CBSE

Additional Practice Questions
Subject: Mathematics (041)
Class: XII 2023-24

Time Allowed: 3 Hours   **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

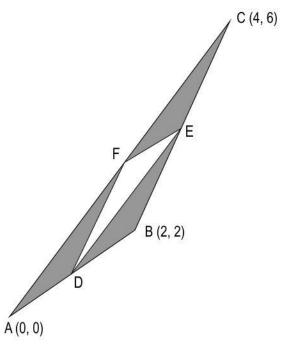| Serial No.	Question	Marks
1	For any 2×2 matrix P, which of the following matrices can be Q such that PQ = QP? (a) $\begin{bmatrix} 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) (No such matrix exists as matrix multiplication is not commutative.)	1

2   V is a matrix of order 3 such that $|\text{adj } V| = 7$.   1

Which of these could be $|V|$?

(a) 7^2
(b) 7
(c) $\sqrt{7}$
(d) $\sqrt[3]{7}$

3   The points D, E and F are the mid-points of AB, BC and CA respectively.   1

C (4, 6)

E

F

B (2, 2)

D

A (0, 0)

(Note: The figure is not to scale.)

What is the area of the shaded region?

(a) 2 sq units
(b) $\frac{3}{2}$ sq units
(c) $\frac{1}{2}$ sq units
(d) $(2\sqrt{26} - 1)$ sq units

4   If $f(x) = \cos^{-1}\sqrt{x}, 0 < x < 1$, which of the following is equal to $f'(x)$?   1

(a) $\dfrac{-1}{\sqrt{1-x}}$

(b) $\dfrac{1}{\sqrt{1-x}}$

(c) $\dfrac{1}{2\sqrt{x(1-x)}}$

(d) $\dfrac{-1}{2\sqrt{x(1-x)}}$

5. A function $f: \mathbb{R} \to \mathbb{R}$ is defined by: 1

$$f(x) = \begin{cases} e^{-2x}, & x < \ln\frac{1}{2} \\ 4, & \ln\frac{1}{2} \le x \le 0 \\ e^{-2x}, & x > 0 \end{cases}$$

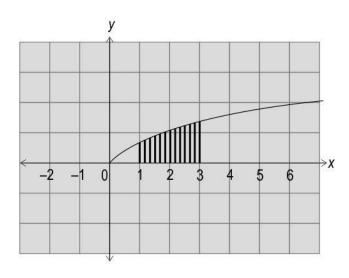Which of the following statements is true about the function at the point $x = \ln\frac{1}{2}$?

(a) $f(x)$ is not continuous but differentiable.
(b) $f(x)$ is continuous but not differentiable.
(c) $f(x)$ is neither continuous nor differentiable.
(d) $f(x)$ is both continuous as well as differentiable.

6. In which of these intervals is the function $f(x) = 3x^2 - 4x$ strictly decreasing?   1

(a) $(-\infty, 0)$
(b) $(0, 2)$
(c) $\left(\frac{2}{3}, \infty\right)$
(d) $(-\infty, \infty)$

7. Which of these is equal to $\int e^{(x\log 5)} e^x \, dx$, where C is the constant of integration?   1

(a) $\dfrac{(5e)^x}{\log 5e} + C$
(b) $\log 5^x + x + C$
(c) $5^x e^x + C$
(d) $(5e)^x \log x + C$

8   Shown below is the curve defined by the equation $y = \log(x + 1)$ for $x \geq 0$.   1

Which of these is the area of the shaded region?

(a) $6\log(2) - 2$
(b) $6\log(2) - 6$
(c) $6\log(2)$
(d) $5\log(2)$

9   In which of the following differential equations is the degree equal to its order?   1

(a) $x^3 \left(\frac{dy}{dx}\right) - \frac{d^3y}{dx^3} = 0$

(b) $\left(\frac{d^3y}{dx^3}\right)^3 + \sin\left(\frac{dy}{dx}\right) = 0$

(c) $x^2 \left(\frac{dy}{dx}\right)^4 + \sin y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$

(d) $\left(\frac{dy}{dx}\right)^3 + x \left(\frac{d^2y}{dx^2}\right) - y^3 \left(\frac{d^3y}{dx^3}\right) + y = 0$

10   Kapila is trying to find the general solution of the following differential   1
       equations.

(i) $xe^{\frac{x}{y}}dx - ye^{\frac{3x}{y}}dy = 0$

(ii) $(2x + 1)\frac{dy}{dx} = 3 - 2y$

(iii) $\frac{dy}{dx} = \sin x - \cos y$

Which of the above become variable separable by substituting $y = b.x$, where
b is a variable?

(a) only (i)
(b) only (i) and (ii)
(c) all - (i), (ii) and (iii)
(d) None of the above

11   For which of these vectors is the projection on the y-axis zero?   1

(i) $2\hat{j}$
(ii) $-5\hat{k}$
(iii) $\hat{i} - 4\hat{k}$

(a) only (i)
(b) only (ii)
(c) only (i) and (ii)
(d) only (ii) and (iii)

12   If $\left(\hat{i} + \lambda\hat{j}\right) \times \left(5\hat{i} + 3\hat{j} + \sigma\hat{k}\right) = 0$, what are the values of λ and σ?   1

(a) $\lambda = \frac{3}{5}, \sigma = 0$
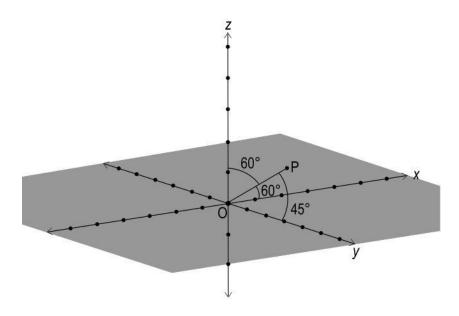(b) $\lambda = \frac{5}{3}, \sigma = 5$
(c) $\lambda = 3, \sigma = 0$
(d) (cannot be found as there are two unknowns and only one equation)

13   A line $\overrightarrow{OP}$ in space, represented by the figure below, has a magnitude of $2\sqrt{2}$ units.   1

Which of these are the direction ratios of $\overrightarrow{OP}$?

(a) $(2, \sqrt{2}, 2)$
(b) $(\sqrt{2}, 2, \sqrt{2})$
(c) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$
(d) $(2\sqrt{2}, 2\sqrt{2}, 2\sqrt{2})$

14   A line m passes through the point (-4, 2, -3) and is parallel to line n, given by: 1

$$\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$$

The vector equation of line m is given by:

$$\vec{r} = (-4\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(p\hat{i} + q\hat{j} + r\hat{k}), \text{ where } \lambda \in \mathbf{R}$$

Which of the following could be the possible values for p, q and r?

(a) $p = 4$, $q = (-2)$, $r = 3$
(b) $p = (-4)$, $q = (-2)$, $r = 3$
(c) $p = (-2)$, $q = 3$, $r = (-6)$
(d) $p = 8$, $q = 4$, $r = (-3)$

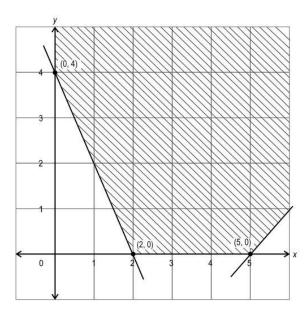15 L_1 and L_2 are two skew lines.   1

How many lines joining L₁ and L₂ can be drawn such that the line is perpendicular to both L₁ and L₂ ?

(a) exactly one
(b) exactly two
(c) infinitely many
(d) (there cannot be a line joining two skew lines such that it is perpendicular to both)

16   A linear programming problem (LPP) along with the graph of its constraints is   1
shown below. The corresponding objective function is Minimize: $Z = 3x + 2y$.
The minimum value of the objective function is obtained at the corner point (2, 0).

The optimal solution of the above linear programming problem _____.

(a) does not exist as the feasible region is unbounded.

(b) does not exist as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.

(c) exists as the inequality $3x + 2y > 6$ has infinitely many points in common with the feasible region.

(d) exists as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.

17   The feasible region of a linear programming problem is bounded. The   1
corresponding objective function is $Z = 6x - 7y$.

The objective function attains _____ in the feasible region.

(a) only minimum
(b) only maximum
(c) both maximum and minimum
(d) either maximum or minimum but not both

18   M and N are two events such that $P(M \cap N) = 0$.   1

Which of the following is equal to $P(M|(M \cup N))$?

(a) $\dfrac{P(M)}{P(N)}$

(b) $\dfrac{P(M \cup N)}{P(M \cup N)}$

(c) $\dfrac{P(M)}{P(M) + P(N)}$

(d) $\dfrac{P(M)}{P(M) \times P(N)}$

19   $X = \{0, 2, 4, 6, 8\}$.   1
P is a relation on X defined by $P = \{(0, 2), (4, 2), (4, 6), (8, 6), (2, 4), (0, 4)\}$.

Based on the above information, two statements are given below - one labelled
Assertion (A) and the other labelled Reason (R). Read the statements carefully
and choose the option that correctly describes statements (A) and (R).

Assertion (A): The relation P on set X is a transitive relation.

Reason (R): The relation P has a subset of the form $\{(a, b), (b, c), (a, c)\}$,
where $a, b, c \in X$.

(a) Both (A) and (R) are true and (R) is the correct explanation for (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation for (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

20   Two statements are given below - one labelled Assertion (A) and the other   1
labelled Reason (R). Read the statements carefully and choose the option that
correctly describes statements (A) and (R).

Assertion (A): The maximum value of the function $f(x) = x^5, x \in [-1, 1]$, is attained at its critical point, $x = 0$.

Reason (R): The maximum of a function can only occur at points where derivative is zero.

(a) Both (A) and (R) are true and (R) is the correct explanation for (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation for (A).
(c) (A) is false but (R) is true.
(d) Both (A) and (R) are false.

SECTION B
(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

Serial No.	Question	Marks
21	Find the domain of the function $y = \cos^{-1}(|x - 1|)$. Show your steps.	2
	OR	
	Draw the graph of the following function:	2
	$y = 2\sin^{-1}(x)$, $-\pi \leq y \leq \pi$	
22	The sum of a matrix and its transpose is $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$. Find one such matrix for which this holds true. Show your work.	2
23	If $x = \cot t$ and $y = \csc^2 t$, find: i) $\frac{dy}{dx}$ ii) $\frac{d^2y}{dx^2}$	2

Show your steps.

24 Iqbal, a data analyst in a social media platform is tracking the number of active   2
    users on their site between 5 pm and 6 pm on a particular day.

    The user growth function is modelled by $N(t) = 1000e^{0.1t}$, where $N(t)$
    represents the number of active users at time t minutes during that period.

    Find how fast the number of active users are increasing or decreasing at 10
    minutes past 5 pm. Show your steps.

    <center>OR</center>

    The population of rabbits in a forest is modelled by the function below: 2

    $P(t) = \frac{2000}{1+e^{-0.5t}}$, where P represents the population of rabbits in t years

    Determine whether the rabbit population is increasing or not, and justify your
    answer.

25 Solve the integral: 2

    $I = \int x(k - x)^{23} dx$, where k is a constant

    Show your steps.

<center>

SECTION C
(This section comprises of short answer type questions (SA) of 3 marks each)

</center>

Serial No.	Question	Marks
		3
26		

Solve the integral:

$$I = \int \frac{3x+5}{x^2+4x+7}dx$$

Show your work.

27   Evaluate the integral: 3

$$\int_0^{\frac{\pi}{2}} \frac{\sin\theta\ d\theta}{(25+\cos\theta)(26+\cos\theta)}$$

Show your steps.

<div align="center">OR</div>

Using the properties of definite integrals, prove the following: 3

$$\int_0^\pi h(\sin x)\ dx = 2\int_0^{\frac{\pi}{2}} h(\sin x)dx,\ \text{where}\ h(\sin x)\ \text{is a function of}\ \sin x.$$

State the property used.

28   When an object is thrown vertically upward, it is under the effect of gravity   3
      and air resistance. For small objects, the force due to air resistance is
      numerically equal to some constant k times v, where v is the velocity of the
      object (in m/s) at time t (s).

      This situation can be modelled as the differential equation shown below.

$$m\frac{dv}{dt} = -F_R - mg$$

where,
m is the mass of the object in kg.
$\frac{dv}{dt}$ is the acceleration of the object in m/s^2.
F_R is the force due to air resistance.
g is the acceleration due to gravity (10 m/s^2).

A tennis ball of mass 0.050 kg is hit upwards with a velocity of 10 m/s. An air
resistance numerically equal to $0.4v$ acts on the ball.

(i) Model the above situation using a differential equation.
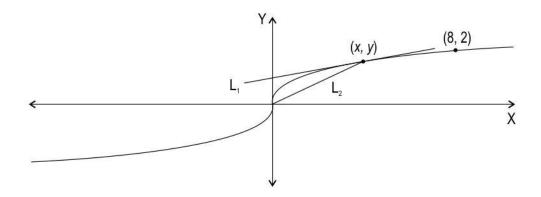(ii) Write an expression for the velocity of the ball in terms of the time.

Show your work.

29   Shown below is a curve.   3

L₁ is the tangent to any point (x, y) on the curve.
L₂ is the line that connects the point (x, y) to the origin.

The slope of L₁ is one third of the slope of L₂.

Find the equation of the curve. Show your work.

<div align="center">OR</div>

Given $x + (y + 1)\frac{dy}{dx} = 2$.   3

(i) Solve the differential equation and show that the solution represents a family of circles.

(ii) Find the radius of a circle belonging to the above family that passes through the origin.

Show your work.

30   Each unit of Product A that a company produces, is sold for Rs 100 with a   3
production cost of Rs 60 and each unit of Product B is sold for Rs 150 with
production cost of Rs 90. On a given day, the company has a budget of Rs
8000 to spend on production. The production process makes it such that they
can only produce a maximum of 100 units each day. Also, the number of
product B produced cannot be more than twice as many of Product A.

Frame a linear programming problem to determine how many units of
Product A and B should the company produce in a day in order to maximize
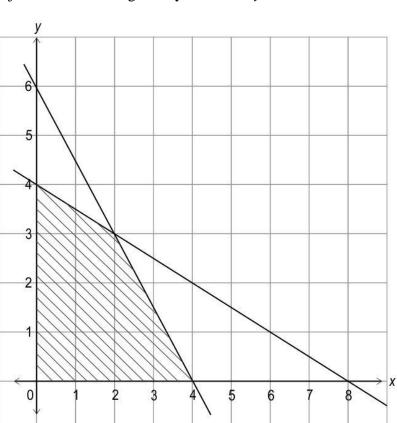their profit?

(Note: No need to find the feasible region and optimal solution.)

OR

Shown below is the feasible region of a maximisation problem whose objective function is given by $Z = 5x + 3y$.   3

i) List all the constraints the problem is subjected to.
ii) Find the optimal solution of the problem.

Show your work.

31   A company follows a model of bifurcating the tasks into the categories shown below.   3

	URGENT	NOT URGENT
IMPORTANT	urgent and important	not urgent but important
NOT IMPORTANT	urgent but not important	not urgent and not important

At the beginning of a financial year, it was noticed that:

♦ 40% of the total tasks were urgent and the rest were not.
♦ half of the urgent tasks were important, and
♦ 30% of the tasks that were not urgent, were not important

What is the probability that a randomly selected task that is not important is urgent? Use Bayes' theorem and show your steps.

SECTION D
(This section comprises of long answer-type questions (LA) of 5 marks each)

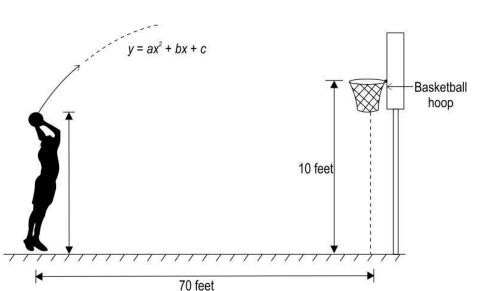| Serial No.	Question	Marks
32	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone. A relation R is defined on the set U = {All people on the Earth} such that R = {(x, y)| the time difference between the time zones x and y reside in is 6 hours}. i) Check whether the relation R is reflexive, symmetric and transitive. ii) Is relation R an equivalence relation? Show your work.	5

A function $f: R - \{-1, 1\} \to R$ is defined by: 5

$$f(x) = \frac{x}{x^2 - 1}$$

i) Check if f is one-one.
ii) Check if f is onto.

Show your work.

33   Abdul threw a basketball in the direction of the basketball hoop which   5
traversed a parabolic path in a vertical plane as shown below.

(Note: The image is for representation purpose only.)

The equation of the path traversed by the ball is $y = ax^2 + bx + c$ with respect
to a xy-coordinate system in the vertical plane. The ball traversed through the
points $(10, 16)$, $(20, 22)$ and $(30, 25)$. The basketball hoop is at a horizontal
distance of 70 feet from Abdul. The height of the basketball hoop is 10 feet
from the floor to the top edge of the rim.

Did the ball successfully go through the hoop? Justify your answer.

*(Hint: Consider the point where Abdul is standing as the origin of the
xy-coordinate system.)*

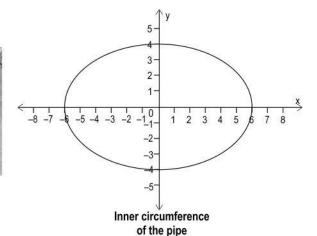34   Shown below are concrete elliptical water pipes, each 10 feet in length.   5

Concrete elliptical pipes

Inner circumference
of the pipe

The graph given above represents the inner circumference of the elliptical pipe, where *x* and *y* are in feet. Assume that the water flows uniformly and fully covers the inner cross-sectional area of the pipe.

Find the volume of water in the pipe at a given instant of time, in terms of π. Use the integration method and show your steps.

(Note: Volume = Area of the base × Height)

35	i) Find the vector and cartesian equations of the straight line passing through the point (-5, 7, -4) and in the direction of (3, -2, 1).	5

ii) Find the point where this straight line crosses the *xy*-plane.

Show your work.

<div align="center">OR</div>

Given below are two lines L_1 and L_2:   5

L_1: $2x = 3y = -z$
L_2: $6x = -y = -4z$

i) Find the angle between the two lines.
ii) Find the shortest distance between the two lines.

Show your work.
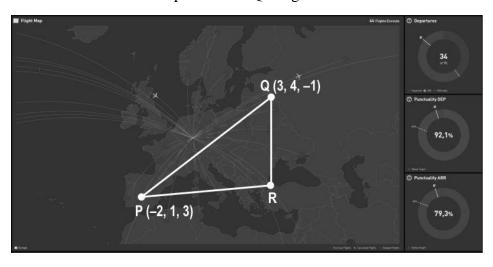
SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

Serial No.	Question	Marks

36 **Answer the questions based on the given information.**

The flight path of two airplanes in a flight simulator game are shown below. The coordinates of the airports P and Q are given.

Airplane 1 flies directly from P to Q.

Airplane 2 has a layover at R and then flies to Q.

The path of Airplane 2 from P to R can be represented by the vector $5\hat{i} + \hat{j} - 2\hat{k}$.

(Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)

i) Find the vector that represents the flight path of Airplane 1. Show your steps.   1

ii) Write the vector representing the path of Airplane 2 from R to Q. Show your steps.   1

iii) What is the angle between the flight paths of Airplane 1 and Airplane 2 just after takeoff? Show your work. 2
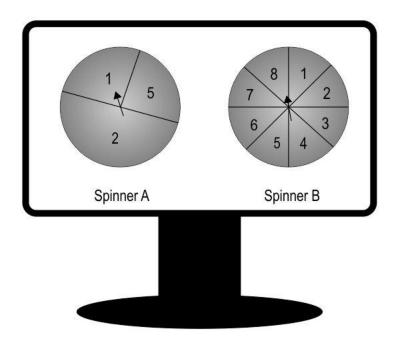
<div align="center">OR</div>

iii) Consider that Airplane 1 started the flight with a full fuel tank. 2

Find the position vector of the point where a third of the fuel runs out if the entire fuel is required for the flight. Show your work.

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37   **Answer the questions based on the given information.**

Rubiya, Thaksh, Shanteri, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both:

Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

♦ Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!
♦ You win a photo frame if Spinner A lands on a value greater than that of Spinner B!

i) Thaksh spun both the spinners, A and B in one of his turns.   1

What is the probability that Thaksh wins a music player in that turn? Show your steps.

ii) Lilly spun spinner B in one of her turns.   1

What is the probability that the number she got is even given that it is a multiple of 3? Show your steps.

iii) Rubiya spun both the spinners.   2

What is the probability that she wins a photo frame? Show your work.

OR

iii) As Shanteri steps up to the screen, the game administrator reveals that for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%.   2

What is the probability that Shanteri gets the number '2'? Show your steps.

38   **Answer the questions based on the given information.**

Two metal rods, R_1 and R_2, of lengths 16 m and 12 m respectively, are insulated at both the ends. Rod R_1 is being heated from a specific point while rod R_2 is being cooled from a specific point.

The temperature (T) in Celsius within both rods fluctuates based on the distance (x) measured from either end. The temperature at a particular point along the rod is determined by the equations $T = (16 - x)x$ and $T = (x - 12)x$ for rods R_1 and R_2 respectively, where the distance x is measured in meters from one of the ends.

i) Find the rate of change of temperature at the mid point of the rod that is being heated. Show your steps.   2

ii) Find the minimum temperature attained by the rod that is being cooled. Show your work.   2