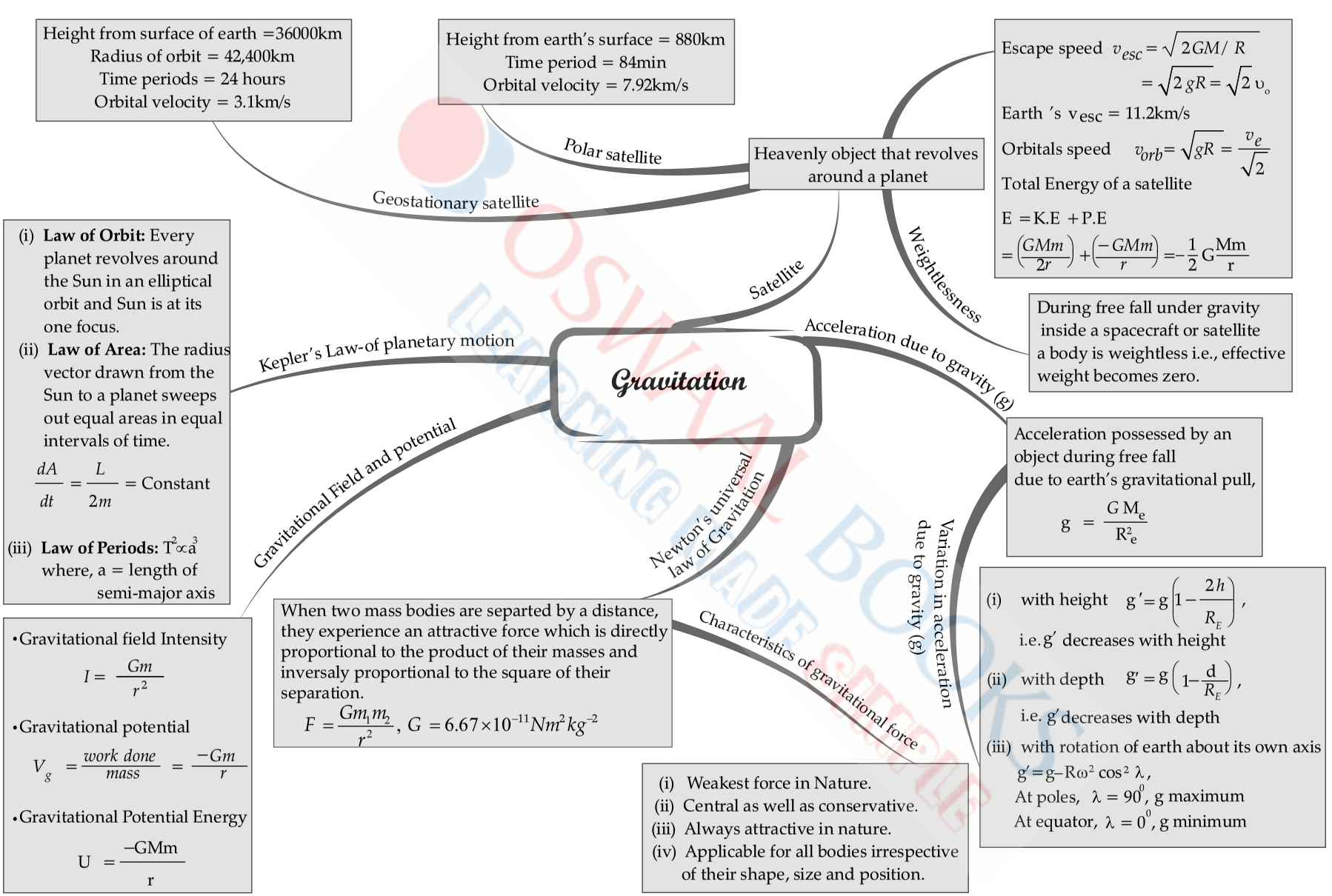


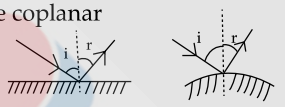
Optics Part-1

Sign Convention

- Pole is taken as origin
- Principle axis as the X-axis
- All distance are measured from origin (or pole)
- All distance measured in the direction of incident ray are taken + ve.
- All distance measured in the direction opposite to the incident ray are taken - ve.

Reflection of Light

- $\angle i = \angle r$
- Incident ray reflected ray and normal to the reflecting surface are coplanar

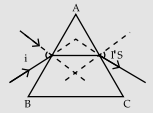


Prism

Angle of deviation $\delta = i - i' - A$

$\delta_{minimum} = 2i - A \quad [i = i']$

$\delta_{minimum} = (\mu - 1)A$, if A is small



Scattering of Light

Light scattered i.e. redirected in different paths when interacts with particle
e.g. sunset, sunrise, colours, blue colour of sky

Spherical Mirror

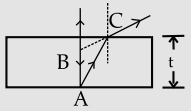
$$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}=\frac{1}{f}$$

Lateral Magnification $= \frac{h_2}{h_1} = -\frac{v}{u}$

Refraction of light

$$\frac{\sin i}{\sin r}=\frac{v_1}{v_2}=\frac{\mu_2}{\mu_1}$$

$$\mu = \frac{real\ depth}{apparent\ depth}$$



$\Delta t = \left(1-\frac{1}{\mu}\right)t$ = image shift

Optical Instruments

Microscope

$M = 1 + \frac{D}{f}$ [image at near point]

$M = D / f$ [image at infinite]

Compound Microscope

$M = \frac{v}{u}\left[\frac{D}{f_e}\right]$ [normal adjustment]

$M = \frac{v}{u}\left(1+\frac{D}{f_e}\right) = -\frac{1}{f_o}\left(1+\frac{D}{f_e}\right)$

For final image at least distance

Telescope

$M = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$ [image at near point]

$M = -\frac{fo}{fe}$ [image at infinite]

Total internal refection

When ray passes from optically denser to rarer medium, if incident angle (i) further increased till $(\theta)_c$ critical angle, entire light is then reflected back to the denser medium again, this process is called T.I.R. It is used in optical fibre.

Critical Angle

- Incident angle (θ_c) for which angle of refraction is 90°

i.e. $\sin\theta_c = 1/\mu$

$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

When ray passes from optically denser to rarer medium.

Refraction on Spherical surface

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

Lateral Magnification

$$m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u} = \frac{R-v}{R-u}$$

Lens Maker's Formula

$$\frac{1}{v}-\frac{1}{u}=\frac{\mu_2-\mu_1}{\mu_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad [If\ \mu_2=\mu, \mu_1=1\ (air)]$$

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ (lens formula)

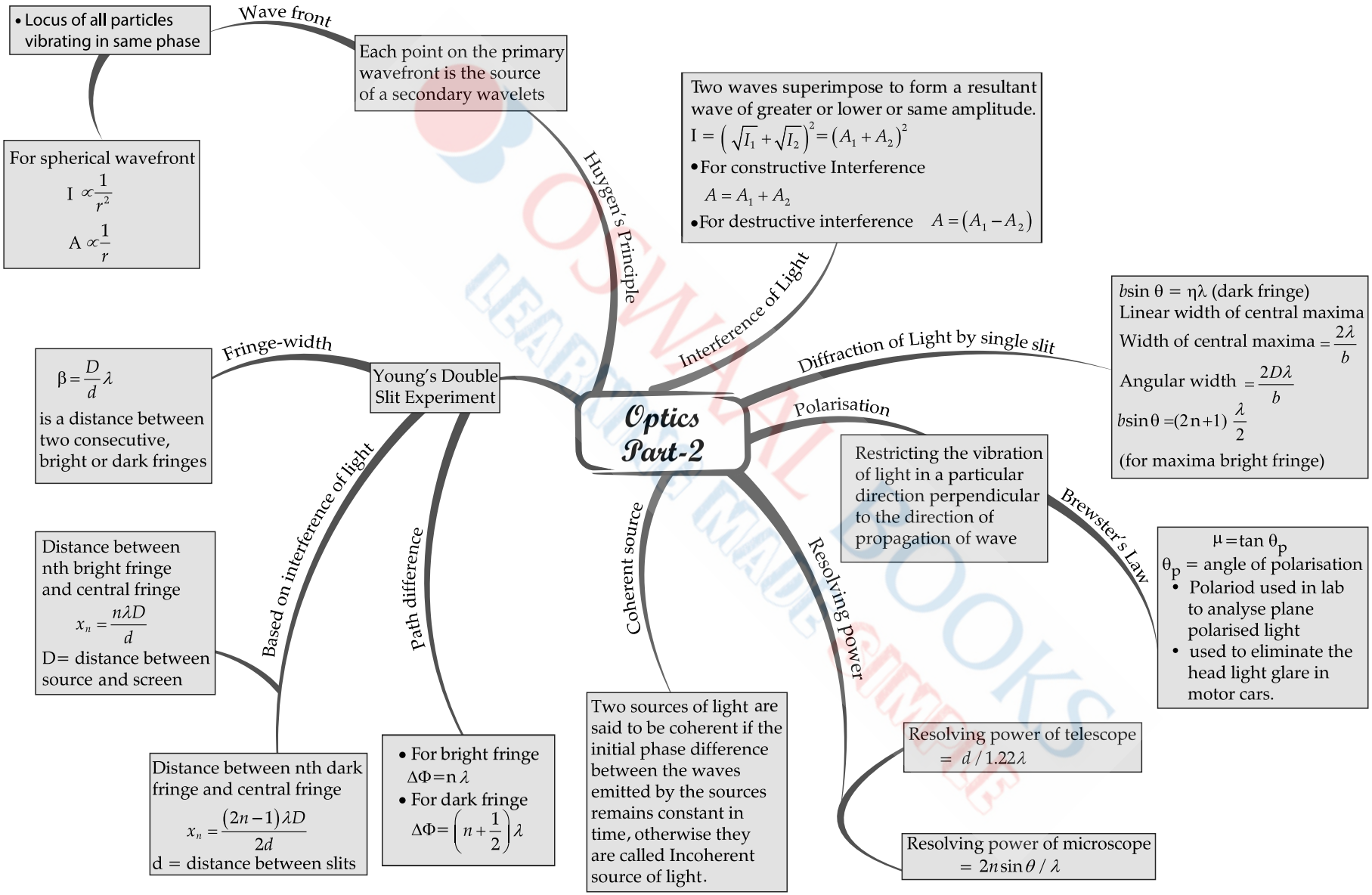
Lateral Magnification $= \frac{h_2}{h_1} = \frac{v}{u}$

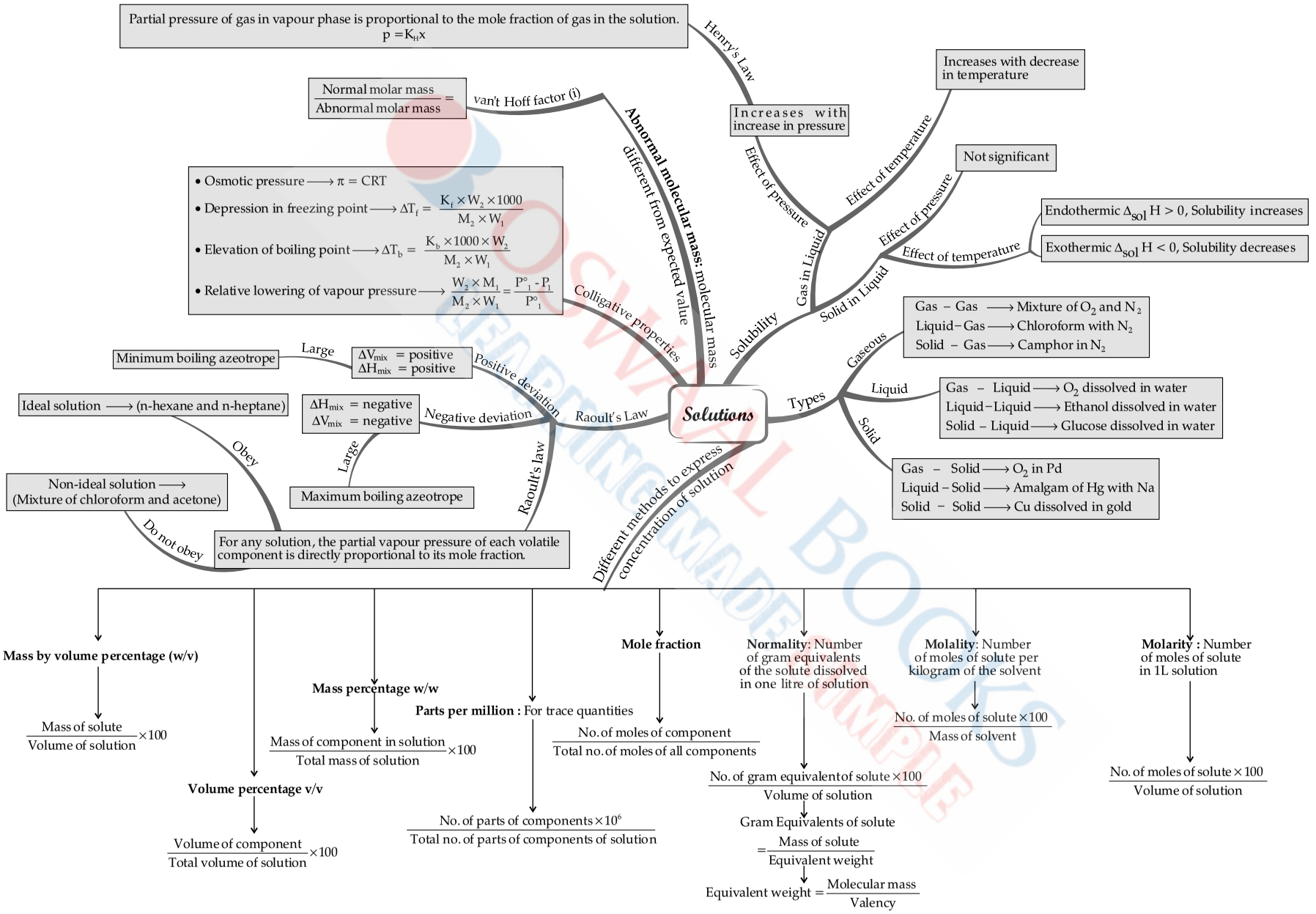
Power of a lens

$$P = \frac{1}{f}$$

[For Combination of lens in Contact]

$$P = \frac{1}{f_1}+\frac{1}{f_2}$$

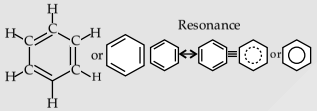

Optics Part-2
Wave front
• Locus of all particles vibrating in same phase
For spherical wavefront
$I \propto \frac{1}{r^2}$
$A \propto \frac{1}{r}$
Each point on the primary wavefront is the source of a secondary wavelets
Huygen's Principle
Interference of Light
Two waves superimpose to form a resultant wave of greater or lower or same amplitude.
$I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$
• For constructive Interference
$A = A_1 + A_2$
• For destructive interference $A = (A_1 - A_2)$
Diffraction of Light by single slit
$b\sin\theta = \eta\lambda$ (dark fringe)
Linear width of central maxima
Width of central maxima $= \frac{2\lambda}{b}$
Angular width $= \frac{2D\lambda}{b}$
$b\sin\theta = (2n+1)\frac{\lambda}{2}$
(for maxima bright fringe)
Polarisation
Restricting the vibration of light in a particular direction perpendicular to the direction of propagation of wave
Brewster's Law
$\mu = \tan\theta_p$
θ_p = angle of polarisation
• Polariod used in lab to analyse plane polarised light
• used to eliminate the head light glare in motor cars.
Resolving power
Resolving power of telescope $= d / 1.22\lambda$
Resolving power of microscope $= 2n\sin\theta / \lambda$
Coherent source
Two sources of light are said to be coherent if the initial phase difference between the waves emitted by the sources remains constant in time, otherwise they are called Incoherent source of light.
Path difference
• For bright fringe $\Delta\Phi = n\lambda$
• For dark fringe $\Delta\Phi = \left(n + \frac{1}{2}\right)\lambda$
Based on interference of light
Distance between nth bright fringe and central fringe
$x_n = \frac{n\lambda D}{d}$
D= distance between source and screen
Distance between nth dark fringe and central fringe
$x_n = \frac{(2n-1)\lambda D}{2d}$
d = distance between slits
Fringe-width
$\beta = \frac{D}{d}\lambda$
is a distance between two consecutive, bright or dark fringes
Young's Double Slit Experiment


Solutions
Types
Gaseous
Gas – Gas ⟶ Mixture of O_2 and N_2
Liquid–Gas ⟶ Chloroform with N_2
Solid – Gas ⟶ Camphor in N_2
Liquid
Gas – Liquid ⟶ O_2 dissolved in water
Liquid–Liquid ⟶ Ethanol dissolved in water
Solid – Liquid ⟶ Glucose dissolved in water
Solid
Gas – Solid ⟶ O_2 in Pd
Liquid – Solid ⟶ Amalgam of Hg with Na
Solid – Solid ⟶ Cu dissolved in gold
Solubility
Gas in Liquid
Henry's Law
Partial pressure of gas in vapour phase is proportional to the mole fraction of gas in the solution.
$p = K_H x$
Increases with increase in pressure
Effect of pressure
Effect of temperature
Increases with decrease in temperature
Solid in Liquid
Effect of pressure
Not significant
Effect of temperature
Endothermic $\Delta_{sol} H > 0$, Solubility increases
Exothermic $\Delta_{sol} H < 0$, Solubility decreases
Abnormal molecular mass: molecular mass different from expected value
$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$ = van't Hoff factor (i)
Colligative properties
• Osmotic pressure ⟶ $\pi = CRT$
• Depression in freezing point ⟶ $\Delta T_f = \frac{K_f \times W_2 \times 1000}{M_2 \times W_1}$
• Elevation of boiling point ⟶ $\Delta T_b = \frac{K_b \times 1000 \times W_2}{M_2 \times W_1}$
• Relative lowering of vapour pressure ⟶ $\frac{W_2 \times M_1}{M_2 \times W_1} = \frac{P^\circ_1 - P_1}{P^\circ_1}$
Raoult's Law
Positive deviation
ΔV_{mix} = positive
ΔH_{mix} = positive
Large
Minimum boiling azeotrope
Negative deviation
ΔH_{mix} = negative
ΔV_{mix} = negative
Large
Maximum boiling azeotrope
Raoult's law
Ideal solution ⟶ (n-hexane and n-heptane)
Obey
Non-ideal solution ⟶ (Mixture of chloroform and acetone)
Do not obey
For any solution, the partial vapour pressure of each volatile component is directly proportional to its mole fraction.
Different methods to express concentration of solution
Mass by volume percentage (w/v)
$\frac{\text{Mass of solute}}{\text{Volume of solution}} \times 100$
Mass percentage w/w
$\frac{\text{Mass of component in solution}}{\text{Total mass of solution}} \times 100$
Volume percentage v/v
$\frac{\text{Volume of component}}{\text{Total volume of solution}} \times 100$
Parts per million : For trace quantities
$\frac{\text{No. of parts of components} \times 10^6}{\text{Total no. of parts of components of solution}}$
Mole fraction
$\frac{\text{No. of moles of component}}{\text{Total no. of moles of all components}}$
Normality: Number of gram equivalents of the solute dissolved in one litre of solution
$\frac{\text{No. of gram equivalent of solute} \times 100}{\text{Volume of solution}}$
Gram Equivalents of solute $= \frac{\text{Mass of solute}}{\text{Equivalent weight}}$
Equivalent weight $= \frac{\text{Molecular mass}}{\text{Valency}}$
Molality: Number of moles of solute per kilogram of the solvent
$\frac{\text{No. of moles of solute} \times 100}{\text{Mass of solvent}}$
Molarity : Number of moles of solute in 1L solution
$\frac{\text{No. of moles of solute} \times 100}{\text{Volume of solution}}$

Hydrocarbons

Compounds of Carbon and Hydrogen

Aromatic Hydrocarbon

- Types: Benzenoids – contain benzene ring, Non-benzenoids – does not contain benzene ring.
- Isomerism: Ortho (o-), Meta (m-), Para (p-)
- Structure: 

- Aromaticity: Planarity, complete delocalisation of the π–electrons in the ring, presence of (4n + 2) π electrons in the ring where n is an integer (n = 0,1,2,.....) (Huckel rule)

Preparation:

- Cyclic polymerisation of ethyne

$$C_6H_5COONa + NaOH \xrightarrow[\Delta]{CaO} C_6H_6 + Na_2CO_3$$

$$C_6H_5OH + Zn \xrightarrow{\Delta} C_6H_6 + ZnO$$

Mechanism of Electrophilic substitution reactions

(i) Generation of Electrophile
(ii) Formation of carbocation intermediate
(iii) Removal of proton

Physical Properties

- Non-polar, usually colourless liquids or solids with characteristic aroma.
- Immiscible with water but miscible with organic solvents.
- Burns with sooty flame.

Chemical Properties

Chemical Properties:

$$\text{Ar–H} + \text{conc. } HNO_3 + \text{conc. } H_2SO_4 \xrightarrow{323\text{-}333K} \text{Ar–NO}_2 + H_2O$$

$$\text{Ar–H} + Cl_2 \xrightarrow{\text{Anhyd. } AlCl_3} \text{Ar–Cl} + HCl \;;\; \text{Ar–H} + H_2SO_4 \xrightarrow{\Delta} \text{Ar–SO}_3H + H_2O$$

$$\text{Ar–H} + CH_3Cl \xrightarrow{\text{Anhyd. } AlCl_3} \text{Ar–CH}_3 + HCl$$

$$\text{Ar–H} + CH_3COCl \xrightarrow{\text{Anhyd. } AlCl_3} \text{Ar–COCH}_3 + HCl$$

Classification

- **Saturated:** Contain C-C and C-H single bonds. (alkanes)
- **Unsaturated:** Contain C-C multiple bonds (alkenes, alkynes)
- **Aromatic**: Contain cyclic compounds

Alkynes (C_nH_{2n-2})

- IUPAC name: replacing 'ane' by the suffix 'yne'.
- Shows position and chain isomerism
- Preparation:

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$$

$$CaO + 3C \longrightarrow CaC_2 + CO$$

$$CaC_2 + 2H_2O \longrightarrow Ca(OH)_2 + C_2H_2$$

$$CH_2Br–CH_2Br + KOH \xrightarrow[-KBr\ -H_2O]{\text{alcohol}} H_2C=CHBr \xrightarrow[-NaBr\ -NH_3]{NaNH_2} CH\equiv CH$$

- Physical Properties:
  - First three members are gases, next eight are liquids and higher ones are solids.
  - Colourless, ethyne has characteristic odour and other are odourless.
  - Lighter than water, immiscible with water but soluble in organic solvents.
  - M.p, b.p. and density increase with increase in molar mass.
- Chemical Properties:

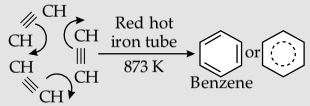
$$HC\equiv CH + Na \longrightarrow HC\equiv CNa + 1/2\,H_2$$

$$HC\equiv CH + H_2 \xrightarrow{Pt/Pd/Ni} [H_2C=CH_2] \xrightarrow{H_2} CH_3–CH_3$$

$$CH_3–C\equiv CH + Br_2 \longrightarrow [CH_3CBr=CHBr] \xrightarrow{Br_2} CH_3–\underset{Br}{\overset{Br}{C}}–\underset{Br}{\overset{Br}{CH}}$$

$$HC\equiv C–H + HBr \longrightarrow [CH_2=CH–Br] \xrightarrow{HBr} \underset{CH_3}{\overset{Br}{CHBr_2}}$$

$$HC\equiv CH + H_2O \xrightarrow[333\ K]{Hg^{2+}/H^+} [CH_2=\underset{OH}{C}–H] \xrightarrow{\text{Isomersation}} CH_3–\underset{O}{C}–H$$

Polymerisation 

Alkenes (C_2H_{2n})

- Shows structural and geometrical isomerism

Preparation:

$$RC\equiv CR' + H_2 \xrightarrow{Pd/C} \underset{H}{\overset{R}{}}C=C\underset{H}{\overset{R'}{}} \quad \text{cis-Alkene}$$

$$RC\equiv CR' + H_2 \xrightarrow{Na/\text{Liquid } NH_3} \underset{H}{\overset{R}{}}C=C\underset{R'}{\overset{H}{}} \quad \text{trans-Alkene}$$

$$H_3C–CH_2X \xrightarrow[\Delta]{Alc.KOH} H_2C=CH_2$$

$$CH_2Br–CH_2Br + Zn \longrightarrow CH_2=CH_2 + ZnBr_2$$

$$CH_3CH_2OH \xrightarrow[\Delta]{\text{conc. } H_2SO_4} CH_2=CH_2 + H_2O$$

- Physical Properties :
  - Ethene is a colourless gas with faint sweet smell.
  - All others are colourless and odourless, insoluble in water but fairly soluble in non-polar solvents.
  - Increase in b.p. with increase in molecular size.
- Chemical Properties:

$$CH_2=CH_2 + Br_2 \xrightarrow{CCl_4} BrCH_2–CH_2Br$$

$$CH_2=CH_2 + HBr \longrightarrow CH_3CH_2Br$$

$$CH_3\text{-}CH=CH_2 + HBr \xrightarrow{\text{Markovnikov rule}} \begin{cases} CH_3\text{-}\underset{Br}{CH}\text{-}CH_3 \\ CH_3\text{-}CH_2\text{-}CH_2\text{-}Br \end{cases}$$

$$H_3C–\underset{CH_3}{C}=CH_2 + H_2O \xrightarrow{H^+} H_3C–\overset{CH_3}{\underset{CH_3}{C^+}} \xrightarrow{} H_3C–\underset{CH_3}{\overset{OH}{C}}–CH_3$$

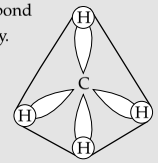
$$CH_3–CH=CH–CH_3 \xrightarrow{KMnO_4/H^+} 2CH_3COOH$$

$$n(CH_2=CH_2) \xrightarrow[\text{Catalyst}]{\text{HighTemp/ Pressure}} -(CH_2–CH_2)_n-$$

$$n(CH_3–CH=CH_2) \xrightarrow[\text{Catalyst}]{\text{HighTemp/Pressure}} -(CH_3–CH_2–CH_2)_n-$$

Alkane (C_nH_{2n+2})

- H-C-H bond angles – 190.5, C-C and C-H bond lengths are 154 pm and 112 pm respectively.
- Shows structural and chain isomerism.



Preparation :

$$CH_2=CH_2 + H_2 \xrightarrow{pd/pt/Ni} CH_3–CH_3$$

Wurtz reaction :

$$CH_3Br + 2Na + BrCH_3 \xrightarrow{\text{Dry ether}} CH_3–CH_3 + 2NaBr$$

$$CH_3COO^-Na^+ + NaOH \xrightarrow[\Delta]{CaO} CH_4 + Na_2CO_3$$

$$2CH_3COONa + 2H_2O \longrightarrow C_2H_6 + 2CO_2 + H_2 + 2NaOH$$

- Physical Properties:

  Non-polar, weak van der Waals forces, colourless, odourless.

  B.P. increases with increases in molecular size.
- Chemical Properties:

$$CH_4 \xrightarrow[hv/-HCl]{+Cl_2} CH_3Cl \xrightarrow[hv/-HCl]{+Cl_2} CH_2Cl_2 \xrightarrow[hv/-HCl]{+Cl_2} CHCl_3 \xrightarrow[hv/-HCl]{+Cl_2} CCl_4$$

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \longrightarrow nCO_2 + (n+1)H_2O$$

$$2CH_4 + O_2 \xrightarrow[100atm]{Cu/523K} 2CH_3OH$$

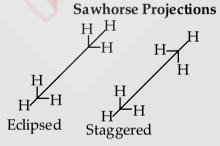
$$CH_4 + O_2 \xrightarrow[\Delta]{Mn_2O_3} HCHO + H_2O$$

$$CH_4(CH_2)CH_3 \xrightarrow{\text{Anhyd. } AlCl_3/HCl} CH_3–\underset{CH_3}{CH}–(CH_2)_2–\underset{CH_3}{CH_2} + CH_3–CH_2–\underset{CH_3}{CH}–CH_2–CH_3$$

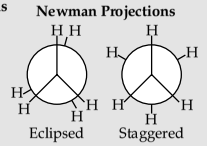
$$CH_4 + H_2O \xrightarrow[\Delta]{Ni} CO + 3H_2$$

$$C_6H_{14} \xrightarrow{733\ K} \begin{cases} C_6H_{12} + H_2 \\ C_4H_8 + C_2H_6 \\ C_3H_6 + C_2H_4 + CH_4 \end{cases}$$

Sawhorse Projections



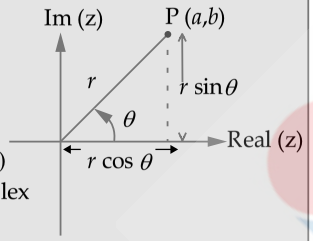
Newman Projections



Complex Numbers & Quadratic Equations

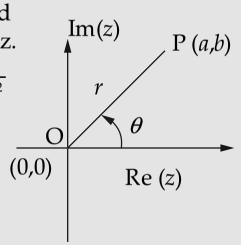
Polar Representation

Let $a = r\cos\theta$
$b = r\sin\theta$
where,
$r = |z|$
and $\theta = \arg(z)$
$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$
The argument 'θ' of complex number $z = a+ib$ is called principal argument of z if
$-\pi < \theta \leq \pi$.



Modulus & Argument of Complex Number

If z= a+ib is a complex number
(i)Distance of z from origin is called as modulus of complex number z.
It is denoted by $r = |z| = \sqrt{a^2+b^2}$
(ii) Angle θ made by OP with +ve direction of X-axis is called argument of z.



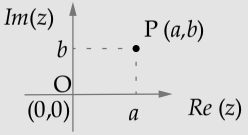
Powers of 'i'

$i = \sqrt{-1}, i^2 = -1$

In general, $i^{4k+r} = \begin{cases} 1; r=0 \\ i; r=1 \\ -1; r=2 \\ -i; r=3 \end{cases}$

Argand Plane

A complex number $z=a+ib$ can be represented by a unique point P(a,b) in the argand plane



$z=a+ib$ is represented by a point P (a,b)

Square root of Complex Number

Let $z=x+iy=\sqrt{a+ib}$, squaring both sides, we get $(x+iy)^2 = a+ib$ i.e. $x^2-y^2=a$, $2xy=b$
solving these equations, we get square root of z.

Multiplicative Inverse of complex Number

For a non-zero complex number $z=a+$ib $(a \neq 0, b \neq 0)$,

there exists a complex number $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$

denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of z

Such that: $(a+ib)\left(\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + 0i = 1$

Solution of Quadratic Equation

- General form of quadratic equation in x is $ax^2+bx+c=0$,
Where $a,b,c \in R$ & $a \neq 0$
The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
(i) If $b^2-4ac > 0$, roots are real and unequal
(ii) If $b^2-4ac = 0$, roots are real and equal
(iii) If $b^2-4ac < 0$, roots are imaginary

Note: • A monomial equation has one root.
• A polynomial equation of degree n has n roots.

Definition of Complex Numbers

A number of the form $a+ib$, where $a,b \in R$ and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

$z = a + ib$ → Imaginary part
↓
Real part

Conjugate of a complex number: For a given complex number $z=a+ib$, its conjugate is defined as $\bar{z} = a - ib$

Algebra of Complex Numbers

Let: $z_1 = a+ib$ and $z_1 = c+id$ be two complex numbers, where $a,b,c,d \in R$ and $i = \sqrt{-1}$

1. Addition: $z_1 + z_2 = (a+ib)+(c+id) = (a+c)+i(b+d)$

2. Subtraction: $z_1 - z_2 = (a+ib)-(c+id) = (a-c)+(b-d)i$

3. Multiplication: $z_1 \cdot z_2 = (a+ib)(c+id)$
$= a(c+id)+ib(c+id)$
$= (ac-bd)+(ad+bc)i$
$(\because i^2 = -1)$

4. Division: $\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id}\cdot\frac{c-id}{c-id}$
$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$

Note: If $a+ib = c+id$
$\Leftrightarrow a = c$ & $b = d$

Sequences and Series

Geometric Mean (GM)

- If a, b, c are in G.P., b is the GM between a & c, $b^2 = ac$, therefore ; $a > 0, c > 0$
- If a, b are two given numbers & $a, G_1, G_2, \ldots, G_n, b$ are in G.P., then $G_1, G_2, G_3 \ldots G_n$ are n GMs between a & b.

$G_1 = a \cdot \left(\frac{b}{a}\right)^{1/n+1}$ $G_2 = a \cdot \left(\frac{b}{a}\right)^{2/n+1}$ ..... $G_n = a \cdot \left(\frac{b}{a}\right)^{n/n+1}$

or $G_1 = ar$, $G_2 = ar^2$, $\ldots$ $G_n = ar^n$ where $r = \left(\frac{b}{a}\right)^{1/n+1}$

Geometric Progression (G.P.)

A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.This constant factor is called **'Common ratio'.** Usually we denote the first term of a G.P. by 'a' and its common ratio by 'r'. The gereral or n^{th} term of G.P. is given by $a_n = ar^{n-1}$
The sum S_n of the first n terms of G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} r > 1 \text{ or } \frac{a(1 - r^n)}{1 - r} r < 1 \text{ if } r \neq 1$$

Sum of infinite terms of G.P. is given by

$$S_\infty = \frac{a}{1 - r}; \ |r| < 1 \text{ or, } S_\infty = \frac{a}{r - 1}; \ r > 1$$

Harmonic Mean

Harmonic Progression (HP)
For the solution of HP, we should follow below steps
Make the reciprocal of each terms of HP
Solve by AP method
Make the reciprocal of AP result
eg: if $\frac{1}{a}$, $\frac{1}{a+d}$, $\frac{1}{a+2d}$--------in HP, then
$a, (a+d), (a+2d)$--------will be in AP
now n^{th} term of AP, $a_n = a+(n-1)d$
So, n^{th} team of HP, = $\frac{1}{a_n} = \frac{1}{a+(n+)d}$
Some Important result
If H is the harmonic mean (H) between a&b, then H= $\frac{2ab}{a+b}$
If there is n harmonic mean between a&b, then n^{th} HM(H_n)= $\frac{ab(n+1)}{na+b}$

Arithmetic Progression (A.P.)

An A.P. is a sequence in which terms increase or decrease regularly by the fixed number (same constant). This fixed number is called 'common difference' of the A.P. If 'a' is the first term & 'd' is the common difference and 'l' is the last term of A.P., then general term or the n^{th} term of the A.P. is given by $a_n = a + (n-1)d$ from starting and $a_n = l - (n-1)d$ from the end.
The sum S_n of the first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$$

Arithmetic Mean (AM)

If three numbers are in A.P., then the middle term is called AM between the other two, so if a, b, c, are in A.P., b is AM of a and c.
AM for any 'n' +ve numbers $a_1, a_2, a_3, \ldots, a_n$ is

$$AM = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

n-Arithmetic Mean between Two Numbers: If a, b are any two given numbers & $a, A_1, A_2, .., A_n, b$ are in A.P. then $A_1, A_2, \ldots, A_n$ are n AM's between a & b.

$$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1} \ldots A_n = a + \frac{n(b-a)}{n+1}$$

or $A_1 = a + d$, $A_2 = a + 2d$, $\ldots$, $A_n = a + nd$,
where $d = \frac{b-a}{n+1}$

Special Sequences

- Sum of first 'n' natural numbers

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

- Sum of squares of first n natural numbers

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- Sum of cubes of first n natural numbers

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right)^2$$

- Sum of first 'n' odd natural numbers

$$\sum_{k=1}^{n} (2k-1) = 1 + 3 + 5 + \ldots + (2n-1) = n^2$$

Sum upto terms of some

In a series,where each term is formed by multiplying the corresponding term of an AP & GP is called Arithmetico-Geometric Series.
Eg: $1 + 3x + 5x^2 + 7x^3 + \ldots$
Here, 1, 3, 5, … are in AP and 1, x, x^2, … are in GP.

Arithmetico Geometric Series

Properties of AM & GM between two quantities

- If A.M.: G.M. of two positive numbers a and b is $m : n$, then $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$
- If A and G be the AM & GM between two positive numbers, then the numbers are $A \pm \sqrt{A^2 - G^2}$

Relationship between AM, GM and HM

Let A, G and H be the AM, GM and HM of two given positive real numbers a & b, respectively. Then,

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2}{a+b}$

So, $G = \sqrt{AH}$ or, $G^2 = AH$