CHAPTER

11 THREE-DIMENSIONAL GEOMETRY

Chapter Objectives

This chapter will help you understand :

➢ ***Three-dimensional geometry :*** *Introduction to coordinate system; Direction cosines and Directional ratios of a line; Equation of line in space; Angle and shortest distance between two lines; Introduction to plane, Co-planarity of two lines; Angle between two planes; Distance of a point from a plane; Angle between a line and a plane.*

Quick Review

❖ Vectors can exist in general n-dimensional space. The general notation for a n-dimensional vector is, $\vec{v} = (a_1, a_2, a_3, \ldots, a_n)$ and each of the axis are called components of the vector.

❖ In physics and mathematics, a pseudo-vector (or axial vector) is a quantity that transforms like a vector under a proper rotation, but in three-dimensional (3D) geometry it gains an additional sign flip under an improper rotation such as a reflection. Geometrically it is the opposite, of equal magnitude but in the opposite direction, of its mirror image. This is as opposed to a true vector, also known, in this context, as a polar vector, which on reflection matches its mirror image.

❖ In 3D, the pseudo-vector, p is associated with the curl of a polar vector or with the cross-product of two polar vectors a and b. The vector p calculated in this way is a pseudo-vector.

❖ Constructing a plane in two-dimensional (2D) is easy, this can be done from either a normal (unit vector) and a point, or from two points in space.

❖ Quadric surfaces in 3D are the graphs of any equation that can be put into the general form :

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Gx + Hy + Iz + J = 0$$

TIPS...

- *Setting up the Matrix for Solving.*
- *Write your equations in standard form.*
- *Transfer the numbers from the system of equations into a matrix.*
- *Draw a large square bracket around full matrix.*

TRICKS...

- *Recognise the form of the solution matrix.*
- *Use scalar multiplication.*
- *Use row-addition or row-subtraction.*
- *Combine row-addition and scalar multiplication in a single step.*
- *Work from top to bottom first.*

Know the Links

☞ *http://tutorial.math.lamar.edu/Classes/CalcII/VectorsIntro.aspx*

☞ *http://docs.godotengine.org/en/3.0/tutorials/math/vectors_advanced.html*

Multiple Choice Questions **(1 mark each)**

Q. 1. Distance of the point (α, β, γ) from y-axis is

(a) β **(b)** $|\beta|$

(c) $|\beta| + |\gamma|$ **(d)** $\sqrt{\alpha^2 + \gamma^2}$

[NCERT Exemp. Ex. 11.3, Q. 29, Page 237]

Ans. Correct option : (d)

Explanation :

The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0, \beta, 0)$.

$\therefore$ Required distance,

$PQ = \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

Q. 2. If the directions cosines of a line are k, k, k, then

(a) $k > 0$ **(b)** $0 < k < 1$

(c) $k = 1$ **(d)** $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

[NCERT Exemp. Ex. 11.3, Q. 30, Page 238]

Ans. Correct option : (d)

Explanation :

Since, direction cosines of a line are k, k and k.

$\therefore l = k$, m = k and n = k

We know that, $l^2 + m^2 + n^2 = 1$

$$\Rightarrow k^2 + k^2 + k^2 = 1$$

$$\Rightarrow \quad k^2 = \frac{1}{3}$$

$$\therefore \quad k = \pm\frac{1}{\sqrt{3}}$$

Q. 3. The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

(a) 1 **(b)** 7

(c) 1/7 **(d)** None of these

[NCERT Exemp. Ex. 11.3, Q. 31, Page 238]

Ans. Correct option : (a)

Explanation :

The distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ form the origin is 1.

[Since $\vec{r}\cdot\hat{n} = d$ is the form of above equation, where d represents the distance of plane from the origin, *i.e.*, d = 1]

Q. 4. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) 10/6√5 **(b)** 4/5√2

(c) 2√3/5 **(d)** √2/10

[NCERT Exemp. Ex. 11.3, Q. 32, Page 238]

Ans. Correct option : (d)

Explanation : We have, the equation of line as

$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

This line is parallel to the vector, $\vec{b} = 3\hat{i}+4\hat{j}+5\hat{k}$

Equation of plane is $2x - 2y + z = 5$.

Normal to the plane is $\vec{n} = 2\hat{i}-2\hat{j}+\hat{k}$.

Its angle between line and plane is 'θ'.

Then,

$$\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|} = \frac{|(3\hat{i}+4\hat{j}+5\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})|}{\sqrt{3^2+4^2+5^2}\sqrt{4+4+1}}$$

$$= \frac{|6-8+5|}{\sqrt{50}\sqrt{9}} = \frac{3}{15\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\sin\theta = \frac{\sqrt{2}}{10}$$

Q. 5. The reflection of the point (α, β, γ) in the *xy*-plane is

(a) (α, β, 0) **(b)** (0, 0, γ)

(c) (–α, –β, γ) **(d)** (α, β, –γ)

[NCERT Exemp. Ex. 11.3, Q. 33, Page 238]

Ans. Correct option : (d)

Explanation :

In *xy*-plane, the reflection of the point (α,β,γ) is $(\alpha,\beta,-\gamma)$.

Q. 6. The area of the quadrilateral *ABCD*, where *A* (0, 4, 1), *B* (2, 3, –1), *C* (4, 5, 0) and *D* (2, 6, 2), is equal to

(a) 9 sq. units **(b)** 18 sq. units

(c) 27 sq. units **(d)** 81 sq. units

[NCERT Exemp. Ex. 11.3, Q. 34, Page 238]

Ans. Correct option : (a)

Explanation :

We have, *A* (0, 4, 1), *B* (2, 3, –1), *C* (4, 5, 0) and *D* (2, 6, 2)

$$\overline{AB} = (2-0)\hat{i}+(3-4)\hat{j}+(-1-1)\hat{k}$$
$$= 2\hat{i}-\hat{j}-2\hat{k}$$
$$\overline{BC} = (4-2)\hat{i}+(5-3)\hat{j}+(0+1)\hat{k}$$
$$= 2\hat{i}+2\hat{j}+\hat{k}$$
$$\overline{CD} = (2-4)\hat{i}+(6-5)\hat{j}+(2-0)\hat{k}$$
$$= -2\hat{i}+\hat{j}+2\hat{k}$$
$$\overline{DA} = (0-2)\hat{i}+(4-6)\hat{i}+(1-2)\hat{k}$$
$$= -2\hat{i}-2\hat{j}-\hat{k}$$

Thus quadrilateral formed is parallelogram.

∴ Area of quadrilateral *ABCD*

$$= |\overline{AB}\times\overline{BC}|$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & -2\\ 2 & 2 & 1\end{vmatrix}$$

$$= |3\hat{i}-6\hat{j}+6\hat{k}|$$

$$= \sqrt{9+36+36}$$

$= 9$ sq. units

Q. 7. The locus represented by $xy + yz = 0$ is

(a) A pair of perpendicular lines

(b) A pair of parallel lines

(c) A pair of parallel planes

(d) A pair of perpendicular planes

[NCERT Exemp. Ex. 11.3, Q. 35, Page 238]

Ans. Correct option : (d)

Explanation :

We have,

$$xy + yz = 0$$

$$\Rightarrow \quad x(y+z) = 0$$

$$\Rightarrow x = 0 \text{ and } y + z = 0$$

Above are equations of planes.

Normal to the plane x = 0 is $\hat{i}$.

And normal to the plane $y + z = 0$ is $\hat{j}+\hat{k}$.

Now, $\hat{i}\cdot(\hat{j}+\hat{k}) = 0$

So, planes are perpendicular.

Q. 8. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(a) √3/2 **(b)** √2/3

(c) 2/7 **(d)** 3/7

[NCERT Exemp. Ex. 11.3, Q. 36, Page 236]

Ans. Correct option : (c)

Explanation :

We have equation of plane as $2x-3y+6z-11=0$.

Normal to the plane is $\vec{n}=2\hat{i}-3\hat{j}+6\hat{k}$.

Also x-axis is along the vector $\vec{a}=\hat{i}+0\hat{j}+0\hat{k}$.

According to the question,

$$\sin\alpha=\frac{|\vec{a}\cdot\vec{n}|}{|\vec{a}||\vec{n}|}$$

$$=\frac{|\hat{i}\cdot(2\hat{i}-3\hat{j}+6\hat{k})|}{\sqrt{1}\sqrt{4+9+36}}=\frac{2}{7}$$

Q. 9. Distance between the two planes : $2x+3y+4z=4$ and $4x+6y+8z=12$ is

(a) 2 units **(b) 4 units**

(c) 8 units **(d) $2/\sqrt{29}$ units**

[NCERT Misc. Ex. Q. 22, Page 499]

Ans. Correct option : (d)

Explanation :

Distance between two parallel planes,

$Ax+By+Cz=d_1$ and $Ax+By+Cz=d_2$ is

$$\left|\frac{d_1-d_2}{\sqrt{A^2+B^2+C^2}}\right|$$

$2x+3y+4z=4$

Comparing with $Ax+By+Cz=d_1$

$A=2, B=3, C=4, d_1=4$

And now, $4x+6y+8z=12$

$2(2x+3y+4z)=12$

Dividing by 2

$2x+3y+4z=6$

Comparing with $Ax+By+Cz=d_2$

$A=2, B=3, C=4, d_2=6$

So,

Distance between the two planes

$$=\left|\frac{4-6}{\sqrt{2^2+3^2+4^2}}\right|=\left|\frac{-2}{\sqrt{4+9+16}}\right|=\frac{2}{\sqrt{29}}$$

Q. 10. The planes : $2x-y+4z=5$ and $5x-2.5y+10z=6$ are

(a) Perpendicular

(b) Parallel

(c) Intersect y-axis

(d) Passes through $\left(0,0,\frac{5}{4}\right)$

[NCERT Misc. Ex. Q. 23, Page 499]

Ans. Correct option : (b)

Explanation :

Angle between two planes $A_1x+B_1y+C_1z=d_1$ and $A_2x+B_2y+C_2z=d_2$ is given by

$$\cos\theta=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$

Given that plane,

$2x-1y+4z=5$

Comparing with $A_1x+B_1y+C_1z=d_1$

$A_1=2, B_1=-1, C_1=4, d_1=5$

$5x-2.5y+10z=16$

Multiplying by 2 on both sides,

$10x-5y+20z=12$

Comparing with $A_2x+B_2y+C_2z=d_2$

$A_2=10, B_2=-5, C_2=20, d_2=12$

$$\text{So, }\cos\theta=\left|\frac{(2\times10)+(-1\times-5)+(4\times20)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{10^2+(-5)^2+20^2}}\right|$$

$$=\left|\frac{20+5+80}{\sqrt{4+1+16}\sqrt{100+25+400}}\right|$$

$$=\left|\frac{105}{\sqrt{21}\sqrt{525}}\right|$$

$$=\left|\frac{105}{\sqrt{21}\times\sqrt{25\times21}}\right|$$

$$=\left|\frac{105}{\sqrt{21}\times5\sqrt{21}}\right|$$

$$=\left|\frac{105}{21\times5}\right|$$

$$=1$$

So, $\cos\theta=1$

$\therefore\ \theta=0°$

Since angle between the planes is 0°.

Therefore, the planes are parallel.

Very Short Answer Type Questions (1 or 2 marks each)

Q. 1. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

[CBSE Board, Foreign Region, 2016]

Ans. $\frac{x}{\sqrt{3}}+\frac{y}{\sqrt{3}}+\frac{z}{\sqrt{3}}=5\sqrt{3}$

or $x+y+z=15$ **[1]**

Q. 2. Find the vector equation of the plane with intercepts 3, −4, 2 on x, y and z-axis respectively.

[CBSE Board, All India Region, 2016]

Ans. $\frac{x}{3}+\frac{y}{-4}+\frac{z}{2}=1$

$$\Rightarrow\vec{r}\cdot(4\hat{i}-3\hat{j}+6\hat{k})=12\ \ or\ \ \vec{r}\cdot\left(\frac{\hat{i}}{3}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}\right)=1$$ **[1]**

Q. 3. Find the coordinates of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

[CBSE Board, All India Region, 2016]

Ans. Equation of line through A (3, 4, 1) and B (5, 1, 6),

$$\frac{x-3}{2}=\frac{y-4}{-3}=\frac{z-1}{5}=k \quad \text{(say)}$$

General point on the line :

$x=2k+3, y=-3k+4, z=5k+1$

Line crosses xz plane, *i.e.*, $y=0$ if $-3k+4=0$

$\therefore\ k=\frac{4}{3}$

Coordinate of required point $=\left(\frac{17}{3},0,\frac{23}{3}\right)$

Angle, which line makes with xz plane :

$$\sin\theta = \left|\frac{2(0)+(-3)(1)+5(0)}{\sqrt{4+9+25}\sqrt{1}}\right|$$
$$= \frac{3}{\sqrt{38}}$$
$$\Rightarrow \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$ **[2]**

Q. 4. Find the vector equation of the line passing through the point A (1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

[CBSE Board, Delhi Region, 2017]

Ans. Equation of given line is $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$.

Its directional ratios $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle$

Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$ **[2]**

Q. 5. A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is perpendicular to the plane $\vec{r}.(3\hat{i} + 4\hat{j} - 5\hat{k}) = 7$. Find the equation of the line in Cartesian and vector forms.

[CBSE Board, Foreign Region, 2017]

Ans. Vector form: $\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$

Cartesian form : $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$ **[2]**

Q. 6. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

[CBSE Board, All India Region, 2017]

Ans. Writing the equations as $\left.\begin{array}{l} 2x - y + 2z = 5 \\ 2x - y + 2z = 8 \end{array}\right\}$

$\Rightarrow$ Distance $= 1$ unit **[1]**

Q. 7. A plane passes through the points (2,0,0), (0,3,0) and (0,0,4). The equation of plane is __________.

[NCERT Exemp. Ex. 11.3, Q. 37, Page 239]

Ans. We know that, equation of a plane that cuts the coordinate axes at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Hence, the equation of plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4) is $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$. **[1]**

Q. 8. The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are ______. [NCERT Exemp. Ex. 11.3, Q. 38, Page 239]

Ans. Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are

$\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$.

i.e., $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$. **[1]**

Q. 9. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is __________.

[NCERT Exemp. Ex. 11.3, Q. 39, Page 239]

Ans. We have equation of line as $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

Line passes through the point $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$ and parallel to the vector $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$.

So, the vector equation will be

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$. **[1]**

Q. 10. The vector equation of the line through the points (3, 4, –7) and (1, –1, 6) is __________.

[NCERT Exemp. Ex. 11.3, Q. 40, Page 239]

Ans. We know that, vector equation of a line that passes through two points $\vec{a}$ and $\vec{b}$ is represented by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

Here, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{a} = 3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 6\hat{k}$

So, the required equation is

$x\hat{i} + y\hat{j} + x\hat{k} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

$\Rightarrow (x-3)\hat{i} + (y-4)\hat{j} + (z+7)\hat{k} = \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$ **[1]**

Q. 11. The Cartesian equation of the plane $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$ is __________.

[NCERT Exemp. Ex. 11.3, Q. 41, Page 239]

Ans. We have, $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$

$\Rightarrow \quad x + y - z = 2$

This is the required Cartesian form. **[1]**

Q. 12. State true/false :

The unit vector normal to the plane $x + 2y + 3z - 6 = 0$ is $\frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$.

[NCERT Exemp. Ex. 11.3, Q. 42, Page 239]

Ans. **True**,

We have equation of plane as $x + 2y + 3z - 6 = 0$.

Normal to the plane is $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Therefore, unit vector normal to the plane is :

$$\hat{n} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{1^2 + 2^2 + 3^2}}$$
$$= \frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$$ **[2]**

Q. 13. State true/false :

The intercepts made by the plane $2x - 3y + 5z + 4 = 0$ on the coordinate axis are –2, 4/3, –4/5.

[NCERT Exemp. Ex. 11.3, Q. 43, Page 239]

Ans. **True**,

We have equation of plane as $2x - 3y + 5z + 4 = 0$

$\Rightarrow \quad 2x - 3y + 5z = -4$

$\Rightarrow \quad \frac{2x}{-4} - \frac{3y}{-4} + \frac{5z}{-4} = 1$

$\Rightarrow \frac{x}{-2} + \frac{y}{\frac{4}{3}} + \frac{z}{\left(-\frac{4}{5}\right)} = 1$

So, the intercepts are $-2, \frac{4}{3}$ and $-\frac{4}{5}$. **[2]**

Q. 14. State true/false :

The angle between the line

$\vec{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0$ is $\sin^{-1}\frac{5}{2\sqrt{91}}$.

[NCERT Exemp. Ex. 11.3, Q. 44, Page 239]

Ans. **False,**

Line $\vec{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ is parallel to the vector $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$.

Normal to the plane is $\vec{n} = 3\hat{i} - 4\hat{j} - \hat{k}$.

Let θ is the angle between line and plane.

Then, $\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$

$= \frac{|(2\hat{i} - \hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - \hat{k})|}{\sqrt{6} \cdot \sqrt{26}}$

$= \frac{|6 + 4 - 1|}{\sqrt{156}} = \frac{9}{2\sqrt{39}}$

$\therefore \quad \theta = \sin^{-1}\frac{9}{2\sqrt{39}}$ **[2]**

Q. 15. State true/false :

The angle between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - \hat{j}) = 4$ is $\cos^{-1}\frac{-5}{\sqrt{58}}$.

[NCERT Exemp. Ex. 11.3, Q. 45, Page 239]

Ans. **False,**

Normal to the plane

$\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$ is $\vec{n}_1 = 2\hat{i} - 3\hat{j} + \hat{k}$

Normal to the plane $\vec{r} \cdot (\hat{i} - \hat{j}) = 4$ is $\vec{n}_2 = \hat{i} - \hat{j}$

$\therefore$ Angle between the planes is given by

$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

$\Rightarrow \cos\theta = \frac{|(2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j})|}{\sqrt{4+9+1}\sqrt{1+1}}$

$\Rightarrow \cos\theta = \frac{|2+3|}{\sqrt{14}\sqrt{2}} = \frac{5}{2\sqrt{7}}$

$\therefore \theta = \cos^{-1}\left(\frac{5}{2\sqrt{7}}\right)$ **[2]**

Q. 16. State true/false :

The line $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$ lies in the plane $\vec{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$.

[NCERT Exemp. Ex. 11.3, Q. 46, Page 239]

Ans. **False,**

We have, $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) = (2+\lambda)\hat{i} + (-3-\lambda)\hat{j} + (-1+2\lambda)\hat{k}$

Position vector of any point on this line is $(2+\lambda)\hat{i} + (-3-\lambda)\hat{j} + (-1+2\lambda)\hat{k}$.

If this point lies on the plane then LHS of the plane is

$\left[(2+\lambda)\hat{i} + (-3-\lambda)\hat{j} + (-1+2\lambda)\hat{k}\right] \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2$

$= 6 + 3\lambda - 3 - \lambda + 1 - 2\lambda + 2 \neq 0$

So, the line does not lie on the plane. **[2]**

Q. 17. State true/false :

The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is $r = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.

[NCERT Exemp. Ex. 11.3, Q. 47, Page 239]

Ans. **True,**

We have, $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

This line is passing through the point (5, –4, 6) and parallel to the vector $3\hat{i} + 7\hat{j} + 2\hat{k}$.

$\therefore$ Its vector form is $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$. **[2]**

Q. 18. State true/false :

The equation of a line, which is parallel to $2\hat{i} + \hat{j} + 3\hat{k}$ and which passes through the point $(5, -2, 4)$ is $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$.

[NCERT Exemp. Ex. 11.3, Q. 48, Page 240]

Ans. **False,**

Line is parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$.

Line passing through the point (5, –2, 4),

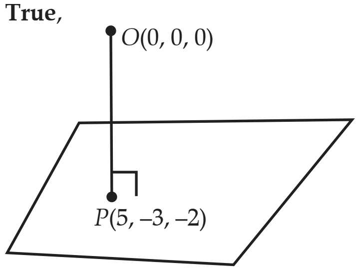
So its equation is $\frac{x-5}{2} = \frac{y+2}{1} = \frac{z-4}{3}$. **[2]**

Q. 19. State true/false :

If the foot of perpendicular drawn from the origin to a plane is (5, –3, –2), then the equation of plane is $\vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$.

[NCERT Exemp. Ex. 11.3, Q. 49, Page 240]

Ans. **True,**



From the figure, normal to the plane is $\vec{n} = \overline{OP} = 5\hat{i} - 3\hat{j} - 2\hat{k}$.

Plane passing through the point $P(5, -3, -2)$.

$\therefore$ Equation of the plane is

$5(x-5) - 3(y+3) - 2(z+2) = 0$ or $5x - 3y - 2z = 38$. **[2]**

Q. 20. If a line makes angles 90°, 135°, 45° with the *x*, *y* and *z*-axes respectively, find its direction cosines.

[NCERT Ex. 11.1, Q. 1, Page 467]

Ans. Direction cosines of a line making angle α with *x*-axis, β with *y*-axis and γ with *z*-axis are l, *m* and *n*.

$l = \cos\alpha, \quad m = \cos\beta, \quad n = \cos\gamma$

Here, $\alpha = 90°, \quad \beta = 135°, \quad \gamma = 45°$,

So, direction cosines are

$l = \cos 90° = 0$

$m = \cos 135° = \cos(180 - 45°) = -\cos 45° = \frac{-1}{\sqrt{2}}$

$n = \cos 45° = \frac{1}{\sqrt{2}}$

Therefore, required direction cosines are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$. **[2]**

Q. 21. Find the direction cosines of a line which makes equal angles with the coordinate axes.

[NCERT Ex. 11.1, Q. 2, Page 467]

Ans. Direction cosines of a line making, α with *x*-axis, β with *y*-axis, and with *z*-axis are *l*, *m* and *n*

$l=\cos\alpha, \quad m=\cos\beta, \quad n=\cos\gamma$

Given the line makes equal angles with the coordinate axes.

So,

$\alpha=\beta=\gamma$ ...(i)

So the, direction cosines are

$l=\cos\alpha, \quad m=\cos\alpha, \quad n=\cos\alpha$

We know that,

$l^2+m^2+n^2=1$ **[1]**

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$ (From Eq. (i))

$$3\cos^2\alpha=\frac{1}{3}$$

$$\cos^2\alpha=\frac{1}{3}$$

$$\cos\alpha=\pm\sqrt{\frac{1}{3}}$$

$$\therefore \quad \cos\alpha=\pm\frac{1}{\sqrt{3}}$$

Therefore, direction cosines are :

$l=\pm\frac{1}{\sqrt{3}}, \quad m=\pm\frac{1}{\sqrt{3}}, \quad n=\pm\frac{1}{\sqrt{3}}$ **[1]**

Q. 22. If a line has the direction ratios –18, 12, –4, then what are its direction cosines?

[NCERT Ex. 11.1, Q. 3, Page 467]

Ans. If direction ratios of a line are a, b and c.

Direction cosines are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

Given that,

Direction ratios = –18, 12, –4

$a=-18, \quad b=12, \quad c=-4$

$$\sqrt{a^2+b^2+c^2}=\sqrt{(-18)^2+12^2+(-4)^2}$$
$$=\sqrt{324+144+16}$$
$$=\sqrt{484}$$
$$=22$$ **[1]**

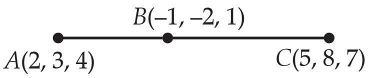
Direction cosines

$$=\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$
$$=\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$$
$$=\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$ **[1]**

Q. 23. Show that the points (2, 3, 4), (–1, –2, 1), (5, 8, 7) are collinear.

[NCERT Ex. 11.1, Q. 4, Page 467]

Ans.



Three points A, B and C are collinear if direction ratios of AB and BC are proportional.

For AB :

A (2, 3, 4)

B (–1, –2, 1)

Direction ratios

$=-1-2, \ -2-3, \ 1-4$

$=-3, \ -5, \ -3$

So, $a_1 = -3$, $b_1 = -5$ and $c_1 = -3$

For BC :

B (–1, –2, 1)

C (5, 8, 7)

Direction ratios

$=5-(-1), \ 8-(-2), \ 7-1$

$=6, \ 10, \ 6$

So, $a_2 = 6$, $b_2 = 10$ and $c_2 = 6$ **[1]**

Now,

$$\frac{a_2}{a_1}=\frac{6}{-3}=-2$$

$$\frac{b_2}{b_1}=\frac{10}{-5}=-2$$

$$\frac{c_2}{c_1}=\frac{6}{-3}=-2$$

Since, $\frac{a_2}{a_1}=\frac{b_2}{b_1}=\frac{c_2}{c_1}=-2$

Therefore A, B and C are collinear. **[1]**

Q. 24. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

[NCERT Ex. 11.2, Q. 4, Page 477]

Ans. Equation of a line passing through a point with position vector $\vec{a}$, and parallel to a vector $\vec{b}$ is

$\vec{r}=\vec{a}+\lambda\vec{b}$

Since line passes through the point (1, 2, 3)

$\vec{a}=1\hat{i}+2\hat{j}+3\hat{k}$

Since line is parallel to $3\hat{i}+2\hat{j}-2\hat{k}$

$b=3\hat{i}+2\hat{j}-2\hat{k}$

Equation of line $\vec{r}=\vec{a}+\lambda\vec{b}$

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$ **[2]**

Q. 25. Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

[NCERT Ex. 11.2, Q. 6, Page 477]

Ans. Equation of a line passing through the point (x_1, y_1, z_1) and parallel to a line having direction ratios a, b, c is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Since the line passes through the points (–2, 4, –5)

$x_1=-2, \quad y_1=4, \quad z_1=-5$

Since the line is parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

$a=3, \quad b=5 \quad c=6$ **[1]**

Therefore, equation of line in Cartesian form is :

$$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$$

$$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$ **[1]**

Q. 26. The Cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its vector form.

[NCERT Ex. 11.2, Q. 7, Page 477]

Ans. Cartesian equation :

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$$

$$\frac{x-5}{3}=\frac{y-(-4)}{7}=\frac{z-6}{2} \quad ...(i)$$

Equation of a line in Cartesian form is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c} \quad ...(ii)$$

Comparing (i) and (ii), we have

$x_1=5, \quad y_1=-4 \quad z_1=6$

And

$a=3, \quad b=7 \quad c=2$ **[1]**

Equation of line in vector form is :

$\vec{r}=\vec{a}+\lambda\vec{b}$

Where

$\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}=5\hat{i}-4\hat{j}+6\hat{k}$ and $b=a\hat{i}+b\hat{j}+c\hat{k}$

$=3\hat{i}+7\hat{j}+2\hat{k}$

Now,

$\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$

Therefore, equation of line in vector form is :

$\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$ **[1]**

Q. 27. Show that the line $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

[NCERT Ex. 11.2, Q. 13, Page 478]

Ans. Two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

and

$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

are perpendicular to each other if

$a_1\,a_2+b_1\,b_2+c_1\,c_2=0$ **[1]**

$$\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$$

$$\frac{x-5}{7}=\frac{y-(-2)}{-5}=\frac{z-0}{1}$$

Comparing with

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1},$$

So,

$x_1=5, \quad y_1=-2 \quad z_1=0$

and $a_1=7, \quad b_1=-5, \quad c_1=1$

Now for :

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$$

$$\frac{x-0}{1}=\frac{y-0}{2}=\frac{z-0}{3}$$

Comparing with

$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2},$$

So,

$x_2=0 \quad y_2=0, \quad z_2=0,$

and $a_2=1, \quad b_2=2, \quad c_2=3$

So, $a_1\,a_2+b_1\,b_2+c_1\,c_2=(7\times1)+(-5\times2)+(1\times3)$

$=7+(-10)+3=0$

Therefore, the two given lines are perpendicular to each other. **[1]**

Q. 28. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

(a) $z=2$ **(b) $x+y+z=1$**

(c) $2x+3y-z=5$ **(d) $5y+8=0$**

[NCERT Ex. 11.3, Q. 1, Page 493]

Ans. (a) For plane

$ax+by+cz=d$

Direction ratios of normal = a, b, c

Direction cosines :

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Distance from the origin $=\frac{d}{\sqrt{a^2+b^2+c^2}}$

Given equation of plane is

$z=2$

$0x+0y+1z=2$

Comparing with $ax+by+cz=d$

$a=0,\ b=0,\ c=1$ and $d=2$

and $\sqrt{a^2+b^2+c^2}=\sqrt{0^2+0^2+1^2}=1$

Direction cosines of the normal to the plane are

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$l=\frac{0}{1},\ m=\frac{0}{1},\ n=\frac{1}{1}$

$l=0,\ m=0,\ n=1$

∴ Direction cosines of the normal to the plane are = (0, 0, 1)

And,

Distance from the origin $=\frac{d}{\sqrt{a^2+b^2+c^2}}=\frac{2}{1}=2.$ **[2]**

(b) For plane

$ax+by+cz=d$

Direction ratios of normal = a, b, c

Direction cosines :

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Distance from the origin $=\frac{d}{\sqrt{a^2+b^2+c^2}}$

Given equation of plane is :

$x+y+z=1$

$1x+1y+1z=1$

Comparing with ax + by + cz = d

$a=1,\ b=1,\ c=1$ and $d=1$

and $\sqrt{a^2+b^2+c^2}=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

Direction cosines of the normal to the plane are

$$l=\frac{a}{\sqrt{a^2+b^2c^2}},\ m=\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$l=\frac{1}{\sqrt{3}},\ m=\frac{1}{\sqrt{3}},\ n=\frac{1}{\sqrt{3}}$

∴ Direction cosines of the normal to the plane are $=\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

And,

Distance from the origin $=\frac{d}{\sqrt{a^2+b^2+c^2}}=\frac{1}{\sqrt{3}}$ **[2]**

(c) For plane

$ax + by + cz = d$

Direction ratios of normal = a, b, c

Direction cosines :

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Distance from the origin $= \dfrac{d}{\sqrt{a^2+b^2+c^2}}$

Given equation of plane is

$2x + 3y - z = 5$

$2x + 3y - 1z = 5$

Comparing with $ax + by + cz = d$

$a = 2,\ b = 3,\ c = -1$ and $d = 5$

and $\sqrt{a^2+b^2+c^2} = \sqrt{2^2+3^2+(-1)^2}$

$= \sqrt{4+9+1}$

$= \sqrt{14}$

Direction cosines of the normal to the plane are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{2}{\sqrt{14}},\ \text{m} = \frac{3}{\sqrt{14}},\ \text{n} = \frac{-1}{\sqrt{14}}$$

$\therefore$ Direction cosines of the normal to the plane are

$$= \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$$

And,

Distance from the origin $= \dfrac{d}{\sqrt{a^2+b^2+c^2}} = \dfrac{5}{\sqrt{14}}$ **[2]**

(d) For plane

$ax + by + cz = d$

Direction ratios of normal = a, b, c

Direction cosines :

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Distance from the origin $= \dfrac{d}{\sqrt{a^2+b^2+c^2}}$

Given equation of plane is

$5\ \ + 8 = 0$

$5\ \ = -8$

$-5\ \ = 8$

$0x - 5y + 0z = 8$

Comparing with $ax + by + cz = d$

$a = 0,\ b = -5,\ c = 0$ and $d = 8$

and $\sqrt{a^2+b^2+c^2} = \sqrt{0^2+(-5)^2+0^2} = \sqrt{25} = 5$

Direction cosines of the normal to the plane are :

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{0}{5},\ m = \frac{-5}{5},\ n = \frac{0}{5}$$

$\therefore$ Direction cosines of the normal to the plane are = (0, –1, 0)

And,

Distance from the origin $= \dfrac{d}{\sqrt{a^2+b^2+c^2}} = \dfrac{8}{5}$ **[2]**

Q. 29. Find the Cartesian equation of the following planes :

(a) $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$ **(b)** $\vec{r}.(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

(c) $\vec{r}.[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$

[NCERT Ex. 11.3, Q. 3, Page 493]

Ans. (a) Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation

$\vec{r}\cdot(\hat{i} + \hat{j} - \hat{k}) = 2$

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(\hat{i} + \hat{j} - \hat{k}) = 2$

$(x \times 1) + (y \times 1) + (z \times -1) = 2$

$x + y - z = 2$

is the Cartesian equation of the given plane. **[2]**

(b) Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation

$\vec{r}\cdot(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

$(x \times 2) + (y \times 3) + (z \times -4) = 1$

$2x + 3y - 4z = 1$

This is the Cartesian equation of the plane. **[2]**

(c) Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation

$\vec{r}\cdot[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$

$x(s - 2t) + y(3-t) + z(2s+t) = 15$

$(s-2t)x + (3-t)y + (2s+t)z = 15$

This is the equation of the plane in Cartesian form. **[2]**

Q. 30. Find the intercepts cut-off by the plane $2x + y - z = 5$. **[NCERT Ex. 11.3, Q. 7, Page 493]**

Ans. The equation of a plane with intercepts a, b, c on x, y, and z-axis respectively is :

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{...(i)}$$

Given, equation of plane is :

$2x + y - z = 5$

Dividing by 5

$$\frac{2x + y - z}{5} = \frac{5}{5}$$

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{5} + \frac{z}{(-5)} = 1$$

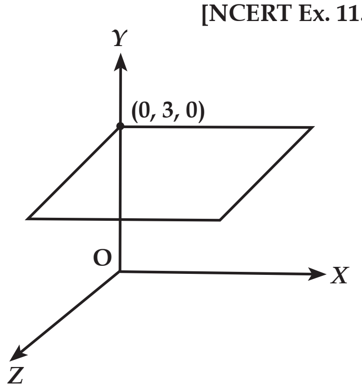
Comparing above equation with (i)

$a = \dfrac{5}{2},\ b = 5,\ c = -5$ **[2]**

Q. 31. Find the equation of the plane with intercept 3 on the *y*-axis and parallel to *ZOX* plane.

[NCERT Ex. 11.3, Q. 8, Page 493]

Ans.



The equation of a plane with intercepts a, b, c on x, y and z-axis respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Given that,

The plane is parallel to ZOX plane as shown

$\therefore$ Intercept on x-axis = 0

So, $a = 0$ and intercept on z-axis = 0

So, $c = 0$

Given that,

Intercept on y-axis = 3

So, $b = 3$

Equation of a plane,

$$\frac{x}{0}+\frac{y}{3}+\frac{z}{0}=1$$

$$0+\frac{y}{3}+0=1$$

$$\frac{y}{3}=1$$

$$y=3$$ **[2]**

Q. 32. In the following cases, find the distance of each of the given points from the corresponding given plane.

Point	**Plane**
(a) (0, 0, 0)	**$3x - 4y + 12z = 3$**
(b) (3, –2, 1)	**$2x - y + 2z + 3 = 0$**
(c) (2, 3, –5)	**$x + 2y - 2z = 9$**
(d) (–6, 0, 0)	**$2x - 3y + 6z - 2 = 0$**

[NCERT Ex. 11.3, Q. 14, Page 494]

Ans. **(a)** The distance of the point (x_1, y_1, z_1) from the plane $Ax + By + Cz = D$ is :

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

Given that, the point is (0, 0, 0).

So, $x_1=0,\ y_1=0,\ z_1=0$

And the equation of plane is :

$$3x-4y+12z=3$$

Comparing with $Ax + By + Cz = D$,

$A=3,\quad B=-4,\quad C=12,\quad D=3$

Now,

Distance of point from the plane is

$$=\left|\frac{(3\times 0)+(-4\times 0)+(12\times 0)-3}{\sqrt{3^2+(-4)^2+12^2}}\right|$$

$$=\left|\frac{0+0+0-3}{\sqrt{9+16+144}}\right|$$

$$=\left|\frac{3}{\sqrt{169}}\right|$$

$$=\frac{3}{13}$$ **[2]**

(b) The distance of the point (x_1, y_1, z_1) from the plane $Ax + By + Cz = D$ is :

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

Given that, the point is (3, –2, 1).

So, $x_1=3,\ y_1=-2,\ z_1=1$

And the equation of plane is

$$2x-y+2z+3=0$$

$$2x-y+2z=-3$$

$$-(2x-y+2z)=3$$

$$-2x+y-2z=3$$

Comparing with $Ax + By + Cz = D$,

$A=-2,\quad B=1,\quad C=-2,\quad D=3$

Now,

Distance of point from the plane is

$$=\left|\frac{(-2\times 3)+(1\times -2)+(-2\times 1)-3}{\sqrt{(-2)^2+1^2+(2)^2}}\right|$$

$$=\left|\frac{(-6)+(-2)+(-2)-3}{\sqrt{4+1+4}}\right|$$

$$=\left|\frac{-13}{\sqrt{9}}\right|$$

$$=\frac{13}{3}$$ **[2]**

(c) The distance of the point (x_1, y_1, z_1) from the plane $Ax + By + Cz = D$ is :

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

Given that, the point is (2, 3, –5).

So, $x_1=2,\ y_1=3,\ z_1=-5$

And the equation of plane is :

$$1x+2y-2z=9$$

Comparing with $Ax + By + Cz = D$,

$A=1,\quad B=2,\quad C=-2,\quad D=9$

Now,

Distance of point from the plane is

$$=\left|\frac{(1\times 2)+(2\times 3)+(-2\times -5)-9}{\sqrt{1^2+2^2+(-2)^2}}\right|$$

$$=\left|\frac{2+6+10-9}{\sqrt{1+4+4}}\right|$$

$$=\left|\frac{18-9}{\sqrt{9}}\right|=\frac{9}{3}=3$$ **[2]**

(d) The distance of the point (x_1, y_1, z_1) from the plane $Ax + By + Cz = D$ is :

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

Given that, the point is (–6, 0, 0).

So, $x_1=-6,\ y_1=0,\ z_1=0$

And the equation of plane is :

$$2x-3y+6z-2=0$$

$$2x-3y+6z=2$$

Comparing with $Ax + By + Cz = D$,

$A=2,\quad B=-3,\quad C=6,\quad D=3$

Now,

Distance of point from the plane

$$=\left|\frac{(2\times -6)+(-3\times 0)+(6\times 0)-2}{\sqrt{2^2+(-3)^2+6^2}}\right|$$

$$=\left|\frac{-12+0+0-2}{\sqrt{4+9+36}}\right|$$

$$=\left|\frac{-14}{\sqrt{49}}\right|=\left|\frac{-14}{7}\right|=|-2|=2$$ **[2]**

Q. 33. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, –1), (4, 3, –1).

[NCERT Misc. Ex. Q. 1, Page 497]

Ans. Two line having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular to each other if

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

Also, a line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) has the direction ratios

$(x_2 - x_1),\ (y_2 - y_1),\ (z_2 - z_1)$

For *AB* :

A (0, 0, 0)

B (2, 1, 1)

Direction ratios

= (2 – 0), (1 – 0), (1 – 0)

= 2, 1, 1

$\therefore a_1 = 2,\ b_1 = 1,\ c_1 = 1$

Now for *CD* :

C (3, 5, –1)

D (4, 3, –1)

Direction ratios

= (4 – 3), (3 – 5), (–1 + 1)

= 1, –2, 0

$\therefore a_2 = 1,\ b_2 = -2$ and $c_2 = 0$

Now,

$$\begin{aligned} a_1 a_2 + b_1 b_2 + c_1 c_2 &= (2\times 1) + (1\times -2) + (1\times 0) \\ &= 2 + (-2) + 0 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

Therefore, the given two lines are perpendicular **[2]**

Q. 34. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1 n_2 - m_2 n_1,\ n_1 l_2 - n_2 l_1,\ l_1 m_2 - l_2 m_1$

[NCERT Misc. Ex. Q. 2, Page 497]

Ans. We know that,

$\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

So, required line is the cross-product of lines having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2

$$\text{Required line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$= \hat{i}(m_1 n_2 - m_2 n_1) - \hat{j}(l_1 n_2 - l_2 n_1) + \hat{k}(l_1 m_2 - l_2 m_1)$

$= (m_1 n_2 - m_2 n_1)\hat{i} - (l_1 n_2 - l_2 n_1)\hat{j} + (l_1 m_2 - l_2 m_1)\hat{k}$

So that, direction cosines = $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1,\ l_1m_2 - l_2m_1$

$\therefore$ Direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1,\ l_1m_2 - l_2m_1$.

Thus proved **[2]**

Q. 35. Find the angle between the lines whose direction ratios are *a, b, c* and *b – c, c – a, a – b*.

[NCERT Misc. Ex. Q. 3, Page 498]

Ans. Angle between the lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Given that, $a_1 = a,\quad b_1 = b,\quad c_1 = c$

and $a_2 = b - c,\quad b_2 = c - a,\quad c_2 = a - b$

$$\text{So, } \cos\theta = \left|\frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}\right|$$

$$= \left|\frac{ab - ac + bc - ab + ca - bc}{\sqrt{a^2+b^2+c^2}\sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ca + a^2 + b^2 - 2ab}}\right|$$

$= 0$

$\therefore \cos\theta = 0$

So, $\theta = 90°$

Therefore, angle between the given pair of lines is 90°. **[2]**

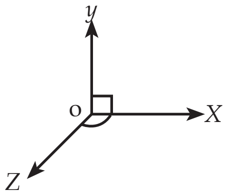
Q. 36. Find the equation of a line parallel to x-axis and passing through the origin.

[NCERT Misc. Ex. Q. 4, Page 498]

Ans. Direction cosines of a line making angle α with *x*-axis, β with *y*-axis and γ with *z*-axis are *l*, *m* and *n*.

x-axis makes an angle 0° with *x*-axis,

90° with *y*-axis and 90° with *z*-axis,



So, $\alpha = 0°,\quad \beta = 90°,\quad \gamma = 90°$

Direction cosines are :

$l = \cos 0°,\quad m = \cos 90°\quad n = \cos 90°$

$l = 1,\qquad m = 0,\qquad n = 0$

$\therefore$ Direction cosines of *x*-axis are 1, 0, 0. **[1]**

Equation of line passing through points (x_1, y_1, z_1) and parallel to a line with direction ratios *a*, *b*, *c* is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Since line passes through origin, *i.e.*, (0, 0, 0),

$x_1 = 1,\quad y_1 = 0,\quad z_1 = 0$

Since line is parallel to x-axis,

$a = 1,\quad b = 0,\quad c = 0$

Equation of line : $\dfrac{x-0}{1} = \dfrac{y-0}{0} = \dfrac{z-0}{0}$

$\dfrac{x}{1} = \dfrac{y}{0} = \dfrac{z}{0}$ **[1]**

Q. 37. If the lines $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-5}$ are perpendicular, find the value of k. **[NCERT Misc. Ex. Q. 6, Page 498]**

Ans. Two lines $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and

$\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$ are perpendicular to each other if

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

Given that,

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$

Comparing with

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$x_1 = 1, \quad y_1 = 2, \quad z_1 = 3$

and $a_1 = -3, \quad b_1 = 2k, \quad c_1 = 2$

Now for :

$$\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$$

Comparing with

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

$x_2 = 1, \quad y_2 = 1, \quad z_2 = 6$

and $a_2 = 3k, \quad b_1 = 1, \quad c_1 = -2,$ **[1]**

Since the two lines are perpendicular,

$$a_1\,a_2 + b_1\,b_2 + c_1\,c_2 = 0$$

$$(-3 \times 3k) + (2k \times 1) + (2 \times -5) = 0$$

$$-9k + 2k - 10 = 0$$

$$-7k = 10$$

$$k = \frac{-10}{7}$$

Therefore, $k = \dfrac{-10}{7}$ **[1]**

Q. 38. Find the equation of the plane passing through (a, b, c) and parallel to the plane $r.(\hat{i} + \hat{j} + \hat{k}) = 2$.

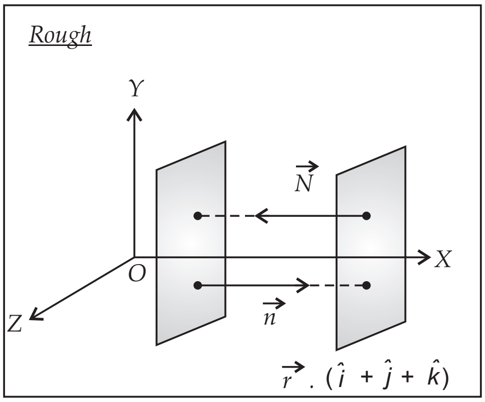
[NCERT Misc. Ex. Q. 8, Page 498]

Ans. The equation of plane passing through (x_1, y_1, z_1) and perpendicular to a line with direction ratio A, B and C is

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

The plane passes through (a, b, c)

So, $x_1 = a, \quad y_1 = b, \quad z_1 = c$



Since both planes are parallel to each other, their normal will be parallel.

$\therefore$ Direction ratios of normal = Direction ratios of normal of $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

Direction ratios of normal = 1, 1, 1

$\therefore A = 1, B = 1, C = 1$

Thus,

Equation of plane in Cartesian form is :

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$1(x - a) + 1(y - b) + 1(z - c) = 0$$

$$x - a + y - b + z - c = 0$$

$$x + y + z - (a + b + c) = 0$$

$$x + y + z = a + b + c$$ **[2]**

Short Answer Type Questions

(3 and 4 marks each)

Q. 1. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

(a) Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

(b) If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can makes $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

[CBSE Board, Delhi Region, 2017]

Ans. $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3$

(a) $c_1 = 1, c_2 = 2$

$[\vec{a}\ \vec{b}\ \vec{c}] = 2 - c_3$

$\because\ \vec{a}, \vec{b}, \vec{c}$ are coplanar $[\vec{a}, \vec{b}, \vec{c}] = 0 \Rightarrow c_3 = 2$

(b) $c_2 = -1, c_3 = 1$

$[\vec{a}\ \vec{b}\ \vec{c}] = c_2 - c_3 = -2 \neq 0$

$\Rightarrow$ No value of c_1 can make $\vec{a}, \vec{b}, \vec{c}$ coplanar. **[4]**

Q. 2. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

[CBSE Board, All India Region, 2017]

Ans. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

$\therefore A$, B and C form a triangle.

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$

$\therefore A$, B and C form a right triangle.

Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ **[4]**

Q. 3. Find the value of λ, if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are coplanar.

[CBSE Board, All India Region, 2017]

Ans. Given points, A, B, C and D are coplanar, if the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, *i.e.*

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}$, $\overrightarrow{AC} = -\hat{i} - 3\hat{j} + 8\hat{k}$, $\overrightarrow{AD} = \hat{i} + (\lambda + 9)\hat{k}$ are coplanar.

i.e.,

$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda - 9 \end{vmatrix} = 0$$

$$-2[-3\lambda + 27] + 4[-\lambda + 17] - 6(3) = 0$$

$\Rightarrow\ \lambda = 2.$ **[4]**

Q. 4. Find the shortest distance between lines

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$$

and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

[NCERT Misc. Ex. Q. 9, Page 498]

Ans. Shortest distance between lines with vector equations

$\vec{r}=\vec{a_1}+\lambda\ \vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\ \vec{b_2}$ is

$$\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

For : $\vec{r}=(6\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k})$

Comparing with $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$,

$\vec{a_1}=6\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b_1}=1\hat{i}-2\hat{j}+2\hat{k}$

Now For :

$\vec{r}=(-4\hat{i}-\hat{k})+\mu(3\hat{i}-2\hat{j}-2\hat{k})$

Comparing with $\vec{r}=\vec{a_2}+\mu\vec{b_2}$,

$\vec{a_2}=4\hat{i}+0\hat{j}+1\hat{k}$ and $\vec{b_2}=3\hat{i}-2\hat{j}-2\hat{k}$

Now, $(\vec{a_2}-\vec{a_1})=(-4\hat{i}+0\hat{j}-1\hat{k})-(6\hat{i}+2\hat{j}+2\hat{k})$

$=(-4-6)\hat{i}+(0-2)\hat{j}+(-1-2)\hat{k}$

$=-10\hat{i}-2\hat{j}-3\hat{k}$

$$(\vec{b_1}\times\vec{b_2})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}$$

$=\hat{i}[(-2\times-2)-(-2\times2)]-\hat{j}[(1\times-2)-(3\times2)]$
$+\hat{k}[(1\times-2)-(3\times-2)]$

$=\hat{i}[4+4]-\hat{j}[-2-6]+\hat{k}[-2+6]$

$=\hat{i}(8)-\hat{j}(-8)+\hat{k}(4)$

$=8\hat{i}+8\hat{j}+4\hat{k}$

Magnitude of $\vec{b_1}\times\vec{b_2}=\sqrt{8^2+8^2+4^2}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{64+64+16}=\sqrt{144}=12$ **[2]**

Also,

$(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})=(8\hat{i}+8\hat{j}+4\hat{k})\cdot(-10\hat{i}-2\hat{j}-3\hat{k})$

$=(8\times-10)+(8\times-2)+4(4\times-3)$

$=80+(-16)+(-12)$

$=-108$

$$\text{Shortest distance}=\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}\times\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$=\left|\frac{-108}{12}\right|=|-9|=9$ **[1]**

Q. 5. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the *YZ*-plane.
[NCERT Misc. Ex. Q. 10, Page 498]

Ans. The equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is

$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

Given, the line passes through

For Points *A* and *B* :

A (5, 1, 6)

$\vec{a}=5\hat{i}+1\hat{j}+6\hat{k}$

B (3, 4, 1)

$\vec{b}=3\hat{i}+4\hat{j}+1\hat{k}$

$(\vec{b}-\vec{a})=(3\hat{i}+4\hat{j}+1\hat{k})-(5\hat{i}+1\hat{j}+6\hat{k})$

$=(3-5)\hat{i}+(4-1)\hat{j}+(1-6)\hat{k}$

$=-2\hat{i}+3\hat{j}-5\hat{k}$

$\therefore \vec{r}=(5\hat{i}+\hat{j}+6\hat{k})+\lambda(-2\hat{i}+3\hat{j}-5\hat{k})$ ...(i)

Let the coordinates of the point where the line crosses the *YZ* plane be (0, *y*, *z*)

So, $\vec{r}=0\hat{i}+y\hat{j}+z\hat{k}$ ...(ii)

Since point lies in the line, it will satisfy the equation,

Putting value of Eq. (ii) in Eq. (i), we have

$0\hat{i}+y\hat{j}+z\hat{k}=5\hat{i}+\hat{j}+6\hat{k}-2\lambda\hat{i}+3\lambda\hat{j}-5\lambda\hat{k}$

$0\hat{i}+y\hat{j}+z\hat{k}=(5-2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(6-5\lambda)\hat{k}$

Two vectors are equal if their corresponding components are equal.

So,

$0=5-2\lambda;\quad y=1+3\lambda;\quad z=6-5\lambda$ **[2]**

Solving

$0=5-2\lambda$

$0=2\lambda$

$\therefore \lambda=\frac{5}{2}$

Now, $y=1+3\lambda=1+3\times\frac{5}{2}=1+\frac{15}{2}=\frac{17}{2}$

and $z=6-5\lambda=6-5\times\frac{5}{2}=6-\frac{25}{2}=\frac{-13}{2}$

Therefore, the coordinates of the required point is $\left(0,\frac{17}{2},\frac{-13}{2}\right)$. **[1]**

Q. 6. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the *ZX*-plane.
[NCERT Misc. Ex. Q. 11, Page 498]

Ans. The equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is :

$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

Given that, the line passes through

A (5, 1, 6)

$\vec{a}=5\hat{i}+1\hat{j}+6\hat{k}$

B (3, 4, 1)

$\vec{b}=3\hat{i}+4\hat{j}+1\hat{k}$

$(\vec{b}-\vec{a})=(3\hat{i}+4\hat{j}+1\hat{k})-(5\hat{i}+1\hat{j}+6\hat{k})$

$=(3-5)\hat{i}+(4-1)\hat{j}+(1-6)\hat{k}$

$=-2\hat{i}+3\hat{j}-5\hat{k}$

$\therefore \vec{r}=(5\hat{i}+\hat{j}+6\hat{k})+\lambda(-2\hat{i}+3\hat{j}-5\hat{k})$ **...(i)**

Let the coordinates of the point where the line crosses the *ZX* plane be (*x*, 0, *z*)

So, $\vec{r}=x\hat{i}+0\hat{j}+z\hat{k}$ **...(ii)**

Since point lies in the line, it will satisfy its equation,

Putting value of Eq. (ii) in Eq. (i)

$x\hat{i}+0\hat{j}+z\hat{k}=5\hat{i}+\hat{j}+6\hat{k}-2\lambda\hat{i}+3\lambda\hat{j}-5\lambda\hat{k}$

$x\hat{i}+0\hat{j}+z\hat{k}=(5-2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(6-5\lambda)\hat{k}$

Two vectors are equal if their corresponding components are equal.

So,

$x=5-2\lambda;\quad 0=1+3\lambda;\quad z=6-5\lambda$ **[2]**

Solving

$$0 = 1 + 3\lambda$$
$$3\lambda = -1$$
$$\therefore \lambda = \frac{-1}{3}$$

Now, $x = 5 - 2\lambda = 5 - 2 \times \frac{-1}{3} = 5 + \frac{2}{3} = \frac{17}{13}$

$z = 6 - 5\lambda = 6 - 5 \times \frac{-1}{3} = 6 + \frac{5}{3} = \frac{23}{3}$ **[1]**

Therefore, the coordinates of the required point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$. **[1]**

Q. 7. Find the coordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane 2*x* + *y* + *z* = 7. [NCERT Misc. Ex. Q. 12, Page 498]

Ans. The equation of a line passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Given that, the line passes through the points

$A(3, -4, -5)$

$\therefore x_1 = 3, y_1 = -4, z_1 = -5$

$B(2, -3, 1)$

$\therefore x_2 = 2, y_2 = -3, z_2 = 1$

So, the equation of line is

$$\frac{x-3}{2-3} = \frac{y-(-4)}{-3-(-4)} = \frac{z-(-5)}{1-(-5)}$$

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k$$

So,

$x = -k + 3; \quad y = k - 4; \quad z = 6k - 5$ ...(i)

Let (x, y, z) be the coordinates of the point where the line crosses the plane $2x + y + z = 7$

Putting value of x, y, z from Eq. (i) in the equation of plane,

$$2x + y + z = 7$$
$$2(-k+3) + (k-4) + (6k-5) = 7$$
$$-2k + 6 + k - 4 + 6k - 5 = 7$$ **[2]**
$$5 \quad -3 = 7$$
$$5 \quad = 7 + 3$$
$$5 \quad 10$$
$$\therefore \quad k = \frac{10}{5} = 2$$

Putting value of k in x, y, z

$x = -k + 3 = -2 + 3 = 1$

$y = k - 4 = 2 - 4 = -2$

$z = 6k - 5 = 6 \times 2 - 5 = 12 - 5 = 7$

Therefore, the coordinate of the required point are (1, –2, 7). **[1]**

Q. 8. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes *x* + 2*y* + 3*z* = 5 and 3*x* + 3*y* + *z* = 0. [NCERT Misc. Ex. Q. 13, Page 498]

Ans. The equation of a plane passing through the point (x_1, y_1, z_1) is given by

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

where *A*, *B* and *C* are the direction ratios of normal to the plane.

Now the plane passes through the point (–1, 3, 2)

So, equation of plane is :

$A(x+1) + B(y-3) + C(z-2) = 0$ **...(i)**

We find the direction ratios of normal to plane, *i.e. A*, *B* and *C*.

Also, the plane is perpendicular to the given two planes.

So, their normal to plane would be perpendicular to normal of both planes.

We know that,

$\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

So, required normal is the cross-product of normal of planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

$$\text{Required normal} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}[2(1) - 3(3)] - \hat{j}[1(1) - 3(3)] + \hat{}[1(3) - 3(2)]$$
$$= \hat{i}(2-9) - \hat{j}(1-9) + \hat{k}(3-6)$$
$$= -7\hat{i} + 8\hat{j} - 3\hat{k}$$

Hence, direction ratios = –7, 8, –3

$\therefore A = -7, B = 8, C = -3$ **[2]**

Putting above values in Eq. (i)

$$A(x+1) + B(y-3) + C(z-2) = 0$$
$$-7k(x+1) + 8k(y-3) - 3k(z-2) = 0$$
$$k[-7(x+1) + 8(y-3) - 3(z-2)] = 0$$
$$-7(x+1) + 8(y-3) - 3(z-2) = 0$$
$$-7x + 8y - 3z - 25 = 0$$
$$0 = 7x - 8y + 3z + 25$$
$$7x - 8y + 3z + 25 = 0$$

Therefore, equation of the required plane is $7x - 8y + 3z + 25 = 0$ **[1]**

Q. 9. If the points (1, 1, *p*) and (–3, 0, 1) be equidistant from the $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, then find the value of *p*. [NCERT Misc. Ex. Q. 14, Page 498]

Ans. The distance of a point with position vector $\vec{a}$ from the plane

$\vec{r} \cdot \vec{n} = d$ is $\left|\frac{\vec{a} \cdot \vec{n} - d}{|\vec{n}|}\right|$

Given, the points are

(i) (1, 1, *p*)

So, $\vec{a_1} = 1\hat{i} + 1\hat{j} + p\hat{k}$

(ii) (–3, 0, 1)

So, $\vec{a_2} = -3\hat{i} + 0\hat{j} + 1\hat{k}$

The equation of plane is :

$$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$$
$$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) = -13$$
$$-\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) = 13$$
$$\vec{r} \cdot (-3\hat{i} - 4\hat{j} + 12\hat{k}) = 13$$

Comparing with $\vec{r} \cdot \vec{n} = d$,

$\vec{n} = -3\hat{i} - 4\hat{j} + 12\hat{k}$ and, $d = 13$

Magnitude of $\vec{n} = \sqrt{(-3)^2 + (-4)^2 + 12^2}$

$$|\vec{n}| = \sqrt{9+16+144}$$
$$= \sqrt{169} = 13$$

(i) Distance of point $\vec{a_1}$ from plane,

$$\left|\frac{\vec{a_1}\cdot\vec{n}-d}{|\vec{n}|}\right| = \left|\frac{(1\hat{i}+1\hat{j}+p\hat{k})\cdot(-3\hat{i}-4\hat{j}+12\hat{k})-13}{13}\right|$$
$$= \left|\frac{(1\times-3)+(1\times-4)+(p\times12)-13}{13}\right|$$
$$= \left|\frac{-3-4+12p-13}{13}\right|$$
$$= \left|\frac{12p-20}{13}\right|$$

(ii) Distance of point $\vec{a_2}$ from plane,

$$\left|\frac{\vec{a_2}\cdot\vec{n}-d}{|\vec{n}|}\right| = \left|\frac{(-3\hat{i}+0\hat{j}+1\hat{k})\cdot(-3\hat{i}-4\hat{j}+12\hat{k})-13}{13}\right|$$
$$= \left|\frac{(-3\times-3)+(0\times-4)+(1\times12)-13}{13}\right|$$
$$= \left|\frac{9+0+12-13}{13}\right|$$
$$= \left|\frac{18}{13}\right| = \frac{8}{13}$$

Since the plane is equi-distance from both the points,

$$\left|\frac{12p-20}{13}\right| = \frac{8}{13}$$
$$|12p-20| = 8$$
$$(12p-20) = \pm 8$$ **[2]**

Solve for both condition one by one :

$$12p-20 = 8$$
$$12p = 8+20$$
$$12p = 28$$
$$p = \frac{28}{12} = \frac{7}{3}$$

Now for,

$$12p-20 = -8$$
$$12p = -8+20$$
$$12p = 12$$
$$p = \frac{12}{12} = 1$$

So,

$p = \frac{7}{3}$ and $p = 1$ **[1]**

Q. 10. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$.

[NCERT Misc. Ex. Q. 7, Page 498]

Ans. The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Given that, the line passes through the point (1, 2, 3).

So, $\vec{a} = 1\hat{i}+2\hat{j}+3\hat{k}$

Finding Normal of Plane :

$$\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$$
$$\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k}) = -9$$
$$-\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k}) = 9$$
$$\vec{r}\cdot(-1\hat{i}-2\hat{j}+5\hat{k}) = 9$$ **[1]**

Comparing with $\vec{r}\cdot\vec{n} = d$,

$$\vec{n} = -\hat{i}-2\hat{j}+5\hat{k}$$

Since line is perpendicular to plane, the line will be parallel to the normal of the plane.

$$\therefore \vec{b} = \vec{n} = -1\hat{i}-2\hat{j}+5\hat{k}$$

Hence,

$$\vec{r} = (1\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-1\hat{i}-2\hat{j}+5\hat{k})$$
$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) - \lambda(\hat{i}+2\hat{j}-5\hat{k})$$

$\therefore$ Vector equation of line is

$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) - \lambda(\hat{i}+2\hat{j}-5\hat{k})$. **[2]**

Q. 11. If the coordinates of the points *A*, *B*, *C* and *D* be (1, 2, 3), (4, 5, 7), (– 4, 3, –6) and (2, 9, 2), respectively, then find the angle between the lines *AB* and *CD*.

[NCERT Misc. Ex. Q. 5, Page 498]

Ans. Angle between a pair of lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

A line passing through $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ has direction ratios $(x_2-x_1), (y_2-y_1), (z_2-z_1)$

For *AB* :

A (1, 2, 3) and *B* (4, 5, 7)

Direction ratios of *AB* :

(4 – 1), (5 – 2) and (7 – 3)

= 3, 3, 4

$\therefore a_1 = 3, b_1 = 3, c_1 = 4$

For *CD* :

C (–4, 3, –6) and *D* (2, 9, 2)

Direction ratios of *CD* :

[2 – (–4)], (9 – 3) and [2 – (–3)]

= 6, 6, 8

$\therefore a_2 = 6, b_2 = 6, c_2 = 8$

Now,

$$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$
$$\cos\theta = \left|\frac{3\times6+3\times6+4\times8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right|$$
$$= \left|\frac{18+18+32}{\sqrt{9+9+16}\sqrt{36+36+64}}\right|$$
$$= \left|\frac{68}{\sqrt{34}\sqrt{136}}\right|$$
$$= \left|\frac{68}{\sqrt{34}\sqrt{4\times34}}\right|$$
$$= \left|\frac{68}{\sqrt{34}\sqrt{4}\times\sqrt{34}}\right|$$
$$= \left|\frac{68}{\sqrt{34}\sqrt{34}\times\sqrt{4}}\right|$$
$$= \left|\frac{68}{34\times2}\right|$$
$$= \left|\frac{68}{68}\right| = 1$$ **[2]**

$\therefore \cos\theta = 1$

So, $\theta = 0°$

Therefore, angle between AB and CD is 0°. **[1]**

Q. 12. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1). **[NCERT Ex. 11.3, Q. 9, Page 493]**

Ans. Equation of a plane passing through the intersection of planes

$A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$

And through the point (x_1, y_1, z_1) is

$(A_1x + B_1y + C_1z = d_1) + \lambda(A_2x + B_2y + C_2z = d_2) = 0$

Given that, plane passes through

For :

$3x - y + 2z - 4 = 0$

i.e. $3x + (-1)y + 2z = 4$

Comparing with

$A_1x + B_1y + C_1z = d_1$

$A_1 = 3, \quad B_1 = -1, \quad C_1 = 2, \quad d_1 = 4$

For :

$x + y + z - 2 = 0$

i.e. $1x + 1y + 1z = 2$

Comparing with

$A_2x + B_2y + C_2z = d_2,$

$A_2 = 1, \quad B_2 = 1, \quad C_2 = 1, \quad d_2 = 2$

Equation of plane is :

$(3x - 1y + 2z - 4) + \lambda(1x + 1y + 1z - 2) = 0$

$3x - y + 2z - 4 + \lambda x + \lambda y + \lambda z - 2\lambda = 0$

$(3+\lambda)x + (-1+\lambda)y + (2+\lambda)z + (-4-2\lambda) = 0$ ...(i)

We now find the value of λ

The plane passes through (2, 2, 1)

Putting (2, 2, 1) in (i), we have

$$(3+\lambda)x + (-1+\lambda)y + (2+\lambda)z + (-4-2\lambda) = 0$$

$$(3+\lambda)\times 2 + (-1+\lambda)\times 2 + (2+\lambda)\times 1 + (-4-2\lambda) = 0$$

$$6 + 2\lambda - 2 + 2\lambda + 2 + \lambda - 4 - 2\lambda = 0$$

$$3\lambda + 2 = 0$$

$$3\lambda = -2$$

$$\therefore \quad \lambda = \frac{-2}{3}$$ **[1]**

Putting value of λ in Eq. (1), we have

$(3+\lambda)x + (-1+\lambda)y + (2+\lambda)z + (-4-2\lambda) = 0$

$$\left[3 + \left(\frac{-2}{3}\right)\right]x + \left[-1 + \left(\frac{-2}{3}\right)\right]y + \left[2 + \left(\frac{-2}{3}\right)\right]z + \left[(-4) - 2\times\frac{-2}{3}\right] = 0$$

$$\left(3 - \frac{2}{3}\right)x + \left(-1 - \frac{2}{3}\right)y + \left(2 - \frac{2}{3}\right)z + \left(-4 + \frac{4}{3}\right) = 0$$

$$\frac{7x}{3} - \frac{5y}{3} + \frac{4z}{3} - \frac{8}{3} = 0$$

$$\frac{1}{3}(7x - 5y + 4z - 8) = 0$$

$$7x - 5y + 4z - 8 = 0$$

$\therefore$ The equation of plane is $7x - 5y + 4z - 8 = 0$ **[2]**

Q. 13. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$, $\vec{r}\cdot(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3). **[NCERT Ex. 11.3, Q. 10, Page 493]**

Ans. The vector equation of plane passing through the intersection of planes $\vec{r}\cdot\vec{n_1} = d_1$ and $\vec{r}\cdot\vec{n_2} = d_2$ and also passes through the point (x_1, y_1, z_1) is $\vec{r}\cdot(\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$

Given that, the plane passes through

(i) $\vec{r}\cdot(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$

Comparing with $\vec{r}\cdot\vec{n_1} = d_1$,

$\vec{n_1} = 2\hat{i} + 2\hat{j} - 3\hat{k}$

and $d_1 = 7$

(ii) $\vec{r}\cdot(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$

Comparing with $\vec{r}\cdot\vec{n_2} = d_2$,

$\vec{n_2} = 2\hat{i} + 5\hat{j} + 3\hat{k}$

and $d_2 = 9$

So, equation of plane is :

$$\vec{r}\cdot\left[(2\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k})\right] = 7 + \lambda.9$$

$$\vec{r}\cdot\left[2\hat{i} + 3\hat{j} - 3\hat{k} + 2\lambda\hat{i} + 5\lambda\hat{j} + 3\lambda\hat{k}\right] = 7 + 9\lambda$$

$$\vec{r}\cdot\left[(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}\right] = 9\lambda + 7 \quad ...(i)$$

Now, to find λ, put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(x\hat{i} + y\hat{j} + z\hat{k})\cdot\left[(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}\right] = 9\lambda + 7$$

$$x(2+2\lambda) + y(2+5\lambda)\hat{j} + z(-3+3\lambda)\hat{k} = 9\lambda + 7 \quad ...(ii)$$

The plane passes through (2, 1, 3)

Putting (2, 1, 3) in Eq. (ii),

$$2(2+2\lambda) + 1(2+5\lambda) + 3(-3+3x) = 9\lambda + 7$$

$$4 + 4\lambda + 2 + 5\lambda + (-9) + 9\lambda = 9\lambda + 7$$

$$18\lambda - 9\lambda = 7 + 3$$

$$9\lambda = 10$$

$$\therefore \quad \lambda = \frac{10}{9}$$ **[2]**

Putting value of λ in Eq. (i),

$$\vec{r}\cdot\left[\left(2 + 2\cdot\frac{10}{9}\right)\hat{i} + \left(2 + 5\cdot\frac{10}{9}\right)\hat{j} + \left(-3 + 3\cdot\frac{10}{9}\right)\hat{k}\right] = 9\cdot\frac{10}{9} + 7$$

$$\vec{r}\cdot\left[\left(2 + \frac{20}{9}\right)\hat{i} + \left(2 + \frac{50}{9}\right)\hat{j} + \left(-3 + \frac{30}{9}\right)\hat{k}\right] = 10 + 7$$

$$\vec{r}\cdot\left[\frac{30}{9}\hat{i} + \frac{68}{9}\hat{j} + \frac{3}{9}\hat{k}\right] = 17$$

$$\frac{1}{9}\vec{r}\cdot\left(38\hat{i} + 38\hat{j} + 3\hat{k}\right) = 17$$

$$\vec{r}\cdot(38\hat{i} + 68\hat{j} + 3\hat{k}) = 17\times 9$$

$$\vec{r}\cdot\left(38\hat{i} + 68\hat{j} + 3\hat{k}\right) = 153$$

Therefore, the vector equation of the required plane is $\vec{r}\cdot\left(38\hat{i} + 68\hat{j} + 3\hat{k}\right) = 153$. **[1]**

Q. 14. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

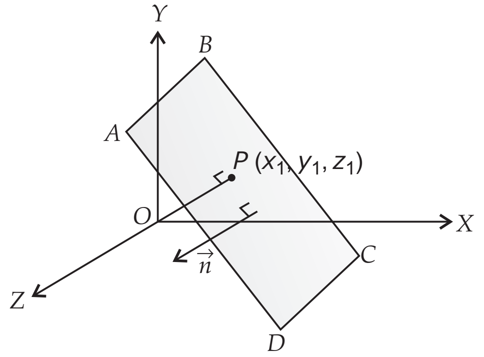
(a) $2x + 3y + 4z - 12 = 0$

(b) $3y + 4z - 6 = 0$

(c) $x + y + z = 1$

(d) $5y + 8 = 0$ **[NCERT Ex. 11.3, Q. 4, Page 493]**

Ans. **(a)** Assume a point $P(x_1, y_1, z_1)$ on the given plane.



Since perpendicular to plane is parallel to normal vector.

Vector $\overrightarrow{OP}$ is parallel to normal vector $\vec{n}$ to the plane.

Given equation of plane is :

$2x+3y+4z-12=0$

$2x+3y+4z=12$

Since, $\overrightarrow{OP}$ and $\vec{n}$ are parallel and their direction ratios are proportional.

Finding direction ratios :

(i) $\overrightarrow{OP}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$

Direction ratios = x_1, y_1, z_1

$\therefore a_1=x_1,\quad b_1=y_1,\quad c_1=z_1$

(ii) $\vec{n}=2\hat{i}+3\hat{j}+4\hat{k}$

Direction ratios = 2, 3, 4

$\therefore a_2=2,\quad b_2=3,\quad c_2=4$

Direction rations are proportional.

So,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=k$$

$$\frac{x_1}{2}=\frac{y_1}{3}=\frac{z_1}{4}=k$$

$x_1=2k,\ y_1=3k,\ z_1=4k$

Also, point P $(x_1, y_1, z_1,)$ lies in the plane.

Putting P $(2k, 3k, 4k)$ in

$$2x+3y+4z=12,$$

$$2(2k)+3(3k)+4(4k)=12$$

$$4k+9k+16k=12$$

$$29k=12$$

$$\therefore \quad k=\frac{12}{29}$$

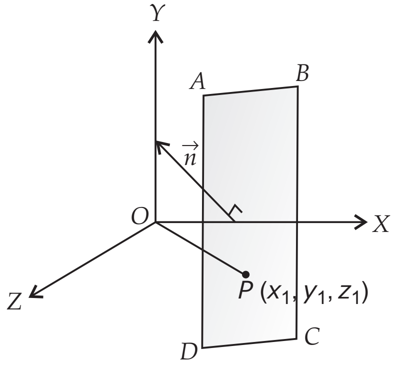
So, $x_1=2k=2\times\frac{12}{29}=\frac{24}{29}$

$y_1=3k=3\times\frac{12}{29}=\frac{36}{29}$

and $z_1=4k=4\times\frac{12}{29}=\frac{48}{29}$

Therefore, coordinate of foot of perpendicular are $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$. **[3]**

(b) Assume a point P (x_1, y_1, z_1) on the given plane.



Since perpendicular to plane is parallel to the normal vector.

Vector $\overrightarrow{OP}$ is parallel to normal vector $\vec{n}$ to the plane.

Given equation of plane is :

$3y+4z-6=0$

$3y+4z=6$

$0x+3y+4z=6$

Since, $\overrightarrow{OP}$ and $\vec{n}$ are parallel and their direction ratios are proportional.

Finding direction ratios :

$\overrightarrow{OP}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$	$\vec{n}=0\hat{i}+3\hat{j}+4\hat{k}$
Direction ratios $=x_1,y_1,z_1$	Direction ratios $=0,3,4$
$\therefore a_1=x_1,\ b_1=y_1,\ c_1=z_1$	$\therefore a_2=0,\ b_2=3,\ c_2=4$

Direction ratios are proportional.

So, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=k$

$$\frac{x_1}{0}=\frac{y_1}{3}=\frac{z_1}{4}=k$$

$x_1=0,\ y_1=3k,\ z_1=4k$

Also, point P (x_1, y_1, z_1) lies in the given plane.

Putting P $(0, 3k, 4k)$ in

$$0x + 3y + 4z = 6,$$

$$0(k) + 3(3k) + 4(4k) = 6$$

$$25k = 6$$

$$k=\frac{6}{25}$$

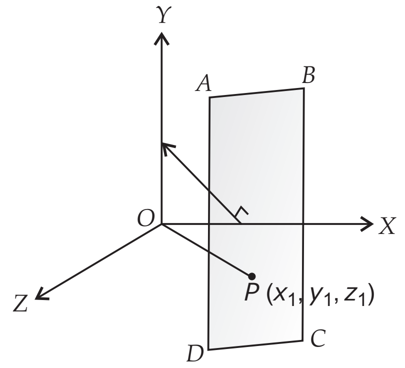
So, $x_1 = 0$

$y_1=3k=3\times\frac{6}{25}=\frac{18}{25}$

$z_1=4k=4\times\frac{6}{25}=\frac{24}{25}$

Therefore, coordinates of foot of perpendicular are $\left(0,\frac{18}{25},\frac{24}{25}\right)$. **[3]**

(c) Assume a point P (x_1, y_1, z_1) on the plane.



Since perpendicular to plane is parallel to the normal vector.

Vector $\overrightarrow{OP}$ is parallel to normal vector $\vec{n}$ to the plane.

Given that, equation of plane is :

$x + y = z = 1$

$1x + 1y + 1z = 1$

Since, $\overrightarrow{OP}$ and $\vec{n}$ are parallel and their direction ratios are proportional.

Finding direction ratios :

$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$	$\vec{n} = 1\hat{i} + 1\hat{j} + 1\hat{k}$
Direction ratios $= x_1, y_1, z_1$	Direction ratios $= 1,1,1$
$\therefore a_1 = x_1,\ b_1 = y_1,\ c_1 = z_1$	$\therefore a_2 = 1,\ b_2 = 1,\ c_2 = 1$

Direction ratios are proportional.

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$

$\frac{x_1}{1} = \frac{y_1}{1} = \frac{z_1}{1} = k$

$x_1 = y_1 = z_1 = k$

Also, point $P(x_1, y_1, z_1)$ lies in the given plane.

Putting $P(k, k, k)$ in

$x + y + z = 1,$

$k + k + k = 1$

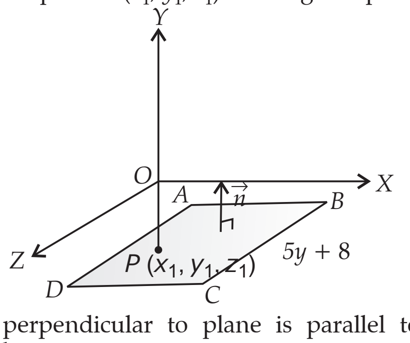
$3k = 1$

$\therefore k = \frac{1}{3}$

So, $x_1 = k = \frac{1}{3},\ y_1 = k = \frac{1}{3},\ z_1 = k = \frac{1}{3}$

Therefore, coordinates of foot of perpendicular are $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. **[3]**

(d) Assume a point $P(x_1, y_1, z_1)$ on the given plane.



Since perpendicular to plane is parallel to the normal vector.

Vector $\overrightarrow{OP}$ is parallel to the normal vector $\vec{n}$ to the plane.

Given that, equation of plane :

$5y + 8 = 0$

$5y = -8$

$-5y = 8$

$0x - 5y + 0z = 8$

Since, $\overrightarrow{OP}$ and $\vec{n}$ are parallel and their direction ratios are proportional.

Finding direction ratios :

$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$	$\vec{n} = 0\hat{i} - 5\hat{j} + 0\hat{k}$
Direction ratios $= x_1, y_1, z_1$	Direction ratios $= 0, -5, 0$
$\therefore a_1 = x_1,\ b_1 = y_1,\ c_1 = z_1$	$\therefore a_2 = 0,\ b_2 = -5,\ c_2 = 0$

Since direction ratios are proportional.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{x_1}{0} = \frac{y_1}{-5} = \frac{z_1}{0} = k$

$x_1 = 0,\ y_1 = -5k,\ z_1 = 0$

Also, point P (x_1, y_1, z_1) lies in the given plane.

Putting (x_1, y_1, z_1) in

$0x - 5y + 0z = 8,$

$0x_1 - 5y_1 + 0z_1 = 8,$

$-5(-5k) = 8$

$25k = 8$

$\therefore \quad k = \frac{8}{25}$

So, $x_1 = 0$

$y_1 = -5k = -5 \times \frac{8}{25} = \frac{-8}{5}$

$z_1 = 0$

$\therefore$ Coordinate of foot of perpendicular $= \left(0, \frac{-8}{5}, 0\right)$. **[3]**

Q. 15. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

[NCERT Misc. Ex. Q. 17, Page 498]

Ans. Equation of a plane passing through the intersection of the places $A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$ is

$(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

Converting equation of planes to Cartesian form to find A_1, B_1, C_1, d_1 and A_2, B_2, C_2, d_2

$\vec{r}\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$	$\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$
$\vec{r}\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) = 4$	$\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) = -5$
	$-\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) = 5$
	$\vec{r}\cdot(-2\hat{i} - \hat{j} + \hat{k}) = 5$
Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,
$\begin{bmatrix}(x\hat{i} + y\hat{j} + z\hat{k}) \\ \cdot(\hat{i} + 2\hat{j} + 3\hat{k})\end{bmatrix} = 4$	$\begin{bmatrix}(x\hat{i} + y\hat{j} + z\hat{k}) \\ \cdot(-2\hat{i} - \hat{j} + \hat{k})\end{bmatrix} = 5$
$\begin{bmatrix}(x \times 1) + (y \times 2) \\ +(z \times 3)\end{bmatrix} = 4$	$(x \times -2) + (y \times -1) = (z \times 1) = 5$
$1x + 2y + 3z = 4$	$-2x - 1y + 1z = 5$

Comparing with	Comparing with
$A_1x+B_1y+C_1z=d_1$	$A_2x+B_2y+C_2z=d_2$
$A_1=1, B_1=2, C_1=3,$	$A_2=-2, B_2=-1, C_2=1,$
$d_1=4$	$d_2=5$

Equation of plane is :

$(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

Putting values, we have

$(1x + 2y + 3z - 4) + \lambda(-2x - 1y + 1z - 5) = 0$

$(1 - 2\lambda)x + (2 - \lambda)y + (3 + \lambda)z + (-4 - 5\lambda) = 0 \quad \ldots(i)$

Now, the plane is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

So, normal to plane $\vec{N}$ will be perpendicular to normal $\vec{n}$ of $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Now,

$\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$

$\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})=-8$

$-\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})=8$

$\vec{r}.(-5\hat{i}-3\hat{j}+6\hat{k})=8$ **[1½]**

As we know that, if two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Finding direction cosines of $\vec{N}$ and $\vec{n}$

$\vec{N}=(1-2\lambda)\hat{i}+(2-\lambda)\hat{j}+(3+\lambda)\hat{k}$	$\vec{n}=-5\hat{i}-3\hat{j}+6\hat{k}$
Direction ratios $=1-2\lambda,$ $2-\lambda, 3+\lambda$	Direction ratios $=-5,-3,6$
$\therefore a_1=1-2\lambda,$	$\therefore a_2=-5,$
$b_1=2-\lambda,$	$b_1=-3,$
$c_1=3+\lambda$	$c_1=6$

Since, $\vec{N}$ is perpendicular to $\vec{n}$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(1 - 2\lambda) \times -5 + (2 - \lambda) \times -3 + (3 + \lambda) \times 6 = 0$

$-5 + 10\lambda - 6 + 3\lambda + 18 + 6\lambda = 0$

$19\lambda + 7 = 0$

$\therefore \quad \lambda = \frac{-7}{19}$

Putting value of λ in Eq. (i), we have

$$(1-2\lambda)x+(2-\lambda)y+(3+\lambda)z+(-4-5\lambda)=0$$

$$\left(1-2\times\frac{-7}{19}\right)x+\left[2-\left(\frac{-7}{19}\right)\right]y+\left[3+\left(\frac{-7}{19}\right)\right]z+\left(-4-5\times\frac{-7}{19}\right)=0$$

$$\left(1+\frac{14}{19}\right)x+\left(1+\frac{7}{19}\right)y+\left(3-\frac{7}{19}\right)z+\left(-4+\frac{35}{19}\right)=0$$

$$\frac{33}{19}x+\frac{45}{19}y+\frac{50}{19}z-\frac{41}{19}=0$$

$$\frac{1}{19}(33x+45y+50z-41)=0$$

$$33x+45y+50z-41=0$$

Therefore, the equation of the plane is $33x + 45y + 50z = 41$. **[1½]**

Q. 16. Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$. [NCERT Misc. Ex. Q. 18, Page 499]

Ans. Given, the equation of line is

$\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$

And the equation of the plane is

$\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$

To find point of intersection of line and plane,

Putting value of $\vec{r}$ from equation of line into equation of plane,

$$\left[(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})\right].(\hat{i}-\hat{j}+\hat{k})=5$$

$$\left[(2\hat{i}-1\hat{j}+2\hat{k}+3\lambda\hat{i}+4\lambda\hat{j}+2\lambda\hat{k})\right].(1\hat{i}-1\hat{j}+1\hat{k})=5$$

$$\left[(2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+2\lambda)\hat{k}\right].(1\hat{i}-1\hat{j}+1\hat{k})=5$$

$$(2+3\lambda)\times1+(-1+4\lambda)\times(-1)+(2+2\lambda)\times1=5$$

[1½]

$2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$

$\lambda + 5 = 5$

$\lambda = 5 - 5$

$\lambda = 0$

So, the equation of line is :

$\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$

$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

Let the point of intersection be (x, y, z).

So, $\quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$x\hat{i}+y\hat{j}+z\hat{k}=2\hat{i}-\hat{j}+2\hat{k}$

Hence, $\quad x=2, y=-1, z=2$

Therefore, the point of intersection is (2, –1, 2).

Now, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

Distance between (2, –1. 2) and (–1, –5, –10)

$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}$

$=\sqrt{(-3)^2+(-4)^2+(-12)^2}$

$=\sqrt{9+16+144}$

$=\sqrt{169}$

$=13.$ **[1½]**

Q. 17. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$. [NCERT Misc. Ex. Q. 19, Page 499]

Ans. The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is

$\vec{r}=\vec{a}+\lambda\vec{b}$

Given that, the line passes through the point (1, 2, 3).

So, $\vec{a}=1\hat{i}+2\hat{j}+3\hat{k}$

Given that, line is parallel to both planes.

$\therefore$ Line is perpendicular to normal of both planes.

i.e. $\vec{b}$ is perpendicular to normal of both planes.

We know that,

$\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

So, $\vec{b}$ is the cross-product of normal of planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

$$\text{Required normal} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$=\hat{i}(-1(1)-1(2))-\hat{j}(1(1)-3(2))+\hat{k}(1(1)-3(-1))$$

$$=\hat{i}(-1-2)-\hat{j}(1-6)+\hat{k}(1+3)$$

$$=-3\hat{i}+5\hat{j}+4\hat{k}$$

Thus, $\vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k}$

Now, putting value of $\vec{a}$ and $\vec{b}$ in formula

$\vec{r} = \vec{a} + \lambda\vec{b}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

Therefore, the equation of the line is $(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$. **[3]**

Q. 18. Find the vector equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

[NCERT Misc. Ex. Q. 20, Page 499]

Ans. The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is

$\vec{r} = \vec{a} + \lambda\vec{b}$

The line passes through the point (1, 2, –4)

So, $\vec{a} = 1\hat{i} + 2\hat{j} - 4\hat{k}$

Given that, line is perpendicular to both lines.

$\therefore$ $\vec{b}$ is perpendicular to both lines.

We know that,

$\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

So, $\vec{b}$ is the cross-product of both lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

$$\text{Required normal} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$= \hat{i}(-16(-5) - 8(7)) - \hat{j}(3(-5) - 3(7)) + \hat{k}(3(8) - 3(-16))$

$= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48)$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

Thus, $\vec{b} = 24\hat{i} + 36\hat{j} + 72\hat{k}$

Now,

Putting value of $\vec{a}$ and $\vec{b}$ in formula

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\therefore \vec{r} = (1\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$

$= (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda 12(2\hat{i} + 3\hat{j} + 6\hat{k})$

$= (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Therefore, the equation of the line is $(\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$. **[3]**

Q. 19. Find the shortest distance between the lines

$$\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} - 2\hat{j} - 3\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(\hat{i} - \hat{j} - 2\hat{k}\right) + \mu\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

[CBSE Board, Delhi Region, 2018]

Ans. Here,

$\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$ and

$\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Now,

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j} + 2\hat{k} - \left(4\hat{i} - \hat{j}\right) = -3\hat{i} + 2\hat{k}$

Also,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = (-10 + 12)\hat{i} - (-5 + 6)\hat{j} + (4 - 4)\hat{k}$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = 2\hat{i} - \hat{j}$

$\Rightarrow \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{4 + 1} = \sqrt{5}$ **[2]**

So, the shortest distance is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \left|\frac{\left(2\hat{i} - \hat{j}\right) \cdot \left(-3\hat{i} + 2\hat{k}\right)}{\sqrt{5}}\right|$$

$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$ units **[2]**

Q. 20. If *O* be the origin and the coordinates of *P* be (1, 2, –3), then find the equation of the plane passing through *P* and perpendicular to *OP*.

[NCERT Misc. Ex. Q. 16, Page 498]

Ans. Equation of plane passing through (x_1, y_1, z_1) and perpendicular to a line with direction ratios *A*, *B*, *C* is

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

The plane passes through *P* (1, 2, –3)

So, $x_1 = 1$, $y_1 = 2$ and $z_1 = -3$

Normal vector to plane = $\overrightarrow{OP}$

where *O* (0, 0, 0), *P* (1, 2, –3)

Direction ratios of $\overrightarrow{OP} = 1 - 0, 2 - 0, -3 - 0$

$= 1, 2, -3$ **[2]**

$\therefore A = 1, B = 2$, and $C = -3$

Equation of plane in Cartesian form is given by

$1(x - 1) + 2(y - 2) + (-3)[z - (-3)] = 0$

$x - 1 + 2y - 4 - 3(z + 3) = 0$

$x - 1 + 2y - 4 - 3z - 9 = 0$

$x + 2y - 3z - 14 = 0$ **[1]**

Q. 21. Find the vector and Cartesian equations of the planes

(a) that passes through the point (1, 0, –2) and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$.

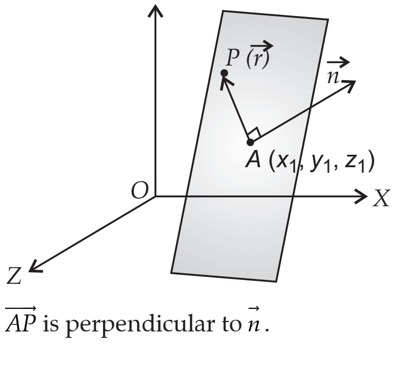
(b) that passes through the point (1, 4, 6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

[NCERT Ex. 11.3, Q. 5, Page 493]

Ans. **(a)** Vector equation of a plane passing through a point (x_1, y_1, z_1) and perpendicular to a line with direction rations *A*, *B*, *C* is :

$\left[\vec{r} - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})\right].(A\hat{i} + B\hat{j} + C\hat{k}) = 0$

Or $(\vec{r} - \vec{a}).\vec{n} = 0$



$\overrightarrow{AP}$ is perpendicular to $\vec{n}$.

So, $\overrightarrow{AP}\cdot\vec{n}=0$

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

Vector Equation

Equation of plane passing through point A whose position vector is $\vec{a}$ and perpendicular to $\vec{n}$ is :

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

Given that,

Plane passes through (1, 0, –2).

So, $\vec{a}=1\hat{i}+0\hat{j}-2\hat{k}$

Normal to plane $=\hat{i}+\hat{j}-\hat{k}$

$=\vec{n}=\hat{i}+\hat{j}-\hat{k}$

Vector equation of plane is :

$$(\vec{r}-\vec{a})\cdot\vec{n}=0$$

$$[\vec{r}-(1\hat{i}+0\hat{j}-2\hat{k})].(\hat{i}+\hat{j}-\hat{k})=0$$

$$[\vec{r}-(\hat{i}-2\hat{k})].(\hat{i}+\hat{j}-\hat{k})=0$$

Cartesian Form (Method 1) :

Vector equation is :

$[\vec{r}-(\hat{i}-2\hat{k})].(\hat{i}+\hat{j}-\hat{k})=0$

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$[(x\hat{i}+y\hat{j}+z\hat{k})-(1\hat{i}+0\hat{j}-2\hat{k})].(1\hat{i}+1\hat{j}-1\hat{k})=0$$

$$[(x-1)\hat{i}+(y-0)\hat{j}+(z-(-2))\hat{k}].(1\hat{i}+1\hat{j}-1\hat{k})=0$$

$$1(x-1)+1(y-0)-1(z+2)=0$$

$$x-1+y-z-2=0$$

$$x+y-z=3$$

So that, the equation of plane in Cartesian form will be,

$x+y-z=3$

Cartesian Form (Method 2) :

Equation of plane passing through (x_1, y_1, z_1) and perpendicular to a line with direction ratios A, B, C is :

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

Since the plane passes through (1, 0, –2)

$x_1=1, y_1=0$ and $z_1=2$

And normal is $\hat{i}+\hat{j}-\hat{k}$

So, direction ratios of line perpendicular to plane = (1, 1, –1)

$\therefore A=1, B=1$ and $C=1$

Therefore, equation of line in Cartesian form is :

$1(x-1)+1(y-0)+1[z-(-2)]=0$

$x+y-z=3$ **[3]**

(b) Vector Equation

Equation of plane passing through point A whose position vector is $\vec{a}$ and perpendicular to $\vec{n}$ is :

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

Given that,

Plane passes through (1, 4, 6).

So, $\vec{a}=1\hat{i}+4\hat{j}+6\hat{k}$

Normal to plane $=\hat{i}-2\hat{j}+\hat{k}$

$\vec{n}=\hat{i}-2\hat{j}+\hat{k}$

Vector equation of plane is :

$(\vec{r}-\vec{a}).\vec{n}=0$

$\left[\vec{r}-(\hat{i}+4\hat{j}+6\hat{k})\right]\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

Cartesian Form (Method 1) :

Vector equation is :

$\left[\vec{r}-(\hat{i}+4\hat{j}+6\hat{k})\right]\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$[(x\hat{i}+y\hat{j}+z\hat{k})-(1\hat{i}+4\hat{j}+6\hat{k})].(1\hat{i}-2\hat{j}+1\hat{k})=0$$

$$[(x-1)\hat{i}+(y-4)\hat{j}+(z-6)\hat{k}].(1\hat{i}+2\hat{j}+1\hat{k})=0$$

$$1(x-1)+(-2)(y-4)+1(z-6)=0$$

$$x-1+2(y-4)+Z-6=0$$

$$x-2y+z+1=0$$

$\therefore$ Equation of plane in Cartesian form is $(x-2y+z+1=0)$.

Cartesian Form (Method 2) :

Equation of plane passing through (x_1, y_1, z_1) and perpendicular to a line with direction ratios A, B, C is :

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

Since the plane passes through (1, 4, 6).

$x_1=1, y_1=4$ and $z_1=6$

And normal plane is $\hat{i}-2\hat{j}+\hat{k}$

So, direction ratios of line perpendicular to plane = (1, –2, 1)

$\therefore A=1, B=-2$ and $C=1$

So that, equations of line in Cartesian form is :

$1(x-1)-2(y-4)+1(x-6)=0$

$x-2y+z+1=0$ **[3]**

Q. 22. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

[NCERT Ex. 11.3, Q. 2, Page 493]

Ans. Vector equation of at a distance 'd' from the origin and normal to the vector $\vec{n}$ is

$\vec{r}\cdot\vec{n}=d$

Unit vector of $\vec{n}=\hat{n}=\frac{1}{|\vec{n}|}(\vec{n})$

Distance from origin $=d=7$

$\vec{n}=3\hat{i}+5\hat{j}-6\hat{k}$

Magnitude of $\vec{n}=\sqrt{3^2+5^2+(-6)^2}$

$|\vec{n}|=\sqrt{9+25+36}=\sqrt{70}$ **[1]**

Now, $\hat{n}=\frac{1}{|\vec{n}|}(\vec{n})=\frac{1}{\sqrt{70}}(3\hat{i}+5\hat{j}-6\hat{k})=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$

Vector equation is :

$$\vec{r}\cdot\hat{n}=d$$

$$\vec{r}\cdot\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$$

So that, the vector equation of the plane is :

$$\vec{r}\cdot\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$$ **[2]**

Q. 23. Find the shortest distance between the lines

$$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})\text{ and}$$

$$\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k}).$$

[NCERT Ex. 11.2, Q. 14, Page 478]

Ans. Shortest distance between the lines with vector equations,

$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$

$$=\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

Given that,

$\left[\begin{array}{l}\vec{r}=(\hat{i}+2\hat{j}+\hat{k})\\+\lambda(\hat{i}-\hat{j}+\hat{k})\end{array}\right]$	$\left[\begin{array}{l}\vec{r}=(2\hat{i}-\hat{j}-\hat{k})\\+\mu(2\hat{i}+\hat{j}+2\hat{k})\end{array}\right]$
Comparing with	Comparing with
$\vec{r}=\overrightarrow{a_1}+\lambda\,\overrightarrow{b_1}$,	$\vec{r}=\overrightarrow{a_2}+\mu\,\overrightarrow{b_2}$,
$\overrightarrow{a_1}=1\hat{i}+2\hat{j}+1\hat{k}$	$\overrightarrow{a_2}=2\hat{i}-1\hat{j}-1\hat{k}$
And $\overrightarrow{b_1}=1\hat{i}-1\hat{j}+1\hat{k}$	And $\overrightarrow{b_2}=2\hat{i}-1\hat{j}+2\hat{k}$

Now,

$$\overrightarrow{a_2}-\overrightarrow{a_1}=(2\hat{i}-1\hat{j}-1\hat{k})-(1\hat{i}+2\hat{j}+1\hat{k})$$
$$=(2-1)\hat{i}+(-1-2)\hat{j}+(-1-1)\hat{k}$$
$$=1\hat{i}-3\hat{j}-2\hat{k}$$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\2&1&2\end{vmatrix}$$

$$=\hat{i}[(-1\times2)-(1\times1)]-\hat{j}[(1\times2)-(2\times1)]+\hat{k}[(1\times1)-(2\times-1)]$$
$$=\hat{i}[-2-1]-\hat{j}[2-2]+\hat{k}[1+2]$$
$$=-3\hat{i}-0\hat{j}+3\hat{k}$$ **[1]**

Magnitude of $(\overrightarrow{b_1}\times\overrightarrow{b_2})=\sqrt{(-3)^2+(0)^2+3^2}$

$$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{9+0+9}=\sqrt{18}=\sqrt{9\times3}=3\sqrt{2}$$

Also,

$$(\overrightarrow{b_1}\times\overrightarrow{b_2})\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})=(-3\hat{i}-0\hat{j}+3\hat{k})\cdot(1\hat{i}-3\hat{j}-2\hat{k})$$
$$=(-3\times1)+(0\times-3)+(3\times-2)$$
$$=-3-0-6=-9$$

So, shortest distance $=\left|\dfrac{(\overrightarrow{b_1}\times\overrightarrow{b_2})\cdot(\overrightarrow{a_2}\times\overrightarrow{a_1})}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$

$$=\left|\frac{-9}{3\sqrt{2}}\right|=\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$$

So that, the shortest distance between the given two lines is $\dfrac{3\sqrt{2}}{2}$. **[2]**

Q. 24. Find the vector and the Cartesian equations of the lines that pass through the origin and (5, –2, 3). **[NCERT Ex. 11.2, Q. 8, Page 477]**

Ans. Vector Equation

Vector equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is :

$$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$$

Given that,

Let two points be A (0, 0, 0) and B (5, –2, 3).

A (0, 0, 0)	B (5, –2, 3)
$\vec{a}=0\hat{i}+0\hat{j}+0\hat{k}$	$\vec{b}=5\hat{i}-2\hat{j}+3\hat{k}$

So, $\vec{r}=(0\hat{i}+0\hat{j}+0\hat{k})+\lambda[(5\hat{i}-2\hat{j}+3\hat{k})-(0\hat{i}+0\hat{j}+0\hat{k})]$

$\vec{r}=\lambda[(5\hat{i}-2\hat{j}+3\hat{k})$ [1½]

Cartesian Equation

Cartesian equation of a line passing through two points $A\ (x_1, y_1, z_1)$ and $B\ (x_2, y_2, z_2)$ is

$$=\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Since the line passes through A (0, 0, 0)

$x_1 = 0, y_1 = 0$ and $z_1 = 0$

And also passes through B (5, –2, 3),

$x_2 = 5, y_2 = -2$ and $z_2 = 3$

Equation of line is :

$$=\frac{x-0}{5-0}=\frac{y-0}{-2-0}=\frac{z-0}{3-0}$$

$$\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$$ **[1½]**

Q. 25. Find the vector and the Cartesian equations of the line that passes through the points (3, –2, –5), (3, –2, 6). **[NCERT Ex. 11.2, Q. 9, Page 478]**

Ans. Vector Equation

Vector equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is :

$$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$$

Given, the two points are

A (3, –2, –5)	B (3, –2, 6)
$\vec{a}=3\hat{i}-2\hat{j}-5\hat{k}$	$\vec{b}=3\hat{i}-2\hat{j}+6\hat{k}$

So,

$$\vec{r}=(3\hat{i}-2\hat{j}-5\hat{k})+\lambda[(3\hat{i}-2\hat{j}+6\hat{k})-(3\hat{i}-2\hat{j}-5\hat{k})]$$
$$=3\hat{i}-2\hat{j}-5\hat{k}+\lambda[(3-3)\hat{i}+(-2-(-2))\hat{j}+(6-(-5))\hat{k})]$$
$$=3\hat{i}-2\hat{j}-5\hat{k}+\lambda[0\hat{i}+0\hat{j}+0\hat{j}+11\hat{k}]$$
$$=3\hat{i}-2\hat{j}-5\hat{k}+\lambda(11\hat{k})$$

So that, the vector equation is $\vec{r}=3\hat{i}-2\hat{j}-5\hat{k}+\lambda(11\hat{k})$ **[1½]**

Cartesian Equation

Cartesian equation of a line passing through two points $A\ (x_1, y_1, z_1)$ and $B\ (x_2, y_2, z_2)$ is :

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Since the line passes through point A (3, –2, –5)

$x_1 = 3, y_1 = -2$ and $z_1 = -5$

And also passes through point B (3, –2, 6)

$x_2 = 3, y_2 = -2$ and $z_2 = 6$

Equation of line is $\dfrac{x-3}{3-3}=\dfrac{y-(-2)}{-2-(-2)}=\dfrac{z-(-5)}{6-(-5)}$

$$\frac{x-3}{0}=\frac{y+2}{0}=\frac{z+5}{11}$$ **[1½]**

Q. 26. Find the angle between the following pairs of lines :

(i) $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ **and** $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$

(ii) $\vec{r}=3\hat{i}+\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}-2\hat{k})$ **and** $\vec{r}=2\hat{i}-\hat{j}-56\hat{k}+\mu(3\hat{i}-5\hat{j}-4\hat{k})$

[NCERT Ex. 11.2, Q. 10, Page 478]

Ans. **(i)** Angle between two vectors :

$$\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$$

And $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is given by

$$\cos\theta=\left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$$

Given that, the pair of lines is :

$\left[\begin{array}{l}\vec{r}=(2\hat{i}-5\hat{j}+\hat{k})\\+\lambda(3\hat{i}+2\hat{j}+6\hat{k})\end{array}\right]$	$\left[\begin{array}{l}\vec{r}=(7\hat{i}-6\hat{k})\\+\mu(\hat{i}+2\hat{j}+2\hat{k})\end{array}\right]$
So, $\vec{a_1}=2\hat{i}-5\hat{j}+1\hat{k}$	So, $\vec{a_2}=7\hat{i}+0\hat{j}-6\hat{k}$
$\vec{b_1}=3\hat{i}+2\hat{j}+6\hat{k}$	$\vec{b_2}=1\hat{i}+2\hat{j}+2\hat{k}$

Now,

$$\vec{b_1}\cdot\vec{b_2}=(3\hat{i}+2\hat{j}+6\hat{k})\cdot(1\hat{i}+2\hat{j}+2\hat{k})$$

$$=(3\times1)+(2\times2)+(6\times2)=3+4+12=19$$

Magnitude of $\vec{b_1}=\sqrt{3^2+2^2+6^2}$

$$|\vec{b_1}|=\sqrt{9+4+36}=\sqrt{49}=7$$

Magnitude of $\vec{b_2}=\sqrt{1^2+2^2+2^2}$

$$|\vec{b_2}|=\sqrt{1+4+4}=\sqrt{9}=3$$

Now,

$$\cos\theta=\left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|=\left|\frac{19}{7\times3}\right|=\frac{19}{21}$$

$$\therefore\quad \theta=\cos^{-1}\left(\frac{19}{21}\right)$$

So that, the angle between the given vectors is $\cos^{-1}\left(\frac{19}{21}\right)$. **[1½]**

(ii) Angle between two vectors,

$r=\vec{a_1}+\ \vec{b_1}$

And $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is given by

$$\cos\theta=\left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$$

Given that, the pair of lines is :

$\left[\begin{array}{l}\vec{r}=(3\hat{i}+\hat{j}-2\hat{k})\\+\lambda(\hat{i}-\hat{j}-2\hat{k})\end{array}\right]$	$\left[\begin{array}{l}\vec{r}=(2\hat{i}-\hat{j}-56\hat{k})\\+\mu(3\hat{i}-5\hat{j}-4\hat{k})\end{array}\right]$
So, $\vec{a_1}=3\hat{i}-1\hat{j}-2\hat{k}$	So, $\vec{a_2}=2\hat{i}+1\hat{j}-56\hat{k}$
$\vec{b_1}=1\hat{i}-1\hat{j}-2\hat{k}$	$\vec{b_2}=3\hat{i}-5\hat{j}-4\hat{k}$

Now,

$$\vec{b_1}\cdot\vec{b_2}=(1\hat{i}-1\hat{j}-2\hat{k})\cdot(3\hat{i}-5\hat{j}-4\hat{k})$$

$$=(1\times3)+(-1\times-5)+(-2\times-4)=3+5+8=16$$

Magnitude of $\vec{b_1}=\sqrt{1^2+(-1)^2+(-2)^2}$

$$|\vec{b_1}|=\sqrt{1+1+4}=\sqrt{6}$$

Magnitude of $\vec{b_2}=\sqrt{3^2+(-5)^2+(-4)^2}$

$$|\vec{b_2}|=\sqrt{9+25+16}=\sqrt{50}=\sqrt{25\times2}=5\sqrt{2}$$

Now,

$$\cos\theta=\left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|=\left|\frac{16}{\sqrt{6}\times5\sqrt{2}}\right|=\left|\frac{16}{\sqrt{3}\times\sqrt{2}\times5\times\sqrt{2}}\right|$$

$$=\left|\frac{16}{\sqrt{3}\times2\times5}\right|=\frac{8}{5\sqrt{3}}$$

$$\therefore \theta=\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

So that, the angle between the given vectors is $\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$. **[1½]**

Q. 27. Find the angle between the following pair of lines :

(i) $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z-3}{-3}$ **and** $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

(ii) $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ **and** $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$

[NCERT Ex. 11.2, Q. 11, Page 478]

Ans. **(i)** Angle between the pair of lines :

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

And $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$	$\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$
$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z-(-3)}{-3}$	$\frac{x-(-2)}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$
Comparing with	Comparing with
$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$	$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$
$x_1=2,\ y_1=1,\ z_1=-3$	$x_2=-2,\ y_2=4,\ z_2=5$
And $a_1=2, b_1=5, c_1=-3$	And $a_2=-1, b_2=8, c_2=4$

Now, $\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$$=\left|\frac{(2\times-1)+(5\times8)+(-3\times4)}{\sqrt{2^2+5^2+(-3)^2}\sqrt{(-1)^2+8^2+4^2}}\right|$$

$$=\left|\frac{-2+40+(-12)}{\sqrt{4+25+9}\sqrt{1+64+16}}\right|$$

$$=\left|\frac{26}{\sqrt{38}\sqrt{81}}\right|$$

$$=\left|\frac{26}{\sqrt{38}\times9}\right|$$

$$=\left|\frac{26}{9\sqrt{38}}\right|$$

So, $\cos\theta=\frac{26}{9\sqrt{38}}$

$$\therefore\quad \theta=\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

So that, the angle between the given lines is $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$. **[1½]**

(ii) Angle between the pair of lines :

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ And

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$

$\frac{x-0}{2}=\frac{y-0}{2}=\frac{z-0}{1}$

Comparing with

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

$x_1=0,\ y_1=0,\ z_1=0$

And $a_1=2,\ b_1=2,\ c_1=1$

$\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-5}{8}$

Comparing with

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

$x_2=5,\ y_2=2,\ z_2=5$

And $a_2=4,\ b_2=1,\ c_2=8$

Now, $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$= \left|\frac{(2\times4)+(2\times1)+(1\times8)}{\sqrt{2^2+2^2+1^2}\times\sqrt{4^2+1^2+8^2}}\right|$

$= \left|\frac{8+2+8}{\sqrt{4+4+1}\sqrt{16+1+64}}\right|$

$= \left|\frac{18}{\sqrt{9}\times\sqrt{81}}\right|$

$= \left|\frac{18}{3\times9}\right|$

$= \left|\frac{2}{3}\right|$

So, $\cos\theta = \frac{2}{3}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$

So that, the angle between the given lines is $\cos^{-1}\left(\frac{2}{3}\right)$. **[1½]**

Q. 28. Find the value of P so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. [NCERT Ex. 11.2, Q. 12]

Ans. Two lines are given by

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

They are at right angles to each other, if

$a_1a_2+b_1b_2+c_1c_2=0$ **[1]**

$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$

$\frac{-(x-1)}{3}=\frac{7(y-2)}{2p}=\frac{z-3}{2}$

$\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$

Comparing with

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

$x_1=1,\ y_1=2,\ z_1=3$

And $a_1=-3,\ b_1=\frac{2p}{7},\ c_1=2$

$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$

$\frac{-7(x-1)}{3p}=\frac{y-5}{1}=\frac{-(z-6)}{5}$

$\frac{x-1}{\frac{-3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$

Comparing with

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$,

$x_2=1,\ y_2=5,\ z_2=6$

And $a_2=\frac{-3p}{7},\ b_2=1,\ c_2=-5$

Since the lines are perpendicular

$a_1a_2+b_1b_2+c_1c_2=0$

$\left(-3\times\frac{-3p}{7}\right)+\left(\frac{2p}{7}\times1\right)+(2\times-5)=0$

$\frac{9p}{7}+\frac{2p}{7}-10=0$

$\frac{11p}{7}=10$

$p=10\times\frac{7}{11}$

$\therefore \quad p=\frac{70}{11}$ **[2]**

Q. 29. Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction $\hat{i}+2\hat{j}-\hat{k}$. [NCERT Ex. 11.2, Q. 5, Page 477]

Ans. Equation of a line passing through a point with position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

$\vec{r}=\vec{a}+\lambda\vec{b}$

Here,

$\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$

And $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

So, $\vec{r}=(2\hat{i}-\hat{j}+4\hat{k})+\lambda(\hat{i}+2\hat{j}-\hat{k})$

$\therefore$ Equation of line in vector form is $(2\hat{i}-\hat{j}+4\hat{k})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ **[1]**

Equation of a line passing though (x_1, y_1, z_1) and parallel to a line having direction ratios a, b, c is

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Since the lines passes through a point with position vector $2\hat{i}-\hat{j}+4\hat{k}$,

$\therefore x_1=2, y_1=-1$ and $z_1=4$

Also, line is in the direction of $\hat{i}+2\hat{j}-\hat{k}$,

Direction ratios : $a=1, b=2$ and $c=-1$

Equation of line in Cartesian form is :

$\frac{x-2}{1}=\frac{y-(-1)}{2}=\frac{z-4}{-1}$

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$ **[2]**

Q. 30. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6). [NCERT Ex. 11.2, Q. 2, Page 477]

Ans. Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular to each other if

$a_1a_2+b_1b_2+c_1c_2=0$

Now, a line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) has the direction ratios :

(x_2-x_1), (y_2-y_1) and (z_2-z_1) **[1]**

A (1, –1, 2)	C (0, 3, 2)
B (3, 4, – 2)	B (3, 5, 6)
Direction ratio:	Direction ratio:
$(3-1), 4-(-1), -2-2$	$(3-0), (5-3), (6-2)$

$= 2, 5, -4$	$= 3, 2, 4$
$\therefore a_1 = 2,\ b_1 = 5,\ c_1 = -4$	$\therefore a_2 = 3,\ b_2 = 2,\ c_2 = 4$

Now,

$$a_1a_2 + b_1b_2 + c_1c_2 = (2\times 3) + (5\times 2) + (-4\times 4)$$
$$= 6 + 10 + (-16) = 16 - 16 = 0$$

So that the given two lines are perpendicular. **[2]**

Q. 31. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (–1, –2, 1), (1, 2, 5).[NCERT Ex. 11.2, Q. 3, Page 477]

Ans. Two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Also, a line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) has the direction ratios :

$(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$ **[1]**

A (4, 7, 8)	C (–1, – 2, 1)
B (2, 3, 4)	B (1, 2, 5)
Direction ratio	Direction ratio
$= 2 - 4, 3 - 7, 4 - 8$	$= 1 - (-1), 2 - (-2), 5 - 1$
$= -2, -4, -4$	$= 2, 4, 4$
$\therefore a_1 = -2,\ b_1 = -4,\ c_1 = -4$	$\therefore a_2 = 2,\ b_2 = 4,\ c_2 = 4$

Now,

$$\frac{a_1}{a_2} = \frac{-2}{2} = -1$$
$$\frac{b_1}{b_2} = \frac{-4}{4} = -1$$
$$\frac{c_1}{c_2} = \frac{-4}{4} = -1$$

Since

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -1$$

So that, the given lines are parallel. **[2]**

Q. 32. Prove that the line through *A* (0, –1, –1) and *B* (4, 5, 1) intersects the line through *C* (3, 9, 4) and *D* (–4, 4, 4). [CBSE Board, Foreign Region, 2016]

Ans. Equation of line $\overline{AB}$

$$\vec{r} = (-\hat{j} - \hat{k}) + \lambda(4\hat{i} + 6\hat{j} + 2\hat{k})$$

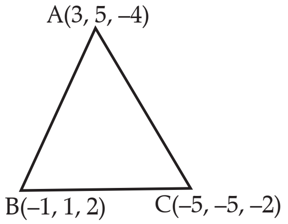
Equation of line $\overline{CD}$

$$\vec{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu(-7\hat{i} - 5\hat{j})$$
$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 10\hat{j} + 5\hat{k}$$
$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$$
$$= 10\hat{i} - 14\hat{j} + 22\hat{k}$$
$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 30 - 140 = 0$$

$\Rightarrow$ Lines intersect **[4]**

Q. 33. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, –4), (–1, 1, 2) and (–5, –5, –2). [NCERT Ex. 11.1, Q. 5, Page 467]

Ans.



Direction ratios of a line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$
$= (x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

Direction cosines

$$= \frac{x_2 - x_1}{PQ},\ \frac{y_2 - y_1}{PQ} \text{ and } \frac{z_2 - z_1}{PQ}$$

Where, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ **[1]**

For *AB* :

A (3, 5, –4)

B (–1, 1, 2)

Direction Ratios

$= -1 - 3, 1 - 5, 2 - (-4)$

$= -4, -4, 6$

$$AB = \sqrt{68}$$
$$= \sqrt{4\times 17} = 2\sqrt{17}$$

Direction cosines

$$= \frac{-4}{2\sqrt{17}},\ \frac{-4}{2\sqrt{17}} \text{ and } \frac{6}{2\sqrt{17}}$$
$$= \frac{-2}{\sqrt{17}},\ \frac{-2}{\sqrt{17}} \text{ and } \frac{3}{\sqrt{17}}$$

For *BC* :

B (–1, 1, 2)

C (–5, –5, –2)

Direction Ratios

$= -5 - (-1), -5 - 1, -2 - 2$

$= -4, -6, -4$

$$BC = \sqrt{68}$$
$$= \sqrt{4\times 17} = 2\sqrt{17}$$

Direction cosines

$$= \frac{-4}{2\sqrt{17}},\ \frac{-6}{2\sqrt{17}} \text{ and } \frac{-4}{2\sqrt{17}}$$
$$= \frac{-2}{\sqrt{17}},\ \frac{-3}{\sqrt{17}} \text{ and } \frac{-2}{\sqrt{17}}$$

For *CA* :

C (–5, –5, –2)

A (3, 5, –4)

Direction Ratios

$= 3 - (-5),\ 5 - (-5),\ -4 - (-2)$

$= 8,\ 10,\ -2$

$$CA = \sqrt{168}$$
$$= \sqrt{4\times 42} = 2\sqrt{42}$$

Direction cosines

$$= \frac{8}{2\sqrt{42}},\ \frac{10}{2\sqrt{42}} \text{ and } \frac{-2}{2\sqrt{42}}$$
$$= \frac{4}{\sqrt{42}},\ \frac{5}{\sqrt{42}} \text{ and } \frac{-1}{\sqrt{42}}$$

[2]

Q. 34. Find the area of a parallelogram *ABCD* whose side AB and the diagonal *AC* are given by the vectors $3\hat{i}+\hat{j}+4\hat{k}$ and $4\hat{i}+5\hat{k}$ respectively.
[CBSE Board, Foreign Region, 2017]

Ans. $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

$=\hat{i}-\hat{j}+\hat{k}$

$$\text{Area}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right| = \text{Magnitude of}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 4\\ 1 & -1 & 1\end{vmatrix}$$

$$=\left|5\hat{i}+\hat{j}-4\hat{k}\right|$$

$$=\sqrt{42}\text{ sq. units}$$ **[4]**

Q. 35. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at 60° to $\overrightarrow{OX}$ and at 45° to *OY* and $\left|\overrightarrow{OA}\right|$ = 10 units.
[NCERT Exemp. Ex. 11.3, Q. 1, Page 235]

Ans. Given that $\overrightarrow{OA}$ is inclined at 60° to *OX* and at 45° to *OY*.

Let $\overrightarrow{OA}$ makes angle α with *OZ*.

$\therefore \cos^2 60° + \cos^2 45° + \cos^2\alpha = 1$ $[\because l^2+m^2+n^2=1]$

$$\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\alpha=1$$

$$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\alpha=1$$

$$\Rightarrow \cos^2\alpha=\frac{1}{4}$$

$$\Rightarrow \cos\alpha=\pm\frac{1}{2}$$

$$\therefore \overrightarrow{OA}=\left|\overrightarrow{OA}\right|\left(\frac{1}{2}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\pm\frac{1}{2}\hat{k}\right)$$

$$=10\left(\frac{1}{2}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\pm\frac{1}{2}\hat{k}\right)$$

$$=5\hat{i}+5\sqrt{2}\hat{j}\pm5\hat{k}$$ **[3]**

Q. 36. Find the vector equation of the line which is parallel to the vector $3\hat{i}-2\hat{j}+6\hat{k}$ and which passes through the point (1, –2, 3).
[NCERT Exemp. Ex. 11.3, Q. 2, Page 235]

Ans. Let $\vec{b}=3\hat{i}-2\hat{j}+6\hat{k}$ and $a=\hat{i}-2\hat{j}+3\hat{k}$.

So, vector equation of the line, which is parallel to the vector $\vec{b}=3\hat{i}-2\hat{j}+6\hat{k}$ and passes through the point $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, is $\vec{r}=\vec{a}+\lambda\vec{b}$

$\therefore\ \vec{r}=(\hat{i}-2\hat{j}+3\hat{k})+\lambda(3\hat{i}-2\hat{j}+6\hat{k})$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})-(\hat{i}-2\hat{j}+3\hat{k})=\lambda(3\hat{i}-2\hat{j}+6\hat{k})$

$\Rightarrow (x-1)\hat{i}+(y+2)\hat{j}+(z-3)\hat{k}=\lambda(3\hat{i}-2\hat{j}+6\hat{k})$ **[3]**

Q. 37. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=$ $=\frac{y-1}{2}=z$ intersect.

Also, find their point of intersection.
[NCERT Exemp. Ex. 11.3, Q. 3, Page 235]

Ans. We have lines

$$L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$$

$$\text{and } L_2:\frac{x-4}{5}=\frac{y-1}{2}=z=\mu$$

Any point on the line L_1 is (2λ + 1, 3λ + 2, 4λ + 3).

Any point on the line L_2 is (5μ + 4, 2μ + 1, μ).

If lines intersect then there exists a value of λ, μ for which

(2λ + 1, 3λ + 2, 4λ + 3) = (5μ + 4, 2μ + 1, μ)

⇒ 2λ + 1 = 5μ + 4, 3λ + 2 = 2μ + 1 and 4λ + 3 = μ

Solving first two equations, we get

λ = – 1, μ = – 1

These values of λ = – 1, μ = – 1 also satisfy the third equation.

That means both lines intersect.

And the point of intersection is (–1, –1, –1). **[3]**

Q. 38. Find the angle between the lines $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+$ $\lambda=(2\hat{i}+\hat{j}+2\hat{k})$ and $\vec{r}=(2\hat{j}-5\hat{k})+\mu(6\hat{i}+3\hat{j}+2\hat{k})$.
[NCERT Exemp. Ex. 11.3, Q. 4, Page 235]

Ans. We have line, $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda(2\hat{i}+\hat{j}+2\hat{k})$, which is parallel to the vector $\vec{b}_1=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{r}=(2\hat{j}-5\hat{k})+\mu(6\hat{i}+3\hat{j}+2\hat{k})$, which is parallel to the vector $\vec{b}_2=6\hat{i}+3\hat{j}+2\hat{k}$.

If θ is an angle between the lines, then

$$\cos\theta=\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

$$=\frac{\left|(2\hat{i}+\hat{j}+2\hat{k})\cdot(6\hat{i}+3\hat{j}+2\hat{k})\right|}{\left|2\hat{i}+\hat{j}+2\hat{k}\right|\left|6\hat{i}+3\hat{j}+2\hat{k}\right|}$$

$$=\frac{|12+3+4|}{\sqrt{9}\sqrt{49}}=\frac{19}{21}$$

$$\therefore\ \theta=\cos^{-1}\frac{19}{21}$$ **[3]**

Q. 39. Prove that the line through *A* (0, –1, –1) and *B* (4, 5, 1) intersects the line through *C* (3, 9, 4) and *D* (–4, 4, 4). **[NCERT Exemp. Ex. 11.3, Q. 5, Page 235]**

Ans. We know that the equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is :

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

So the, equation of line passes through points *A* (0, –1, –1) and B (4, 5, 1) is :

$$\frac{x-0}{4-0}=\frac{y+1}{5+1}=\frac{z+1}{1+1}\text{ or }\frac{x}{4}=\frac{y+1}{6}=\frac{z+1}{2}\quad\ldots\text{(i)}$$

And the equation of the line passes through *C* (3, 9, 4) and *D* (–4, 4, 4) is :

$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}\text{ or }\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}\quad\ldots\text{(ii)}$$

Any point on the line (i) is (4λ, 6λ – 1, 2λ – 1).

Any point on the line (ii) is (–7μ + 3, –5μ + 9, 4).

If lines intersect then there exists a value of λ, μ for which

= (4λ, 6λ – 1, 2λ – 1) ≡ (–7μ + 3, –5μ + 9, 4)

∴ 4λ = –7μ + 3, 6λ – 1 = –5μ + 9, 2λ – 1 = 4

⇒ λ = 5/2

So, from the first equation, μ = –1

Also, these values of λ and μ satisfy the second equation.

So, both lines intersect. **[3]**

Q. 40. Prove that the lines *x* = *py* + *q*, *z* = *ry* + *s* and *x* = *p'y* + *q'*, *z* = *r'y* + *s'* are perpendicular if *pp'* + *rr'* + 1 = 0. **[NCERT Exemp. Ex. 11.3, Q. 6, Page 235]**

Ans. We have line $x = py + q, z = ry + s$

$\Rightarrow \quad y = \frac{x-q}{p}$ and $y = \frac{z-s}{r}$

$\Rightarrow \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$ (i)

Similarly, line $x = p'y + q', z = r'y + s'$

$\Rightarrow \frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'}$ (ii)

Line (i) is parallel to the vector $p\hat{i} + \hat{j} + r\hat{k}$.

Line (ii) is parallel to the vector $p'\hat{i} + \hat{j} + r'\hat{k}$.

Lines are perpendicular,

$\therefore (p\hat{i} + \hat{j} + r\hat{k}) \cdot (p'\hat{i} + \hat{j} + r'\hat{k})$

$\therefore pp' + 1 + rr' = 0.$ **[3]**

Q. 41. Find the equation of a plane which bisects perpendicularly the line joining the points *A* (2, 3, 4) and *B* (4, 5, 8) at right angles.

[NCERT Exemp. Ex. 11.3, Q. 7, Page 235]

Ans. Since, the equation of a plane is bisecting perpendicular to the line joining the points *A* (2, 3, 4) and *B* (4, 5, 8) at right angles.

So, mid-point of *AB* is $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$ *i.e.,* (3,4,6)

Also normal to the plane,

$\vec{n} = (4-2)\hat{i} + (5-3)\hat{j} + (8-4)\hat{k}$

$= 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, the required equation of the plane is $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\Rightarrow \left[(x-3)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}\right] \cdot (2\hat{i} + 2\hat{j} + 4\hat{k}) = 0$

$[\because \vec{a} = 3\hat{i} + 4\hat{j} + 6\hat{k}]$

$\Rightarrow 2x - 6 + 2y - 8 + 4z - 24 = 0$

$\Rightarrow \quad x + y + 2z = 19$ **[3]**

Q. 42. Find the equation of a plane which is at a distance 3√3 units from origin and the normal to which is equally inclined to coordinate axis.

[NCERT Exemp. Ex. 11.3, Q. 8, Page 235]

Ans. Since, normal to the plane is equally inclined to the coordinate axis.

So that, $\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

So, the normal is $\vec{n} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ and plane is at a distance of $3\sqrt{3}$ units from the origin.

The equation of plane is $\vec{r} \cdot \hat{n} = 3\sqrt{3}$.

[Since vector equation of the plane at a distance *p* from the origin is $\vec{r} \cdot \hat{n} = p$]

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = 3\sqrt{3}$

$\Rightarrow \quad \frac{3}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$

$\therefore \quad x + y + z = 9$ **[3]**

Q. 43. If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), find the equation of the plane.

[NCERT Exemp. Ex. 11.3, Q. 9, Page 235]

Ans. Since, the line drawn from the point *B* (–2, –1, –3) meets a plane at right angle, at the point 4 (1, –3, 3). So, the plane passes through the point 4 (1, –3, 3).

Also normal to plane is $\overrightarrow{AB} = -3\hat{i} + 2\hat{j} - 6\hat{k}$.

So, the equation of required plane is :

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \qquad$ where $\vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$

$\Rightarrow \left[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 3\hat{j} + 3\hat{k})\right] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$

$\Rightarrow \left[(x-1)\hat{i} + (y+3)\hat{j} + (z-3)\hat{k}\right] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$

$\Rightarrow -3x + 3 + 2y + 6 - 6z + 18 = 0$

$\therefore 3x - 2y + 6z - 27 = 0$ **[3]**

Q. 44. Find the equation of the plane through the points (2, 1, 0), (3, –2, –2) and (3, 1, 7).

[NCERT Exemp. Ex. 11.3, Q. 10, Page 235]

Ans. We know that, the equation of a plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(-21) - (y-1)(9) + z(3) = 0$

$\therefore 7x + 3y - z = 17$ **[3]**

Q. 45. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

[NCERT Exemp. Ex. 11.3, Q. 11, Page 236]

Ans. Given equation of the line is : $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = \lambda$

So, direction ratios of the line are (2, 1, 1) ≡ (a_1, b_1, c_1)

Any point on the given line is *p* (2λ + 3, λ + 3, λ)

So, direction ratios of *OP* are :

(2λ + 3, λ + 3, λ) ≡ (a_2, b_2, c_2)

Now, angle between given line and *OP* is $\frac{\pi}{3}$.

$\because \quad \cos\frac{\pi}{3} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\therefore \quad \frac{1}{2} = \frac{(4\lambda + 6) + (\lambda + 3) + (\lambda)}{\sqrt{6}\sqrt{(2\lambda + 3)^2 + (\lambda + 3)^2 + \lambda^2}}$

$\Rightarrow \quad \frac{\sqrt{6}}{2} = \frac{6\lambda + 9}{\sqrt{6\lambda^2 + 18\lambda + 18}}$

$\Rightarrow \sqrt{\lambda^2 + 3\lambda + 3} = 2\lambda + 3$

$\Rightarrow \quad \lambda^2 + 3\lambda + 3 = 4\lambda^2 + 9 + 12\lambda$

$\Rightarrow \quad \lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow (\lambda + 1)(\lambda + 2) = 0$

$\therefore \quad \lambda = -1, -2$

So, the direction ratios are 1, 2, –1 and –1, 1, –2.

Also, both the required lines pass through origin.

So, the equations of required lines are $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$. **[3]**

Q. 46. Find the angle between the lines whose direction cosines are given by the equations $l+m+n=0$, $l^2+m^2-n^2=0$.

[NCERT Exemp. Ex. 11.3, Q. 12, Page 236]

Ans. Given that,

$l+m+n=0, l^2+m^2-n^2=0$

Eliminating n from both the equations, we have

$l^2+m^2-(l+m)^2=0$

$\Rightarrow l^2+m^2-l^2-m-2ml^2=0$

$\Rightarrow 2lm=0$

$\Rightarrow lm=0$

$\Rightarrow l=0$ or $m=0$

If $l=0$, we have $m+n=0$ and $m^2-n^2=0$

$\Rightarrow l=0, m=\lambda, n=-\lambda$

If $m=0$, we have $l+m=0$ and $l^2-m^2=0$

$\Rightarrow l=-\lambda, m=0, n=\lambda$

So, the vectors parallel to these given lines are $\hat{a}=\hat{j}-\hat{k}$ and $\hat{b}=-\hat{i}=\hat{k}$.

If angle between the lines is 'θ', then

$$\cos\theta=\frac{|\vec{a}\cdot\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{1}{\sqrt{2}\cdot\sqrt{2}}$$

$$\Rightarrow \cos\theta=\frac{1}{2}$$

$$\therefore \quad \theta=\frac{\pi}{3}$$ **[3]**

Q. 47. If a variable line in two adjacent positions has direction cosines l, m, n and $l+\delta l, m+\delta m, n+\delta n$, show that the small angle $\delta\theta$ between the two positions is given by $\delta\theta^2=\delta l^2+\delta m^2+\delta n^2$

[NCERT Exemp. Ex. 11.3, Q. 13, Page 236]

Ans. We have l, n, n and $l+\delta l, m+\delta m, n+\delta n$ as direction cosines of a variable line in two different positions.

$\therefore l^2+m^2+n^2=1$ ....(i)

And $(l+\delta l)^2+(m+\delta m)^2+(n+\delta n)^2=1$ ....(ii)

$\Rightarrow l^2+m^2+n^2+\delta l^2+\delta m^2+\delta n^2+2(l\delta l+m\delta m+n\delta n)=1$

$\Rightarrow \delta l^2+\delta m^2+\delta n^2=-2(l\delta l+m\delta m+n\delta n)$ [$\because l^2+m^2+n^2=1$]

$$\Rightarrow l\delta l+m\delta m+n\delta n=\frac{-1}{2}(\delta l^2+\delta m^2+\delta n^2) \quad ...(iii)$$

Now $\vec{a}$ and $\vec{b}$ are unit vectors along a line with direction cosines l, m, n and $(l+\delta l)$, $(m+\delta m)$ and $(n+\delta n)$, respectively.

$\therefore \quad \vec{a}=l\hat{i}+m\hat{j}+n\hat{k}$ and

$\vec{b}=(l+\delta l)\hat{i}+(m+\delta m)\hat{j}+(n+\delta n)\hat{k}$

$$\Rightarrow \cos\delta\theta=l(l+\delta l)+m(m+\delta m)+n(n+\delta n)$$

$$=(l^2+m^2+n^2)+(l\delta l+m\delta m+n\delta n)$$

$$=1-\frac{1}{2}(\delta l^2+\delta m^2+\delta n^2) \text{ [Using Eq. (iii)]}$$

$$\Rightarrow 2(1-\cos\delta\theta)=(\delta l^2+\delta m^2=\delta n^2)$$

$$\Rightarrow 2.2\sin^2\frac{\delta\theta}{2}=\delta l^2+\delta m^2+\delta n^2$$

$$\Rightarrow 4\left(\frac{\delta\theta}{2}\right)^2=\delta l^2+\delta m^2+\delta n^2$$

[Since $\frac{\delta\theta}{2}$ is small, $\sin\frac{\delta\theta}{2}=\frac{\delta\theta}{2}$]

$$\Rightarrow \delta\theta^2=\delta l^2+\delta m^2+\delta n^2$$ **[3]**

Q. 48. O is the origin and A is (a, b, c). Find the direction cosines of the line OA and the equation of plane through A at right angle to OA.

[NCERT Exemp. Ex. 11.3, Q. 14, Page 236]

Ans. Direction ratios of OA are a, b and c.

$\therefore$ Direction ratios of line OA are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}} \text{ and } \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

Also,

$\vec{n}=\overrightarrow{OA}=a\hat{i}+b\hat{j}+c\hat{k}$

The equation of plane passes through (a, b, c) and perpendicular to OA is given by

$a(x-a)+b(y-b)+c(z-c)=0$

$\Rightarrow ax+by+cz=a^2+b^2+c^2$ **[3]**

Q. 49. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a', b', c', respectively, from the origin, prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$$

[NCERT Exemp. Ex. 11.3, Q. 15, Page 236]

Ans. Consider OX, OY, OZ and OX', OY', OZ' are two systems of rectangular axes.

Let their corresponding equation of plane be :

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad ...(i)$$

$$\text{And } \frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1 \quad ...(ii)$$

Also, the length of perpendicular from origin to equations (i) and (ii) must be same.

$$\therefore \frac{\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{\frac{0}{a'}+\frac{0}{b'}+\frac{0}{c'}-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$$ **[3]**

Q. 50. Find the angle between the planes whose vector equations are $\vec{r}(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.

[NCERT Ex. 11.3, Q. 12, Page 494]

Ans. Angle between two planes $\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by

$$\cos\theta=\left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$$

Given, the two planes are :

$\vec{r}(2\hat{i}+2\hat{j}-3\hat{k})=5$	$\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$
Comparing with $\vec{r}.\vec{n_1}=\vec{d_1}$,	Comparing with $\vec{r}.\vec{n_2}=\vec{d_2}$,
$\vec{n_1}=2\hat{i}+2\hat{j}-3\hat{k}$	$\vec{n_2}=2\hat{i}+2\hat{j}-3\hat{k}$
Magnitude of $\vec{n_1}$	Magnitude of $\vec{n_2}$
$=\sqrt{2^2+2^2+(-3)^2}$	$=\sqrt{3^2+(-3)^2+(5)^2}$
$\lvert\vec{n_1}\rvert=\sqrt{4+4+9}=\sqrt{17}$	$\lvert\vec{n_2}\rvert=\sqrt{9+9+25}=\sqrt{43}$

So, $\cos\theta = \left|\frac{(2\hat{i}+2\hat{j}-3\hat{k}).(3\hat{i}-3\hat{j}+5\hat{k})}{\sqrt{17}\times\sqrt{43}}\right|$

$= \left|\frac{(2\times 3)+(2\times -3)+(-3\times 5)}{\sqrt{17\times 43}}\right|$

$= \left|\frac{6-6-15}{\sqrt{731}}\right|$

$= \left|\frac{-15}{\sqrt{731}}\right|$

So, $\cos\theta = \frac{15}{\sqrt{731}}$

$\therefore \theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$

Therefore, the angle between the planes is $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$. **[3]**

Q. 51. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

(a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

(b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$

(c) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

(d) $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

(e) $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

[NCERT Ex. 11.3, Q. 13, Page 494]

Ans. (a) Given that, the two planes are :

$7x+5y+6z+30=0$	$3x-y-10z+4=0$
$7x+5y+6z=-30$	$3x-y-10z=-4$
$-(7x+5y+6z)=30$	$-(3x-y-10z)=4$
$-7x-5y-6z=30$	$-3x+y+10z=4$
Comparing with	Comparing with
$A_1x+B_1y+C_1z=d_1$	$A_2x+B_2y+C_2z=d_2$
Direction ratios of	Direction ratios of
normal $=-7,-5,-6$	normal $=-3,1,10$
$A_1=-7,\ B_1=-5,\ C_1=-6$	$A_2=-3,\ B_2=1,\ C_2=10$

Check Parallel

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are parallel if

$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

So, $\frac{A_1}{A_2}=\frac{-7}{-3}=\frac{7}{3},\quad \frac{B_1}{B_2}=\frac{-5}{1}=-5,\quad \frac{C_1}{C_2}=\frac{-6}{10}=\frac{-3}{5}$

Since $\frac{A_1}{A_2}\neq\frac{B_1}{B_2}\neq\frac{C_1}{C_2}$

So, these two normal planes are not parallel.

$\therefore$Given that, two planes are not parallel.

Check Perpendicular

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are perpendicular if

$A_1A_2 + B_1B_2 + C_1C_2 = 0$

$= \left|\frac{21-5-60}{\sqrt{49+25+36}\sqrt{9+1+100}}\right|$

$= \left|\frac{-44}{\sqrt{110}\sqrt{110}}\right|$

$= \left|\frac{-44}{110}\right| = \left|\frac{-2}{5}\right| = \frac{2}{5}$

Hence, $\cos\theta = \frac{2}{5}$

$\therefore \qquad \theta = \cos^{-1}\left(\frac{2}{5}\right)$

Hence, angle between two planes is $\cos^{-1}\left(\frac{2}{5}\right)$ **[3]**

(b) Given that, the two planes are :

$2x+y+3x-2=0$	$x-2y+5=0$
$2x+1y+3z=2$	$1x-2y=-5$
	$-1x+2y=5$
	$-1x+2y+0z=5$
Comparing with	Comparing with
$A_1x+B_1y+C_1z=d_1$	$A_2x+B_2y+C_2z=d_2$
Direction ratios of	Direction ratios of
normal $=2,1,3$	normal $=-1,2,0$
$A_1=2,\ B_1=1,\ C_1=3$	$A_2=-1,\ B_2=2,\ C_2=0$

Check Parallel

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are parallel if

$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

So, $\frac{A_1}{A_2}=\frac{2}{-1}=-2,\quad \frac{B_1}{B_2}=\frac{1}{2},\quad \frac{C_1}{C_2}=\frac{3}{0}$

Since, direction ratios are not proportional, these two normal planes are not parallel.

$\therefore$ Given that, two planes are not parallel.

Check Perpendicular

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are perpendicular if

$A_1A_2 + B_1B_2 + C_1C_2 = 0$

Now,

$A_1A_2+B_1B_2+C_1C_2=(2\times -1)+(1\times 2)+(3\times 0)$

$=-2+2+0$

$=0$

Since $A_1A_2+B_1B_2+C_1C_2=0$

These two normal planes are perpendicular.

Since normal are perpendicular, planes are perpendicular. **[3]**

(c) Given that, the two planes are :

$2x-2y+4z+5=0$	$3x-3y+6z-1=0$
$2x-2y+4z=-5$	$3x-3y+6z=1$
$-2x+2y-4z=5$	
Comparing with	Comparing with
$A_1x+B_1y+C_1z=d_1$	$A_2x+B_2y+C_2z=d_2$
Direction ratios of	Direction ratios of
normal $=-2,2,-4$	normal $=3,-3,6$
$A_1=-2,\ B_1=2,\ C_1=-4$	$A_2=3,\ B_2=-3,\ C_2=6$

Check Parallel

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are parallel if

$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

Here, $\frac{A_1}{A_2}=\frac{-2}{3}=-2,$

$$\frac{B_1}{B_2} = \frac{2}{-3} = \frac{-2}{3},$$

$$\frac{C_1}{C_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since, $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{-2}{3}$

Therefore, these two normal planes are parallel.

Since normal are parallel, the two planes are parallel. **[3]**

(d) Given that, the two planes are :

$2x - y + 3z - 1 = 0$	$2x - y + 3z + 3 = 0$
$2x - y + 3z = 1$	$2x - y + 3z = -3$
	$-2x + y - 3z = 3$
Comparing with	Comparing with
$A_1x + B_1y + C_1z = d_1$	$A_2x + B_2y + C_2z = d_2$
Direction ratios of	Direction ratios of
normal $= 2, -1, 3$	normal $= -2, 1, -3$
$A_1 = 2,\ B_1 = -1,\ C_1 = 3$	$A_2 = -2,\ B_2 = 1,\ C_2 = -3$

Check Parallel

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are parallel if

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

Here, $\frac{A_1}{A_2} = \frac{2}{-2} = -1$

$$\frac{B_1}{B_2} = \frac{-1}{1} = -1$$

$$\frac{C_1}{C_2} = \frac{3}{-3} = -1$$

Since $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = -1,$

Therefore, these two normal planes are parallel.

Since normal are parallel, the two planes are parallel. **[3]**

(e) Given that, the two planes are :

$4x + 8y + z - 8 = 0$	$y + z - 4 = 0$
$4x + 8y + z = 8$	$y + z = 4$
$4x + 8y + 1z = 8$	$0x + 1y + 1z = 4$
Comparing with	Comparing with
$A_1x + B_1y + C_1z = d_1$	$A_2x + B_2y + C_2z = d_2$
Direction ratios of	Direction ratios of
normal $= 2, -1, 3$	normal $= -2, 1, -3$
$A_1 = 2,\ B_1 = -1,\ C_1 = 3$	$A_2 = -2,\ B_2 = 1,\ C_2 = -3$

Check Parallel

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are parallel if

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

So, $\frac{A_1}{A_2} = \frac{4}{0} = -1, \quad \frac{B_1}{B_2} = \frac{8}{1} = 8, \quad \frac{C_1}{C_2} = \frac{1}{1} = 1$

Since $\frac{A_1}{A_2} \neq \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$

So, these two normal planes are not parallel.

$\therefore$ Given that, two planes are not parallel.

Check Perpendicular

Two lines with direction ratios A_1, B_1, C_1 and A_2, B_2, C_2 are perpendicular if

$A_1A_2 + B_1B_2 + C_1C_2 = 0$

$$A_1A_2 + B_1B_2 + C_1C_2 = (4 \times 0) + (8 \times 1) + (1 \times 1)$$
$$= 0 + 8 + 1$$
$$= 9$$

Since, $A_1A_2 + B_1B_2 + C_1C_2 \neq 0$

Therefore, these two normal planes are not perpendicular.

Hence, the given two planes are not perpendicular.

Finding angle

Now, the angle between two planes $A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$ is given by

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

$$= \left| \frac{(4 \times 0) + (8 \times 1) + (1 \times 1)}{\sqrt{4^2 + 8^2 + 1^2}\sqrt{0^2 + 1^2 + 1^2}} \right|$$

$$= \left| \frac{0 + 8 + 1}{\sqrt{16 + 64 + 1}\sqrt{0 + 1 + 1}} \right|$$

$$= \left| \frac{9}{\sqrt{18}\sqrt{2}} \right| = \frac{9}{9\sqrt{2}} = \frac{1}{\sqrt{2}}$$

So, $\cos\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45°$

Therefore, the angle between the given two planes is 45°. **[3]**

Q. 52. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar.

[CBSE Board, Delhi Region, 2016]

Ans. Given that,

$\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar.

$$\therefore \left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 0$$

i.e., $(\vec{a} + \vec{b}).\left\{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\right\} = 0$

$$(\vec{a} + \vec{b}).\left\{(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a})\right\} = 0$$

$$\vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{b} \times \vec{a}) + \vec{a}.(\vec{c} \times \vec{a})$$
$$+ \vec{b}.(\vec{b} \times \vec{c}) + \vec{b}.(\vec{b} \times \vec{a}) + \vec{b}.(\vec{c} \times \vec{a}) = 0$$

$$2\left[\vec{a}, \vec{b}, \vec{c}\right] = 0 \text{ or } \left[\vec{a}, \vec{b}, \vec{c}\right] = 0$$

Thus, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar. **[4]**

Q. 53. Find the vector and Cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines :

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \text{ and}$$

$$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

[CBSE Board, Delhi Region, 2016]

Ans. Vector equation of the required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu[(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})]$$

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda[(2\hat{i} + 3\hat{j} + 6\hat{k})]$$

In Cartesian form,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

[4]

Long Answer Type Questions (5 and 6 marks each)

Q. 1. Find the coordinate of the point P where the line through *A* (3, –4, –5) and *B* (2, –3, 1) crosses the plane passing through three points *L* (2, 2, 1), *M* (3, 0, 1) and *N* (4, –1, 0). Also, find the ratio in which *P* divides the line segment *AB*.

[CBSE Board, Delhi Region, 2016]

Ans. Equation of the line *AB* :

$$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda$$

Equation of plane *LMN* :

$$\begin{pmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{pmatrix}=0$$

$2(x-2)+1(y-2)+1(z-1)=0$ or $2x+y+z-7=0$

Any point on line *AB* is $(-\lambda+3,\ \lambda-4,\ 6\lambda-5)$.

If this point lies on plane, then

$2(-\lambda+3)+(\lambda-4)+(6\lambda-5)-7=0$

$\Rightarrow \quad 5\lambda=10$

$\Rightarrow \quad \lambda=2$

$\therefore P$ is $(1,-2,7)$.

Let P divides AB into .

$\Rightarrow 1=\dfrac{2K+3}{K+1}\Rightarrow K=-2$ *i.e.*, *P* divides, *AB* externally into $2:1$. **[6]**

Q. 2. Find the distance of the point $(-1,-5,-10)$ from the point of intermission of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\times(\hat{i}-\hat{j}+\hat{k})=5$.

[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$$\vec{r}=2\hat{i}-\hat{j}+2k+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$$

$$\Rightarrow \vec{r}=(2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}$$

Substitute $\vec{r}=(2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}$ in

$\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5.$

$\therefore \left[(2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}\right]\cdot(\hat{i}-\hat{j}+\hat{k})=5$

$\Rightarrow \quad (2+3\lambda)-(-1+4\lambda)\hat{j}+(2+2\lambda)=5$

$\Rightarrow \quad 2+3\lambda+1-4\lambda+2+2\lambda=5$

$\Rightarrow \quad \lambda+5=5$

$\Rightarrow \quad \lambda=0$

Substituting $\lambda=0$ in

$2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}$,

We get $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

Therefore, the coordinates of the points are $2,-1,2$ and $-1,-5,-10$.

The distance between the two points is given by

$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$=\sqrt{9+16+144}$

$=\sqrt{169}$

$=13$ units. **[6]**

Q. 3. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to x-axis.

[NCERT Misc. Ex. Q. 15 , Page 498]

Ans. Equation of a plane passing through the intersection of two planes

$A_1x+B_1y+C_1z=d_1$ and $A_2x+B_2y+C_2z=d_2$ is

$(A_1x+B_1y+C_1z-d_1)+\lambda(A_2x+B_2y+C_2z-d_2)=0.$

Converting equation of planes to Cartesian form to find $A_1,\ B_1,\ C_1,\ d_1$ and $A_2,\ B_2,\ C_2,\ d_2$. **[1]**

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$

Putting,

$\vec{r}=\hat{xi}+y\hat{j}+\hat{zk}$

$(\hat{xi}+y\hat{j}+\hat{zk})\cdot(\hat{i}+\hat{j}+\hat{k})=1$

$(x\times1)+(y\times1)+(z\times1)=1$

$1x+1y+1z=1$

Comparing with

$A_1x+B_1y+C_1z=d_1$

$A_1=1,\ B_1=1,\ C_1=1,\ d_1=1$

Now,

$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$

$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=-4$

$-\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=4$

$\vec{r}\cdot(-2\hat{i}-3\hat{j}+\hat{k})=4$

Putting, $\vec{r}=\hat{xi}+y\hat{j}+z\hat{k}$,

$(\hat{xi}+y\hat{j}+z\hat{k})\cdot(-2\hat{i}-3\hat{j}+1\hat{k})=4$

$(x\times-2)+(y\times-3)+(z\times1)=4$

$-2x-3y+1z=4$

Comparing with

$A_2x+B_2y+C_2z=d_2$

$A_2=-2,\ B_2=-3,\ C_2=1,\ d_2=4$

Equation of plane is :

$(A_1x+B_1y+C_1z-d_1)+\lambda(A_2x+B_2y+C_2z-d_2)=0$

$(1x+1y+1z-1)+\lambda(-2x-3y+1z-4)=0$

$(1-2\lambda)x+(1-3\lambda)y+(1+\lambda)z+(-1-4\lambda)=0$ **(i)**

Also, the plane is parallel to *x*-axis.

So, normal vector $\overrightarrow{N}$ to the plane is perpendicular to *x*-axis. **[1]**

As we know that,

Two lines with direction ratios $a_1,\ b_1,\ c_1$ and $a_2,\ b_2,\ c_2$ are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$.

Finding direction ratio normal and x-axis :

$\overrightarrow{N}=(1-2\lambda)\hat{i}+(1-3\lambda)\hat{j}+(1+\lambda)\hat{k}$

Direction ratios

$=(1-2\lambda),\ (1-3\lambda),\ (1+\lambda)\hat{k}$

$\therefore a_1=1-2\lambda,\ b_1=1-3\lambda,\ c_1=1+\lambda$

Now, for

$\overrightarrow{OX} = 1\hat{i} + 0\hat{j} + 0\hat{k}$

Direction ratios $= 1,\ 0,\ 0$

$\therefore a_2 = 1,\ b_2 = 0,\ c_2 = 0$ **[2]**

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(1-2\lambda)\times 1 + (1-3\lambda)\times 0 + (1+\lambda)\times 0 = 0$$
$$(1-2\lambda) + 0 + 0 = 0$$
$$1 = 2\lambda$$
$$\therefore \qquad \lambda = \frac{1}{2}$$

Putting value of λ in equation (i), we get

$$\left(1-2\cdot\frac{1}{2}\right)x + \left(1-3\cdot\frac{1}{2}\right)y + \left(1+\frac{1}{2}\right) + \left(-1-4\cdot\frac{1}{2}\right) = 0$$
$$(1-1)x + \left(1-\frac{3}{2}\right)y + \left(1+\frac{1}{2}\right)z + (-1-2) = 0$$
$$0x - \frac{1}{2}y + \frac{3}{2}z - 3 = 0$$
$$0x - \frac{1}{2}y + \frac{3}{2}z = 3$$
$$-y + 3z = 6$$
$$0 = y - 3z + 6$$
$$y - 3z + 6 = 0$$

Therefore, the equation of the plane is $y - 3z + 6 = 0$. **[2]**

Q. 4. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. [NCERT Ex. 11.3, Q. 11, Page 493]

Ans. Equation of a plane passing through the insertion of planes $A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$

$$(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$$

Given that the plane passes through,

$x + y + z = 1$

$1x + 1y + 1z = 1$

Comparing with

$A_1x + B_1y + C_1z = d_1$

$A_1 = 1,\ B_1 = 1,\ C_1 = 1,\ d_1 = 1$

For, $2x + 3y + 4z = 5$

Comparing with

$A_2x + B_2y + C_2z = d_2$

$A_2 = 2,\ B_2 = 3,\ C_2 = 4,\ d_2 = 5$

So, the equation of plane is :

$$(A_1x + B_1y + C_1z = d_1) + \lambda(A_2x + B_2y + C_2z = d_2) = 0$$

Putting values

$$(1x + 1y + 1z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$
$$x + y + z - 1 + 2\lambda x + 3\lambda y + 4\lambda z - 5\lambda = 0$$
$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z + (-1-5\lambda) = 0 \quad \text{...(i)}$$

Also, the plane is perpendicular to the plane $x - y + z = 0$.

So, the normal vector $\vec{N}$ to the plane is perpendicular to the normal vector of $x - y + z = 0$.

As we know that, two lines with direction ratios $a_1,\ b_1,\ c_1$ and $a_2,\ b_2,\ c_2$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$. **[2½]**

$\vec{N} = (1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}$

Direction ratio $= 1+2\lambda,\ 1+3\lambda,\ 1+4\lambda$

$\therefore a_1 = 1+2\lambda,\ b_1 = 1+3\lambda,\ c_1 = 1+4\lambda$

For, $\vec{n} = 1\hat{i} - 1\hat{j} + 1\hat{k}$

Direction ratio $= 1,\ -1,\ 1$

$\therefore a_2 = 1,\ b_2 = -1,\ c_2 = 1$

Since, $\vec{N}$ is perpendicular to $\vec{n}$,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(1+2\lambda)\times 1 + (1+3\lambda)\times -1 + (1+4\lambda)\times 1 = 0$$
$$1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$$
$$1 + 3\lambda = 0$$
$$-1 = 3\lambda$$
$$\therefore \qquad \lambda = \frac{-1}{3}$$

Putting value of **λ** in equation (i), we get

$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z + (-1-5\lambda) = 0$$
$$\left(1+2\times\frac{-1}{3}\right)x + \left(1+3\times\frac{-1}{3}\right)y + \left(1+4\times\frac{-1}{3}\right)z + \left(-1-5\times\frac{-1}{3}\right) = 0$$
$$\left(1-\frac{2}{3}\right)x + (1-1)y + \left(1-\frac{4}{3}\right)z + \left(-1+\frac{5}{3}\right) = 0$$
$$\frac{1}{3}x + 0y - \frac{1}{3}z + \frac{2}{3} = 0$$
$$\frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$$
$$\frac{1}{3}(x - z + 2) = 0$$
$$x - z + 2 = 0$$

Therefore, the equation of plane is $x - z + 2 = 0$. **[2½]**

Q. 5. Find the equations of the planes that pass through three points.

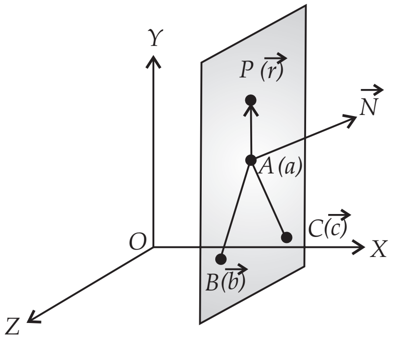
(a) (1, 1, –1), (6, 4, –5), (–4, –2, 3)

(b) (1, 1, 0), (1, 2, 1), (–2, 2, –1)

[NCERT Ex. 11.3, Q. 6, Page 493]

Ans. **(a)** Vectors equation of plane passing through three points with position vectors $\vec{a},\ \vec{b},\ \vec{c}$ is

$$(\vec{r} - \vec{a})\cdot\left[(\vec{b} - \vec{a})\times(\vec{c} - \vec{a})\right] = 0$$



Vectors perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC}$

$= \overrightarrow{AB} \times \overrightarrow{AC}$

So, $\vec{N} = \overrightarrow{AB} \times \overrightarrow{AC}$

Also, $\overrightarrow{AP}$ is perpendicular to $\vec{N}$,

So, $\overrightarrow{AP}\cdot\vec{N} = 0$

$$(\vec{r} - \vec{a})\cdot\vec{N} = 0$$
$$(\vec{r} - \vec{a})\cdot(\overrightarrow{AB} \times \overrightarrow{AC}) = 0$$
$$(\vec{r} - \vec{a})\cdot\left[(\vec{b} - \vec{a})\times(\vec{c} - \vec{a})\right] = 0$$

Vector equation of plane passing through three points with position vectors $\vec{a},\vec{b},\vec{c}$ is

$(\vec{r}-\vec{a})\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$

Now, plane passes through the points

$A\ (1,\ 1,\ -1)$

$\vec{a}=1\hat{i}+1\hat{j}-1\hat{k}$

$B\ (6,\ 4,\ -5)$

$\vec{b}=6\hat{i}+4\hat{j}-5\hat{k}$

$C\ (-4,\ -2,\ 3)$

$\vec{c}=-4\hat{i}-2\hat{j}+3\hat{k}$

$(\vec{b}-\vec{a})=(6\hat{i}+4\hat{j}-5\hat{k})-(1\hat{i}+1\hat{j}-1\hat{k})$

$=(6-1)\hat{i}+(4-1)\hat{j}+(-5-(-1))\hat{k}$

$=5\hat{i}+3\hat{j}-4\hat{k}$

$(c-a)=(-4\hat{i}-2\hat{j}+3\hat{k})-(1\hat{i}+1\hat{j}-1\hat{k})$

$=(-4-1)\hat{i}+(-2-1)\hat{j}+(-3-(-1))\hat{k}$

$=-5\hat{i}-3\hat{j}+4\hat{k}$

$$(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5&3&-4\\-5&-3&4\end{vmatrix}$$

$$=-\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5&3&-4\\5&3&-4\end{vmatrix}$$

$=0$

As we know that if two rows of determinant are same, the value of determinant is zero.

This implies, the three points are collinear.

$\therefore$ Vector equation of plane is :

$\left[\vec{r}-(\hat{i}+\hat{j}-\hat{k})\right]\cdot\vec{0}=0$

Since, the above equation is satisfied for all values of $\vec{r}$.

Therefore, there will be infinite planes through the given three collinear points. **[5]**

(b) Vectors equation of plane passing through three points with position vectors $\vec{a},\vec{b},\vec{c}$ is

$(\vec{r}-\vec{a})\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$

Now, the plane passing through the points

$A\ (1,\ 1,\ 0)$

$\vec{a}=1\hat{i}+1\hat{j}+0\hat{k}$

$B\ (1,\ 2,\ 1)$

$\vec{b}=1\hat{i}+2\hat{j}+1\hat{k}$

$C\ (-2,\ 2,\ -1)$

$\vec{c}=-2\hat{i}+2\hat{j}-1\hat{k}$

$(\vec{b}-\vec{a})=(1\hat{i}+2\hat{j}+1\hat{k})-(1\hat{i}+1\hat{j}+0\hat{k})$

$=(1-1)\hat{i}+(2-1)\hat{j}+(1-0)\hat{k}$

$=0\hat{i}+1\hat{j}+1\hat{k}$

$(\vec{c}-\vec{a})=(-2\hat{i}+2\hat{j}+1\hat{k})-(1\hat{i}+1\hat{j}+0\hat{k})$

$=(-2-1)\hat{i}+(2-1)\hat{j}+(-1-0)\hat{k}$

$=-3\hat{i}+1\hat{j}-1\hat{k}$

$$(\vec{b}\times\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&1\\-3&1&-1\end{vmatrix}$$

$=\hat{i}\left[(1\times-1)-(1\times1)\right]-\hat{j}\left[(0\times-1)-(-3\times1)\right]$
$+\hat{k}\left[(0\times1)-(-3\times1)\right]$

$=\hat{i}(-1-1)-\hat{j}(0+3)+\hat{k}(0+3)$

$=-2\hat{i}-3\hat{j}+3\hat{k}$

$\therefore$ Vector equation of plane is :

$\left[\vec{r}-(1\hat{i}+1\hat{j}+0\hat{k})\right]\cdot(-2\hat{i}-3\hat{j}+3\hat{k})=0$

$\left[\vec{r}-(\hat{i}+\hat{j})\right]\cdot(-2\hat{i}-3\hat{j}+3\hat{k})=0$

$\left[\vec{r}-(\hat{i}+\hat{j})\right]\cdot(-2\hat{i}-3\hat{j}+3\hat{k})=0$

Finding Cartesian equation :

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\left[\vec{r}-(\hat{i}+\hat{j})\right]\cdot(-2\hat{i}-3\hat{j}+3\hat{k})=0$

$\left[(x\hat{i}+y\hat{j}+z\hat{k})-(\hat{i}+\hat{j})\right]\cdot(-2\hat{i}-3\hat{j}+3\hat{k})=0$

$\left[(x-1)\hat{i}+(y-1)\hat{j}+z\hat{k}\right].(-2\hat{i}-3\hat{j}+3\hat{k})=0$

$-2(x-1)+(-3)(y-1)+3(z)=0$

$-2x+2-3y+3+3z=0$

$2x+3y-3z=5$

$\therefore$ Equation of plane in Cartesian form is $2x+3y-3z=5$. **[5]**

Q. 6. Find the shortest distance between the lines $\dfrac{x+1}{7}=\dfrac{y+1}{-6}=\dfrac{z+1}{1}$ and $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z-7}{1}$.

[NCERT Ex. 11.2, Q. 15, Page 478]

Ans. Method I : By Cartesian Method

Shortest distance between two lines,

$l_1:\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$

$l_2:\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$

$$\text{is}\left|\frac{\begin{vmatrix}x_2-x_1&y_2-y_1&z_2-z_1\\a_1&b_1&c_1\\a_2&b_2&c_2\end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}\right|$$

[1]

Now solve for,

$\dfrac{x+1}{7}=\dfrac{y+1}{-6}=\dfrac{z+1}{1}$

$\dfrac{x-(-1)}{7}=\dfrac{y-(-1)}{-6}=\dfrac{z-(-1)}{1}$

Comparing with

$l_1:\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$

$x_1=-1,\ y_1=-1,\ z_1=-1$

and $a_1=7,\ b_1=-6,\ c_1=1$

Now solve for,

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{x-7}{1}$$

Comparing with

$$l_2:\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

$$x_2=3,\ y_2=5,\ z_2=7,$$

and $a_2=1,\ b_2=-2,\ c_2=1$

$$d=\left|\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}\right|$$ **[2]**

$$=\left|\frac{\begin{vmatrix}3-(-1) & 5-(-1) & 7-(-1)\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}}{\sqrt{[7(-2)-1(-6)]^2+[-6(1)-(-2)1]^2+[1(1)-1(7)]^2}}\right|$$

$$d=\left|\frac{\begin{vmatrix}4 & 6 & 8\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}}{\sqrt{(-14+6)^2+(-6+2)^2+(1-7)^2}}\right|$$

$$=\left|\frac{\begin{vmatrix}4 & 6 & 8\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}}{\sqrt{(8)^2+(-4)^2+(-6)^2}}\right|$$

$$=\left|\frac{\begin{vmatrix}4 & 6 & 8\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}}{\sqrt{116}}\right|$$

$$=\left|\frac{4(-6(1)-(-2)1)-6(7(1)-1(1))+8(7(-2)-1(-6))}{\sqrt{116}}\right|$$

$$=\left|\frac{4(-6+2)-6(7-1)+8(-14+6)}{\sqrt{116}}\right|=\left|\frac{-16-36-64}{\sqrt{116}}\right|$$

$$=\left|\frac{-116}{\sqrt{116}}\right|=\left|-\sqrt{116}\right|=\sqrt{116}=\sqrt{4\times 29}=2\sqrt{29}$$ **[2]**

Q. 7. Find the shortest distance between the lines whose vector equations are $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

[NCERT Ex. 11.2, Q. 16, Page 478]

Ans. Shortest distance between the lines with vector equations

$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is

$$\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

Given that,

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$$

Comparing with $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$,

$\vec{a_1}=1\hat{i}+2\hat{j}+3\hat{k}$ and $b_1=1\hat{i}-3\hat{j}+2\hat{k}$

Similarly, $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$

Comparing with

$$\vec{r}=\vec{a_2}+\mu\vec{b_2}$$

$$\vec{a_2}=4\hat{i}+5\hat{j}+6\hat{k}$$

and $\vec{b_2}=2\hat{i}+3\hat{j}+1\hat{k}$

Now, $(\vec{a_2}-\vec{a_1})=(4\hat{i}+5\hat{j}+6\hat{k})-(1\hat{i}+2\hat{j}+3\hat{k})$

$$=(4-1)\hat{i}+(5-2)\hat{j}+(6-3)\hat{k}$$

$$=3\hat{i}+3\hat{j}+3\hat{k}$$

$$(\vec{b_1}\times\vec{b_2})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$$=\hat{i}[(-3\times 1)-(-3\times 2)]-\hat{j}[(1\times 1)-(2\times 2)]+\hat{k}[(1\times 3)-(2\times -3)]$$

$$=\hat{i}[-3-6]-\hat{j}[1-4]+\hat{k}[3+6]$$

$$=\hat{i}(-9)-\hat{j}(-3)+\hat{k}(-9)$$

$$=-9\hat{i}+3\hat{j}+9\hat{k}$$

Magnitude of $(\vec{b_1}\times\vec{b_2})=\sqrt{(-9)^2+3^2+9^2}$

$$|\vec{b_1}\times\vec{b_2}|=\sqrt{81+9+81}$$

$$=\sqrt{171}=\sqrt{9\times 19}=3\sqrt{19}$$ **[2½]**

Also, $(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})$

$$=(-9\hat{i}+3\hat{j}+9\hat{k})\cdot(3\hat{i}+3\hat{j}+3\hat{k})$$

$$=(-9\times 3)+(3\times 3)+(9\times 3)$$

$$=-27+9+27$$

$$=9$$

So, shortest distance $=\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{\vec{b_1}\times\vec{b_2}}\right|$

$$=\left|\frac{9}{3\sqrt{19}}\right|=\frac{3}{\sqrt{19}}$$

Therefore, shortest distance between the given two lines is $\frac{3}{\sqrt{19}}$. **[2½]**

Q. 8. Find the shortest distance between the lines whose vectorequationsare $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$.

[NCERT Ex. 11.2, Q. 17, Page 478]

Ans. Shortest distance lines with vector equation $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is

$$\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$$
$$=1\hat{i}-t\hat{i}+t\hat{j}-2\hat{j}+3\hat{k}-2+\hat{k}$$
$$=(1\hat{i}-2\hat{j}+3\hat{k})+t(-1\hat{i}+1\hat{j}-2\hat{k})$$

Compare with $\vec{r}=\vec{a_1}+t\vec{b_1}$, $\vec{a_1}=1\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=-1\hat{i}+1\hat{j}-2\hat{k}$

$$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$$
$$=s\hat{i}+1\hat{i}+2s\hat{j}-1\hat{j}-2s\hat{k}-1\hat{k}$$
$$=(1\hat{i}-1\hat{j}-1\hat{k})+s(1\hat{i}+2\hat{j}-2\hat{k})$$

Comparing with $\vec{r}=\vec{a_2}+s\vec{b_2}$,

$\vec{a_2}=1\hat{i}-1\hat{j}-1\hat{k}$ and $\vec{b_2}=1\hat{i}+2\hat{j}-2\hat{k}$ **[1]**

Now, $(\vec{a_2}-\vec{a_1})=(1\hat{i}-1\hat{j}-1\hat{k})-(1\hat{i}-2\hat{j}+3\hat{k})$

$$=(1-1)\hat{i}+(-1+2)\hat{j}+(-1-3)\hat{k}$$
$$=0\hat{i}+1\hat{j}-4\hat{k}$$

$$(\vec{b_1}\times\vec{b_2})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -2\\ 1 & 2 & -2\end{vmatrix}$$

$$=\hat{i}[(1\times-2)-(2\times-2)]-\hat{j}[(-1\times-2)(1\times-2)]+\hat{k}[(-1\times2)-(1\times1)]$$
$$=\hat{i}[-2+4]-\hat{j}[2+2]A+\hat{k}[-2-1]$$
$$=2\hat{i}-4\hat{j}-3\hat{k}$$

Magnitude of $(\vec{b_1}\times\vec{b_2})=\sqrt{2^2+(-4)^2+(-3)^2}$

$$|\vec{b_1}\times\vec{b_2}|=\sqrt{4+16+9}=\sqrt{29}$$

Also, $(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})$

$$=(2\hat{i}-4\hat{j}-3\hat{k})\cdot(0\hat{i}+1\hat{j}-4\hat{k})$$
$$=(2\times0)+(-4\times1)+(-3\times-4)$$
$$=-0+(-4)+12$$
$$=8$$ **[2]**

So, shortest distance $=\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(a_2-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$

$$=\left|\frac{8}{\sqrt{29}}\right|=\frac{8}{\sqrt{29}}$$

Therefore, shortest distance between the given two lines is $\frac{8}{\sqrt{29}}$. **[2]**

Q. 9. Show that the three lines with direction cosines $\frac{12}{13},\frac{-3}{13},\frac{-4}{13};\frac{4}{13},\frac{12}{13},\frac{3}{13};\frac{3}{13},\frac{-4}{13},\frac{12}{13}$ are mutually perpendicular. [NCERT Ex. 11.2, Q. 1, Page 477]

Ans. Two lines with directional cosines l_1,m_1,n_1 and l_2,m_2,n_2 are perpendicular to each other if

$$l_1l_2+m_1m_2+n_1n_2=0$$

Line 1 :

$$l_1=\frac{12}{13},m_1=\frac{-3}{13},n_1=\frac{-4}{13}$$

Line 2 :

$$l_2=\frac{4}{13},m_2=\frac{12}{13},n_2=\frac{3}{13}$$

Now find,

$$\Rightarrow l_1l_2+m_1m_2+n_1n_2$$
$$=\left(\frac{12}{13}\times\frac{4}{13}\right)+\left(\frac{-3}{13}\times\frac{12}{13}\right)+\left(\frac{-4}{13}\times\frac{3}{13}\right)$$
$$=\frac{48}{169}+\left(\frac{-36}{169}\right)+\left(\frac{-12}{169}\right)$$
$$=\frac{48-36-12}{169}=\frac{48-48}{169}=0$$ **[1]**

$$l_1l_2+m_1m_2+n_1n_2=0$$

Hence, two lines are perpendicular.

Line 2 :

$$l_2=\frac{4}{13},m_2=\frac{12}{13},n_2=\frac{3}{13}$$

Line 3 :

$$l_3=\frac{3}{13},m_3=\frac{-4}{13},n_3=\frac{12}{13}$$

Now,

$$\Rightarrow l_2l_3+m_2m_3+n_2n_3$$
$$=\left(\frac{4}{13}\times\frac{3}{13}\right)+\left(\frac{12}{13}\times\frac{-4}{13}\right)+\left(\frac{3}{13}\times\frac{12}{13}\right)$$
$$=\frac{12}{169}+\left(\frac{-48}{169}\right)+\frac{36}{169}$$
$$=\frac{12-48+36}{169}$$
$$=\frac{48-48}{169}$$
$$=0$$

$$\therefore l_2l_3+m_2m_3+n_2n_3=0$$

Hence, two lines are perpendicular. **[2]**

Line 3 :

$$l_3=\frac{3}{13},m_3=\frac{-4}{13},n_3=\frac{12}{13}$$

Line 1 :

$$l_1=\frac{12}{13},m_1=\frac{-3}{13},n_1=\frac{-4}{13}$$

Now,

$$\Rightarrow l_1l_3+m_1m_3+n_1n_3$$
$$=\left(\frac{3}{13}\times\frac{12}{13}\right)+\left(\frac{-4}{13}\times\frac{-3}{13}\right)+\left(\frac{12}{13}\times\frac{-4}{13}\right)$$
$$=\frac{36}{169}+\frac{12}{169}+\left(\frac{-48}{169}\right)$$
$$=\frac{36+12-48}{169}$$
$$=\frac{48-48}{169}$$
$$=0$$

$$\therefore l_1l_3+m_1m_3+n_1n_3=0$$

Hence, two lines are perpendicular.

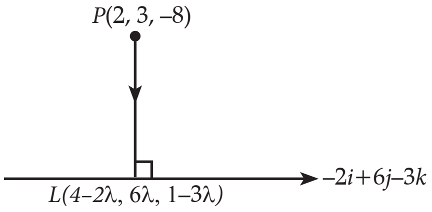
Therefore, the given three lines are mutually perpendicular. **[2]**

Q. 10. Find the foot of perpendicular from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line. [NCERT Exemp. Ex. 11.3, Q. 16, Page 236]

Ans. We have equation of line as $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

$\Rightarrow \frac{4-x}{-2}=\frac{y}{6}=\frac{1-z}{-3}=\lambda$

$\Rightarrow x=-2\lambda+4,\ y=6\lambda$ and $z=-3\lambda+1$



Let the foot of perpendicular from point $P\ (2,3-8)$ on the line is $L\ (4-2\lambda,\ 6\lambda,\ 1-3\lambda)$.

Then the direction ratios of PL are proportional to $(4-2\lambda-2,\ 6\lambda-3,\ 1-3\lambda+8)$ or $(2-2\lambda,\ 6\lambda-3,\ 9-3\lambda)$.

Also, direction ratios of line are $-2, 6, -3$.

Since, PL is perpendicular to the given line.

$\therefore -2(2-2\lambda)+6(6\lambda-3)-3(9-3\lambda)=0$

$\Rightarrow \quad -4+4\lambda+36\lambda-18-27+9\lambda=0$

$\Rightarrow \quad 49\lambda=49$

$\Rightarrow \quad \lambda=1$

So, the coordinates of L are $L\ (4-2\lambda,\ 6\lambda,\ 1-3\lambda)\equiv(2,6,-2)$.

Also, length of PL $=\sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}$

$=\sqrt{0+9+36}=3\sqrt{5}$ units **[5]**

Q. 11. Find the distance of a point (2, 4, –1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$. [NCERT Exemp. Ex. 11.3, Q. 17, Page 236]

Ans. We have, equation of line as $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$

$x=\lambda-5,\ y=4\lambda-3,\ z=6-9\lambda$

Let the coordinates of L are $(\lambda-5,\ 4\lambda-3,\ 6-9\lambda)$.

Then, direction ratios of PL are $(\lambda-5-2,\ 4\lambda-3-4,\ 6-9\lambda+1)$ or $(\lambda-7,\ 4\lambda-7,\ 7-9\lambda)$.

Also, the direction ratios of the given line are $1,4,-9$.

Since, PL is perpendicular to the given line.

$\therefore (\lambda-7)\cdot 1+(4\lambda-7)\cdot 4+(7-9\lambda)\cdot(-9)=0$

$\Rightarrow \quad \lambda-7+16\lambda-28+81\lambda-63=0$

$\Rightarrow \quad 98\lambda=98$

$\Rightarrow \quad \lambda=1$

So, the coordinates of L are $(\lambda-5,\ 4\lambda-3,\ 6-9\lambda)\equiv(-4,1,-3)$.

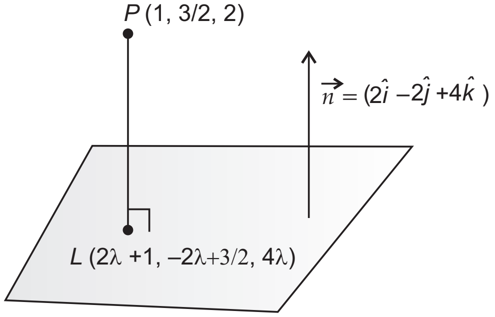
$\therefore$ Also $PL=\sqrt{(-4-2)^2+(1-4)^2+(-3+1)^2}$

$=\sqrt{36+9+4}=7$ units **[5]**

Q. 12. Find the length and the foot of perpendicular from the point (1, $\frac{3}{2}$, 2) to the plane $2x-2y+4z+5=0$. [NCERT Exemp. Ex. 11.3, Q. 18, Page 236]

Ans. Equation of the given plane is $2x-2y+4z+5=0$.

Normal to the plane is $\vec{n}=2\hat{i}-2\hat{j}+4\hat{k}$.



So, the equation of line through $P\left(1,\ \frac{3}{2},\ 2\right)$ and parallel to $\vec{n}$ is given by

$\frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}=\lambda$

Any point on this line is $\left(2\lambda+1,-2\lambda+\frac{3}{2},4\lambda+2\right)$.

If this point lies on the given plane (point L), then

$2(2\lambda+1)-2\left(-2\lambda+\frac{3}{2}\right)+4(4\lambda+2)+5=0$

$\Rightarrow \quad 4\lambda+2+4\lambda-3+16\lambda+8+5=0$

$\Rightarrow \quad 24\lambda=-12$

$\Rightarrow \quad \lambda=\frac{-1}{2}$

$\therefore$ Required foot of perpendicular

$\left(2\lambda+1,-2\lambda+\frac{3}{2},4\lambda+2\right)=\left(0,\frac{5}{2},0\right)\left(\text{Putting } \lambda=-\frac{1}{2}\right)$

$\therefore$ Required length of perpendicular

$=\sqrt{(1-0)^2+\left(\frac{3}{2}-\frac{5}{2}\right)^2+(2-0)^2}$

$=\sqrt{1+1+4}=\sqrt{6}$ units **[5]**

Q. 13. Find the equations of the line passing through the point (3, 0, 1) and parallel to the planes $x+2y=0$ and $3y-z=0$. [NCERT Exemp. Ex. 11.3, Q. 19, Page 236]

Ans. Equation of two planes are $x+2y=0$ and $3y-z=0$.

Normal to the planes are $\vec{n_1}=\hat{i}+2\hat{j}$ and $\vec{n_2}=3\hat{j}-\hat{k}$, respectively.

Since, required line is parallel to the given two planes, it is perpendicular to $\vec{n_1}$ and $\vec{n_2}$.

Therefore, line is parallel to the vector

$\vec{b}=\vec{n_1}\times\vec{n_2}$

$=\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 3 & -1 \end{vmatrix}$

$=-2\hat{i}+\hat{j}+3\hat{k}$

So, the equation line passes through the point $(3,0,1)$ and is also parallel to the point. The parallel to the given two plane is $\frac{x-3}{-2}=\frac{y}{1}=\frac{z-1}{3}$. **[5]**

Q. 14. Find the equation of the plane through the points (2, 1, –1) and (–1, 3, 4), and perpendicular to the plane $x - 2y + 4z = 10$.

[NCERT Exemp. Ex. 11.3, Q. 20, Page 237]

Ans. The equation of the plane passing through the points $(2,1,-1)$ is

$a(x-2)+b(y-1)+c(z+1)=0$ ...(i)

Since, this plane passes through the points $(-1,3,4)$.

$\therefore a(-1-2)+b(3-1)+c(4+1)=0$

$\Rightarrow -3a+2b+5c=0$ ...(ii)

Since, the plane in equation (i) is perpendicular to the plane $x-2y+4z=10$.

$\therefore 1\cdot a-2\cdot b+4\cdot c=0$

$\Rightarrow a-2b+4c=0$ ...(iii)

On solving equations (ii) and (iii) by cross-multiplication method, we get

$$\frac{a}{8+10}=\frac{-b}{-17}=\frac{c}{4}=\lambda$$

$\Rightarrow a=18\lambda,\ b=17\lambda,\ c=4\lambda$

From equation (i), we have

$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$

$\Rightarrow \quad 18x-36+17y-17+4z+4=0$

$\therefore \quad 18x+17y+4z=49$ **[5]**

Q. 15. Find the shortest distance between the lines given by $\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$ and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

[NCERT Exemp. Ex. 11.3, Q. 21, Page 237]

Ans. We have,

$\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$

$=8\hat{i}-9\hat{j}+10\hat{k}+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$

$\Rightarrow \vec{a_1}=8\hat{i}-9\hat{j}+10\hat{k}$ and $\vec{b_1}=(3\hat{i}-16\hat{j}+7\hat{k})$ ...(i)

Also, $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

$\vec{a_2}=15\hat{i}+29\hat{j}+5\hat{k}$ and $\vec{b_2}=(3\hat{i}+8\hat{j}-5\hat{k})$ ...(ii)

Now, shortest distance between two lines is given by

$$=\left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$\therefore\ \vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}$$

$=\hat{i}(80-56)-\hat{j}(-15-21)+\hat{k}(24+48)$

$=24\hat{i}+36\hat{j}+72\hat{k}$

$\Rightarrow |\vec{b_1}\times\vec{b_2}|=\sqrt{24^2+36^2+72^2}$

$=12\sqrt{2^2+3^2+6^2}$

$=84$

Now, $(\vec{a_2}-\vec{a_1})=(15-8)\hat{i}+(29+9)\hat{j}+(5-10)\hat{k}$

$=7\hat{i}+38\hat{j}-5\hat{k}$

$\therefore$ Shortest distance

$$=\left|\frac{(24\hat{i}+36\hat{j}+72\hat{k})\cdot(7\hat{i}+38\hat{j}-5\hat{k})}{84}\right|$$

$$=\left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\cdot(7\hat{i}+38\hat{j}-5\hat{k})}{7}\right|$$

$$=\left|\frac{14+144-30}{7}\right|=14$$ **[5]**

Q. 16. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

[NCERT Exemp. Ex. 11.3, Q. 22, Page 237]

Ans. The equation of a plane passing through the lines of intersection of planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is $(x+2y+3z-4)+\lambda(2x+y-z+5)=0$.

$\Rightarrow x(1+2\lambda)+y(2+\lambda)+z(-\lambda+3)-4+5\lambda=0$ ...(i)

Also, this is perpendicular to the plane $5x+3y+6z+8=0$.

$\therefore\ 5+10\lambda+6+3\lambda+18-6\lambda=0$

$$\Rightarrow \qquad \lambda=\frac{-29}{7}$$

Putting this value of λ in equation (i), we get equation of plane as :

$51x+15y-50z+173=0$ **[5]**

Q. 17. The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $ax+by\pm\left(\sqrt{a^2+b^2}\tan\alpha\right)z=0$.

[NCERT Exemp. Ex. 11.3, Q. 23, Page 237]

Ans. Given that,

Plans are $ax+by=0$ ...(i)

and $z=0$ ...(ii)

$\therefore$ Equation of any plane passing through the line of intersection of planes in equations (i) and (ii) may be taken as,

$ax+by+kz=0$

The directional cosines of a normal to the plane in equation (iii) are :

$$\frac{a}{\sqrt{a^2+b^2+k^2}},\frac{b}{\sqrt{a^2+b^2+k^2}},\frac{k}{\sqrt{a^2+b^2+k^2}}$$

The directional cosines of a normal to the plane in equation (i) are $\frac{a}{\sqrt{a^2+b^2}},\frac{b}{\sqrt{a^2+b^2}},0$

Since the angle between the planes in equation (i) and (ii) is α,

$$\therefore \cos\alpha=\frac{a\cdot a+b\cdot b+k\cdot 0}{\sqrt{a^2+b^2+k^2}\sqrt{a^2+b^2}}=\sqrt{\frac{a^2+b^2}{\sqrt{a^2+b^2+k^2}}}$$

$\Rightarrow k^2\cos^2\alpha=a^2(1-\cos^2\alpha)+b^2(1-\cos^2\alpha)$

$$\Rightarrow \qquad k^2=\frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}$$

$\Rightarrow \qquad k=\pm\sqrt{a^2+b^2}\tan\alpha$

Putting the value of k in equation (iii), we get equation of plane as $ax+by\pm z\sqrt{a^2+b^2}\tan\alpha=0$. **[5]**

Q. 18. Find the equation of the plane through the line of intersection of $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$. Hence find whether

the plane thus obtained contains the line $x-1=2y-4=3z-12$.

[CBSE Board, Delhi Region, 2017]

Ans. Equation of family of planes,

$$\vec{r}\cdot\left[\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}\right)\right]=1-4\lambda$$

$$\Rightarrow \vec{r}\cdot\left[(2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right]=1-4\lambda \quad ...(i)$$

Plane in equation (i) is perpendicular to $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)+8=0$.

$$2(2+\lambda)-1(-3-\lambda)+1(4)=0$$

$$\Rightarrow \qquad \lambda=-\frac{11}{3}$$

Substituting $\lambda=-\frac{11}{3}$ in equation (i), we get

$$\vec{r}\cdot\left(-\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+4\hat{k}\right)=\frac{47}{3}$$

$$\Rightarrow \vec{r}\cdot\left(-5\hat{i}+2\hat{j}+12\hat{k}\right)=47 \quad \text{[Vector equation]}$$

$$-5x+2y+12z-47=0 \quad \text{[Cartesian equation]} \quad (ii)$$

Line $\frac{x-1}{1}=\frac{y-2}{\frac{1}{2}}=\frac{z-2}{\frac{1}{3}}$ lies on the plane in equation (i) at point $P\ (1,\ 2,\ 4)$ satisfies the equation (ii) and $a_1a_2+b_1b_2+c_1c_2=-5+1+4=0$.

$\Rightarrow$ Line is perpendicular to normal plane.

$\therefore$ Plane contains the given line. **[6]**

Q. 19. Find the vector and Cartesian equations of a line passing through (1, 2, −4) and perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

[CBSE Board, Delhi Region, 2017]

Ans. Equation of L_1 passing through the points $(1, 2, -4)$ is

$$L_1:\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c}$$

$$L_2:\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$$

$$L_3:\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

$$\because L_1\perp L_2 \Rightarrow 3a-16b+7c=0$$

$$L_1\perp L_3 \Rightarrow 3a+8b-5c=0$$

Solving we get,

$$\frac{a}{24}=\frac{b}{36}=\frac{c}{72}$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$$

$\therefore$ Required Cartesian equation of line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$.

Vector equation $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$. **[6]**

Q. 20. Find the vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$. Hence find whether the plane thus obtained contains the line $\frac{x+2}{5}=\frac{y-3}{4}=\frac{z}{5}$ or not.

[CBSE Board, Foreign Region, 2017]

Ans. Equation of the plane passing through the intersecting of planes is :

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$

$$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-(1+5\lambda)=0 \quad ...(i)$$

This plane is perpendicular to $x-y+z=0$.

$$\therefore 1(1+2\lambda)-1(1+3\lambda)+1(1+4\lambda)=0$$

$$\Rightarrow \lambda=\frac{-1}{3}$$

$\therefore$ Equation of plane is :

$$(x+y+z-1)-\frac{1}{3}(2x+3y+4z-5)=0$$

$$\Rightarrow x-z+2=0$$

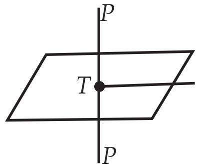
Vector form of plane as $\vec{r}\cdot\left(\hat{i}-\hat{k}\right)+2=0$.

Yes, lies line on plane as $(-2,3,0)$ satisfies $\vec{r}\cdot\left(\hat{i}-\hat{k}\right)+2=0$ and normal to plane is perpendicular to the given line is $1(5)+0(4)-1(5)=0$. **[6]**

Q. 21. Find the image P' of the point P having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$. Hence find the length of PP'.

[CBSE Board, Foreign Region, 2017]

Ans. Let PT is perpendicular to the given plane.



Let position vector of T is $\vec{b_1}=a\hat{i}+b\hat{j}+c\hat{k}$.

$$\therefore \overrightarrow{PT}=(a-1)\hat{i}+(b-3)\hat{j}+(c-4)\hat{k}$$

$$\overrightarrow{PT}\parallel\vec{n}\ \text{(normal)}$$

$$\therefore \frac{a-1}{-2}=\frac{b-3}{1}=\frac{c-4}{-1}=\lambda$$

$$\Rightarrow \quad a=-2\lambda+1,\ b=\lambda+3 \text{ and } c=-\lambda+4$$

$$\therefore \quad \vec{b_1}=(-2\lambda+1)\hat{i}+(\lambda+3)\hat{j}+(-\lambda+4)\hat{k}$$

$\vec{b_1}$ lies on plane.

$$\therefore \left[(-2\lambda+1)\hat{i}+(\lambda+3)\hat{j}+(-\lambda+4)\hat{k}\right]\cdot\left(-2\hat{i}+\hat{j}-\hat{k}\right)=3$$

$$\Rightarrow \qquad \lambda=1$$

$$\therefore \vec{b_1}=-\hat{i}+4\hat{j}+3\hat{k}$$

Let position vector of P' is $\vec{c_1}=x\hat{i}+y\hat{j}+z\hat{k}$.

Using Section Formula, we have

$$\vec{c_1}=-3\hat{i}+5\hat{j}+2\hat{k}$$

Also, $PP'=\sqrt{24}$ or $2\sqrt{6}$. **[6]**

Q. 22. Find the equation of the plane through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}\cdot\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$ whose perpendicular distance from origin is unity.

[NCERT Exemp. Ex. 11.3, Q. 24, Page 237]

Ans. Given that,

Planes are $\vec{r}\cdot\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}\cdot\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$.

Equation of family of planes passing through the intersection of these planes is

$$\vec{r}\cdot\left(\hat{i}+3\hat{j}\right)-6+\lambda\left[\vec{r}\cdot\left(3\hat{i}-\hat{j}-4\hat{k}\right)\right]=0$$

$$\Rightarrow \vec{r}\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}+\hat{k}(-4\lambda)\right]=6 \quad ...(i)$$

$$\Rightarrow \frac{\vec{r}\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}+\hat{k}(-4\lambda)\right]}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}$$

$$\Rightarrow \frac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}$$

Since, the perpendicular distance from the origin is unity.

$$\therefore \quad \frac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}=1$$

$$\Rightarrow \quad (1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2=36$$

$$\Rightarrow \quad 1+9\lambda^2+6\lambda+9+\lambda^2-6\lambda+16\lambda^2=36$$

$$\Rightarrow \quad \lambda^2=1$$

$$\therefore \quad \lambda=\pm1$$

$\therefore$ Using equation (i), the required plane is :

$$\vec{r}\cdot\left[(1\pm3)i+(3\mp1)j+(\mp4)\right]=6$$

$$\Rightarrow \vec{r}\cdot\left(4\hat{i}+2\hat{j}-4\hat{k}\right)=6 \text{ and } \vec{r}\cdot\left(-2\hat{i}+4\hat{j}+4\hat{k}\right)=6$$

Or, $4x+2y-4z-6=0$ and $-2x+4y+4z-6=0$ **[5]**

Q. 23. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

[NCERT Misc. Ex. Q. 21, Page 499]

Ans. Distance of the points (x_1, y_1, z_1) from the plane $Ax + By + Cz = D$ is

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

The equation of a plane having intercepts, a, b, c on the x, y and z-axis, respectively is :

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Comparing with $Ax + By + Cz = D$,

$$A=\frac{1}{a}, B=\frac{1}{b}, C=\frac{1}{c}, D=1$$ **[2]**

Given that, the plane is at a distance of 'p' units from the origin.

So, the points are $O(0, 0, 0)$.

So, $x_1 = 0, y_1 = 0$, and $z_1 = 0$

Now,

$$\text{Distance}=\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

Putting values, we have

$$p=\left|\frac{\frac{1}{a}\times0+\frac{1}{b}\times0+\frac{1}{c}\times0-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}\right|=\left|\frac{0+0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|$$

$$p=\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|$$

$$\frac{1}{p}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

Squaring both sides, we have

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

Thus proved. **[3]**

Q. 24. Show that the points $(\hat{i}-\hat{j}+3\hat{k})$ and $3(\hat{i}+\hat{j}+\hat{k})$ are equidistant from the plane $\vec{r}\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9=0$ and lies on opposite side of it.

[NCERT Exemp. Ex. 11.3, Q. 25, Page 237]

Ans. To show that these given points $(\hat{i}-\hat{j}+3\hat{k})$ and $3(\hat{i}+\hat{j}+\hat{k})$ are equidistant from the plane. $\vec{r}\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9=0$.

$\because$ We have to prove that mid-points of these points lie on the plane. Now mid-point of the given plane is $2\hat{i}+\hat{j}+3\hat{k}$.

On substituting $\vec{r}$ by the mid-point in a plane, we get

$$\text{LHS}=(2\hat{i}+\hat{j}+3\hat{k})\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9$$
$$=10+2-21+9$$
$$=0$$
$$=\text{RHS}$$

So that, these two points lie on opposite sides of the plane are equidistant from the plane. **[5]**

Q. 25. $\overrightarrow{AB}=3\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{CD}=-3\hat{i}+2\hat{j}+4\hat{k}$ are two vectors. The position vectors of the points *A* and *C* are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$, respectively. Find the position vector of a point *P* on the line *AB* and a point *Q* on the line *CD* such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

[NCERT Exemp. Ex. 11.3, Q. 26, Page 237]

Ans. We have,

$$\overrightarrow{AB}=3\hat{i}-\hat{j}+\hat{k} \text{ and } \overrightarrow{CD}=-3\hat{i}+2\hat{j}+4\hat{k}$$

Also, the position vectors of A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$, respectively.

Since, $\overrightarrow{PQ}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. So, P and Q will be foot of perpendicular to both the lines that pass through A and C.

Now, equation of the line through A and parallel to the vector $\overrightarrow{AB}$ is,

$$\vec{r}=(6\hat{i}+7\hat{j}+4\hat{k})+\lambda(3\hat{i}-\hat{j}+\hat{k}) \quad \text{...(i)}$$

And the line passing through C and parallel to the vector $\overrightarrow{CD}$ is given by

$$\vec{r}=-9\hat{j}+2\hat{k}+\mu(-3\hat{i}+2\hat{j}+4\hat{k}) \quad \text{...(ii)}$$

Let $P\,(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$ is any point on the first line and Q be any point on second line is given by $(-3\mu,-9+2\mu,2+4\mu)$.

$$\therefore \overrightarrow{PQ}=(-3\mu,-6-3\lambda)\hat{i}+(2\mu+\lambda-16)\hat{j}+(4\mu-\lambda-2)\hat{k}$$

If $\overrightarrow{PQ}$ is perpendicular to the first line, then

$$3(-3\mu-6-3\lambda)-(2\mu+\lambda-16)+(4\mu-\lambda-2)=0$$

$$\Rightarrow \quad -7\mu-11\lambda-4=0 \quad \text{...(iii)}$$

If $\overrightarrow{PQ}$ is perpendicular to the second line, then

$$-3(-3\mu-6-3\lambda)+2(2\mu+\lambda-16)+4(4\mu-\lambda-2)=0$$

$$\Rightarrow \quad 29\mu+7\lambda-22=0 \quad \text{...(iv)}$$

On solving equations (iii) and (iv), we get

$$\mu=1 \text{ and } \lambda=-1$$

$$\therefore \quad \overrightarrow{OP}=3\hat{i}+8\hat{j}+3\hat{k} \quad \text{[From (i)]}$$

and $\overrightarrow{OQ}=-3\hat{i}-7\hat{j}+6\hat{k}$ [From (ii)] **[5]**

Q. 26. Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles.

[NCERT Exemp. Ex. 11.3, Q. 27, Page 237]

Ans. We have,

$2l + 2m - n = 0$ ...(i)

And $mn + nl + lm = 0$ ...(ii)

Eliminating m from the both equations, we get

$$\Rightarrow \left(\frac{n-2l}{2}\right)n + nl + l\left(\frac{n-2l}{2}\right) = 0$$

$$\Rightarrow \frac{n^2 - 2nl + 2nl + nl - 2l^2}{2} = 0$$

$$\Rightarrow n^2 + nl - 2l^2 = 0$$

$$\Rightarrow (n+2l)(n-l) = 0$$

$$\Rightarrow n = -2l \text{ and } n = l$$

$\therefore \quad m = -2l,\ m = \frac{-v}{2}$ [From Eq. (i)]

Thus, the direction ratios of two are proportional to $l, -2l, -2l$ and $l, \frac{-l}{2}, l$

Or directional ratios are 1, –2, –2 and 2, –1, 2

Therefore, angle between vectors is given by

$$\cos\theta = \frac{(\hat{i} - 2\hat{j} - 2\hat{k})\cdot(2\hat{i} - \hat{j} + 2\hat{k})}{|\hat{i} - 2\hat{j} - 2\hat{k}||2\hat{i} - \hat{j} + 2\hat{k}|} = \frac{2+2-4}{3\cdot 3} = 0$$

$$\therefore \quad \theta = \frac{\pi}{2}$$ **[5]**

Q. 27. If l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$ makes equal angles with them.

[NCERT Exemp. Ex. 11.3, Q. 28, Page 237]

Ans. Let,

$$\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$$
$$\vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$$
$$\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$$
$$\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$$

Also, let α, β and γ are the angles between $\vec{a}$ and $\vec{d}$, $\vec{b}$ and $\vec{d}$, $\vec{c}$ and $\vec{d}$:

$$\therefore \cos\alpha = l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3)$$
$$= l_1^2 + l_1l_2 + l_1l_3 + m_1^2 + m_1m_2 + m_1m_3 + n_1^2 + n_1n_2 + n_1n_3$$
$$= (l_1^2 + m_1^2 + n_1^2) + (l_1l_2 + l_1l_3 + m_1m_2 + m_1m_3 + n_1n_2 + n_1n_3)$$
$$= 1 + 0 = 1$$

Similarly, cos β = 1 and cos γ = 1

⇒ cos α = cos β = cos γ

⇒ α = β = γ

Thus proved **[5]**

Q. 28. Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + 0 - z + 5 = 0$ and whose x-intercept is twice its z-intercept. Hence write the vector equation of a plane passing through the point (2, 3, –1) and parallel to the plane obtained above.

[CBSE Board, Foreign Region, 2016]

Ans. Equation of family of planes passing through two given planes :

$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0$

$\Rightarrow (1 + 2k)x + (2 + k)y + (3 - k)z = 4 - 5k$ ...(i)

$$\Rightarrow \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition,

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)}$$

$$\Rightarrow k = \frac{4}{5} \text{ or } \frac{1}{5}$$

For $k = \frac{1}{5}$, equation of plane is $7x + 11y + 14z = 15$.

For $k = \frac{4}{5}$, equation of plane is $13x + 14y + 11z = 0$.

Equation of plane passing through the points (2, 3, –1) and parallel to the plane is :

$7(x - 2) + 11(y - 3) + 14(z + 1) = 0$

$\Rightarrow 7x + 11y + 14z = 33$

Vector form : $\vec{r}\cdot(7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ **[6]**

Q. 29. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\vec{r}\cdot(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$. Also find image of P in the plane. [CBSE Board, All India Region, 2016]

Ans. Line passing through 'P' and perpendicular to plane is :

$$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$$

General point on line is :

$$\vec{r} = (2 + 2\lambda)\hat{i} + (3 + \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$$

For some $\lambda \in R$, $\vec{r}$ is the foot of perpendicular, say Q, from P to the plane, since it lies on plane.

$$\therefore [(2+2\lambda)\hat{i} + (3+\lambda)\hat{j} + (4+3\lambda)\hat{k}]\cdot(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$$

$$4 + 4\lambda + 3 + \lambda + 12 + 9\lambda - 26 = 0$$

$$\lambda = \frac{1}{2}$$

$$\therefore \text{Foot of perpendicular} = Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right).$$

Let $P'(a\hat{i} + b\hat{j} + c\hat{k})$ be the image of P in the plane, then Q is the mid-point of PP'.

$$\therefore Q\left(\frac{a+2}{2}\hat{i} + \frac{b+3}{2}\hat{j} + \frac{c+4}{2}\hat{k}\right)$$

$$= Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right)$$

$$\Rightarrow \frac{a+2}{2} = 3,\ \frac{b+3}{2} = \frac{7}{2},\ \frac{c+4}{2} = \frac{11}{2}$$

$$\Rightarrow a = 4,\ b = 4 \text{ and } c = 7$$

$$\therefore \quad P'(4\hat{i} + 4\hat{j} + 7\hat{k})$$

Perpendicular distance of P from plane

$$= PQ\sqrt{(2-3)^2 + \left(3 - \frac{7}{2}\right)^2 + \left(4 - \frac{11}{2}\right)^2} = \sqrt{\frac{7}{2}}.$$ **[6]**

Some Commonly Made Errors

- The dot product gives us a scalar, not another vector. The products are added together, not put into vector components.
- Note that there is very little difference between the two-dimensional (2D) and three-dimensional (3D) formulae above. To get from the 3D formula to the 2D formula all we did is to take out the third component/coordinate. Because of this, most of the formulae here are given only in their 3D version. If we need them in their 2D form we can easily modify the 3D form.
- When two lines are perpendicular, the angle between the lines is 90° which gives the condition of perpendicular as :
  - $l_1l_2 + m_1m_2 + n_1n_2 = 0$
  - **Or this implies,**
  - $a_1a_2 + 7_1b_2 + c_1c_2 = 0.$
- Similarly, when two lines are parallel, the angle between them, *i.e.* $\theta = 0°$.
  - **This gives** $l_1/l_2 = m_1/m_2 = n_1/n_2$
  - **This also gives** $a_1/a_2 = b_1/b_2 = c_1/c_2$
  - **So don't confuse with the applied conditions on both case.**

EXPERT ADVICE

- *Practice questions from previous year's question papers, sample papers and model papers within the time-frame you will have at the final exam.*
- *Try the given problems with the conventional methods first, and then look into the short-cut methods given. This makes it evident for you, the lesser labour involved, in comparison to the conventional methods.*
- *Don't be in a rush to solve problems. In Board Question Papers, both speed and strike-rate matter. You need to be quick as well as accurate to achieve high scores. High speed with low accuracy can actually ruin your results.*
- *More from rigid reliance on rules without understanding (rule-oriented study) to an understanding of mathematical concepts and flexibility in problem solving (concept-oriented study).*
- *Focus on solving as many problems as you can, rather than just reading theories, formulae and solutions.*

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