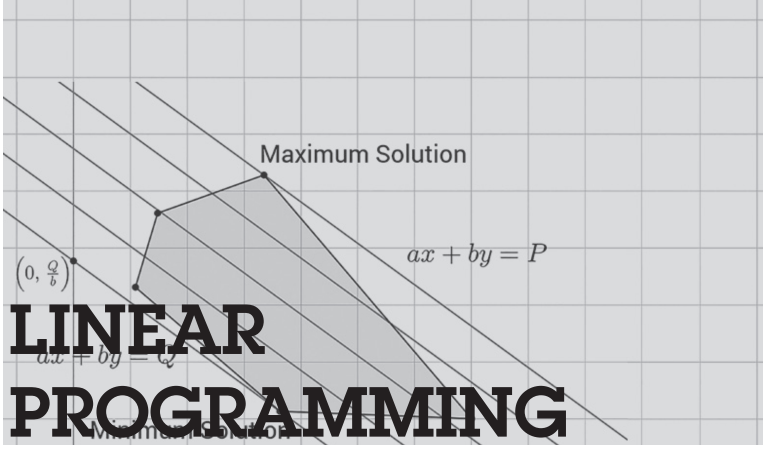
CHAPTER

12 LINEAR PROGRAMMING



Chapter Objectives

This chapter will help you understand :

- *Linear programming problem and its Mathematical formulation.*
- *Graphical method of solving linear programming problems.*
- *Different types of linear programming problems.*

Quick Review

- A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables (called objective function) subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints). Variables are sometimes called decision variables and are non-negative.
- The common region determined by all the constraints including the non-negative constraints $x \geq 0, y \geq 0$ of a linear programming problem is called the feasible region (or solution region) for the problem.
- Points within and on the boundary of the feasible region represent feasible solutions of the constraints.
  Any point outside the feasible region is an infeasible solution.
- Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

TIPS...

- *Minimizes the objective of the linear optimization problem.*
- *Understand the concept of optimization to learn linear programming problems.*

TRICKS...

- *Try to learn a systematic procedure for generating and testing vertex solutions to a linear program.*
- *Objective function represents how the decision variables affect the cost or value that is to be optimized.*

- If the feasible region is unbounded, then a maximum or a minimum may not exist. However, if it exists, it must occur at a corner point of *R*.
- The following Theorems are fundamental in solving linear programming problems :
  Theorem 1 : Let *R* be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When *Z* has an optimal value (maximum or minimum), where the variables *x* and *y* are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
  Theorem 2 : Let *R* be the feasible region for a linear programming problem, and let $Z = ax + by$ be the objective function. If *R* is bounded, then the objective function *Z* has both a maximum and a minimum value on *R* and each of these occurs at a corner point (vertex) of *R*.

Know the Links

- *http://www.learncbse.in/ncert-solutions-for-class-12th-maths-chapter-12-linear-programming/*
- *https://www.khanacademy.org/math/in-in-grade-12-ncert/in-in-linear-programming*
- *http://www.purplemath.com/modules/linprog.html*

Multiple Choice Questions

(1 mark each)

Q. 1. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is $Z = 4x + 3y$.
Compare the quantity in Column *A* and Column *B*

Column *A*	Column *B*
Maximum of Z	325

(a) The quantity in column *A* is greater.
(b) The quantity in column *B* is greater.

(c) The two quantities are equal.
(d) The relationship cannot be determined on the basis of the information supplied.

[NCERT Exemp. Ex. 12.3, Q. 26, Page 254]

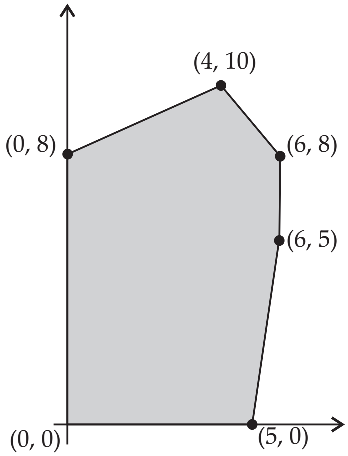
Ans. Correct option : (b)

Explanation :

Corner points	Corresponding value of $Z = 4x + 3y$
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 ← Maximum
(60, 0)	240

Hence, maximum value of $Z = 300 < 325$
So, the quantity in column B is greater.

Q. 2. The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



(a) (0, 0) **(b) (0, 8)**
(c) (5, 0) **(d) (4, 10)**

[NCERT Exemp. Ex. 12.3, Q. 27, Page 255]

Ans. Correct option : (b)

Explanation :

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(5, 0)	15 ← Maximum
(6, 5)	–2
(6, 8)	–14
(4, 10)	–28
(0, 8)	–32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (–32).

Q. 3. Refer to Q.2 of multiple choice questions, maximum of Z occurs at

(a) (5, 0) **(b) (6, 5)**
(c) (6, 8) **(d) (4, 10)**

[NCERT Exemp. Ex. 12.3, Q. 28, Page 255]

Ans. Correct option : (a)

Explanation : Maximum of Z occurs at (5, 0).

Q. 4. Refer to Q.2 of multiple choice questions, (Maximum value of Z + Minimum value of Z) is equal to

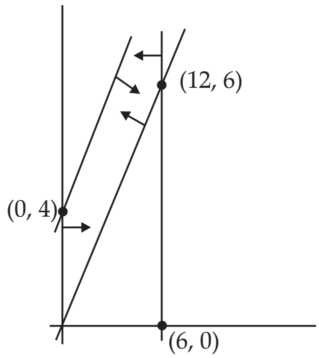
(a) 13 **(b) 1**
(c) –13 **(d) –17**

[NCERT Exemp. Ex. 12.3, Q. 29, Page 255]

Ans. Correct option : (d)

Explanation : Maximum value of Z + Minimum value of $Z = 15 - 32 = -17$

Q. 5. The feasible region for an LPP is shown in the given Figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is



(a) 0 **(b) 8**
(c) 12 **(d) –18**

[NCERT Exemp. Ex. 12.3, Q. 30, Page 255]

Ans. Correct option : (c)

Explanation :

The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner points	Corresponding value of $F = 3x - 4y$
(0, 0)	0
(12, 6)	12 ← Maximum
(0, 4)	–16 ← Minimum

Hence, the maximum value of F is 12.

Q. 6. Refer to Q.5 of multiple choice questions, minimum value of F is

(a) 0 **(b) –16**
(c) 12 **(d) does not exist**

[NCERT Exemp. Ex. 12.3, Q. 31, Page 256]

Ans. Correct option : (b)

Explanation : Minimum value of F is -16 at (0, 4).

Q. 7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function.
The minimum value of F occurs at

(a) (0, 2) only
(b) (3, 0) only
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only
(d) any point on the line segment joining the points (0, 2) and (3, 0)

[NCERT Exemp. Ex. 12.3, Q. 32, Page 256]

Ans. Correct option : (d)

Explanation :

Corner points	Corresponding value of $F = 4x+6y$
(0, 2)	12 ← Minimum
(3, 0)	12 ← Minimum
(6, 0)	24
(6, 8)	72 ← Maximum
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

Q. 8. Refer to Q. 7 above, Maximum of F – Minimum of F =

(a) 60 (b) 48
(c) 42 (d) 18

[NCERT Exemp. Ex. 12.3, Q. 33, Page 256]

Ans. Correct option : (a)

Explanation : Maximum of F – Minimum of F = 72 − 12 = 60

Q. 9. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) $p = 2q$ **(b)** $p = q/2$
(c) $p = 3q$ **(d)** $p = q$

[NCERT Exemp. Ex. 12.3, Q. 34, Page 256]

Ans. Correct option : (b)

Explanation :

Corner points	Corresponding value of $Z = px + qy;\ p, q > 0$
(0, 3)	$3q$
(1, 1)	$p + q$
(3, 0)	$3p$

So, condition of p and q, so that the minimum of Z occurs at (3, 0) and (1, 1) is

$$p+q=3p$$
$$\Rightarrow 2p=q$$
$$\therefore \quad p=\frac{q}{2}$$

Q. 10. In a LPP, the linear inequalities or restrictions on the variables are called _________.

[NCERT Exemp. Ex. 12.3, Q. 35, Page 257]

Ans. Linear constraints

Q. 11. In a LPP, the objective function is always _________

[NCERT Exemp. Ex. 12.3, Q. 36, Page 257]

Ans. Linear

Q. 12. If the feasible region for a LPP is _________, then the optimal value of the objective function $Z = ax + by$ may or may not exist.

[NCERT Exemp. Ex. 12.3, Q. 37, Page 257]

Ans. Unbounded

Q. 13. In a LPP if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same _________ value.

[NCERT Exemp. Ex. 12.3, Q. 38, Page 257]

Ans. Maximum

Q. 14. A feasible region of a system of linear inequalities is said to be _________ if it can be enclosed within a circle. [NCERT Exemp. Ex. 12.3, Q. 39, Page 257]

Ans. Bounded

Q. 15. A corner point of a feasible region is a point in the region which is the _________ of two boundary lines. [NCERT Exemp. Ex. 12.3, Q. 40, Page 257]

Ans. Intersection

Q. 16. The feasible region for an LPP is always a ________ polygon.

[NCERT Exemp. Ex. 12.3, Q. 41, Page 257]

Ans. Convex

Q. 17. If the feasible region for a LPP is unbounded, maximum or minimum of the objective function $Z = ax + by$ may or may not exist.
State is it True or False.

[NCERT Exemp. Ex. 12.3, Q. 42, Page 257]

Ans. True

Q. 18. Maximum value of the objective function $Z = ax + by$ in a LPP always occurs at only one corner point of the feasible region.
State is it True or False.

[NCERT Exemp. Ex. 12.3, Q. 43, Page 257]

Ans. False

Q. 19. In a LPP, the minimum value of the objective function $Z = ax + by$ is always 0 if origin is one of the corner point of the feasible region.
State is it True or False.

[NCERT Exemp. Ex. 12.3, Q. 44, Page 257]

Ans. False

Q. 20. In a LPP, the maximum value of the objective function $Z = ax + by$ is always finite.
State is it True or False.

[NCERT Exemp. Ex. 12.3, Q. 45, Page 257]

Ans. True

Q. 21. The corner points of the feasible region determined by the following system of linear inequalities : $2x + y \le 10$, $x + 3y \le 15$, $x, y \ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(a) $p = q$ **(b)** $p = 2q$
(c) $p = 3q$ **(d)** $q = 3p$

[NCERT Ex. 12.2, Q. 11, Page 520]

Ans. Correct option : (d)

Explanation :

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points : (3, 4) and (0, 5).

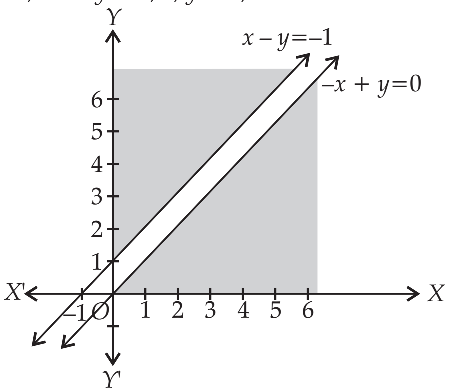
∴ Value of Z at (3, 4) = Value of Z at (0, 5)

$$\Rightarrow p(3)+q(4)=p(0)+q(5)$$
$$\Rightarrow \quad 3p+4q=5q$$
$$\therefore \quad q=3p$$

Very Short Answer Type Questions (2 marks each)

Q. 1. Maximise $Z = x + y$, subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$.
Show that the minimum of Z occurs at more than two points [NCERT Ex. 12.1, Q. 10, Page 514]

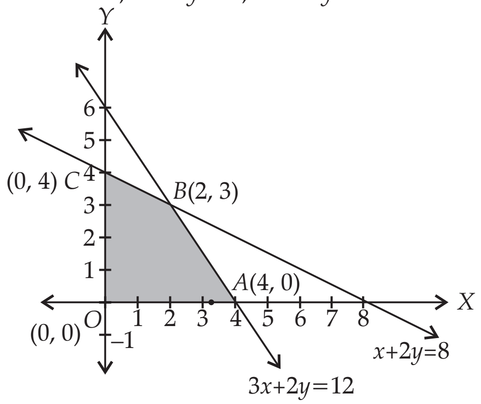
Ans. The region determined by the constraints, $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$, is as follows :



There is no feasible region and thus, Z has no maximum value. **[2]**

Q. 2. Minimise $Z = -3x + 4y$
Subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$.
Solve the Linear Programming Problems graphically. [NCERT Ex. 12.1, Q. 2, Page 514]

Ans. The feasible region determined by the system of constraints, $x + 2y \le 8$, $3x + 2y \le 12$.



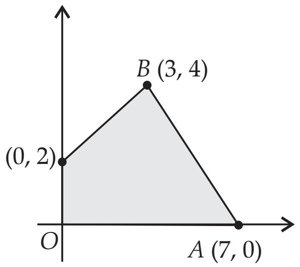
The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3) and C (0, 4).

The values of Z at these corner points are as follows :

Corner point	$Z = -3x + 4y$	
O (0, 0)	0	
A (4, 0)	–12	← Minimum
B (2, 3)	6	
C (0, 4)	16	

Therefore, the minimum value of Z is –12 at the point (4, 0). **[2]**

Q. 3. Feasible region (shaded) for a LPP is shown in given figure. Maximise $Z = 5x + 7y$.



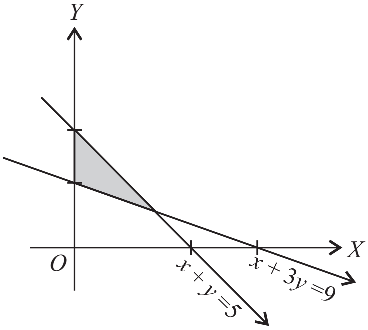
[NCERT Exemp. Ex. 12.3, Q. 6, Page 250]

Ans. The shaded region is bounded and has coordinates of corner points as (0, 0), (7, 0), (3, 4) and (0, 2). Also, $Z = 5x + 7y$.

Corner Points	Corresponding value of $z = 5x + 7y$
(0, 0)	0
(7, 0)	35
(3, 4)	43 ← Maximum
(0, 2)	14

Hence z is maximum at (3, 4) and its maximum value is 43. **[2]**

Q. 4. The feasible region for a LPP is shown in given figure. Find the minimum value of $Z = 11x + 7y$.



[NCERT Exemp. Ex. 12.3, Q. 7, Page 251]

Ans. From the figure, it is clear that feasible region is bounded with coordinates of corner points as (0, 3), (3, 2) and (0, 5). Here, $Z = 11x + 7y$.

$\because x + 3y = 9$ and $x + y = 5$

$2y = 4$

$y = 2$ and $x = 3$

So, intersection points of $x + y = 5$ and $x + 3y = 9$ is (3, 2).

Corner points	Corresponding value of $Z = 11x + 7y$
(0, 3)	21 ← Minimum
(3, 2)	47
(0, 5)	35

Hence, the minimum value of Z is 21 at (0, 3). **[2]**

Q. 5. Refer to Q. 4 of very short answer type questions Find the maximum value of Z.
[NCERT Exemp. Ex. 12.3, Q. 8, Page 251]

Ans. Z is maximum at (3, 2) and its maximum value is 47. **[2]**

Q. 6. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs. 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

[NCERT Exemp. Ex. 12.3, Q. 15, Page 253]

Ans. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h.

Therefore, cost on petrol is $2x + 3y$.

Since, he has to spend Rs. 120 at most on petrol.

$\therefore\ 2x + 3y \le 120$ …(i)

Also, he has at most 1 hours' time.

$\therefore \frac{x}{50} + \frac{y}{80} \le 120$

$\Rightarrow 8x + 5y \le 400$ …(ii)

Also, we have $x \ge 0, y \ge 0$ [Non-negative constraints]

Thus, required LPP to travel maximum distance by him is

Maximise $Z = x + y$, subject to $2x + 3y \le 120$, $8x + 5y \le 400$, $x \ge 0$, $y \ge 0$ **[2]**

Q. 7. Two tailors, A and B, earn < 300 and < 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

[CBSE Board, All India Region, 2017]

Ans. Let A works for x days and B for y days.

$\therefore$ LPP is :

Minimize $C = 300x + 400y$

Subject to :

$6x + 10y \ge 60$

$4x + 4y \ge 32$

$x \ge 0, y \ge 0$ **[2]**

Q. 8. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs. 100 and that on a bracelet is Rs. 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced. [CBSE Board, All India Region, 2017]

Ans. Let x necklaces and y bracelets are manufactured.

$\therefore$ LPP is

Maximise profit, $P = 100x + 300y$

Subject to constraints

$x + y \le 24$

$1/2x + y \le 16$

or $x + 2y \le 32$

$x, y \ge 1$ **[2]**

Q. 9. A company produces two types of goods A and B that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can procure a maximum of 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs.40 and that of type B Rs. 50, formulate LPP to maximize profit.

[CBSE Board, Foreign Scheme, 2017]

Ans. Let number of goods $A = x$ units, number of goods $B = y$ units

LPP is :

Maximise profit, $P = 40x + 50y$

Subject to following :

$3x + y \le 9$

$x + 2y \le 8$

$x \ge 0, y \ge 0$ **[2]**

Short Answer Type Questions (3 marks each)

Q. 1. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints : $2x + y \le 6$, $x \le 2$, $x \ge 0$, $y \ge 0$. [NCERT Exemp. Ex. 12.3, Q. 1, Page 250]

Ans. We have,

Maximise $Z = 11x + 7y$ …(i)

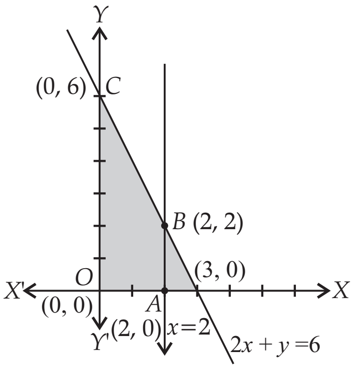
Subject to the constraints,

$2x + y \le 6$ …(ii)

$x \le 2$ …(iii)

$x \ge 0, y \ge 0$ …(iv)

We see that, the feasible region as shaded determined by the system of constraints (ii) to (iv) is $OABC$ and is bounded. So, now, we shall use corner point method to determine the maximum value of Z.



Corner points	Corresponding value of $Z = 11x + 7y$
(0, 0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	42 ← Maximum

Hence, the maximum value of Z is 42 at (0, 6). **[3]**

Q. 2. Maximise $Z = 3x + 4y$, subject to the constraints : $x + y \le 1, x \ge 0, y \ge 0$.

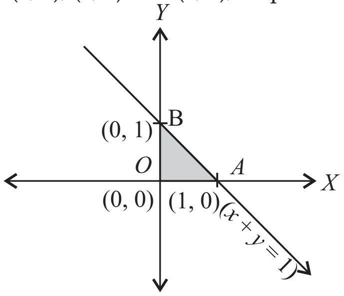
[NCERT Exemp. Ex. 12.3, Q. 2, Page 250]

Ans. Maximise $Z = 3x + 4y$,

Subject to the constraints,

$x + y \le 1, x \ge 0, y \ge 0$.

The shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0, 0), (1, 0) and (0, 1), respectively.



Corner points	Corresponding value of $Z = 3x + 4y$
(0, 0)	0
(1, 0)	3
(0, 1)	4 ← Maximum

Hence, the maximum value of Z is 4 at (0, 1). **[3]**

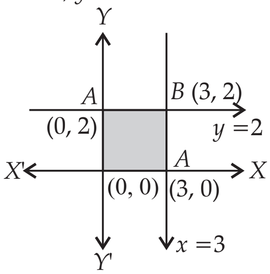
Q. 3. Maximise the function $Z = 11x + 7y$, subject to the constraints : $x \le 3, y \le 2, x \ge 0, y \ge 0$.

[NCERT Exemp. Ex. 12.3, Q. 3, Page 250]

Ans. Maximise $Z = 11x + 7y$,

Subject to the constraints,

$x \le 3, y \le, 2, x \ge 0, y \ge 0$.



The shaded region as shown in the figure as $OABC$ is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2) and (0, 2), respectively.

Corner points	Corresponding value of $Z = 11x + 7y$
(0, 0)	0
(3, 0)	33
(3, 2)	47 ← Maximum
(0, 2)	14

Hence, Z is maximum at (3, 2) and its maximum value is 47. **[3]**

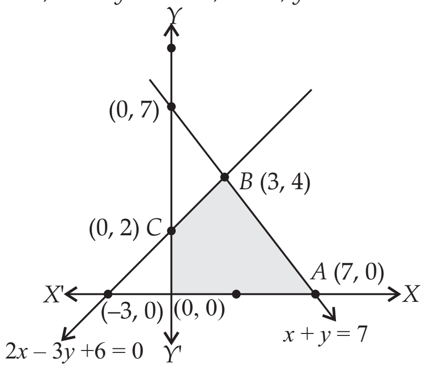
Q. 4. Minimise $Z = 13x - 15y$ subject to the constraints : $x + y \le 7, 2x - 3y + 6 \ge 0, x \ge 0, y \ge 0$.

[NCERT Exemp. Ex. 12.3, Q. 4, Page 250]

Ans. Minimise $Z = 13x - 15y$

Subject to the constraints,

$x + y \le 7, 2x - 3y + 6 \ge 0, x \ge 0, y \ge 0$.

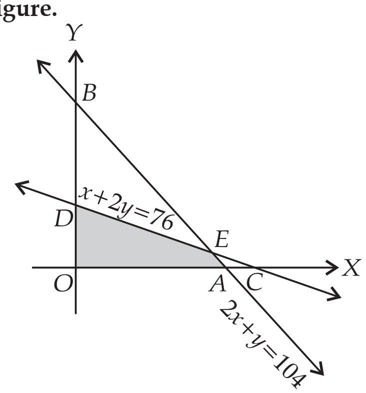


The shaded region shown as $OABC$ is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2), respectively.

Corner points	Corresponding value of $Z = 13x - 15y$
(0, 0)	0
(7, 0)	91
(3, 4)	–21
(0, 2)	–30 ← Minimum

Hence, the minimum value of Z is (–30) at (0, 2). **[3]**

Q. 5. Determine the maximum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in given figure.



[NCERT Exemp. Ex. 12.3, Q. 5, Page 250]

Ans. As clear from the graph, corner points are O, A, E and D with coordinates (0, 0), (52, 0), (144, 16) and (0, 38), respectively. Also, given region is bounded.

Here, $Z = 3x + 4y$

$\because$ $2x + y = 104$ and $2x + 4y = 152$

$\Rightarrow -3y = -48$

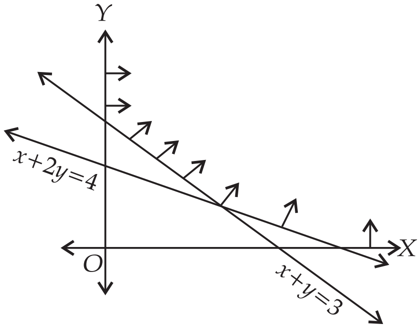
$\Rightarrow \quad y = 16$ and $x = 44$

Corner points	Corresponding value of $Z = 3x + 4y$
(0, 0)	0
(52, 0)	156

(44, 16)	196 ← Maximum
(0, 38)	152

Hence, Z is at (44, 16) is maximum and its maximum value is 196. **[3]**

Q. 6. The feasible region for a LPP is shown in Fig. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists.

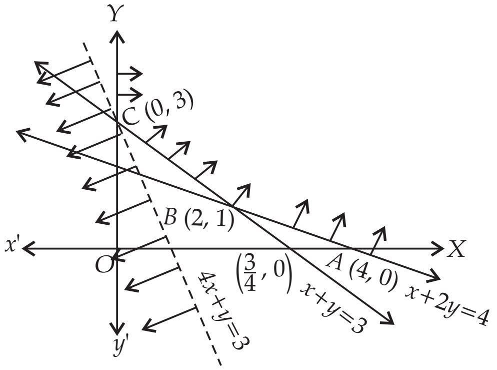


[NCERT Exemp. Ex. 12.3, Q. 9, Page 251]

Ans. From the shaded region, it is clear that feasible region is unbounded with the corner points A (4, 0), B (2, 1) and C (0, 3).

Also, we have $Z = 4x + y$.

[Since, $x + 2y = 4$ and $x + y = 3 \Rightarrow y = 1$ and $x = 2$]

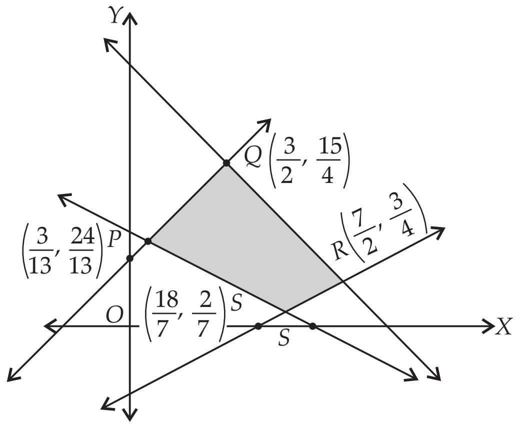


Corner points	Corresponding value of $Z = 4x + y$
(4, 0)	16
(2, 1)	9
(0, 3)	3 ← Minimum

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that, the region is unbounded, therefore 3 may or may not be the minimum value of Z. To decide this issue, we graph the inequality $4x + y < 3$ and check whether the resulting open half plan has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value 3 at (0, 3). **[3]**

Q. 7. In given Figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x + 2y$.



[NCERT Exemp. Ex. 12.3, Q. 10, Page 252]

Ans. From the shaded bounded region, it is clear that the coordinates of corner points are $\left(\frac{3}{13}, \frac{24}{13}\right), \left(\frac{18}{7}, \frac{2}{7}\right), \left(\frac{7}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{15}{4}\right)$.

Also, we have to determine maximum and minimum value of $Z = x + 2y$.

Corner points	Corresponding value of $Z = x + 2y$
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{3}{13} + \frac{48}{13} = \frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{18}{7} + \frac{4}{7} = \frac{22}{7} = 3\frac{1}{7}$ ← Minimum
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{7}{2} + \frac{6}{4} = \frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{3}{2} + \frac{30}{4} = \frac{36}{4} = 9$ ← Maximum

Hence, the maximum and minimum values of Z are 9 and $3\frac{1}{7}$, respectively. **[3]**

Q. 8. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.

[NCERT Exemp. Ex. 12.3, Q. 11, Page 252]

Ans. Let the manufacturer produces x units of type A circuits and y units of type B circuits.

From the given information, we have following corresponding constraints table.

	Type $A(x)$	Type $B(y)$	Maximum stock
Resistors	20	10	200
Transis-tors	10	20	120

Capacitors	10	30	150
Profit	₹50	₹60	

Thus, we see that total profit Z = $50x + 60y$ (in ₹).

Now, we have the following mathematical model for the given problem.

Maximise $Z = 50x + 60y$ …(i)

Subject to the constraints,

$20x + 10y \le 200$ **[Resistors constraint]**

$\Rightarrow 2x + y \le 20$ …(ii)

and $10x + 20y \le 120$ **[Transistors constraint]**

$\Rightarrow x + 2y \le 12$ …(iii)

and $10x + 30y \le 150$ **[Capacitors constraint]**

$\Rightarrow x + 3y \le 15$ …(iv)

and $x \ge 0, y \ge 0$ **[Non-negative constraint]** …(v)

So, maximise $Z = 50x + 60y$, subject to $2x + y \le 20$, $x + 2y \le 12$, $x + 3y \le 15$, $x \ge 0, y \ge 0$. **[3]**

Q. 9. A firm has to transport 1200 packages using large vans which can carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimise cost.

[NCERT Exemp. Ex. 12.3, Q. 12, Page 252]

Ans. Let the firm has x number of large vans and y number of small vans. From the given information, we have following corresponding constraint table.

	Large vans (x)	**Small vans (y)**	**Maximum/ Minimum**
Packages	200	80	1,200
Cost	400	200	3,000

Thus, we see that objective function for minimum cost is $Z = 400x + 200y$.

Subject to constraints

$200x + 80y \ge 1{,}200$ **[Packages constraint]**

$\Rightarrow 5x + 2y \ge 30$ …(i)

and $400x + 200y \le 3{,}000$ **[Cost constraint]**

$\Rightarrow 2x + y \le 15$ …(ii)

and $x \le y$ **[Van constraint]** …(iii)

and $x \ge 0, y \ge 0$ **[Non-negative constraint]** …(iv)

Thus, required LPP to minimise cost is minimise $Z = 400x + 200y$, subject to $5x + 2y \ge 30$.

$2x + y \le 15$

$x \le y$

$x \ge 0, y \ge 0$ **[3]**

Q. 10. A company manufactures two types of screws *A* and *B*. All the screws have to pass through a threading machine and a slotting machine. *A* box of Type *A* screws requires 2 minutes on the threading machine and 3 minutes on the slotting machine. *A* box of type *B* screws requires 8 minutes of threading on the threading machine and 2 minutes on the slotting machine. In a week, each machine is available for 60 hours.

On selling these screws, the company gets a profit of Rs. 100 per box on type *A* screws and Rs. 170 per box on type *B* screws. Formulate this problem as a LPP given that the objective is to maximise profit.

[NCERT Exemp. Ex. 12.3, Q. 13, Page 252]

Ans. Let the company manufactures x boxes of type A screws and y boxes of type B screws. From the given information, we have following corresponding constraint table.

	Type $A(x)$	**Type $B(y)$**	**Maximum time available on each machine in a week**
Time required for screws on threading machine	2	8	60 × 60 (min)
Time required for screws on slotting machine	3	2	60 × 60 (min)
Profit	₹100	₹170	

Thus, we see that objective function for maximum profit is $Z = 100x + 170y$.

Subject to constraints

$2x + 8y \le 60 \times 60$ **[Time constraint for threading machine]**

$\Rightarrow x + 4y \le 1800$ …(i)

and $3x + 2y \le 60 \times 60$ **[Time constraint for slotting machine]**

$\Rightarrow 3x + 2y \le 3600$ …(ii)

Also, $x \ge 0, y \ge 0$ …(iii)

[Non-negative constraints]

$\therefore$ Required LPP is,

Maximise $Z = 100x + 170y$

Subject to constraints $x + 4y \le 1800$, $3x + 2y \le 3600$, $x \ge 0, y \ge 0$. **[3]**

Q. 11. A company manufactures two types of sweaters : type *A* and type *B*. It costs Rs. 360 to make a type A sweater and Rs. 120 to make a type *B* sweater. The company can make at most 300 sweaters and spend at most Rs. 72000 a day. The number of sweaters of type *B* cannot exceed the number of sweaters of type *A* by more than 100. The company makes a profit of Rs. 200 for each sweater of type *A* and Rs. 120 for every sweater of type *B*. Formulate this problem as a LPP to maximise the profit to the company.

[NCERT Exemp. Ex. 12.3, Q. 14, Page 253]

Ans. Let the company manufactures x number of type A sweaters and y number of type B sweaters.

From the given information we see that cost to make a type A sweater is ₹360 and cost to make a type B sweater is ₹120.

Also, the company spend at most ₹72,000 a day.

$\therefore 360x + 120y \le 72{,}000$

$\Rightarrow 3x + y \le 600$ …(i)

Also, company can make at most 300 sweaters.

$\therefore x + y \le 300$ …(ii)

Further, the number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100 *i.e.*,

$x + 100 \ge y$

$\Rightarrow x - y \geq -100$ …(iii)

Also, we have non-negative constraints for x and y *i.e.*, $x \geq 0, y \geq 0$ …(iv)

Hence, the company makes a profit of ₹200 for each sweater of type A and ₹120 for each sweater of type B *i.e.*,

Profit (Z) = $200x + 120y$

Thus, the required LPP to maximise the profit is

Maximise $Z = 200x + 120y$ is subject to constraints.

$3x + y \leq 600$

$x + y \leq 300$

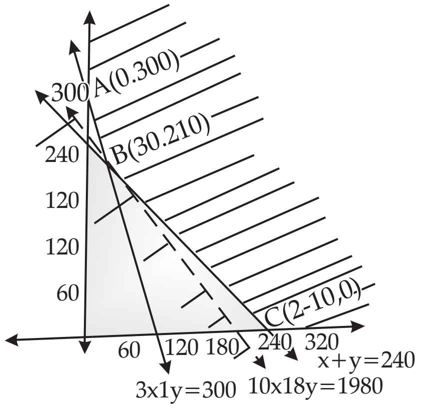
$x - y \geq -100$

$x \geq 0, y \geq 0$ **[3]**

Q. 12. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹10 per kg and 'B' cost ₹8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

[CBSE Board, All India Region, 2016]

Ans.



Let x kg of fertiliser A be used

and y kg of fertiliser B be used

Then the linear programming problem is :

Minimise cost : $Z = 10x + 8y$

Subject to $\dfrac{12x}{100} + \dfrac{4y}{100} \geq 12$

$\Rightarrow 3x + y \geq 300$

$\dfrac{5x}{100} + \dfrac{5y}{100} \geq 12$

$\Rightarrow x + y \geq 240$

$x, y \geq 0$

Correct Graph :

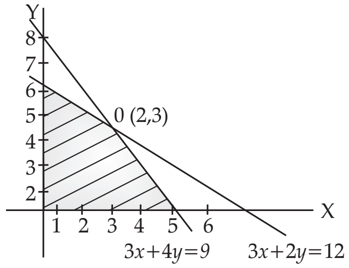
Value of Z at corners of the unbounded region ABC :

Corner	Value of Z
$A(0,300)$	₹ 2,400
$B(30,210)$	₹ 1,980 (Minimum)
$C(240,0)$	₹ 2,400

The region of $10x + 8y < 1,980$ or $5x + 4y < 990$ has no point in common to the feasible region. Hence, minimum cost = ₹1980 at $x = 30$ and $y = 210$ **[4]**

Q. 13. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs. 7 profit and that of B at a profit of Rs. 4. Find the production level per day for maximum profit graphically. [CBSE Board, Delhi Region, 2016]

Ans.



Let production of A, B (per day) be x, y respectively.

Maximise $P = 7x + 4y$

$3x + 2y \leq 12$

Subject $3x + y \leq 9$

$x \geq 0, y \geq 0$

Correct Graph :

$P(A) = 24$

$P(B) = 26$

$P(C) = 21$

$\therefore$ 2 units of product A and 3 units of product B for maximum profit. **[3]**

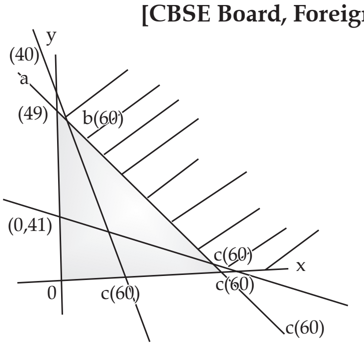
Q. 14. In order to supplement daily diet, a person wishes to take X and Y tablets. The contents (in milligrams per tablet) of iron, calcium and vitamins in X and Y are given as below :

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is Rs. 2 and Rs.1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost? Make an LPP and solve graphically.

[CBSE Board, Foreign Scheme, 2016]

Ans.



Let x tablets of type X and y tablets of type Y are taken.

Minimise $C = 2x + y$

Subjected to

$6x + 2y \geq 18$

$3x + 3y \geq 21$

$2x + 4y \geq 16$

$x, y \geq 0$

Correct Graph :

$Cl_{A\,(0,\,9)} = 9$

$Cl_{B\,(1,\,6)} = 8 \leftarrow$ Minimum value

$Cl_{C\,(6,\,1)} = 13$

$Cl_{D\,(8,\,0)} = 16$

$2x + y < 8$ does not pass through unbound region.

Thus, minimum value of $C = 8$ at $x = 1, y = 6$. **[4]**

Q. 15. Maximise $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Solve the above LPP graphically.

[CBSE Board, All India Region, 2017]

Ans. $Z = x + 2y$

Subject to the constraints $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$

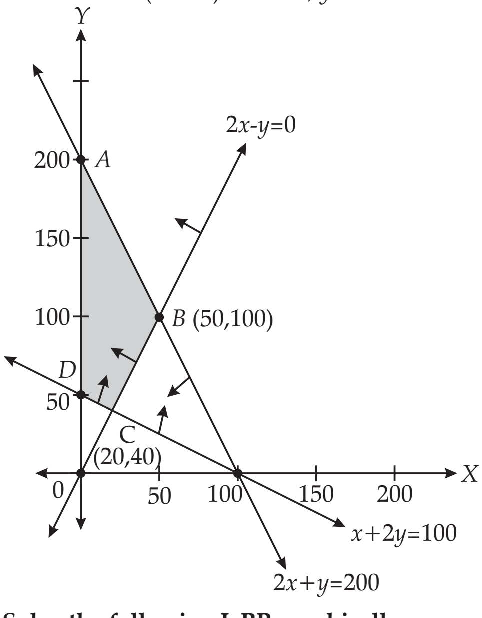
$Z(A) = 0 + 400 = 400$

$Z(B) = 50 + 200 = 250$

$Z(C) = 20 + 80 = 100$

$Z(D) = 0 + 100 = 100$

$\therefore$ Maximum (= 400) at $x = 0, y = 200$



[4]

Q. 16. Solve the following L.P.P. graphically :

Minimise $Z = 5x + 10y$

Subject to $x + 2y \leq 120$

Constraints $x + y \geq 60$

$x - 2y \geq 0$

and $x, y \geq 0$ [CBSE Board, Delhi Region, 2017]

Ans. $Z = 5x + 10y$

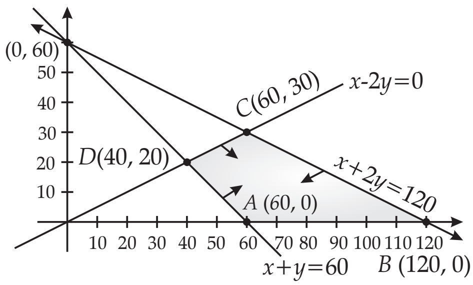
$Z|\ A(60, 0) = 300$

$Z|\ B(120, 0) = 600$

$Z|\ C(60, 30) = 600$

$Z|\ D(40, 20) = 400$

Minimum value of $Z = 300$ at $x = 60, y = 0$



[4]

Q. 17. A toy company manufactures two types of dolls, A and B. Market research and available resources have indicated that the combined production level should not exceed 1,200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

[NCERT Misc. Ex. Q.10, Page 528]

Ans. Let x and y be the number of dolls of type A and B respectively that are produced per week.

The given problem can be formulated as follows :

Maximise $Z = 12x + 16y$ …(i)

Subject to the constraints,

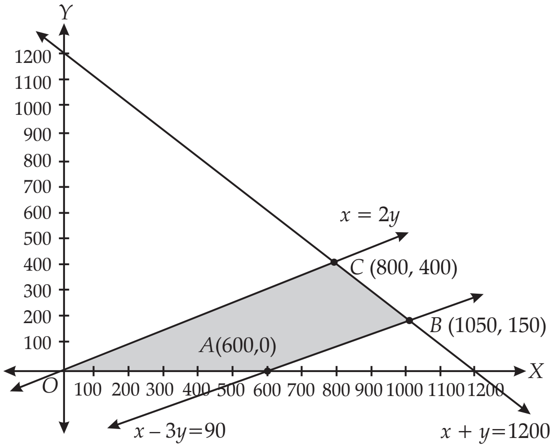
$x + y \leq 1{,}200$ …(ii)

$y \leq \dfrac{x}{2} \Rightarrow x \geq 2y$ …(iii)

$x - 3y \leq 600$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the system of constraints is as follows :



The corner points are A (600, 0), B (1,050, 150), and C (800, 400).

The values of Z at these corner points are as follows :

Corner points	$Z = 12x + 16y$	
A (600, 0)	7,200	
B (1,050, 150)	15,000	
C (800, 400)	16,000	$\leftarrow$ Maximum

The maximum value of Z is 16,000 at (800, 400).

Thus, 800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16,000. **[3]**

Q. 18. A fruit grower can use two types of fertilizer in his garden, brand *P* and brand *Q*. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

kg per bag		
	Brand *P*	**Brand *Q***
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

[NCERT Misc. Ex. Q.8, Page 527]

Ans. Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows :

Minimise $Z = 3x + 3.5y$ …(i)

Subject to the constraints,

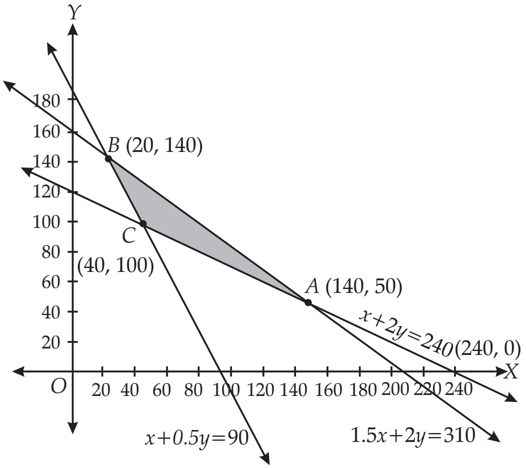
$x + 2y \geq 240$ …(ii)

$x + 0.5y \geq 90$ …(iii)

$1.5x + 2y \leq 310$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the system of constraints is as follows :



The corner points are A (240, 50), B (20, 140), and C (40, 100).

The values of Z at these corner points are as follows.

Corner points	$Z = 3x + 3.5y$	
A (140, 50)	595	
B (20, 140)	550	
C (40, 100)	470	← Minimum

The minimum value of Z is 470 at (40, 100).

Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimise the amount of nitrogen.

The minimum amount of nitrogen added to the garden is 470 kg. **[3]**

Q. 19. Refer to Q. 18 of short answer type questions, If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

[NCERT Misc. Ex. Q.9, Page 528]

Ans. Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows.

Maximise $Z = 3x + 3.5y$ …(i)

Subject to the constraints,

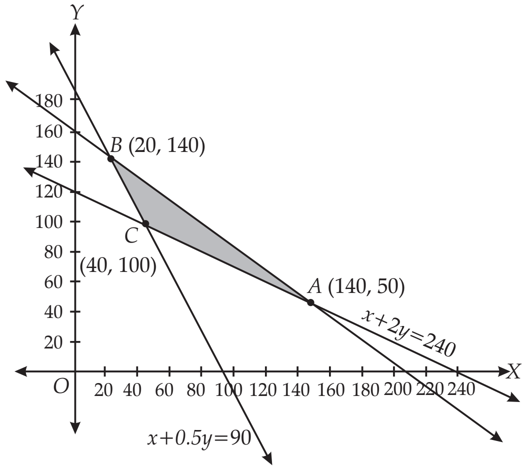
$x + 2y \geq 240$ …(ii)

$x + 0.5y \geq 90$ …(iii)

$1.5x + 2y \leq 310$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the system of constraints is as follows.



The corner points are A (140, 50), B (20, 140) and C (40, 100).

The values of Z at these corner points are as follows.

Corner point	$Z = 3x + 3.5y$	
A (140, 50)	595	← Maximum
B (20, 140)	550	
C (40, 100)	470	← Minimum

The maximum value of z is 595 at (140, 50).

Thus, 140 bags of brand P and 50 bags of brand Q should be used to maximise the amount of nitrogen.

The maximum amount of nitrogen added to the garden is 595 kg. **[3]**

Q. 20. Two godowns *A* and *B* have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, *D*, *E* and *F* whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops is given in the following table :

Transportation cost per quintal (in Rs.)		
From/To	***A***	***B***
D	6	4
E	3	2
F	2.50	3

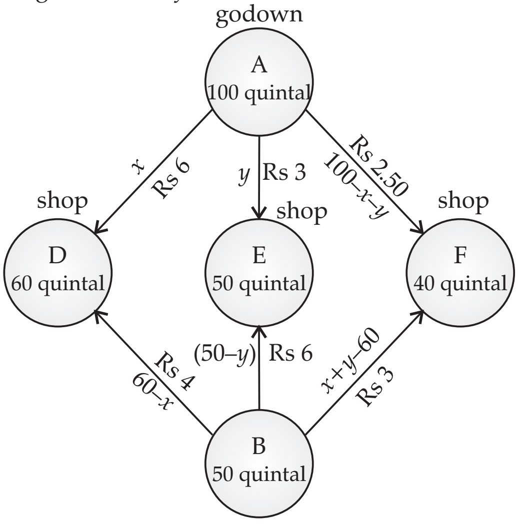
How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost? [NCERT Misc. Ex. Q.6, Page 527]

Ans. Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows :



$x \geq 0, y \geq 0$, and $100 - x - y \geq 0$

$\Rightarrow x \geq 0, y \geq 0$, and $x + y \leq 100$

$60 - x \geq 0, 50 - y \geq 0$, and $x + y - 60 \geq 0$

$\Rightarrow x \leq 60, y \leq 50$, and $x + y \geq 60$

Total transportation cost Z is given by,

$$Z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$$

$$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$$

$$= 2.5x + 1.5y + 410$$

The given problem can be formulated as,

Minimise $Z = 2.5x + 1.5y + 410$ …(i)

Subject to the constraints,

$x + y \leq 100$ ….(ii)

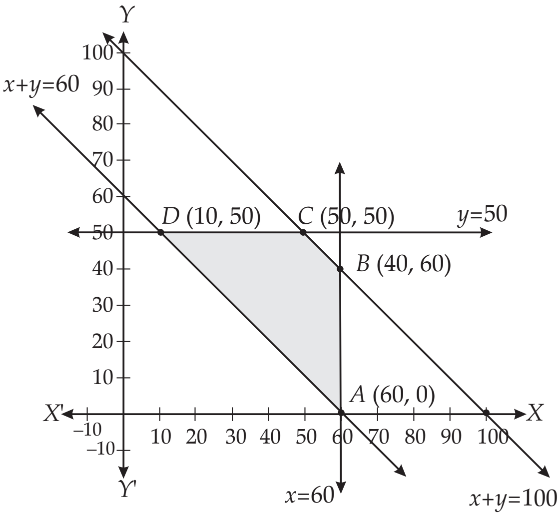
$x \leq 60$ …(iii)

$y \leq 50$ …(iv)

$x + y \geq 60$ …(v)

$x, y \geq 0$ …(vi)

The feasible region determined by the system of constraints is as follows :



The corner points are A (60, 0), B (60, 40), C (50, 50) and D (10, 50).

The values of z at these corner points are as follows :

Corner point	$Z = 2.5x + 1.5y + 410$	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	← Minimum

The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E and F is 10 quintals, 50 quintals and 40 quintals, respectively and from B to D, E and F is 50 quintals, 0 quintals and 0 quintals, respectively.

The minimum cost is Rs. 510. **[3]**

Q. 21. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

[NCERT Misc. Ex. Q.5, Page 526]

Ans. Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows :

Maximise $Z = 1{,}000x + 600y$ …(i)

Subject to the constraints,

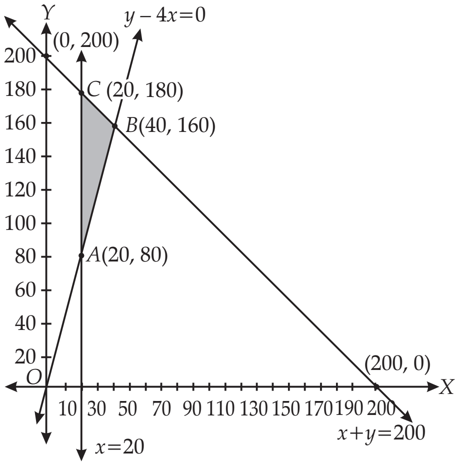
$x + y \leq 200$ …(ii)

$x \geq 20$ …(iii)

$y - 4x \geq 0$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the constraints is as follows :



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of Z at these corner points are as follows :

Corner point	$Z = 1{,}000x + 600y$	
A (20, 80)	68,000	
B (40, 160)	1,36,000	$\rightarrow$ Maximum
C (20, 180)	1,28,000	

The maximum value of Z is 1,36,000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximise the profit and the maximum profit is Rs. 1,36,000. **[3]**

Q. 22. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Types of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

[NCERT Misc. Ex. Q.4, Page 526]

Ans. Let x and y toys of type A and B respectively be manufactured in a day.

The given problem can be formulated as follows :

Maximise $Z = 7.5x + 5y$ …(i)

Subject to the constraints,

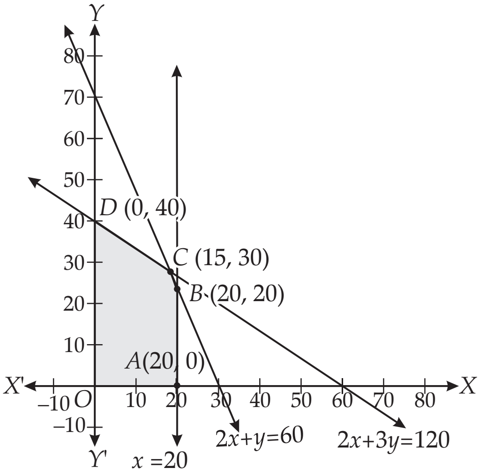
$2x + y \leq 60$ …(ii)

$x \leq 20$ …(iii)

$2x + 3y \leq 120$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the constraints is as follows :



The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30) and D (0, 40).

The values of Z at these corner points are as follows :

Corner point	$Z = 7.5x + 5y$	
A (20, 0)	150	
B (20, 20)	250	
C (15, 30)	262.5	$\leftarrow$ Maximum
O (0, 40)	200	

The maximum value of z is 262.5 at (15, 30).

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximise the profit. **[3]**

Q. 23. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food are given below :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

[NCERT Misc. Ex. Q.3, Page 526]

Ans. Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows :

Minimize $Z = 16x + 20y$ …(i)

Subject to the constraints,

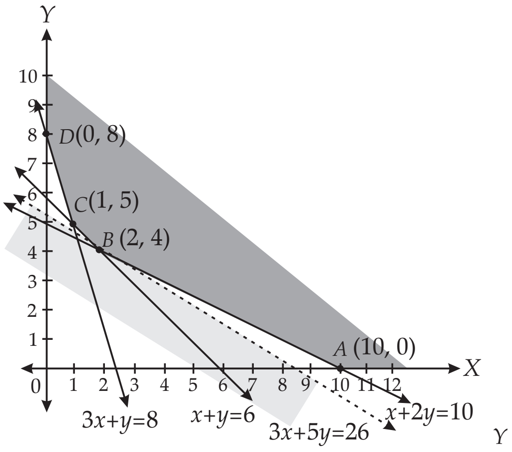
$x + 2y \geq 10$ …(ii)

$x + y \geq 6$ …(iii)

$3x + y \geq 8$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the system of constraints is as follows :



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5) and D (0, 8).

The values of Z at these corner points are as follows :

Corner point	$Z = 16x + 20y$	
A (10, 0)	160	
B (2, 4)	112	← Minimum
C (1, 5)	116	
D (0, 8)	160	

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $4x + 5y < 28$

Therefore, the minimum value of Z is 112 at (2, 4).

Thus, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs. 112. **[3]**

Q. 24. A farmer mixes two brands *P* and *Q* of cattle feed. Brand *P*, costing Rs. 250 per bag, contains 3 units of nutritional element *A*, 2.5 units of element *B* and 2 units of element *C*. Brand *Q* costing Rs. 200 per bag contains 1.5 units of nutritional element *A*, 11.25 units of element *B*, and 3 units of element *C*. The minimum requirements of nutrients *A*, *B* and *C* are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

[NCERT Misc. Ex. Q.2, Page 526]

Ans. Let the farmer mix x bags of brand P and y bags of brand Q.

The given information can be compiled in a table as follows :

	Vitamin A (units/bags)	Vitamin B (units/bags)	Vitamin C (units/bag)	Cost (Rs./bag)
Food *P*	3	2.5	2	250
Food *Q*	1.5	11.25	3	200
Requirement (units/bag)	18	45	24	

The given problem can be formulated as follows :

Minimize $Z = 250x + 200y$ …(i)

Subject to the constraints,

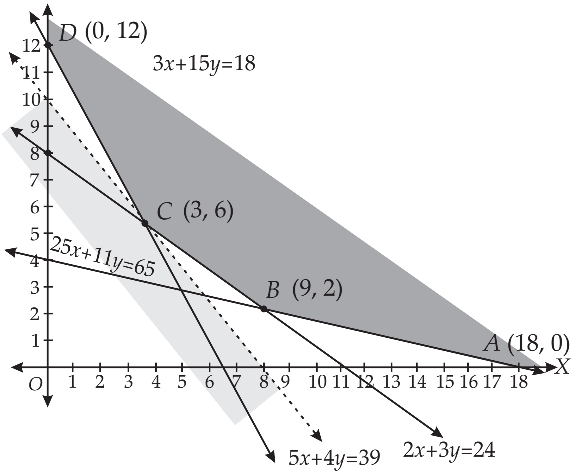
$3x + 1.5y \geq 18$ …(ii)

$2.5x + 11.25y \geq 45$ …(iii)

$2x + 3y \geq 24$ …(iv)

$x, y \geq 0$ …(v)

The feasible region determined by the system of constraints is as follows :



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6) and D (0, 12).

The values of Z at these corner points are as follows :

Corner point	$Z = 250x + 200y$	
A (18, 0)	4,500	
B (9, 2)	2,659	
C (3, 6)	1,950	← Minimum
D (0, 12)	2,400	

As the feasible region is unbounded, therefore, 1,950 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $250x + 200y < 1{,}950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 39$

Therefore, the minimum value of Z is 1,950 at (3, 6).

Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs. 1,950. **[3]**

Q. 25. A dietician has to develop a special diet using two foods *P* and *Q*. Each packet (containing 30 g) of food *P* contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food *Q* contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet? [NCERT Misc. Ex. Q.1, Page 525]

Ans. Let the diet contain x and y packets of foods P and Q respectively. Therefore, $x \geq 0$ and $y \geq 0$.

The mathematical formulation of the given problem is as follows :

Maximise $Z = 6x + 3y$ ...(i)

Subject to the constraints,

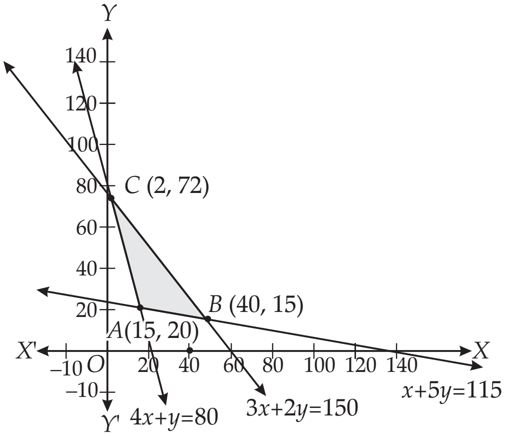
$4x + y \geq 80$ ...(ii)

$x + 5y \geq 115$ ...(iii)

$3x + 2y \leq 150$ ...(iv)

$x, y \geq 0$ ...(v)

The feasible region determined by the system of constraints is as follows :



The corner points of the feasible region are *A* (15, 20), *B* (40, 15) and *C* (2, 72).

The values of *Z* at these corner points are as follows :

Corner point	$Z = 6x + 3y$	
A (15, 20)	150	
B (40, 15)	285	← Maximum
C (2, 72)	228	

Thus, the maximum value of z is 285 at (40, 15).

Therefore, to maximise the amount of vitamin A in the diet, 40 packets of food *P* and 15 packets of food *Q* should be used.

The maximum amount of vitamin A in the diet is 285 units. **[3]**

Long Answer Type Questions **(5 or 6 marks)**

Q. 1. Reshma wishes to mix two types of food *P* and *Q* in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food *P* costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food *P* contains 3 units/kg of Vitamin A and 5 units/kg of Vitamin B while food *Q* contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture. [NCERT Ex. 12.2, Q. 1, Page 519]

Ans. Let the mixture contain x kg of food *P* and y kg of food *Q*.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs./ kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Therefore, the constraints are

$3x + 4y \geq 8$

$5x + 2y \geq 11$

Total cost, *Z*, of purchasing food is, $Z = 60x + 80y$

The mathematical formulation of the given problem is

Minimise, $Z = 60x + 80y$ ...(i)

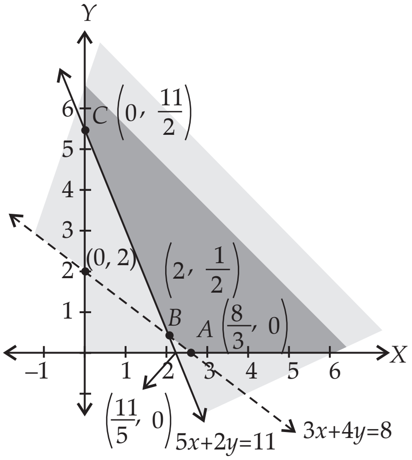
Subject to the constraints,

$3x + 4y \geq 8$ ...(ii)

$5x + 2y \geq 11$ ...(iii)

$x, y \geq 0.$ ...(iv)

The feasible region determined by the system of constraints is as follow :



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3},0\right), B\left(2,\frac{1}{2}\right)$, and $C\left(0,\frac{11}{2}\right)$.

The values of Z at these corner points are as follows :

Corner point	$Z = 60x + 80y$	
$A\left(\frac{8}{3},0\right)$	160	
$B\left(2,\frac{1}{2}\right)$	160	← Minimum
$C\left(0,\frac{11}{2}\right)$	440	

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 4y < 8$

Therefore, the minimum cost of the mixture will be Rs. 160 at the line segment joining the points $\left(\frac{8}{3},0\right)$ and $\left(2,\frac{1}{2}\right)$. **[5]**

Q. 2. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

[NCERT Ex. 12.2, Q. 2, Page 519]

Ans. Let there be x cakes of first kind and y cakes of second kind.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows :

	Flour (g)	Fat (g)
Cakes of first kind, x	200	25
Cakes of second kind, y	100	50
Availability	5,000	1,000

$\therefore 200x + 100y \leq 5{,}000$

$\Rightarrow \quad 2x + y \leq 50$

$25x + 50y \leq 1{,}000$

$\Rightarrow \quad x + 2y \leq 40$

Total numbers of cakes, Z, that can be made are, $Z = x + y$

The mathematical formulation of the given problem is

Maximise, $Z = x + y$ …(i)

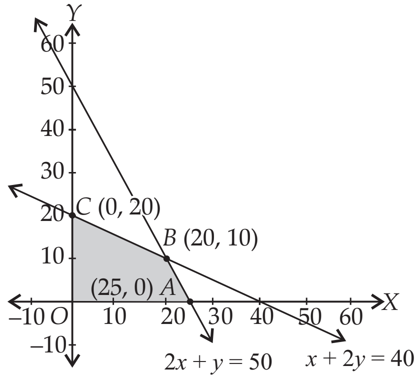
Subject to the constraints,

$2x + y \leq 50$ …(ii)

$x + 2y \leq 40$ …(iii)

$x, y \geq 0$ …(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (25, 0), B (20, 10), O (0, 0) and C (0, 20).

The values of Z at these corner points are as follows :

Corner point	$Z = x + y$	
A (25, 0)	25	
B (20, 10)	30	→ Maximum
C (0, 20)	20	
O (0, 0)	0	

Thus, the maximum number of cakes that can be made are 30 (20 of one kind and 10 of the other kind.) **[5]**

Q. 3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity.

[NCERT Ex. 12.2, Q. 3, Page 519]

Ans. **(i)** Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time is not available for more than 42 hours.

$\therefore 1.5x + 3y \leq 42$ …(i)

The craftsman's time is not available for more than 24 hours.

$\therefore 3x + y \leq 24$ …(ii)

The factory is to work at full capacity. Therefore,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we obtain

$x = 4$ and $y = 12$

Thus, 4 rackets and 12 bats must be made.

(ii) The given information can be compiled in a table as follows :

	Tennis Racket	Cricket Bat	Availability
Machine time (h)	1.5	3	42
Craftsman's time (h)	3	1	24

$\therefore$ $1.5x + 3y \leq 42$
$3x + y \leq 24$
$x, y \geq 0$

The profit on a racket is Rs 20 and on a bat Rs 10.

$\therefore$ $Z = 20x + 10y$

The mathematical formulation of the given problem is

Maximise, $Z = 20x + 10y$ ...(i)

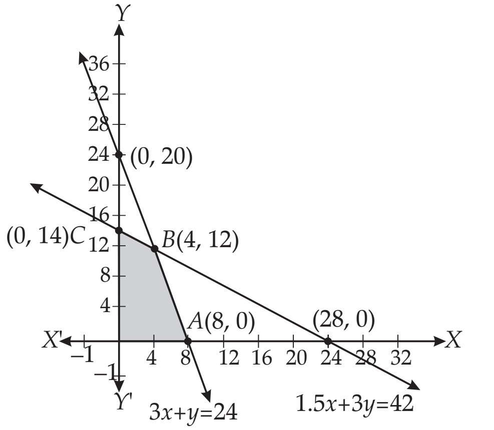
Subject to the constraints,

$1.5x + 3y \leq 42$ ...(ii)

$3x + y \leq 24$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (8, 0), B (4, 12), C (0, 14) and O (0, 0).

The values of Z at these corner points are as follows :

Corner point	$Z = 20x + 10y$	
A (8, 0)	160	
B (4, 12)	200	← Maximum
C (0, 14)	140	
O (0, 0)	0	

Thus, the maximum profit of the factory when it works to its full capacity is Rs. 200. **[5]**

Q. 4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day? [NCERT Ex. 12.2, Q. 4, Page 519]

Ans. Let the manufacturer produce x packages of nuts and y packages of bolts.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Nuts	Bolts	Availability
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs. 7. Therefore, the constraints are

$x + 3y \leq 12$

$3x + y \leq 12$

Total profit, $Z = 17.5x + 7y$

The mathematical formulation of the given problem is

Maximise, $Z = 17.5x + 7y$ ...(i)

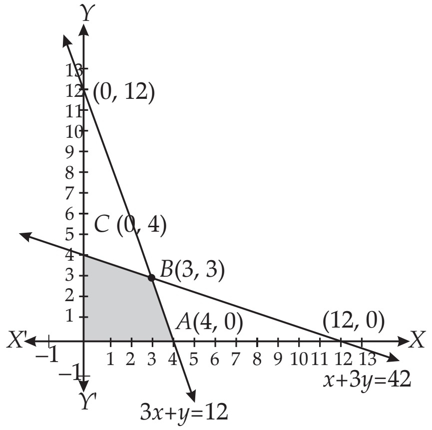
Subject to the constraints,

$x + 3y \leq 12$ ...(ii)

$3x + y \leq 12$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (4, 0), B (3, 3), and C (0, 4).

The values of Z at these corner points are as follows :

Corner point	$Z = 17.5x + 7y$	
O (0, 0)	0	
A (4, 0)	70	
B (3, 3)	73.5	← Maximum
C (0, 4)	28	

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50. **[5]**

Q. 5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machine to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machine to manufacture a package of screws B. Each machine is available for

at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs. 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

[NCERT Ex. 12.2, Q. 5, Page 519]

Ans. Let the factory manufacture x screws of type A and y screws of type B on each day.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Screw A	Screw B	Availability
Automatic machine (min)	4	6	$4 \times 60 = 240$
Hand-operated machine (min)	6	3	$4 \times 60 = 240$

The profit on a package of screw A is Rs. 7 and on the package of screws B is Rs. 10. Therefore, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem is

Maximise, $Z = 7x + 10y$ ...(i)

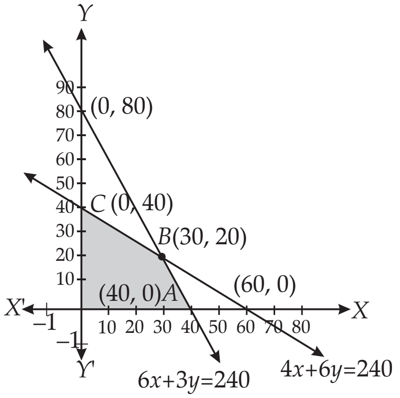
Subject to the constraints,

$4x + 6y \leq 240$ ...(ii)

$6x + 3y \leq 240$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (40, 0), B (30, 20) and C (0, 40). The values of Z at these corner points are as follows :

Corner point	$Z = 7x + 10y$	
A (40, 0)	280	
B (30, 20)	410	← Maximum
C (0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs. 410. **[5]**

Q. 6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 5 and that from a shade is Rs. 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

[NCERT Ex. 12.2, Q. 6, Page 520]

Ans. Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Lamps	Shades	Availability
Grinding/cutting machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs. 5 and on the shades is Rs. 3. Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, $Z = 5x + 3y$

The mathematical formulation of the given problem is

Maximise, $Z = 5x + 3y$ ...(i)

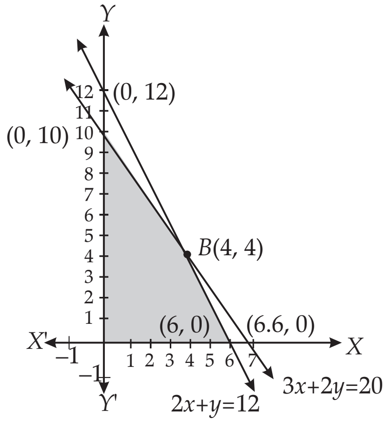
Subject to the constraints,

$2x + y \leq 12$ ...(ii)

$3x + 2y \leq 20$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (6, 0), B (4, 4) and C (0, 10). The values of Z at these corner points are as follows :

Corner point	$Z = 5x + 3y$	
A (6, 0)	30	
B (4, 4)	32	← Maximum
C (0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximise his profit. **[5]**

Q. 7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs. 5 each for type *A* and Rs. 6 each for type *B* souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

[NCERT Ex. 12.2 ,Q. 7, Page 520]

Ans. Let the company manufacture x souvenirs of type A and y souvenirs of type B.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Type *A*	**Type *B***	**Availability**
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on Type A souvenirs is Rs. 5 and on Type B souvenirs is Rs. 6. Therefore, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ *i.e.*, $5x + 4y \leq 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem is

Maximise, $Z = 5x + 6y$ ...(i)

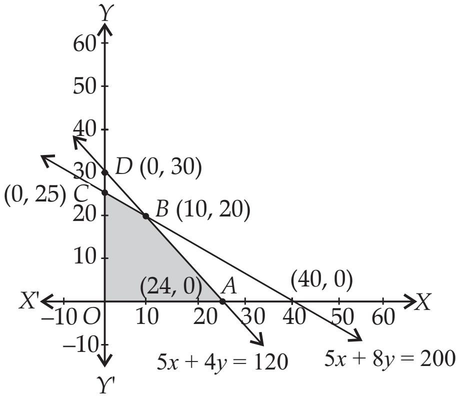
Subject to the constraints,

$5x + 8y \leq 200$ ...(ii)

$5x + 4y \leq 120$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows :

Corner point	$Z = 5x + 6y$	
A (24, 0)	120	
B (8, 20)	160	→ Maximum
C (0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs. 160. **[5]**

Q. 8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

[NCERT Ex. 12.2, Q. 8, Page 520]

Ans. Let the merchant stock x desktop models and y portable models.

Therefore, $x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs. 25,000 and of a portable model is Rs. 4,000. However, the merchant can invest a maximum of Rs. 70 lakhs.

$\therefore 25{,}000x + 40{,}000y \leq 70{,}00{,}000$

$5x + 8y \leq 1{,}400$

The monthly demand of computers will not exceed 250 units.

$\therefore x + y \leq 250$

The profit on a desktop model is Rs. 4,500 and the profit on a portable model is Rs. 5,000.

Total profit, $Z = 4{,}500x + 5{,}000y$

Thus, the mathematical formulation of the given problem is

Maximum, $Z = 4{,}500x + 5{,}000y$ ...(i)

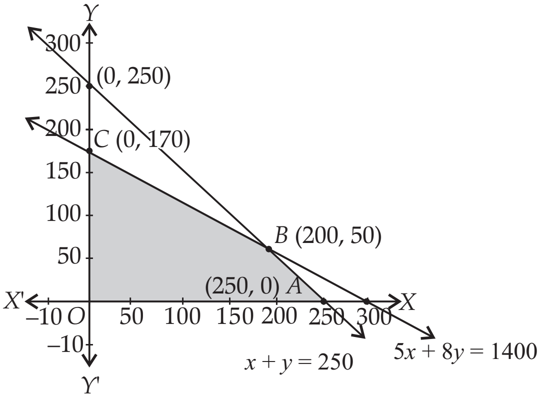
Subject to constraints,

$5x + 8y \leq 1400$ ...(ii)

$x + y \leq 250$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the system of constraints is as follows :



The corner points are A (250, 0), B (200, 50) and C (0, 175).

The values of Z at these corner points are as follows :

Corner point	$Z = 4{,}500x + 5{,}000y$	
A (250, 0)	11,25,000	
B (200, 50)	11,50,000	→ Maximum
C (0, 175)	8,75,000	

The maximum value of Z is 11,50,000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 11,50,000. **[5]**

Q. 9. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs. 6/kg and F_2 costs Rs. 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

[NCERT Ex. 12.2, Q. 10, Page 520]

Ans. Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Nitrogen (%)	Phosphoric acid (%)	Cost (Rs./kg)
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$\therefore$ 10% of x + 5% of $y \geq 14$

$$\frac{x}{10}+\frac{y}{20}\geq 14$$

$$2x + y \geq 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

$\therefore$ 6% of x + 10% of $y \geq 14$

$$\frac{6x}{100}+\frac{10y}{100}\geq 14$$

$$3x + 5y \geq 700$$

Total cost of fertilisers, $Z = 6x + 5y$

The mathematical formulation of the given problem is :

Minimise, $Z = 6x + 5y$ …(i)

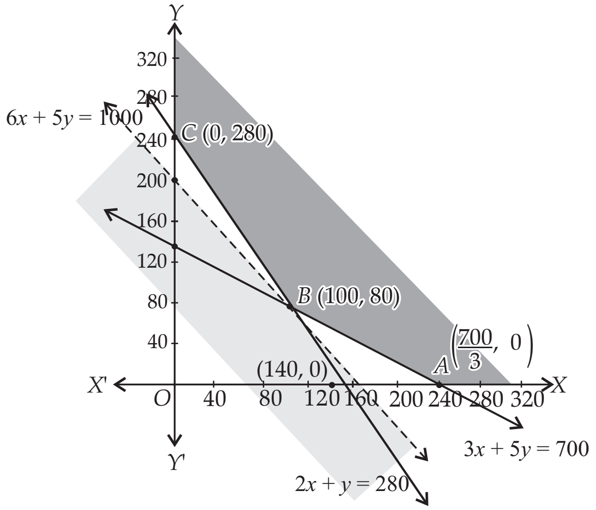
Subject to the constraints,

$2x + y \geq 280$ …(ii)

$3x + 5y \geq 700$ …(iii)

$x, y \geq 0$ …(iv)

The feasible region determined by the system of constraints is as follows :



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3}, 0\right)$, B (100, 80) and C (0, 280).

The values of Z at these points are as follows :

Corner point	$Z = 6x + 5y$	
$A\left(\frac{700}{3}, 0\right)$	1,400	
B (100, 80)	1,000	→ Minimum
C (0, 280)	1,400	

As the feasible region is unbounded, therefore, 1,000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1,000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with

$6x + 5y < 1,000$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 should be used to minimise the cost. The minimum cost is Rs 1,000. **[5]**

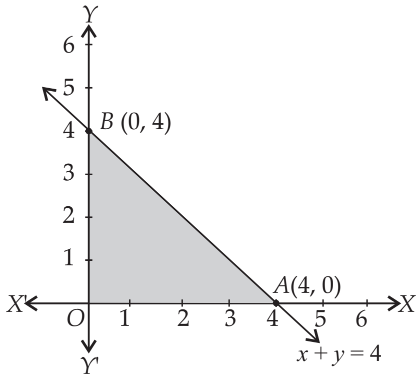
Q. 10. Maximise, $Z = 3x + 4y$

Subject to the constraints : $x + y \leq 4, x \geq 0, y \geq 0$.

Solve the Linear Programming Problem graphically :

[NCERT Ex. 12.1, Q. 1, Page 513]

Ans. The feasible region determined by the constraints, $x + y \leq 4, x \geq 0, y \geq 0$, is as follows :



The corner points of the feasible region are O (0, 0), A (4, 0) and B (0, 4). The values of Z at these points are as follows :

Corner point	$Z = 3x + 4y$	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	← Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4). **[5]**

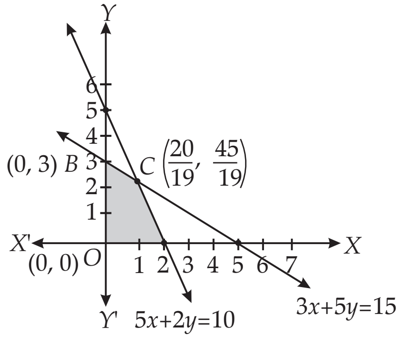
Q. 11. Maximise Z = 5x + 3y

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

Solve the Linear Programming Problem graphically.

[NCERT Ex. 12.1, Q. 3, Page 514]

Ans. The feasible region determined by the system of constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$, are as follows :



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3) and C $\left(\frac{20}{19}, \frac{45}{19}\right)$.

The values of Z at these corner points are as follows :

Corner point	$Z = 5x + 3y$	
0 (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
C$\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$	← Maximum

Therefore, the maximum values of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$. **[5]**

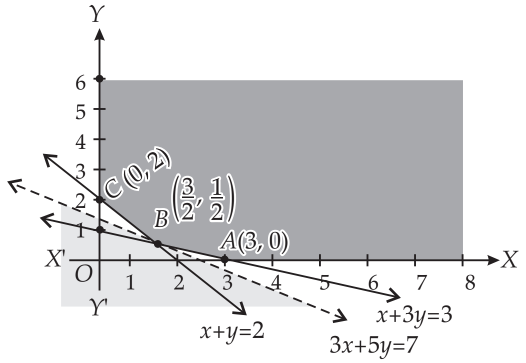
Q. 12. Minimise, Z = 3x + 5y

Such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

Solve the Linear Programming Problem graphically :

[NCERT Ex. 12.1, Q. 4, Page 514]

Ans. The feasible region determined by the system of constraints, $x + 3y \geq 3$, $x + y \geq 2$ and $x, y \geq 0$, is as follows :



It can be seen that the feasible region is unbounded. The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2}, \frac{1}{2}\right)$ and C (0, 2).

The values of Z at these corner points are as follows :

Corner point	$Z = 3x + 5y$	
A (3, 0)	9	
$B\left(\frac{3}{2}, \frac{1}{2}\right)$	7	← Minimum
C (0, 2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality, $3x + 5y < 7$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 5y < 7$ Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$. **[5]**

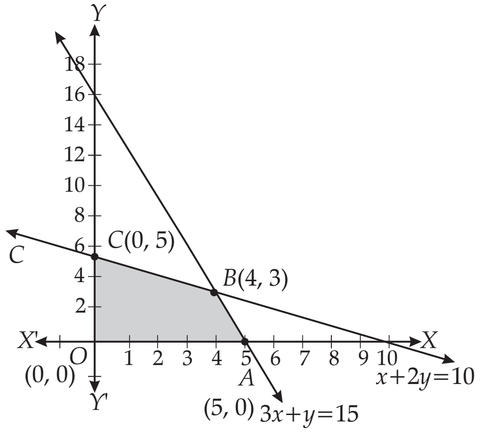
Q. 13. Maximise $Z = 3x + 2y$

Subject to : $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.

Solve the Linear Programming Problem graphically

[NCERT Ex. 12.1, Q. 5, Page 514]

Ans. The feasible region determined by the constraints, $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, and $y \geq 0$, is as follows :



The corner points of the feasible region are A (5, 0), B (4, 3) and C (0, 5).

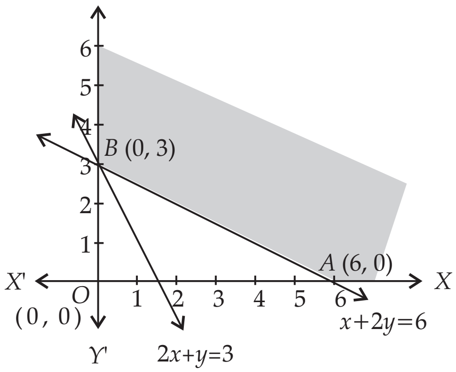
The values of Z at these corner points are as follows :

Corner point	$Z = 3x + 2y$	
A (5, 0)	15	
B (4, 3)	18	→ Maximum
C (0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3). **[5]**

Q. 14. Minimise, Z = $x + 2y$
Subject to : $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$.
Solve the Linear Programming Problem graphically
[NCERT Ex. 12.1, Q. 6, Page 514]

Ans. The feasible region determined by the constraints, $2x + y \geq 3, x + 2y \geq 6, x \geq 0$ and $y \geq 0$, is as follows :



The corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows :

Corner point	$Z = x + 2y$
A (6, 0)	6
B (0, 3)	6

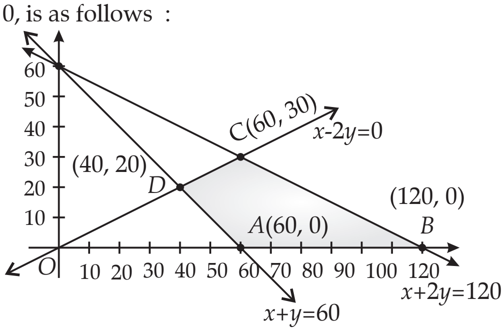
It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line $x + 2y = 6$, then $Z = 6$

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$ **[5]**

Q. 15. Minimise and Maximise, Z = $5x + 10y$
Subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.
Show that the minimum of Z occurs at more than two points [NCERT Ex. 12.1, Q. 7, Page 514]

Ans. The feasible region determined by the constraints, $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0$ and $y \geq 0$, is as follows :



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30) and D (40, 20).

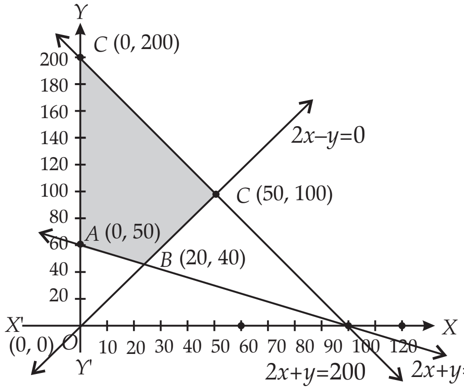
The values of Z at these corner points are as follows :

Corner point	$Z = 5x + 10y$	
A (60, 0)	300	← Minimum
B (120, 0)	600	← Maximum
C (60, 30)	600	← Maximum
D (40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30). **[5]**

Q. 16. Minimise and Maximise, Z = $x + 2y$
Subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200$; $x, y \geq 0$.
Show that the minimum of Z occurs at more than two points. [NCERT Ex. 12.1, Q. 8, Page 514]

Ans. The feasible region determined by the constraints, $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x \geq 0$ and $y \geq 0$, is as follows :



The corner points of the feasible region are A (0, 50), B (20, 40), C (50, 100) and D (0, 200).

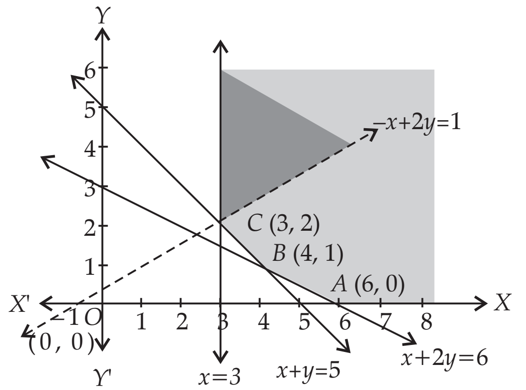
The values of Z at these corner points are as follows :

Corner points	$Z = x + 2y$	
A (0, 50)	100	← Minimum
B (20, 40)	100	← Minimum
C (50, 100)	250	
D (0, 200)	400	← Maximum

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40). **[5]**

Q. 17. Maximise Z = $-x + 2y$,
Subject to the constraints :
$x \geq 3, x + y \geq 5, x + 2y \leq 6, y \geq 0$.
Show that the minimum of Z occurs at more than two points. [NCERT Ex. 12.1, Q. 9, Page 514]

Ans. The feasible region determined by the constraints, $x \geq 3, x + y \geq 5, x + 2y \geq 6$ and $y \geq 0$, is as follows :



It can be seen that the feasible region is unbounded. The values of Z at corner points A (6, 0), B (4, 1) and C (3, 2) are as follows :

Corner point	$Z = -x + 2y$
A (6, 0)	$Z = -6$
B (4, 1)	$Z = -2$
C (3, 2)	$Z = 1$

As the feasible region is unbounded, therefore, Z = 1 may or may not be the maximum value.

For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half plane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

Therefore, Z = 1 is not the maximum value, Z has no maximum value. **[5]**

Q. 18. An oil company has two depots *A* and *B* with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, *D*, *E* and *F* whose requirements are 4500 L, 3000 L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table :

Distance in (km)		
From/To	***A***	***B***
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

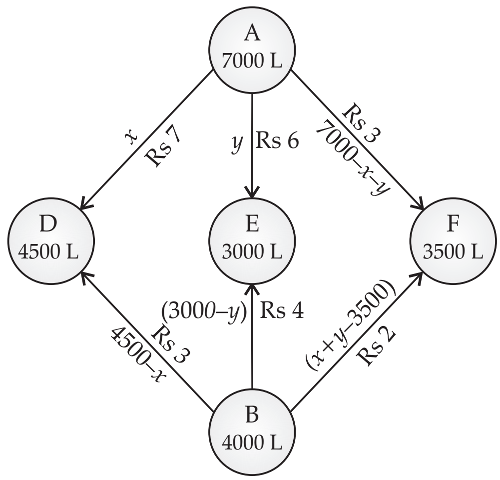
[NCERT Misc. Ex. Q.7, Page 527]

Ans. Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, $(7,000 - x - y)$ will be supplied from A to petrol pump F.

The requirements at petrol pump D is 4,500 L. Since x L are transported from depot A, the remaining $(4,500 - x)$ L will be transported from petrol B.

Similarly, $(3,000 - y)$ L and $3,500 - (7,000 - x - y) = (x + y - 3,500)$ L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows :



$x \geq 0$, $y \geq 0$, and $(7,000 - x - y) \geq 0$

$\Rightarrow x \geq 0$, $y \geq 0$, and $x + y \leq 7,000$

$4,500 - x \geq 0$, $3,000 - y \geq 0$, and $x + y - 3,500 \geq 0$

$\Rightarrow x \leq 4,500$, $y \leq 3,000$, and $x + y \geq 3,500$

Cost of transporting 10 L of petrol = Rs 1

Cost of transporting 1 L of petrol = Re $\frac{1}{10}$

Therefore, total transportation cost is given by,

$$Z = \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$

$$= 0.3x + 0.1y + 3,950$$

The problem can be formulated as follows :

Minimise, $Z = 0.3x + 0.1y + 3,950$ …(i)

Subject to the constraints,

$x + y \leq 7,000$ …(ii)

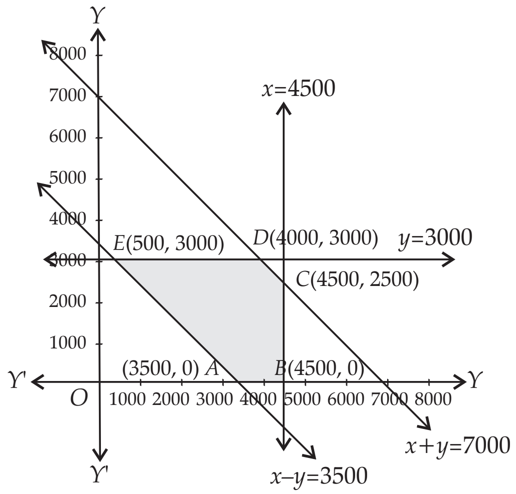
$x \leq 4,500$ …(iii)

$y \leq 3,000$ …(iv)

$x + y \geq 3,500$ …(v)

$x, y \geq 0$ …(vi)

The feasible region determined by the constraints is as follows :



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000) and E (500, 3000).

The values of Z at these corner points are as follows :

Corner point	$Z = 0.3x + 0.1y + 3950$	
A (3500, 0)	5,000	
B (4500, 0)	5,300	
C (4500, 2500)	5,550	
D (4000, 3000)	5,450	
E (500, 3000)	4,400	← Minimum

The minimum value of Z is 4,400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E and F respectively.

The minimum transportation cost is Rs. 4,400. **[5]**

Q. 19. Solve the following LPP graphically :

Maximise, $Z = 1000x + 600y$

Subject to the constraints :

$x + y \leq 200$

$x \geq 20$

$y - 4x \geq 0$

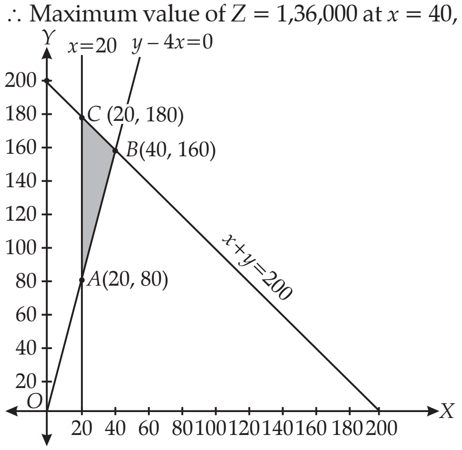
$x, y \geq 0$. [CBSE Board, Foreign Scheme, 2017]

Ans. $Z(A) = 6{,}80{,}000$

$Z(B) = 1{,}36{,}000$

$Z(C) = 1{,}28{,}000$

$\therefore$ Maximum value of $Z = 1{,}36{,}000$ at $x = 40$, $y = 160$



[6]

Q. 20. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of Re 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit.

[CBSE Board, Delhi Region, 2018]

Ans. Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore, $x \geq 0$ and $y \geq 0$. We can write the two equations as given below :

$4x + 6y \leq 4 \times 60$

$\Rightarrow 4x + 6y \leq 240$

and

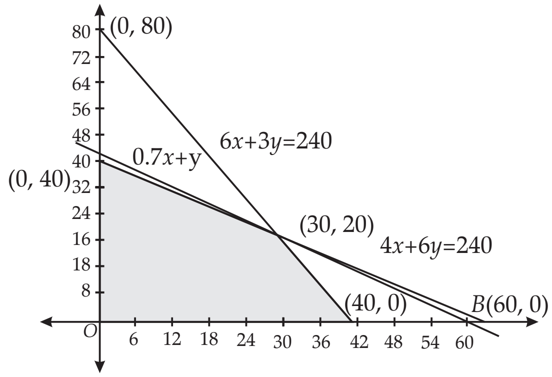
$6x + 3y \leq 4 \times 60$

$\Rightarrow 6x + 3y \leq 240$

Now, the total profit for A and B are 70 paise and 1 Re.

Therefore, we have to maximise $0.7x + y$

First we draw the graph of $x \geq 0$, $y \geq 0$, $4x + 6y \leq 240$ and $6x + 3y \leq 240$



The corner points of the feasible region are (0, 0), (40, 0), (30, 20), *and* (0, 40).

The values of Z at these corner points are as follows :

Corner point	$Z = 0.7x + y$
(0, 0)	0
(40, 0)	28
(30, 20)	41 ← Maximum
(0, 40)	40

The maximum value of Z is 41 at (30, 20). **[6]**

Q. 21. Refer to Q .8 of short answer type questions, How many of circuits of Type A and of Type B, should be produced by the manufacturer so as to maximise his profit? Determine the maximum profit.

[NCERT Exemp. Ex. 12.3, Q. 16, Page 253]

Ans. Maximise $Z = 50x + 60y$,

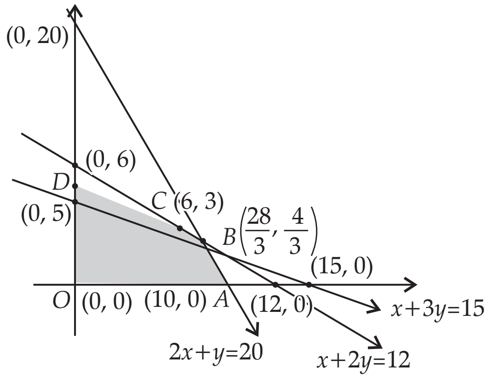
Subject to constraints,

$2x + y \leq 20$, $x + 2y \leq 12$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$

From the shaded region it is clear that the feasible region determined by the system of constraints is OABCD and is bounded and the coordinates of corner points are (0, 0), (10, 0), $\left(\frac{28}{3}, \frac{4}{3}\right)$, (6, 3) and (0, 5), respectively.

[Since, $x + 2y = 12$ and $2x + y = 20 \Rightarrow x = \frac{28}{3}, y = \frac{4}{3}$

and $x + 3y = 15$ and $x + 2y = 12 \Rightarrow y = 3$ and $x = 6$]

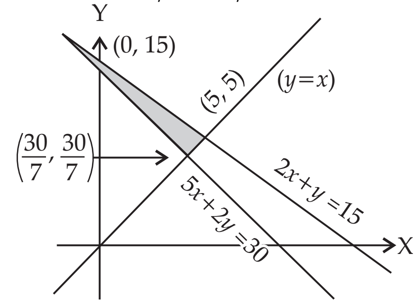


Corner points	Corresponding value of $Z = 50x + 60y$
(0, 0)	0
(10, 0)	500
$\left(\frac{28}{3}, \frac{4}{3}\right)$	$\frac{1400}{3}+\frac{240}{3}=\frac{1640}{3}=546.66$ ← Maximum
(6, 3)	480
(0, 5)	300

Since, the manufacturer is required to produce two types of circuits A and B and it is clear that parts of resistor, transistor and capacitor cannot be in fraction, so the required maximum profit is 480 where circuits of type A is 6 and circuits of type B is 3. **[5]**

Q. 22. Refer to Q.9 of short answer type questions, What will be the minimum cost?
[NCERT Exemp. Ex. 12.3, Q. 17, Page 253]

Ans. We have minimise $Z = 400x + 200y$,
Subject to constraints,
$5x + 2y \geq 30, 2x + y \leq 15, x \leq y, x \geq 0, y \geq 0.$
On solving $x - y = 0$ and $5x + 2y = 30$,
we get : $y = \frac{30}{7}, x = \frac{30}{7}$



On solving $x - y = 0$ and $2x + y = 15$, we get $x = 5$, $y = 5$

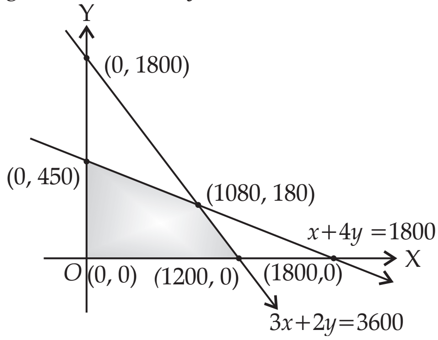
So, from the shaded feasible region it is clear that coordinates of corner points are (0, 15), (5, 5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner point	Corresponding value of $Z = 400x + 200y$
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7}$ = 2571.43 ← Minimum

Hence, the minimum cost is ₹ 2,571.43. **[5]**

Q. 23. Refer to Q. 10 of short answer type questions, Solve the linear programming problem and determine the maximum profit to the manufacturer.
[NCERT Exemp. Ex. 12.3, Q. 18, Page 253]

Ans. Maximise $Z = 100x + 170y$
Subject to constraints,
$3x + 2y \leq 3600, x + 4y \leq 1800, x \geq 0, y \geq 0$
From the shaded feasible region it is clear that the coordinates of corner points are (0, 0), (1200, 0), (1080, 180) and (0, 450).
On solving $x + 4y = 1800$ and $3x + 2y = 3600$, we get $x = 1080$ and $y = 180$

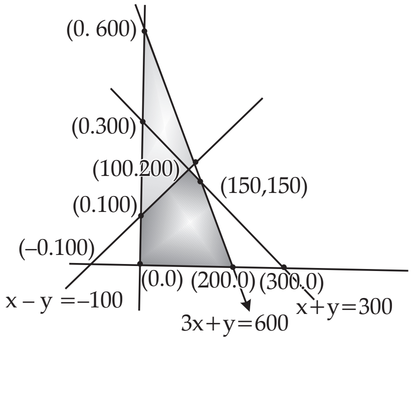


Corner points	Corresponding value of $Z = 100x + 170y$
(0, 0)	0
(1200, 0)	1200 × 100 = 0
(1080, 180)	100 × 1,080 + 170 × 180 = 1,38,600 ← Maximum
(0, 450)	0 + 170 × 450 = 76,500

Hence, the maximum profit to the manufacturer is 1,38,600. **[5]**

Q. 24. Refer to Q. 11 of short answer type questions, How many sweaters of each type should the company make in a day to get a maximum profit? What is the maximum profit?
[NCERT Exemp. Ex. 12.3, Q. 19, Page 253]

Ans. we have maximise $Z = 200x + 120y$
Subject to constraints,
$x + y \leq 300, 3x + y \leq 600, x - y \geq -100, x \geq 0, y \geq 0.$
On solving $x + y = 300$ and $3x + y = 600$, we get : $x = 150, y = 150$
On solving $x - y = -100$ and $x + y = 300$, we get : $x = 100, y = 200$



From the shaded feasible region it is clear that coordinates of corner points are (0, 0), (200, 0), (150, 150), (100, 200) and (0, 100).

Corner points	Corresponding value of $Z = 200x + 120y$
(0, 0)	0
(200, 0)	40,000
(150, 150)	$150 \times 200 + 120 \times 150 = 48{,}000 \leftarrow$ Maximum
(100, 200)	$100 \times 200 + 120 \times 200 = 44{,}000$
(0, 100)	$120 \times 100 = 12{,}000$

Hence, 150 sweaters of each type made by company and maximum profit = Rs. 48,000. **[5]**

Q. 25. Refer to Q. 6 of very short answer type questions, Determine the maximum distance that the man can travel.

[NCERT Exemp. Ex. 12.3, Q. 20, Page 253]

Ans. Maximise, $Z = x + y$,

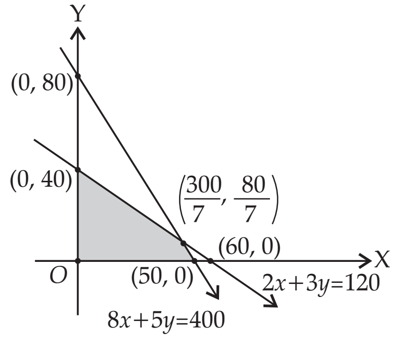
Subject to constraints,

$2x + 3y \le 120$, $8x + 5y \le 400$, $x \ge 0$, $y \ge 0$

On solving, we get :

$8x + 5y = 400$ and $2x + 3y = 120$, we get :

$x = \frac{300}{7}, y = \frac{80}{7}$



From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (50, 0), $\left(\frac{300}{7}, \frac{80}{7}\right)$ and (0, 40).

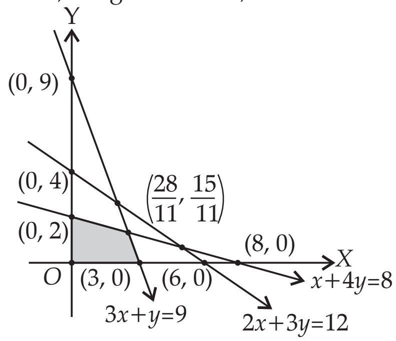
Corner points	Corresponding value of $Z = x + y$
(0, 0)	0
(50, 0)	50
$\frac{300}{7}$ $\frac{80}{7}$	$\frac{380}{7} = 54\frac{2}{7}$ km $\leftarrow$ Maximum
(0, 40)	40

Hence, the maximum distance that the man can travel is $54\frac{2}{7}$ km. **[5]**

Q. 26. Maximise $Z = x + y$, subject to $x + 4y \le 8$, $2x + 3y \le 12$, $3x + y \le 9$, $x \ge 0$, $y \ge 0$.

[NCERT Exemp. Ex. 12.3, Q. 21, Page 253]

Ans. Here, the given LPP is,



Maximise, $Z = x + y$

Subject to constraints,

$x + 4y \le 8$, $2x + 3y \le 12$, $3x + y \le 9$, $x \ge 0$, $y \ge 0$.

On solving $x + 4y = 8$ and $3x + y = 9$, we get :

$x = \frac{28}{11}, y = \frac{15}{11}$.

From the feasible region, it is clear that coordinates of corner points are (0, 0), (3, 0), $\left(\frac{28}{11}, \frac{15}{11}\right)$ and (0, 2).

Corner points	Value of $Z = x + y$
(0, 0)	0
(3, 0)	3
$\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{43}{11} = 3\frac{10}{11}$ $\leftarrow$ Maximum
(0, 2)	2

Hence, the maximum value is $3\frac{10}{11}$. **[5]**

Q. 27. A manufacturer produces two Models of bikes-Model *X* and Model *Y*. Model *X* takes a 6 man-hours to make per unit, while Model *Y* takes 10 man-hours per unit. There is a total of 450 man-hour available per week. Handling and Marketing costs are Rs. 2000 and Rs. 1000 per unit for Models *X* and *Y* respectively. The total funds available for these purposes are Rs. 80,000 per week. Profits per unit for Models *X* and *Y* are Rs. 1000 and Rs. 500, respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit. [NCERT Exemp. Ex. 12.3, Q. 22, Page 253]

Ans. Let the manufacturer produces x number of models X and y number of model Y bikes. Model X takes 6 man-hours to make per unit and model Y takes 10 man-hours to make per unit.

There is total of 450 man-hour available per week.

$\therefore\ 6x + 10y \le 450$

$\Rightarrow\ 3x + 5y \le 225$ …(i)

For models X and Y, handling and marketing costs are Rs. 2,000 and Rs. 1,000, respectively, total funds available for these purposes are Rs. 80,000 per week.

$\therefore\ 2{,}000x + 1{,}000y \le 80{,}000$

$\Rightarrow\ 2x + y \le 80$ …(ii)

Also, $x \ge 0$, $y \ge 0$

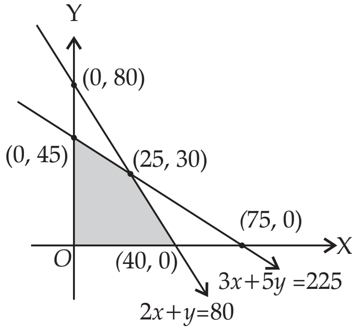
Hence, the profits per unit for models X and Y are Rs. 1,000 and Rs. 500, respectively.

$\therefore$ Required LPP is

Maximise, $Z = 1000x + 500y$

Subject to constraints,

$3x + 5y \leq 225, 2x + y \leq 80, x \geq 0, y \geq 0$



From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (40, 0), (25, 30) and (0, 45).

On solving $3x + 5y = 225$ and $2x + y = 80$, we get :

$x = 25, y = 30$

Corner points	Value of $Z = 1000x + 500y$
(0, 0)	0
(40, 0)	40,000← Maximum
(25, 30)	25,000 + 15,000 = 40,000 ← Maximum
(0, 45)	22,500

So, the manufacturer should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of Rs. 40,000.

Since, in question it is asked that each model bikes should be produced. **[5]**

Q. 28. In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below :

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs. 2 and Re 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

[NCERT Exemp. Ex. 12.3, Q. 23, Page 254]

Ans. Let the person takes x units of tablet X and y units of tablet Y.

So, from the given information, we have

$6x + 2y \geq 18 \Rightarrow 3x + y \geq 9$ ...(i)

$3x + 3y \geq 21 \Rightarrow x + y \geq 7$ ...(ii)

and $2x + 4y \geq 16 \Rightarrow x + 2y \geq 8$ ...(iii)

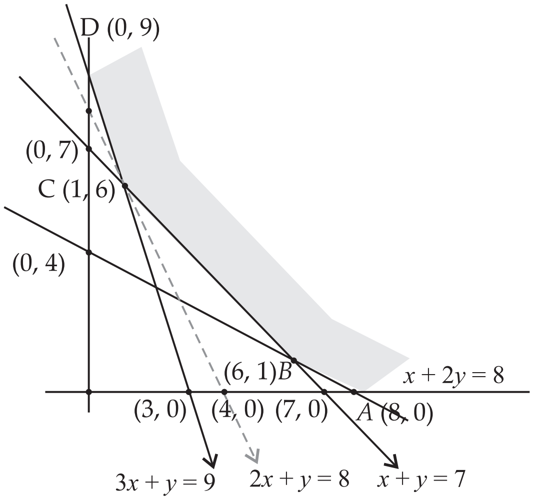
Also, we know that here, $x \geq 0, y \geq 0$ ...(iv)

The price of each tablet of X and Y is Rs. 2 and Rs. 1, respectively.

So, the corresponding LPP is minimise $Z = 2x + y$,

Subject to constraints,

$3x + y \geq 9, x + y \geq 7, x + 2y \geq 8, x \geq 0, y \geq 0$



From the shaded graph, we see that for the shown unbounded region, we have coordinates of corner points A, B, C and D as (8, 0), (6, 1), (1, 6) and (0, 9), respectively.

[On solving $x + 2y = 8$ and $x + y = 7$, we get $x = 6$, $y = 1$ and on solving $3x + y = 9$ and $x + y = 7$, we get $x = 1, y = 6$]

Corner points	Value of $Z = 2x + y$
(8, 0)	16
(6, 1)	13
(1, 6)	8 ← Minimum
(0, 9)	9

Thus, we see that 8 is the minimum value of Z at the corner point (1, 6). Here, we see that the feasible region is unbounded. Therefore, 8 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality

$2x + y < 8$ ...(v)

And check whether the resulting open half has points in common with feasible region or not.

If it has common point, then 8 will not be the minimum value of Z, otherwise 8 will be the minimum value of Z.

Thus, from the graph it is clear that, it has no common point.

Therefore, $Z=2x+ y$ has 8 as minimum value subject to the given constraints.

Hence, the person should take 1 unit of X tablet and 6 units of Y tablets to satisfy the given requirements and at the minimum cost of Rs. 8.

Q. 29. A company makes 3 model of calculators : A, B and C at factory I and factory II. The company has orders for at least 6400 calculators of model A, 4000 calculators of model B and 4800 calculators of model C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every

day; at factory II, 40 calculators of model *A*, 20 of model *B* and 40 of model *C* are made every day. It costs Rs. 12000 and Rs. 15000 each day to operate factory I and II, respectively. Find the number of days each factory should operate to minimise the operating costs and still meet the demand.

[NCERT Exemp. Ex. 12.3, Q. 24, Page 254]

Ans. Let the factory I operate for x days and the factory II operate for y days.

At factory I, 50 calculators of model *A* and at factory II, 40 calculators of model *A* are made every day. Also, company has ordered for at least 6,400 calculators of model *A*.

$\therefore 50x + 40y \geq 6{,}400$

$\Rightarrow 5x + 4y \geq 640.$ ...(i)

Also, at factory I 50 calculators of model *B* and at factory II, 20 calculators of model *B* are made every day.

Since, the company has ordered at least 4,000 calculators of model *B*.

$\therefore 50x + 20y \geq 4{,}000$

$\Rightarrow 5x + 2y \geq 400$ ...(ii)

Similarly, for model *C*, $30x + 40y \geq 4{,}800$

$\Rightarrow 3x + 4y \geq 480$ ...(iii)

Also, $x \geq 0, y \geq 0$ ...(iv)

[Since, x and y are non-negative.]

It costs Rs. 12,000 and Rs. 15,000 each day to operate factories I and II, respectively.

$\therefore$ Corresponding LPP is,

Minimise, $Z = 12{,}000x + 15{,}000y$,

Subject to constraints,

$5x + 4y \geq 640$

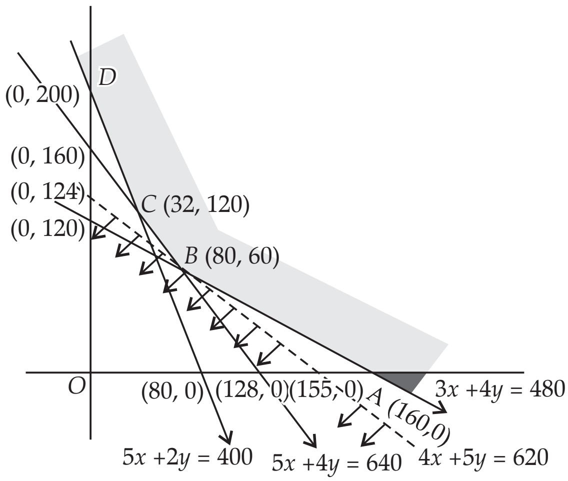
$5x + 2y \geq 400$

$3x + 4y \geq 480$

$x \geq 0, y \geq 0$

On solving $3x + 4y = 480$ and $5x + 4y = 640$, we get $x = 80, y = 60$

On solving $5x + 4y = 640$ and $5x + 2y = 400$, we get $x = 32, y = 120$



Thus, from the graph, it is clear that feasible region is unbounded and the coordinates of corner points *A*, *B*, *C* and *D* are (160, 0), (80, 60), (32, 120) and (0, 200), respectively.

Corner points	Value of $Z = 12{,}000x + 15{,}000y$
(160, 0)	$160\times 12{,}000 = 1920{,}000$
(80, 60)	$(80 \times 12 + 60 \times 15) \times 1{,}000 = 18{,}60{,}000 \leftarrow$ Minimum
(32, 120)	$(32 \times 12 + 120 \times 15) \times 1{,}000 = 21{,}84{,}000$
(0, 200)	$0 + 200 \times 15{,}000 = 30{,}00{,}000$

From the above table, it is clear that for given unbounded region the minimum value of *Z* may or may not be 18,60,000.

Now, for deciding this, we graph the inequality

$12{,}000x + 15{,}000y < 18{,}60{,}000$

$\Rightarrow 4x + 5y < 620$

And check whether the resulting open half plane has points in common with feasible region or not.

Thus, as shown in the figure, it has no common points so, $Z = 12{,}000x + 15{,}000y$ has minimum value 18,60,000.

So, number of days factory I should be operated is 80 and number of days factory II should be operated is 60 for the minimum cost and satisfying the given constraints. **[5]**

Q. 30. Maximise and Minimise $Z = 3x - 4y$ subject to $x - 2y \leq 0, -3x + y \leq 4, x - y \leq 6, x, y \geq 0$

[NCERT Exemp. Ex. 12.3, Q. 25, Page 254]

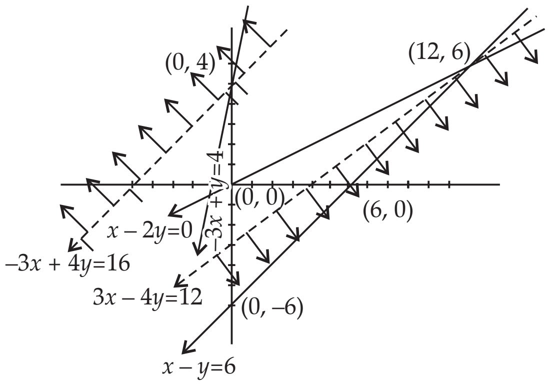
Ans. Given LPP is,

Maximise and minimise $Z = 3x - 4y$

Subject to constraints,

$x - 2y \leq 0, -3x + y \leq 4, x - y \leq 6, x, y \geq 0.$

[On solving $x - y = 6$ and $x - 2y = 0$, we get $x = 12$, $y = 6$]



From the shown graph, for the feasible region, we see that it is unbounded and coordinates of corner points are (0, 0), (12, 6) and (0, 4).

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(0, 4)	$-16 \leftarrow$ Minimum
(12, 6)	$12 \leftarrow$ Maximum

For given unbounded region the minimum value of *Z* may or may not be –16. So, for deciding this, we graph the inequality.

$3x - 4y < -16$

And check whether the resulting open half plane has common points with feasible region or not.

Thus, from the figure it shows that it has common points with feasible region, so it does not have any minimum value.

Also, similarly for maximum value, we graph the inequality $3x - 4y > 12$ and see that resulting open half plane has no common points with the feasible region and hence maximum value 12 exist for $Z = 3x - 4y$. **[5]**

Q. 31. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit food and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements. [NCERT Ex. 12.2, Q. 9, Page 520]

Ans. Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows :

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Require-ment	80	100	

The cost of food F_1 is Rs. 4 per unit and of Food F_2 is Rs. 6 per unit. Therefore, the constraints are :

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise, $Z = 4x + 6y$ ...(i)

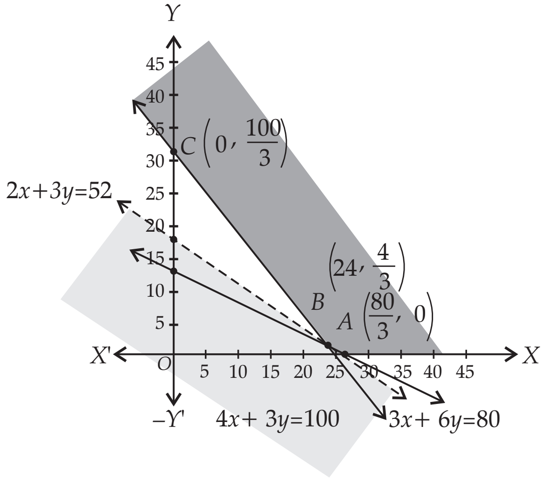
Subject to the constraints,

$3x + 6y \geq 80$ ...(ii)

$4x + 3y \geq 100$ ...(iii)

$x, y \geq 0$ ...(iv)

The feasible region determined by the constraints is as follows :



It can be seen that the feasible region is unbounded. The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right), B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$.

The corner points are

$A\left(\frac{80}{3}, 0\right), B\left(24, \frac{4}{3}\right)$, and $C\left(0, \frac{100}{3}\right)$.

The values of Z at these corner points are as follows :

Corner points	$Z = 4x + 6y$	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be ₹ 104. **[5]**

Some Commonly Made Errors

- Error in solving the linear programming problems through graphical method.
- Errors in identifying the feasible region.
- Students get confused in plotting the constraints.
- Errors in finding the optimal solution.

EXPERT ADVICE

- *For Linear Programming problems, proper shading of the feasible region is very important. If the region is unbounded, don't forget to draw the kink line.*
- *Note that the main constraints are written as ≤ for the standard maximum problem and ≥ for the standard minimum problem.*
- *Calculate the value of the objective function at each of the vertices to determine which of them has the maximum or minimum values. It must be taken into account the possible non-existence of a solution if the compound is not bounded.*
- *For all linear programming, the decision variables should always take non-negative values, which means the values for decision variables should be greater than or equal to 0.*

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