = 0, 1, 2, \ldots, n$
Similarly, in an experiment of tossing a coin three times, we have $n = 3$ and random variable X can take values $r = 0,, 1, 2$ and 3 with $p = \frac{1}{2}$ and $q = \frac{1}{2}$

X	0	1	2	3
$P(X)$	${}^3C_0\, q^3$	${}^3C_1\, pq^2$	${}^3C_2\, p^2q$	${}^3C_3\, p^3$

So, we see that in the experiment of tossing a coin three times, we have random variable X which can take values 0, 1, 2 and 3 with parameters $n = 3$ and $P = \frac{1}{2}$.

Therefore, it is said to have a Binomial distribution. **[3]**

Q. 47. A and B are two events such that $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$. Find :
(i) $P(A|B)$ (ii) $P(B|A)$
(iii) $P(A'|B)$ (iv) $P(A'|B')$
[NCERT Exemp. Ex. 13.3, Q. 7, Page 272]

Ans. Here,

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}$$

(i) $$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

(ii) $$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

(iii) $$P(A'/B) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B) - P(A\cap B)}{P(B)} = \frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}$$

(iv) $$P(A'/B') = \frac{P(A'\cap B')}{P(B')} = \frac{1-P(A\cup B)}{1-P(B)}$$

$$= \frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$= \frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}} = \frac{1-\frac{14}{24}}{\frac{2}{3}} = \frac{\frac{10}{24}}{\frac{2}{3}} = \frac{5}{8}$$ **[3]**

Q. 48. Let E_1 and E_2 be two independent events such that $p(E_1) = p_1$ and $P(E_2) = p_2$. Describe in words of the events whose probabilities are :
(i) p_1p_2 (ii) $(1 - p_1)p_2$
(iii) $1 - (1 - p_1)(1 - p_2)$ (iv) $p_1 + p_2 - 2p_1p_2$
[NCERT Exemp. Ex. 13.3, Q. 9, Page 272]

Ans. Given that,
$P(E_1) = p_1$ and $P(E_2) = p_2$

(i) $p_1p_2 \Rightarrow P(E_1)\cdot P(E_2) = P(E_1 \cap E_2)$

So, E_1 and E_2 occur simultaneously.

(ii) $(1-p_1)p_2 = P(E_1')\cdot P(E_2) = P(E_1' \cap E_2)$

So, E_1 does not occur and E_2 occurs.

(iii) $1 - (1 - p_1)(1 - p_2)$

$$= 1 - P(E_1')P(E_2')$$
$$= 1 - P(E_1' \cap E_2') = 1 - [1 - P(E_1 \cup E_2)] = P(E_1 \cup E_2)$$

So, either E_1 or E_2 or both E_1 and E_2 occurs.

(iv) $p_1 + p_2 - 2p_1p_2$

$$= P(E_1) + P(E_2) - 2P(E_1)\cdot P(E_2)$$
$$= P(E_1) + P(E_2) - 2P(E_1 \cap E_2) = P(E_1 \cup E_2) - P(E_1 \cap E_2)$$

So, either E_1 or E_2 occurs but not both. **[3]**

Q. 49. A discrete random variable X has the probability distribution given as below :

X	0.5	1	1.5	2
$P(X)$	k	k^2	$2k^2$	K

(i) Find the value of k
(ii) Determine the mean of the distribution
[NCERT Exemp. Ex. 13.3, Q. 10, Page 272]

Ans.

X	0.5	1	1.5	2
$P(X)$	k	k^2	$2k^2$	K

(i) We know that, $\sum_{i=1}^{n} P_i = 1$, where $P_i \geq 0$

$$\Rightarrow P_1 + P_2 + P_3 + P_1 = 1$$
$$\Rightarrow k + k^2 + 2k^2 + k = 1$$
$$\Rightarrow 3k^2 + 2k - 1 = 0$$
$$\Rightarrow (3k-1)(k+1) = 0$$
$$\Rightarrow k = \frac{1}{3} \text{ (as } k \text{ is} \geq 0)$$

(ii) Mean of the distribution $(\mu) = E(X) = \sum_{i=1}^{n} x_i P_i$

$$= 0.5(k) + 1(k)^2 + 1.5(2k^2) + 2(k)$$
$$= 4k^2 + 2.5k$$
$$= 4\cdot\frac{1}{9} + (2.5)\frac{1}{3} = \frac{4+7.5}{9} = \frac{23}{18}$$ **[3]**

Q. 50. Prove that :
(i) $P(A) = P(A\cap B) + P(A\cap B')$
(ii) $P(A\cup B) = P(A\cap B) + P(A\cap \overline{B}) + P(\overline{A}\cap B)$
[NCERT Exemp. Ex. 13.3, Q. 11, Page 272]

Ans. (i) $\because P(A) = P(A\cap B) + P(A\cap \overline{B})$

$$\therefore \text{ RHS} = P(A\cap B) + P(A\cap \overline{B})$$
$$= P(A)\cdot P(B) + P(A)\cdot P(\overline{B})$$
$$= P(A)[P(B) + P(\overline{B})]$$
$$= P(A)[P(B) + 1 - P(B)] \qquad [\because P(\overline{B}) = 1 - P(B)]$$
$$= P(A) = \text{LHS}$$

Hence proved.

(ii) $\because P(A\cup B) = P(A\cap B) + P(A\cap \overline{B}) + P(\overline{A}\cap B)$

$$\therefore \text{ RHS} = P(A)\cdot P(B) + P(A)\cdot P(\overline{B}) + P(\overline{A})\cdot P(B)$$

$= P(A)\cdot P(B) + P(A)\cdot[1-P(B)] + [1-P(A)]P(B)$

$= P(A)\cdot P(B) + P(A) - P(A)\cdot P(B) + P(B) - P(A)\cdot P(B)$

$= P(A) + P(B) - P(A)\cdot P(B)$

$= P(A) + P(B) - P(A\cap B)$

$= P(A\cup B) = \text{LHS}$

Hence proved.

[3]

Q. 51. If X is the number of tails in three tosses of a coin, determine the standard deviation of X.

[NCERT Exemp. Ex. 13.3, Q. 12, Page 272]

Ans. Given that, random variable X is the number of tails in three tosses of a coin.

So, $X = 0, 1, 2, 3$.

$\Rightarrow P(X=x) = {}^nC_x (p)^x q^{n-x}$,

where $n = 3$, $p = ½$, $q = 1/2$ and $x = 0, 1, 2, 3$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$XP(X)$	0	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{8}$
$X^2P(X)$	0	$\frac{3}{8}$	$\frac{3}{2}$	$\frac{9}{8}$

We know that, Var.$(X) = E(X^2) - [E(X)]^2$ (i)

where, $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$ and $E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$\therefore E(X^2) = \sum_{i=1}^{n} x_i^2 P(X_i) = 0 + \frac{3}{8} + \frac{3}{2} + \frac{9}{8} = \frac{24}{8} = 3$

and

$[E(X)]^2 = \left[\sum_{i=0}^{n} x_i P(x_i)\right]^2 = \left[0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8}\right]^2 = \left[\frac{12}{8}\right]^2 = \frac{9}{4}$

$\therefore \text{Var.}(X) = 3 - \frac{9}{4} = \frac{3}{4}$ [By using Eq. (i)]

and standard deviation of

$$X = \sqrt{\text{Var.}(X)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

[3]

Q. 52. In a dice game, a player pays a stake of Re 1 for each throw of a die. She receives Rs 5 if the die shows a 3, Rs 2 if the die shows a 1 or 6, and nothing otherwise. What is the player's expected profit per throw over a long series of throws?

[NCERT Exemp. Ex. 13.3, Q. 13, Page 272]

Ans. Let X is the random variable of profit per throw.

X	–1	1	4
$P(X)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Since, she loses ₹1 on getting any of 2, 4 or 5.

So, at $X = -1$, $P(X) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Similarly, at $X = 1$, $P(X) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

[if die shows of either 1 or 6]

and at $X = 4$, $P(X) = \frac{1}{6}$ [if die shows a 3]

$\therefore$ Player's expected profit $= E(X) = \Sigma(X) = \Sigma XP(X)$

$= -1\times\frac{1}{2} + 1\times\frac{1}{3} + 4\times\frac{1}{6} = \frac{-3+2+4}{6} = \frac{1}{2} = ₹0.50$

[3]

Q. 53. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

[NCERT Exemp. Ex. 13.3, Q. 14, Page 273]

Ans. Three dice are thrown.

$\therefore n(S) = 6^3 = 216$

Let E_1 is the event when the sum of numbers on the dice was six and E_2 is the event when three two's occurs.

$\Rightarrow$ $E_1 = \{(1, 1, 4), (1, 2, 3), (1, 3, 2), (1, 4, 1), (2, 1, 3), (2, 2, 2), (2, 3, 1), (3, 1, 2), (3, 2, 1), (4, 1, 1)\}$

$\Rightarrow n(E_1) = 10$ and $n(E_1) = 1$

Also, $(E_1 \cap E_2) = 1$

$\therefore\ P(E_1/E_2) = \frac{P(E_1\cap E_2)}{P(E_1)} = \frac{1/216}{10/216} = \frac{1}{10}$

[3]

Q. 54. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

[NCERT Exemp. Ex. 13.3, Q. 16, Page 273]

Ans. Let E_1 is the event that ball transferred from the first bag is white and E_2 is the event that the ball transferred from the first bag is black.

Also, E is the event that the ball drawn from the second bag is white.

$\therefore\ P(E_1) = \frac{4}{9}$ and $P(E_2) = \frac{5}{9}$

and $P(E/E_1) = \frac{10}{17}$, $P(E/E_2) = \frac{9}{17}$

$\therefore$ Using total probability theorem,

$P(E) = P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2)$

$= \frac{4}{9}\cdot\frac{10}{17} + \frac{5}{9}\cdot\frac{9}{17} = \frac{40+45}{153} = \frac{85}{153} = \frac{5}{9}$

[3]

Q. 55. Ten coins are tossed. What is the probability of getting at least 8 heads?

[NCERT Exemp. Ex. 13.3, Q. 21, Page 273]

Ans. Let X is the random variable for getting a head.

Here, $n = 10$, $r \geq 8$, $p = \frac{1}{2}$, $q = \frac{1}{2}$

Now $P(X = r) = {}^nC_r p^r q^{n-r}$

$\therefore\ P(X = r) = P(r = 8) + P(r = 9) + P(r = 10)$

$= {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$

$= \frac{10!}{8!2!}\left(\frac{1}{2}\right)^{10} + 10\cdot\left(\frac{1}{2}\right)^{10} + 1\cdot\left(\frac{1}{2}\right)^{10}$

$= \left(\frac{1}{2}\right)^{10}\left[\frac{10\times 9}{2} + 10 + 1\right] = 56\cdot\left(\frac{1}{2}\right)^{10} = \frac{7}{128}$

[3]

Q. 56. The probability of a man hitting a target is 0.25. He shoots 7 times. What is the probability of his hitting at least twice?

[NCERT Exemp. Ex. 13.3, Q. 22, Page 273]

Ans. Here,

$n = 7,\ p = 0.25 = \frac{1}{4},\ q = 1 - \frac{1}{4} = \frac{3}{4}$ and $r \geq 2$,

where, $P(X) = {}^nC_r (p)^r (q)^{n-1}$

$\therefore P(X = r \geq 2)$

$= 1 - [P(r = 0) + P(r = 1)]$

$= 1 - \left[{}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^7 + {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{7-1} \right]$

$= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{3}{4} + \frac{7}{4}\right) = \frac{4547}{8192}$ **[3]**

Q. 57. Consider the probability distribution of a random variable X :

X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15

Calculate (i) $V\left(\frac{X}{2}\right)$ (ii) Variance of X.

[NCERT Exemp. Ex. 13.3, Q. 24, Page 274]

Ans. We have,

X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15
$XP(X)$	0	0.25	0.6	0.6	0.60
$X^2P(X)$	0	0.25	1.2	1.8	2.40

Var.$(X) = E(X^2) - [E(X)]^2$

where, $E(X) = \mu = \sum_{i=1}^{n} x_i P_i(x_i)$

and $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$

$\therefore\ E(X) = 0 + 0.25 + 0.6 + 0.6 + 0.60 = 2.05$

$E(X^2) = 0 + 0.25 + 1.2 + 1.8 + 2.40 = 5.65$

(i) $V\left(\frac{X}{2}\right) = \frac{1}{4} V(X) = \frac{1}{4}[5.65 - (2.05)^2]$

$= \frac{1}{4}[5.65 - 4.2025] = \frac{1}{4} \times 1.4475 = 0.361875$

(ii) $V(X) = 1.4475$ **[3]**

Q. 58. The probability distribution of a random variable X is given below :

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(i) Determine the value of k.

(ii) Determine $P(X \leq 2)$ and $P(X > 2)$

(iii) Find $P(X \leq 2) + P(X > 2)$

[NCERT Exemp. Ex. 13.3, Q. 25, Page 274]

Ans. We have,

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(i) $\sum_{i=1}^{n} P_i = 1$, $i = 1, 2, \ldots, n$ and $P_i \geq 0$

$\therefore\ k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$

$\Rightarrow\ 8k + 4k + 2k + k = 1$

$\therefore\ k = \frac{8}{15}$

(ii) $P(X \leq 2) = P(0) + P(1) + P(2)$

$= k + \frac{k}{2} + \frac{k}{4} = \frac{7k}{4} = \frac{7}{4} \cdot \frac{8}{15} = \frac{14}{15}$

and $P(X > 2) = P(3) = \frac{k}{8} = \frac{1}{8} \cdot \frac{8}{15} = \frac{1}{15}$

(iii) $P(X \leq 2) + P(X > 2) = \frac{14}{15} + \frac{1}{15} = 1$ **[3]**

Q. 59. For the following probability distribution, determine standard deviation of the random variable X.

X	2	3	4
$P(X)$	0.2	0.5	0.3

[NCERT Exemp. Ex. 13.3, Q. 26, Page 274]

Ans. We have,

X	2	3	4
$P(X)$	0.2	0.5	0.3
$XP(X)$	0.4	1.5	1.2
$X^2P(X)$	0.8	4.5	4.8

We know that, standard deviation of $X = \sqrt{\text{Var.}(X)}$

where, $\text{Var.}(X) = E(X^2) - [E(X)]^2$

$= \sum_{i=1}^{n} x_i^2 P(X_i) - \left[\sum_{i=1}^{n} x_i P_i\right]^2$

$\therefore\ \text{Var.}(X) = [0.8 + 4.5 + 4.8] - [0.4 + 1.5 + 1.2]^2$

$= 10.1 - (3.1)^2 = 10.1 - 9.61 = 0.49$

$\therefore$ Standard deviation of

$X = \sqrt{\text{Var.}X} = \sqrt{0.49} = 0.7$ **[3]**

Q. 60. A biased die is such that $P(4) = 1/10$ and other scores being equally likely. The die is tossed twice. If X is the 'number of fours seen', find the variance of the random variable X.

[NCERT Exemp. Ex. 13.3, Q. 27, Page 274]

Ans. Since, X = Number of fours seen

On tossing two die, $X = 0, 1, 2$

Also, $P(4) = \frac{1}{10}$ and

$P(X = 0) = P(\text{not } 4) \cdot P(\text{not } 4) = \frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$

$P(X=1) = P(\text{not } 4)\cdot P(4) + P(4)\cdot P(\text{not } 4) = \frac{9}{10}\cdot\frac{1}{10}$

$+\frac{1}{10}\cdot\frac{9}{10} = \frac{18}{100}$

$P(X=2) = P(4)\cdot P(4) = \frac{1}{10}\cdot\frac{1}{10} = \frac{1}{100}$

Thus, we get following table :

X	0	1	2
$P(X)$	$\frac{81}{100}$	$\frac{18}{100}$	$\frac{1}{100}$
$XP(X)$	0	$\frac{18}{100}$	$\frac{2}{100}$
$X^2P(X)$	0	$\frac{18}{100}$	$\frac{4}{100}$

$\therefore\ \text{Var}(X) = E(X^2) - [E(X)]^2$

$= \Sigma X^2P(X) - [\Sigma XP(X)]^2$

$= \left[0 + \frac{18}{100} + \frac{4}{100}\right] - \left[0 + \frac{18}{100} + \frac{2}{100}\right]^2$

$= \frac{22}{100} - \left(\frac{22}{100}\right)^2 = \frac{11}{50} - \frac{1}{25} = \frac{18}{100} = 0.18$

[3]

Q. 61. A die is thrown three times. Let X be 'the number of twos seen'. Find the expectation of X.
[NCERT Exemp. Ex. 13.3, Q. 28, Page 274]

Ans. We have,

X = number of twos seen

So, on throwing a die three times, we will have X = 0, 1, 2, 3

$\therefore\ P(X=0) = P(\text{not } 2)\cdot P(\text{not } 2)\cdot P(\text{not } 2)$

$= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{125}{216}$

$P(X=1) = 3\cdot P(\text{not } 2)\cdot P(\text{not } 2)\cdot P(2)$

$= 3\frac{5}{6}\frac{5}{6}\frac{1}{6} = \frac{25}{72}$

$P(X=2) = 3\cdot P(\text{not } 2)\cdot P(2)\cdot P(2)$

$= 3\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{5}{72}$

$P(X=3) = P(2)\cdot P(2)\cdot P(2)$

$= \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{216}$

We know that,

$E(X) = \Sigma X\ P(X)$

$= 0\cdot\frac{125}{216} + 1\cdot\frac{25}{72} + 2\cdot\frac{5}{72} + 3\cdot\frac{1}{216}$

$= \frac{75+30+3}{216} = \frac{108}{216} = \frac{1}{2}$

[3]

Q. 62. Two biased dice are thrown together. For the first die $P(6) = 1/2$, the other scores being equally likely while for the second die, $P(1) = 2/5$ and the other scores are equally likely. Find the probability distribution of 'the number of ones seen'.
[NCERT Exemp. Ex. 13.3, Q. 29, Page 274]

Ans. For first die,

$P(6) = \frac{1}{2}$ and $P(\text{not } 6) = \frac{1}{2}$

$\Rightarrow\ P(1) + P(2) + P(3) + P(4) + P(5) = \frac{1}{2}$

$\Rightarrow\ P(1) = \frac{1}{10}$ and $P(\text{not } 1) = \frac{9}{10}$

$[\because P(1) = P(2) = P(3) = P(4) = P(5)]$

For second die,

$P(1) = \frac{2}{5}$ and $P(\text{not } 1) = 1 - \frac{2}{5} = \frac{3}{5}$

Let X = Number of one's seen

$P(X=0) = P(\text{not 1 on first die})\cdot P(\text{not 1 on second die})$

$= \frac{9}{10}\cdot\frac{3}{5} = \frac{27}{50} = 0.54$

$P(X=1) = P(\text{1 on first die})\cdot P(\text{not 1 on second die})$
$+P(\text{1 on second die})\cdot P(\text{not 1 on first die})$

$= \frac{1}{10}\cdot\frac{3}{5} + \frac{9}{10}\cdot\frac{2}{5} = \frac{21}{50} = 0.42$

$P(X=2) = P(\text{1 on first die})\cdot P(\text{1 on seond die})$

$= \frac{1}{10}\cdot\frac{2}{5} = 0.04$

[3]

Hence, the required probability distribution is as below :

X	0	1	2
$P(X)$	0.54	0.42	0.04

Q. 63. Two probability distributions of the discrete random variable X and Y are given below.

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E(Y^2) = 2E(X)$.
[NCERT Exemp. Ex. 13.3, Q. 30, Page 275]

Ans. We have,

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

$\therefore\quad E(X) = \Sigma XP(X) = 0\cdot\frac{1}{5} + 1\cdot\frac{2}{5} + 2\cdot\frac{1}{5} + 3\cdot\frac{1}{5} = \frac{7}{5}$

$\Rightarrow\ 2E(X) = \frac{14}{5}$ ...(i)

$$E(Y^2) = \Sigma y^2 P(Y)$$

$$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{10} + 4 \cdot \frac{2}{5} + 9 \cdot \frac{1}{10} = \frac{3}{10} + \frac{8}{5} + \frac{9}{10} = \frac{28}{10}$$

$$\Rightarrow E(Y^2) = \frac{14}{5} \quad \text{...(ii)}$$

From equations (i) and (ii), we have

$E(Y^2) = 2E(X)$ **Hence proved.** **[3]**

Q. 64. A factory produces bulbs. The probability that any one bulb is defective is 1/50 and they are packed in boxes of 10. From a single box, find the probability that

(i) none of the bulbs is defective

(ii) exactly two bulbs are defective

(iii) more than 8 bulbs work properly

[NCERT Exemp. Ex. 13.3, Q. 31, Page 275]

Ans. Let X is the random variable which denotes that a bulb is defective.

Here, $n = 10$, $p = \frac{1}{50}$, $q = \frac{49}{50}$ and

$P(X = r) = {}^nC_r p^r q^{n-r}$

(i) None of the bulbs is defective, *i.e.*, $r = 0$

$$\therefore P(X = r = 0) = {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} = \left(\frac{49}{50}\right)^{10}$$

(ii) Exactly two bulbs are defective, *i.e.*, $r = 2$

$$\therefore P(X = r = 2) = {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8 = \frac{10!}{8!2!}\left(\frac{1}{50}\right)^2 \cdot \left(\frac{49}{50}\right)^8$$

$$= \frac{45 \times (49)^8}{50^{10}}$$

(iii) More than 8 bulbs work properly, *i.e.*, there is less than 2 bulbs which are defective.

So, $r < 2 \Rightarrow r = 0, 1$

$\therefore P(X = r < 2)$

$$= {}^{10}C_{10}\left(\frac{1}{50}\right)^2\left(\frac{49}{50}\right)^{10} + {}^{10}C_1 \cdot \frac{1}{50} \cdot \left(\frac{49}{50}\right)^9$$

$$= \left(\frac{49}{50}\right)^{10} + \frac{10}{50} \cdot \left(\frac{49}{50}\right)^9 = \left(\frac{49}{50} + \frac{10}{50}\right)\left(\frac{49}{50}\right)^9 = \frac{59 \times 49^9}{50^{10}}$$

[3]

Q. 65. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

[NCERT Exemp. Ex. 13.3, Q. 33, Page 275]

Ans. We have,

Parameters	Blood group 'O'	Other than blood group 'O'
(i) Number of people	30%	70%
(ii) Percentage of left-handed people	6%	10%

Event, E_1 = the person selected is of blood group O

Event, E_2 = the person selected is of other than blood group O

Event, E = the selected person is left handed

$\therefore$ $P(E_1) = 0.3$, $P(E_2) = 0.7$

$P(E/E_1) = 0.06$ and $P(E/E_2) = 0.1$

$\therefore$ Using Bayes' theorem,

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$

$$= \frac{0.3 \times 0.06}{0.3 \times 0.06 + 0.7 \times 0.1} = \frac{0.018}{0.018 + 0.07} = \frac{9}{44}$$

[3]

Q. 66. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.

[NCERT Exemp. Ex. 13.3, Q. 35, Page 275]

Ans. Let X is random variable score obtained when a die is thrown twice.

$\therefore$ $X = 1, 2, 3, 4, 5, 6$

$n(S) = 36$

$\therefore P(X = 1) = \frac{1}{36}$ (as (1, 1))

$P(X = 2) = \frac{3}{36}$ (as (1, 2), (2, 1), (2, 2))

$P(X = 3) = \frac{5}{36}$ (as (1, 3), (2, 3), (3, 1), (3, 2), (3, 3))

Similarly,

$$P(X = 4) = \frac{7}{36}, \; P(X = 5) = \frac{9}{36}, \; P(X = 6) = \frac{11}{36}$$

So, the required distribution is,

X	1	2	3	4	5	6
$P(X)$	1/36	3/36	5/36	7/36	9/36	11/36

$\therefore$ Mean $\{E(x)\} = \Sigma XP(X)$

$$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$$

[3]

Q. 67. The random variable X can take only the values 0, 1, 2. Given that $P(X = 0) = P(X = 1) = p$ and that $E(X^2) = E[X]$, find the value of p.

[NCERT Exemp. Ex. 13.3, Q. 36, Page 275]

Ans. Since, $X = 0, 1, 2$ and $P(X)$ at $X = 0$ and 1 is p, let at $X = 2$, $P(X)$ is x.

$\Rightarrow p + p + x = 1$

$\Rightarrow x = 1 - 2p$

So we have, the following distribution.

X	0	1	2
$P(X)$	p	p	$1 - 2p$

$\therefore$ $E(X) = \Sigma XP(X)$

$$= 0 \cdot p + 1 \cdot p + 2(1 - 2p)$$

$$= p + 2 - 4p = 2 - 3p$$

and $E(X^2) = \Sigma X^2 P(X)$

$$= 0 \cdot p + 1 \cdot p + 4 \cdot (1 - 2p)$$

$$= p + 4 - 8p = 4 - 7p$$

It is given that,

$$E(X^2) = E(X)$$

$\Rightarrow 4 - 7p = 2 - 3p$

$\Rightarrow \quad 4p = 2$

$\therefore \quad p = \frac{1}{2}$

[3]

Q. 68. Find the variance of the distribution :

x	0	1	2	3	4	5
$P(x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

[NCERT Exemp. Ex. 13.3, Q. 37, Page 276]

Ans. We have,

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$
$XP(X)$	0	$\frac{5}{18}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{4}{9}$	$\frac{5}{18}$
$X^2P(X)$	0	$\frac{5}{18}$	$\frac{8}{9}$	$\frac{3}{2}$	$\frac{16}{9}$	$\frac{25}{18}$

$\therefore$ Variance $= E(X^2) - [E(X)]^2$

$$= \sum X^2P(X) - [\sum XP(X)]^2$$

$$= \left[0 + \frac{5}{18} + \frac{8}{9} + \frac{3}{2} + \frac{16}{9} + \frac{25}{18}\right] - \left[0 + \frac{5}{18} + \frac{4}{9} + \frac{1}{2} + \frac{4}{9} + \frac{5}{18}\right]^2$$

$$= \frac{105}{18} - \frac{35^2}{18^2} = \frac{18 \times 105 - 35 \times 35}{18^2} = \frac{19 \times 35}{324} = \frac{665}{324}$$

[3]

Q. 69. *A* and *B* throw a pair of dice, alternately. *A* wins the game if he gets a total of 6 and *B* wins if she gets a total of 7. It *A* starts the game, find the probability of winning the game by *A* in third throw of the pair of dice.

[NCERT Exemp. Ex. 13.3, Q. 38, Page 276]

Ans. A and B throw a pair of dice.

A wins if he gets a total of 6.

$\therefore A = \{(2, 4), (1, 5), (5, 1), (4, 2), (3, 3)\}$

B wins if she gets a total of 7.

$\therefore B = \{(2, 5), (1, 6), (6, 1), (5, 2), (3, 4), (4, 3)\}$

$\therefore P(A) = \frac{5}{36}$ and $P(B) = \frac{1}{6}$

It is given that A is starting the game and we have to find the probability that A wins in third throw

Required probability $= P(A') \cdot P(B') \cdot P(A)$

$$= \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} = \frac{775}{7776}$$

[3]

Q. 70. Two dice are tossed. Find whether the following two events *A* and *B* are independent : $A = \{(x, y) : x + y = 11\}$, $B = \{(x, y) : x \neq 5$ where (x, y) denotes a typical sample point.

[NCERT Exemp. Ex. 13.3, Q. 39, Page 276]

Ans. We have,

$A = \{(x, y) : x + y = 11\}$

$\therefore A = \{(5, 6), (6, 5)\}$

and $B = \{(x, y) : x \neq 5\}$

Clearly $A \cap B = \phi$

So, A and B are mutually exclusive, hence not independent. **[3]**

Q. 71. An urn contains m white and n black balls. *A* ball is drawn at random and is put back into the urn along with *k* additional balls of the same colour as that of the ball drawn. *A* ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on *k*.

[NCERT Exemp. Ex. 13.3, Q. 40, Page 276]

Ans. Let,

Event, E_1 = First ball drawn of white colour

Event, E_2 = First ball drawn of black colour

and Event, E = Second ball drawn of white colour

$\therefore P(E_1) = \frac{m}{m+n}$ and $P(E_2) = \frac{n}{m+n}$

Also, $P(E/E_1) = \frac{m+k}{m+n+k}$ and $P(E/E_2) = \frac{m}{m+n+k}$

$\therefore$ Using total probability theorem, we have

$$P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2)$$

$$= \frac{m}{m+n} \cdot \frac{m+k}{m+n+k} + \frac{n}{m+n} \cdot \frac{m}{m+n+k}$$

$$= \frac{m(m+k)+nm}{(m+n+k)(m+n)} = \frac{m(m+k+n)}{(m+n+k)(m+n)} = \frac{m}{m+n}$$

Hence, the probability of drawing a white ball does not depend on k. **[3]**

Long Answer Type Questions

(5 or 6 marks each)

Q. 1. *A* black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

[NCERT Ex. 13.1, Q. 10, Page 539]

Ans. Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space S has $6 \times 6 = 36$ number of elements.

(a) Let,

A : Obtaining a sum greater than 9

$= \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

B : Black die results in a 5.

$= \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\therefore A \cap B = \{(5, 5), (5, 6)\}$

The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5, is given by $P(A|B)$.

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{2}{36}}{\frac{6}{36}}=\frac{2}{6}=\frac{1}{3}$$

(b) E : Sum of the observations is 8.

$= \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

F : Red die resulted in a number less than 4.

$$=\begin{Bmatrix}(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),\\(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),\\(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\end{Bmatrix}$$

$\therefore E\cap F=\{(5,3),(6,2)\}$

$$P(F)=\frac{18}{36} \text{ and } P(E\cap F)=\frac{2}{36}$$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by $P(E|F)$.

Therefore,

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{2}{36}}{\frac{18}{36}}=\frac{2}{18}=\frac{1}{9}$$

[5]

Q. 2. A fair die is rolled. Consider events $E = \{1,3,5\}$, $F = \{2,3\}$ and $G = \{2,3,4,5\}$
Find
(i) $P(E|F)$ and $P(F|E)$
(ii) $P(E|G)$ and $P(G|E)$
(iii) $P((E\cup F)|G)$ and $P((E\cap F)|G)$
[NCERT Ex. 13.1, Q. 11, Page 539]

Ans. When a fair die is rolled, the sample space S will be

$S = \{1, 2, 3, 4, 5, 6\}$

It is given that,

$E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$

$$\therefore P(E)=\frac{3}{6}=\frac{1}{2}$$

$$P(F)=\frac{2}{6}=\frac{1}{3}$$

$$P(G)=\frac{4}{6}=\frac{2}{3}$$

(i) $E\cap F=\{3\}$

$$\therefore P(E\cap F)=\frac{1}{6}$$

$$\therefore\ P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{2}$$

$$P(F|E)=\frac{P(E\cap F)}{P(E)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$$

(ii) $E\cap G=\{3,\ 5\}$

$$\therefore P(E\cap G)=\frac{2}{6}=\frac{1}{3}$$

$$\therefore\ P(E|G)=\frac{P(E\cap G)}{P(G)}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$$

$$P(G|E)=\frac{P(E\cap G)}{P(E)}=\frac{\frac{1}{3}}{\frac{1}{2}}=\frac{2}{3}$$

(iii) $E\cup F=\{1,\ 2,\ 3,\ 5\}$

$(E\cup F)\cap G=\{1,\ 2,\ 3,\ 5\}\cap\{2,\ 3,\ 4,\ 5\}=\{2,\ 3,\ 5\}$

$E\cap F=\{3\}$

$(E\cap F)\cap G=\{3\}\cap\{2,\ 3,\ 4,\ 5\}=\{3\}$

$$\therefore\quad P(E\cup G)=\frac{4}{6}=\frac{2}{3}$$

$$P((E\cup F)\cap G)=\frac{3}{6}=\frac{1}{2}$$

$$P(E\cap F)=\frac{1}{6}$$

$$P((E\cap F)\cap G)=\frac{1}{6}$$

$$\therefore P((E\cup F)|G)=\frac{P((E\cup F)\cap G)}{P(G)}$$

$$=\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$$

$$P((E\cap F)|G)=\frac{P((E\cap G)\cap G)}{P(G)}$$

$$=\frac{\frac{1}{6}}{\frac{2}{3}}=\frac{1}{6}\times\frac{3}{2}=\frac{1}{4}$$

[5]

Q. 3. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(i) the youngest is a girl,
(ii) at least one is a girl?
[NCERT Ex. 13.1, Q. 12, Page 539]

Ans. Let b and g represent the boy and the girl child, respectively. If a family has two children, the sample space will be

$S = \{(b, b), (b, g), (g, b), (g, g)\}$

Let A be the event that both children are girls.

$\therefore\quad A=\{(g,\ g)\}$

(i) Let B be the event that the youngest child is a girl.

$\therefore\quad B=[(b,g),(g,g)]$

$\Rightarrow A\cap B=\{(g,g)\}$

$$\therefore\quad P(B)=\frac{2}{4}=\frac{1}{2}$$

$$P(A\cap B)=\frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by $P(A|B)$.

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event that at least one child is a girl.

$\therefore \quad C=\{(b, g), (g, b), (g, g)\}$

$\Rightarrow A\cap C=\{g, g\}$

$\Rightarrow \quad P(C)=\frac{3}{4}$

$P(A\cap C)=\frac{1}{4}$

The conditional probability that both are girls, given that at least one child is a girl, is given by $P(A|C)$.

Therefore, $P(A|C)=\dfrac{P(A\cap C)}{P(C)}=\dfrac{\frac{1}{4}}{\frac{3}{4}}=\dfrac{1}{3}$ **[5]**

Q. 4. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

[NCERT Ex. 13.1, Q. 13, Page 539]

Ans. The given data can be tabulated as follows :

	True/false	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us denote E = easy questions, M = multiple choice questions, D = difficult questions and T = True/False questions

Total number of questions = 1400

Total number of multiple choice questions = 900

Therefore, probability of selecting an easy multiple choice question is

$$P(E\cap M)=\frac{500}{1400}=\frac{5}{14}$$

Probability of selecting a multiple choice question, $P(M)$, is

$$\frac{900}{1400}=\frac{9}{14}$$

$P(E|M)$ represents the probability that a randomly selected question will be an easy question, given that it is a multiple choice question.

$$\therefore P(E|M)=\frac{P(E\cap M)}{P(M)}=\frac{\frac{5}{14}}{\frac{9}{14}}=\frac{5}{9}$$

Therefore, the required probability is $\frac{5}{9}$. **[5]**

Q. 5. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

[NCERT Ex. 13.1, Q. 14, Page 539]

Ans. When dice is thrown, number of observations in the sample space = 6 × 6 = 36

Let A be the event that the sum of the numbers of the dice is 4 and B be the event that the two numbers appearing on throwing the two dice are different.

$\therefore A = \{(1, 3), (2, 2), (3, 1)\}$

$$B=\begin{Bmatrix}(1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6)\\ (2, 1) & (2, 3) & (2, 4) & (2, 5) & (2, 6)\\ (3, 1) & (3, 2) & (3, 4) & (3, 5) & (3, 6)\\ (4, 1) & (4, 2) & (4, 3) & (4, 5) & (4, 6)\\ (5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 6)\\ (6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5)\end{Bmatrix}$$

$A\cap B=\{(1, 3), (3, 1)\}$

$$\therefore P(B)=\frac{30}{36}=\frac{5}{6} \text{ and } P(A\cap B)=\frac{2}{36}=\frac{1}{18}$$

Let $P(A|B)$ represents the probability that the sum of the numbers on the dice is 4, given that two numbers appearing on throwing the two dice are different.

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{18}}{\frac{5}{6}}=\frac{1}{15}$$

Therefore, the required probability is $\frac{1}{15}$. **[5]**

Q. 6. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

[NCERT Ex. 13.1, Q. 15, Page 539]

Ans. The outcomes of the given experiment can be represented by the following tree diagram.

The sample space of the experiment is,

$$S=\begin{Bmatrix}(1, H),(1, T),(2, H),(2, T),(3, 1)(3, 2),(3, 3),\\ (3, 4),(3, 5),(3, 6),\\ (4, H),(4, T),(5, H),(5, T),(6, 1),(6, 2),(6, 3),\\ (6, 4),(6, 5),(6, 6)\end{Bmatrix}$$

Let A be the event that the coin shows a tall and B be the event that at least one die shows 3.

$\therefore A = \{(1, T), (2, T), (4, T), (5, T)\}$

$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$

$\Rightarrow \quad A\cap B= \phi$

$\therefore \quad P(A\cap B) = 0$

Then, $P(B) = P(\{3, 1\}) + P(\{3, 2\}) + P(\{3, 3\}) + P(\{3, 4\}) + P(\{3, 5\}) + P(\{3, 6\}) + P(\{6, 3\})$

$$=\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}=\frac{7}{20}$$

Probability of the event that the coin shows a tail, given that at least one die shows 3, is given by $P(A|B)$.

Therefore,

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0}{\frac{7}{20}}=0$$

[5]

Q. 7. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

[NCERT Ex. 13.3, Q. 2, Page 556]

Ans. Let E_1 and E_2 be the events of selecting first bag and second bag, respectively.

$$\therefore \quad P(E_1)=P(E_2)=\frac{1}{2}$$

Let A be the event of getting a red ball.

$\Rightarrow P(A|E_1)=P$(drawing a red ball from first bag)

$$=\frac{4}{8}=\frac{1}{2}$$

$\Rightarrow P(A|E_2)=P$(drawing a red ball from Second bag)

$$=\frac{2}{8}=\frac{1}{4}$$

The probability of drawing a ball from the first bag, given that it is red, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}}=\frac{\frac{1}{4}}{\frac{3}{8}}=\frac{2}{3}$$

[5]

Q. 8. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?

[NCERT Ex. 13.3, Q. 4, Page 556]

Ans. Let E_1 and E_2 be the respective events that the student knows the answer and he guesses the answer.

Let A be the event that answer is correct. Therefore,

$$P(E_1)=\frac{3}{4} \text{ and } P(E_2)=\frac{1}{4}$$

The probability that the student answered correctly, given that he knows the answer, is 1.

$\therefore P(A|E_1) = 1$

Probability that the student answered correctly, given that he guessed, is $\frac{1}{4}$.

$$\therefore P(A|E_2)=\frac{1}{4}$$

The probability that the student knows the answer, given that he answered it correctly, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{3}{4}\cdot 1}{\frac{3}{4}\cdot 1+\frac{1}{4}\cdot\frac{1}{4}}=\frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}}=\frac{\frac{3}{4}}{\frac{13}{16}}=\frac{12}{13}$$

[5]

Q. 9. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (*i.e.* if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

[NCERT Ex. 13.3, Q. 5, Page 556]

Ans. Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Sine E_1 and E_2 are events complimentary to each other,

$\therefore P(E_1) + P(E_2) = 1$

$\Rightarrow P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1)=0.1\%=\frac{0.1}{100}=0.001$$

$P(A|E_1) = P$(result is positive given the person has disease) = 99% = 0.99

$P(A|E_2) = P$(result is positive given that the person has no disease) = 0.5% = 0.005 probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By suing Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{0.001\times 0.99}{0.001\times 0.99+0.999\times 0.005}$$

$$=\frac{0.00099}{0.00099+0.004995}=\frac{0.00099}{0.005985}$$

$$=\frac{990}{5985}=\frac{110}{665}=\frac{22}{133}$$

[5]

Q. 10. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

[NCERT Ex. 13.3, Q. 6, Page 555]

Ans. Let E_1, E_2 and E_3 be the respective events of choosing a two-headed coin, and an unbiased coin.

$$\therefore P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$\therefore$ $P(A|E_1) = P$(coin showing heads, given that it is a two-headed coin) = 1

Probability of heads coming up, given that it is a biased coin = 75%

$\therefore$ $P(A|E_2) = P$(coin showing heads, given that it is a biased coin) = $\frac{75}{100} = \frac{3}{4}$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$\therefore$ $P(A|E_3) = P$(coin showing heads, given that it is an unbiased coin) = $\frac{1}{2}$

The probability that the coin is two headed, given that it shows heads, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$$

[5]

Q. 11. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

[NCERT Ex. 13.3, Q. 7, Page 555]

Ans. Let E_1, E_2 and E_3 be the respective events that the driver is a scooter driver, a car driver and a truck driver.

Let A be the event that the person meets with an accident.

There are 2000 scooter drivers, 4000 car drivers and 6000 truck drivers.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$P(E_1) = P$(scooter driver) $= \frac{2000}{12000} = \frac{1}{6}$

$P(E_2) = P$(car driver) $= \frac{4000}{12000} = \frac{1}{3}$

$P(E_3) = P$(truck driver) $= \frac{6000}{12000} = \frac{1}{2}$

$P(A|E_1) = P$(scooter driver met with an accident) $= 0.01 = \frac{1}{100}$

$P(A|E_2) = P$(car driver met with an accident) $= 0.03 = \frac{3}{100}$

$P(A|E_3) = P$(truck driver met with an accident) $= 0.15 = \frac{15}{100}$

The probability that the driver is a scooter driver, given that he met with an accident, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{6}\cdot\frac{1}{100} + \frac{1}{3}\cdot\frac{3}{100} + \frac{1}{2}\cdot\frac{15}{100}} = \frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{6} + 1 + \frac{15}{2}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{104}{12}} = \frac{1}{6}\times\frac{12}{104} = \frac{1}{52}$$

[5]

Q. 12. *A* factory has two machines *A* and *B*. Past record shows that machine *A* produced 60% of the items of output and machine *B* produced 40% of the items. Further, 2% of the items produced by machine *A* and 1% produced by machine *B* were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine *B*?

[NCERT Ex. 13.3, Q. 8, Page 555]

Ans. Let E_1 and E_2 be the respective events of items produced by machines A and B.

Let X be the event that the produced item was found to be defective.

$\therefore$ Probability of items produced by machine A,

$P(E_1) = 60\% = \frac{3}{5}$

Probability of items produced by machine B, $P(E_2)$

$= 40\% = \frac{2}{5}$

Probability that machine A produced defective items, $P(X|E_1) = 2\% = \frac{2}{100}$

Probability that machine B produced defective items, $P(X|E_2) = 1\% = \frac{1}{100}$

The probability that the randomly selected item was from machine B, given that it is defective, is given by $P(E_2|X)$.

By using Bayes' theorem, we obtain

$$P(E_2|A) = \frac{P(E_2)\cdot P(X|E_2)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2)}$$

$$= \frac{\frac{2}{5}\cdot\frac{1}{100}}{\frac{3}{5}\cdot\frac{2}{100} + \frac{2}{5}\cdot\frac{1}{100}} = \frac{\frac{2}{500}}{\frac{6}{500} + \frac{2}{500}} = \frac{2}{8} = \frac{1}{4}$$

[5]

Q. 13. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find

the probability that the new product introduced was by the second group.

[NCERT Ex. 13.3, Q. 9, Page 555]

Ans. Let E_1 and E_2 be the respective events that the first group and the second group win the competition. Let A be the event of introducing a new product.

$P(E_1)$ = Probability that the first group wins the competition = 0.6

$P(E_2)$ = Probability that the second group wins the competition = 0.4

$P(A|E_1)$ = Probability of introducing a new product if the first group wins = 0.7

$P(A|E_2)$ = Probability of introducing a new product if the second group wins = 0.3

The probability that the new product is introduced by the second group is given by $P(E_2|A)$.

By using Bayes' theorem, we obtain

$$P(E_2 | A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{0.4\times 0.3}{0.6\times 0.7 + 0.4\times 0.3} = \frac{0.12}{0.42+0.12}$$

$$= \frac{0.12}{0.54} = \frac{12}{54} = \frac{2}{9}$$

[5]

Q. 14. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

[NCERT Ex. 13.3, Q. 10, Page 555]

Ans. Let E_1 be the event that the outcome on the die is 5 or 6 and E_2 be the event the outcome on the die is 1, 2, 3, or 4.

$$\therefore P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let A be the event of getting exactly one head.

$P(A|E_1)$ = Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 $= \frac{3}{8}$

$P(A|E_2)$ = Probability of getting exactly one head in a single throw of coin if she gets 1, 2, 3, or 4 $= \frac{1}{2}$

The probability that the girl threw 1, 2, 3, or 4 with the die, if she obtained exactly one head, is given by $P(E_2|A)$.

By using Bayes' theorem, we obtain

$$P(E_2 | A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{3}{8} + \frac{2}{3}\cdot\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{3}{8}+1\right)} = \frac{1}{\frac{11}{8}} = \frac{8}{11}$$

[5]

Q. 15. *A* manufacturer has three machine operators *A*, *B* and *C*. The first operator *A* produces 1% defective items, where as the other two operators *B* and *C* produce 5% and 7% defective items respectively. *A* is on the job for 50% of the time, *B* is on the job for 30% of the time and *C* is on the job for 20% of the time. *A* defective item is produced, what is the probability that it was produced by *A*?

[NCERT Ex. 13.3, Q. 11, Page 555]

Ans. Let E_1, E_2 and E_3 be the respective events of the time consumed by machines *A*, *B* and *C* for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X | E_1) = 1\% = \frac{1}{100}$$

$$P(X | E_2) = 5\% = \frac{5}{100}$$

$$P(X | E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by *A* is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1 | X) = \frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2) + P(E_3)\cdot P(X|E_3)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100} + \frac{3}{10}\cdot\frac{5}{100} + \frac{1}{5}\cdot\frac{7}{100}} = \frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2}+\frac{3}{2}+\frac{7}{5}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{17}{5}} = \frac{5}{34}$$

[5]

Q. 16. *A* card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

[NCERT Ex. 13.3, Q. 12, Page 555]

Ans. Let E_1 and E_2 be the respective events of choosing a diamond card and a card which is not diamond.

Let *A* denotes the lost card.

Out of 52 cards, 13 cards are diamond and 39 cards are not diamond. Therefore,

$$P(E_1) = \frac{13}{52} = \frac{1}{4}$$

$$P(E_2) = \frac{39}{52} = \frac{3}{4}$$

When one diamond card is lost, there are 12 diamond cards out of 51 cards.

Two cards can be drawn out of 12 diamond cards in ${}^{12}C_2$ ways.

Similarly, 2 diamond cards can be drawn out of 51 cards in ${}^{51}C_2$ ways. The probability of getting two cards, when one diamond card is lost, is given by $P(A|E_1)$.

$$P(A|E_1)=\frac{^{12}C_2}{^{51}C_2}=\frac{12!}{2!\times10!}\times\frac{2!\times49!}{51!}=\frac{11\times12}{50\times51}=\frac{22}{425}$$

When the lost card is not a diamond, there are 13 diamond cards out of 51 cards.

Two cards can be drawn out of 13 diamond cards in $^{13}C_2$ ways whereas 2 cards can be drawn out of 51 cards in $^{51}C_2$ ways.

The probability of getting two cards, when one card is lost which is not diamond, is given by $P(A|E_2)$.

$$P(A|E_2)=\frac{^{13}C_2}{^{51}C_2}=\frac{13!}{2!\times11!}\times\frac{2!\times49!}{51!}=\frac{12\times13}{50\times51}=\frac{26}{425}$$

The probability that the lost card is diamond is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{1}{4}\cdot\frac{22}{425}}{\frac{1}{4}\cdot\frac{22}{425}+\frac{3}{4}\cdot\frac{26}{425}}=\frac{\frac{1}{425}\left(\frac{22}{4}\right)}{\frac{1}{425}\left(\frac{22}{4}+\frac{26\times3}{4}\right)}$$

$$=\frac{\frac{11}{2}}{25}=\frac{11}{50}$$

[5]

Q. 17. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

[NCERT Ex. 13.4, Q. 6, Page 570]

Ans. It is given that out of 30 bulbs, 6 are defective.

$\Rightarrow$ Number of non-defective bulbs = 30 – 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$\therefore$ $P(X = 0) = P$(4 non-defective and 0 defective)

$$= {^4C_0}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\frac{4}{5}=\frac{256}{625}$$

$P(X = 1) = P$(3 non-defective and 1 defective)

$$= {^4C_1}\cdot\left(\frac{1}{5}\right)\cdot\left(\frac{4}{5}\right)^3=\frac{256}{625}$$

$P(X = 2) = P$(2 non-defective and 2 defective)

$$= {^4C_2}\cdot\left(\frac{1}{5}\right)^2\cdot\left(\frac{4}{5}\right)^2=\frac{96}{625}$$

$P(X = 3) = P$(1 non-defective and 3 defective)

$$= {^4C_3}\cdot\left(\frac{1}{5}\right)^3\cdot\left(\frac{4}{5}\right)=\frac{16}{625}$$

$P(X = 4) = P$(0 non-defective and 4 defective)

$$= {^4C_4}\cdot\left(\frac{1}{5}\right)^4\cdot\left(\frac{4}{5}\right)^0=\frac{1}{625}$$

[5]

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Q. 18. *A* coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

[NCERT Ex. 13.4, Q. 7, Page 570]

Ans. Let the probability of getting a tail in the biased coin be x.

$\therefore \quad P(T) = x$

$\Rightarrow \quad P(H) = 3x$

For a biased coin, $P(T) + P(H) = 1$

$\Rightarrow \quad x + 3x = 1$

$\Rightarrow \quad 4x = 1$

$\Rightarrow \quad x=\frac{1}{4}$

$\therefore \quad P(T)=\frac{1}{4}$ and $P(H)=\frac{3}{4}$

When the coin is tossed twice, the sample space is {*HH, TT, HT, TH*}.

Let X be the random variable representing the number of tails.

$$\therefore P(X=0)=P(\text{no tail})=P(H)\times P(H)=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$$

$P(X = 1) = P$(one tail) $P(HT) + P(TH)$

$$=\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{3}{4}=\frac{3}{16}+\frac{3}{16}=\frac{3}{8}$$

$P(X = 2) = P$(two tails) $= P(TT) =\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$ **[5]**

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Q. 19. Find the mean number of heads in three tosses of a fair coin. [NCERT Ex. 13.4, Q. 10, Page 570]

Ans. Let X denotes the success of getting heads.

Therefore, the sample space is

S = {*HHH, HHT, HTH, HTT, THH, THT, TTH, TTT*}

It can be seen that X can be take the value of 0, 1, 2, or 3.

$$\therefore P(X=0)=P(TTT)$$
$$=P(T)\cdot P(T)\cdot P(T)$$
$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$

$$\therefore P(X=1)=P(HTT)+P(THT)+P(TTH)$$
$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

$$\therefore P(X=2)=P(HHT)+P(HTH)+P(THH)$$
$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

$$\therefore P(X=3)=P(HHH)$$
$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of $XE(X), \mu = \sum x_i P(X_i)$

$$= 0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$$

$$= \frac{3}{8}+\frac{3}{4}+\frac{3}{8}=\frac{3}{2}=1.5$$

[5]

Q. 20. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find $E(X)$.

[NCERT Ex. 13.4, Q. 12, Page 570]

Ans. The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For $X = 2$, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For $X = 3$, the possible observations are (1, 3), (2, 3), (3, 1) and (3, 2).

$$\therefore P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For $X = 4$, the possible observations are (1, 4), (2, 4), (3, 4), (4, 2) and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For $X = 5$, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2) and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For $X = 6$, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 5), (6, 4), (6, 3), (6, 2) and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows :

X	2	3	4	5	6
$P(X)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, $E(X) = \Sigma X_i P(X_i)$

$$= 2\cdot\frac{1}{15}+3\cdot\frac{2}{15}+4\cdot\frac{1}{5}+5\cdot\frac{4}{15}+6\cdot\frac{1}{3}$$

$$= \frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2=\frac{70}{15}=\frac{14}{3}$$

[5]

Q. 21. Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

[NCERT Ex. 13.4, Q. 13, Page 570]

Ans. When two fair dice are rolled, $6 \times 6 = 36$ observations are obtained.

$$P(X = 2) = P(1, 1) = \frac{1}{36}$$

$$P(X = 3) = P(1, 2) + P(2, 1) = \frac{2}{36}=\frac{1}{18}$$

$$P(X = 4) = P(1, 3) + P(2, 2) + P(3, 1) = \frac{3}{36}=\frac{1}{12}$$

$$P(X = 5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1) = \frac{4}{36}=\frac{1}{9}$$

$$P(X = 6) = P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) = \frac{5}{36}$$

$$P(X = 7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) = \frac{6}{36}=\frac{1}{6}$$

$$P(X =8) = P(2, 6) + P(3, 5) + P(4, 4) + P(5, 3) + P(6, 2) = \frac{5}{36}$$

$$P(X = 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3) = \frac{4}{36}=\frac{1}{9}$$

$$P(X = 10) = P(4, 6) + P(5, 5) + P(6, 4) = \frac{3}{36}=\frac{1}{12}$$

$$P(X = 11) = P(5, 6) + P(6, 5) = \frac{2}{36}=\frac{1}{18}$$

$$P(X= 12) = P(6, 6) = \frac{1}{36}$$

Therefore, the required probability distribution is as follows :

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Then, $E(X) = \sum X_i P(X_i)$

$$= 4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}$$

$$+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$= \frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$= \frac{987}{18}=\frac{329}{6}=54.833$$

Then,

$\text{Var.}(X) = E(X^2) - [E(X)]^2$

$= 54.833 - (7)^2$

$= 54.833 - 49$

$= 5.833$

$\therefore$ Standard deviation

$= \sqrt{\text{Var.}(X)} = \sqrt{5.833} = 2.415$

[5]

Q. 22. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

[NCERT Ex. 13.4, Q. 11, Page 571]

Ans. Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

$\therefore$ $P(X = 0) = P$(not getting six on any of the dice)

$$= \frac{25}{36}$$

$P(X = 1) = P$(six on first die and no six on second die) + P(no six on first die and six on second die)

$$= 2\left(\frac{1}{6} \times \frac{5}{6}\right) = \frac{10}{36}$$

$P(X = 2) = P$(six on both the dice) $= \frac{1}{36}$

Therefore, the required probability distribution is as follows :

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of $X = E(X) = \Sigma X_i P(X_i)$

$$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{1}{3}$$

[5]

Q. 23. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of *X*.

[NCERT Ex. 13.4, Q. 14, Page 570]

Ans. There are 15 students in the class. Each student has the same change to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows :

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X=14) = \frac{2}{15}, P(X=15) = \frac{1}{15}, P(X=16) = \frac{2}{15},$$

$$P(X=16) = \frac{3}{15},$$

$$P(X=18) = \frac{1}{15}, P(X=19) = \frac{2}{15}, P(X=20) = \frac{3}{15},$$

$$P(X=21) = \frac{1}{15}$$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of $X = E(X)$

$$= \Sigma X_i P(X_i)$$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15}$$

$$+ 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$$

$$= \frac{263}{15} = 17.53$$

$$E(X^2) = \Sigma X_i^2 P(X_i)$$

$$= (14)^2 \cdot \frac{2}{15} + (15)^2 \cdot \frac{1}{15} + (16)^2 \cdot \frac{2}{15} + (17)^2 \cdot \frac{3}{15} + (18)^2 \cdot \frac{1}{15}$$

$$+ (19)^2 \cdot \frac{2}{15} + (20)^2 . \frac{3}{15} + (21)^2 \cdot \frac{1}{15}$$

$$= \frac{1}{15} \cdot (392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441)$$

$$= 312.2$$

$\therefore$ Variance $(X) = E(X^2) - [E(X)]^2$

$$= 312.2 - \left(\frac{263}{15}\right)^2$$

$$= 312.2 - 307.4177$$

$$= 4.7823 \approx 4.78$$

Standard deviation $= \sqrt{\text{Variance}(X)}$

$$= \sqrt{4.78}$$

$$= 2.186 \approx 2.19$$

[5]

Q. 24. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(i) 5 successes? (ii) at least 5 successes?

(iii) at most 5 successes?

[NCERT Ex. 13.5, Q. 1, Page 576]

Ans. The repeated tosses of a die are Bernoulli trials. Let X denotes the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p = \frac{3}{6} = \frac{1}{2}$

$$\therefore \qquad q = 1 - p = \frac{1}{2}$$

X has a binomial distribution.

Therefore, $P(X = x) = {}^nC_{n-x} q^{n-x} p^x$, where $n = 0, 1, 2 ... n$

$$= {}^6C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x = {}^6C_x \left(\frac{1}{2}\right)^6$$

(i) P(5 successes) $= P(X = 5)$

$$= {}^6C_5 \left(\frac{1}{2}\right)^6 = 6 \cdot \frac{1}{64} = \frac{3}{32}$$

(ii) P(at least 5 successes) $= P(X \geq 5)$

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6 = 6 \cdot \frac{1}{64} + 1 \cdot \frac{1}{64} = \frac{7}{64}$$

(iii) P(at most 5 successes) $= P(X \leq 5)$

$$= 1 - P(X > 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 = 1 - \frac{1}{64} = \frac{63}{64}$$

[5]

Q. 25. *A* pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes. **[NCERT Ex. 13.5, Q. 2, Page 577]**

Ans. The repeated tosses of a pair of dice are Bernoulli trials. Let X denotes the number of times of getting doublets in an experiment of throwing two dice simultaneously 4 times.

Probability of getting doublets in a single throw of the pair of dice is

$$p = \frac{6}{36} = \frac{1}{6}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Clearly, X has the binomial distribution with $n = 4$, $p = \frac{1}{6}$, and $q = \frac{5}{6}$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x, \text{ where } x = 0, 1, 2, 3 \ldots n$$

$$= {}^4C_x \left(\frac{5}{6}\right)^{4-x} \cdot \left(\frac{1}{6}\right)^x = {}^4C_x \cdot \frac{5^{4-x}}{6^4}$$

Therefore,

$$\text{P(2 successes)} = \text{P}(X=2)$$

$$= {}^4C_2 \cdot \frac{5^{4-2}}{6^4} = 6 \cdot \frac{25}{1296} = \frac{25}{216}$$

[5]

Q. 26. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

[NCERT Ex. 13.5, Q. 3, Page 577]

Ans. Let X denotes the number of defective items in a sample of 10 items drawn successively. Since the drawing is done with replacement the trials are Bernoulli trials.

$$\Rightarrow p = \frac{5}{100} = \frac{1}{20}$$

$$\therefore q = 1 - \frac{1}{20} = \frac{19}{20}$$

X has a binomial distribution with $n = 10$ and $p = \frac{1}{20}$

$$P(X = x) = {}^nC_x q^{n-x} p^x, \text{ where } x = 0, 1, 2 \ldots n$$

$$= {}^{10}C_x \left(\frac{19}{20}\right)^{10-x} \cdot \left(\frac{1}{20}\right)^x$$

P(not more than 1 defective item) $= P(X \le 1)$

$= P(X = 0) + P(X = 1)$

$$= {}^{10}C_0 \left(\frac{19}{20}\right)^{10} \cdot \left(\frac{1}{20}\right)^0 + {}^{10}C_1 \left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)^1$$

$$= \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left(\frac{29}{20}\right) = \left(\frac{29}{20}\right) \cdot \left(\frac{19}{20}\right)^9$$

[5]

Q. 27. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(i) none **(ii) not more than one**

(iii) more than one **(iv) at least one**

will fuse after 150 days of use.

[NCERT Ex. 13.5, Q. 5, Page 577]

Ans. Let X represents the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that,

$p = 0.05$

$\therefore \quad q = 1 - p = 1 - 0.05 = 0.95$

X has a binomial distribution with $n = 5$ and $p = 0.05$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x, \text{ where } x = 1, 2, \ldots n$$

$$= {}^5C_x (0.95)^{5-x} \cdot (0.05)^x$$

(i) P (none) $= P(X = 0)$

${}^5C_0 (0.95)^5 (0.05)^0$

$= 1 \times (0.95)^5$

$= (0.95)^5$

(ii) P (not more than one) $= P(X \le 1)$

$= P(X = 0) + P(X = 1)$

${}^5C_0(0.95)^5 \times (0.05)^0 + {}^5C_1(0.95)^4 \times (0.05)^1$

$1 \times (0.95)^5 + 5 \times (0.95)^4 \times (0.05)$

$= (0.95)^5 + (0.25)(0.95)^4$

$= (0.95)^4 [0.95 + 0.25]$

$= (0.95)^4 \times 1.2$

(iii) P (more than 1) $= P(X > 1)$

$= 1 - P(X \le 1)$

$= 1 - P$(not more than 1)

$= 1 - (0.95)^4 \times 1.2$

(iv) P(at least one) $= P(X \ge 1)$

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^5C_0 (0.95)^5 \times (0.05)^0$

$= 1 - 1 \times (0.95)^5$

$= 1 - (0.95)^5$

[5]

Q. 28. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

[NCERT Ex. 13.5, Q. 7, Page 577]

Ans. Let X represents the number of correctly answered questions out of 20 questions.

The repeated tosses of a coin are Bernoulli trails. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$$\therefore p = \frac{1}{2}$$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

X has a binomial distribution with $n = 20$ and $p = \frac{1}{2}$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x, \text{ where } x = 0, 1, 2, \ldots n$$

$$= {}^{20}C_x \left(\frac{1}{2}\right)^{20-x} \cdot \left(\frac{1}{2}\right)^x = {}^{20}C_x \left(\frac{1}{2}\right)^{20}$$

P(at least 12 questions answered correctly) = $P(X \geq 12)$

$= P(X = 12) + P(X = 13) + ...+ P(X = 20)$

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{20} + ... + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20} \cdot \left[{}^{20}C_{12} + {}^{20}C_{13} + ... + {}^{20}C_{20}\right]$$

[5]

Q. 29. Suppose X has a binomial distribution $B(6,1/2)$. Show that $X = 3$ is the most likely outcome.
[NCERT Ex. 13.5, Q. 8, Page 577]

Ans. X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Then, $P(X = x) = {}^nC_x q^{n-x} p^x$

$$= {}^6C_x\left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^{x} = {}^6C_x\left(\frac{1}{2}\right)^{6}$$

It can be seen that $P(X = x)$ will be maximum, if ${}^6C_x$ will be maximum. Then,

$$^6C_0 = {}^6C_6 = \frac{6!}{0! \cdot 6!} = 1$$

$$^6C_1 = {}^6C_5 = \frac{6!}{1! \cdot 5!} = 6$$

$$^6C_2 = {}^6C_4 = \frac{6!}{2! \cdot 4!} = 15$$

$$^6C_3 = \frac{6!}{3! \cdot 3!} = 20$$

The value of ${}^6C_3$ is maximum. Therefore, for $x = 3$, $P(X = x)$ is maximum.

Thus, $X = 3$ is the most likely outcome. **[5]**

Q. 30. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100. What is the probability that he will win a prize
(a) at least once (b) exactly once
(c) at least twice? [NCERT Ex. 13.5, Q. 10, Page 577]

Ans. Let X represents the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\therefore P(X = x) {}^nC_x q^{n-x} p^x = {}^{50}C_x\left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^{x}$$

(a) P(winning at least once) = $P(X \geq 1)$

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$$= 1 - {}^{50}C_0\left(\frac{99}{100}\right)^{50}$$

$$= 1 - 1 \cdot \left(\frac{99}{100}\right)^{50} = 1 - \left(\frac{99}{100}\right)^{50}$$

(b) P(winning exactly once) = $P(X = 1)$

$$= {}^{50}C_1\left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49} = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P(at least twice) = $P(X \geq 2)$

$= 1 - P(X < 2)$

$= 1 - P(X \leq 1)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= [1 - P(X = 0)] - P(X = 1)$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right) = 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

[5]

Q. 31. Find the probability of throwing at most 2 sixes in 6 throws of a single die.
[NCERT Ex. 13.5, Q. 12, Page 578]

Ans. The repeated tossing of the die are Bernoulli trials. Let X represents the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die,

$$p = \frac{1}{6}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Clearly, X has a binomial distribution with $n = 6$

$$\therefore q(X = x) = {}^nC_x q^{n-x} p^x = {}^6C_x\left(\frac{5}{6}\right)^{6-x} \cdot \left(\frac{1}{6}\right)^{x}$$

P(at most 2 sixes) = $P(X \leq 2)$

$P(X = 0) + P(X = 1) + P(X = 2)$

$$= {}^6C_0\left(\frac{5}{6}\right)^{6} + {}^6C_1 \cdot \left(\frac{5}{6}\right)^{5} \cdot \left(\frac{1}{6}\right) + {}^6C_2\left(\frac{5}{6}\right)^{4} \cdot \left(\frac{1}{6}\right)^{2}$$

$$= 1 \cdot \left(\frac{5}{6}\right)^{6} + 6 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{5} + 15 \cdot \frac{1}{36} \cdot \left(\frac{5}{6}\right)^{4}$$

$$= \left(\frac{5}{6}\right)^{6} + \left(\frac{5}{6}\right)^{5} + \frac{5}{12} \cdot \left(\frac{5}{6}\right)^{4}$$

$$= \left(\frac{5}{6}\right)^{4}\left[\left(\frac{5}{6}\right)^{2} + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right)\right]$$

$$= \left(\frac{5}{6}\right)^{4} \cdot \left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right]$$

$$= \left(\frac{5}{6}\right)^{4} \cdot \left[\frac{25 + 30 + 15}{36}\right]$$

$$= \frac{70}{36} \cdot \left(\frac{5}{6}\right)^{4} = \frac{35}{18} \cdot \left(\frac{5}{6}\right)^{4}$$

[5]

Q. 32. Three bags contain a number of red and white balls as follows : Bag 1 :3 red balls, Bag 2 :2 red

balls and 1 white ball Bag 3 :3 white balls. The probability that bag *i* will be chosen and a ball is selected from it is *i*/6, *i* = 1, 2, 3. What is the probability that

(i) a red ball will be selected?

(ii) a white ball is selected?

[NCERT Exemp. Ex. 13.3, Q. 41, Page 276]

Ans. **Bag 1 :** 3 red balls and 0 white ball.

Bag 2 : 2 red balls and 1 white ball.

Bag 3 : 0 red ball and 3 white balls.

Let E_1, E_2 and E_3 be the events that Bag 1, Bag 2 and Bag 3 is selected and a ball is chosen from it.

$$P(E_1)=\frac{1}{6}, P(E_2)=\frac{2}{6} \text{ and } P(E_3)=\frac{3}{6}$$

(i) Let E be the event that a red ball is selected.

Then, probability that red ball will be selected

$P(E) = P(E_1)\, P(E|E_1) + P(E_2)\, P(E|E_2) + P(E_3)\, P(E|E_3)$

$$=\frac{1}{6}\cdot\frac{3}{3}+\frac{2}{6}\cdot\frac{2}{3}+\frac{3}{6}\cdot 0=\frac{7}{18}$$

(ii) Let E be the event that a white ball is selected

$\therefore P(E') = P(E_1)\, P(E'|E_1) + P(E_2)\, P(E'|E_2) + P(E_3)\, P(E'|E_3)$

$$=\frac{1}{6}\cdot 0+\frac{2}{6}\cdot\frac{1}{3}+\frac{3}{6}\cdot 1=\frac{1}{9}+\frac{3}{6}=\frac{11}{18}$$

[5]

Q. 33. Refer to Question 196 above. If a white ball is selected, what is the probability that it came from

(i) Bag 2 **(ii) Bag 3**

[NCERT Exemp. Ex. 13.3, Q. 42, Page 276]

Ans. Referring to the previous solution, using Bayes' theorem, we have

(i)

$$P(E_2/E')=\frac{P(E_2)\cdot P(E'/E_2)}{P(E_1)\cdot P(E'/E_1)+P(E_2)\cdot P(E'/E_2)+P(E_3)\cdot P(E'/E_3)}$$

$$=\frac{\frac{2}{6}\cdot\frac{1}{3}}{\frac{1}{6}\cdot 0+\frac{2}{6}\cdot\frac{1}{3}+\frac{3}{6}\cdot 1}=\frac{\frac{2}{18}}{\frac{2}{18}+\frac{3}{6}}=\frac{2}{11}$$

(ii)

$$P(E_3/E')=\frac{P(E_3)\cdot P(E'/E_3)}{P(E_1)\cdot P(E'/E_1)+P(E_2)\cdot P(E'/E_2)+P(E_3)\cdot P(E'/E_3)}$$

$$=\frac{\frac{3}{6}\cdot 1}{\frac{1}{6}\cdot 0+\frac{2}{6}\cdot\frac{1}{3}+\frac{3}{6}\cdot 1}=\frac{\frac{3}{6}}{\frac{2}{18}+\frac{3}{6}}=\frac{9}{11}$$

[5]

Q. 34. *A* shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 :4 :2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(i) of a randomly chosen seed to germinate

(ii) that it will not germinate given that the seed is of type A_3,

(iii) that it is of the type A_2 given that a randomly chosen seed does not germinate.

[NCERT Exemp. Ex. 13.3, Q. 43, Page 276]

Ans. We have, $A_1 : A_2 : A_3 = 4 : 4 : 2$

$$P(A_1)=\frac{4}{10}, P(A_2)=\frac{4}{10} \text{ and } P(A_3)=\frac{2}{10}$$

where A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that a seed germinates and E' be the event that a seed does not germinate.

$$\therefore \quad P(E/A_1)=\frac{45}{100}, P(E/A_2)=\frac{60}{100}$$

$$\text{and } P(E/A_3)=\frac{35}{100}$$

$$\text{and } P(E'/A_1)=\frac{55}{100}, P(E'/A_2)=\frac{40}{100}$$

$$\text{and } P(E'/A_3)=\frac{65}{100}$$

(i) $\therefore P(E) = P(A_1)\, P(E/A_1) + P(A_2)\, P(E/A_2) + P(A_3)\, P(E/A_3)$

$$=\frac{4}{10}\cdot\frac{45}{100}+\frac{4}{10}\cdot\frac{60}{100}+\frac{2}{10}\cdot\frac{35}{100}$$

$$=\frac{180}{1000}+\frac{240}{1000}+\frac{70}{1000}=\frac{490}{1000}=0.49$$

(ii)

$$P(E'/A_3)=1-P(E/A_3)=1-\frac{35}{100}=\frac{65}{100}$$

(iii)

$$P(A_2/E')=\frac{P(A_2)\cdot P(E'/A_2)}{P(A_1)\cdot P(E'/A_1)+P(A_2)\cdot P(E'/A_2)+P(A_3)\cdot P(E'/A_3)}$$

$$=\frac{\frac{4}{10}\cdot\frac{40}{100}}{\frac{4}{10}\cdot\frac{55}{100}+\frac{4}{10}\cdot\frac{40}{100}+\frac{2}{10}\cdot\frac{65}{100}}=\frac{16}{51}$$

[5]

Q. 35. *A* letter is known to have come either from *TATA NAGAR* or from *CALCUTTA*. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from *TATA NAGAR*.

[NCERT Exemp. Ex. 13.3, Q. 44, Page 277]

Ans. Also, let E be the event that on the letter, two consecutive letters TA are visible.

$$\therefore P(E_1)=\frac{1}{2} \text{ and } P(E_2)=\frac{1}{2}$$

Also when two consecutive letters are visible is the case of 'TATANAGAR', we have following sets of possible consecutive letters,

$\{TA, AT, TA, AN, NA, AG, GA, AR\}$

In the case of '*CALCUTTA*, we have following set of possible consecutive letters,

$\{CA, AL, LC, CU, UT, TT, TA\}$

$$\therefore P(E/E_1)=\frac{2}{8} \text{ and } P(E/E_2)=\frac{1}{7}$$

$$\therefore P(E_1/E)=\frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)}$$

$$=\frac{\frac{1}{2}\cdot\frac{2}{8}}{\frac{1}{2}\cdot\frac{2}{8}+\frac{1}{2}\cdot\frac{1}{7}}=\frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{14}}=\frac{\frac{1}{8}}{\frac{22}{8\times 14}}=\frac{\frac{1}{8}}{\frac{11}{56}}=\frac{7}{11}$$

[5]

Q. 36. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the Ist bag; but it shows up any other number, a ball is chosen from the second bag. Find the probability of choosing a black ball

[NCERT Exemp. Ex. 13.3, Q. 45, Page 277]

Ans. Since,

Bag I = {3 black, 4 white balls},

Bag II = {4 black, 3 white balls}

Let E_1 be the event that bag I is selected and E_2 Be the event that bag II is selected.

Let E be the event that black ball is chosen.

$$\therefore P(E_1)=\frac{2}{6}=\frac{1}{3} \text{ and } P(E_2)=1-\frac{1}{3}=\frac{2}{3}$$

$$\text{and } P(E/E_1)=\frac{3}{7} \text{ and } P(E/E_2)=\frac{4}{7}$$

$$\therefore P(E)=P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)$$

$$=\frac{1}{3}\cdot\frac{3}{7}+\frac{2}{3}\cdot\frac{4}{7}=\frac{11}{21}$$

[5]

Q. 37. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls, and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn.

[NCERT Exemp. Ex. 13.3, Q. 46, Page 277]

Ans. We have,

U_1 = {2 white, 3 black balls}

U_2 = {3 white, 2 black balls}

and U_3 = {4 white, 1 black balls}

$$\therefore P(U_1)=P(U_2)=P(U_3)=\frac{1}{3}$$

Let E_1, E_2 and E_3 be the event that a ball is chosen from an urn U_1, U_2 and U_3, respectively.

$$\therefore P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$$

Now, let E be the event that white ball is drawn.

$$\therefore P(E/E_1)=\frac{2}{5}P(E/E_2)=\frac{3}{5},P(E/E_3)=\frac{4}{5}$$

Now using Bayes' Theorem,

$$P(E_2/E)=\frac{P(E_2)\cdot P(E/E_2)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)+P(E_3)\cdot P(E/E_3)}$$

$$=\frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{1}{3}\cdot\frac{2}{5}+\frac{1}{3}\cdot\frac{3}{5}+\frac{1}{3}\cdot\frac{4}{5}}=\frac{\frac{3}{15}}{\frac{2}{15}+\frac{3}{15}+\frac{4}{15}}=\frac{3}{9}=\frac{1}{3}$$

[5]

Q. 38. By examining the chest *X* ray, the probability that *TB* is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have *TB* is 0.001. In a certain city, 1 in 1000 people suffers from *TB*. *A* person is selected at random and is diagnosed to have *TB*. What is the probability that he actually has *TB*?

[NCERT Exemp. Ex. 13.3, Q. 47, Page 277]

Ans. Let,

E_1 = Event that person has *TB*

E_2 = Event that person does not have *TB*

E = Event that the person is diagnosed to have *TB*

$$\therefore \quad P(E_1)=\frac{1}{1000}=0.001, P(E_2)=\frac{999}{1000}=0.999$$

$$P(E/E_1)=0.99 \text{ and } P(E/E_2)=0.001$$

$$P(E_1/E)=\frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)}$$

$$=\frac{0.001\times 0.99}{0.001\times 0.99+0.999\times 0.001}$$

$$=\frac{990}{990+999}=\frac{110}{110+111}=\frac{110}{221}$$

[5]

Q. 39. An item is manufactured by three machines *A*, *B* and *C*. Out of the total number of items manufactured during a specified period, 50% are manufactured on *A*, 30% on *B* and 20% on *C*. 2% of the items produced on A and 2% of items produced on *B* are defective, and 3% of these produced on *C* are defective. All the items are stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine *A*?

[NCERT Exemp. Ex. 13.3, Q. 48, Page 277]

Ans.

$$P(E_1)=\frac{50}{100}=\frac{1}{2},$$

$$P(E_2)=\frac{30}{100}=\frac{3}{10}$$

$$\text{and } P(E_3)=\frac{20}{100}=\frac{1}{5}$$

$$P(E/E_1)=P(E/E_2)=\frac{2}{100}=\frac{1}{50}$$

$$\text{and } P(E/E_3)=\frac{3}{100}$$

$$P(E_1/E)=\frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot(E/E_1)+P(E_2)\cdot P(E/E_2)+P(E_3)\cdot P(E/E_3)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{50}}{\frac{1}{2}\cdot\frac{1}{50}+\frac{3}{10}\cdot\frac{1}{50}+\frac{1}{5}\cdot\frac{3}{100}}$$

$$=\frac{\frac{1}{100}}{\frac{5}{500}+\frac{3}{500}+\frac{3}{500}}=\frac{5}{11}$$

[5]

Q. 40. Let *X* be a discrete random variable whose probability distribution is defined as follows :

$$P(X=x)=\begin{cases} k(x+1) \text{ for } x=1,2,3,4 \\ 2kx \quad\quad \text{for } x=5,6,7 \\ 0 \quad\quad\quad \text{otherwise} \end{cases}$$

where *k* is a constant. Calculate (i) the value of *k* (ii) *E* (*X*) (iii) Standard deviation of *X*.

[NCERT Exemp. Ex. 13.3, Q. 49, Page 277]

Ans. We have,

$$P(X=x)=\begin{cases}k(x+1) & \text{for } x=1,2,3,4\\ 2kx & \text{for } x=5,6,7\\ 0 & \text{otherwise}\end{cases}$$

Thus, we have

X	1	2	3	4	5	6	7	Other-wise
P(X)	$2k$	$3k$	$4k$	$5k$	$10k$	$12k$	$14k$	0
XP(X)	$2k$	$6k$	$12k$	$20k$	$50k$	$72k$	$95k$	0
X2P(X)	$2k$	$12k$	$36k$	$80k$	$250k$	$432k$	$686k$	0

(i) Since, $\Sigma P_i = 1$

$\Rightarrow k(2+3+4+5+10+12+14)=1$

$\therefore \quad k=\dfrac{1}{50}$

(ii) $\because E(X) = \Sigma XP(X)$

$\therefore E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k + 0$

$$= 260k = 260\times\frac{1}{50}=\frac{26}{5}=5.2$$

(iii) We know that,

$$\text{Var }(X)=[E(X^2)]-[E(X)]^2$$
$$=\Sigma X^2P(X)-[\Sigma\{XP(X)\}]^2$$
$$=[2k+12k+36k+80k+250k+432k$$
$$+686k+0]-[5.2]^2$$
$$=[1498k]-27.04=\left[1498\times\frac{1}{50}\right]-27.04$$
$$=29.96-27.04=2.92$$

We know that, standard deviation of $X=\sqrt{\text{Var }(X)}=\sqrt{2.92}$ **[5]**

$$=1.7088\cong 1.7$$

Q. 41. The probability distribution of random variable *X* is given as under :

X	1	2	4	$2A$	$3A$	$5A$
$P(X)$	$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{1}{5}}$	$\mathbf{\frac{3}{25}}$	$\mathbf{\frac{1}{10}}$	$\mathbf{\frac{1}{25}}$	$\mathbf{\frac{1}{25}}$

Calculate :

(i) The value of *A* if *E*(*X*) = 2.94

(ii) Variance of *X*.

[NCERT Exemp. Ex. 13.3, Q. 50, Page 278]

Ans. **(i)** We have, $\Sigma XP(X)=\dfrac{1}{2}+\dfrac{2}{5}+\dfrac{12}{25}+\dfrac{2A}{10}+\dfrac{3A}{25}+\dfrac{5A}{25}$

$$=\frac{25+20+24+10A+6A+10A}{50}=\frac{69+26A}{50}$$

Since, $E(X) = \Sigma XP(X)$

$$\Rightarrow 2.94=\frac{69+26A}{50}$$
$$\Rightarrow 26A=50\times 2.94-69$$
$$\Rightarrow \quad A=\frac{147-69}{26}=\frac{78}{26}=3$$

(ii) We know that,

$$\text{Var.}(X)=E(X^2)-[E(X)]^2$$
$$=\Sigma X^2P(X)-[\Sigma(X)]^2$$
$$=\frac{1}{2}+\frac{4}{5}+\frac{48}{25}+\frac{4A^2}{10}+\frac{9A^2}{25}+\frac{25A^2}{25}-[E(X)]^2$$
$$=\frac{25+40+96+20A^2+18A^2+50A^2}{50}-[E(X)]^2$$
$$=\frac{161+88A^2}{50}-[E(X)]^2$$
$$=\frac{161+88\times(3)^2}{50}-[E(X)]^2$$
$$=\frac{953}{50}-[2.94]^2 \qquad [\because E(X)=2.94]$$
$$=19.06-8.6436=10.4164$$

[5]

Q. 42. The probability distribution of a random variable *x* is given as under :

$$P(X=x)=\begin{cases}kx^2 & \text{for } x=1,2,3\\ 2kx & \text{for } x=4,5,6\\ 0 & \text{otherwise}\end{cases}$$

Where *k* is a constant Calculate.

(i) *E*(X) **(ii) *E*(3X^2)**

(iii) *P*(X ≥ 4)

[NCERT Exemp. Ex. 13.3, Q. 51, Page 278]

Ans.

X	1	2	3	4	5	6	Otherwise
$P(X)$	k	$4k$	$9k$	$8k$	$10k$	$12k$	0

We know that $\Sigma P_i = 1$

$\Rightarrow \quad 44k=1$

$\therefore \quad k=\dfrac{1}{44}$

$\therefore \quad \Sigma XP(X)=k+8k+27k+32k+50k+72k+0$

$$=190k=190\times\frac{1}{44}=\frac{95}{22}$$

(i) So, $E(X)=\Sigma XP(X)=\dfrac{95}{22}=4.32$

(ii) Also, $E(X^2) = \Sigma X^2P(X) = k + 16k + 81k + 128k + 250k + 432k$

$$=908k=908\times\frac{1}{44}=20.636\cong 20.64$$

$\therefore \quad E(3X^2) = 3E(X^2) = 3 \times 20.64 = 61.92$

(iii) $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$$=8k+10k+12k=30k=30\times\frac{1}{44}=\frac{15}{22}$$ **[5]**

Q. 43. *A* bag contains (2*n* + 1) coins. It is known that n of these coins have a head on both sides where as the rest of the coins are fair. *A* coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is 31/42, determine the value of n.

[NCERT Exemp. Ex. 13.3, Q. 52, Page 278]

Ans. Given n coins have head on both sides and $(n + 1)$ coins are fair coins.

Let,

E_1 = Event that an unfair coin is selected

E_2 = Event that a fair coin is selected

H = Event that the toss result in a head

$\therefore \quad P(E_1) = \frac{n}{2n+1}$ and $P(E_2) = \frac{n+1}{2n+1}$

Also, $P(H/E_1) = 1$ and $P(H/E_2) = \frac{1}{2}$

$$\therefore \quad P(H) = P(E_1)\cdot P(H/E_1) + P(E_2)\cdot P(H/E_2)$$

$$= \frac{n}{2n+1}\cdot 1 + \frac{n+1}{2n+1}\cdot\frac{1}{2}$$

$$\Rightarrow \quad \frac{31}{42} = \frac{2n+n+1}{2(2n+1)} \quad \text{(Given)}$$

$$\Rightarrow \quad \frac{31}{42} = \frac{3n+1}{4n+2}$$

$$124n + 62 = 126n + 42$$

$$2n = 20$$

$$n = 10$$

[5]

Q. 44. Two cards are drawn successively without replacement from a well shuffled deck of cards. Find the mean and standard variation of the random variable X where X is the number of aces.

[NCERT Exemp. Ex. 13.3, Q. 53, Page 278]

Ans. Let X denotes a random variable of number of aces.

$\therefore X = 0, 1, 2$

Now, $P(X=0) = \frac{48}{52}\cdot\frac{47}{51} = \frac{2256}{2652}$

$$P(X=1) = \frac{48}{52}\cdot\frac{4}{51} + \frac{4}{52}\cdot\frac{48}{51} = \frac{384}{2652}$$

$$P(X=2) = \frac{4}{52}\cdot\frac{3}{51} = \frac{12}{2652}$$

X	0	1	2
$P(X)$	$\frac{2256}{2652}$	$\frac{384}{2652}$	$\frac{12}{2652}$
$XP(X)$	0	$\frac{384}{2652}$	$\frac{24}{2652}$
$X^2P(X)$	0	$\frac{384}{2652}$	$\frac{48}{2652}$

We know that,

Mean $(\mu) = E(X) = \Sigma XP(X)$

$$= 0 + \frac{384}{2652} + \frac{24}{2652} = \frac{408}{2652} = \frac{2}{13}$$

Also, Var.$(X) = E(X^2) - [E(X)]^2$

$$= \Sigma X^2P(X) - [E(X)]^2$$

$$= \left[0 + \frac{384}{2652} + \frac{48}{2652}\right] - \left(\frac{2}{13}\right)^2$$

$$= \frac{432}{2652} - \frac{4}{169}$$

$$= 0.1629 - 0.0237 = 0.1392$$

$\therefore$ Standard deviation $= \sqrt{\text{Var.}(X)} = \sqrt{0.1392} \cong 0.373$ (approx.) **[5]**

Q. 45. *A* die is tossed twice. *A* 'success' is getting an even number on a toss. Find the variance of the number of successes.

[NCERT Exemp. Ex. 13.3, Q. 54, Page 278]

Ans. At $X = 0, P(X=0) = {}^2C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{2-0} = \frac{1}{4}$

At $X = 1, P(X=1) = {}^2C_0\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{2-1} = 2\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

At $X = 2, P(X=2) = {}^2C_0\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{2-2} = \frac{1}{4}$

Thus,

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$XP(X)$	0	$\frac{1}{2}$	$\frac{1}{2}$
$X^2P(X)$	0	$\frac{1}{2}$	1

$$\therefore \quad \sum XP(X) = 0 + \frac{1}{2} + \frac{1}{2} = 1 \quad \text{...(i)}$$

$$\text{and } \sum X^2P(X) = 0 + \frac{1}{2} + 1 = \frac{3}{2} \quad \text{...(ii)}$$

$$\because \quad \text{Var.}(X) = E(X^2) - [E(X)]^2$$

$$= \sum X^2P(X) - [\sum XP(X)]^2$$

$$= \frac{3}{2} - (1)^2 = \frac{1}{2}$$

[By usnig Eqs. (i) and (ii)]

[5]

Q. 46. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on two cards drawn. Find the mean and variance of X.

[NCERT Exemp. Ex. 13.3, Q. 55, Page 278]

Ans. Here,

S = {(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 4), (4, 2), (2, 5), (5, 2), (3, 4), (4, 3), (3, 5), (5, 3), (5, 4), (4, 5)}.

$\Rightarrow n(S) = 20$

Let random variable be X which denotes the sum of the numbers on two cards drawn.

$\therefore X = 3, 4, 5, 6, 7, 8, 9$

At $X = 3$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 4$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 5$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 6$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 7$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 8$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 9$, $P(X) = \frac{2}{20} = \frac{1}{10}$

$\therefore$ Mean, $E(X) = \Sigma XP(X) = \frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10}$

$= \frac{3+4+10+12+14+8+9}{10} = 6$

Also, $\Sigma X^2P(X) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{5} + \frac{81}{10}$

$= \frac{9+16+50+72+98+64+81}{10} = 39$

$\therefore$ Var.$(X) = \Sigma X^2P(X) - [\Sigma XP(X)]^2$

$= 39 - (6)^2 = 39 - 36 = 3$

[5]

Q. 47. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.
[CBSE Board, Delhi Region, 2016]

Ans. Let,

X = Number of red balls

X :	0	1	2	3	4
$P(X)$:	${}^4C_0\left(\frac{1}{3}\right)^4 = \frac{1}{81}$	${}^4C_1\left(\frac{1}{3}\right)^3\frac{2}{3} = \frac{8}{81}$	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81}$	${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{32}{81}$	${}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{81}$
$XP(X)$:	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$
$X^2P(X)$:	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$

Mean $= \Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$

Variance $= \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$ **[5]**

Q. 48. Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from A to B.
[CBSE Board, Foreign Scheme, 2016]

Ans. Let,

H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 black ball, transferred

H_3 be the event 2 black and 1 black ball transferred

E be the event that ball drawn from B is red.

$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$ $\quad P(E/H_1) = \frac{6}{10}$

$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28}$ $\quad P(E/H_2) = \frac{5}{10}$

$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$ $\quad P(E/H_3) = \frac{4}{10}$

$$P(H_1/E) = \frac{\frac{3}{28}\times\frac{6}{10}}{\frac{3}{28}\times\frac{6}{10} + \frac{15}{28}\times\frac{5}{10} + \frac{10}{28}\times\frac{4}{10}} = \frac{18}{133}$$

[5]

Q. 49. Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution. **[CBSE Board, All India Region, 2016]**

Ans. Let,

X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4

P = Probability of a bad orange $= \frac{1}{5}, q = 1 - p = \frac{4}{5}$

$\therefore$ Probability distribution is :

X :	0	1	2	3	4
P(X) :	${}^4C_0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1\frac{1}{5}\left(\frac{4}{5}\right)^3 = \frac{256}{625}$	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$	${}^4C_3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right) = \frac{16}{625}$	${}^4C_4\left(\frac{1}{5}\right)^4 = \frac{1}{625}$

$$\text{Mean } (\mu) = \sum X.P(X)$$

$$= 0\times\frac{256}{625} + 1\times\frac{256}{625} + 2\times\frac{96}{625} + 3\times\frac{16}{625} + 4\times\frac{1}{625} = \frac{4}{5}$$

$$\text{Variance } (\sigma^2) = \sum x^2 \cdot P(x) - [\sum x \cdot P(x)]^2$$

$$= 0\times\frac{256}{625} + \frac{1\times 256}{625} + \frac{4\times 96}{625} + \frac{9\times 16}{625} + \frac{16}{625} - \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$ **[5]**

Some Commonly Made Errors

- Students get confused between dependent and independent events.
- Errors in the implementation of Bayes' formula.
- Error in taking the possible outcome.
- They get confused between intersection and union.

EXPERT ADVICE

- *Understand the concept of conditional probability and its properties.*
- *A Probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence. In order to understand probability distribution, it is important to understand variables, random variables and some notation.*
- *Try to learn the assumptions of binomial distribution.*
- *Practice some questions based on Bayes' theorem.*

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