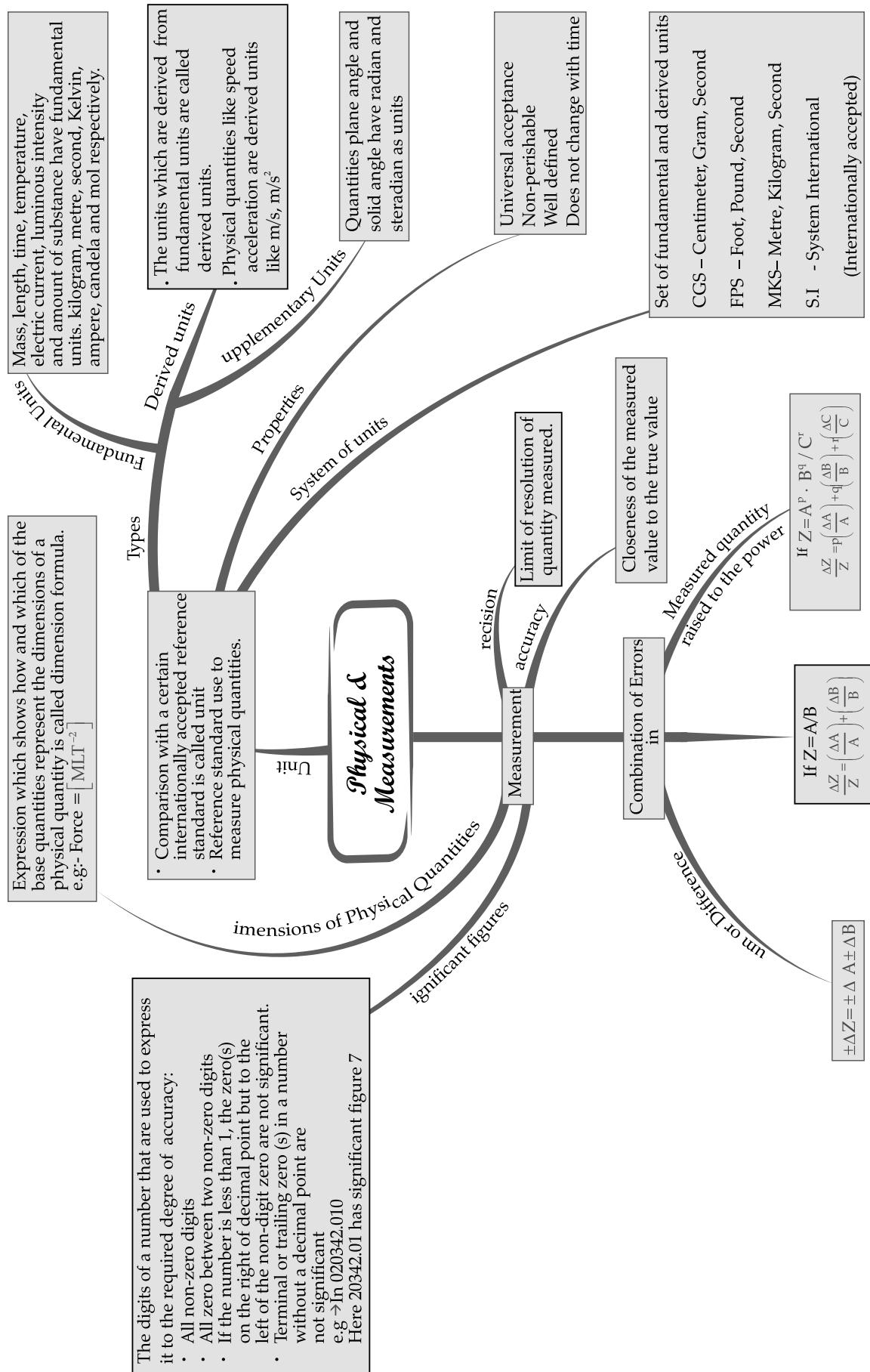


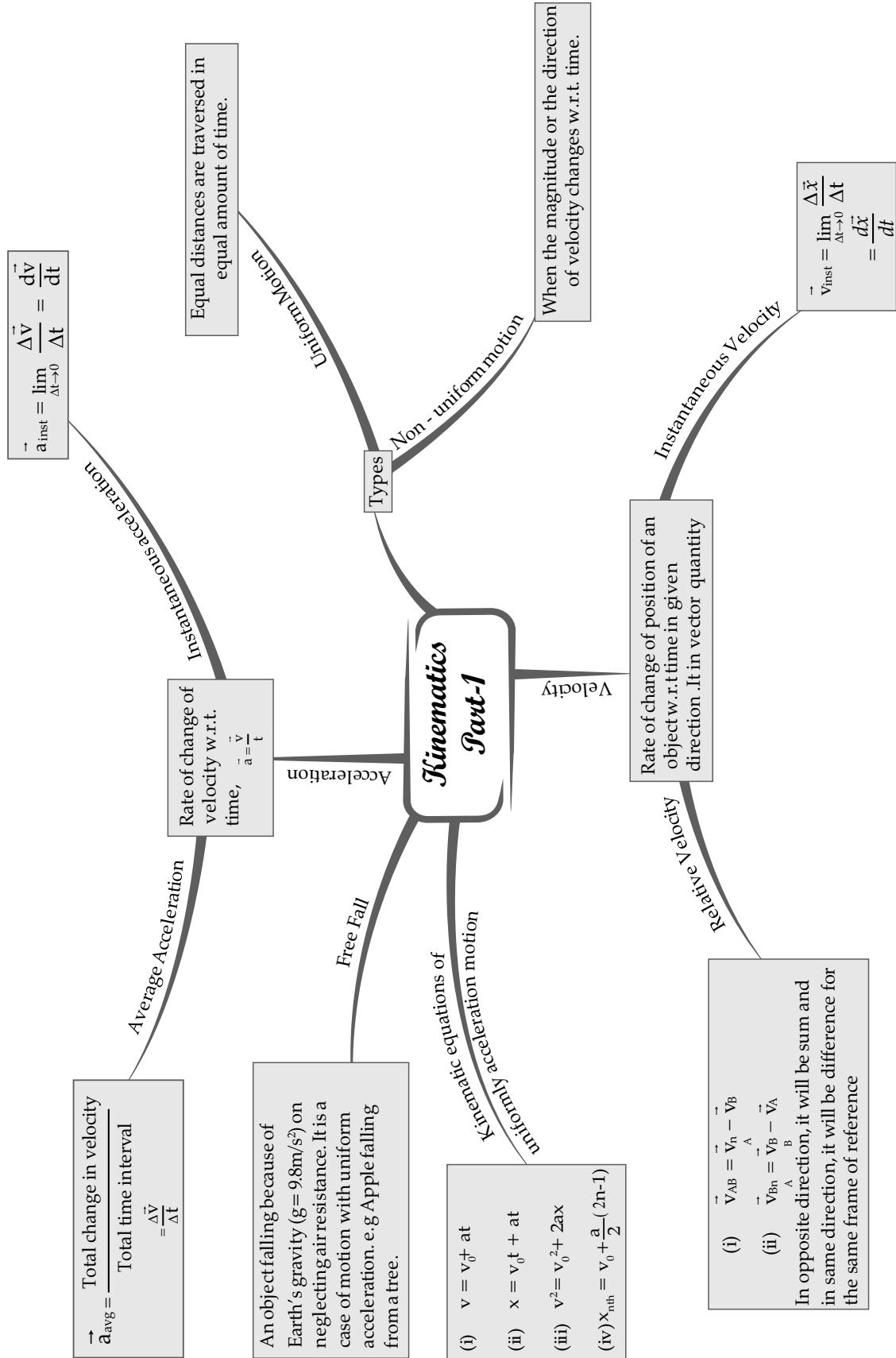
# **PHYSICS**

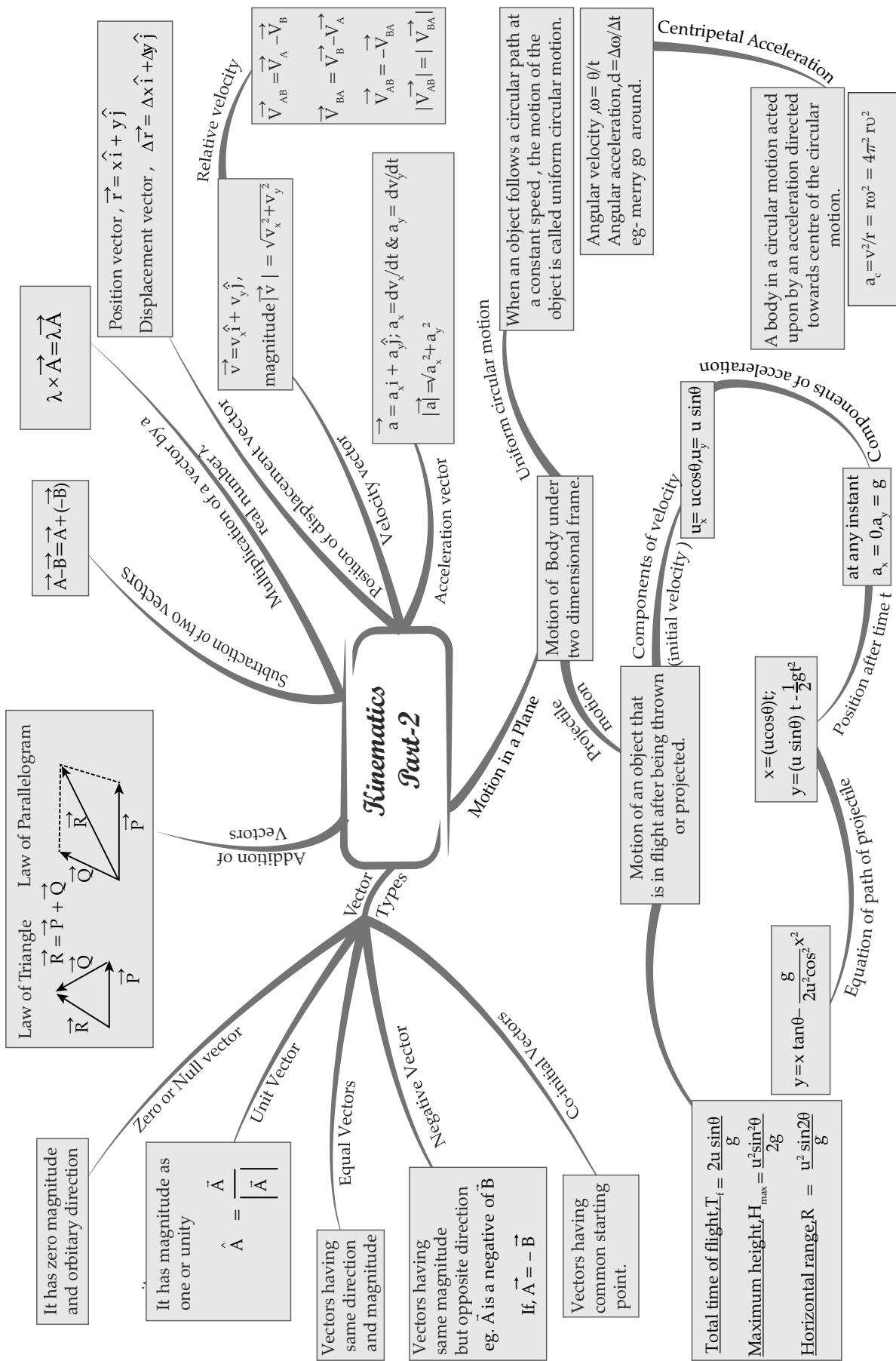
## **MIND MAPS**

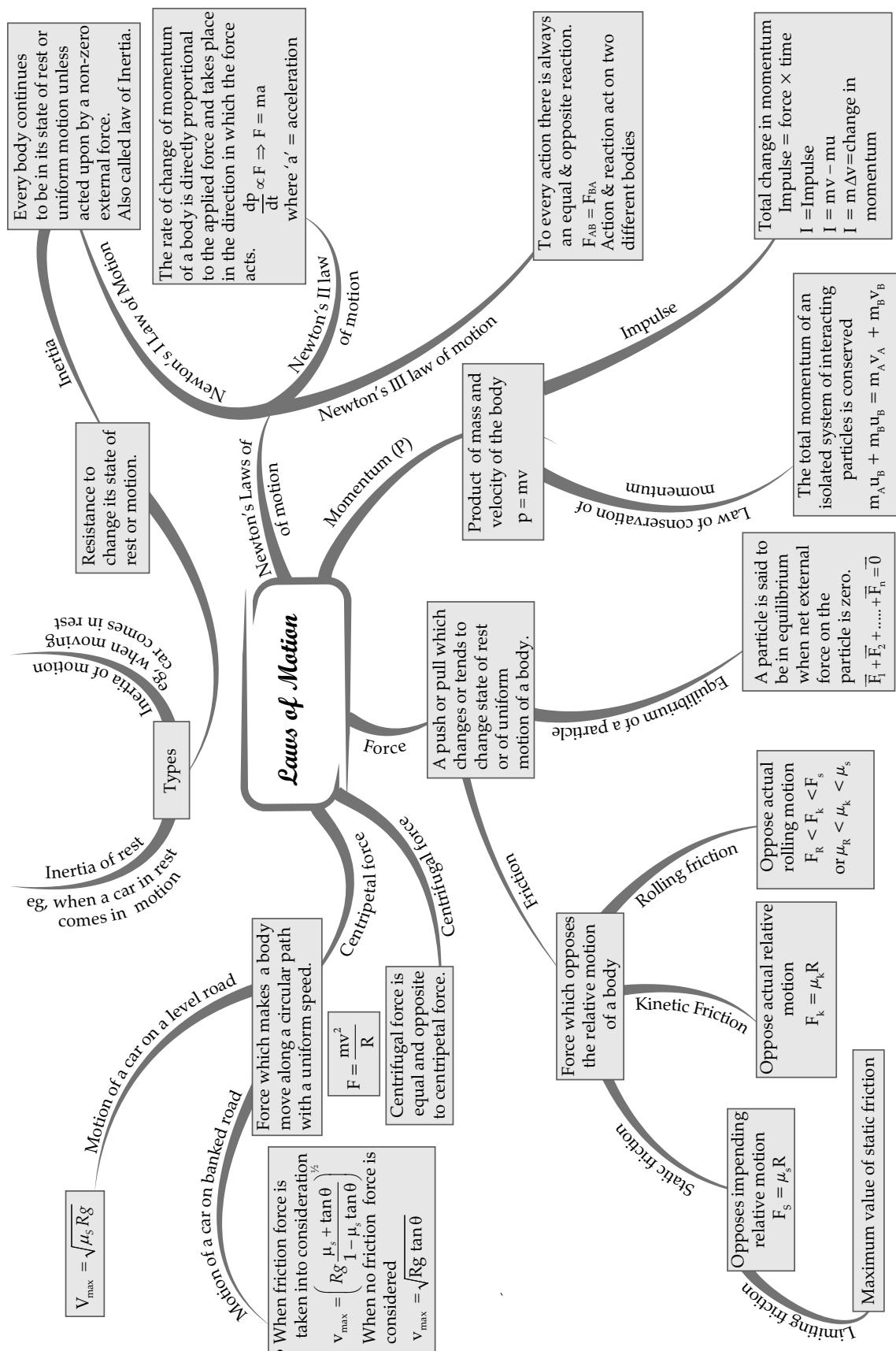
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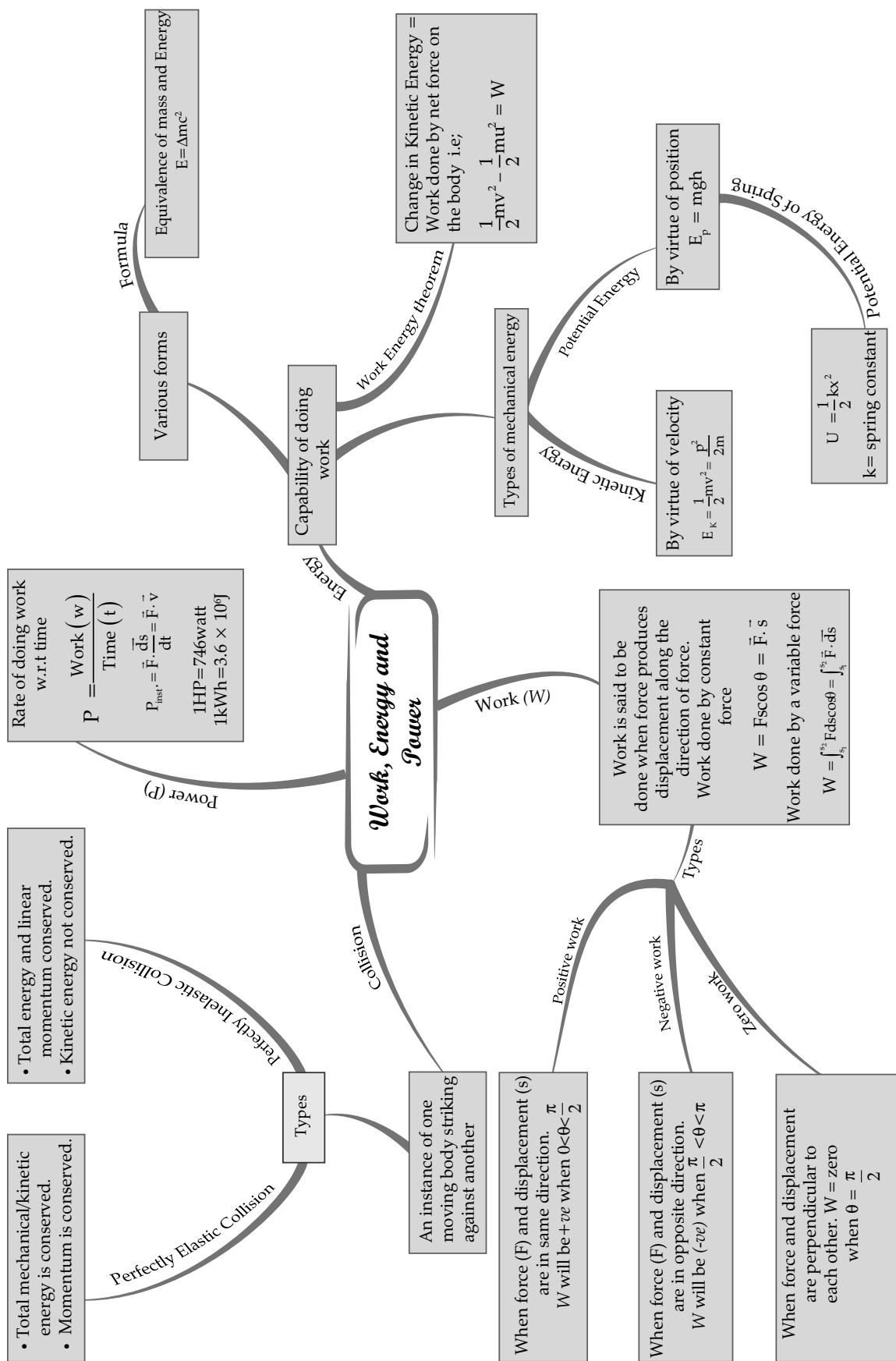
## **MNEMONICS**

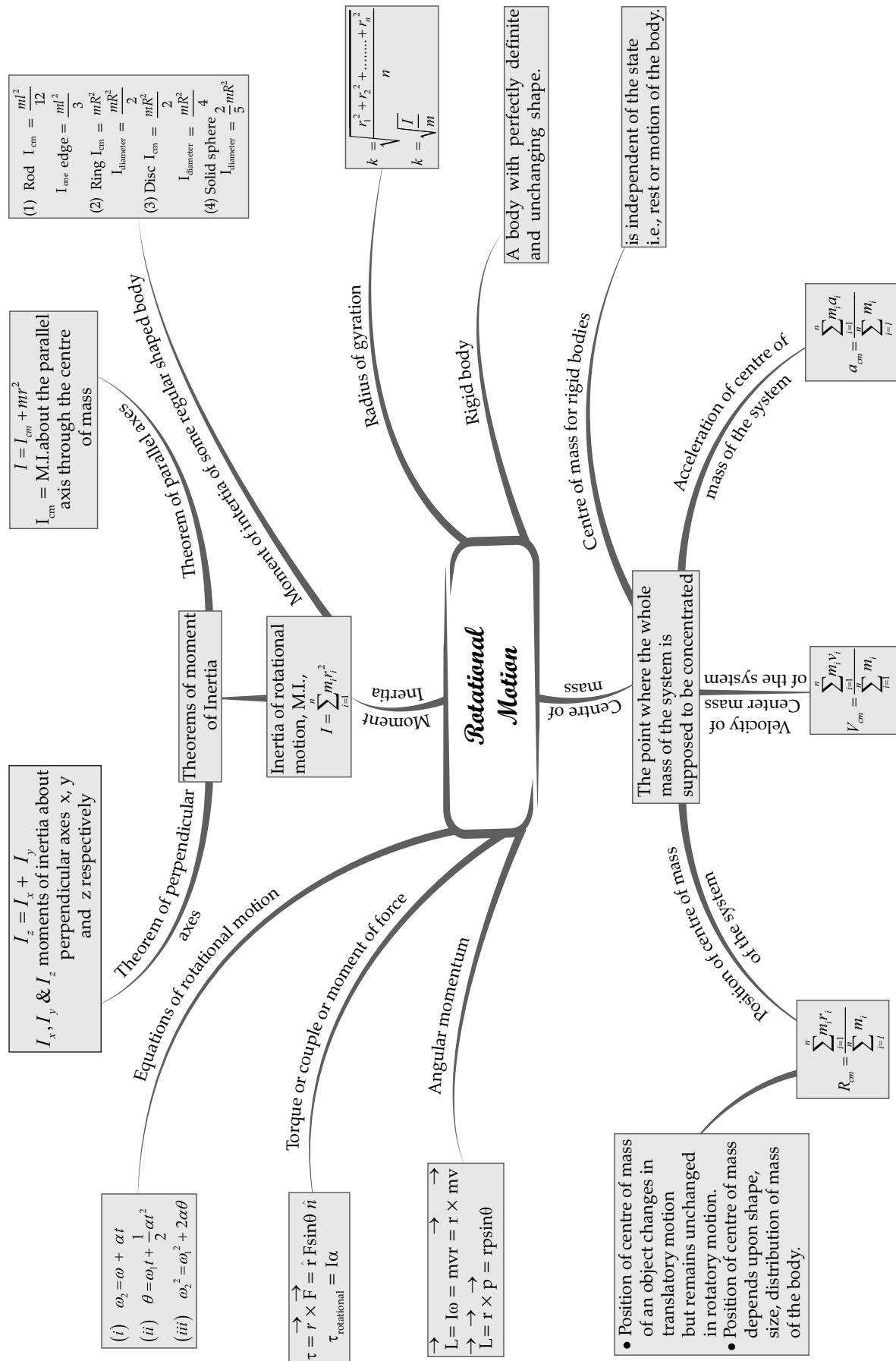


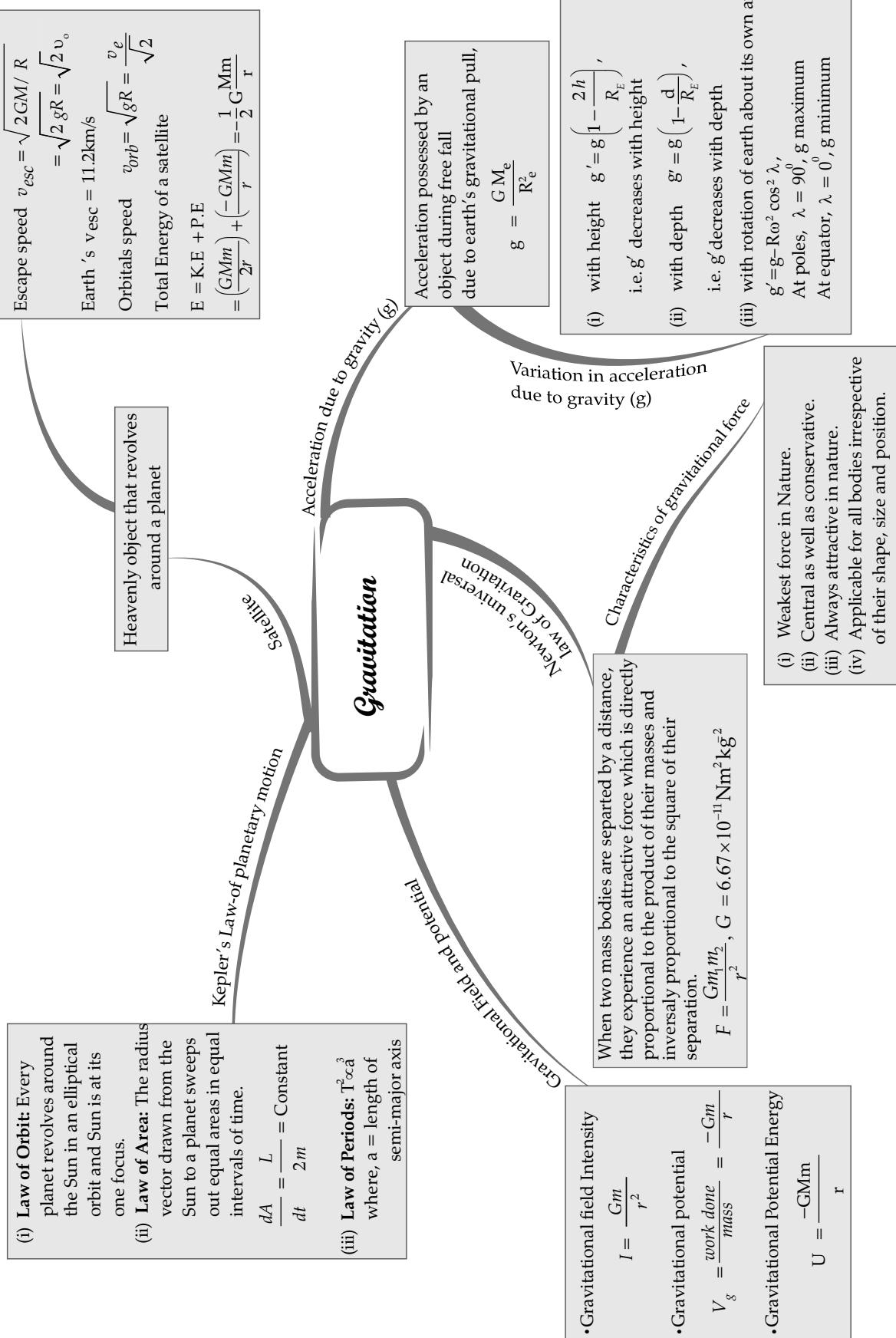


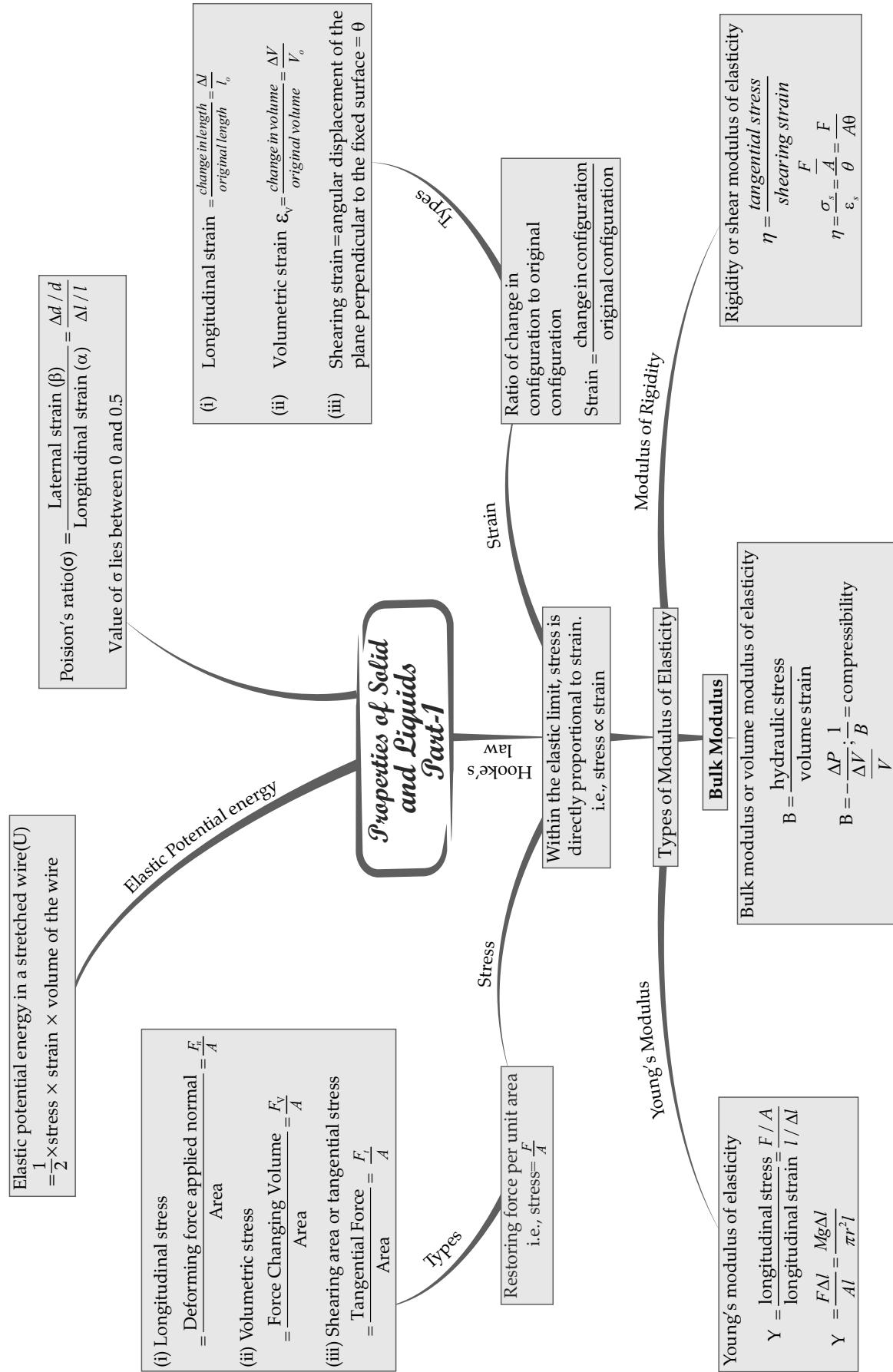


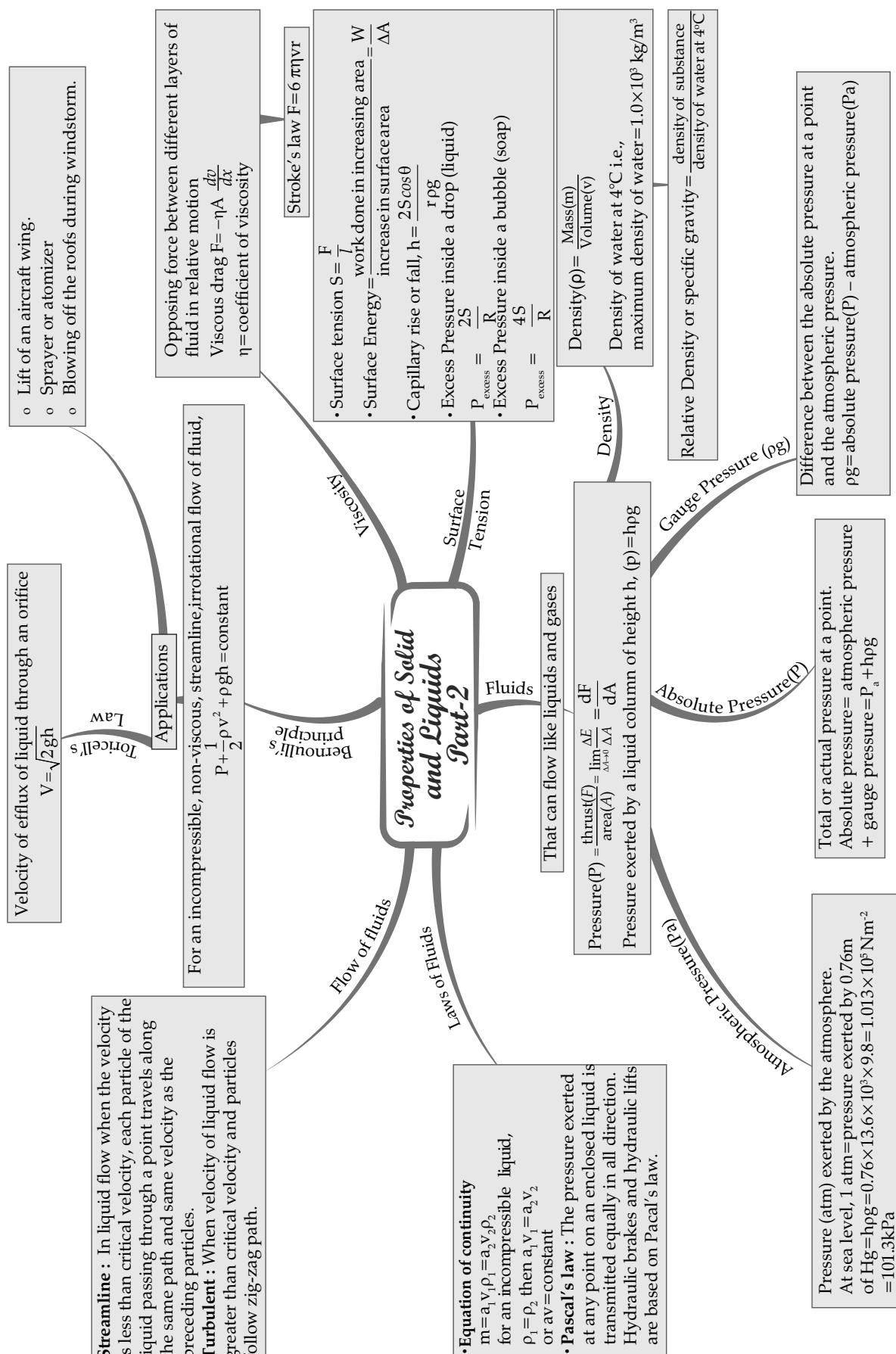


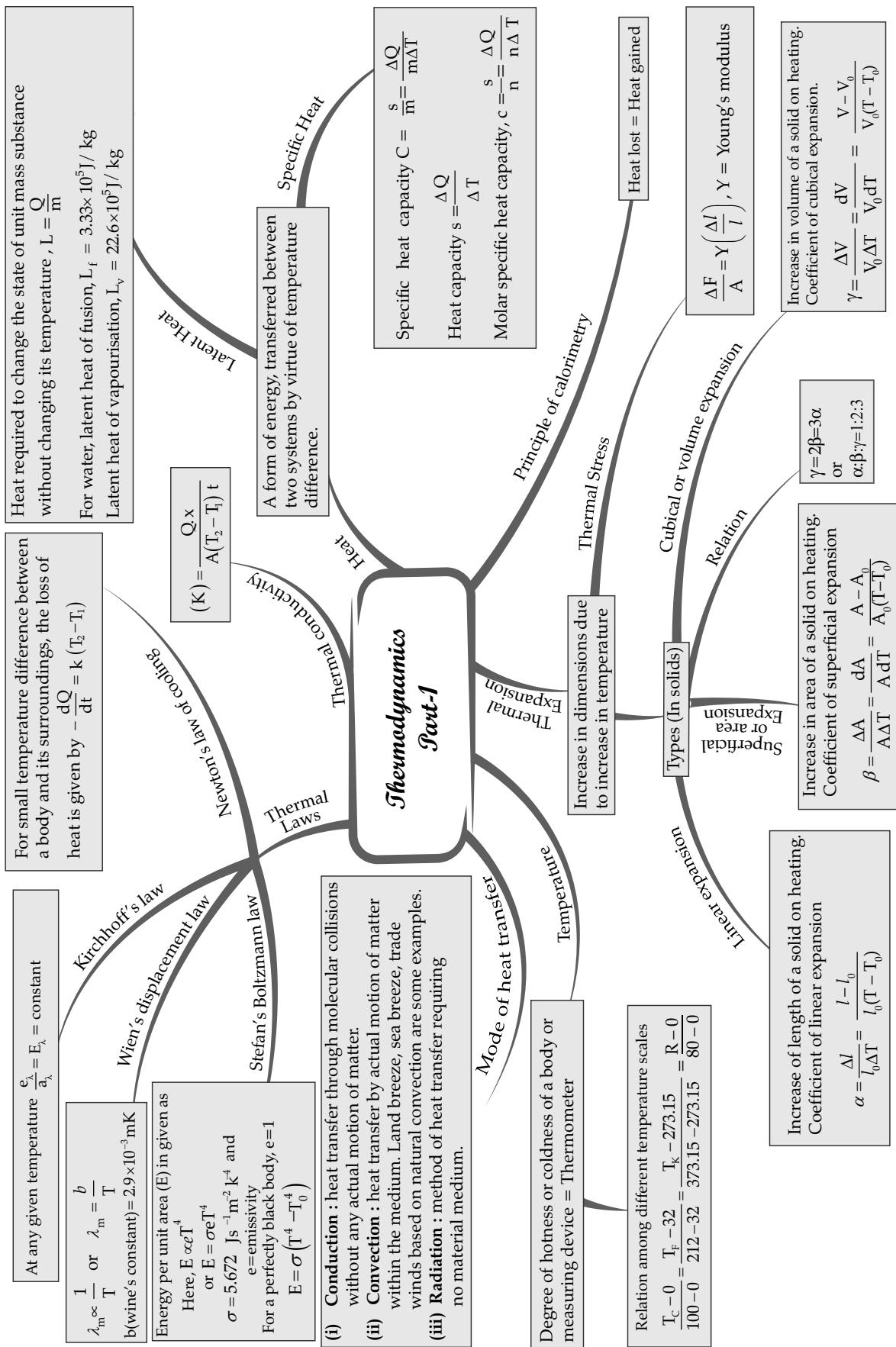


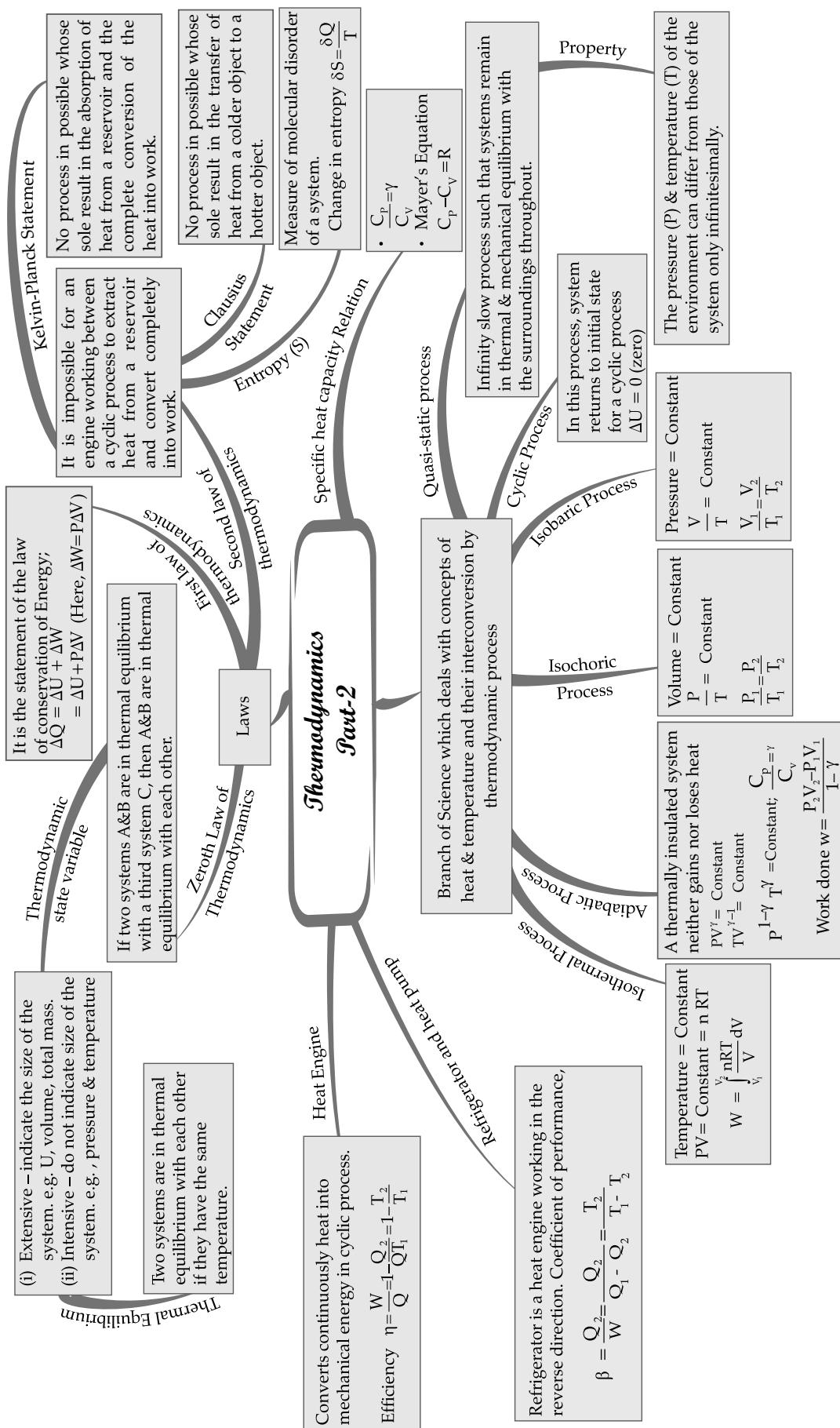


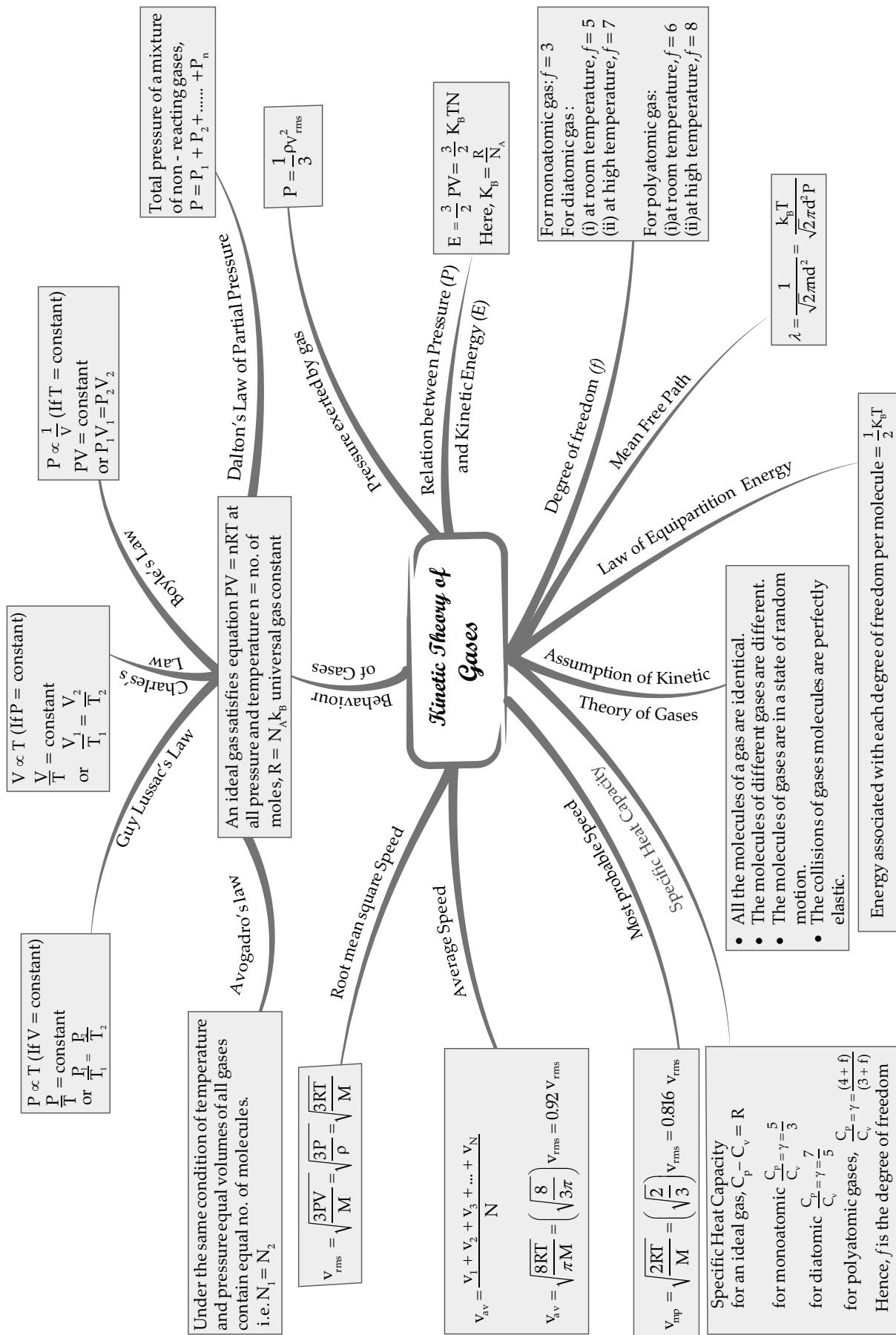


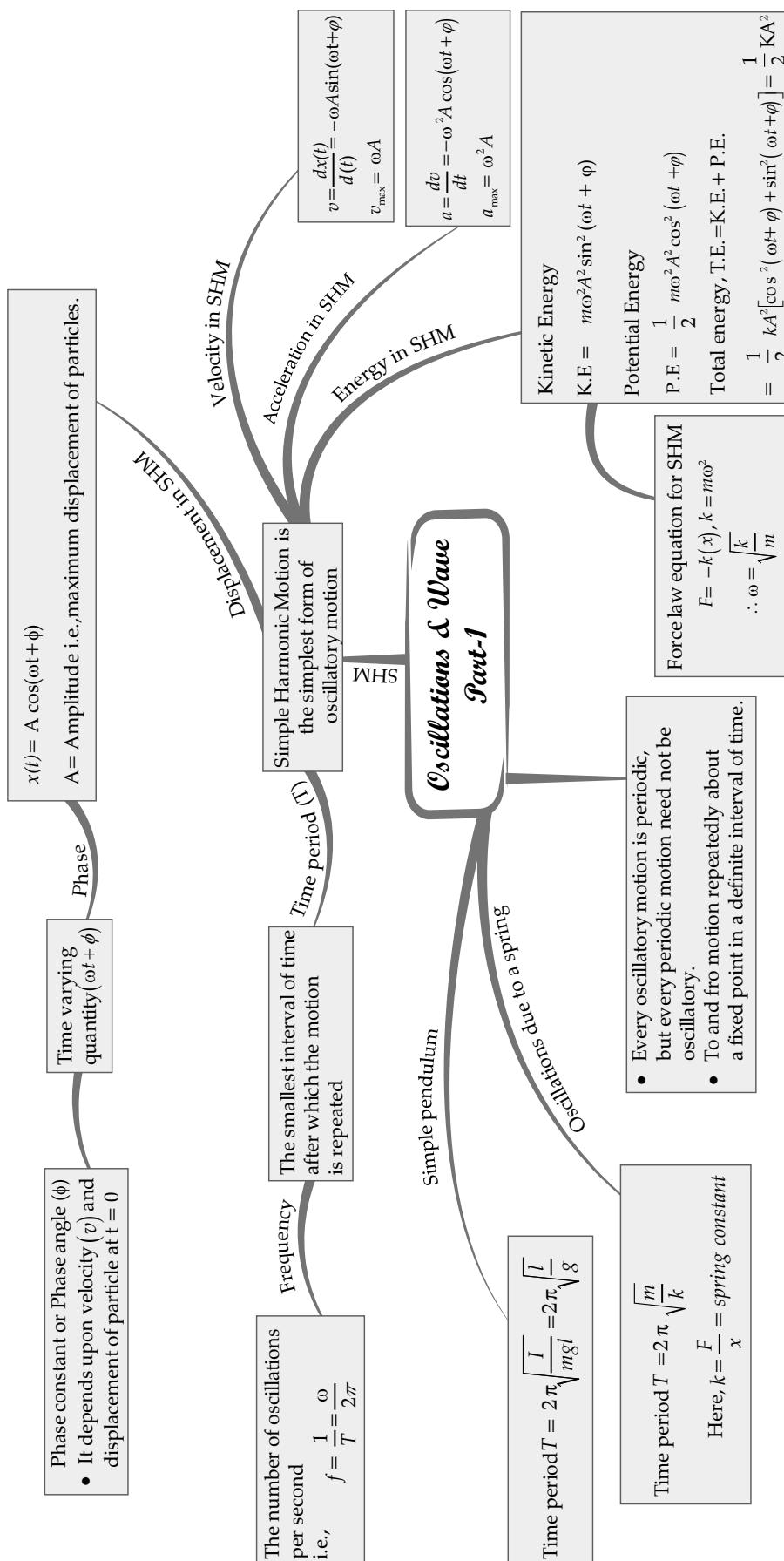


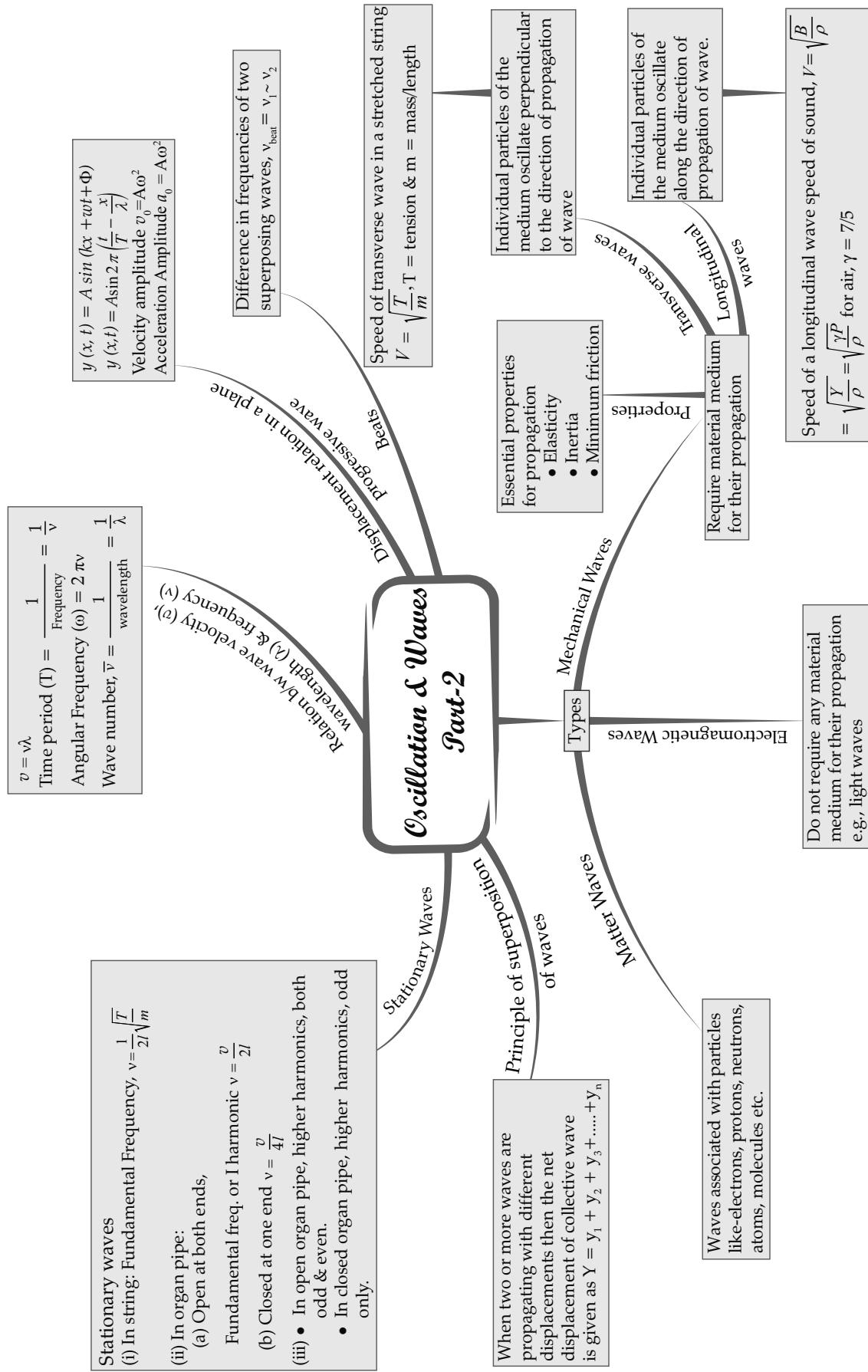


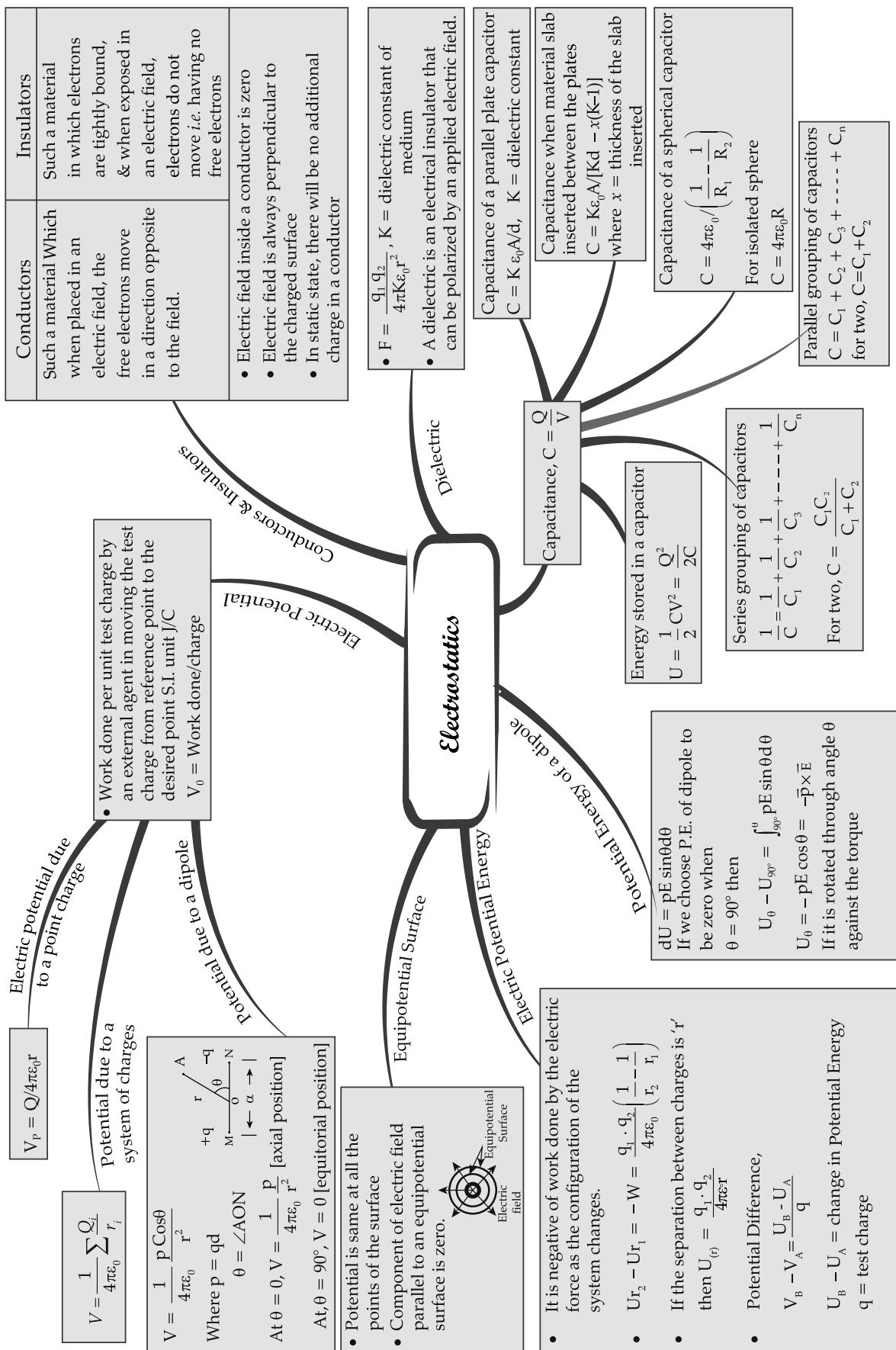


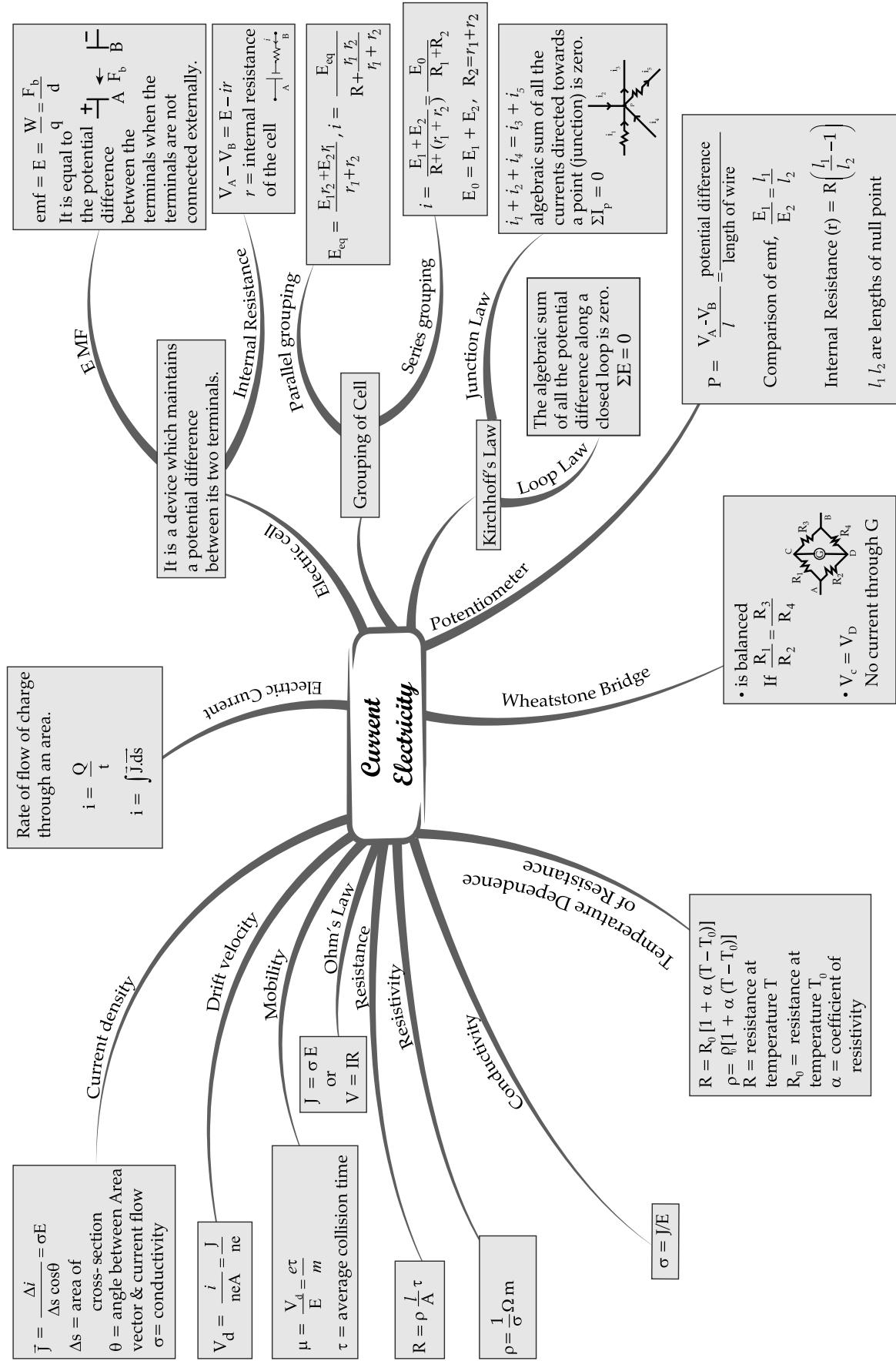


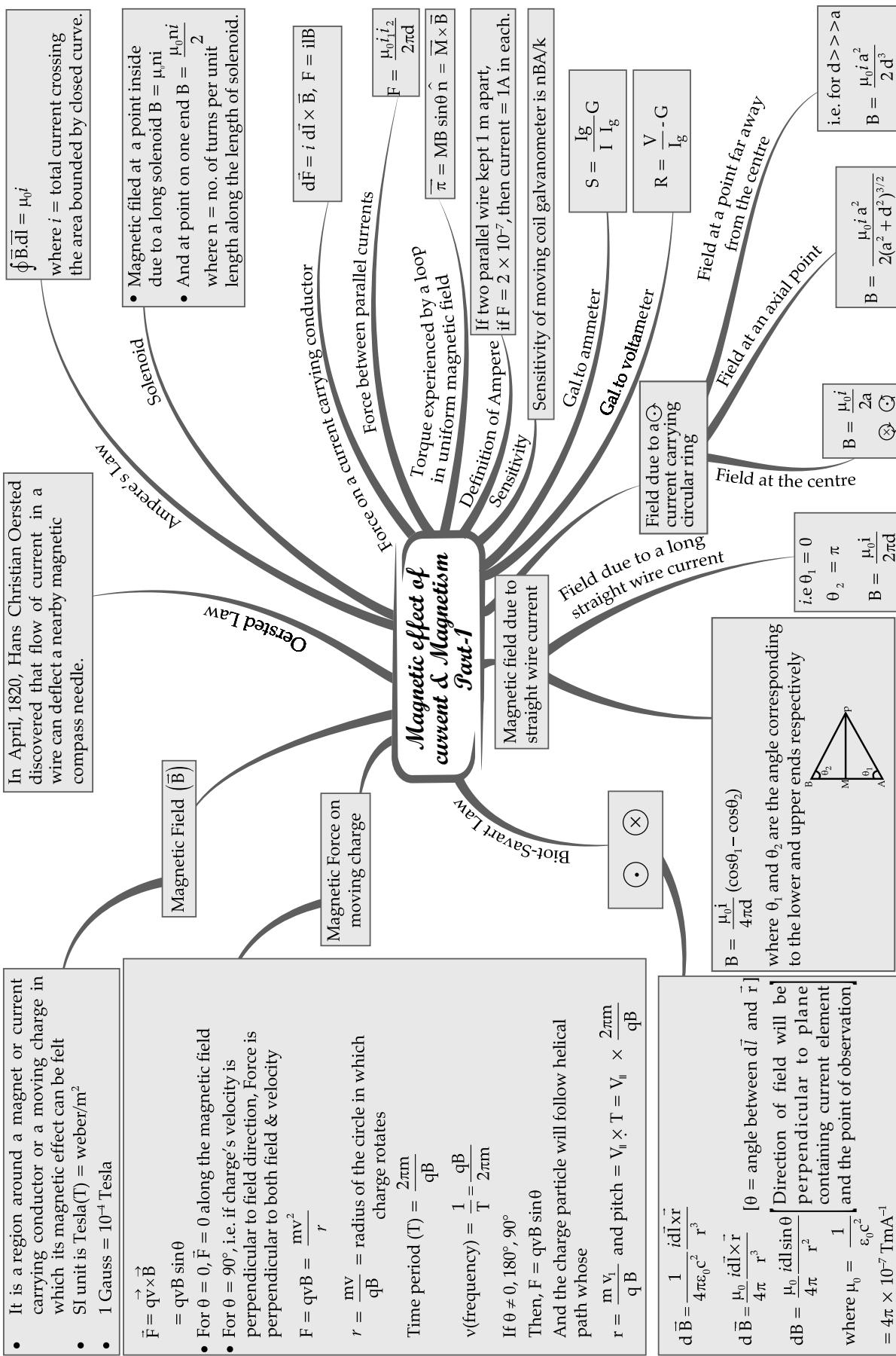


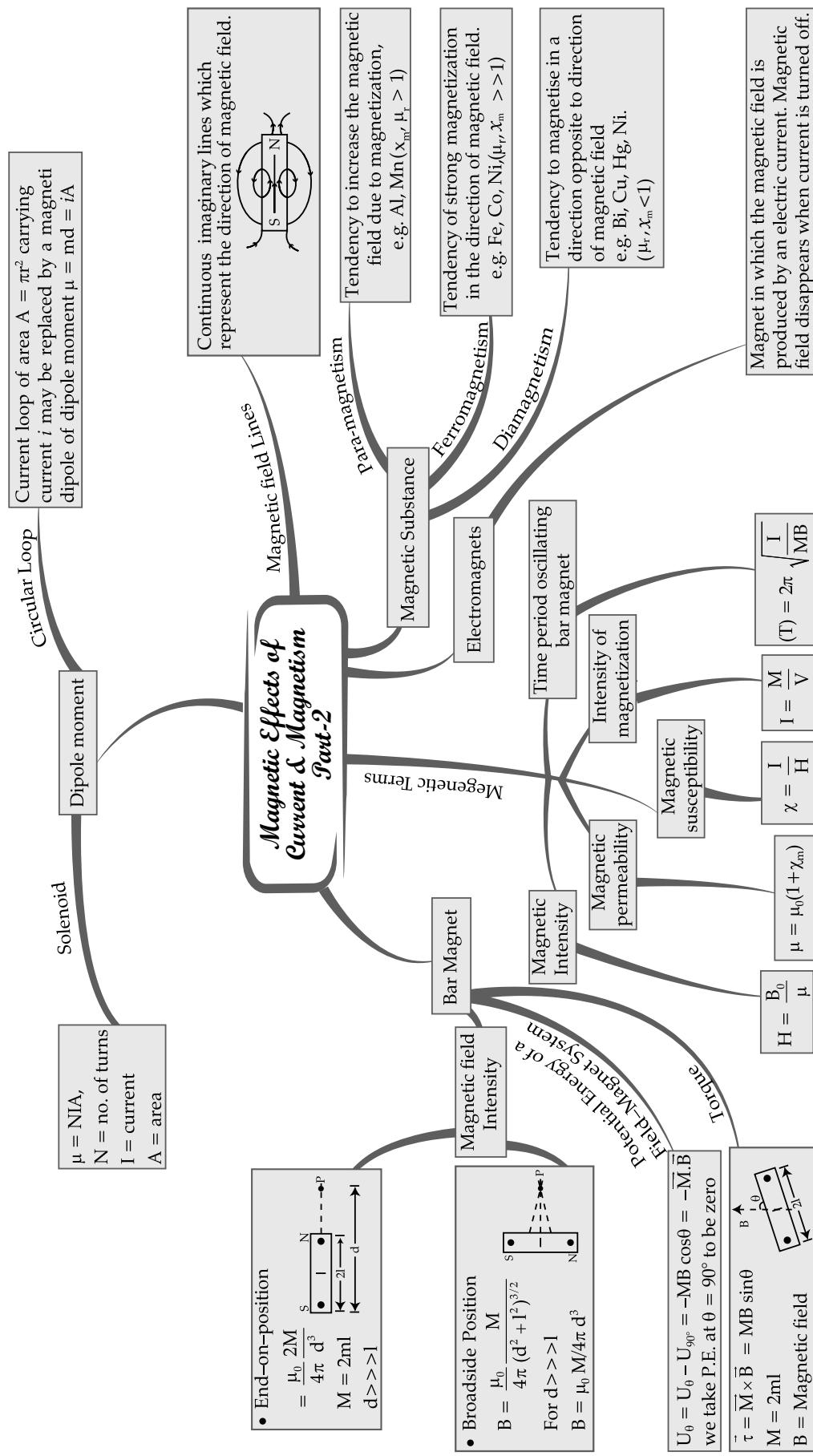


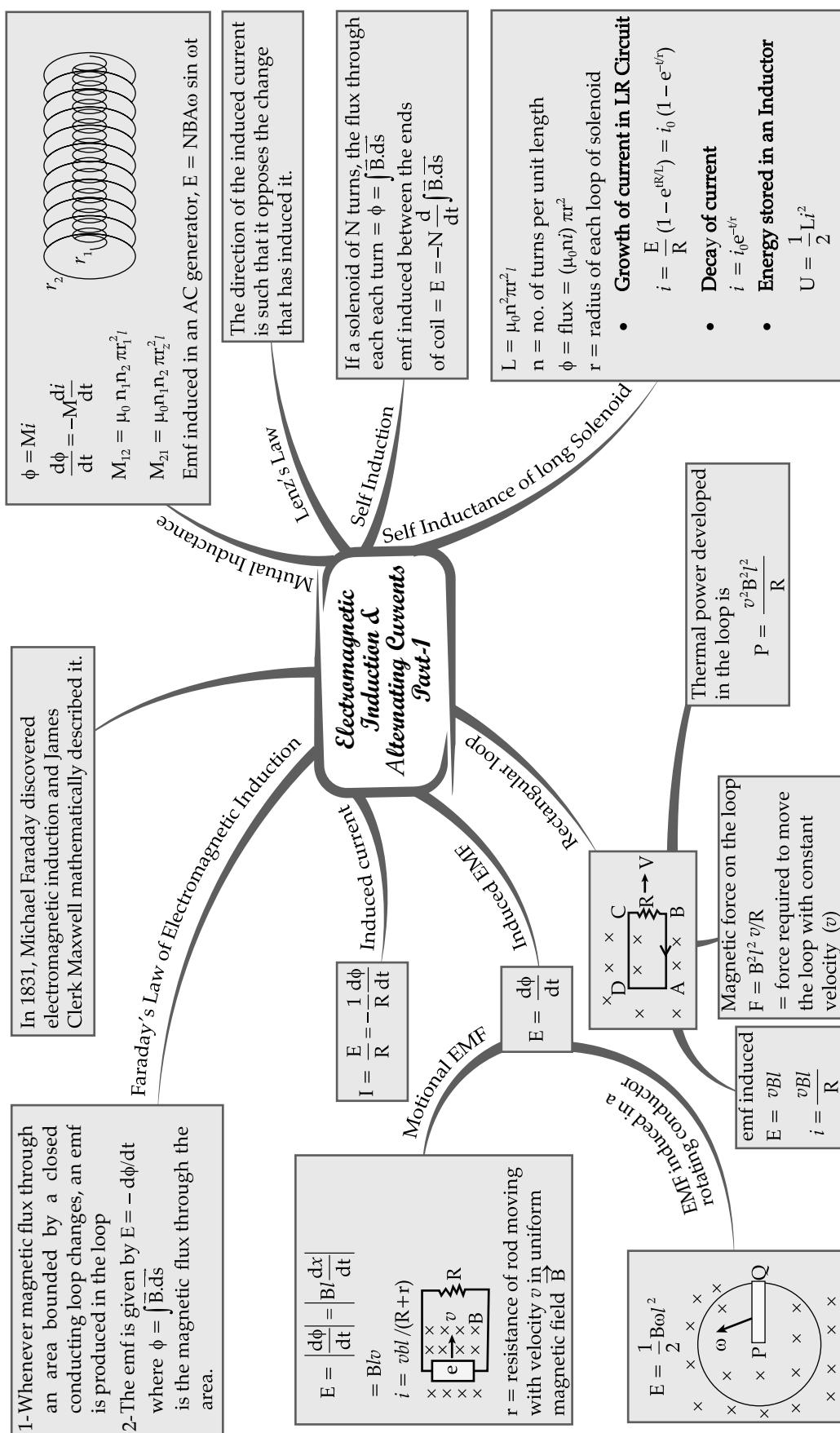


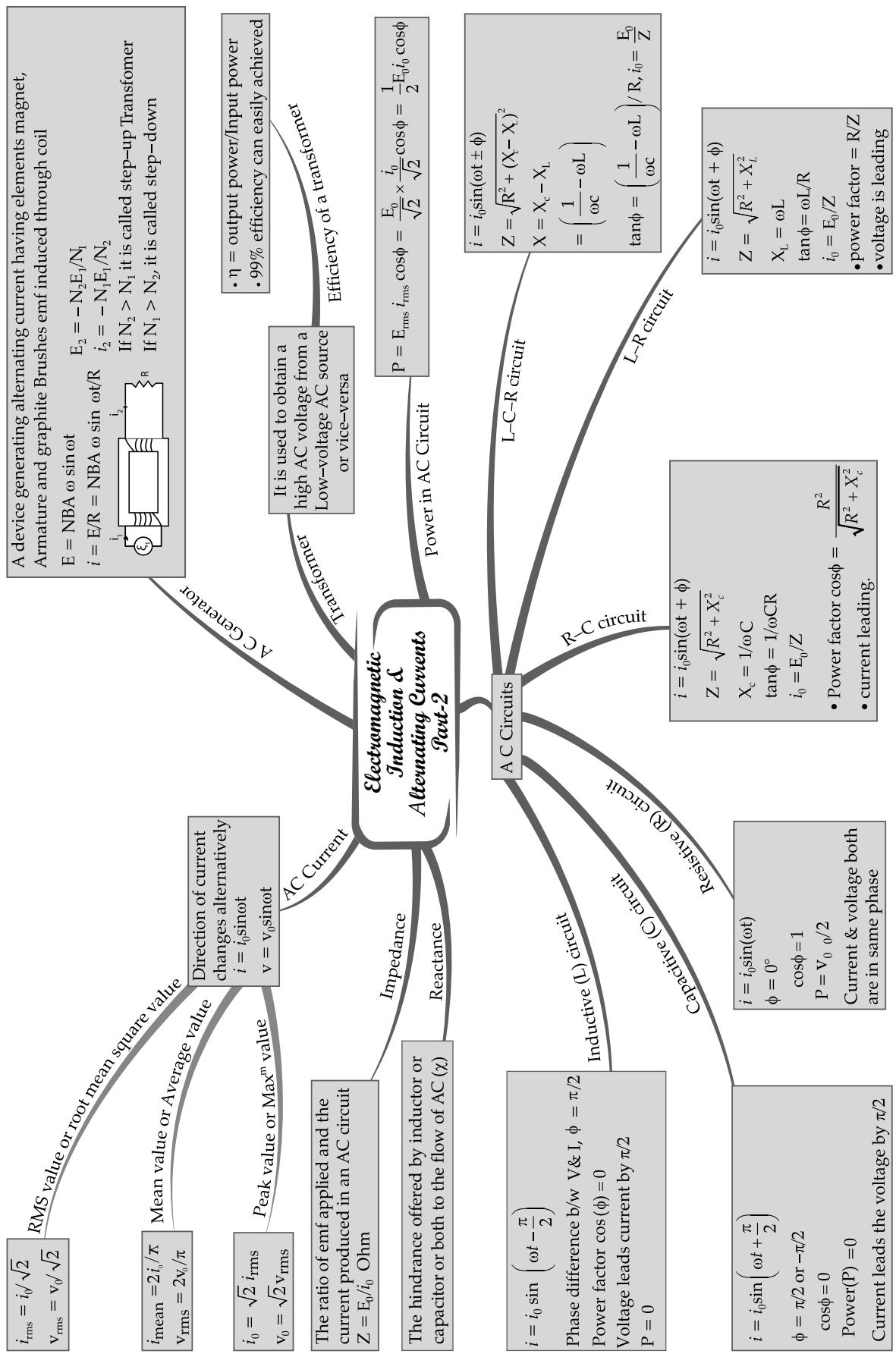


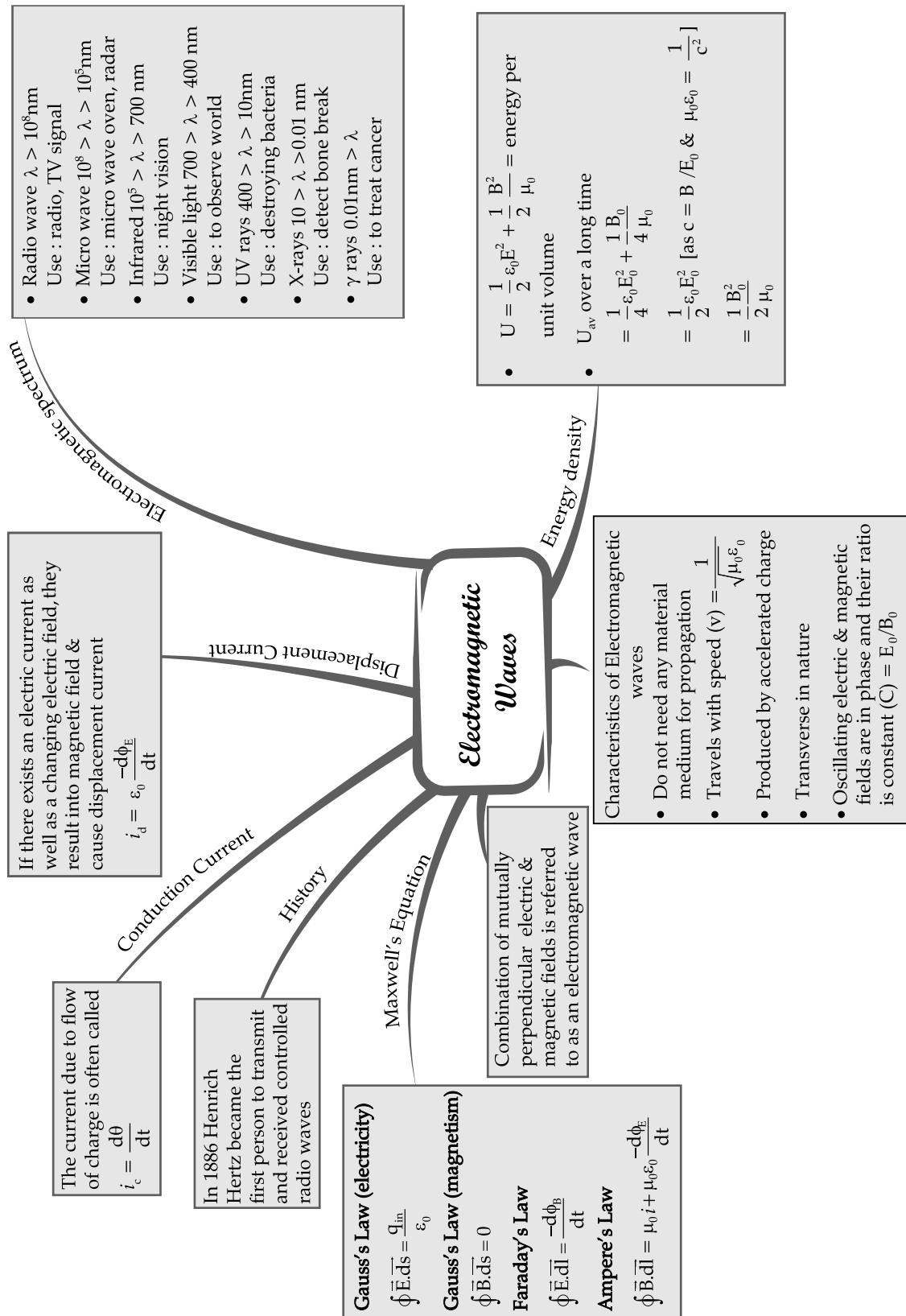


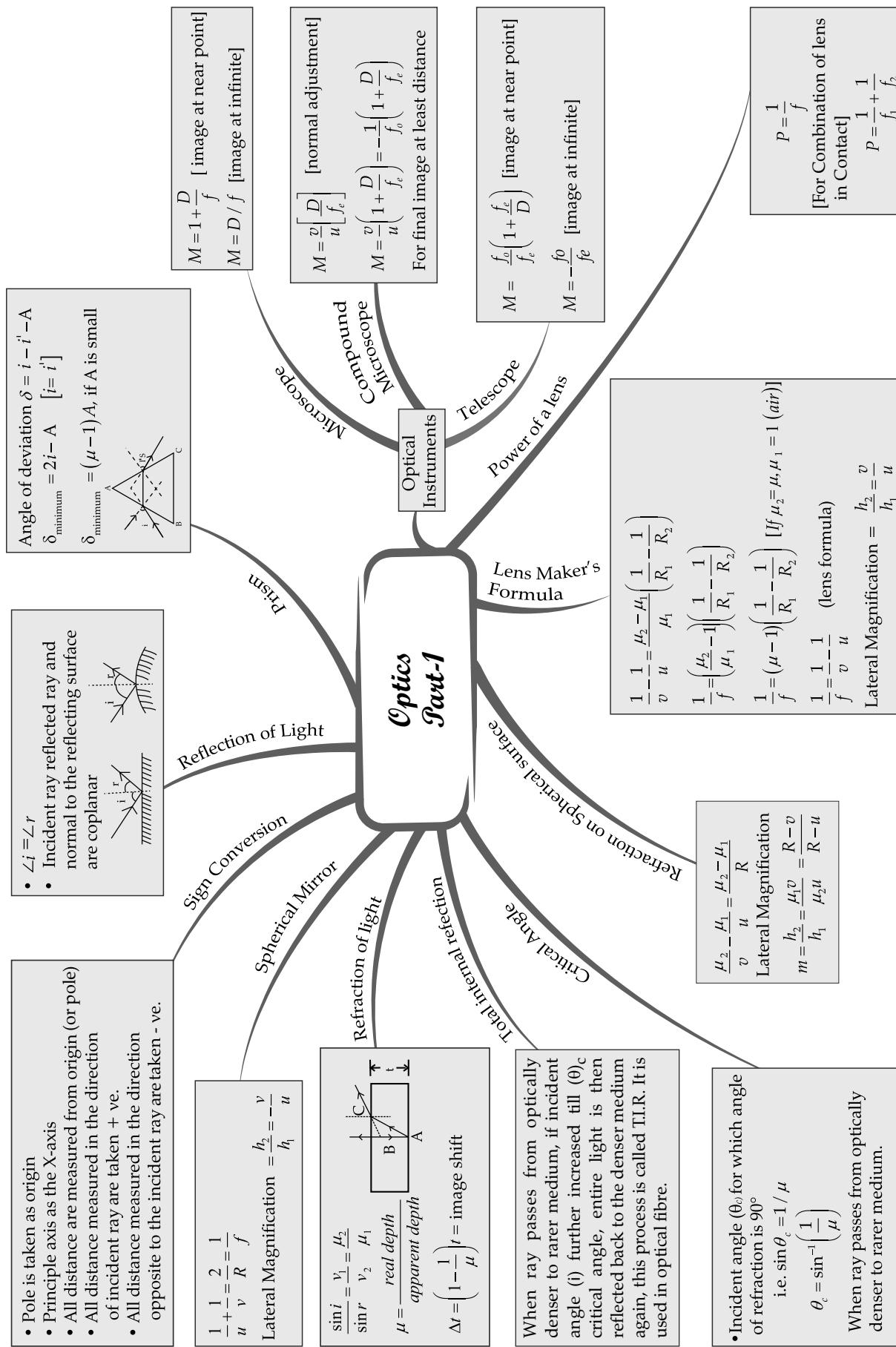


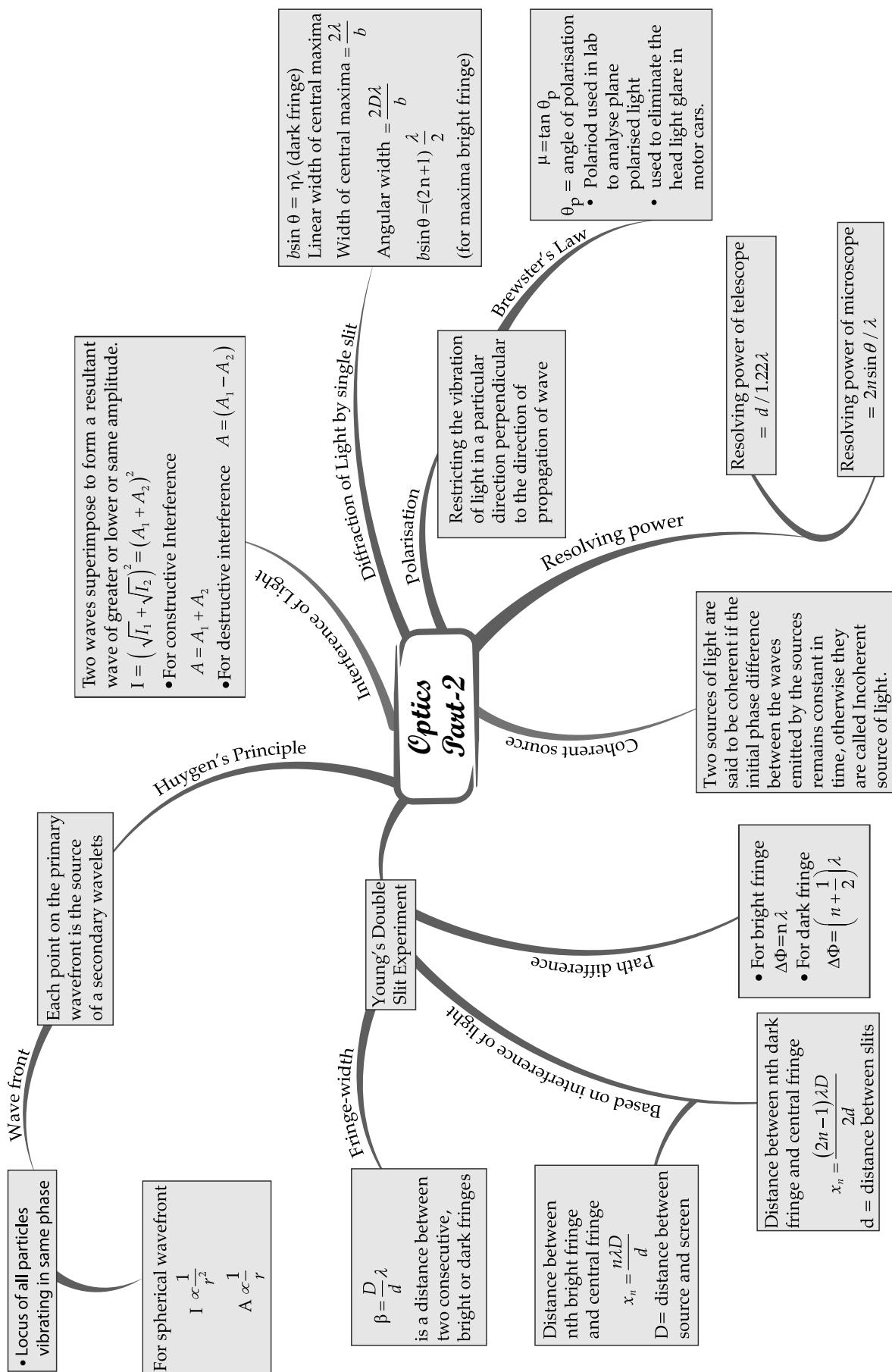












Einstein, after an average academic career put forward quantum theory of light in 1905 while working as a grade III technical officer in a patent office.

- Light has both wave character as well as particle
- Interference can be explained by wave nature
- When light is of sufficiently low wavelength, it behaves as particle
- Light particles having definite energy and definite linear momentum are called "photons"
- Energy of each photon =  $h\nu = hc/\lambda$
- Momentum of each photon =  $h/\lambda = E/c$

**Dual Nature of Radiation**

**photoelectric effect**

## Dual Nature of Matter & Radiation

All matter can exhibit wave-like behaviour e.g. beam of electrons can be diffracted like a water wave

Waves

Contribution

**de-Broglie relation**

$$\lambda = \frac{h}{p}$$

$\lambda$  = wavelength associated with particle or de-Broglie wavelength  
 $p$  = momentum

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK_{max}}}$$

A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Scattered beam of electrons is received by detector.

Results :  $\lambda = \text{de Broglie's wavelength}$

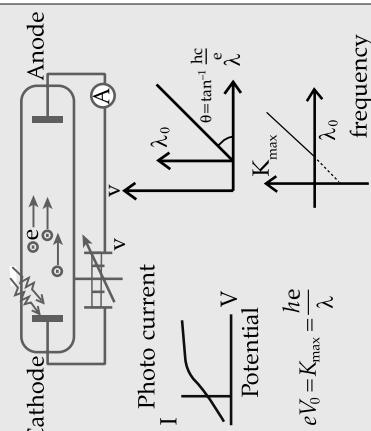
$$= h/p$$

$$= 1227/\sqrt{v} \text{ nm}$$

$$= 1227/\sqrt{54} \text{ nm}$$

$$= 0.167 \text{ nm} = 1.67 \text{ \AA}$$

This experiment verifies the wave nature of electrons & relation with de-Broglie wavelength.



**Hertz and Lenard's Observation**

**Davisson-Germer Experiment**

**Einstein's Photoelectric equation**

**Photoelectric Effect**

**Dual Nature of Radiation**

**Contribution**

**de-Broglie relation**

**Photo current**

**Potential**

**Frequency**

- $K_{max} = E - \phi = eV_0$
- If  $\lambda = \lambda_0 = hc/\phi$   
 $K_{max} = 0, i.e.$   
Electron may just come out.
- If  $\lambda > \lambda_0$   
i.e.  $E < \phi$   
no electron will come out
- If  $\lambda \leq \lambda_0$   
Photoelectric effect takes place this  $\lambda_0$   
•  $\lambda_0$  = depends on metal used

- When light of sufficient small wavelength is incident on metal surface, electrons are ejected from the metal, the phenomenon is called photoelectric effect.
- Ejected electrons are called photoelectrons
- Minimum energy equal to work function ( $\phi$ ) must be given to an electron so as to bring it out of the metal

$$• K_{max} = E - \phi = eV_0$$

$$= \frac{hc}{\lambda} - \phi, \quad V_0 = \text{stopping potential}$$

$K_{max}$  = maximum kinetic energy of ejected electrons

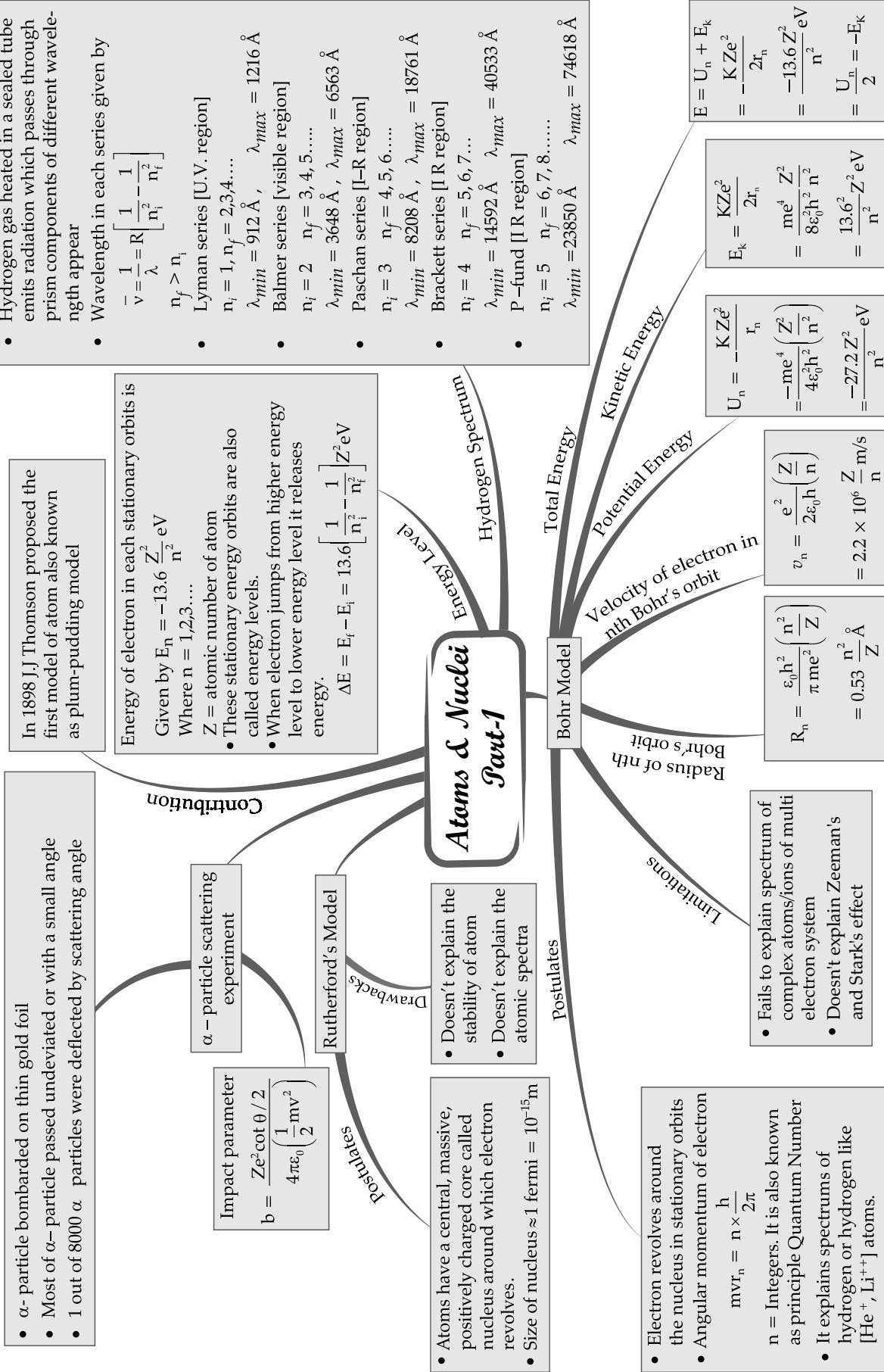
Here,  $\lambda_0 = hc/\phi$

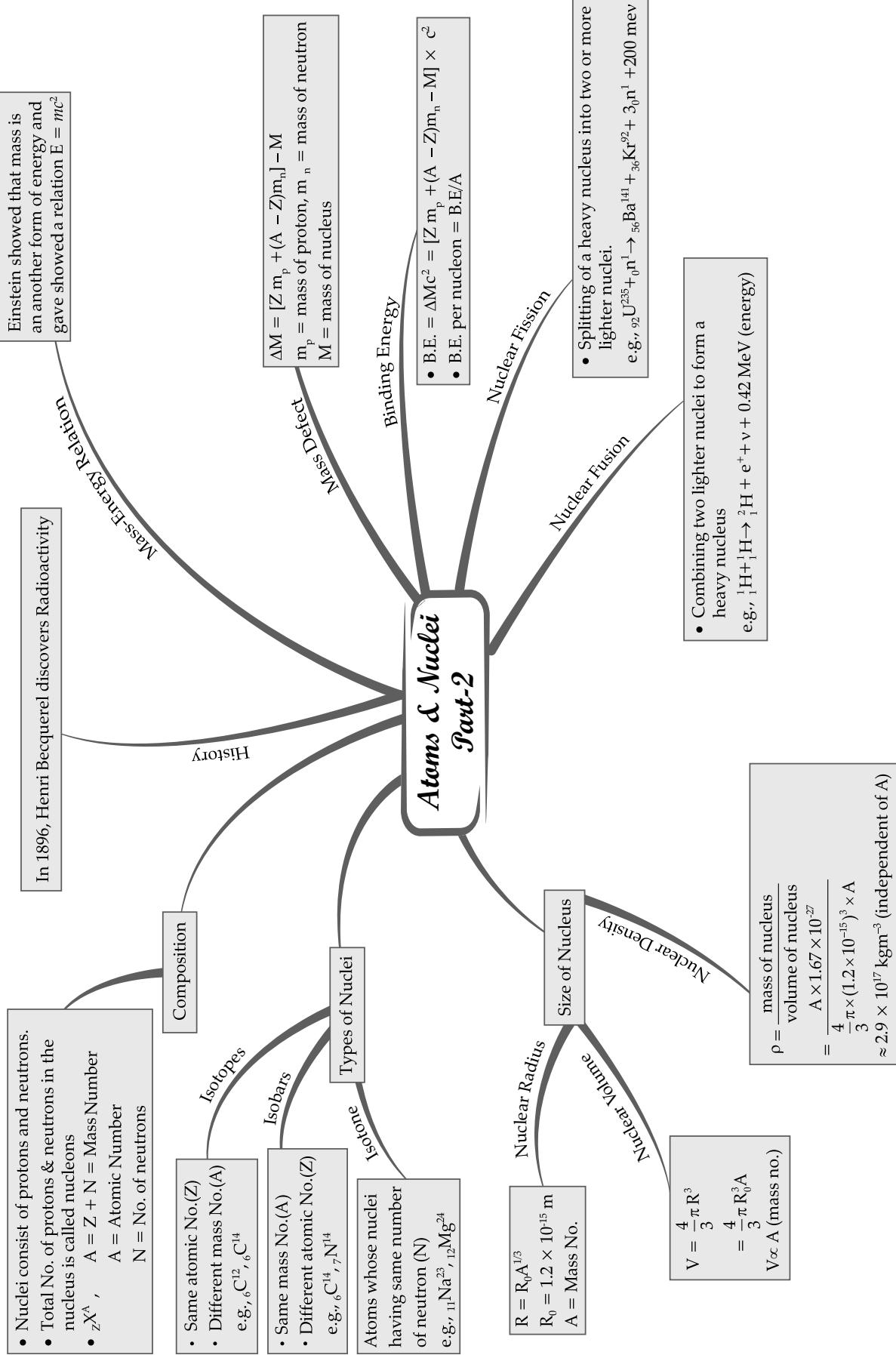
$\lambda_0$  = Threshold Wavelength

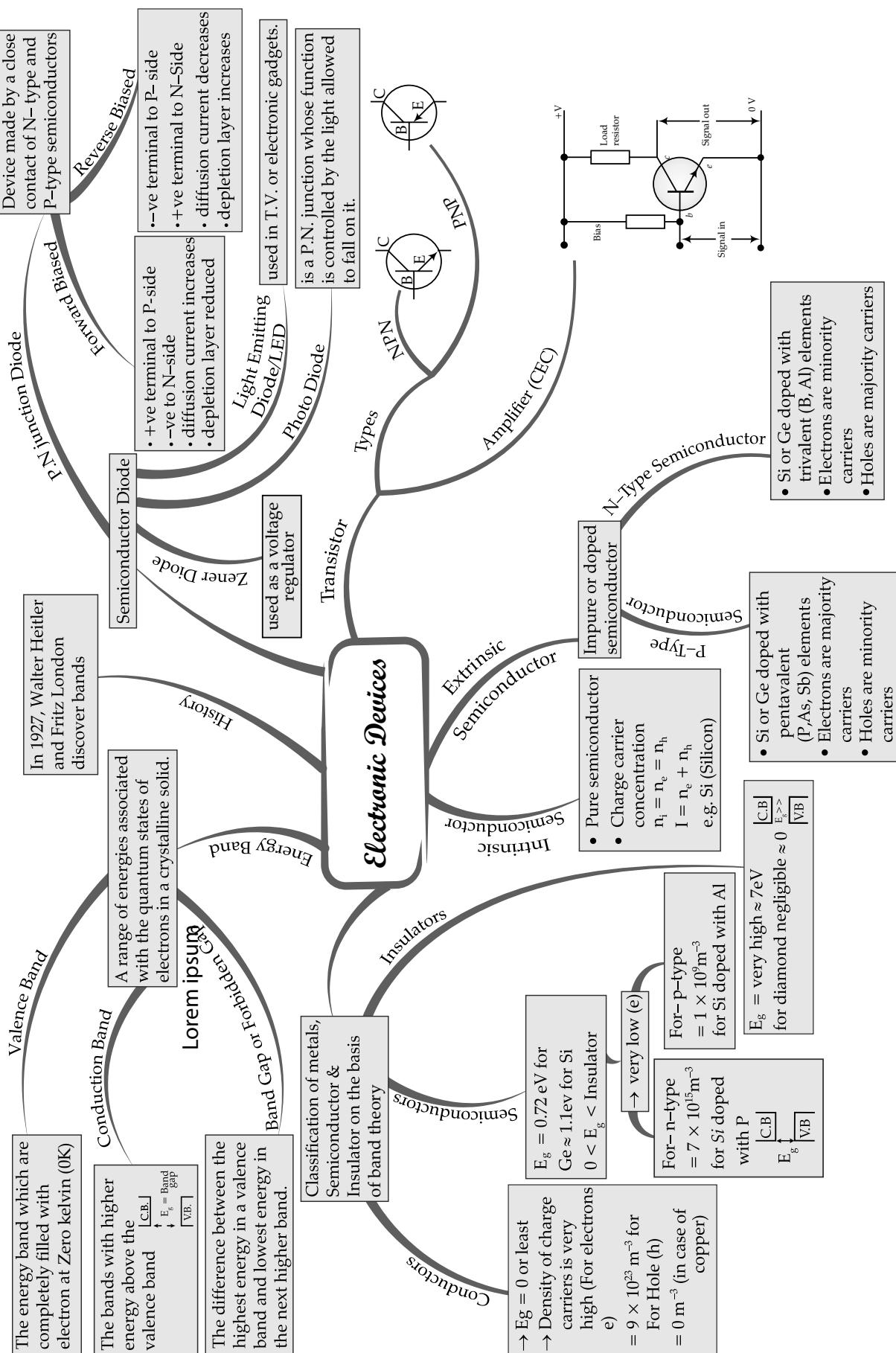
$$\lambda_0 = c/\lambda_0 = \phi/h$$

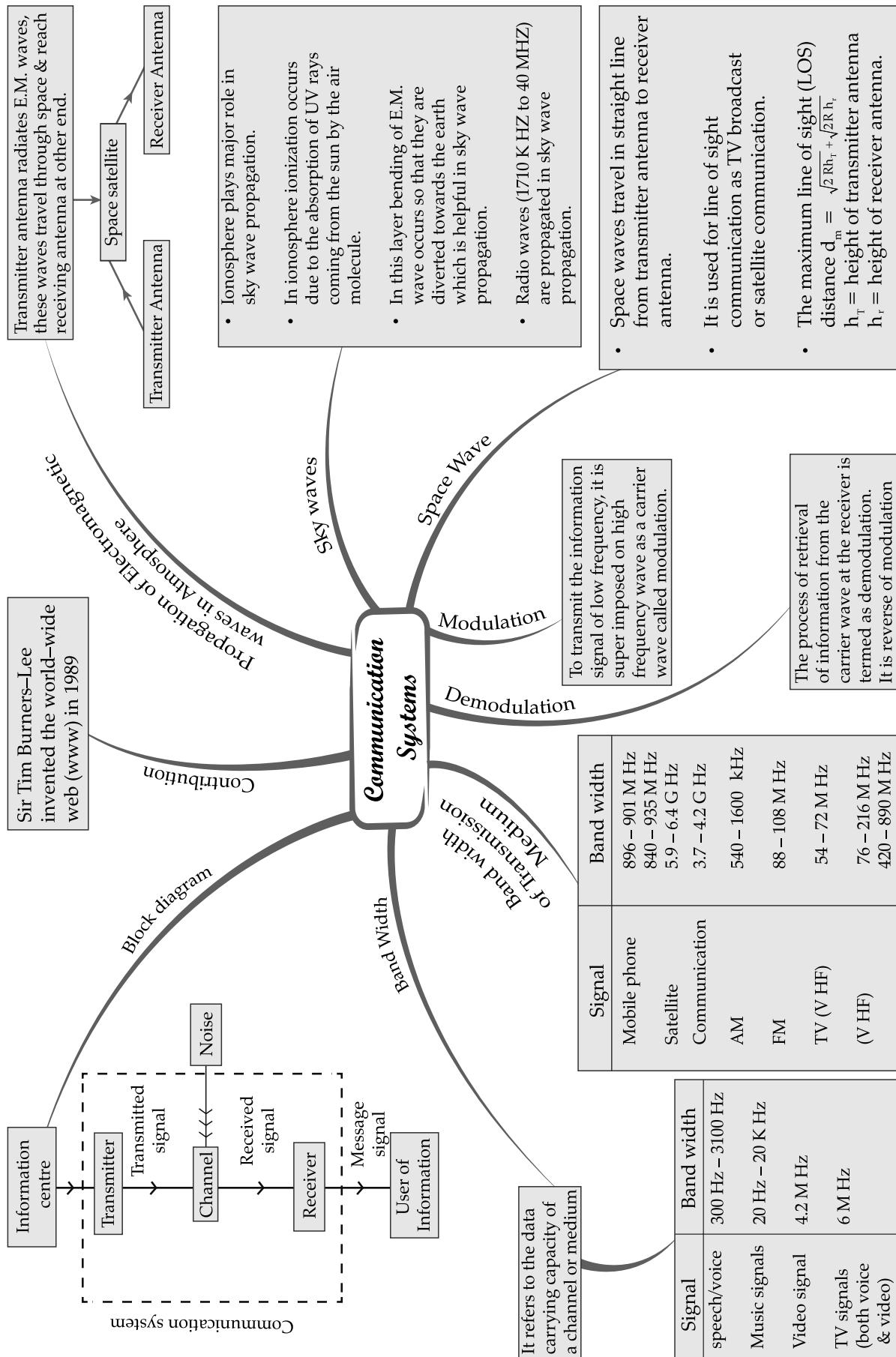
$\lambda_0$  = Threshold frequency

$$K_{max} = \lambda_0(\nu - \nu_0)$$



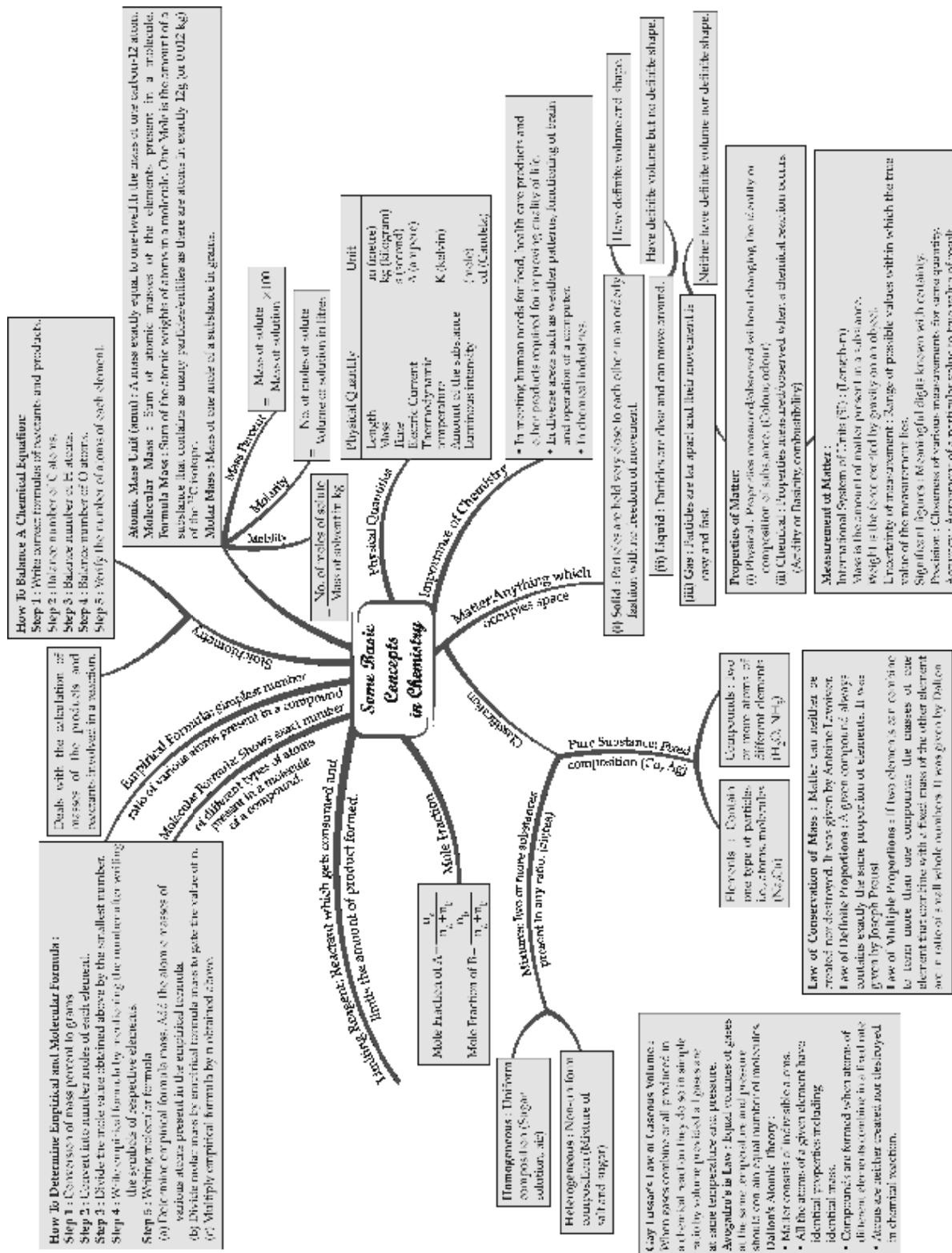


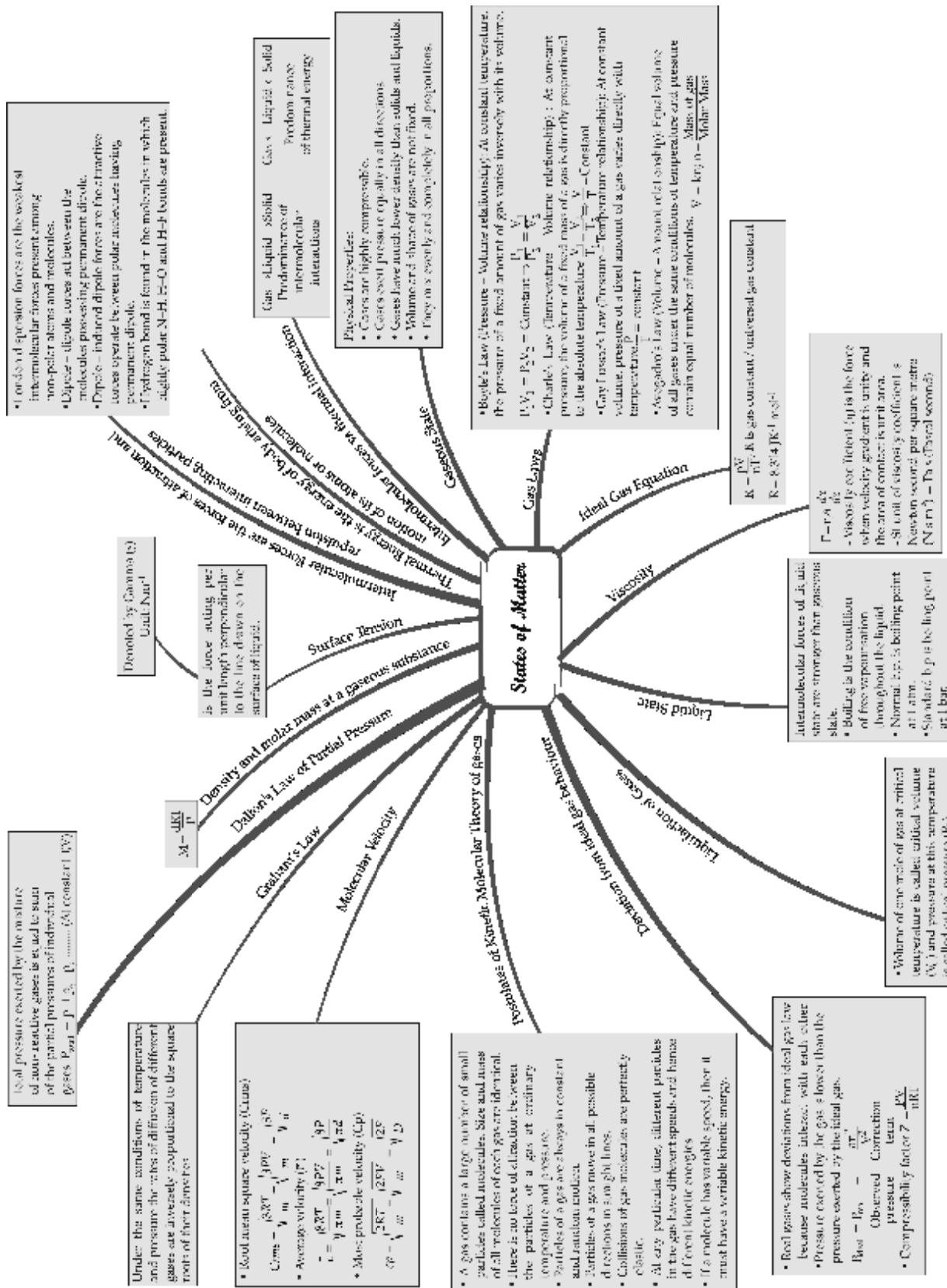


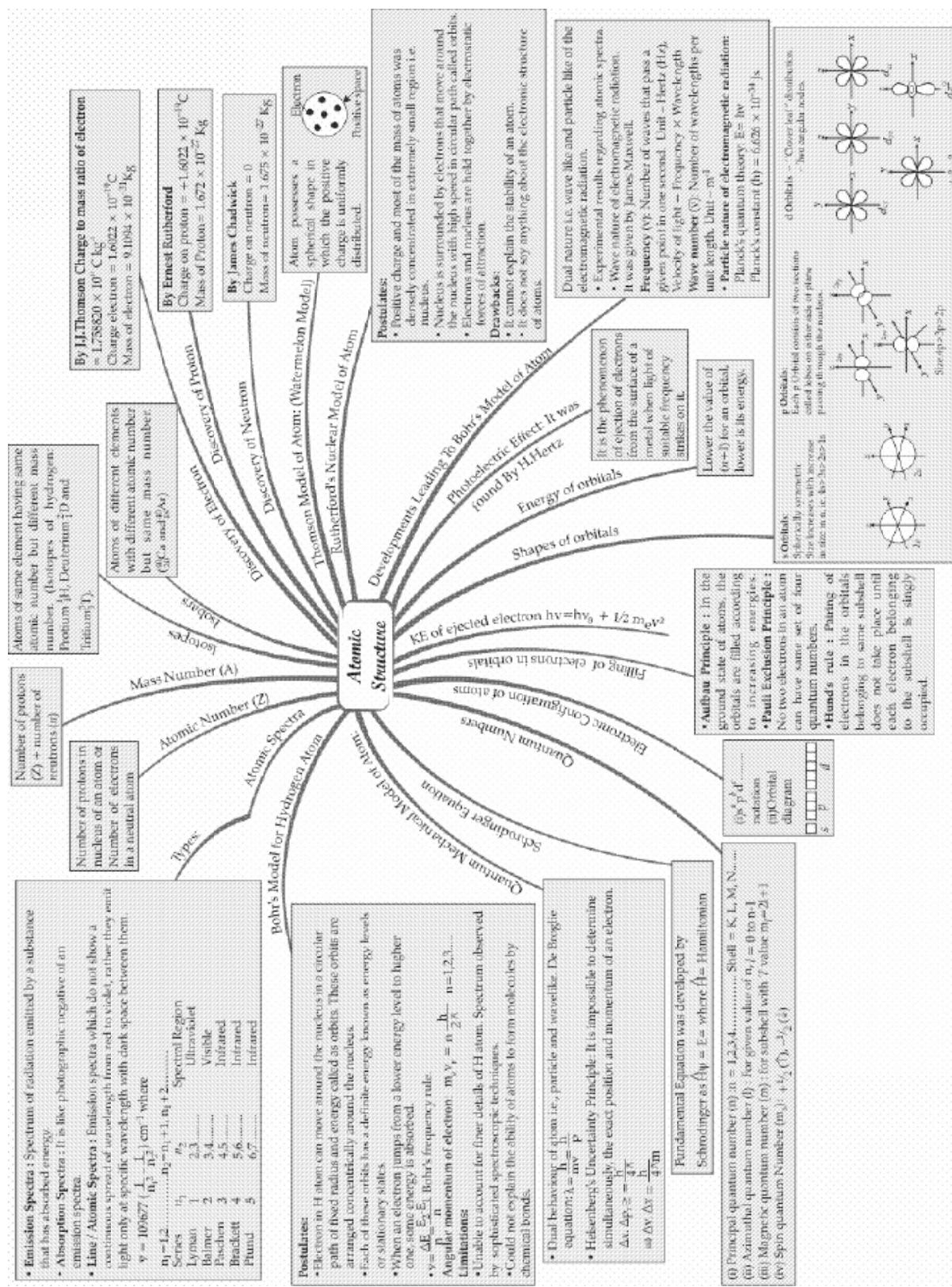


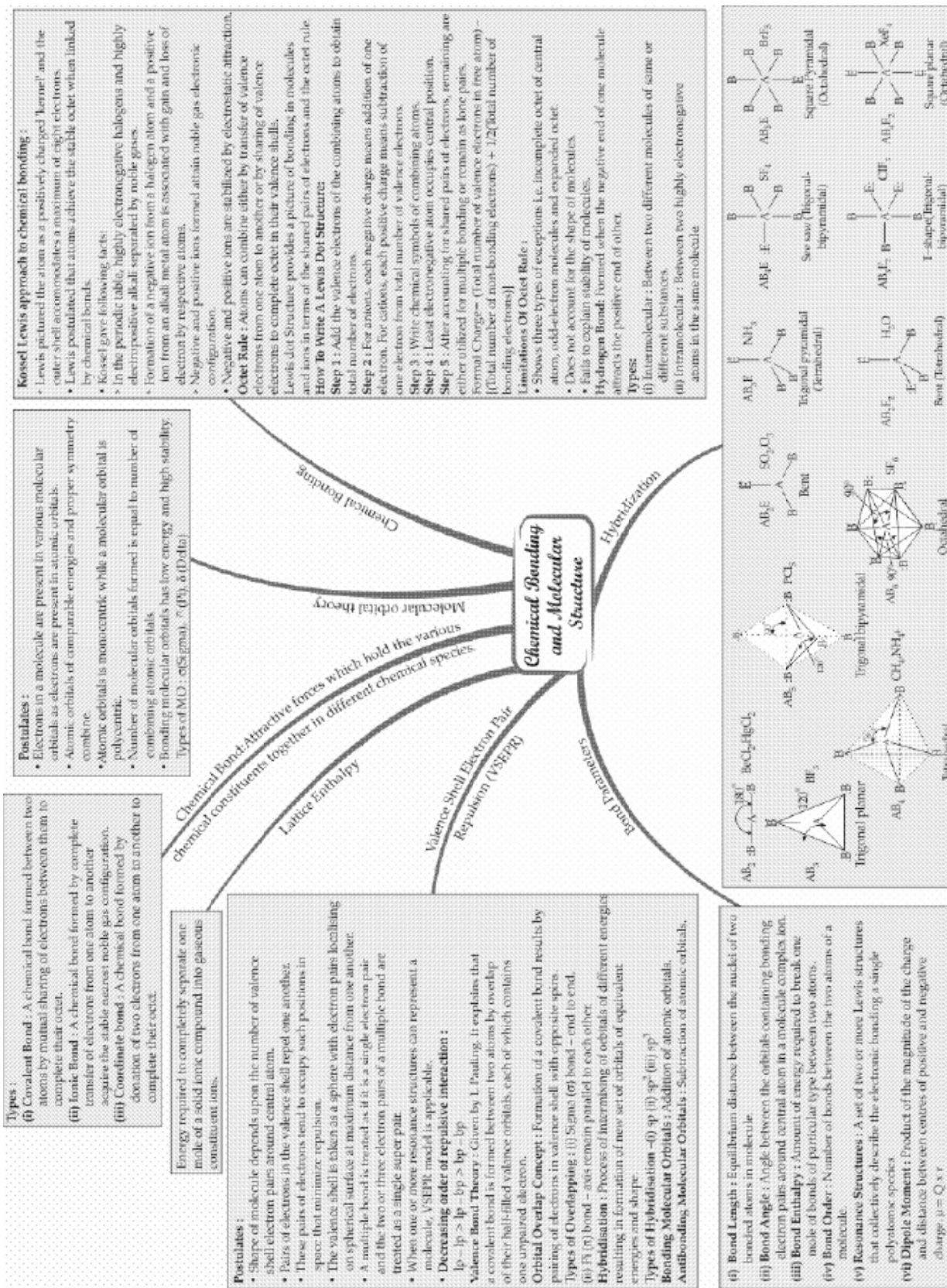


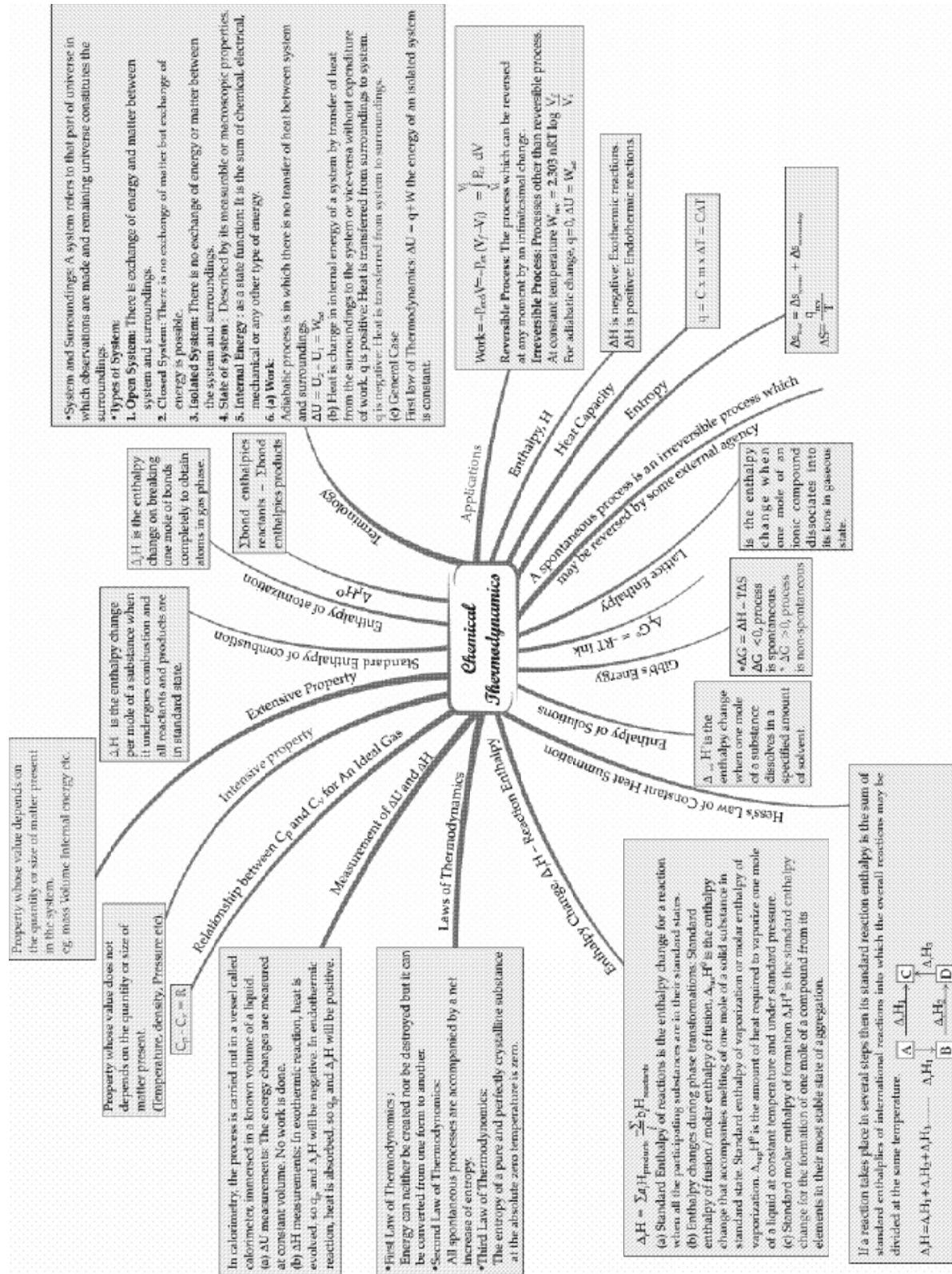
# **CHEMISTRY MIND MAPS & MNEMONICS**

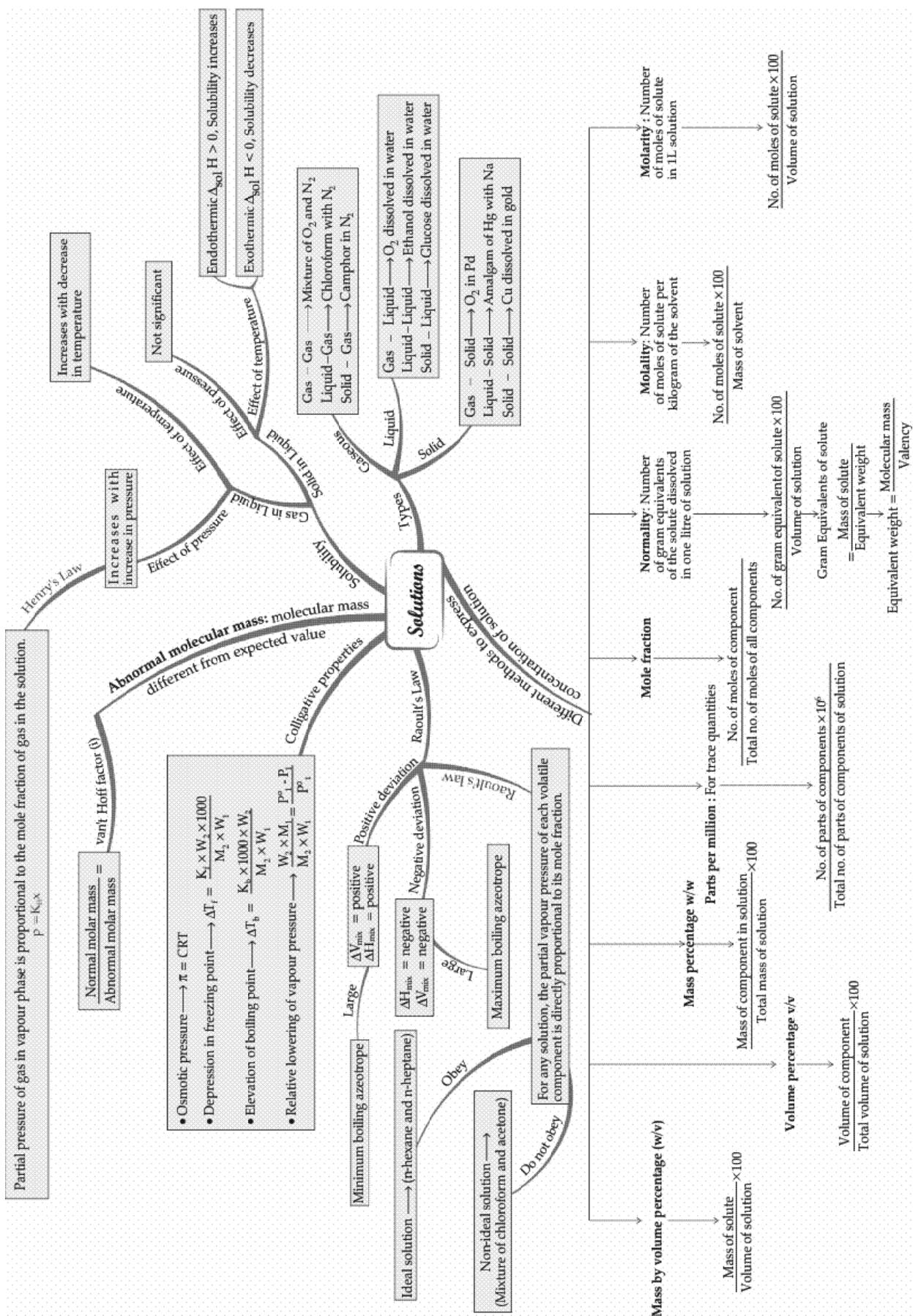


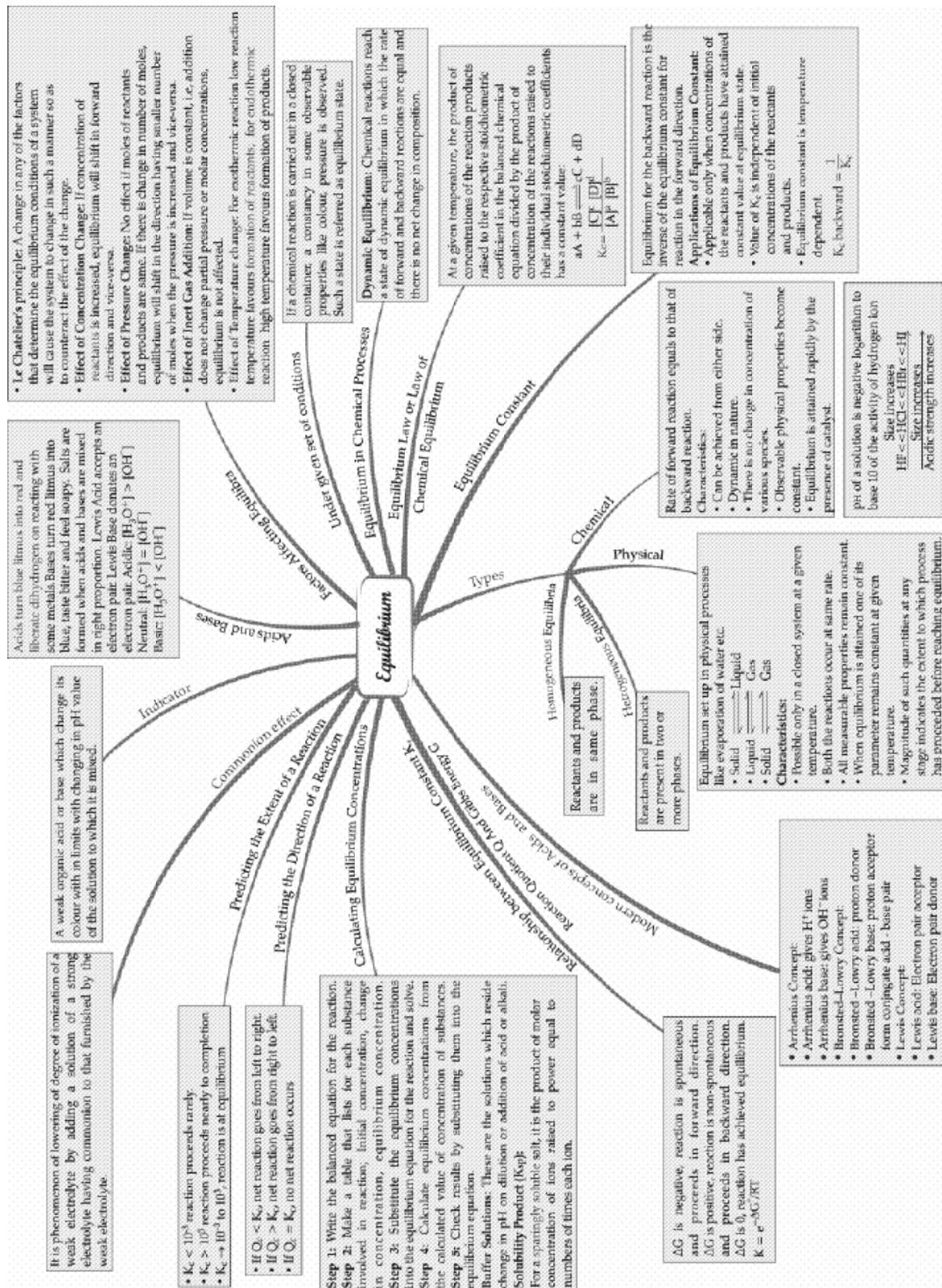


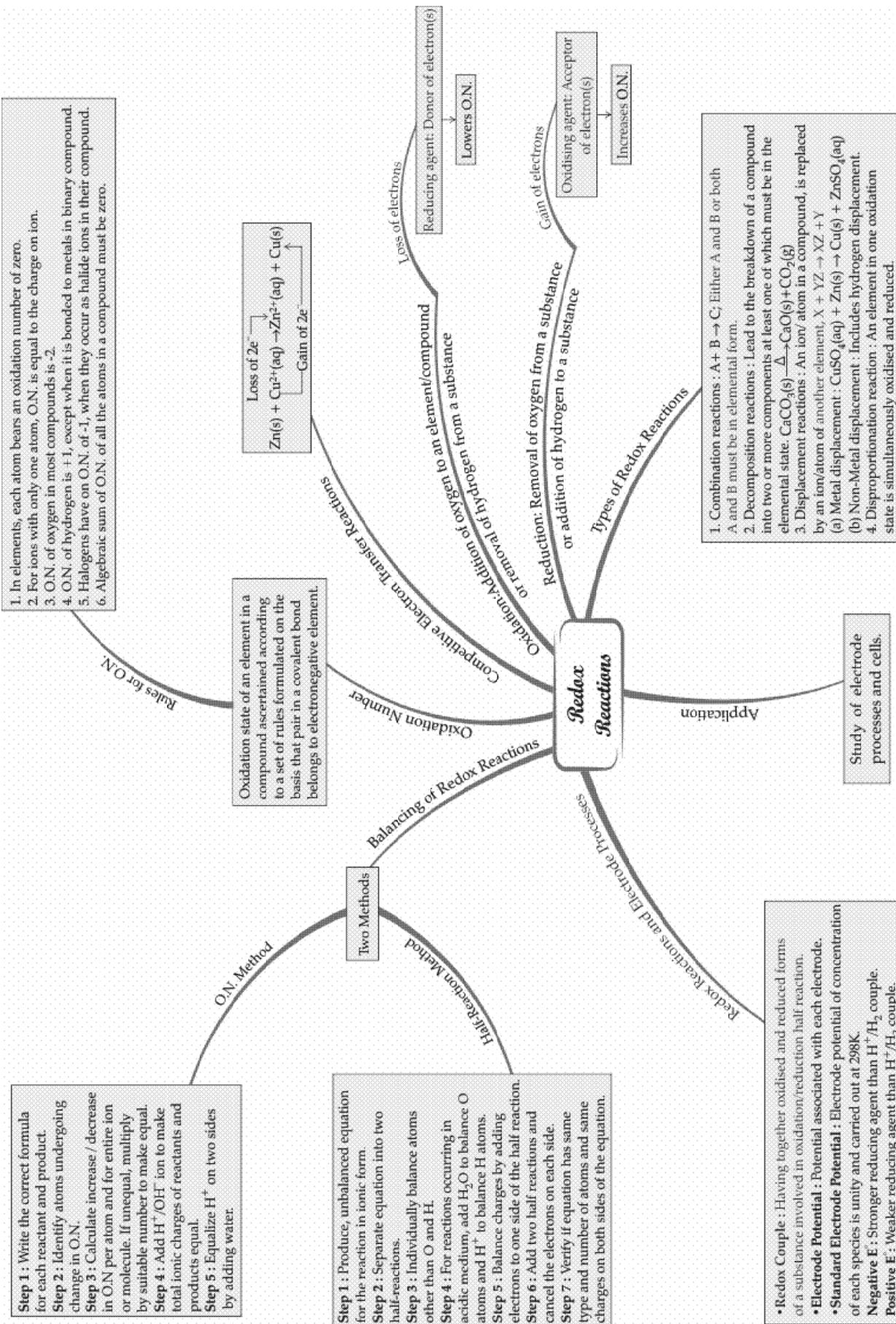


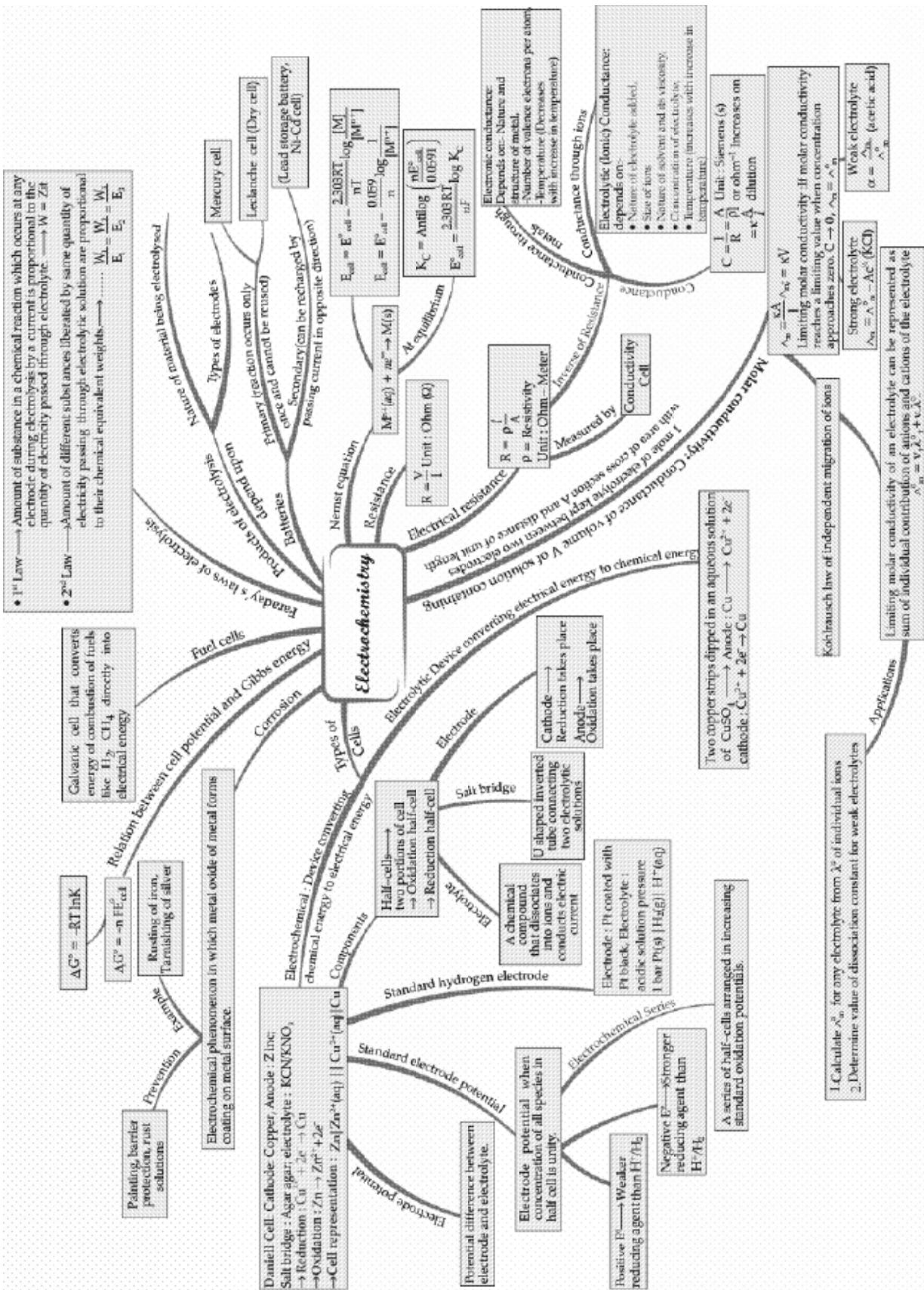


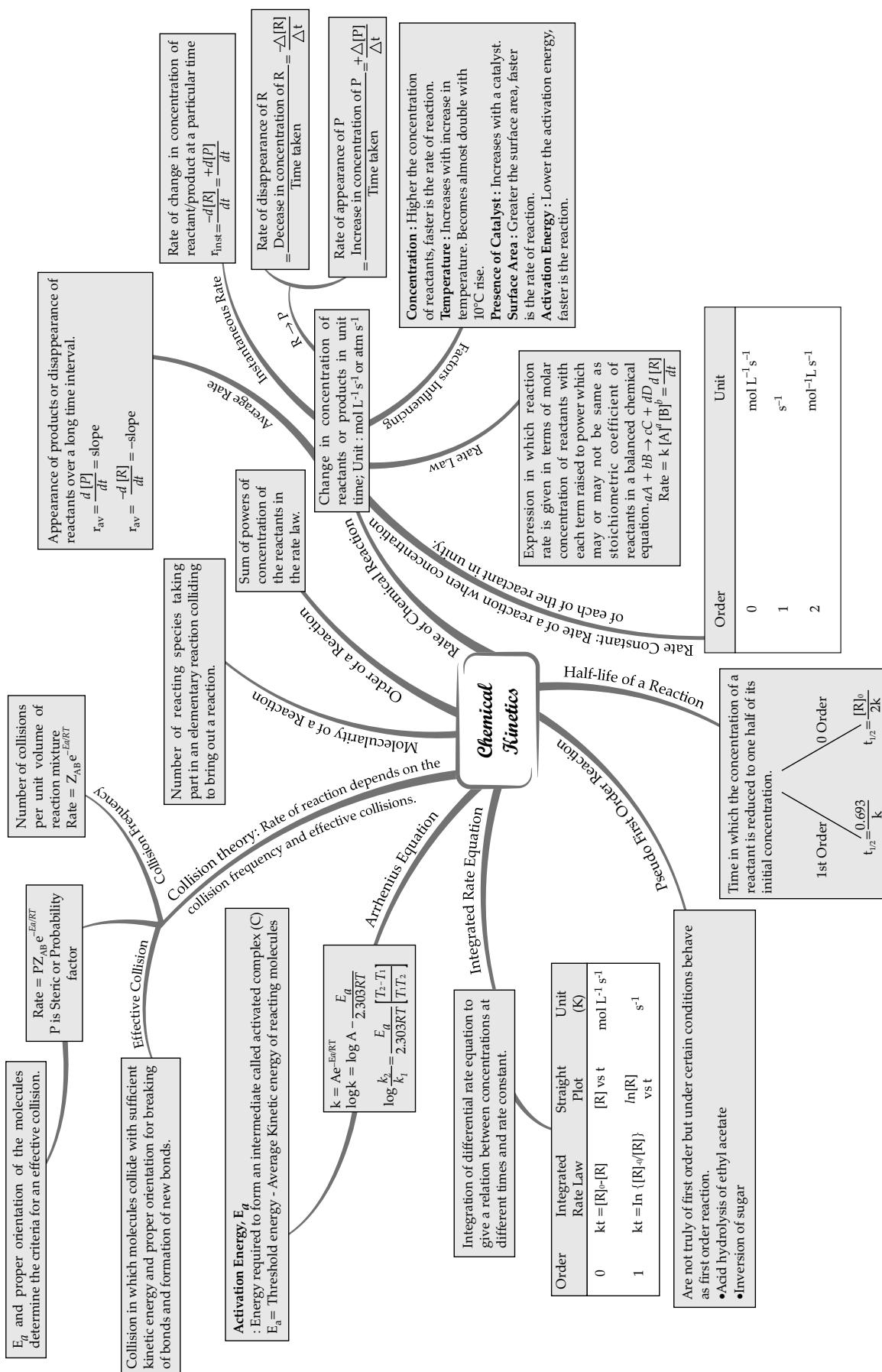


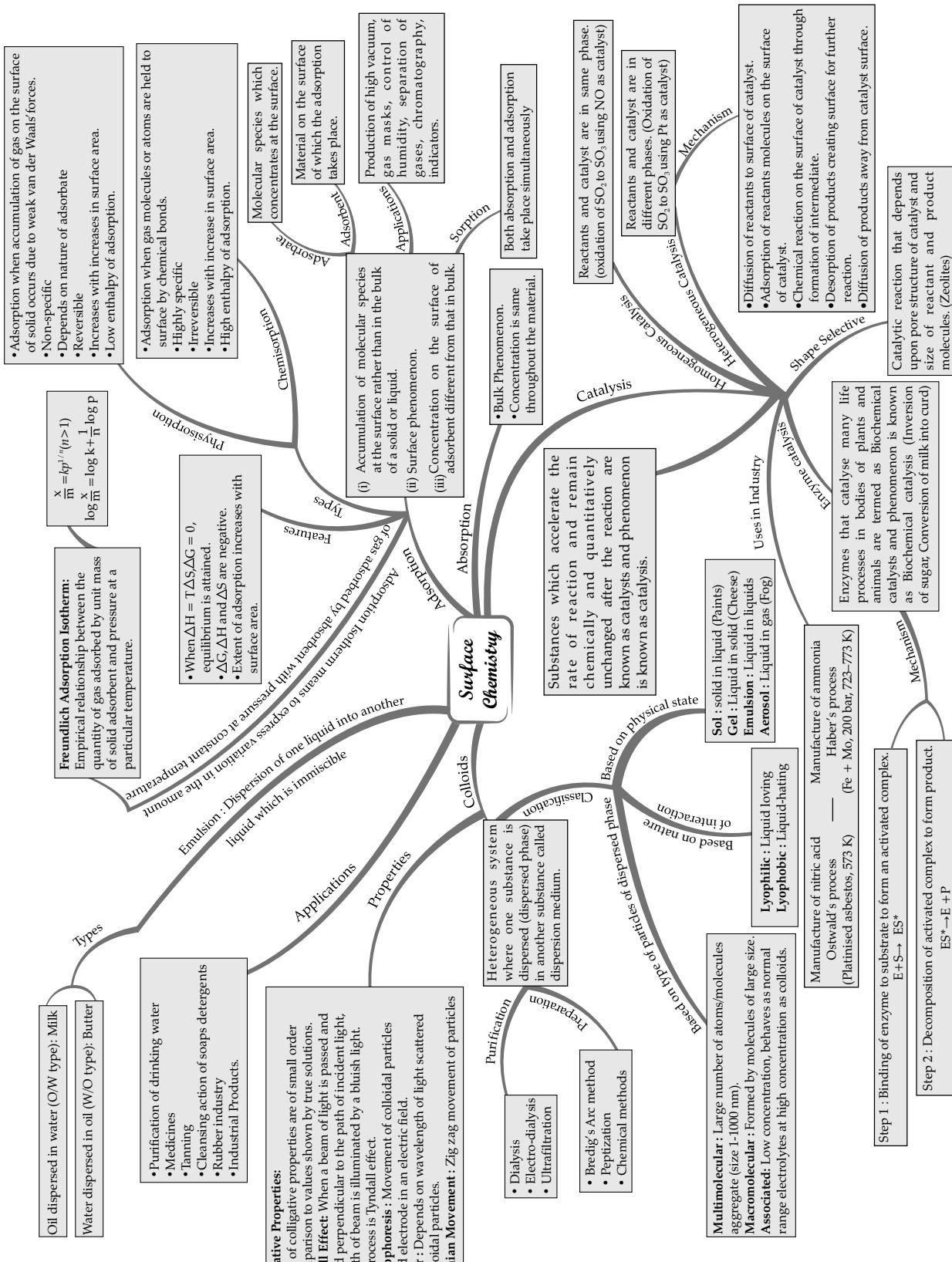


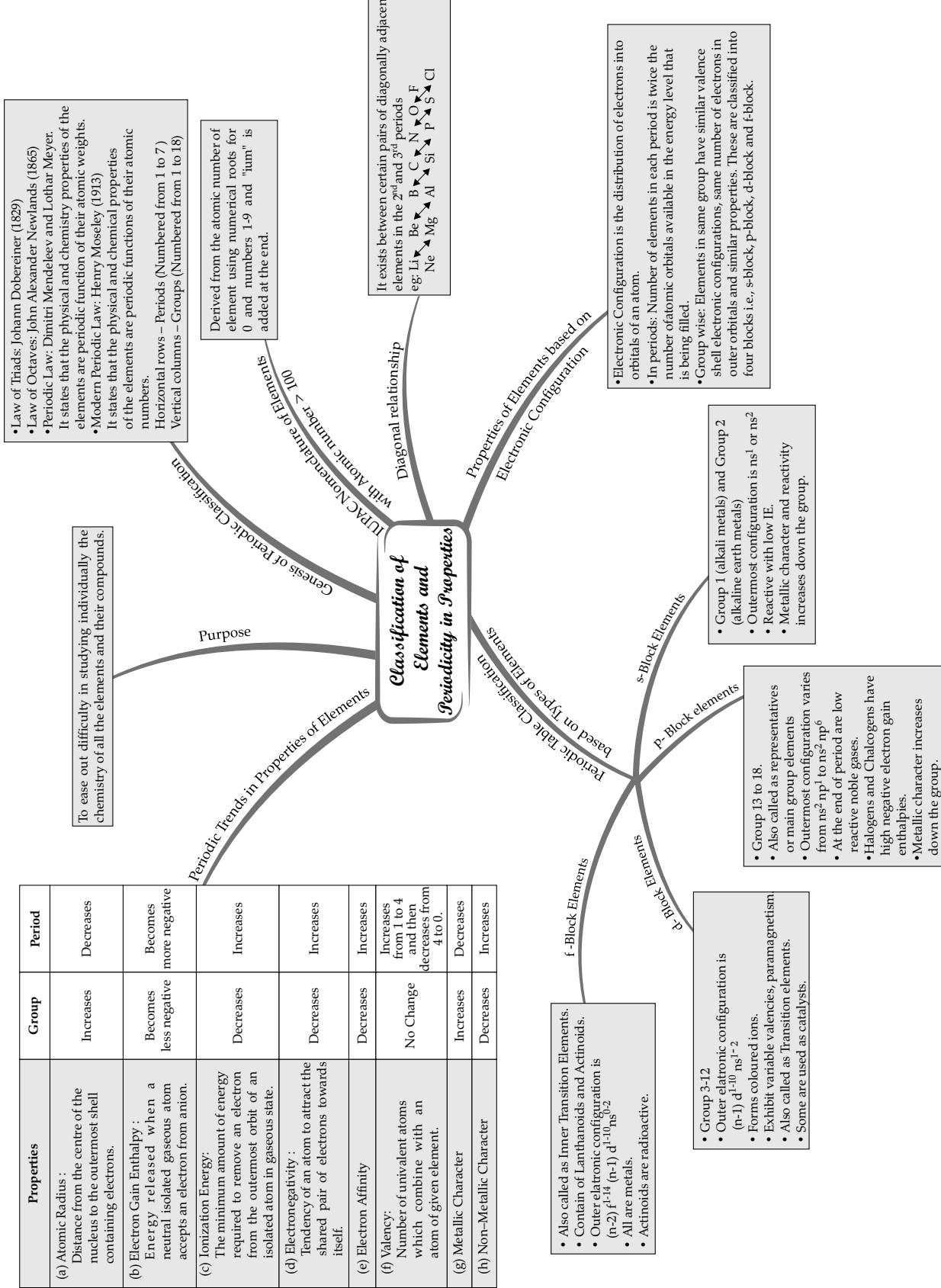


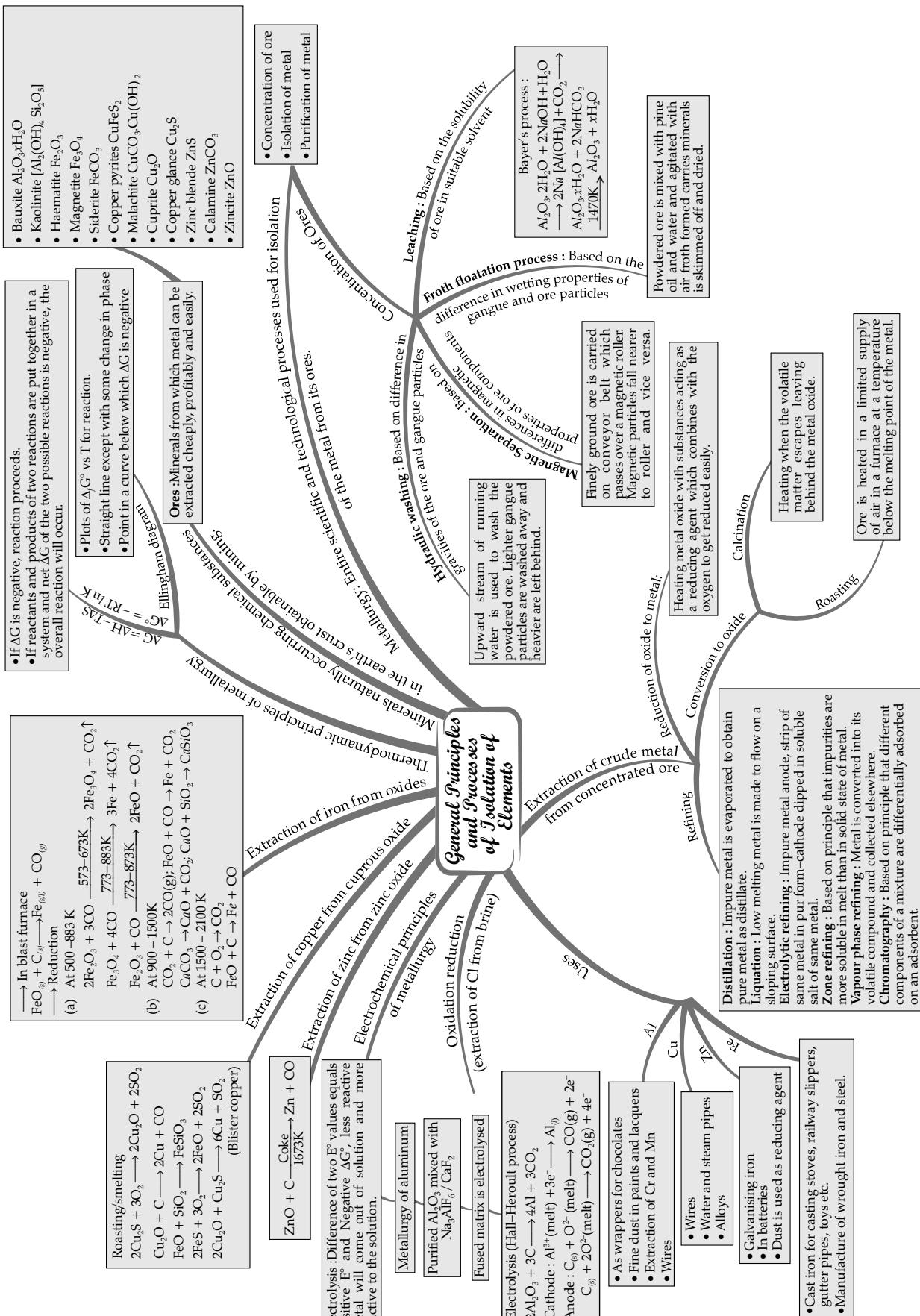


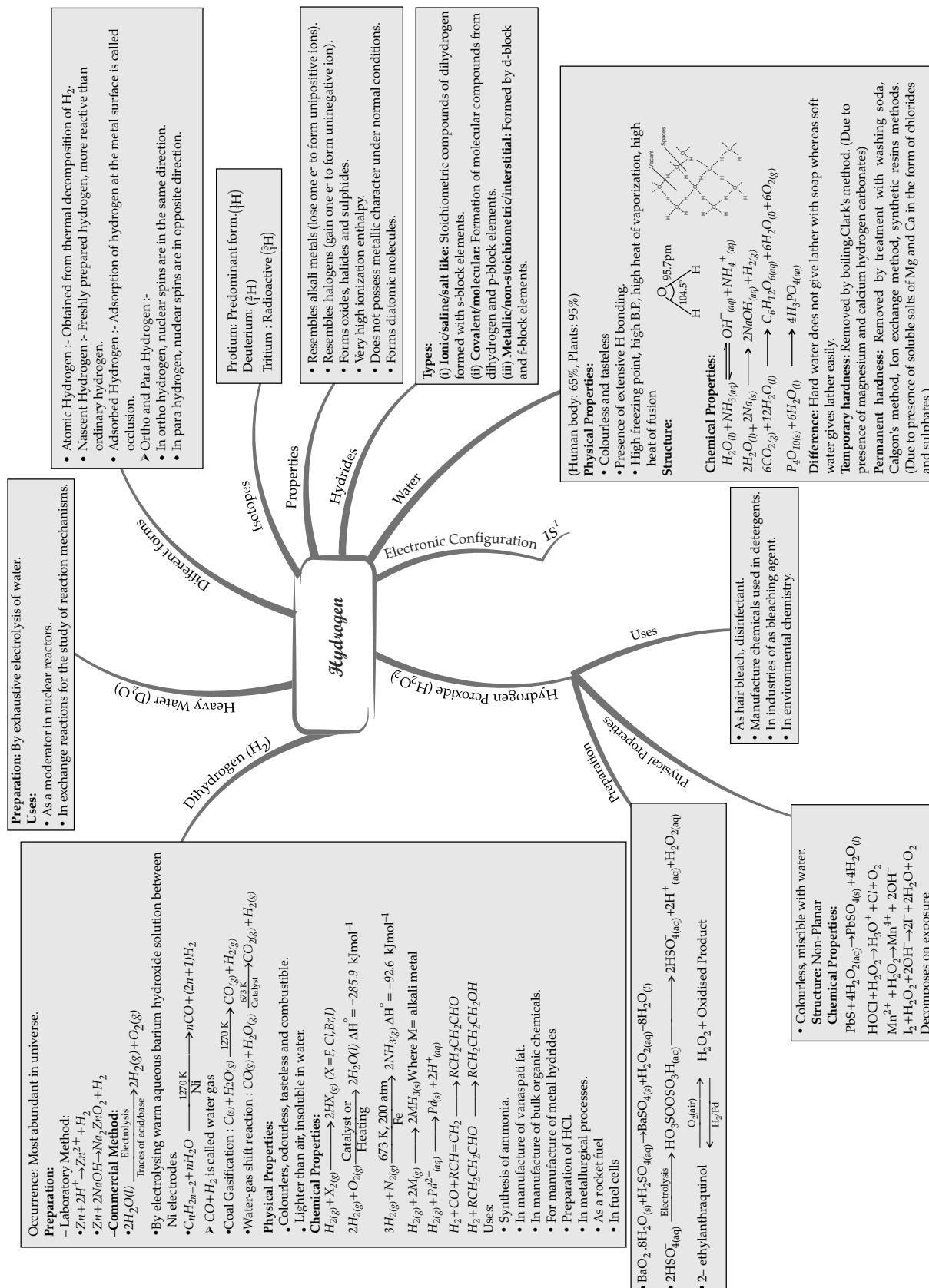


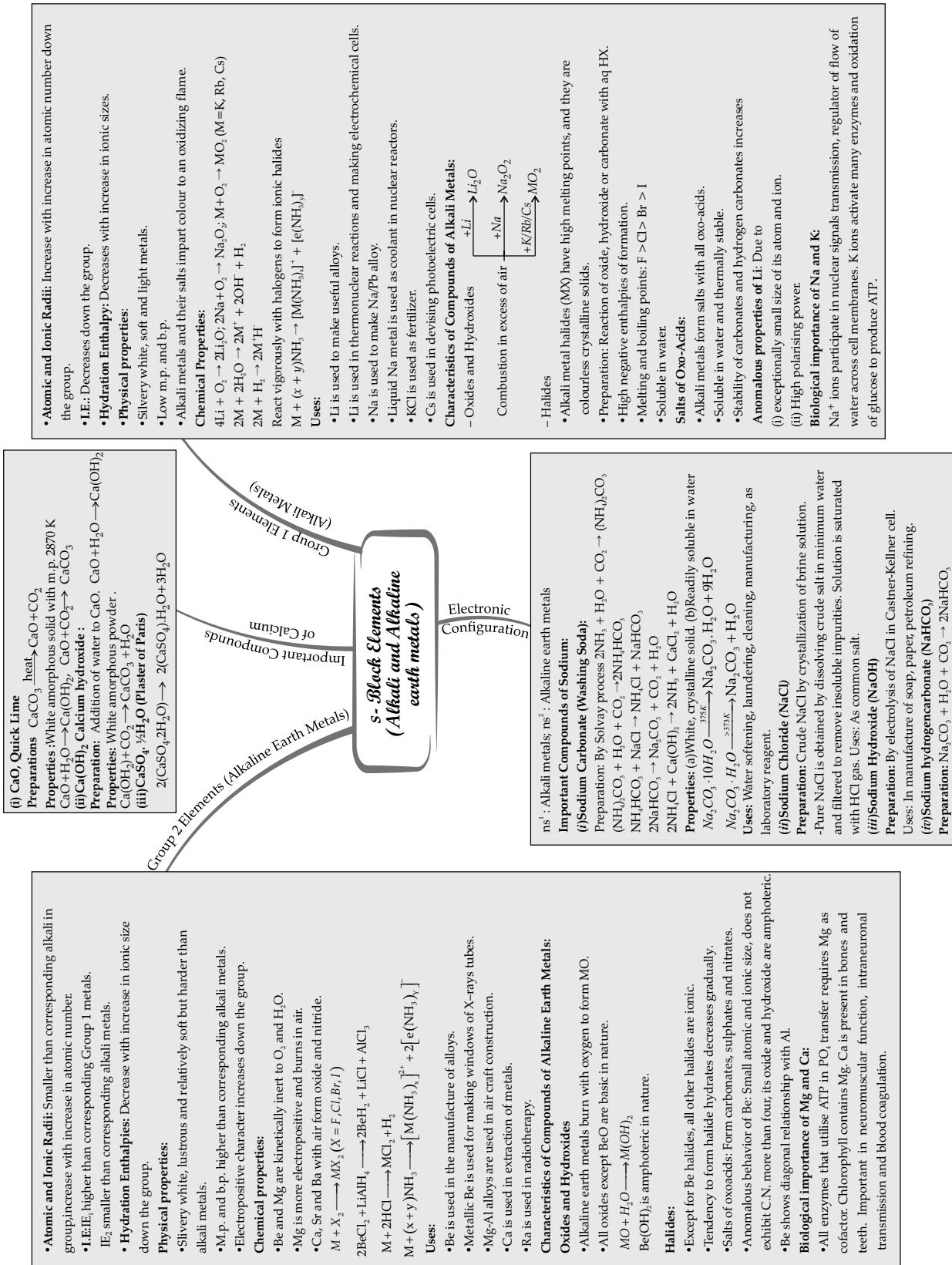


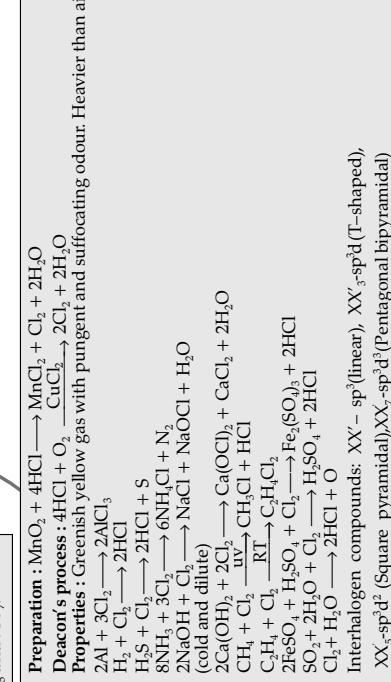
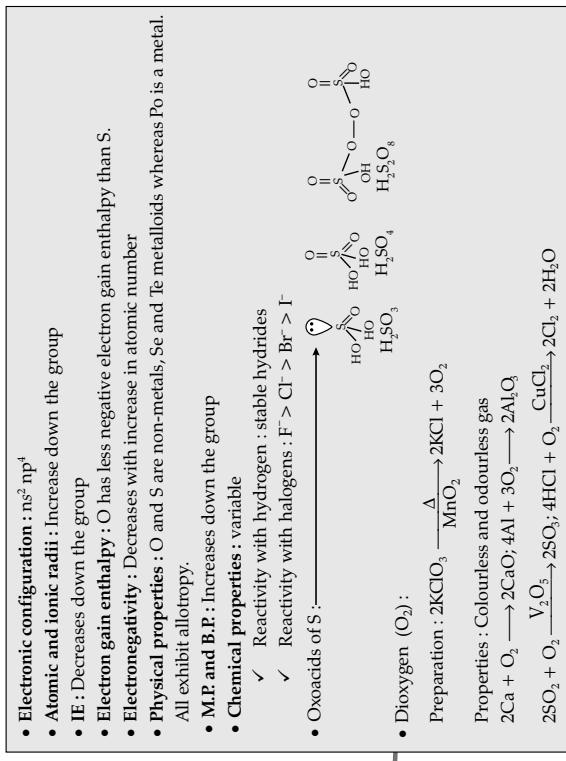
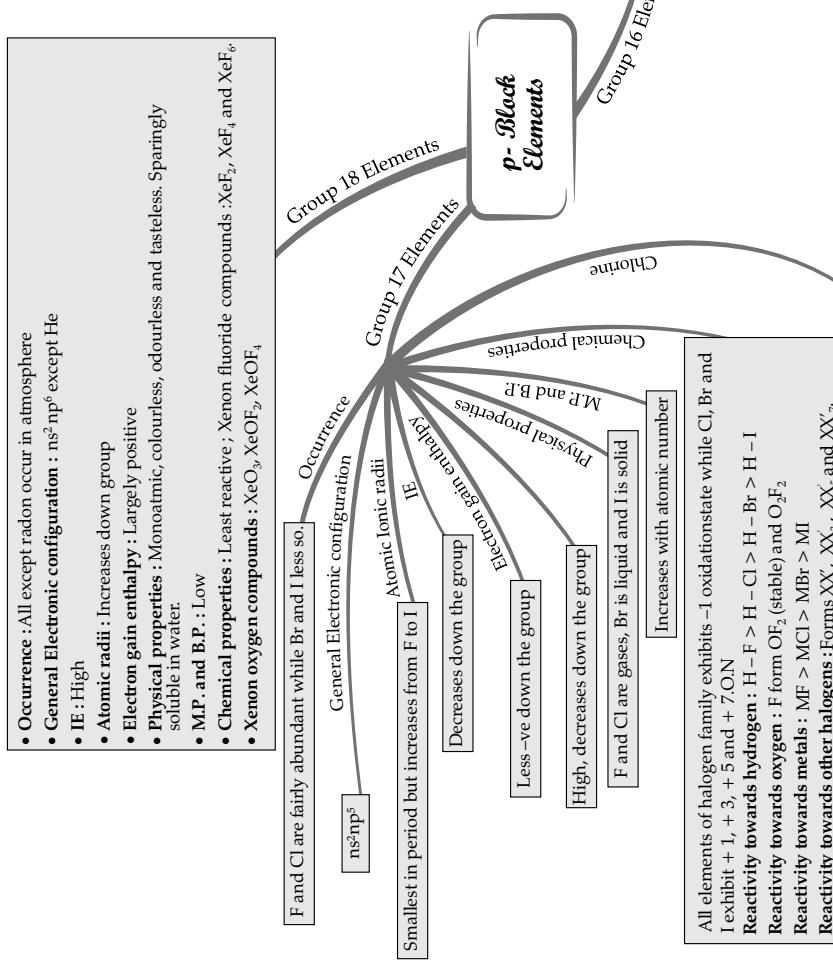


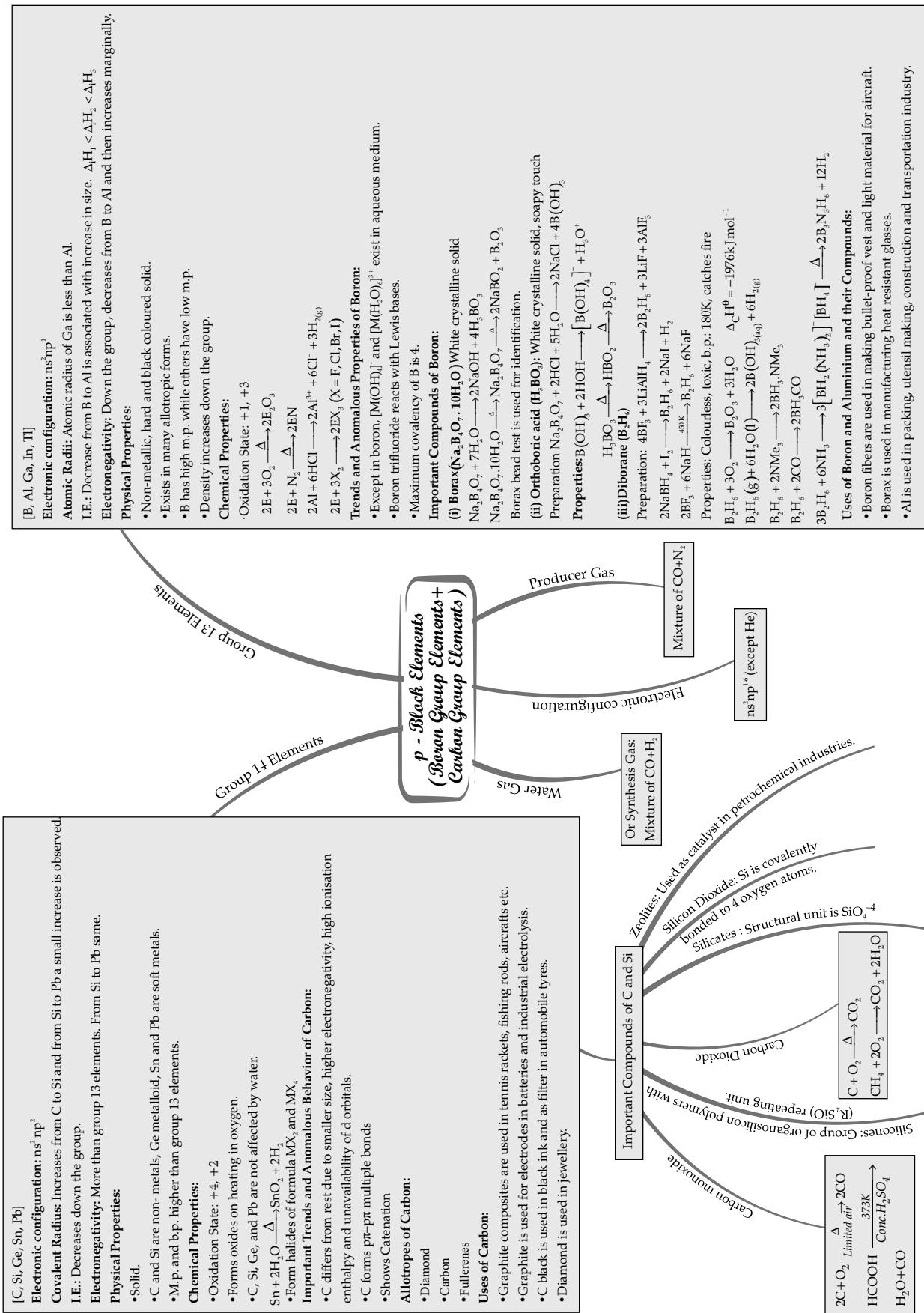


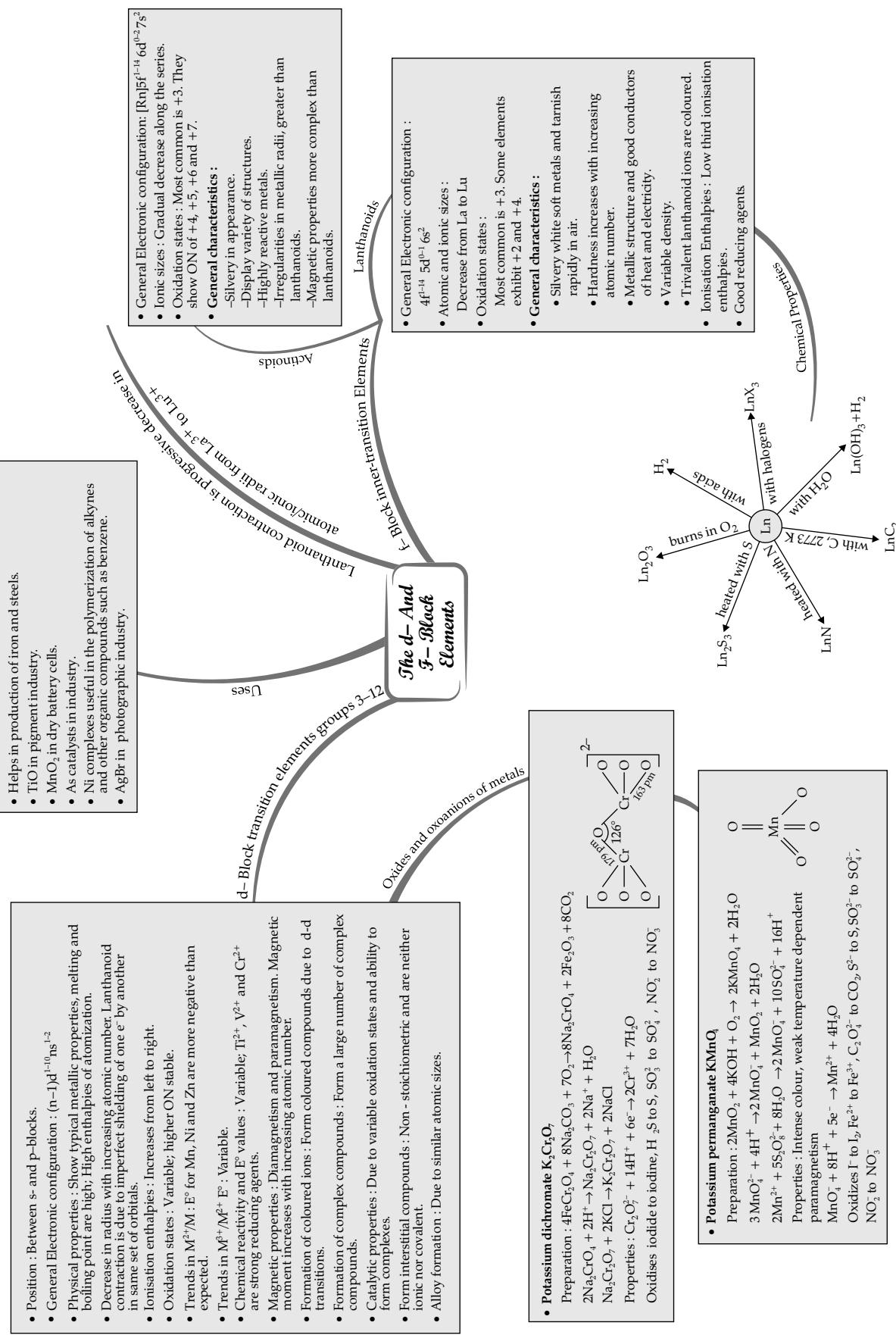


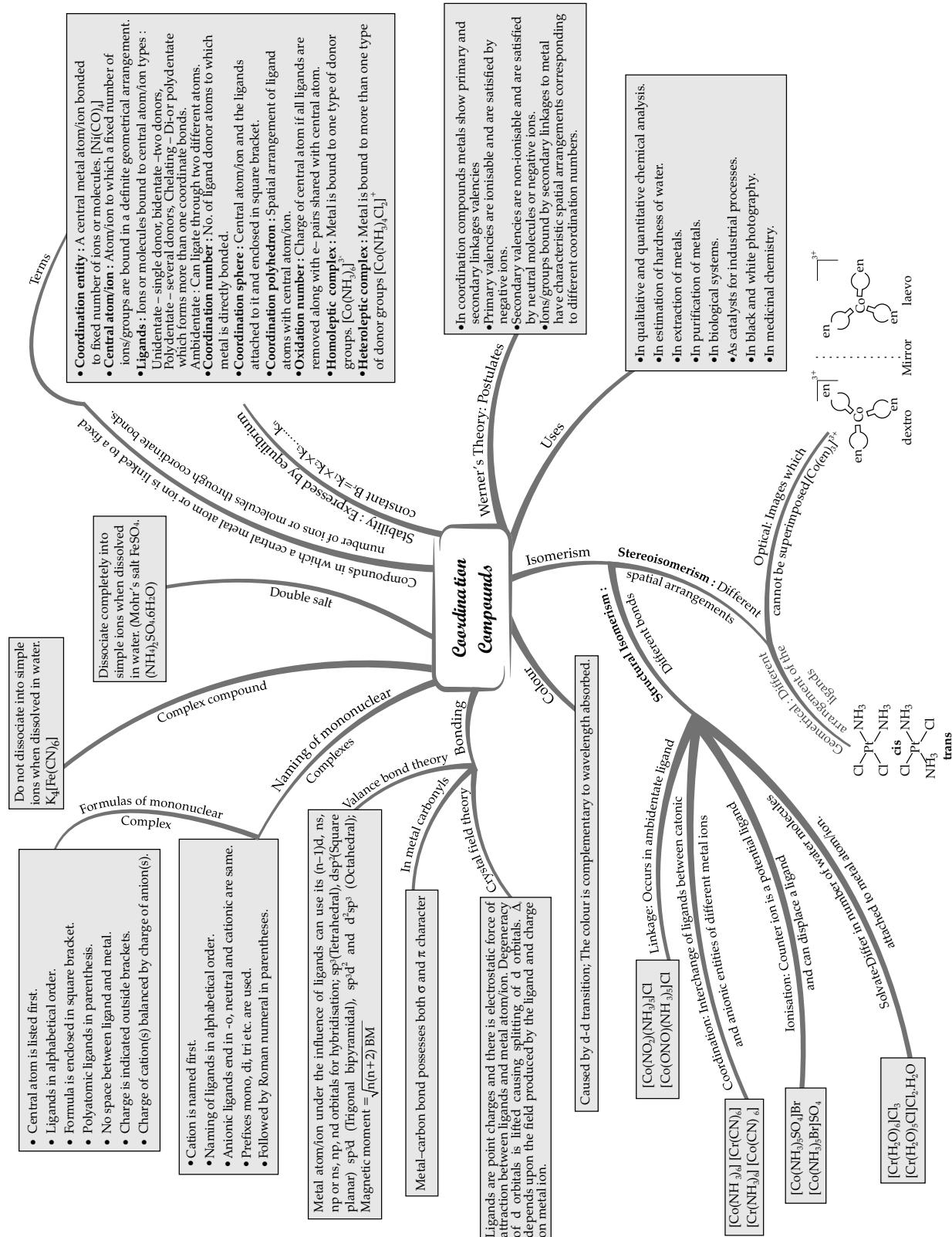


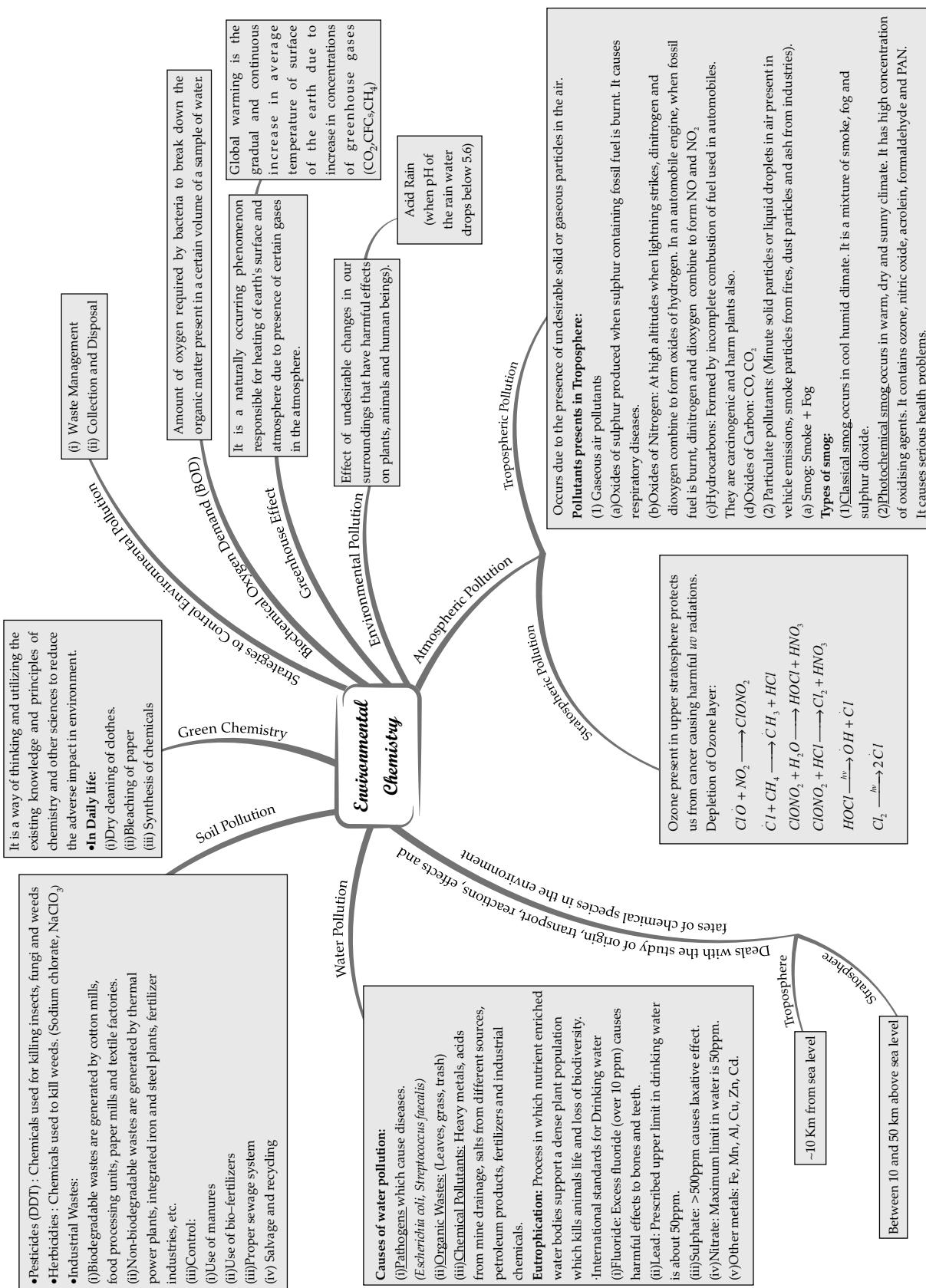


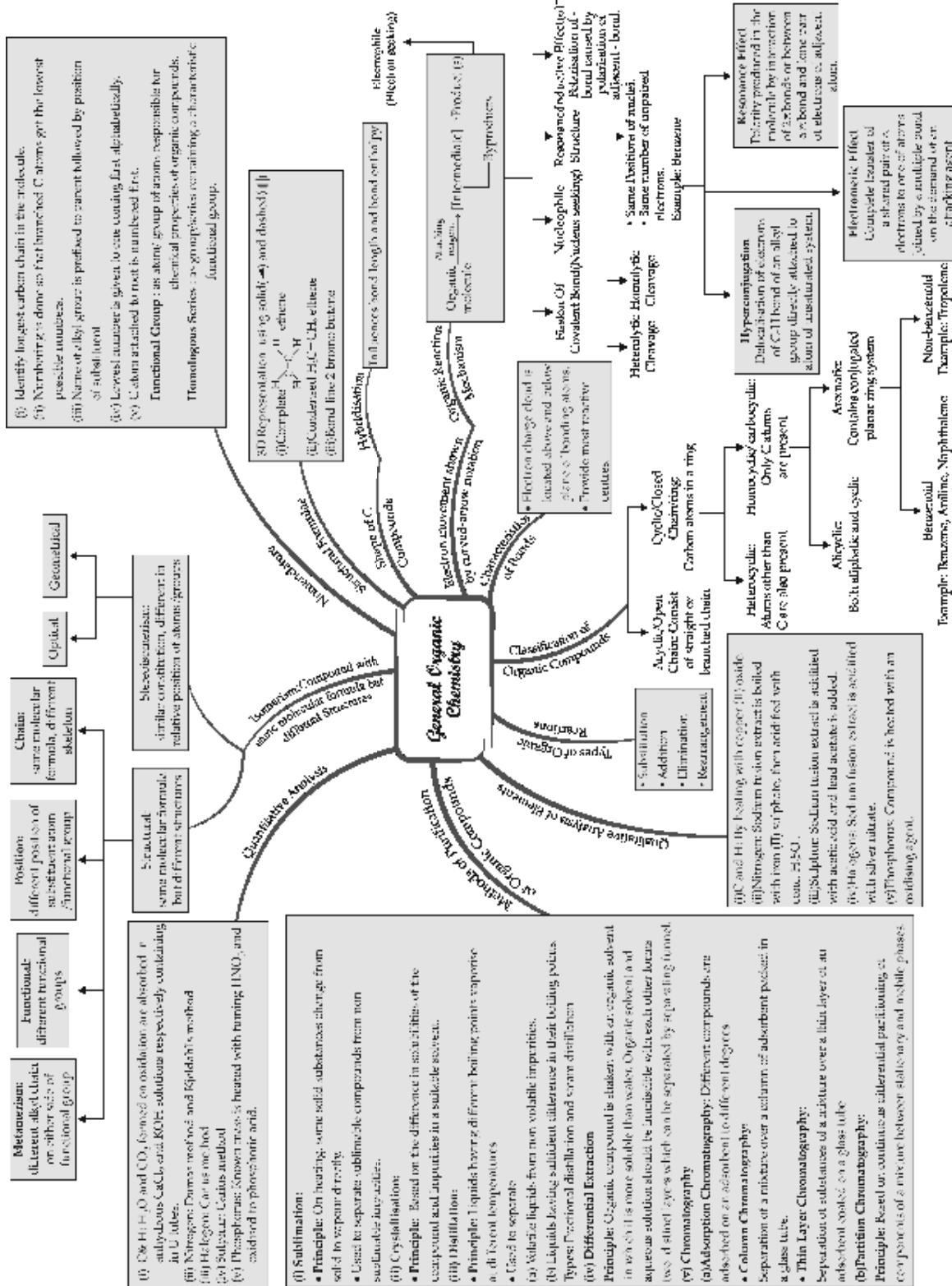


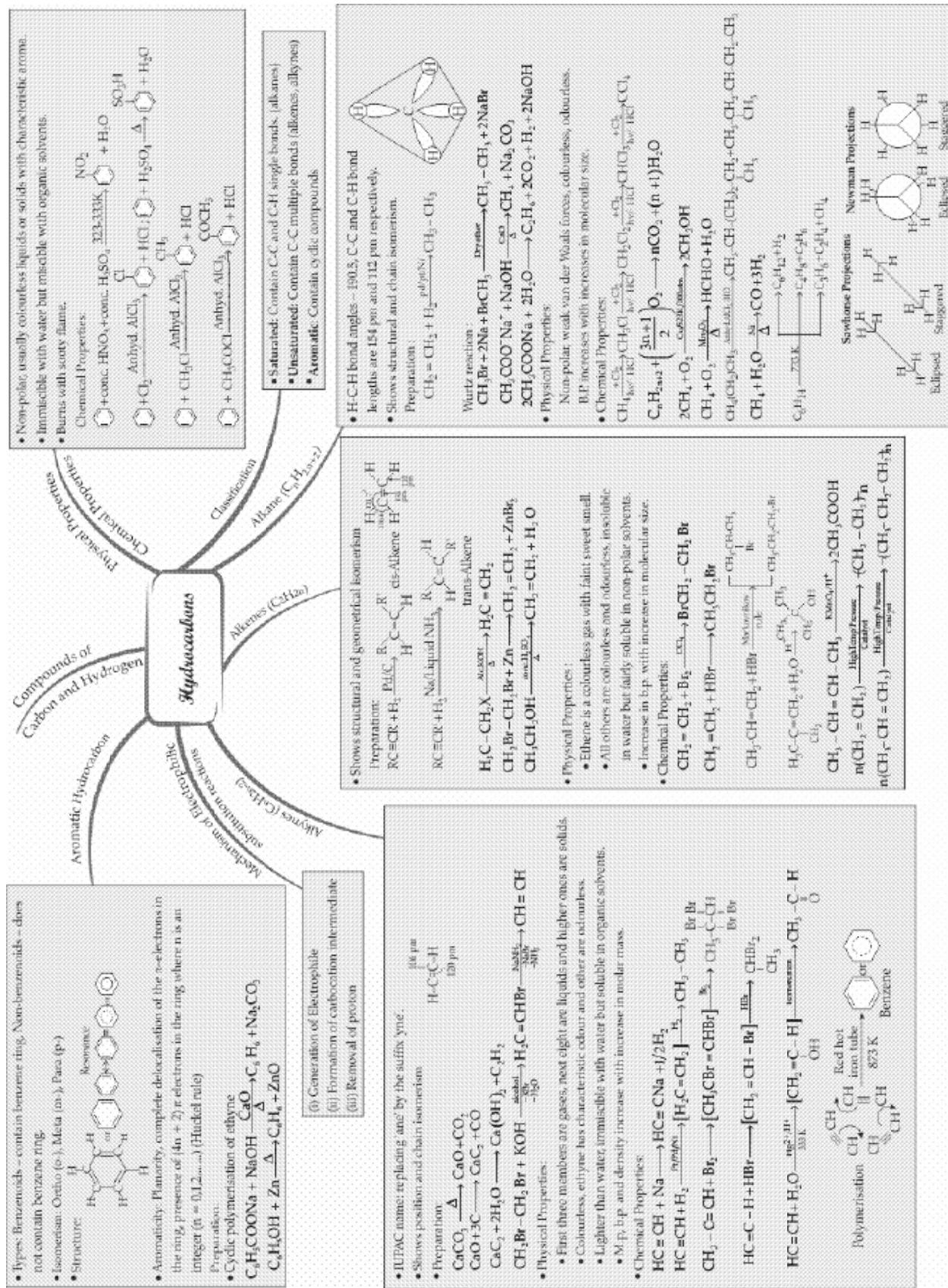


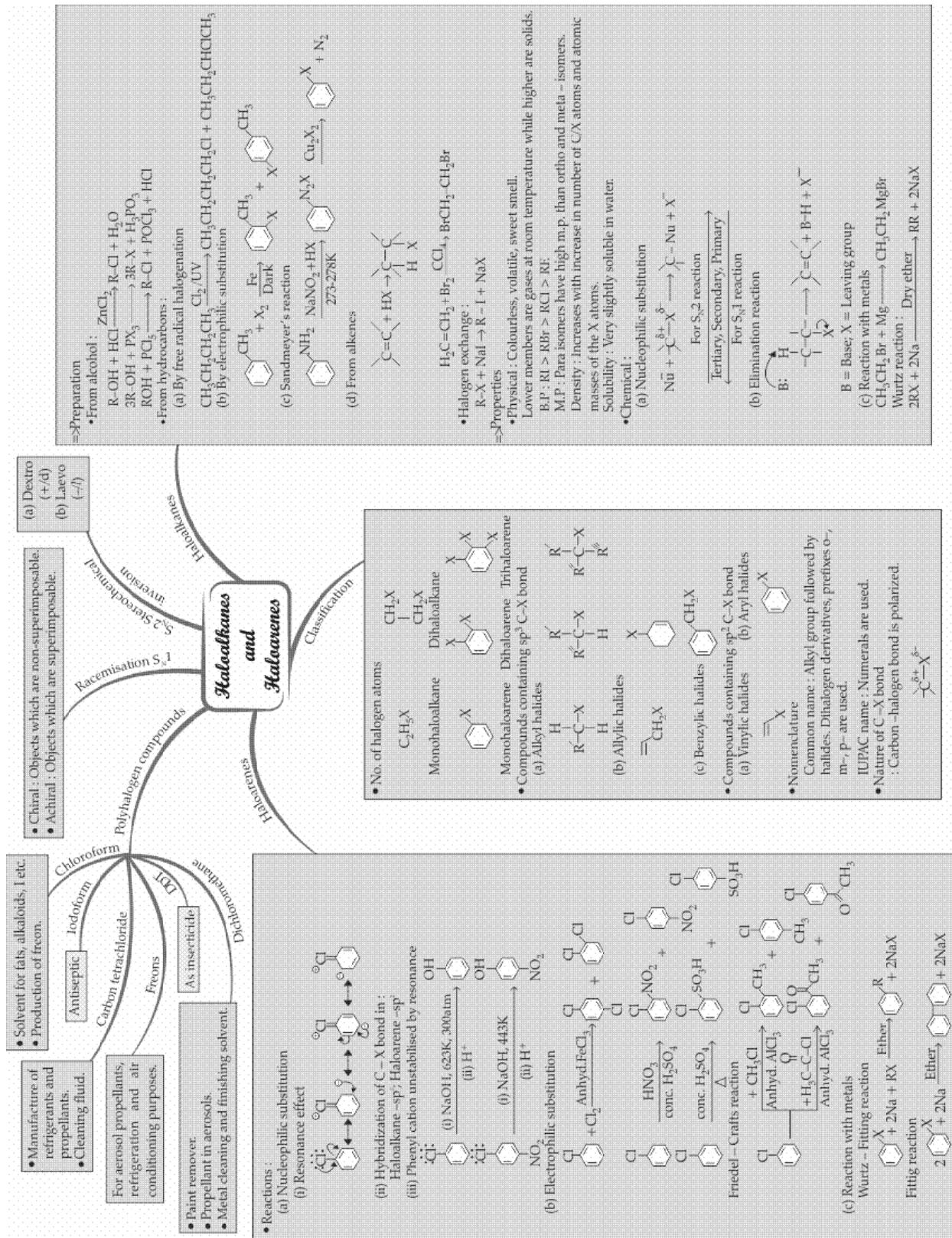


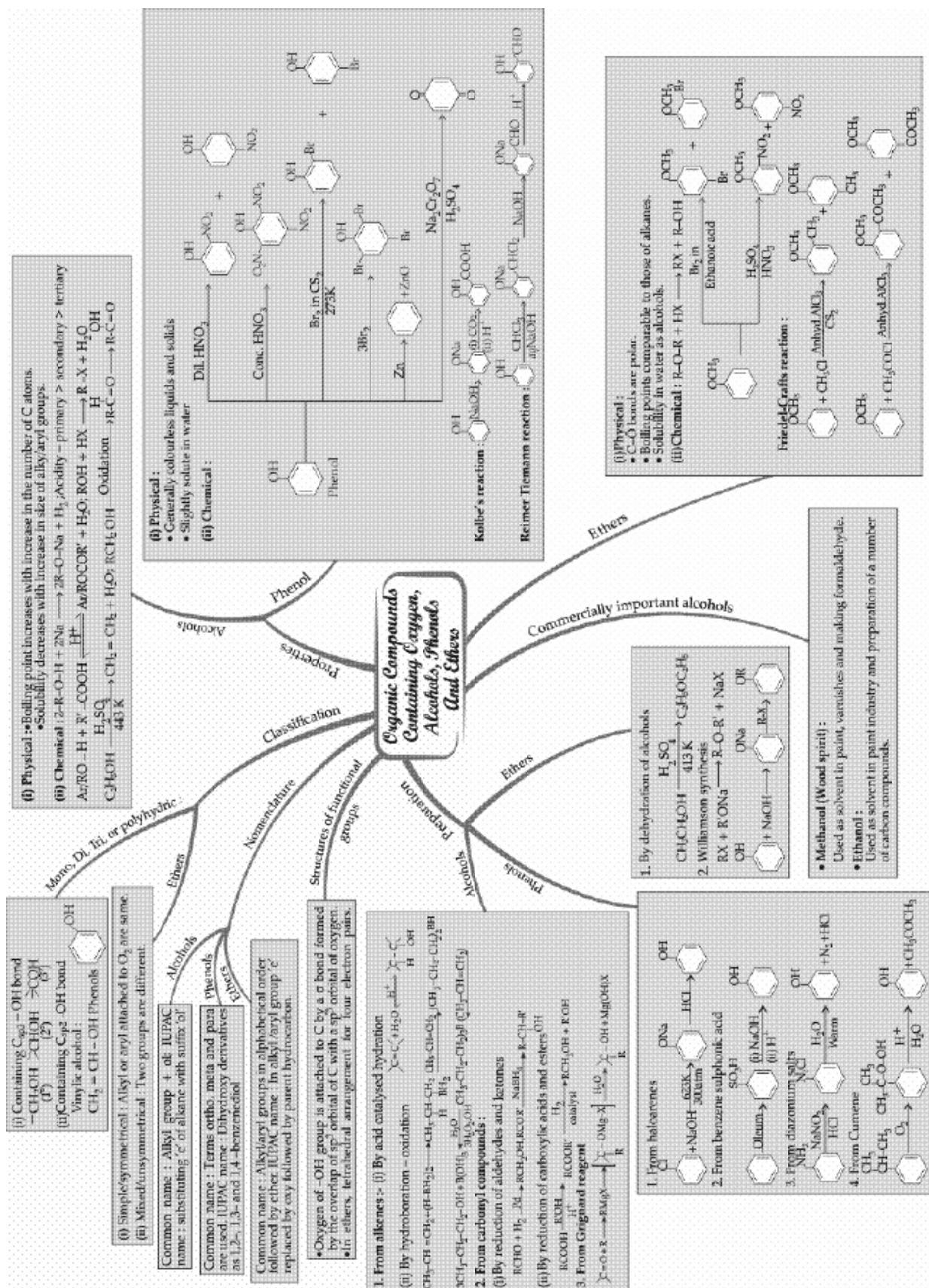


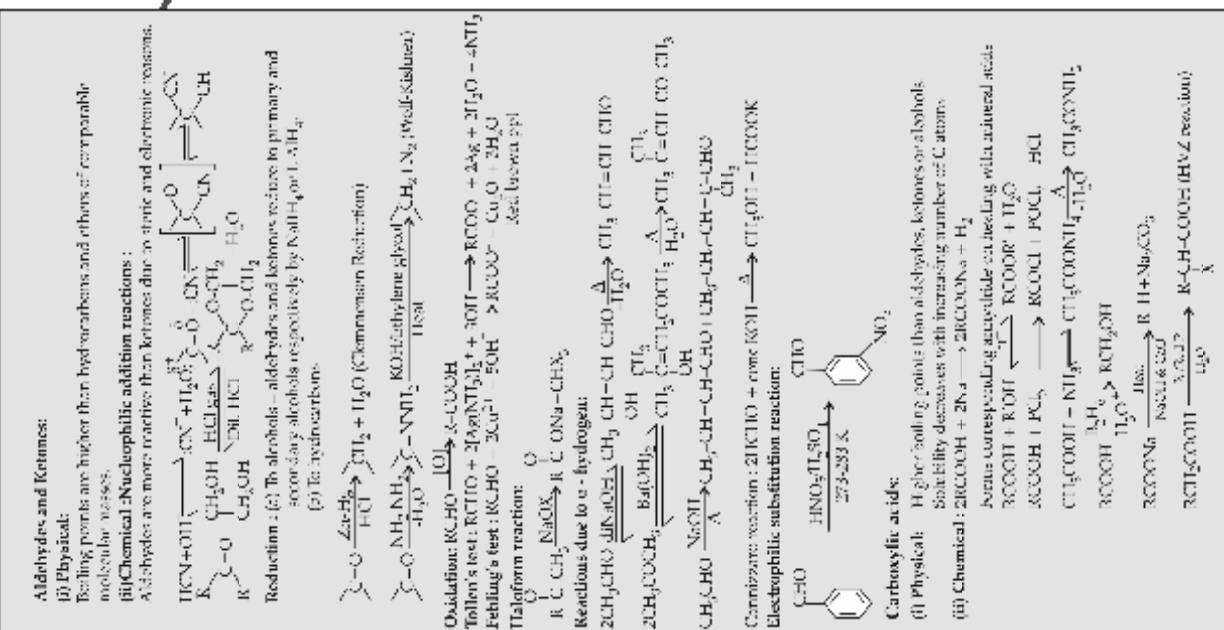
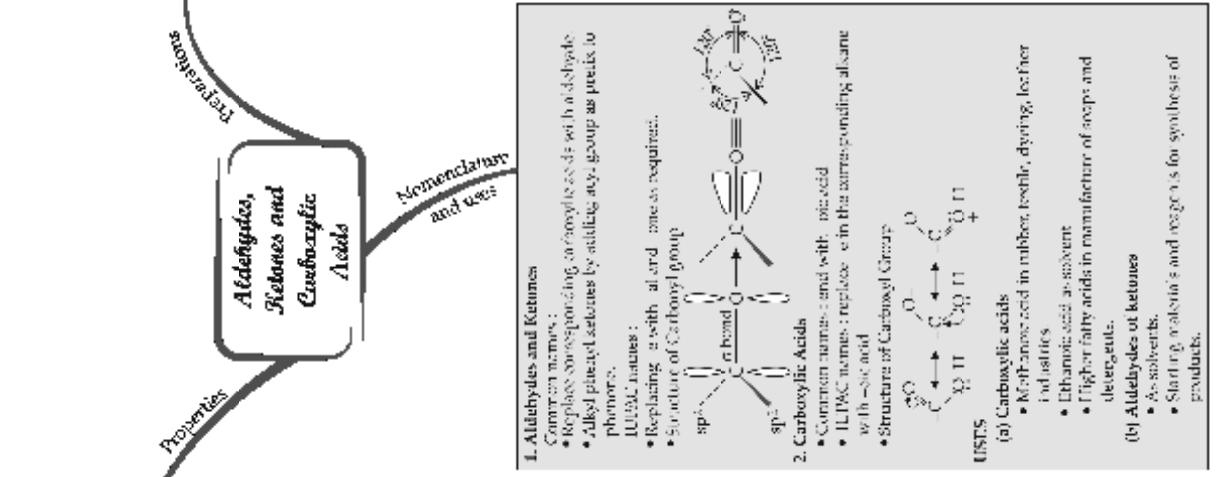
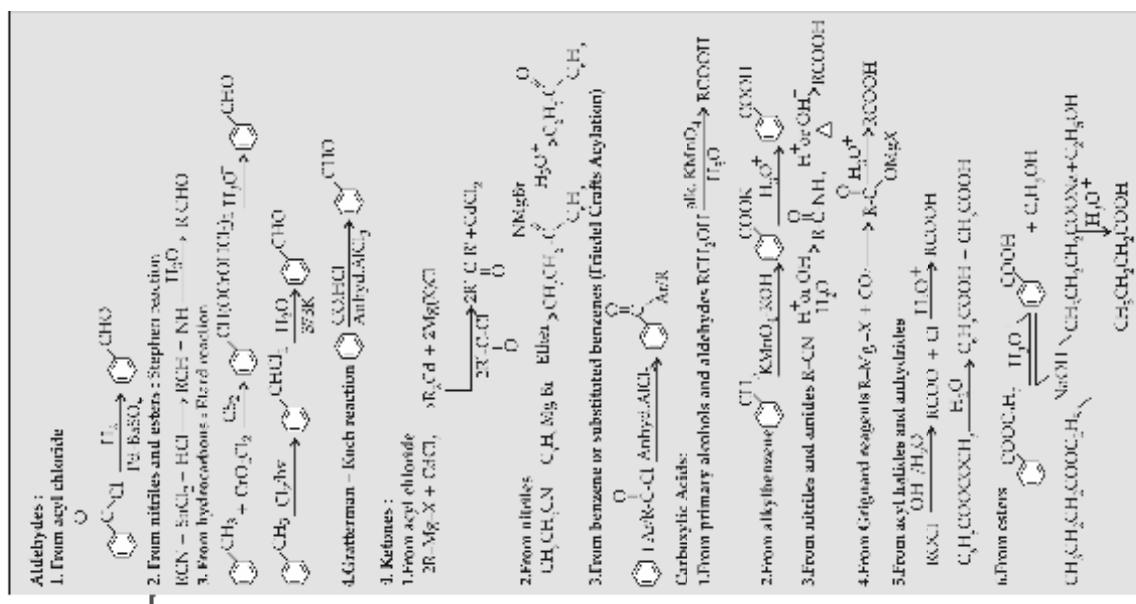


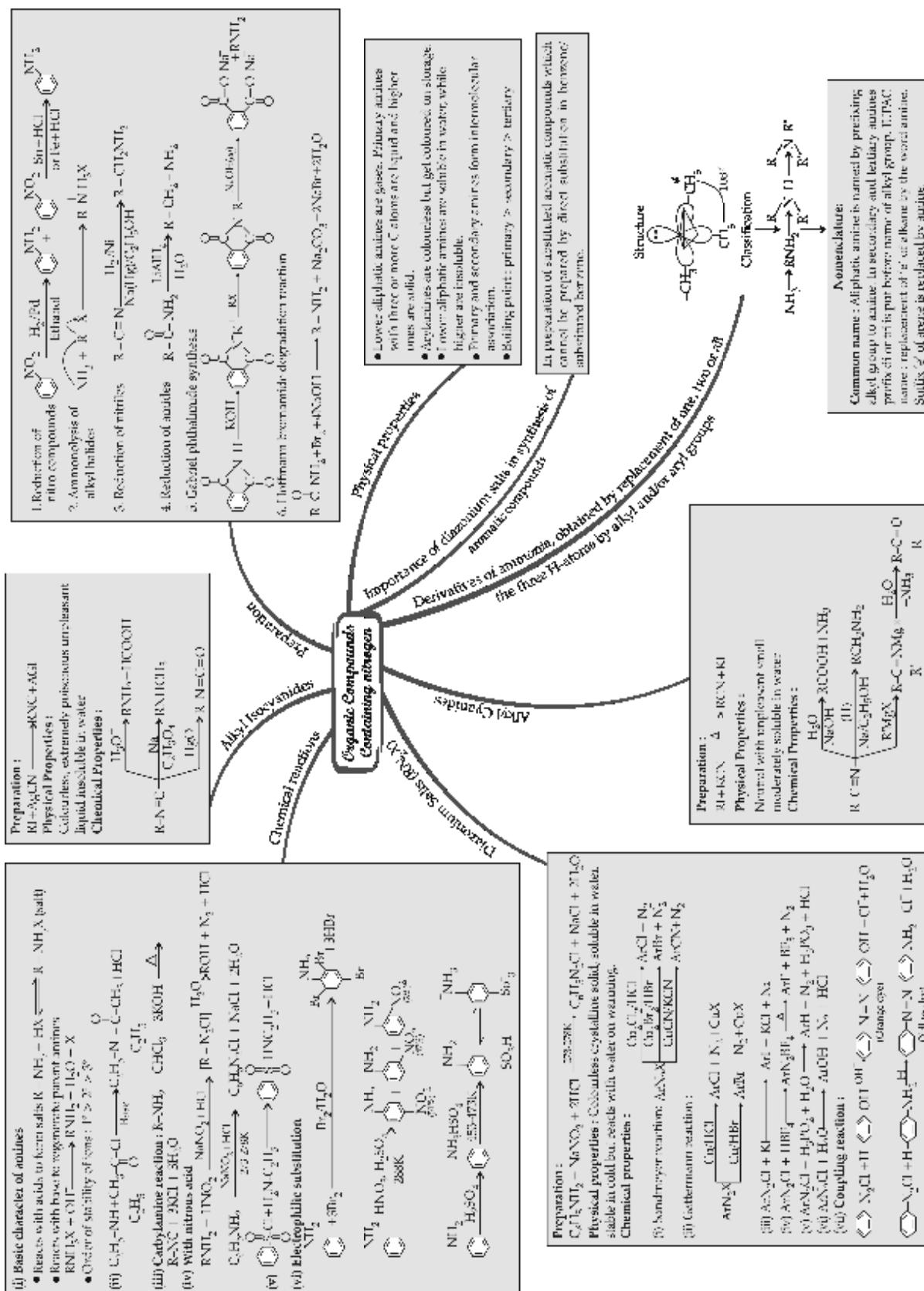


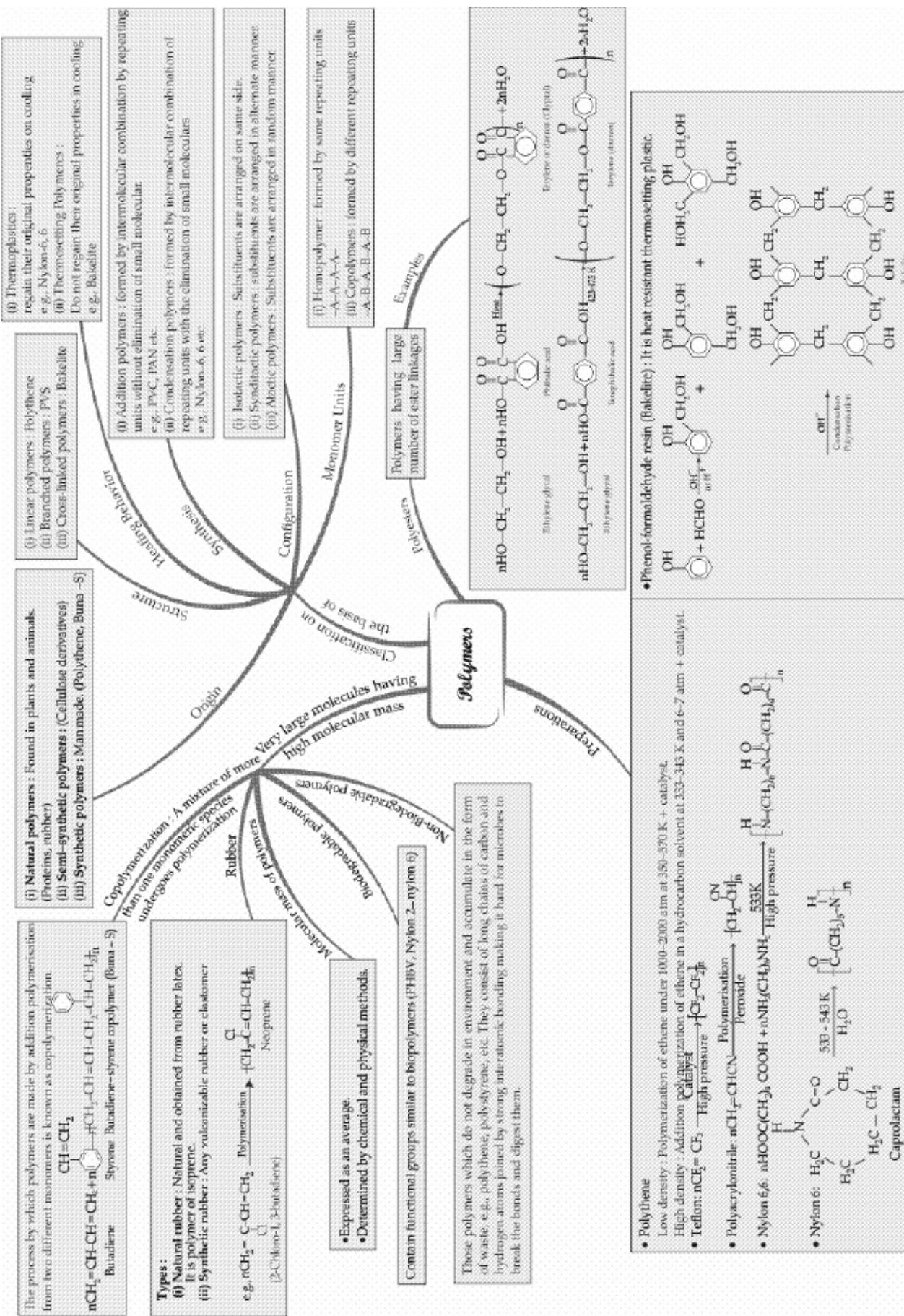












**Carbohydrates**

- Polymers of  $\alpha$ -amino acids)
- Amino acids contain -NH<sub>2</sub> and -COOH group.

**Classification:**

- On the basis of relative number of -NH<sub>2</sub> and -COOH group:
  - (i) Neutral : Equal number of -NH<sub>2</sub> and -COOH group.
  - (ii) Basic : More number of -NH<sub>2</sub> than -COOH group.
  - (iii) Acidic : More number of -COOH than -NH<sub>2</sub> group.
- On the basis of place of synthesis:
  - (i) Essential - cannot be synthesized in the body.
  - (ii) Non-essential - synthesized in the body.
- On the basis of shape:
  - (i) Fibrous : Fibre-like structure
  - (ii) Globular : Spherical
  - Structure :  $\text{H}_2\text{N}-\text{CH}_2-\text{[CO-NH]}-\text{CH-COOH}$

**Denaturation of proteins :**  
When a protein in its native form is subjected to physical change, globules unfold, helix get uncold and protein loses its biological activity.

**Enzymes**

**Biomolecules**

**DNA Fingerprinting: Unique sequence of bases on DNA**

**Nucleic Acids**

**Hormones**

**Vitamins**

**Chemical substances produced by ductless gland called endocrine gland.  
e.g., Adrenalin, thyroxine etc.**

**Importance:**

- Form a major portion of food.
- As storage molecules.
- Cellulose forms cell wall of bacteria and plants.
- Starch : Large number of monosaccharides units joined by glycosidic linkages.
- Cellulose
- Glycogen

**Structure :**

- Disaccharides : Linkage between 2 monosaccharides- Glycosidic linkage (Sucrose, maltose)
- Polysaccharides : Large number of monosaccharides units joined by glycosidic linkages.
- Starch : Polymer of  $\alpha$ -glucose with two components amylose and amylopectin

**Chromosomes :** Particles in nucleus responsible for heredity. Chromosomes are made up of proteins and nucleic acid.

**Two types :** Deoxyribonucleic acid (DNA), Ribonucleic acid (RNA)

**Composition :** In DNA, sugar is  $\beta$ -D-2-deoxyribose whereas in RNA is  $\beta$ -D-ribose. DNA contains A,G,C,T whereas RNA has A,G,C,U.

**Structure :**

- Nucleoside : Formed by attachment of a base to 1' of sugar
- Nucleotide : Formed by link to phosphoric acid at 5' of sugar

Base      Base  
—Sugar—Phosphate—[Sugar—Phosphate]—Sugar—

**Types of RNA :** m-RNA, r-RNA, t-RNA

**Biological Functions :**

- Chemical basis of heredity.
- Responsible for identity of different species of organisms.
- Nucleic acids are responsible for protein synthesis in cell.

**Optically active polyhydroxy aldehydes or ketones or compounds which produce such units on hydrolysis.**

**Classification:**

- Monosaccharides (Aldehyde group -aldehyde, Keto group -ketose)

**Glucose : Preparation :**

- (a) From sucrose :  $\text{C}_12\text{H}_{22}\text{O}_{11} + \text{H}_2\text{O} \xrightarrow{\text{H}^+} \text{C}_6\text{H}_{12}\text{O}_6 + \text{C}_6\text{H}_{12}\text{O}_6$
- (b) From starch :  $(\text{C}_6\text{H}_{10}\text{O}_5)_n + n\text{H}_2\text{O} \xrightarrow[\text{H}_2\text{O}]{\text{HI}, \Delta, 393\text{K}, 2-3 \text{ atm}} \text{C}_6\text{H}_{12}\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_3$

**Structure:**

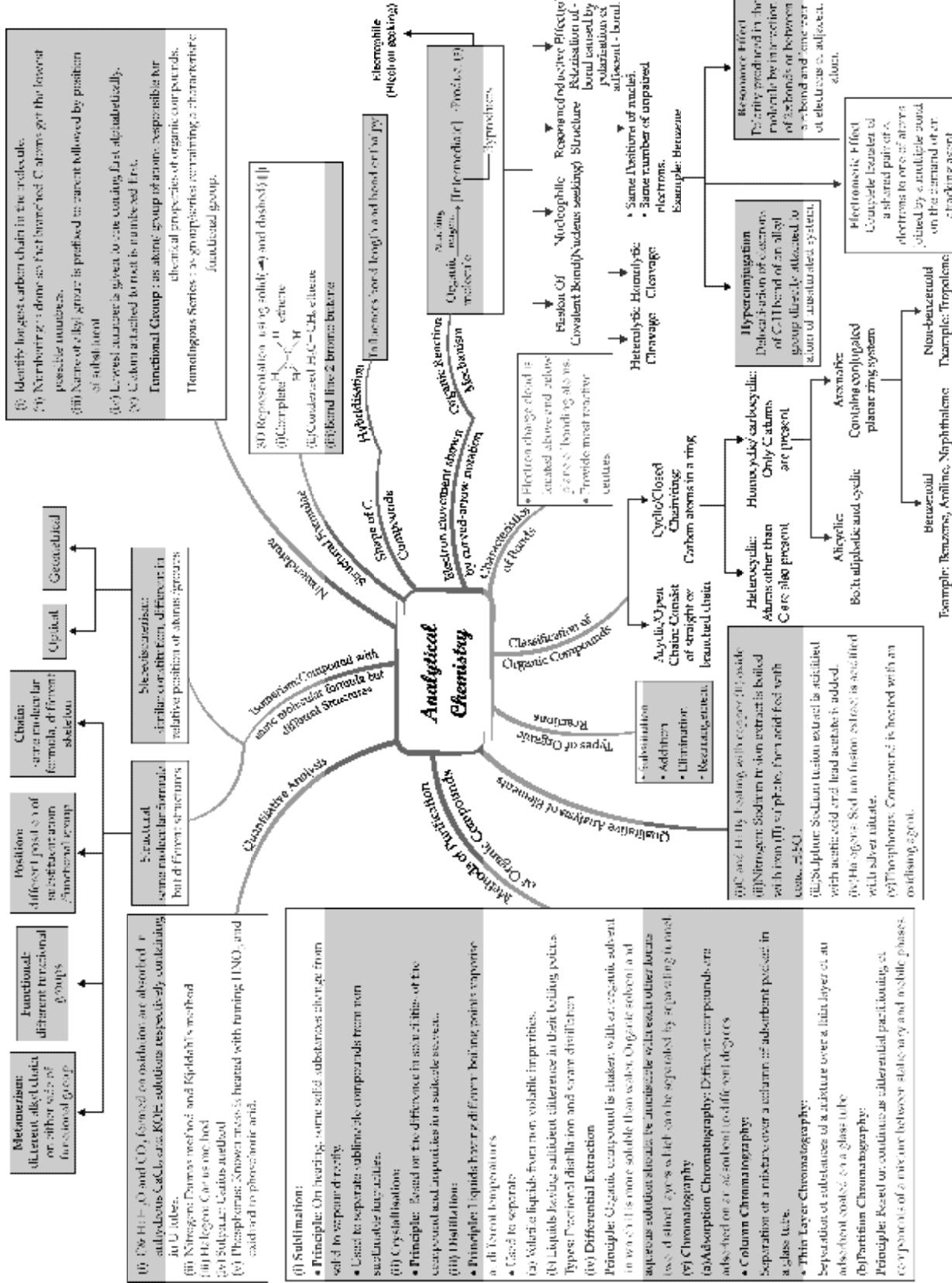
**Cyclic Structure**

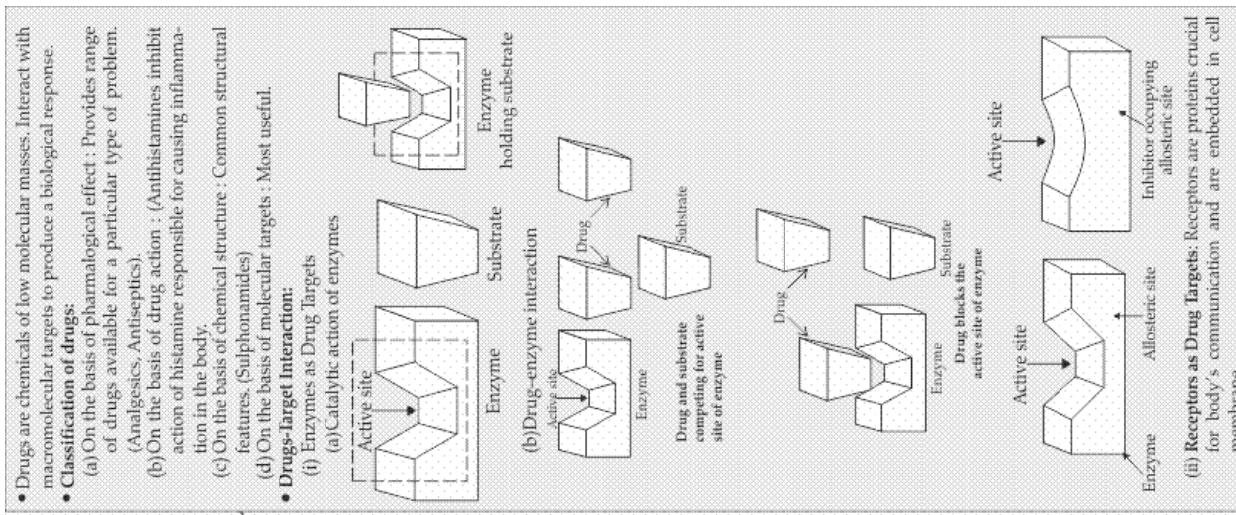
**Structure of Fructose**

**Importance:**

- Form a major portion of food.
- As storage molecules.
- Cellulose forms cell wall of bacteria and plants.
- Starch : Large number of monosaccharides units joined by glycosidic linkages.
- Cellulose
- Glycogen

**Mechanism :** Reduces the magnitude of activation energy





**Purpose:**

- For their preservation
- Enhancing their appeal
- Adding nutritive value

- Artificial Sweetening Agents : Natural sweeteners (sucrose), artificial sweeteners (Aspartane, Saccharin)
- Food Preservatives : Prevent spoilage of food due to microbial growth. (Table salt, sugar)

- (i) Soap (Saponification)**
- $$\text{Glycerol ester of stearic acid (fat)} + \text{Sodium hydroxide} \longrightarrow \text{Sodium Stearate} + \text{Glycerol}$$
- (ii) Synthetic Detergents :**
- Anionic detergents : Sodium salts of sulphonated long chain alcohols or hydrocarbons. (Sodium salts of alkyl benzene sulphonates)
  - Cationic detergents : Quaternary ammonium salts of amines with acetates, chlorides or bromides as anions. (Cetyltrimethylammonium bromide)
  - Non-ionic Detergents : Non-ionic type.

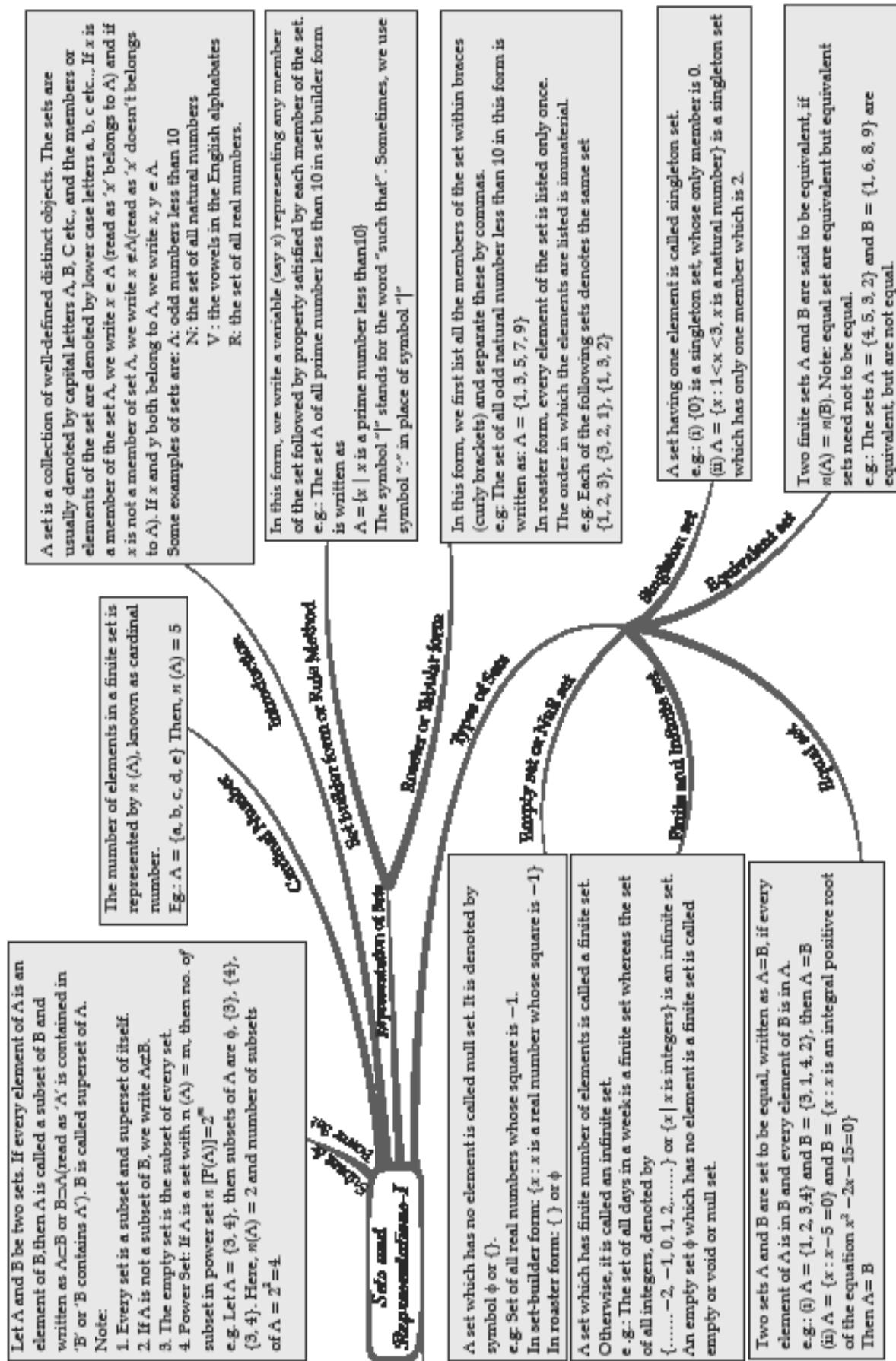


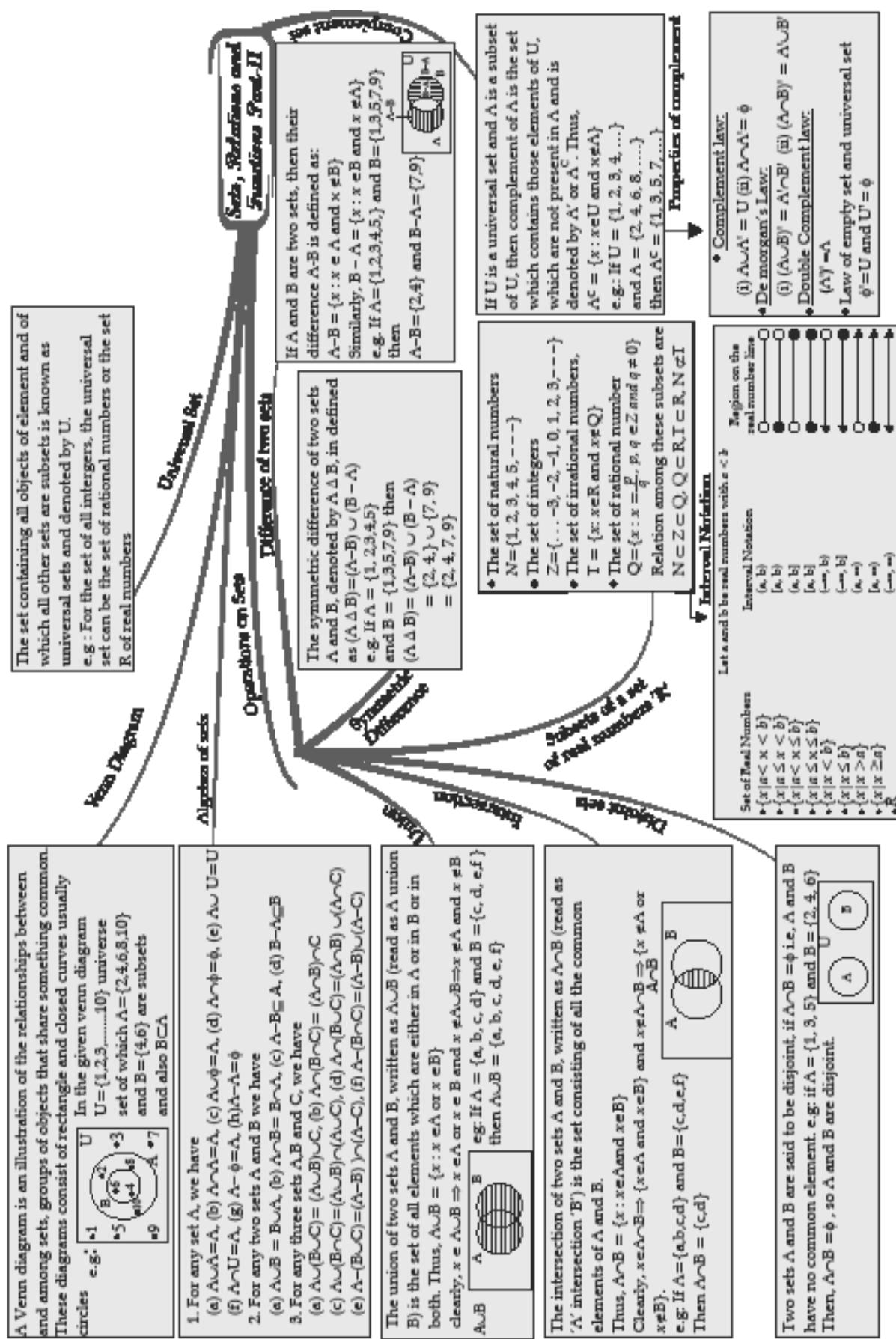
# **MATHEMATICS**

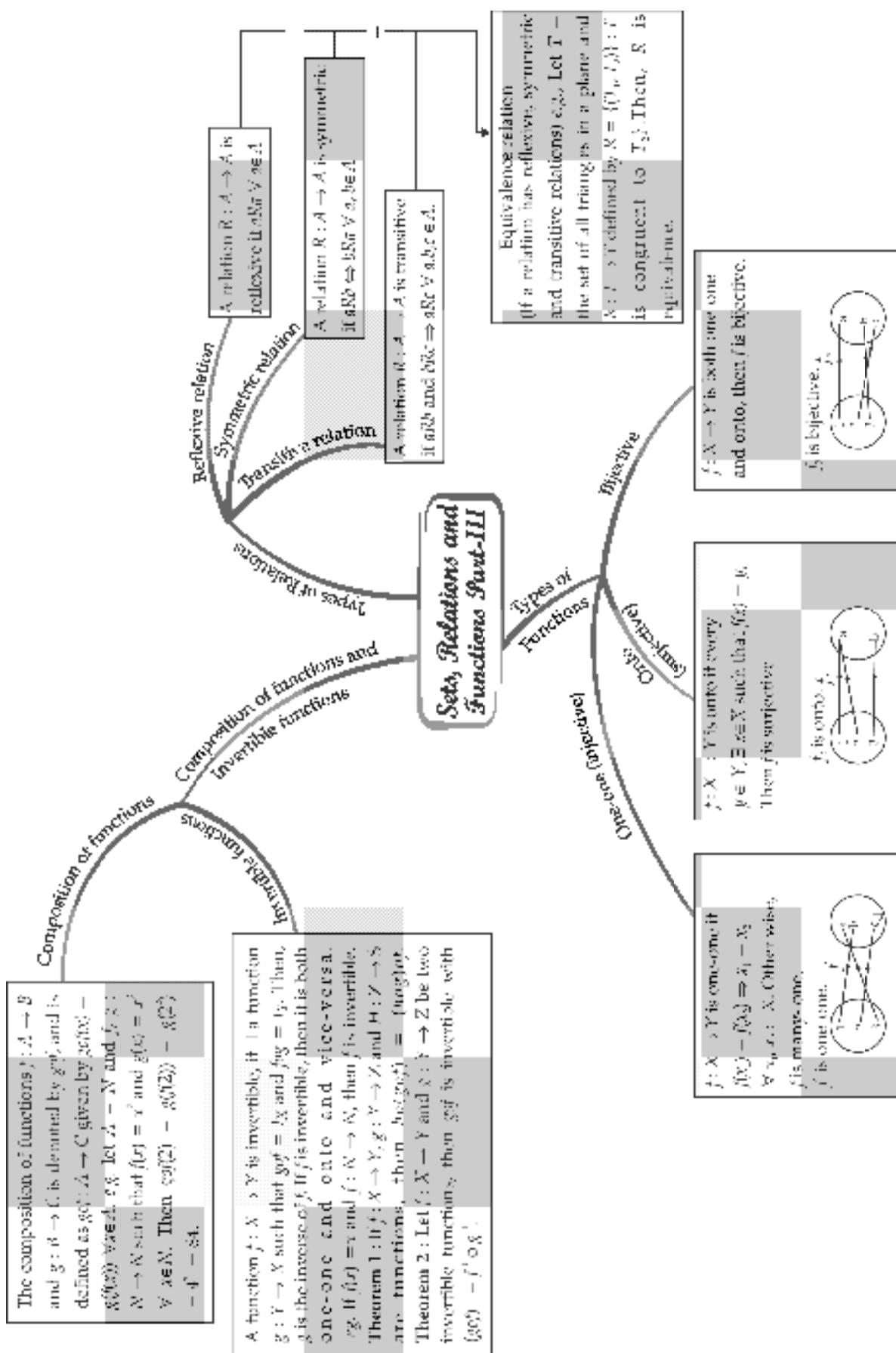
## **MIND MAPS**

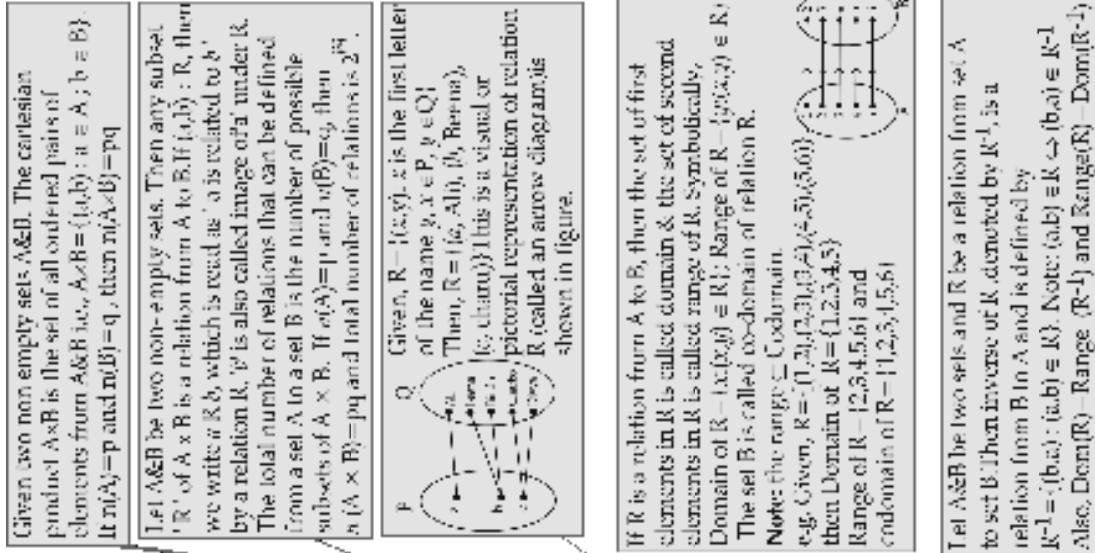
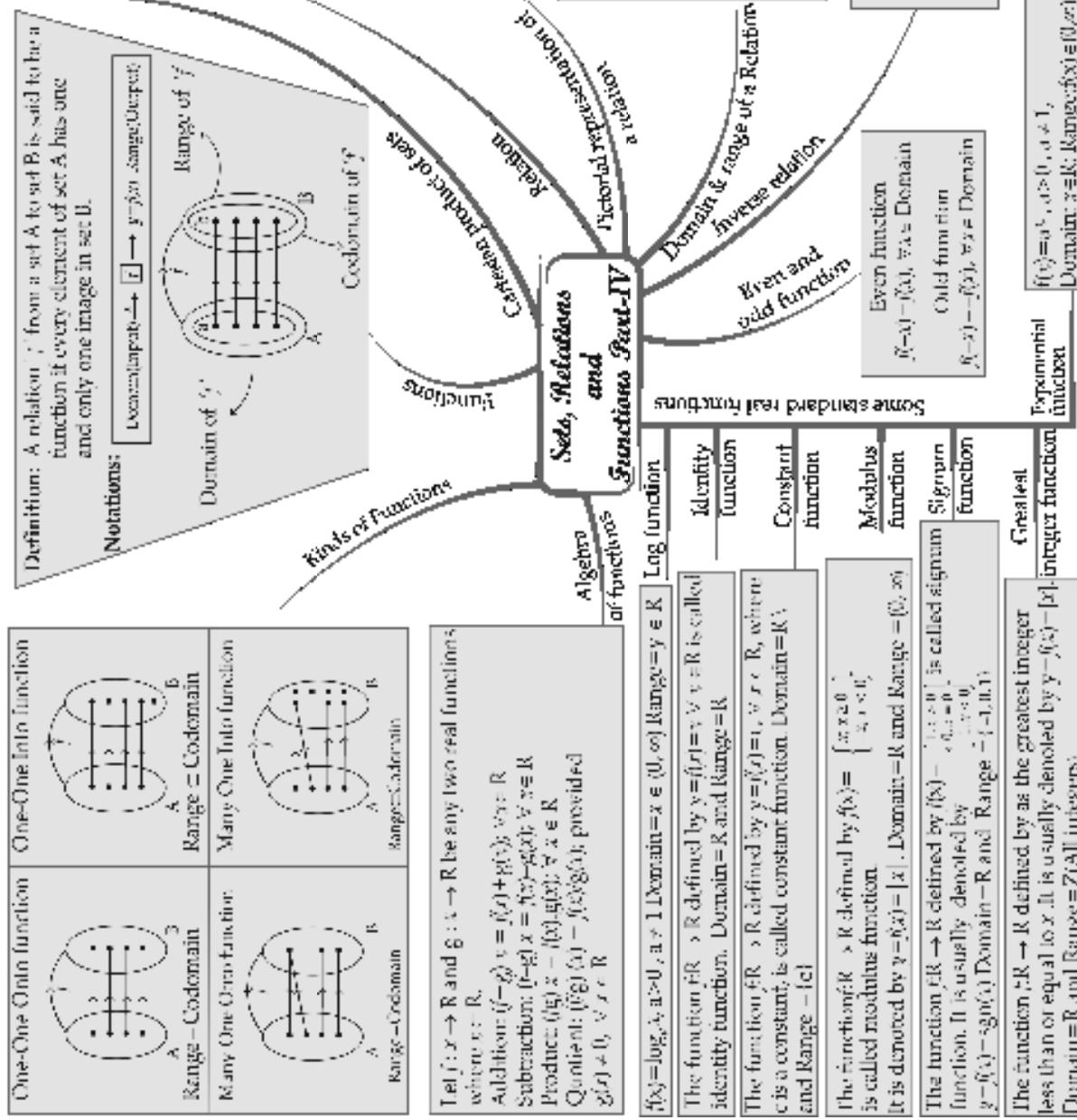
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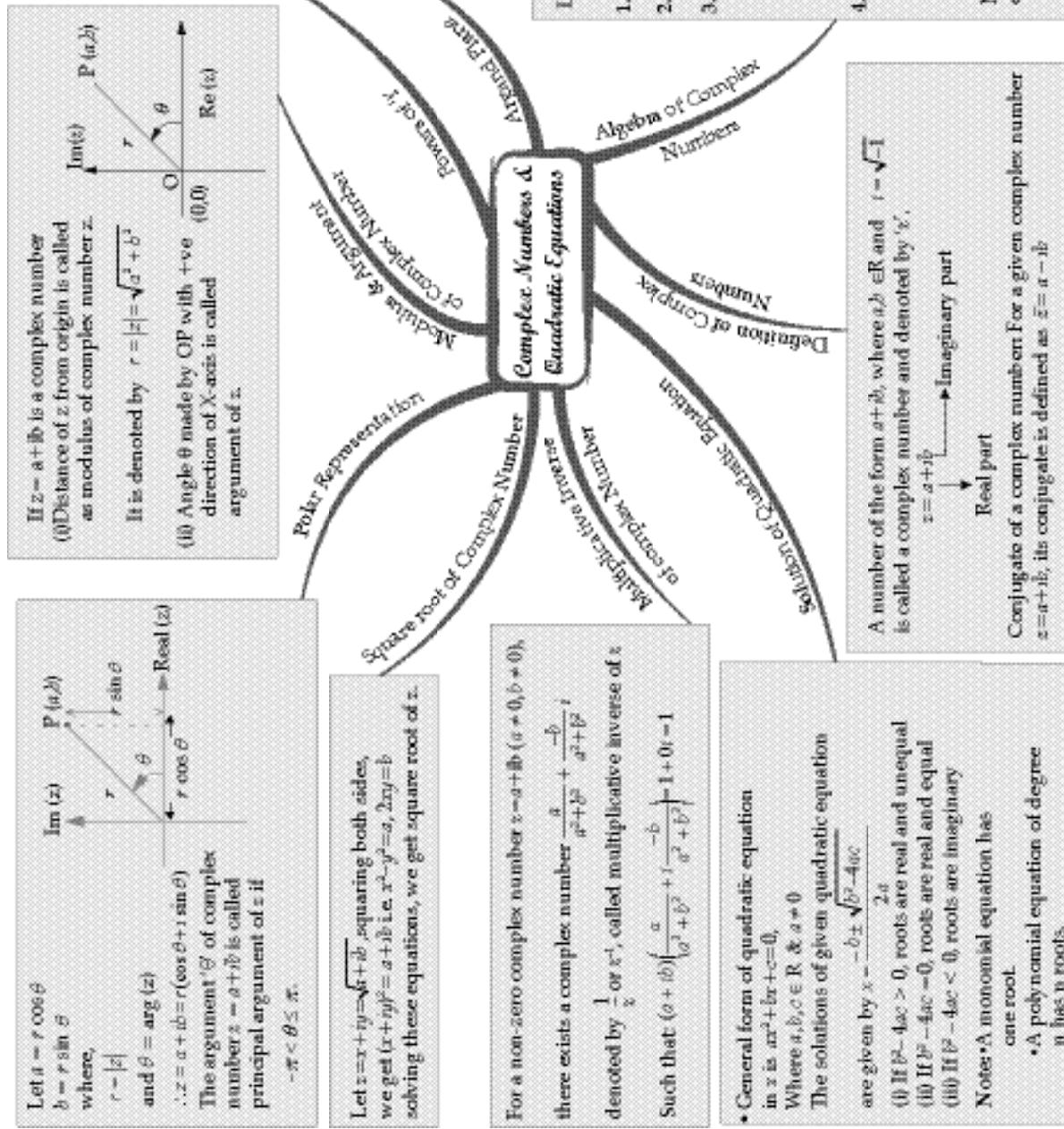
## **MNEMONICS**

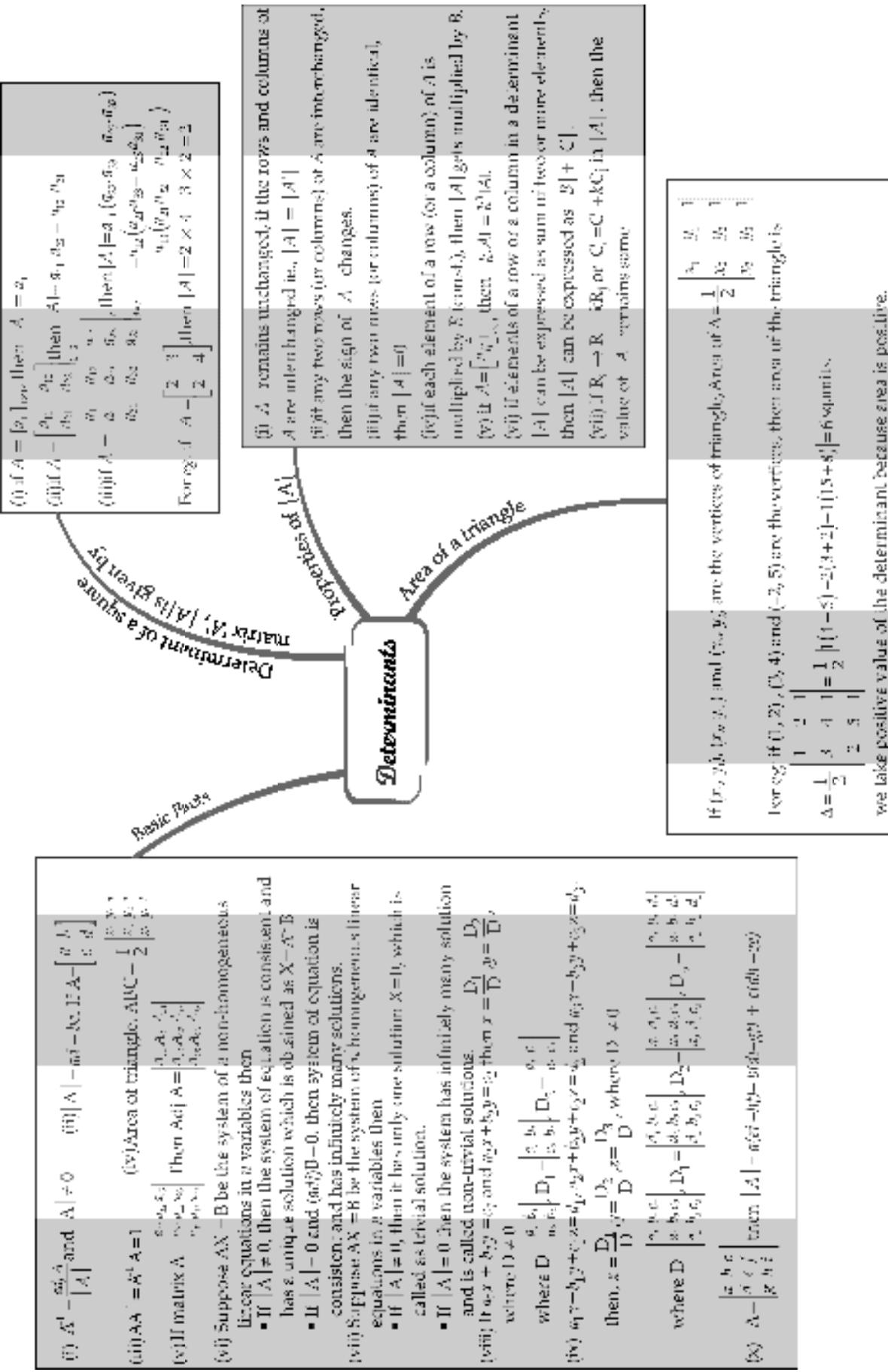


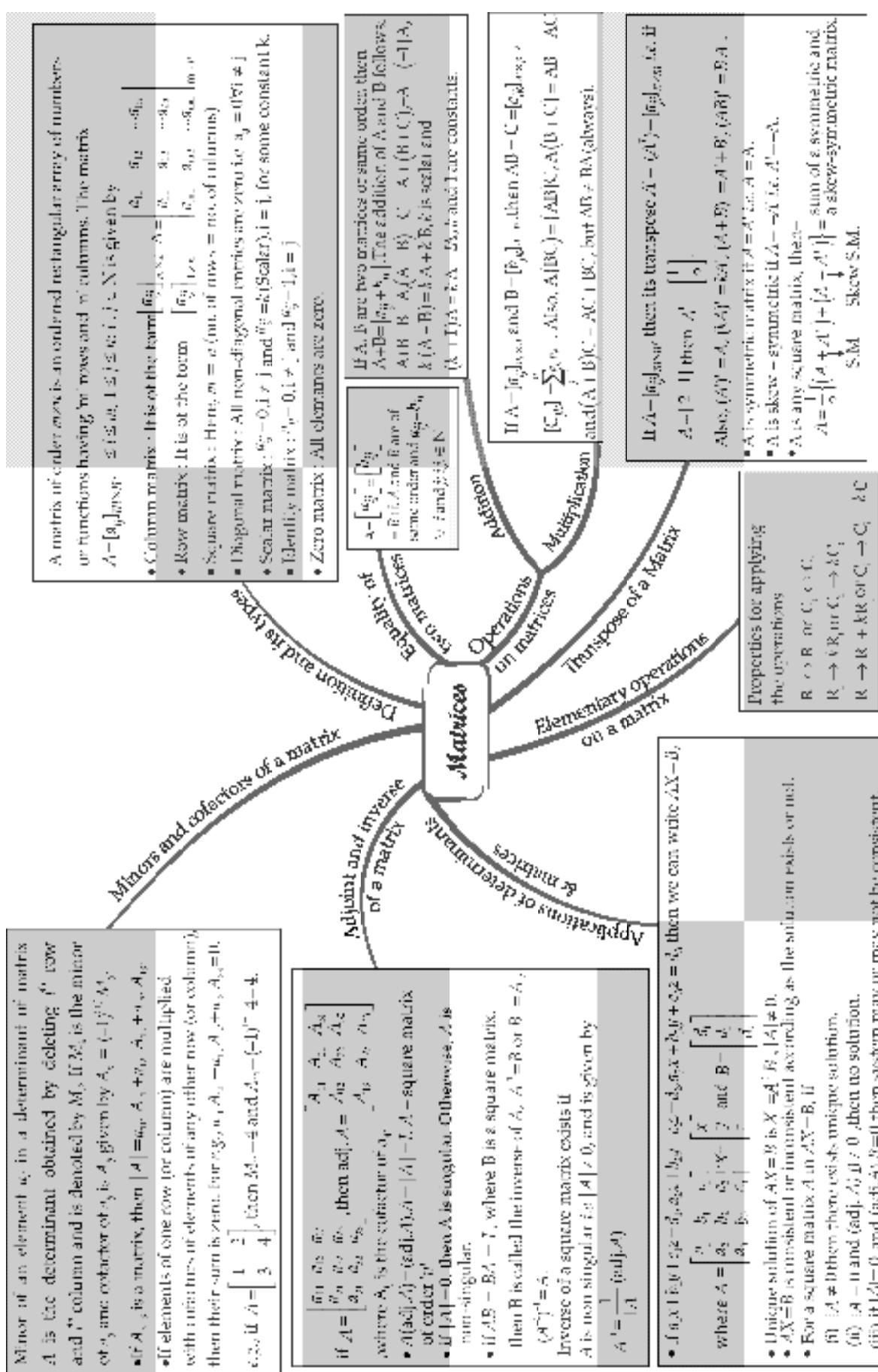


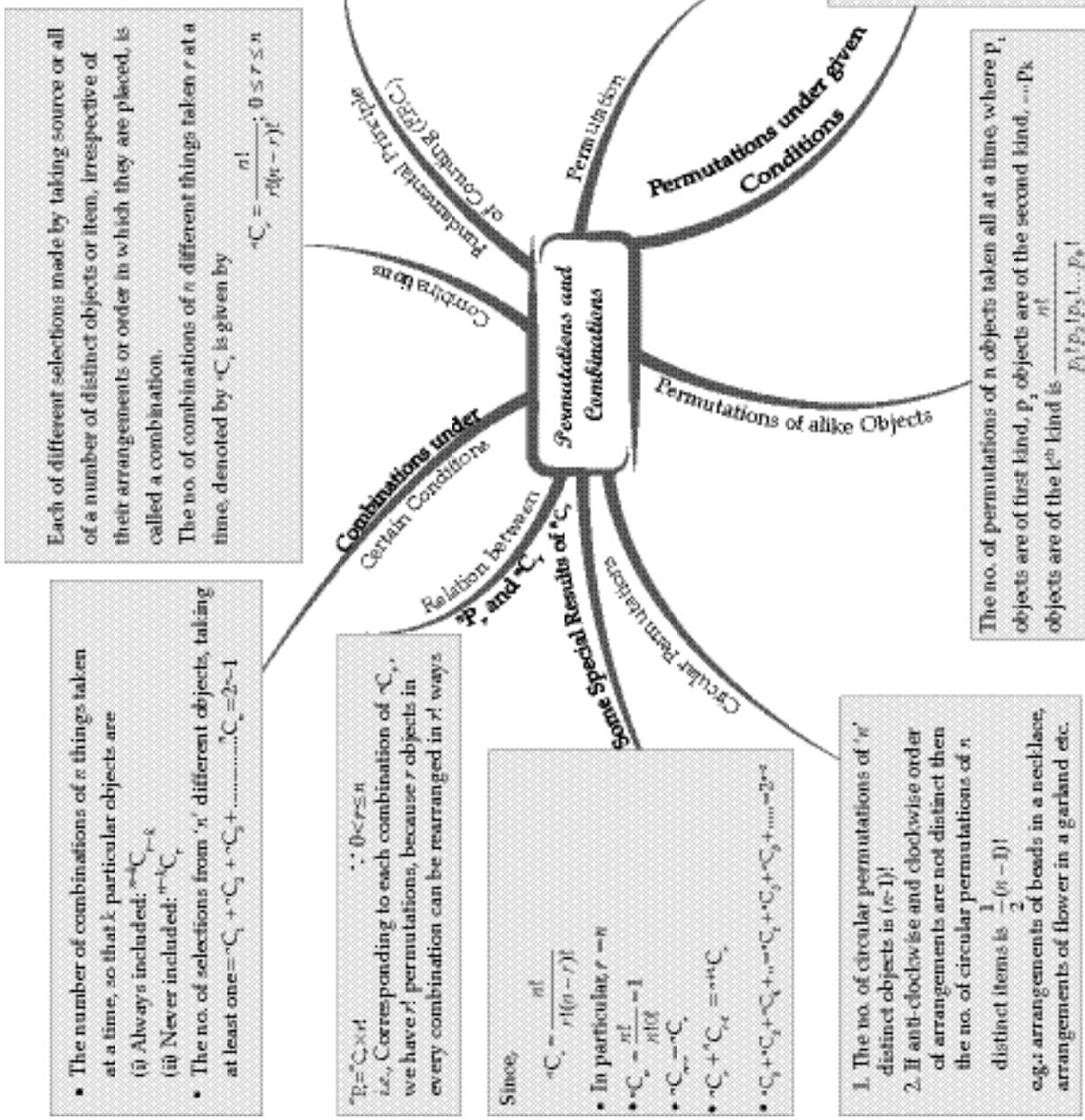


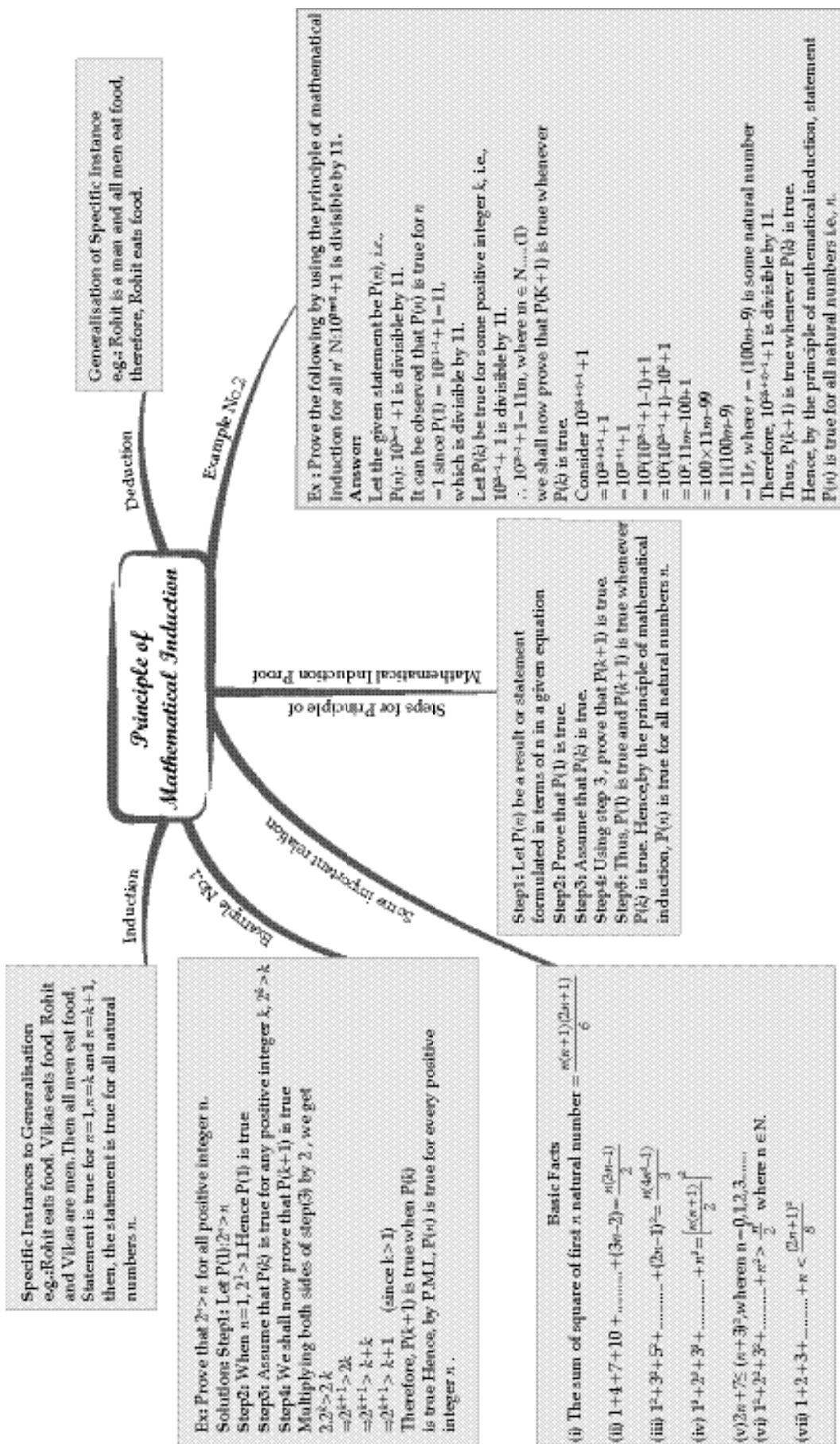












The general term of an expansion  $(a+b)^n$  is  $T_{r+1} = {}^nC_r a^{n-r} b^r$ ,  $0 \leq r \leq n, r \in N$

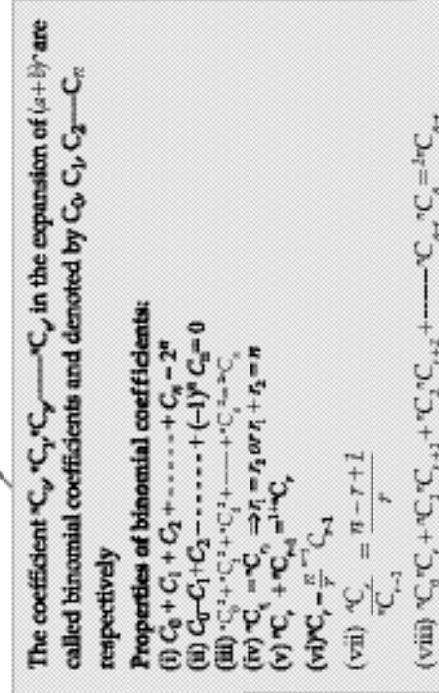
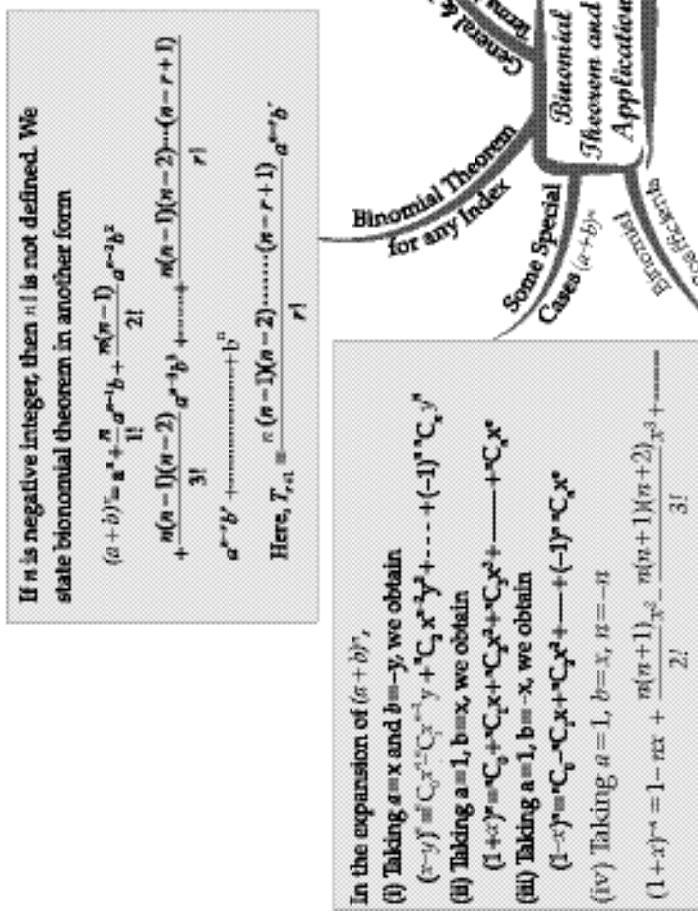
**Middle Terms:**

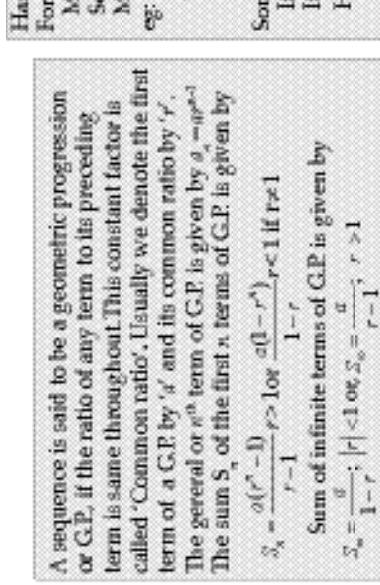
1. In  $(a+b)^n$ , if  $n$  is even, then the no. of terms in the expansion is odd. Therefore, there is only one middle term and it is  $\binom{n+2}{2}$  term.
2. In  $(a+b)^n$ , if  $n$  is odd then the no. of terms in the expansion is even. Therefore, there are two middle terms and those are  $\binom{n+1}{2}$  and  $\binom{n+3}{2}$  terms.

If  $a, b \in R$  and  $n \in N$  then

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n$$

- Remarks: If the index of the binomial is  $n$  then the expansion contains  $n+1$  terms.
- In each term, the sum of indices of  $a$  and  $b$  is always  $n$ .
- Coefficients of the terms in binomial expansion equidistant from both the ends are equal.

$$(a-b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + (-1)^n {}^nC_n a^0 b^n$$




- If  $a, b, c$  are in G.P,  $b$  is the GM between  $a$  &  $c$ .  $b^2 = ac$ , therefore  $; a > 0, c > 0$

If  $a, b$  are two given numbers &  $a, G_1, G_2, \dots, G_n, b$  are in G.P., then  $G_1, G_2, G_3, \dots, G_n$  are  $n$  GMs between  $a$  &  $b$ .

$$G_1 = a \cdot \left( \sqrt[n+1]{b/a} \right)^{n+1}, G_2 = a \cdot \left( \sqrt[n+2]{b/a} \right)^{n+2}, \dots, G_n = a \cdot \left( \sqrt[n+1]{b/a} \right)^{n+1}$$

or  $G_1 = ar, G_2 = ar^2, \dots, G_n = ar^n$  where  $r = \left[ \sqrt[n+1]{b/a} \right]$

In a series, where each term is formed by multiplying the corresponding term of an AP & GP is called Arithmetico-Geometric Series.

$$\text{Eg: } 1 + 3x + 7x^2 + \dots$$

Here,  $1, 3, 5, \dots$  are in AP and  $1, x, x^2, \dots$  are in GP

- If AM, GM, of two positive numbers  $a$  and  $b$  is  $m : n$ , then  $a : b = \left( m + \sqrt{mn^2 - n^2} \right) : \left( mn - \sqrt{mn^2 - n^2} \right)$

- Let A, G and H be the AM, GM and HM of two given positive real numbers  $a$  &  $b$ , respectively. Then,

$$d = \frac{a+b}{2}, G = \sqrt{ab} \text{ and } H = \frac{2}{\frac{1}{a} + \frac{1}{b}}$$

$$\text{So, } G = \sqrt{AH} \text{ or, } G^2 = AH$$

**Harmonic Progression (HP)**  
For the solution of HP, we should follow below steps  
Make the reciprocal of each terms of HP  
Solve by AP method  
Make the reciprocal of AP result  
eg: if  $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$  in HP, then  
 $\frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{7}{5}, \dots$  will be in AP  
now  $n^{th}$  term of AP,  $a_n = a + (n-1)d$   
So,  $n^{th}$  term of HP,  $= \frac{1}{a + (n-1)d}$   
Some important result  
If H is the harmonic mean (H) between  $a$  &  $b$ , then  $H = \frac{2ab}{a+b}$   
If there is  $n$  harmonic mean between  $a$  &  $b$ , then  $H^n$

$$H^n = \frac{a(r^n - 1)}{r^n - 1} ; r > 1 \text{ or } H^n = \frac{a(1 - r^n)}{1 - r} ; r < 1$$

Sum of infinite terms of GP is given by

$$S_\infty = \frac{a}{1-r}; |r| < 1 \text{ or } S_\infty = \frac{a}{r-1}; r > 1$$

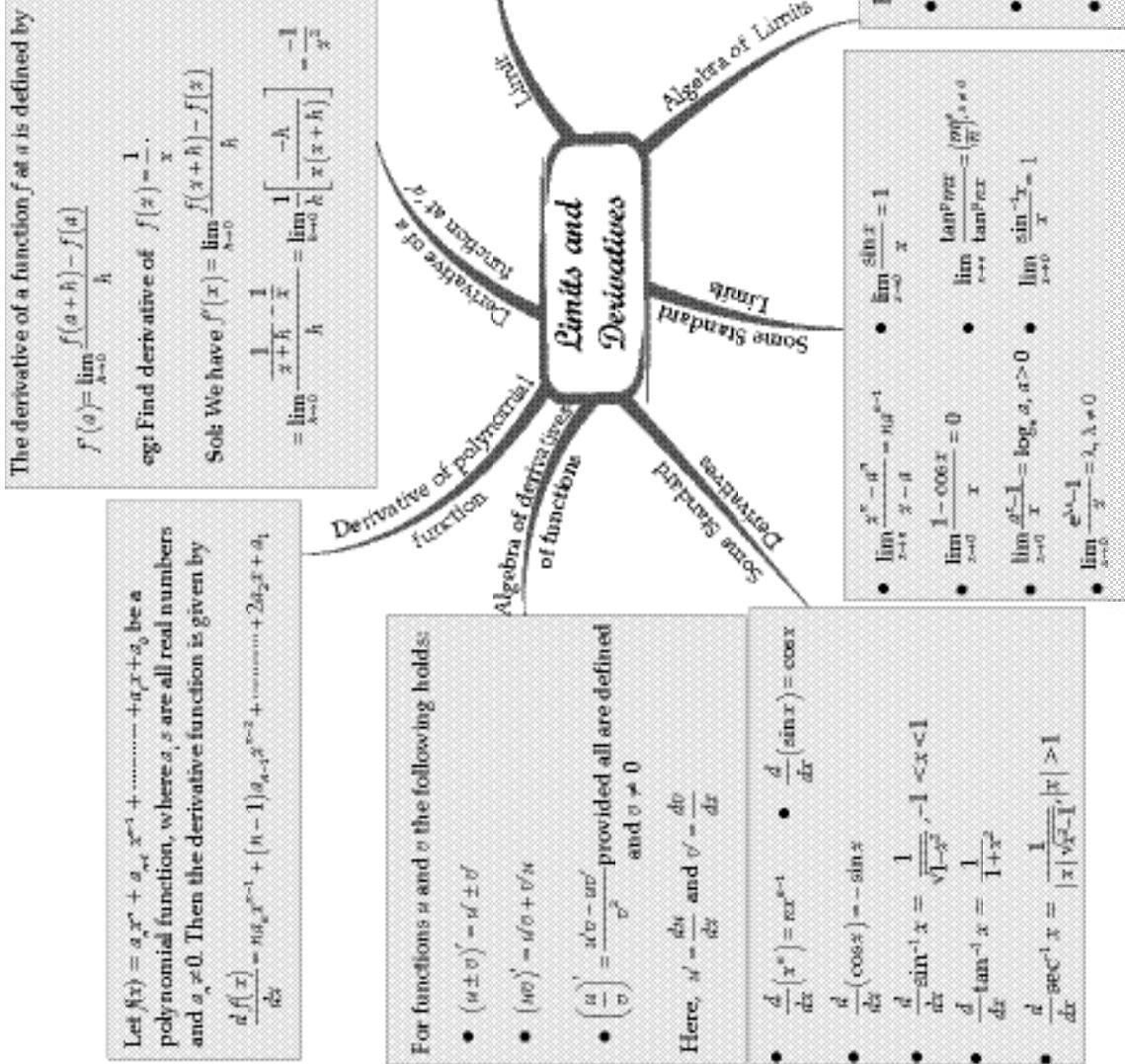
An AP is a sequence in which terms increase or decrease regularly by the fixed number (same constant). This fixed number is called 'common difference' of the AP. If 'a' is the first term & 'd' is the common difference and 'T' is the last term of AP, then general term or the  $n^{th}$  term of the AP is given by  $a_n = a + (n-1)d$  from starting and  $a_n = l - (n-1)d$  from the end.  
The sum  $S_n$  of the first  $n$  terms of an AP is given by

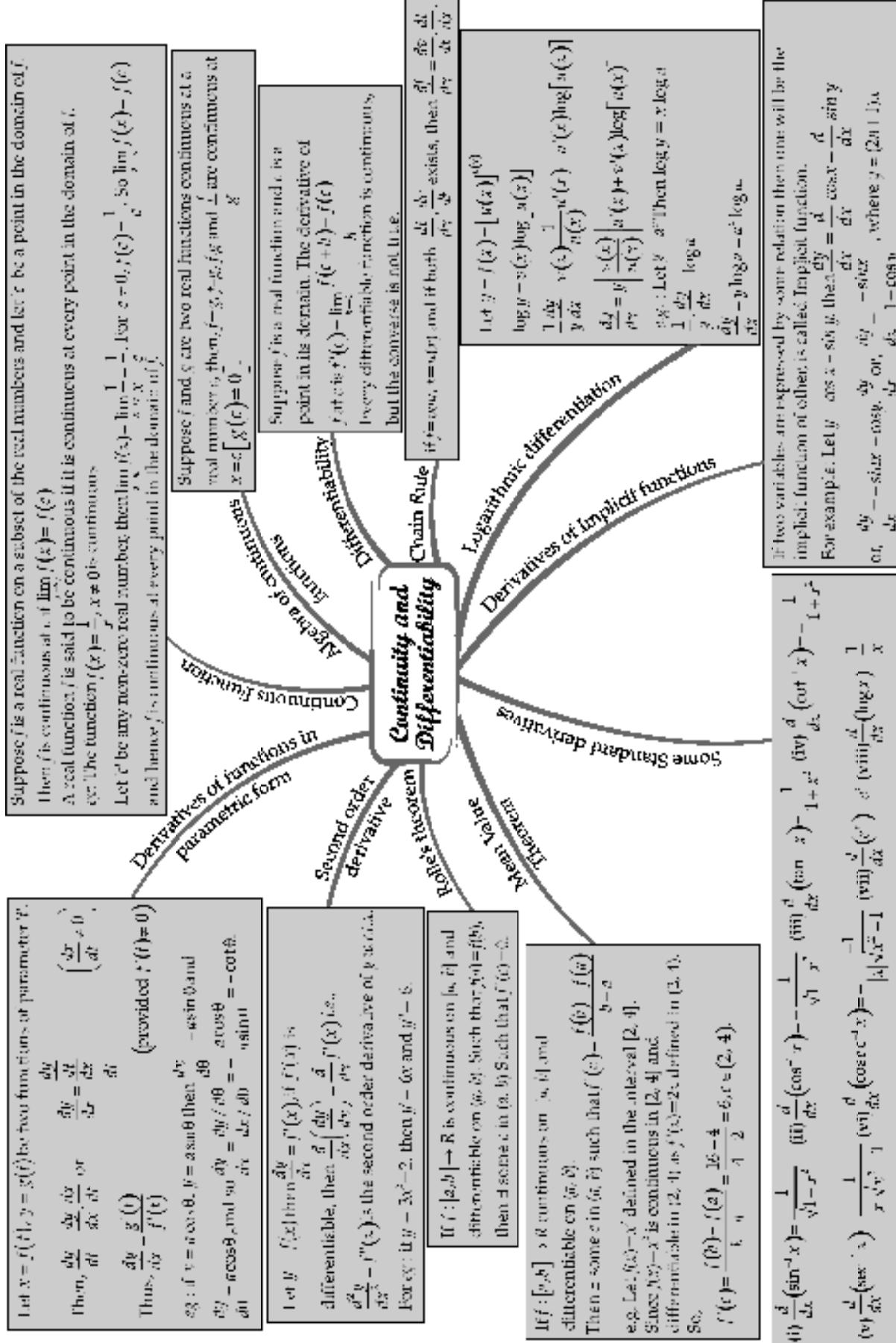
$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n(a+l)}{2}$$

If three numbers are in AP, then the middle term is called AM between the other two, so if  $a, b, c$  are in AP,  $b$  is AM of  $a$  and  $c$ .  
AM for any ' $n'$  +ve numbers  $a_1, a_2, a_3, \dots, a_n$  is

$$\text{AM} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

**n-Arithmetic Mean between Two Numbers:** If  $a, b$  are any two given numbers &  $a, A_1, A_2, \dots, A_n, b$  are in AP then  $A_1, A_2, \dots, A_n$  are  $n$  AM's between  $a$  &  $b$ .  
 $A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, \dots, A_n = a + \frac{n(b-a)}{n+1}$   
or  $A_1 = a+d, A_2 = a+2d, \dots, A_n = a+nd$ ,  
where  $d = \frac{b-a}{n+1}$





Let  $y = f(x)$ .  $\Delta x$  be a small increment in  $x$  and  $\Delta y$  be the small increment in  $y$  corresponding to the increment in  $x$ , i.e.,  $\Delta y = f(x + \Delta x) - f(x)$ . Then,  $\Delta y$  is given by  $\frac{dy}{dx}$  (i.e., if  $dy = \frac{dy}{dx} dx$ ,  $\Delta y$  is approximated of  $\Delta y$  when  $dx = \Delta x$  is relatively small and denoted by  $\Phi$  say).

e.g.— Let us approximate  $\sqrt{36.6}$ . To do this, we take  
 $y = \sqrt{x}$ ,  $x = 36$ ,  $\Delta x = 0.6$  then,  $\Delta y = \frac{dy}{dx} \Delta x = \frac{\sqrt{x}}{2\sqrt{x}} \Delta x = \frac{\sqrt{36}}{2\sqrt{36.6}} \times 0.6 = -0.015$

$$\therefore \sqrt{36.6} = 6 - 0.015 = 5.985$$

Now,  $\Phi$  is approximately  $\Delta y$  and is given by  

$$\frac{dy}{dx} \Big|_{x=36} = \frac{1}{2\sqrt{x}}(0.6) = \frac{1}{2\sqrt{36}}(0.6) = 0.05.$$

$$\therefore \text{So, } \sqrt{36.6} \approx 6 + 0.05 = 6.05.$$

A point  $C$  in the domain of  $f$  at which either  $f'(C) = 0$  or  $f$  is not differentiable is called a critical point of  $f$ .



- Let  $f$  be a function defined on a given interval  $I$ , twice differentiable at  $C$ . Then
- (i)  $x = C$  is a point of local maxima if  $f'(C) = 0$  and  $f''(C) < 0$ ,  $f''(C)$  is local maximum of  $f$ .
  - (ii)  $x = C$  is a point of local minima if  $f'(C) = 0$  and  $f''(C) > 0$ ,  $f''(C)$  is local minimum of  $f$ .
  - (iii) If  $f'(C) = 0$  and  $f''(C) = 0$ , then the test fails.

Let  $f$  be continuous at a critical point  $C$  in open interval. Then (i) if  $f'(x) > 0$  for every  $x$  in the open interval  $(x_1, x_2)$  where  $x_1 < C < x_2$ , then  $f$  is increasing in  $(x_1, x_2)$  and  $f$  is decreasing in  $(x_2, x_3)$  where  $x_3 > C$ . Then  $f'(x) = x^2 - 3x + 4$  is parallel to the  $x$ -axis and its equation is  $y = x^2 - 3x + 4$ .

The equation of tangent at  $(x_1, y_1)$  to the curve  $y = f(x)$  is given by  

$$(y - y_1) = \frac{dy}{dx} \Big|_{x=x_1} (x - x_1)$$

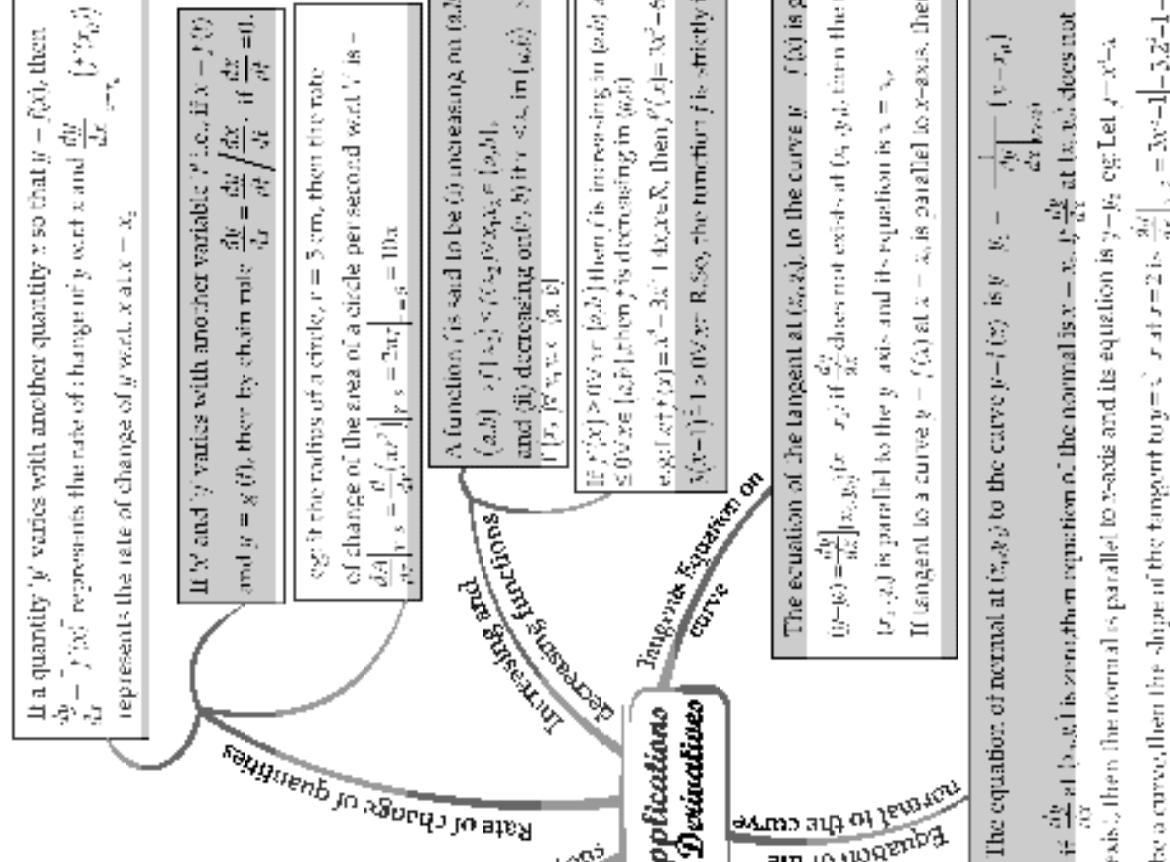
$$\therefore$$
 If  $\frac{dy}{dx}$  does not exist at  $(x_1, y_1)$ , then the tangent at  $(x_1, y_1)$  is parallel to the  $y$ -axis and its equation is  $y = y_1$ .

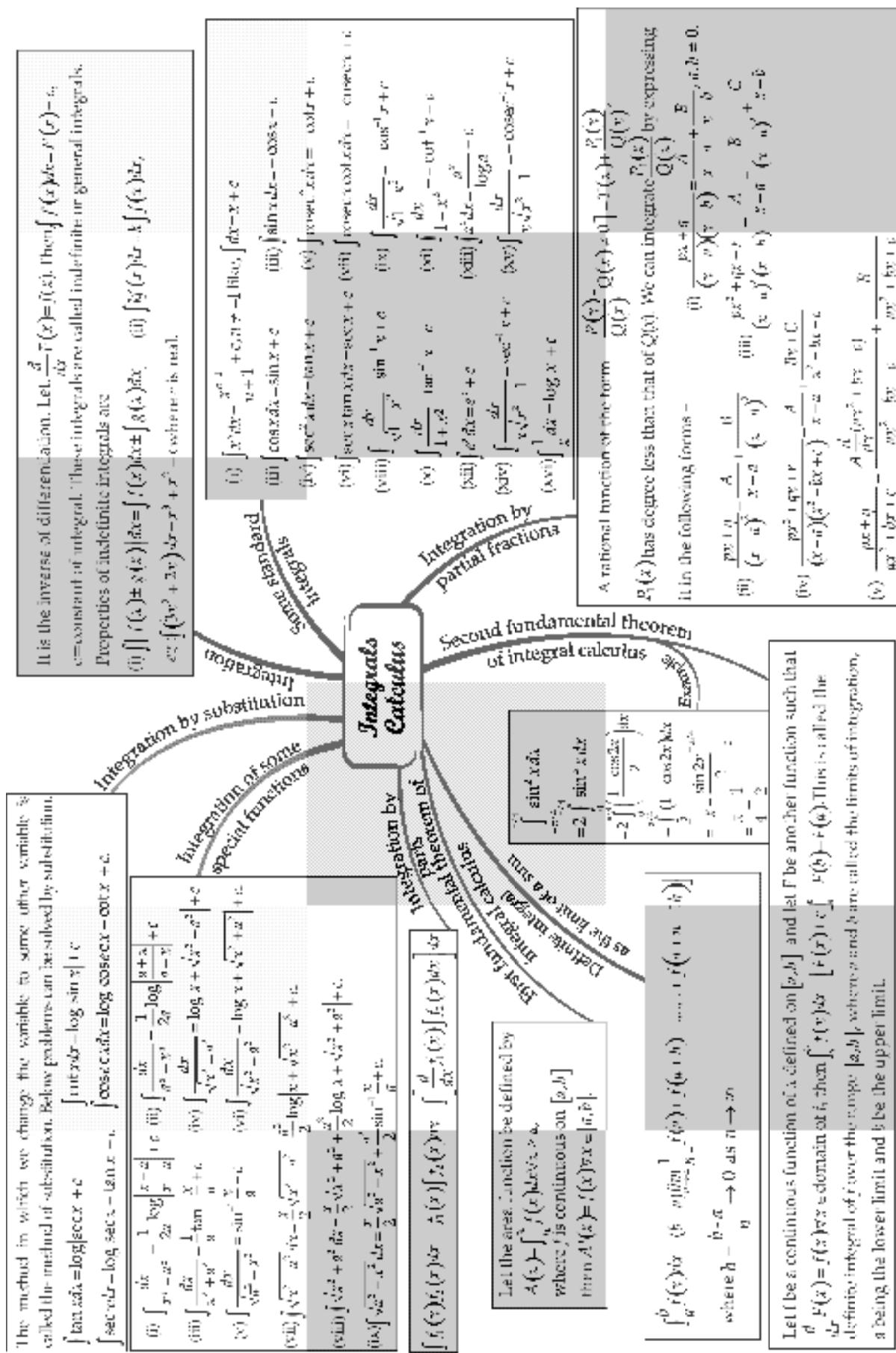
If tangent to a curve  $y = f(x)$  at  $x = x_0$  is parallel to  $x$ -axis, then  $\frac{dy}{dx}|_{x=x_0} = 0$ .

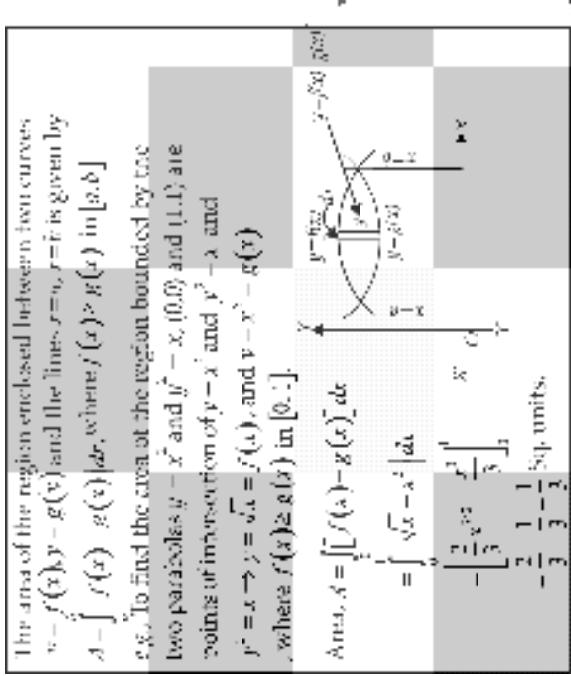
The equation of normal at  $(x_1, y_1)$  to the curve  $y = f(x)$  is given by  

$$(y - y_1) = -\frac{1}{\frac{dy}{dx}|_{x=x_1}} (x - x_1)$$

$$\therefore$$
 If  $\frac{dy}{dx}|_{x=x_1}$  is zero then equation of the normal is  $x = x_1$ . If  $\frac{dy}{dx}|_{x=x_1}$  does not exist, then the normal is parallel to  $x$ -axis and its equation is  $y = y_1$ ; e.g. Let  $y = x^{1/3}$  be a curve, then the slope of the tangent to  $y = x^{1/3}$  at  $x = 216$ ,  $\frac{dy}{dx}|_{x=216} = 3x^{1/3}|_{x=216} = 3 \cdot 216^{1/3} = 3 \cdot 6 = 18$ .







Some fundamental properties of definite integral are:

- Value of integration is independent of change of variable  $\int_a^b f(x) dx = \int_{\alpha}^{\beta} f(t) dt$

- If the limits of definite integral are interchanged then, its value changes only by minus sign i.e.

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx$$

$$= \int_a^c f(x) dx - \int_a^b f(x) dx$$

- If  $f(x)$  is an odd function, then  $\int_{-a}^a f(x) dx = 0$

- If  $f(x) \leq g(x)$  on the interval  $[a, b]$ , then  $\int_a^b f(x) dx \leq \int_a^b g(x) dx$

Properties of Definite Integrals

### Applications of the Integrals

If  $f(x) > g(x)$  in  $[a, b]$  and  $f(x) > h(x)$  in  $[a, b]$ , then the area is

$$A = \int_a^b [f(x) - g(x)] dx + \int_a^b [g(x) - h(x)] dx$$

Area bounded by  
two curves

Area bounded by  
three curves

The area of the region bounded by the curve  $y = f(x)$  and the lines  $x=a$ ,  $x=b$  is given by

$$A = \int_a^b f(x) dx$$

e.g. To find the area of the region bounded by the two parabolas  $y = x^2$  and  $y^2 = 4x$  and  $y^2 = x$  and  $y^2 = x \Rightarrow y = \sqrt{x}$ , where  $f(x) \geq g(x)$  in  $[0, 1]$ .

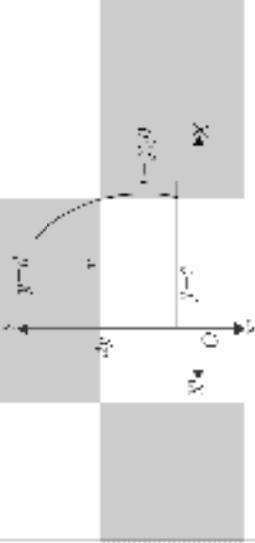
Area,  $A = \int_0^1 y^2 dx = \int_0^1 x dx = \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3}$  sq. units.

The area of the region bounded by the curve  $y = f(x)$ ,  $y=0$  and the lines  $x=a$ ,  $x=b$  is given by

$$A = \int_a^b y dx$$

e.g.: the area bounded by  $y=x^2$ ,  $y=0$  and the lines  $x=2$  and  $x=3$  is,

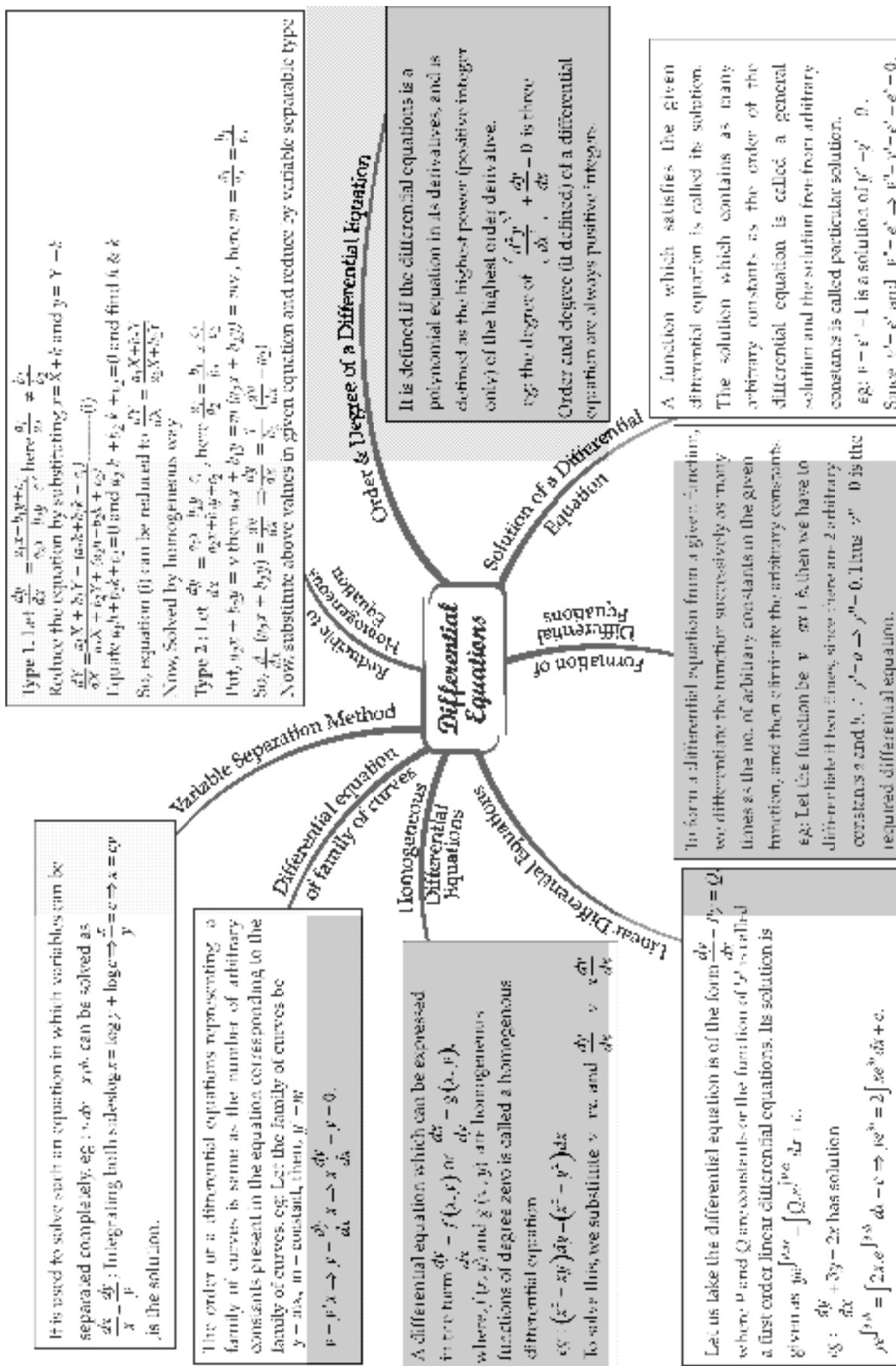
$$A = \int_2^3 x^2 dx = \frac{1}{3}x^3 \Big|_2^3 = \frac{1}{3}(27-8) = \frac{19}{3}$$
 sq. units.



The area of the region bounded by the curve  $y = f(x)$ ,  $y=0$  and the lines  $x=a$ ,  $x=b$  is given by  $A = \int_a^b y dx$  or  $\int_a^b f(x) dx$ .

e.g.: the area bounded by  $y=x^2 - x^3$ ,  $y=0$  and the lines  $x=1$  and  $x=2$  is

$$\int_1^2 x^2 - x^3 dx = \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_1^2 = \frac{1}{3}[2^3 - 1^3] - \frac{1}{4}[2^4 - 1^4] = \frac{15}{4}$$
 sq. units.

1. When two lines are parallel their slopes are equal. Thus, any line parallel to  $y = mx + c$  is of the type  $y = mx + d$ , where  $d$  is any parameter  
 2. Two lines  $ax + by + c = 0$  and  $a'x + b'y + c' = 0$  are parallel if  $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$   
 3. The distance between two parallel lines with equations  $ax + by + c_1 = 0$  and  $ax + by + c_2 = 0$  is  $d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

The equation of bisectors of the angles between the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  is given by  

$$y = \frac{a_1x + b_1y + c_1 \pm a_2x + b_2y + c_2}{\sqrt{a_1^2 + b_1^2} \pm \sqrt{a_2^2 + b_2^2}}$$

1. When two lines of the slope  $m_1$  &  $m_2$  are at right angles, the Product of their slope is  $-1$ , i.e.,  $m_1m_2 = -1$ . Thus, any line perpendicular to  $y = mx + c$  is of the form  $y = -\frac{1}{m}x + d$  where  $d$  is any parameter.  
 2. Two lines  $ax + by + c = 0$  and  $a'x + b'y + c' = 0$  are perpendicular if  $aa' + bb' = 0$ . Thus, any line perpendicular to  $ax + by + c = 0$  is of the form  $bx - ay + k = 0$ , where  $k$  is any parameter.

1. The image of a point  $(x_1, y_1)$  about a line  $ax + by + c = 0$  is:  

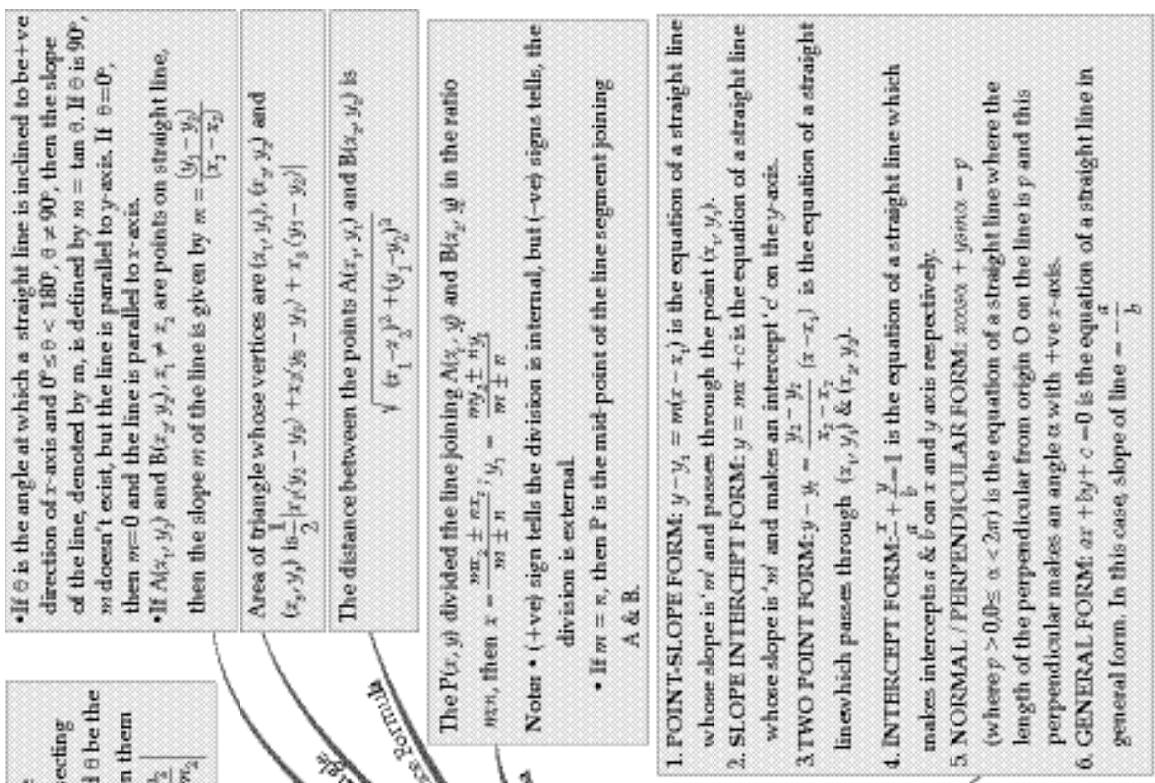
$$(a(x+x_1), b(y+y_1))$$
 where,  $r = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$
2. Perpendicular distances from a point  $(x_1, y_1)$  on the line  $ax+by+c=0$  is:  

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$
3. The foot of  $\perp$  from a point  $(x_1, y_1)$  on the line  $ax + by + c = 0$  is  

$$(xr_1, yr_1)(br_1, ar_1)$$
 where  $r = \frac{-(ax_1+by_1+c)}{a^2+b^2}$

The point of intersection between the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  is  

$$\left( \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)$$



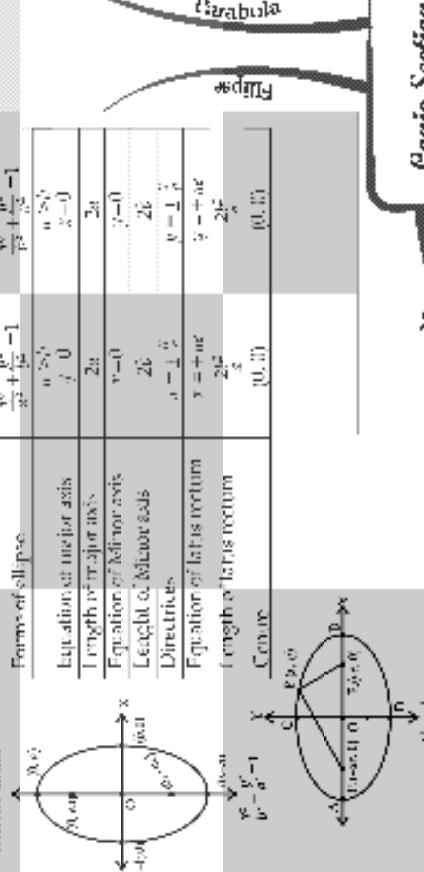
- An ellipse is the set of all points in a plane, that the sum of their distances from two fixed points in the plane is constant.

The two fixed points are called the 'foci' of the ellipse.

The midpoint of line segment joining foci is called the 'centre' of the ellipse.

The line segment through the foci of the ellipse is called 'major axis'.

The line segment through centre & perpendicular to major axis is called minor axis.



$$\text{Here, } a^2 \text{ and } b^2 = a^2(1-e^2), \text{ see 1}$$

- A hyperbola is the set of all points in a plane, that the difference of whose distances from two fixed points in the plane is a constant.

The two fixed points are called the 'foci' of the hyperbola.

The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.

The line through the foci and perpendicular to transverse axis is called 'conjugate axis'.

Points at which a hyperbola intersects transverse axis are called 'vertices'.

Forms of the Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Equation of transverse axis	$y=0$	$x=0$
Equation of conjugate axis	$x=0$	$y=0$
Length of transverse axis	$2a$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm ae)$
Equation of latus rectum	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Centre	$(0, 0)$	$(0, 0)$

Here,  $a^2 = a^2(e^2 - 1)$ ,  $e > 1$

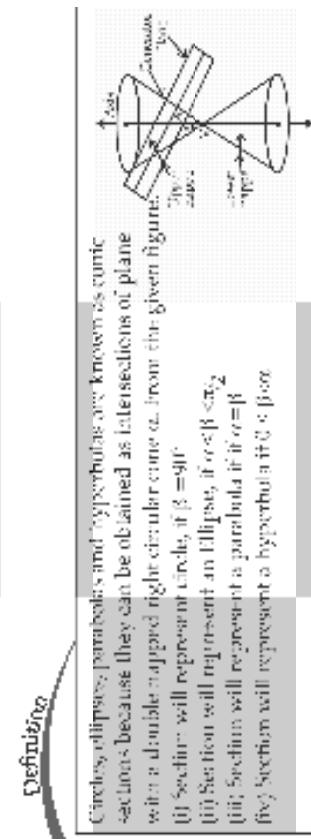
- A parabola is the set of all points in a plane that are equidistant from a fixed point in the plane, Fixed point is called 'focus' of parabola. Fixed point F is called the 'vertex'. A line through focus & perpendicular to axis of parabola is called 'axis'. Point of intersection of parabola w/ axis is called 'vertex'.

#### Main facts about the parabola

Focus of Parabola	$y^2 = 4ax$	$y^2 = -4ax$
Axis	$y=0$	$y=0$
Directrix	$x=-a$	$x=a$
Vertex	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$
Length of latus rectum	$4a$	$4a$
Equations of latus rectum	$x=a$	$x=-a$

#### Definition

#### Conic Sections

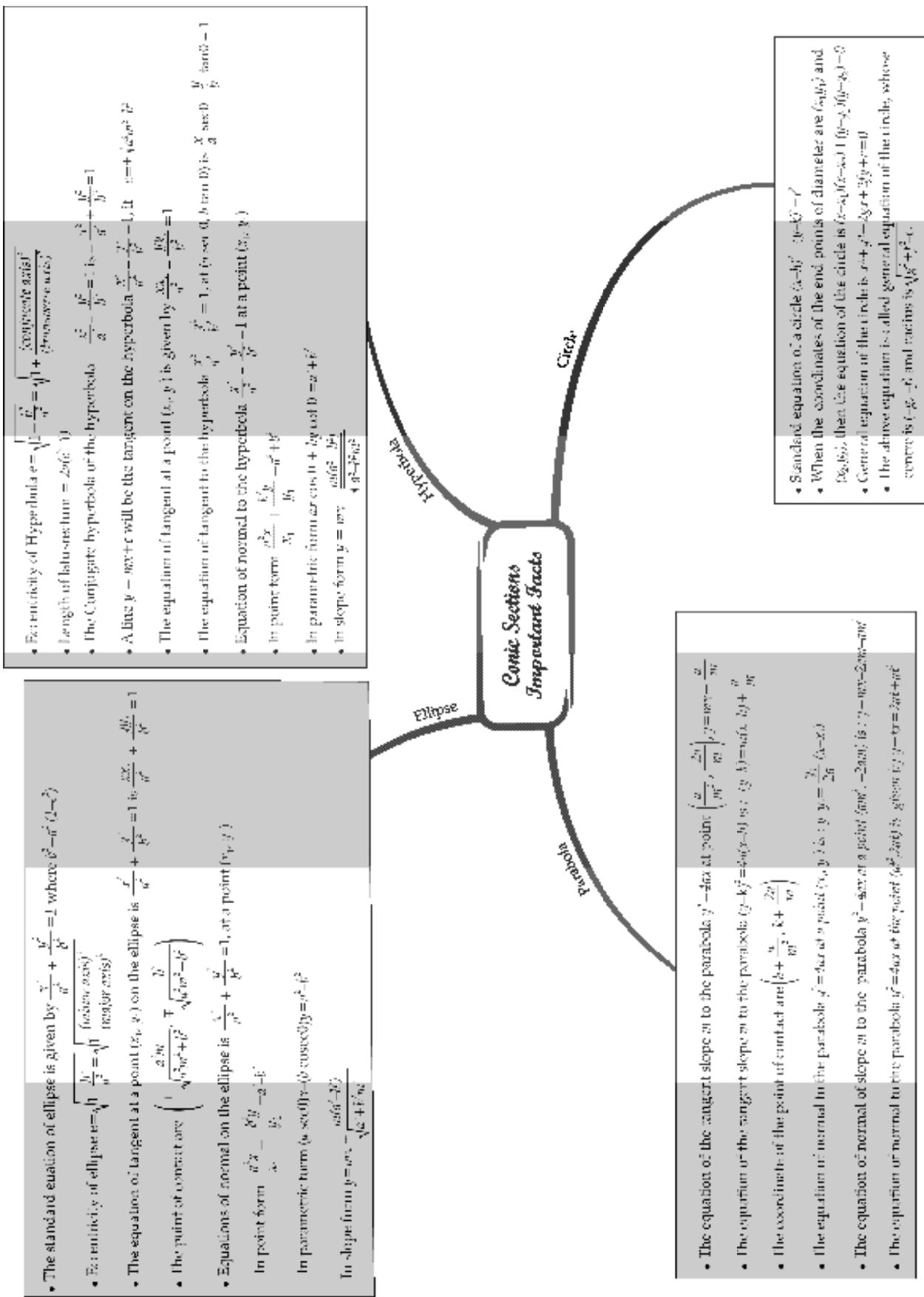


Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double-napped right circular cone  $\alpha$ . From the given figure, (i) 'Section' will represent a circle, if  $\beta = 90^\circ$ .  
 (ii) Section will represent an Ellipse, if  $0^\circ < \beta < 90^\circ$ .  
 (iii) Section will represent a parabola, if  $\beta = 90^\circ$ .  
 (iv) Section will represent a hyperbola, if  $\beta < 0^\circ$ .

A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre  $(h, k)$  and the radius  $r$  is  

$$(x-h)^2 + (y-k)^2 = r^2$$
  
 The general equation of circle is  $x^2 + y^2 + 2gx + 2fy + c = 0$   
 its centre is  $(-g, -f)$  and radius  $r = \sqrt{g^2 + f^2 - c}$



The coordinates of the centroid of the triangle, whose vertices are  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are  $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$ .

**eg:** The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, -5, 7) and (-1, 7, -6), respectively, find the coordinates of the point C.

Sol: Let the coordinates of C be  $(x, y, z)$ , and the coordinates of the centroid G be (1, 1, 1). Then  $\frac{x+3-1}{3} = 1$ , i.e.,  $x=1$ ;

$$\frac{y+7+7}{3} = 1, \text{i.e., } y=1;$$

$$\frac{z+7-6}{3} = 1, \text{i.e., } z=2.$$
 So, C (x, y, z) = (1, 1, 2).

The coordinates of the midpoint of the line segment joining two points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  are  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ .

**eg:** Find the midpoint of the line joining two points P(1, -3, 4) and Q(-4, 1, 2).

Sol: Coordinates of the midpoint of the line joining the points P & Q are  $\left(\frac{1-4}{2}, \frac{-3+1}{2}, \frac{4+2}{2}\right)$  i.e.  $\left(\frac{-3}{2}, -1, 3\right)$

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The axes are called x, y and z-axes.
- The three planes determined by the pair of axes are the coordinate planes, called xy and xz planes.
- The three coordinate planes divide the space into eight parts known as octants.
- The coordinates of a point P in 3D Geometry is always written in the form of triplet like (x, y, z). Here, x, y and z are the distances from xy, xz and yz planes, respectively.
- eg • Any point on x-axis is : (x, 0, 0)
  - Any point on y-axis is : (0, y, 0)
  - Any point on z-axis is : (0, 0, z)

Distance between two points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**eg:** Find the distance between the points P(1, -3, 4) and (-4, 1, 2).

Sol: The distance PQ between the points P & Q is given by

$$PQ = \sqrt{(-4 - 1)^2 + (1 + 3)^2 + (2 - 4)^2}$$

$$= \sqrt{25 + 16 + 4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

## Three Dimensional Geometry-I

### Section Formula

The coordinates of the point R which divides the line segment joining two points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  internally and externally in the ratio m : n are given by

$$\left(\frac{mx_1 + nx_2}{m+n}, \frac{my_1 + ny_2}{m+n}, \frac{mz_1 + nz_2}{m+n}\right) \text{ & } \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

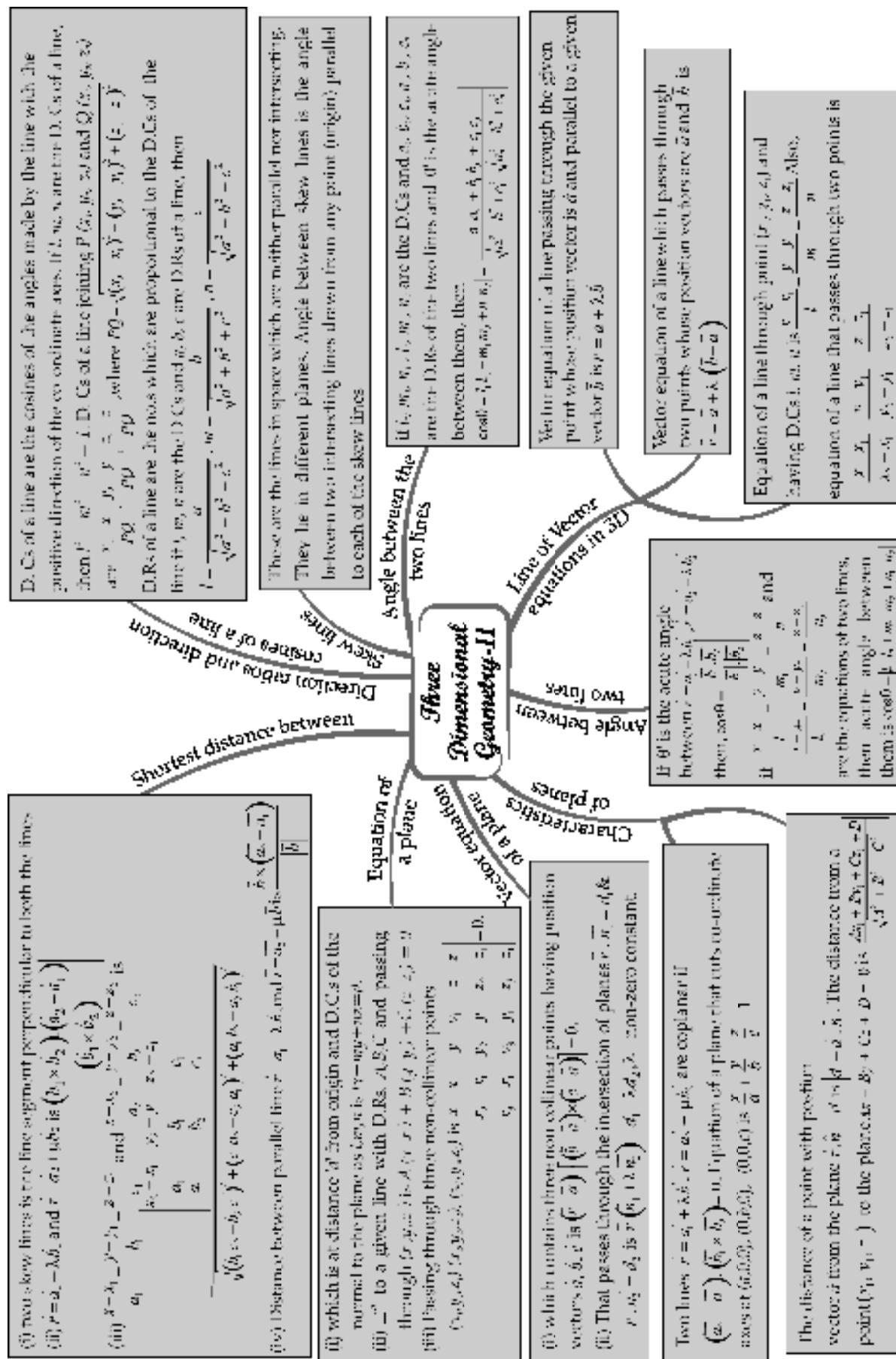
respectively.

**eg:** Find the coordinates of the point which divides the line segment joining the points (1, -2, 3) and (3, 4, -5) in the ratio 2:3 internally.

Sol: Let P(x, y, z) be the point which divides line segment joining A (1, -2, 3) and B (3, 4, -5) internally in the ratio 2:3. Therefore,

$$x = \frac{2(3) + 3(1)}{2+3} = \frac{9}{5}, y = \frac{2(4) + 3(-2)}{2+3} = \frac{2}{5}, z = \frac{2(-5) + 3(3)}{2+3} = \frac{-1}{5}$$

Thus, the required point is  $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$ .



A quantity that has both magnitude and direction is called a vector. The distance between the initial and terminal points of a vector is called its magnitude. Magnitude of vector  $\vec{AB}$  is  $|AB|$ .

For a given vector  $a$ , the vector  $\hat{a} = \frac{a}{|a|}$  gives the unit vector in the direction of  $a$ . If  $a^2 = \vec{AA}'$ , then  $\hat{a} = \frac{\vec{AA}'}{|AA'|} = \vec{AA}'$ , which is a unit vector.

The position vector of a point  $P(x, y, z)$  is  $\vec{OP} = xi + yj + zk$  and its magnitude is  $|OP| = \sqrt{x^2 + y^2 + z^2}$ . e.g. Position vector of  $P(2, 3, 5)$  is  $2\hat{i} + 3\hat{j} + 5\hat{k}$  and its magnitude is  $\sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$ .

**Properties**

- $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
- $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a}(\vec{b} \times \vec{c}) - \vec{b}(\vec{a} \times \vec{c})$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$  where  $\theta$  is the angle between them, then their scalar product  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

If  $\vec{a}, \vec{b}$  are the vectors and  $\hat{n}$  is the unit vector between them, then their scalar product  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$   
 $\Rightarrow \cos \theta = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}| |\vec{b}|}$

$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$  is a unit vector perpendicular to line joining  $a, b$ .

**Properties**

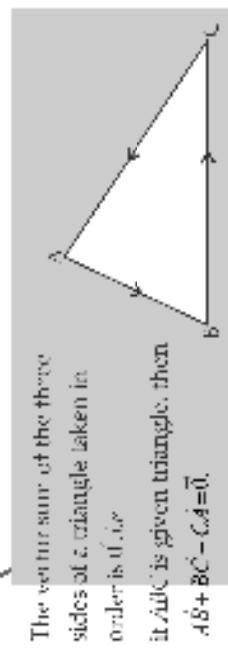
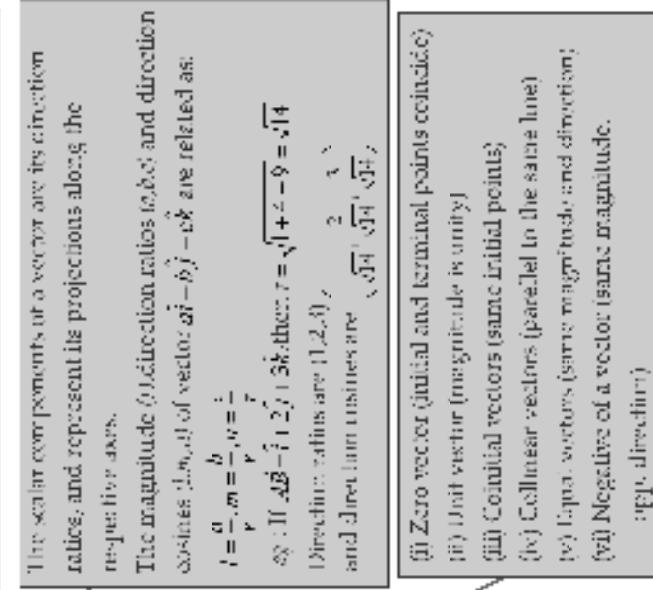
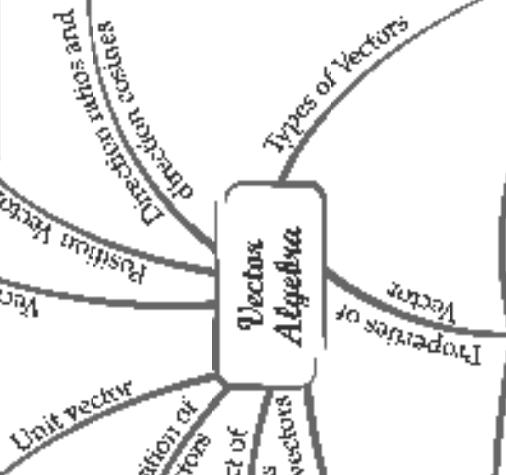
- $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
- $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a}(\vec{b} \times \vec{c}) - \vec{b}(\vec{a} \times \vec{c})$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$  and  $\hat{n}$  is any scalar then-

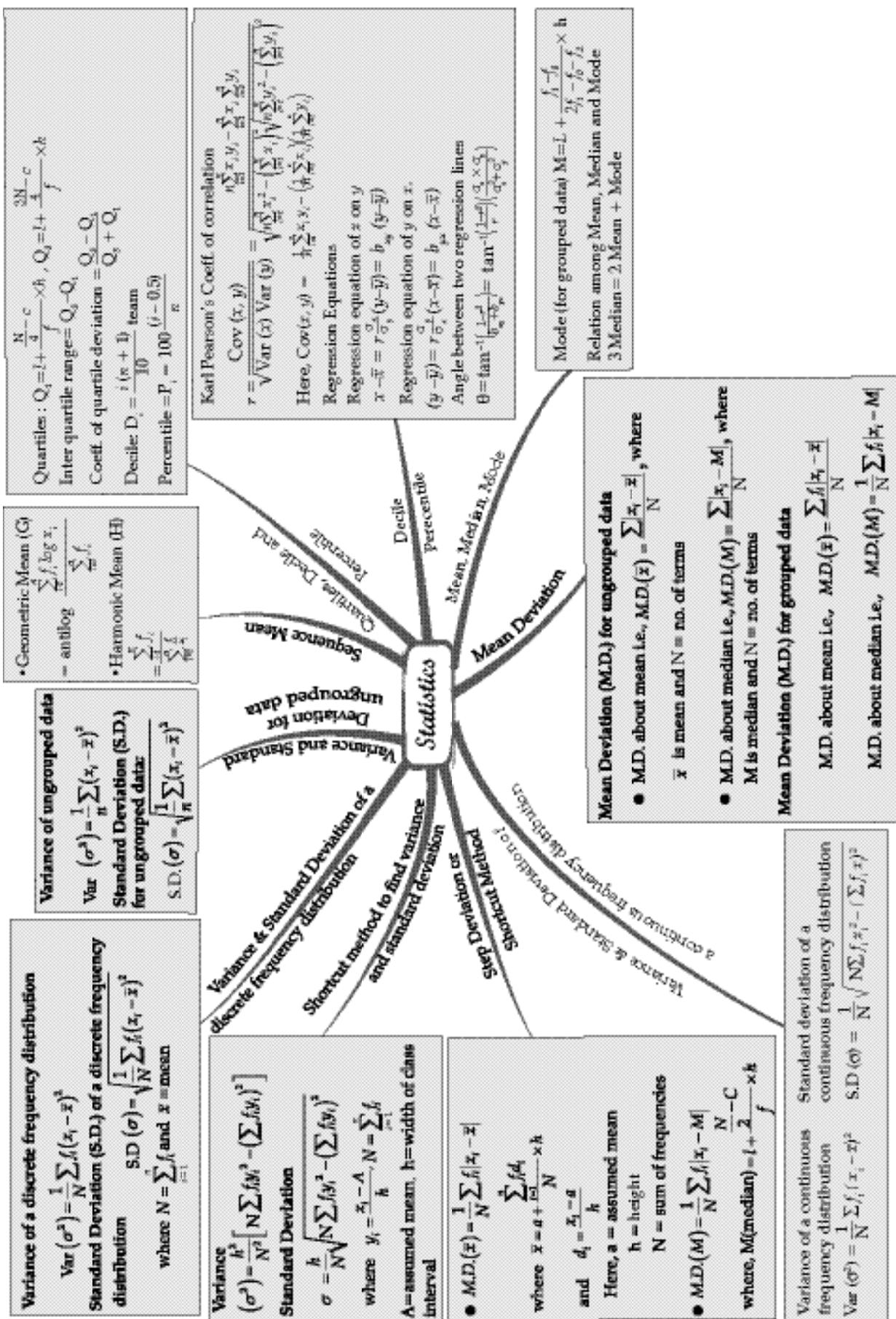
- (i)  $\lambda \vec{a} \times \vec{b} = \lambda(\vec{a} \times \vec{b}) = (\lambda \vec{a}) \times \vec{b}$
- (ii)  $\lambda \vec{a} \times \vec{b} = (\vec{a} \times \lambda \vec{b}) = \vec{a} \times (\lambda \vec{b})$
- (iii)  $\vec{a} \times \vec{b} = a_1 b_1 - a_2 b_2 - a_3 b_3$  and

$$\text{(iv)} \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

If  $\vec{AB}, \vec{AC}$  are the given vectors, then  $\vec{AB} \times \vec{AC} = \vec{AD}$

## Vector Algebra





Events A & B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event i.e. they cannot occur simultaneously.  
eg: A die is thrown. Event A = All even outcomes & event B = All odd outcomes. then,  
A & B are mutually exclusive events, they cannot occur simultaneously.

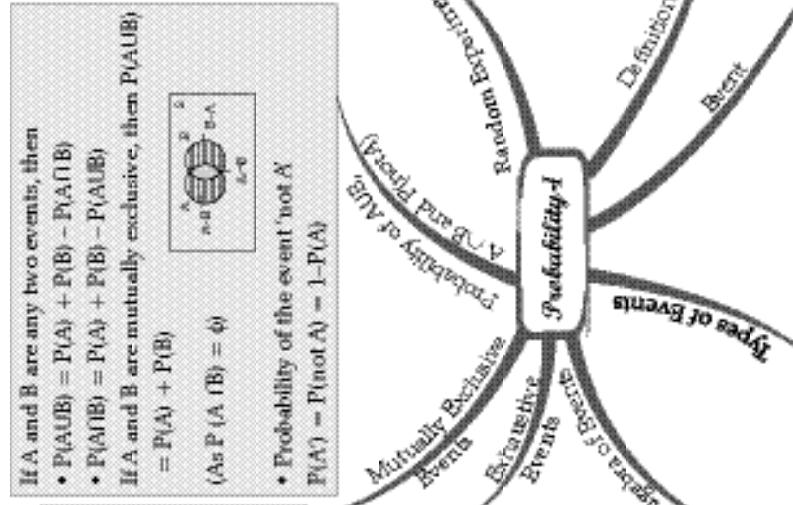
Note: Simple events of a sample space are always mutually exclusive.

Many events that together form sample space are called exhaustive events.

eg: A die is thrown. Event A = All even outcomes and event B = All odd outcomes.

Event A & B together forms exhaustive events as it forms sample space.

- Event A or B or (A<sub>c</sub> ∪ B)  
 $A ∪ B = \{w : w ∈ A \text{ or } w ∈ B\}$
- Event A and B or (A ∩ B)  
 $A ∩ B = \{w : w ∈ A \text{ and } w ∈ B\}$
- Event A but not B or (A - B)  
 $A - B = A ∩ B'$



An Experiment is called random experiment if it satisfies the following two conditions:

- It has more than one possible outcome.
- It is not possible to predict the outcome in advance.

Outcome A possible result of a random experiment is called its outcome.

Sample Space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by symbol 'S'.  
eg: In a toss of a coin, sample space is Head & Tail, i.e., S = {H, T}.

Sample Point: Each element of the Sample Space is called a sample point.

eg: In a toss of a coin, head is a sample point  
Equally Likely Outcomes: All outcome with equal probability.

Probability is the measure of uncertainty of various phenomenon, numerically, It can have positive value from 0 to 1.

$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

eg: Probability of getting an even no. in a throw of a die.  
Sol. Here, favourable outcomes = {2, 4, 6}  
Total no. of outcomes = {1, 2, 3, 4, 5, 6}

Probability =  $\frac{3}{6} = \frac{1}{2}$

It is the set of favourable outcomes. Any subset E of a sample space S is called an event.

eg: Event of getting an even number (outcome) in a throw of a die.

Occurrence of event: The event E of a sample space 'S' is said to have occurred if the outcome w of the experiment is such that  $w ∈ E$ . If the outcome w is such that  $w ∉ E$ , we say that event E has not occurred.

• Impossible and Sure Event: The empty set  $\emptyset$  is called an impossible event, where as the whole sample space 'S' is called 'Sure event'.

eg: In a rolling of a die, impossible event is that number more than 6 and event of getting number less than or equal to 6 is sure event.

• Simple Events: If an event has only one sample point of a sample space, it is called a 'simple event'.  
eg: In rolling of a die, simple event could be the event of getting number 4.

• Compound Events: If an event has more than one sample point, it is called a 'compound event'.  
eg: In a rolling of a die, compound event could be event of getting an even number.

• Complementary Event: Complement event to A = 'not A'  
eg: If an event A = Event of getting odd number in a throw of a die i.e., {1, 3, 5} then,  
complementary event to A = Event of getting an even number in a throw of a die, i.e., {2, 4, 6}  
 $A' = \{w : w ∈ S \text{ and } w ∉ A\} = S - A$  (where S is the sample space)

