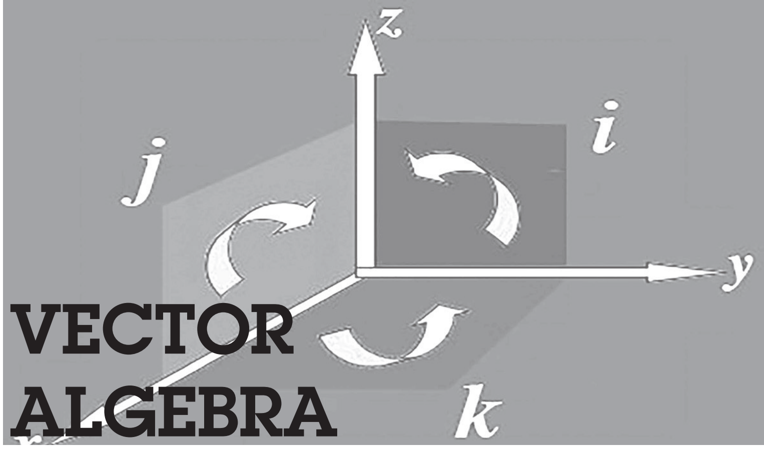
CHAPTER

10 VECTOR ALGEBRA



Chapter Objectives

This chapter will help you understand :

- ***Vector algebra :*** *Introduction; Types of vectors; Properties of vectors - Addition and Multiplication of vectors, Product of two vectors.*

Quick Review

- Linear algebra, mathematical discipline that deals with vectors and matrices and, more generally, with vector spaces and linear transformations.
- Vector, in mathematics, a quantity that has both magnitude and direction but not position.
- In their modern form, vectors appeared late in the 19th century when Josiah Willard Gibbs and Oliver Heaviside (of the United States and Britain, respectively) independently developed vector analysis to express the new laws of electro-magnetism discovered by the Scottish physicist James Clerk Maxwell. Since that time, vectors have become essential in physics, mechanics, electrical engineering and other sciences to describe forces mathematically.
- A unit vector is a vector of unit length. It is sometimes denoted by replacing the arrow on a vector with a symbol " ^ " or just adding a symbol " ^ " on a bold-faced character.
- The value of the triple product is equal to the volume of the parallelepiped constructed from the vectors.

TIPS...

- *Understand the concept of zero vector and unit vector.*
- *Study the parallelogram law of vector addition.*
- *Understand the concept of section formula and projection of a vector on a line.*

TRICKS...

- *Addition of two vectors is accomplished by laying the vectors head to tail in sequence to create a triangle.*
- *If necessary, use the Pythagorean theorem to find the magnitude of the resultant vector and select a suitable trigonometric function to find the angle that the resultant vector makes with the x-axis.*

Know the Links

- *http://tutorial.math.lamar.edu/Classes/CalcII/VectorsIntro.aspx*
- *http://docs.godotengine.org/en/3.0/tutorials/math/vectors_advanced.html*

Multiple Choice Questions/True or False

(1 mark each)

Q. 1. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

[NCERT Misc. Ex. Q. 16, Page 459]

Ans. Correct option : (b)

Explanation :

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore \vec{a}.\vec{b} \geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0 \qquad \left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]$

$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$.

Q. 2. Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

(a) $\theta = \frac{\pi}{4}$ **(b)** $\theta = \frac{\pi}{3}$

(c) $\theta = \frac{\pi}{2}$ **(d)** $\theta = \frac{2\pi}{3}$

[NCERT Misc. Ex. Q. 17, Page 459]

Ans. Correct option : (d)

Explanation :

Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ be the angle between them.

Then, $|\vec{a}+\vec{b}| = |\vec{b}| = 1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}| = 1$.

$$|\vec{a}+\vec{b}| = 1$$
$$\Rightarrow (\vec{a}+\vec{b})^2 = 1$$
$$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$$
$$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}\vec{b} = 1$$
$$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2 = 1$$
$$\Rightarrow 1^2+2|\vec{a}||\vec{b}|\cos\theta+1^2 = 1$$
$$\Rightarrow \cos\theta = -\frac{1}{2}$$
$$\Rightarrow \theta = \frac{2\pi}{3}$$

So that, $|\vec{a}+\vec{b}|$ is a unit vector if $\theta = \frac{2\pi}{3}$.

Q. 3. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is

(a) 0 **(b)** –1

(c) 1 **(d)** 3

[NCERT Misc. Ex. Q. 18, Page 459]

Ans. Correct option : (c)

Explanation :

$$\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$$
$$=\hat{i}.\hat{i}+\hat{j}.(-\hat{j})+\hat{k}.\hat{k}$$
$$=1-\hat{j}.\hat{j}+1$$
$$=1-1+1$$
$$=1$$

Q. 4. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to

(a) 0 **(b)** $\pi/4$

(c) $\pi/2$ **(d)** π

[NCERT Misc. Ex. Q. 19, Page 459]

Ans. Correct option : (b)

Explanation :

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$$
$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$
$$\Rightarrow \cos\theta = \sin\theta \quad [\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}]$$
$$\Rightarrow \tan\theta = 1$$
$$\Rightarrow \theta = \frac{\pi}{4}$$

So that, $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

Q. 5. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\pi/6$ **(b)** $\pi/4$

(c) $\pi/3$ **(d)** $\pi/2$

[NCERT Ex. 10.4, Q. 11, Page 455]

Ans. Correct option : (b)

Explanation :

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$.

We know that $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\ \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\times\vec{b}$ is a unit vector if $|\vec{a}\times\vec{b}| = 1$.

$$|\vec{a}\times\vec{b}| = 1$$
$$\Rightarrow ||\vec{a}||\vec{b}|\sin\theta\hat{n}| = 1$$
$$\Rightarrow |\vec{a}||\vec{b}||\sin\theta| = 1$$
$$\Rightarrow 3\times\frac{\sqrt{2}}{3}\times\sin\theta = 1$$
$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \theta = \frac{\pi}{4}$$

So that, $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

Q. 6. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively *is*

(a) $\frac{1}{2}$ **(b)** 1

(c) 2 **(d)** 4

[NCERT Ex. 10.4, Q. 12, Page 455]

Ans. Correct option : (c)

Explanation :

The position vectors of vertices A, B, C and D of rectangle ABCD are given as :

$$\overrightarrow{OA} = -\hat{i}+-\hat{j}+\hat{k}\ \overrightarrow{OB} = \hat{i}+\frac{1}{2}\hat{j}+4\hat{k},$$
$$\overrightarrow{OC} = \hat{i}-\frac{1}{2}\hat{j}+4\hat{k},\ \overrightarrow{OD} = -\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$$

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of the given rectangle are given as :

$$\overrightarrow{AB} = (1+1)\hat{i}+\left(\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k} = 2\hat{i}$$
$$\overrightarrow{BC} = (1-1)\hat{i}+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k} = -\hat{j}$$
$$\therefore \overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$= \hat{k}(-2) = -2\hat{k}$

$\Rightarrow \left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = 2$

Now, it is known that the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $\left|\vec{a} \times \vec{b}\right|$.

So that, the area of the given rectangle is $\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = 2$sq. units.

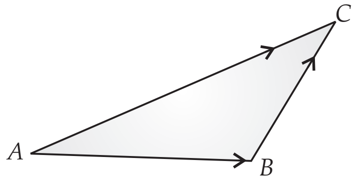
Q. 7. In triangle ABC (Figure), which of the following is not true :

(a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

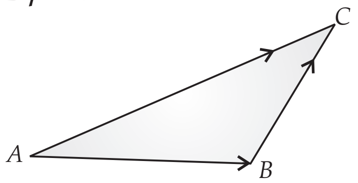
(d) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$



[NCERT Ex. 10.2, Q. 18, Page 441]

Ans. Correct option : (c)

Explanation :



Applying the triangle law of addition in the above triangle, we have

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ ...(i)

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$ ...(ii)

$\therefore$ The equation given in alternative (a) is true.

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\therefore$ The equation given in alternative (b) is true.

From equation (ii), we have

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

The equation given in alternative (d) is true.

Now, consider the equation given in alternative (c) :

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$ ...(iii)

For equations (i) and (iii), we have :

$\overrightarrow{AC} = \overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$

$\Rightarrow 2\overrightarrow{AC} = \vec{0}$

$\Rightarrow \overrightarrow{AC} = \vec{0}$, which is not true.

So that, the equation given in alternative (c) is incorrect.

Q. 8. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(a) $\lambda = 1$

(b) $\lambda = -1$

(c) $a = |\lambda|$

(d) $a = 1/|\lambda|$ **[NCERT Ex. 10.3, Q. 18, Page 448]**

Ans. Correct option : (d)

Explanation :

Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$.

Now,

$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda||\vec{a}| = 1$

$\Rightarrow |\vec{a}| = \dfrac{1}{|\lambda|}$ $[\lambda \neq 0]$

$\Rightarrow a = \dfrac{1}{|\lambda|}$ $[|\vec{a}| = a]$

So that, vector $\lambda\vec{a}$ is a unit vector if $a = \dfrac{1}{|\lambda|}$.

Q. 9. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect :

(a) $\vec{b} = \lambda\vec{a}$**, for some scalar** λ

(b) $\vec{a} = \pm\vec{b}$

(c) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(d) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes [NCERT Ex. 10.2, Q. 19, Page 441]

Ans. Correct option : (d)

Explanation :

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then,

$\vec{b} = \lambda\vec{a}$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$

$\Rightarrow \dfrac{b_1}{a_1} = \dfrac{b_2}{a_2} = \dfrac{b_3}{a_3} = \lambda$

So that, the respective components of $\vec{a}$ and $\vec{b}$ are proportional. However, vectors $\vec{a}$ and $\vec{b}$ can have different directions. Hence, the statement given in option (d) is incorrect.

Q. 10. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(a) $\hat{i} - 2\hat{j} + 2\hat{k}$ **(b)** $\dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ **(d)** $9(\hat{i} - 2\hat{j} + 2\hat{k})$

[NCERT Exemp. Ex. 10.3, Q. 19, Page 216]

Ans. Correct option : (c)

Explanation :

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by $\dfrac{\vec{a}}{|\vec{a}|}$.

$= \dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9.

$=9\,\dfrac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

$=3(\hat{i}-2\hat{j}+2\hat{k})$

Q. 11. The position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3 :1 is :

(a) $\dfrac{3\vec{a}-2\vec{b}}{2}$ (b) $\dfrac{7\vec{a}-8\vec{b}}{4}$

(c) $\dfrac{3\vec{a}}{4}$ (d) $\dfrac{5\vec{a}}{4}$

[NCERT Exemp. Ex. 10.3, Q. 20, Page 217]

Ans. Correct option : (d)

Explanation :

Let the position vector of the R divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$.

$\therefore$ Position vector, $R=\dfrac{3(\vec{a}+\vec{b})+1(2\vec{a}-3\vec{b})}{3+1}$

Since, the position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ration $m:n$ internally, is given by $\dfrac{m\vec{q}+n\vec{p}}{m+n}$.

$\therefore R=\dfrac{5\vec{a}}{4}$

Q. 12. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is :

(a) $-\hat{i}+12\hat{j}+4\hat{k}$

(b) $5\hat{i}+2\hat{j}-4\hat{k}$

(c) $-5\hat{i}+2\hat{j}+4\hat{k}$

(d) $\hat{i}+\hat{j}+\hat{k}$

[NCERT Exemp. Ex. 10.3, Q. 21, Page 217]

Ans. Correct option : (c)

Explanation :

Required vector $=(-3-2)\hat{i}+(7-5)\hat{j}+(4-0)\hat{k}$

$=-5\hat{i}+2\hat{j}+4\hat{k}$

Q. 13. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b}=2\sqrt{3}$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{2}$ (d) $\dfrac{5\pi}{2}$

[NCERT Exemp. Ex. 10.3, Q. 22, Page 217]

Ans. Correct option : (b)

Explanation :

Here, $|\vec{a}|=\sqrt{3},\ |\vec{b}|=4$ and $\vec{a}.\vec{b}=2\sqrt{3}$ [Given]

We know that,

$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow\ 2\sqrt{3}=\sqrt{3}.4.\cos\theta$

$\Rightarrow \cos\theta=\dfrac{2\sqrt{3}}{4\sqrt{3}}=\dfrac{1}{2}$

$\therefore\quad \theta=\dfrac{\pi}{3}$

Q. 14. Find the value of λ such that the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ are orthogonal.

(a) 0 (b) 1

(c) $\dfrac{3}{2}$ (d) $\dfrac{-5}{2}$

[NCERT Exemp. Ex. 10.3, Q. 23, Page 217]

Ans. Correct option : (d)

Explanation :

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal, *i.e.,* $\vec{a}.\vec{b}=0$

$\therefore (2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+3\hat{k})=0$

$\Rightarrow\quad 2+2\lambda+3=0$

$\therefore\quad \lambda=\dfrac{-5}{2}$

Q. 15. The value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is

(a) 2/3 (b) 3/2

(c) 5/2 (d) 2/5

[NCERT Exemp. Ex. 10.3, Q. 24, Page 217]

Ans. Correct option : (a)

Explanation :

Let $\vec{a}=3\hat{i}-6\hat{j}+\hat{k}$ and $\vec{b}=2\vec{i}-4\vec{j}+\lambda\hat{k}$

Since, $\vec{a}\,||\,\vec{b}$

$\Rightarrow\ \dfrac{3}{2}=\dfrac{-6}{-4}=\dfrac{1}{\lambda}$

$\Rightarrow\ \lambda=\dfrac{2}{3}$

Q. 16. The vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$ respectively, then the area of triangle OAB is

(a) 340 (b) $\sqrt{25}$

(c) $\sqrt{229}$ (d) $\dfrac{1}{2}\sqrt{229}$

[NCERT Exemp. Ex. 10.3, Q. 25, Page 217]

Ans. Correct option : (d)

Explanation :

Area of $\Delta OAB=\dfrac{1}{2}\left|\overline{OA}\times\overline{OB}\right|$

$=\dfrac{1}{2}\left|(2\hat{i}-3\hat{j}+2\hat{k})\times(2\hat{i}+3\hat{j}+\hat{k})\right|$

$=\dfrac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$

$=\dfrac{1}{2}\left|\left[\hat{i}(-3-6)-\hat{j}(2-4)+\hat{k}(6+6)\right]\right|$

$=\dfrac{1}{2}\left|-9i+2\hat{j}+12\hat{k}\right|$

Area of ΔOAB

$=\dfrac{1}{2}\sqrt{(81+4+144)}$

$=\dfrac{1}{2}\sqrt{229}$

Q. 17. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

(a) $\vec{a}^2$ (b) $3\vec{a}^2$

(c) $4\vec{a}^2$ (d) $2\vec{a}^2$

[NCERT Exemp. Ex. 10.3, Q. 26, Page 218]

Ans. **Correct option :** (d)

Explanation :

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \quad \vec{a}^2 = x^2 + y^2 + z^2$$

$$\therefore \quad \vec{a}\times\hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}[0] - \hat{j}[-z] + \hat{k}[-y]$$

$$= z\hat{j} - y\hat{k}$$

$$\therefore \quad (\vec{a}\times\hat{i})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k})$$

$$= y^2 + z^2$$

Similarly, $(\vec{a}\times\hat{j})^2 = x^2 + z^2$ and $(\vec{a}\times\hat{k})^2 = x^2 + z^2$

$$(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y^2$$

$$= 2(x^2 + y^2 + z^2) = 2\vec{a}^2$$

Q. 18. If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 12$, then the value of $|\vec{a}\times\vec{b}|$ is

(a) 5 **(b)** 10

(c) 14 **(d)** 16

[NCERT Exemp. Ex. 10.3, Q. 27, Page 218]

Ans. **Correct option :** (d)

Explanation :

Here, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 12$ [Given]

$$\therefore \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$12 = 10 \times 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{12}{20} = \frac{3}{5}$$

$$\Rightarrow \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}}$$

$$\sin\theta = \pm\frac{4}{5}$$

$$\therefore \quad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$

$$= 10 \times 2 \times \frac{4}{5}$$

$$= 16$$

Q. 19. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar, if

(a) $\lambda = -2$ **(b)** $\lambda = 0$

(c) $\lambda = 1$ **(d)** $\lambda = -1$

[NCERT Exemp. Ex. 10.3, Q. 28, Page 218]

Ans. **Correct option :** (a)

Explanation :

Let $\vec{a} = \lambda\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \lambda\hat{k}$

For $\vec{a}$, $\vec{b}$ and $\vec{c}$ to be coplanar,

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$$

$$\Rightarrow \quad \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$$

$$\Rightarrow \quad \lambda^3 - 6\lambda - 4 = 0$$

$$\Rightarrow \quad (\lambda + 2)(\lambda^2 - 2\lambda - 2) = 0$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm\sqrt{12}}{2}$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm 2\sqrt{3}}{2} = 1\pm\sqrt{3}$$

Q. 20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

(a) 1 **(b)** 3

(c) –3/2 **(d)** None of these

[NCERT Exemp. Ex. 10.3, Q. 29, Page 218]

Ans. **Correct option :** (c)

Explanation :

We have, $\vec{a} + \vec{b} + \vec{c} = 0$ and $\vec{a}^2 = 1$, $\vec{b}^2 = 1$, $\vec{c}^2 = 1$

$$\therefore \quad (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c}) = 0$$

$$\Rightarrow \vec{a}^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{b}^2 + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}^2 = 0$$

$$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a},\ \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{b} \text{ and } \vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{c}]$$

$$\Rightarrow 1 + 1 + 1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$$

Q. 21. The projection vector of $\vec{a}$ on $\vec{b}$ is

(a) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\vec{b}$ **(b)** $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ **(d)** $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\hat{b}$

[NCERT Exemp. Ex. 10.3, Q. 30, Page 218]

Ans. **Correct option :** (a)

Explanation :

Projection vector of $\vec{a}$ on $\vec{b}$ is given by,

$$= \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\vec{b}$$

$$= \left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)\cdot\vec{b}$$

Q. 22. If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

(a) 0 **(b)** 1

(c) –19 **(d)** 38

[NCERT Exemp. Ex. 10.3, Q. 31, Page 218]

Ans. **Correct option :** (c)

Explanation :

Here, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}^2 = 4$, $\vec{b}^2 = 9$, $\vec{c}^2 = 25$

$$\therefore \quad (\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a}^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{b}^2 + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}^2 = \vec{0}$$

$$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0 \; [\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$$

$\Rightarrow\ 4+9+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\dfrac{-38}{2}=-19$

Q. 23. If $|\vec{a}|=4$ and $-3\le\lambda\le 2$, then the range of $|\lambda\vec{a}|$ is

(a) [0, 8] **(b) [–12, 8]**

(c) [0, 12] **(d) [8, 12]**

[NCERT Exemp. Ex. 10.3, Q. 32, Page 218]

Ans. Correct option : (c)

Explanation :

We have, $|\vec{a}|=4$ and $-3\le\lambda\le 2$

$\therefore\ |\lambda\vec{a}|=|\lambda||\vec{a}|=\lambda|4|$

$\Rightarrow |\lambda\vec{a}|=|-3|4=12,$ at $\lambda=-3$

$|\lambda\vec{a}|=|0|4=0,$ at $\lambda=0$

and $|\lambda\vec{a}|=|2|4=8,$ at $\lambda=2$

So, the range of $|\lambda\vec{a}|$ is [0,12].

Q. 24. The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ is

(a) one **(b) two**

(c) three **(d) infinite**

[NCERT Exemp. Ex. 10.3, Q. 33, Page 218]

Ans. Correct option : (b)

Explanation :

The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$ (say)

i.e., $\vec{c}=\pm(\vec{a}\times\vec{b})$.

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

Q. 25. Answer the following as true or false.

(a) $\vec{a}$ and $\overrightarrow{-a}$ are collinear.

(b) Two collinear vectors are always equal in magnitude.

(c) Two vectors having same magnitude are collinear.

(d) Two collinear vectors having the same magnitude are equal. **[NCERT Ex. 10.1, Q. 5, Page 428]**

Ans. Correct option : (a)

Explanation :

(a) True

Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line.

(b) False

Collinear vectors are those vectors that are parallel to the same line.

(c) False

It is not necessary for two vectors having the same magnitude to be parallel to the same line.

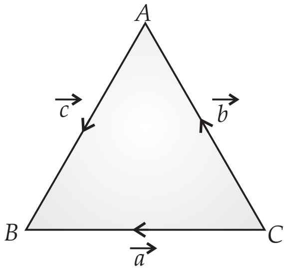
(d) False

Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

Very Short Answer Type Questions **[1 or 2 marks each]**

Q. 1. If $\vec{a}=\vec{b}+\vec{c}$, then is it true that $|\vec{a}|=|\vec{b}|+|\vec{c}|$? Justify your answer. **[NCERT Misc. Ex. Q. 4, Page 458]**

Ans. In ΔABC, let $\overrightarrow{CB}=\vec{a}$, $\overrightarrow{CA}=\vec{b}$, and $\overrightarrow{AB}=\vec{c}$ (As shown in the following figure).



Now, by the triangle law of vector addition, we have $\vec{a}=\vec{b}+\vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\vec{a}|<|\vec{b}|+|\vec{c}|$

Hence, it is not true that $|\vec{a}|=|\vec{b}|+|\vec{c}|$. **[2]**

Q. 2. Find the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector. **[NCERT Misc. Ex. Q. 5, Page 458]**

Ans. $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector if $|x(\hat{i}+\hat{j}+\hat{k})|=1$.

Now,

$|x(\hat{i}+\hat{j}+\hat{k})|=1$

$\Rightarrow \sqrt{x^2+x^2+x^2}=1$

$\Rightarrow \sqrt{3x^2}=1$

$\Rightarrow \sqrt{3}x=1$

$\Rightarrow x=\pm\dfrac{1}{\sqrt{3}}$

So that, the required value of x is $\pm\dfrac{1}{\sqrt{3}}$. **[2]**

Q. 3. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

[NCERT Misc. Ex. Q. 6, Page 458]

Ans. We have,

$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$\vec{c}=\vec{a}+\vec{b}$

$=(2+1)\hat{i}+(3-2)\hat{j}+(-1+1)\hat{k}$

$=3\hat{i}+\hat{j}$

$\therefore |\vec{c}|=\sqrt{3^2+1^2}$

$=\sqrt{9+1}$

$= \sqrt{10}$

$\therefore \quad \hat{c} = \frac{\vec{c}}{|\vec{c}|}$

$= \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$\pm 5 \cdot \hat{c} = \pm \frac{5.1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \pm \frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}}{2}\hat{j}$. **[2]**

Q. 4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.
[NCERT Misc. Ex. Q. 7, Page 458]

Ans. We have,

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$$2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$$
$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$
$$= 3\hat{i} - 3\hat{j} + 2\hat{k}$$

$$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2}$$
$$= \sqrt{9 + 9 + 4}$$
$$= \sqrt{22}$$

Hence, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is :

$$= \frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}$$
$$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}}$$
$$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}.$$ **[2]**

Q. 5. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.
[NCERT Misc. Ex. Q. 1, Page 458]

Ans. If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the *x*-axis.

Therefore, for θ = 30° :

$$\vec{r} = \cos 30°\hat{i} + \sin 30°\hat{j}$$
$$= \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$. **[2]**

Q. 6. Find the scalar components and magnitude of the vector joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2).
[NCERT Misc. Ex. Q. 2, Page 458]

Ans. The vector joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) can be obtained by,

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $[(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)]$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. **[2]**

Q. 7. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.
[NCERT Ex. 10.4, Q. 1, Page 454]

Ans. We have,

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$
$$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21)$$
$$= 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2}$$
$$= \sqrt{2 \times (19)^2}$$
$$= 19\sqrt{2}$$ **[2]**

Q. 8. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$. **[NCERT Ex. 10.3, Q. 1, Page 447]**

Ans. It is given that,

$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and, $\vec{a} \cdot \vec{b} = \sqrt{6}$

Now, we know that,

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$$\therefore \quad \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$$
$$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$
$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the given vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$. **[2]**

Q. 9. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. **[NCERT Ex. 10.3, Q. 2, Page 447]**

Ans. The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2}$$
$$= \sqrt{1 + 4 + 9}$$
$$= \sqrt{14}$$
$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2}$$
$$= \sqrt{9 + 4 + 1}$$
$$= \sqrt{14}$$

Now,

$$\vec{a} \cdot \vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k})$$
$$= 1.3 + (-2)(-2) + 3.1$$
$$= 3 + 4 + 3$$
$$= 10$$

Also, we know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore \quad 10 = \sqrt{14}\sqrt{14}\cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{5}{7}\right)$

[2]

Q. 10. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a}\cdot\vec{b} = 0$. But the converse need not be true, Justify your answer with an example.[NCERT Ex. 10.3, Q. 14, Page 448]

Ans. Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then, $\vec{a}\cdot\vec{b} = 2.3 + 4.3 + 3(-6)$
$= 6 + 12 - 18$
$= 0$

We now observe that :

$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2}$
$= \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2}$
$= \sqrt{54}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true. **[2]**

Q. 11. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$. [NCERT Ex. 10.3, Q. 3, Page 447]

Ans. Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$.

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}\left(\vec{a}\cdot\vec{b}\right) = \frac{1}{\sqrt{1+1}}\left[1.1 + (-1)(1)\right]$$
$$= \frac{1}{\sqrt{2}}(1-1)$$
$$= 0$$

Hence, the projection of vector $\vec{a}$ on $\vec{b}$ is 0. **[2]**

Q. 12. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. [NCERT Ex. 10.3, Q. 4, Page 447]

Ans. Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $b = 7\hat{i} - \hat{j} + 8\hat{k}$.

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}\left(\vec{a}\cdot\vec{b}\right) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}}\left[1(7) + 3(-1) + 7(8)\right]$$
$$= \frac{7 - 3 + 56}{\sqrt{49 + 1 + 64}}$$
$$= \frac{60}{\sqrt{114}}$$

[2]

Q. 13. Show that each of the given three vectors is a unit vector :

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}),\ \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) \text{ and } \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}).$$

Also, show that they are mutually perpendicular to each other. [NCERT Ex. 10.3, Q. 5, Page 447]

Ans. Let,

$$\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$
$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$
$$\vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$
$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$$
$$\vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$
$$= \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}.$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$$
$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}}$$
$$= 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$$
$$= \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}}$$
$$= 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2}$$
$$= \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}}$$
$$= 1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a}\cdot\vec{b} = \frac{2}{7}\times\frac{3}{7} + \frac{3}{7}\times\left(\frac{-6}{7}\right) + \frac{6}{7}\times\frac{2}{7}$$
$$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49}$$
$$= 0$$

$$\vec{b}\cdot\vec{c} = \frac{3}{7}\times\frac{6}{7} + \left(\frac{-6}{7}\right)\times\frac{2}{7} + \frac{2}{7}\times\left(\frac{-3}{7}\right)$$
$$= \frac{18}{49} - \frac{12}{49} - \frac{6}{49}$$
$$= 0$$

$$\vec{c}\cdot\vec{a} = \frac{6}{7}\times\frac{2}{7} + \frac{2}{7}\times\frac{3}{7} + \left(\frac{-3}{7}\right)\times\frac{6}{7}$$
$$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49}$$
$$= 0$$

Hence, the given three vectors are mutually perpendicular to each other. **[2]**

Q. 14. Show that $(\vec{a} - \vec{b})\times(\vec{a} + \vec{b}) = 2(\vec{a}\times\vec{b})$.

[NCERT Ex. 10.4, Q. 4, Page 454]

Ans. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$

$=\left(\vec{a}-\vec{b}\right)\times\vec{a}+\left(\vec{a}-\vec{b}\right)\times\vec{b}$ [By distributivity of vector product over addition]

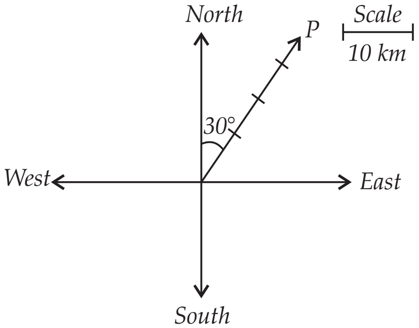
$=\vec{a}\times\vec{a}-\vec{b}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{b}$ [Again, by distributivity of vector product over addition]

$=\vec{0}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-\vec{0}$

$=2\vec{a}\times\vec{b}$ **[2]**

Q. 15. Represent graphically a displacement of 40 km, 30° east of north. [NCERT Ex. 10.1, Q. 1, Page 428]

Ans.



Here, vector $\overrightarrow{OP}$ represents the displacement of 40 km, 30° east of north. **[1]**

Q. 16. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$. [NCERT Misc. Ex. Q. 11, Page 458]

Ans. Let a vector be equally inclined to axes OX, OY, and OZ at angle a.

Then, the direction cosines of the vector are cos *a*, cos *a*, and cos *a*.

Now,

$\cos^2 a+\cos^2 a+\cos^2 a=1$

$\Rightarrow \quad 3\cos^2 a=1$

$\Rightarrow \quad \cos a=\frac{1}{\sqrt{3}}$

So that, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$. **[2]**

Q. 17. The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if ________.
[NCERT Exemp. Ex. 10.3, Q. 34, Page 218]

Ans. If vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors, then

$\vec{a}\cdot(\vec{a}+\vec{b})=|\vec{a}||\vec{a}+\vec{b}|\cos\theta$

$\vec{a}\cdot(\vec{a}+\vec{b})=a\sqrt{a^2+b^2}\cos\theta$

$\Rightarrow \cos\theta=\frac{\vec{a}\cdot(\vec{a}+\vec{b})}{a\sqrt{a^2+b^2}}$ ...(i)

and $\vec{b}\cdot(\vec{a}+\vec{b})=|\vec{b}|\cdot|\vec{a}+\vec{b}|\cos\theta$

$\vec{b}\cdot(\vec{a}+\vec{b})=b\sqrt{a^2+b^2}\cos\theta$ [Since, θ should be same.]

$\Rightarrow \cos\theta=\frac{\vec{b}\cdot(\vec{a}+\vec{b})}{b\sqrt{a^2+b^2}}$ ...(ii)

From equations (i) and (ii), we have :

$\frac{\vec{a}\cdot(\vec{a}+\vec{b})}{a\sqrt{a^2+b^2}}=\frac{\vec{b}\cdot(\vec{a}+\vec{b})}{b\sqrt{a^2+b^2}}$

$\Rightarrow \quad \frac{\vec{a}}{|\vec{a}|}=\frac{\vec{b}}{|\vec{b}|}$

$\therefore \hat{a}=\hat{b}\Rightarrow a$ and b are equal vectors. **[2]**

Q. 18. If $\vec{r}\cdot\vec{a}=0$, $\vec{r}\cdot\vec{b}=0$ and $\vec{r}\cdot\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}\cdot(\vec{b}\times\vec{c})$ is ________.
[NCERT Exemp. Ex. 10.3, Q. 35, Page 219]

Ans. Since, $\vec{r}$ is a non-zero vector. So, we can say that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are in a same plane.

$\therefore \vec{a}\cdot(\vec{b}\times\vec{c})=0$

[Since, angles between $\vec{a}$, $\vec{b}$ and $\vec{c}$ are zero, *i.e.*, $\theta = 0$] **[1]**

Q. 19. The vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$ are the adjacent sides of a parallelogram. The acute angle between its diagonals is ________.
[NCERT Exemp. Ex. 10.3, Q. 36, Page 219]

Ans. We have, $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$

$\therefore \vec{a}+\vec{b}=2\hat{i}-2\hat{j}$ and $\vec{a}-\vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$

Now, let θ is the acute angle between the diagonals $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

$$\cos\theta=\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$$

$$=\frac{(2\hat{i}-2\hat{j})\cdot(4\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{8}\sqrt{16+4+16}}$$

$$=\frac{8+4}{2\sqrt{2}\cdot 6}$$

$$=\frac{1}{\sqrt{2}}$$

$\therefore \quad \theta=\frac{\pi}{4}$ $\left[\because \cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}\right]$ **[2]**

Q. 20. The values of k, for which $|k\,\vec{a}|<|\vec{a}|$ and $k\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are _______.
[NCERT Exemp. Ex. 10.3, Q. 37, Page 219]

Ans. We have, $|k\,\vec{a}|<\vec{a}|$ and $k\,\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$

$\therefore \ |k\,\vec{a}|<|\vec{a}|$

$\Rightarrow |k||\vec{a}|<|\vec{a}|$

$\Rightarrow \ |k|<1$

$\Rightarrow -1<k<1$

Also, since $k\,\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$, then we see that at $k=\frac{-1}{2}$, $k=\vec{a}+\frac{1}{2}\vec{a}$ becomes a null vector and then it will not be parallel to $\vec{a}$.

So, $k = \vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true when $k \in [-1, 1]\, k \neq \frac{-1}{2}$. **[2]**

Q. 21. The value of the expression $\left|\vec{a}\times\vec{b}\right|^2 + (\vec{a}\cdot\vec{b})^2$ is ______________.

[NCERT Exemp. Ex. 10.3, Q. 37, Page 219]

Ans.

$$\left|\vec{a}\times\vec{b}\right|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2\left|\vec{b}\right|^2 \sin^2\theta + (\vec{a}\cdot\vec{b})^2$$
$$= |\vec{a}|^2\left|\vec{b}\right|^2 (1-\cos^2\theta) + (\vec{a}\cdot\vec{b})^2$$
$$= |\vec{a}|^2\left|\vec{b}\right|^2 - |\vec{a}|^2\left|\vec{b}\right|^2\cos^2\theta + (\vec{a}\cdot\vec{b})^2$$
$$= |\vec{a}|^2\left|\vec{b}\right|^2 - (\vec{a}\cdot\vec{b})^2 + (\vec{a}\cdot\vec{b})^2$$
$$\left|\vec{a}\times\vec{b}\right|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2\left|\vec{b}\right|^2$$

[2]

Q. 22. If $\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}\cdot\vec{b}\right|^2 = 144$ and $|\vec{a}| = 4$, then $\left|\vec{b}\right|$ is equal to ______________.

[NCERT Exemp. Ex. 10.3, Q. 39, Page 219]

Ans. $\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}\cdot\vec{b}\right|^2 = 144 = |\vec{a}|^2\cdot\left|\vec{b}\right|^2$

$$|\vec{a}|^2\left|\vec{b}\right|^2 = 144$$
$$\left|\vec{b}\right|^2 = \frac{144}{|\vec{a}|^2}$$
$$= \frac{144}{16}$$
$$= 9$$
$$\left|\vec{b}\right| = 3$$

[2]

Q. 23. If $\vec{a}$ is any non-zero vector, then $(\vec{a}\cdot\hat{i})\cdot\hat{i} + (\vec{a}\cdot\hat{j})\cdot\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$ is equal to ______________.

[NCERT Exemp. Ex. 10.3, Q. 40, Page 219]

Ans. Let

$$\vec{a} = a_1\,\hat{i} + a_2\,\hat{j} + a_3\,\hat{k}$$

$\therefore$ $\vec{a}\cdot\hat{i} = a_1$, $\vec{a}\cdot\hat{j} = a_2$ and $\vec{a}\cdot\hat{k} = a_3$

$\therefore$ $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k} = a_1\,\hat{i} + a_2\,\hat{j} + a_3\,\hat{k} = \vec{a}$ **[2]**

Q. 24. Find the magnitude of each of the two vectors a and b, having the same magnitude such that the angle between them is 60° and their scalar product is 9/2.

[CBSE Board, All India Region, 2018]

Ans. Suppose $\vec{a}$ and $\vec{b}$ be the two vectors.

Now, $\vec{a}\cdot\vec{b} = |\vec{a}|\left|\vec{b}\right|\cos\theta$

We are given that $|\vec{a}| = \left|\vec{b}\right|$, $\theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{9}{2}$

$\therefore$ $\frac{9}{2} = |\vec{a}||\vec{a}|\cos 60°$

$\Rightarrow$ $\frac{9}{2} = |\vec{a}|^2\left(\frac{1}{2}\right)$

$\Rightarrow |\vec{a}|^2 = 9$

$\Rightarrow$ $|\vec{a}| = 3 = \left|\vec{b}\right|$ **[1]**

Q. 25. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ find sin θ

[CBSE Board, All India Region, 2018]

Ans. Using cross-product of two vectors, we have

$$\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)\cdot\left(3\hat{i} - 2\hat{j} + \hat{k}\right) = \left|\hat{i} - 2\hat{j} + 3\hat{k}\right|\left|3\hat{i} - 2\hat{j} + \hat{k}\right|\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)\cdot\left(3\hat{i} - 2\hat{j} + \hat{k}\right)}{\left|\hat{i} - 2\hat{j} + 3\hat{k}\right|\left|3\hat{i} - 2\hat{j} + \hat{k}\right|}$$

$$\Rightarrow \quad \cos\theta = \frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}$$

$$\Rightarrow \quad \cos\theta = \frac{10}{14} = \frac{5}{7}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$$
$$= \sqrt{1-\frac{25}{49}}$$
$$= \sqrt{\frac{24}{49}}$$
$$= \frac{2\sqrt{6}}{7}$$

[2]

Q. 26. Given that $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}\times\vec{b} = \vec{0}$. what can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

[NCERT Ex. 10.4, Q. 6, Page 454]

Ans. $\vec{a}\cdot\vec{b} = 0$

Then,

(i) Either $|\vec{a}| = 0$ or $\left|\vec{b}\right| = 0$, or $\vec{a}\perp\vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero.)

$\vec{a}\times\vec{b} = 0$

(ii) Either $|\vec{a}| = 0$ or $\left|\vec{b}\right| = 0$, or $\vec{a}\parallel\vec{b}$ (in case $\vec{a}$ and are non-zero.)

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $\left|\vec{b}\right| = 0$. **[2]**

Q. 27. Write the distance of the point (3, – 5, 12) from x-axis. [CBSE Board, Delhi Region, 2017]

Ans. The distance of the point (3, –5, 12) from x-axis will be :

$\sqrt{(-5)^2 + (12)^2} = 13$ **[1]**

Q. 28. If $\vec{a}$, $\vec{b}$, $\vec{c}$, are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then write the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

[CBSE Board, Foreign Scheme, 2016]

Ans.

$$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$$
$$\Rightarrow |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$
$$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$$

[1]

Q. 29. If $\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}\cdot\vec{b}\right|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $\left|\vec{b}\right|$. [CBSE Board, Foreign Scheme, 2016]

Ans. $a^2\, b^2 sin^2\theta + a^2\, b^2\cos^2\theta = 400$

$$\Rightarrow \quad \left|\vec{b}\right| = 4$$

[1]

Q. 30. Evaluate the product $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$.
[NCERT Ex. 10.3, Q. 7, Page 448]

Ans. $\left(3\vec{a}-5\vec{b}\right)\cdot\left(2\vec{a}+7\vec{b}\right)$

$= 3\vec{a}\cdot 2\vec{a}+3\vec{a}\cdot 7\vec{b}-5\vec{b}\cdot 2\vec{a}-5\vec{b}\cdot 7\vec{b}$

$= 6\vec{a}\cdot\vec{a}+21\vec{a}\cdot\vec{b}-10\vec{a}\cdot\vec{b}-35\vec{b}\cdot\vec{b}$

$= 6|\vec{a}|^2+11\vec{a}\cdot\vec{b}-35|\vec{b}|^2$

[2]

Q. 31. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is ½.
[NCERT Ex. 10.3, Q. 8, Page 448]

Ans. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that,

$|\vec{a}|=|\vec{b}|$, $\vec{a}\cdot\vec{b}=\dfrac{1}{2}$, and $\theta=60°$ …(i)

We know that,

$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\therefore \quad \dfrac{1}{2}=|\vec{a}||\vec{b}|\cos 60° \quad$ [Using Eq. (i)]

$\Rightarrow \quad \dfrac{1}{2}=|\vec{a}|^2\times\dfrac{1}{2}$

$\Rightarrow |\vec{a}|^2=1$

$\Rightarrow \quad |\vec{a}|=|\vec{b}|=1$

[2]

Q. 32. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$.
[NCERT Ex. 10.3, Q. 9, Page 448]

Ans. $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$

$\Rightarrow \vec{x}\cdot\vec{x}+\vec{x}\cdot\vec{a}-\vec{a}\cdot\vec{x}-\vec{a}\cdot\vec{a}=12$

$\Rightarrow \quad |\vec{x}|^2-|\vec{a}|^2=12$

$\Rightarrow \quad |\vec{x}|^2-1=12 \left[\because |\vec{a}|=1 \text{ as } \vec{a} \text{ is a unit vector.}\right]$

$\Rightarrow \quad |\vec{x}|^2=13$

$\therefore \quad |\vec{x}|=\sqrt{13}$

[2]

Q. 33. Write the position vector of the point which divides the join of points with position vectors $3\vec{a}-2\vec{b}$ and $2\vec{a}+3\vec{b}$ in the ratio 2:1.
[CBSE Board, All India Region, 2016]

Ans. $\dfrac{2(2\vec{a}+3\vec{b})+1(3\vec{a}-2\vec{b})}{2+1}$

$=\dfrac{7}{3}\vec{a}+\dfrac{4}{3}\vec{b}$ (or enternal division may also be considered) **[1]**

Q. 34. Find the position vector of a point which divides the join of points with position vectors $\vec{a}-2\vec{b}$ and $2\vec{a}+\vec{b}$ externally in the ratio 2 :1.
[CBSE Board, Delhi Region, 2016]

Ans. Getting position vector as $2(2\vec{a}+\vec{b})-1(\vec{a}-2\vec{b})$

$=3\vec{a}+4\vec{b}$ **[1]**

Q. 35. If $|\vec{a}|=|\vec{b}|$, then necessarily it implies $\vec{a}=\pm\vec{b}$. State whether it is True or False.
[NCERT Exemp. Ex. 10.3, Q. 41, Page 219]

Ans. True,

If $\quad |\vec{a}|=|\vec{b}|$

$\Rightarrow \quad \vec{a}=\pm\vec{b}$

So, it is a true statement. **[1]**

Q. 36. Position vector of a point $\vec{P}$ is a vector whose initial point is origin. State whether it is True or False. [NCERT Exemp. Ex. 10.3, Q. 42, Page 219]

Ans. True,

Since, $\vec{P}=\overrightarrow{OP}=$ displacement of vector $\vec{P}$ from origin. **[1]**

Q. 37. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are orthogonal.[NCERT Exemp. Ex. 10.3, Q. 43, Page 219]

Ans. True,

Since, $\quad |\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Rightarrow \quad |\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$

$\Rightarrow \quad 2|\vec{a}||\vec{b}|=-2|\vec{a}||\vec{b}|$

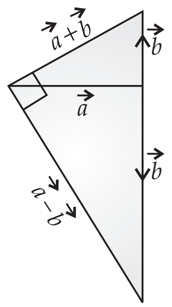
$\Rightarrow \quad 4|\vec{a}||\vec{b}|=0$

$\Rightarrow \quad |\vec{a}||\vec{b}|=0 \quad [\because \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos 90°=0]$

Hence, $\vec{a}$ and $\vec{b}$ are orthogonal. **[2]**

Q. 38. The formula $(\vec{a}+\vec{b})^2=\vec{a}^2+\vec{b}^2+2\vec{a}\times\vec{b}$ is valid for non-zero vectors $\vec{a}$ and $\vec{b}$.
[NCERT Exemp. Ex. 10.3, Q. 44, Page 219]

Ans. False



$(\vec{a}+\vec{b})^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$

$=\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}$ **[2]**

Q. 39. If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}\cdot\vec{b}=0$.[NCERT Exemp. Ex. 10.3, Q. 45, Page 219]

Ans. False

If $\vec{a}.\vec{b}=0$, then $\vec{a}\,.\,\vec{b}=|\vec{a}||\vec{b}|\cos 90°$

Hence, angle between $\vec{a}$ and $\vec{b}$ is 90°, which is not possible in a rhombus.

Since, angle between adjacent sides in a rhombus is not equal to 90°. **[2]**

Q. 40. Write two different vectors having same magnitude. [NCERT Ex. 10.2, Q. 2, Page 440]

Ans. Consider $\vec{a}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ and $\vec{b}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)$.

It can be observed that,

$|\vec{a}|=\sqrt{1^2+(-2)^2+3^2}$

$=\sqrt{1+4+9}$

$=\sqrt{14}$

and $|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2}$

$= \sqrt{4+1+9}$

$= \sqrt{14}$

Hence, $\vec{a}$ and $\vec{b}$ are two different vectors having the same magnitude. The vectors are different because they have different directions. **[2]**

Q. 41. Write two different vectors having same direction. **[NCERT Ex. 10.2, Q. 3, Page 440]**

Ans. Consider $\vec{p} = (\hat{i} + \hat{j} + \hat{k})$ and $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$.

The direction cosines of $\vec{p}$ are given by,

$$l = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}},$$

$$m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}},$$

and $n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$

The direction cosines of $\vec{q}$ are given by

$$l = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

and $n = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

The direction cosines of $\vec{p}$ and $\vec{q}$ are the same. Hence, the two vectors have the same direction. **[2]**

Q. 42. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal. **[NCERT Ex. 10.2, Q. 4, Page 440]**

Ans. The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required value of x and y are 2 and 3 respectively. **[2]**

Q. 43. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (– 5, 7). **[NCERT Ex. 10.2, Q. 5, Page 440]**

Ans. The vector with the initial point $P(2, 1)$ and terminal point $Q(-5, 7)$ can be given by,

$$\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$$

$$\Rightarrow \overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$$

Hence, the required scalar components are –7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$. **[2]**

Q. 44. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ **[NCERT Ex. 10.2, Q. 6, Page 440]**

Ans. The given vectors are :

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$$\therefore \quad \vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k}$$

$$= 0 \cdot \hat{i} - 4\hat{j} - 1 \cdot \hat{k}$$

$$= -4\hat{j} - \hat{k}$$

[2]

Q. 45. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ **[NCERT Ex. 10.2, Q. 7, Page 440]**

Ans. The unit $\hat{a}$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \frac{\vec{a}}{|a|}$.

$$|\vec{a}| = \sqrt{1^2+1^2+2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

[2]

Q. 46. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively. **[NCERT Ex. 10.2, Q. 8, Page 440]**

Ans. The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{3^2+3^2+3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of $\overrightarrow{PQ}$ is :

$$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

[2]

Q. 47. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$ **[NCERT Ex. 10.2, Q. 9, Page 440]**

Ans. The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{1^2+1^2} = \sqrt{2}$$

Hence, the unit vector in the direction of $(\vec{a}+\vec{b})$ is :

$$\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=\frac{\hat{i}+\hat{k}}{\sqrt{2}}$$
$$=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$$

[2]

Q. 48. Find a vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$. which has magnitude 8 units.
[NCERT Ex. 10.2, Q. 10, Page 440]

Ans. Let, $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$.

$$\therefore \quad |\vec{a}|=\sqrt{5^2+(-1)^2+2^2}$$
$$=\sqrt{25+1+4}$$
$$=\sqrt{30}$$
$$\therefore \quad \hat{a}=\frac{\hat{a}}{|\vec{a}|}$$
$$=\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a}=8\left(\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{30}}\right)$$
$$=\frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}$$

[2]

Q. 49. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear. **[NCERT Ex. 10.2, Q. 11, Page 440]**

Ans. Let, $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=-4\hat{i}+6\hat{j}-8\hat{k}$.

It is observed that,

$$\vec{b}=-4\hat{i}+6\hat{j}-8\hat{k}$$
$$=-2(2\hat{i}-3\hat{j}+4\hat{k})$$
$$=-2\vec{a}$$
$$\therefore \vec{b}=\lambda\vec{a}$$

Where,

$\lambda=-2$

Hence, the given vectors are collinear. **[2]**

Q. 50. Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}$.
[NCERT Ex. 10.2, Q. 12, Page 440]

Ans. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$. Therefore,

$$|\vec{a}|=\sqrt{1^2+2^2+3^2}$$
$$=\sqrt{1+4+9}$$
$$=\sqrt{14}$$

Hence, the direction cosines of $\vec{a}$ are

$$\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right).$$

[2]

Q. 51. Find the direction cosines of the vector joining the points A (1, 2, –3) and B (–1, –2, 1), directed from A to B. **[NCERT Ex. 10.2, Q. 13, Page 440]**

Ans. The given points are A (1, 2, –3) and B (–1, –2, 1).

$$\therefore \overrightarrow{AB}=(-1-1)\hat{i}+(-2-2)\hat{j}+\{1-(-3)\}\hat{k}$$
$$\Rightarrow \overrightarrow{AB}=-2\hat{i}-4\hat{j}+4\hat{k}$$
$$\therefore |\overrightarrow{AB}|=\sqrt{(-2)^2+(-4)^2+4^2}$$
$$=\sqrt{4+16+16}$$
$$=\sqrt{36}$$
$$=6$$

Hence, the direction cosines of $\overrightarrow{AB}$ are

$$\left(-\frac{2}{6},-\frac{4}{6},\frac{4}{6}\right)=\left(-\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right).$$

[2]

Q. 52. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY and OZ.
[NCERT Ex. 10.2, Q. 14, Page 440]

Ans. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$.

Then,

$$|\vec{a}|=\sqrt{1^2+1^2+1^2}$$
$$=\sqrt{3}$$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$.

Now, let α, β and γ be the angles formed by $\vec{a}$ with the positive directions of *x*, *y*, and *z* axes.

Then, we have,

$$\cos\alpha=\frac{1}{\sqrt{3}}$$
$$\cos\beta=\frac{1}{\sqrt{3}}$$
$$\cos\gamma=\frac{1}{\sqrt{3}}.$$

Hence, the given vector is equally inclined to axes *OX*, *OY*, and *OZ*. **[2]**

Q. 53. Find the position vector of the midpoint of the vector joining the points *P* (2, 3, 4) and *Q* (4, 1, –2).
[NCERT Ex. 10.2, Q. 16, Page 441]

Ans. The position vector of mid-point *R* of the vector joining points *P* (2, 3, 4) and *Q* (4, 1, –2) is given by,

$$\overline{OR}=\frac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2}$$
$$=\frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$
$$=\frac{6\hat{i}+4\hat{j}+2\hat{k}}{2}$$
$$=3\hat{i}+2\hat{j}+\hat{k}$$

[2]

Q. 54. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.
[NCERT Ex. 10.3, Q. 11, Page 448]

Ans.
$$\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right)\cdot\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)=\left(\begin{array}{l}|\vec{a}|^2\vec{b}\cdot\vec{b}-|\vec{a}||\vec{b}|\vec{b}\cdot\vec{a}+\\ |\vec{b}||\vec{a}|\vec{a}\cdot\vec{b}-|\vec{b}|^2\vec{a}\cdot\vec{a}\end{array}\right)$$
$$=|\vec{a}|^2|\vec{b}|^2-|\vec{b}|^2|\vec{a}|^2$$
$$=0$$

Hence, $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ are perpendicular to each other. **[2]**

Q. 55. If $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$, then what can be concluded about the vector $\vec{b}$? **[NCERT Ex. 10.3, Q. 12, Page 448]**

Ans. It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$.

Now,

$$\vec{a}\cdot\vec{a}=0$$

$$\Rightarrow |\vec{a}|^2=0$$

$$\Rightarrow |\vec{a}|=0$$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector. **[2]**

Short Answer Type Questions (3 and 4 marks each)

Q. 1. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=15$. [NCERT Misc. Ex. Q. 12, Page 458]

Ans. Let $\vec{d}=d_1\hat{i}+d_2\hat{j}+d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have :

$\vec{d}\cdot\vec{a}=0$

$\Rightarrow d_1+4d_2+2d_3=0$ ...(i)

And,

$\vec{d}\cdot\vec{b}=0$

$\Rightarrow 3d_1-2d_2+7d_3=0$ ...(ii)

Also, it is given that :

$\vec{c}\cdot\vec{d}=15$

$\Rightarrow 2d_1-d_2+4d_3=15$ ...(iii)

On solving equations (i), (ii), and (iii), we get :

$$d_1=\frac{160}{3}$$

$$d_2=-\frac{5}{3}$$

and $d_3=\dfrac{70}{3}$

$$\therefore \quad \vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}$$

$$=\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$. **[3]**

Q. 2. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. [NCERT Misc. Ex. Q. 13, Page 458]

Ans. $\Rightarrow \left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Therefore, unit vector along $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$ is given as :

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ with this unit vector is 1.

$$\Rightarrow \left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\Rightarrow \lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\Rightarrow \lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$\Rightarrow 8\lambda=8$$

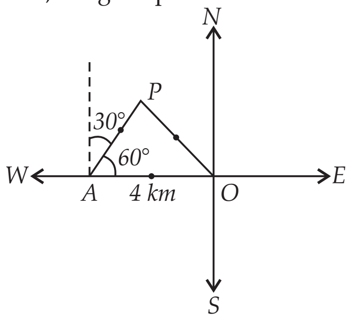
$$\Rightarrow \lambda=1$$

Hence, the value of λ is 1. **[3]**

Q. 3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. [NCERT Misc. Ex. Q. 3, Page 458]

Ans. Let O and B be the initial and final positions of the girl, respectively.

The, the girl's position can be shown as :



Now, we have :

$$\overrightarrow{OA}=-4\hat{i}$$

$$\overrightarrow{AB}=\hat{i}\left|\overrightarrow{AB}\right|\cos 60°+\hat{j}\left|\overrightarrow{AB}\right|\sin 60°$$

$$=\hat{i}3\times\frac{1}{2}+\hat{j}3\times\frac{\sqrt{3}}{2}$$

$$=\frac{3}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have :

$$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$$

$$=\left(-4\hat{i}\right)+\left(\frac{3}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}\right)$$

$$=\left(-4+\frac{3}{2}\right)\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$$

$$=\left(\frac{-8+3}{2}\right)\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$$

$$=\frac{-5}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is :

$=\frac{-5}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$. **[3]**

Q. 4. Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ [NCERT Ex. 10.4, Q. 5, Page 454]

Ans. $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$

$$\Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 27\\ 1 & \lambda & \mu\end{vmatrix}=0\hat{i}+0\hat{j}+0\hat{k}$$

$$\Rightarrow \hat{i}(6\mu-27\lambda)-\hat{j}(2\mu-27)+\hat{k}(2\lambda-6)=0\hat{i}+0\hat{j}+0\hat{k}$$

On comparing the corresponding components, we have :

$6\mu-27\lambda=0$

$2\mu-27=0$

$2\lambda-6=0$

Now,

$2\lambda-6=0$

$\Rightarrow \lambda=3$

$2\mu-27=0$

$\Rightarrow \mu=\frac{27}{2}$

Hence, $\lambda=3$ and $\mu=\frac{27}{2}$. **[3]**

Q. 5. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ and $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$. [NCERT Ex. 10.4, Q. 2, Page 454]

Ans. We have,

$$\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$$

$$\text{and } \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{a}+\vec{b}=4\hat{i}+4\hat{j}$$

$$\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 4 & 0\\ 2 & 0 & 4\end{vmatrix}$$

$$=\hat{i}(16)-\hat{j}(16)+\hat{k}(-8)$$

$$=16\hat{i}-16\hat{j}-8\hat{k}$$

$$\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|=\sqrt{16^2+(-16)^2+(-8)^2}$$

$$=\sqrt{2^2\times8^2+2^2\times8^2+8^2}$$

$$=8\sqrt{2^2+2^2+1}$$

$$=8\sqrt{9}$$

$$=8\times3$$

$$=24$$

Hence, the unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is given by the relation,

$$=\pm\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|}$$

$$=\pm\frac{16\hat{i}-16\hat{j}-8\hat{k}}{24}$$

$$=\pm\frac{2\hat{i}-2\hat{j}-\hat{k}}{3}$$

$$=\pm\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|}$$

[3]

Q. 6. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find angle ABC. [angle ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$]. [NCERT Ex. 10.3, Q. 15, Page 448]

Ans. The vertices of ΔABC are given as A (1, 2, 3), B (–1, 0, 0), and C (0, 1, 2).

Also, it is given that $\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

$$\overrightarrow{BA}=\{1-(-1)\}\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}$$

$$=2\hat{i}+2\hat{j}+3\hat{k}$$

$$\overrightarrow{BC}=\{0-(-1)\}\hat{i}+(1-0)\hat{j}+(2-0)\hat{k}$$

$$=\hat{i}+\hat{j}+2\hat{k}$$

$$\overrightarrow{BA}\cdot\overrightarrow{BC}=(2\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+2\hat{k})$$

$$=2\times1+2\times1+3\times2$$

$$=2+2+6$$

$$=10$$

$$|\overrightarrow{BA}|=\sqrt{2^2+2^2+3^2}$$

$$=\sqrt{4+4+9}$$

$$=\sqrt{17}$$

$$|\overrightarrow{BC}|=\sqrt{1+1+2^2}$$

$$=\sqrt{6}$$

Now, it is know that :

$$\overrightarrow{BA}\cdot\overrightarrow{BC}=|\overrightarrow{BA}||\overrightarrow{BC}|\cos(\angle ABC).$$

$$\therefore\quad 10=\sqrt{17}\times\sqrt{6}\cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC)=\frac{10}{\sqrt{17}\times\sqrt{6}}$$

$$\Rightarrow \angle ABC=\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

[3]

Q. 7. Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, –1) are collinear. [NCERT Ex. 10.3, Q. 16, Page 448]

Ans. The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, –1).

$$\overrightarrow{AB}=(2-1)\hat{i}+(6-2)\hat{j}+(3-7)\hat{k}$$

$$=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{BC}=(3-2)\hat{i}+(10-6)\hat{j}+(-1-3)\hat{k}$$

$$=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{AC}=(3-1)\hat{i}+(10-2)\hat{j}+(-1-7)\hat{k}$$

$$=2\hat{i}+8\hat{j}-8\hat{k}$$

$$|\overrightarrow{AB}|=\sqrt{1^2+4^2+(-4)^2}$$

$$=\sqrt{1+16+16}$$

$$=\sqrt{33}$$

$$|\overrightarrow{BC}|=\sqrt{1^2+4^2+(-4)^2}$$

$$=\sqrt{1+16+16}$$

$$=\sqrt{33}$$

$$|\overrightarrow{AC}|=\sqrt{2^2+8^2+8^2}$$

$$=\sqrt{4+64+64}$$

$$=\sqrt{132}$$

$$=2\sqrt{33}$$

$$\therefore |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$$

Hence, the given points A, B, and C are collinear. **[3]**

Q. 8. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} + 4\hat{k}$ form the vertices of a right-angled triangle. [NCERT Ex. 10.3, Q. 17, Page 448]

Ans. Let vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B, and C respectively.

i.e., $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$

and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Now, vectors $\overrightarrow{AB}, \overrightarrow{BC}$, and $\overrightarrow{AC}$ represent the sides of ΔABC.

i.e., $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$$\therefore \overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k}$$
$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$
$$\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k}$$
$$= 2\hat{i} - \hat{j} + \hat{k}$$
$$\overrightarrow{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k}$$
$$= -\hat{i} + 3\hat{j} + 5\hat{k}$$
$$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2}$$
$$= \sqrt{1+4+36}$$
$$= \sqrt{41}$$
$$|\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2 + 1^2}$$
$$= \sqrt{4+1+1}$$
$$= \sqrt{6}$$
$$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2}$$
$$= \sqrt{1+9+25}$$
$$= \sqrt{35}$$
$$\therefore |\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 6 + 35$$
$$= 41 |\overrightarrow{AB}|^2$$

Hence, ΔABC is a right-angled triangle. **[3]**

Q. 9. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. [NCERT Ex. 10.3, Q. 10, Page 448]

Ans. The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$$
$$= (2-\lambda)i + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$$

If $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$
$$\Rightarrow \left[(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)k\right] \cdot (3\hat{i} + \hat{j}) = 0$$
$$\Rightarrow (2-\lambda)3 + (2+2\lambda)1 + (3+\lambda)0 = 0$$
$$\Rightarrow -\lambda + 8 = 0$$
$$\Rightarrow \lambda = 8$$

Hence, the required value of λ is 8. **[3]**

Q. 10. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$. [NCERT Ex. 10.3, Q. 6, Page 448]

Ans.
$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$
$$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$$
$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$
$$\Rightarrow \left(8|\vec{b}|\right)^2 - |\vec{b}|^2 = 8 \quad \left[\because |\vec{a}| = 8|\vec{b}|\right]$$
$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$
$$\Rightarrow 63|\vec{b}|^2 = 8$$
$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$
$$\Rightarrow |\vec{b}|^2 = \sqrt{\frac{8}{63}} \quad \left[\begin{array}{l}\text{Magnitude of a} \\ \text{vector is non-} \\ \text{negative.}\end{array}\right]$$
$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$
$$|\vec{a}| = 8|\vec{b}|$$
$$= \frac{8 \times 2\sqrt{2}}{3\sqrt{7}}$$
$$= \frac{16\sqrt{2}}{3\sqrt{7}}$$

[3]

Q. 11. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$. [NCERT Ex. 10.4, Q. 7, Page 454]

Ans. We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$
$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$
$$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

Now, $\vec{a} \times (\vec{b} + \vec{c}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$

$$= \hat{i}\left[a_2(b_3 + c_3) - a_3(b_2 + c_2)\right] - \hat{j}\left[a_1(b_3 + c_3) - a_3(b_1 + c_1)\right] + \hat{k}\left[a_1(b_2 + c_2) - a_2(b_1 + c_1)\right]$$
$$= \hat{i}\left[a_2b_3 + a_2c_3 - a_3b_2 + a_3c_2\right] + \hat{j}\left[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1\right] + \hat{k}\left[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1\right] \quad \text{...(i)}$$
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
$$= \hat{i}\left[a_2b_3 - a_3b_2\right] + \hat{j}\left[b_1a_3 - a_1b_3\right] + \hat{k}\left[a_1b_2 - a_2b_1\right] \quad \text{...(ii)}$$
$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$
$$= \hat{i}\left[a_2c_3 - a_3c_2\right] + \hat{j}\left[a_3c_1 - a_1c_3\right] + \hat{k}\left[a_1c_2 - a_2c_1\right] \text{...(iii)}$$

[1½]

On adding equations (ii) and (iii), we get :

$(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})=\hat{i}[a_2b_3+a_2c_3-a_3b_2-a_3c_2]+\hat{j}[b_1a_3+a_3c_1-a_1b_3-a_1c_3)+\hat{k}[a_1b_2+a_1c_2-a_2b_1-a_2c_1]$ ...(iv)

Now, from equations (i) and (iv), we have :

$\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

Hence, the given result is proved. **[1½]**

Q. 12. If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\times\vec{b}=\vec{0}$. Is the converse true? Justify your answer with an example. [NCERT Ex. 10.4, Q. 8, Page 454]

Ans. Take any parallel non-zero vectors so that $\vec{a}\times\vec{b}=\vec{0}$.

Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=4\hat{i}+6\hat{j}+8\hat{k}$.

Then,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 4 & 6 & 8\end{vmatrix}$$

$=\hat{i}(24-24)-\hat{j}(16-16)+\hat{k}(12-12)$

$=0\hat{i}+0\hat{j}+0\hat{k}$

$=\hat{0}$

It can now be observed that :

$|\vec{a}|=\sqrt{2^2+3^2+4^2}$

$=\sqrt{29}$

$\therefore \vec{a}\neq\vec{0}$

$|\vec{b}|=\sqrt{4^2+6^2+8^2}$

$=\sqrt{116}$

$\therefore \vec{b}\neq\vec{0}$

Hence, the converse of the given statement need not be true. **[3]**

Q. 13. Find the area of the triangle with vertices *A* (1, 1, 2), *B* (2, 3, 5) and *C* (1, 5, 5). [NCERT Ex. 10.4, Q. 9, Page 454]

Ans. The vertices of triangle *ABC* are given as *A* (1, 1, 2), *B* (2, 3, 5) and *C* (1, 5, 5).

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of ΔABC are given as :

$\overrightarrow{AB}=(2-1)\hat{i}+(3-1)\hat{j}+(5-2)\hat{k}$

$=\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{BC}=(1-2)\hat{i}+(5-3)\hat{j}+(5-5)\hat{k}$

$=-\hat{i}+2\hat{j}$

Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}|$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & 2 & 0\end{vmatrix}$$

$=\hat{i}(-6)-\hat{j}(3)+\hat{k}(2+2)$

$=-6\hat{i}-3\hat{j}+4\hat{k}$

$|\overrightarrow{AB}\times\overrightarrow{BC}|=\sqrt{(-6)^2+(-3)^2+4^2}$

$=\sqrt{36+9+16}$

$=\sqrt{61}$

Hence, the area of ΔABC is $\frac{\sqrt{61}}{2}$ sq. units. **[3]**

Q. 14. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

[NCERT Ex. 10.4, Q. 10, Page 455]

Ans. The area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a}\times\vec{b}|$.

Adjacent sides are given as :

$\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

$=\hat{i}(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2)$

$=20\hat{i}+5\hat{j}-5\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{20^2+5^2+5^2}$

$=\sqrt{400+25+25}$

$=15\sqrt{2}$

Hence, the area of the given parallelogram is $15\sqrt{2}$ sq. units. **[3]**

Q. 15. Show that the points A, B and C with position vectors, $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively form the vertices of a right angled triangle. [NCERT Ex. 10.2, Q. 17, Page 441]

Ans. Position vectors of points A, B, and C are respectively given as :

$\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ *and* $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$

$\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ *and* $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$

$\therefore \overrightarrow{AB}=\vec{b}-\vec{a}$

$=(2-3)\hat{i}+(-1+4)\hat{j}+(1+4)\hat{k}$

$=-\hat{i}+3\hat{j}+5\hat{k}$

$\overrightarrow{BC}=\vec{c}-\vec{b}=(1-2)\hat{i}+(-3+1)\hat{j}+(-5-1)\hat{k}$

$=-\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{CA}=\vec{a}-\vec{c}=(3-1)\hat{i}+(-4+3)\hat{j}+(-4+5)\hat{k}$

$=2\hat{i}-\hat{j}+\hat{k}$

$\therefore |\overrightarrow{AB}|^2=(-1)^2+3^2+5^2$

$=1+9+25$

$=35$

$|\overrightarrow{BC}|^2=(-1)^2+(-2)^2+(-6)^2$

$=1+4+36$

$=41$

$|\overrightarrow{CA}|^2=2^2+(-1)^2+1^2$

$=4+1+1$

$=6$

$|\overrightarrow{AB}|^2+|\overrightarrow{CA}|^2=35+6$

$=41$

$=|\overrightarrow{BC}|^2$

Hence, ABC is a right-angled triangle. **[3]**

Q. 16. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2 :1

(i) internally (ii) externally

[NCERT Ex. 10.2, Q. 15, Page 440]

Ans. The position vector of point R dividing the line segment joining two points P and Q in the ratio m : n is given by :

Internally :

$$\frac{m\vec{b}+n\vec{a}}{m+n}$$

Externally :

$$\frac{m\vec{b}-n\vec{a}}{m-n}$$

Position vectors of P and Q are given as :

$\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2 :1 is given by,

$$\overrightarrow{OR}=\frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1}$$

$$=\frac{(-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$

$$=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

[2]

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\overrightarrow{OR}=\frac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}$$

$$=(-2\hat{i}+2\hat{j}+2\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$$

$$=-3\hat{i}+3\hat{k}$$

[1]

Q. 17. Compute the magnitude of the following vectors :

$$\vec{a}=\hat{i}+\hat{j}+k;\ \vec{b}=2\hat{i}-7\hat{j}-3\hat{k};\ \vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$$

[NCERT Ex. 10.2, Q. 1, Page 440]

Ans. The given vectors are :

$$\vec{a}=\hat{i}+\hat{j}+k$$

$$\vec{b}=2\hat{i}-7\hat{j}-3\hat{k}$$

$$\vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$$

$$|\vec{a}|=\sqrt{(1)^2+(1)^2+(1)^2}$$

$$=\sqrt{3}$$

$$|\vec{b}|=\sqrt{(2)^2+(-7)^2+(-3)^2}$$

$$=\sqrt{4+49+9}$$

$$=\sqrt{62}$$

$$|\vec{c}|=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$=\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$$

$$=1$$

[3]

Q. 18. Classify the following measures as scalars and vectors :

(i) 10 kg (ii) 2 meters north-west

(iii) 40° (iv) 40 watt

(v) 10^{-19} coulomb (vi) 20 m/s^2

[NCERT Ex. 10.1, Q. 2, Page 428]

Ans. (i) 10 kg is a scalar quantity because it involves only magnitude.

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.

(iii) 40° is a scalar quantity as it involves only magnitude.

(iv) 40 watts is a scalar quantity as it involves only magnitude.

(v) 10^{-19} Coulomb is a scalar quantity as it involves only magnitude.

(vi) 20 m/s^2 is a vector quantity as it involves magnitude as well as direction. **[3]**

Q. 19. Classify the following as scalar and vector quantities :

(i) Time period (ii) Distance

(iii) Force (iv) Velocity

(v) Work done [NCERT Ex. 10.1, Q. 3, Page 428]

Ans. (i) Time period is a scalar quantity as it involves only magnitude.

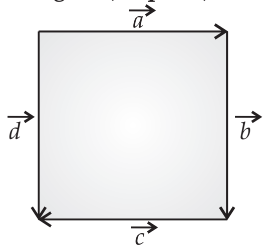
(ii) Distance is a scalar quantity as it involves only magnitude.

(iii) Force is a vector quantity as it involves both magnitude and direction.

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.

(v) Work done is a scalar quantity as it involves only magnitude. **[3]**

Q. 20. In Figure (a square), identify the following vectors.



(i) Co-initial (ii) Equal

(iii) Collinear but not equal [NCERT Ex. 10.1, Q. 4, Page 428]

Ans. (i) Vectors $\vec{a}$ and $\vec{d}$ are co-initial because they have the same initial point.

(ii) Vectors $\vec{b}$ and $\vec{d}$ are equal because they have the same magnitude and direction.

(iii) Vectors $\vec{a}$ and $\vec{c}$ are collinear but not equal. This is because although they are parallel, their directions are not same. **[3]**

Q. 21. Prove that $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$, if and only if $\vec{a},\vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0},\vec{b}\neq\vec{0}$..

[NCERT Misc. Ex. Q. 15, Page 459]

Ans. $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=|\vec{a}|^2+|\vec{b}|^2$

$\Leftrightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2$ [Distributive property of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ [$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (Scalar product is commutative)]

$\Leftrightarrow 2\vec{a}\cdot\vec{b}=0$

$\Leftrightarrow \vec{a}\cdot\vec{b}=0$

$\therefore \vec{a}$ and $\vec{b}$ are perpendicular.

$\left[\vec{a}\neq\vec{0},\vec{b}\neq\vec{0}\ \text{(Given)}\right]$ **[3]**

Q. 22. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

[CBSE Board, Foreign Scheme, 2016]

Ans. $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ ...(i)

$\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ ...(ii)

From equations (i) and (ii), we have

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=\vec{d}\times(\vec{b}-\vec{c})$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow (\vec{a}-\vec{d})\,||\,(\vec{b}-\vec{c})$ **[4]**

Q. 23. Find the unit vector in the direction of sum of vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{j}+\hat{k}$.

[NCERT Exemp. Ex. 10.3, Q. 1, Page 215]

Ans. Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$.

We have $\vec{c}=\vec{a}+\vec{b}$

$=2\hat{i}-\hat{j}+\hat{k}+2\hat{j}+\hat{k}=2\hat{i}+\hat{j}+2\hat{k}$

$\therefore$ Unit vector in the direction of,

$$\vec{c}=\frac{\vec{c}}{|\vec{c}|}=\frac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{2^2+1^2+2^2}}=\frac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{9}}$$

$$\vec{c}=\frac{2\hat{i}+\hat{j}+2\hat{k}}{3}$$ **[4]**

Q. 24. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, find the unit vector in the direction of

(i) $6\vec{b}$ **(ii) $2\vec{a}-\vec{b}$**

[NCERT Exemp. Ex. 10.3, Q. 2, Page 215]

Ans. Given that,

Here, $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$

(i) Since, $6\vec{b}=12\hat{i}+6\hat{j}-12\hat{k}$

$\therefore$ Unit vector in the direction of $6\vec{b}=\dfrac{6\vec{b}}{|6\vec{b}|}$

$$=\frac{12\hat{i}+6\hat{j}-12\hat{k}}{\sqrt{12^2+6^2+12^2}}=\frac{6(2\hat{i}+\hat{j}-2\hat{k})}{\sqrt{324}}=\frac{6(2\hat{i}+\hat{j}+2\hat{k})}{18}=\frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$$ **[2]**

(ii) Since, $2\vec{a}-\vec{b}=2(\hat{i}+\hat{j}+2\hat{k})-(2\hat{i}+\hat{j}-2\hat{k})$

$=2\hat{i}+2\hat{j}+4\hat{k}-2\hat{i}-\hat{j}+2\hat{k}$

$=\hat{j}+6\hat{k}$

$\therefore$ Unit vector in the direction of,

$$2\vec{a}-\vec{b}=\frac{2\vec{a}-\vec{b}}{|2\vec{a}-\vec{b}|}=\frac{\hat{j}-6\hat{k}}{\sqrt{1+36}}=\frac{1}{\sqrt{37}}(\hat{j}+6\hat{k})$$ **[2]**

Q. 25. Find a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q have coordinates (5, 0, 8) and (3, 3, 2), respectively.[NCERT Exemp. Ex. 10.3, Q. 3, Page 215]

Ans. Since, the coordinates of P and Q are (5, 0, 8) and (3, 3, 2), respectively.

$\therefore \overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}$

$=(3\hat{i}+3\hat{j}+2\hat{k})-(5\hat{i}+0\hat{j}+8\hat{k})$

$=-2\hat{i}+3\hat{j}-6\hat{k}$ **[2]**

$\therefore$ Unit vector in the direction of,

$$\overrightarrow{PQ}=\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\frac{-2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{2^2+3^2+6^2}}=\frac{-2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{49}}=\frac{-2\hat{i}+3\hat{j}-6\hat{k}}{7}$$ **[2]**

Q. 26. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that $BC = 1.5\,BA$.

[NCERT Exemp. Ex. 10.3, Q. 4, Page 215]

Ans. Since, $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$

$\overrightarrow{BA}=\overrightarrow{OA}-\overrightarrow{OB}=\vec{a}-\vec{b}$

and $1.5\overrightarrow{BA}=1.5(\vec{a}-\vec{b})$

Since, $\overrightarrow{BC}=1.5\overrightarrow{BA}$

$=1.5(\vec{a}-\vec{b})$

$\overrightarrow{OC}-\overrightarrow{OB}=1.5\vec{a}-1.5\vec{b}$

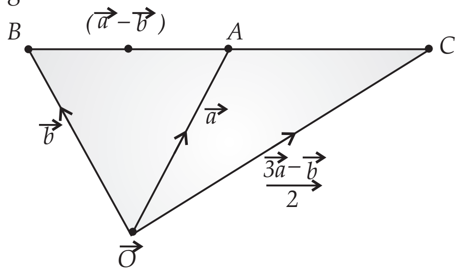
$\overrightarrow{OC}=1.5\vec{a}-1.5\vec{b}+\vec{b}$ $[\because \overrightarrow{OB}=\vec{b}]$

$$= 1.5\vec{a} - 0.5\vec{b}$$

$$= \frac{3\vec{a} - \vec{b}}{2}$$

[2]

Graphically, explanation of the above solution is given below :



[1]

Q. 27. Using vectors, find the value of k such that the points (k, –10, 3), (1, –1, 3) and (3, 5, 3) are collinear.
[NCERT Exemp. Ex. 10.3, Q. 5, Page 215]

Ans. A B C

Let the points are A (k, –10, 3), B (1, –1, 3) and C (3, 5, 3).

So, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= (\hat{i} - \hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$$

$$= (1-k)\hat{i} + (-1+10)\hat{j} + (3-3)\hat{k}$$

$$= (1-k)\hat{i} + 9\hat{j} + 0\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{(1-k)^2 + (9)^2 + 0}$$

$$= \sqrt{(1-k)^2 + 81}$$

Similarly, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + 3\hat{k})$$

$$= 2\hat{i} + 6\hat{j} + 0\hat{k}$$

$$\therefore \quad |\overrightarrow{BC}| = \sqrt{2^2 + 6^2 + 0}$$

$$= 2\sqrt{10}$$

and $\quad \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OB}$

$$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$$

$$= (3-k)\hat{i} + 15\hat{j} + 0\hat{k}$$

$$|\overrightarrow{AC}| = \sqrt{(3-k)^2 + 225}$$

[2]

If A, B and C are collinear, then sum of modulus of any two vectors will be equal to the modulus of third vectors.

For $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$,

$$\sqrt{(1-k)^2 + 81} + 2\sqrt{10} = \sqrt{(3-k)^2 + 225}$$

$$\Rightarrow \quad \sqrt{(3-k)^2 + 225} - \sqrt{(1-k)^2 + 81} = 2\sqrt{10}$$

$$\Rightarrow \sqrt{9 + k^2 - 6k + 225} - \sqrt{1 + k^2 - 2k + 81} = 2\sqrt{10}$$

$$\Rightarrow \quad \sqrt{k^2 - 6k + 234} - 2\sqrt{10} = \sqrt{k^2 - 2k + 82}$$

$$\Rightarrow k^2 - 6k + 234 + 40 - 2\sqrt{k^2 - 6k + 234} \cdot 2\sqrt{10} = k^2 - 2k + 82$$

$$\Rightarrow k^2 - 6k + 234 + 40 - k^2 + 2k - 82 = 4\sqrt{10}\sqrt{k^2 + 234 - 6k}$$

$$\Rightarrow \quad -4k + 192 = 4\sqrt{10}\sqrt{k^2 + 234 - 6k}$$

$$\Rightarrow \quad -k + 48 = \sqrt{10}\sqrt{k^2 + 234 - 6k}$$

On squaring both sides, we get

$$\Rightarrow \quad (k^2 + 4k) = +16 \times 16 - 260 \quad \begin{bmatrix} \text{Divided by 9} \\ \text{both sides} \end{bmatrix}$$

$$\Rightarrow \quad k^2 + 4k = -4$$

$$k^2 + 4k + 4 = 0$$

$$\Rightarrow \quad (k+2)^2 = 0$$

$$\therefore \quad k = -2$$

[2]

Q. 28. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.
[NCERT Exemp. Ex. 10.3, Q. 6, Page 215]

Ans. We have, $|\vec{r}| = 2\sqrt{3}$

Since, $\vec{r}$ is equally inclined to the three axes, $\vec{r}$ so direction cosines of the unit vector $\vec{r}$ will be same *i.e.*, l = m = n.

We know that,

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow l^2 + l^2 + l^2 = 1$$

$$\Rightarrow \quad l^2 = \frac{1}{3}$$

$$\Rightarrow \quad l = \pm\left(\frac{1}{\sqrt{3}}\right)$$

So, $\quad \hat{r} = \pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}$

$$\therefore \quad \vec{r} = \hat{r}|\vec{r}| \quad \left[\because \hat{r} = \frac{\vec{r}}{|\vec{r}|}\right]$$

$$= \left[\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right]2\sqrt{3}$$

$$\left[\because |r| = 2\sqrt{3}\right]$$

$$= \pm 2\hat{i} \pm 2\hat{j} \pm 2\hat{k}$$

$$= \pm 2(\hat{i} + \hat{j} + \hat{k})$$

[4]

Q. 29. A vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3, –6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.
[NCERT Exemp. Ex. 10.3, Q. 7, Page 215]

Ans. Here, $|\vec{r}| = 14$, $\vec{a} = 2k$, $\vec{b} = 3k$ and $\vec{c} = -6k$

$\therefore$ Direction cosines l, m and n are :

$$l = \frac{\vec{a}}{|\vec{r}|}$$

$$= \frac{2k}{14}$$

$$= \frac{k}{7}$$

$$m = \frac{\vec{b}}{|\vec{r}|} = \frac{3k}{14}$$

and $n = \dfrac{\vec{c}}{|\vec{r}|}$

$$= \frac{-6k}{14} = \frac{-3k}{7}$$

Also, we know that,

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \frac{k^2}{49} + \frac{9k^2}{196} + \frac{9k^2}{49} = 1$$

$$\Rightarrow \frac{4k^2 + 9k^2 + 36k^2}{196} = 1$$

$$\Rightarrow \quad k^2 = \frac{196}{49} = 4$$

$$\Rightarrow \quad k = \pm 2$$

So, the direction cosines (l, m and n) are :

$\frac{2}{7}, \frac{3}{7}$ and $\frac{-6}{7}$.

[Since, $\vec{r}$ makes an acute angle with x-axis]

$\because \vec{r} = \hat{r}.|\vec{r}|$

$\therefore \vec{r} = (l\hat{i} + m\hat{j} + n\hat{k})|\vec{r}|$

$$= \left(\frac{\pm 2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right).14$$

$$= \pm 4\hat{i} + 6\hat{j} - 12\hat{k}$$

[4]

Q. 30. Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.
[NCERT Exemp. Ex. 10.3, Q. 8, Page 215]

Ans. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 3\hat{k}$

So, any vector perpendicular to both the vector $\vec{a}$ and $\vec{b}$ is given by,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & -1 & 3 \end{vmatrix}$$

$$= \hat{i}(-3+2) - \hat{j}(6-8) + \hat{k}(-2+4)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k}$$

$$= \vec{r} \quad \text{[Say]}$$

A vector of magnitude 6 in the direction of $\vec{r}$

$$= \frac{\vec{r}}{|\vec{r}|}.6 = \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}}.6$$

$$= \frac{-6}{3}\hat{i} + \frac{12}{3}\hat{j} + \frac{12}{3}\hat{k}$$

$$= -2\hat{i} + 4\hat{j} + 4\hat{k}$$

[4]

Q. 31. Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.[NCERT Exemp. Ex. 10.3, Q. 9, Page 215]

Ans. Let $\vec{a} = 2\hat{i} - \hat{j} = \hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$

We know that, angle between two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k})(3\hat{i} + 4\hat{j} - \hat{k})}{\sqrt{4+1+1}\sqrt{9+16+1}}$$

$$= \frac{6-4-1}{\sqrt{6}\sqrt{26}}$$

$$= \frac{1}{2\sqrt{39}}$$

$$\theta = \cos^{-1}\left(\frac{1}{2\sqrt{39}}\right)$$

[3]

Q. 32. If $\vec{a} + \vec{b} + \vec{c} = 0$, then show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Interpret the result geometrically.
[NCERT Exemp. Ex. 10.3, Q. 10, Page 215]

Ans. Since, $\vec{a} + \vec{b} + \vec{c} = 0$

$$\vec{b} = -\vec{c} - \vec{a}$$

Now, $\vec{a} \times \vec{b} = \vec{a} \times (-\vec{c} - \vec{a})$

$$= \vec{a} \times (-\vec{c}) + \vec{a} \times (-\vec{a})$$

$$= -\vec{a} \times \vec{c}$$

$$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a} \quad \text{...(i)}$$

Also, $\vec{b} \times \vec{c} = (-\vec{c} - \vec{a}) \times \vec{c}$

$$= (\vec{c} \times \vec{c}) + (-\vec{a} \times \vec{c})$$

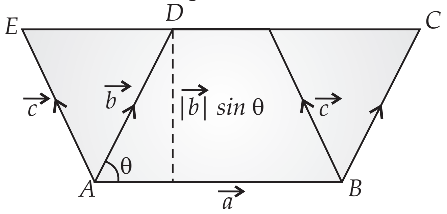
$$= -\vec{a} \times \vec{c}$$

$$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \quad \text{...(ii)}$$

From equations (i) and (ii), we have

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

Geometrical interpretation of the result :



If $ABCD$ is a parallelogram such that $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$. These adjacent sides are making angle θ between each other, then we say that,

Area of parallelogram $ABCD = |\vec{a}||\vec{b}||\sin\theta| = |\vec{a} \times \vec{b}|$

Since, parallelogram on the same base and between the same parallels is equal in area.

We can say that

$$|\vec{a} \times \vec{b}| = |\vec{a} \times \vec{c}| = |\vec{b} \times \vec{c}|$$

This also implies that,

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

So, areas of the parallelograms formed by taking any two sides represented by $\vec{a}, \vec{b}$ and $\vec{c}$ as adjacent are equal. **[3]**

Q. 33. Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.
[NCERT Exemp. Ex. 10.3, Q. 11, Page 215]

Ans. Here, $a_1 = 3, a_2 = 1, a_3 = 2$ and $b_1, = 2, b_2 = -2, b_3 = 4$

We know that,

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$= \frac{3\times 2 + 1\times(-2) + 2\times 4}{\sqrt{3^2+1^2+2^2}\sqrt{2^2+(-2)^2+4^2}}$$

$$= \frac{6-2+8}{\sqrt{14}\sqrt{24}}$$

$$= \frac{12}{2\sqrt{14}\sqrt{6}}$$

$$= \frac{6}{\sqrt{84}}$$

$$= \frac{6}{2\sqrt{21}}$$

$$= \frac{3}{\sqrt{21}}$$

$$\therefore\ \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\frac{9}{21}}$$

$$= \sqrt{\frac{12}{21}}$$

$$= \frac{2\sqrt{3}}{\sqrt{3}\sqrt{7}}$$

$$= \frac{2}{\sqrt{7}}$$

[3]

Q. 34. If A, B, C, and D are the points with position vectors $\hat{i}+\hat{j}-\hat{k}, 2\hat{i}-\hat{j}+3\hat{k}, 2\hat{i}-3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ respectively, then find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

[NCERT Exemp. Ex. 10.3, Q. 12, Page 216]

Ans. Here,

$\overrightarrow{OA} = \hat{i}+\hat{j}-\hat{k}$

$\overrightarrow{OB} = 2\hat{i}-\hat{j}+3\hat{k}$

$\overrightarrow{OC} = 2\hat{i}-3\hat{k}$

and $\overrightarrow{OD} = 3\hat{i}-2\hat{j}+\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA}$

$= (2-1)\hat{i}+(-1-1)\hat{j}+(3+1)\hat{k}$

$= \hat{i}-2\hat{j}+4\hat{k}$

and $\overrightarrow{CD} = \overrightarrow{OD}-\overrightarrow{OC}$

$= (3-2)\hat{i}+(-2-0)\hat{j}+(1+3)\hat{k}$

$= \hat{i}-2\hat{j}+4\hat{k}$

So, the projection of $\overrightarrow{AB}$ along,

$$\overrightarrow{CD} = \overrightarrow{AB}\cdot\frac{\overrightarrow{CD}}{|\overrightarrow{CD}|}$$

$$= \frac{(\hat{i}-2\hat{j}+4\hat{k}).(\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{1^2+2^2+4^2}}$$

$$= \frac{1+4+16}{\sqrt{21}}$$

$$= \frac{21}{\sqrt{21}}$$

$= \sqrt{21}$ units **[3]**

Q. 35. Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B (2, –1, 4) and C (4, 5, –1).

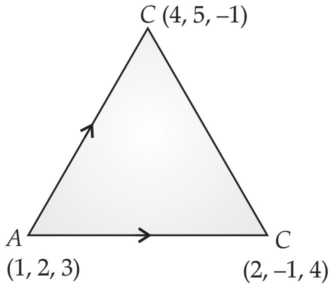
[NCERT Exemp. Ex. 10.3, Q. 13, Page 216]

Ans. Here, $\overrightarrow{AB} = (2-1)\hat{i}+(-1-2)\hat{j}+(4-3)\hat{k}$

$= \hat{i}-3\hat{j}+\hat{k}$

and $\overrightarrow{AC} = (4-1)\hat{i}+(5-2)\hat{j}+(-1-3)\hat{k}$

$= 3\hat{i}+3\hat{j}-4\hat{k}$



$$\therefore\quad \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 3 & 3 & -4\end{vmatrix}$$

$$= \hat{i}(12-3)-\hat{j}(-4-3)+\hat{k}(3+9)$$

$$= 9\hat{i}+7\hat{j}+12\hat{k}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 3 & 3 & -4\end{vmatrix}$$

$$= \hat{i}(12-3)-\hat{j}(-4-3)+\hat{k}(3+9)$$

$$= 9\hat{i}+7\hat{j}+12\hat{k}$$

and $|\overrightarrow{AB}\times\overrightarrow{AC}| = \sqrt{9^2+7^2+12^2}$

$= \sqrt{81+49+144}$

$= \sqrt{274}$

$\therefore$Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

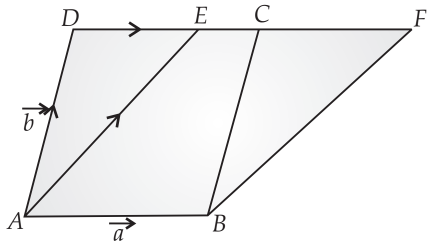
$= \frac{1}{2}\sqrt{274}$ sq. units

[3]

Q. 36. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area. [NCERT Exemp. Ex. 10.3, Q. 14, Page 216]

Ans. Let $ABCD$ and $ABFE$ are parallelograms on the same base AB and between the same parallel lines AB and DF.

Here, $AB||CD$ and $AE||BF$



Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$

$\therefore$ Area of parallelogram, $ABCD = \vec{a}\times\vec{b}$

Now, area of parallelogram $ABFF = \overrightarrow{AB}\times\overrightarrow{AE}$

$= \overrightarrow{AB}\times(\overrightarrow{AD}+\overrightarrow{DE})$

$= \overrightarrow{AB}\times(\vec{b}+k\vec{a})$ $\left[\text{Let } \overrightarrow{DE} = k\vec{a}, \text{ where } k \text{ is scalar}\right]$

$= \vec{a}\times(\vec{b}+k\vec{a})$

$= (\vec{a}\times\vec{b}) + (\vec{a}\times k\vec{a})$
$= (\vec{a}\times\vec{b}) + k(\vec{a}\times\vec{a})$
$= (\vec{a}\times\vec{b})$ $\qquad [\because \vec{a}\times\vec{a} = 0]$
= Area of parallelogram *ABCD*
Thus, proved.
[3]

Q. 37. Let $\vec{a}=4\vec{i}+5\vec{j}-\vec{k}, \vec{b}=\vec{i}-4\vec{j}+5\vec{k}$ and $\vec{c}=3\vec{i}+\vec{j}-\vec{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a}=21$.
[CBSE Board, All India Region, 2018]

Ans. We are given that $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$.

$\vec{d}.\vec{b} = 0$ and $\vec{d}.\vec{c} = 0$

Let us suppose $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

Now,

$$\vec{d}.\vec{b} = 0$$
$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}-4\hat{j}+5\hat{k}) = 0$$
$$\Rightarrow \quad x - 4y + 5z = 0 \qquad \ldots(i)$$
$$\vec{d}.\vec{c} = 0$$
$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(3\hat{i}+\hat{j}-\hat{k}) = 0$$
$$\Rightarrow \quad 3x + y - z = 0 \qquad \ldots(ii)$$
$$\vec{d}.\vec{a} = 21$$
$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(4\hat{i}+5\hat{j}-\hat{k}) = 21$$
$$\Rightarrow \quad 4x + 5y - z = 21 \qquad \ldots(iii)$$

Solving equations (i), (ii) and (iii), we get

$$x = \frac{1}{3}, y = \frac{16}{3} \text{ and } z = \frac{13}{3}$$
$$\therefore \quad \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$$
[4]

Q. 38. If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vectors $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$, Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.
[CBSE Board, Delhi Region, 2017]

Ans. $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}.\vec{b} = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ $\qquad \ldots(i)$

Let α, β and γ be the angles made by $(\vec{a}+\vec{b}+\vec{c})$ with $\vec{a},\vec{b}$ and $\vec{c}$ respectively.

$|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}.\vec{b} = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$

$(\vec{a}+\vec{b}+\vec{c}).\vec{a} = |\vec{a}+\vec{b}+\vec{c}||\vec{a}|\cos\alpha$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

Similarly, $\beta = \cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

and $\gamma = \cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

Using equation (i), we get α = β = γ

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3|\vec{a}|^2$ [Using Eq. (i)]

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}|\vec{a}|$

$$\therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \beta = \gamma$$
[4]

Q. 39. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 6$.
[CBSE Board, Delhi Region, 2017]

Ans. Let, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$; $\vec{a}.\vec{c} = 6$

$\Rightarrow 2x + y - z = 6$

Now, $\vec{a}\times\vec{c} = \vec{b}$

$$\Rightarrow \begin{vmatrix} i & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = 4\hat{i} - 7\hat{j} + \hat{k}$$

$\Rightarrow \hat{i}(z+y) - \hat{j}(2z+x) + \hat{k}(2y-x) = 4\hat{i} - 7\hat{j} + \hat{k}$

$\Rightarrow z + y = 4,\ 2z + x = 7,\ 2y - x = 1$

Solving and getting $x = 3,\ y = 2,\ z = 2$

$\vec{c} = 3\hat{i} + 2\hat{i} + 2\hat{k}$
[4]

Q. 40. Find the position vector of a point *R* which divides the line joining two points *P* and *Q* whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ. [NCERT Misc. Ex. Q. 9, Page 458]

Ans. It is given that $\overrightarrow{OP} = 2\vec{a}+\vec{b},\ \overrightarrow{OQ} = \vec{a}-3\vec{b}$.

It is given that point *R* divides a line segment joining two points *P* and *Q* externally in the ratio 1:2. Then, on using the Section Formula, we get :

$$\overrightarrow{OR} = \frac{2(2\vec{a}+\vec{b}) - (\vec{a}-3\vec{b})}{2-1}$$
$$= \frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1}$$
$$= 3\vec{a}+5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a}+5\vec{b}$.

Position vector of the mid-point of RQ $= \dfrac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$

$$= \frac{(\vec{a}-3\vec{b}) + (3\vec{a}+5\vec{b})}{2}$$
$$= 2\vec{a}+\vec{b}$$
$$= \overrightarrow{OP}$$

Hence, P is the mid-point of the line segment RQ. **[3]**

Q. 41. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.
[NCERT Misc. Ex. Q. 10, Page 458]

Ans. Adjacent sides of a parallelogram are given as : $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$.

$$\vec{a}+\vec{b} = (2+1)\hat{i} + (-4-2)\hat{j} + (5-3)\hat{k}$$
$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Thus, the unit vector parallel to the diagonal is :

$$\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}}$$
$$=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}$$
$$=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$$
$$=\frac{3}{7}\hat{i}$$
$$=\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$

$\therefore$ Area of parallelogram $ABCD=|\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$
$$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$
$$=22\hat{i}+11\hat{j}$$
$$=11(2\hat{i}+\hat{j})$$
$$\therefore |\vec{a}\times\vec{b}|=11\sqrt{2^2+1^2}$$
$$=11\sqrt{5}$$

Hence, the area of the parallelogram is $11\sqrt{5}$ sq. units. **[3]**

Long Answer Type Questions (5 or 6 marks each)

Q. 1. Show that the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC. [NCERT Misc. Ex. Q. 8, Page 458]

Ans. The given points are $A\ (1,-2,-8)$, $B\ (5,0,-2)$ and C $(11,3,7)$.

$$\overrightarrow{AB}=(5-1)\hat{i}+(0+2)\hat{j}+(-2+8)\hat{k}$$
$$=4\hat{i}+2\hat{j}+6\hat{k}$$
$$\overrightarrow{BC}=(11-5)\hat{i}+(3-0)\hat{j}+(7+2)\hat{k}$$
$$=6\hat{i}+3\hat{j}+9\hat{k}$$
$$\overrightarrow{AC}=(11-1)\hat{i}+(3+2)\hat{j}+(7+8)\hat{k}$$
$$=10\hat{i}+5\hat{j}+15\hat{k}$$
$$|\overrightarrow{AB}|=\sqrt{4^2+2^2+6^2}$$
$$=\sqrt{16+4+36}$$
$$=\sqrt{56}$$
$$=2\sqrt{14}$$
$$|\overrightarrow{BC}|=\sqrt{6^2+3^2+9^2}$$
$$=\sqrt{36+9+81}$$
$$=\sqrt{126}$$
$$=3\sqrt{14}$$
$$|\overrightarrow{AC}|=\sqrt{10^2+5^2+15^2}$$
$$=\sqrt{100+25+225}$$
$$=\sqrt{350}$$
$$=5\sqrt{14}$$
$$\therefore |\overrightarrow{AC}|=|\overrightarrow{AB}|+|\overrightarrow{BC}|$$

Thus, the given points A, B and C are collinear.
Now, let points B divide AC in the ratio λ :1. Then, we have :

$$\overrightarrow{OB}=\frac{\lambda\overrightarrow{OC}+\overrightarrow{OA}}{(\lambda+1)}$$

$$\Rightarrow \quad 5\hat{i}-2\hat{k}=\frac{\lambda(11\hat{i}+3\hat{j}+7\hat{k})+(\hat{i}-2\hat{j}-8\hat{k})}{\lambda+1}$$

$$\Rightarrow \quad (\lambda+1)(5\hat{i}-2\hat{k})=11\lambda\hat{i}+3\lambda\hat{i}+7\lambda\hat{k}+\hat{i}-2\hat{j}-8\hat{k}$$
$$\Rightarrow 5(\lambda+1)\hat{i}-2(\lambda+1)\hat{k}=(11\lambda+1)\hat{i}+(3\lambda-2)\hat{j}+(7\lambda-8)\hat{k}$$

On equating the corresponding components, we get :
$$5(\lambda+1)=11\lambda+1$$
$$\Rightarrow 5\lambda+5=11\lambda+1$$
$$\Rightarrow 6\lambda=4$$
$$\Rightarrow \lambda=\frac{4}{6}=\frac{2}{3}$$

Hence, point B divides AC in the ratio 2 :3. **[5]**

Q. 2. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. [NCERT Ex. 10.3, Q. 13, Page 448]

Ans. It is given that $\vec{a}+\vec{b}+\vec{c}=0$.

$$\therefore \vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{a}\cdot\vec{0}$$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=\vec{a}\cdot\vec{0}$ [Distributivity of scalar product over addition]

$\Rightarrow 1+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ …(i)

$$\begin{bmatrix}\vec{a}\cdot\vec{a}=|\vec{a}|\cdot|\vec{a}|\cos 0°=1\\ (\vec{a}\text{ is unit vector}\Rightarrow|\vec{a}|=1)\end{bmatrix}$$

$$\therefore \vec{b}\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{b}\cdot\vec{0}$$
$$\Rightarrow \vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=\vec{b}\cdot\vec{0}$$

$\Rightarrow \vec{b}\cdot\vec{a}+1+\vec{b}\cdot\vec{c}=0$ …(ii) $\left[\vec{b}\cdot\vec{b}=1\right]$

$$\therefore \vec{c}\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{c}\cdot\vec{0}$$
$$\Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}.\vec{c}=\vec{c}\cdot\vec{0}$$

$\Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+1=0$ …(iii) $\left[\vec{b}\cdot\vec{b}=1\right]$

From equations (i), (ii) and (iii), we have
$$\left(1+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}\right)+\left(\vec{b}.\vec{a}+1+\vec{b}\cdot\vec{c}\right)+\left(\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+1\right)$$
$$=0+0+0$$
$$\Rightarrow\left(3+\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{a}\right)+\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}\right)+\left(\vec{c}\cdot\vec{a}+\vec{b}\cdot\vec{c}\right)$$
= 0 [Scalar product is commutative.]

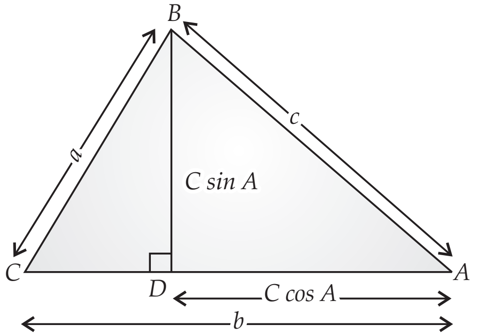
$$\Rightarrow 3+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-3}{2}$$

[5]

Q. 3. Prove that in any triangle ABC, $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$, where a, b, c are the magnitudes of the sides opposite to the vertices A, B, C, respectively.
[NCERT Exemp. Ex. 10.3, Q. 15, Page 216]

Ans. Here, components of C are $c\cos A$ and $c\sin A$ is drawn.



Since, $\overrightarrow{CD}=b-c\cos A$
In ΔBDC,

$$a^2=(b-c\cos A)^2+(c\sin A)^2$$

$$\Rightarrow a^2=b^2+c^2\cos^2 A-2bc\cos A+c^2\sin^2 A$$

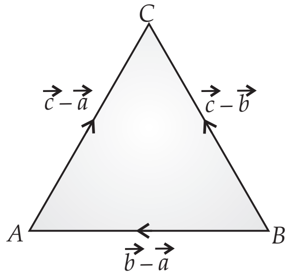
$$\Rightarrow 2bc\cos A=b^2-a^2+c^2(\cos^2 A+\sin^2 A)$$

$$\therefore \cos A=\frac{b^2+c^2-a^2}{2bc}$$

[5]

Q. 4. If $\vec{a},\vec{b}$ and $\vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}[\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}]$ gives the vector area of the triangle. Hence deduce the condition that the three points $\vec{a},\vec{b}$ and $\vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle. [NCERT Exemp. Ex. 10.3, Q. 16, Page 216]

Ans. Since, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the vertices of a ABC as shown below.



$$\therefore \text{ Area of } \Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}$ and $\overrightarrow{AC}=\vec{c}-\vec{a}$

$$\therefore \text{ Area of } \Delta ABC=\frac{1}{2}\left|\vec{b}-\vec{a}\times\vec{c}-\vec{a}\right|$$

$$=\frac{1}{2}\left|\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}\right|$$

$$=\frac{1}{2}\left|\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{0}\right|$$

$$=\frac{1}{2}\left|\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right|$$

For three points to be collinear, area of the ΔABC should be equal to zero.
This is the required condition for collinearity of three points $\vec{a}$, $\vec{b}$ and $\vec{c}$.
Let $\hat{n}$ be the unit vector normal to the plane of the ΔABC.

$$\therefore \hat{n}=\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}$$

$$=\frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}$$

[5]

Q. 5. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.
[NCERT Ex. 10.4, Q. 3, Page 454]

Ans. Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$$\Rightarrow \vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

Since $\vec{a}$ is a unit vector, $|\vec{a}|=1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have :

$$\cos\frac{\pi}{3}=\frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2}=a_1 \quad \left[|\vec{a}|=1\right]$$

$$\cos\frac{\pi}{4}=\frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}}=a_2 \quad \left[|\vec{a}|=1\right]$$

Also, $\cos\theta=\dfrac{a_3}{|\vec{a}|}$,

$$\Rightarrow a_3=\cos\theta$$

Now,

$$|a|=1$$

$$\Rightarrow \sqrt{a_1^2+a_2^2+a_3^2}=1$$

[3]

$$\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\theta=1$$

$$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\theta=1$$

$$\Rightarrow \frac{3}{4}+\cos^2\theta=1$$

$$\Rightarrow \cos^2\theta=1-\frac{3}{4}=\frac{1}{4}$$

$$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$$

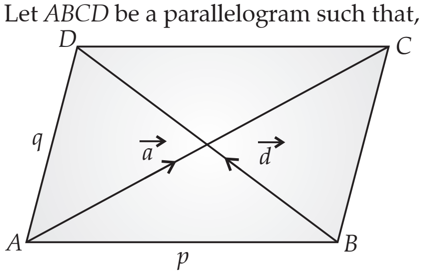
$$\therefore a_3=\cos\frac{\pi}{3}=\frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$. **[2]**

Q. 6. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

[NCERT Exemp. Ex. 10.3, Q. 17, Page 216]

Ans. Let $ABCD$ be a parallelogram such that,



$\overrightarrow{AB} = \vec{p},\ \overrightarrow{AD} = \vec{q} \Rightarrow \overrightarrow{BC} = \vec{q}$

By triangle law of addition, we get

$\overrightarrow{AC} = \vec{p} + \vec{q} = \vec{a}$ [Say] ...(i)

Similarly, $\overrightarrow{BD} = -\vec{p} + \vec{q} = \vec{b}$ [Say] ...(ii)

On adding equations (i) and (ii), we get :

$$\vec{a} + \vec{b} = 2\vec{q} \Rightarrow \vec{q} = \frac{1}{2}(\vec{a} + \vec{b})$$

On subtracting Eq. (ii) from Eq. (i), we get :

$$\vec{a} - \vec{b} = 2\vec{p} \Rightarrow \vec{p} = \frac{1}{2}(\vec{a} - \vec{b})$$

Now,

$$\vec{p}\times\vec{q} = \frac{1}{4}(\vec{a} - \vec{b})\times(\vec{a} + \vec{b})$$

$$= \frac{1}{4}(\vec{a}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{a} - \vec{b}\times\vec{b})$$

$$= \frac{1}{4}[\vec{a}\times\vec{b} + \vec{a}\times\vec{b}]$$

$$= \frac{1}{2}(\vec{a}\times\vec{b})$$

So, area of parallelogram $ABCD = |\vec{p}\times\vec{q}| = \frac{1}{2}|\vec{a}\times\vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

$$= \frac{1}{2}\left|(2\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+3\hat{j}-\hat{k})\right|$$

$$= \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{matrix} \right\|$$

$$= \frac{1}{2}\left|[\hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)]\right|$$

$$= \frac{1}{2}\left|-2\hat{i}+3\hat{j}+7\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{4+9+49}$$

$$= \frac{1}{2}\sqrt{62} \text{ sq. units}$$

[5]

Q. 7. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$, then find a vector $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$.

[NCERT Exemp. Ex. 10.3, Q. 18, Page 216]

Ans. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Also $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$

For $\vec{a}\times\vec{c} = \vec{b}$,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j}-\hat{k}$$

$$\Rightarrow \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x) = \hat{j}-\hat{k}$$

$\therefore\ z - y = 0$ ...(i)

$x - z = 1$ ...(ii)

$x - y = 1$ ...(iii)

Also, $\vec{a}\cdot\vec{c} = 3$

$$(\hat{i}+\hat{j}+\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = 3$$

$\Rightarrow x + y + z = 3$ ...(iv)

On adding equations (ii) and (iii), we get :

$2x - y - z = 2$ ...(v)

On solving equations (iv) and (v), we get :

$$x = \frac{5}{3}$$

$$\therefore \quad y = \frac{5}{3} - 1 = \frac{2}{3} \text{ and } z = \frac{2}{3}$$

Now, $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$$= \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$$

[5]

Q. 8. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

[NCERT Misc. Ex. Q. 14, Page 458]

Ans. Since $\vec{a}, \vec{b}$, and $\vec{c}$ are mutually perpendicular vectors, we have

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0.$

It is given that,

$|\vec{a}| = |\vec{b}| = |\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a}, \vec{b}$ and $\vec{c}$ at angles θ_1, θ_2, and θ_3 respectively.

Then, we have :

$$\cos\theta_1 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{\vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} \qquad [\because \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a} = 0]$$

$$= \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}$$

$$= \frac{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}$$

$$= \frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|} \quad [\because \vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{b}=0]$$

$$= \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_3 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$= \frac{\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{c}|}$$

$$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{c}|} \quad [\because \vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=0]$$

$$= \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now, as $|\vec{a}|=|\vec{b}|=|\vec{c}|$, $\cos\theta_1=\cos\theta_2=\cos\theta_3$.

$\therefore \theta_1=\theta_2=\theta_3$

Hence, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. **[5]**

Some Commonly Made Errors

- The dot product gives us a scalar, not another vector. The products are added together, not put into vector components.
- When you add vectors, use vector addition. That means resolving vectors into components. Too many people just add the magnitudes of the vectors without realising that they should be adding components instead.

EXPERT ADVICE

- *Don't be in a rush to solve problems. In Board question papers, both speed and strike-rate matter. You need to be quick as well as accurate to achieve high scores. High speed with low accuracy can actually ruin your results.*
- *More from rigid reliance on rules without understanding (rule-oriented study) to an understanding of mathematical concepts and flexibility in problem solving (concept-oriented study).*
- *Practice questions from previous year papers, sample papers and model papers within the time frame you will have at the final exam.*
- *Try the given problems with the conventional methods first and then look into the shortcut methods given. This makes it evident to you, the lesser labour involved, in comparison to the conventional methods.*
- *Focus on solving as many problems as you can, rather than just reading theories, formulae and solutions.*

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