Sample QuestionPaper-II (2023-24)
CLASS-XII
MATHEMATICS (041)

TIME: 3 Hours **MM.80**

General Instructions:

- This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has **18** MCQ's and **02** Assertion-Reason based questions of **1** mark each.
- **Section B** has **5** Very Short Answer (VSA)-type questions of **2** marks each.
- **Section C** has **6** Short Answer (SA)-type questions of **3** marks each.
- **Section D** has **4** Long Answer (LA)-type questions of **5** marks each.
- **Section E** has **3** source based/case based/passage based/integrated units of assessment of **4** marks each with sub-parts.

	Section –A **(Multiple Choice Questions)** **Each question carries 1 mark**
Q1.	The value of x – y + z from the following equation is $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ (a) - 3 (b) - 1 (c) 1 (d) 3
Q2.	If A be a 3 × 3 square matrix such that $A(adj\ A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then the value of $\lvert Adj\ A \rvert$ is (a) 5 (b) 25 (c) 125 (d) 625
Q3.	If A and B are symmetric matrices of same order, then $\left(AB^T - 2\,BA^T\right)$ is a (a) Skew symmetric matrix (b)Symmetric matrix (c) Neither Symmetric matrix nor Skew symmetric matrix (d) Null matrix
Q4.	In the interval (1,2) the function $f(x) = 2\lvert x-1\rvert + 3\lvert x-2\rvert$ is (a) Strictly Increasing (b) Strictly Decreasing (c) Neither Increasing nor Decreasing (d) Remains constant
Q5.	If the set A contains 5 elements and the set B contains 6 elements, then the number of both one-one and onto mapping from A to B is (a) 720 (b) 120 (c) 30 (d) 0
Q6.	The sum of order & degree of the differential equation $\frac{d^3y}{dx^3} = (1 + \frac{dy}{dx})^5$ is (a) 3 (b) 4 (c) 5 (d) 8

Q7.	The solution set of the inequation $3x+2y>3$ is (a) half plane containing the origin (b) half plane not containing the origin (c) the point being on the line $3x+2y=3$ (d) None of these
Q8.	The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is (a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$ (c) $\sqrt{18}$ (d) $\sqrt{52}$
Q9.	The value of $\int_{-\pi/2}^{\pi/2} x^3 \sin^4 x \, dx$ is (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi^2}{4}$
Q10.	If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $A^{-1}=kA$, then k is equal to (a) 19 (b) 1/19 (c) -1/19 (d) - 19
Q11.	The corner points of the feasible region for the Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let the objective function is $Z=4x+6y$ then the minimum value of the objective function occurs at (a) (0, 2) only (b) (3, 0) only (c) The mid-point on the line segment joining the points (0,2) and (3,0) (d) Any point on the line segment joining the points (0,2) and (3,0)
Q12.	If the projection of $\lambda\hat{i}+\hat{j}+4\hat{k}$ on $2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units, then the value of λ is equal to (a) - 9 (b) - 5 (c) 5 (d) 9
Q13.	If $A=\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}$ then $(AB)^{-1}$ is equal to (a) $\begin{bmatrix}15 & -19\\ -26 & 33\end{bmatrix}$ (b) $\begin{bmatrix}11 & -14\\ -29 & 37\end{bmatrix}$ (c) $\begin{bmatrix}37 & 14\\ 29 & 11\end{bmatrix}$ (d) $\begin{bmatrix}37 & -14\\ -29 & 11\end{bmatrix}$
Q14.	In a hockey match, both teams A and B scored same number of goals up to the end of this game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, then probability of B winning the match is. (a) 1 / 6 (b) 5 / 6 (c) 5/11 (d) 6 / 11

Q15. The value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{2}{5}$

Q16. The integrating factor of the differential equation $\frac{dy}{dx}\left(x\log x\right)+y=2\log x$ is

(a) e^x (b) $\log x$

(c) $\log(\log x)$ (d) x

Q17. The function $f\left(x\right)=x^x$ has a stationary point at

(a) $x = e$ (b) $x=\frac{1}{e}$

(c) $x = 1$ (d) $x=\sqrt{e}$

Q18. The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are

(a) 3, 6, 1 (b) 3, 6, -1

(c) 2, 1, 6 (d) 2, 1, -6

ASSERTION-REASON BASED QUESTIONS

The following questions consist of two statements – Assertion (A) and Reason (R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true and R is not the correct explanation for A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A)** :The Differential coefficient of $\sec\left(\tan^{-1}x\right)$ with respect to x is $\frac{x}{\sqrt{1+x^2}}$

Reason (R) :The Differential coefficient of the function with respect to $x$is the first order derivative of the function .

Q20. **Assertion (A)** : The vector equation of the line passing through the points (6,-4,5) and (3,4,1) is $\vec{r}=(6\hat{i}-4\hat{j}+5\hat{k})+\lambda(-3\hat{i}+8\hat{j}+4\hat{k})$.

Reason (R) : The vector equation of the line passing through the points $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$.

Section – B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

Q21. If $\cos^{-1}\alpha+\cos^{-1}\beta+\cos^{-1}\gamma=3\pi$, then find the value of $\alpha\left(\beta+\gamma\right)-\beta\left(\gamma+\alpha\right)+\gamma\left(\alpha+\beta\right)$.

OR

Reduce $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ where $\frac{\pi}{2}<x<\pi$ in to simplest form.

Q22. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?

OR

The volume of the cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

Q23. If $\vec{a}+\vec{b}+\vec{c}=0$, $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$ and $\left|\vec{c}\right|=7$ then find the angle between $\vec{a}$ and $\vec{b}$

Q24. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $5\,units$ from the point (1,3,3).

Q25. Find the area of the region bounded by the curve $y^2=4x$, y-axis and line $y=3$.

	Section – C **(This section comprises of short answer type questions (SA) of 3 marks each)**
Q26.	Solve the following Linear Programming Problem graphically: Minimize Z = 3x + 9y Subject to the constraints $x + 3y \leq 60$ $x + y \geq 10$ $x \leq y$ $x \geq 0, y \geq 0$.
Q27.	Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained, Find the probability distribution of X. **OR** A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occur is 1/3. Find the probability of the occurrence of A.
Q28.	Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\, dx$ **OR** Find $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\, d\phi$
Q29.	Solve the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$. **OR** Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$
Q30.	Draw a rough sketch of the curve $y = 1 + \|x+1\|$, $x = -3$, $x = 3$, $y = 0$ and find the area of the region bounded by them using integration.
Q31.	If $x = a\sin t - b\cos t, y = a\cos t + b\sin t$, then $prove\, that$ $\frac{d^2y}{dx^2} = -(\frac{x^2+y^2}{y^3})$.
	Section – D **(This section comprises of long answer type questions (LA) of 5 marks each)**
Q32.	If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1} and hence solve the system of equations $x + 2y - 3z = -4$;$2x + 3y + 2z = 14$; $3x - 3y - 4z = -15$
Q33.	Find the equations of the lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$ each. **OR** Find the equation of the line which intersect the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).
Q34.	Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\, dx$. **OR** Evaluate $\int_0^{\pi} \log(1+\cos x)\, dx$

Q35. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers. Prove that f is one- one and onto function.

Section – E

(This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Q36. The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated number of electric vehicles in use at any time t is given by the function

$$V(t) = t^3 - 3t^2 + 3t - 100$$

Where t represents time and t = 1, 2, 3, --------- corresponds to year 2021, 2022, 2023 ------- respectively.

Based on the above information answer the following:

(i) Can the above function be used to estimate number of vehicles in the year 2020? Justify.

(ii) Find the estimated number of vehicles in the year 2040.

(iii) Prove that the function V(t) is an increasing function.

Q37. Senior students tend to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the class XII.

Using above information answer the following:

(i). Find the conditional probability that a student attains A grade given that he is not 100 % regular student.

(ii) Find the probability of attaining A grade by the students of class XII

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(iii) Find the probability that student is 100% regular given that he attains A grade.

	OR Find the probability that student is irregular given that he attains A grade.
Q38.	In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meters of water.  Using above information answer the following: (i) Find the minimum surface area of the tank. (ii) Find the percentage increase in volume of the tank, if size of square base of tank become twice and height remains same.