JEE (Main) SOLVED PAPER

2023

08^{th} April Shift 1

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. A cylindrical wire of mass (0.4 ± 0.01)g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be:

(A) 4% **(B)** 1%
(C) 3.5% **(D)** 5%

Q. 2. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is : (neglect air speed. Speed of sound in air = 330 ms^{-1})

(A) 400 Hz **(B)** 388 Hz
(C) 200 Hz **(D)** 412 Hz

Q. 3. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(A) 300 N **(B)** Zero
(C) 100 N **(D)** 200 N

Q. 4. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(A) 120 km^2 **(B)** 1549 km^2
(C) 4868 km^2 **(D)** 3942 km^2

Q. 5. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

(A) To produce large deflecting torque on the coil
(B) To bring the coil to rest quickly
(C) To oscillate the coil in magnetic field for longer period of time
(D) To make the magnetic field radial

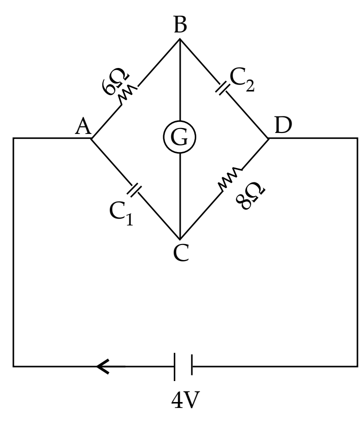
Q. 6. Dimension of $\frac{1}{\mu_0\varepsilon_0}$ should be equal to

(A) $\frac{T}{L}$ **(B)** $\frac{T^2}{L^2}$
(C) $\frac{L}{T}$ **(D)** $\frac{L^2}{T^2}$

Q. 7. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is (g = 10 m/s^2)

(A) $2:\sqrt{3}$ **(B)** $\sqrt{3}:2$
(C) 4 : 9 **(D)** 1 : 1

Q. 8. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(A) $\frac{5}{4}$ **(B)** 1
(C) $\frac{4}{5}$ **(D)** $\frac{3}{4}$

Q. 9. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(A) Helical path with the axis along magnetic field B

(B) Straight along the direction of magnetic field B

(C) Helical path with the axis perpendicular to the direction of magnetic field B

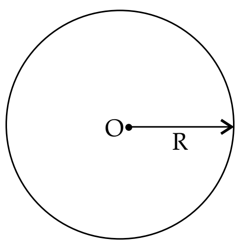
(D) Circular path

Q. 10. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$):

(A) 1 : 43 **(B)** 43 : 1

(C) 1 : 1849 **(D)** 1 : 1

Q. 11. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



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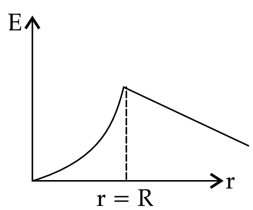

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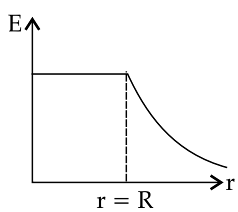

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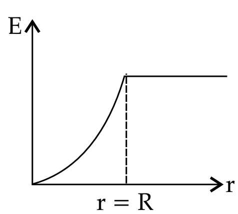

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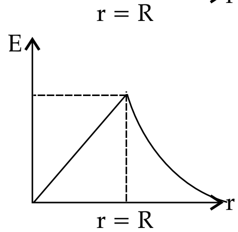


Q. 12. For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon $(b_s) = -a_1 A^{2/3}$.

B. The Coulomb contribution to the binding energy $b_c = -a_2 \frac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $bv = a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the most appropriate answer from the options given below:

(A) B, C only **(B)** A, B, C, D only

(C) B, C, E only **(D)** C, D only

Q. 13. At any instant the velocity of a particle of mass 500 g is $(2t\hat{i} + 3t^2\hat{j})$ ms^{-1}. If the force acting on the particle at t = 1s is $(\hat{i} + x\hat{j})$ N . Then the value of x will be:

(A) 2 **(B)** 6

(C) 3 **(D)** 4

Q. 14. Given below are two statements:

Statement I: If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$.

Statement II: The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(A) Both Statement I and Statement II are incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Statement I is correct but Statement II is incorrect

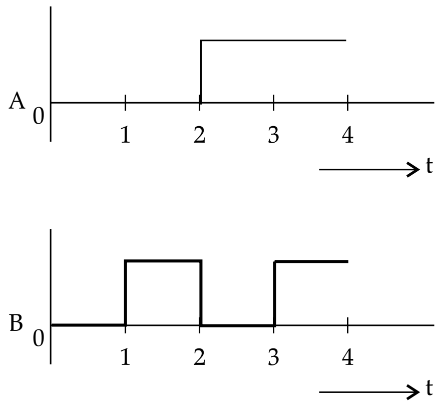
(D) Both Statement I and Statement II are correct

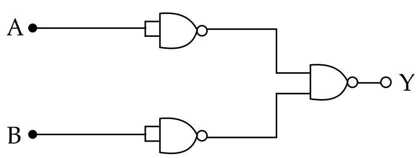
Q. 15. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is:

(A) $\frac{5A}{2}$ **(B)** $\frac{\sqrt{5}A^2}{2}$

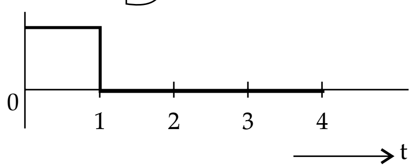
(C) $\frac{\sqrt{5}A}{4}$ **(D)** $\frac{\sqrt{5}A}{2}$

Q. 16. For the logic circuit shown, the output waveform at Y is:





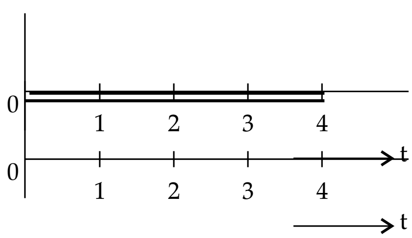
(A)



(B)

(C)

(D)



Q. 17. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(A) 5600 **(B)** 2800
(C) 11200 **(D)** 8400

Q. 18. Given below are two statements:
Statement I: If heat is added to a system, its temperature must increase.
Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.
In the light of the above statements, choose the correct answer from the options given below
(A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Q. 19. An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be:

(A) 3 cm^3 **(B)** 4 cm^3
(C) 2 cm^3 **(D)** 5 cm^3

Q. 20. In a reflecting telescope, a secondary mirror is used to:
(A) Make chromatic aberration zero
(B) Reduce the problem of mechanical support
(C) Move the eyepiece outside the telescopic tube
(D) Remove spherical aberration

Section B

Q. 21. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is ________ %

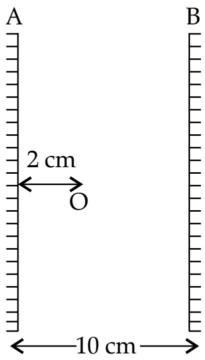
Q. 22. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is _________ MeV.

Q. 23. An electric dipole of dipole moment 6.0×10^{-6} Cm placed in a uniform electric field of 1.5×10^3 NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be _________ mJ.

Q. 24. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is _________ Hz.

Q. 25. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}$ MR^2, where R is the radius and M is the mass of the semicircular ring. The value of x will be _________.

Q. 26. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is ________ cm



Q. 27. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^3 Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is __________ A.

Q. 28. A current of 2 A through a wire of cross-sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is ________ $\times 10^{-6}$ ms^{-1}

(given, charge on electron = 1.6×10^{-19} C).

Q. 29. An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be __________ mA.

Q. 30. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. TGe co-efficient of viscosity of the solution (neglect density of air) is __________ poise (given, g = 10 ms^{-2}).

Chemistry

Section A

31. The reaction

$$\frac{1}{2}H_2(g)+AgCl(s) \rightleftharpoons H^+(aq)+Cl^-(aq)+Ag(s)$$

Occurs in which of the given galvanic cell.

(1) Pt $|H_2(g)|HCl(sol^n)|AgNO_3(sol^n)|$Ag

(2) Pt $|H_2(g)|HCl(sol^n)|AgCl(s)|$Ag

(2) Pt $|H_2(g)|KCl(sol^n)|AgCl(s)|$Ag

(4) Ag $|AgCl(s)|KCl(sol^n)|AgNO_3|$Ag

32. Sulphur (S) containing amino acids from the following are:

(a) isoleucine

(b) cysteine

(c) lysine

(d) methionine

(e) glutamic acid

(1) b, c, e **(2)** a, d

(3) a, b, c **(4)** b, d

33. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $K_3[Co(CN)_6]$ **(2)** $[Ni(NH_3)_6]Cl_2$

(3) $[Co(H_2O)_6]Cl_2$ **(4)** $Na_3[CoCl_6]$

34. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu **(2)** Au

(3) Pb **(4)** Sn

35. The correct order of spin only magnetic moments for the following complex ions is

(1) $[CoF_6]^{3-}<[MnBr_4]^{2-}<[Fe(CN)_6]^{3-}<[Mn(CN)_6]^{3-}$

(2) $[Fe(CN)_6]^{3-}<[CoF_6]^{3-}<[MnBr_4]^{2-}<[Mn(CN)_6]^{3-}$

(3) $[MnBr_4]^{2-}<[CoF_6]^{3-}<[Fe(CN)_6]^{2-}<[Mn(CN)_6]^{3-}$

(4) $[Fe(CN)_6]^{3-}<[Mn(CN)_6]^{3}<[CoF_6]^{3-}<[MnBr_4]^{2-}$

36. The water gas on reacting with cobalt as a catalyst forms

(1) Methanoic acid **(2)** Methanal

(3) Ethanol **(4)** Methanol

37. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

What is the value of x?

(1) 12 **(2)** 10

(3) 2 **(4)** 6

38. What is the purpose of adding gypsum to cement?

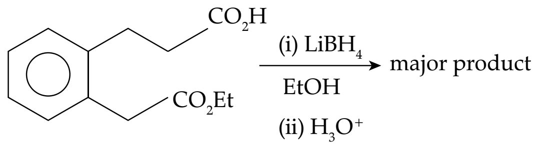
(1) To give a hard mass

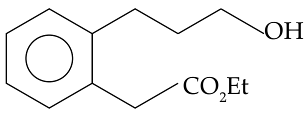
(2) To speed up the process of setting

(3) To facilitate the hydration of cement

(4) To slow down the process of setting

39. The major product formed in the following reaction is:



(1)


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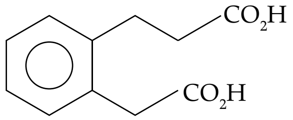

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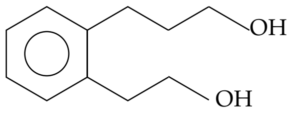
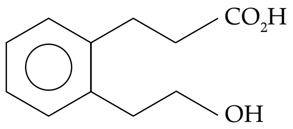


(4)


40. Match list I with list II:

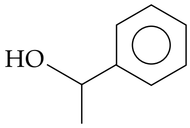
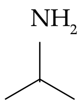
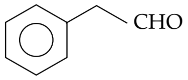
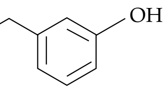
List I (species)	List II (Maximum allowed concentration in ppm in drinking water)
A. F^-	I. <50 ppm
B. SO_4^{2-}	II. <5 ppm
C. NO_3^-	III. <2 ppm
D. Zn	IV. <500 ppm

(1) A-III, B-II, C-I, D-IV
(2) A-II, B-I, C-III, D-IV
(3) A-IV, B-III, C-II, D-I
(4) A-I, B-II, C-III, D-IV

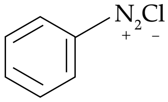
41. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti: 22, V : 23, Cr : 24, Mn : 25, Fe : 26)
(1) Fe (III) **(2)** V (IV)
(3) Ti (III) **(4)** Mn (VII)

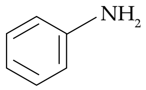
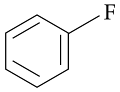
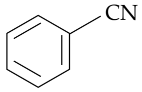
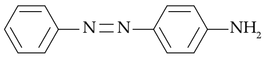
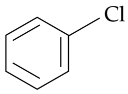
42. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: Butan-1-ol has higher boiling point than ethoxyethane.
Reason R: Extensive hydrogen bonding leads to stronger association of molecules.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are true but R is not the correct explanation of A
(2) Both A and R are true and R is the correct explanation of A
(3) A is false but R is true
(4) A is true but R is false

43. Match List I with List II:

List I (Reagents used)	List II (Compound with Functional group detected)
A. Alkaline solution of copper sulphate and sodium citrate	I. 
B. Neutral $FeCl_3$ solution	II. 
C. Alkaline chloroform solution	III. 
D. Potassium iodide and sodium hypochloride	IV. 

Choose the correct answer from the options given below:
(1) A-III, B-IV, C-II, D-I
(2) A-II, B-IV, C-III, D-I
(3) A-IV, B-I, C-II, D-III
(4) A-III, B-IV, C-I, D-II

44. Match List I with List II:
 is reacted with reagents in List I to form products in List II.

List I (Reagent)	List II (Product)
A. 	I. 
B. HBF_4, Δ	II. 
C. Cu, HCl	III. 
D. CuCN/KCN	IV. 

Choose the correct answer from the options given below:
(1) A-I, B-III, C-IV, D-II **(2)** A-III, B-I, C-II, D-IV
(3) A-III, B-I, C-IV, D-II **(4)** A-IV, B-III, C-II, D-I

45. Match List I with List II:

List I	List II
A. Saccharin	I. High potency sweetener
B. Aspartame	II. First artificial sweetening agent
C. Alitame	III. Stable at cooking temperature
D. Sucralose	IV. Unstable at cooking temperature

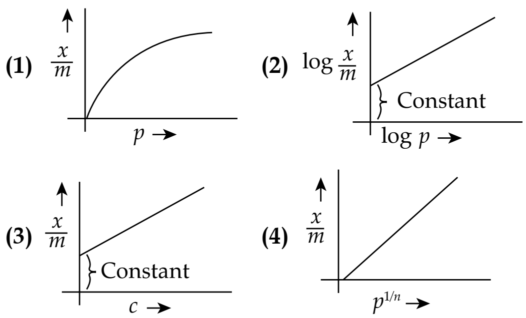
Choose the correct answer from the options given below:
(1) A-II, B-III, C-IV, D-I
(2) A-II, B-IV, C-I, D-III
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-IV, C-III, D-I

46. The correct order of electronegativity for given elements is:
(1) P > Br > C > At **(2)** C > P > At > Br
(3) Br > P > At > C **(4)** Br > C > At > P

47. Given below are two statements:
Statement I: Lithium and Magnesium do not form superoxide
Statement II: The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct
(3) Both statement I and Statement II are correct
(4) Both statement I and Statement II are incorrect

48. Which of the following represent the Freundlich adsorption isotherms?

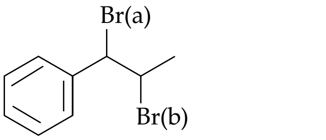


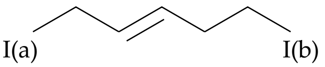
49. Which halogen is known to cause the reaction given below:

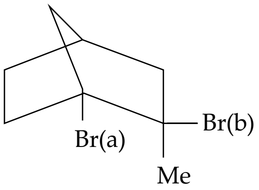
$2Cu^{2+} + 4X^- \rightarrow Cu_2X_2(s) + X_2$

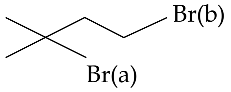
(1) All halogens (2) Only chlorine
(3) Only Bromine (4) Only Iodine

50. Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C, & D)

A. 

B. 

C. 

D. 

(1) A-Br(a) ; B-I(a) ; C-Br(b) ; D-Br(a)
(2) A-Br(b) ; B-I(a) ; C-Br(a) ; D-Br(a)
(3) A-Br(b) ; B-I(b) ; C-Br(b) ; D-Br(b)
(4) A-Br(a) ; B-I(a) ; C-Br(a) ; D-Br(a)

Section B

51. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanal and propanone is __________ g mol^{-1} (Molar mass of C : 12 g mol^{-1}, H : 1 gmol^{-1})

52. XeF_4 reacts with SbF_5 to form

$[XeF_m]^{n+}[SbF_y]^{z-}$

$m + n + y + z =$ __________

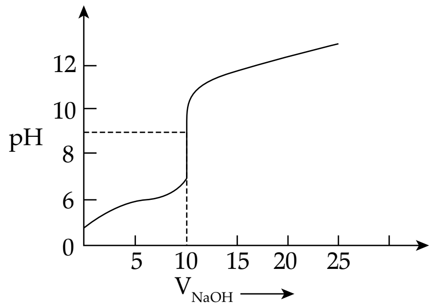
53. The number of following statements which is/are incorrect is __________

(1) Line emission spectra are used to study the electronic structure.
(2) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet.
(3) An absorption spectrum is like the photographic negative of an emission spectrum.
(4) The element helium was discovered in the sun by spectroscopic method.

54. The titration curve of a weak acid vs. strong base with phenolphthalein as indictor is shown below.

The $K_{phenolphthalein} = 4 \times 10^{-10}$

Given: log 2 = 0.3

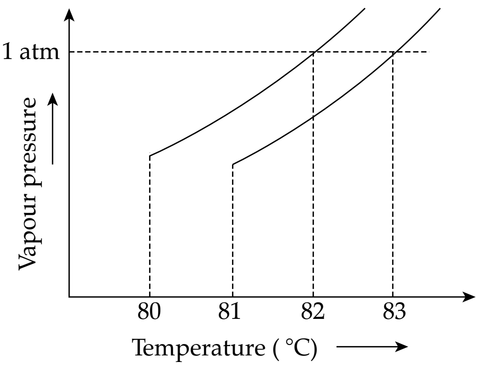


The number of following statement/s which is/are correct about phenolphthalein is __________

(1) It can be used as an indicator for the titration of weak acid with weak base.
(2) It begins to change colour at pH = 8.4
(3) It is a weak organic base
(4) It is colourless in acidic medium

55. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is _______ J K^{-1}(Nearest integer).

56. The vapour pressure vs. temperature curve for a solution solvent system is shown below:



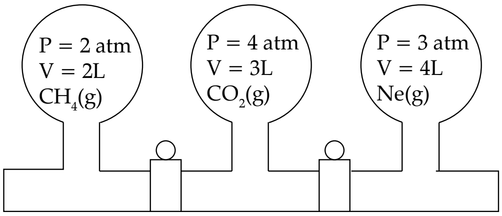
The boiling point of the solvent is ______ °C

57. 0.5 g of an organic compound (X) with 60% carbon will produce ______ × 10^{-1} g of CO_2 on complete combustion.

58. The number of following factors which affect the percent covalent character of the ionic bond is_____

(1) Polarising power of cation
(2) Extent of distortion of anion
(3) Polarisability of the anion
(4) Polarising power of anion

59.



Three bulbs are filled with CH_4, CO_2 and Ne as shown the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be________atm. (Nearest integer)

60. The number of given statement/s which is/are correct is______

(1) The stronger the temperature dependence of the rate constant, the higher is the activation energy.
(2) If a reaction has zero activation energy, its rate is independent of temperature.
(3) The stronger the temperature dependence of the rate constant, the smaller is the activation energy
(4) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Mathematics

Section A

61. The area of the region $\{(x, y): x^2 \le y \le 8 - x^2, y \le 7\}$ is

(A) 24 **(B)** 21
(C) 20 **(D)** 18

62. Let $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, If

$P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

(A) 2004 **(B)** 2007
(C) 2005 **(D)** 2006

63. Negation of $(p \rightarrow q) \rightarrow (q \rightarrow p)$ is

(A) $(-q) \wedge p$ **(B)** $p \vee (\sim q)$
(C) $(\sim p) \vee q$ **(D)** $q \wedge (\sim p)$

64. Let C(α, β) be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$,
$4y - 3x = 17$ and
$x + 7y = 61$.
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(A) 18 **(B)** 15
(C) 16 **(D)** 17

65. Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(A) $\frac{155}{8}$ **(B)** 21

(C) 19 **(D)** $\frac{169}{8}$

66. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

(A) 752 **(B)** 772
(C) 782 **(D)** 792

67. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(A) 5481 **(B)** 3654
(C) 2436 **(D)** 1817

68. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q.

Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is

(A) 325 **(B)** 346
(C) 296 **(D)** 317

69. Let $S_K = \frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D ∈ N and A has least value. Then

(A) A + B is divisible by D
(B) A + B = 5 (D –C)
(C) A + C + D is not divisible by B
(D) A + B + D is divisible by 5

70. The shortest distance between the lines

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(A) $2\sqrt{6}$ **(B)** $3\sqrt{6}$

(C) $6\sqrt{3}$ **(D)** $6\sqrt{2}$

71. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.
(A) 16800 **(B)** 14800
(C) 18000 **(D)** 33600

72. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}$, $6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to
(A) 49 **(B)** 36
(C) 25 **(D)** 16

73. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.
(A) $\frac{5}{14}$ **(B)** $\frac{3}{7}$
(C) $\frac{9}{28}$ **(D)** $\frac{2}{7}$

74. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation
(A) $x^2 + 3x - 4 = 0$ **(B)** $x^2 + 7x + 12 = 0$
(C) $x^2 + x - 12 = 0$ **(D)** $x^2 + 2x - 3 = 0$

75. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to __________.
(A) 24 **(B)** 9
(C) 18 **(D)** 15

76. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is
(A) $7\,(720)^2$ **(B)** 720
(C) $7\,(360)^2$ **(D)** $126\,(5!)^2$

77. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to
(A) $\frac{-2}{3}$ **(B)** $\frac{2}{9}$
(C) $\frac{-1}{3\sqrt{3}}$ **(D)** $\frac{2}{3\sqrt{3}}$

78. If the eqation of the plane containing the line $x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ is $ax + by + cz = 4$, then $(a - b + c)$ is equal to
(A) 22 **(B)** 24
(C) 20 **(D)** 18

79. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj (adj (adj } 2A))| = (16)^n$, then n is equal to
(A) 8 **(B)** 9
(C) 12 **(D)** 10

80. Let $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx, x>0.$ $\lim_{x\to\infty} I(x) = 0$, then I(1) is equal to
(A) $\frac{e+1}{e+2}-\log_e(e+1)$ **(B)** $\frac{e+2}{e+1}+\log_e(e+1)$
(C) $\frac{e+2}{e+1}-\log_e(e+1)$ **(D)** $\frac{e+1}{e+2}+\log_e(e+1)$

Section B

81. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = $(x, y) \in A \times A$: $x - y$ is odd positive integer or $x - y = 2$}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ______.

82. Let $[t]$ denote the greatest integer $\leq t$, If the constant term in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to _____.

83. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$ is

84. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x \log_e x^2)\,dy = 0$, $x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to ______.

85. Let $\vec{a}=6\hat{i}+9\hat{j}+12\hat{k}, \vec{b}=\alpha\hat{i}+11\hat{j}-2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$. If $\vec{a}.\vec{c}=-12$, $\vec{c}.(\hat{i}-2\hat{j}+\hat{k})=5$, then $\vec{c}(\hat{i}+\hat{j}+\hat{k})$ is equal to ________.

86. The largest natural number n such that $3n$ divides 66! is ______.

87. If a_0 is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}, n = 1, 2, 3, ...,$ then a is equal to ________.

88. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to ________.

89. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to ________.

80. Let $[t]$ denote the greatest integer $\leq t$. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} (8\,[\text{cosec } x] - 5\,[\cot x])\,dx$ is equal to ________.

Answer Key

Physics

Q. No.	Answer	Topic Name	Chapter Name
1	A	Error	Unit and dimension
2	A	Doppler's effect	Sound wave
3	D	Variation of g	Gravitation
4	D	Range	Communication system
5	B	Galvanometer	Magnetism
6	D	Dimensional equation	Units & Dimension
7	C	Range of projectile	Motion in 2D
8	C	RC circuit	Electric current
9	A	Force on moving charge in magnetic field	Moving charge and magnetism
10	D	Debroglie wavelength	Dual nature of matter
11	D	electric field due to charged sphere	Electrostatics
12	D	Binding energy	Nuclear physics
13	C	Momentum	Newton's second law
14	A	Energy of satellite	Gravitation
15	D	Resultant vector	Vectors
16	B	Logic gates	Semiconductors
17	A	Energy stored in stretched rod	Elasticity
18	D	Thermodynamic process	Thermodynamics
19	D	Isothermal process	Thermodynamics
20	C	Telescope	Ray optics
21	[125]	Kinetic energy	Work, Energy and Power
22	[121]	Binding energy	Nuclear physics
23	[18]	Dipole in uniform electric field	Electric dipole
24	[900]	Organ pipe	Sound wave
25	[1]	MI of semicircular ring	Rotational motion
26	[18]	Plane mirror	Ray optics
27	[2]	Solenoid	Electromagnetism
28	[25]	Drift velocity	Electric current
29	[9]	LC circuit	Electromagnetic induction
30	[10]	Terminal velocity	Fluid mechanics

Chemistry

Q No	Answer	Topic's Name	Chapter's Name
31	(2)	Electrochemical Cell	Electrochemistry
32	(4)	Proteins and Polysaccharide	Biomolecules
33	(1)	Crystal Field Theory	Coordination Compounds
34	(4)	Concentration of Ores	General Principles and Processes of Isolation of Elements
35	(4)	Crystal Field Theory	Coordination Compounds
36	(4)	Catalysis	Surface Chemistry
37	(2)	Applications of Oxidation Number	Redox Reaction
38	(4)	Some Important Compounds of Calcium	S-Block Elements
39	(4)	Reactions of Carboxylic Acid	Aldehydes, Ketones & Carboxylic Acids
40	(1)	Water Pollution	Environmental Chemistry
41	(4)	Some Transition Elements	d & f Block Elements
42	(2)	Physical Properties	Alcohols, Phenols & Ethers
43	(1)	Detection of Functional Group	Salt Analysis
44	(3)	Naming Reaction	Nitrogen Containing Compounds
45	(2)	Chemicals in Food	Chemistry In Everyday Life
46	(4)	Trends in Physical Properties	Classification of Elements and Periodicity in Properties
47	(3)	Trends in Physical Properties	Classification of Elements and Periodicity in Properties
48	(BONUS)	Freundlich Isothermal	Surface Chemistry
49	(4)	Some Transition Elements	d & f Block Elements
50	(1)	Fundamental Concepts in Organic Reaction Mechanism	Organic Chemistry - Some Basic Principles and Techniques
51	[70]	Law of Chemical Combinations	Some Basic Concepts of Chemistry
52	[11]	Some Transition Elements	d & f Block Elements
53	[1]	Evidence for Quantized Electronic Energy Levels	Atomic Structure
54	[2]	Hydrolysis of Salts and the pH of their Solutions	Ionic Equilibrium
55	[1200]	Adiabatic	Thermodynamics
56	[82]	Vapour Pressure of Solutions of Liquids in Liquids	Solutions
57	[11]	Stoichiometry	Some Basic Concepts of Chemistry
58	[3]	Ionic And Electrovalent Bond	Chemical Bonding & Molecular Structure
59	[3]	Ideal Gas Equation	States of Matter
60	[2]	Effect of Temperature and Catalyst on Rate of a Reaction	Chemical Kinetics

Mathematics

Q. No.	Answer	Topic Name	Chapter Name
61	C	Area between the curves	Integral Calculus
62	C	Algebra of matrices	Matrices
63	D	Negation of a statement	Mathematical Reasoning
64	D	Circumcentre	Straight line
65	C	Cube root of unity	Cubic Equation
66	D	r things out of n things	Permutation and Combination
67	B	Coefficient of a term	Binomial theorem
68	A	Parabola	Conic Section
69	A	Sum of n terms	Sequences and series
70	B	Shortest distance	Three dimensional geometry
71	A	Number of ways	Permutation and Combination
72	B	Collinearity	Vector algebra
73	A	Conditional probability	Probability
74	B	Roots of equation	Complex numbers
75	C	Limits of trigonometry	Limits
76	D	Number of ways	Permutation and Combination
77	B	Higher order derivatives	Differentiability
78	A	Equation of plane	Three dimensional geometry
79	D	Adjoint	Matrices and Determinants
80	C	Indefinite Integral	Integral Calculus
81	[19]	Symmetric relation	Relation and Function
82	[1275]	General term	Binomial theorem
83	[9]	Plane	Three dimensional geometry
84	[3]	Linear Differential Equation	Differential equation
85	[11]	Algebra of vectors	Vector algebra
86	[31]	Remainder theorem	Binomial theorem
87	[5]	Maxima/Minima	Application of derivatives
88	[25]	Mean, Variance	Statistics
89	[2]	Circle	Conic Section
90	[14]	Definite Integral	Integral Calculus

JEE (Main) SOLVED PAPER

2023

08^{th} April Shift 1

ANSWERS WITH EXPLANATIONS

Physics

Section A

1. **Option (A) is correct.**

$$\rho = \frac{m}{V} = \frac{m}{\pi r^2 l}$$

Now the relative error is given by

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta l}{l}\right)$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{max} = \left(\frac{0.01}{0.4}\right) + 2\left(\frac{0.03}{6}\right) + \left(\frac{0.04}{8}\right)$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{max} = 0.04$$

Now % error in density = $\left(\frac{\Delta\rho}{\rho}\right) \times 100\%$

= 0.04 × 100% = 4%

2. **Option (A) is correct.**

The relative velocity between the train and passanger is zero. Therefore there would not be Doppler's effect. And hence frequency heard by the passanger would be same 400 Hz.

3. **Option (D) is correct.**

Given $w = mg$ = 400 N

$$d = \frac{R}{2}$$

At depth d,

$$g' = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow g' = g\left(1 - \frac{R}{2R}\right) \Rightarrow g' = g\left(\frac{1}{2}\right) = \frac{g}{2}$$

So,

$$w' = mg' = \frac{mg}{2}$$

$$w' = \frac{400}{2} = 200 \text{ N}$$

4. **Option (D) is correct.**

Given values

$h_T = 98$ m

$h_R = 0$

R = 6400 km

Now

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_r}$$

$$= \sqrt{2 \times 6400 \times 10^3 \times 98} + 0$$

$$d = \frac{112}{\sqrt{2}} \text{km}$$

Now area = πd^2

$$= \frac{22}{7} \times \left(\frac{112}{\sqrt{2}}\right)^2 = 3942 \text{ km}^2$$

5. **Option (B) is correct.**

By making fixed core with non magnetic material, the coil is brought quickly at rest because due to motion of the coil eddy current develops.

6. **Option (D) is correct.**

Since,

$$c^2 = \frac{1}{\mu_0 \varepsilon_0} \quad \text{(where } c = \text{speed of light)}$$

$$\Rightarrow \left[\frac{1}{\mu_0\varepsilon_0}\right] = [c^2]$$

$$\Rightarrow \left[\frac{1}{\mu_0\varepsilon_0}\right] = [L^2T^{-2}] = \left[\frac{L^2}{T^2}\right]$$

7. **Option (C) is correct.**

Range of a projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Now $$R_A = \frac{(40)^2 \sin(2\times 30)}{g} \quad ...(i)$$

$$R_B = \frac{(60)^2 \sin(2\times 60)}{g} \quad ...(ii)$$

From (i) & (ii)

$$\frac{R_A}{R_B} = \frac{(40)^2 \sin(2\times 30)}{(60)^2 \sin(2\times 60)} = \frac{4}{9}$$

8. **Option (C) is correct.**

At a steady state, no current would be flowing in capacitor circuit.

Required = 6 + 2 + 8 = 16 Ω

From Ohm's law

$$i = \frac{V}{R} = \frac{4}{16} = \frac{1}{4} \text{A}$$

Voltage across AC

$$V_{AC} = i(6+2) = \frac{1}{4} \times 8 = 2 \text{ V}$$

Voltage across BD

$V_{BD} = i(2+8) = \frac{1}{4}\times 10 = 2.5$ V

Now $\frac{V_{AC}}{V_{BD}} = \frac{2}{2.5} = \frac{4}{5}$

9. Option (A) is correct.

As the velocity vector has two components. One is along magnetic field and other is perpendicular to it. Due to perpendicular component of velocity, the charge particle moves in circular path.

And the component of velocity is along the magnetic field remains unchanged.

Therefore the particle will move in helical path along magnetic field.

10. Option (D) is correct.

Debroglie wavelength (λ) is given by :

$\lambda = \frac{h}{mv}$ (h = plank's const.)

According to the question,

$\lambda_e = \lambda p$

$\Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_p v_p}$

$\Rightarrow \frac{h}{p_e} = \frac{h}{p_p}$ ($\because$ P = mv)

$\Rightarrow P_e = P_p \Rightarrow \frac{P_P}{P_e} = \frac{1}{1}$

11. Option (D) is correct.

Electric field due to uniformly charged solid sphere is given by

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $r \geq R$

&

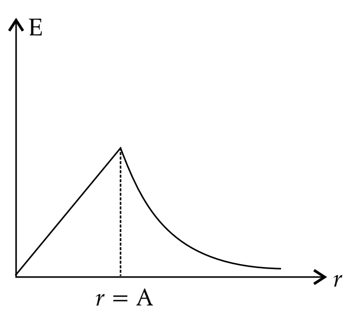
$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$ $r \leq R$

Therefore

$E \propto r$ when $r \leq R$

and $E \propto \frac{1}{r^2}$ when $r \geq R$

So



12. Option (D) is correct.

Mass number $A \propto r^3 \Rightarrow r \propto A^{\frac{1}{3}}$

Now, surface energy per nucleon $\propto \frac{r^2}{A} \propto \frac{A^{\frac{2}{3}}}{A} \propto \frac{1}{A^{\frac{1}{3}}}$ is

$= -\frac{a_2 Z(Z-1)}{(A)^{\frac{1}{3}}}$

And volume energy ∝ A

Based on above statement, the correct option is (D).

13. Option (C) is correct.

Given value,

m = 500 g = 0.5 kg

$\vec{v} = (2t\hat{i} + 3t^2\hat{j})$

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 6t\hat{j}$

acceleration at t = 1 sec.

$\vec{a} = 2\hat{i} + 6\hat{j}$

Now,

$\vec{F} = m\vec{a} = 0.5(2\hat{i} + 6\hat{j}) = \hat{i} + 3\hat{j}$

$\vec{F} = \hat{i} + x\hat{j}$

Therefore, $x = 3$

14. Option (A) is correct.

Kinetic energy of satellite $= \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Now, potential energy of the satellite

$U = -\frac{GMm}{2r}$

Total energy = K.E + U

$= \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{6Mm}{2r}$

Therefore, potential energy = 2 × total energy

and, kinetic energy = |Total energy|

15. Option (D) is correct.

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow |\vec{F}_{net}| = \sqrt{A^2 + \left(\frac{A}{2}\right)^2 + 2A + \frac{A}{2}\times\cos 90°}$ ($\because \theta = 90°$)

$\Rightarrow |\vec{F}_{net}| = \sqrt{A^2 + \frac{A^2}{4}}$

$\Rightarrow |\vec{F}_{net}| = \sqrt{\frac{5A^2}{4}}$

$\Rightarrow |\vec{F}_{net}| = \sqrt{5}\frac{A}{2}$

16. Option (B) is correct.

For the given circuit the truth table would be :

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

The truth table shows that the output graph is given by :

17. Option (A) is correct.

Given values,

$y = 7 \times 10^{10}\,N/m^2$

strain (E) = 0.04% $= \frac{0.04}{100}$

Now

$$\text{Energy} = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2$$

$$\Rightarrow \text{Energy} = \frac{1}{2}\left(\frac{\Delta l}{l}\right)^2 YAl$$

$$\Rightarrow \frac{\text{Energy}}{Al} = \frac{1}{2}Y(E)^2$$

$$\Rightarrow \frac{E}{V} = \frac{1}{2}\times 7\times 10^{10}\times\frac{0.04\times 0.04}{100\times 100} = 56\times 10^2$$

(where volume = A × l)

18. Option (D) is correct.

Statement-I: From I[st] law of thermodynamics

$$\Delta Q = \Delta u + w$$

If heat is supplied ot the system and converted into work done.

Then, $\Delta u < 0$. Hence, $\Delta T < 0$

Statement I is false

Statement-II: Work done in thermodynamics system is given by

$$w = \int P dV$$

Therefore, to get the positive work done volume of the system must increase

Statement-II is true

19. Option (D) is correct.

$P = P_0 + \rho g h$

$= 10^5\,Pa + 10^3 \times 10 \times 40 = 5 \times 10^5\,Pa$

In isothermal process

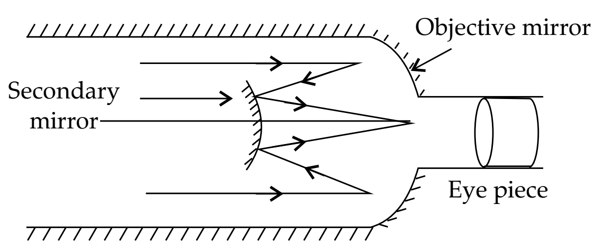
$PV = P_0V_0$

$\Rightarrow 5 \times 10^5\,Pa \times 1\,cm^3 = 10^5\,Pa \times V_0$

$$\Rightarrow V_0 = \frac{5\times 10^5\,Pa\times 1cm^3}{5\times 10^5\,Pa}$$

$\Rightarrow V_0 = 5\,cm^3$

20. Option (C) is correct.



Here the secondary mirror is used to move the eyepiece outside the telescope & it has advantage of a large focal length in a short telescope.

Section B

21. The correct answer is (125).

$$K.E. = \frac{P^2}{2m}$$

$$K.E._{initial} = \frac{P^2}{2m}$$

$$K.E._{final} = \frac{(1.5P)^2}{2m} = 2.25\frac{P^2}{2m}$$

$$\%\text{ increase in K.E.} = \frac{2.25\frac{P^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\times 100$$

% increase in K.E. = 1.25 × 100 = 125%

22. The correct answer is (121).

Binding energy (Initial) = 242 × 7.6 MeV

Binding energy (final)

= 121 × 8.1 MeV + 121 × 8.1 MeV

= 242 × 8.1 MeV

Gain in binding energy.

= Binding energy final – Binding energy initial

= 242 × 8.1 – 242 × 7.6

= 242 (8.1 – 7.6)

= 242 × 0.5 = 121 MeV

23. The correct answer is (18).

Work done in rotating the dipole = $V_f - V_i$

Now, $V_f = -PE\cos(180°)$

$V_i = -PE\cos 0°$

Therefore, $W = V_f - V_i$

$= (-PE\cos 180°) - (-PE\cos 0°)$

$= 2PE$

$= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^3 = 18$ mJ

24. The correct answer is (900).

In an open organ pipe, the condition for second hormonies is :

The length of organ pipe = wavelength

$$h = \lambda$$

Now frequency = $f = \frac{v}{\lambda}$

$$\Rightarrow f = \frac{360}{L} = \frac{360}{\frac{40}{100}} = \frac{360\times 100}{40}$$

$\Rightarrow f = 900$ Hz

25. The correct answer is (1).

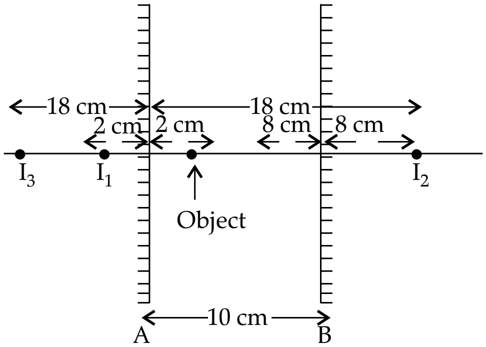
Moment of inertia of a semicircular ring about its centre and perpendicular to the plane of ring is given by MR^2

Therefore, $MR^2 = \frac{1}{x}MR^2$

$\therefore x = 1$

26. The correct answer is (18).

Therefore the second nearest image behind mirror A is at 18 cm



27. The correct answer is (2).

$B = \mu_0 ni$

and $H = \dfrac{B}{\mu_0}$

$\Rightarrow H = \dfrac{\mu_0 ni}{\mu_0} = ni$

Now, $i = \dfrac{H}{h} = \dfrac{1.6\times10^3\times10^{-2}}{8} = 2\,A$

28. The correct answer is (25).

$V_d = \dfrac{I}{neA}$

$\Rightarrow V_d = \dfrac{2}{2\times10^{28}\times1.6\times10^{-19}\times25\times10^{-6}}$

$\Rightarrow V_d = 25\times10^{-6}$ m/s

29. The correct answer is (9).

Maximum energy stored in capacitor = Maximum energy stored in inductor which is given by :

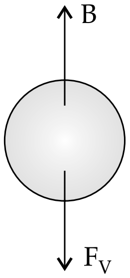
$\dfrac{1}{2}Li_{max}^2 = \dfrac{1}{2}\dfrac{Q_{max}^2}{C}$

$\Rightarrow i_{max}^2 = \dfrac{Q_{max}^2}{LC}$

$\Rightarrow i_{max} = \sqrt{\dfrac{Q_{max}^2}{LC}} = \dfrac{Q_{max}}{\sqrt{LC}}$

$= \dfrac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}} = 9$ mA

30. The correct answer is (10).



From Newton's 1st law

$F_{net} = 0$

V = Constant

As the bubble moves with constant velocity, so net force must be zero.

$\therefore B = F_V$

$\Rightarrow \dfrac{4}{3}\pi R^3\rho g = 6\pi\eta RV$

$\Rightarrow \eta = \dfrac{4\pi R^3\rho g}{3\times6\pi RV}$

$\Rightarrow \eta = \dfrac{2R^2\rho g}{9V}$

$\Rightarrow \eta = \dfrac{2\times(3\times10^{-3})^2\times1750\times10}{9\times0.35\times10^{-2}}$

$\Rightarrow \eta = 10$ Poise

Chemistry

SECTION - A

31. Option (2) is correct.

Explanation:

H_2 gets oxidised to HCl in the galvanic cell.

Pt, H_2 |HCl(*sol*) forms anode.

AgCl(*s*)|Ag forms cathode.

$\therefore$ The cell can be written as

Pt,$H_2(g)$|HCl(*sol*)||AgCl(*s*)|Ag

32. Option (4) is correct.

Explanation:

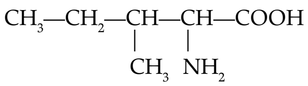
Isoleucine is an α-amino acid that is used in the biosynthesis of proteins. It contains an α amino group, an α-carboxylic acid group, and a hydrocarbon side chain with a branch. It is classified as a non-polar, uncharged, branched-chain, aliphatic amino acid.

Lysine is an α-amino acid that is a precursor to many proteins. It contains an α-amino group, an alpha-carboxylic acid group, and a side chain lysyl, classifying it as a basic, charged, aliphatic amino acid. It is encoded by the codons AAA and AAG.

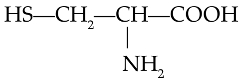
Glutamic acid is an amino acid used to form proteins.

Cysteine and methionine are sulphur containing amino acids. Amino acids get linked to one another by peptide bond formation and form a polypeptide chain of proteins. Hence cysteine and methionine are found in several proteins.

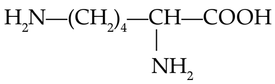
(a) isoleucine :



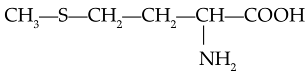
(b) cysteine :



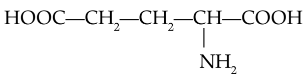
(c) lysine :



(d) methionine :



(e) glutamic acid :



33. Option (1) is correct.

Explanation:

$[Co(CN)_6]^{3-}$ and $[Co(H_2O)_6]^{3+}$ are diamagnetic but first one is more stable as Δ_0 is high for first complex.

$K_3[Co(CN)_6]$

$+3 + x - 6 = 0$

$\boxed{x = +3}$

$\Downarrow$

$Co^{+3} \rightarrow 3d^6$

$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$\therefore$ CN^- is SFL so pairing occur so

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{}\boxed{}$

$U_e = 0$

$\downarrow$

So diamagnetic in nature

34. Option (4) is correct.

Explanation:

Alkali-acid leaching is an effective method used to purify graphite and remove silicate minerals. In this study, the dissolution behavior and mechanism of sericite in alkali-acid leaching were investigated.

In this method, the ore is treated with aqueous alkali to form a soluble complex. For example, bauxite, an important ore of aluminium is heated with a solution of sodium hydroxide or sodium carbonate in the temperature range 470–520 K at 35 atm to form soluble sodium meta- aluminate leaving behind the impurities, iron oxide and titanium oxide

$$Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l) \rightarrow 2Na[Al(OH)_4](aq)$$

The hot solution is decanted, cooled, and diluted. This solution is neutralised by passing gas, to form hydrated Al_2O_3 precipitate.

$$2Na[Al(OH)_4](aq) + CO_2(g) \rightarrow Al_2O_3 \cdot xH_2O(s) + 2NaHCO_3(aq)$$

The precipitate is filtered off and heated around 1670K to get pure alumina Al_2O_3.

35. Option (4) is correct.

Explanation:

Br^- is a weak ligand while CN^- is a strong ligand. More the number of unpaired electrons, more the value of magnetic moment.

$[CoF_6]^{3-}$

$\downarrow$

Co^{+3}

F^- is WFL

So no pairing

$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$U_e = 4$

$[MnBr_4]^{2-}$

$\downarrow$

Mn^{+2}

Br^- is WFL

no pairing

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$U_e = 5$

$[Fe(CN)_6]^{3-}$

$\downarrow$

Fe^{3+}

CN^- is SFL

pairing occur

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$\downarrow$

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{}\boxed{}$

$U_e = 1$

$[Mn(CN)_6]^{3-}$

$\downarrow$

Mn^{+3}

CN^- is SFL

no pairing

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}$

$\downarrow$

$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}$

$U_e = 2$

$[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

36. Option (4) is correct.

Explanation:

Water gas shift reaction is defined as the reaction between carbon and water vapor to form carbon monoxide and hydrogen. The mixture of carbon monoxide and hydrogen is known as water gas. This reaction is more catalyzed on iron catalysts and merely catalyzed by cobalt catalysts. The chemical equation for the formation of water gas shift reaction follows:

$$C + H_2O \rightleftharpoons CO + H_2$$

$$(CO + H_2) + H_2 \xrightarrow[\substack{ZnO,\ Cr_2O_3 \\ \downarrow \\ \text{Catalyst}}]{700K} CH_3OH$$

37. Option (2) is correct.

Explanation:

Given reaction, $I_2 \rightarrow IO_3^- + I^-$

Disproportionation reaction is the reaction in which an element under go both oxidation and reduction.

$$\overset{(0)}{I}_2 \rightarrow \overset{(+5)}{I}O_3^- + \overset{(-1)}{I}$$

I has oxidised from O to +5 and reduced from 0 to –1.

n factor of IO_3^- and I^- in the given redox reaction are 5 and 1 respectively. Therefore, will always react in the molar ratio 1 : 5 to get I_2.

$$IO_3^- + 6H^+ + 5I^- \rightarrow 3I_2 + 3H_2O$$

To get 6 molar I_2, multiple equation by 2

$$2IO_3^- + 12H^+ + 10I^- \rightarrow 6I_2 + 6H_2O$$

So, $x = 10$

38. Option (4) is correct.

Explanation:

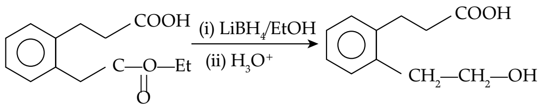
Gypsum ($CaSO_4.2H_2O$) plays an important role in controlling the rate of hardening of the cement. During cement manufacturing process upon cooling of clinker a small amount of gypsum is added during the final grinding process. Gypsum controls the setting of the cement and if not added the cement will set immediately leaving no time for concrete placing.

39. Option (4) is correct.

Explanation:

$NaBH_4$ (Sodium borohydride) is a weak reducing agent, it reduces aldehydes/ketones but not esters, it produces alcohols through reduction.

ketones reduces to 2° alcohol.



Note: Lithium borohydride is commonly used for selective reduction of esters and lactones to the corresponding alcohol.

40. Option (1) is correct.

Explanation:

NO_3^- The maximum limit of nitrate NO_3^- in drinking water is 50 ppm and its source is fertilisers, if the maximum limit is increased in water it will cause methemoglobinemia (blue baby syndrome SO_4^{2-}. The maximum limit of sulphate (SO_4^{2-}) according to WHO is 500 pm and its source are acid rain, industries. Excess SO_4^{2-} has laxative effect. F^- The maximum limit of fluoride (F^-) is about 1.5 ppm. Its higher concentration converts enamel to more harder fluorapatite. Concentration (>2ppm) causes brown mottling of teeth and high concentration (>10 ppm) are harmful for bones and teeth. SO_4^{2-} (100 ppm) and NO_3^- (50 ppm) in water is suitable for drinking but the concentration of F^- (10 ppm makes water unsuitable for dirnking.

Zinc is one of the important trace elements that play a vital role in the physiological and metabolic process of many organisms. Nevertheless, higher concentrations of zinc can be toxic to the organism.

It plays an important role in protein synthesis and is a metal which shows fairly low concentration in surface water due to its restricted mobility from the place of rock weathering or from the natural sources.

According to the standards maximum prescribed limit of Zn in drinking water is 5 ppm.

41. Option (4) is correct.

Explanation:

When a mixture containing chloride ion is heated with $K_2Cr_2O_7$ and concentrated H_2SO_4, deep orange-red fumes of chromyl chloride (CrO_2Cl_2) are formed

$$K_2Cr_2O_7 + 4NaCl + 6H_2SO_4 \rightarrow 2KHSO_4 + 4NaHSO_4 + 2CrO_2Cl_2\uparrow + 3H_2O$$

Orange-red fumes

So in this case, X in CrO_2Cl_2.

Oxidation state of Cl = –1, O = –2, Cr = x

$x + 2 \times (-2) + 2 \times (-1) = 0 \Rightarrow x = +6$

$CrO_2Cl_2 \rightarrow$ Chromyl chloride

$\Downarrow$

$$\left.\begin{array}{l} Cr^{+6} \rightarrow 4s^0\,3d^0 \\ Mn(vii) \rightarrow Mn^{+7} \\ \qquad\quad \Downarrow \\ \qquad\quad 4s^0\,3d^0 \end{array}\right] \text{Same}$$

42. Option (2) is correct.

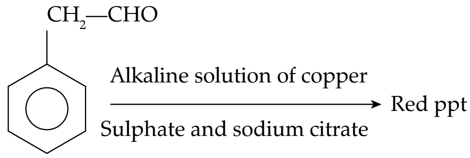
Explanation:

Both are functional isomers. However, intermolecular hydrogen bonding is present in butan-1-ol molecules while it is absent in the molecules of diethyl ether. Therefore, boiling point of alcohol (390 K) is higher as compared to that of ether (308 K).

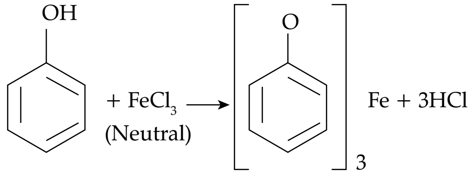
43. Option (1) is correct.

Explanation:

(A) Fehling's solution is used to distinguish between aldehyde and ketone functional groups. Aldehydes oxidize to give a positive result but ketones won't react to the test (except for α-hydroxy ketones). Fehling's test is used as a general test for determining monosaccharides and other reducing sugars.



(B)



(C) This reaction is a chemical test for detection of primary amines, in which the amine is heated with alcoholic potassium hydroxide and chloroform. If a primary amine is present, the isocyanide is formed. The reaction is known as carbylamine reaction.

$$RNH_2 + CHCl_3 + 3KOH \rightarrow RN^+ \equiv C^- + 3KCl + 3H_2O$$

When a methyl ketone (even acetaldehyde) is reacted with halogen in aqueous sodium hydroxide, the ketone gets oxidised to the sodium salt of acid with one carbon less than ketone and at the same time haloform (CHX_3) also gets formed.

$$2NaOH + X_2 \rightarrow \underset{\text{Sodium halide}}{NaX} + \underset{\text{Sodium hypohalite}}{NaOX}$$

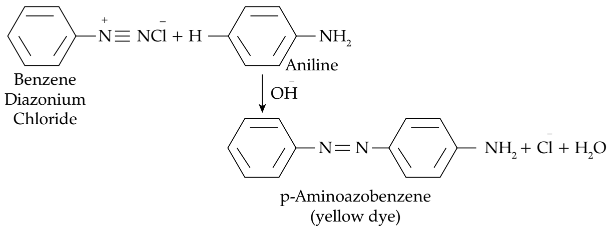
The hydroxide ion acts as a nucleophile and attacks the electrophilic carbon which is doubly bonded to oxygen. This carbon-oxygen double bond becomes a single bond making the oxygen atom anionic.

$$\overset{+}{K}\overset{-}{I} + NaOCl \xrightarrow{CH_3-CH(OH)-Ph} \text{Haloform reaction}$$

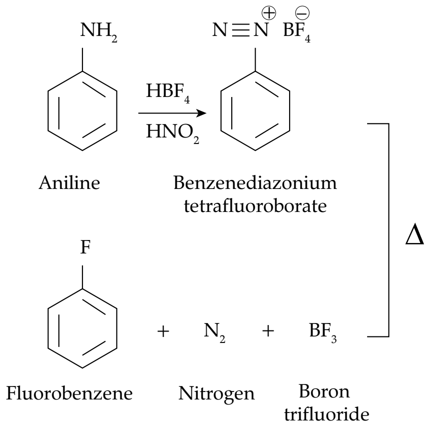
44. Option (3) is correct.

Explanation:

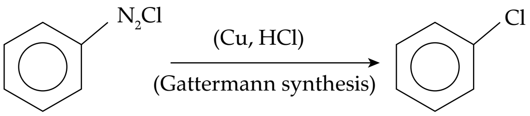
(A) Coupling Reaction:



(B) Balz–Schiemann reaction:



(C) Gattermann Reaction → In this reaction, chlorine or bromine can be introduced in the benzene ring by treating the benzene diazonium salt solution with corresponding halogen acid in the presence of copper powder.



(D) In the given reaction when benzene diazonium salt is treated with KCN and copper powder, it forms aryl nitrile or benzyl nitrile as a product.

45. Option (2) is correct.

Explanation:

A. Saccharin is first popular artificial sweetening agent. It is 550 times as sweet as cane sugar. It is used as intake by diabetic person.

B. Aspartame is 100 times as sweet as cane sugar. It is used in cold foods and soft drinks because it is unstable at cooking temperature.

C. Alitame is 2000 times as sweet compared to cane sugar. It has excellent stability at high temperature. So, can be used in cooking and baking.

D. Sucralose is 600 times as sweet as cane sugar. It is heat stable and finds its use in baked goods.

46. Option (4) is correct.

Explanation:

Electronegativity is a measure of an atom's ability to attract shared electrons to itself. On the periodic table, electronegativity generally increases as we move from left to right across a period due to increase in effective nuclear charge.

C (2.5)
P (2.1) $\Rightarrow$ Br > C > At > P
Br (2.8)
At (2.2)

47. Option (3) is correct.

Explanation:

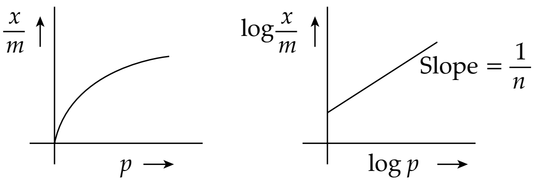
Lithium and Magnesium does not form superoxides because they are very small in size.

48. Option (1 & 2) is correct.

Explanation:

The equation $\frac{x}{m} = kp^{1/n}$ represents Freundlich adsorption isotherm. It is an empirical relationship between the amount of gas adsorbed by a given amount of solid adsorbent surface and pressure of the gas at a particular temperature.

$\frac{x}{m} = kp^{1/n}$ $\qquad$ $\log\frac{x}{m} = \log K + \frac{1}{n}\log p$



49. Option (4) is correct.

Explanation:

$2Cu^{+2} + 4X_2 \rightarrow Cu_2X_2 + X_2$ and the same applies to CuX. On the other hand, all Cu(II) halides are known except the iodide. In this case, Cu^{2+} oxidises I^- to I_2:

$2Cu^{2+} + 4I^- \rightarrow Cu_2I_2(s) + I_2$

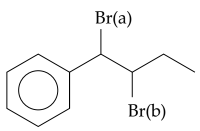
However, many copper (I) compounds are unstable in aqueous solution and undergo disproportionation.

$2Cu^+ \rightarrow Cu^{2+} + Cu$

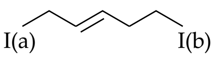
50. Option (1) is correct.

Explanation:

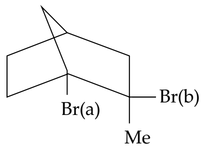
Organic compounds are more reactive towards SN 1, if the carbocation is stable compared to others. Reactivity in reactions depends upon the stability of the carbocation intermediate.

(A) 

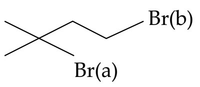
→ Br(a), Becuase formed intermediate carbocation get stable by conjugation with phenyl ring

(B) 

→ I(a), Becuase formed intermediate carbocation become more stable by conjugation

(C) 

→ Br(b), Becuase we can't remove Br(a) from bridge head carbon (Bredt's rule

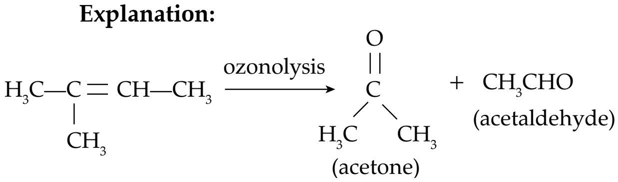
(D) 

$\rightarrow$ Br(a), because formed intermediate 3° carbocation is more stable (stability of carbocation 3° > 2° > 1°)

SECTION - B

51. Correct answer is [70].

Explanation:



Hydrocarbon (X) is 2-methyl- but-2-ene (C_5H_{10})
And the molecular mass is 5 (12) + 10 (1) = 70.

52. Correct answer is [11].

Explanation:

XeF_4 reacts with SbF_5 which is a lewis acid and forms adduct. The reaction is
$XeF_4 + SbF_5$ $[XeF_3]^+[SbF_6]^-$
The cation is T-shaped and the anion is octahedral.
$m + n + x + y = 3 + 1 + 6 + 1 = 11$
Xenon fluoride act as F^- donor.

53. Correct answer is [1].

Explanation:

The emission spectra of atoms in the gaseous phase do not show a continuous spread of wavelength from red to violet, rather they emit light only at specific wavelengths with dark spaces between them.

54. Correct answer is [2].

Explanation:

Phenolphthalein is used as an indicator in titrations involving weak acids and strong bases because it changes color at the point of neutralization. At a pH of 8.2-9.8, phenolphthalein is colorless, but at a pH above 10, it becomes pink. This change in color allows for a visual indication of neutralisation. acid and base. Additionally, phenolphthalein has a relatively low detection limit, making it useful for determining the end point of a titration accurately.

55. Correct answer is [1200].

Explanation:

Adiabatic wall {no heat exchange between system and surrounding}
$Cv \times \Delta T = E \times t$
$Cv \times 5 = 60 \times 100$
$Cv = 1200$

56. Correct answer is [82].

Explanation:

At any temperature, the vapour pressure of the solution is lower than that of the pure solvent. Hence vapour pressure- temperature curve of solution lies below that of solvent.

The more volatile liquid evaporates fast as compared to the less volatile liquid at a low temperature because the volume increases with respect to temperature so it has a low boiling point.

57. Correct answer is [11].

Explanation:

Mass of Carbon = 12
Molar Mass of $CO_2 = 12 + (16 \times 2) = 44$
Mass of Compound = 0.5 g

$$\%\text{ of C} = \frac{\text{Molar mass of C} \times \text{Mass Of } CO_2}{\text{Mass Of Compound} \times \text{Molar Mass Of } CO_2}$$

$$\frac{60}{100} = \frac{12 \times x}{0.5 \times 44}$$

$1.1 = x$
$x = 11 \times 10^{-1}$

58. Correct answer is [3].

Explanation:

Percent covalent character of the ionic bond depends upon
(1) Polarising power of cation
(2) Extent of distortion of anion
(3) Polarisability of the anion
Every ionic compound having some percentage of covalent character according to Fajan's rule. The percentage of ionic character in a compound having covalent character, can also be calculated by the following equation.
The percent ionic character

$$= \frac{\text{observed dipole moment}}{\text{Calculated dipole moment assuming 100\% ionic bond}} \times 100$$

59. Correct answer is [3].

Explanation:

Dalton's law of partial pressure states that whenever two or more gases, which do not react chemically, are enclosed in vessel, the total pressure is equal to sum of partial pressure of each gas.
From Dalton's partial pressure law,
$P_f V_f = P_1 V_1 + P_2 V_2 + P_3 V_3$
$P_f \times 9 = 2 \times 2 + 4 \times 3 + 3 \times 4$
$P_f = \dfrac{28}{9} = 3.11 = 3$

60. Correct answer is [2].

Explanation:

Clearly, if $E_a = 0$, K is temperature independent
if $E_a > 0$, K increase with increase in temperature
if $E_a < 0$, K decrease with increase in temperature

- Rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.
- Higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.
- The pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.
  a. A high activation energy usually implies a slow reaction.
  b. $k = P \times Z \times e^{-E_a/RT}$
  c. The pre-exponential factor (A = P×Z) is independent of the activation energy and the energy of molecules.

Mathematics

Section A

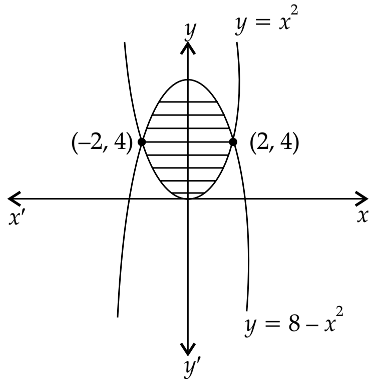
61. Option (C) is correct.

The given curves are

$x^2 \le y, y \le 8 - x^2; y \le 7$

On solving, we get

$x^2 = 8 - x^2$



$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

So, area $= 2\left[\int_0^4 \sqrt{y}\,dy + \int_4^7 \sqrt{8-y}\,dy\right]$

$= 2\left\{\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 + \left[\frac{-(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^7\right\}$

$= 2\times\frac{2}{3}\{[4^{3/2} - 0] + (-(1)^{3/2} + (4)^{3/2})\}$

$= \frac{4}{3}\{8 - 1 + 8\} = \frac{4}{3}\times 15 = 20$ sq. units

HINT:

Draw the graph of both curves, then find the bounded region and proceed.

62. Option (C) is correct.

Here, P $= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Here, PP^T $= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ \frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$||$ P^{T}P = I

$\because$ Q = PAP^T

$\Rightarrow$ Q^{2007} = (PAP^T) (PAP^T)......... 2007 time

= PA^{2007}P^T

As, A $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

Hence, P^{T}Q^{2007} P = A^{2007} $= \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a = 1, b = 2007, c = 0, d = 1$

$\therefore 2a + b - 3c - 4d = 2(1) + 2007 - 3(0) - 4(1)$

$= 2 + 2007 - 4 = 2005$

HINT:

Transpose the given matrix and multiply the matrices to solve further.

63. Option (D) is correct.

Given: $(p \to q) \to (q \to p)$

Negation of above statement is

$\sim[(p \to q) \to (q \to p)]$

$\equiv \sim [\sim p \to q \wedge q \to p]$

$\equiv\ p \to q \wedge \sim q \to p$

$\equiv \sim p \vee q \wedge q \wedge \sim p]$

$\equiv q \wedge (\sim p)$

HINT:

The negation of a statement is the opposite of the given mathematical statement.

64. Option (D) is correct.

We have,

$4x + 3y = 69$ ...(i)

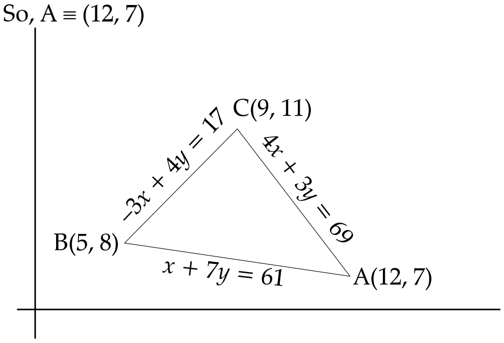
$4y - 3x = 17$ ...(ii)

$x + 7y = 61$ ...(iii)

On solving (i) and (iii), we get

$x = 12$, and $y = 7$

So, A ≡ (12, 7)



On solving (ii) and (iii), we get

$x = 5$ and $y = 8$

So, $B \equiv (5, 8)$

Hence, circumcentre $\equiv \left(\frac{12+5}{2}, \frac{7+8}{2}\right)$

$\equiv \left(\frac{17}{2}, \frac{15}{2}\right)$

$\therefore\ \alpha = \frac{17}{2}, \beta = \frac{15}{2}$

$\therefore\ (\alpha - \beta)^2 + (\alpha + \beta) = \left(\frac{17}{2} - \frac{15}{2}\right)^2 + \left(\frac{17}{2} + \frac{15}{2}\right)$

$= (1)^2 + (16) = 17$

HINT:

Circumcentre of a right triangle is the midpoint of hypotenuse of the triangle.

65. Option (C) is correct.

Given cubic equation is :

$x^3 + bx + c = 0$

$\because$ α, β, γ are the roots of above equation.

And $\beta\gamma = 1 = -\alpha$

So, product of roots $= -c$

$\Rightarrow \alpha\beta\gamma = -c$

$\Rightarrow (-1)(1) = -c$

$\Rightarrow c = 1$

Since, $\alpha = -1$ is the root. So,

$\Rightarrow -1 - b + c = 0$

$\Rightarrow c - b = 1$

$\Rightarrow 1 - b = 1 \Rightarrow b = 0$

The given equation becomes $x^3 + 1 = 0$

So, roots are $-1, -\omega, -\omega^2$

$\therefore\ b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$= 0 + 2 - 3(-1)^3 - 6(-\omega)^3 - 8(-\omega^2)^3$

$= 2 + 3 + 6\omega^3 + 8\omega^6$

$= 5 + 6 + 8 = 19$

HINT:

For a cubic equation, $ax^3 + bx^2 + cx + d = 0$

Sum of roots $= \frac{-b}{a}$,

Product of roots taken two at a time $= \frac{c}{a}$

Product of roots $= \frac{-d}{a}$

66. Option (D) is correct.

Since, $n(A) = 5$, $n(B) = 2$

$\Rightarrow n(A \times B) = n(A) \times n(B)$

$= 5 \times 2 = 10$

So, number of subsets having 3 elements $= {}^{10}C_3$

Number of subsets having 4 elements $= {}^{10}C_4$

Number of subsets having 5 elements $= {}^{10}C_5$

Number of subsets having 6 elements $= {}^{10}C_6$

$\therefore$ No. of subsets $= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

$= 120 + 210 + 252 + 210 = 792$

HINT:

No of subsets having r elements out of total n elements $= {}^nC_r$

67. Option (B) is correct.

Given: ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$= 1 : 5 : 20$

$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{1}{5}$

$\Rightarrow \frac{r}{(n-r+1)} = \frac{1}{5}$

$\Rightarrow 5r = n - r + 1$

$\Rightarrow n = 6r - 1$ ...(i)

Also, $\frac{n}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{20} = \frac{1}{20}$

$\Rightarrow \frac{(r+1)}{(n-r)} = \frac{1}{4}$

$\Rightarrow 4r + 4 = n - r$

$\Rightarrow n = 5r + 4$ ...(ii)

From (i) and (ii), we get

$6r - 1 = 5r + 4$

$\Rightarrow r = 5$

So, $n = 5(5) + 4 = 29$

So, coefficient of 4th terms $= {}^nC_3 = {}^{29}C_3$

$= \frac{29!}{3!26!} = \frac{29 \times 28 \times 27}{3 \times 2} = 3654$

HINT:

In the expansion of $(a + b)^n$,

the general term is $T_{r+1} = {}^nC_r (a)^{n-r} b^r$

68. Option (A) is correct.

$y^2 = 20x$, $y = mx + c$

Put value of x

$y^2 = 20\left(\frac{y-c}{m}\right)$

$\Rightarrow y^2 - \frac{20}{m}y + \frac{20}{m}c = 0$ ...(i)

Since, centroid = (10, 10)

So, $\frac{y_1 + y_2 + 0}{3} = 10$

$\Rightarrow y_1 + y_2 = 30$

From (1),

Sum of roots $= \frac{20}{m} = 30 \Rightarrow m = \frac{2}{3}$

Also, $c - m = 6 \Rightarrow c = 6 + \frac{2}{3} = \frac{20}{3}$

Now, the equation is :

$y^2 - \frac{20}{2} \times 3y + \frac{20}{2} \times 3 \times \frac{20}{3} = 0$

$\Rightarrow y^2 - 30y + 200 = 0$

$\Rightarrow y^2 - 20y - 10y + 200 = 0$

$\Rightarrow (y - 20)(y - 10) = 0$

$\Rightarrow y = 10, 20 \Rightarrow x = 5, x = 20$

$\therefore$ P $\equiv$ (5, 10), Q $\equiv$ (20, 20)

So, $(PQ)^2 = (20 - 5)^2 + (20 - 10)^2$

$= 225 + 100 = 325$

HINT:

Centroid of a triangle having vertices (a, b), (c, d) & (e, f) is $\left(\frac{a+c+e}{3}, \frac{b+d+f}{3}\right)$

69. Option (A) is correct.

$\because S_k = \frac{1+2+...+k}{k}$

$= \frac{k(k+1)}{2k} = \frac{k+1}{2}$

$\Rightarrow S_k^2 = \left(\frac{k+1}{2}\right)^2 = \frac{k^2+1+2k}{4}$

$\Rightarrow \sum_{j=1}^{n} S_j^2 = \frac{1}{4}\left[\sum_{j=1}^{n} k^2 + \sum_{j=1}^{n} 1 + 2\sum_{j=1}^{n} k\right]$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + \frac{2n(n+1)}{2}\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{24}[2n^2 + 3n + 1 + 6 + 6n + 6]$

$= \frac{n}{24}[2n^2 + 9n + 13]$

On comparing, we get

A = 24, B = 2, C = 9, D = 13

(1) A + B = 24 + 2 = 26, divisible by 13

(2) A + B = 26

5 (D – C) = 5 (13 – 9) = 20

$\therefore 26 \neq 20$

(3) A + C + D = 46, which is divisible by 2

(4) A + B + D = 39, which is not divisible by 5.

HINT:

$1 + 2 + ... + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + ... n^2 = \frac{n(n+1)(2n+1)}{6}$

70. Option (B) is correct.

The given lines are :

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$

So, $\vec{b}_1 = 4\hat{i} + 5\hat{j} + 3\hat{k}$

$\vec{b}_2 = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{a}_1 = 4\hat{i} - 2\hat{j} - 3\hat{k}, \vec{a}_2 = \hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$

$= (10-12)\hat{i} - (8-9)\hat{j} + (16-15)\hat{k}$

$= -2\hat{i} + \hat{j} + \hat{k}$

Shortest distance, $d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \left|\frac{(3\hat{i} - 5\hat{j} - 7\hat{k}).(-2\hat{i} + \hat{j} + \hat{k})}{\sqrt{4+1+1}}\right|$

$= \left|\frac{-6-5-7}{\sqrt{6}}\right| = \frac{18}{\sqrt{6}} = 3\sqrt{6}$ units

HINT:

Shortest distance between two lines is:

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

71. Option (A) is correct.

In the given word,

vowels are : I, E, E, E, E

Consonants are : N, D, P, N, D, N, C

So, number of words $= \frac{8!}{3!2!} \times \frac{5!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4}{2} \times 5 = 16800$

HINT:

Out of n objects, if r things are same, so number of ways $= \frac{n!}{r!}$

72. Option (B) is correct.

Given: Points with position vectors

$\alpha\hat{i} + 10\hat{j} + 13\hat{k}, 6\hat{i} + 11\hat{j} + 11\hat{k}$

and $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear.

So, $\frac{\alpha - 6}{6 - \frac{9}{2}} = \frac{10-11}{11-\beta} = \frac{13-11}{11+8}$

$\Rightarrow \frac{2(\alpha-6)}{3} = \frac{-1}{11-\beta} = \frac{2}{19}$

$\Rightarrow \frac{2}{3}(\alpha - 6) = \frac{2}{19}$

$\Rightarrow 19\alpha - 114 = 3 \Rightarrow 19\alpha = 117$

$\Rightarrow \alpha = \frac{117}{19}$

And, $\frac{-1}{11-\beta} = \frac{2}{19}$

$\Rightarrow -19 = 22 - 2\beta$

$\Rightarrow 2\beta = 41$

$\Rightarrow \beta = \frac{41}{2}$

$$\therefore (19\alpha - 6\beta)^2 = \left(19 \times \frac{117}{19} - \frac{6 \times 41}{2}\right)^2$$

$= (117 - 123)^2 = 36$

HINT:

If point $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$, $(\alpha_3, \beta_3, \gamma_1)$ are collinear, then $\dfrac{\alpha_1 - \alpha_2}{\alpha_2 - \alpha_3} = \dfrac{\beta_1 - \beta_2}{\beta_2 - \beta_3} = \dfrac{\gamma_1 - \gamma_2}{\gamma_2 - \gamma_3}$.

73. Option (A) is correct.

Given: $P(A) = \dfrac{20}{100} = \dfrac{2}{10}$

$P(B) = \dfrac{30}{100} = \dfrac{3}{10}$; $P(C) = \dfrac{50}{100} = \dfrac{5}{10}$

Let E $\rightarrow$ Event that the bolt is defective.

So, $P(E/A) = \dfrac{3}{100}, P\left(\dfrac{E}{B}\right) = \dfrac{4}{100}, P\left(\dfrac{E}{C}\right) = \dfrac{2}{100}$

So, P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

$$= \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{3}{100} \times \frac{2}{10} + \frac{4}{100} \times \frac{3}{10} + \frac{2}{100} \times \frac{5}{10}}$$

$$= \frac{10}{6 + 12 + 10} = \frac{10}{28} = \frac{5}{14}$$

HINT:

Conditional probability P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

74. Option (B) is correct.

Given: $|z + 2| = z + 4(1 + i)$

Also, $z = \alpha + i\beta$

$\therefore |z + 2| = |\alpha + i\beta + 2| = (\alpha + i\beta) + 4 + 4i$

$\Rightarrow |(\alpha + 2) + i\beta| = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \sqrt{(\alpha + 2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \beta + 4 = 0 \Rightarrow \beta = -4$

Now, $(\alpha + 2)^2 + \beta^2 = (\alpha + 4)^2$

$\Rightarrow \alpha^2 + 4 + 4\alpha + \beta^2 = \alpha^2 + 16 + 8\alpha$

$\Rightarrow 4 + 4\alpha + 16 = 16 + 8\alpha$

$\Rightarrow 4\alpha = 4 \Rightarrow \alpha = 1$

So, $\alpha + \beta = -3$ and $\alpha\beta = -4$

$\therefore$ Required equation is

$x^2 - (-3 - 4)x + (-3)(-4) = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

75. Option (C) is correct.

$$\lim_{x \to 0}\left[\left(\frac{1 - \cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x + 1))^5}\right)\right]$$

$$= \lim_{x \to 0}\left[\frac{1 - \cos^2(3x)}{9x^2} \times \frac{9x^2}{\cos^3(4x)}\right] \times \frac{\frac{\sin^3 4x}{(4x)^3} \times 64x^3}{\left[\frac{\log_e(2x + 1)}{2x}\right]^5 \times (2x)^5}$$

$$= \left[\frac{1 \times 9 \times 1}{(1)}\right] \times \left[\frac{1 \times 64}{1 \times 32}\right]$$

$= 9 \times 2 = 18$

HINT:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

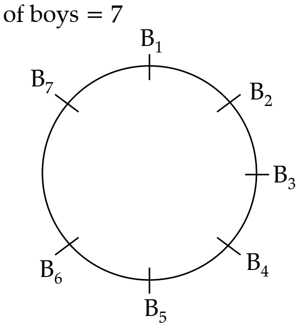
$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

76. Option (D) is correct.

We have,

Number of girls = 5

Number of boys = 7



So, number of ways of arranging boys around the table = 6! and 5 girls can be arranged in 7 gaps in ${}^7P_5$ ways

$\therefore$ Required no. of ways $= 6! \times {}^7P_5$

$= 126 \times (5!)^2$

77. Option (B) is correct.

$$f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$$

$$= \frac{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x - 1}{\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x}$$

$$= \frac{\cos\left(x - \frac{\pi}{4}\right) - 1}{\sin\left(x - \frac{\pi}{4}\right)}$$

$$= \frac{-2\sin^2\left(\frac{x}{2}-\frac{\pi}{8}\right)}{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}$$

$$\Rightarrow f(x) = -\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$\Rightarrow f'(x) = -\frac{1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$\Rightarrow f''(x) = -\frac{1}{2}.2\sec\left(\frac{x}{2}-\frac{\pi}{8}\right).\sec\left(\frac{x}{2}-\frac{\pi}{8}\right)\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\times\frac{1}{2}$$

$$= -\frac{1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right).\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

Now, $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$

$$= -\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)\times\frac{-1}{2}\sec^2\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)\times\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)$$

$$= \frac{1}{2}\tan^2\left(\frac{\pi}{6}\right)\times\sec^2\frac{\pi}{6}$$

$$= \frac{1}{2}\times\frac{1}{3}\times\frac{4}{3} = \frac{2}{9}$$

HINT:

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec^2 x) = 2\sec^2 x.\tan x$$

78. Option (A) is correct.

Equation of plane P containing the given lines is

$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$

$\Rightarrow (1 + 2\lambda)\, x + (2 + \lambda)y + (3 - \lambda)\, z + (-4 + 5\,\lambda) = 0$

Now, plane P is perpendicular to plane P'

$$\vec{r} = (\hat{i}-\hat{j}) + \lambda(\hat{i}+\hat{j}+\hat{k}) + \mu(\hat{i}-2\hat{j}+3\hat{k})$$

So, normal to plane P' is

$$\vec{n} = (\hat{i}+\hat{j}+\hat{k})\times(\hat{i}-2\hat{j}+3\hat{k})$$

$$\Rightarrow \vec{n} = 5\hat{i}-2\hat{j}-3\hat{k}$$

$\therefore$ P and P' are perpendicular

$\therefore 5\,(1 + 2\lambda) - 2(2 + \lambda) - 3\,(3 - \lambda) = 0$

$\Rightarrow 5 + 10\lambda - 4 - 2\lambda - 9 + 3\lambda = 0$

$\Rightarrow 11\lambda = 8 \Rightarrow \lambda = \frac{8}{11}$

$$\therefore P: \left(1+\frac{16}{11}\right)x + \left(2+\frac{8}{11}\right)y + \left(3-\frac{8}{11}\right)z + \left(5\times\frac{8}{11}-4\right) = 0$$

i.e., $27x + 30y + 25z = 4$

which is same as $ax + by + cz = 4$

$\therefore a = 27, b = 30$ and $c = 25$

$\Rightarrow a - b + c = 27 - 30 + 25 = 22$

HINT:

When two planes are perpendicular, then dot product of their normals is zero.

79. Option (D) is correct.

We have,

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(4-1) - 1(2-0) + 0$$

$= 6 - 2 = 4$

So, $|2A| = 2^3\,|A| = 8 \times 4 = 32$

Now, $|\text{adj}\,(\text{adj}\,(\text{adj}\,2A))| = |2A|^{(n-1)^3}$

$= (32)^{2^3} = 32^8$

$\Rightarrow 16^n = (32)^8 = 2^8 \times 16^8$

$\Rightarrow 16^n = 16^{2+8} \Rightarrow n = 10$

HINT:

(1) $|kA| = k^n\,|A|$

(2) $|\text{adj}\,A| = |A|^{n-1}$

80. Option (C) is correct.

$$I = \int \frac{x+1}{x(1+xe^x)^2}dx$$

Put $1 + xe^x = t \Rightarrow xe^x = t - 1$

$\Rightarrow (xe^x + e^x)\,dx = dt$

$\Rightarrow e^x\,(x + 1)\,dx = dt$

$$\therefore I = \int\frac{dt}{e^x.xt^2} = \int\frac{dt}{(t-1)t^2}$$

Let $\frac{1}{t^2(t-1)} = \frac{A}{(t-1)} + \frac{Bt+C}{t^2}$

$\Rightarrow 1 = At^2 + (Bt + C)\,(t - 1)$

Comparing coefficients of t^2, t and constant terms, we get

$A + B = 0, C - B = 0, -C = 1$

On solving above equations, we get

$C = -1, =B, A = 1$

$$\therefore\ I = \int\frac{1}{t-1}dt + \int\frac{-t-1}{t^2}dt$$

$$= \int\frac{1}{t-1}dt - \int\frac{1}{t}dt - \int\frac{1}{t^2}dt$$

$$= \log|t-1| - \log|t| + \frac{1}{t} + C$$

$$\Rightarrow I = \log|xe^x| - \log|1+xe^x| + \frac{1}{1+xe^x} + c$$

$$= \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$$

Now, $\lim_{x\to\infty} I(x) = 0$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C\right\} = 0$$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left(\frac{e^x}{\frac{1}{x}+e^x}\right)+\frac{\frac{1}{x}}{\frac{1}{x}+e^x}+C\right\}$$

$\Rightarrow 0 + 0 + C = 0 \Rightarrow C = 0$

$$\therefore I(x) = \log\left|\frac{xe^x}{1+xe^x}\right|+\frac{1}{1+xe^x}$$

$$\Rightarrow I(1)=\log\left|\frac{e}{1+e}\right|+\frac{1}{1+e}=1-\log(1+e)+\frac{1}{1+e}$$

$$=\frac{2+e}{1+e}-\log|1+e|$$

HINT:

(1) $\lim_{x\to\infty}\frac{1}{x}=0$

(2) $\log 1 = 0$

81. The correct answer is (19).

We have, A = {0, 3, 4, 6, 7, 8, 9, 10}

Case I: $x - y$ is odd, if one is odd and one is even and $x > y$.

$\therefore$ Possibilites are {(3, 0), (4, 3), (6, 3), (7, 6), (7, 4), (7, 0), (8, 7), (8, 3), (9, 8), (9, 6), (9, 4), (9, 0), (10, 9), (10, 7), (10, 3)}

No. of cases = 15

Case II: $x - y = 2$

$\therefore$ Possibilities are {(6, 4), (8, 6), (9, 7), (10, 8)}

$\therefore$ No. of cases = 4

So, minimum ordered pair to be added = 15 + 4 = 19

HINT:

Any relations said to be symmetric if $(a, b) \in R$ $\Rightarrow (b, a) \in R$

82. The correct answer is (1275).

Let T_{r+1} be the constant term.

$$T_{r+1} = {}^7C_r(3x^2)^{7-r}\left(\frac{-1}{2x^5}\right)^r$$

For constant term, power of x should be zero.

i.e., $14 - 2r - 5r = 0$

$\Rightarrow 14 = 7r \Rightarrow r = 2$

Now, constant term = α

$$\Rightarrow {}^7C_2(3)^5\left(\frac{-1}{2}\right)^2=\alpha$$

$$\Rightarrow 21\times 243\times\frac{1}{4}=\alpha$$

$\Rightarrow [\alpha] = [1275.75] = 1275$

HINT:

Let $(a + b)^n$, then $T_{r+1} = {}^nC_r\, a^{n-r} . b^r$

83. The correct answer is (9).

Since $\left(\frac{5}{2},1,\lambda\right)$ and (–2, 0, 1) are equidistant from plane $2x + 3y - 6z + 7 = 0$

$$\therefore \left|\frac{2\left(\frac{5}{2}\right)+3(1)-6(\lambda)+7}{\sqrt{2^2+3^2+6^2}}\right|=\left|\frac{2(-2)+3(0)-6(1)+7}{\sqrt{2^2+3^2+6^2}}\right|$$

$\Rightarrow |5 + 3 - 6\lambda + 7| = |-4 -6 + 7|$

$\Rightarrow |15 - 6\lambda| = |-3|$

$\Rightarrow 15 - 6\lambda = \pm 3$

$\Rightarrow 15 - 6\lambda = 3$ or $15 - 6\lambda = -3$

$\Rightarrow 6\lambda = 12$ or $6\lambda = 18$

$\Rightarrow \lambda = 2$ or $\lambda = 3$

$\because \lambda_1 > \lambda_2$

$\therefore \lambda_1 = 3$ and $\lambda_2 = 2$

So, point will be (1, 2, 3)

Let $M_0 = (1, 2, 3)$

M_1 is the point through which line passes i.e, (5, 1, –7)

and $\vec{s}=\hat{i}+2\hat{j}+2\hat{k}$

$\therefore \overrightarrow{M_0M_1}=4\hat{i}-\hat{j}-10\hat{k}$

Now, required distance = $\left|\frac{\overrightarrow{M_0M_1}\times\vec{s}}{|\vec{s}|}\right|$

$$=\frac{|(4\hat{i}-\hat{j}-10\hat{k})\times(\hat{i}+2\hat{j}+2\hat{k})|}{\sqrt{1+4+4}}$$

$$=\frac{|18\hat{i}-18\hat{j}+9\hat{k}|}{3}=9$$

HINT:

Distance of a point (a, b, c) from a plane $px + qy + rz + s = 0$ is $\frac{|ap+bq+cr+s|}{\sqrt{p^2+q^2+r^2}}$

84. The correct answer is (3).

The given differential equation is,

$(y - 2\log x)\,dx + (x\log x^2)\,dy = 0$

$$\Rightarrow \frac{dy}{dx}=\frac{(2\log x-y)}{2x\log x}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{2x\log x}=\frac{1}{x}$$

It is a linear differential equation.

$$\therefore \text{I.F.} = e^{\int\frac{1}{2x\log x}dx}$$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore \text{I.F.}=e^{\int\frac{1}{2t}dt}=e^{\log(t)^{\frac{1}{2}}}=\sqrt{t}=\sqrt{\log x}$$

So, required solution is,

$$y\sqrt{\log x}=\int\frac{\sqrt{\log x}}{x}dx$$

$$\log x = v \Rightarrow \frac{1}{x}dx = dv$$

$$\Rightarrow y\sqrt{\log x}=\int\sqrt{v}\,dv + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2v^{3/2}}{3} + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2}{3}(\log x)^{3/2} + C$$

Now, this curve passes through $\left(e, \frac{4}{3}\right)$ and (e^4, α)

$$\therefore \frac{4}{3}\sqrt{\log e} = \frac{2}{3}(\log e)^{3/2} + C$$

$$\Rightarrow C = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

Also, $\alpha\sqrt{\log e^4} = \frac{2}{3}(\log e^4)^{3/2} + \frac{2}{3}$

$$\Rightarrow 2\alpha = \frac{2}{3}\times(4)^{3/2} + \frac{2}{3} = \frac{16}{3} + \frac{2}{3} = \frac{18}{3}$$

$\Rightarrow \alpha = 3$

HINT:

Reduce the given differential equation to linear differential equation and find its solution.

85. The correct answer is (11).

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Now, $\vec{a}.\vec{c} = -12$

$\Rightarrow 6C_1 + 9C_2 + 12C_3 = -12$ ...(i)

Also, $\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$

$\Rightarrow C_1 - 2C_2 + C_3 = 5$ ...(ii)

Now, $\vec{a}\times\vec{c} = \vec{a}\times\vec{b}$

$\Rightarrow \vec{a}\times(\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a}$ is parallel to $(\vec{c} - \vec{b})$

$\Rightarrow \vec{a} = \lambda(\vec{c} - \vec{b})$

$$\Rightarrow 6\hat{i} + 9\hat{j} + 12\hat{k} = \lambda(c_1 - \alpha)\hat{i} + \lambda(c_2 - 11)\hat{j} + \lambda(c_3 + 2)\hat{k}$$

On comparing, we get

$$c_1 = \frac{6}{\lambda} + \alpha, c_2 = \frac{9}{\lambda} + 11, c_3 = \frac{12}{\lambda} - 2$$

Put there values in (ii), we get

$$\frac{6}{\lambda} + \alpha - \frac{18}{\lambda} - 22 + \frac{12}{\lambda} - 2 = 5$$

$\Rightarrow \alpha = 29$

From (i) and values of C_1, C_2, C_3, and α we have

$$6\left(\frac{6}{\lambda} + 29\right) + 9\left(\frac{9}{\lambda} + 11\right) + 12\left(\frac{12}{\lambda} - 2\right) = -12$$

$$\Rightarrow \frac{261}{\lambda} = -261 \Rightarrow \lambda = -1$$

So, $C_1 = 23$, $C_2 = 2$, $C_3 = -14$

$$\therefore \vec{c}.(\hat{i} + \hat{j} + \hat{k}) = (23\hat{i} + 2\hat{j} + -14\hat{k}).(\hat{i} + \hat{j} + \hat{k})$$

$= 23 + 2 - 14 = 11$

HINT:

$$\vec{a}\times\vec{c} = \vec{a}\times\vec{b} \Rightarrow \vec{a} \,||\, (\vec{c} - \vec{b}) \Rightarrow a = \lambda(\vec{c} - \vec{b})$$

86. The correct answer is (31).

We have,

$$\left[\frac{66}{3}\right] = 22$$

$$\left[\frac{66}{3^2}\right] = 7$$

$$\left[\frac{66}{3^3}\right] = 2$$

Highest powers of 3 is greater than 66. So, their g.i.f. is always 0

$\therefore$ Required natural number = 22 + 7 + 2 = 31

87. The correct answer is (5).

Let $y = \dfrac{x^3}{x^4 + 147}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^4 + 147)\times 3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

$$= \frac{3x^6 + 441x^2 - 4x^6}{(x^4 + 147)^2} = \frac{441x^2 - x^6}{(x^4 + 147)^2}$$

For maxima/minima, put $\frac{dy}{dx} = 0$

$\Rightarrow 441\,x^2 - x^6 = 0 \Rightarrow x^4 = 441$

$\Rightarrow x = \pm\sqrt{21}, \pm\sqrt{21}i$

Now, by descrates rule on number line we have

+ − + −

$-\sqrt{21}$ 0 $\sqrt{21}$

Since sign changes from negative to positive at 0.

$\therefore$ Maximum value of is at $x = \sqrt{21} = 4.58$

Now, 4 < 4.5 < 5

$$\therefore y \text{ at } x = 4 = \frac{64}{403} = 0.159$$

$$y \text{ at } x = 5 = \frac{125}{772} = 0.162$$

So, y is maximum at $x = 5$

$\therefore \alpha = 5$

HINT:

For maximum value, find $\frac{dy}{dx}$ and then observe the change in signs using decrates rule.

88. The correct answer is (25).

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
x	$x - 9$	$(x - 9)^2$
y	$y - 9$	$(y - 9)^2$
10	1	1
12	3	9

6	–3	9
12	3	9
4	–5	25
8	–1	1
$x+y+92$		$(x-9)^2+(y-9)^2+54$

Now, mean $(\bar{x}) = 9$

$$\Rightarrow \frac{x+y+52}{8} = 9$$

$\Rightarrow x + y = 20$ ...(i)

Also, variance = 9.25

$$\Rightarrow \frac{(x-9)^2+(y-9)^2+54}{8} = 9.25$$

$\Rightarrow x^2 + y^2 + 81 + 81 - 2 \times 9(x + y) = 20$

$\Rightarrow x^2 + y^2 - 18 \times 20 = -142$

$\Rightarrow x^2 + y^2 = 218$

$\Rightarrow x^2 + (20 - x)^2 = 218$

$\Rightarrow x^2 + 400 + x^2 - 40x = 218$

$\Rightarrow 2x^2 - 40x + 182 = 0$

$$\Rightarrow x = \frac{40 \pm 12}{4}$$

$\Rightarrow x = 13$ or $x = 7 \Rightarrow y = 7$ or $y = 13$

But $x > y$

$\therefore x = 13$ and $y = 7$

So, $3x - 2y = 39 - 14 = 25$

HINT:

(1) Mean = $\dfrac{\Sigma x_i}{n}$

(2) Variance = $\dfrac{\Sigma(x_i - \bar{x})^2}{n}$

89. The correct answer is (2).

We have,

$C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$

$C_1 : (x - 2)^2 + (y - 1)^5 - 5 = \alpha - 5$

$C_1 : (x - 2)^2 + (y - 1)^2 = (\sqrt{\alpha})^2$

So, centre and radius of C_1 are (2, 1) and $\sqrt{\alpha}$ respectively

Now, image of (2, 1) along the line $y = 2x + 1$ is,

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{2^2+(-1)^2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$

$$\Rightarrow x = \frac{-6}{5} \text{ and } y = \frac{13}{5}$$

Now, $\left(\frac{-6}{5}, \frac{13}{5}\right)$ will be the centre of C_2

$$\therefore f = \frac{6}{5} \text{ and } g = \frac{-13}{5}$$

Now, radius of $C_2 = r = \sqrt{f^2 + g^2 - \frac{36}{5}}$

$$\Rightarrow r = \sqrt{\frac{36}{25} + \frac{169}{25} - \frac{36}{5}} = 1$$

$\because r = 1$ so, $\alpha = 1$

$\therefore \alpha + r = 1 + 1 = 2$

HINT:

Image of a point (x_1, y_1) w.r.t. $ax + by + c = 0$ is (x, y), then

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{(a^2+b^2)}$$

90. The correct answer is (14).

$$\text{Let I} = \frac{2}{\pi}\int_{\frac{\pi}{6}}^{5\frac{\pi}{6}} \{8[\operatorname{cosec} x] - 5[\cot x]\}dx$$

$$= \frac{2}{\pi}\left[8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\operatorname{cosec} x]dx - 5\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x]dx\right]$$

$$= \frac{2}{\pi}\left[8\int_{\pi/6}^{5\pi/6} dx - 5\left\{\int_{\pi/6}^{\pi/4} dx + \int_{\pi/4}^{\pi/2} 0.dx + \int_{\pi/2}^{3\pi/4} (-1)dx + \int_{3\pi/4}^{5\pi/6} (-2)dx\right\}\right]$$

$$= \frac{2}{\pi}\left[8 \times \left(\frac{5\pi}{6} \frac{-\pi}{6}\right) - 5\left\{\left(\frac{\pi}{4} - \frac{\pi}{6}\right) - \left(\frac{3\pi}{4} - \frac{\pi}{2}\right)\right\} - 2\left(\frac{5\pi}{6} - \frac{3\pi}{4}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{16\pi}{3} + \frac{5\pi}{3}\right] = 14$$

HINT:

Check the graph of [cosec x] and [cot x].

JEE (Main) SOLVED PAPER

2023
08^{th} April Shift 2

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying the load is 250 cm^2. The maximum pressure the smaller piston would have to bear is [Assume $g = 10\ m/s^2$] :

(A) $2 \times 10^{+5}$ Pa **(B)** $20 \times 10^{+6}$ Pa
(C) $200 \times 10^{+6}$ Pa **(D)** $2 \times 10^{+6}$ Pa

Q. 2. The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth center is increased by eight times to its initial value, then the new angular momentum will be-

(A) 8 L **(B)** 3 L
(C) 4 L **(D)** 9 L

Q. 3. The waves emitted when a metal target is bombarded with high energy electrons are

(A) Microwaves **(B)** X-rays
(C) Radio Waves **(D)** Infrared rays

Q. 4. Match List I with List II:

	LIST-I		LIST-II
A.	Torque	**I.**	$ML^{-2}T^{-2}$
B.	Stress	**II.**	$ML^{2}T^{-2}$
C.	Pressure gradient	**III.**	$ML^{-1}T^{-1}$
D.	Coefficient of viscosity	**IV.**	$ML^{-1}T^{-2}$

Choose the correct answer from the options given below:

(A) A-III, B-IV, C-I, D-II
(B) A-II, B-I, C-IV, D-III
(C) A-IV, B-II, C-III, D-I
(D) A-II, B-IV, C-I, D-III

Q. 5. Give below are two statements
Statement I : Area under velocity- time graph gives the distance travelled by the body in a given time.
Statement II : Area under acceleration- time graph is equal to the change in velocity- in the given time.
In the light of given statement, choose the correct answer from the options given below

(A) Both Statement I and Statement II are true.
(B) Statement I is correct but Statement II is false.
(C) Both Statement I and and Statement II are false
(D) Statement I is incorrect but Statement II is true.

Q. 6. The power radiated from a linear antenna of length l is proportional to (Given, λ = Wavelength of wave):

(A) $\frac{l}{\lambda}$ **(B)** $\frac{l^2}{\lambda}$
(C) $\frac{l}{\lambda^2}$ **(D)** $\left(\frac{l}{\lambda}\right)^2$

Q. 7. Electric potential at a point 'P' due to a point charge of 5×10^{-9}C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0} = 9\times10^{+9}\,Nm^2C^{-2}$)

(A) 3 cm **(B)** 9 cm
(C) 0.9 cm **(D)** 90 cm

Q. 8. The acceleration due to gravity at height h above the earth if $h << R$ (Radius of earth) is given by

(A) $g' = g\left(1 - \frac{h^2}{2R^2}\right)$ **(B)** $g' = g\left(1 - \frac{h}{2R}\right)$

(C) $g' = g\left(1 - \frac{2h^2}{R^2}\right)$ **(D)** $g' = g\left(1 - \frac{2h}{R}\right)$

Q. 9. An emf of 0.08 V is induced in a metal rod of length 10 cm held normal to a uniform magnetic field of 0.4 T, when moves with a velocity of:

(A) 2 ms^{-1} **(B)** 20 ms^{-1}
(C) 3.2 ms^{-1} **(D)** 0.5 ms^{-1}

Q. 10. Work done by a Carnot engine operating between temperatures 127°C and 27°C is 2 kJ. The amount of heat transferred to the engine by the reservoir is:

(A) 2 kJ **(B)** 4 kJ
(C) 2.67 kJ **(D)** 8 kJ

Q. 11. The width of fringe is 2 mm on the screen in a double slits experiment for the light of wavelength of 400 nm. The width of the fringe for the light of wavelength 600 nm will be:

(A) 1.33 mm **(B)** 3 mm
(C) 2 mm **(D)** 4 mm

Q. 12. The temperature at which the kinetic energy of oxygen molecules becomes double than its value at 27°C is

(A) 1227°C **(B)** 627°C
(C) 327°C **(D)** 927°C

Q. 13. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Electromagnets are made of soft iron.

Reason R : Soft iron has high permeability and low retentivity.

In the light of above, statements, chose the most appropriate answer from the options given below

(A) A is correct but R is not correct
(B) Both A and R are correct and R is the correct explanation of A
(C) Both A and R are correct but R is NOT the correct explanation of A
(D) A is not correct but R is correct

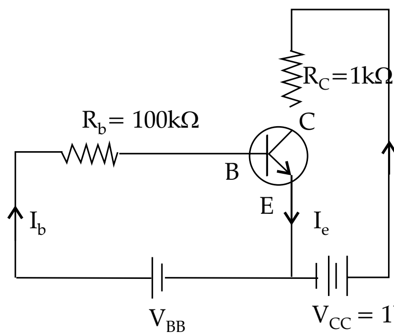
Q. 14. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be.

(A) 10 m **(B)** 200 m
(C) $10\sqrt{2}$ m **(D)** 5 m

Q. 15. A bullet of mass 0.1 kg moving horizontally with speed 400 ms^{-1} hits a wooden block of mass 3.9 kg kept on a horizontal rough surface. The bullet gets embedded into the block and moves 20 m before coming to rest. The coefficient of friction between the block and the surface is _______.
(Given $g = 10$ m/s^2)

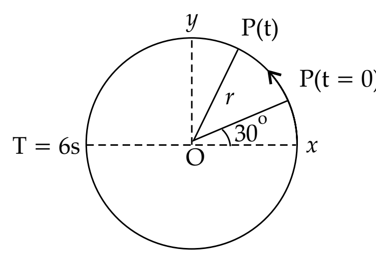
(A) 0.90 **(B)** 0.65
(C) 0.25 **(D)** 0.50

Q. 16. For a given transistor amplifier circuit in CE configuration $V_{CC} = 1$ V, $R_C = 1$ kΩ, $R_b = 100$ kΩ and $\beta = 100$. Value of base current I_b is



(A) $I_b = 100$ μA **(B)** $I_b = 10$ μA
(C) $I_b = 0.1$ μA **(D)** $I_b = 1.0$ μA

Q. 17. For particle P revolving round the centre O with radius of circular path r and angular velocity ω, as shown in below figure, the projection of OP on the x-axis at time t is



(A) $x(t) = r\cos\left(\omega t + \frac{\pi}{6}\right)$

(B) $x(t) = r\cos\left(\omega t - \frac{\pi}{6}\omega\right)$

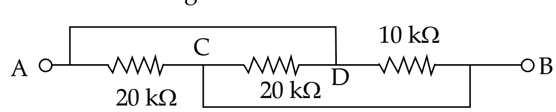
(C) $x(t) = r\cos(\omega t)$

(D) $x(t) = r\sin\left(\omega t + \frac{\pi}{6}\right)$

Q. 18. A radio active material is reduced to 1/8 of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days the initial amount of the material is

(A) 64 g **(B)** 40 g
(C) 32 g **(D)** 256 g

Q. 19. The equivalent resistance between A and B as shown in figure is:



(A) 20 kΩ **(B)** 30 kΩ
(C) 5 kΩ **(D)** 10 kΩ

Q. 20. In photo electric effect

A. The photocurrent is proportional to the intensity of the incident radiation.

B. Maximum Kinetic energy with which photoelectrons are emitted depends on the intensity of incident light.

C. Max K.E with which photoelectrons are emitted depends on the frequency of incident light.

D. The emission of photoelectrons require a minimum threshold intensity of incident radiation.

E. Max. K.E of the photoelectrons is independent of the frequency of the incident light.

Choose the correct answer from the options given below:

(A) B and C only **(B)** A and C only
(C) A and E only **(D)** A and B only

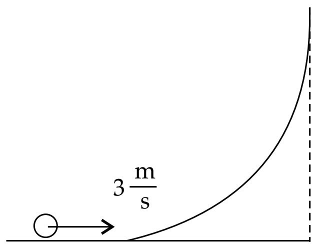
Section B

Q. 21. A 600 pF capacitor is charged by 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. Electrostatic energy lost in the process is ________ μJ

Q. 22. A series combination of resistor of resistance 100 Ω, inductor of inductance 1 H and capacitor of capacitance 6.25 μF is connected to an ac source. The quality factor of the circuit will be ________

Q. 23. The number density of free electrons in copper is nearly 8×10^{28} m^{-3}. A copper wire has its area of cross section = 2×10^{-6} m^2 and is carrying a current of 3.2 A. The drift speed of the electrons is ________ $\times 10^{-6}$ ms^{-1}

Q. 24. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity 3 m/s (as shown in figure). Maximum height with respect to the initial position covered by it will be ________ cm (take, g = 10 m/s^2)



Q. 25. A steel rod of length 1 m and cross sectional area 10^{-4} m^2 is heated from 0°C to 200°C without being allowed to extend or bend. The compressive tension produced in the rod is ________ × 10^4N. (Given Young's modulus of steel = 2×10^{11} Nm^{-2}, coefficient of linear expansion = $10^{-5}K^{-1}$)

Q. 26. The ratio of magnetic field at the centre of a current carrying coil of radius r to the magnetic field at distance from the centre of coil on its axis is $\sqrt{x}:1$. The value of x is ________

Q. 27. The ratio of wavelength of spectral lines H_α and H_β in the Balmer series is $\frac{x}{20}$. The value of x is ________.

Q. 28. Two transparent media having refractive indices 1.0 and 1.5 are separated by a spherical refracting surface of radius of curvature 30 cm. The centre of curvature of surface is towards denser medium and a point object is placed on the principle axis in rarer medium at a distance of 15 cm from the pole of the surface. The distance of image from the pole of the surface is ________ cm.

Q. 29. A guitar string of length 90 cm vibrates with a fundamental frequency of 120 Hz. The length of the string producing a fundamental frequency of 180 Hz will be ________ cm.

Q. 30. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. Now a force of 2 N acts on the body in the direction of its motion for 5 s. The increase in the Kinetic energy of the body is ________ J.

Chemistry

Section A

31. The statement/s which are true about antagonists from the following is/are :

A. They bind to the receptor site

B. Get transferred inside the cell for their action

C. Inhibit the natural communication of the body

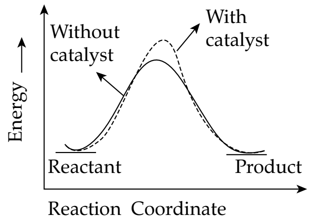
D. Mimic the natural messenger.

Choose the correct answer from the options given below:

(1) A and B **(2)** A and C
(3) A, C and D **(4)** B only

32. The correct reaction profile diagram for a positive catalyst reaction.

(1)



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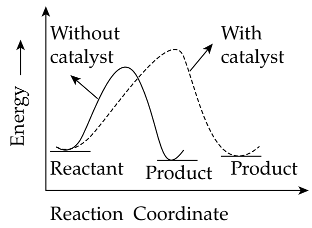
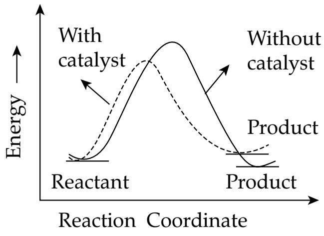
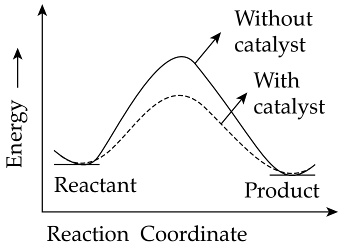


(3)



(4)



33. Given below are two statements: One is labelled as **Assertion A** and other is labelled as **Reason R**

Assertion A: Sodium is about 30 times as abundant as potassium in the oceans.

Reason R: Potassium is bigger in size than sodium.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are true but R is NOT the correct explanation of A
(2) A is true but R is false
(3) A is false but R is true
(4) Both A and R are true and R is the correct explanation of A

34. Which of these reactions is not a part of breakdowns of ozone in stratosphere?

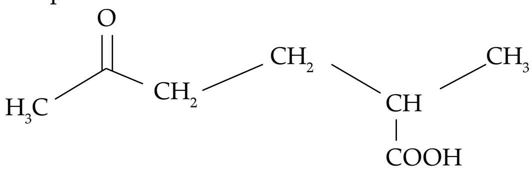
(1) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(2) $\dot{C}l(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$

(3) $2Cl\dot{O} \rightarrow ClO_2(g) + \dot{C}l(g)$

(4) $Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$

35. The correct IUPAC nomenclature for the following compound is :



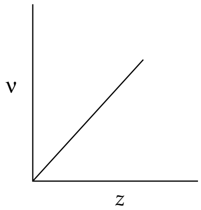
(1) 2-Methyl-5-oxohexanoic acid
(2) 2-Formyl-5-methylhexan-6-oic acid
(3) 5-Formyl-2-methylhexanoic acid
(4) 5-Methyl-2-oxohexan-6-oic acid

36. Henry Moseley studied characteristic X-ray spectra of elements. The graph which represents his observation correctly is

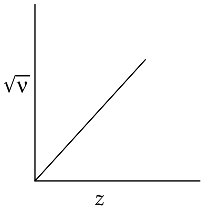
Given ν = frequency of X-ray emitted

z = atomic number

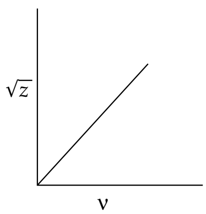
(1)



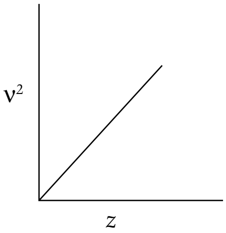
(2)



(3)



(4)



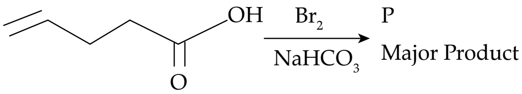
37. Match list I with list II

List I Coordination complex	List II Number of unpaired electrons
A. $[Cr(CN)_6]^{3-}$	I. 0
B. $[Fe(H_2O)_6]^{2+}$	II. 3
C. $[Co(NH_3)_6]^{3+}$	III. 2
D. $[Ni(NH_3)_6]^{2+}$	IV. 4

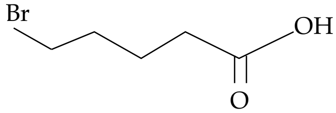
Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III
(2) A-IV, B-III, C-II, D-I
(3) A-II, B-I, C-IV, D-III
(4) A-III, B-IV, C-I, D-II

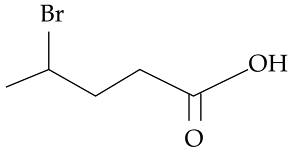
38. Major product 'P' formed in the following reaction is:

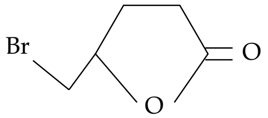


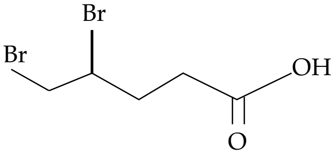
(1)



(2)



(3) 

(4) 

39. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be as close as to

(1) 2 **(2)** 1
(3) 3 **(4)** 4

40. Match list I with list II

List I Natural amino acid	List II One letter code
A. Glutamic acid	I. Q
B. Glutamine	II. W
C. Tyrosine	III. E
D. Tryptophan	IV. Y

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II
(2) A-IV, B-III, C-I, D-II
(3) A-II, B-I, C-IV, D-III
(4) A-III, B-IV, C-I, D-II

41. Which of the following have same number of significant figures?

A. 0.00253 B. 1. 0003
C. 15.0 D. 163

Choose the correct answer from the options given below

(1) B and C only **(2)** A, B and C only
(3) A, C and D only **(4)** C and D only

42. Given below are two statements:

Statement I: Methyl orange is a weak acid.

Statement II: The benzenoid form of methyl orange is more intense/deeply coloured than the quinonoid form.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) Both statement I and Statement II are incorrect
(2) Both statement I and Statement II are correct
(3) Statement I is correct but statement II is incorrect
(4) Statement I is incorrect but statement II is correct

43. The descending order of acidity for the following carboxylic acid is –

A. CH_3COOH B. $F_3C–COOH$
C. $ClCH_2–COOH$ D. $FCH_2–COOH$
E. $Br–CH_2COOH$

Choose the correct answer from the options given below:

(1) D > B > A > E > C
(2) B > D > C > E > A
(3) E > D > B > A > C
(4) B > C > D < E > A

44. In Hall-Heroult process, the following is used for reducing Al_2O_3:

(1) Magnesium **(2)** Graphite
(3) Na_3AlF_6 **(4)** CaF_2

45. Arrange the following gases in increasing order of van der waals constant 'a'

A. Ar B. CH_4
C. H_2O D. C_6H_6

Choose the correct options from the following

(1) A, B, C and D **(2)** B, C, D and A
(3) C, D, B and A **(4)** D, C, B and A

46. Given below are two statement:

Statement I: In redox titration, the indicators used are sensitive to change in pH of the solution.

Statement II: In acid-base titration, the indicators used are sensitive to change in oxidation potential.

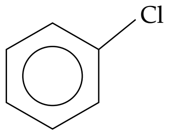
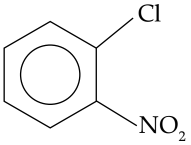
In the light of the above statement, choose the most appropriate answer from the options given below

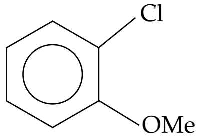
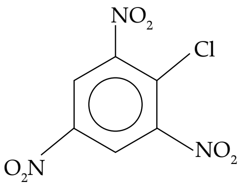
(1) Both statement I and Statement II are incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are correct

47. Which of the following can reduce decomposition of H_2O_2 on exposure to light

(1) Dust **(2)** Urea
(3) Glass containers **(4)** Alkali

48. The correct order of reactivity of following haloarenes towards nucleophilic substitution with aqueous NaOH is

A. 
B. 

C. 
D. 

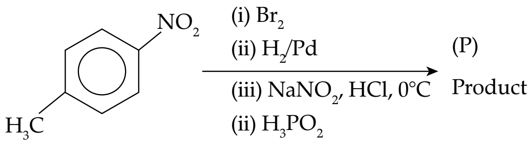
Choose the correct answer from the options given below:

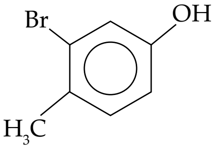
(1) D > B > A > C **(2)** A > B > D > C
(3) C > A > D > B **(4)** D > C > B > A

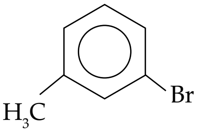
49. A compound 'X' when treated with phthalic anhydride in presence of concentrated H_2SO_4 yields 'Y'. 'Y' is used as an acid/base indicator. 'X' and 'Y' are respectively.

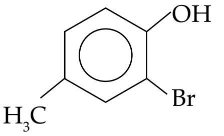
(1) Anisole, methyl orange
(2) Toluidine, Phenolphthalein
(3) Carbolic acid, Phenolphthalein
(4) Salicylaldehyde, Phenolphthalein

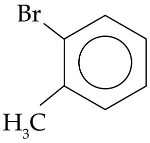
50. The product (P) formed from the following multistep reaction is:



(1) 

(2) 

(3) 

(4) 

Section B

51. The observed magnetic moment of the complex $[Mn(NCS)_6]^{x-}$ is 6.06 BM. The numerical value of x is ________.

52. For complete combustion of ethene.
$C_2H_4(g)+3O_2(g) \rightarrow 2CO_2(g)+2H_2O(l)$
The amount of heat produced as measured in bomb calorimeter is 1406 kJ mol^{-1} at 300K. The mimimum value of TΔS needed to reach equilibrium is (-)________kJ (Nearest integer)

53. The solubility product of $BaSO_4$ is 1×10^{-10} at 298 K. The solubility of $BaSO_4$ in 0.1 M $K_2SO_4(aq)$ solute is________$\times10^{-9}$ g L^{-1} (Nearest integer)
Given: Molar mass of $BaSO_4$ is 233 g mol^{-1}

54. The number of atomic orbitals from the following having 5 radial nodes is_________
7s, 7p, 6s, 8p, 8d

55. The number of incorrect statement from the following is________
(1) The electrical work that a reaction can perform at constant pressure and temperature is equal to the Gibbs energy
(2) E°_{cell} is dependent on the pressure
(3) $\dfrac{dE^\circ_{cell}}{dT}=\dfrac{\Delta S^\circ}{nF}$
(4) A cell is operating reversibly if the cell potential is exactly balanced by an opposing source of potential difference

56. Coagulating value of the electrolytes $AlCl_3$ and NaCl for As_2S_3 are 0.09 and 50.04 respectively. The coagulating power of $AlCl_3$ is x times the coagulating power of NaCl. The value of x is______

57. If the boiling points of two solvents X and Y (having same molecular weights) are in the ratio 2 : 1 and their enthalpy of vaporizations are in the ratio 1 : 2, then the boiling point elevation constant of X is m times the boiling point elevation constant of Y. The value of m is________(nearest integer)

58. The number of species from the following carrying a single lone pair on central atom Xenon is_______
XeF_5^+, XeO_3, XeO_2F_2, XeF_5^-, XeO_3F_2, $XeOF_4$, XeF_4

59. The ratio of sigma and π bonds present in pyrophosphoric acid is________

60. The sum of oxidation state of the metals in $Fe(CO)_5$, VO^{2+} and WO_3 is_______

Mathematics

Section A

61. Let
$$A=\left\{\theta\in(0,2\pi):\frac{1+2i\sin\theta}{1-i\sin\theta}\text{ is purely imaginary}\right\}.$$
Then the sum of the elements in A is
(A) π **(B)** 3π
(C) 4π **(D)** 2π

62. Let P be the plane passing through the line $\dfrac{x-1}{1}=\dfrac{y-2}{-3}=\dfrac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, γ) then $\alpha+\beta+\gamma$ is equal to
(A) 12 **(B)** 9
(C) 10 **(D)** 11

63. If $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$. $A^{-1}=\alpha A+BI$ and $\alpha+\beta=-2$, then $4\alpha^2+\beta^2+\lambda^2$ is equal to :
(A) 14 **(B)** 12
(C) 19 **(D)** 10

64. The area of the quadrilateral ABCD with vertices A(2,1,1), B (1,2, 5), C(–2,–3, 5) and D (1, –6, –7) is equal to
(A) 54 **(B)** $9\sqrt{38}$
(C) 48 **(D)** $8\sqrt{38}$

65. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by
(A) 34 but not by 14 **(B)** 14 but not by 34
(C) Both 14 and 34 **(D)** Neither 14 nor 34

66. Let O be the origin and OP and OQ be the tangents to the circle $x^2+y^2-6x+4y+8=0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha,\frac{1}{2}\right)$, then a value of α is.
(A) $-\dfrac{1}{2}$ **(B)** $\dfrac{5}{2}$
(C) 1 **(D)** $\dfrac{3}{2}$

67. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 +.... and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(A) 11260 **(B)** 11280
(C) 11290 **(D)** 11310

68. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$, then k is equal to

(A) β **(B)** 2α
(C) 2β **(D)** α

69. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)k$, is equal to

(A) 1890 **(B)** 945
(C) 2835 **(D)** 5670

70. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$
$$-x + (\tan\theta)y + \sqrt{7}z = 0$$
$$x + y + (\tan\theta)z = 0$$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in S}\theta$ is equal to

(A) 20 **(B)** 40
(C) 30 **(D)** 10

71. For $a, b \in \mathbb{Z}$ and $|a - b| \le 10$, let the angle between the plane P : $ax + y - z = b$ and the line $l : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(A) 85 **(B)** 48
(C) 25 **(D)** 32

72. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a + b + c)$ is equal to

(A) 4 **(B)** 12
(C) 6 **(D)** 0

73. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(A) $10^3 - 10$ **(B)** $11^3 - 11$
(C) $12^3 - 12$ **(D)** $13^3 - 13$

74. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {$(x, y) \in A \times A : x + y = 7$} is

(A) Symmetric but neither reflexive nor transitive
(B) Transitive but neither symmetric nor reflexive
(C) An equivalence relation
(D) Reflexive but neither symmetric nor transitive

75. If the probability that the random variable X takes values x is given by $P(X=x) = k(x+1)3^{-x}$, $x = 0, 1, 2, 3, ...,$ where k is a constant, then $P(X \ge 2)$ is equal to

(A) $\frac{7}{27}$ **(B)** $\frac{11}{18}$
(C) $\frac{7}{18}$ **(D)** $\frac{20}{27}$

76. The integral $\int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to

(A) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$ **(B)** $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$
(C) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$ **(D)** $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

77. The value of $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is

(A) 27 **(B)** 54
(C) 18 **(D)** 36

78. Let A (0, 1), B(1, 1) and C (1, 0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{3}, 0)$, , where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(A) 6 **(B)** 8
(C) $\frac{9}{2}$ **(D)** 12

79. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(A) $p \wedge (\sim q)$ **(B)** $p \wedge (q \wedge (\sim p))$
(C) $p \vee (q \vee (\sim p))$ **(D)** $p \wedge q$

80. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $m + n$ is equal to

(A) 316 **(B)** 317
(C) 315 **(D)** 314

Section B

81. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions $f : R \to S$ such that $f(a) \neq 1$ is equal to _____.

82. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5[x + 2] - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then $m^2 + mn + n^2$ is equal to ________.

83. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1) on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

84. If domain of the function $\log_e\left(\dfrac{6x^2+5x+1}{2x-1}\right) + \cos^{-1}\left(\dfrac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then, $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to

85. Let the area enclosed by the lines $x + y = 2$, $y = 0$, $x = 0$ and the curve $f(x) = \min\left\{x^2 + \dfrac{3}{4}, 1 + [x]\right\}$ where $[x]$ deontes the greatest integer $\leq x$, be A. Then the value of 12A is ______.

86. Let $0 < z < y < x$ be three real numbers such that $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \dfrac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _________.

87. Let the solution curve $x = x(y)$, $0 < y < \dfrac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y \, dx - (1 + 3x \log_e(\cos y)) \sin y \, dy = 0$ satisfy $x\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2\log_e 2}$. If $x\left(\dfrac{\pi}{6}\right) = \dfrac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to

88. Let $[t]$ denote the greatest integer function. If $\int_0^{24} [x^2]dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to _________.

89. The ordinates of the points P and Q on the parabola with focus (3.0) and directrix $x = -3$ are in the ratio 3 : 1. If R (α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\dfrac{\beta^2}{\alpha}$ is equal to ________.

90. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2 & x \geq 1 \end{cases}$ is differentiable for all $x > 0$. Then $\dfrac{8f'(8)}{f'\left(\dfrac{1}{8}\right)}$ is equal to _____.

Answer Key

Physics

Q. No.	Answer	Topic Name	Chapter Name
1	D	Pascal's law	Fluid mechanics
2	B	Satellight	Gravitation
3	B	X ray	EM waves
4	D	Dimension	Units & Dimensions
5	D	v-t and a-t graph	Motion in One Dimension
6	D	Power of antenna	Communication System
7	D	Electric potential	Electrostatics
8	D	Acceleration due to gravity	Gravitation
9	A	Induced EMF	Electromagnetic Induction
10	D	Carnot engine	Thermodynamics
11	B	YDSE	Wave optics
12	C	Kinetic energy of gas	Kinetic theory of gasses
13	B	Electromagnet	Electromagnetism
14	D	Projectile	Motion in 2D
15	C	Momentum conservation	Collision
16	B	Transistor	Semiconductors
17	A	Phasor	Circular Motion
18	D	Half life	Nuclear Physics
19	C	Resistance circuit	Electric Current
20	B	Kinetic Energy of photoelectron	Photoelectric Effect
21	[6]	Energy in capacotors	Capacitors
22	[4]	RLC circuit	Electromagnetic Induction
23	[125]	Drift velocity	Electric current
24	[75]	Rotational Kinetic Energy	Rotational Motion
25	[4]	Thermal stresses	Elasticity
26	[8]	Magnetic field due to current carrying loop	Magnetism
27	[27]	Hydrogen spectra	Atoms
28	[30]	Refraction through spherical surfaces	Ray optics
29	[60]	Fundamental Frequency	Sound waves
30	[30]	Change in Kinetic Energy	Work, Energy and Power

Chemistry

Q No	Answer	Topic's Name	Chapter's Name
31	(2)	Drug-Target Interaction	Chemistry in Everyday Life
32	(4)	Catalysis	Surface Chemistry
33	(1)	Alkali Metals	s-Block Elements
34	(3)	Atmospheric Pollution	Environmental Chemistry
35	(1)	Nomenclature of Organic Compounds	Organic Chemistry - Some Basic Principles and Techniques
36	(2)	Dual Nature of Radiation And Matter	Atomic Structure
37	(1)	Crystal Field Theory	Coordination Compounds
38	(3)	Chemical Reactions of Alcohols And Phenols	Alcohols, Phenols & Ethers
39	(1)	Some Important Compounds of Calcium	s-Block Elements
40	(1)	Amino Acids	Biomolecules
41	(3)	Significant Figures	Some Basic Concepts of Chemistry
42	(3)	Qualitative Analysis	Organic Chemistry - Some Basic Principles and Techniques
43	(2)	Chemical Reactions of Carboxylic Acids	Aldehydes, Ketones and Carboxylic Acids
44	(2)	Electrochemical Principles of Metallurgy	General Principles and Processes of Isolation of Elements
45	(1)	Behaviour of Real Gases	States of Matter
46	(1)	Titrations	Redox Reactions
47	(2)	Hydrogen Peroxide	Hydrogen
48	(1)	Reactions of Haloarenes	Haloalkanes And Haloarenes
49	(3)	Reactions of Phenols	Alcohols, Phenols & Ethers
50	(4)	Reactions of Nitro Compounds	Amines
51	[4]	Some Transition Elements	d & f Block Elements
52	[1411]	Bomb Calorimetery	Thermochemistry
53	[233]	Solubility Equilibria	Ionic Equilibrium
54	[3]	Shapes of Orbitals and Nodes	Atomic Structure
55	[1]	Nernst Equation	Electrochemistry
56	[556]	Properties of Colloids	Surface Chemistry
57	[8]	Colligative Properties and Determination of Molar Mass	Solutions
58	[4]	Valence Bond Theory	Chemical Bonding And Molecular Structure
59	[6]	Oxoacids of Phosphorus	p-Block Elements
60	[10]	Oxidation Number	Redox Reactions

Mathematics

77	Answer	Topic Name	Chapter Name
78	C	General form	Complex Numbers
79	C	Equation of plane	Three Dimensional Geometry
80	A	Charactersictic equation	Matrices and Determinants
81	D	Area of quadrilateral	Vector Algebra
82	A	Remainder theorem	Binomial Theorem
83	B	Circumcircle	Circle
84	C	Special series	Sequences and Series
85	B	Limits of trigonometry	Limits
86	D	Number of words	Permutation and Combination
87	A	System of linear equations	Matrices and Determinants
88	D	Distance of a point from a plane	Three Dimensional Geometry
89	B	Scalar triple product	Vector Algebra
90	C	General term	Binomial Theorem
74	A	Equivalence relation	Relation and Function
75	A	Probaility distribuution	Probability
76	D	Indefinite Integral	Integral Calculus
77	D	Trigonometric relations	Trigonometry
78	B	Incentre of triangle	Parabola
79	D	Equivalent statement	Mathematical Reasoning
80	B	Mean, Variance	Statistics
81	[180]	Number of onto fuctions	Relation and Function
82	[9]	Roots of equation	Quadratic equations
83	[11]	Equation of plane	Three dimensional geometry
84	[20]	Domain of a function	Function
85	[17]	Area between the curves	Integral Calculus
86	[150]	A.P., G.P.	Sequences and series
87	[12]	Linear differential equation	Differential equations
88	[6]	Definite Integral	Integral Calculus
89	[16]	Parabola	Conic Section
90	[309]	First derivative	Differentiability

JEE (Main) SOLVED PAPER

2023
08^{th} April Shift 2

ANSWERS WITH EXPLANATIONS

Physics

Section A

1. Option (D) is correct.

Given, $m = 5000$ kg

$A = 250\text{ cm}^2 = 250 \times 10^{-4}\text{ m}^2$

$F = mg = 5000 \times 10 = 50000$ N

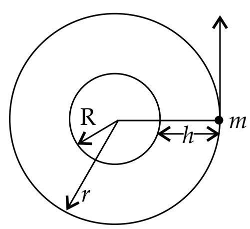
From Pascal's law pressure would be same at both ends of piston.

$$P = \frac{F}{A}$$

$$P = \frac{50000}{250 \times 10^{-4}} = 2 \times 10^6\ Pa$$

2. Option (B) is correct.

Angular momentum is given by



$L = mvr$

where $v = \sqrt{\frac{GM}{r}}$

$$L = m\sqrt{\frac{GM}{r}} \times r = m\sqrt{\frac{GMr^2}{r}} = \sqrt{GMr}$$

$$L \propto r^{\frac{1}{2}}$$

Now, the new distance from centre $= r + 8r = 9r$

New angular momentum

$$L' \propto (9r)^{\frac{1}{2}}$$

Therefore,

$$\frac{L'}{L} = \frac{(r)^{\frac{1}{2}}}{(9r)^{\frac{1}{2}}} = \frac{1}{3}$$

$\Rightarrow L' = 3L$

3. Option (B) is correct.

When target metal is bombarded with high energy electron then X-rays are emitted.

4. Option (D) is correct.

A. $\vec{\tau} = \vec{r} \times \vec{F}$

$[\tau] = [L]\,[MLT^{-2}] = ML^2T^{-2}$

B. $\sigma = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

C. $\frac{\Delta P}{\Delta x} = \frac{\left[\frac{F}{A}\right]}{L} = ML^{-2}T^{-2}$

D. $F = 6\,\pi\eta rV$

$\Rightarrow MLT^{-2} = [\eta]\,L^2\,T^{-1}$

$\Rightarrow [\eta] = ML^{-1}\,T^{-1}$

5. Option (D) is correct.

$\vec{S} = \int \vec{V}dt$

Therefore area under velocit time graph gives displacement.

Hence statement-I is false.

Area under acceleration time graph gives change in velocity.

$$a = \frac{dV}{dt}$$

$\Rightarrow dV = adt$

$\Rightarrow \int dV = \int a\,dt$

Hence statement-II is correct.

6. Option (D) is correct.

Radiating power of linear antenna is given by,

$$P = \frac{\pi}{12} I_{02}Z_0\left(\frac{l}{\lambda}\right)^2$$

Hence,

$$P \propto \left(\frac{l}{\lambda}\right)^2$$

7. Option (D) is correct.

Given, $q = 5 \times 10^{-9}$ C

$V = 50V$

$$V = k\frac{q}{r} \qquad \left(k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,\frac{Nm^2}{C^2}\right)$$

$$\Rightarrow 50 = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{r}$$

$$\Rightarrow r = \frac{9 \times 5}{10} = 0.9\,\text{m} = 90\text{ cm}$$

8. Option (D) is correct.

$$g = \frac{GM}{R^2} \quad ...(i)$$

Now acceleration due to gravity at height h is given by

$$g' = \frac{GM}{(R+h)^2} = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \quad \left(\because \text{from(i)}\, g = \frac{GM}{R^2}\right)$$

$$\Rightarrow g' = g\left(1+\frac{h}{R}\right)^{-2}$$

$$\Rightarrow g' = g\left(1-\frac{2h}{R}\right) \text{ from binomial expansion.}$$

9. Option (A) is correct.

When a metal rod is held normal moving in uniform magnetic field then the induced emf is given by

Induced E = Blv

$$\Rightarrow \upsilon = \frac{E}{Bl} = \frac{0.08}{0.4\times\frac{10}{100}}$$

$\upsilon = 0.08 \times 100/0.4 \times 10 = 2$ m/s

10. Option (D) is correct.

Efficiency of a carnot engine is given by

$$\eta = 1-\frac{T_2}{T_1} = \frac{W}{Q_1}$$

where $T_1 = 127 + 273 = 400K$

$T_2 = 27 + 273 = 300K$

Therefore,

$$\frac{W}{Q_1} = 1-\frac{T_2}{T_1} = 1-\frac{300}{400}$$

$$\Rightarrow \frac{2\times10^3 J}{Q_1} = \frac{1}{4}$$

$\Rightarrow Q_1 = 8 \times 10^3$ J = 8 kJ

11. Option (B) is correct.

Given, $\beta_1 = 2$mm, $\lambda_1 = 400$ nm

$\beta_2 = ?$, $\lambda_2 = 600$ nm

Fringe width $(\beta) = \frac{D\lambda}{d}$

$$\beta_1 = \frac{D\lambda_1}{d}$$

$$\beta_2 = \frac{D\lambda_2}{d}$$

$$\Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \frac{2mm}{\beta_2} = \frac{400\text{ nm}}{600\text{ nm}} = \frac{2}{3}$$

$$\Rightarrow \beta_2 = \frac{2\times3}{2}\text{mm} = 3\text{mm}$$

12. Option (C) is correct.

Kinetic energy is given by

$$K = \frac{f}{2}KT$$

$K \propto T$

Therefore,

$$\frac{K_1}{K_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{1}{2} = \frac{(27+273)}{T_2}$$

$\Rightarrow T_2 = 2 \times 300K$

$\Rightarrow T_2 = 600$ K = 327 C

13. Option (B) is correct.

Soft iron has high permeability and low retentivity. Therefore, iron is used to make electromagnet.

So, both A & R are correct and R is the correct explanation of A.

14. Option (D) is correct.

$$y = x - \frac{x^2}{20} \quad ...(i)$$

$$\frac{dy}{dx} = 0 \quad \text{(For maximum height)}$$

$$\Rightarrow \frac{dy}{dx} = 1-\frac{2x}{20} = 0$$

$\Rightarrow 20 - 2x = 0$

$\Rightarrow x = 10$

Putting the value of x in (i) for y_{max}.

$$y = x - \frac{x^2}{20}$$

$$\Rightarrow y_{max} = 10 - \frac{100}{20}$$

$= 10 - 5 = 5$ m

15. Option (C) is correct.

Momentum of bullet before collision

$P_i = 0.1 \times 400 = 40$ N-s

Momentum after collision

$P_f = (0.1 + 3.9)v$

Applying conservation of momentum

$P_i = P_f$

$\Rightarrow 40 = (0.1 + 3.9)v$

$\Rightarrow \upsilon = 10$m/s

Now, $v^2 = u^2 + 2as$

$\Rightarrow 0 = (10)^2 - 2 \times a \times 20$ ($u = 10$ m/s after collision)

$\Rightarrow 40a = 100$

$$\Rightarrow a = \frac{10}{4} = 2.5 \text{ m/s}^2$$

So, F = ma

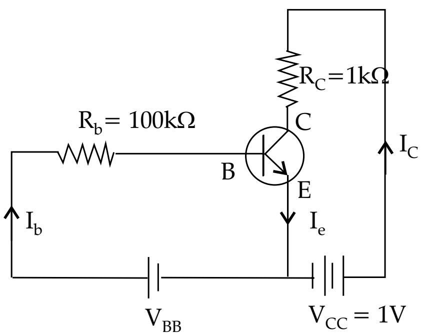
$\Rightarrow \mu mg = ma$

$\Rightarrow a = \mu g$

$$\Rightarrow \mu = \frac{a}{g} = \frac{2.5}{10} = 0.25$$

16. Option (B) is correct.

In saturation mode $V_{CE} = 0$



Now

$V_{CC} - I_cR_c = 0$ (from KVL)

$$\Rightarrow I_b = \frac{V_{CC}}{R_C} = \frac{1}{1\times10^3} = 10^{-3}\,A$$

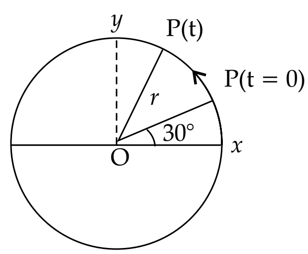
Given that

$$\beta = 100 = \frac{I_c}{I_b}$$

$$\Rightarrow I_b = \frac{I_c}{100} = \frac{10^{-3}}{100} = 10^{-5}\,A$$

$\Rightarrow I_b = 10\ \mu A$

17. Option (A) is correct.



After time t, the angular displacement will be

$\theta = \omega t$

Total angular displacement from x-axis.

$$\theta_{total} = \omega t + \frac{\pi}{6} \qquad \left(\because 30° = \frac{\pi}{6}\right)$$

Now, OP has two component

The horizontal component will be the projection along x-axis

$$= r\cos(\theta_{Total}) = r\cos\left(\omega t + \frac{\pi}{6}\right)$$

18. Option (D) is correct.

$$m = m_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{m_0}{8} = m_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

$\Rightarrow n = 3$

3 days = 3 half life

1 day = 1 half life

Now

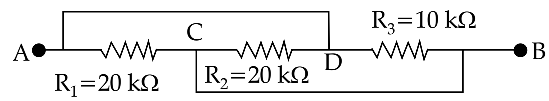
$$m = m_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow 8\times10^{-3} = m_0\left(\frac{1}{2}\right)^5$$

$\Rightarrow m_0 = 32 \times 8 \times 10^{-3}$

$\Rightarrow m_0 = 256$ g

19. Option (C) is correct.



Potential across $R_1 = V_A - V_C$ $(\because V_B = V_C)$

$= V_A - V_B$

Potential across $R_2 = V_D - V_C$ $(\because V_D = V_A)$

$= V_A - V_B$

Potential across $R_3 = V_D - V_B$ $(\because V_B = V_C)$

$= V_A - V_B$

It means all the resistance are in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1+1+2}{20}$$

$$\Rightarrow Req = \frac{20}{4} = 5\ k\Omega$$

20. Option (B) is correct.

Intensity of incident light ∝ photo current

So, A statement is correct

Now, $K.E_{max} = h\nu - \phi$

∴ K.E. depends upon frequency

So, C statement is correct.

Section B

21. The correct answer is (6)

$$U_i = \frac{1}{2}CV^2$$

$$U_i = \frac{1}{2}\times(600\times10^{-12})\times(200)^2$$

$U_i = 12\ \mu J$

Charge on capacitor ⇒ Q = CV

$Q = 600 \times 10^{-12} \times 200 = 12 \times 10^{-8}$ C

When connected to another uncharged capacitor

$$Q' = \frac{Q}{2} = \frac{12\times10^{-8}}{2} = 6\times10^{-8}C$$

$$U_f = 2\times\frac{1}{2}\times\frac{Q'^2}{C} = \frac{Q'^2}{C}$$

$$\Rightarrow U_f = \frac{(6\times10^{-8})}{600\times10^{-12}} = 6\ \mu J$$

$\Delta E = U_f - V_i = (12 - 6)\ \mu J = 6\ \mu J$

22. The correct answer is (4).

$R = 100\ \Omega$

$L = 1H$

$C = 6.25 \times 10^{-6}$ F

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 6.25 \times 10^{-6}}}$$

$\Rightarrow \omega = 400/s$

Now

$$Q_{factor} = \frac{\omega L}{R} = \frac{400 \times 1}{100} = 4$$

23. The correct answer is (125).

$n = 8 \times 10^{28}\ m^{-3}$

$A = 2 \times 10^{-6}\ m^2$

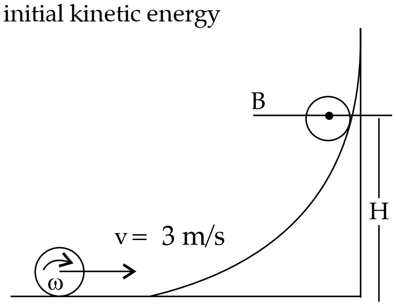
$I = 3.2A$

$$V_d = \frac{i}{enA} = \frac{3.2}{1.6 \times 10^{-19} \times 8 \times 10^{28} \times 2 \times 10^{-6}}$$

$\Rightarrow V_d = 125 \times 10^{-6}$ m/s

24. The correct answer is (75).

Total initial kinetic energy



$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$v = R\omega$ (for pure rolling)

$$K.EC = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{3}mR^2 \times \frac{v^2}{R^2} = \frac{5}{6}mv^2$$

Energy remains conserve during whole journey.

$K.E._i + P.E._i = K.E._f + P.E._f$

$$\Rightarrow \frac{5}{2}mv^2 = mgH \qquad (\because K.E._f = 0)$$

$$\Rightarrow H = \frac{5}{6} \times \frac{v^2}{g}$$

$$= \frac{5 \times (3)^2}{6 \times 10}$$

$$= \frac{15}{20}\ m = 0.75\ m = 75\ cm$$

25. The correct answer is (4).

$$\frac{\sigma}{E} = Y$$

$$\Rightarrow \sigma = YE = Y\frac{\Delta l}{l}$$

$$\Rightarrow \sigma = Y = \frac{l \alpha \Delta T}{l}$$

$\Rightarrow \sigma = Y\ \alpha\ \Delta T$

Now,

$$\sigma = \frac{F}{A}$$

$\Rightarrow F = \sigma A = YA\ \alpha\ \Delta T$

$\Rightarrow F = 2 \times 10^{11} \times 10^{-4} \times 10^{-5} \times 200$

$= 4 \times 10^4$

$x = 4$

26. The correct answer is (8).

Magnetic field due to current carrying coil on axis at distance d.

$$B_a = \frac{\mu_0 I r^2}{2(r^2 + d^2)^{\frac{3}{2}}}$$

Given that $d = r$

$$\text{Now, } B_a = \frac{\mu_0 I r^2}{2(2r^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{4\sqrt{2}r}$$

Magnetic field at centre of current carrying coil

$$B_c = \frac{\mu_0 I}{2r}$$

$$\frac{B_c}{B_a} = \frac{\mu_0 I}{2r} \times \frac{4\sqrt{2}r}{\mu_0 I} = \frac{2\sqrt{2}}{1} = \frac{\sqrt{8}}{1}$$

$x = 8$

27. The correct answer is (27).

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \text{ for balmer series } n = 2$$

Now, for H_α, $m = 3$ and for H_β, $m = 4$

$$\text{Here, } \frac{1}{\lambda_{H_\alpha}} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

$$\& \ \frac{1}{\lambda_{H_\beta}} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}$$

So,

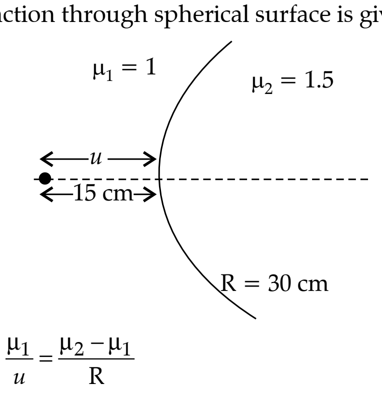
$$\frac{\frac{1}{\lambda_{H_\alpha}}}{\frac{1}{\lambda_{H_\beta}}} = \frac{\frac{5R}{36}}{\frac{3R}{16}}$$

$$\Rightarrow \frac{\lambda_{H_\alpha}}{\lambda_{H_\beta}} = \frac{27}{20}$$

$\therefore x = 27$

28. The correct answer is (30).

Refraction through spherical surface is given by:



$$\frac{\mu_2}{V} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1.5}{V} - \frac{1}{-15} = \frac{1.5-1}{30}$$

$$\Rightarrow \frac{1.5}{V} + \frac{1}{15} = \frac{1}{60}$$

$$\Rightarrow \frac{1.5}{V} + \frac{1}{60} - \frac{1}{15} = \frac{1-4}{60}$$

$$\Rightarrow \frac{1.5}{V} = -\frac{3}{60} = -\frac{1}{20}$$

$\Rightarrow V = -20 \times 1.5 = -30$ cm

29. The correct answer is (60).

$$l = \frac{\lambda}{2}$$

$\Rightarrow \lambda = 2l$

Now, $\upsilon = f\lambda$

$$\Rightarrow f_1 = \frac{\upsilon}{\lambda} = \frac{\upsilon}{2l_1}$$

$$\Rightarrow f_1 \propto \frac{1}{l_1}$$

$$\Rightarrow f_2 \propto \frac{1}{l_2}$$

Here,

$$\frac{f_1}{f_2} = \frac{l_2}{l_1}$$

$$\Rightarrow \frac{120}{180} = \frac{l_2}{90}$$

$\Rightarrow l_2 = 60$ cm

30. The correct answer is (30).

Given, $P_i = 10$ kg ms^{-1}

Now from Newton's second law

$$F = \frac{dP}{dt}$$

$\Rightarrow F.dt = dP$

$\Rightarrow 2 \times 5 = P_f - P_i$

$\Rightarrow 10 = P_f - 10$

$\Rightarrow P_f = 20$ kg ms^{-1}

Now, $P_i = mV_i$

$10 = 5 \times V_i$

$\Rightarrow V_i = 2$ ms^{-1}

and $P_f = 20 = mV_f$

$\Rightarrow 20 = 5 \times V_f$

$\Rightarrow V_f = 4$ ms^{-1}

$$\Delta K.E. = K.E._f - K.E._i = \frac{1}{2} \times 5[(2)^2 - (4)^2]$$

$$= \frac{1}{2} \times 5 \times 12 = 30J$$

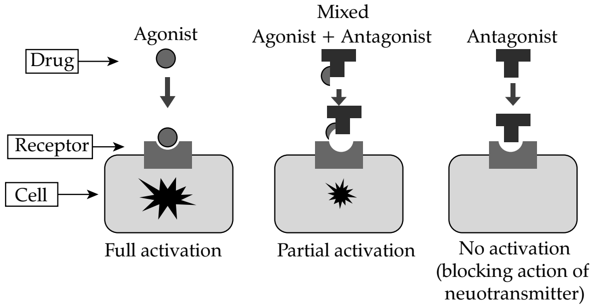
Chemistry

SECTION - A

31. Option (2) is correct.

Explanation:

Drugs that bind to the receptor site and inhibit its natural function are called antagonists. These are useful when blocking of message is required.



32. Option (4) is correct.

Explanation:

In the presence of a catalyst, the activation energy is reduced between reactant and product. The intermediate formed is faster in presence of catalyst followed by an alternate and short pathway.

Always remember that the activation energy in a reaction is always positive. We know that the energy changes which result from a chemical reaction can either be positive, negative, or even zero, but it is also true that in all of the cases, an energy barrier has to be overcome before any reaction takes place. It should be noted that higher the activation energy, slower will be the chemical reaction.

33. Option (1) is correct.

Explanation:

Sodium is the most abundant alkali metal in the earth's crust. Sodium and potassium are the seventh and eighth most abundant elements by weight in the earth's crust. Owing to its high reactivity, sodium is found in nature only as a compound and never as the free element.

Potassium atom is in 4th period and sodium is in 3rd period, the number of outermost orbits is greater in potassium than sodium. So, the nuclear attractions decreases in potassium which ultimately leads to the larger size of it.

34. Option (3) is correct.

Explanation:

In the stratosphere, ozone is a product of the action of UV radiations on dioxygen as:

(i) $O_2(g) \xrightarrow{UV} O(g) + O(g)$

(ii) $O_2(g) + O(g) \xleftrightarrow{UV} O_3(g)$

Reaction (ii) indicates the dynamic equilibrium existing between the production and de-composition of ozone molecules. Any factor that disturbs the equilibrium may cause depletion of ozone layer by its decomposition. One such factor is the release of chlorofluorocarbon compounds (CFCs). These are non-reactive, non-flammable molecules that are used in refrigerators, air conditioners, plastics, and electronic industries. Once released CFCs mix with atmospheric gases and reach the stratosphere, where they are decomposed by UV radiations.

(iii) $CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + CF_2\dot{C}l(g)$

The chlorine free radical produced in reaction (iii) reacts with ozone as:

(iv) $\dot{C}l(g) + O_3(g) \rightarrow \dot{C}lO(g) + O_2(g) + \dot{C}l(g)$

The radicals further react with atomic oxygen to produce more chlorine radicals as:

(v) $\dot{C}lO(g) + \dot{O}(g) \rightarrow \dot{C}l(g) + O_2(g)$

(vi) The regeneration of causes a continuous breakdown of ozone present in the stratosphere, damaging the ozone layer.

35. Option (1) is correct.

Explanation:

Functional group as suffix or prefix:When an organic compound contains two or more different functional groups

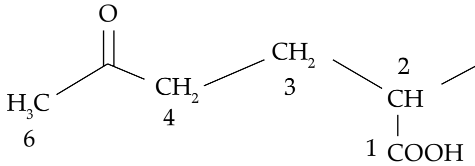
1. Highest priority:
   1.1 Use suffix
2. Other functional groups (act as a substituent):
   2.1 use as prefix

The choice of the principal functional group is made on the basis of the order of preferences.

Note:

$-R$, $-C_6H_5$, $-X$ (F, Cl, Br, I), $-NO$, $-NO_2$, $-OR$ (alkoxy) do not have any priority order and considered as alkyl substituents while numbering.

The IUPAC name of the compound is 2-Methyl-5-oxohexanoic acid



36. Option (2) is correct.

Explanation:

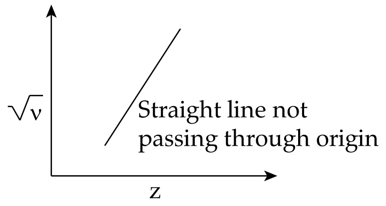
Moseley's Law describes the relationship between atomic number and frequency of a spectral line of characteristic X-rays.

The atomic number of an element can be related to the square root of the frequency of a spectral line of characteristic X-rays using Moseley's law. Moseley's law is an empirical law concerning the characteristic X-rays emitted by atoms. It states that the square root of the frequency of emitted X-rays is approximately proportional to the atomic number.

Mathematically, the law can be represented using the equation $\sqrt{\nu} = z$ where ν is the frequency of the emitted X-ray.The final equation of Moseley's law can be written as

$\nu = A(z - b)^2$

where A and b are constants depending upon the X-ray emission. Moseley measured and plotted the X-ray frequencies of about 40 elements of the periodic table following his law and observed the graph to be a straight line. The plot of the graph with atomic numbers on the x-axis and the square root of frequencies on the y-axis furnished a straight line not passing through the origin.



37. Option (1) is correct.

Explanation:

For option A:

The six cyanide ligands are arranged around the chromium ion in an octahedral geometry

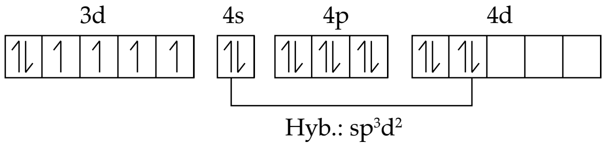
Cr^{+3} : $3d^3$

$CN^- \rightarrow$ SFL $\Rightarrow$ No. of unpaired electrons = 3

For option B:

$Fe[(H_2O)_6]^{2+}$ is a high spin complex and contains 4 unpaired electrons. And H_2O being a weak field ligand do not pair up electrons and due to the presence of unpaired electrons, it is paramagnetic in nature. Its magnetic moment is 5.4 B.M. It has pale green color.

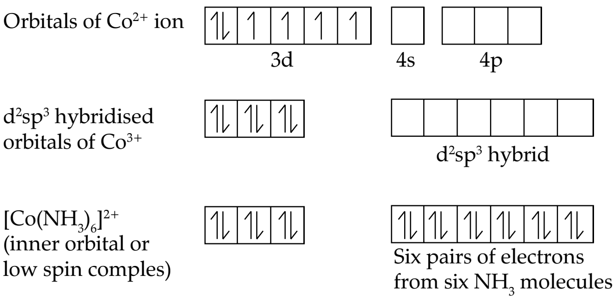
Fe^{2+} in $[Fe(H_2O)_6]^{2+}$



For option C:

In $[Co(NH_3)_6]^{3+}$ the oxidation state of cobalt is +3.Ammonia is a strong field ligand so it pair up 4 unpaired electron and free up 2 3−d orbitals. These 3−d orbitals are involved in hybridisation with one 4S and three 4P orbitals forming an inner orbital complex, so hybridisation of $[Co(NH_3)_6]^{3+}$ is d^2sp^3

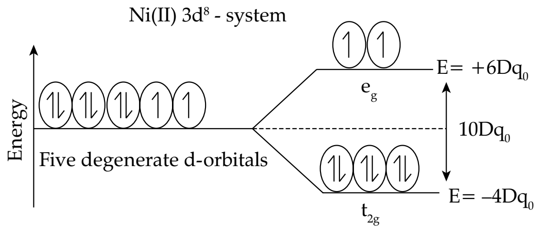
Since it has no unpaired electrons, $[Co(NH_3)_6]^{3+}$ is diamagnetic.



For option D

In case of $[Ni\ (NH_3)_6]^{2+}$ ion, ligand NH_3 act as a weak field ligand as crystal field stabilization energy is less than pairing energy. That is, 10Dq < P.

Therefore, under the influence of octahedral crystal field, the electronic configuration is $t_{2g}^{\ 6}\ e_g^{\ 2}$.

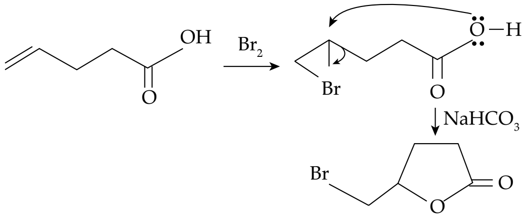


From the above electronic configuration, it has been found that the complex has two unpaired electrons. Hence the complex $[Ni\ (NH_3)_6]^{2+}$ is paramagnetic.

38. Option (3) is correct.

Explanation:

Addition of Bromine (Br_2) To Alkenes is stereoselective, giving "Anti" Addition Stereo-chemistry. The stereospecificity of bromine addition can be explained by considering the anti-addition or trans-addition, alkene to form a flat carbocation. Then the bromide ion would attack the bottom face of the alkene. Thus, anti-addition to cis-2 butene leads to the formation of an enantiomer.



39. Option (1) is correct.

Explanation:

The raw materials required for the manufacture of cement are lime stone, stone and clay. Lime stone or calcium carbonate ($CaCO_3$) provides calcium oxide. (CaO) Clay is hydrated aluminium silicate, ($Al_2O_3.2SiO_2.2H_2O$) and it provides alumina as well as silica. A small amount of gypsum, $CaSO_4.2H_2O$ is also required. It is added in calculated quantity in order to adjust the rate of setting of cement.

Manufacture : Cement is made by strongly heating a mixture of lime stone and clay in a rotatory kiln. Lime stone and clay are finely powdered and a little water is added to get a thick paste called slurry. The slurry is fed into a rotatory kiln from the top through the hopper.

The hot gases produce a temperature of about 1770–1870 K in the kiln. At this high temperature the lime stone and clay present in slimy combine to form cement in the form of small pieces called clinker. This clinker is mixed with 2–3% by weight of gypsum ($CaSO_4.2H_2O$) to regulate the setting time and is then ground to an exceedingly fine powder.

$$\text{Limestone} + \text{Clay} \xrightarrow[\text{Clinker}]{1770-1870\text{K}} \text{Cement} + CO_2\uparrow$$

When mixed with water the cement reacts to form gelatinous mass which sets to a hard mass when three dimensional cross lines are formed between silica oxygen silica and silica oxygen aluminium as ....Si - O - Si....... and Si - O - Al........

Composition of cement:

CaO = 50–60%

SiO_2 = 20–25%

Al_2O_3= 5–10%

MgO = 2–3%

Fe_2O_3= 1–2%

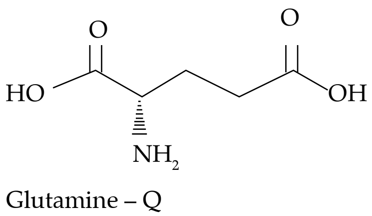
SO_3 = 1–2%

For a good quality cement the ratio of silica (SiO_2) and alumina (Al_2O_3) should be between 2.5 to 4.0. Similarly the ratio of lime (CaO) to the total oxide mixtures consisting of SiO_2, Al_2O_3 and Fe_2O_3 should be roughly 2:1:1, If lime is in excess, the cement cracks during setting. On the other hand, if lime is less than the required, the cement is weak in strength. Therefore, a proper composition of cement must be maintained to get cement of good quality.

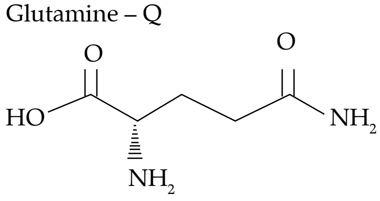
40. Option (1) is correct.

Explanation:

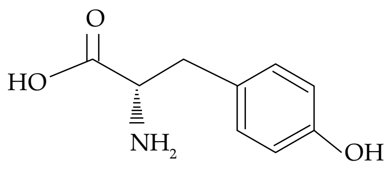
A. Glutamic acid – E



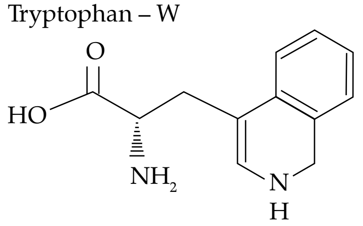
B. Glutamine – Q



C. Tyrosine – Y



D. Tryptophan – W



41. Option (3) is correct.

Explanation:

The number of single digits that are important in the coefficient of an expression in scientific notation is termed as significant number.

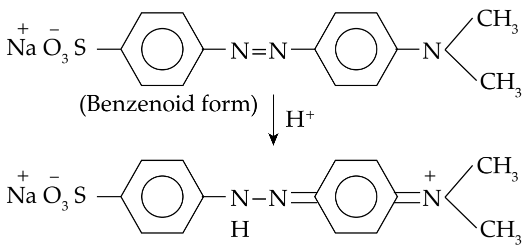
0.00253, 15.0, 163

All have three significant figures.

42. Option (3) is correct.

Explanation:

Methyl orange is a weak acid. So, statement- 1 is correct. In acidic medium, it exists in quinonoid form which is red in colour and in alkaline medium it exists in benzenoid form which is yellow in colour. Since red is more deeply coloured than yellow, so statement- 2 is wrong.



Red color (quinonoid form)

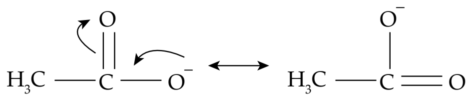
43. Option (2) is correct.

Explanation:

Acidic strength depends on the stability of the conjugate base after the release of H^+ ion.

Comparing the stabilities of the conjugate bases

1. Among the conjugate bases, CH_3COO^- is more stable than $CH_3CH_2O^-$ because the negative charge developed in CH_3COO^- is stabilised by two equivalent resonating structures.



Resonating structures
of acetate ion

Electron withdrawing substituent increases the acidity by increasing the ionic character of O–H by inductive effect. Electronegativity decreases in the order F > Cl > Br and hence –I effect also decreases in the same order

Acidity α stability of conjugate base

Stability order – $F_3C-COO^- > F-CH_2-COO^- > Cl-CH_2-COO^- > Br-CH_2-COO^- > CH_3COO^-$

44. Option (2) is correct.

Explanation:

The electrolysis of alumina by Hall and Heroult's process is carried by using a fused mixture of alumina and cryolite along with minor quantities of aluminium fluoride and fluorspar. The addition of cryolite and fluorspar increases the electrical conductivity of alumina and lowers the fusion temperature. Also at anode, alumina reacts with fluorine to give oxygen. The liberated oxygen reacts with carbon to form CO and CO_2. These gases are liberated at anode. In case of Hall's process, reduction of Al_2O_3 to Al can be done using graphite. In electrolytic reduction of alumina, oxygen gas is evolved at the anode which oxidises the carbon (graphite) anode to CO and further, to CO_2 .As a result ,the graphite anode is consumed and needs to be replaced gradually.

45. Option (1) is correct.

Explanation:

Van der Waals' constant 'a' is a measure of intermolecular forces of attractions. Stronger is the force of attraction, greater will be 'a' and easily a gas can be liquified. Vander waal force depends on molecular size and molecular mass. The value of Vander waal constant 'a' is as follows

Ar – 1.355

CH_4 – 2.283

H_2O – 5.536

C_6H_6 – 18.24

46. Option (1) is correct.

Explanation:

The indicators used in redox reactions are sensitive to change in oxidation potential. The ideal oxidation-reduction indicators have an oxidation potential intermediate between the values for the solution being titrated and the titrant and these show sharp readily detectable colour change.

Acid-Base Titration: An acid-base titration involves a neutralization reaction between the analyte (the solution with the unknown concentration) and the acidic or basic titrant.

47. Option (2) is correct.

Explanation:

H_2O_2 decomposes slowly on exposure to light

$2H_2O_2(l) \rightarrow 2H_2O(l) + O_2(g)$

In the presence of metal surfaces or traces of alkali (present in glass containers), the above reactions is catalysed. It is, therefore stored in wax-lined glass or plastic vessels in dark. Urea be added as a stabiliser. It is kept away from dust because dust can induce explosive decomposition of the compound.

48. Option (1) is correct.

Explanation:

Nucleophilic substitution reaction:-

It is basically a chemical reaction in which, an electron rich nucleophile will attack the positively charged electrophile and will replace with a leaving group.

Rate of Subtitution $\propto$ Presence of EWG at O/P position

R $\propto$ -M group present at o/p position

$$\text{Rate} \propto \text{EWG} \propto \frac{1}{\text{EDG}}$$

$-NO_2$ is an EWG while

–OMe is an EDG

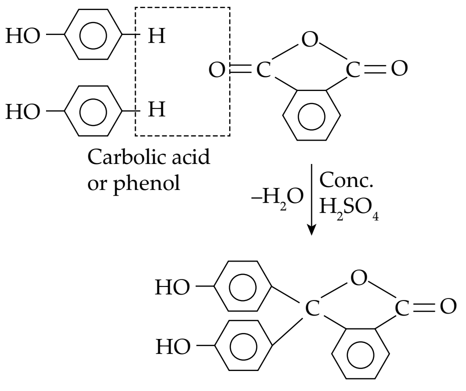
49. Option (3) is correct.

Explanation:

The reaction between phenol and phthalic anhydride in the presence of sulphuric acid is a well- known reaction for the production of phenolphthalein.

Sulphuric acid in this reaction acts as a dehydrating agent.

Phenolphthalein is renowned as the indicator dye. Phenolphthalein can be synthesized by the condensation of phthalic anhydride with two equivalents of phenol under acidic condition, hence is named as Phenolphthalein by Adolf von Baeyer.

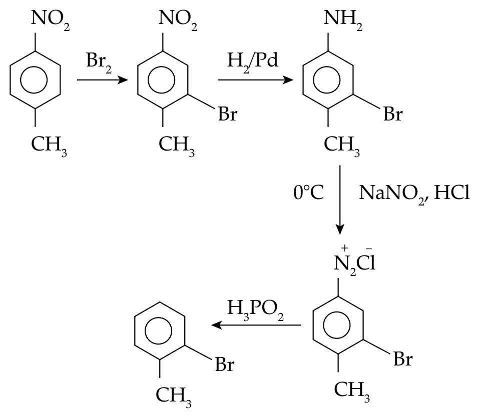


Phenolphthalein

50. Option (4) is correct.

Explanation:

Conversion of 4-nitrotoluene to 2-bromotoluene can be achieved by the following steps, which are shown below:



SECTION - B

51. Correct answer is [4].

Explanation:

Magnetic moment (μ) is given as $\mu = \sqrt{n}\,(n+2)$

$[Mn(NCS)_6]^{x-}$ Number of unpaired electron = 5

So, Mn must be in +2 oxidation state (Mn^{+2})

$\Rightarrow 2 + (-6) = -x$

$\Rightarrow -4 = -x$

$\Rightarrow x = 4$

52. Correct answer is [1411].

Explanation:

Amount of heat produced in the bomb calorimeter $= -1406\ \text{kJmol}^{-1}$

Enthalpy of a combustion reaction is:

$\Delta H = \Delta U + \Delta n_g RT$

Where, ΔU = internal energy

Δn_g = moles of gas (products -reactants), R = Gas constant=, T = Temperature in K

As per the equation,

$\Delta n_g = 2-4 = -2$

$\Delta G = \Delta H - T\Delta S$ at equilibrium:-

$\Delta G = 0$

$T\Delta S = \Delta H = \Delta U + \Delta n_g RT$

$= -1406 + (-2)\ 8.3 \times 300 \times 10^{-3} = 1410.98 \approx 1411$

53. Correct answer is [233].

Explanation:

$BaSO_4$ ionizes completely in solution as:

$$BaSO_{4(s)} \rightleftharpoons \underset{s}{Ba^{2+}_{(aq)}} + \underset{s}{SO^{2-}_{4(aq)}}$$

In presence of 0.1 M K_2SO_4(aq)

Let S mole/litre $BaSO_4$ is dissolved

$K_{SP} = [Ba^{2+}]\,[SO_4^{2-}]$

$1 \times 10^{-10} = S(S + 0.1) = S \times 0.1$

as S<<0.1 so 0.1 + S = 0.1

$S = 10^{-9}$ M

S(in g/l) $= 233 \times 10^{-9}$

54. Correct answer is [3].

Explanation:

Number of radial nodes $= n - l - 1$

where, n is the Principal quantum number and l is the Azimuthal quantum number.

For 6S $\rightarrow 6 - 0 - 1 = 5$,

7P $\rightarrow 7 - 1 - 1 = 5$

8d $\rightarrow 8 - 2 - 1 = 5$

55. Correct answer is [1].

Explanation:

The cell potential (E_{cell}) of a reaction is related as

$\Delta G = -nF\,E_{cell}$

where ΔG represents maximum useful electrical work.

n = no. of moles of electrons exchanged during the reaction.

For reversible cell reaction

$d(\Delta G) = (\Delta V)dp - (\Delta S)dT$

At const. P, $d(\Delta G) = -(\Delta S)dT$

At const. P, $\Delta G = \Delta H - T\Delta S$ (i)

$\therefore\ \Delta G = \Delta H + T(d(\Delta G)\Delta T)P$ (ii)

$\left(\frac{dE_{cell}}{dT}\right)P$ is known as temperature coefficient of the emf of the cell.

From equations (i) and (ii)

$$-\Delta S = \frac{d\Delta G}{dT} = \frac{d - nFE}{dT}$$

$$\Delta S = nF = \frac{dF}{dT}$$

$$\text{or } \frac{dE}{dT} = \frac{\Delta S}{dF}$$

56. Correct answer is [556].

Explanation:

The precipitation of colloidal solution through induced aggregation by the addition of suitable electrolyte is called coagulation or flocculation.

The minimum concentration of electrolyte in millimoles required to cause coagulation of one litre of colloidal solution is called coagulation value.

It needs to be noted that the coagulation of a colloidal solution by an electrolyte does not take place until the added electrolyte has certain minimum concentration in the solution.

Coagulation power is inversely proportional to coagulation value.

$$\text{Coagulating power} \propto \frac{1}{\text{Coagulation Value}}$$

Coagulation power of $AlCl_3$ = Coagulation power of NaCl

$$\frac{\text{Coagulation power of } AlCl_3}{\text{Coagulation power of NaCl}}$$

$$= \frac{\text{Coagulation value of NaCl}}{\text{Coagulation value of } AlCl_3} = \frac{50.04}{0.09} = 556$$

57. Correct answer is [8].

Explanation:

The boiling point of a substance is the temperature at which the vapor pressure of the liquid equals the pressure surrounding the liquid and the liquid changes into a vapor. The boiling point of a liquid varies depending upon the surrounding environmental pressure.

Molal elevation constant:

It is defined as the elevation in boiling point when the molality of the solution is unity i.e 1 mole of the solute is dissolved in 1 kg of the solvent. The units are degree/molality i.e K/m

Molal elevation constant from enthalpy of vapourisation

$$K_b = \frac{RT_b^2}{1000 l_v}$$

$$l_v = \frac{\Delta_{vap}H}{M}$$

T_b = boiling point of liquid

l_v = latent heat of vaporization per gram of the solvent

$\Delta_{vap}H$ = latent heat of vaporization per mole of the solvent

M = molecular mass of the solvent

$$\frac{(K_b)_x}{(K_b)_y} = \frac{(T_b^2 M)_x}{(T_b^2 M)_y} \times \frac{(\Delta H)_y}{(\Delta H)_x} = \left(\frac{2}{1}\right)^2 \times \left(\frac{2}{1}\right) = \frac{8}{1}$$

58. Correct answer is [4].

Explanation:

The lone pairs are the valence electrons which do not take part in the bonding. Determine the valence electrons involved in the molecules and then subtract the total number of bonding electrons from the valence electrons to calculate the number of lone pairs.

Lone pairs = $\frac{1}{2}$(Valence e^- in the molecule − Bonding e^- in the molecule)

The lone pairs are a pair of valence electrons that are not shared by another atom in the covalent bond, it is also termed as the unshared pair or the non-bonding pair. The lone pairs are in the outermost shell of atoms. These pairs of electrons are not used in chemical bonding.

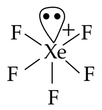
The lone pairs can find out by knowing the geometry of the molecule.

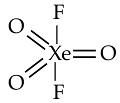
Step 1) Count all the number of valence electrons in the molecule.

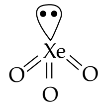
Step 2) Count the total number of atoms that are bonded to the central atom and multiply it by 8 so that all the atoms complete the octet.

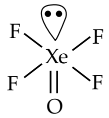
Step 3) Find the number of lone pairs by subtracting the valence electrons and bonded atoms from the total valence electrons.

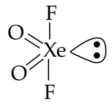
Step 4) now we divide the lone pair electrons found in step 3) by 2 to get the number of lone pairs on the central atom.

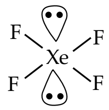
XeF_5^+ 

XeO_3F_2 

XeO_3 

$XeOF_4$ 

XeO_2F_2 

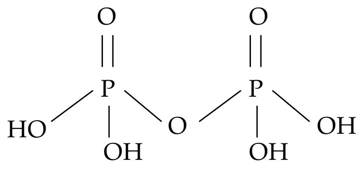
XeF_4 

59. Correct answer is [6].

Explanation:

We know that phosphorus on oxidation and condensation forms various oxoacids and the condensation of two molecules of phosphoric acid results in the pyrophosphoric acid formation that has a formula of $(H_4P_2O_7)$

We also know that σ and π bonds are formed by overlapping atomic orbitals. End to end overlapping of atomic orbitals results in sigma bonds formation and pi bond is formed by sideways overlapping of atomic orbitals.



Pyrophosphoric acid

Number of σ bond = total number of atom – 1
= 13 – 1 = 12

Number of π bond = 2

$$\frac{\sigma}{\pi} = \frac{12}{2} = 6$$

60. Correct answer is [10].

Explanation:

The charge on a complex is the sum of the oxidation state of the metal center and the charges on the ligands.

CO is a neutral ligand. Hence, the charge on the ligand is 0. The total charge on the complex is also 0.

The oxidation number of Fe in the complex, $Fe(CO)_5$ is as follows;

(Charge on Fe) + 5(Charge on CO) = 0

(Charge on Fe) + 5(0) = 0

(Charge on Fe) = 0

Thus, the oxidation number of Fe in $Fe(CO)_5$ is 0.

For VO^{2+}

Here the oxidation state of vanadium is +4.

The steps include:

$x - 2 = 2$

$x = +4$

Therefore, the oxidation state of vanadium in VO^{2+} is 4.

For WO_3

Tungsten trioxide (WO_3) consists of one tungsten atom and three oxygen atoms. Tungsten is a d-block metal from group 6 and has an oxidation state +6 in the compound.

So, Sum of oxidation state = 0 + 4 + 6 = 10

Mathematics

Section A

1. Option (C) is correct.

Here, $z = \dfrac{1+2i\sin\theta}{1-i\sin\theta}\times\dfrac{1+i\sin\theta}{1+i\sin\theta}$

$$\frac{1+i\sin\theta+2i\sin\theta-2\sin^2\theta}{1-i^2\sin^2\theta}$$

$$=\frac{(1-2\sin^2\theta)+i(3\sin\theta)}{1+\sin^2\theta}$$

z is purely imaginary, so Re $z = 0$

$$\Rightarrow \frac{1-2\sin^2\theta}{1+\sin^2\theta}=0$$

$$\Rightarrow 2\sin^2\theta = 1 \Rightarrow \sin^2\theta = \frac{1}{2}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\therefore A = \left[\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right] \qquad \because \theta\in(0,2\pi)$$

$$\therefore \text{Sum} = \frac{\pi+3\pi+5\pi+7\pi}{4} = \frac{16\pi}{4} = 4\pi$$

HINT:

For a complex number, $z = a + ib$, if z is purely imaginary, then Re $z = 0 \Rightarrow a = 0$

2. Option (C) is correct.

Equation of line : $\dfrac{x-1}{1}=\dfrac{y-2}{-3}=\dfrac{z+5}{7}$

Let B ≡ (2, 4, –3)

A (1, 2, –5) P (1, –3, 7)

So, $\overline{AB} = (2-1)\hat{i}+(4-2)\hat{j}+(-3+5)\hat{k}$

$=\hat{i}+2\hat{j}+2\hat{k}$

$$\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 7\\ 1 & 2 & 2\end{vmatrix} = (-6-14)\hat{i}-(2-7)\hat{j}+(2+3)\hat{k}$$

$=-20\hat{i}+5\hat{j}+5\hat{k}$

$=-5(4\hat{i}-\hat{j}-\hat{k})$

$\therefore$ Eqn. of plane is :

$4(x-1) + (-1)(y-2) - 1(z+5) = 0$

$\Rightarrow 4x - 4 - y + 2 - z - 5 = 0$

$\Rightarrow 4x - y - z - 7 = 0$

Image of point (–1, 3, 4) is (α, β, γ)

So, $\dfrac{\alpha+1}{4}=\dfrac{\beta-3}{-1}=\dfrac{\gamma-4}{-1}=\dfrac{-2(-4-3-4-7)}{16+1+1}=2$

$\Rightarrow \alpha = 7, \beta = 1, \gamma = 2$

So, $\alpha + \beta + \gamma = 10$

HINT:

Equation of plane passing through the line and a point can be find by using the normal vector.

3. Option (A) is correct.

$$A = \begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$$

$\Rightarrow |A - xI| = 0$

$$\Rightarrow \begin{vmatrix}1-x & 5\\ \lambda & 10-x\end{vmatrix}=0$$

$\Rightarrow (1-x)(10-x) - 5\lambda = 0$

$\Rightarrow 10 - 11x + x^2 - 5\lambda = 0$

Also, $\Rightarrow A^{-1} = \alpha A + \beta I$

$\Rightarrow \alpha A^2 + \beta A - I = 0$

and $A^2 - 11A + (10 - 5\lambda) I = 0$

On solving, we get

On solving, we get

$\alpha = \frac{1}{5}, \beta = -\frac{11}{5}$

So, $5\lambda - 10 = 5 \Rightarrow \lambda = 3$

$\therefore 4\alpha^2 + \beta^2 + \lambda^2$

$= 4\left(\frac{1}{25}\right) + \left(\frac{121}{25}\right) + 9$

$= \frac{125}{25} + 9 = 14$

HINT:

The characteristic equation is :

$|A - xI| = 0$

64. Option (D) is correct.

Here $\overline{AC} = (-2-2)\hat{i} + (-3-1)\hat{j} + (5-1)\hat{k}$

$= -4\hat{i} - 4\hat{j} + 4\hat{k}$

$\overline{BD} = (1-1)\hat{i} + (-6-2)\hat{j} + (-7-5)\hat{k}$

$= -8\hat{j} - 12\hat{k}$

So, area of quadrilateral $= \frac{1}{2} | |\overline{AC} \times \overline{BD}|$

$$= \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -4 & 4 \\ 0 & -8 & -12 \end{matrix}\right\|$$

$= \frac{1}{2} |(48+32)\hat{i} - (48-0)\hat{j} + (32-0)\hat{k}|$

$= \frac{1}{2} |80\hat{i} - 48\hat{j} + 32\hat{k}|$

$= \frac{1}{2} 16 |15\hat{i} - 3\hat{j} + 2\hat{k}|$

$= 8\sqrt{25+9+4} = 8\sqrt{38}$ sq units.

HINT:

Area of quadrilateral = Half of product of diagonal vectors.

65. Option (A) is correct.

The given expression is divisible by 6 and 17.

Also, $25^{190} - 8^{190}$ is not divisible by 7

but $19^{190} - 2^{190}$ is divisible by 7,

So, $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by 34 but not by 14.

66. Option (B) is correct.

Centre (3, –2)

Equation of circumcircle is

$x(x-3) + y(y+2) = 0$

$\Rightarrow x^2 - 3x + y^2 + 2y = 0$

Since $\left(\alpha, \frac{1}{2}\right)$ is on the circle

So $\alpha^2 - 3\alpha + \frac{1}{4} + 1 = 0$

$\Rightarrow 4\alpha^2 - 12\alpha + 5 = 0$

$\Rightarrow \alpha = \frac{12 \pm \sqrt{144-80}}{8}$

$= \frac{12 \pm \sqrt{64}}{8} = \frac{12 \pm 8}{8}$

$\alpha = \frac{20}{8}, \frac{4}{8} = \frac{5}{2}, \frac{1}{2}$

HINT:

Equation of circumcircle whose diametric points are (a, b) & (c, d) is $(x-a)(x-c) + (y-b)(y-d) = 0$

67. Option (C) is correct.

Let $S_n = 5 + 8 + 14 + 23 + ... + a_n$

and $S_n = 0 + 5 + 8 + 14 + ... + a_n$

On subtracting, we get

$0 = 5 + 3 + 6 .... - a_n$

$\Rightarrow a_n = 5 + 3 + 6 + 9 + ... (n-1)$ terms

$= 5 + \left[\frac{(n-1)}{2}(6 + (n-2)3)\right]$

$= 5 + \left[\frac{(n-1)}{2}(6 + 3n - 6)\right]$

$= 5 + \frac{(n-1)(3n)}{2}$

$= \frac{10 + 3n^2 - 3n}{2}$

So, $a_{40} = \frac{3(40)^2 - 3(40) + 10}{2}$

$= \frac{4800 - 120 + 10}{2} = 2345$

Now, $S_n = \sum_{k=1}^{n} a_k$

$\Rightarrow S_{30} = \frac{3\sum_{n=1}^{30} n^2 - 3\sum_{n=1}^{30} n + 10\sum_{n=1}^{30} 1}{2}$

$= \frac{3 \times (30)(30+1)(60+1)}{12} - \frac{3 \times 30 \times 31}{4}$

$\frac{+10 \times 30}{2}$

$= \frac{28365 - 1395 + 300}{2} = \frac{27270}{2}$

$= 13635$

$\therefore S_{30} - a_{40} = 13635 - 2345 = 11290$

HINT:

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$\sum_{k=1}^{n} k^2 = \frac{k(k+1)(2k+1)}{6}$

68. Option (B) is correct.

Since, α, β are roots of $ax^2 + bx + 1 = 0$

Replace $x \to \dfrac{1}{x}$

$$\frac{a}{x^2}+\frac{b}{x}+1=0 \Rightarrow x^2+bx+a=0$$

So, $\dfrac{1}{\alpha},\dfrac{1}{\beta}$ are the roots

Now, $\lim\limits_{x\to\frac{1}{\alpha}}\left[\dfrac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right]^{\frac{1}{2}}$

$$= \lim_{x\to\frac{1}{\alpha}}\left[\frac{2\sin^2\left(\frac{x^2+bx+a}{2}\right)}{2(1-\alpha x)^2}\right]^{\frac{1}{2}}$$

$$= \lim_{x\to\frac{1}{\alpha}}\left[\frac{2\sin^2\frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}}{4\times 2\alpha^2\frac{\left(x-\frac{1}{\alpha}\right)^2\left(x-\frac{1}{\beta}\right)^2}{4}}\left(x-\frac{1}{\beta}\right)^2\right]^{\frac{1}{2}}$$

$$= \lim_{x\to\frac{1}{\alpha}}\left[\pm\frac{1}{2}\frac{\sin\frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}}{\alpha\frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}}\left(x-\frac{1}{\beta}\right)\right]$$

$$=\frac{1}{2\alpha}\left(\frac{-1}{\alpha}+\frac{1}{\beta}\right)$$

$$\Rightarrow \frac{1}{k}\left[\frac{1}{\beta}-\frac{1}{\alpha}\right]=\frac{1}{2\alpha}\left[\frac{1}{\beta}-\frac{1}{\alpha}\right]$$

$\Rightarrow k = 2\alpha$

HINT:

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

69. Option (D) is correct.

Total number of words $= \dfrac{11!}{2!2!2!}$

Number of words in which C and S are together

$$=\frac{10!}{2!2!2!}\times 2!$$

So, required number of words

$$=\frac{11!}{2!2!2!}-\frac{10!}{2!2!}$$

$$=\frac{11\times 10!}{2!2!2!}-\frac{10!}{2!2!}$$

$$=\frac{10!}{2!2!}\left[\frac{11}{2}-1\right]=\frac{10!}{2!2!}\times\frac{9}{2}$$

$= 5670 \times 6!$

$\Rightarrow k\,(6!) = 5670 \times 6!$

$\Rightarrow k = 5670$

HINT:

Out of n objects if r things are same, then number of ways $= \dfrac{n!}{r!}$

70. Option (A) is correct.

Since, the given system has a non trivial solution,

So, $\Delta = 0$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$$\Rightarrow 1(\tan^2\theta-\sqrt{7})-1(-\tan\theta-\sqrt{7}) + \sqrt{3}(-1-\tan\theta)=0$$

$$\Rightarrow \tan^2\theta-\sqrt{7}+\tan\theta+\sqrt{7}-\sqrt{3}-\sqrt{3}\tan\theta = 0$$

$$\Rightarrow \tan\theta(\tan\theta-\sqrt{3})+1(\tan\theta-\sqrt{3})=0$$

$$\Rightarrow \tan\theta=\sqrt{3} \quad \text{or} \quad \tan\theta=-1$$

$$\therefore \theta=\left\{\frac{\pi}{3},\frac{-2\pi}{3},\frac{-\pi}{4},\frac{3\pi}{4}\right\}$$

So, $\dfrac{120}{\pi}\sum\limits_{\theta\in S}\theta=\dfrac{120}{\pi}\left\{\dfrac{4\pi-8\pi-3\pi+9\pi}{12}\right\}$

$$=\frac{120}{\pi}\left[\frac{2\pi}{12}\right]=20$$

HINT:

For a system of linear equation having non trivial solution, $\Delta = 0$

71. Option (D) is correct.

We have, $\theta = \cos^{-1}\dfrac{1}{3}$

$$\Rightarrow \cos\theta=\frac{1}{3}$$

$$\therefore \sin\theta=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$$

The given plane line and are

$ax + y - z = b$ & $x - 1 = a - y = z + 1$

$$\therefore \sin\theta = \frac{a.1+(1)(-1)+(-1)(1)}{\sqrt{a^2+1^2+1^2}\sqrt{1^2+1^2+1^2}}$$

$$\Rightarrow \frac{a-1-1}{\sqrt{a^2+2}\sqrt{3}}=\frac{2\sqrt{2}}{3}$$

$$\Rightarrow 3(a-2)=2\sqrt{6}\sqrt{a^2+2}$$

$$\Rightarrow 9\,(a^2 + 4 - 4a) = 24\,(a^2 + 2)$$

$\Rightarrow 9a^2 + 36 - 36a = 24a^2 + 48$
$\Rightarrow 15a^2 + 36a + 12 = 0$
$\Rightarrow 5a^2 + 12a + 4 = 0$
$\Rightarrow 5a^2 + 10a + 2a + 4 = 0$
$\Rightarrow 5a(a + 2) + 2(a + 2) = 0$
$\Rightarrow a = \frac{-2}{5}, -2$

So, $a = -2$ $\because a \in Z$
Hence, the eqn. of plane is $-2x + y - z - b = 0$

Now, $d = \left|\frac{-12-6-4-b}{\sqrt{4+1+1}}\right| = 3\sqrt{6}$

$\Rightarrow |-(b + 22)| = 18$
$\Rightarrow b = 18 - 22 = -4$
$\therefore a^4 + b^2 = (-2)^4 + (-4)^2$
$= 16 + 16 = 32$

HINT:
Distance of a point (a_1, b_1, c_1) from the plane $ax + by + cz + d = 0$ is $d = \left|\frac{aa_1 + bb_1 + cc_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

72. Option (B) is correct.
Since, $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are coplanar.

So, $[\vec{u}_1\ \vec{u}_2\ \vec{u}_3] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(b - 1) - 1(1 - c) + a(1 - bc) = 0$
$\Rightarrow b - 1 - 1 + c + a - abc = 0$
$\Rightarrow a + b + c - 2 = abc$ ...(i)
Also, $[\vec{v}_1\ \vec{v}_2\ \vec{v}_3] = 0$

$$\Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$\Rightarrow (a + b)[bc + ba + c^2 + ca - ab] - c[ac + a^2 - ab] + c[ab - b^2 - bc] = 0$
$\Rightarrow abc + ac^2 + a^2c + b^2c + bc^2 + abc - ac^2 - a^2c + abc + abc - b^2c - bc^2 = 0$
$\Rightarrow 4abc = 0 \Rightarrow abc = 0$ ...(ii)
So, $a + b + c - 2 = 0$ [from (i)]
$\Rightarrow a + b + c = 2$
$\Rightarrow 6(a + b + c) = 12$

HINT:
If three non-zero vectors are coplanar, then their scalar triple product is zero.

73. Option (C) is correct.

General term of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is:

$T_{r+1} = {}^{11}C_r (2x^2)^{11-r} \left(\frac{1}{2x}\right)^r$
$= {}^{11}C_r 2^{11-r} x^{22-2r} 2^{-r} x^{-r}$
$= {}^{11}C_r 2^{11-r} x^{22-3r}$
Now, $22 - 2r = 10$ and $22 - 3r = 7$
$\Rightarrow 3r = 12$ $\Rightarrow 3r = 15$
$\Rightarrow r = 4$ $\Rightarrow r = 5$
$\therefore$ Coeff. of $x^{10} = {}^{11}C_4 . 2^{11-8} = {}^{11}C_4 \times 8$
Coeff. of $x^7 = {}^{11}C_5 . 2^{11-10} = {}^{11}C_4 \times 2$
Now, required difference
$= {}^{11}C_4 \times 8 - {}^{11}C_5 \times 2$
$= \frac{11\times10\times9\times8\times7!}{4!\times7!}\times8 - \frac{11\times10\times9\times8\times7\times6!\times2}{5!\,6!}$
$= \frac{11\times10\times9\times8\times8}{24} - \frac{11\times10\times9\times8\times7\times2}{120}$
$= 11 \times 10 \times 8 \times 3 - 11 \times 3 \times 4 \times 7$
$= 11 \times 3 \times 4 [20 - 7]$
$= 11 \times 12 \times 13 = (12 - 1) \times 12 \times (12 + 1)$
$= 12(12^2 - 1) = 12^3 - 12$

HINT:
General term of $(a + b)^n$ is
$T_{r+1} = {}^nC_r a^{n-r} b^r$

74. Option (A) is correct.
Here, A = {1, 2, 3, 4, 5, 6, 7}
Since, $x + y = 7 \Rightarrow y = 7 - x$
So, R = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
$\because (a, b) \in R \Rightarrow (b, a) \in R$
$\therefore$ R is symmetric only.

HINT:
For a relation,
if $(a, a) \in R \Rightarrow$ R is reflexive
if $(a, b) \in R \Rightarrow (b, a) \in R$ So, R is symmetric
if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So, R is transitive

75. Option (A) is correct.
As, we know that sum of all the probabilities = 1

So, $\sum_{x=1}^{\infty} P(X = x) = 1$

$\Rightarrow k[1 + 2.3^{-1} + 3.3^{-2} + ...\infty] = 1$

Let S = $1 + \frac{2}{3} + \frac{3}{3^2} + ... + \infty$

$\Rightarrow \frac{S}{3} = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + ... + \infty$

On subtracting, we get

$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + ... + \infty$

$\Rightarrow \frac{2S}{3} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}}$

$\Rightarrow \frac{2S}{3} = \frac{3}{2}$

$\Rightarrow S = \frac{9}{4}$

So, $k \times \frac{9}{4} = 1 \Rightarrow k = \frac{4}{9}$

Now, P (X ≥ 2) = 1 – P (X < 2)
= 1 – P (X = 0) – P (X = 1)

$= 1 - \frac{4}{9}(1) - \frac{4}{9} \times \frac{2}{3}$

$= 1 - \frac{4}{9} - \frac{8}{27} = \frac{27-12-8}{27} \quad = \frac{7}{27}$

Sum of probabilities = 1

$\sum_{x=0}^{\infty} P(X = x) = 1$

76. Option (D) is correct.

Note: Given integral is wrong it may be

$\int\left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right] \ln\left(\frac{ex}{2}\right) dx$

Let I = $\int\left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right] \ln\left(\frac{ex}{2}\right) dx$

$= \int\left[e^{x \ln x - x \ln 2} + e^{x \ln 2 - x \ln x}\right] dx$

Let $x \ln x - x \ln 2 = t$

$(\ln x + 1 - \ln 2)dx = dt$

$\Rightarrow \ln\left(\frac{ex}{2}\right) dx = dt$

$\therefore I = \int\left[e^t - e^{-t}\right] dt$

$= e^t + e^{-t} + c$

$= \left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + c$

77. Option (D) is correct.

$4 \cos^2\theta - 1 = 4 (1 - \sin^2\theta) - 1$

$= 3 - 4 \sin^2\theta$

$= \frac{3 \sin\theta - 4 \sin^3\theta}{\sin\theta}$

$= \frac{\sin 3\theta}{\sin\theta}$

So, $36 (4 \cos^2 9° - 1) (4 \cos^2 27° - 1) (4 \cos^2 81° - 1) (4 \cos^2 243° - 1)$

$= 36\left[\frac{\sin 27°}{\sin 9°} \times \frac{\sin 81°}{\sin 27°} \times \frac{\sin 243°}{\sin 81°} \times \frac{\sin 729°}{\sin 243°}\right]$

$= 36\left[\frac{\sin 729°}{\sin 9°}\right] = 36 \times 1 = 36$

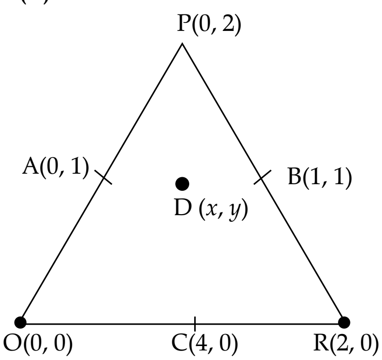
HINT:

Use the formula:

$\sin^2\theta + \cos^2\theta = 1$

$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta$

78. Option (B) is correct.



So, $D \equiv \left(\frac{4}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right)$

$\equiv \left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$

$= \left(\frac{2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}, \frac{2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}\right)$

$\equiv (2-\sqrt{2}, 2-\sqrt{2})$

$\because y^2 = 4ax$

$(2-\sqrt{2})^2 = 4a(2-\sqrt{2})$

$\Rightarrow 4a = 2-\sqrt{2} \Rightarrow a = \frac{2-\sqrt{2}}{4}$

$\Rightarrow \frac{1}{2} - \frac{\sqrt{2}}{4} = \alpha + \beta\sqrt{2}$

$\Rightarrow \alpha = \frac{1}{2}, \beta = \frac{-1}{4}$

So, $\frac{\alpha}{\beta^2} = \frac{\frac{1}{2}}{\frac{1}{16}} = 8$

HINT:

The incentre of a triangle is the intersection point of all the three interior angle bisectors of the triangle.

79. Option (D) is correct.

$(p \wedge (\sim q)) \vee (\sim p)$

$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$\equiv T \wedge (\sim q \vee \sim p)$

$\equiv \sim q \vee \sim p$ negation $p \wedge q$

HINT:

$a \vee \sim a \equiv T$

$\sim a \vee \sim b \equiv b \wedge a$

80. Option (B) is correct.

Since, Mean = $\frac{9}{2}$

$\Rightarrow \Sigma x = \frac{9}{2} \times 12 = 54$

Also, variance = 4

$\Rightarrow \frac{\sum x^2}{12} = \left[\frac{\sum x_i}{12}\right]^2 = 4$

$\Rightarrow \frac{\sum x^2}{12} = 4 + \frac{81}{4} = \frac{97}{4}$

$\Rightarrow \sum x^2 = 291$

$\sum x' = 54 - (9 + 10) + 7 + 14$

$= 54 - 19 + 21 = 56$

and $\sum x^2 = 291 - (81 + 100) + 49 + 196$

$= 291 - 181 + 49 + 196 = 355$

So, $\sigma^2_{\text{new}} = \frac{\sum x^2_{\text{new}}}{12} - \left(\frac{\sum x_{\text{new}}}{12}\right)^2$

$= \frac{355}{12} - \left(\frac{56}{12}\right)^2$

$= \frac{4260 - 3136}{144} = \frac{1124}{144} = \frac{281}{36}$

$= \frac{m}{n}$

$\Rightarrow m = 281$ & $n = 36$

$\Rightarrow m + n = 281 + 36 = 317$

HINT:

Mean $= \frac{\sum x}{n}$

Variance $(\sigma^2) = \frac{\sum x^2}{n} - \left[\frac{\sum x}{n}\right]^2$

Section B

81. The correct answer is (180).

Total number of onto functions

$= \frac{5!}{3!2!} \times 4!$

$= \frac{5 \times 4}{2} \times 24 = 240$

When $f(a) = 1$, number of onto functions

$= 4! + \frac{4!}{2!2!} \times 3!$

$= 24 + 36 = 60$

So, required number of onto functions

$= 240 - 60 = 180$

82. The correct answer is (9).

The givne eqn is : $x^2 - 12x + [x] + 31 = 0$

$\Rightarrow \{x\} - x = x^2 - 12x + 31$

$\Rightarrow \{x\} = x^2 - 11x + 31$

So, $0 \le x^2 - 11x + 31 < 1$

$\Rightarrow x^2 - 11x + 30 \le 0$

$\Rightarrow (x - 5)(x - 6) < 0$

$\Rightarrow x \in (5, 6)$

$\therefore [x] = 5$

$\therefore x^2 - 12x + 5 + 31 = 0$

$\Rightarrow x^2 - 12x + 36 = 0$

$\Rightarrow (x - 6)^2 = 0 \Rightarrow x = 6$

Hence, $x \in \phi$ $\quad (\because x \in (5, 6))$

$\therefore m = 0$

Another equation is $x^2 - 5[x + 2] - 4 = 0$

Case I: $x \ge -2$

$x^2 - 5x - 14 = 0 \Rightarrow x = 7, -2$

Case II: $x < -2$

$x^2 + 5x + 6 = 0 \Rightarrow x = -3 - 2$

$\therefore x \in \{-3, -2, 7\}$

$\therefore n = 3$

Hence, $m^2 + mx + n^2 = 0 + 0 + 9 = 9$

HINT:

The relation between the greatest integer function and fractional part is :

$[x] = x - \{x\}$

83. The correct answer is (11).

Equation of plane P_2 passing through (2, –1, 0), (2, 0, –1) and (5, 1, 1) is

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow (x - 5)(4 - 1) - (y - 1)(6 - 3) + (z - 1)(3 - 6) = 0$

$\Rightarrow 3x - 15 - 3y + 3 - 3z + 3 = 0$

$\Rightarrow 3x - 3y - 3z - 9 = 0$

$\Rightarrow x - y - z = 3$ ...(i)

Now, direction ratios of line of intersection of P_1 and P_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 3 & -1 & -7 \end{vmatrix}$$

$= \hat{i}(7 - 1) - \hat{j}(-7 + 3) + \hat{k}(-1 + 3)$

$= 6\hat{i} + 4\hat{j} + 2\hat{k}$

At $z = 0$, $x - y = 3$ [from (i)]

$3x - y = 11$

on solving, we get

$x = 4$ and $y = 1$

So, equation of line is

$\frac{x - 4}{6} = \frac{y - 1}{4} = \frac{z - 2}{6} = k$

$\therefore (\alpha, \beta, \gamma) = (6k + 4, 4k + 1, 2k)$

$\Rightarrow (6)(\alpha - 7) + 4(\beta - 4) + 2(\gamma + 1) = 0$

$\Rightarrow 6(6k + 4 - 7) + 4(4k + 1 - 4) + 2(2k + 1) = 0$

$\Rightarrow 36k - 18 + 16k - 12 + 4k + 4 = 0$

$\Rightarrow 56k = 26 \Rightarrow k = \dfrac{1}{2}$

So, $\alpha = 7$, $\beta = 3$ and $\gamma = 1$

$\therefore \alpha + \beta + \gamma = 7 + 3 + 1 = 11$

HINT:

Equation of plane passing through (a, b, c), (d, c, f) and (g, h, i) is

$$\begin{vmatrix} x-h & y-h & z-i \\ g-a & h-b & i-e \\ g-d & h-e & i-f \end{vmatrix} = 0$$

84. The correct answer is (20).

Domain of $\log_e \left(\dfrac{6x^2+5x+1}{2x-1}\right)$

So, $\dfrac{6x^2+5x+1}{2x-1} > 0$

$\Rightarrow \dfrac{(3x+1)(2x+1)}{2x-1} > 0$

$\Rightarrow x \in \left(\dfrac{-1}{2}, \dfrac{-1}{3}\right) \cup \left(\dfrac{1}{2}, \infty\right)$

For domain of $\cos^{-1}\left(\dfrac{2x^2-3x+4}{3x-5}\right)$ domain of $\cos^{-1} x \to [-1, 1]$

$-1 \le \dfrac{2x^2-3x+4}{3x-5} \le 1$

$\dfrac{2x^2-1}{3x-5} \ge 0$ and $\dfrac{2x^2-6x+9}{3x-5} \le 0$

$\Rightarrow x \in \left[\dfrac{-1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right] \cup \left(\dfrac{5}{3}, \infty\right)$

So, common domain is $\left(\dfrac{-1}{2}, \dfrac{-1}{3}\right) \cup \left[\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}\right]$

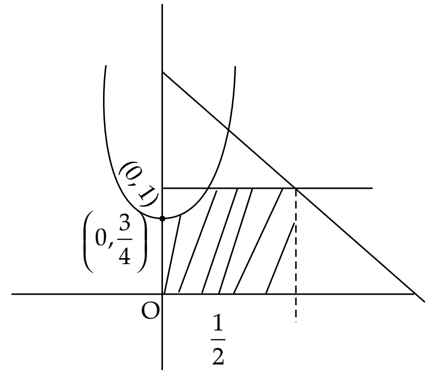
$\therefore 18(\alpha^2+\beta^2+\gamma^2+\delta^2) = 18\left(\dfrac{1}{4}+\dfrac{1}{9}+\dfrac{1}{4}+\dfrac{1}{2}\right)$

$= 18\left(\dfrac{9+4+9+18}{36}\right) = \dfrac{1}{2}(40) = 20$

HINT:

For $\log_e x$, $x > 0$ and $-1 \le \cos^{-1} x \le 1$

85. The correct answer is (17).



Required area $= \left[\displaystyle\int_0^{\frac{1}{2}} \left(x^2 + \frac{3}{4}\right) dx\right] + \left[\frac{1}{2}\left(\frac{3}{2} + \frac{1}{2}\right) \times 1\right]$

$= \left[\dfrac{x^3}{3} + \dfrac{3x}{4}\right]_0^{\frac{1}{2}} + 1$

$= \dfrac{1}{24} + \dfrac{3}{8} - 0 + 1 = \dfrac{1+9+24}{24} = \dfrac{34}{24} = \dfrac{17}{12}$

So, $12A = 12 \times \dfrac{17}{12} = 17$

HINT:

Find the common region bounded by all the given curves and then using integration, find the required area.

86. The correct answer is (150).

$\because \dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ are in A.P.

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{z} = \dfrac{2}{y}$ ...(i)

and $x, \sqrt{2}y, z$ are in G.P.

$\Rightarrow 2y^2 = xz$ ...(ii)

from (i), $\dfrac{2}{y} = \dfrac{x+z}{xz} = \dfrac{x+z}{2y^2}$

$\Rightarrow 4y = x + z$

Also, $xy + yz + zx = \dfrac{3}{\sqrt{2}} xyz$

$y(4y) + xz = \dfrac{3}{\sqrt{2}}(2y^2)y$

$\Rightarrow 4y^2 + 2y^2 = 3\sqrt{2}y^3$

$\Rightarrow 6y^2 = 3\sqrt{2}y^3 \Rightarrow y = \sqrt{2}$

$\therefore 3(x+y+z)^2 = 3(5y)^2 = 3(5\sqrt{2})^2$

$= 150$

HINT:

$a, b, c \to$ A.P. $\Rightarrow a + c = 2b$
$a, b, c \to$ G.P. $\Rightarrow b^2 = ac$

87. The correct answer is (12).

Given:

(cos y), (ln (cos y))^2 dx = (1 + 3x ln cos y) sin$y$ dy

$$\Rightarrow \frac{dx}{dy} = \frac{(1+3x\ln\cos y)\sin y}{(\ln\cos y)^2\cos y}$$

$$= \tan y\left[\frac{1}{(\ln\cos y)^2} + \frac{3x}{\ln\cos y}\right]$$

$$\Rightarrow \frac{dx}{dy} - \left(\frac{3\tan y}{\ln\cos y}\right)x = \frac{\tan y}{(\ln\cos y)^2}$$

which is a linear differential equation.

$$\text{I.F.} = e^{-\int\frac{3\tan y}{\ln\cos y}dy} = (\ln\cos y)^3 \qquad \text{I.F.} = e^{\int P.dx}$$

So, the solution is :

$$x \times (\ln\cos y)^3 = \int\left((\ln\cos y)^3 \times \frac{\tan y}{(\ln\cos y)^2}\right)dy$$

$$x \times (\ln\cos y)^3 = \frac{-(\ln\cos y)^2}{2} + C$$

At $y = \frac{\pi}{3}$,

$$\frac{1}{2\ln 2} \times \left(\ln\left(\frac{1}{2}\right)\right)^3 = -\frac{\left(\ln\left(\frac{1}{2}\right)\right)^2}{2} + C$$

$\Rightarrow$ C = 0

$$\text{So, } x \times \ln^3\cos y = \frac{-\ln^2\cos y}{2}$$

$$\text{At } y = \frac{\pi}{6}, x \times \left(\ln\left(\frac{\sqrt{3}}{2}\right)\right)^3 = -\frac{1}{2}\left(\ln\left(\frac{\sqrt{3}}{2}\right)\right)^2$$

$$\Rightarrow x = -\frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{1}{2[\ln\sqrt{3} - \ln 2]} = \frac{-1}{2\left[\frac{1}{2}\ln 3 - \ln 2\right]}$$

$$= \frac{-1}{2\left[\frac{\ln 3 - \ln 4}{2}\right]} = \frac{1}{\ln 4 - \ln 3}$$

$\Rightarrow m = 4, n = 3$

$\Rightarrow mn = 12$

HINT:

For a linear differential equation, $\frac{dx}{dy} + P(y)x = Q(y)$,

the solution is x × I.F. = $\int \text{I.F.} \times Q(y)\,dy$

where I.P. = $e^{\int P(y)dy}$

88. The correct answer is (6).

$$\int_0^{2.4}[x^2]dx = \int_0^1[x^2]dx + \int_1^{\sqrt{2}}[x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}}[x^2]dx + \int_{\sqrt{3}}^{2}[x^2]dx + \int_2^{\sqrt{5}}[x^2]dx + \int_{\sqrt{5}}^{2.4}[x^2]dx$$

$$= \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx + \int_2^{\sqrt{5}} 4dx + \int_{\sqrt{5}}^{2.4} 5dx$$

$$0 + [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^{2} + 4[x]_2^{\sqrt{5}} + 5[x]_{\sqrt{5}}^{2.4}$$

$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} + 4\sqrt{5} - 8 + 12 - 5\sqrt{5}$$

$$= -\sqrt{2} - \sqrt{3} - \sqrt{5} + 9$$

$\therefore \alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

So, $\alpha + \beta + \gamma + \delta = 9 - 1 - 1 - 1 = 6$

HINT:

The greater integer value is that integeral value which is less than or equal to that number.

89. The correct answer is (16).

Give parabola is : $y^2 = 12x$ $\quad$ ($\because a = 3$)

So, P $\equiv (at_1^2, 2at_1)$

Q $\equiv (at_2^2, 2at_2)$

So, point R $(\alpha, \beta) \equiv (at_1t_2, a(t_1 + t_2))$

$\equiv ((3t)(3t), 3(t + 3t)) = (9t^2, 12t)$

$$\therefore \frac{\beta^2}{\alpha} = \frac{144t^2}{9t^2} = 16$$

HINT:

For equation of parabola $y^2 = 4ax$, focus is $(a, 0)$

90. The correct answer is (309).

$$\text{Here, } f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$$

$\because f(x)$ is differentiable at $x > 0$

So, $f(x)$ is differentiable at $x = 1$

$f(1^-) = f(1) = f(1^+)$

$$3 + k\sqrt{2} = m + k^2 \qquad ...(i)$$

$f'(1^-) = f'(1^+)$

$$6(1) + \frac{k}{2\sqrt{1+1}} = 2m(1)$$

$$\Rightarrow 6+\frac{k}{2\sqrt{2}}=2m \quad \text{...(ii)}$$

Using (i) and (ii),

$$3+k\sqrt{2}=3+\frac{k}{4\sqrt{2}}+k^2$$

$$\Rightarrow k^2+k\left[\frac{1}{4\sqrt{2}}-\sqrt{2}\right]=0$$

$$\Rightarrow k\left[k+\frac{1-8}{4\sqrt{2}}\right]=0 \Rightarrow k=0,\frac{7}{4\sqrt{2}}$$

for $k=\frac{7}{4\sqrt{2}}, m=3+\frac{\frac{7}{4\sqrt{2}}}{4\sqrt{2}}$

$$=3+\frac{7}{32}=\frac{96+7}{32}=\frac{103}{32}$$

So, $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}=\frac{8\times\left[2\times\frac{103}{32}\times 8\right]}{6\times\frac{1}{8}+\frac{7}{4\sqrt{2}}\times 2\sqrt{918}}$

$$=\frac{412}{\frac{3}{4}+\frac{7}{12}}=\frac{412}{\frac{9+7}{12}}=\frac{412\times 12}{16}=309$$

HINT:

$f(x)$ is differentiable at $x=a$, if $f'(a^-)=f'(a^+)$

JEE (Main) SOLVED PAPER

2023
24^{th} Jan Shift 1

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of circular loop and at a distance *r* from the center of the loop on its axis is:

(1) $2\sqrt{2}:1$ **(2)** $1:3\sqrt{2}$
(3) $1:\sqrt{2}$ **(4)** $3\sqrt{2}:2$

Q. 2. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth R_e = 6400 km):

(1) 8 N **(2)** 4.9 N
(3) 9.8 N **(4)** 19.6 N

Q. 3. Two long straight wires *P* and *Q* carrying equal current 10 A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F_1. If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire P will be:

(1) $\frac{F_1}{8}$ (2) $8\ F_1$
(3) $10\ F_1$ (4) $\frac{F_1}{10}$

Q. 4. Given below are two statements:

Statement I: The temperature of a gas is –73°C. When the gas is heated to 527°C, the root mean square speed of the molecules is doubled.

Statement II: The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Q. 5. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is:

(1) 272 m **(2)** 68 m
(3) 192 m **(4)** 136 m

Q. 6. Given below are two statements:

Statement I: If the Brewster's angle for the light propagating from air to glass is θ_B, then the Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2}-\theta_B$.

Statement II: The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Q. 7. A 100 m long wire having cross-sectional area 6.25×10^{-4} m² and Young's modulus is 10^{10}Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be:

(1) 4×10^{-3} m **(2)** 6.25×10^{-3} m
(3) 6.25×10^{-6} m **(4)** 4×10^{-4}m

Q. 8. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant k. What will be the equivalent distance between charges in air for the same electrostatic force?

(1) $2d\sqrt{k}$ **(2)** $1.5\,d\sqrt{k}$
(3) $d\sqrt{k}$ **(4)** $k\sqrt{d}$

Q. 9. Consider the following radioactive decay process

$$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

The mass number and the atomic number of A_6 are given by:

(1) 210 and 84
(2) 210 and 82
(3) 211 and 80
(4) 210 and 80

Q. 10. From the photoelectric effect experiment, following observations are made. Identify which of these are correct.

A. The stopping potential depends only on the work function of the metal.
B. The saturation current increases as the intensity of incident light increases.
C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.
D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

(1) A, C, D only **(2)** B, C only
(3) B only **(4)** A, B, D only

Q. 11. Given below are two statements:

Statement I: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

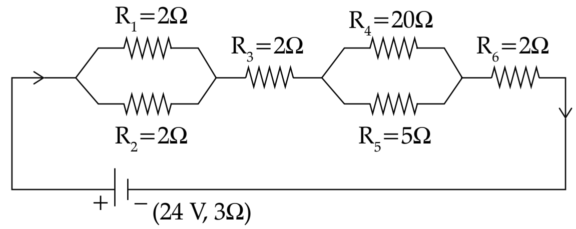
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Q. 12. 1 g of a liquid is converted to vapour at 3×10^5 P_a pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be:

(1) 432000 J **(2)** 4320 J
(3) 4800 J **(4)** 4.32×10^8 J

Q. 13. As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of 3Ω. The currents through the resistors R_4 and R_5 are I_4, and I_5 respectively. The values of I_4 and I_5 are:



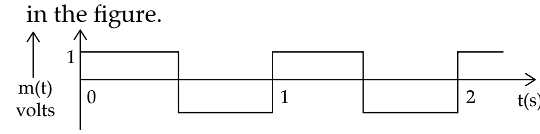
(1) $I_4 = \frac{2}{5}$ and $I_5 = \frac{8}{5}$A

(2) $I_4 = \frac{24}{5}$A and $I_5 = \frac{6}{5}$A

(3) $I_4 = \frac{8}{5}$A and $I_5 = \frac{2}{5}$A

(4) $I_4 = \frac{6}{5}$A and $I_5 = \frac{24}{5}$A

Q. 14. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as c(t) = 2sin(8πt) volts, the modulation index is:

(1) $\frac{1}{4}$ **(2)** $\frac{1}{2}$

(3) 1 **(4)** $\frac{1}{3}$

Q. 15. A conducting circular loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) emf = 1mV **(2)** emf = 5mV
(3) emf = 100mV **(4)** emf = 10mV

Q. 16. In $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by :

(ω - angular frequency):

(1) $\omega(\bar{E}\times\bar{K})$ **(2)** $\omega(\vec{K}\times\vec{E})$

(3) $\bar{K}\times\bar{E}$ **(4)** $\frac{1}{\omega}(\bar{K}\times\bar{E})$

Q. 17. Match List I with List II:

	LIST I		LIST II
A.	Planck's constant (h)	I.	$[M^1 L^2 T^{-2}]$
B.	Stopping potential (V_s)	II.	$[M^1 L^1 T^{-1}]$
C.	Work function (∅)	III.	$[M^1 L^2 T^{-1}]$
D.	Momentum (p)	IV.	$[M^1 L^2 T^{-3} A^{-1}]$

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II
(2) A-III, B-I, C-II, D-IV

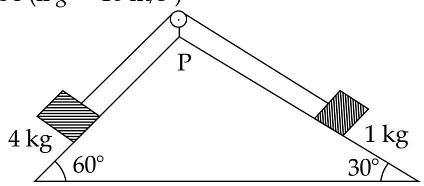
(3) A-II, B-IV, C-III, D-I
(4) A-III, B-IV, C-I, D-II

Q. 18. A travelling wave is described by the equation $y(x, t) = [0.05\sin(8x - 4t)]$m

The velocity of the wave is: [all the quantities are in SI unit]

(1) 8 ms^{-1} **(2)** 4 ms^{-1}
(3) 0.5 ms^{-1} **(4)** 2 ms^{-1}

Q. 19. As per given figure, a weightless pulley P is attached on a double inclined frictionless surfaces. The tension in the string (massless) will be (if g = 10 m/s²)



(1) $(4\sqrt{3}+1)\,N$ **(2)** $4(\sqrt{3}+1)\,N$
(3) $(4\sqrt{3}-1)\,N$ **(4)** $4(\sqrt{3}-1)\,N$

Q. 20. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R Assertion

A: Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason: The current in the forward bias is more than the current in the reverse bias for a p - n junction diode.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false
(2) A is false but R is true
(3) Both A and R are true and R is the correct explanation of A
(4) Both A and R are true but R is NOT the correct explanation of A

Section B

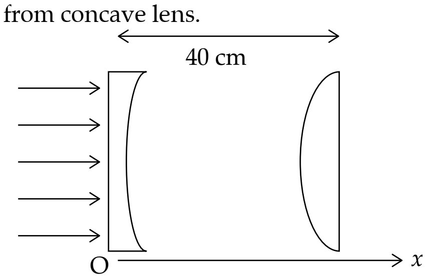
Q. 21. Vectors $a\hat{i}+b\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}+4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $x/2$. The value of x is

Q. 22. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6×10^{-27} kg and radius of nucleus is $1.5 \times 10^{-15}\ A^{1/3}$ m. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is

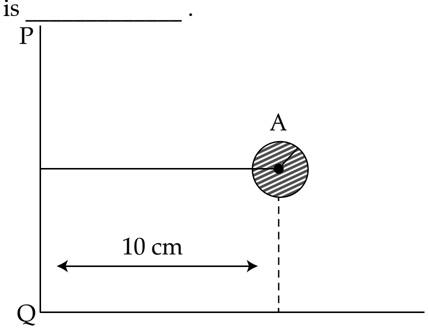
Q. 23. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n \times 10^{-3}\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\Omega$m. The value of n is

Q. 24. A stream of a positively charged particles having $\frac{q}{m} = 2 \times 10^{11}\frac{C}{kg}$ and velocity $\vec{v}_0 = 3 \times 10^7\ \hat{i}$ m/s is deflected by an electric field $1.8\hat{j}$ kV/m. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is ___mm

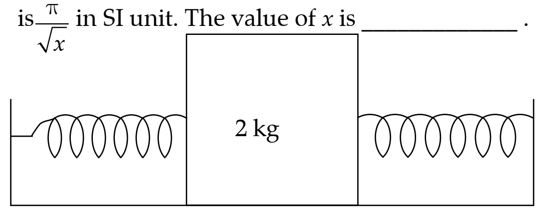
Q. 25. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance = __cm, from concave lens.



Q. 26. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is ____________ .



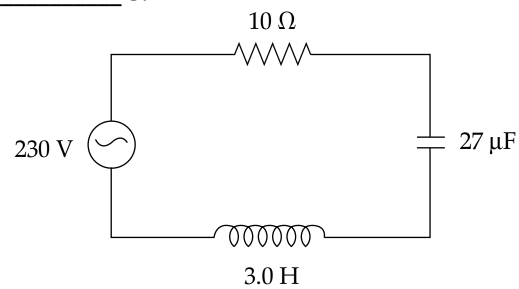
Q. 27. A block of a mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is ____________ .



Q. 28. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is d $\times 10^{-3}$ cm. The value of d will be ____________ if coefficient of linear expansion of the metal is 1.6×10^{-5}/°C.

Q. 29. In the circuit shown in the figure, the ratio of the quality factor and the band width is

_____________ S.

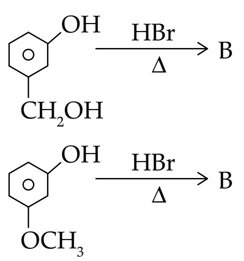


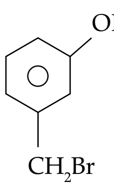
Q. 30. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5^{th} second. The force acted on the body is _____________ N.

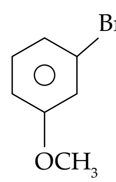
Chemistry

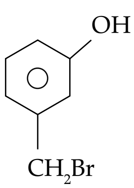
Section A

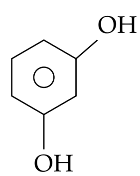
Q. 31. 'A' and 'B' formed in the following set of reactions are:

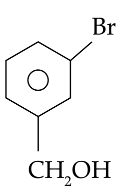


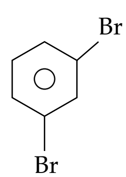
(1) A = 

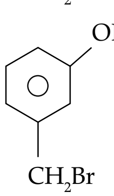
, B = 

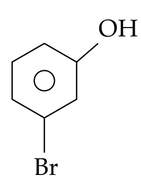
(2) A = 

, B = 

(3) A = 

, B = 

(4) A = 

, B = 

Q. 32. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by

A. Liquid water

B. Ice

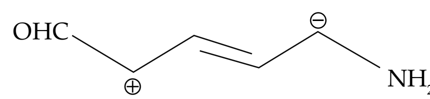
C. Impure water

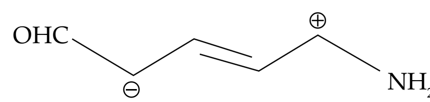
Choose the correct answer from the options given below:

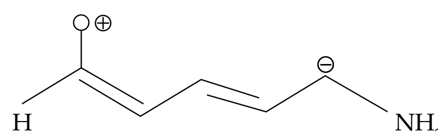
(1) B > A > C **(2)** A > B > C

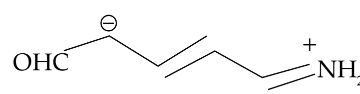
(3) A = B > C **(4)** C > B > A

Q. 33. Increasing order of stability of the resonance structures is:

A. 

B. 

C. 

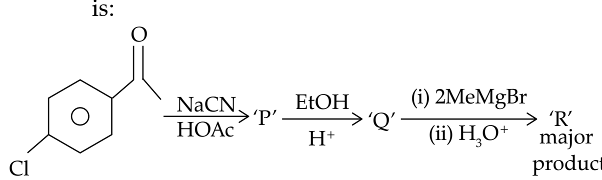
D. 

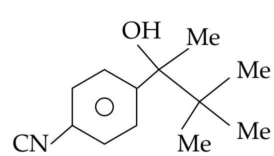
Choose the correct answer from the options given below:

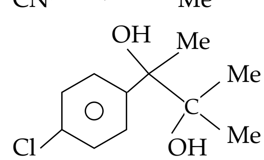
(1) D, C, A, B **(2)** D, C, B, A

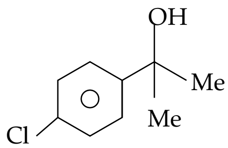
(3) C, A, B, D **(4)** C,D, B, A

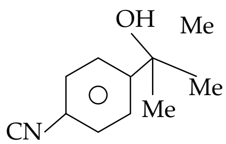
Q. 34. 'R' formed in the following sequence of reactions is:



(1) 

(2) 

(3) 

(4) 

Q. 35. The primary and secondary valencies of cobalt respectively in $[Co(NH_3)_5Cl]Cl_2$ are:

(1) 3 and 6 **(2)** 2 and 6
(3) 3 and 5 **(4)** 2 and 8

Q. 36. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is:

(1) Co^{2+} **(2)** Ni^{2+}
(3) Fe^{2+} **(4)** Cu^{2+}

Q. 37. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF_2 and NH_4F. What is 'A' ?

(1) $(NH_4)_2BeF_4$ **(2)** H_3NBeF_3
(3) $(NH_4)Be_2F_5$ **(4)** $(NH_4)BeF_3$

Q. 38. Match List I with List II

	List I		List II
A.	Reverberatory furnace	**I.**	Pig Iron
B.	Electrolytic cell	**II.**	Aluminum
C.	Blast furnace	**III.**	Silicon
D.	Zone Refining furnace	**IV.**	Copper

Choose the correct answer from the options given below:

(1) A-IV, B-II, C-I, D-III
(2) A-I, B-III, C-II, D-IV
(3) A-III, B-IV, C-I, D-II
(4) A-I, B-IV, C-II, D-III

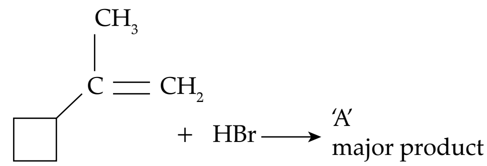
Q. 39. Match List I with List II

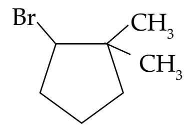
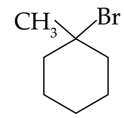
	List I		List II
A.	Chlorophyll	**I.**	Na_2CO_3
B.	Soda ash	**II.**	$CaSO_4$
C.	Dentistry, Ornamental work	**III.**	Mg^{2+}
D.	Used in white washing	**IV.**	$Ca(OH)_4$

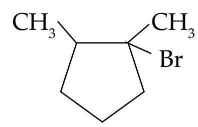
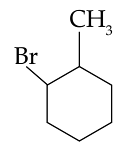
Choose the correct answer from the options given below:

(1) A-II, B-I, C-III, D-IV
(2) A-III, B-I, C-II, D-IV
(3) A-II, B-III, C-IV, D-I
(4) A-III, B-IV, C-I, D-II

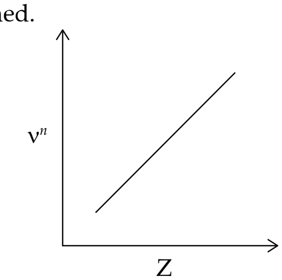
Q. 40. In the following given reaction, ' A ' is



(1)  **(2)** 

(3)  **(4)** 

Q. 41. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power " n " i.e. ν^n of X-rays emitted is plotted against atomic number " Z ", following graph is obtained.



The value of " n " is

(1) 3 **(2)** 2
(3) 1 **(4)** $\frac{1}{2}$

Q. 42. Given below are two statements:

Statement I: Noradrenaline is a neurotransmitter.

Statement II: Low level of noradrenaline is not the cause of depression in human.

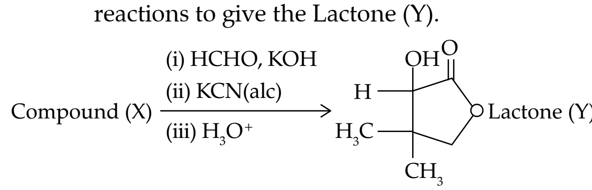
In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is correct but Statement II is incorrect
(2) Both Statement I and Statement II are correct
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

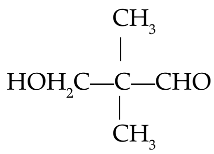
Q. 43. Which of the Phosphorus oxoacid can create silver mirror from $AgNO_3$ solution?

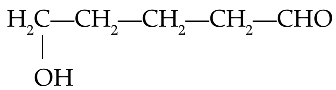
(1) $(HPO_3)_n$ **(2)** $H_4P_2O_6$
(3) $H_4P_2O_5$ **(4)** $H_4P_2O_7$

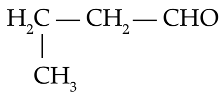
Q. 44. Compound (X) undergoes following sequence of reactions to give the Lactone (Y).

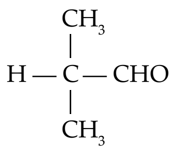


Compound (X) is

(1) 

(2) 

(3) 

(4) 

Q. 45. Order of Covalent bond;

A. KF > KI; LiF > KF

B. KF < KI; LiF > KF

C. $SnCl_4 > SnCl_2$; CuCl > NaCl

D. LiF > KF; CuCl < NaCl

E. KF < KI; CuCl > NaCl

Choose the correct answer from the options given below:

(1) C, E only **(2)** B, C, E only

(3) A, B only **(4)** B, C only

Q. 46. Which of the following is true about freons?

(1) These are radicals of chlorine and chlorine monoxide

(2) These are chemicals causing skin cancer

(3) These are chlorofluorocarbon compounds

(4) All radicals are called freons

Q. 47. In the depression of freezing point experiment

A. Vapour pressure of the solution is less than that of pure solvent

B. Vapour pressure of the solution is more than that of pure solvent

C. Only solute molecules solidify at the freezing point

D. Only solvent molecules solidify at the freezing point

Choose the most appropriate answer from the options given below:

(1) A and C only **(2)** A only

(3) A and D only **(4)** B and C only

Q. 48. **Statement I:** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.

Statement II: For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the correct answer from the options given below:

(1) Options 1. Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Q. 49. **Assertion A:** Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.

Reason R: I^- is a good nucleophile as well as a good leaving group. In the light of the above statements, choose the correct answer from the options given below

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Q. 50. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is

(1) Cr^{2+} **(2)** Ti^{2+}

(3) V^{2+} **(4)** Mn^{2+}

Section B

Q. 51. When $Fe_{0.93}O$ is heated in presence of oxygen, it converts to Fe_2O_3. The number of correct statement/s from the following is__________ .

A. The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively.

C. $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions

D. The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively

Q. 52. The number of correct statement/s from the following is __________ .

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperature

D. A plot of ln K VS $\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$

Q. 53. For independent processes at 300 K

Process	Δ H/kJ mol^{-1}	Δ S/J K^{-1}
A	-25	-80
B	-22	40
C	25	-50
D	22	20

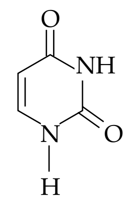
The number of non-spontaneous processes from the following is

Q. 54. 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1M solution?

Given: Molar Mass of Na, O and H is 23,16 and 1 g mol^{-1} respectively

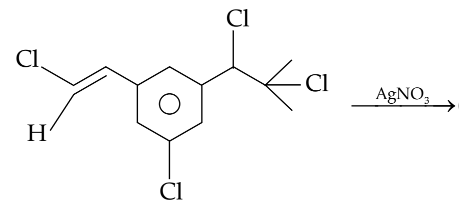
Q. 55. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is nm. (Nearest integer)

Q. 56. Uracil is a base present in RNA with the following structure. % of N in uracil is



Q. 57. The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of 0.2M CH_3COONa solution is mixed with 25 mL of 0.02M CH_3COOH solution, the pH of the resultant solution is found to be equal to 5. The value of x is__________ .

Q. 58. Number of moles of AgCl formed in the following reaction is



(A)+X AgCl↓

Q. 59. The d-electronic configuration of $[CoCl_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of "m" and "number of unpaired electrons" is __________ .

Q. 50. At 298 K, a 1 litre solution containing 10mmol of $Cr_2O_7^{2-}$ and 100mmol of Cr^{3+} shows a pH of 3.0.

Given: $Cr_2O_7^{2-} \rightarrow Cr^{3+}$; $E^0 = 1.330$ V and $\frac{2.303RT}{F} = 0.059$ V

The potential for the half cell reaction is $x \times 10^{-3}$ V. The value of x is __________ .

Mathematics

Section A

Q. 61. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(A) 2 **(B)** $\frac{3}{2}$

(C) 1 **(D)** $-\frac{2}{3}$

Q. 62. $\lim_{t \to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + ... + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(A) n^2 **(B)** $\frac{n(n+1)}{2}$

(C) n **(D)** $n^2 + n$

Q. 63. Let α be a root of the equation $(a - c)x^2 + (b - a)x (c - b) = 0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular.

Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

(A) 12 **(B)** 9
(C) 3 **(D)** 6

Q. 64. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is:

(A) 9 **(B)** $\frac{22}{3}$

(C) $\frac{23}{3}$ **(D)** $\frac{25}{3}$

Q. 65. Let $p, q \in \mathbb{R}$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$, $i = \sqrt{-1}$. Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

(A) $x^2 - 4x - 1 = 0$ **(B)** $x^2 - 4x + 1 = 0$
(C) $x^2 + 4x - 1 = 0$ **(D)** $x^2 + 4x + 1 = 0$

Q. 66. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(A) 21 **(B)** 18
(C) 20 **(D)** 19

Q. 67. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

(A) –6 **(B)** 12
(C) 6 **(D)** 2

Q. 68. The relation R = $\{(a, b): gcd\,(a, b) = 1, 2a \neq b, a, b \in Z\}$ is:

(A) reflexive but not symmetric
(B) transitive but not reflexive
(C) symmetric but not transitive
(D) neither symmetric nor transitive

Q. 69. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(A) 4 **(B)** 3
(C) 1 **(D)** 2

Q. 70. Let $y = y(x)$ be the solution of the differential equation $x^3dy + (xy - 1)\,dx = 0$, $x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to

(A) 1 **(B)** e
(C) 3 **(D)** $2 - e$

Q. 71. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2B$, then

(A) $A^2 = I$ or $B = I$ **(B)** $A^2B = I$
(C) $AB = I$ **(D)** $A^2B = BA^2$

Q. 72. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has :

(A) a unique solution in $(-\infty, 1)$
(B) no solution
(C) exactly two solutions in $(-\infty, \infty)$
(D) a unique solution in $(-\infty, \infty)$

Q. 73. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segment AB lie on a parabola with the

(A) length of latus rectum $\frac{3}{2}$
(B) directrix $4x = -3$
(C) length of latus rectum 2
(D) directrix $4x = 3$

Q. 74. Let Ω be the sample space and $A \subseteq \Omega$ be an event.
Given below are two statements :
(S_1) : If P(A) = 0, then A = ϕ
(S_2): If P(A) = 1, then A = Ω
Then

(A) both (S_1) and (S_2) are true
(B) only (S_1) is true
(C) only (S_2) is true
(D) both (S_1) and (S_2) are false

Q. 75. The value of $\sum_{r=0}^{22} {}^{22}C_r \, {}^{23}C_r$ is

(A) ${}^{44}C_{23}$ **(B)** ${}^{45}C_{23}$
(C) ${}^{44}C_{22}$ **(D)** ${}^{45}C_{24}$

Q. 76. The distance of the point (–1, 9, –16) from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(A) 31 **(B)** $13\sqrt{2}$
(C) $20\sqrt{2}$ **(D)** 26

Q. 77. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to:

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ **(D)** $\frac{\pi}{2}$

Q. 78. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Then at $x = 0$

(A) f is continuous but not differentiable
(B) f and f' both are continuous
(C) f' is continuous but not differentiable
(D) f is continuous but f' is not continuous

Q. 79. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(A) $(\sim Q) \vee P$
(B) $((\sim P) \vee Q) \wedge (\sim Q)$
(C) $(\sim P) \vee Q$
(D) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Q. 80. The distance of the point (7, −3, −4) from the plane passing through the points (2, −3,1), (−1,1, −2) and (3, −4,2) is :

(A) 5 **(B)** 4
(C) $5\sqrt{2}$ **(D)** $4\sqrt{2}$

Section B

Q. 81. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S = $\{x + \lambda : x$ is an integer solution of E$\}$ is

Q. 82. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is

Q. 83. The shortest distance between the lines $\frac{x-2}{3}$

$= \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to __________ .

Q. 84. Suppose $\sum_{r=0}^{2023} r^2\, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is __________ .

Q. 85. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is

Q. 86. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is __________ .

Q. 87. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is __________ .

Q. 88. The 4^{th} term of a GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbb{N}$.. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is_____ .

Q. 89. Let C be the largest circle centred at (2,0) and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\,\alpha^2$ is equal to __________ .

Q. 90. The value of $12\int_0^3 \left|x^2 - 3x + 2\right| dx$ is __________ .

Answer Key

Physics

Q. No.	Answer	Topic Name	Chapter Name
1	1	Magnetic Field Intensity due to Current Carrying Circular Coil	Moving Charges and Magnetism
2	1	Variation in Acceleration Due to Gravity	Gravitation
3	2	Force Between Two Current Carrying Straight Parallel Conductor	Moving Charges and Magnetism
4	3	RMS Speed	Kinetic Theory of Gases
5	1	Projectile Motion	Motion in a Plane
6	2	Total Internal Reflection	Ray Optics
7	1	Young's Modulus	Mechanical Properties of Solids
8	3	Coulomb's Law	Electric Charges and Fields
9	4	Radioactive Decay	Nuclei
10	3	Photoelectric Effect	Dual Nature of Radiation and Matter
11	3	Apparent Weight in Elevator/Lift	Laws of Motion
12	2	First Law of Thermodynamics	Thermodynamics
13	1	Equivalent Resistance	Current Electricity
14	2	Modulation	Communication Systems
15	4	Induced EMF	Electromagnetic Induction
16	4	Propogation of EMW	Electromagnetic Waves
17	4	Dimensions	Units and Measurements
18	3	Wave Equation	Waves
19	2	Motion of Connected Bodies	Laws of Motion
20	4	p-n Junction Diode	Semiconductor Electronics

21	[1]	Vectors	Mathematical Tools
22	[11]	Nuclear Density	Nuclei
23	[2]	Resistivity	Current Electricity
24	[2]	Electric Force	Electric Charges and Fields
25	[120]	Combination of Thin Lenses	Ray Optics
26	[110]	Moment of Inertia	System of Particles and Rotational Motion
27	[5]	SHM	Oscillations
28	[12]	Linear Expansion	Thermal Propoerties of Matter
29	[10]	LCR Circuit	Alternating Current
30	[40]	Kinetic Energy	Work, Energy and Power

Chemistry

Q. No.	Answer	Topic Name	Chapter Name
31	2	Properties of alcohol	Alcohol phenol and ether
32	1	Extent of hydrogen bonding	Hydrogen
33	3	Stability of resonating structure	General organic chemistry
34	2	Properties of carbonyl compounds	Aldehyde and ketone
35	1	Werner theory	Coordination compounds
36	2	Identification of basic radical	Qualitative analysis
37	1	Properties of alkaline earth metals compounds	s-block
38	1	Types of purification methods	metallurgy
39	2	Uses of S block element	s-block
40	3	Electrophillic addition reaction of alkene	Hydrocarbon
41	4	Properties of X-ray	Structure of atom
42	1	Anti depression drug	Chemistry in everyday life
43	3	Oxyacid of phosphorous	p-block
44	4	Properties of carbonyl compounds	Aldehyde and ketone
45	2	Fajan's rule	Chemical bonding and molecular structure
46	3	Freons	Halo Alkane and halo arene
47	3	Colligative properties	Liquid solution
48	2	Properties of colloidal solution	Surface chemistry
49	3	Properties of alkyl halide	Halo Alkane and halo arene
50	3	Magnetic moment of transition metals	d and f block
51	[4]	Oxidation number,equivalent weight and percentage composites	Solid state
52	[3]	Arrhenius equation	Chemical kinetics
53	[2]	Spontaneity of reaction	Thermodynamics and thermochemistry
54	[180]	Molarity on dilution	Liquid solution

55	[492]	Hydrogen spectrum	Structure of atom
56	[25]	Percentage composition in uracil	Biomolecules
57	[10]	pH calculation	Ionic equilibrium
58	[2]	Reactivity of halogen containing compounds	Halo Alkane and halo arene
59	[7]	Calculation of number of unpaired electron	Coordination chemistry
60	[917]	Nernst Equation	Electrochemistry

Mathematics

Q. No.	Answer	Topic Name	Chapter Name
61	C	Triple Products	Vector Algebra
62	C	Methods of Evaluation of Limits	Limits
63	C	Quadratic Equation and its Solution	Quadratic Equations
64	A	Area Bounded by Curves	Area under Curves
65	B	Euler's law	Complex Numbers
66	C	System of linear equations	Matrices and Determinants
67	D	Arithmetic Progressions	Sequences and Series
68	D	Equivalence Relations	Set Theory and Relations
69	B	Scalar and Vector Products	Vector Algebra
70	A	Solution of Linear Differential Equations	Differential Equations
71	D	Operations on Matrices	Matrices and Determinants
72	D	Quadratic Equations and its solution	Quadratic Equations
73	D	Interaction Between Two Conics	Hyperbola
74	A	Basics of Probability	Probability
75	B	Properties of Binomial Coefficients	Binomial Theorem
76	D	Intersection of a Line and a Plane	Three Dimensional Geometry
77	A	Basics of Inverse Trigonometric Functions	Inverse Trigonometric Functions
78	D	Differentiability of a Function	Continuity and Differentiability
79	C	Logical Operations	Mathematical Reasoning
80	C	Plane and a Point	Three Dimensional Geometry
81	[5]	Algebra of Functions	Function
82	[7]	Properties of Ellipse	Ellipse
83	[14]	Skew Lines	Three Dimensional Geometry
84	[1012]	Properties of Binomial Coefficients	Binomial Theorem
85	[2]	Properties of Definite Integrals	Definite Integration
86	[60]	Permutations	Permutations and Combinations
87	[546]	Combinations	Permutations and Combinations
88	[12]	Geometric Progressions	Sequences and Series
89	[118]	Interaction between Two Conics	Ellipse

JEE (Main) PHYSICS SOLVED PAPER

2023

24^{th} Jan Shift 1

ANSWERS WITH EXPLANATIONS

Physics

Section A

1. Option (1) is correct.

Magnetic field at of a coil, $B_1 = \frac{\mu_0 I}{2r}$

magnetic field on axis of coil at a distance,

$$B_2 = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I r^2}{2(r^2+r^2)^{3/2}} \quad (\text{as } x = r)$$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I r^2}{2(2\sqrt{2}r^3)}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = 2\sqrt{2} : 1$$

2. The correct option is (1)

Given, $mg = 18$ N,

At a height of h , $g' = g\frac{R^2}{(R+h)^2}$

$$\Rightarrow \quad g' = g\frac{6400^2}{(6400+3200)^2} = \frac{4}{9}g$$

Weight, $W = mg' = \frac{4}{9}mg$

$$\Rightarrow \quad W = \frac{4}{9} \times 18 = 8\text{N}$$

3. The correct option is (2)

Force per unit length between two parallel conductors is given by, $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2r}$

As l =constant, $I_1 = I_2 = I$, So $F \propto \frac{I^2}{r}$

$$\frac{F_1}{F_2} = \frac{\frac{I^2}{r}}{\frac{(2I)^2}{r/2}} = \frac{1}{8}$$

$\Rightarrow F_2 = F_1 = 8:1$

4. The correct option is (3)

Since, $V_{rms} = \sqrt{T}$

$$\frac{v_{rms1}}{v_{rms2}} = \frac{\sqrt{273+(-73)}}{\sqrt{273+(527)}} = \frac{\sqrt{200}}{\sqrt{800}} = \frac{1}{2}$$

$$\Rightarrow \quad v_{rms_2} = 2v_{rms_1}$$

Hence statement 1 is true.

Now, $KE_{trans} = \frac{3}{2}nRT = \frac{3}{2}PV$

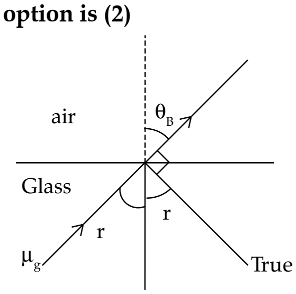
Hence, Statement 2 is false.

5. The correct option is (1)

Max height, $H = \frac{v^2}{2g} = 136$ m (given)

Max range, $R = \frac{v^2}{g} = 136 \times 2 = 272$m

6. The correct option is (2)



In reflection, $< I = < r = \theta_B$

In refraction, $r = \frac{\pi}{2} - \theta_B$

So, statement I is correct.

From glass to air, $\mu_g \sin i_B = \cos i_B$

$\Rightarrow \tan i_B = \frac{1}{\mu_g}$ or $i_B = \tan^{-1} = \left(\frac{1}{\mu_g}\right)$

Clearly, we can see statement II is incorrect.

7. The correct option is (1)

Given, l =100 m, $A = 6.25 \times 10^{-4}$ m^2

$F = 250$ N

$Y = 10^{10}$ Nm^2

We know, Stress = Y strain

$$\Rightarrow \quad \frac{F}{A} = Y\frac{\Delta l}{l}$$

$$\Rightarrow \quad \frac{250}{6.25}\times 10^{-4} = 10^{10}\frac{\Delta l}{100}$$

$$\Rightarrow \quad \Delta l = 4\times 10^{-3}\text{m}$$

8. The correct option is (3)

Force in air, $F_1 = \frac{kq_1q_2}{d^2}$

Force in dielectric, $F_2 = \frac{kq_1q_2}{Kr^2}$

As $F_1 = F_2$

$\Rightarrow$ $r = d\sqrt{K}$

9. The correct option is (4)

In α : A = –4 and z = –2

In β^-: z = +1 and β^+: z = –1

$${}^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 .$$

$$\xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

Mass number will be changed only due to α-particles.

$A_f = 218 - 2\alpha = 218 - 2\times 4 = 210$

Atomic number will be changed due to α, β^+ and β^- particles.

$Z_f = 84 - 2\alpha - \beta^+ - \beta^{-1} = 84 - 2\times 2 - (-1) - 1 = 80$

10. The correct option is (3)

Stopping potential or max KE depends upon frequency of light not on intensity. Wave theory cannot explain photoelectric effect. So statement A,C and D are false.

11. The correct option is (3)

When weight (force) is balanced a=0 and v = constant. Hence, statement I is true. When elevator goes down it means acceleration acts downwards. So

$mg - N = ma$

Or $N = m(g - a)$, it means weight will reduce. Hence, statement II is false.

12. The correct option is (2)

Heat supplied,

$Q = P\Delta V = 3\times 10^5 \times 1600\times 10^{-6}$

Q = 4800J

From 1st law of thermodynamics

$Q = \Delta u + w$

$\Delta u = Q - \frac{Q}{10} = \frac{9}{10}Q$

$\Delta u = \frac{9}{10}\times 4800 = 4320\text{J}$

13. The correct option is (1)

$R_{eq} = R_1 || R_2 + R_3 + R_4 || R_5 + R_6 + r_{internal}$

$= \frac{2\times 2}{2+2} + 2 + \frac{20\times 5}{25} + 2 + 3 = 12\,\Omega$

$I_{eq} = \frac{V}{R_{eq}} = \frac{24}{12} = 2\text{A}$

$I_4 = \frac{5}{20+5}\times 2 = \frac{2}{5}\text{A}$

$I_5 = \frac{20}{20+5}\times 2 = \frac{8}{5}\text{A}$

14. The correct option is (2)

Given, $A_m = 1$, $A_C = 2$

Modulation index, $\mu = \frac{A_m}{A_c} = \frac{1}{2}$

15. The correct option is (4)

$\varepsilon = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt}$

$\varepsilon = -A\frac{\Delta B}{\Delta t} = -\frac{\pi R^2(0-B)}{\Delta t}$

$$\varepsilon = -\frac{\pi\left(\frac{10}{\sqrt{\pi}}\times 10^{-2}\right)^2 \times 0.5}{0.5} = 10m\text{V}$$

16. The correct option is (4)

Magnitude of $B = \frac{E}{C} = \frac{K}{\Omega}E$

In emw, $\vec{E}, \vec{B}$, and $\vec{K}$, all three are mutually perpendicular to each other. And B will be given by $\vec{K}\times\vec{E}$.

17. The correct option is (4)

Plank constant $h = \frac{E}{v} = \frac{[M^1L^2T^{-2}]}{[T^{-1}]} = [M^1L^2T^{-1}]$

Stopping potential, $[V] = \frac{[W]}{[q]} = [M^1L^2T^{-3}A^{-1}]$

Work function , $[W] = [M^1L^2T^{1}]$

Momentum, $[P] = [M^1L^2T^{1}]$

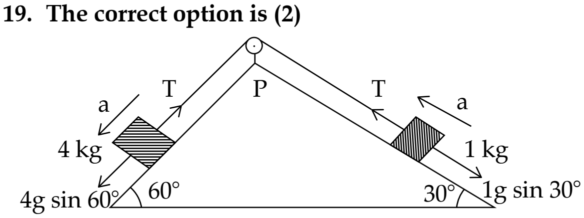
18. The correct option is (3)

On comparing given eq. with standard eq.

$y = A\sin(kx - \omega t)$, we get

$V = \frac{\omega}{k} = \frac{4}{8} = 0.5\ \text{ms}^{-1}$

19. The correct option is (2)



For 1 kg block

$T - mg\sin 30° = ma$

$T - 1\times 10\times \frac{1}{2} = a$

$T - 5 = a$ (i)

For 4 kg block

$mg\sin 60° - T = ma$

$4\times 10\,\frac{\sqrt{3}}{2} - T = 4a$

$20\sqrt{3} - T = 4a$ (ii)

From Eqn. (i) and (ii)

$20\sqrt{3} - T = 4(T-5)$

$20\sqrt{3} + 20 = 5T$

$T = 4(\sqrt{3}+1)$ N

20. The correct option is (4)

Photodiode detect light or optic signal and operate in reverse bias. In reverse bias current is very less as compare to forward bias unless breakdown occurs. Hence, both statements are correct.

Section B

21. The correct answer is [1]

As vectors are perpendicular so, $\vec{A}.\vec{B} = 0$

$(a\hat{i} + b\hat{j} + \hat{k}).(2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$

$\Rightarrow$ 2a – 3b = –4 …(i)

Given 3a + 2b = 7 …(ii)

From (i) and (ii), a = 1 and b = 2

$\frac{a}{b} = \frac{1}{2} = \frac{x}{2} \Rightarrow x = 1$

22. The correct answer is [11]

$\rho_{\text{nucleus}} = \frac{M}{V} = \frac{mA}{\frac{4}{3}\pi R^3 A}$

$\rho_{\text{nucleus}} = \frac{3\times1.6\times10^{-27}}{4\times3.14\times(1.5\times10^{-15})^3}$ – 11×10^{14} kg/m^3

$\rho_{\text{water}} = 10^3$ kg/m^3

Now, $\frac{\rho_{\text{nucleus}}}{\rho_{\text{Water}}} = 11 \times 10^{13}$, on comparing with $n \times 10^{13}$, we get n = 11.

23. The correct answer is [2]

We know $R = \rho\frac{l}{A} = \rho\frac{l}{A} = \rho\frac{l}{\pi(r_2^2 - r_1^2)}$

$R = 2.4 \times 10^{-8}\frac{3.14}{3.14(4^2 - 2^2)} = 2 \times 10^{-3}\ \Omega$

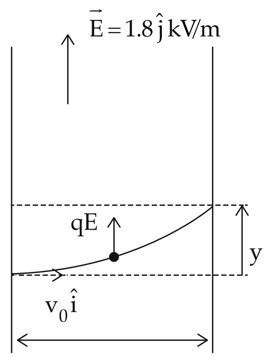
On comparing above value with $n \times 10^{-3}$, we get n = 2

24. The correct answer is [2]

$Y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2$

$\Rightarrow y = = \frac{1}{2}\frac{qE}{m}\left(\frac{l}{v}\right)^2$ $(\because v = l/t)$

$\Rightarrow y = \frac{1}{2} \times 2 \times 10^{11} \times 1.8 \times 10^3\left(\frac{0.1}{3\times10^7}\right)^2 = 2$ mm



25. The correct answer is [120]

Focal length of the combination is given by,

$f = \frac{R}{\mu - 1} = \frac{30}{1.75 - 1} = 40$ cm

Focal length of concave surface f_1 = -40 cm

For the rays coming from infinity, it will form image at its focus. So, v_1= -40 and it will become object for 2nd lens (convex).

Focal length of convex surface f_2 = + 40 cm

$\frac{1}{f} = \frac{1}{v_2} - \frac{1}{u}$

$\Rightarrow \frac{1}{40} = \frac{1}{v_2} - \frac{1}{-80}$

$\Rightarrow$ v_2 = 80 cm, image will form right side of convex lens.

Image will form at, v = 80 + 40 cm (distance between two lens) = 120 cm from concave lens.

26. The correct answer is [110]

$I_{PQ} = I_{CM} + md^2$

$\Rightarrow mk^2 = \frac{2}{5}mR^2 + md^2$

$\Rightarrow k = \sqrt{\frac{2}{5}5^2 + 10^2} = \sqrt{110}$cm

On comparing with $\sqrt{x}$, we get x = 110

27. The correct answer is [5]

$K_{eq} = k_1 + k_2 = 20 + 20 = 40$ N/m

Time period, $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{1}{20}} = \pi\sqrt{\frac{1}{5}}$

On comparing with $\frac{\pi}{\sqrt{x}}$, we get x = 5

28. The correct answer is [12]

Linear expansion, $l' = l(1 + \alpha\Delta T)$

$l' = 5(1 + 1.6 \times 10^{-5} \times (177 - 27))$

$l' = 12 \times 10^{-3}$ cm,

On comparing with $d \times 10^{-3}$, we get

$d = 12$

29. The correct answer is [10]

Quality factor is given by, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Bandwidth of LCR circuit, $\omega = \frac{R}{L}$

Now, $\frac{Q}{\omega} = \frac{L}{R^2}\sqrt{\frac{L}{C}}$

$= \frac{3}{100}\sqrt{\frac{3}{27\times10^{-6}}} = 10$

30. The correct answer is [40]

Here , m = 2kg u = 0 ms^{-1}

T = 5s KE = 10^4J

$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\times10^4}{2}} = 100$ ms^{-1}

$a = \frac{v - u}{t} = \frac{100 - 0}{5} = 20$ ms^{-1}

Force acting on the body, F = ma = 2 × 20 = 40 N

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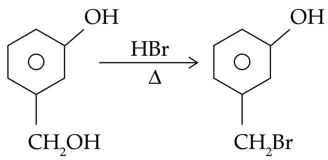
Chemistry

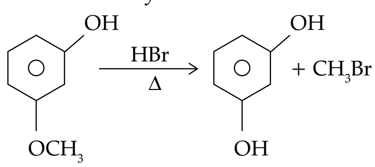
Section A

31. Option (2) is correct.



(A)

In the first step, the -OH of -CH_2-OH group is substituted by Br atom.



(B)

In this reaction, the reaction of ether takes place with HBr in which the deavage of ether taken place in such a way that aromatic alcohol is formed.

32. Option (1) is correct.

Among liquid water, ice and impure water, the extent of H-bonding is highly observed in ice. In ice, one water molecule is surrounded by four other water molecule to from a cage like structure while in liquid water the extent of H-bonding is less than ice.

In impure water, due to the presence of impurity extent of H-bonding decreases and it has least H-bonding. The decreasing order of H-bonding is as follows.

Ice > Liquid water > Impure water
(B) (A) (C)

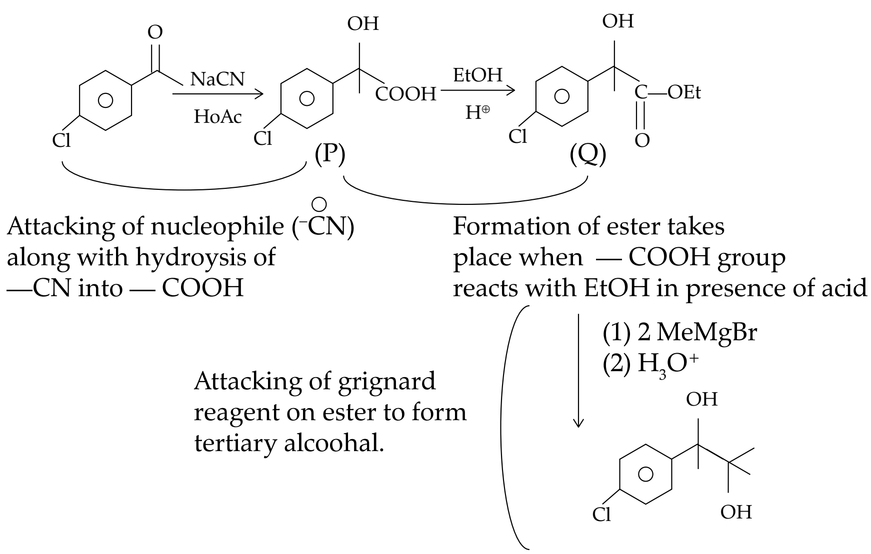
33. Option (3) is correct.

The correct answer should be C<A<B<D

$\mathrm{H-\overset{O^+}{\overset{|}{C}}=C-CH=CH-\overset{\ominus}{C}H_2-NH_2}$ (C) $<$ $\mathrm{H-\overset{O}{\overset{\|}{C}}-\underset{\oplus}{C}H=CH-\overset{\ominus}{C}H-NH_2}$ (A) $<$

$\mathrm{H-\overset{O}{\overset{\|}{C}}-\underset{\ominus}{C}H-CH=CH-\overset{\oplus}{C}H-NH_2}$ (B) $<$ $\mathrm{H-\overset{O}{\overset{\|}{C}}-\overset{\ominus}{C}H-CH=CH-CH=\overset{\oplus}{N}H_2}$ (D)

34. Option (2) is correct.



35. Option (1) is correct.

$[Co(NH_3)_5 Cl] Cl_2$

According to Werner's theory, the species which satisfied the coordination numbers are termed as secondary valency while the species which satisfied the oxidation number are termed as primary valency. Here five NH_3 and one Cl^- ion Satisfying the secondary valency while 3 Cl^- ion satisfying the primary valency.

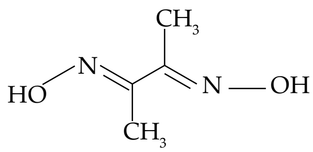
The secondary valency are 6 while primary valency are 3.

36. Option (2) is correct.

The metal ion which gives a brilliant red precipitate on addition of dimethyl glyoxime is Ni^{2+} ion.

$$Ni^{2+} + 2\,DMG \longrightarrow [Ni(DMG)_2]$$

Dimethyl glyoxime = (DMG)



37. Option (1) is correct.

The compound which is formed on reaction of BeO with ammonia and hydrogen fluoride is $(NH_4)\,BeF_4$.

$Beo + 2NH_3 + 4HF \rightarrow (NH_4)_2\,BeF_4 + H_2O$

$(NH_4)_2\,BeF_4 \rightarrow BeF_2 + NH_4F$

38. Option (1) is correct.

The correct matching of the given process with metal are as follows:

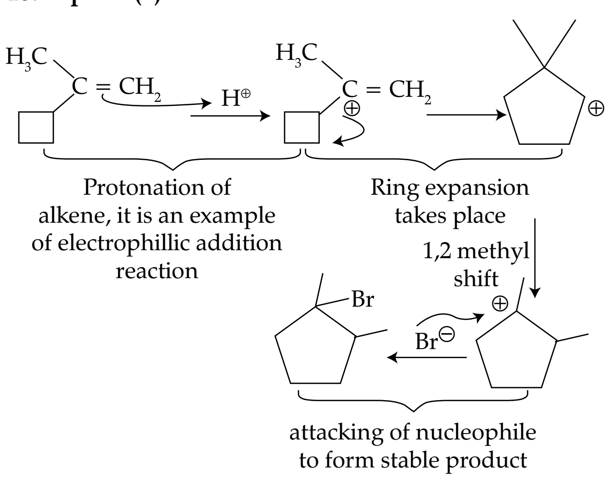
	List –I		List –II
A.	Reverberatory furnace	**IV.**	Copper
B.	Electrolytic cell	**II.**	Aluminium
C.	Blast furnace	**I.**	pig iron
D.	Zone refining furnace	**III.**	Silicon

39. Option (2) is correct.

The correct matching for the given substance along with the compound are as follows –

	List –I		List-II
A.	Chlorophyll	**III.**	Mg^{2+}
B.	Soda ash	**I.**	Na_2CO_3
C.	Dentistry, ornamental work	**II.**	$CaSO_4$
D.	Used in White washing	**IV.**	$Ca(OH)_2$

40. Option (3) is correct.



41. Option (4) is correct.

According to Henry Moseley

$\sqrt{\nu} \propto Z\text{-}b$

From the formula

$\sqrt{\nu} \propto Z$

Or $\nu^n \propto Z$

Or $\nu^{\frac{1}{2}} \propto Z$

So $n = \dfrac{1}{2}$

42. Option (1) is correct.

Statement I is correct, noradrenaline is a neurotransmitter,

Statement II is incorrect, low level of noradrenaline is not the cause of depression in human, this statement is not correct.

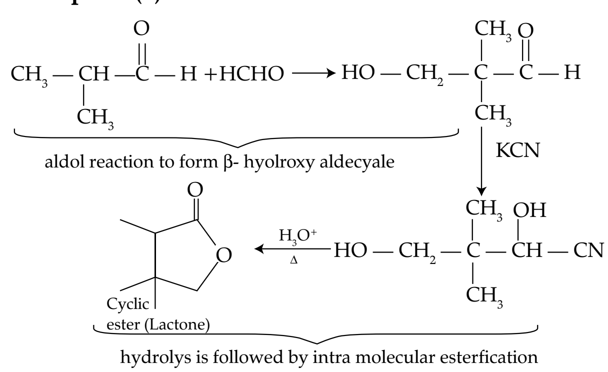
43. Option (3) is correct.

The silver mirror test is given by P^{+1} and P^{3+} because both P^{3+} and P^{+1} can be easily oxidized into P^{5+}

Compound	Oxidation state of P
$(HPO_3)_n$	+5
$H_4P_2O_5$	+3
$H_4P_2O_6$	+4
$H_4P_2O_7$	+5

Here $H_4P_2O_5$ can form silver mirror from $AgNO_3$ solution. Here $H_4P_2O_5$ contain P-H bond which can reduce $AgNO_3$ to Ag.

44. Option (4) is correct.



45. Option (2) is correct.

According to Fajan's rule, the ionic compound which is formed by small cation and large anion has more covalent character than other ionic comp. according to Fajan's rule

A. KF > KI – False LiF > KF – True

B. KF < KI – True LiF > KF – True

C. $SnCl_4$ > $SnCl_2$ – True C_4Cl > NaCl –True

D. LiF > KF – True C_4Cl >NaCl – False

E. KF < KI – True C_4Cl > NaCl – True

Order B,C and E is correct. So, option (2) is correct.

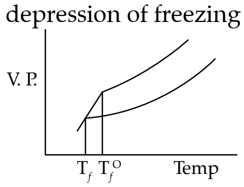
46. Option (3) is correct.

The chlorofluorocarbon compound of methane and ethane are collectively called Freon's. They are stable, unreactive, non-toxic gas.

47. Option (3) is correct.

A – On addition of non-volatile solute in the pure solvent the vapour pressure of pure solvent decrease, therefore vapour pressure of solution is less than that of pure solvent.

D – Similarly only addition of solute partide only can affect the depression of freezing point.



T_f^O = freezing point of solvent

T_f = freezing point of solution

48. Option (2) is correct.

Statement I : In colloidal solution, the size of partides is very big compared to true solution. Therefore, the colloidal particle has lower value of colligative properties.

Statement II : The potential different developed between two layers i.e., fixed and diffused layer of opposite charge is called zeta potential.

Statement I is correct but statement II is not correct.

49. Option (3) is correct.

The rate of hydrolysis of an alkyl chloride is slow because Cl^- is not a better nucleophile and leaving group compared to I^-.

The nucleophilicity of I^- is greater than Cl^- because the basic nature of I^- is less than Cl^-.

50. Option (3) is correct.

1. $Cr^{+2}:[Ar], 3d^4, 4s^0 n=4, \mu=\sqrt{4(4+2)}=\sqrt{24}=4.89BM$

2. $Mn^{+2}:[Ar], 3d^5, 4s^0 n=5, \mu=\sqrt{5(5+2)}=\sqrt{35}=5.91BM$

3. $V^{+2}:[Ar], 3d^3, 4s^0 n=3, \mu=\sqrt{3(3+2)}=\sqrt{15}=3.87BM$

4. $Ti^{+2}:[Ar], 3d^2, 4s^0 n=2, \mu=\sqrt{2(2+2)}=\sqrt{8}=2.82BM$

Section B

51. The correct answer is [4]

(A) The oxidation state of Fe in $Fe_{0\text{-}93}O_{1.0}$ and Fe_2O_3 are $\frac{200}{3}$ & +3 receptively.

First we need to find nf

$$nf = \left(3-\frac{200}{93}\right)\times 0.93$$

$$nf = 0.79$$

So, the equivalent weight of $Fe_{0.93}O_{1.0}$ is

$$\frac{\text{Molecular weight}}{0.79}$$

(B) Let the mole of $Fe^{2+} = x$

The mole of $Fe^{3+} = 0.93-x$

So $+2 \times x + (0.93 - x) \times 3 = 2$

$x = 0.79$

So, mole of Fe^{2+} = 0.79 mole

Mole of Fe^{3+} =0.14 mole

(C) In the given $Fe_{0.93}O_{1.0}$, the number of metal ion are less than that of O^{2-} ion ∴ it is a metal deficient compound.

(D) % of $Fe^{2+} = \frac{0.79}{0.93}\times 100 = 85\%$

% of $Fe^{3+} = \frac{0.14}{0.93}\times 100 = 15\%$

All the four statement are correct.

52. The correct answer is [3]

A is correct, from $K = Ae^{-Ea/RT}$

If E_a increases the value of K decreases.

B is correct, from $u = \frac{K_T+10}{K_T}$

If value of activation energy is higher, than the value of u is also high.

as $(u \times E_a)$

C is wrong, from $\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{T_2\text{-}T_1}{T_1T_2}\right)$

The value of K depends upon temperature so it can be increases as decreases'.

D is correct, $\text{Ln } K = L_n A - \frac{E_a}{RT}$

Here the plot of ln K VS $\frac{1}{T}$ is a straight line

With slope equal to $\frac{-E_a}{R}$.

Out of the four statement, there statement are correct.

53. The correct answer is [2]

From $\Delta G = \Delta H - T\Delta S$

A. $\Delta G = -25 \times 10^3 - (300 \times -80)$

$= -25000 + 24000 = -1000$ J/Mol

ΔG = –Ve (spontaneous process)

B. $\Delta G = -22 \times 10^3 - (300\times 40)$

$= -22000 - 12000$

$= -34000$ J/mol

ΔG = –Ve (spontaneous process)

C. $\Delta G = 25 \times 10^3 - (300\times -50)$

$= 25000+15000 = 40000$ J/mol

ΔG = +Ve (Non spontaneous)

D. $\Delta G = 22 \times 10^3 - (300 \times 20)$

$= 22000 - 6000 = 18000$ J/mol

$\Delta G = +Ve$ (Non Spontaneous)

Process C & D are non-spontaneous

54. The correct answer is [180]

$$\text{Molarity} = \frac{\text{mass of NaOH}}{\text{molar mass of NaOH}} \times \frac{1000}{\text{Volume (inmL)}}$$

$$\text{Molarity} = \frac{5\text{ gm}}{40\text{ gm/mol}} \times \frac{1000}{450} = 0.278\text{ M}$$

From $M_1 V_1 = M_2 V_2$ (dilution law)

$0.278\text{ M} \times V_1 = 0.1\text{ M} \times 500\text{ mL}$

$V_1 = 180$ mL

55. The correct answer is [492]

For Pascher series $(4 \to 3)$

First line $\lambda_1 = 720$ nm.

For second line $5 \to 3$

$$\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For 1st line $\frac{1}{(\lambda_1)_p} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{16}\right)$

$n_1 = 3$ $n_2 = 4$

For 2nd line

$$\frac{1}{(\lambda_2)_p} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{25}\right)$$

$n_1 = 3$ $n_2 = 4$

$$\frac{(\lambda_2)_p}{(\lambda_1)_p} = \frac{\frac{7}{16 \times 9}}{\frac{16}{25 \times 9}} = \frac{25 \times 7}{16 \times 16}$$

$$(\lambda_2)_p = \frac{175}{256} \times (\lambda_1)_p$$

$$= \frac{175}{256} \times 720\text{ nm} = 492.2\text{ nm}$$

≈ 492 nm (nearest integer)

56. The correct answer is [25]

Molar mass of uracil $(C_4 H_4 N_4 O_2)$

$= 4(C) \times 12 + 4(H) \times 1 + 2(N) \times 14 + 2(O) \times 16$

$= 48 + 4 + 28 + 32 = 112$ gm

$$\%\text{ of N-atom} = \frac{\text{mass of N}}{\text{Total mass}} \times 100$$

$$= \frac{28}{112} \times 100 = (25\%)$$

57. The correct answer is [10]

From Hendersen Hassel Bach equation

$$\text{pH} = \text{pK}_q + \log\left[\frac{\text{salt}}{\text{acid}}\right]$$

Given :

pH = 5

[Salt] = 0.2 M [acid] = 0.02 M

By putting the value

$$5 = \text{pk}_a + \log\frac{0.2}{0.02}$$

$5 = \text{pk}_a + \log 10$

$5 = \text{pk}_a + 1$

$\text{pk}_a = 4$

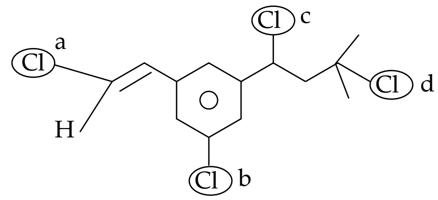
Or ka = (antilog) 4

ka $= 10^{-4}$

Or ka $= 10 \times 10^{-5}$

So $x = 10$

58. The correct answer is [2]



Tow of four chlorine atoms give AgCl on reaction with $AgNO_3$. (c) and (d) will give reaction with $AgNO_3$ while (a) and (b) do not give reaction with $AgNO_3$ because they are highly stable and do not undergo substitution reaction.

59. The correct answer is [7]

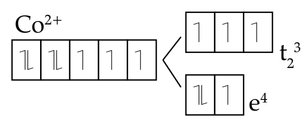
In $[CoCl_4]^{2-}$ Let oxidation state of Co = x

Oxidation state of Co $\Rightarrow x - 4 = -2$

$\Rightarrow x = +2$

Configuration of Co = $3d^7 4s^2$

$Co^{2+} = 3d^7$



The value of m = 4 and n = 3 so the sum of m + n $\Rightarrow$ 4+3 = (7)

60. The correct answer is [917]

The balanced chemical ionic equation.

$Cr_2O_7^{2-} + 14H^+ + 6e^- \to 2Cr^{3+} + 7H_2O$

$$\text{From } E = E^\circ - \frac{0.059}{n}\log\frac{\left[Cr^{3+}\right]^2}{\left[Cr_2O_7^{2-}\right]\left[H^+\right]^{14}}$$

Given $E^\circ = 1.330$V

n = 6

pH = 3, $[H^+] = 10^{-3}$

mmol of $Cr_2O_7^{2-}$ = 10 mmol

$= 10 \times 10^{-3}$ mol $= 10^{-2}$ mol

mmol of Cr^{3+} = 100 mmol

$= 100 \times 10^{-3}$ mol $= 10^{-1}$ mol.

By putting the values.

$$E = 1.33 - \frac{0.059}{6}\log\frac{(0.1)^2}{(10^{-2})(10^{-3})^{14}}$$

$E = 0.917 \times 10^{-3}$

$x = 917$

Mathematics

Section A

61. Option (C) is correct.

Given, $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$

Taking dot product with $\vec{v}$

$\Rightarrow (\vec{v} \times \vec{w}).\vec{v} = (\vec{u} + \lambda\vec{v}).\vec{v}$

$\Rightarrow 0 = \vec{u}.\vec{v} + \lambda\vec{v}.\vec{v}$

$\Rightarrow 0 = (\hat{i} - \hat{j} - 2\hat{k}).(2\hat{i} + \hat{j} - \hat{k}) + \lambda |\vec{v}|^2$

$\Rightarrow 0 = 2 - 1 + 2 + \lambda (4 + 1 + 1)$

$\Rightarrow \lambda = \frac{-3}{6} = \frac{-1}{2}$

So, $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v} = \vec{u} - \frac{1}{2}\vec{v}$

Taking dot product with $\vec{w}$

$\Rightarrow (\vec{v} \times \vec{w}).\vec{w} = \vec{u}.\vec{w} - \frac{1}{2}\vec{v}.\vec{w}$

$\Rightarrow 0 = \vec{u}.\vec{w} - \frac{1}{2}(2)$

$\Rightarrow \vec{u}.\vec{w} = 1$

HINT:

(1) $(\vec{a} \times \vec{b}).\vec{b} = 0$

(2) Take dot product to obtain the desired Dot - product

62. Option (C) is correct.

Let $l = \lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$

$= \lim_{t\to 0} n\left[\left(\frac{1}{n}\right)^{\text{cosec}^2 t} + \left(\frac{2}{n}\right)^{\text{cosec}^2 t} + \ldots + \left(\frac{n}{n}\right)^{\text{cosec}^2 t}\right]^{\sin^2 t}$

$= n \lim_{t\to 0}\left[\sum_{r=1}^{n-1}\left(\frac{r}{n}\right)^{\text{cosec}^2 t} + 1\right]^{\sin^2 t} = n \lim_{t\to 0}\left[\sum_{r=1}^{n-1} 0 + 1\right]^{\sin^2 t}$

$\left[\text{as} \left(\frac{r}{n}\right)^{\text{cosec}^2 t} \to 0 \because 0 < \frac{r}{n} < 1 \;\&\; \text{cosec}^2 t \to \infty\right]$

$= n$

HINT:

(1) As $n \to \infty, \frac{r}{n} \to 0,$ for $0 < r < n$

(2) As $x \to 0, \text{cosec}^2 x \to \infty$

63. Option (C) is correct.

Let $P = \begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$

Given that P is singular

$\Rightarrow |P| = 0$

$\Rightarrow \alpha^2 (c - b) - \alpha (c - a) + 1(b - a) = 0$

$\Rightarrow \alpha^2 (c - b) + \alpha (a - c) + (b - a) = 0$

Put $\alpha = 1$, $c - b + a - c + b - a = 0$

So, $\alpha = 1$ is a root

Now, $\sum \frac{(a-c)^2}{(b-a)(c-b)} = M$ (say)

$= \sum \frac{(a-c)^2}{(b-a)(c-b)}.\left(\frac{a-c}{a-c}\right) = \sum \frac{(a-c)^3}{(b-a)(c-b)(a-c)}$

$= \frac{(a-c)^3 + (c-b)^3 + (b-a)^3}{(b-a)(c-b)(a-c)}$

Here, $(a - c) + (c - b) + (b - a) = 0$

$\Rightarrow (a - c)^3 + (c - b)^3 + (b - a)^3 = 3 (a - c) (c - b) (b - a)$

$\therefore M = \frac{3(a-c)(c-b)(b-a)}{(b-a)(c-b)(a-c)}$

$\Rightarrow M = 3$

HINT:

(1) If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$

(2) $ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

64. Option (A) is correct.

Let $C_1 : y^2 + 4x = 4$ & $C_2 : y - 2x = 2$

Solving C_1 & C_2

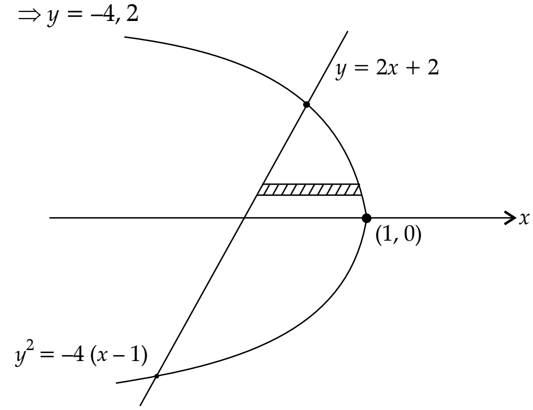
$\Rightarrow y^2 + 4\left(\frac{y}{2} - 1\right) = 4$

$\Rightarrow y^2 + 2y - 4 = 4$

$\Rightarrow y^2 + 2y - 8 = 0$

$\Rightarrow (y - 2)(y + 4) = 0$

$\Rightarrow y = -4, 2$



$$\text{Area} = \int (x_2 - x_1)dy$$

$$\Rightarrow \text{Area} = \int_{-4}^{2}\left\{\left(\frac{4-y^2}{4}\right)-\left(\frac{y-2}{2}\right)\right\}dy$$

$$\Rightarrow \text{Area} = \int_{-4}^{2}\left(2-\frac{y}{2}-\frac{y^2}{4}\right)dy$$

$$\Rightarrow \text{Area} = \left[2y-\frac{y^2}{4}-\frac{y^3}{12}\right]_{-4}^{2}$$

$$\Rightarrow \text{Area} = 2(2-(-4))-\frac{1}{4}(4-16)-\frac{1}{12}(8-(-64))$$

$\Rightarrow$ Area = 12 + 3 – 6

$\Rightarrow$ Area = 9 sq. units

HINT:

(1) Solve both the curves to get the point of intersection.

(2) Plot both the curves & think of a strip either horizontal or vertical and then integrate.

65. Option (B) is correct.

Given $2^{199}(p+iq)=(1-\sqrt{3}i)^{200}$

$$\Rightarrow \frac{2^{199}}{2^{200}}(p+iq)=\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{200}$$

$$\Rightarrow p+iq = 2\left(\text{C is}\left(-\frac{\pi}{3}\right)\right)^{200}$$

$$\Rightarrow p+iq = 2\left(e^{-i\frac{\pi}{3}}\right)^{200}$$

$$\Rightarrow p+iq = 2e^{-i\frac{200\pi}{3}}$$

$$\Rightarrow p+iq = 2\left(\cos\left(\frac{200\pi}{3}\right)-i\sin\left(\frac{200\pi}{3}\right)\right)$$

$$\Rightarrow p+iq = 2\left[\cos\left(66\pi+\frac{2\pi}{3}\right)-i\sin\left(66\pi+\frac{2\pi}{3}\right)\right]$$

$$\Rightarrow p+iq = 2\left[\cos\left(\frac{2\pi}{3}\right)-i\sin\left(\frac{2\pi}{3}\right)\right]$$

$$\Rightarrow p+iq = 2\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$\Rightarrow p=-1, q=-\sqrt{3}$

Now, $p + q + q^2 = -1-\sqrt{3}+(-\sqrt{3})^2 = 2-\sqrt{3}$

$p-q+q^2 = -1-(-\sqrt{3})+(-\sqrt{3})^2 = 2+\sqrt{3}$

Equation whose roots are $p+q+q^2$ & $p-q+q^2$ having

Sum of roots (S) = $(2-\sqrt{3})+(2+\sqrt{3}) = 4$

and product of roots (P) = $(2-\sqrt{3})(2+\sqrt{3}) = 4-3 = 1$

Required quadratic equation is $x^2-(S)x+P=0$

$\Rightarrow x^2-4x+1=0$

HINT:

(1) Use $\cos\theta + i\sin\theta = e^{i\theta}$

(2) Quadratic equation whose roots are α and β is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

66. Option (C) is correct.

System of equations is,

$$x+y+z=1$$
$$2x+Ny+2z=2$$
$$3x+3y+Nz=3$$

For unique solution, $\Delta \neq 0$

$$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix} \neq 0$$

$\Rightarrow 1(N^2-6)-(2N-6)+(6-3N)\neq 0$

$\Rightarrow N^2-5N+6\neq 0$

$\Rightarrow (N-2)(N-3)\neq 0$

Therefore $N\neq 2, N\neq 3$

So, favourable cases are $\{1, 4, 5, 6\}$

Total cases $\equiv \{1, 2, 3, 4, 5, 6\}$

$$\text{Hence, probability} = \frac{\text{Number of favourable cases}}{\text{Total cases}}$$

$\Rightarrow \frac{k}{6}=\frac{4}{6} \Rightarrow k=4$

So, sum of all possible values of k & N

$= 4 + (1 + 4 + 5 + 6) = 20$

HINT:

(1) For system of linear equations have unique solution $\Delta \neq 0$

(2) Probability = $\dfrac{\text{Number of favourable cases}}{\text{Total number of cases}}$

67. Option (D) is correct.

Given than, $x^{pq^2}=y^{qr}=z^{p^2r}$ ...(1)

Also, 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P.

$\Rightarrow 3+\frac{1}{2}=3\log_y x$

$\Rightarrow \log_y x=\frac{7}{6}$

$\Rightarrow x=(y)^{7/6} \Rightarrow x^6=y^7$

$3\log_z y = 3+2\left(\frac{1}{2}\right)=4$ ...(2)

$\Rightarrow \log_z y=\frac{4}{3}$

$\Rightarrow y=(z)^{4/3}$

$\Rightarrow y^3=z^4$ ...(3)

$7\log_x(z) = 3+3\left(\frac{1}{2}\right)=\frac{9}{2}$

$\Rightarrow \log_x(z) = \dfrac{9}{14}$

$\Rightarrow z = (x)^{9/14}$

$\Rightarrow z^{14} = x^9$ ...(4)

Now from (1), we have

$$x^{pq^2} = \left(x^{\frac{6}{7}}\right)^{qr} = \left(x^{\frac{9}{14}}\right)^{p^2r}$$

$$\Rightarrow pq^2 = \frac{6}{7}qr = \frac{9}{14}p^2r$$

$$\Rightarrow pq = \frac{6}{7}r, q^2 = \frac{9}{14}pr$$

Also, $r = pq + 1$

$$\Rightarrow r = \frac{6}{7}r + 1$$

$$\Rightarrow \frac{r}{7} = 1 \Rightarrow r = 7$$

Now, $q^2 = \dfrac{9}{14}pr$

$$\Rightarrow q(q^2) = \left(\frac{9}{14}r\right)(pq)$$

$$\Rightarrow q^3 = \left(\frac{9}{14}\right)r\left(\frac{6}{7}\right)r$$

$$\Rightarrow q^3 = \frac{9\times 6}{14\times 7}\times(7)^2$$

$\Rightarrow q^3 = 27$

$\Rightarrow q = 3$

And, $pq = \dfrac{6}{7}r$

$$\Rightarrow p = \frac{6}{7}\times\frac{7}{3}$$

$\Rightarrow p = 2$

So, $r - p - q$

$= 7 - 2 - 3 = 2$

HINT:

(1) For an A.P., $T_n = a + (n-1)d$, where a & d are first term & common difference respectively.

(2) $\log_b(a) = c \Rightarrow a = b^c$

(3) If $a^b = a^q \Rightarrow p = q$

68. Option (D) is correct.

$R = \{(a, b) : gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$

Reflexive:

Check for (a, a)

$gcd(a, a) = a$

So, R is not reflexive

Symmetric:

$(a, b) \in R \Rightarrow gcd\,(a, b) = 1$

Now check for (b, a)

$gcd\,(b, a) = gcd\,(a, b) = 1$

But $gcd\,(1, 2) = gcd\,(2, 1) = 1$

But $b \neq 2a$, So R is not symmetric.

Transitive:

Consider, $gcd(2, 3) = 1$

$\Rightarrow (2, 3) \in R$

Now, $gcd\,(3, 4) = 1$

$\Rightarrow (3, 4) \in R$

Again, $gcd\,(2, 4) = 2 \neq 1$

$\Rightarrow (2, 4) \notin R$

$\Rightarrow$ R is not Transitive.

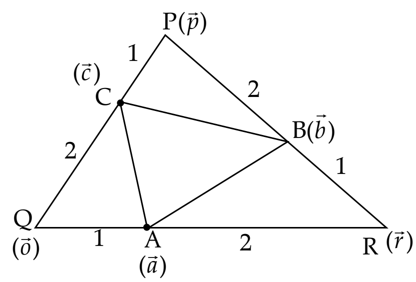
HINT:

(1) R is reflexive if $(a, a) \in R\ \forall\ a \in A$

(2) If $a, b \in A, (a, b) \in R \Rightarrow (b, a) \in R$, then R is symmetric.

(3) If $a, b, c \in A, (a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$, then R is transitive.

69. Option (B) is correct.



Let position vector of Q, P, and R be $\vec{o}, \vec{p}$ and $\vec{r}$ respectively.

Again, let position vector of points A, B, C be $\vec{a}, \vec{b}$ & $\vec{c}$ respectively.

Using section formula

$$\vec{a} = \frac{\vec{r}}{3}, \vec{b} = \frac{\vec{p}+2\vec{r}}{3}, \vec{c} = \frac{2\vec{p}}{3}$$

$$\text{Area of } \Delta PQR = \left|\frac{1}{2}|\overline{QP}\times\overline{QR}|\right| = \frac{1}{2}|\vec{r}\times\vec{p}|$$

$$\text{Area of } \Delta ABC = \frac{1}{2}|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|$$

$$= \frac{1}{2}\left|\left(\frac{\vec{r}}{3}\times\left(\frac{\vec{p}+2\vec{r}}{3}\right)\right) + \left(\frac{\vec{p}+2\vec{r}}{3}\right)\times\left(\frac{2\vec{p}}{3}\right) + \left(\frac{2\vec{p}}{3}\times\frac{\vec{r}}{3}\right)\right|$$

$$= \frac{1}{2}\left|\frac{\vec{r}\times\vec{p}}{9} + 4\left(\frac{\vec{r}\times\vec{p}}{9}\right) + \frac{2}{9}(\vec{p}\times\vec{r})\right|, \text{ as } \vec{r}\times\vec{r} = 0$$

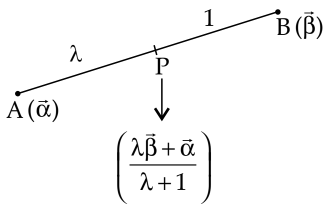
$$= \frac{1}{18}|\vec{r}\times\vec{p} + 4(\vec{r}\times\vec{p}) - 2(\vec{r}\times\vec{p})| \text{ as } \vec{p}\times\vec{r} = -(\vec{r}\times\vec{p})$$

$$= \frac{|\vec{r}\times\vec{p}|}{6}$$

So, $\dfrac{\text{area of } \Delta PQR}{\text{area of } \Delta ABC} = 3$

HINT:

(1)



(2) Think of calculating the area of triangle using cross product.

70. Option (A) is correct.

$x^3 dy + (xy - 1)\,dx = 0$

Also, $y\left(\dfrac{1}{2}\right) = 3 - e \;\&\; x > 0$

Now, $x^3\dfrac{dy}{dx} + xy - 1 = 0$

$\Rightarrow x^3\dfrac{dy}{dx} = 1 - xy$

$\Rightarrow x^3\dfrac{dy}{dx} + xy = 1$

$\Rightarrow \dfrac{dy}{dx} + \left(\dfrac{1}{x^2}\right)y = \dfrac{1}{x^3}$

This is a linear differential equation

I.F. $= e^{\int \frac{1}{x^2}dx} = e^{\frac{-1}{x}}$

So, differential equation becomes,

$ye^{\frac{-1}{x}} = \int e^{\frac{-1}{x}}.\dfrac{1}{x^3}dx$

Put $\dfrac{-1}{x} = t \;\Rightarrow\; \dfrac{1}{x^2}dx = dt$

R.H.S. $= \int -\underbrace{t}_{\text{I}}\underbrace{e^t}_{\text{II}}dt$

Integrating by parts

R.H.S. $= -\,[te^t - \int e^t dt] \;= -te^t + e^t + c$

So, solution of differential equation is

$ye^{\frac{-1}{x}} = -e^{\frac{-1}{x}}\left(\dfrac{-1}{x} - 1\right) + c$

$\Rightarrow y = \left(\dfrac{1}{x} + 1\right) + ce^{\frac{1}{x}}$

At $x = \dfrac{1}{2}, y = 3 - e$

$\Rightarrow 3 - e = \dfrac{1}{\left(\frac{1}{2}\right)} + 1 + ce^{\frac{1}{\left(\frac{1}{2}\right)}}$

$\Rightarrow 3 - e = (2 + 1) + ce^2$

$\Rightarrow ce^2 = -e$

$\Rightarrow c = \dfrac{-1}{e}$

So, $y = \left(\dfrac{1}{x} + 1\right) + \left(-\dfrac{1}{e}\right)e^{\frac{1}{x}}$

Now, at $x = 1$, $y = \left(\dfrac{1}{1} + 1\right) + \left(\dfrac{-1}{e}\right)e^{\frac{1}{1}}$

$\Rightarrow y = 2 - 1 = 1$

HINT:

(1) Convert given differential equation to linear differential equation & find its integrating factor to obtain the solution

(2) $\int \underbrace{f(x)}_{\text{I}}\underbrace{g(x)}_{\text{II}}\,dx = (\int g(x)dx)f(x) - \int f'(x)(\int g(x)dx)dx$

Using integration by parts

71. Option (D) is correct.

Given, $A^2 + B = A^2B$ ...(1)

$\Rightarrow A^2 - A^2B + B = 0$

$\Rightarrow A^2(I - B) - (-B + I - I) = 0$

$\Rightarrow A^2(I - B) - (I - B) = -I$

$\Rightarrow (A^2 - I)(I - B) = -I$

$\Rightarrow (I - A^2)(I - B) = I$

$\Rightarrow (I - B)(I - A^2) = I$

$\Rightarrow I - B - A^2 + BA^2 = I$

$\Rightarrow BA^2 - B - A^2 = 0$ ...(2)

$(1) + (2) \Rightarrow A^2B = BA^2$

HINT:

(1) $-(-A) = A$ **(2)** $A(BC^2) = ABC^2$

(3) $(A^2B)C = A^2BC$

72. Option (D) is correct.

$x^2 - 4x + [x] + 3 = x[x]$, where [.] = GIF

$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$

$\Rightarrow (x - 1)(x - 3) = (x - 1)[x]$

$\Rightarrow (x - 1)(x - 3 - [x]) = 0$

$\Rightarrow x = 1, x - 3 = [x]$

$\Rightarrow x = 1, x - [x] = 3$

$\Rightarrow x = 1, \{x\} = 3$, where {.} = Fractional part function

But $\{x\} \in [0, 1)$

So, $\{x\} \neq 3$

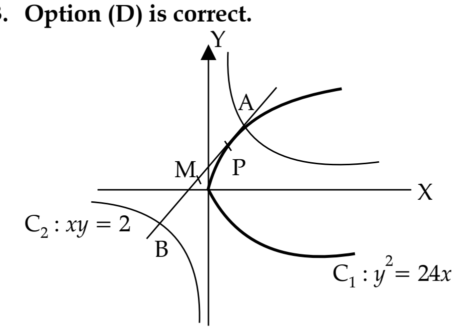
$\Rightarrow x = 1$

HINT:

(1) Take terms containing GIF on R.H.S. & factorize

(2) For $y = \{x\}$, where {.} = FPF, $y \in [0, 1)$

73. Option (D) is correct.



$P \equiv (at^2, 2at)$, here $a = 6$

$\Rightarrow P \equiv (6t^2, 12t)$

Tangent to parabola at P(t)

$\Rightarrow ty = x + 6t^2$ ...(1)

Let M (h, k) be the mid-point of chord AB to hyperbola $xy = 2$

$AB \equiv \frac{x}{h} + \frac{y}{k} = 2$ ...(2)

Comparing (1) & (2), we get

$$\frac{-1}{\left(\frac{1}{h}\right)} = \frac{t}{\left(\frac{1}{k}\right)} = \frac{6t^2}{2}$$

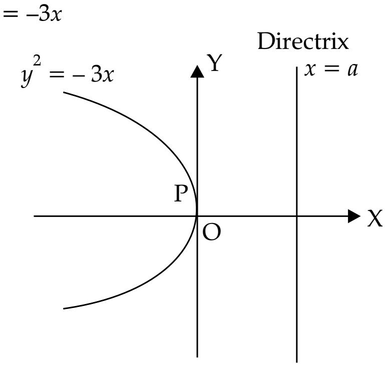
$\Rightarrow -h = tk = 3t^2$

$\Rightarrow h = -3t^2, k = 3t$

So, $h = -3\left(\frac{k}{3}\right)^2$

$\Rightarrow k^2 = -3h$

$\Rightarrow y^2 = -3x$



But $4a = 3$

$\Rightarrow a = \frac{3}{4}$

$\therefore$ Directrix is $x = \frac{3}{4}$

HINT:

(1) Write equation of tangent to parabola in parametric form.

(2) Write equation of chord to rectangular hyperbola $xy = c^2$ whose middle point is given as (h, k).

74. Option (A) is correct.

Let $\Omega \equiv \{1, 2, 3, 4, 5, 6\}$ i.e., they are outcome of throwing a dice.

Let A $\equiv$ Getting a number 7

Now, $P(A) = \frac{\text{Favourable cases}}{\text{Total cases}} = \frac{0}{6} = 0$

But $A = \phi$

$\Rightarrow S_2$ is true.

B $\equiv$ Getting a number < 7

$P(B) = \frac{\text{Favourable cases}}{\text{Total cases}}$

$\Rightarrow P(B) = \frac{6}{6} = 1$

Since, $B = \Omega$

$\Rightarrow S_2$ is true.

HINT:

(1) Think of a case that can never happen.

(2) Think of a case that will always happen.

75. Option (B) is correct.

Let $S = \sum_{r=0}^{22} ({}^{22}C_r)({}^{23}C_r)$

Consider, $(1 + x)^{22} = {}^{22}C_0 x^0 + {}^{22}C_1 x^1 + \ldots + {}^{22}C_{22} x^{22}$ ...(1)

Again, $(x + 1)^{23} = {}^{23}C_0 x^{23} + {}^{23}C_1 x^{22} + \ldots + {}^{23}C_{23} x^0$ ...(2)

(1) × (2) gives

$(1 + x)^{45} = ({}^{22}C_0 x^0 + {}^{22}C_1 x^1 + \ldots + {}^{22}C_{22} x^{22}) \times ({}^{23}C_0 x^{23} + {}^{23}C_1 x^{22} + \ldots + {}^{23}C_{23} x^0)$ ...(3)

Again, $S = \sum_{r=0}^{22} ({}^{22}C_{22-r})({}^{23}C_r)$,

Using ${}^nC_r = {}^nC_{n-r}$

$\Rightarrow S = ({}^{22}C_{22})({}^{23}C_0) + ({}^{22}C_{21})({}^{23}C_1) + \ldots + ({}^{22}C_0)({}^{23}C_{22})$

From (3), comparing coefficient of x^{23} on both sides

or $S = {}^{45}C_{23}$

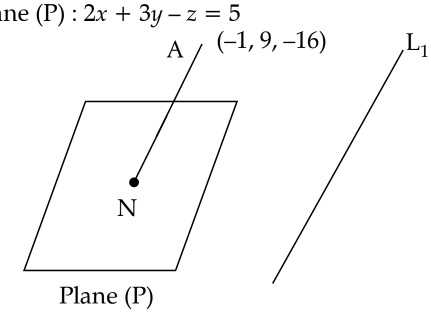
HINT:

(1) Use ${}^nC_r = {}^nC_{n-r}$

(2) Write expansion of $(1 + x)^n$ & $(x + 1)^n$. Multiply both series and compare coefficient of power of x to obtain the sum.

76. Option (D) is correct.

Plane (P) : $2x + 3y - z = 5$



where $A \equiv (-1, 9, -16)$ & $L_1 \equiv \frac{x+4}{3} = \frac{y-2}{-4} = \frac{z-3}{12}$

Equation of line AN is,

$$\frac{x-(-1)}{3} = \frac{y-9}{-4} = \frac{z-(-16)}{12} = \lambda \text{ (say)}$$

$\Rightarrow x = 3\lambda - 1, y = -4\lambda + 9, z = 12\lambda - 16$

This point lies on plane (P)

$\Rightarrow 2(3\lambda - 1) + 3(-4\lambda + 9) - (12\lambda - 16) = 5$

$\Rightarrow -18\lambda + 41 = 5 \Rightarrow \lambda = 2$

So, $N \equiv (3(2) - 1, -4(2) + 9, 12(2) - 16)$

$\Rightarrow N \equiv (5, 1, 8)$

$$AN = \sqrt{(5-(-1))^2+(1-9)^2+(8-(-16))^2}$$

$$= \sqrt{36+64+576} = \sqrt{676} = 26$$

HINT:

(1) Write equation of line passing through (–1, 9, –16) & parallel to $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$

(2) Find point where it intersects the plane and then the required distance.

77. Option (A) is correct.

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}(\sqrt{3}+1)}\right) + \sec^{-1}\left(\sqrt{\frac{4(2+\sqrt{3})}{3(2+\sqrt{3})}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{4}{3}}\right)$$

$$= \frac{\pi}{6} + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$$

HINT:

Take common & simplify the given expressions.

78. Option (D) is correct.

Given: $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$

As we know function is said to be continuous at a point if limiting value of the function at that point is equal to the functional value of the function at that point.

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0 = f(0)$

$\therefore$ $f(x)$ is continuous at $x = 0$.

As we know function is said to be differentiable at a point if LHD = RHD at that point.

Now, L.H.D. at $x = 0$, $f'(0^-) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h}$

$$= \lim_{h\to 0} \frac{-h^2 \sin\left(\frac{1}{h}\right) - 0}{-h} = \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$$

Now, R.H.D. at $x = 0$, $f'(0^+) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$$

$\because$ L.H.D. = R.H.D. at $x = 0$

$\therefore$ $f(x)$ is differentiable at $x = 0$

Now, $f'(x) = \begin{cases} 2x \sin\left(\frac{1}{x}\right) + x^2 \cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$

$f'(x) = \begin{cases} 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$

$\because$ limit of $f'(x) = 0$ oscillates

$\therefore$ $f'(x)$ is not continuous at $x = 0$

HINT:

(1) Function $f(x)$ is said to be continuous at a point $x = a$ if $\lim_{x\to a} f(x) = f(a)$

(2) Function is said to be differentiable at the point if L.H.D. = R.H.D. at that point.

79. Option (C) is correct.

As we know $A \Rightarrow B \equiv \sim A \vee B$

So, $\sim(\sim P \wedge Q) \Rightarrow (\sim(P \vee Q)$

$= \sim[\sim(\sim P \wedge Q)] \vee (\sim P \vee Q)$

$= (\sim P \wedge Q) \vee (\sim P \vee Q)$

$= [\sim P \vee (\sim P \vee Q)] \wedge [Q \vee (\sim P \vee Q)]$

$= [\sim P \vee Q] \wedge [\sim P \vee Q] \equiv \sim P \vee Q$

HINT:

(1) Use $A \Rightarrow B = \sim A \vee B$

(2) Use $\sim(A \vee B) = \sim A \wedge \sim B$

80. Option (C) is correct.

As we know equation of plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \end{vmatrix} = 0$$

So, equation of plane passing through (2, –3, 1), (–1, 1, –2) and (3, –4, 2) is given by

$$\begin{vmatrix} x-2 & y+3 & z-1 \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(4-3) - (y+3)(-3+3) + (z-1)(3-4) = 0$

$\Rightarrow x - 2 - z + 1 = 0$

$\Rightarrow x - z - 1 = 0$

Now, distance of the point (7, –3, –4) from plane $x - z - 1 = 0$ is

$$d = \left|\frac{7-(-4)-1}{\sqrt{2}}\right| \Rightarrow d = 5\sqrt{2}$$

HINT:

(1) Equation of plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \end{vmatrix} = 0$$

(2) Distance of the point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

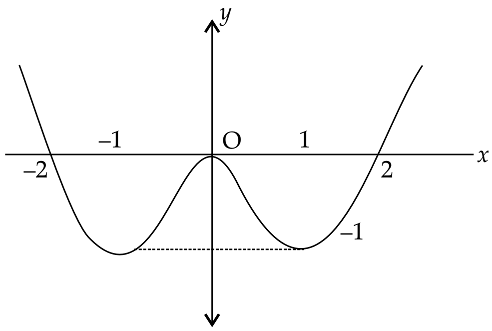
81. The correct answer is (5).

Given, E : $|x|^2 - 2|x| + |\lambda - 3| = 0$

$S = \{x + \lambda : x$ is an integer solution of E$\}$.

So, $|x|^2 - 2|x| = -|\lambda - 3|$

Lets draw the graph of $f(x) = |x|^2 - 2|x|$



It is clear from the figure, $-1 \le |x|^2 - 2|x| < \infty$ and $-|\lambda - 3| \le 0$

So, given equation holds only when $|\lambda - 3| \le 1$ and $x \in [-2, 2]$

$\Rightarrow -1 \le \lambda - 3 \le 1 \Rightarrow 2 \le \lambda \le 4$

For $x = 0, \lambda = 3$

For $x = \{-1, 1\}, \lambda = 4$ or 2

For $x = \{-2, 2\}, \lambda = 3$

So, largest element in the set S is 5

HINT:

(1) Write given equation as $|x|^2 - 2|x| = -|\lambda - 3|$ and draw the graph of $|x|^2 - 2|x|$ and analyse further using the concept of modulus function.

(2) Quadratic function $f(x) = x^2 + bx + c$; $a > 0$ has minimum value at $x = \frac{-b}{2a}$.

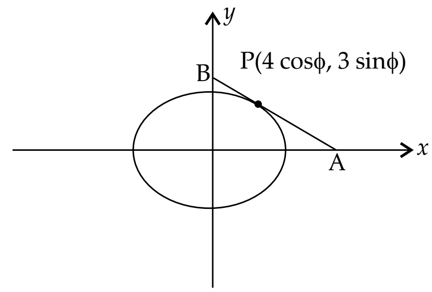
82. The correct answer is (7).

Given equation of curve is $9x^2 + 16y^2 = 144$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$$

As we know equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point $(a\cos\phi, b\sin\phi)$ is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$

So, equation of tangent to given ellipse at $(4\cos\phi, 3\sin\phi)$ is $\frac{x}{4}\cos\phi + \frac{y}{3}\sin\phi = 1$



So, coordinates of A = $(4\sec\phi, 0)$

Coordinates of B = $(0, 3\,\text{cosec}\,\phi)$

Now, AB = $\sqrt{16\sec^2\phi + 9\,\text{cosec}^2\phi}$

$\Rightarrow$ AB = $\sqrt{16(1+\tan^2\phi) + 9(1+\cot^2\phi)}$

$\Rightarrow$ AB = $\sqrt{25 + (4\tan\phi)^2 + (3\cot\phi)^2}$

$\Rightarrow$ AB = $\sqrt{25 + (4\tan\phi - 3\cot\phi)^2 + 24}$

$\Rightarrow (AB)_{min} = \sqrt{25 + 24} = 7$

HINT:

(1) Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point $(a\cos\phi, b\sin\phi)$ is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$

(2) Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$.

83. The correct answer is (14).

Given lines $L_1 : \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$

$L_2 : \frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$

L_1 can be written as in vector from

$\vec{r} = (2\hat{i} - \hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 2\hat{k})$

L_2 can be written as in vector from

$\vec{r} = (6\hat{i} + \hat{j} - 8\hat{k}) + \mu(3\hat{i} - 2\hat{j})$

As we know shortest distance between two lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$$

Here, $\vec{a} = 2\hat{i} - \hat{j} + 6\hat{k}$,

$\vec{b} = 6\hat{i} + \hat{j} - 8\hat{k}$,

$\vec{p} = 3\hat{i} + 2\hat{j} + 2\hat{k}$,

$\vec{q} = 3\hat{i} - 2\hat{j}$

Now, $\vec{p}\times\vec{q} = \begin{vmatrix} i & j & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}$

$\Rightarrow \vec{p}\times\vec{q} = 4\hat{i}+6\hat{j}-12\hat{k} = 2(2\hat{i}+3\hat{j}-6\hat{k})$

Now, $\vec{b}-\vec{a} = 4\hat{i}+2\hat{j}-14\hat{k} = 2(2\hat{i}+\hat{j}-7\hat{k})$

So, $(\vec{b}-\vec{a}).(\vec{p}\times\vec{q}) = 4[4+3+42] = 196$

And $|\vec{p}\times\vec{q}| = 2\sqrt{4+9+36} = 14$

$\therefore$ Shortest distance between given lines is $d = \left|\frac{196}{14}\right| = 14$

HINT:

(1) Write the given equation of line in vector form and use distance between two lines $\vec{r} = \vec{a}+\lambda\vec{p}$ and $\vec{r} = \vec{b}+\mu\vec{q}$ is given by $d = \left|\frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

(2) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

84. The correct answer is (1012).

Given: $\sum_{r=0}^{2023} r^2\, {}^{2023}C_r = 2023\times\alpha\times 2^{2022}$

Let A = $\sum_{r=0}^{2023} r^2\, {}^{2023}C_r$

$\Rightarrow$ A = $\sum_{r=0}^{2023} r^2 \frac{2023}{r}\, {}^{2022}C_{r-1}$ $\left\{\because {}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}\right\}$

$\Rightarrow$ A = $\sum_{r=1}^{2023} r(2023)\, {}^{2022}C_{r-1}$

$\Rightarrow$ A = $2023\left\{\sum_{r=1}^{2023}(r-1)\, {}^{2022}C_{r-1} + \sum_{r=1}^{2023} {}^{2022}C_{r-1}\right\}$

$\Rightarrow$ A = 2023 $\{2022\ 2^{2021} + 2^{2022}\}$

$\{\because {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + ... + n\,{}^nC_n = n\,2^{n-1}$ and ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n\}$

$\Rightarrow$ A = $2023 \times 2022 \times 2^{2021} + 2^{2022} \times 2023$

$\Rightarrow$ A = $2023 \times 2^{2022}(1011 + 1)$

$\Rightarrow$ A = $1012 \times 2023 \times 2^{2022}$

$\Rightarrow 2023 \times \alpha \times 2^{2022} = 1012 \times 2023 \times 2^{2022}$

$\Rightarrow \alpha = 1012$

HINT:

(1) Simplify given expression using ${}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}$

(2) Use ${}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + ... + n\,{}^nC_n = n2^{n-1}$

(3) Use ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

85. The correct answer is (2).

Let I = $\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ ...(1)

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\cos x)^{2023}+(\sin x)^{2023}}dx$ ...(2)

(1) + (2), 2I = $\int_0^{\frac{\pi}{2}} dx$

$\Rightarrow$ I = $\frac{1}{2}\left(\frac{\pi}{2}-0\right) \Rightarrow$ I = $\frac{\pi}{4}$

So, $\frac{8}{\pi}(\mathrm{I}) = \frac{8}{\pi}.\frac{\pi}{4} = 2$

HINT:

Use property: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

86. The correct answer is (60).

$1 \rightarrow 3$ times

$2 \rightarrow 2$ times

$3 \rightarrow 2$ times

$4 \rightarrow 2$ times

_ X _ X _ X _ X _

O E O E O E O E O

O - odd, E - even

Number of ways for even digits to occupy even places

$= \frac{4!}{2!2!} = \frac{24}{2\times 2} = 6$

Total number of 9 digit numbers = $(6)\left(\frac{5!}{3!2!}\right)$

$= (6)\left(\frac{120}{6\times 2}\right) = 60$

HINT:

Number of ways of arranging 'm' alike & 'n' distinct objects in a line are $\frac{(m+n)!}{m!}$

87. The correct answer is (546).

Total courses = 12

5 Language Courses (LA) | 7 Other Course (OT)

Number of ways

= {O(LA), 5 (OT)} + {1(LA), 4 (OT)}

+ {2 (LA), 3 (OT)}

$\Rightarrow ({}^5C_0)({}^7C_5) + ({}^5C_1)({}^7C_4) + ({}^5C_2)({}^7C_3)$

$\Rightarrow (1)({}^7C_2) + (5)({}^7C_3) + \left(\frac{5\times4}{2}\right)({}^7C_3)$

$= \left(\frac{7\times6}{2}\right)+(5)\left(\frac{7\times6\times5}{6}\right)+(10)\left(\frac{7\times6\times5}{6}\right)$

$= 21 + 175 + 350 = 546$

HINT:

(1) Make cases like (zero language, 5 other) or so on.
(2) Use ${}^nC_r = {}^nC_{n-r}$

88. The correct answer is (12).

Let first term of G.P. be 'a'.

Given, $T_4 = a\left(\frac{1}{m}\right)^3 = 500$

$\Rightarrow \frac{a}{m^3} = 500$

$\Rightarrow a = 500\, m^3$

Consider, $S_n - S_{n-1}$

$= a\left(\frac{1-r^n}{1-r}\right) - a\left(\frac{1-r^{n-1}}{1-r}\right)$, where $r = \frac{1}{m}$

$= \frac{a}{(1-r)}[1-r^n-1+r^{n-1}] = \frac{ar^{n-1}}{(1-r)}(1-r) = ar^{n-1}$

$\Rightarrow S_n - S_{n-1} = \frac{a}{m^{n-1}} \Rightarrow S_n - S_{n-1} = \frac{500m^3}{m^{n-1}}$

$\Rightarrow S_n - S_{n-1} = 500\, m^{4-n}$

Given, $S_6 - S_5 > 1$

$\Rightarrow 500\, m^{4-6} > 1$

$\Rightarrow \frac{500}{m^2} > 1$ ...(1)

Again, $S_7 - S_6 < \frac{1}{2}$

$\Rightarrow 500\, m^{4-7} < \frac{1}{2}$

$\Rightarrow \frac{500}{m^3} < \frac{1}{2}$ ...(2)

(1) $\Rightarrow m^2 < 500$

(2) $\Rightarrow m^3 > 1000$

So, $m \in \{11, 12, 13, ..., 22\}$

$\therefore$ Number of possible values of m is 12.

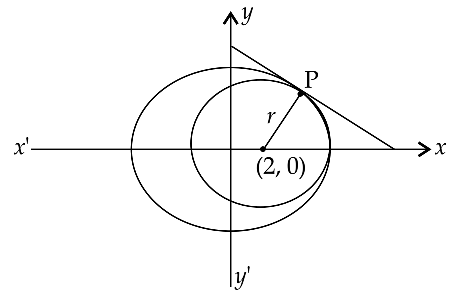
HINT:

(1) n^{th} term of a G.P. is $T_n = ar^{n-1}$, where 'a' and 'r' are first term and common ratio respectively.
(2) Sum of 'n' terms of a G.P.

$$S_n = a\left(\frac{1-r^n}{1-r}\right), \text{ if } r < 1$$

89. The correct answer is (118).

Let ellipse be E: $\frac{x^2}{36}+\frac{y^2}{16}=1$



Circle (C) $\equiv (x-2)^2 + (y-0)^2 = r^2$

For C to be largest possible circle, its radius has to be maximum.

Point P on ellipse $\equiv (6\cos\theta, 4\sin\theta)$

T = 0

Normal at P on ellipse should also be normal to circle as ellipse and circle are touching each other at P.

Normal $\equiv 6x\sec\theta - 4y\,\text{cosec}\theta = 36 - 16$

It also passes through centre of circle i.e., (2, 0)

$\Rightarrow 12\sec\theta = 20$

$\Rightarrow \cos\theta = \frac{3}{5}$

So, $\sin\theta = \frac{4}{5}$

$P \equiv \left(6\times\frac{3}{5}, 4\times\frac{4}{5}\right) \equiv \left(\frac{18}{5}, \frac{16}{5}\right)$

Now, $r = \sqrt{\left(2-\frac{18}{5}\right)^2+\left(0-\frac{16}{5}\right)^2}$

$\Rightarrow r = \sqrt{\frac{64}{25}+\frac{256}{25}}$

$\Rightarrow r = \frac{\sqrt{320}}{5} = \frac{8\sqrt{5}}{5} \Rightarrow r = \frac{8}{\sqrt{5}}$

Now $(1, \alpha)$ lies on c.

$\therefore \sqrt{(2-1)^2+(0-\alpha)^2} = \frac{8}{\sqrt{5}}$

$1 + \alpha^2 = \frac{64}{5} \Rightarrow \alpha^2 = \frac{64}{5} - 1 = \frac{59}{5}$

$10\alpha^2 = \frac{59}{5} \times 10 = 118$

HINT:

(1) Think of common normal and remember that normal of circle passes through its centre.
(2) Normal to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at P $(a\cos\theta, b\sin\theta)$ is $(a\sec\theta)\,x - (b\,\text{cosec}\theta)\,y = a^2 - b^2$
(3) Finally find radius of circle and make it equal to the distance between $(1, \alpha)$ and (2, 0).

90. The correct answer is (22).

Let $I = \int_0^3 |x^2 - 3x + 2| dx$

$$|x^2 - 3x + 2| = \begin{cases} x^2 - 3x + 2 & x \in (-\infty, 1] \cup [2, \infty) \\ -(x^2 - 3x + 2), & x \in (1, 2) \end{cases}$$

So, $I = \int_0^1 (x^2 - 3x + 2) dx - \int_1^2 (x^2 - 3x + 2) dx + \int_2^3 (x^2 - 3x + 2) dx$

$$\Rightarrow I = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3$$

$$\Rightarrow I = \left[\frac{1}{3} - \frac{3}{2} + 2\right] - \left[\left(\frac{8}{3} - \frac{1}{3}\right) - \frac{3}{2}(4-1) + 2(2-1)\right] + \left[\left(\frac{27}{3} - \frac{8}{3}\right) - \frac{3}{2}(9-4) + 2(3-2)\right]$$

$$\Rightarrow I = \frac{5}{6} + \frac{1}{6} + \frac{5}{6} = \frac{11}{6}$$

So, $12I = 12\left(\frac{11}{6}\right) = 22$

HINT:

(1) $|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$

(2) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

JEE (Main) SOLVED PAPER

2023
24^{th} Jan Shift 2

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion: A pendulum clock when taken to Mount Everest becomes fast.

Reason: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) A is not correct but R is correct

Q. 2. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $\nu = r^{a} \rho^{b} s^{c}$. The values of a, b and c respectively are

(1) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$ **(2)** $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$
(3) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ **(4)** $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

Q. 3. Given below are two statements:

Statement I : Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II : Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if $h = d$.

In the light of above statements, choose the most appropriate answer from the options given below

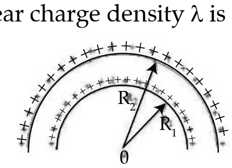
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but statement II is correct
(3) Both Statement I and II are correct
(4) Statement I is correct but statement II is incorrect

Q. 4. A long solenoid is formed by winding 70 turns cm^{-1} . If 2.0 A current flows, then the magnetic field produced inside the solenoid is __________

$[\mu_0 = 4\pi \times 10^{7}\ TmA^{-1}]$

(1) 88×10^{-4} T **(2)** 352×10^{-4} T
(3) 176×10^{-4} T **(4)** 1232×10^{-4} T

Q. 5. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is :



(1) $\frac{\lambda}{2\varepsilon_0}$ **(2)** $\frac{\lambda}{4\varepsilon_0}$
(3) $\frac{2l}{e_0}$ **(4)** $\frac{\lambda}{\varepsilon_0}$

Q. 6. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is :

(1) 6×10^6 km **(2)** 3×10^6 km
(3) 3×10^7 km **(4)** 6×10^7 km

Q. 7. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is (given, $h = 4 \times 10^{-15}$eVs) :

(1) 99.3 nm **(2)** 941 nm
(3) 974 nm **(4)** 94.1 nm

Q. 8. A cell of emf 90 V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be:

(1) 90 V **(2)** 45 V
(3) 80 V **(4)** 40 V

Q. 9. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be:

(1) −1 **(2)** 3
(3) 2 **(4)** 1

Q. 10. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

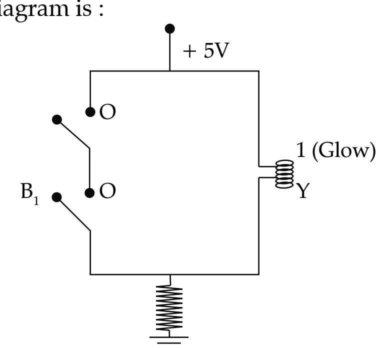
$E_x = E_0 \sin(kz - \omega t)$

$B_y = B_o \sin(kz - \omega t)$

Then the correct relation between E_o and B_o is given by

(1) $E_0 B_0 = \omega k$ **(2)** $E_0 = kB_0$
(3) $kE_0 = \omega B_0$ **(4)** $\omega E_0 = kB_0$

Q. 11. The logic gate equivalent to the given circuit diagram is :



(1) NAND **(2)** OR
(3) AND **(4)** NOR

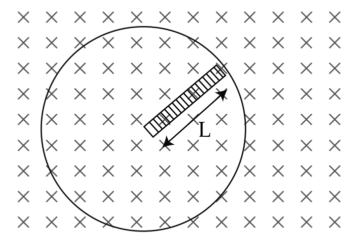
Q. 12. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is :

(1) $\frac{25}{21}$ **(2)** $\frac{35}{27}$
(3) $\frac{21}{25}$ **(4)** $\frac{27}{35}$

Q. 13. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called:

(1) Spherical aberration
(2) Polarisation
(3) Chromatic aberration
(4) Scattering

Q. 14. A metallic rod of length ' L ' is rotated with an angular speed of ' ω ' normal to a uniform magnetic field ' B ' about an axis passing through one end of rod as shown in figure. The induced emf will be:



(1) $\frac{1}{4} B L^2 \omega$ **(2)** $\frac{1}{2} B^2 L^2 \omega$
(3) $\frac{1}{4} B^2 L \omega$ **(4)** $\frac{1}{2} B L^2 \omega$

Q. 15. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength:

(1) $\lambda_a < \lambda_p < \lambda_e$ **(2)** $\lambda_a = \lambda_p = \lambda_e$
(3) $\lambda_a > \lambda_p > \lambda_e$ **(4)** $\lambda_a > \lambda_p < \lambda_e$

Q. 16. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason

Assertion A : Steel is used in the construction of buildings and bridges.

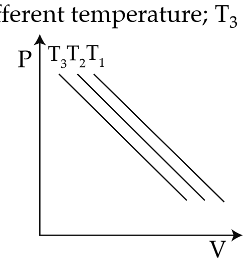
Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below

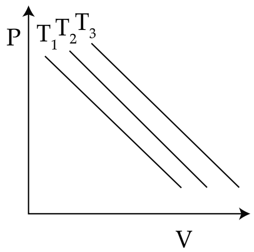
(1) Both A and R are correct and R is the correct explanation of A
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is correct but R is not correct
(4) A is not correct but R is correct

Q. 17. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as:

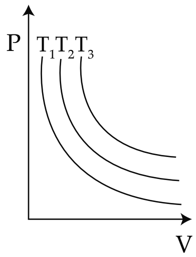
(1)



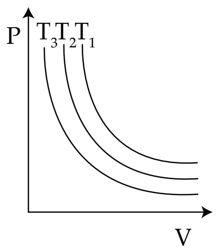
(2)



(3)



(4)



Q. 18. Match List I with List II

	List I		List II
A.	AM Broadcast	**I.**	88 – 108MHz
B.	FM Broadcast	**II.**	540 – 1600kHz
C.	Television	**III.**	3.7 – 4.2GHz
D.	Satellite Communication	**IV.**	54MHz – 890MHz

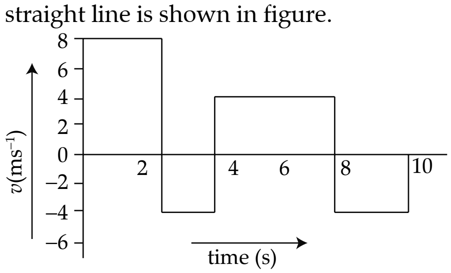
Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III
(2) A-I, B-III, C-II, D-IV
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-III, C-I, D-IV

Q. 19. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5rad/s. Then the ratio of extension in the spring to its natural length will be :

(1) 2:5 **(2)** 1: 1
(3) 2: 3 **(4)** 1: 2

Q. 20. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10 s is :

(1) 1: 1 **(2)** 1: 2
(3) 1: 3 **(4)** 1: 4

Section B

Q. 21. A body of mass 1 kg begins to move under the action of a time dependent force, $\vec{F} = (t\hat{i} + 3t^2\,\hat{j})$ N where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2s$, will be _________ W.

Q. 22. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be ________ cm

(Given refractive index of water $= \frac{4}{3}$)

Q. 23. The energy released per fission of nucleus of ^{240}X is 200MeV. The energy released if all the atoms in 120 g of pure ^{240}X undergo fission is ______ $\times 10^{25}$MeV (Given $N_A = 6 \times 10^{23}$)

Q. 24. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2}$ = _______ (Both I_1 and I_2 are about the axis of the cylinder)

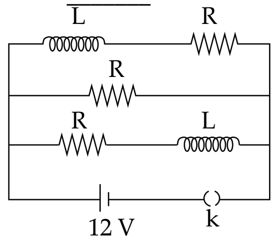
Q. 25. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$ pF. The value of x is _________

Q. 26. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of x is __________

Q. 27. A mass m attached to free end of a spring executes SHM with a period of 1 s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass m is _____ kg.

Q. 28. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is _________ %

Q. 29. Three identical resistors with resistance R = 12Ω and two identical inductors with self inductance L = 5mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be _______ A.



Q. 30. A Spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is (Given, $g = 9.8$ m/s ^{2} and $\pi = \frac{22}{7}$)

Chemistry

Section A

Q. 31. Identify the correct statements about alkali metals.

A. The order of standard reduction potential ($M^+ \mid M$) for alkali metal ions is Na > Rb > Li.
B. CsI is highly soluble in water.
C. Lithium carbonate is highly stable to heat.
D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.
E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below:

(1) C and E only **(2)** A, B and E only
(3) A, B, D only **(4)** A and E only

Q. 32. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is not correct but R is correct
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) Both A and R are correct but R is NOT the correct explanation of A

Q. 33. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p(mmHg)	50	100	200	400
relative $t_{1/2}$(s)	4	2	1	0.5

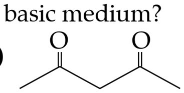
The order of the reaction is

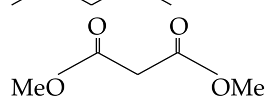
(1) 0 (zero) **(2)** 0.5
(3) 1 **(4)** 2

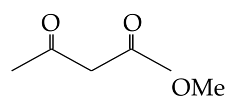
Q. 34. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to

(1) Carbon dioxide **(2)** Sulphur trioxide
(3) Sulphur dioxide **(4)** Hydrogen sulphide

Q. 35. Which will undergo deprotonation most readily in basic medium?

(a) 

(b) 

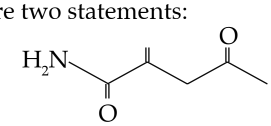
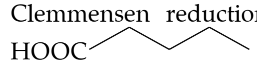
(c) 

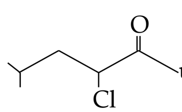
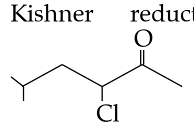
(1) c only **(2)** a only
(3) Both a and c **(4)** b only

Q. 36. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is

(1) d^2sp^3 and paramagnetic
(2) sp^3d^2 and diamagnetic
(3) d^2sp^3 and diamagnetic
(4) sp^3d^2 and paramagnetic

Q. 37. Given below are two statements:

Statement 1:  under Clemmensen reduction conditions will give 

Statement 2:  under Wolff-Kishner reduction condition will give 

In the light of the above statements, choose the correct answer from the options given below:

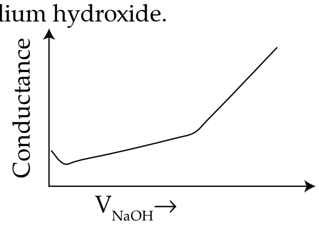
(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

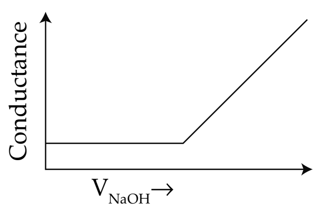
Q. 38. Which of the following cannot be explained by crystal field theory?
(1) The order of spectrochemical series
(2) Stability of metal complexes
(3) Magnetic properties of transition metal complexes
(4) Colour of metal complexes

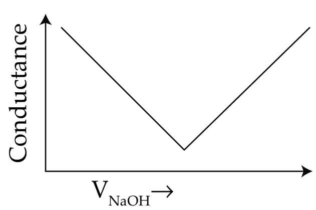
Q. 39. The number of s-electrons present in an ion with 55 protons in its unipositive state is
(1) 8 **(2)** 10
(3) 9 **(4)** 12

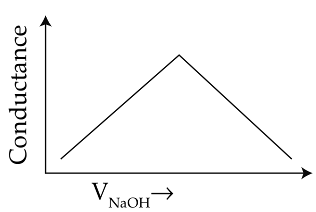
Q. 40. Which one amongst the following are good oxidizing agents?
(A) Sm^{2+} **(B)** Ce^{2+}
(C) Ce^{4+} **(D)** Tb^{4+}
Choose the most appropriate answer from the options given below:
(1) D only **(2)** C only
(3) C and D only **(4)** A and B only

Q. 41. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

(1)


(2)


(3)


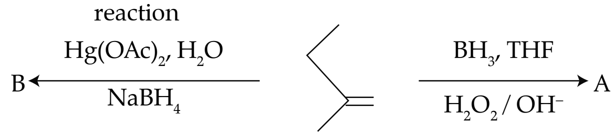
(4)


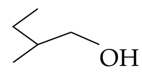
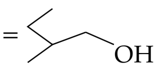
Q.42. Match List I with List II

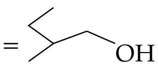
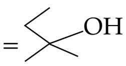
	List I Type		List II Name
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III.	Seldane
D.	Antibiotic	IV.	Ampicillin

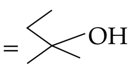
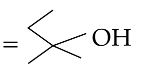
Choose the correct answer from the options given below:
(1) A-I, B-III, C-II, D-IV
(2) A-IV, B-III, C-II, D-I
(3) A-I, B-II, C-III, D-IV
(4) A-II, B-I, C-III, D-IV

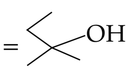
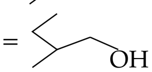
Q 43. Find out the major products from the following reaction



(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Q 44. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R
Assertion: Benzene is more stable than hypothetical cyclohexatriene
Reason : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are correct and R is the correct explanation of A
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is false but R is true
(4) A is true but *R* is false

Q. 45. In which of the following reactions the hydrogen peroxide acts as a reducing agent?
(1) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$
(2) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$
(3) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$
(4) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

Q. 46. Given below are two statements:
Statement I : Pure Aniline and other arylamines are usually colourless.
Statement II : Arylamines get coloured on storage due to atmospheric reduction
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are correct

Q. 47. Correct statement is:
(1) An average human being consumes nearly 15 times more air than food

(2) An average human being consumes 100 times more air than food

(3) An average human being consumes equal amount of food and air

(4) An average human being consumes more food than air

Q. 48. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : N_2; N_2^+ ; O_2; O_2^+ ?

(1) 2,1,0,1 **(2)** 0, 1, 0, 1

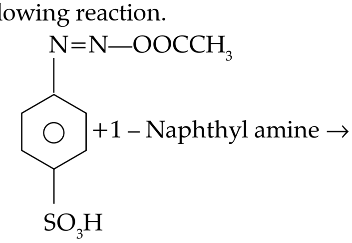
(3) 0,1,0,1 **(4)** 2,1,2,1

Q. 49. The metal which is extracted by oxidation and subsequent reduction from its ore is:

(1) Ag **(2)** Fe

(3) Cu **(4)** Al

Q. 50. Choose the correct colour of the product for the following reaction.

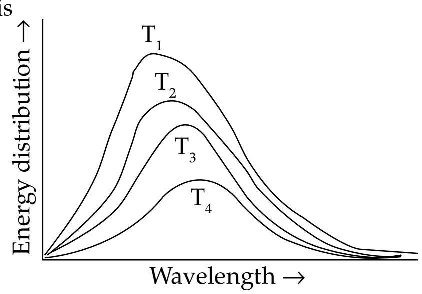


(1) White **(2)** Red

(3) Blue **(4)** Yellow

Section B

Q. 51. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statement/s from the following is



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{v_1} = \frac{T_2}{v_2} = \frac{T_3}{v_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Q. 52. The number of units, which are used to express concentration of solutions from the following is______ Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

Q. 53. The number of statement/s which are the characteristics of physisorption is______________

A. It is highly specific in nature

B. Enthalpy of adsorption is high

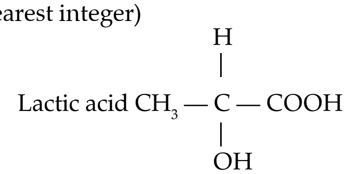
C. It decreases with increase in temperature

D. It results into unimolecular layer

E. No activation energy is needed

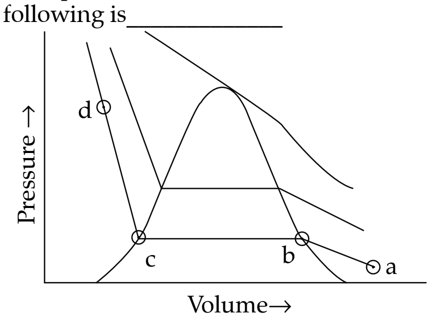
Q. 54. Sum of π - bonds present in peroxodisulphuric acid and pyrosulphuric acid is:

Q. 55. If the pKa of lactic acid is 5, then the pH of 0.005M calcium lactate solution at 25°C is ______ $\times 10^{-1}$ (Nearest integer)



Q. 56. The total pressure observed by mixing two liquids A and B is 350 mmHg when their mole fractions are 0.7 and 0.3 respectively. The total pressure become 410 mmHg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is________ mm Hg. (Nearest integer) Consider the liquids and solutions behave ideally.

Q. 57. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is_____________



A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

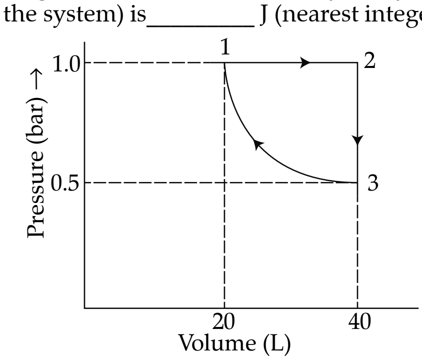
C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) to (c), the amount of liquid decreases

Q. 58. Maximum number of isomeric monochloro derivatives which can be obtained from 2, 2, 5, 5 tetramethylhexane by chlorination is ____________.

Q. 59. Total number of tripeptides possible by mixing of valine and proline is____________

Q. 60. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is_________ J (nearest integer)



Mathematics

Section A

Q. 61. If, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in \mathbb{R}$ then

(A) $f(1) + f(2) + f(3) = f(0)$
(B) $2f(0) - f(1) + f(3) = f(2)$
(C) $3f(1) + f(2) = f(3)$
(D) $f(3) - f(2) = f(1)$

Q. 62. If the system of equations

$$x + 2y + 3z = 3$$
$$4x + 3y - 4z = 4$$
$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to:

(A) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ **(B)** $\left(-\frac{72}{5}, -\frac{21}{5}\right)$
(C) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ **(D)** $\left(\frac{72}{5}, \frac{21}{5}\right)$

Q. 63. If, $f(x) = \frac{2^{2x}}{2^{2x} + 2}, x \in R$, then

$f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots + f\left(\frac{2022}{2023}\right)$ is equal to

(A) 1011 **(B)** 2010
(C) 1010 **(D)** 2011

Q. 64. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 . (\hat{i} + \hat{j} + \hat{k})$ is

(A) 7 **(B)** 9
(C) 6 **(D)** 11

Q. 65. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)\, dx + 3xy dy = 0, y(1) = 1$. Then $6y^2(e)$ is equal to

(A) $2e^2$ **(B)** $3e^2$
(C) e^2 **(D)** $\frac{3}{2}e^2$

Q. 66. The locus of the mid points of the chords of the circle C_1: $(x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$
(C) $\frac{\pi}{6}$ **(D)** $\frac{3\pi}{4}$

Q. 67. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(A) 0 **(B)** 3
(C) 4 **(D)** 2

Q. 68. Let A be a 3 × 3 matrix such that $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$ Then $|A^{-1}\text{adj}A|$ is equal to

(A) $\sqrt{6}$ **(B)** $2\sqrt{3}$
(C) 12 **(D)** 1

Q. 69. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}} dx$

(A) 2π **(B)** $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

Q. 70. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(A) 125 **(B)** 225
(C) 150 **(D)** 120

Q. 71. If $({}^{30}C_1)^2 + 2\,({}^{30}C_2)^2 + 3\,({}^{30}C_3)^2 + \ldots. + 30\,({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to:

(A) 30 **(B)** 10
(C) 60 **(D)** 15

Q. 72. Let the plane containing the line of intersection of the planes $P_1 : x + (\lambda + 4)y + z = 1$ and P_2: $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is

(A) $4\sqrt{6}$ **(B)** $3\sqrt{6}$
(C) $5\sqrt{6}$ **(D)** $2\sqrt{6}$

Q. 73. Let $f(x)$ be a function such that $f(x + y) = f(x).f(y)$ for all, $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(A) 9 **(B)** 6
(C) 8 **(D)** 7

Q. 74. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to:

(A) 210 **(B)** 220
(C) 200 **(D)** 105

Q. 75. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda) y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(A) 4 **(B)** 2
(C) $\sqrt{6}$ **(D)** $2\sqrt{2}$

Q. 76. Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

(A) $p \vee (p \wedge q)$ **(B)** $p \vee (p \wedge (\sim q))$
(C) $(\sim p) \vee q$ **(D)** $p \vee ((\sim p) \wedge q)$

Q. 77. The set of all values of a for which $\lim_{x \to a} ([x - 5] - [2x + 2]) = 0$, where $[\alpha]$ denotes the greatest integer less than or equal to α is equal to

(A) $[-7.5, -6.5)$ **(B)** $[-7.5, -6.5]$
(C) $(-7.5, -6.5]$ **(D)** $(-7.5, -6.5)$

Q. 78. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(A) 3 **(B)** 1
(C) –1 **(D)** 5

Q. 79. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(A) 168 **(B)** 220
(C) 120 **(D)** 48

Q. 80. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

(A) $-\frac{1}{2}(\sqrt{3}-i)$ **(B)** $-\frac{1}{2}(1-i\sqrt{3})$

(C) $\frac{1}{2}(1-i\sqrt{3})$ **(D)** $\frac{1}{2}(\sqrt{3}+i)$

Section B

Q. 81. If the shortest distance between the lines $\dfrac{x+\sqrt{6}}{2}=\dfrac{y-\sqrt{6}}{3}=\dfrac{z-\sqrt{6}}{4}$ and $\dfrac{x-\lambda}{3}=\dfrac{y-2\sqrt{6}}{4}=\dfrac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is

Q. 82. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is

Q. 83. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$

Then $\sum_{\theta \in s} \sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to

Q. 84. If $\dfrac{1^3+2^3+3^3+\ldots \text{ up to } n \text{ terms}}{1.3+2.5+3.7+\ldots\text{up to } n \text{ terms}}=\dfrac{9}{5}$, then the value of n is

Q. 85. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\dfrac{3}{x^2}\right)^n, x \neq 0, n \in \mathrm{N}$, be 376. Then the coefficient of x^4 is

Q. 86. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P(2, a) be the centroid of Δ ABC. Then $(BC)^2$ is equal to

Q. 87. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}, \vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$,

$\vec{a}.\vec{c}=7, 2\vec{b}.\vec{c}+43=0,\ \vec{a}\times\vec{c}=\vec{b}\times\vec{c}$.

Then $|\vec{a}.\vec{b}|$ is equal to

Q. 88. The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is

Q. 89. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to

Q. 90. Lef f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that $f(x) > 0$ and

$f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e, \forall x \in \left[0,\frac{\pi}{2}\right]$.

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

Answer Key

Physics

Q. No.	Answer	Topic Name	Chapter Name
1	4	Time Period of Simple Pendulum	Oscillations
2	3	Dimensions	Units and Measurements
3	4	Variation in Acceleration Due to Gravity	Gravitation
4	3	Solenoid	Moving Charges and Magnetism
5	1	Electric Potential due to Continuous Charge Distribution	Electrostatic Potential and Capacitance
6	2	Kepler's Law	Gravitation
7	4	Line Spectrum of Hydrogen	Atoms
8	2	Ohm's Law	Current Electricity
9	3	Vectors	Mathematical Tools
10	3	Representation of EMW	Electromagnetic Waves
11	3	Logic Gates	Communication Systems
12	1	Specific Heats	Thermodynamics
13	3	Refractive Index	Ray Optics
14	4	Motional EMF	Electromagnetic Induction
15	1	de-Broglie Relation	Dual Nature of Radiation and Matter
16	1	Elasticity	Mechanical Properties of Solids
17	3	Ideal Gas Equation	Kinetic Theory of Gases
18	1	Bandwidth of Signals	Communication Systems
19	3	Rotational Motion	System of Particle and Rotational Motion
20	3	Graphical Representation of Motion	Motion in a Line
21	[100]	Power	Work, Energy and Power
22	[54]	Lens Maker's Formula	Ray Optics
23	[6]	Nuclear Fission	Nuclei
24	[32]	Moment of Inertia	System of Particle and Rotational Motion
25	[105]	Parallel Plate Capacitor	Electrostatic Potential and Capacitance
26	[9]	Force on Current Carrying Wire Placed in Magnetic Field	Moving Charges and Magnetism
27	[1]	SHM	Oscillations
28	[44]	Resistivity	Current Electricity
29	[3]	Electric Circuit	Current Electricity
30	[7]	Terminal Velocity/ Stoke's Law	Fluid Mechanics

Chemistry

Q. No.	Answer	Topic Name	Chapter Name
31	4	Properties of s-Block	s-Block
32	3	Reduction Potential of s-Block	s-Block
33	4	Order of Reaction	Chemical kinetics

34	3	Properties of potassium dichromate	d & f Block
35	2	Deprotonation reaction in carbonyl compounds	Aldehyde and ketone
36	3	Hybridisation and magnetic behaviour of transition metal complexes	Coordination chemistry
37	2	reduction of carbonyl compounds	Aldehyde and Ketone
38	1	Crystal field theory	Coordination chemistry
39	2	Calculation of number of s electron	Periodic classification of elements
40	3	Oxidising nature of f Block	d & f Block
41	1	Conductometric titration	Electrochemistry
42	3	Classification of drugs	Chemistry in everyday life
43	2	Hydration of alkene	Hydrocarbons
44	1	Stability of benzene ring	Aromatic hydrocarbon
45	3	Properties of hydrogen peroxide	Hydrogen
46	3	Properties of nitrogen containing compounds	Amines
47	1	Introduction of balanced diet	Chemistry in everyday life
48	2	Molecular orbital theory	Chemical bonding and molecular structure
49	1	Extraction of metal	Metallurgy
50	2	Properties of aromatic amines	Amines
51	[2]	Black body radiation	Structure of atom
52	[5]	Concentration terms	Liquid solution
53	[2]	Properties of physical adsorption	Surface chemistry
54	[8]	Calculation of number of pie bonds in oxyacid of phosphorus	p-Block elements
55	[85]	pH of salt solution	Ionic Equilibrium
56	[314]	Vapour pressure of solution	Liquid solution
57	[2]	Andrews isotherm	States of matter
58	[3]	Calculation of number of isomers	Isomerism
59	[8]	Calculation of number of tripeptides	Biomoelcules
60	[620]	Calculation of work done	Thermodynamics and thermochemistry

Mathematics

Q. No.	Answer	Topic Name	Chapter Name
61	B	Higher Order Derivatives	Differential Calculus
62	C	System of linear equations	Matrices and Determinants
63	A	Algebra of Functions	Function
64	A	Scalar and Vector Products	Vector Algebra
65	A	Linear Differential Equations	Differential Equations
66	B	Interaction between Circle and a Line	Circle
67	A	Quadratic Equation and its Solution	Quadratic Equations
68	B	Adjoint of a Matrix	Matrices and Determinants
69	A	Basics of Definite Integration	Definite Integration
70	D	Permutations	Permutation and Combination
71	D	Properties of Binomial Coefficients	Binomial Theorem

72	B	Plane and a Point	Three Dimensional Geometry
73	D	Geometric Progressions	Sequences and Series
74	A	Measures of Dispersion	Statistics
75	D	Tangent to a Parabola	Parabola
76	C	Logical Operations	Mathematical Reasoning
77	D	Algebra of Limits	Limits
78	D	Lines in 3D	Three Dimensional Geometry
79	A	Permutations	Permutations and Combinations
80	A	Representation of Complex Numbers	Complex Numbers
81	[384]	Skew Lines	Three Dimensional Geometry
82	[432]	Bayes' Theorem	Probability
83	[2]	Trigonometric Equations	Trigonometric Equations and Inequalities
84	[5]	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
85	[405]	Binomial Theorem for Positive Integral Index	Binomial Theorem
86	[122]	Interaction between Two Lines	Point and Straight Line
87	[8]	Scalar and Vector Products	Vector Algebra
88	[13]	Algebra of Relations	Set Theory and Relations
89	[36]	Area Bounded by Curves	Area under Curves
90	[27]	Variable Separable Form	Differential Equations

JEE (Main) SOLVED PAPER 2023

24th Jan Shift 2

ANSWERS WITH EXPLANATIONS

Physics

Section A

1. Option (4) is correct.

As height increases, g decreases. So statement II is true.

Time period of simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

or $T \propto \frac{1}{\sqrt{g}}$, On Everest as g is less than g at surface So, time period of pendulum will increase. So statement I is false.

2. Option (3) is correct.

Given, $v = r^a \rho^b s^c$

$[T^{-1}] = [L]^a[M^1L^{-3}]^b[M^1T^{-2}]^c$

$[M^0L^0T^{-1}] = [M^{b+c}L^{a-3b}T^{-2c}]$

On comparing dimensions of both sides,

$-2c = -1 \quad \Rightarrow c = -\frac{1}{2}$

$b+c = 0$

$\Rightarrow \quad b = -c = -\frac{1}{2}$

$a-3b = 0$

$\Rightarrow \quad a = 3b \quad \Rightarrow a = -\frac{3}{2}$

3. Option (4) is correct.

Acceleration due to gravity decrease as we go up or down from the surface, but decrease in gravity with height is more or faster as compare with depth.

4. Option (3) is correct.

$B = \mu_0 ni$ (For a solenoid)

$B = 4\times\frac{22}{7}\times10^{-7}\times70\times100\times2 = 176\times10^{-4}T$

5. Option (1) is correct.

Electric potential due to circular arc (linear charge)

$V_C = V_1+V_2$

$\Rightarrow \quad = K\frac{q_1}{r_1}+K\frac{q_2}{r_2}$

$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{\lambda\pi R_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{\lambda\pi R_2}{r_2} = \frac{\lambda}{2\varepsilon_0}$

6. Option (2) is correct.

According of Kepler's law, $T^2 \propto R^3$

$\Rightarrow \quad \left(\frac{T_e}{T_p}\right)^2 = \left(\frac{R_e}{R_p}\right)^3$

$\Rightarrow \quad \left(\frac{1}{2.83}\right)^2 = \left(\frac{1.5\times10^6}{R_p}\right)^3$

$\Rightarrow \quad R_p = 1.5\times10^6\times2.83^{\frac{3}{2}}$

$\Rightarrow \quad R_p = 3\times10^6 \text{km}$

7. Option (4) is correct.

Here, $n_1 = 1$ $n_2 = 4$

$E_4 = -\frac{13.6}{n^2} = -\frac{13.6}{16} = -0.85\text{eV}$

$E_1 = -\frac{13.6}{n^2} = -\frac{13.6}{1} = -13.6\text{eV}$

Now $\quad E_4 - E_1 = \frac{hc}{\lambda}$

$\Rightarrow \quad 0.85-(-13.6) = \frac{(4\times10^{15}\times3\times10^8\times10^9)}{\lambda}$

$\Rightarrow \quad \lambda = 94.1 \text{ nm}$

8. Option (2) is correct.

Both resistors have same value so, voltage supplied will be equally divided across two resistors. Resistance of voltmeter will have no impact here, as it gets connected in parallel with resistors.

9. Option (3) is correct.

As vectors are perpendicular, $\vec{P}.\vec{Q} = 0$

$\Rightarrow \quad 2m\hat{j}+m\hat{k}).(4\hat{i}-2\hat{j}+m\hat{k}) = 0$

$\Rightarrow \quad 1\times4 + 2m\times-2 + m\times m = 0$

$\Rightarrow \quad m^2 - 4m + 4$

$\Rightarrow \quad m = 2$

10. Option (3) is correct.

Since the relation between the electric and magnetic field is given as= $\frac{E_0}{B_0} = \frac{\omega}{k}$

11. Option (1) is correct.

We can see two switches are in series, so given circuit will behave as either AND or NAND gate.

Now if we see, there is earthing, so if switches are open then also blub will glow. If both switches are closed, there will be a short circuit condition, so bulb won't glow. Hence, given circuit is NAND gate.

12. Option (1) is correct.

For monoatomic gas $\quad \gamma_1 = \frac{5}{3} = 1.6$

For diatomic gas $\quad \gamma_2 = \frac{7}{5} = 1.4$

$\Rightarrow \quad \frac{\gamma_1}{\gamma_2} = \frac{\frac{5}{3}}{\frac{7}{5}} = \frac{25}{21}$

13. Option (3) is correct.

When a ray of white light pass through convex lens, different colors of light converges at different points on the principle axis after refraction. This is due to the difference in refractive index for different colors.

14. Option (4) is correct.

Motional emf is given by, $\int_0^L B\omega x\,dx$

$$\Rightarrow \quad \varepsilon = B\omega\int_0^L x\,dx$$

$$\Rightarrow \quad \varepsilon = B\omega\left[\frac{x^2}{2}\right]_0^L = \frac{1}{2}B\omega L^2$$

15. Option (1) is correct.

De-Broglie wavelength is given by, $\lambda = \frac{h}{\sqrt{2mK}}$

If K (kinetic energy) is constant, $\lambda \propto \frac{1}{\sqrt{m}}$

$\because m_\alpha > m_\beta > m_\gamma$

$\because \lambda_\alpha < \lambda_\beta < \lambda_\gamma$

16. Option (1) is correct.

Steel is more elastic and have high strength, so it is used in construction industry.

17. Option (3) is correct.

For an ideal gas, $PV = nRT$

For isothermal process, $P \propto \frac{1}{V}$, As V increases, P reduces nonlinearly

18. Option (1) is correct.

Usually satellite communication is done through higher frequency bands. It is a data based question.

19. Option (3) is correct.

Centripetal force acting on body, $m\omega^2(l + \Delta l) = k\Delta l$

$$\Rightarrow \quad \frac{l}{\Delta l} = \frac{k}{m\omega^2} - 1$$

$$\Rightarrow \quad \frac{l}{\Delta l} = \frac{12.5}{0.2\times 25} - 1 = \frac{3}{2}$$

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{2}{3}$$

20. Option (3) is correct.

Area under v-t gives displacement, distance

Displacement = $8 \times 2 - 4 \times 2 + 4 \times 4 - 2 \times 4 = 16$m

Distance = $8 \times 2 + 4\times 2 + 4\times 4 + 2\times 4 = 48$ m

$\Rightarrow$ Displacement : Distance = 1 : 3

Section B

21. The correct answer is [100]

$$\vec{v} = \int \frac{F}{m}dt$$

$$\Rightarrow \quad \vec{v} = \int \frac{(t\hat{i} + 3t^2\hat{j})}{m}dt$$

$$\Rightarrow \quad \vec{v} = \int_0^2 t\,dt\,\hat{i} + \int_0^2 3t^2\,dt\,\hat{j} \qquad [\because m = 1]$$

Power, $p = \vec{F}.\vec{v} = \left(t\hat{i} + 3t^2\hat{j}\right)\cdot\left(\frac{t^2}{2}\hat{i} + t^3\hat{j}\right)$

$$\Rightarrow \quad P = \frac{t^3}{2} + 3t^5$$

$$\Rightarrow \quad P = \frac{8^2}{2} + 3\times 32 = 100$$

22. The correct answer is [54]

From lens maker's formula, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

In air, $\frac{1}{18} = (1.5\text{-}1)\,\frac{2}{R}$ ....(i)

In water, $\frac{1}{f} = \left(\frac{1.5}{\frac{4}{3}} - 1\right)\left(\frac{2}{R}\right) = \frac{1}{8}\left(\frac{2}{R}\right) = \frac{1}{4R}$ .....(ii)

On dividing (i) by (ii)

On $\frac{f}{18} = \frac{0.5}{0.5/4}$

$\Rightarrow$ Focal length in water, $f = 18 \times 4 = 72$cm

$\Rightarrow$ Change in focal length = 72 – 18 = 54 cm

23. The correct answer is [6]

Energy released on fission of 1 nucleus, 200MeV

Total number of nuclei = $\frac{120}{240}\times 6\times 10^{23} = 3\times 10^{23}$

Total energy released = $3 \times 10^{23} \times 200 = 6\times 10^{25}$ MeV

24. The correct answer is [32]

$I_1 = \frac{MR^2}{2}$ (MI, axis of cylinder)

Mass of cylinder of radius R, $M = \rho\pi R^2$

Mass of carved out cylinder, $M' = \rho V'$

$\Rightarrow M' = \rho\pi\left(\frac{R}{2}\right)^2 \times \frac{L}{2}$

$\Rightarrow M' = \frac{M}{8}$

MI of carved out cylinder, $I_2 = \frac{M'R'^2}{2}$

$\Rightarrow I_2 = \frac{\frac{M}{8}\times\left(\frac{R}{2}\right)^2}{2} = \frac{1}{32}\frac{MR^2}{2}$

$\Rightarrow I_1 : I_2 = 32:1$

25. The correct answer is [105]

In air, capacitance $\quad C = \frac{\varepsilon_0 A}{d}$

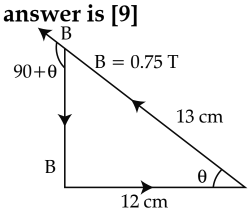
In dielectric $\quad C' = K\frac{\varepsilon_0 A}{2d}$

$\Rightarrow \quad C' = K\frac{C}{2}$

$\Rightarrow \quad C' = 3.5 \times \frac{15}{2} = \frac{105\text{pF}}{4}$

On comparing with $\frac{x}{4}$ pF, we get x = 105

26. The correct answer is [9]



$$\vec{F} = i\left(\vec{dl} \times \vec{8}\right)$$

$\Rightarrow \quad 2 \times 5 \times 10^{-2} \times 0.75 \times \sin(90 + \theta)$

$\Rightarrow \quad = 0.075 \times \frac{12}{13} \qquad [\because \sin(90 + \theta) = \cos\theta]$

$\Rightarrow \quad = \frac{9}{130}$

On comparing with, , $\frac{x}{130}$, we get x = 9

27. The correct answer is [1]

Time period of simple pendulum, $T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{m}{k}} = 1$ …….(i)

Now, $T = 2\pi\sqrt{\frac{m+3}{k}} = 2$ ……(ii)

On dividing (ii) by (i), we get $\sqrt{\frac{m+3}{m}} = 2$

On squaring both sides, m+3= 4m

$\Rightarrow \quad m = 1$ kg

28. The correct answer is [44]

As length becomes 1.2 times, area will become $\frac{1}{1.2}$

$R' = \rho \times \frac{1.2l}{\frac{A}{1.2}} = 1.2^2 \rho^2 \frac{l}{A} = 1.44R$

It means resistance is increasing by 44%.

29. The correct answer is [3]

Long after closing the switch, inductors will not oppose change in current and will act like short circuit. So, all three resistors will be in parallel.

$R_{eq} = 12 \,||\, 12 \,||\, 12 = \frac{12}{3} = 4\Omega$

As per ohm's law, $I = \frac{V}{R} = \frac{12}{4} = 3A$

30. The correct answer is [7]

Terminal velocity, $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{n}$

$\Rightarrow \quad v_T = \frac{2}{9}\frac{10^{-6} \times (10.5 - 1.5) \times 10^3 \times 9.8}{9.8 \times 0.1}$

$\Rightarrow \quad v_T = 2 \times 10^{-2}\ ms^{-1}$

Viscose force, $F = 6\pi\eta r v$

$\Rightarrow \quad F = 6 \times \frac{22}{7} \times 9.8 \times 0.1 \times 10^{-3} \times 2 \times 10^{-2}$

$\Rightarrow \quad F = 3696 \times 10^{-7}$

On comparing with 3696×10^{-x}, we get $x = 7$

Chemistry

Section A

31. Option (4) is correct.

Statement (A) is correct

The standard reduction potential of the elements are –

Li	Na	Rb
–3.237	–2.898	–3.079

From the above data, the order of standard reduction potential (M^+/M) for Alkali: metals is Na > Rb > Li

Statement (D) is correct

In conc. liquid ammonia solution, potassium get dissolved to form blue colour and Paramagnetic solution.

Statement (E) is correct

All alkali metal hydrides are ionic solids with high melting point. This is due to their large size and very low ionisation enthalpy.

From the given option, option (4) is correct.

A and E only.

32. Option (3) is correct.

Assertion is correct

The standard reduction potential of alkaline earth metal are as follows-

SRP	Be	Mg	Ca	Sr	Ba	Ra
V	–1.97	–2.36	–2.84	–2.89	–2.92	–2.92

From the above data it is clear that the beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason is correct –

Due to small size of beryllium ion, the value of hydration energy is very high and similarly due to the small size of beryllium ion, the enthalpy of atomisation is also very high

33. Option (4) is correct.

The relation between half life of the reaction and the pressure is given as follows

$t_{\frac{1}{2}} \,\alpha\, (P_0)^{1-n}$

For two condition –

$$\frac{\left(t_{\frac{1}{2}}\right)_1}{\left(t_{\frac{1}{2}}\right)_2} = \frac{(P_0)_1^{1-n}}{(P_0)_2^{1-n}}$$

Given : $\left(t_{\frac{1}{2}}\right)_1 = 4$ s $\quad \left(t_{\frac{1}{2}}\right)_2 = 2$s

$(P_0)_1 = 50$ mmHg $\quad (P_0)_2 = 100$mmHg

By putting these values in the formula

$$\left(\frac{4}{2}\right) = \left(\frac{50}{100}\right)^{1-n}$$

$$2 = \left(\frac{1}{2}\right)^{1-n}$$

Or $(2)^1 = (2)^{n-1}$

n – 1 =1

or $\boxed{n = 2}$

The order of the reaction is 2.

34. Option (3) is correct.

The reaction of potassium dichromate ($K_2Cr_2O_7$) with sulphur dioxide (SO_2) in the presence of dilute acid gives green colour of chromic sulphate and sulphate ion.

In this reaction, potassium dichromate act as an oxidizing agent which oxidises sulphur dioxide into sulphate ion and itself get reduced to chromic sulphate, which has green colour.

The reaction between potassium dichromate and sulphur dioxide is shown as follows

$$Cr_2O_7^{2-} + 2H^+ + 3SO_2 \rightarrow \underset{\text{(Green)}}{2Cr^3} + 3SO_4^{2-} + H_2O$$

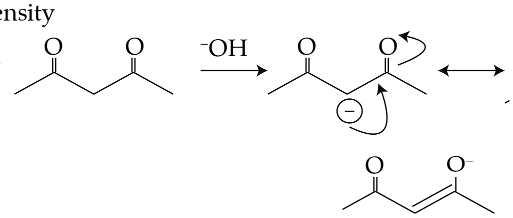
35. Option (2) is correct.

Among the given compounds, that compound will undergo deprotonation in basic medium which forms a stable carbanion.

The stability of carbanion can be increased by an electron withdrawing group.

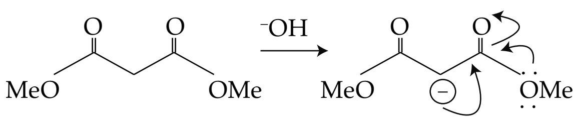
Here compound A contains electron withdrawing group while compounds B and C are surrounded with electron donating groups which decreases the stability of carbanion by donating their electron density

A.



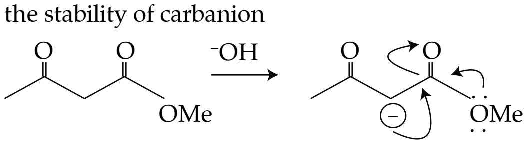
enolate ion

B.



Here, cross conjugation takes place which decrease the stability of carbanion

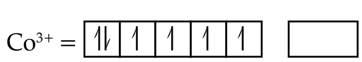
C.



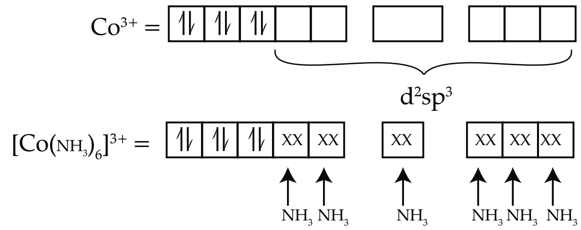
Here, cross conjugation takes place which decrease the stability of carbanion. As the carbanion is stabilised only A compound,thus it will undergo deprotonation most readily in basic medium.

36. Option (3) is correct.

In the complex $[Co(NH_3)_6]^{+3}$ the oxidation state of Co is +3, electronic configuration of Co^{3+} = [Ar] $3d^6$ $4s^o$



Being a strong ligand NH_3 pair the unpaired e^- and vacant the d- orbital for it.

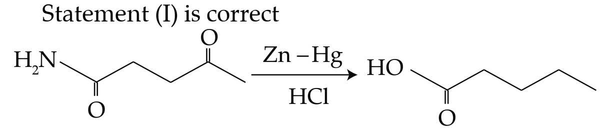


From above it is clear that no unpaired e^- is present in the complex

Therefor it is diamagnetic in nature and $\mu = 0$.

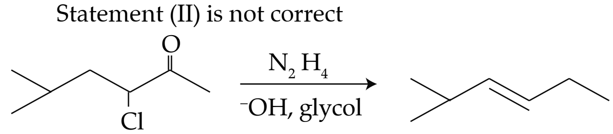
37. Option (2) is correct.

Statement (I) is correct



The hydrolysis of an amide in presence of an acid gives carboxylic acid, therefore this reaction is correct.

Statement (II) is not correct



In the wolf Kishner reduction along with carbonyl group, halogen will atom also get eliminated to form a double bond. Therefore, the statement (II) is not correct.

38. Option (1) is correct.

By the help of experimental value of Δ,

Crystal field theory introduces spectrochemical series but unable to explain its order. Because as per CFT, anionic ligands should exert greatest splitting effect. However they lie lower on lower end of the spectrochemical series.

Similarly crystal field theory explain the stability, colour and magnetic properties of transition metal complexes.

39. Option (2) is correct.

An ion which contains 55 proton in its unipositive state is Cs.

The electronic configuration of Cs $\Rightarrow [Xe]6s^1$

On losing one electron it become Cs^+

The configuration of Cs^+ will be

$1s^2$	$2s^2$	$2p^6$	$3s^2$	$3p^6$	$3d^{10}$	$4s^2$	$4p^6$	$4d^{10}$	$5s^2$	$5p^6$
$\downarrow$	$\downarrow$		$\downarrow$			$\downarrow$			$\downarrow$	
2+	2	+	2		+	2		+	2	$=10e^-$

The number of s-electron present in Cs^+ ion = $10e^-$.

40. Option (3) is correct.

The electronic configuration of the ions are as follows

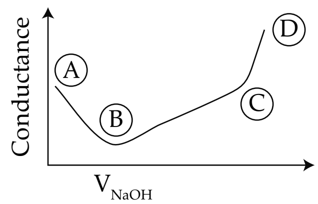
A. Sm^{2+}: [Xe] $4f^6$
B. Ce^{2+}: [Xe] $4f^2$
C. Ce^{4+}: [Xe] $4f^0$
D. Tb^{4+}: [Xe] $4f^7$

Out of the following ions Ce^{4+} and Tb^{4+} act as good oxidizing agent. Because they can be readily converted to their +3 oxidation states.

41. Option (1) is correct.

The reaction of benzoic acid with sodium hydroxide is shown as follows-

$C_6H_5COOH + NaOH \rightarrow C_6H_5COONa + H_2O$



From point A → B

Free H^+ ions are replaced by Na^+ which decreases conductance.

From point B → C

Undissociated benzoic acid reacts with strong base, NaOH and forms a salt, CH_3COONa which is a strong electrolyte and helps in the increment of conductance.

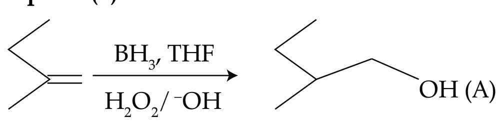
From point C→D

Once the equivalence point is reached, the excess NaOH will further increase the conductance.

42. Option (3) is correct.

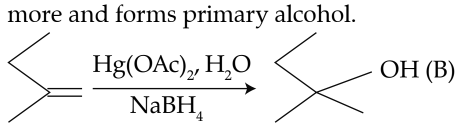
A.	Antifertility drug	i.	Norethindrone
B.	Tranquilizer	ii.	Meprobomate
C.	Antihistamine	iii.	Seldane
D.	Anti-biotic	iv.	Ampicillin

43. Option (2) is correct.



The above reaction addition of water takes place according to anti- Markonikov's Rule

Where the negative part of the reagent attached to that carbon where number of hydrogen atom are more and forms primary alcohol.



In the above reaction addition of water takes place according to Markonikov's Rule where the negative part of the reagent attached to that carbon where number of hydrogen atom are less and forms secondary alcohol.

44. Option (1) is correct.

50. Option (2) is correct.

Assertion is true

Benzere is more stable than hypothetical cyclohexatriene because in benzene . This is due to delocalioation of π e^- s resulting in extra stability.

The delocalised π e^- cloud in bonding molecular orbital of carbon atom which increases the attraction of π e^- towards the nuclei of carbon atoms.

45. Option (3) is correct.

Reducing agent is the reagent in which the element itself gets oxidise and reduces other by losing electron.

Here in option (3), H_2O_2 oxidises itself into O_2 and acts as a reducing agent.

$$\underset{+1}{HOCl} + \underset{-1}{H_2O_2} \longrightarrow H_3O^+ + \underset{-1}{Cl^-} + \underset{0}{O_2}$$

reduction

oxidation

46. Option (3) is correct.

Statement (I) is correct

Pure aniline and other arylamines are usually colourless so it is a correct statement.

Statement (II) is incorrect.

Arylamine or aniline gets coloured or storage due to action of air and light i.e; when oxidation of these compounds occur.

47. Option (1) is correct.

An average human being requires nearly 12-15 times more air than the food.

48. Option (2) is correct.

The electronic e^- configuration of species given are as follows:

N_2 : $\sigma_{1s}^2, \sigma^*_{1s}{}^2, \sigma_{2s}^2, \pi_{2px}^2 = \pi_{2py}^2, \sigma_{2pz}^2$

number of unpaired e^- present in HOMO = 0

N_2^+ $\sigma_{1s}^2, \sigma^*_{1s}{}^2, \sigma_{2s}^2, \sigma^*_{2s}{}^2, \pi_{2px}^2 = \pi_{2py}^2, \sigma_{2pz}^1$

Number of unpaired e^- present in HOMO = 1

O_2 $\sigma_{1s}^2, \sigma^*_{1s}{}^2, \sigma_{2s}^2, \sigma^*_{2s}{}^2, \sigma_{2pz}^2\ \pi_{2px}^2, \pi_{2py}^2, \pi^*_{2px}{}^1 = \pi^*_{2py}{}^1$

Number of unpaired e^- present in HOMO = 2

O_2^+ $\sigma_{1s}^2, \sigma^*_{1s}{}^2, \sigma_{2s}^2, \sigma^*_{2s}{}^2, \sigma_{2Pz}^2\ \pi_{2Px}^2, \pi_{2Py}^2, \pi^*_{2Px}{}^2 = \pi^*_{2Py}{}^1$

Number of unpaired e^- present in HOMO = 1

The number of unpaired e- in HOMO of the following species : N_2, N_2^+, O_2, O_2^+ ⇒ 0, 1, 2, 1

49. Option (1) is correct.

The metal which is extracted by oxidation & then subsequent reduction from its ore is silver (Ag).

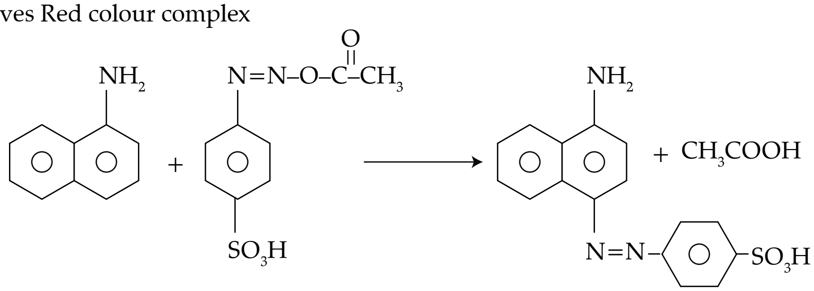
Oxidation reaction :

$4\,Ag + 8\,CN^- + O_2 + 2H_2O \rightarrow 4\,[Ag(CN)_2]^- + 4\,OH^-$

Reduction reaction :

$2\,[Ag(CN)_2]^- + Zn \rightarrow 2\,Ag\downarrow + [Zn(CN)_4]^{2-}$

It gives Red colour complex



(Red Colour)

Section B

51. The correct answer is [2]

The spectrum of black body radiation is explained using quantization of enengy. With increase in temperature, peak of spectrum shifts to shorter wavelength or higher frequency.

So statements. C and E are correct.

52. The correct answer is [5]

The number of units which are used to express concentration of solution are –

Mass percentage, mole fraction, molarity, ppm, molality.

53. The correct answer is [2]

The characteristic of physical adsorption are

It is not specific in nature. The enthalpy of adsorption is low.

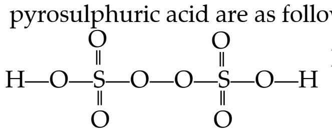
It decreases with increase in temperature.

It forms multi-layers, less or no activation energy is required.

Out of the given statements only C and E are correct.

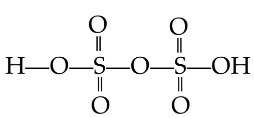
54.The correct answer is [8]

The structure of peroxodisulphuric acid and pyrosulphuric acid are as follows



Peroxodisulphuric acid

No. of π– bonds = 4



Pyrosulphuric acid

No. of π- bond = 4

Total no. of π- bonds = 4+4 = 8

55. The correct answer is [85]

Given [calcium lactate] = 0.005M

$Ca(LaC)_2$	$\rightarrow$	Ca^{2+}	+	$2\,LaC^{\ominus}$
0.005		—		—
—		0.005		0.005×2

[lactate ion] = 0.005 × 2 = 0.010 M

Calcium lactate is a salt of weak acid and strong base Therefore, it under go anionic hydrolysis.

The pH of the weak acid and strong base salt solution can be calculated as follows

$$pH = \left(\frac{14 + pka + \log C}{2}\right)$$

$$= \frac{14 + 5 + \log 0.01}{2}$$

$$pH = 8.5$$

$$= 85 \times 10^{-1}.$$

56. The correct answer is [314]

Let vapour pressure of pure liquid $A = P_A^0$

Let vapour pressure of pure liquid $B = P_B^0$

For case 1

$X_A = 0.7$ and $X_B = 0.3$

$P_{solution} = 350$ mmHg

So $P_A^0 \times 0.7 + P_B^0 \times 0.3 = 350$ ….(1)

For case 2

$X_A = 0.2$ and $X_B = 0.8$

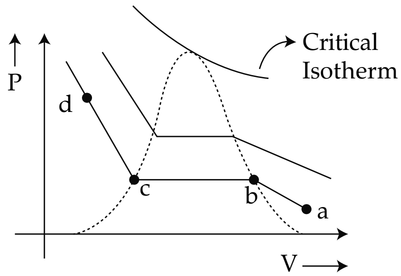
$P_{solution} = 410$ mmHg

So $P_A^0 \times 0.2 + P_B^0 \times 0.8 = 410$ ….(2)

On solving (1) & (2)

$P_A^0 = 314$ mmHg $P_B^0 = 434$ mmHg

57. The correct answer is [2]



At point (a) CO_2 exits as a gas.

At point (b) due to increase in pressure, the volume decreases and at point b liquefaction of CO_2 starts.

The liquification starts from point b and continous till point C. At point b the first particle of gas will be converted to liquid. At point c ,all the particles will convert into liquid.Thus liquid and gaseous carbon dioxide coexist betwwen points(b) and (c)

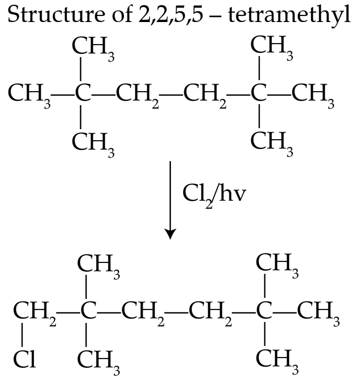
As the liquification starts from point b ,therefore the volume of liquid increases from pt b to c.

So from above point (A) and (C) are correct.

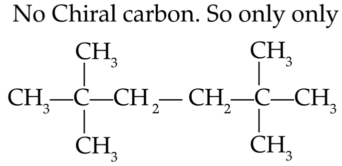
58. The correct answer is [3]

The number of isomeric morochloro derivatives which can be obtained from 2,2,5,5- tetramethyl hexane by chlorination are (3)

Structure of 2,2,5,5 – tetramethyl hexane



No Chiral carbon. So only only 1 product is obtained.



One chiral carbon is there. So two enantiomers would be obtained.

Total number of products including isomers = 3

59. The correct answer is [8]

Number of tripeptides = 8

Number of amino acid = 2

Possible products = 3 (tripeptide)

No. of products = $2^3 = 8$

(1) Val – val – val

(2) Pro – pro- pro

(3) Val- pro – pro

(4) Pro-val-pro

(5) Val-val-pro

(6) Val-pro-val

(7) Pro-pro-val

(8) Pro-val-val

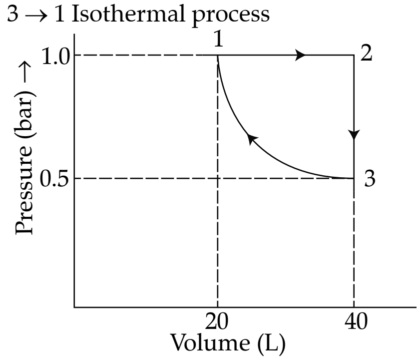
60. The correct answer is [620]

The process

$1 \rightarrow 2$ Isobaric process

$2 \rightarrow 3$ Isochoric process

$3 \rightarrow 1$ Isothermal process



In Isochoric process work done =0

Thus Total work done For a process

$$W = W_{1\to2} + W_{2\to3} + W_{3\to1}$$

$$= \left[-p(V_2 - V_1) + 0 + \left(-PV_1 \ln\left(\frac{V_2}{V_1}\right)\right)\right]$$

$$= \left[-1\times(40-20) + 0 + \left[-1\times 20 \ln\left(\frac{20}{40}\right)\right]\right]$$

$$= -20+20 \ln 2$$

$$= -20+20\times 2.303 \times 0.3$$

$$= -6.2 \text{ bar L}$$

$$1 \text{ L bar} = 100 \text{ J approx}$$

$$|W| = 6.2 \times 100 \text{ J} = 620 \text{ J}$$

Mathematics

Section A

61. Option (B) is correct.

Given, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in \mathbb{R}$

Let $f'(1) = p, f''(2) = q$ and $f'''(3) = r$

$\Rightarrow f(x) = x^3 - px^2 + qx - r$

$\Rightarrow f'(x) = 3x^2 - 2px + q$

$\Rightarrow f''(x) = 6x - 2p$

$\Rightarrow f'''(x) = 6$

So, $f'''(3) = 6 = r$

Now, $f'(1) = 3(1)^2 - 2p(1) + q$

$\Rightarrow p = 3 - 2p + q$

$\Rightarrow 3p = 3 + q$ ...(i)

And $f''(2) = 6(2) - 2p$

$\Rightarrow q = 12 - 2p$

$\Rightarrow 2p + q = 12$ ...(ii)

On solving equation (i) and equation (ii), we get

$p = 3, q = 6$

$\therefore f(x) = x^3 - 3x^2 + 6x - 6$

So, $f(0) = -6, f(1) = -2, f(2) = 2, f(3) = 12$

Now, $2f(0) - f(1) + f(3) = 2(-6) - (-2) + 12$

$= 2 = f(2)$

HINT:

Find $f'(x), f''(x)$ and $f'''(x)$ using $\frac{d}{dx}(x^n) = nx^{n-1}$ and solve further.

62. Option (C) is correct.

Given: System of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

As we know for infinite many solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & -4 \\ 8 & 4 & -\lambda \end{vmatrix} = 0$

$\Rightarrow 1(-3\lambda + 16) - 2(-4\lambda + 32) + 3(16 - 24) = 0$

$\Rightarrow -3\lambda + 16 + 8\lambda - 64 - 24 = 0$

$\Rightarrow 5\lambda - 72 = 0$

$\Rightarrow \lambda = \frac{72}{5}$

Now, $\Delta_3 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 8 & 4 & 9+\mu \end{vmatrix} = 0$

$\Rightarrow 1(27 + 3\mu - 16) - 2(36 + 4\mu - 32) + 3(16 - 24) = 0$

$\Rightarrow 11 + 3\mu - 8\mu - 8 - 24 = 0$

$\Rightarrow -5\mu - 21 = 0$

$\Rightarrow \mu = -\frac{21}{5}$

HINT:

Consider, $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

To solve this system we first define the following determinants

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

System of linear equations have infinite solutions if $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

63. Option (A) is correct.

Given: $f(x) = \frac{2^{2x}}{2^{2x} + 2}, x \in R$

$\Rightarrow f(x) = \frac{4^x}{4^x + 2}$

Now, $f(1 - x) = \frac{4^{(1-x)}}{4^{(1-x)} + 2}$

$\Rightarrow f(1 - x) = \frac{4}{4 + 2.4^x}$

$\Rightarrow f(1 - x) = \frac{2}{2 + 4^x}$

So, $f(x) + f(1 - x) = \frac{4^x}{4^x + 2} + \frac{2}{4^x + 2} = 1$

Let $A = f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$

$\Rightarrow A = f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{2021}{2023}\right) + ...$
$+ f\left(\frac{1011}{2023}\right) + f\left(\frac{1012}{2023}\right)$

$\Rightarrow A = 1 + 1 + 1 + ...$ up to 1011 terms

$\{\because f(x) + f(1 - x) = 1\}$

$\Rightarrow A = 1011$

HINT:

Use $f(x) + f(1 - x) = 1$ and solve further.

64. Option (A) is correct.

Given: $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\because$ $\vec{\beta}_1$ is parallel to $\vec{\alpha}$

$\Rightarrow \beta_1 = \mu(4\hat{i} + 3\hat{j} + 5\hat{k}); \mu \in R$

Also given that $\vec{\beta}_2$ is perpendicular to α

$\Rightarrow \vec{\beta}_2\,\alpha = 0$

Since, $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\Rightarrow \vec{\beta} = \mu\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow \vec{\beta}.\vec{\alpha} = \mu|\vec{\alpha}^2| + \vec{\beta}_2.\vec{\alpha}$

$\Rightarrow (4\hat{i} + 3\hat{j} + 5\hat{k}).(\hat{i} + 2\hat{j} - 4\hat{k}) = \mu(\sqrt{16+9+25})^2 + 0$

$\Rightarrow 4 + 6 - 20 = \mu(50)$

$\Rightarrow \mu = -\dfrac{1}{5}$

Now, $\vec{\beta} = -\dfrac{1}{5}\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow 5\vec{\beta}_2 = 5\vec{\beta} + \vec{\alpha}$

$\Rightarrow 5\vec{\beta}_2 = 5(\hat{i} + 2\hat{j} - 4\hat{k}) + (4\hat{i} + 3\hat{j} + 5\hat{k})$

$\Rightarrow 5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

Now, $5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

HINT:

(1) Use if $\vec{a}$ is parallel to $\vec{b}$, then $\vec{b} = k\vec{a}; k \in R$

(2) Use if $\vec{u}$ is perpendicular to $\vec{v}$, then $\vec{u}.\vec{v} = 0$

(3) If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

65. Option (A) is correct.

Given, differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$

$\Rightarrow 3xy\dfrac{dy}{dx} - 3y^2 = -x^2$

$\Rightarrow y\dfrac{dy}{dx} - \dfrac{y^2}{x} = -\dfrac{x}{3}$

$\Rightarrow 2y\dfrac{dy}{dx} - \dfrac{2y^2}{x} = -\dfrac{2x}{3}$

Let $y^2 = v$

$\Rightarrow 2y\dfrac{dy}{dx} = \dfrac{dv}{dx}$

So, $\dfrac{dv}{dx} - \dfrac{2v}{x} = \dfrac{-2x}{3}$ which is linear differential equation

Now, I.F. $= e^{\int -\frac{2}{x}dx}$

$\Rightarrow$ I.F. $= e^{-2\ln x}$

$\Rightarrow$ I.F. $= e^{\ln x^{-2}} \Rightarrow$ I.F. $= \dfrac{1}{x^2}$

Now, solution of linear differential equation is

$v\,(\text{I.F.}) = \int \dfrac{-2x}{3}(\text{I.F.})\,dx + c$

$\Rightarrow v\dfrac{(1)}{(x^2)} = \int \dfrac{-2x}{3} \times \dfrac{1}{x^2}dx + c$

$\Rightarrow \dfrac{v}{x^2} = -\dfrac{2}{3}\int \dfrac{1}{x}dx + c$

$\Rightarrow \dfrac{v}{x^2} = -\dfrac{2}{3}\ln x + c$

$\Rightarrow \dfrac{y^2}{x^2} = -\dfrac{2}{3}\ln x + c$ $\quad\because y(1) = 1$

$\Rightarrow c = 1$

So, $\dfrac{y^2}{x^2} = -\dfrac{2}{3}\ln x + 1$

$\Rightarrow y^2 = -\dfrac{2}{3}x^2\ln x + x^2$

$\Rightarrow y^2(e) = -\dfrac{2}{3}e^2\ln e + e^2$

$\Rightarrow y^2(e) = \dfrac{e^2}{3}$

$\Rightarrow 6y^2(e) = 2e^2$

HINT:

(1) Convert given differential equation into linear differential equation by substituting $y^2 = v$.

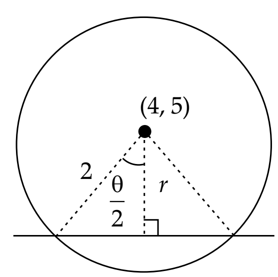
(2) Solution of linear differential equation $\dfrac{dy}{dx} + Px = Q$ is given by $y(\text{I.F.}) = \int Q(\text{I.F.})dx + c$, where I.F. $= e^{\int P dx}$

66. Option (B) is correct.

Given: Circle c_1: $(x-4)^2 + (y-5)^2 = 4$

$\Rightarrow$ Centre $= (4, 5)$ and radius $= 2$

Also given that $\theta_1 = \dfrac{\pi}{3}, \theta_3 = \dfrac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$



So, $\cos\left(\dfrac{\theta_i}{2}\right) = \dfrac{r_i}{2}$

$\Rightarrow r_i = 2\cos\left(\frac{\theta_i}{2}\right)$ $[\because r_1^2 = r_2^2 + r_3^2]$

$\Rightarrow \cos^2\frac{\theta_1}{2} = \cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_3}{2}$

$\Rightarrow \cos^2\left(\frac{\pi}{6}\right) = \cos^2\left(\frac{\theta_2}{3}\right) + \cos^2\left(\frac{\pi}{3}\right)$

$\Rightarrow \frac{3}{4} = \frac{1}{4} + \cos^2\frac{\theta_2}{2}$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \frac{\theta_2}{2} = \frac{\pi}{4}$

$\Rightarrow \theta_2 = \frac{\pi}{2}$

HINT:

(1) Use radius of locus will be perpendicular from centre of the circle to chord.

(2) Use perpendicular drawn from centre to the chord bisects the chord.

67. Option (A) is correct.

Given: $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$\Rightarrow 3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Let $x + \frac{1}{x} = v$

$\Rightarrow 3\,[v^2 - 2] - 2v + 5 = 0$

$\Rightarrow 3v^2 - 2v - 1 = 0$

$\Rightarrow (3v + 1)(v - 1) = 0$

$\Rightarrow v = 1, -\frac{1}{3}$

As we know $x + \frac{1}{x} \geq 2$ or $x + \frac{1}{x} \leq -2$

But $x + \frac{1}{x} = v = 1, -\frac{1}{3}$

So, no real solution of the given equation is possible.

HINT:

(1) Convert given equation into quadratic equation by substituting $x + \frac{1}{x} = v$ and solve further.

(2) Use $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

68. Option (B) is correct.

Given $|\text{adj (adj (adj A))}| = 12^4$

$\Rightarrow |A|^{(n-1)^3} = 12^4$

$\Rightarrow |A|^{(2)^3} = 12^4$

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A| = \sqrt{12}$

Now, $|A^{-1}\,\text{adj}A| = |A^{-1}|\,|\text{adj } A|$

$= \frac{1}{|A|}|A|^2$ $\{\because |\text{adj } A\} = |A|^{n-1};$

where n = order of matrix A$\}$

$= |A| = \sqrt{12} = 2\sqrt{3}$

HINT:

(1) Use $|\text{adj (adj (adj A))}| = |A|^{(n-1)^3}$; where n = order of square matrix.

(2) Use $|\text{adj } A| = |A|^{n-1}$; where n = order of square matrix.

(3) Use $|AB| = |A|\,|B|$

69. Option (A) is correct.

Let $I = \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}}dx$

As we know $\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

$\Rightarrow I = \frac{48}{2}\left[\sin^{-1}\left(\frac{2x}{3}\right)\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$\Rightarrow I = 24\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$\Rightarrow I = 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] \Rightarrow I = 24\left[\frac{\pi}{12}\right] \Rightarrow I = 2\pi$

HINT:

(1) Use $\int_a^b f(x)dx = F(b) - F(a)$,

where $\int f(x)dx = F(x) + c$

(2) Use $\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

70. Option (D) is correct.

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

$\because$ Sum of all the elemetns in each row and in each column is 1

$\therefore$ In every row and every column there would be exactly one 1 and four zeroes.

So, number of required matrices

$= {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

HINT:

(1) In every row and every column there would be exactly one 1 and four zeroes.

(2) Recall multiplication principle of counting.

71. Option (D) is correct.

Given, $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3\,({}^{30}C_3)^2 + ... + 30\,({}^{30}C_{30})^2$

$= \dfrac{\alpha 60!}{(30!)^2}$

Let $P = 0\,({}^{30}C_0)^2 + 1\,({}^{30}C_1)^2 + 2\,({}^{30}C_2)^2 + ... + 30({}^{30}C_{30})^2$ ...(i)

$P = 30\,({}^{30}C_{30})^2 + 29\,({}^{30}C_{29})^2 + 28\,({}^{30}C_{28})^2 + ... + 0({}^{30}C_0)^2$ ...(ii)

Adding equation (i) and equation (ii), we get

$2P = 30\left[\left({}^{30}C_0^2\right)+\left({}^{30}C_1^2\right)+\left({}^{30}C_2^2\right)+....+\left({}^{30}C_{30}^2\right)\right]$

As we know $\sum_{r=0}^{n}\left({}^{n}C_r\right)^2 = {}^{2n}C_n$

So, $P = 15\ {}^{60}C_{30}$

$\Rightarrow P = 15\ \dfrac{60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

HINT:

(1) Let $P = 0\,({}^{30}C_0)^2 + 1\,({}^{30}C_1)^2 + ... + 30\,({}^{30}C_{30})^2$ and make another equation by reversing the term and add both the equations.

(2) Use $\sum_{r=0}^{n}\left({}^{n}C_r\right)^2 = {}^{2n}C_n$

72. Option (B) is correct.

Given planes P_1: $x + (\lambda + 4)\,y + z = 1$

P_2: $2x + y + z = 2$

Equation of plane containing the line of intersection of the plane P_1 and P_2 is given by

$P: [x + (\lambda + 4)\,y + z - 1] + k\,[2x + y + z - 2] = 0$

$\because$ Plane P passes through (0, 1, 0)

$\Rightarrow \lambda + 4 - 1 + k\,(1 - 2) = 0$

$\Rightarrow \lambda - k + 3 = 0$ ...(i)

Plane P also passes through (1, 0, 1)

$\Rightarrow 1 + k\,(2 + 1 - 2) = 0$

$\Rightarrow k = -1$

Put the value of $k = -1$ in equation (i), we get

$\lambda = -4$

So, point $(2\lambda, \lambda, -\lambda) = (-8, -4, 4)$

Now, distance of $(-8, -4, 4)$ from plane P_2 is

$$d = \left|\frac{2(-8) - 4 + 4 - 2}{\sqrt{2^2 + 1^2 + 1^2}}\right|$$

$$\Rightarrow d = \left|\frac{-18}{\sqrt{6}}\right|$$

$$\Rightarrow d = 3\sqrt{6}$$

HINT:

(1) Equation of plane containing the line of intersection of the plane P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$.

(2) Perpendicular distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

73. Option (D) is correct.

Given: $f(x + y) = f(x).\,f(y)$

$\Rightarrow f(x) = p^x$ $[\because f(1) = 3]$

$\Rightarrow p = 3$

So, $f(x) = 3^x$

Also given that $\sum_{k=1}^{n} f(k) = 3279$

$\Rightarrow f(1) + f(2) + ... + f(n) = 3279$

$\Rightarrow 3 + 3^2 + ... + 3^n = 3279$

$\Rightarrow \dfrac{3(3^n - 1)}{3 - 1} = 3279$

$\Rightarrow 3^n = 2187$

$\Rightarrow 3^n = 3^7$

$\Rightarrow n = 7$

HINT:

(1) Consider $f(x) = P^x$ and find the value of P by given condition.

(2) Sum of GP whose first term is a, common ratio $= r$ is given by $S = \dfrac{a(r^n - 1)}{r - 1}; r > 1$ where n = number of terms.

74. Option (A) is correct.

Given $a_1, a_2, a_3, a_4, a_5, a_6$ are in A.P. and $a_1 + a_3 = 10$

And mean of $a_1, a_2, a_3, a_4, a_5, a_6 = \dfrac{19}{2}$

So, $\dfrac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = \dfrac{19}{2}$

$\Rightarrow a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$

Let common difference of AP be d.

So, $\dfrac{6}{2}(2a_1 + 5d) = 57$

$\{\because$ Sum of n terms of AP is given by $\dfrac{n}{2}[2a + (n-1)d]$

where a = first term and d = common difference$\}$

$\Rightarrow 2a_1 + 5d = 19$ ...(i)

[$\because a_1 + a_3 = 10$]

$\Rightarrow a_1 + a_1 + 2d = 10$

$\Rightarrow 2a_1 + 2d = 10$

$\Rightarrow a_1 + d = 5$ ...(ii)

On solving equation (i) and equation (ii), we get

$a_1 = 2$ and $d = 3$

Now, variance $= \sigma^2 = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$$\Rightarrow \sigma^2 = \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$\Rightarrow \sigma^2 = \frac{699}{6} - \frac{361}{4}$$

$$\Rightarrow \sigma^2 = \frac{105}{4} \Rightarrow 8\sigma^2 = 210$$

HINT:

(1) mean of $a_1, a_2, a_3, ..., a_n$ is given by
$x = \dfrac{a_1 + a_2 + a_3 + ... + a_n}{n}$

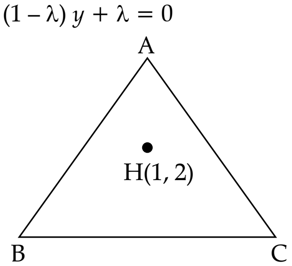
(2) Sum of n terms of AP is given by
$S_n = \dfrac{n}{2}[2a + (n-1)d]$; where a = first term and d = common difference

(3) Variance $= \dfrac{\sum x_i^2}{n} - (\bar{x})^2$

75. Option (D) is correct.

Given: Equation of AB : $(\lambda + 1)x + \lambda y = 4$

AC : $\lambda x + (1 - \lambda)y + \lambda = 0$



$\because$ Vertex A lies on y-axis

$\therefore$ x-coordinate of point A = 0

So, $x = 0$ will satisfy the equation of AB and AC

So, from equation of AB, $y = \dfrac{4}{\lambda}$

And from equation of AC, $y = \dfrac{\lambda}{\lambda - 1}$

So, $\dfrac{4}{\lambda} = \dfrac{\lambda}{\lambda - 1}$

$\Rightarrow 4\lambda - 4 = \lambda^2$

$\Rightarrow (\lambda - 2)^2 = 0$

$\Rightarrow \lambda = 2$

So, A = (0, 2)

Now, AB : $3x + 2y = 4$ and AC : $2x - y = -2$

Slope of AB, $m_{AB} = -\dfrac{3}{2}$

$\because \mu(1, 2)$ is orthocentre of ΔABC

$\therefore m_{CH} \cdot m_{AB} = -1$

$\Rightarrow m_{CH} = \dfrac{2}{3}$

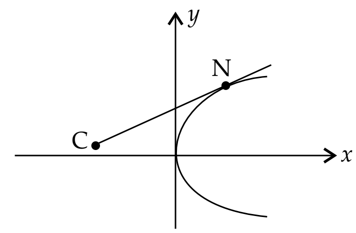
Let the coordinates of point C be (P, 2P + 2)

$\Rightarrow \dfrac{2P + 2 - 2}{P - 1} = \dfrac{2}{3}$

$\Rightarrow P = -\dfrac{1}{2}$

$\therefore C = \left(-\dfrac{1}{2}, 1\right)$

Given equation of parabola is $y^2 = 6x$



Now, equation of tangent to the parabola $y^2 = 6x$ in parametric form is given by $ty = x + \dfrac{3}{2}t^2$.

$\because$ Tangent is passing through $C\left(-\dfrac{1}{2}, 1\right)$

$\therefore t = -\dfrac{1}{2} + \dfrac{3}{2}t^2$

$\Rightarrow 3t^2 - 2t - 1 = 0$

$\Rightarrow (3t + 1)(t - 1) = 0 \Rightarrow t = 1$

So, coordinates of point of contact N = $(at^2, 2at)$

$= \left(\dfrac{3}{2}, 3\right)$

Now, NC $= \sqrt{\left(\dfrac{3}{2} + \dfrac{1}{2}\right)^2 + (3 - 1)^2}$

$\Rightarrow$ NC $= \sqrt{4 + 4} = 2\sqrt{2}$

HINT:

(1) Find the coordinates of point A by using the condition of vertex A lies on y-axis.

(2) Find the coordinates of point B and C by using the definition of orthocentre.

(3) Equation of tangent to parabola $y^2 = 4ax$ in parametric form is given by $ty = x + at^2$

76. Option (C) is correct.

As we know $A \Rightarrow B = \sim A \vee B$

So, $p \Rightarrow \sim q = \sim p \vee \sim q$

Now, $p \wedge (p \Rightarrow \sim q) = p \wedge (\sim p \vee \sim q)$

$= (p \wedge \sim p) \vee (p \wedge \sim q) = F \vee (p \wedge \sim q)$

Now, $\sim [p \wedge (p \Rightarrow \sim q)] = \sim [F \vee (p \wedge \sim q)]$

$= \sim F \wedge \sim (p \wedge \sim q) = T \wedge (\sim p \vee q) = \sim p \vee q$

HINT:

(1) Use $A \Rightarrow B = \sim A \vee B$

(2) Use $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

77. Option (D) is correct.

Given, $\lim_{x \to a}([x-5]-[2x+2]) = 0$

$\Rightarrow [a-5]-[2a+2] = 0$

$\Rightarrow [a]-5-[2a]-2 = 0$

$\Rightarrow [a]-[2a] = 7$ ...(i)

If $a \in z$, we have $a = -7$

For $a \in (-7.5, -7)$, $[a]-[2a] = -8+15 = 7$

So, $a \in (-7.5, -7)$ satisfy the given equation.

For $a \in (-7, -6.5)$, $[a]-[2a] = -7+14 = 7$

So, $a \in (-7, -6.5)$ satisfy the given equation

At $a = -7.5$

$[a]-[2a] = -8+15 = 7$

So, $a = -7.5$ satisfy the equation (i)

Now, at $a = -6.5$

$[a]-[2a] = -7+13 = 6$

So, $a = -6.5$ doesn't satisfy the equation (i)

$\because x \to a$

$\therefore a \neq -6.5$ or -7.5

So, $a \in (-7.5, -6.5)$

HINT:

Solve given limit using the definition of greatest integer function.

78. Option (D) is correct.

Let the normals of the plane $x + 2y + z = 0$ and $3y - z = 3$ be $\vec{n}_1$ & $\vec{n}_2$

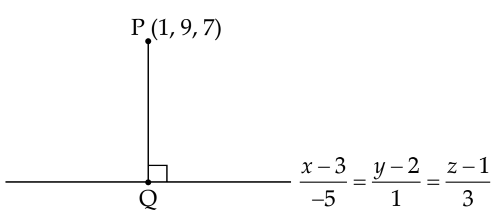
$\Rightarrow \vec{n}_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{n}_2 = 3\hat{j} - \hat{k}$

And the direction ratio of the line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-2-3) - \hat{j}(-1+0) + \hat{k}(3-0) = -5\hat{i} + \hat{j} + 3\hat{k}$

So the equation of the line passing through (3, 2, 1) is

$\dfrac{x-3}{-5} = \dfrac{y-2}{1} = \dfrac{z-1}{3} = k$



Let the coordinates of point Q be $(-5k+3, k+2, 3k+1)$

Now, direction ratios of PQ $= -5k + 3 - 1, k + 2 - 9, 3k + 1 - 7$

$= -5k + 2, k - 7, 3k - 6$

$\because$ PQ $\perp$ Line

So, $(-5k+2)(-5) + (k-7)(1) + (3k-6)3 = 0$

$\Rightarrow 35k = 35$

$\Rightarrow k = 1$

$\therefore$ Foot of perpendicular Q $= (-5+3, 1+2, 3+1)$

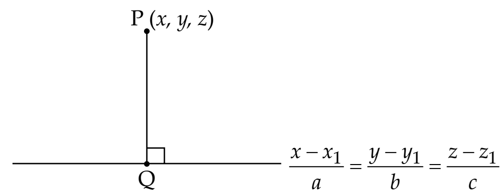
$= (-2, 3, 4)$

So, $\alpha + \beta + \gamma = -2 + 3 + 4 = 5$

HINT:

(1) Direction ratio of the line will be $\vec{n}_1 \times \vec{n}_2$; where $\vec{n}_1$ and $\vec{n}_2$ are normal vectors of given planes.

(2)



Assume coordinates of point Q on the line in parametric form and find the value of unknown using PQ $\perp$ line

79. Option (A) is correct.

Given digits : 3, 5, 6, 7, 8

All five digits number is greater than 7000

So, number of five digits number = 5! = 120

For 4 digits number greater than 7000

For 1000th place we can take only 7 or 8 from given digits and for remaining places we can take any digit from given digits.

So, number of 4 digits number greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

$\therefore$ Number of integer, greater than 7000

$= 120 + 48 = 168$

HINT:

First find number of 5 digits numbers and then find 4 digit numbers of taking 7 or 8 on 1000th place using the fundamental principle of counting.

80. Option (A) is correct.

Let $A = \left(\dfrac{1 + \sin\left(\frac{2\pi}{9}\right) + i\cos\left(\frac{2\pi}{9}\right)}{1 + \sin\left(\frac{2\pi}{9}\right) - i\cos\left(\frac{2\pi}{9}\right)} \right)^3$

$\Rightarrow A = \left(\dfrac{1 + \cos\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) + i\sin\left(\frac{\pi}{2} - \frac{2\pi}{9}\right)}{1 + \cos\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) - i\sin\left(\frac{\pi}{2} - \frac{2\pi}{9}\right)} \right)^3$

$$\Rightarrow A = \left(\frac{1+\cos\left(\frac{5\pi}{18}\right)+i\sin\left(\frac{5\pi}{18}\right)}{1+\cos\left(\frac{5\pi}{18}\right)-i\sin\left(\frac{5\pi}{18}\right)}\right)^3$$

$$\Rightarrow A = \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}{2\cos^2\frac{5\pi}{36}-2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}\right)^3$$

$$\Rightarrow A = \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$\Rightarrow A = \left(\frac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$$

$$\Rightarrow A = \left(e^{i\frac{5\pi}{18}}\right)^3 \Rightarrow A = e^{i\frac{5\pi}{6}}$$

$$\Rightarrow A = \cos\frac{5\pi}{6}+i\sin\left(\frac{5\pi}{6}\right)$$

$$\Rightarrow A = -\frac{\sqrt{3}}{2}+i\frac{1}{2}$$

HINT:

(1) Simplify given expression using trigonometric identities and try to convert given expression as $\cos\theta + i\sin\theta$ in numerator and denominator and then solve further using Euler form.

(2) Use $e^{i\theta} = \cos\theta + i\sin\theta$

81. The correct answer is (384).

Given lines L_1: $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$

$$\Rightarrow L_1: \vec{r} = (-\sqrt{6}\hat{i}+\sqrt{6}\hat{j}+\sqrt{6}\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$$

And L_2: $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$

$$\Rightarrow L_2: \vec{r} = (\lambda\hat{i}+2\sqrt{6}\hat{j}-2\sqrt{6}\hat{k})+\mu(3\hat{i}+4\hat{j}+5\hat{k})$$

As we know shortest distance between two lines

$\vec{r} = \vec{a}+\lambda\vec{p}$ and $\vec{r} = \vec{b}+\mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

So, $\vec{a} = -\sqrt{6}\hat{i}+\sqrt{6}\hat{j}+\sqrt{6}\hat{k}$

$\vec{b} = \lambda\hat{i}+2\sqrt{6}\hat{j}-2\sqrt{6}\hat{k}$

$\vec{p} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{q} = 3\hat{i}+4\hat{j}+5\hat{k}$

Now, $\vec{b}-\vec{a} = (\lambda+\sqrt{6})\hat{i}+\sqrt{6}\hat{j}-3\sqrt{6}\hat{k}$

$$\vec{p}\times\vec{q} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}$$

$\Rightarrow \vec{p}\times\vec{q} = -\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow |\vec{p}\times\vec{q}| = \sqrt{1+4+1} = \sqrt{6}$

Now, $(\vec{b}-\vec{a}).(\vec{p}\times\vec{q}) = -\lambda-\sqrt{6}+2\sqrt{6}+3\sqrt{6} = -\lambda+4\sqrt{6}$

So, shortest distance $= \left|\frac{-\lambda+4\sqrt{6}}{\sqrt{6}}\right| = 6$

$\Rightarrow |-\lambda+4\sqrt{6}| = 6\sqrt{6}$

$\Rightarrow -\lambda+4\sqrt{6} = \pm 6\sqrt{6}$

$\Rightarrow \lambda = 4\sqrt{6} \mp 6\sqrt{6}$

$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$

Sum of all possible values of $\lambda = -2\sqrt{6}+10\sqrt{6} = 8\sqrt{6}$

$\therefore (8\sqrt{6})^2 = 384$

HINT:

(1) Write the given equation of line vector form and use distance between two lines $\vec{r} = \vec{a}+\lambda\vec{p}$ and $\vec{r} = \vec{b}+\mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

(2) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

82. The correct answer is (432).

Given, Urn A contains 4 Red, 6 Black

Urn B contains 5 Red, 5 Black

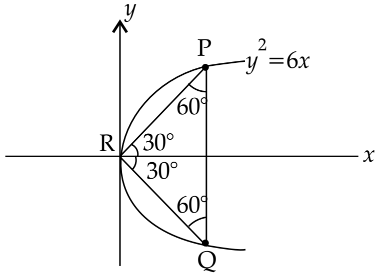
Urn C contains λ Red, 4 Black

Also P(Red ball from urn C) = 0.4

$$\Rightarrow \frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\times\frac{\lambda}{\lambda+4}} = \frac{4}{10}$$

$$\Rightarrow \frac{\frac{\lambda}{\lambda+4}}{\frac{2}{10}+\frac{\lambda}{\lambda+4}} = \frac{4}{10} \Rightarrow 24\lambda = 144 \Rightarrow \lambda = 6$$

So, equation of parabola is $y^2 = 6x$



Let parametric coordinates of point P be $\left(\frac{3}{2}t^2, 3t\right)$

Now, slope of PR = tan30°

$\Rightarrow \frac{3t}{\frac{3}{2}t^2} = \frac{1}{\sqrt{3}} \quad \Rightarrow t = 2\sqrt{3}$

$\therefore$ Coordinates of P = $(18, 6\sqrt{3})$

Now, PR = $\sqrt{(18)^2 + (6\sqrt{3})^2}$

$\Rightarrow$ PR = $\sqrt{432} \Rightarrow (\text{PR})^2 = 432$

HINT:

(1) Find the value of λ using Bayes theorem.

(2) **Bayes Theorem:** Let ε_1, ε_2, ε_n be a set of events associated with a sample space S, where all the events ε_1, ε_2,.. ε_n have non zero probability of occurence and they form a partition of S. Let B be any event associated with S, then according to Bayes theorem.

$$\text{P}\left(\frac{\varepsilon_i}{\text{B}}\right) = \frac{\text{P}(\varepsilon_i).\text{P}(\text{B}|\varepsilon_i)}{\sum_{k=1}^{n}\text{P}(\varepsilon_k)\text{P}(\text{A}|\varepsilon_k)}; k = 1, 2, ..., n$$

(3) Parametric coordinates of any point on parabola $y^2 = 4ax$ is $(at^2, 2at)$

83. The correct answer is (2).

Given: S = {θ ∈[0, 2 λ); tan (π cos θ) + tan (π sin θ) = 0}

So, tan (π cos θ) = – tan (π sin θ)

⇒ tan (π cos θ) = tan (–π sin θ)

As we know if tan θ = tan α, then θ = $n\pi$ + α; $n \in$ I

∴ π cos θ = $n\pi$ – π sin θ; $n \in$ I

⇒ π cos θ+ π sin θ = $n\pi$

⇒ cos θ + sin θ = n

Since, $-\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$

∴ n = –1, 0, 1

Case 1: If n = –1

cos θ + sin θ = –1

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$

$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4}$

$\Rightarrow \theta = 2k\pi + \pi$ or $\theta = 2k\pi - \frac{\pi}{2}$

$\Rightarrow \theta = \pi, \frac{3\pi}{2}$

Case-2: If n = 0

cos θ + sin θ = 0

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = 0$

$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{2}$

$\Rightarrow \theta = 2k\pi + \frac{3\pi}{4}$ or $\theta = 2k\pi - \frac{\pi}{4}$

$\Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

Case-3: If n = 1

$\cos\theta + \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4}$

$\Rightarrow \theta = 2k\pi + \frac{\pi}{2}$ or $\theta = 2k\pi$

$\Rightarrow \theta = \frac{\pi}{2}, 0$

$\therefore \theta = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

So, $\sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Now, $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$

HINT:

(1) General solution of tanθ = tanα is θ= $n\pi$ + α; $\alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $n \in$ I

(2) Use $-\sqrt{a^2 + b^2} \le a\sin x + b\cos x \le \sqrt{a^2 + b^2}$

84. The correct answer is (5).

Given: $\frac{1^3 + 2^3 + 3^3 + ... \text{ upto } n \text{ terms}}{1.3 + 2.5 + 3.7 + ... \text{upto } n \text{ terms}} = \frac{9}{5}$

As we know, sum of cubes of n natural numbers

$= \left\{\frac{n(n+1)}{2}\right\}^2$

$\Rightarrow \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\sum_{x=1}^{n} x(2x+1)} = \frac{9}{5}$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\sum_{x=1}^{n}(2x^2+x)} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\sum_{x=1}^{n}x^2+\sum_{x=1}^{n}x} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\left\{\frac{n(n+1)(2n+1)}{6}\right\}+\left\{\frac{n(n+1)}{2}\right\}} = \frac{9}{5}$$

{∵ Sum of squares of n natural numbers $= \frac{n(n+1)(2n+1)}{6}$ and sum of n natural numbers $= \frac{n(n+1)}{2}$}

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{n(n+1)\left\{\frac{2n+1}{3}+\frac{1}{2}\right\}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+2+3)}{6}} = \frac{9}{5} \Rightarrow \frac{3n(n+1)}{2(4n+5)} = \left(\frac{9}{5}\right)$$

$\Rightarrow 5n^2 + 5n = 24n + 30$
$\Rightarrow 5n^2 - 19n - 30 = 0$
$\Rightarrow 5n^2 + 6n - 25n - 30 = 0$
$\Rightarrow n(5n + 6) - 5(5n + 6) = 0$
$\Rightarrow (n - 5)(5n + 6) = 0$
$\Rightarrow n = 5$

HINT:

(1) Use $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

(2) Use $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

(3) Use $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

85. The correct answer is (405).

Given: Sum of coefficients of first 3 terms of $\left(x - \frac{3}{x^2}\right)^n = 376$

General term of given expansion is

$$T_{r+1} = {}^nC_r x^{n-r}\left(\frac{-3}{x^2}\right)^r.$$

So, coefficients of first three terms are ${}^nC_0, -3\,{}^nC_1, 9\,{}^nC_2$

$\therefore\ {}^nC_0 - 3\,{}^nC_1 + 9\,{}^nC_2 = 376$

$$\Rightarrow 1 - 3n + \frac{9n(n-1)}{2} = 376$$

$\Rightarrow 3n^2 - 5n - 250 = 0$
$\Rightarrow (3n + 25)(n - 10) = 0$
$\Rightarrow n = 10, \frac{-25}{3}$ (not possible)

For coefficient of x^4, $n - 3r = 4$
$\Rightarrow 10 - 3r = 4$
$\Rightarrow r = 2$

$\therefore$ Coefficient of $x^4 = {}^{10}C_2(-3)^2 = \frac{10\times 9}{2\times 1}\times 9 = 405$

HINT:

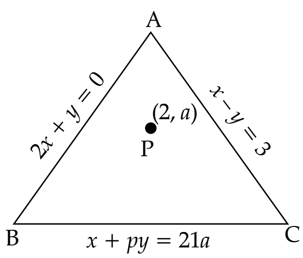
General term of binomial expansion $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

86. The correct answer is (122).

Given equation of sides are

AB : $2x + y = 0$ ...(i)
BC : $x + py = 21a$ ...(ii)
CA : $x - y = 3$ ...(iii)

Centroid of ΔABC = P $(2, a)$



On solving equation (i) and equation (iii), we get
A = (1, –2)

Let the coordinates of point B be $(m, -2m)$ and coordinates of point C be $(n + 3, n)$

∵ Centroid of Δ ABC = $(2, a)$

$$\Rightarrow \frac{m+n+3+1}{3} = 2 \text{ and } \frac{n-2m-2}{3} = a$$

$\Rightarrow m + n = 2$ ...(iv)
& $n - 2m = 3a + 2$ ...(v)

Put $n = 2 - m$ from equation (iv) to (v), we get
$m = -a$

Point B satisfy the equation of BC
So, $m - 2mp = 21a$
$\Rightarrow m(1 - 2p) = 21a$
$\Rightarrow 2p - 1 = 21$
$\Rightarrow p = 11$

Point C also satisfy the equation of BC
So, $n + 3 + p(n) = 21a$
$\Rightarrow 12n + 3 = -21m$

$\Rightarrow 12n + 21m + 3 = 0$ ...(vi)

On solving equation (iv) and equation (vi), we get

$m = -3, n = 5$

$\therefore$ B = (–3, 6) and C = (8, 5)

Now, BC = $\sqrt{(11)^2 + 1^2}$

$\Rightarrow$ BC = $\sqrt{122}$

$\Rightarrow BC^2 = 122$

HINT:

(1) Find the coordinates of A by solving equation of side AB and AC.
(2) Find the coordinates of B and C using given condition and solve further.

87. The correct answer is (8).

Given: $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$

$\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$

$\vec{a}.\vec{c} = 7$

$2\vec{b}.\vec{c} + 43 = 0$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

$\because \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

$\Rightarrow (\vec{a} - \vec{b}) \times \vec{c} = 0$

$\Rightarrow (\vec{a} - \vec{b}) || \vec{c}$

$\Rightarrow \vec{c} = k(\vec{a} - \vec{b}) \Rightarrow \vec{c} = k(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$

$\because \vec{a}.\vec{c} = 7$

$\Rightarrow k(-2 + 14 + 2\lambda^2) = 7$

$\Rightarrow k(2\lambda^2 + 12) = 7$ ...(i)

Also, $2\vec{b}.\vec{c} = -43$

$\Rightarrow 2k(-6 - 35 - 2\lambda^2) = -43$

$\Rightarrow 2k(-41 - 2\lambda^2) = -43$ ...(ii)

From equation (i) and equation (ii), we get

$$\frac{2\lambda^2 + 12}{2(41 + 2\lambda^2)} = \frac{7}{43}$$

$\Rightarrow 43(\lambda^2 + 6) = 7(2\lambda^2 + 41)$

$\Rightarrow 29\lambda^2 = 29$

$\Rightarrow \lambda^2 = 1$

Now, $\vec{a}.\vec{b} = 3 - 10 - \lambda^2 = -8$

$\Rightarrow |\vec{a}.\vec{b}| = 8$

HINT:

(1) Use if $\vec{a}$ is parallel to $\vec{b}$, then $\vec{a} = k\vec{b}; k \in R$

(2) If $\vec{a} = \vec{a}_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \vec{b}_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

88. The correct answer is (13).

Given: Relation R = {(a, b), (b, c), (b, d)} on set {a, b, c, d} for a relation to be equivalence relation, it must be reflexive, symmetric and transitive.

For reflexive relation, (a, a), (b, b), (c, c), (d, d) must be added in relation R.

So, R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d)}

For symmetric relation, if $(x, y) \in R \Rightarrow (y, x) \in R$

Now, as $(a, b) \in R \Rightarrow (b, a) \in R$

and $(b, c) \in R \Rightarrow (c, b) \in R$

and $(b, d) \in R \Rightarrow (d, b) \in R$

So, R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (b, a), (c, b), (d, b)}

For transitive relation, if $(x, y) \in R$ and $(y, z) \in R$ $\Rightarrow (x, z) \in R$

So, R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (b, a), (c, b), (d, b), (a, c), (a, d), (c, a), (c, d), (d, c), (d, a)}

So, total number of elements added = 13

HINT:

(1) For a relation to be equivalence relation, it must be reflexive, symmetric and transitive.
(2) For reflexive relation, $(x, x) \in R$.
(3) For symmetric relation, if $(x, y) \in R \Rightarrow (y, x) \in R$
(4) For transitive relation, if $(x, y) \in R$ & (y, z) R $\Rightarrow (x, z) \in R$

89. The correct answer is (36).

Given curves $y^2 - 2y = -x$ and $x + y = 0$

Now, $y^2 - 2y + 1 = -x + 1$

$\Rightarrow (y - 1)^2 = -(x - 1)$ ...(i)

Let's find intersecting points of both curves

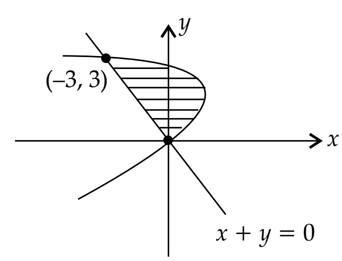
$y^2 - 2y - y = 0$

$\Rightarrow y^2 - 3y = 0$

$\Rightarrow y = 0, 3$

$\Rightarrow x = 0, -3$

$\therefore$ Intersecting points are (0, 0) and (–3, 3)



So, required area = $\int_0^3 \{(2y - y^2) - (-y)\}dy$

$\Rightarrow A = \int_0^3 (3y - y^2)dy$

$\Rightarrow A = \left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$

$\Rightarrow A = \dfrac{27}{2} - 9$

$\Rightarrow A = \dfrac{9}{2}$

$\Rightarrow 8A = 36$

HINT:

Draw the figure of both curves and identify the bounded region and use the concept of vertical strip and solve further.

90. The correct answer is (27).

Given: $f(x) + \int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \dfrac{\pi}{2}\right]$ ...(i)

Differentiate the above equation, we get

$f'(x) + f(x)\sqrt{1-[\log_e f(x)]^2} = 0$

$\Rightarrow \dfrac{dy}{dx} + y\sqrt{1-(\log_e y)^2} = 0$

$\Rightarrow \dfrac{dy}{dx} = -y\sqrt{1-\log_e^2 y}$

$\Rightarrow \int \dfrac{dy}{y\sqrt{1-\log_e^2 y}} = \int -1\,dx$

Let $\log_e y = u$

$\Rightarrow \dfrac{1}{y}dy = du$

$\Rightarrow \int \dfrac{du}{\sqrt{1-u^2}} = -x + c$

$\Rightarrow \sin^{-1} u = -x + c$

$\Rightarrow \sin^{-1} \log_e y = -x + c$

Put $x = 0$ in equation (i), we get

$f(0) = e$ i.e., $y(0) = e$

So, at $x = 0$, $\sin^{-1}(1) = c$

$\Rightarrow c = \dfrac{\pi}{2}$

$\therefore \sin^{-1} \log_e y = -x + \dfrac{\pi}{2}$

$\Rightarrow \log_e y = \sin\left(\dfrac{\pi}{2} - x\right)$

$\Rightarrow \log_e y = \cos x$

At $x = \dfrac{\pi}{6}$, $\log_e f\left(\dfrac{\pi}{6}\right) = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

So, $\left[6\log_e f\left(\dfrac{\pi}{6}\right)\right]^2 = \left[6 \times \dfrac{\sqrt{3}}{2}\right]^2 = 27$

HINT:

(1) Differentiate given equation using newton leibnitz rule and solve further differential equation using variable separable form.

(2) If $I(x) = \int_{g(x)}^{h(x)} \phi(x)dx$, then

$I'(x) = \phi(h(x))h'(x) - \phi(g(x))g'(x)$

JEE (Main) SOLVED PAPER 2022

29^{th} June Shift 1

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at $t = 0$ s. Ball B is thrown vertically down with an initial velocity 'u' at $t = 2$ s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1} . [use $g = 10\ ms^{-2}$] :

(A) 10 **(B)** 15

(C) 20 **(D)** 30

Q. 2. A body of mass M at rest explodes into three pieces, in the ratio of masses 1:1:2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be:

(A) 15 ms^{-1} **(B)** 25 ms^{-1}

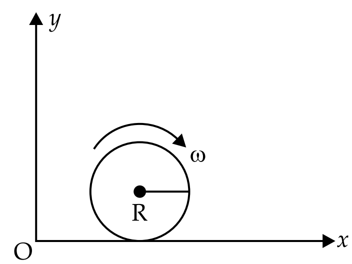
(C) 35 ms^{-1} **(D)** 50 ms^{-1}

Q. 3. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci ?

(A) 30 days **(B)** 35 days

(C) 40 days **(D)** 25 days

Q. 4. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3} R^2 \omega$. The value of a will be :



(A) 2 **(B)** 3

(C) 5 **(D)** 4

Q. 5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be : (**Given:** gas constant $R = 8.3 JK^{-1} mol^{-1}$)

(A) 249 J **(B)** 415 J

(C) 498 J **(D)** 830 J

Q. 6. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to :

(A) $\sqrt{L_1 L_2}$ **(B)** $\frac{L_1 + L_2}{2}$

(C) $2 L_1 - L_2$ **(D)** $3L_1 - 2L_2$

Q. 7. Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason **R.**

Assertion A : The photoelectric effect does not takes place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Q. 8. A particle of mass 500 gm is moving in a straight line with velocity $v = bx^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is : (Take $b = 0.25\ \text{m}^{-3/2}\ \text{s}^{-1}$).

(A) 2 J **(B)** 4 J

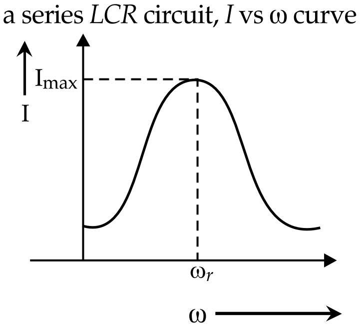
(C) 8 J **(D)** 16 J

Q. 9. A charge particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charge particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charge particle

(A) 1 : 1 **(B)** 1 : 2

(C) 2 : 1 **(D)** 1 : 4

Q. 10. For a series *LCR* circuit, *I* vs ω curve is shown :



(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

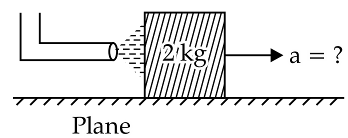
(d) At ω_r, impedance of the circuit is 0 .

Choose the most appropriate answer from the options given below :

(A) (a) and (d) only **(B)** (b) and (d) only

(C) (a) and (c) only **(D)** (b) and (c) only

Q. 11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be :



(A) 3 **(B)** 6

(C) 5 **(D)** 4

Q. 12. In Van der Waals equation $\left[P + \frac{a}{V^2}\right][V - b] =$ RT ; P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to:

(A) $\frac{P}{V}$ **(B)** $\frac{V}{P}$

(C) PV **(D)** PV^3

Q. 13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be :

(A) $\sin^{-1}\left(\frac{3}{5}\right)$ **(B)** $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$ **(D)** $\cos^{-1}\left(\frac{1}{3}\right)$

Q. 14. The escape velocity of a body on a planet '*A*' is 12 kms^{-1}. The escape velocity of the body on another planet '*B*', whose density is four times and radius is half of the planet '*A*', is :

(A) 12 kms^{-1} **(B)** 24 kms^{-1}

(C) 36 kms^{-1} **(D)** 6 kms^{-1}

Q. 15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :

(A) $\frac{1}{\sqrt{3}}$ **(B)** $\frac{1}{2}$

(C) $\sqrt{3}$ **(D)** 1