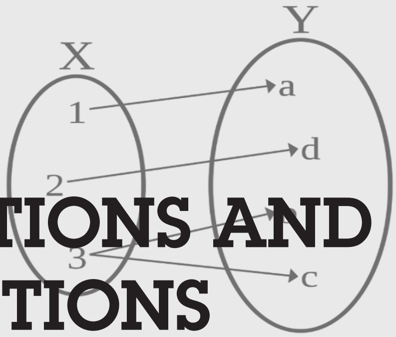
CHAPTER

1 RELATIONS AND FUNCTIONS



Chapter Objectives

This chapter will help you understand :

➢ ***Relations and functions*** *: Introduction; Types of relations; Types of functions; Composition of functions and Invertible functions; and Binary operations.*

Quick Review

❖ Modern set theory formulated by George Cantor is fundamental for the whole of Mathematics.

❖ The reformulation of geometry in terms of analysis, and the invention of set theory by Cantor, eventually led to the much more general modern concept of a function as a single-valued mapping from one set to another.

❖ Historically, the concept was elaborated with the infinitesimal calculus at the end of 17th century, and, until the 19th century.

❖ Mathematicians of the 18th century typically regarded a function as being defined by an analytic expression.

❖ In the 19th century, the demands of the rigorous development of analysis by Weierstrass.

TIPS...

- *Revise the important formulae for the chapter Relations and Functions and then practice the important questions.*
- *Understand the mapping using diagram.*
- *Learn all the properties of law's applicable in functions and relation.*

Know the Links

☞ *https://en.wikibooks.org/wiki/Discrete_Mathematics/Functions_and_relations*

☞ *www.ltcconline.net/greenl/courses/152a/functgraph/relfun.htm*

☞ *www.state.nj.us/education/archive/frameworks/math/math9.pdf*

TRICKS...

- *To evaluate a function, f(x), for a particular value of x, just substitute that value everywhere you see an x.*
- *For many functions, you can solve it by plugging the given values into the function.*
- *To solve* SAT *functions with special symbols, look for how the function is defined in an equation.*
- *Plug the values into the functions by replacing the strange symbol and solve it.*

Multiple Choice Questions **(1 mark each)**

Q. 1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b $\forall a, b \in T$. Then R is

(a) reflexive but not transitive

(b) transitive but not symmetric

(c) equivalence

(d) None of these

[NCERT Exemp. Ex. 1.3, Q. 28, Page 13]

Ans. Correct option : (c)

Explanation **:** Consider that aRb, if a is congruent to $b, \forall a, b \in T$.

Then, $aRa \Rightarrow a \cong a$,

Which is true for all $a \in T$

So, R is reflexive, …(i)

Let $aRb \Rightarrow a \cong b$

$\Rightarrow b \cong a \Rightarrow b \cong a$

$\Rightarrow bRa$

So, R is symmetric. …(ii)

Let aRb and bRc

$\Rightarrow b \cong b$ and $b \cong c$

$\Rightarrow a \cong c \Rightarrow aRc$

So, R is transitive. …(iii)

Hence, R is equivalence relation.

Q. 2. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is

(a) symmetric but not transitive.
(b) transitive but not symmetric.
(c) neither symmetric nor transitive.
(d) both symmetric and transitive.

[NCERT Exemp. Ex. 1.3, Q. 29, Page 13]

Ans. Correct option : (b)

Explanation : $aRb \Rightarrow a$ is brother of b.
This does not mean b is also a brother of a as b can be a sister of a.
Hence, R is not symmetric.
$aRb \Rightarrow a$ is brother of b
and $bRc \Rightarrow b$ is a brother of c.
So, a is brother of c.
Hence, R is transitive.

Q. 3. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

(a) 1 **(b) 2**
(c) 3 **(d) 5**

[NCERT Exemp. Ex. 1.3, Q. 30, Page 14]

Ans. Correct option : (d)

Explanation : Given that, $A = \{1,2,3\}$
Now, number of equivalence relations are as follows:

$$R_1 = \{(1,1),(2,2),(3,3)\}$$
$$R_2 = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$$
$$R_3 = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$$
$$R_4 = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$$
$$R_5 = \{(1,2,3) \Leftrightarrow A \times A = A^2\}$$

$\therefore$ Maximum number of equivalence relation on the set $A = \{1, 2, 3\} = 5$

Q. 4. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

(a) reflexive. **(b) transitive.**
(c) symmetric. **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 31, Page 14]

Ans. Correct option : (b)

Explanation : R on the set $\{1, 2, 3\}$ is defined by $R = \{(1, 2)\}$
It is clear that R is transitive.

Q. 5. Let us define a relation R in R as aRb if $a \geq b$. Then R is

(a) an equivalence relation.
(b) reflexive, transitive but not symmetric.
(c) symmetric, transitive but not reflexive.
(d) neither transitive nor reflexive but symmetric.

[NCERT Exemp. Ex. 1.3, Q. 32, Page 14]

Ans. Correct option : (b)

Explanation : Given that, aRb *if* $a \geq b$
$\Rightarrow aRa \Rightarrow a \geq a$ which is true
Let aRb, $a \geq b$, then $b \geq a$ which is not true as R is not symmetric.
But aRb and bRc
$\Rightarrow a \geq b$ and $b \geq c$
$\Rightarrow a \geq c$
Hence, R is transitive.

Q. 6. Let $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)\}$. Then R is

(a) reflexive but not symmetric
(b) reflexive but not transitive
(c) symmetric and transitive
(d) neither symmetric, nor transitive

[NCERT Exemp. Ex. 1.3, Q. 33, Page 14]

Ans. Correct option : (a)

Explanation : Given that $A = \{1, 2, 3\}$
and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)\}$.
$\because \quad (1, 1), (2, 2), (3, 3) \in R$
Hence, R is reflexive.
$(1, 2) \in R$ but $(2, 1) \notin R$
Hence, R is not symmetric.
$(1, 2) \in R$ and $(2, 3) \in R$
$\Rightarrow (1, 3) \in R$
Hence, R is transitive.

Q. 7. The identity element for the binary operation $*$ defined on $Q \sim \{0\}$ as
$$a * b = \frac{ab}{2}, \forall a, b \in Q \sim \{0\} \textit{ is } 0$$

(a) 1 **(b) 0**
(c) 2 **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 34, Page 14]

Ans. Correct option : (c)

Explanation :
Given that $a * b = \dfrac{ab}{2}, \forall a, b \in Q \sim \{0\}$
Let e be the identity element for $*$.
$$\therefore \quad a * e = \frac{ae}{2} (a * e = e * a = a)$$
$$\Rightarrow \quad a = \frac{ae}{2}$$
$$\Rightarrow \quad e = 2$$

Q. 8. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(a) 720 **(b) 120**
(c) 0 **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 35, Page 14]

Ans. Correct option : (c)

Explanation : We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is
$n!$ if m $= n$
0, if $m \neq n$
Given that, $m = 5$ and $n = 6$
$\therefore \quad m \neq n$
Number of one-one and onto mapping = 0

Q. 9. Let $A = \{1, 2, 3, ...n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is

(a) $^n\mathrm{P}_2$ **(b) $2^n – 2$**
(c) $2^n – 1$ **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 36, Page 14]

Ans. Correct option : (a)

Explanation : Given that $A = \{1, 2, 3, ...n\}$ and $B = \{a, b\}$
We know that, if A and B are two non-empty finite

sets containing m and n elements, respectively, then the number of surjection from A into B is

$^nC_m \times m!$ *if* $n \geq m$

0, *if* $m < n$

Here, $m = 2$

$$^nC_2 \times 2! = \frac{n!}{2!(n-2)!} \times 2!$$
$$= {}^nP_2$$
$$= \frac{n(n-1)(n-2)!}{2 \times 1(n-2)} \times 2!$$
$$= n^2 - n$$

Q. 10. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}, \forall \in R$ Then f is

(a) one-one
(b) onto
(c) bijective
(d) f is not defined

[NCERT Exemp. Ex. 1.3, Q. 37, Page 15]

Ans. Correct option : (d)

Explanation :

We have, $f(x) = \frac{1}{x}, \ \forall x \in R$

For $x = 0$, $f(x)$ is not defined.

Hence, $f(x)$ is a not define function.

Q. 11. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$.

Then $g\ o\ f$ is

(a) $\frac{3x^2-5}{9x^4-30x^2+26}$ (b) $\frac{3x^2-5}{9x^4-6x^2+26}$

(c) $\frac{3x^2}{x^4+2x^2-4}$ (d) $\frac{3x^2}{9x^4+30x^2-2}$

[NCERT Exemp. Ex. 1.3, Q. 38, Page 15]

Ans. Correct option : (a)

Explanation :

Given that $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$

$$g\,of = g\{f(x)\} = g(3x^2-5)$$
$$= \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+25+1}$$
$$= \frac{3x^2-5}{9x^4-30x^2+26}$$

Q. 12. Which of the following functions from Z into Z are bijections?

(a) $f(x) = x^3$ **(b) $f(x) = x + 2$**

(c) $f(x) = 2x + 1$ (d) **$f(x) = x^2 + 1$**

[NCERT Exemp. Ex. 1.3, Q. 39, Page 15]

Ans. Correct option : (b)

Explanation : For bijection on Z, $f(x)$ must be one-one and onto.

Function $f(x) = x^2 + 1$ is many-one as $f(1) = f(-1)$

Range of $f(x) = x^3$ is not Z for $x \in Z$.

Also $f(x) = 2x + 1$ takes only values of type $= 2k + 1$ for $x = k \in Z$

But $f(x) = x + 2$ takes all integral values for $x \in Z$

Hence $f(x) = x + 2$ is bijection of Z.

Q. 13. Let $f : R \to R$ be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is

(a) $(x + 5)^{1/3}$ **(b) $(x - 5)^{1/3}$**

(c) $(5 - x)^{1/3}$ **(d) $(5 - x)$**

[NCERT Exemp. Ex. 1.3, Q. 40, Page 15]

Ans. Correct option : (b)

Explanation :

Given that, $f(x) = x^3 + 5$

Let,

$$y = x^3 + 5$$
$$x^3 = y - 5$$
$$x = (y-5)^{\frac{1}{3}}$$
$$f^{-1}(x) = (x-5)^{\frac{1}{3}}$$

Q. 14. Let $f : A \to B$ and $g : B \to C$ be the bijective functions. Then $(g\ o\ f)^{-1}$ is

(a) $f^{-1}\ o\ g^{-1}$ **(b) $f\ o\ g$**

(c) $g^{-1}\ o\ f^{-1}$ **(d) $g\ o\ f$**

[NCERT Exemp. Ex. 1.3, Q. 41, Page 15]

Ans. Correct option : (a)

Explanation : Given that, $f : A \to B$ and $g : B \to C$ be the bijective functions.

$(f^{-1}\ o\ g^{-1})\ o\ (g\ o\ f) = f^{-1}\ o\ (g^{-1}\ o\ g\ o\ f)$

$= f^{-1}\ o\ (g^{-1}\ o\ g\ o\ f)$

[As composition of functions is associative.]

$(f^{-1}\ o\ g^{-1})\ o\ (g\ o\ f) = (f^{-1}\ o\ I_B\ o\ f)$

[where I_B is identity function on B]

$= (f^{-1}\ o\ I_B)\ o\ f$

$= f^{-1}\ o\ f$

$= I_A$

Thus, $(g\ o\ f)^{-1} = f^{-1}\ o\ g^{-1}$

Q. 15. Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $f(x) = \frac{3x+2}{5x-3}$, Then

(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$

(c) $(fof)x = -x$ (d) $f^{-1}(x) = \frac{1}{19}f(x)$

[NCERT Exemp. Ex. 1.3, Q. 42, Page 15]

Ans. Correct option : (a)

Explanation :

Given that, $f(x) = \frac{3x+2}{5x-3}$

Let $\quad y = \frac{3x+2}{5x-3}$

$$3x + 2 = 5xy - 3y$$
$$\Rightarrow x(3-5y) = -3y - 2$$
$$\Rightarrow \quad x = \frac{-(3y+2)}{-(5y-3)}$$
$$\Rightarrow \quad x = \frac{3y+2}{5y-3}$$
$$\Rightarrow \quad f^{-1}(x) = \frac{3x+2}{5x-3}$$
$$f^{-1}(x) = f(x)$$

Q. 16. Let $f : [0, 1] \to [0, 1]$ be defined by

$$\mathbf{f(x)} = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$$ **then $(f\ o\ f)x$ is**

(a) constant **(b) $1 + x$**
(c) x **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 43, Page 15]

Ans. Correct option : (c)

Explanation :

Given that $f:[0, 1]\to[0, 1]$ be defined by

$$f(x)=\begin{cases} x, & \text{if } x \text{ is rational.} \\ 1-x, & \text{if } x \text{ is irrational.}\end{cases}$$

$$\therefore (fof)x = f(f(x)) = x$$

Q. 17. Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

(a) R **(b) $[1, \infty)$**
(c) $[4, \infty)$ **(d) $[5, \infty)$**

[NCERT Exemp. Ex. 1.3, Q. 44, Page 16]

Ans. Correct option : (b)

Explanation **:**

Given that, $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5$

$\Rightarrow y = x^2 - 4x + 4 + 1$

$= (x-2)^2 + 1$

$\Rightarrow (x-2)^2 = y - 1$

$\Rightarrow x - 2 = \sqrt{y-1}$

$\Rightarrow x = 2 + \sqrt{y-1}$

$\therefore y - 1 \geq 0,\ y \geq 1$

Range $= [1, \infty)$

Q. 18. Let $f : N \to R$ be the function defined by $f(x) = \dfrac{2x-1}{2}$. and $g : Q \to R$ be another function defined by $g(x) = x + 2$. Then $(g\ o\ f)$ 3/2 is

(a) 1 **(b) 1**
(c) 7/2 **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 45, Page 16]

Ans. Correct option : (d)

Explanation :

Given that $f(x) = \dfrac{2x-1}{2}$ and $g(x) = x + 2$

$$(g\,o\,f)\left(\frac{3}{2}\right) = g\left[f\left(\frac{3}{2}\right)\right] = g\left(\frac{2\times\frac{3}{2}-1}{2}\right)$$

$= g(1) = 1 + 2 = 3$

Q. 19. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$$ **Then $f(-1) + f(2) + f(4)$ is**

(a) 9 **(b) 14**
(c) 5 **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 46, Page 16]

Ans. Correct option : (a)

Explanation :

Given that $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$

$f(-1) + f(2) + f(4)$

$= 3(-1) + (2)^2 + 2\times 4$

$= -3 + 4 + 8$

$= 9$

Q. 20. Let $f : R \to R$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(a) $\dfrac{\pi}{4}$ **(b) $\left\{n\pi + \dfrac{\pi}{4} : n \in Z\right\}$**
(c) does not exist **(d) None of these**

[NCERT Exemp. Ex. 1.3, Q. 47, Page 16]

Ans. Correct option : (a)

Explanation :

Given that, $f(x) = \tan x$

Let $y = \tan x$

$\Rightarrow x = \tan^{-1} y$

$\Rightarrow f^{-1}(x) = \tan^{-1} x$

$\Rightarrow f^{-1}(1) = \tan^{-1} 1$

$\Rightarrow \tan^{-1}\tan\dfrac{\pi}{4} = \dfrac{\pi}{4}$ $\left[\because \tan\dfrac{\pi}{4} = 1\right]$

Q. 21. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer :

(a) R is reflexive and symmetric but not transitive.

(b) R is reflexive and transitive but not symmetric.

(c) R is symmetric and transitive but not reflexive.

(d) R is an equivalence relation.

[NCERT Ex. 1.1, Q. 15, Page 7]

Ans. Correct option : (b)

Explanation : Let R be the relation in the set {1, 2, 3, 4} is given by:

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

(a) (1, 1), (2, 2), (3, 3), (4, 4) ∈ R
Therefore, R is reflexive.

(b) (1, 2) ∈ R but (2, 1) ∉ R.
Therefore, R is not symmetric.

(c) If (1, 3) ∈ R and (3, 2) ∈ R then (1, 2) ∈ R.
Therefore, R is transitive.

Q. 22. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.

(a) (2, 4) ∈ R **(b) (3, 8) ∈ R**
(c) (6, 8) ∈ R **(d) (8, 7) ∈ R**

[NCERT Ex. 1.1, Q. 16, Page 7]

Ans. Correct option : (c)

Explanation : $R = \{(a, b) : a = b - 2, b > 6\}$

Now,

Since $b > 6$, (2, 4) ∉ R

Also, as $3 \neq 8 - 2$,

∴ (3, 8) ∉ R

And, as $8 \neq 7 - 2$

∴ (8, 7) ∉ R

Now, consider (6, 8),

We have $8 > 6$ and also, $6 = 8 - 2$.

∴ (6, 8) ∈ R

Q. 23. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

(a) f is one-one onto.

(b) f is many-one onto.

(c) f is one-one but not onto.
(d) f is neither one-one nor onto.

[NCERT Ex. 1.2, Q. 11, Page 11]

Ans. Correct option : (d)

Explanation : We know that $f : R \to R$ is defined as $f(x) = x^4$.

Let $x, y \in R$ such that $f(x) = f(y)$.

$\Rightarrow \quad x^4 = y^4$

$\Rightarrow \quad x = \pm y$

$\therefore \quad f(x) = f(y)$

does not imply that $x = y$.

For example, $f(1) = f(-1) = 1$

$\therefore$ f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that $f(x) = 2$.

$\therefore$ f is not onto.

Hence, function f is neither one-one nor onto.

Q. 24. Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

(a) f is one- one onto
(b) f is many-one onto
(c) f is one-one but not onto
(d) f is neither one-one nor onto

[NCERT Ex. 1.2, Q. 12, Page 11]

Ans. Correct option : (a)

Explanation : $f : R \to R$ is defined as $f(x) = 3x$.

Let $x, y \in R$ such that $f(x) = f(y)$.

$\Rightarrow \quad 3x = 3y$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one.

Also, for any real number (y) in co-domain R, there exists $\frac{y}{3}$ in R such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$.

$\therefore$ f is onto.

Hence, function f is one-one and onto.

Q. 25. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x)$ is

(a) $x^{1/3}$ **(b)** x^3
(c) x **(d)** $(3 - x^3)$

[NCERT Ex. 1.3, Q. 13, Page 19]

Ans. Correct option : (c)

Explanation :

$f : R \to R$ is given as $f(x) = (3 - x^3)^{\frac{1}{3}}$.

$$f(x) = (3 - x^3)^{\frac{1}{3}}$$

$$fof(x) = f(f(x))$$

$$= f\left((3 - x^3)^{\frac{1}{3}}\right)$$

$$= \left[3 - \left((3 - x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= \left[3 - (3 - x^3)\right]^{\frac{1}{3}}$$

$$= (x^3)^{\frac{1}{3}}$$

$$= x$$

$$fof(x) = x$$

Q. 26. Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be function defined as $f(x) = \frac{4x}{3x + 4}$. The inverse of f is map g : Range $f \to R - \left\{-\frac{4}{3}\right\}$ given by

(a) $g(y) = \frac{3y}{3 - 4y}$ **(b)** $g(y) = \frac{4y}{4 - 3y}$

(c) $g(y) = \frac{4y}{3 - 4y}$ **(d)** $g(y) = \frac{3y}{4 - 3y}$

[NCERT Ex. 1.3, Q. 14, Page 19]

Ans. Correct option : (b)

Explanation :

It is given that $f : R - \left\{-\frac{4}{3}\right\} \to R$ is defined as $f(x) = \frac{4x}{3x + 4}$.

Let y be an arbitrary element of range f.

Then, there exists $x \in R - \left\{-\frac{4}{3}\right\}$ such that $y = f(x)$.

$\Rightarrow \quad y = \frac{4x}{3x + 4}$

$\Rightarrow \; 3xy + 4y = 4x$

$\Rightarrow x(4 - 3y) = 4y$

$\Rightarrow \quad x = \frac{4y}{4 - 3y}$

Let us define g : Range $f \to R - \left\{-\frac{4}{3}\right\}$ as $g(y) = \frac{4y}{4 - 3y}$.

Now,

$(g\ o\ f)(x) = f(f(x))$

$$= g\left(\frac{4x}{3x + 4}\right)$$

$$= \frac{4\left(\frac{4x}{3x + 4}\right)}{4 - 3\left(\frac{4x}{3x + 4}\right)}$$

$$= \frac{16x}{12x + 16 - 12x}$$

$$= \frac{16x}{16}$$

$$= x$$

And, $(f\ o\ g)(y) = f(g(y)) = f\left(\frac{4y}{4 - 3y}\right)$

$$= \frac{4\left(\frac{4y}{4 - 3y}\right)}{3\left(\frac{4y}{4 - 3y}\right) + 4} = \frac{16y}{12y + 16 - 12y} = \frac{16y}{16} = y$$

Therefore, $g\,o\,f = I_{R - \left\{-\frac{4}{3}\right\}}$ and $f\,o\,g = I_{\text{Range} f}$

Thus, g is the inverse of f *i.e.*, $f^{-1} = g$.

Hence, the inverse of f is the map g: Range $f \to R - \left\{-\frac{4}{3}\right\}$, which is given by $g(y) = \frac{4y}{4 - 3y}$.

Q. 27. Consider *a* binary operation $*$ on *N* defined as $a * b = a^3 + b^3$. Choose the correct answer.

(a) Is $*$ both associative and commutative?

(b) Is $*$ commutative but not associative?

(c) Is $*$ associative but not commutative?

(d) Is $*$ neither commutative nor associative?

[NCERT Ex. 1.4, Q. 13, Page 26]

Ans. Correct option : (b)

Explanation : On N, the operation $*$ is defined as $a * b = a^3 + b^3$.

For, $a, b, \in N$, we have

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$

[Addition is commutative in N.]

Therefore, the operation $*$ is commutative.

It can be observed that,

$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756$

And

$1* (2 * 3) = 1 * (2^3 + 3^3) = 1* (8 + 27) = 1*35 = 1^3 + 35^3 = 1 + 42875 = 42876$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in N$. Therefore, the operation $*$ is not associative.

Hence, the operation $*$ is commutative, but not associative.

Q. 28. Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(a) 1 **(b) 2**

(c) 3 **(d) 4**

[NCERT Misc. Ex. Q. 16, Page 30]

Ans. Correct option : (a)

Explanation : The given set is $A = \{1, 2, 3\}$.

The smallest relation containing (1, 2) and (1, 3), which is reflexive and symmetric, but not transitive is given by:

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)\}$

This is because relation R is reflexive as (1, 1), (2, 2), (3, 3) $\in R$.

Relation R is symmetric since (1, 2), (2, 1) $\in R$ and (1, 3), (3, 1) $\in R$.

But relation R is not transitive as (3, 1), (1, 2) $\in R$, but (3, 2) $\notin R$.

Now, if we add any two pairs (3, 2) and (2, 3) (or both) to relation R, then relation R will become transitive.

Hence, the total number of desired relations is one.

Q. 29. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is

(a) 1 **(b) 2**

(c) 3 **(d) 4**

[NCERT Misc. Ex. Q. 17, Page 31]

Ans. Correct option : (b)

Explanation : It is given that $A = \{1, 2, 3\}$.

The smallest equivalence relation containing (1, 2) is given by,

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

Now, we are left with only four pairs, *i.e.,* (2, 3), (3, 2), (1, 3), and (3, 1).

If we odd any one pair [say (2, 3)] to R_1, then for symmetry we must add (3, 2). Also, for transitivity we are required to add (1, 3) and (3, 1).

Hence, the only equivalence relation ($>R_1$) is the universal relation.

This shows that the total number of equivalence relations containing (1, 2) is two.

Q. 30. Number of binary operations on the set $\{a, b\}$ are

(a) 10 **(b) 16**

(c) 20 **(d) 8**

[NCERT Misc. Ex. Q. 19, Page 31]

Ans. Correct option : (b)

Explanation :

A binary operation $*$ on $\{a, b\}$ is a function from $\{a, b\} \times \{a, b\} \rightarrow \{a, b\}$ *i.e.,* $*$ is a function from $\{(a, a), (a, b), (b, a), (b, b)\} \rightarrow \{a, b\}$.

Hence, the total number of binary operations on the set $\{a, b\}$ is 2^4 *i.e.,* 16.

Q. 31. Let $*$ be binary operation defined on *R* by $a * b = 1 + ab$, $\forall\ a, b \in R$. Then the operation $*$ is

(a) commutative but not associative

(b) associative but not commutative

(c) neither commutative nor associative

(d) both commutative and associative

[NCERT Exemp. Ex. 1.3, Q. 27, Page 13]

Ans. Correct option : (a)

Explanation : Given that $a * b = 1 + ab, \forall a,\ b \in R$

$a * b = ab + 1 = b * a$

So, $*$ is a commutative binary operation.

Also $a * (b * c) = a * (1 + bc) = 1 + a(1 + bc)$

$a * (b * c) = 1 + a + abc$ ...(i) ...(i)

$(a * b) * c = (1 + ab) * c$

$= 1 + (1 + ab)c$

$= 1 + c + abc$ ...(ii)

From Eqs. (i) and (ii), we have

$a * (b * c) \neq (a * b) * c$

So, $*$ is not associative.

Hence, $*$ is commutative but not associative.

Very Short Answer Type Questions (1 or 2 marks each)

Q. 1. State True or False in the given statement.

An integer *m* is said to be related to another integer n, if *m* is *a* integral multiple of *n*. This relation in *Z* is reflexive, symmetric and transitive.

[NCERT Exemp. Ex. 1.3, Q. 56, Page 17]

Ans. False, the given relation is reflexive and transitive but not symmetric. **[1]**

Q. 2. State True or False in the given statement.

Every function is invertible.

[NCERT Exemp. Ex. 1.3, Q. 61, Page 17]

Ans. False, only bijective functions are invertible. **[1]**

Q. 3. State True or False in the given statement.

A binary operation on a set has always the identity element. [NCERT Exemp. Ex. 1.3, Q. 62, Page 17]

Ans. **False,** '+' is a binary operation on the set N but it has no identity element. **[1]**

Q. 4. If $a * b$ denotes the larger 'a' and 'b' and if $a \; o \; b = (a * b) + 3$, then write the value of (5) o 10, where $*$ and o are binary operations.
[CBSE Board, Delhi Region, 2018]

Ans. Given that,

$a \; o \; b = (a * b) + 3$

Here $a * b$ denotes the larger of 'a' and 'b'.

Substitute $a = 5$ and $b = 10$ in $a \; o \; b = (a * b) + 3$

$5 \; o \; 10 = (5 * 10) + 3$

$= 10 + 3$ $\quad [5 * 10 = 10]$

$= 13$ **[1]**

Q. 5. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.
[NCERT Ex. 1.1, Q. 2, Page 5]

Ans. $R = \{(a, b) : a \le b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \in R$, Since, $\frac{1}{2} > \left(\frac{1}{2}\right)^2$.

$\therefore R$ is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$ But, 4 is not less than 12.

$\therefore (4, 1) \notin R$

$\therefore R$ is not symmetric. **[1]**

Now,

$(3, 2), (2, 1.5) \in R$

[as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$]

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive. **[1]**

Q. 6. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.
[NCERT Ex. 1.1, Q. 3, Page 5]

Ans. Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as : $R = \{(a, b) : b = a + 1\}$

$\therefore R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$ we can find $(a, a) \notin R$, where $a \in A$.

For instance,

$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

$\therefore R$ is not reflexive. **[1]**

It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.

$\therefore R$ is not symmetric.

Now, $(1, 2), (2, 3) \in R$

But, $(1, 3) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive. **[1]**

Q. 7. Show that the relation R in R defined as $R = \{(a, b) : a \le b\}$, is reflexive and transitive but not symmetric. **[NCERT Ex. 1.1, Q. 4, Page 5]**

Ans. $R = \{(a, b) : a \le b\}$

Clearly $(a, a) \in R$

[as $a = a$]

$\therefore R$ is reflexive.

Now, $(2, 4) \in R$ (as $2 < 4$)

But, $(4, 2) \notin R$ as 4 is greater than 2.

$\therefore R$ is not symmetric. **[1]**

Now, let $(a, b), (b, c) \in R$.

Then, $a \le b$ and $b \le c$

$\Rightarrow a \le c$

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence R is reflexive and transitive but not symmetric. **[1]**

Q. 8. Check whether the relation R in R defined by $R = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive. **[NCERT Ex. 1.1, Q. 5, Page 5]**

Ans. $R = \{(a, b) : a \le b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \in R$, Since, $\frac{1}{2} > \left(\frac{1}{2}\right)^3$.

$\therefore R$ is not reflexive.

Now, $(1, 2) \in R$

[As $1 < 2^3 = 8$]

But, $(2, 1) \notin R$

[as $2^3 > 1$]

$\therefore R$ is not symmetric. **[1]**

$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$,

Since $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive. **[1]**

Q. 9. Show that the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.
[NCERT Ex. 1.1, Q. 6, Page 6]

Ans. Let $A = \{1, 2, 3\}$.

A relation R on A is defined as $R = \{(1, 2), (2, 1)\}$.

It is clear that $(1, 1), (2, 2), (3, 3) \notin R$.

$\therefore R$ is not reflexive. **[½]**

Now, as $(1, 2) \in R$ and $(2, 1) \in R$, then R is symmetric.

Now, $(1, 2)$ and $(2, 1) \in R$

However, $(1, 1) \notin R$

$\therefore R$ is not transitive.

Hence, R is symmetric but neither reflexive nor transitive. **[1½]**

Q. 10. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$. **[NCERT Ex. 1.1, Q. 14, Page 6]**

Ans. **Part I** : $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$

(i) It is clear that $L_1 \parallel L_1$ *i.e.*, $(L_1, L_2) \in R$. Therefore, R is reflexive.

(ii) If $L_1 \parallel L_2$ and $L_2 \parallel L_1$ then $(L_1, L_2) \in R$. Therefore, R is symmetric.

(iii) If $L_1 \parallel L_2$ and $L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3$. Therefore, R is transitive.

Therefore, R is an equivalent relation. **[1½]**

Part II : All the lines related to the line $y = 2x + 4$ and $y = 2x + k$, where k is a real number. **[½]**

Q. 11. Prove that the Greatest Integer Function $f : R \to R$ given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x. [NCERT Ex. 1.2, Q. 3, Page 10]

Ans. $f : R \to R$ is given by, $f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$.

$\therefore \quad f(1.2) = f(1.9)$, but $1.2 \neq 1.9$.

$\therefore f$ is not one-one.

Now, consider $0.7 \in R$.

It is known that $f(x) = [x]$ is always an integer. Thus, there does not exist any element $x \in R$ such that $f(x) = 0.7$.

$\therefore f$ is not onto.

Hence, the greatest integer function is neither bijective. **[2]**

Q. 12. Show that the Modulus Function $f : R \to R$ given by $f(x)=|x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative. [NCERT Ex. 1.2, Q. 4, Page 11]

Ans. Modulus Function $f : R \to R$, given by $f(x) = |x|$.

$$\text{Now } |x| = \begin{cases} -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

$\Rightarrow f$ contians $(-1,1),(1,1),(-2,2),(2,2)$ **[1]**

Thus, negative integers are not images of any element. Therefore, f is not one-one.

Also second set R contains some negative numbers which are not images of any real number.

$\therefore f$ is not onto. **[1]**

Q. 13. Show that the Signum Function $f : R \to$ R, given by

$$f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$$

is neither one-one nor onto.

[NCERT Ex. 1.2, Q. 5, Page 11]

Ans. Signum Function $f : R \to R$, given by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

$f(1) = f(2) = 1$

$\Rightarrow f(x_1) = f(x_2) = 1$ for $n > 0$

$\Rightarrow \quad x_1 \neq x_2 \ \therefore f$ is not one-one. **[1]**

Except –1, 0, 1 no other members of co-domain of f has any pre-image its domain.

$\therefore f$ is not onto.

Therefore, f is neither one-one nor onto. **[1]**

Q. 14. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one. [NCERT Ex. 1.2, Q. 6, Page 11]

Ans. It is given that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$.

$f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$.

$\therefore f(1) = 4, f(2) = 5, f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

Q. 15. Let A and B be sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is bijective function. [NCERT Ex. 1.2, Q. 8, Page 11]

Ans. $f : A \times B \to B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in$ A×B such that $f(a_1, b_1) = f(a_2, b_2)$.

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow \quad b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\therefore f$ is one-one. **[1]**

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$.

[By definition of f]

$\therefore f$ is onto.

Hence, f is bijective. **[1]**

Q. 16. Let $f : \{1, 3, 4\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof. [NCERT Ex. 1.3, Q. 1, Page 18]

Ans. The functions $f : \{1, 3, 4\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 3\}$ are defined as $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$.

$g\,o\,f(1) = g[f(1)] = g(2) = 3$ [as $f(1) = 2$ and $g(2) = 3$]

$g\,o\,f(3) = g[f(3)] = g(5) = 1$ [as $f(3) = 5$ and $g(5) = 1$]

$g\,o\,f(4) = g[f(4)] = g(1) = 3$ [as $f(4) = 1$ and $g(1) = 3$]

$\therefore g\,o\,f = \{(1, 3), (3, 1), (4, 3)\}$ **[2]**

Q. 17. Find $g\,o\,f$ and $f\,o\,g$, if

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

[NCERT Ex. 1.3, Q. 3, Page 18]

Ans. (i) $f(x) = |x|$ and $g(x) = |5x-2|$

$\therefore g\,o\,f(x) = g(f(x)) = g(|x|) = |5|x|-2|$

$f\,o\,g(x) = f(g(x)) = f(|5x-2|) = ||5x-2|| = |5x-2|$ **[1]**

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

$\therefore g\,o\,f(x) = g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$

$f\,o\,g(x) = f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$ **[1]**

Q. 18. If $f(x) = \dfrac{(4x+3)}{(6x-4)}, x \neq \dfrac{2}{3}$, show that $f\,o\,f(x) = x$, for all $x \neq \dfrac{2}{3}$. What is the inverse of f? [NCERT Ex. 1.3, Q. 4, Page 18]

Ans. It is given that $f(x) = \dfrac{(4x+3)}{(6x-4)}, x \neq \dfrac{2}{3}$.

$$(fof)(x) = f(f(x))$$

$$= f\left(\frac{4x+3}{6x-4}\right)$$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16}$$

$= \frac{34x}{34}$

$= x$ **[1½]**

Therefore, $f \, o \, f(x) = x$, for all $x \neq \frac{2}{3}$.

$\Rightarrow f \, o \, f = I_x$

Hence, the given function f is invertible and the inverse of f is f itself. [½]

Q. 19. Let $f : X \to Y$ be an invertible function. Show that f has unique inverse. [Hint : suppose g_1 and g_2 are two inverses of f. Then for all $y \in Y$, $f \, o \, g_1(y) = I_Y(y) = f \, o \, g_2(y)$. Use one – one ness of f].

[NCERT Ex. 1.3, Q. 10, Page 19]

Ans. Let $f : X \to Y$ be an invertible function.

Also, suppose f has two inverses (say g_1 and g_2)

Then, for all $y \in Y$, we have

$f \, o \, g_1(y) = I_Y(y) = f \, o \, g_2(y)$

$\Rightarrow \quad f(g_1(y)) = f(g_2(y))$

$\Rightarrow \quad g_1(y) = g_2(y)$

[as f is invertible $\Rightarrow f$ is one-one]

$\Rightarrow \quad g_1 = g_2$

[as g is one-one]

Hence, f has a unique inverse. **[2]**

Q. 20. Let $f : X \to Y$ be an invertible function. Show that the inverse of f^{-1} is f, *i.e.*, $(f^{-1})^{-1} = f$.

[NCERT Ex. 1.3, Q. 12, Page 19]

Ans. Let $f : X \to Y$ be an invertible function.

Then, there exists a function g : $Y \to X$ such that $g \, o \, f = I_X$ and $f \, o \, g = I_Y$.

Here, $f^{-1} = g$.

Now, $g \, o \, f = I_X$ and $f \, o \, g = I_Y$

$\Rightarrow \quad f^{-1} of = I_X$ and $f \, o \, f^{-1} = I_Y$

Hence, $f^{-1} : Y \to X$ is invertible and f is the inverse of f^{-1} *i.e.*, $(f^{-1})^{-1} = f$. **[2]**

Q. 21. Consider the binary operation $\wedge$ on the set {1, 2, 3, 4, 5} defined by $a \wedge b = \min \{a, b\}$.

Write the operation table of the operation $\wedge$.

[NCERT Ex. 1.4, Q. 3, Page 24]

Ans. The binary operation $\wedge$ on the set {1, 2, 3, 4, 5} is defined as $a \wedge b = \min \{a, b\}$ for all $a, b \in \{1, 2, 3, 4, 5\}$. **[1]**

Thus, the operation table for the given operation $\wedge$ can be given as:

^	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

[1]

Q. 22. Consider a binary operation $*$ on the set {1, 2, 3, 4, 5} given by the following multiplication table.

(i) Compute $(2 * 3) * 4$ and $2 * (3 * 4)$

(ii) Is $*$ commutative?

(iii) Compute $(2 * 3) * (4 * 5)$.

(Hint : Use the table given below)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

[NCERT Ex. 1.4, Q. 4, Page 25]

Ans. (i) $(2 * 3) * 4 = 1 * 4 = 1$

$2 * (3 * 4) = 2 * 1 = 1$ [½]

(ii) For every $a, b \in \{1, 2, 3, 4, 5\}$, we have $a * b = b * a$. Therefore, the operation $*$ is commutative. [½]

(iii) $(2 * 3) = 1$ and $(4 * 5) = 1$

$\therefore (2 * 3) * (4 * 5) = 1 * 1 = 1$ **[1]**

Q. 23. Let $*'$ be the binary operation on the set {1, 2, 3, 4, 5} defined by $a *' b$ = H.C.F. of a and b. Is the operation $*'$ same as the operation $*$ defined in Exercise 1.4, Q. 4 above? Justify your answer.

[NCERT Ex. 1.4, Q. 5, Page 25]

Ans. The binary operation $*'$ on the set {1, 2, 3 4, 5} is defined as $a *' b$ = H.C.F of a and b. **[1]**

The operation table for the operation $*'$ can be given as:

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

[½]

We observe that the operation tables for the operations $*$ and $*'$ are the same.

Thus, the operation $*'$ is same as the operation$*$. [½]

Q. 24. Is $*$ defined on the set {1, 2, 3, 4, 5} by $a * b$ = L.C.M. of a and b a binary operation? Justify your answer. [NCERT Ex. 1.4, Q. 7, Page 25]

Ans. The operation $*$ on the set A = {1, 2, 3, 4, 5} is defined as $a * b$ = L.C.M. of a and b. [½]

$2 * 3$ = L.C.M. of 2 and 3 = 6. But 6 does not belong to the given set. So, the given operation $*$ is not a binary operation. [½]

Q. 25. Let $*$ be the binary operation on N defined by $a * b$ = H.C.F. of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

[NCERT Ex. 1.4, Q. 8, Page 25]

Ans. The binary operation $*$ on N is defined as : $a * b$ = H.C.F. of a and b

It is known that,

H.C.F. of a and b = H.C.F. of b and a for all $a, b \in N$.

$\therefore a * b = b * a$

Thus, the operation $*$ is commutative. **[1]**

For $a, b, c \in$ N, we have

$(a * b) * c$ = (H.C.F. of a and b) $* c$ = H.C.F. of a, b and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of $a, b,$ and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation $*$ is associative.

Now, an element $e \in N$ will be the identity for the operation $*$ if $a * e = a = e*$ a for all $a \in N$.

But this relation is not true for any $a \in N$.

Thus, the operation $*$ does not have any identity in N. **[1]**

Q. 26. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation $*$ on a set N, $a * a = a \ \forall \ a \in N$.

(ii) If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

[NCERT Ex. 1.4, Q. 12, Page 26]

Ans. **(i)** Define an operation $*$ on N as $a * b = a + b \ \forall \ a, b \in N$

Then, in particular, for $b = a = 3$, we have

$3 * 3 = 3 + 3 = 6 \neq 3$

Therefore, statement (i) is false. **[1]**

(ii) R.H.S. $= (c * b) * a$

$= (b * c) * a$

[$*$ is commutative]

$= a * (b * c)$

[Again, as $*$ is commutative.]

= L.H.S.

$\therefore a * (b * c) = (c * b) * a$

Therefore, statement (ii) is true. **[1]**

Q. 27. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$. **[NCERT Misc. Ex. Q. 3, Page 29]**

Ans. It is given that $f : R \to R$ is defined as

$$f(x) = x^2 - 3x + 2.$$

$$f(f(x)) = f(x^2 - 3x + 2)$$

$$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$$

$$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$$

$$= x^4 - 6x^3 + 10x^2 - 3x$$ **[2]**

Q. 28. Give examples of two functions $f : N \to$ Z and g : Z $\to$ Z such that gof is injective but g is not injective. (Hint : Consider $f(x) = x$ and $g(x) = |x|$).

[NCERT Misc. Ex. Q. 6, Page 29]

Ans. Define $f : N \to Z$ as $f(x) = x$ and g : Z $\to$ Z as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that,

$(-1) = |-1| = 1$

$(1) = |1| = 1$

$\therefore \ (-1) = g(1)$, but $-1 \neq 1$.

$\therefore \ g$ is not injective. **[1]**

Now, g o$f : N \to$ Z is defined as

$g\,o(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in N$ such that $g \circ f(x) = g \circ f(y)$.

$\Rightarrow |x| = |y|$

Since x and $y \in N$, both are positive.

$\therefore \ |x| = |y| \Rightarrow x = y$

Hence, $g \circ f$ is injective. **[1]**

Q. 29. Given examples of two functions $f : N \to N$ and $g : N \to N$ such that gof is onto but f is not onto. (Hint : Consider $f(x) = x + 1$ and

$$g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

[NCERT Misc. Ex. Q. 7, Page 29]

Ans. Define $f : N \to N$ by, $f(x) = x + 1$

And, $g : N \to N$ by,

$$g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

We first show that g is not onto. **[1]**

For this, consider element 1 in co-domain of N. It is clear that this element is not an image of any of the elements in domain N.

$\therefore f$ is not onto.

Now, $g \circ f : N \to N$ is defined by $g\ o\ f(x) = g(f(x))$ $= g(x + 1) = x + 1 - 1 = x$

$[x \in N \Rightarrow x+1 > 1]$

Then, it is clear that for $y \in N$, there exists $x = y \in N$ such that $g \circ f(x) = y$.

Hence, $g \circ f$ is onto. **[1]**

Q. 30. Given a non-empty set X, consider $P(X)$ which is the set of all subsets of X.

Define the relation R in $P(X)$ as follows:

For subsets A, B in $P(X)$, ARB if and only if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify you answer. **[NCERT Misc. Ex. Q. 8, Page 29]**

Ans. We know that every set is a subset of itself, ARA for all $A \in P(X)$.

$\therefore R$ is reflexive.

Let $ARB \Rightarrow A \subset B$.

This cannot be implied to $B \subset A$.

For instance, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then it cannot be implied that B is related to A.

$\therefore R$ is not symmetric. **[1]**

Further, if ARB and BRC, then $A \subset B$ and $B \subset C$.

$\Rightarrow A \subset C$

$\Rightarrow ARC$

$\therefore R$ is transitive.

Hence, R is not an equivalence relation as it is not symmetric. **[1]**

Q. 31. Given a non-empty set X, consider the binary operation $* : P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B \ \forall \ A, B$ in $P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation$*$.

[NCERT Misc. Ex. Q. 9, Page 30]

Ans. It is given the binary operation $*$:

$P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B \ \forall \ A, B$ in $P(X)$

We know that $A \cap X = A = X \cap A$ for all $A \in P(X)$

$\Rightarrow A * X = A = X * A$ for all $A \in P(X)$

Thus, X is the identity element for the given binary operation $*$. **[1]**

Now, an element $A \in P(X)$ is invertible if there exists $B \in P(X)$ such that

$A * B = X = B * A$

[As X is the identity element.]

or

$A \cap B = X = B \cap A$

This case is possible only when $A = X = B$.

Thus, X is the only invertible element in $P(X)$ with respect to the given operation$*$.

Hence, the given result is proved. **[1]**

Q. 32. Find the number of all onto functions from the set {1, 2, 3, …, n} to itself.

[NCERT Misc. Ex. Q. 10, Page 30]

Ans. Onto functions from the set {1, 2, 3, …, n} to itself is simply a permutation on n symbols 1, 2, …, n.

Thus, the total number of onto maps from {1, 2, …, n} to itself is the same as the total number of permutations on n symbols 1, 2, …, n, which is n. **[1]**

Q. 33. Let S = {a, b, c} and T = {1, 2, 3}. Find F^{-1} of the following functions F from S to T, if it exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$

(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

[NCERT Misc. Ex. Q. 11, Page 30]

Ans. $S = \{a, b, c\}$, $T = \{1, 2, 3\}$

(i) $F : S \to T$ is defined as $F = \{(a, 3), (b, 2), (c, 1)\}$

$\Rightarrow F(a) = 3, F(b) = 2, F(c) = 1$

Therefore, $F^{-1} : T \to S$ is given by $F^{-1} = \{(3, a), (2, b), (1, c)\}$. **[1]**

(ii) $F : S \to T$ is defined as $F = \{(a, 2), (b, 1), (c, 1)\}$

Since $F(b) = F(c) = 1$, F is not one-one.

Hence, F is not invertible *i.e.,* F^{-1} does not exist. **[1]**

Q. 34. Given a non-empty set X, let $* : P(X) \times P(X) \to P(X)$ be defined as $A * B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set ϕ is the identity for the operation $*$ and all the elements A of $P(X)$ are invertible with $A^{-1} = A$.

(Hint : $(A - \phi) \cup (\phi - A) = A$ and $(A - A) \cup (A - A) = A * A = \phi$). [NCERT Misc. Ex. Q. 13, Page 30]

Ans. It is given that $* : P(X) \times P(X) \to P(X)$ is defined as $A * B = (A - B) \cup (B - A)$ $A, B \in P(X)$.

Let $A \in P(X)$. Then, we have:

$A * \phi = (A - \phi) \cup (\phi - A) = A \cup \phi = A$

$\phi * A = (\phi - A) \cup (A - \phi) = \phi \cup A = A$

Therefore, $A * \phi = A = \phi * A, A \in P(X)$ **[1]**

Thus, ϕ is the identity element for the given operation$*$.

Now, an element $A \in P(X)$ will be invertible if there exists $B \in P(X)$ such that $A * B = \phi = B * A$.

(As $\forall$ is the identity element)

Now, we observed that $A*A = (A - A) \cup (A - A) = \phi \cup \phi = \phi$ ϕ $A \in P(X)$.

Hence, all the elements A of $P(X)$ are invertible with $A^{-1} = A$. **[1]**

Q. 35. Let $f : R \to R$ be the Signum Function defined as

$$f(x)=\begin{cases}\mathbf{1}, & x>0\\ \mathbf{0}, & x=0\\ \mathbf{-1}, & x<1\end{cases}$$ **and $g : R \to R$ be the Greatest Integer Function given by $g(x) = [x]$, where $[x]$ is greatest integer less than or equal to x. Then does $f \circ g$ and $g \circ f$ coincide in (0, 1]?**

[NCERT Misc. Ex. Q. 18, Page 31]

Ans. It is clear that $g\,o\,f : R \to R$ and $f\,o\,g : R \to \mathrm{R}$

Consider $x = \frac{1}{2}$ which lie on (0, 1)

Now, $(g\,o\,f)\left(\frac{1}{2}\right) = g\left\{f\left(\frac{1}{2}\right)\right\}$

$= g(1) = [1] = 1$

And $(f\,o\,g)\left(\frac{1}{2}\right) = f\left\{g\left(\frac{1}{2}\right)\right\}$

$= f\left(\left[\frac{1}{2}\right]\right) = f(0) = 0$

$\Rightarrow$ $g\,o\,f \neq fog$ in (0,1]

No, $f\,o\,g$ and $g\,o\,f$ don't coincide in (0, 1). **[2]**

Q. 36. Let A = {a, b, c} and the relation R be defined on A as follows:

$R = \{(a, a), (b, c), (a, b)\}$. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

[NCERT Exemp. Ex. 1.3, Q. 1, Page 11]

Ans. Given relation,

$R = \{(a, a), (b, c), (a, b)\}$.

To make R reflexive we must add (b, b) and (c, c) to R. Also, to make R transitive we must add (a, c) to R.

So, minimum number of ordered pairs to be added is 3. **[1]**

Q. 37. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then, write D.

[NCERT Exemp. Ex. 1.3, Q. 2, Page 11]

Ans. Given that,

Real Valued Function $f(x)$ is such that

$f(x) = \sqrt{25 - x^2}$

Since $f(x)$ is real valued we must have,

$25 - x^2 \geq 0$

$\Rightarrow$ $x^2 \leq 25$

$\Rightarrow$ $-5 \leq x \leq 5$

$\Rightarrow$ Domain, $D = [-5, 5]$ **[1]**

Q. 38. Let $f, g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2, \forall x \in R$, respectively. Then, find $g\,o\,f$.

[NCERT Exemp. Ex. 1.3, Q. 3, Page 11]

Ans. We have,

$f(x) = 2x + 1$ and $g(x) = x^2 - 1, \forall x \in R$

$\therefore gof = g(f(x))$

$= g(2x + 1)$

$= (2x + 1)^2 - 2$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$ **[1]**

Q. 39. Let $f : R \to R$ be the function defined by $f(x) = 2x - 3$ $\forall x \in R$. write f^{-1}.

[NCERT Exemp. Ex. 1.3, Q. 4, Page 11]

Ans. Given $f(x) = 2x - 3, \forall x \in \mathrm{R}$

Now, let $a, b \in R$ such that

$f(a) = f(b)$

$\Rightarrow 2a - 3 = 2b - 3$

$\Rightarrow$ $a = b$

$\Rightarrow f(x)$ is one-one function. **[1]**

Also, if $x, y \in R$

Such that

$f(x) = y \Rightarrow 2x - 3 = y$

$\Rightarrow x = \dfrac{y+3}{2} = g(y) \forall y \in R$

$\Rightarrow$ $f(x)$ is onto function and therefore, Bijective implies $f(x)$ has an inverse function.

Let f^{-1} denotes the inverse of $f(x)$.

Then

$f^{-1}(x) = g(x) = \dfrac{x+3}{2} \forall x \in R$ **[1]**

Q. 40. If $A = \{a, b, c, d\}$ and the function $f = \{(a, b), (b, d), (c, a), (d, c)\}$, write f^{-1}.
[NCERT Exemp. Ex. 1.3, Q. 5, Page 11]

Ans. Let $f : A \to A$ then Inverse f^{-1} is such that $f^{-1} = (b, a), (d, b)(a, c), (c, d)$

$f^{-1} : A \to A$ and $f^{-1} = (b, a), (d, b)(a, c), (c, d)$ **[1]**

Q. 41. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, write $f(f(x))$. **[NCERT Exemp. Ex. 1.3, Q. 6, Page 11]**

Ans. Given that,

$f(x) = x^2 - 3x + 2$

$$\begin{aligned} f(f(x)) &= f(x^2 - 3x + 2) \\ &= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2 \\ &= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2 \\ &= x^4 + 10x^2 - 6x^3 - 3x \\ f(f(x)) &= x^4 - 6x^3 + 10x^2 - 3x \end{aligned}$$ **[1]**

Q. 42. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ *a* function? If g is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β.
[NCERT Exemp. Ex. 1.3, Q. 7, Page 11]

Ans. Given that,

$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$.

Here, each element of domain has unique image. So, g is a function.

Now given that, $g(x) = ax + \beta$

$g(1) = 1$

$\Rightarrow \quad \alpha + \beta = 1$ ...(i)

$g(2) = 3$

$\Rightarrow 2\alpha + \beta = 3$ ...(ii)

Solving (i) and (ii), we get

$\alpha = 2, \beta = -1$

$\therefore \quad g(x) = 2x - 1$

Above function satisfies (3, 5) and (4, 7). **[2]**

Q. 43. Are the following sets of ordered pairs of functions? If so, examine whether the mapping is injective or surjective.

(i) $\{(x, y) : x$ is a person, y is the mother of $x\}$.

(ii) $\{(a, b) : a$ is a person, b is an ancestor of a$\}$.
[NCERT Exemp. Ex. 1.3, Q. 8, Page 11]

Ans. (i) We have $\{(x, y) : x$ is a person, y is the mother of $x\}$.

Clearly, each person 'x' has only one biological mother. So above set of ordered pair is a function. Now more than one person may have same mother. So function is many one and surjective.

It represents a function. Here, the images of distinct elements of x under f are not distinct, so it is not injective but it is surjective. **[1]**

(ii) We have $\{(a, b) : a$ is a person, b is an ancestor of $a\}$.

Clearly, any person 'a' has more than one ancestor. Here, each element of domain does not have a unique image. So, it does not represent function. **[1]**

Q. 44. If the mappings f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, write $f \circ g$.
[NCERT Exemp. Ex. 1.3, Q. 9, Page 11]

Ans.

Given that,

$f = \{(1,2),(3,5),(4,1)\}$

And $\quad g = \{(2,3),(5,1),(1,3)\}$,

Now, $f \circ g(2) = f\{g(2)\} = f(3) = 5$

$$\begin{aligned} f \circ g(5) &= f\{g(5)\} \\ &= f(1) = 2 \\ f \circ g(1) &= f\{g(1)\} \\ &= f(3) = 5 \\ f \circ g &= \{(2,5),(5,2),(1,5)\} \end{aligned}$$ **[2]**

Q. 45. Let C be the set of complex numbers. Prove that the mapping $f : C \to R$ given by $f(z) = |z|, \forall z \in C$, is neither one-one nor onto.
[NCERT Exemp. Ex. 1.3, Q. 10, Page 11]

Ans. We have $f : C \to R$ such that $f(z) = |z|, \forall z \in C$

Now $\quad f(3+4i) = |3+4i|$

$= \sqrt{3^2 + 4^2} = 5$

And $\quad f(3-4i) = |3-4i|$

$= \sqrt{3^2 + (-4)^2} = 5$

Thus $f(z)$ is many-one function.

Also $\quad |z| \geq 0, \forall z \in C$

But co-domain given is 'R'

Hence $f(z)$ is not onto function. **[2]**

Q. 46. Let the function $f : R \to R$ be defined by $f(x) = \cos x, \forall x \in R$. Show that f is neither one-one nor onto.
[NCERT Exemp. Ex. 1.3, Q. 11, Page 11]

Ans. Given function,

$f(x) = \cos x, \forall x \in R$

Now, $\quad f\left(\dfrac{\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$

$\Rightarrow \quad f\left(\dfrac{-\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$

$\Rightarrow \quad f\left(\dfrac{\pi}{2}\right) = f\left(\dfrac{-\pi}{2}\right)$

But $\quad \dfrac{\pi}{2} \neq \dfrac{-\pi}{2} = 0$ **[1]**

So, $f(x)$ is not one-one function.

Now, $f(x) = \cos x, \forall x \in R$ is not onto function as there is no pre-image for any real number.

Which does not belong to the intervals $[-1, 1]$, the range of $\cos x$. **[1]**

Q. 47. Let $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$. Find whether the following subsets of $X \times Y$ are functions from X to Y or not.

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

(ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $h = \{(1,4), (2, 5), (3, 5)\}$

(iv) $k = \{(1,4), (2, 5)\}$
[NCERT Exemp. Ex. 1.3, Q. 12, Page 11]

Ans. We have $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$

$\therefore \quad X \times Y = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

f is not a function because f has not unique image. **[½]**

(ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

g is a function as each element of the domain has unique image. **[½]**

(iii) $h = \{(1, 4), (2, 5), (3, 5)\}$

It is clear that h is a function. **[½]**

(iv) $k = \{(1, 4), (2, 5)\}$.

k is not a function as 3 has does not have any image under the mapping. **[½]**

Q. 48. If functions $f : A \to B$ and $g : B \to A$ satisfy $g \, o \, f = I_A$, then show that f is one-one and g is onto.

[NCERT Exemp. Ex. 1.3, Q. 13, Page 11]

Ans. Given that,

$f : A \to B$ and $g : B \to A$ satisfy $g \, o \, f = I_A$,

$\because \quad gof = I_A$

$\Rightarrow \; gof\{f(x_1)\} = gof\{f(x_2)\}$

$\Rightarrow \quad g(x_1) = g(x_2)$

As we know that

$[\because gof = I_A]$

So,

$\therefore x_1 = x_2$

Clearly, g is inverse of function f. So f is one-one and onto. Hence g is also one-one and onto.

Hence, f is one-one function and g is onto function. **[2]**

Q. 49. Let $f : R \to R$ be the function defined by $f(x) = \dfrac{1}{2 - \cos x}$, $\forall x \in R$. Then, find the range of f.

[NCERT Exemp. Ex. 1.3, Q. 14, Page 12]

Ans. We have,

$$f : R \to R, f(x) = \frac{1}{2 - \cos x}, \forall x \in R.$$

$$\text{Let } y = \frac{1}{2 - \cos x}$$

$$\Rightarrow \; 2y - y\cos x = 1$$

$$\Rightarrow \quad \cos x = \frac{2y - 1}{y}$$

$$\Rightarrow \quad \cos x = 2 - \frac{1}{y}$$ **[1]**

Now, we know that range of cos term is $-1 \le \cos x \le 1$

$$\Rightarrow -1 \le 2 - \frac{1}{y} \le 1$$

$$\Rightarrow -3 \le -\frac{1}{y} \le -1$$

$$\Rightarrow \; 1 \le \frac{1}{y} \le 3$$

$$\Rightarrow \; \frac{1}{3} \le y \le 1$$

So, range is $\left[\dfrac{1}{3}, 1\right]$ **[1]**

Q. 50. Let R be relation defined on the set of natural number N as follows : $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

[NCERT Exemp. Ex. 1.3, Q. 17, Page 12]

Ans. We have,

$R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$.

Domain $= \{1, 2, 3, \ldots, 20\}$ $\{\because y \in N\}$

$\therefore R = \{(1, 39), (2, 37), (3, 35), \ldots, (19, 3), (20, 1)\}$

$\therefore$ Range $= \{1, 3, 5, \ldots, 39\}$

R is not reflexive as $(2, 2) \notin R$ as $2 \times 2 + 2 \neq 41$

Also R is not symmetric.

As $(1, 39) \in R$ but $(39, 1) \notin R$

Further R is not transitive.

As $(11, 19) \in R$, $(19, 3) \in R$; but $(11, 3) \notin R$

Hence, R is neither reflexive, nor symmetric and nor transitive. **[2]**

Q. 51. Given $A = \{2, 3, 4\}$, $B = \{2, 5, 6, 7\}$. Construct an example of each of the following:

(a) an injective mapping from A to B.

(b) a mapping from A to B which is not injective.

(c) a mapping from B to A.

[NCERT Exemp. Ex. 1.3, Q. 18, Page 12]

Ans. Given that,

$A = \{2, 3, 4\}$, $B = \{2, 5, 6, 7\}$

(i) Let $f : A \to B$ denotes a mapping

$$f = \{(x, y) : y = x + 3\}$$

Or $f = \{(2,5), (3,6), (4,7)\}$, which is an injective mapping. **[1]**

(ii) Let $g : A \to B$ denote a mapping such that $g = \{(2, 2), (3, 2), (4, 5)\}$, which is not an injective mapping. [Because 5 have two pre-images.] **[½]**

(iii) Let $h : B \to A$ denote a mapping such that $h = \{(2, 2), (5, 3), (6, 4), (7, 4)\}$, which is one of the mappings from B to A. **[½]**

Q. 52. Using the definition, prove that the function $f : A \to B$ is invertible if and only if f is both one-one and onto. [NCERT Exemp. Ex. 1.3, Q. 24, Page 13]

Ans. A function $f : A \to B$ is defined to be invertible, if there exists a function $g : B \to A$. Such that $g \, o \, f = I_A$ and fog $= I_B$. The function is called the inverse of f and this is denoted by f^{-1}.

A function $f : A \to B$ is invertible if f is a bijective function. **[2]**

Q. 53. Give an example of a map

(i) which is one-one but not onto

(ii) which is not one-one but onto

(iii) which is neither one-one nor onto.

[NCERT Exemp. Ex. 1.3, Q. 19, Page 12]

Ans. (i) Let $f : N \to N$, be a mapping defined by $f(x) = 2x$

Which is one-one

For $f(x) = f(x_2)$

$\Rightarrow \; 2x_1 = 2x_2$

$x_1 = x_2$

Further f is not onto, as for $1 \in N$. There does not exist any x in N such that $f(x) = 2x$.

(ii) Let $f : R \to [0, \infty)$ be a mapping defined by $f(x) = |x|$. Clearly $f(x)$ is not one-one as $f(1) = f(-1)$. But $|x| \ge 0$. $\therefore$ f is onto.

(iii) The mapping $f : R \to R$ defined as $f(x) = x^2$, is neither one-one not onto. **[2]**

Q. 54. Let the relation R be defined in N by aRb if $2a + 3b = 30$. Then $R =$ ....

[NCERT Exemp. Ex. 1.3, Q. 48, Page 16]

Ans. Given that, $2a + 3b = 30$

$3b = 30 - 2a$

$b = \dfrac{30-2a}{3} = 10 - \dfrac{2a}{3}$

For $a = 3, b = 8$

$a = 6, b = 6$

$a = 9, b = 4$

$a = 12, b = 2$

$R = \{(3,8), (6,6), (9,4), (12,2)\}$ **[2]**

Q. 55. Let the relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 8$. Then R is given by........

[NCERT Exemp. Ex. 1.3, Q. 49, Page 16]

Ans. Given $A = \{1, 2, 3, 4, 5\}$

$R = \{(a, b) : |a^2 - b^2| < 8\}$

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 3), (3, 4), (4, 4), (5, 5)\}$ **[2]**

Q. 56. Let $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$. Then $g\, o\, f =$ ______ and $f\, o\, g =$ ______.

[NCERT Exemp. Ex. 1.3, Q. 50, Page 16]

Ans. Given that, $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$

$\therefore\ g\, o\, f(1) = g\{f(1)\} = g(2) = 3$

$g\, o\, f(3) = g\{f(3)\} = g(5) = 1$

$g\, o\, f(4) = g\{f(4)\} = g(1) = 3$

$\therefore\ g\, o\, f(x) = \{(1, 3), (3,1), (4, 3)\}$

Now, $f\, o\, g(2) = f\{g(2)\} = f(3) = 5$

$f\, o\, g(5) = f\{g(5)\} = f(1) = 2$

$f\, o\, g(1) = f\{g(1)\} = f(3) = 5$

$f\, o\, g = \{(2, 4), (5, 2), (1, 5)\}$ **[2]**

Q. 57. Let $f : R \to R$ be defined by $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then $(fofof)\,(x) =$ ....

[NCERT Exemp. Ex. 1.3, Q. 51, Page 17]

Ans. Given that, $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$(fofof)(x) = f[f\{f(x)\}]$

$$= f\left[f\left(\frac{x}{\sqrt{1+x^2}}\right)\right]$$

$$= f\left[\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}\right]$$

$$= f\left[\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{\frac{(1+2x^2)}{(1+x^2)}}}\right] = f\left[\frac{x}{\sqrt{1+2x^2}}\right]$$

$$= \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}} = \frac{\frac{x}{\sqrt{1+2x^2}}}{\frac{\sqrt{1+3x^2}}{\sqrt{1+2x^2}}}$$

$$= \frac{x}{\sqrt{1+3x^2}} = \frac{x}{\sqrt{3x^2+1}}$$ **[2]**

Q. 58. If $f(x) = (4 - (x-7)^3\}$, then $f^{-1}(x) =$ ....

[NCERT Exemp. Ex. 1.3, Q. 52, Page 17]

Ans. Given that, $f(x) = [4 - (x - 7)^3]$

Let $y = [4 - (x - 7)^3]$

$(x - 7)^3 = 4 - y$

$\Rightarrow\quad (x - 7) = (4 - y)^{1/3}$

$\Rightarrow\quad x = 7 + (4 - y)^{1/3}$

$f^{-1}(x) = 7 + (4 - x)^{1/3}$ **[2]**

Q. 59. State True or False for the statement.
Let $R = \{(3, 1), (1, 3), (3, 3)\}$ be a relation defined on the set $A = \{1, 2, 3\}$. Then R is symmetric, transitive but not reflexive.

[NCERT Exemp. Ex. 1.3, Q. 53, Page 17]

Ans. False,

Given that, $R = \{(3, 1), (1, 3), (3, 3)\}$ be defined on the set $A = \{1, 2, 3\}$

$(1, 1) \notin R$

So, R is not reflexive. $(3, 1) \notin R, (1, 3) \in R$

Here, R is symmetric.

Since, $(3, 1) \in R, (1, 3) \in R$

But $(1, 1) \notin R$

Hence, R is not transitive. **[2]**

Q. 60. State True or False for the statement.
Let $f : R \to R$ be the function defined by $f(x) = \sin(3x + 2)\ \forall x \in$ R. Then f is invertible.

[NCERT Exemp. Ex. 1.3, Q. 54, Page 17]

Ans. False,

Given $f(x) = \sin(3x + 2)\ \forall x \in R$. is not one-one function for all $x \in R$.

So, f is not invertible. **[2]**

Q. 61. State True or False for the statement.
Every relation which is symmetric and transitive is also reflexive.

[NCERT Exemp. Ex. 1.3, Q. 55, Page 17]

Ans. False,

Let R be a relation defined by $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$ on the set $A = \{1, 2, 3\}$.

It is clear that $(3, 3) \in R$.

So, it is not reflexive. **[2]**

Q. 62. State True or False for the statement.
Let $A = \{0, 1\}$ and N be the set of natural numbers. Then the mapping $f : N \to A$ defined by $f(2n - 1) = 0, f(2n) = 1, \forall n \in N$, is onto.

[NCERT Exemp. Ex. 1.3, Q. 57, Page 17]

Ans. True,

Given, $A=\{0, 1\}$

$f(2n - 1) = 0, f(2n)=1, \forall\ n \in N$

So, the mapping $f : N \to A$ is onto. **[2]**

Q. 63. State True or False for the statement.
The relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{\{1, 1), (1, 2), (2, 1), (3, 3)\}$ is reflexive, symmetric and transitive.

[NCERT Exemp. Ex.1.3, Q.58, Page 17]

Ans. False,

Given that,

$R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$

$(2, 2) \notin R$

So, R is not reflexive. **[2]**

Q. 64. State True or False for the statement. The composition of functions is commutative.
[NCERT Exemp. Ex. 1.3, Q. 59, Page 17]

Ans. False,

Let $f(x) = x^2$

And $g(x) = x + 1$

$f \circ g(x) = f\{g(x)\} = f(x+1)$

$=(x + 1)^2 = x^2 + 2x + 1$

$g \circ f(x) = g\{f(x)\} = g(x^2) = x^2 + 1$

$\therefore f \circ g(x) \neq gof(x)$ **[2]**

Q. 65. State True or False for the statement. The composition of functions is associative.
[NCERT Exemp. Ex. 1.3, Q. 60, Page 17]

Ans. True,

Let $f(x) = x$, $g(x) = x + 1$

And $h(x) = 2x - 1$

Then, $fo\{goh(x)\} = f[g\{h(x)\}]$

$= f\{g(2x - 1)\}$

$= f(2x - 1) + 1$

$= f(2x) = 2x$

$\therefore (fog)\ oh\ (x) = (fog)\{h(x)\}$

$= (fog)(2x - 1)$

$= f\{g(2x - 1)\}$

$= f(2x - 1 + 1)$

$= f(2x) = 2x$ **[2]**

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x, y) : x$ and y have same number of pages$\}$ is an equivalence relation. [NCERT Ex. 1.1, Q. 7, Page 6]

Ans. Set A is the set of all books in the library of a college.

$R = \{x, y) : x$ and y have the same number of pages$\}$

Now, R is reflexive since $(x, x) \in R$ as x and x has the same number of pages.

Let $(x, y) \in R \Rightarrow x$ and y have the same number of pages.

$\Rightarrow y$ and x have the same number of pages.

$\Rightarrow (y, x) \in R$

$\therefore R$ is symmetric. **[1½]**

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ and y and have the same number of pages and y and z have the same number of pages.

$\Rightarrow x$ and z have the same number of pages.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation. **[1½]**

Q. 2. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of 2, 4$\}$. [NCERT Ex. 1.1, Q. 8, Page 6]

Ans. $A = \{1, 2, 3, 4, 5\}$ and $R = \{(a, b) : |a-b|$ is even$\}$

It is clear that for any element $a \in A$, we have $|a-a| = 0$ (which is even).

$\therefore R$ is reflexive.

Let $(a, b) \in R$.

$\Rightarrow |a - b|$ *is even*

$\Rightarrow |-(a - b)| = |b - a|$ is also even

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow |a - b|$ is even and $|b - c|$ is even.

$\Rightarrow (a - b)$ is even and $(b - c)$ is even.

$\Rightarrow (a - c) = (a - b) + (b - c)$ is even.

[Sum of two even integers is even]

$\Rightarrow |a - b|$ *is even.*

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation. **[1½]**

Now, all elements of the set $\{1, 2, 3\}$ are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set $\{2, 4\}$ are related to each other as all the elements of this subset are even.

Also, no element of the subset $\{1, 3, 5\}$ can be related to any element of $\{2, 4\}$ as all elements of $\{1, 3, 5\}$ are odd and all elements of $\{2, 4\}$ are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even. [As $1 - 2$, $1 - 4$, $3 - 2$, $3 - 4$, $5 - 2$ and $5 - 4$ all are odd] **[1½]**

Q. 3. Show that the relation R in the set A of points in a plane given by $R = \{(P, Q)$: Distance of the point P from the origin is same as the distance of the point Q from the origin$\}$, is an equivalence relation. Further, show that the set of all point related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.
[NCERT Ex. 1.1, Q. 11, Page 6]

Ans. $R = \{(P, Q)$: Distance of point P from the origin is the same as the distance of point Q from the origin$\}$

Clearly, $(P, P) \in R$ since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

$\therefore R$ is reflexive.

Now, Let $(P, Q) \in R$.

$\Rightarrow$ The distance of point P from the origin is the same as the distance of point Q from the origin.

$\Rightarrow$ The distance of point Q from the origin is the same as the distance of point P from the origin.

$\Rightarrow (Q, P) \in R$

$\therefore R$ is symmetric.

Now, let $(P, Q), (Q, S) \in R$.

$\Rightarrow$ The distance of points P and Q from the origin is the same and also, the distance of points Q and S from the origin is the same.

$\Rightarrow$ The distance of points P and S from the origin is the same.

$\Rightarrow (P, S) \in R$

$\therefore$ R is transitive.

Therefore, R is an equivalence relation. **[2]**

The set of all points related to $P \neq (0, 0)$ will be those points whose distance from the origin is the same as the distance of point P from the origin.

In other words, if O (0, 0) is the origin and $OP = k$, then the set of all points related to P is at a distance of k from the origin.

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P. **[1]**

Q. 4. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

[NCERT Ex. 1.1, Q. 12, Page 6]

Ans. $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$

R is reflexive since every triangle is similar to itself.

Further,

If $(T_1, T_2) \in R$, then T_1 is similar to T_2.

$\Rightarrow T_2$ is similar to T_1.

$\Rightarrow (T_2, T_1) \in R$

$\therefore$ R is symmetric. **[1]**

Now,

Let $(T_1, T_2), (T_2, T_3) \in R$.

$\Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to T_3.

$\Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive.

Thus, R is an equivalence relation. **[1]**

Now,

We can observe that $\frac{3}{4} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$

$\therefore$ The corresponding sides of triangles T_1 and T_3 are in the same ratio.

Then, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3. **[1]**

Q. 5. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

[NCERT Ex. 1.1, Q. 13, Page 6]

Ans. $R = \{(P_1, P_2) : P_1$ and P_2 have same the number of sides$\}$

R is reflexive,

Since $(P_1, P_2) \in R$, as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides.

$\Rightarrow P_2$ and P_1 have the same number of sides.

$\Rightarrow (P_2, P_1) \in R$

$\therefore$ R is symmetric. **[1]**

Now,

Let $(P_1, P_2), (P_2, P_3) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides.

Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$\Rightarrow (P_1, P_3) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation. **[1]**

The elements in A related to the right-angled triangle (T) with sides 3, 4 and 5 are those polygons which have 3 sides. (Since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles. **[1]**

Q. 6. Show that the function $f : R \to R*$ defined by $f(x) = 1/x$ is one-one and onto, where $R*$ is the set of all non-zero real numbers. Is the result true, if the domain $R*$ is replaced by N with co-domain being same as $R*$? [NCERT Ex. 1.2, Q. 1, Page 10]

Ans. $$f(x) = \frac{1}{x}, f : R \to R_*$$

Part I : $f(x_1) = \frac{1}{x_1}$ and $f(x_2) = \frac{1}{x_2}$

If $f(x_1) = f(x_2)$ then $\frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow \quad x_1 = x_2$

$\therefore f$ is one-one function.

$$f(x) = \frac{1}{x}$$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad x = \frac{1}{y}$

$\Rightarrow \quad f\left(\frac{1}{y}\right) = y \therefore f$ is onto function.

[2]

Part II : When domain R is replaced by N, co-domain R_* remaining the same, then,

$f = N \to R$

If $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{1}{n_1} = \frac{1}{n_2}$

$\Rightarrow \quad n_1 = n_2$ where $n_1, n_2 \in N$

$\therefore f$ is one-one function.

But every real number belonging to co-domain may not have a pre-image in N.

as $3 \in R$

$g(3) = \frac{1}{3} \notin N \therefore f$ is not onto function.

[1]

Q. 7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f : R \to R$ defined by $f(x) = 3 - 4x$

(ii) $f : R \to R$ defined by $f(x) = 1 + x^2$

[NCERT Ex. 1.2, Q. 7, Page 11]

Ans. (i) $f : R \to R$ is defined as $f(x) = 3 - 4x$.

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow \quad -4x_1 = -4x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one function.

For any real number (y) in R, there exists $\dfrac{3-y}{4}$ in R such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y$$

$\therefore$ f is onto function.

Hence, f is bijective. **[1½]**

(ii) $f : R \to R$ is defined as $f(x) = 1 + x^2$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow \quad x_1 = \pm x_2$

$\therefore \quad f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

For example, $f(1) = f(-1) = 2$

$\therefore$ f is not one-one function.

Consider an element -2 in co-domain R.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in R$.

Thus, there does not exist any x in domain R such that $f(x) = -2$.

$\therefore$ f is not onto function.

Hence, f is neither one-one function nor onto function. **[1½]**

Q. 8. Let $f : N \to N$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N.$$

State whether the function f is bijective. Justify your answer. [NCERT Ex. 1.2, Q. 9, Page 11]

Ans. $f : N \to N$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N.$$

It can be observed that:

$f(1) = \dfrac{1+1}{2} = 1$ and $f(2) = \dfrac{2}{2} = 1$ [By definition of f]

$\therefore$ $f(1) = f(2)$, where $1 \neq 2$.

Therefore, f is not one-one function.

Consider a natural number (n) in co-domain N.

Case I : n is odd.

Therefore, $n = 2r + 1$ for some $r \in N$.

Then, there exists $4r + 1 \in N$ such that

$f(4r+1) = \dfrac{4r+1+1}{2} = 2r + 1$. **[2]**

Case II : n is even.

Therefore, $n = 2r$ for some $r \in N$.

Then, there exists $4r \in N$ such that $f(4r) = \dfrac{4r}{2} = 2r$.

Therefore, f is onto function.

f is not one-one but it is onto function.

Hence, f is not a bijective function. **[1]**

Q. 9. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. [NCERT Ex. 1.2, Q. 10, Page 11]

Ans. $A = R - \{3\}$, $B = R - \{1\}$

$f : A \to B$ is defined as $f(x) = \left(\dfrac{x-2}{x-3}\right)$.

Let $x, y \in A$ such that $f(x) = f(y)$.

$\Rightarrow \quad \dfrac{x-2}{x-3} = \dfrac{y-2}{y-3}$

$\Rightarrow \quad (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow \quad -3x - 2y = -3y - 2x$

$\Rightarrow \quad 3x - 2x = 3y - 2y$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one function. **[1½]**

Let $y \in B = R - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$\Rightarrow \quad \dfrac{x-2}{x-3} = y$

$\Rightarrow \quad x - 2 = xy - 3y$

$\Rightarrow x(1-y) = -3y + 2$

$\Rightarrow \quad x = \dfrac{2-3y}{1-y} \in A \quad [y \neq 1]$

Thus, for any $y \in B$, there exists $\dfrac{2-3y}{1-y} \in A$,

Such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\dfrac{2-3y}{1-y}\right) - 2}{\left(\dfrac{2-3y}{1-y}\right) - 3}$$

$$= \frac{2 - 3y - 2 + 2y}{2 - 3y - 3 + 3y}$$

$$= \frac{-y}{-1}$$

$$= y.$$

$\therefore$ f is onto function.

Hence, function f is one-one function and onto function. **[1½]**

Q. 10. Let f, g and h be functions from R to R. Show that

(i) $(f + g)\,o\,h = f\,o\,h + g\,o\,h$

(ii) $(f.g)o\,h = (f\,o\,h).(g\,o\,h)$

[NCERT Ex. 1.3, Q. 2, Page 18]

Ans. To prove : $(f + g)o\,h = f\,o\,h + g\,o\,h$

LHS $= [(f + g)o\,h](x)$

$= (f + g)[h(x)]$

$= f[h(x)] + g[h(x)]$

$= (f\,o\,h)(x) + (g\,o\,h)(x)$

$= \{(f\,o\,h)(x) + (g\,o\,h)\}(x) = RHS$

$\therefore$ $\{(f + g)o\,h\}(x) = \{(f\,o\,h)(x) + (g\,o\,h)\}(x)$

[For all $x \in R$]

Hence, $(f + g)o\,h = f\,o\,h + g\,o\,h$ **[1½]**

To prove : $(f.g)o\,h = (f\,o\,h).(g\,o\,h)$

LHS $= [(f.g)oh](x)$

$= (f.g)[h(x)]$

$= f[h(x)] \,.\, g[h(x)]$

$= (foh)(x) \,.\, (goh)(x)$

$= \{(foh).(goh)\}(x) = RHS$

$\therefore \quad [(f.g)oh](x) = \{(foh).(goh)\}(x)$ [For all $x \in R$]

Hence, $(f.g)oh = (foh).(goh)$ **[1½]**

Q. 11. State with reason whether following functions have inverse

(i) $f : \{1, 2, 3, 4\} \to \{10\}$ with
$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g : \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with
$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h : \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ with
$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

[NCERT Ex. 1.3, Q. 5, Page 18]

Ans. (i) $f : \{1, 2, 3, 4\} \to \{10\}$ defined as $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

From the given definition of f, we can see that f is a many-one function as

$f(1) = f(2) = f(3) = f(4) = 10$

$\therefore f$ is not one-one function.

Hence, function f does not have an inverse function. **[½]**

(ii) $g : \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ defined as

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

From the given definition of g, it is seen that g is a many-one function as

$g(5) = g(7) = 4$.

$\therefore g$ is not one-one function.

Hence, function g does not have an inverse function. **[½]**

(iii) $h : \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ defined as

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

It is seen that all distinct elements of the sets $\{2, 3, 4, 5\}$ have distinct images under h.

$\therefore$ Function h is one-one function.

Also, h is onto function since for every element y of the set $\{7, 9, 11, 13\}$, there exists an element x in the set $\{2, 3, 4, 5\}$, such that $h(x) = y$.

Thus, h is a one-one function and onto function.

Hence, h has an inverse function. **[2]**

Q. 12. Show that $f : [-1, 1] \to R$, given by $f(x) = x/(x + 2)$ is one-one. Find the inverse of the function $f : [-1, 1] \to$ Range f.

(Hint : For $y \in$ Range f, $y = f(x)=x/(x + 2)$, for some x in $[-1, 1]$, *i.e.*, $x = 2y/(1 - y)$).

[NCERT Ex. 1.3, Q. 6, Page 18]

Ans. $f : [-1, 1] \to R$ is given as $f(x)=x(x + 2)$

For one-one function

Let $f(x) = f(y)$

$\Rightarrow x/x + 2 = y/y+2$

$\Rightarrow xy + 2x = xy +2y$

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

$\therefore f$ is a one-one function.

It is clear that $f : [-1, 1] \to$ Range f is onto function. **[1]**

$\therefore f : [-1, 1] \to$ Range f is one-one function and onto function and therefore, the inverse of the function $f : [-1, 1] \to$ Range f exists.

Let g : Range $f \to [-1, 1]$ be the inverse of f.

Let y be an arbitrary element of range f.

Since $f : [-1, 1] \to$ Range f is onto, we have

$y = f(x)$ for some $x \in [-1, 1]$

$\Rightarrow y = x/x + 2$

$\Rightarrow xy + 2y = x$

$\Rightarrow x(1 - y) = 2y$

$\Rightarrow x = 2y/1 - y, y \neq 1$

Now, let us define g : Range $f \to [-1, 1]$ as

$g(y) = 2y/1 - y, y \neq 1$ **[1]**

Now,

$$\text{Now, } (g\,o\,f)(x) = g(f(x)) = g\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{1-\frac{x}{x+2}} = \frac{2x}{x+2-x} = \frac{2x}{2} = x$$

$$(f\,o\,g)(y) = f(g(y)) = f\left(\frac{2y}{1-y}\right) = \frac{\frac{2y}{1-y}}{\frac{2y}{1-y}+2} = \frac{2y}{2y+2-2y} = \frac{2y}{2} = y$$

Therefore, $gof = I_{[-1, 1]}$ and $fog = I$Range f

Therefore, $f^{-1} = g$

Therefore, $f^{-1}(y) = \dfrac{2y}{1-y}, y \neq 1$ **[1]**

Q. 13. Consider $f : R \to R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

[NCERT Ex. 1.3, Q. 7, Page 18]

Ans. $f : R \to R$ is given by, $f(x) = 4x + 3$

For one-one function

Let $f(x) = f(y)$

$\Rightarrow 4x + 3 = 4y + 3$

$\Rightarrow 4x = 4y$

$\Rightarrow x = y$

$\therefore f$ is a one-one function.

For onto function,

For $y \in R$, let $y = 4x + 3$.

$\Rightarrow x = y - 3/4 \in R$

Therefore, for any $y \in R$, there exists $x = y - 3/4 \in R$, such that

$$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$$

Therefore, f is onto function.

Thus, f is one-one and onto functions and therefore, f^{-1} exists. **[2]**

Let us define $g : R \to R$ by $g(x) = \dfrac{x-3}{4}$

Now, $(g\,o\,f)(x) = g(f(x))$
$= g(4x+3)$
$= \dfrac{(4x+3)}{4}$
$= x$
$(f\,o\,g)(y) = f(g(y))$
$= f\left(\dfrac{y-3}{4}\right)$
$= 4\left(\dfrac{y-3}{4}\right)+3$
$= y-3+3$
$= y$

Therefore, $g\,o\,f = f\,o\,g = I_R$

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = g(y) = \dfrac{y-3}{4}$. **[1]**

Q. 14. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

[NCERT Ex. 1.3, Q. 8, Page 18]

Ans. $f: R_+ \to [4, \infty)$ is given as $f(x) = x^2 + 4$.

For one-one function,

Let $f(x) = f(y)$

$\Rightarrow x^2 + 4 = y^2 + 4$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad x = y$ [As $x = y \in R_+$]

$\therefore f$ is a one-one function. **[1]**

For onto function,

For $y \in [4, \infty)$, let $y = x^2 + 4$

$\Rightarrow x^2 = y - 4 \geq 0$ [As $y \geq 4$]

$\Rightarrow x = \sqrt{y-4} \geq 0$

Therefore, for any $y \in [4, \infty)$, there exists $x = \sqrt{y-4} \in R_+$, such that $f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y - 4 + 4 = y$

$\therefore f$ is onto function.

Thus, f is one-one and onto functions and therefore, f^{-1} exists. **[1]**

Let us define,

$g: [4, \infty) \to R+$ by $g(y) = \sqrt{y-4}$

Now, $(g\,o\,f)(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$

And $(f\,o\,g)(y) = f(g(y)) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y - 4 + 4 = y$

$\therefore \quad g\,o\,f = f\,o\,g = I_R$

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = g(y) = \sqrt{y-4}$ **[1]**

Q. 15. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right).$$

[NCERT Ex. 1.3, Q. 9, Page 19]

Ans. $f: R_+ \to [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$.

Let y be an arbitrary element of $[-5, \infty)$.

Let $y = 9x^2 + 6x - 5$

$\Rightarrow \quad y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$

$\Rightarrow y + 6 = (3x+1)^2$

$\Rightarrow 3x + 1 = \sqrt{y+6}$ [As $y \geq -5 \Rightarrow y + 6 > 0$]

$\Rightarrow \quad x = \dfrac{\sqrt{y+6}-1}{3}$

Therefore, f is onto, thereby range $f = [-5, \infty)$. **[1]**

Let us define $g: [-5, \infty) \to R+$ as $g(y) = \dfrac{\sqrt{y+6}-1}{3}$.

We now have:

$(g\,o\,f)(x) = g(f(x))$
$= g(9x^2+6x-5)$
$= g\left((3x+1)^2-6\right)$
$= \dfrac{\sqrt{(3x+1)^2-6+6}-1}{3}$
$= \dfrac{3x+1-1}{3}$
$= x$

And, $(fog)(y) = f(g(y))$
$= f\left(\dfrac{\sqrt{y+6}-1}{3}\right)$
$= \left[3\left(\dfrac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$
$= \left(\sqrt{y+6}\right)^2 - 6$
$= y+6-6$
$= y$

Therefore, $g\,o\,f = IR$ and $f\,o\,g = I_{(-5,\infty)}$ **[1½]**

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = g(y) = \dfrac{\sqrt{y+6}-1}{3}$. **[½]**

Q. 16. Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$ Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

[NCERT Ex. 1.4, Q. 11, Page 25]

Ans. check commutativity :

$*$ is commutative if

$(a, b) * (c, d) = (c, d) * (a, b)$

$\forall a, b, c, d \in R$

$(a,b)*(c,d)$	$(c,d)*(a,b)$
$=(a+c,b+d)$	$=(c+a,d+b)$
	$=(a+c,b+d)$

Since

$(a, b) * (c, d) = (c, d) * (a, b)\ \forall\, a, b, c, d \in R$

$*$ is commutative.

$(a, b) * (c, d) = (a + c, b + d)$ **[½]**

check associativity :

$*$ is associative if

$(a, b) * ((c, d) * (x, y)) = ((a, b) * (c, d)) * (x, y)$

$\forall a, b, c, d, x, y \in R$

$(a,b)*((c,d)*(x,y))$	$((a,b)*(c,d))*(x,y)$
$=(a+b)*(c+x,d+y)$	$=(a+c,b+d)*(x,y)$
$=(a+c+x,b+d+y)$	$=(a+c+x,b+d+y)$

Since

$(a, b) * ((c, d) * (x, y)) = ((a, b) * (c, d)) * (x, y)$

$\forall a, b, c, d, x, y \in R$

$*$ is associative

$(a, b) * (c, d) = (a + c, b + d)$ **[½]**

Identity element

e is identity of $*$ if

$(a, b) * e = e * (a, b) = (a, b)$ [e is the identity of $*$ if $a * e = e * a = a$]

Where $e = (x, y)$

So, $(a, b) * (x, y) = (x, y) * (a, b) = (a, b)$

$(a + x, b + y) = (x + a, b + y) = (a, b)$

Now, $(a + x, b + y)$ (a, b)

On comparing both, we have

$$\underbrace{\begin{array}{c|c} a+x=a & b+y=b \\ x=a-a=0 & y=b-b \\ x=0 & y=0 \end{array}}$$

Since $A = N \times N$

x and y are natural numbers

Since 0 is not natural number.

Identity element does not exist.

Therefore, the operation $*$ does not have any identity element. **[2]**

Q. 17. Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $g\ o\ f = f\ o\ g = I_R$.

[NCERT Misc. Ex. Q. 1, Page 29]

Ans. It is given that $f : R \to R$ is defined as $f(x) = 10x + 7$.

One-one function:

Let $f(x) = f(y)$, where $x, y \in R$.

$\Rightarrow 10x + 7 = 10y + 7$

$\Rightarrow x = y$

Therefore, f is a one-one function.

Onto function:

For $y \in R$, let $y = 10x + 7$.

$$\Rightarrow x = \frac{y-7}{10} \in R$$

Therefore, for any $y \in R$, there exists $x = \frac{y-7}{10} \in R$ such that

$$f(x) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y.$$

Therefore, f is onto function.

Therefore, f is one-one function and onto function.

Thus, f is an invertible function. **[2]**

Let us define $g : R \to R$ as $g(y) = \frac{y-7}{10}$.

Now, we have:

$$g\,o\,f(x) = g(f(x)) = g(10x+7) = \frac{(10x+7)-7}{10} = \frac{10x}{10} = 10$$

And,

$$f\,o\,g(y) = f(g(y)) = f\left(\frac{y-7}{10}\right)$$

$$= 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$\therefore$ $g\ o\ f = IR$ and $f\ o\ g = I_R$

Hence, the required function $g : R \to R$ is defined as

$g(y) = \frac{y-7}{10}$. **[1]**

Q. 18. Show that the function $f : R \to R$ given by $f(x) = x^3$ is injective. [NCERT Misc. Ex. Q. 5, Page 29]

Ans. $f : R \to R$ given by $f(x) = x^3$ (given)

$f(x) = x^3$

We need to check injective (one-one function)

$f(x_1) = (x_1)^3$

$f(x_2) = (x_2)^3$

Putting

$f(x_1) = f(x_2)$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow \quad x_1 = x_2$

Since if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore$ It is one-one (injective) **[1½]**

For one-one function

Suppose $f(x) = f(y)$, where $x, y \in R$.

$\Rightarrow x^3 = y^3$ ...(i)

Now, we need to show that $x = y$.

Suppose $x \neq y$, their cubes will also not be equal.

$\Rightarrow x^3 \neq y^3$

However, this will be a contradiction to (i).

$\therefore x = y$

Hence, f is injective. **[1½]**

Q. 19. Consider the binary operations $* : R \times R \to R$ and $o : R \times R \to R$ defined as $a*b = |a - b|$ and $a\ o\ b = a$, $\forall\ a, b \in$ R. Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that $\forall\ a, b, c \in R$, $a*(b\ o\ c) = (a * b)\ o\ (a * c)$. [If it is so, we say that the operation $*$ distributes over the operation o]. Does o distribute over $*$? Justify your answer.

[NCERT Misc. Ex. Q. 12, Page 30]

Ans. It is given that $* : R \times R \to$ and $o : R \times R \to R$ is defined as $a * b = |a - b|$ and $a\ o\ b = a$, $\forall\ a, b \in R$

For $a, b \in R$, we have

$a * b = |a - b|$ and $b * a = |b - a| = |-(a - b)| = |a - b|$

$\therefore a*b = b*a$

Hence, the operation $*$ is commutative. **[½]**

It can be observed that $(1*2)*3 = (|1-2|)*3 = 1*3 = |1-3| = 2$

and

$1*(2 * 3) = 1*(|2-3|) = 1*1 = |1-1| = 0$

$\therefore (1 * 2)*3 \neq 1*(2 * 3)$ where $1, 2, 3 \in R$.

Hence, the operation $*$ is not associative. **[½]**

Now, consider the operation o

It can be observed that $1\ o\ 2 = 1$ and $2\ o\ 1 = 2$.

$\therefore$ $1\ o\ 2 \neq 2\ o\ 1$ where $1, 2 \in R$.

Hence, the operation o is not commutative.

Let $a, b, c \in R$. Then, we have

$(a\ o\ b)\ o\ c = a\ o\ c = a$

and

$a\ o\ (b\ o\ c) = a\ o\ b = a$

$\therefore$ $a\ o\ b)\ o\ c = a\ o\ (b\ o\ c)$, where $a, b, c \in R$

Hence, the operation o is associative. **[1]**

Now, let $a, b, c \in R$, then we have,

$a * (b\ o\ c) = a * b = |a - b|$

$(a * b)\ o\ (a * c) = (|a - b|)o(|a - c|) = |a - b|$

Hence, $a * (b\ o\ c) = (a * b)\ o\ (a * c)$.

Now,

$1\ o\ (2 * 3) = 1\ o\ (|2-3|) = 1\ o\ 1 = 1$

$(1\ o\ 2) * (1\ o\ 3) = 1 * 1 = |1-1| = 0$

$\therefore$ $1\ o\ (2 * 3) \neq (1\ o\ 2) * (1\ o\ 3)$ where $1, 2, 3 \in R$

Hence, the operation o does not distribute over $*$. **[1]**

Q. 20. Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as $a * b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-c & \text{if } a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a.

[NCERT Misc. Ex. Q. 14, Page 30]

Ans. Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation $*$ on X is defined as:

$$a * b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

An element $e \in X$ is the identity element for the operation $*$, if

$a * e = a = e * a \ \forall\ a \in X$ **[½]**

For $a \in X$, we observed that:

$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$

$0 * a = 0 + a = a \quad [a \in X \Rightarrow 0 + a < 6]$

$\therefore a * 0 = a = 0 * a \ \forall\ a \in X$

Thus, 0 is the identity element for the given operation $*$. **[½]**

An element $a \in X$ is invisible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$i.e., \begin{cases} a+b=0=b+a, & \text{if } a+b<6 \\ a+b-6=0=b+a-6, & \text{if } a+b \geq 6 \end{cases}$$

i.e., $a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a. $a \in X$.

Hence, the inverse of an element $a \in X, a \neq 0$ is $6 - a$

i.e., $a^{-1} = 6 - a$. **[2]**

Q. 21. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2|x - 1/2| - 1, x \in$ A. Are f and g equal? Justify your answer. (Hint : One may note that two function $f : A \to B$ and $g : A \to B$ such that $f(a) = g(a)\ \forall\ a \in A$, are called equal functions).

[NCERT Misc. Ex. Q. 15, Page 30]

Ans. Function f and g are equal if

$f(a) = g(a)$ for all $a \in A$,

i.e., $f(a) = g(a)$ for all $a \in A\ \{-1, 0, 1, 2\}$ **[½]**

x	$f(x) = x^2 - x$	$g(x) = 2\left\lvert x - \frac{1}{2}\right\rvert - 1$	Is $f(x) = g(x)$
–1	$f(-1) = (-1)^2 - (-1)$ $= 1 + 1$ $= 2$	$g(-1) = 2\left\lvert(-1) - \frac{1}{2}\right\rvert - 1$ $= 2\left\lvert\frac{-3}{2}\right\rvert - 1$ $= 2\left(\frac{3}{2}\right) - 1$ $= 3 - 1 = 2$	Yes
0	$f(0) = (0)^2 - (0)$ $= 0 - 0$ $= 0$	$g(0) = 2\left\lvert 0 - \frac{1}{2}\right\rvert - 1$ $= 2\left\lvert\frac{-1}{2}\right\rvert - 1$ $= 2\left(\frac{1}{2}\right) - 1$ $= 1 - 1$ $= 0$	Yes
1	$f(1) = (1)^2 - (1)$ $= 0 - 0$ $= 0$	$g(0) = 2\left\lvert 1 - \frac{1}{2}\right\rvert - 1$ $= 2\left\lvert\frac{-1}{2}\right\rvert - 1$ $= 2\left(\frac{1}{2}\right) - 1$ $= 1 - 1$ $= 0$	Yes
2	$f(2) = (2)^2 - (2)$ $= 4 - 2$ $= 2$	$g(2) = 2\left\lvert 2 - \frac{1}{2}\right\rvert - 1$ $= 2\left\lvert\frac{3}{2}\right\rvert - 1$ $= 2\left(\frac{3}{2}\right) - 1$ $= 3 - 1$ $= 2$	Yes

[2]

Since $f(a) = g(a)$ for all $a \in A\ \{-1, 0, 1, 2\}$

Hence, the functions f and g are equal. **[½]**

Q. 22. Let n be a fixed positive integer. Define a relation R in Z as follows $\forall\, a, b \in Z$ aRb if and only if $a - b$ is divisible by n. Show that R is an equivalence relation. [NCERT Exemp. Ex. 1.3, Q. 15, Page 12]

Ans. Given that, $\forall\, a, b \in Z$ aRb if and only $a - b$ is divisible by n.

Now,

$aRa \Rightarrow (a - a)$ is divisible by n, which is true for any integer a as '0' is divisible by n.

Hence, R is reflexive

$a\,R\,b$

$\Rightarrow$ a – b is divisible by n.

$\Rightarrow -(b - a)$ is divisible by n.

$\Rightarrow (b - a)$ is divisible by n.

$\Rightarrow bRa$

Hence, R is symmetric. **[1½]**

Let aRb and bRc

$\Rightarrow (a - b)$ is divisible by n and $(b - c)$ is divisible by n.

$\Rightarrow (a - b) + (b - c)$ is divisible by n.

$\Rightarrow (a - c)$ is divisible by n,

$\Rightarrow aRc$

Hence, R is transitive.

So, R is an equivalence relation. **[1½]**

Q. 23. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being:

(a) reflexive, transitive but not symmetric

(b) symmetric but neither reflexive nor transitive

(c) reflexive, symmetric and transitive

[NCERT Exemp. Ex. 1.3, Q. 16, Page 12]

Ans. Given that, $A = \{1, 2, 3\}$.

(i) Let $R_1 = \{(1, 1), (1, 2), (1, 3), (2, 3), (2, 2), (1, 3), (3, 3)\}$

R_1 is reflexive, since, (1, 1) (2, 2) (3, 3) lie in R_1.

R_1 is transitive, since, $(1, 2) \in R_1$, $(2, 3) \in R_1$

$\Rightarrow (1, 3) \in R_1$

Now $(1, 2) \in R_1$

$\Rightarrow (2, 1) \notin R_1$.

So, it is not symmetric. **[½]**

(ii) Let $R_2 = \{(1, 2), (2, 1)\}$
Now, $(1, 2) \in R_2$, $(2, 1) \in R_2$
So, it is symmetric.
Clearly R_2 is not reflexive as $(1, 1) \notin R_2$.
Also R_2 is not transitive as $(1, 2) \in R_2$, $(2, 1) \in R_2$ but $(1, 1) \notin R_2$ **[1½]**

(iii) Let $R_3 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
R_3 is reflexive as $(1, 1)\ (2, 2)\ (3, 3) \in R_1$.
R_3 is symmetric as $(1, 2), (1, 3), (2, 3) \in R_1 \Rightarrow (2, 1), (3, 1), (3, 2) \in R_1$
Hence, R_3 is reflexive, symmetric and transitive. **[1]**

Q. 24. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f : A \to B$ be defined by $f(x) = \dfrac{x-2}{x-3}\ \forall x \in A$.
Then show that f is bijective.
[NCERT Exemp. Ex. 1.3, Q. 20, Page 12]

Ans. Given that,
$A = R - \{3\}, B = R - \{1\}$

$f : A \to B$ is defined by $f(x) = \dfrac{x-2}{x-3}, \forall x \in R$

$$\therefore \quad f(x) = \frac{x-3+1}{x-3} = 1 + \frac{1}{x-3}$$

Let $\quad f(x_1) = f(x_2)$

$$\Rightarrow \quad 1 + \frac{1}{x_1 - 3} = 1 + \frac{1}{x_2 - 3}$$

$$\Rightarrow \quad \frac{1}{x_1 - 3} = \frac{1}{x_2 - 3}$$

$$\Rightarrow \quad x_1 = x_2$$

So, $f(x)$ is an injective function **[1½]**

Now let $y = \dfrac{x-2}{x-3}$

$$\Rightarrow \quad x - 2 = xy - 3y$$

$$\Rightarrow \quad x(1-y) = 2 - 3y$$

$$\Rightarrow \quad x = \frac{2-3y}{1-y}$$

$$\Rightarrow \quad x = \frac{3y-2}{y-1}$$

$\Rightarrow y \in R - \{1\} = B$

Hence, $f(x)$ is onto or surjective function.
Hence, $f(x)$ is a bijective function. **[1½]**

Q. 25. Let $A = \{1, 2, 3, \ldots 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].
[NCERT Exemp. Ex. 1.3, Q. 23, Page 13]

Ans. Given that $A = \{1, 2, 3, \ldots 9\}$
$(a, b)\ R\ (c, d)\ a + d = b + c$ for $(a, b) \in A \times A$ and $(c, d) \in A \times A$.
Let $(a, b)\ R\ (a, b)$
$\Rightarrow a + b = b + a, \forall\ a, b \in A$
Which is true for any $a, b \in A$
Hence, R is reflexive. **[1]**
Let $(a, b)\ R\ (c, d)$
$a + d = b + c$
$c + b = d + a \Rightarrow (c, d)\ R\ (a, b)$
So, R is symmetric. **[½]**
Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$
$a + d = b + c$ and $c + f = d + e$
$a + d = b + c$ and $d + e = c + f\ (a + d) - (d + e) = (b + c) - (c + f)$
$(a - e) = b - f$
$a + f = b + e$
$(a, b)\ R\ (e, f)$ **[1]**
Now, equivalence class containing [(2, 5) is {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}]
{(2, 5)} is {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}. **[½]**

Q. 26. Functions $f, g : R \to R$ are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find
(i) $f \circ g$ **(ii)** $g \circ f$
(iii) $f \circ f$ **(iv)** $g \circ g$
[NCERT Exemp. Ex. 1.3, Q. 25, Page 13]

Ans. Given that,
$f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$,

(i) $f \circ g = f\{g(x)\} = f(2x - 3)$
$= (2x - 3)^2 + 3(2x - 3) + 1$
$= 4x^2 + 9 - 12x + 6x - 9x + 1$
$= 4x^2 + 9 - 15x + 1$
$= 4x^2 - 15x + 10$ **[1]**

(ii) $g \circ f = g\{f(x)\} = g(x^2 + 3x + 1)$
$= 2(x^2 + 3x + 1) - 3$
$= 2x^2 + 6x + 2 - 3$
$= 2x^2 + 6x - 1$ **[½]**

(iii) $f \circ f = f\{f(x)\} = f\{x^2 + 3x + 1\}$
$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$
$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$
$= x^4 + 6x^3 + 14x^2 + 15x + 5$ **[1]**

(iv) $g \circ g = g\{g(x)\} = g(2x - 3)$
$= 2(2x - 3) - 3$
$= 4x - 6 - 3 = 4x - 9$ **[½]**

Q. 27. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative
(i) $a * b = a - b, \forall\ a, b \in \mathbf{Q}$
(ii) $a * b = a^2 + b^2, \forall\ a, b \in \mathbf{Q}$
(iii) $a * b = a + ab, \forall\ a, b \in \mathbf{Q}$
(iv) $a * b = (a - b)^2, \forall\ a, b \in \mathbf{Q}$
[NCERT Exemp. Ex. 1.3, Q. 26, Page 13]

Ans. Given that $*$ be the binary operation defined on Q.

(i) $a * b = a - b, \forall a, b \in Q$ and $b * a = b - a$
So, $a * b \neq b * a$ $\quad [\because b - a \neq a - b]$
Hence, $*$ is not commutative. **[1]**

(ii) $a * b = a^2 + b^2$
$b * a = b^2 + a^2$
So, $*$ is commutative. [since, '+' is on rational is commutative.] **[1]**

(iii) $a * b = a + ab$
$b * a = b + ab$
Clearly, $a + ab \neq b + ab$
So, $*$ is not commutative. **[½]**

(iv) $a * b = (a - b)^2, \forall a, b \in Q$
$b * a = (b - a)^2$
$\because (a - b)^2 = (b - a)^2$
Hence, $*$ is commutative. **[½]**

Long Answer Type Questions

(5 or 6 marks each)

Q. 1. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements relates to 1. Also write the equivalence class [2].

[CBSE Board, Delhi Region, 2016]

Ans. Given : $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Also, $R = \{(a, b) : a, b \in A, |a - b|\}$

For reflexive,

Substitute $a = b$

$R = |b - b| = 0$ which is divisible by 4.

Therefore, R is reflexive. **[1]**

For symmetric,

Substitute $a = b$ and $b = a$

$R = |b - a| = |a - b|$ which is divisible by 4.

Therefore, R is symmetric. **[1]**

For transitive,

If $|a - b|$ is divisible by 4 *i.e.*, $a - b = \pm 4k$

Also, If $|b - c|$ is divisible by 4 *i.e.*, $b - c = \pm 4k$

$\therefore$ $|a - c| = \pm 4k \pm 4k$ is also divisible by 4.

Therefore, R is transitive. **[2]**

Now, set of elements related to 1 is $\{(1, 1), (1, 5), (1, 9), (5,1), (9,1)\}$

Let $(x, 2) \in R$

$|x - 2| = 4k$ where $k \le 3$

$\therefore \quad x = 2, 6\ 10$

Therefore, the equivalent class [2] is $\{2, 6, 10\}$. **[2]**

Q. 2. Show that the function $f : R \to R$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto. Also if $g : R \to R$ is defined as $g(x) = 2x - 1$, find fog(x). **[CBSE Board, Delhi Region, 2016]**

Ans. Given that,

$$f(x) = \frac{x}{x^2+1}$$

Let $\quad y = \dfrac{x}{x^2+1}$

$\Rightarrow \quad yx^2 + y = x$

$\Rightarrow yx^2 + y - x = 0$

$\Rightarrow \quad x = -\dfrac{1 \pm \sqrt{1-4y^2}}{2y}$

Since for any value of x we will get two values for y. Hence, $f(x)$ is many-one function not one-one function. **[2]**

Since x is real number.

Hence, $1 - 4y^2 \ge 0$

$\Rightarrow (1+2y)(1-2y) \ge 0$

$\therefore \dfrac{-1}{2} \le y \le \dfrac{1}{2}$

Therefore, we always get the value of y as $\left[-\dfrac{1}{2}\ \dfrac{1}{2}\right]$.

Hence, $f(x)$ is not onto function. **[2]**

Now, $g(x) = 2x - 1$

$$fog(x) = \frac{2x-1}{(2x-1)^2+1}$$

$$= \frac{2x-1}{4x^2+1-4x+1}$$

$$= \frac{2x-1}{4x^2-4x+2}$$

[2]

Q. 3. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\dfrac{\sqrt{y+6}-1}{3}\right)$. Hence Find

(i) f^{-1} (10)

(ii) y if $f^{-1}(y) = \dfrac{4}{3}$,

where R_+ is the set of all non-negative real numbers. **[CBSE Board, Delhi Region, 2017]**

Ans. Clearly $f^{-1}(y) = g(y) : [-5, \infty] \to R+$ and,

$$\text{fog}(y) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5$$

$$= y$$

[2]

And $(g\ o\ f)(x) = g(9x^2 + 6x - 5)$

$$= \frac{\sqrt{9x^2+6x+1-1}}{3} = x$$

[2]

$\therefore g = f^{-1}$ **[1]**

(i) $f^{-1}(10) = \dfrac{\sqrt{16}-1}{3} = 1$ **[½]**

(ii) $f \quad (y) = - \Rightarrow y = 19$ **[½]**

Q. 4. Discuss the commutativity and associativity of binary operation '$*$' defined on $A = Q \sim \{1\}$ by the rule $a * b = a - b + ab$ for all $a, b \in A$. Also find the identity element of $*$ in A and hence find the invertible elements of A.

[CBSE Board, Delhi Region, 2017]

Ans. $\quad a * b = a - b + ab\ \forall a, b \in A = Q \sim$ [1]

$b * a = b - a + ba$

$(a * b) \ne b * a \Rightarrow *$ is not commutative. **[1½]**

$(a * b) * c = (a - b + ab) * c$

$= a - b - c + ab + ac - bc + abc$

$a * (b * c) = a * (b - c + bc)$

$= a - b + c + ab - ac - bc + abc$

$(a * b) * c \ne a * (b * c)$ **[1½]**

$\Rightarrow *$ is not associative.

Existence of identity,

$a * e = a - e + ae = a$

$e * a = e - a + ea = a$

$\Rightarrow \ e(a - 1) = 0$

$\Rightarrow e(1 + a) = 2a$

$\Rightarrow \quad e = 0$

$\Rightarrow \quad e = \dfrac{2a}{1+a}$ **[1½]**

$\because e$ is not unique.

$\therefore$ No identity element exists.

$a * b = e = b * a$

$\therefore$ No identity element exists. [1½]

$\Rightarrow$ Inverse element does not exist.

Q. 5. Consider $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.
[CBSE Board, All India Region, 2017]

Ans. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ [½]

$$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$$

$$\Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$$

$$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$$

$$\Rightarrow 16(x_1 - x_2) - 9(x_1 - x_2) = 0$$

$$\Rightarrow x_1 - x_2 = 0$$

$$\Rightarrow x_1 = x_2$$

Hence f is a one-one function. [2]

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$$3xy + 4y = 4x + 3$$

$$\Rightarrow 4x - 3xy = 4y - 3$$

$$\Rightarrow x = \frac{4y-3}{4-3y}$$

$\therefore \forall y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is onto function and so bijective [2]

And $f^{-1}(y) = \frac{4y-3}{4-3y}; y \in R - \left\{\frac{4}{3}\right\}$ [1]

$f^{-1}(0) = -\frac{3}{4}$ [½]

And

$$f^{-1}(x) = 2$$

$$\Rightarrow \frac{4x-3}{4-3x} = 2$$

$$\Rightarrow 4x - 3 = 8 - 6x$$

$$\Rightarrow 10x = 11$$

$$\Rightarrow x = \frac{11}{10}$$ [½]

Q. 6. Let $A = Q \times Q$ and let $*$ be *a* binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to $*$ on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.
[CBSE Board, All India Region, 2017]

Ans. $(a, b) * (c, d) = (ac, b + ad)$; $(a, b), (c, d) \in A$

$(c, d) * (a, b) = (ca, d + bc)$

Since $b + ad \neq d + bc$

$\Rightarrow *$ is not commutative [1½]

For associativity, we have,

$[(a, b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f)$
$= (ace, b + ad + acf)$

$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf)$
$= (ace, b + ad + acf)$ [1½]

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element is A.

Then $(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow (ae, b + af) = (a, b) = (ae, f + be)$

$\Rightarrow e = 1, f = 0 \Rightarrow (1, 0)$ is the identity element [1½]

(ii) Let (c, d) be the inverse element for (a, b)

$\Rightarrow (a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow (ac, b + ad) = (1, 0) = (ac, d + bc)$

$\Rightarrow ac = 1$

$\Rightarrow c = \frac{1}{a}$ and $b + ad = 0$

$\Rightarrow d = -\frac{b}{a}$ and $d + bc = 0$

$\Rightarrow d = -bc = -b\left(\frac{1}{a}\right)$

$\Rightarrow \left(\frac{1}{a}, \frac{b}{a}\right), a \neq 0$ is the inverse of $(a, b) \in A$ [1½]

Q. 7. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A$.

Show that f is bijective. Also, find

(i) x, if $f^{-1}(x) = 4$

(ii) $f^{-1}(7)$. **[CBSE Board, Foreign Scheme, 2017]**

Ans. Let $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1-2)(x_2-3) = (x_1-3)(x_2-2)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow x_1 = x_2$$

Hence f is a one-one function [2]

Let $y = \frac{x-2}{x-3}$ for $y \in R - \{1\}$

$$\Rightarrow x = \frac{3y-1}{y-1}; y \neq 1$$

$\therefore \forall y \in R - \{1\}, x \in R - \{3\}$

i.e., Range of f = co-domain of f.

Hence f is onto function and so bijective. [2]

Also, $f^{-1}(x) = \frac{3x-2}{x-1}; x \neq 1$ [1]

Now, $f^{-1}(x) = 4$

$\Rightarrow \frac{3x-2}{x-1} = 4$ [½]

$\Rightarrow x = 2$

And $f^{-1}(7) = \frac{19}{6}$ [½]

Q. 8. Let $A = R \times R$ and let $*$ be a binary operation on A defined by

$(a, b) * (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in R \times R$.

(i) Show that $*$ is commutative on A.

(ii) Show that $*$ is associative on A.

(iii) Find the identity element of $*$ in A.
[CBSE Board, Foreign Scheme, 2017]

Ans. (i) $(a, b) * (c, d) = (ad + bc, bd)$

Now, $(c, d) * (a, b) = (cb + da, db) = (ad + bc, bd)$
$= (a, b) * (c, d)$

$\Rightarrow *$ is commutative. **[2]**

(ii) $[(a, b) * (c, d)] * (e, f) = (ad + bc, bd) * (e, f)$
$= (adf + bef + bde, bdf)$
$(a, b) * [(c, d) * (e, f)] = (a, b) * (cf + de, df) = (adf + bef + bde, bdf)$
$\Rightarrow *$ is associative. **[2]**

Let (e_1, e_2) be the identity element of A.
$\Rightarrow (a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$
$\Rightarrow (ae_2 + be_1, be_2) = (a, b) = (e_1b + e_2a, e_2b)$
$\Rightarrow \quad ae_2 + be_1 = a$ and $be_2 = b$
$\Rightarrow \quad e_1 = 0, e_2 = 1$
$\Rightarrow (0, 1)$ is the identity on A. **[2]**

Q. 9. Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

[CBSE Board, Delhi Region, 2016]

Ans. Let $x_1, x_2 \in \mathrm{N}$ and $f(x_1) = f(x_2)$

$$\Rightarrow \quad 9x_1^2 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$
$$\Rightarrow \quad 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$$
$$\Rightarrow \quad (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$$
$$\Rightarrow \quad x_1 - x_2 = \quad \text{or } x_1 = x_2$$

as $(9x_1 + 9x_2 + 6) \neq 0, x_1, x_2 \in \mathrm{N}$

$\therefore f$ is a one-one function **[2]**

$f : N \to S$ is onto function as co-domain = Range **[1]**

Hence f is invertible.

$$y = 9x^2 + 6x - 5$$
$$= (3x+1)^2 - 6$$
$$\Rightarrow \quad x = \frac{\sqrt{y+6} - 1}{3}$$
$$\therefore f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}, y \in S$$ **[2]**

$$f^{-1}(43) = \frac{\sqrt{49} - 1}{3} = 2$$ **[½]**

$$f^{-1}(163) = \frac{\sqrt{169} - 1}{3} = 4$$ **[½]**

Q. 10. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

[CBSE Board, All India Region, 2016]

Ans. $\Rightarrow (x + 3)2x - (-2)(-3x) = 8$ [½]
$\Rightarrow \quad 6x^2 + 6x - 6x = 8$
$\Rightarrow \quad x = 2$ [½]

Q. 11. Show that the binary operation $*$ on $A = R - \{-1\}$ defined as $a*b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A. Also find the identity element of $*$ in A and prove that every element of A is invertible.

[CBSE Board, All India Region, 2016]

Ans. Commutativity : For any elements $a, b \in A$
$a * b = a + b + ab = b + a + ba = b * a$.
Hence $*$ is commutative. **[1½]**

Associativity : For any three elements $a, b, c, \in A$
$a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc$
$(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc$ **[1½]**
$\therefore a * (b * c) = (a * b) * c$,
Hence $*$ is Associative.

Identity element : Let $e \in A$ be the identity element then $a * e = e * a = a$
$\Rightarrow a + e + ae = e + a + ea = a \Rightarrow e(1 + a) = 0$, as $a \neq -1$
$e = 0$ is the identity. **[1½]**

Invertibility : Let $a, b \in A$ so that 'b' is inverse of a
$a * b = b * a = e$
$\Rightarrow a + b + ab = b + a + ba = 0$
As $a \neq -1, b = \dfrac{-a}{1+a} \in A$.
Hence every element of A is invertible. **[1½]**

Q. 12. If $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$. Then find $f \circ g$ and $g \circ f$. Hence find $f \circ g(-3)$, $f \circ g(5)$ and $g \circ f(-2)$.

[CBSE Board, Foreign Scheme, 2016]

Ans. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall x \in R$

$(fog)(x) = f(g(x))$ **[1]**
$= ||x| - x| + |x| - x$ **[1]**
$(\text{gof})(x) = g(f(x))$ **[½]**
$= ||x| + x| - |x| - x$ **[½]**
$(f \circ g)(-3) = 12$ **[1]**
$(f \circ g)(5) = 0$ **[1]**
$(g \circ f)(-2) = 0$ [1]

Q. 13. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3...13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as $R = \{(x, y) : y = x + 5$ and $x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y$ is divisible by $x\}$

(iv) Relation R in the set Z of all integers defined as $R = \{(x, y) : x - y$ is an integer$\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y) : x$ and y work at the same place$\}$

(b) $R = \{(x, y) : x$ and y live in the same locality$\}$

(c) $R = \{(x, y) : x$ is exactly 7 cm taller than $y\}$

(d) $R = \{(x, y) : x$ is wife of $y\}$

(e) $R = \{(x, y) : x$ is father of y$\}$

[NCERT Ex. 1.1, Q. 1, Page 5]

Ans. (i) $A = \{1, 2, 3 \ldots 13, 14\}$
$R = \{(x, y) : 3x - y = 0\}$
$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
R is not reflexive since $(1, 1), (2, 2) \ldots (14, 14) \notin R$.
Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. $[3(3) - 1 \neq 0]$
Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$. $[3(1) - 9 \neq 0]$. Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) $R = \{(x, y) : y = x + 5 \text{ and } x < 4\} = \{(1, 6), (2, 7), (3, 8)\}$
It is clear that $(1, 1) \notin R$.

$\therefore$ R is not reflexive.

$(1, 6) \in R$ But, $(1, 6) \notin R$.

$\therefore$ R is not symmetric.

Now, since there is no pair in R such that (x, y) and $(y, z) \in R$, then (x, z) cannot belong to R.

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y \text{ is divisible by } x\}$

We know that any number (x) is divisible by itself.

So, $(x, x) \in R$

$\therefore R$ is reflexive.

Now,

$(2, 4) \in R$ [as 4 is divisible by 2].

But, $(4, 2) \notin R$. [as 2 is not divisible by 4].

$\therefore$ R is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible by x and z is divisible by y.

$\therefore$ z is divisible by x.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) $R = \{(x, y) : x - y \text{ is an integer}\}$

Now, for every $x \in Z$, $(x, x) \in R$ as $x - x = 0$ is an integer.

$\therefore$ R is reflexive.

Now, for every $x, y \in Z$, if $(x, y) \in R$, then $x - y$ is an integer.

$\Rightarrow -(x - y)$ is also an integer.

$\Rightarrow (y - x)$ is an integer.

$\therefore$ $(y, x) \in R$

$\therefore$ R is symmetric.

Now,

Let (x, y) and $(y, z) \in R$, where $x, y, z \in Z$.

$\Rightarrow (x - y)$ and $(y - z)$ are integers.

$\Rightarrow x - z = (x - y) + (y - z)$ is an integer.

$\therefore$ $(x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric and transitive.

(v) **(a)** $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$

$\Rightarrow (x, x) \in R$ [as x and x work at the same place]

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow y$ and x work at the same place.

$\Rightarrow (y, x) \in R$.

$\therefore$ R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow x$ and y work at the same place and y and z work at the same place.

$\Rightarrow x$ and z work at the same place.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric and transitive.

(b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$

Clearly, $(x, x) \in R$ as x and x is the same human being.

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

$\Rightarrow y$ and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore$ R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ and y live in the same locality and y and z live in the same locality.

$\Rightarrow x$ and z live in the same locality.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric and transitive.

(c) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$

Now, $(x, x) \notin R$

Since human being x cannot be taller than himself.

$\therefore$ R is not reflexive.

Now, let $(x, y) \in R$.

$\Rightarrow x$ is exactly 7 cm taller than y.

Then, y is not taller than x.

[Since, y is 7 cm smaller than x]

$\therefore$ $(y, x) \notin$ R

Indeed, if x is exactly 7 cm taller than y, then y is exactly 7 cm shorter than x.

$\therefore$ R is not symmetric.

Now,

Let $(x, y), (y, z) \in R$.

$\Rightarrow x$ is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

$\Rightarrow x$ is exactly 14 cm taller than z.

$\therefore$ $(x, z) \notin R$

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) $R = \{(x, y) : x \text{ is the wife of } y\}$

Now,

$(x, x) \notin R$

Since x cannot be the wife of herself.

$\therefore$ R is not reflexive.

Now, let $(x, y) \in R$

$\Rightarrow x$ is the wife of y.

Clearly y is not the wife of x.

$\therefore$ $(y, x) \notin R$

Indeed, if x is the wife of y, then y is the husband of x.

$\therefore$ R is not transitive.

Let $(x, y), (y, z) \in R$

$\Rightarrow x$ is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

$\therefore$ $(x, z) \notin R$

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) $R = \{(x, y) : x \text{ is the father of } y\}$

$(x, x) \notin R$

As x cannot be the father of himself.

$\therefore$ R is not reflexive.

Now, let $(x, y) \notin R$.

$\Rightarrow x$ is the father of y.

$\Rightarrow y$ cannot be the father of y.

Indeed, y is the son or the daughter of y.

$\therefore$ $(y, x) \notin R$

$\therefore$ R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \notin R$.
$\Rightarrow x$ is the father of y and y is the father of z.
$\Rightarrow x$ is not the father of z.
Indeed, x is the grandfather of z.
$\therefore (x, z) \notin R$
$\therefore R$ is not transitive.
Hence, R is neither reflexive, nor symmetric, nor transitive. **[5]**

Q. 14. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by
(i) $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$
(ii) $R = \{(a, b) : a = b\}$
is an equivalence relation. Find the set of all elements related to 1 in each case.
[NCERT Ex. 1.1, Q. 9, Page 6]

Ans. $A = \{x \in Z : 0 \leq x \leq 12\} = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$
(i) $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$
For any element $a \in A$, we have $(a, a) \in R$ as $|a - a| = 0$ is a multiple of 4.
$\therefore R$ is reflexive.
Now, let $(a, b) \in R \Rightarrow |a - b|$ is a multiple of 4.
$\Rightarrow |-(a - b)| = |b - a|$ is a multiple of 4.
$\Rightarrow (b, a) \in R$
$\therefore R$ is symmetric.
Now, let $(a, b), (b, c) \in R$.
$\Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.
$\Rightarrow (a - b)$ is a multiple of 4 and $(b - c)$ is a multiple of 4.
$\Rightarrow (a - c) = (a - b) + (b - c)$ is a multiple of 4.
$\Rightarrow |a - c|$ is a multiple of 4.
$\Rightarrow (a, c) \in R$
$\therefore R$ is transitive.
Hence, R is an equivalence relation.
The set of elements related to 1 is $\{1, 5, 9\}$ as
$|1 - 1| = 0$ is a multiple of 4.
$|5 - 1| = 4$ is a multiple of 4.
$|9 - 1| = 8$ is a multiple of 4. **[2½]**

(ii) $R = \{(a, b) : a = b\}$
For any element a $\in A$, we have $(a, a) \in R$, since $a = a$.
$\therefore R$ is reflexive.
Now, let $(a, b) \in R$.
$\Rightarrow a = b$
$\Rightarrow b = a$
$\Rightarrow (b, a) \in R$
$\therefore R$ is symmetric.
Now, let $(a, b) \in R$ and $(b, c) \in R$.
$\Rightarrow a = b$ and $b = c$
$\Rightarrow a = c$
$\Rightarrow (a, c) \in R$
$\therefore R$ is transitive.
Hence, R is an equivalence relation.
The elements in R that are related to 1 will be those elements from set A which are equal to 1.
Hence, the set of elements related to 1 is $\{1\}$. **[2½]**

Q. 15. Given an example of a relation. Which is:
(i) symmetric but neither reflexive nor transitive.
(ii) transitive but neither reflexive nor symmetric.
(iii) reflexive and symmetric but not transitive.
(iv) reflexive and transitive but not symmetric.
(v) symmetric and transitive but not reflexive.
[NCERT Ex. 1.1, Q. 10, Page 6]

Ans. (i) Let $A = \{5, 6, 7\}$.
Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.
Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.
Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.
$\Rightarrow (5, 6), (6, 5) \in R$, but $(5, 5) \notin R$
$\therefore R$ is not transitive.
Hence, relation R is symmetric but not reflexive or transitive. **[1]**

(ii) Consider a relation R in R defined as:
$R = \{(a, b) : a < b\}$
For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than itself.
In fact, $a = a$.
$\therefore R$ is not reflexive.
Now, $(1, 2) \in R$ (as $1 < 2$)
But, 2 is not less than 1.
$\therefore (2, 1) \notin R$
$\therefore R$ is not symmetric.
Now, let $(a, b), (b, c) \in R$.
$\Rightarrow a < b$ and $b < c$
$\Rightarrow a < c$
$\Rightarrow (a, c) \in R$
$\therefore R$ is transitive.
Hence, relation R is transitive but not reflexive and symmetric. **[1]**

(iii) Let $A = \{4, 6, 8\}$.
Define a relation R on A as
$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$
Relation R is reflexive since for every a $\in$ A, $(a, a) \in R$
i.e., $\{(4, 4), (6, 6), (8, 8)\} \in R$.
Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.
Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.
Hence, relation R is reflexive and symmetric but not transitive. **[1]**

(iv) Define a relation R in R as:
$R = \{a, b) : a^3 \geq b^3\}$
Clearly $(a, a) \in R$ as $a^3 = a^3$.
$\therefore R$ is reflexive.
Now, $(2, 1) \in R$ [as $2^3 \geq 1^3$]
But, $(1, 2) \notin R$ [as $1^3 < 2^3$]
$\therefore R$ is not symmetric.
Now, Let $(a, b), (b, c) \in R$.
$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$
$\Rightarrow a^3 \geq c^3$
$\Rightarrow (a, c) \in R$
$\therefore R$ is transitive.
Hence, relation R is reflexive and transitive but not symmetric. **[1]**

(v) Let $A = \{-5, -6\}$.
Define a relation R on A as
$R = \{(-5, -6), (-6, -5), (-5, -5)\}$
Relation R is not reflexive as $(-6, -6) \notin R$.
Relation R is symmetric as $(-5, -6) \in R$ and $(-6, -5) \in R$.
It is seen that $(-5, -6), (-6, -5) \in R$. Also, $(-5, -5) \in R$.
$\therefore$ The relation R is transitive.
Hence, relation R is symmetric and transitive but not reflexive. **[1]**

Q. 16. Check the injectivity and surjectivity of the following functions:

(i) $f : N \to N$ given by $f(x) = x^2$

(ii) $f : Z \to Z$ given by $f(x) = x^2$

(iii) $f : R \to R$ given by $f(x) = x^2$

(iv) $f : N \to N$ given by $f(x) = x^3$

(v) $f : Z \to Z$ given by $f(x) = x^3$

[NCERT Ex. 1.2, Q. 2, Page 10]

Ans. (i) $f : N \to N$ is given by $f(x) = x^2$

It is seen that for $x, y \in$ N, $f(x) = f(y)$

$\Rightarrow x^2 = y^2 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in N$. But, there does not exist any x in N such that $f(x) = x^2 = 2$.

$\therefore f$ is not surjective.

Hence, function f is injective but not surjective. **[1]**

(ii) $f : Z \to Z$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in Z$. But, there does not exist any element $x \in$ Z such that

$f(x) = -2$ or $x^2 = -2$.

$\therefore f$ is not surjective.

Hence, function f is neither injective, nor surjective. **[1]**

(iii) $f : R \to R$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in R$. But, there does not exist any element $x \in R$ such that

$f(x) = -2$ or $x^2 = -2$.

$\therefore f$ is not surjective.

Hence, function f is neither injective nor surjective. **[1]**

(iv) $f : N \to N$ given by $f(x) = x^3$

It is seen that for $x, y \in N$, $f(x) = f(y)$

$\Rightarrow x^3 = y^3$

$\Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in N$. But, there does not exist any element $x \in N$ such that

$f(x) = 2$ or $x^3 = 2$.

$\therefore f$ is not surjective

Hence, function f is injective but not surjective. **[1]**

(v) $f : Z \to Z$ is given by $f(x) = x^3$

It is seen that for $x, y \in$ Z, $f(x) = f(y)$

$\Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in Z$. But, there does not exist any element $x \in Z$ such that

$f(x) = 2$ or $x^3 = 2$.

$\therefore f$ is not surjective.

Hence, function f is injective but not surjective. **[1]**

Q. 17. Consider $f : \{1, 2, 3\} \to \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$. [NCERT Ex. 1.3, Q. 11, Page 19]

Ans. Function $f : \{1, 2, 3\} \to \{a, b, c\}$ is given by $f(1) = a$, $f(2) = b$, and $f(3) =$ c

If we define $g : \{a, b, c\} \to \{1, 2, 3\}$ as $g(a) = 1$, $g(b) = 2$, $g(c) = 3$.

We have ,

$(f \, o \, g)(a)=f(g(a)) = f(1) = a$

$(f \, o \, g)(b)=f(g(b)) = f(2) = b$

$(f \, o \, g)(c)=f(g(c)) = f(3) = c$

And $(g \, o \, f)(1) = g(f(1)) = f(a) = 1$

$(g \, o \, f)(2) = g(f(2)) = f(b) = 2$

$(g \, o \, f)(3) = g(f(3)) = f(c) = 3$

$\therefore g \, o \, f = I_X$ and $f \, o \, g = I_Y$, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

Thus, the inverse of f exists and $f^{-1} = g$.

$\therefore f^{-1}: \{a, b, c\} \to \{1, 2, 3\}$ is given by $f^{-1}(a) = 1, f^{-1}(b) = 2, f^{-1}(c) = 3$ **[2½]**

Let us now find the inverse of f^{-1} *i.e.*, find the inverse of g.

If we define $h : \{1, 2, 3\} \to \{a, b, c\}$ as $h(1) = a$, $h(2) = b$, $h(3) = c$

We have,

$(g \, o \, h)(1) = g(h(1)) = g(a)=1$

$(g \, o \, h)(2) = g(h(2)) = g(b)=2$

$(g \, o \, h)(3) = g(h(3)) = g(c)=3$

And $(h \, o \, g)(a) = h(g(a))=h(1)=$a

$(h \, o \, g)(b) = h(g(b)) = h(2) = b$

$(h \, o \, g)(c) = h(g(c)) = h(3) = c$

$\therefore g \, o \, h = I_X$ and $h \, o \, g = I_Y$, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

Thus, the inverse of g exists and $g^{-1} = h \Rightarrow (f^{-1})^{-1} = h$.

It can be noted that $h = f$.

Hence, $(f^{-1})^{-1} = f$. **[2½]**

Q. 18. Determine whether or not each of the definition of given below gives a binary operation.

In the event that $*$ is not a binary operation, give justification for this.

(i) On Z^+, define $*$ by $a * b = a - b$

(ii) On Z^+, define $*$ by $a * b = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^+, define $*$ by $a * b = |a - b|$

(v) On Z^+, define $*$ by $a * b = a$

[NCERT Ex. 1.4, Q. 1, Page 24]

Ans. (i) On $Z+$, $*$ is defined by $a * b = a - b$.

It is not a binary operation.

As the image of (1, 2) under $*$ is $1 * 2 = 1 - 2 = -1 \notin Z+$. **[1]**

(ii) On Z^+, $*$ is defined by $a * b = ab$.

It is seen that for each $a, b \in Z+$, there is a unique element ab in Z^+.

This means that $*$ carries each pair (a, b) to a unique element $a * b = ab$ in Z^+.

Therefore, $*$ is a binary operation. **[1]**

(iii) On R, $*$ is defined by $a * b = ab^2$.

It is seen that for each $a, b \in R$, there is a unique element ab^2 in R.

This means that $*$ carries each pair (a, b) to a unique element $a * b = ab^2$ in R.

Therefore, $*$ is a binary operation. **[1]**

(iv) On Z^+, $*$ is defined by $a * b = |a - b|$.

It is seen that for each $a, b \in Z^+$, there is a unique element $|a - b|$ in Z^+.

This means that $*$ carries each pair (a, b) to a unique element $a * b = |a - b|$ in Z^+.

Therefore, $*$ is a binary operation. **[1]**

(v) On Z^+, $*$ is defined by $a * b = a$.

It is seen that for each $a, b \in Z^+$, there is a unique element a in Z^+.
This means that $*$ carries each pair (a, b) to a unique element $a * b = a$ in Z^+.
Therefore, $*$ is a binary operation. **[1]**

Q. 19. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative.
(i) On Z, define $a * b = a - b$
(ii) On Q, define $a * b = ab + 1$
(iii) On Q, define $a * b = ab/2$
(iv) On $Z+$, define $a * b = 2^{ab}$
(v) On $Z+$, define $a * b = a^b$
(vi) On $R - \{-1\}$, define $a * b = a/b + 1$
[NCERT Ex. 1.4, Q. 2, Page 24]

Ans. (i) On Z, $*$ is defined by $a * b = a - b$.
It can be observed that $1 * 2 = 1 - 2 = -1$ and $2 * 1 = 2 - 1 = 1$.
$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in Z$
Hence, the operation $*$ is not commutative.
Also, we have
$(1 * 2) * 3 = (1 - 2) * 3 = -1 * 3 = -1 - 3 = -4$ $1 * (2 * 3) = 1 * (2 - 3) = 1 * -1 = 1 - (-1) = 2$
$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in Z$ Hence, the operation $*$ is not associative. **[1]**

(ii) On Q, defined by $a * b = ab + 1$
<u>**check commutativity :**</u>
$*$ is commutative if
$a * b = b * a$

$$\begin{array}{l|l} a * b & b * a \\ = ab + 1 & = ba + 1 \\ & = ab + 1 \end{array}$$

Since
$a * b = b * a \;\forall\; a, b \in Q$
$*$ is commutative.
<u>**check associativity :**</u>
$*$ is associative if
$(a * b) * c = a * (b * c)$

$$\begin{array}{l|l} (a*b)*c & a*(b*c) \\ = (ab+1)*c & = a*(bc+1) \\ = (ab+1)c+1 & = a(bc+1)+1 \\ = abc+c+1 & = abc+a+1 \end{array}$$

Since $(a * b) * c \neq a * (b * c)$
$*$ is not an associative binary operation. **[1]**

(iii) On Q, $*$ is defined by $a * b = ab/2$
<u>**check commutativity:**</u>
$*$ is commutative if
$a * b = b * a$

$$\begin{array}{l|l} a*b & b*a \\ = \dfrac{ab}{2} & = \dfrac{ba}{2} \\ & = \dfrac{ab}{2} \end{array}$$

Since
$a * b = b * a \;\forall\; a, b \in Q$
$*$ is commutative.
check associativity :
$*$ is associative if
$(a * b) * c = a * (b * c)$

$$\begin{array}{l|l} (a*b)*c & a*(b*c) \\ = \left(\dfrac{ab}{2}\right)*c & = a*\left(\dfrac{bc}{2}\right) \\ = \dfrac{\frac{ab}{2}\times c}{2} & = \dfrac{a\times\frac{bc}{2}}{2} \\ = \dfrac{abc}{4} & = \dfrac{abc}{4} \end{array}$$

Since $(a * b) * c = a * (b * c) \;\forall\; a, b, c \in Q$
$*$ is an associative binary operation. **[1]**

(iv) On Z^+, $*$ is defined by $a * b = 2^{ab}$.
$a * b = b * a$
check commutativity :
$*$ is commutative if

$$\begin{array}{l|l} a*b & b*a \\ = 2^{ab} & = 2^{ba} \\ & = 2^{ab} \end{array}$$

Since
$a * b = b * a \;\forall\; a, b, c \in Z^+$
$*$ is commutative
check associativity :
$*$ is associative if
$(a * b) * c = a * (b * c)$

$$\begin{array}{l|l} (a*b)*c & a*(b*c) \\ = (2^{ab})*c & = a*(2^{bc}) \\ = 2^{2^{ab}c} & = 2^{a2^{bc}} \end{array}$$

Since $(a * b) * c \neq a * (b * c)$
$*$ is not an associative binary operation. **[1]**

(v) On Z^+, $*$ is defined by $a * b = a^b$.
<u>**Check commutativity:**</u>
$*$ is commutative if
$a * b = b * a$

$$\begin{array}{l|l} a*b & b*a \\ = a^b & = b^a \end{array}$$

Since
$a * b \neq b * a$
$*$ is not commutative.
<u>**Check associativity:**</u>
$*$ is associative if
$(a * b) * c = a * (b * c)$

$$\begin{array}{l|l} (a*b)*c & a*(b*c) \\ = (a^b)*c & = a*(2^{bc}) \\ = (a^b)^c & = 2^{a2^{bc}} \end{array}$$

<u>**Example**</u>
Let a = 2, b = 3, c = 4

$$\begin{array}{l|l} (a*b)*c & a*(b*c) \\ = (2*3)*4 & = 2*(3*4) \\ = (2^3)*4 & = 2*(3^4) \\ = 8*4 & = 2*81 \\ = 8^4 & = 2^{81} \end{array}$$

Since $(a * b) * c \neq a * (b * c)$
$*$ is not an associative binary operation. **[½]**

(vi) On R, $* - \{-1\}$ is defined by $a * b = a/b+1$
<u>**check commutativity :**</u>
$*$ is commutative if

$a * b = b * a$

$a*b$	$b*a$
$=\dfrac{a}{b+1}$	$=\dfrac{b}{a+1}$

Since

$a * b \neq b * a$

$*$ is not commutative.

check associativity :

$*$ is associative if

$(a * b) * c = a *(b * c)$

$(a*b)*c$	$a*(b*c)$
$=\left(\dfrac{a}{b+1}\right)*c$	$=a*\left(\dfrac{b}{c+1}\right)$
$=\dfrac{\frac{a}{b+1}}{c}$	$=\dfrac{a}{\frac{b}{c+1}}$
$=\dfrac{a}{c(b+1)}$	$=\dfrac{a(c+1)}{b}$

Since $(a * b) * c \neq a * (b * c)$

$*$ is not an associative binary operation. **[½]**

Q. 20. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $*$ commutative?

(iii) Is $*$ associative?

(iv) Find the identity of $*$ in N.

(v) Which elements of N are invertible for the operation $*$? [NCERT Ex. 1.4, Q. 6, Page 25]

Ans. The binary operation $*$ on N is defined as $a * b$ = L.C.M. of a and b.

(i) $5 * 7$ = L.C.M. of 5 and 7 = 35

$20 * 16$ = L.C.M of 20 and 16 = 80 **[1]**

(ii) It is known that,

L.C.M of a and b = L.C.M of b and a for all $a, b \in N$.

$\therefore a * b = b * a$

Thus, the operation $*$ is commutative. **[1]**

(iii) For $a, b, c \in N$, we have

$(a * b) * c$ = (L.C.M of a and 6) $* c$ = LCM of a, b and c

$a * (b * c) = a *$ (LCM of b and c) = L.C.M of a, b and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation $*$ is associative. **[1]**

(iv) It is known that,

L.C.M. of a and 1 = a = L.C.M. 1 and a for all $a \in N$.

$\Rightarrow a * 1 = a = 1 * 6$ for all $a \in N$.

Thus, 1 is the identity of $*$ in N. **[1]**

(v) An element a in N is invertible with respect to the operation $*$ if there exists an element b in N, such that $a * b = e = b * a$.

Here, $e = 1$

This means that,

L.C.M of a and b = 1 = L.C.M of b and a

This case is possible only when a and b are equal to 1.

Thus, 1 is the only invertible element of N with respect to the operation $*$. **[1]**

Q. 21. Let $*$ be a binary operation on the set Q of rational numbers as follows:

(i) $a * b = a - b$ **(ii) $a * b = a^2 + b^2$**

(iii) $a * b = a + ab$ **(iv) $a * b = (a - b)^2$**

(v) $a * b = ab/4$ **(vi) $a * b = ab^2$**

Find which of the binary operations are commutative and which are associative.

[NCERT Ex. 1.4, Q. 9, Page 25]

Ans. (i) On Q, the operation $*$ is defined as $a * b = a - b$.

It can be observed that : for $2, 3, 4 \in Q$

$2 * 3 = 2 - 3 = -1$ and $3 * 2 = 3 - 2 = 1$.

$2 * 3 \neq 3 * 2$

Thus, the operation $*$ is not commutative.

It can also be observed that:

$(2 * 3)*4 = (-1)*4 = -1 -4 = -5$ and $2*(3 * 4) = 2*(-1) = 2-(-1) = 3$.

$(2 * 3)*4 \neq 2*(3 * 4)$

Thus, the operation $*$ is not associative. **[1]**

(ii) On Q, the operation $*$ is defined as $a * b = a^2 + b^2$.

For $a, b \in Q$, we have:

$a * b = a^2 + b^2 = b^2 + a^2 = b * a$

Therefore, $a * b = b * a$

Thus, the operation $*$ is commutative.

It can be observed that:

$(1 * 2) * 3 = (1^2 + 2^2)*3 = (1 + 4)*4 = 5 * 4 = 5^2 + 4^2 = 41$

$1*(2 * 3) = 1*(2^2 + 3^2) = 1*(4 + 9) = 1*13 = 1^2 + 13^2 = 169$

$\therefore (1 * 2)*3 \neq 1 * (2 * 3)$; where $1, 2, 3 \in Q$

$1^2 + 13^2 = 170$

Thus, the operation $*$ is not associative. **[1]**

(iii) On Q, the operation $*$ is defined as $a * b = a + ab$.

It can be observed that:

$1 * 2 = 1 + 1 \times 2 = 1 + 2 = 3$

$2 * 1 = 2 + 2 \times 1 = 2 + 2 = 4$

$\therefore 1 * 2 \neq 2 * 1$: where $1, 2 \in Q$

Thus, the operation $*$ is not commutative.

It can also be observed that:

$(1 * 2)*3 = (1+1 \times 2)*3 = 3*3 = 3 + 3 \times 3 = 3 + 9 = 12$

$1*(2 * 3) = 1*(2 + 2 \times 3) = 1 * 8 = 1 + 1 \times 8 = 9$

$\therefore (1*2) *3 \neq 1 * (2*3)$; where $1, 2, 3 \in Q$

Thus, the operation $*$ is not associative. **[1]**

(iv) On Q, the operation $*$ is defined by $a * b = (a - b)^2$.

For $a, b \in Q$, we have:

$a * b = (a - b)^2$

$b * a = (b - a)^2 = [-(a - b)]^2 = (a - b)^2$

Therefore, $a * b = b * a$

Thus, the operation $*$ is commutative.

It can be observed that:

$(1 * 2)*3 = (1 - 2)^2 *3 = (-1)^2 *3 = 1*3 = (1 - 3)^2 = (-2)^2 = 4$

$1*(2 * 3) = 1 *(2 - 3)^2 = 1* (-1)^2 = 1*1 = (1-1)^2 = 0$

$\therefore (1 * 2)*3 \neq 1*(2 * 3)$; where $1, 2, 3 \in Q$

Thus, the operation $*$ is not associative. **[½]**

(v) On Q, the operation $*$ is defined as $a * b = \dfrac{ab}{4}$.

For $a, b \in Q$, we have:

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

Therefore, $a * b = b * a$

Thus, the operation $*$ is commutative.

For $a, b, c \in Q$, we have:

$$(a*b)*c=\frac{ab}{4}*c=\frac{\frac{ab}{4}\cdot c}{4}=\frac{abc}{16}$$

$$a*(b*c)=a*\frac{bc}{4}=\frac{a\cdot\frac{bc}{4}}{4}=\frac{abc}{16}$$

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation $*$ is associative. **[½]**

(vi) On Q, the operation $*$ is defined as $a * b = ab^2$

It can be observed that for 2, 3 $\in$ Q,:

$2 * 3 = 2.3^2 = 18$ and $3 * 2 = 3.2^2 = 12$

Hence $2 * 3 \neq 3 * 2$

$$\frac{1}{2}*\frac{1}{3}=\frac{1}{2}\cdot\left(\frac{1}{3}\right)^2$$
$$=\frac{1}{2}\cdot\frac{1}{9}$$
$$=\frac{1}{18}$$

$$\therefore \frac{1}{2}*\frac{1}{3}\neq\frac{1}{3}*\frac{1}{2};\ \text{where } \frac{1}{2},\frac{1}{3}\in Q$$

Thus, the operation $*$ is not commutative.

It can also be observed that for 1, 2, 3 $\in Q$:

$(1 * 2) * 3 = (1.2^2)*3 = 4 * 3 = 4.3^2 = 36$

$1*(2 * 3) = 1*(2.3^2) = 1*18 = 1.18^2 = 324$

$(1 * 2)*3 \neq 1*(2 *3)$

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}$$
$$=\left[\frac{1}{2}\cdot\left(\frac{1}{3}\right)^2\right]*\frac{1}{4}$$
$$=\frac{1}{18}*\frac{1}{4}$$
$$=\frac{1}{18}\cdot\left(\frac{1}{4}\right)^2$$
$$=\frac{1}{18\times16}$$

$$\therefore\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right);\ \text{where } \frac{1}{2},\frac{1}{3},\frac{1}{4}\in Q$$

Thus, the operation $*$ is not associative. Hence, the operations defined in (ii), (iv) and (v) are commutative and the operation defined in (v) is associative. **[½]**

Q. 22. Find which of the operations given above has identity. [NCERT Ex. 1.4, Q. 10, Page 25]

Ans. An element $e \in Q$ will be the identity element for the operation $*$

If $a * e = a = e * a$, for all $a \in$ Q.

(i) $a * b = a - b$

This operation is not commutative. Hence it does not have identity element.

(ii) $a * b = a^2 + b^2$

If $a * e = a$, then $a^2 + e^2 = a$. for $a = -2$, $(-2)^4 + e^2 = 4 + e^2 \neq -2$.

Hence there is no identity element.

(iii) $a * b = a + ab$

This operation is not commutative. Hence it does not have identity element.

(iv) $a * b = (a - b)^2$

If $a * e = a$, then $(a - e)^2 = a$. A square is always positive, so for $a = -2$, $(-2 -e)^2 \neq -2$.

Hence there is no identity element.

(v) $a * b = ab/4$

If $a * e = a$, then $ab/4 = a$, hence $e = 4$ is the identity element.

$a * 4 = 4 * a = 4a/ 4 = a$.

(vi) $a * b = ab^2$

This operation is not commutative. Hence it does not have identity element.

Therefore only (v) has an identity element.

However, there is no such element $e \in Q$ with respect to each of the six operations satisfying the above condition. Thus, none of the six operations has identity. **[5]**

Q. 23. Let f : $W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers. [NCERT Misc. Ex. Q. 2, Page 29]

Ans. It is given that,

$$f: W\rightarrow \text{is defined as } f(n)=\begin{cases} n-1, & \text{if } n \text{ is odd}\\ n+1, & \text{if } n \text{ is even}\end{cases}$$

For one-one function,

Let $f(n) = f(m)$.

It can be observed that if n is odd and m is even, then we will have $n - 1 = m + 1$.

$\Rightarrow n - m = 2$

However, this is impossible.

Similarly, the possibility of n being even and m being odd can also be ignored under a similar argument.

$\therefore$ Both n and m must be either odd or even. Now, if both n and m are odd,

Then, we have

$f(n) = f(m)$

$\Rightarrow n - 1 = m - 1$

$\Rightarrow \quad n = m$

Again, if both n and m are even,

$f(n) = f(m)$

$\Rightarrow n + 1 = m + 1$

$\Rightarrow \quad n = m$

$\therefore f$ is one-one function.

For onto function,

It is clear that any odd number $2r + 1$ in co-domain N is the image of $2r$ in domain N and any even number $2r$ in co-domain N is the image of $2r + 1$ in domain N.

$\therefore f$ is onto function.

Hence, f is an invertible function. **[2½]**

Let us define,

$$g: W\rightarrow W \text{ as } g(m)=\begin{cases} m+1, & \text{if } m \text{ is even}\\ m-1, & \text{if } m \text{ is odd}\end{cases}$$

Now, when n is odd function.

$(n)=g(f(n))=g(n-1)=n-1+1=n$ and

When n is even $(n)=g(f(n))=g(n+1)=n+1-1=n$

Similarly,

When m is odd function.

$(m) = f(g(m))=f(m-1)=m-1+1 = m$ and

When m is even $(m)=f(g(m))=f(m+1)=m+1-1=m$

$\therefore g\ o\ f = I_W$ and $f\ o\ g = I_W$

Thus, f is invertible and the inverse of f is given by $f^{-1} = g$, which is the same as f. Hence, the inverse of f is f itself. **[2½]**

Q. 24. Show that function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = x/1+|x|$, $x \in R$ is one-one and onto function. [NCERT Misc. Ex. Q. 4, Page 29]

Ans. It is given that $f : R \to \{x \in R : -1 < x < 1\}$ is defined as $f(x) = \dfrac{x}{1+|x|}, x \in R$

Suppose $f(x) = f(y)$, where $x, y \in R$.

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

It can be observed that if x is positive and y is negative, then we have:

$$\frac{x}{1+x} = \frac{y}{1-y}$$

$$\Rightarrow \; 2xy = x - y$$

Since x is positive and y is negative:
$x > y \Rightarrow x - y > 0$
But, $2xy$ is negative.
Then, $2xy \neq x - y$.
Thus, the case of x being positive and y being negative can be ruled out.
Under a similar argument, x being negative and y being positive can also be ruled out.
$\therefore$ x and y have to be either positive or negative. **[2½]**
When x and y are both positive, we have

$$f(x) = f(y)$$
$$\Rightarrow \quad \frac{x}{1+x} = \frac{y}{1+y}$$
$$\Rightarrow \quad x + xy = y + xy$$
$$\Rightarrow \quad x = y$$

When x and y are both negative, we have

$$f(x) = f(y)$$
$$\Rightarrow \quad \frac{x}{1-x} = \frac{y}{1-y}$$
$$\Rightarrow \; x - xy = y - xy$$
$$\Rightarrow \quad x = y$$

Therefore, f is one-one function.
Now, let $y \in R$ such that $-1 < y < 1$.

If y is negative, then there exists $x = \dfrac{y}{1+y} \in R$ such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\dfrac{y}{1+y}\right)}{1+\left|\dfrac{y}{1+y}\right|} = \frac{\dfrac{y}{1+y}}{1+\left(\dfrac{-y}{1+y}\right)} = \frac{y}{1+y-y} = y$$

If y is positive, then there exists $x = \dfrac{y}{1-y} \in R$ such that

$$f(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\dfrac{y}{1-y}\right)}{1+\left|\dfrac{y}{1-y}\right|} = \frac{\dfrac{y}{1-y}}{1+\dfrac{y}{1-y}} = \frac{y}{1-y+y} = y$$

Therefore, f is onto function.
Hence, f is one-one function and onto function. **[2½]**

Q. 25. Let $A = [-1, 1]$. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

(i) $f(x) = \dfrac{x}{2}$ **(ii)** $g(x) = |x|$

(iii) $h(x) = x|x|$ **(iv)** $k(x) = x^2$

[NCERT Exemp. Ex. 1.3, Q. 21, Page 12]

Ans. Given that,
$$A = [-1, 1]$$

(i) $f(x) = \dfrac{x}{2}$

Let $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1}{2} = \frac{x_2}{2}$$
$$\Rightarrow \quad x_1 = x_2$$

So, $f(x)$ is one-one function.

Now, let $y = \dfrac{x}{2}$

$\Rightarrow x = 2y \notin A, \forall\, y \in A$
As for $y = 1 \in A$, $x = 2 \notin A$
So, $f(x)$ is not onto function.
Also, $f(x)$ is not bijective as it is not onto function. **[1½]**

(ii) $g(x) = |x|$

Let $g(x_1) = g(x_2)$
$$|x_1| = |x_2|$$
$$x_1 = \pm x_2$$

So, $g(x)$ is not one-one function.
Now, $y = |x| \Rightarrow x = \pm y \notin A, \forall y \in A$

So, $g(x)$ is not onto function, and also, $g(x)$ is not bijective. **[1]**

(iii) $h(x) = x|x|$

$$\Rightarrow x_1|x_1| = x_2|x_2|$$
$$\Rightarrow \quad x_1 = x_2$$

So, $h(x)$ is one-one function.
Now, let $y = x|x|$

$\Rightarrow y = x \;\in A \;\forall x \in A$
So, $h(x)$ is onto function also, $h(x)$ is a bijective. **[1]**

(iv) $k(x) = x^2$

Let $k(x_1) = k(x^2)$

$$\Rightarrow \quad x_1^2 = x_2^2$$
$$\Rightarrow \quad \pm x_1 = \pm x_2$$

Thus, $k(x)$ is not one-one function.

Now, let $y = x^2$

$\Rightarrow x = \sqrt{y} \notin A, \forall y \in A$

As for $y = -1, x = \sqrt{-1} \notin A$

Hence, $k(x)$ is neither one-one function nor onto function. **[1½]**

Q. 26. Each of the following defines *a* relation on *N*:

(i) x is greater than y, $x, y \in N$

(ii) $x + y = 10$, $x, y \in N$

(iii) xy is square of an integer $x, y \in N$

(iv) $x + 4y = 10$ $x, y \in N$.

Determine which of the above relations are reflexive, symmetric and transitive.

[NCERT Exemp. Ex. 1.3, Q. 22, Page 12]

Ans. (i) x is greater than y; $x, y \in N$

If $(x, x) \in R$, then $x > x$, which is not true for any $x \in N$.

Let $(x, y) \in R$

$\Rightarrow xRy$

$\Rightarrow x > y$

$\Rightarrow y > x$, which is not true for any $x, y \in N$

Thus, R is not symmetric.

Let xRy and yRz

$\Rightarrow x > y$ and $y > z$

$\Rightarrow x > z$

$\Rightarrow xRz$

So, R is transitive. **[1½]**

(ii) $x + y = 10$; $x\ y \in N$

$\therefore R = \{(x, y); x + y = 10, x, y \in N\}$

$\therefore R = \{(1, 9), (2, 8), (3, 7), (4, 6) (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$

Clearly $(1, 1) \notin R$

So, R is not reflexive.

$(x, y) \in R \Rightarrow (y, x) \in R$

Thus, R is not symmetric.

Now $(1, 9) \in R$, $(9, 1) \in R$, but $(1, 1) \notin R$

Hence, R is not transitive. **[1]**

(iii) Given xy, is square of an integer $x, y \in N$

$\therefore R = \{(x, y) : xy$ is a square of an integer $x, y \in N\}$

Clearly $(x, x) \in R, \forall x \in N$

As x^2 is square of an integer for any $x \in N$

Hence, R is reflexive.

If $(x, y) \in R \Rightarrow (y, x) \in R$

So, R is symmetric

Now if xy is square of an integer and yz is square of an integer.

Then, let $xy = m^2$ and $yz = n^2$ for some $m, n \in Z$

$$\Rightarrow x = \frac{m^2}{y} \text{ and } z = \frac{x^2}{y}$$

$$\Rightarrow xz = \frac{m^2n^2}{y^2}, \text{ which is square of an integer.}$$

So, R is transitive. **[1½]**

(iv) $x + 4y = 10$; $x, y \in N$

$R = \{(x, y) : x + 4y = 10; x, y \in N\}$

$\therefore R = \{(2, 2), (6, 1)\}$

Clearly $(1, 1) \notin R$

Thus, R is not reflexive.

$(6, 1) \in R$ but $(1, 6) \notin R$

Hence, R is not symmetric.

$(x, y) \in R \Rightarrow x + 4y = 10$

And $(y, z) \in R$

$\Rightarrow y + 4z = 10$

$\Rightarrow x - 16z = -30$

$(x, z) \notin R$

So, R is not transitive. **[1]**

Some Commonly Made Errors

- Generally, students get confused to find the domain of a variety of functions.
- Students does not define correct state of the Domain.
- Students always confuse in isolating the Variable.
- Students confuse in the domain of a function with a square root when there are multiple solutions.

EXPERT ADVICE

- *The domain of a function is the set of numbers that can go in to a given function.*
- *The domain is the full set of x-values that can be plugged into a function to produce a y-value.*
- *The set of possible y-values is called the Range.*
- *The type of function will determine the best method for finding a domain.*
- *The proper notation for the domain is easy to learn.*
- *The format for expressing the domain is an open bracket/parenthesis, followed by the two endpoints of the domain separated by a comma, followed by a closed bracket/parenthesis.*

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