JEE (Main) MATHEMATICS SOLVED PAPER

2023

08^{th} April Shift 1

General Instructions :

(i) There are 30 questions in this section.
(ii) Section A consists of 20 Multiple choice questions and Section B consists of 10 Numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.
(iii) There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted questions.
(iv) For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
(v) Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
(vi) All calculations/ written work should be done in the rough sheet which is provided with Question Paper.

Section A

Q. 1. The area of the region $\{(x, y): x^2 \le y \le 8 - x^2, y \le 7\}$ is

(A) 24 **(B)** 21
(C) 20 **(D)** 18

Q. 2. Let $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. . If P^T

$Q^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

(A) 2004 **(B)** 2007
(C) 2005 **(D)** 2006

Q. 3. Negation of $(p \to q) \to (q \to p)$ is

(A) $(-q) \wedge p$ **(B)** $p \vee (\sim q)$
(C) $(\sim p) \vee q$ **(D)** $q \wedge (\sim p)$

Q. 4. Let C(α, β) be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$,
$4y - 3x = 17$ and
$x + 7y = 61$.
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(A) 18 **(B)** 15
(C) 16 **(D)** 17

Q. 5. Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(A) $\frac{155}{8}$ **(B)** 21
(C) 19 **(D)** $\frac{169}{8}$

Q. 6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

(A) 752 **(B)** 772
(C) 782 **(D)** 792

Q. 7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(A) 5481 **(B)** 3654
(C) 2436 **(D)** 1817

Q. 8. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q.

Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is

(A) 325 **(B)** 346
(C) 296 **(D)** 317

Q. 9. Let $S_K = \frac{1 + 2 + ... + K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D ∈ N and A has least value. Then

(A) A + B is divisible by D
(B) A + B = 5 (D –C)
(C) A + C + D is not divisible by B
(D) A + B + D is divisible by 5

Q. 10. The shortest distance between the lines

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(A) $2\sqrt{6}$ **(B)** $3\sqrt{6}$
(C) $6\sqrt{3}$ **(D)** $6\sqrt{2}$

Q. 11. The number of arrangements of the letters of the word “INDEPENDENCE” in which all the

vowels always occur together is.
(A) 16800 (B) 14800
(C) 18000 (D) 33600

Q. 12. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}$, $6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to
(A) 49 (B) 36
(C) 25 (D) 16

Q. 13. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.
(A) $\frac{5}{14}$ (B) $\frac{3}{7}$
(C) $\frac{9}{28}$ (D) $\frac{2}{7}$

Q. 14. If for $z = \alpha + i\beta$, $|z+2| = z + 4(1+i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation
(A) $x^2 + 3x - 4 = 0$ (B) $x^2 + 7x + 12 = 0$
(C) $x^2 + x - 12 = 0$ (D) $x^2 + 2x - 3 = 0$

Q. 15. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to __________.
(A) 24 (B) 9
(C) 18 (D) 15

Q. 16. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is
(A) $7\,(720)^2$ (B) 720
(C) $7\,(360)^2$ (D) $126\,(5!)^2$

Q. 17. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to
(A) $\frac{-2}{3}$ (B) $\frac{2}{9}$
(C) $\frac{-1}{3\sqrt{3}}$ (D) $\frac{2}{3\sqrt{3}}$

Q. 18. If the eqation of the plane containing the line $x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ is $ax + by + cz = 4$, then $(a - b + c)$ is equal to
(A) 22 (B) 24
(C) 20 (D) 18

Q. 19. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj}\ (\text{adj}\ (\text{adj}\ 2A))| = (16)^n$, then n is equal to
(A) 8 (B) 9
(C) 12 (D) 10

Q. 20. Let $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx, x>0.$ $\lim_{x\to\infty} I(x)=0$, then I(1) is equal to
(A) $\frac{e+1}{e+2}-\log_e(e+1)$ (B) $\frac{e+2}{e+1}+\log_e(e+1)$
(C) $\frac{e+2}{e+1}-\log_e(e+1)$ (D) $\frac{e+1}{e+2}+\log_e(e+1)$

Section B

Q. 21. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R =$(x, y) \in A \times A$: $x - y$ is odd positive integer or $x - y = 2$}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ______.

Q. 22. Let $[t]$ denote the greatest integer $\le t$, If the constant term in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to _____.

Q. 23. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2},1,\lambda\right)$ and (–2, 0, 1) are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$ is

Q. 24. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)\,dy = 0$, $x > 1$ passes through the points $\left(e,\frac{4}{3}\right)$ and (e^4, α), then α is equal to ______.

Q. 25. Let $\vec{a}=6\hat{i}+9\hat{j}+12\hat{k}, \vec{b}=\alpha\hat{i}+11\hat{j}-2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$. If $\vec{a}.\vec{c}=-12$, $\vec{c}.(\hat{i}-2\hat{j}+\hat{k})=5$, then $\vec{c}(\hat{i}+\hat{j}+\hat{k})$ is equal to ________.

Q. 26. The largest natural number n such that $3n$ divides 66! is ______.

Q. 27. If a_0 is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}, n = 1, 2, 3, ...,$ then a is equal to ________.

Q. 28. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to ________.

Q. 29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to ________.

Q. 30. Let $[t]$ denote the greatest integer $\leq t$. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} (8\,[\text{cosec}\ x] - 5\,[\cot x])\,dx$ is equal to ________.

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	C	Area between the curves	Integral Calculus
2	C	Algebra of matrices	Matrices
3	D	Negation of a statement	Mathematical Reasoning
4	D	Circumcentre	Straight line
5	C	Cube root of unity	Cubic Equation
6	D	r things out of n things	Permutation and Combination
7	B	Coefficient of a term	Binomial theorem
8	A	Parabola	Conic Section
9	A	Sum of n terms	Sequences and series
10	B	Shortest distance	Three dimensional geometry
11	A	Number of ways	Permutation and Combination
12	B	Collinearity	Vector algebra
13	A	Conditional probability	Probability
14	B	Roots of equation	Complex numbers
15	C	Limits of trigonometry	Limits
16	D	Number of ways	Permutation and Combination
17	B	Higher order derivatives	Differentiability
18	A	Equation of plane	Three dimensional geometry
19	D	Adjoint	Matrices and Determinants
20	C	Indefinite Integral	Integral Calculus
21	[19]	Symmetric relation	Relation and Function
22	[1275]	General term	Binomial theorem
23	[9]	Plane	Three dimensional geometry
24	[3]	Linear Differential Equation	Differential equation
25	[11]	Algebra of vectors	Vector algebra
26	[31]	Remainder theorem	Binomial theorem
27	[5]	Maxima/Minima	Application of derivatives
28	[25]	Mean, Variance	Statistics
29	[2]	Circle	Conic Section
30	[14]	Definite Integral	Integral Calculus

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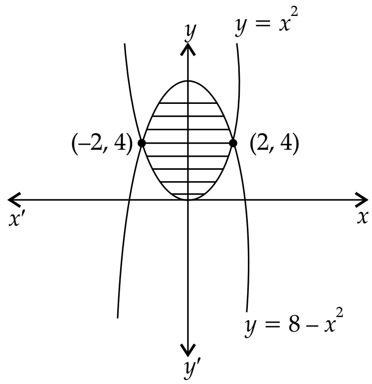
Solutions

Section A

1. Option (C) is correct.
The given curves are
$x^2 \le y, y \le 8 - x^2; y \le 7$
On solving, we get
$x^2 = 8 - x^2$



$\Rightarrow x^2 = 4$
$\Rightarrow x = \pm 2$

So, area $= 2\left[\int_0^4 \sqrt{y}\,dy + \int_4^7 \sqrt{8-y}\,dy\right]$

$$= 2\left\{\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 + \left[\frac{-(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^7\right\}$$

$$= 2 \times \frac{2}{3}\left\{[4^{3/2} - 0] + (-(1)^{3/2} + (4)^{3/2})\right\}$$

$$= \frac{4}{3}\{8 - 1 + 8\} = \frac{4}{3} \times 15 = 20 \text{ sq. units}$$

HINT:
Draw the graph of both curves, then find the bounded region and proceed.

2. Option (C) is correct.

Here, $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Here, $PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$$= \begin{bmatrix} \frac{3}{4} + \frac{1}{4} & -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ \frac{-\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

|| $P^TP = I$
$\because Q = PAP^T$
$\Rightarrow Q^{2007} = (PAP^T)\,(PAP^T)$......... 2007 time
$= PA^{2007}P^T$

As, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

Hence, $P^TQ^{2007}P = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

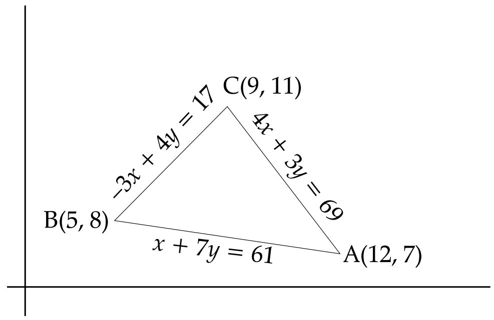
$\Rightarrow a = 1, b = 2007, c = 0, d = 1$
$\therefore 2a + b - 3c - 4d = 2\,(1) + 2007 - 3(0) - 4(1)$
$= 2 + 2007 - 4 = 2005$

HINT:
Transpose the given matrix and multiply the matrices to solve further.

3. Option (D) is correct.
Given: $(p \to q) \to (q \to p)$
Negation of above statement is
$\sim[(p \to q) \to (q \to p)]$
$\equiv \sim [\sim p \to q \wedge q \to p]$
$\equiv\ p \to q \wedge \sim q \to p$
$\equiv \sim p \vee q \wedge q \wedge \sim p]$
$\equiv q \wedge (\sim p)$

HINT:
The negation of a statement is the opposite of the given mathematical statement.

4. Option (D) is correct.
We have,
$4x + 3y = 69$ ...(i)
$4y - 3x = 17$ ...(ii)
$x + 7y = 61$ ...(iii)
On solving (i) and (iii), we get
$x = 12$, and $y = 7$
So, $A \equiv (12, 7)$



On solving (ii) and (iii), we get

$x = 5$ and $y = 8$

So, $B \equiv (5, 8)$

Hence, circumcentre $\equiv \left(\frac{12+5}{2}, \frac{7+8}{2}\right)$

$\equiv \left(\frac{17}{2}, \frac{15}{2}\right)$

$\therefore \alpha = \frac{17}{2}, \beta = \frac{15}{2}$

$\therefore (\alpha - \beta)^2 + (\alpha + \beta) = \left(\frac{17}{2} - \frac{15}{2}\right)^2 + \left(\frac{17}{2} + \frac{15}{2}\right)$

$= (1)^2 + (16) = 17$

HINT:

Circumcentre of a right triangle is the midpoint of hypotenuse of the triangle.

5. Option (C) is correct.

Given cubic equation is :

$x^3 + bx + c = 0$

$\because \alpha, \beta, \gamma$ are the roots of above equation.

And $\beta\gamma = 1 = -\alpha$

So, product of roots $= -c$

$\Rightarrow \alpha\beta\gamma = -c$

$\Rightarrow (-1)(1) = -c$

$\Rightarrow c = 1$

Since, $\alpha = -1$ is the root. So,

$\Rightarrow -1 - b + c = 0$

$\Rightarrow c - b = 1$

$\Rightarrow 1 - b = 1 \Rightarrow b = 0$

The given equation becomes $x^3 + 1 = 0$

So, roots are $-1, -\omega, -\omega^2$

$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$= 0 + 2 - 3(-1)^3 - 6(-\omega)^3 - 8(-\omega^2)^3$

$= 2 + 3 + 6\omega^3 + 8\omega^6$

$= 5 + 6 + 8 = 19$

HINT:

For a cubic equation, $ax^3 + bx^2 + cx + d = 0$

Sum of roots $= \frac{-b}{a}$,

Product of roots taken two at a time $= \frac{c}{a}$

Product of roots $= \frac{-d}{a}$

6. Option (D) is correct.

Since, $n(A) = 5, n(B) = 2$

$\Rightarrow n(A \times B) = n(A) \times n(B)$

$= 5 \times 2 = 10$

So, number of subsets having 3 elements $= {}^{10}C_3$

Number of subsets having 4 elements $= {}^{10}C_4$

Number of subsets having 5 elements $= {}^{10}C_5$

Number of subsets having 6 elements $= {}^{10}C_6$

$\therefore$ No. of subsets $= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

$= 120 + 210 + 252 + 210 = 792$

HINT:

No of subsets having r elements out of total n elements $= {}^nC_r$

7. Option (B) is correct.

Given: ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$= 1 : 5 : 20$

$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{1}{5}$

$\Rightarrow \frac{r}{(n-r+1)} = \frac{1}{5}$

$\Rightarrow 5r = n - r + 1$

$\Rightarrow n = 6r - 1$ ...(i)

Also, $\frac{n}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{20} = \frac{1}{20}$

$\Rightarrow \frac{(r+1)}{(n-r)} = \frac{1}{4}$

$\Rightarrow 4r + 4 = n - r$

$\Rightarrow n = 5r + 4$ ...(ii)

From (i) and (ii), we get

$6r - 1 = 5r + 4$

$\Rightarrow r = 5$

So, $n = 5(5) + 4 = 29$

So, coefficient of 4^{th} terms $= {}^nC_3 = {}^{29}C_3$

$= \frac{29!}{3!26!} = \frac{29 \times 28 \times 27}{3 \times 2} = 3654$

HINT:

In the expansion of $(a + b)^n$, the general term is $T_{r+1} = {}^nC_r (a)^{n-r} b^r$

8. Option (A) is correct.

$y^2 = 20x, y = mx + c$

Put value of x

$y^2 = 20\left(\frac{y-c}{m}\right)$

$\Rightarrow y^2 - \frac{20}{m}y + \frac{20}{m}c = 0$ ...(i)

Since, centroid = (10, 10)

So, $\frac{y_1 + y_2 + 0}{3} = 10$

$\Rightarrow y_1 + y_2 = 30$

From (1),

Sum of roots $= \frac{20}{m} = 30 \Rightarrow m = \frac{2}{3}$

Also, $c - m = 6 \Rightarrow c = 6 + \frac{2}{3} = \frac{20}{3}$

Now, the equation is :

$y^2 - \frac{20}{2} \times 3y + \frac{20}{2} \times 3 \times \frac{20}{3} = 0$

$\Rightarrow y^2 - 30y + 200 = 0$

$\Rightarrow y^2 - 20y - 10y + 200 = 0$

$\Rightarrow (y - 20)(y - 10) = 0$

$\Rightarrow y = 10, 20 \Rightarrow x = 5, x = 20$

$\therefore$ P ≡ (5, 10), Q ≡ (20, 20)

So, $(PQ)^2 = (20 - 5)^2 + (20 - 10)^2$

= 225 + 100 = 325

HINT:

Centroid of a triangle having vertices (a, b), (c, d) & (e, f) is $\left(\frac{a+c+e}{3}, \frac{b+d+f}{3}\right)$

9. Option (A) is correct.

$\because S_k = \frac{1+2+...+k}{k}$

$= \frac{k(k+1)}{2k} = \frac{k+1}{2}$

$\Rightarrow S_k^2 = \left(\frac{k+1}{2}\right)^2 = \frac{k^2+1+2k}{4}$

$\Rightarrow \sum_{j=1}^{n} S_j^2 = \frac{1}{4}\left[\sum_{j=1}^{n} k^2 + \sum_{j=1}^{n} 1 + 2\sum_{j=1}^{n} k\right]$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + \frac{2n(n+1)}{2}\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{24}[2n^2 + 3n + 1 + 6 + 6n + 6]$

$= \frac{n}{24}[2n^2 + 9n + 13]$

On comparing, we get

A = 24, B = 2, C = 9, D = 13

(1) A + B = 24 + 2 = 26, divisible by 13

(2) A + B = 26

5 (D – C) = 5 (13 – 9) = 20

$\therefore 26 \neq 20$

(3) A + C + D = 46, which is divisible by 2

(4) A + B + D = 39, which is not divisible by 5.

HINT:

$1 + 2 + ... + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + ... n^2 = \frac{n(n+1)(2n+1)}{6}$

10. Option (B) is correct.

The given lines are :

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$

So, $\vec{b}_1 = 4\hat{i} + 5\hat{j} + 3\hat{k}$

$\vec{b}_2 = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{a}_1 = 4\hat{i} - 2\hat{j} - 3\hat{k}, \vec{a}_2 = \hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$

$= (10 - 12)\hat{i} - (8 - 9)\hat{j} + (16 - 15)\hat{k}$

$= -2\hat{i} + \hat{j} + \hat{k}$

Shortest distance, $d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \left|\frac{(3\hat{i} - 5\hat{j} - 7\hat{k}).(-2\hat{i} + \hat{j} + \hat{k})}{\sqrt{4+1+1}}\right|$

$= \left|\frac{-6-5-7}{\sqrt{6}}\right| = \frac{18}{\sqrt{6}} = 3\sqrt{6}$ units

HINT:

Shortest distance between two lines is:

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

11. Option (A) is correct.

In the given word,

vowels are : I, E, E, E, E

Consonants are : N, D, P, N, D, N, C

So, number of words $= \frac{8!}{3!2!} \times \frac{5!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4}{2} \times 5 = 16800$

HINT:

Out of n objects, if r things are same, so number of ways $= \frac{n!}{r!}$

12. Option (B) is correct.

Given: Points with position vectors

$\alpha\hat{i} + 10\hat{j} + 13\hat{k}, 6\hat{i} + 11\hat{j} + 11\hat{k}$

and $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear.

So, $\frac{\alpha - 6}{6 - \frac{9}{2}} = \frac{10-11}{11-\beta} = \frac{13-11}{11+8}$

$\Rightarrow \frac{2(\alpha - 6)}{3} = \frac{-1}{11-\beta} = \frac{2}{19}$

$\Rightarrow \frac{2}{3}(\alpha - 6) = \frac{2}{19}$

$\Rightarrow 19\alpha - 114 = 3 \Rightarrow 19\alpha = 117$

$\Rightarrow \alpha = \frac{117}{19}$

And, $\frac{-1}{11-\beta} = \frac{2}{19}$

$\Rightarrow -19 = 22 - 2\beta$

$\Rightarrow 2\beta = 41$

$\Rightarrow \beta = \frac{41}{2}$

$\therefore (19\alpha - 6\beta)^2 = \left(19 \times \frac{117}{19} - \frac{6 \times 41}{2}\right)^2$

$= (117 - 123)^2 = 36$

HINT:

If point $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$, $(\alpha_3, \beta_3, \gamma_1)$ are collinear, then $\frac{\alpha_1 - \alpha_2}{\alpha_2 - \alpha_3} = \frac{\beta_1 - \beta_2}{\beta_2 - \beta_3} = \frac{\gamma_1 - \gamma_2}{\gamma_2 - \gamma_3}$.

13. Option (A) is correct.

Given: $P(A) = \frac{20}{100} = \frac{2}{10}$

$P(B) = \frac{30}{100} = \frac{3}{10}$; $P(C) = \frac{50}{100} = \frac{5}{10}$

Let E $\rightarrow$ Event that the bolt is defective.

So, $P(E/A) = \frac{3}{100}, P\left(\frac{E}{B}\right) = \frac{4}{100}, P\left(\frac{E}{C}\right) = \frac{2}{100}$

So, P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

$$= \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{3}{100} \times \frac{2}{10} + \frac{4}{100} \times \frac{3}{10} + \frac{2}{100} \times \frac{5}{10}}$$

$$= \frac{10}{6+12+10} = \frac{10}{28} = \frac{5}{14}$$

HINT:

Conditional probability P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

14. Option (B) is correct.

Given: $|z + 2| = z + 4(1 + i)$

Also, $z = \alpha + i\beta$

$\therefore |z + 2| = |\alpha + i\beta + 2| = (\alpha + i\beta) + 4 + 4i$

$\Rightarrow |(\alpha + 2) + i\beta| = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \sqrt{(\alpha+2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \beta + 4 = 0 \Rightarrow \beta = -4$

Now, $(\alpha + 2)^2 + \beta^2 = (\alpha + 4)^2$

$\Rightarrow \alpha^2 + 4 + 4\alpha + \beta^2 = \alpha^2 + 16 + 8\alpha$

$\Rightarrow 4 + 4\alpha + 16 = 16 + 8\alpha$

$\Rightarrow 4\alpha = 4 \Rightarrow \alpha = 1$

So, $\alpha + \beta = -3$ and $\alpha\beta = -4$

$\therefore$ Required equation is

$x^2 - (-3 - 4)x + (-3)(-4) = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

15. Option (C) is correct.

$$\lim_{x\to 0}\left[\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right]$$

$$= \lim_{x\to 0}\left[\frac{1-\cos^2(3x)}{9x^2} \times \frac{9x^2}{\cos^3(4x)}\right] \times \frac{\frac{\sin^3 4x}{(4x)^3} \times 64x^3}{\left[\frac{\log_e(2x+1)}{2x}\right]^5 \times (2x)^5}$$

$$= \left[\frac{1 \times 9 \times 1}{(1)}\right] \times \left[\frac{1 \times 64}{1 \times 32}\right]$$

$= 9 \times 2 = 18$

HINT:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

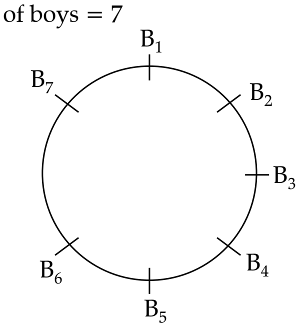
$$\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$$

16. Option (D) is correct.

We have,

Number of girls = 5

Number of boys = 7



So, number of ways of arranging boys around the table = 6! and 5 girls can be arranged in 7 gaps in ${}^7P_5$ ways

$\therefore$ Required no. of ways $= 6! \times {}^7P_5$

$= 126 \times (5!)^2$

17. Option (B) is correct.

$$f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$$

$$= \frac{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x - 1}{\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x}$$

$$= \frac{\cos\left(x - \frac{\pi}{4}\right) - 1}{\sin\left(x - \frac{\pi}{4}\right)}$$

$$= \frac{-2\sin^2\left(\frac{x}{2} - \frac{\pi}{8}\right)}{2\sin\left(\frac{x}{2} - \frac{\pi}{8}\right)\cos\left(\frac{x}{2} - \frac{\pi}{8}\right)}$$

$$\Rightarrow f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$\Rightarrow f'(x) = -\frac{1}{2}\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$\Rightarrow f''(x) = -\frac{1}{2}.2\sec\left(\frac{x}{2} - \frac{\pi}{8}\right).\sec\left(\frac{x}{2} - \frac{\pi}{8}\right) \tan\left(\frac{x}{2} - \frac{\pi}{8}\right) \times \frac{1}{2}$$

$$= -\frac{1}{2}\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right).\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

Now, $f\left(\frac{7\pi}{12}\right) f''\left(\frac{7\pi}{12}\right)$

$$= -\tan\left(\frac{7\pi}{24} - \frac{\pi}{8}\right) \times \frac{-1}{2}\sec^2\left(\frac{7\pi}{24} - \frac{\pi}{8}\right) \times \tan\left(\frac{7\pi}{24} - \frac{\pi}{8}\right)$$

$$= \frac{1}{2}\tan^2\left(\frac{\pi}{6}\right) \times \sec^2\frac{\pi}{6}$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{4}{3} = \frac{2}{9}$$

HINT:

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec^2 x) = 2\sec^2 x.\tan x$$

18. Option (A) is correct.

Equation of plane P containing the given lines is

$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$

$\Rightarrow (1 + 2\lambda)\, x + (2 + \lambda)y + (3 - \lambda)\, z + (-4 + 5\, \lambda) = 0$

Now, plane P is perpendicular to plane P'

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$

So, normal to plane P' is

$\vec{n} = (\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} - 2\hat{j} + 3\hat{k})$

$\Rightarrow \vec{n} = 5\hat{i} - 2\hat{j} - 3\hat{k}$

$\therefore$ P and P' are perpendicular

$\therefore 5\,(1 + 2\lambda) - 2(2 + \lambda) - 3\,(3 - \lambda) = 0$

$\Rightarrow 5 + 10\lambda - 4 - 2\lambda - 9 + 3\lambda = 0$

$\Rightarrow 11\lambda = 8 \Rightarrow \lambda = \frac{8}{11}$

$$\therefore P: \left(1 + \frac{16}{11}\right)x + \left(2 + \frac{8}{11}\right)y + \left(3 - \frac{8}{11}\right)z + \left(5 \times \frac{8}{11} - 4\right) = 0$$

i.e., $27x + 30y + 25z = 4$

which is same as $ax + by + cz = 4$

$\therefore a = 27, b = 30$ and $c = 25$

$\Rightarrow a - b + c = 27 - 30 + 25 = 22$

HINT:

When two planes are perpendicular, then dot product of their normals is zero.

19. Option (D) is correct.

We have,

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(4-1) - 1(2-0) + 0$$

$= 6 - 2 = 4$

So, $|2A| = 2^3\,|A| = 8 \times 4 = 32$

Now, $|\text{adj (adj (adj } 2A))| = |2A|^{(n-1)^3}$

$= (32)^{2^3} = 32^8$

$\Rightarrow 16^n = (32)^8 = 2^8 \times 16^8$

$\Rightarrow 16^n = 16^{2+8} \Rightarrow n = 10$

HINT:

(1) $|kA| = k^n\,|A|$

(2) $|\text{adj } A| = |A|^{n-1}$

20. Option (C) is correct.

$$I = \int \frac{x+1}{x(1 + xe^x)^2}dx$$

Put $1 + xe^x = t \Rightarrow xe^x = t - 1$

$\Rightarrow (xe^x + e^x)\, dx = dt$

$\Rightarrow e^x\,(x + 1)\, dx = dt$

$$\therefore I = \int \frac{dt}{e^x.xt^2} = \int \frac{dt}{(t-1)t^2}$$

Let $\frac{1}{t^2(t-1)} = \frac{A}{(t-1)} + \frac{Bt + C}{t^2}$

$\Rightarrow 1 = At^2 + (Bt + C)\,(t - 1)$

Comparing coefficients of t^2, t and constant terms, we get

$A + B = 0, C - B = 0, -C = 1$

On solving above equations, we get

$C = -1, =B, A = 1$

$$\therefore\ I = \int \frac{1}{t-1}dt + \int \frac{-t-1}{t^2}dt$$

$$= \int \frac{1}{t-1}dt - \int \frac{1}{t}dt - \int \frac{1}{t^2}dt$$

$$= \log|t-1| - \log|t| + \frac{1}{t} + C$$

$$\Rightarrow I = \log|xe^x| - \log|1 + xe^x| + \frac{1}{1 + xe^x} + c$$

$$= \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$$

Now, $\lim_{x\to\infty} I(x) = 0$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C\right\} = 0$$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left(\frac{e^x}{\frac{1}{x}+e^x}\right) + \frac{\frac{1}{x}}{\frac{1}{x}+e^x} + C\right\}$$

$\Rightarrow 0 + 0 + C = 0 \Rightarrow C = 0$

$$\therefore I(x) = \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x}$$

$$\Rightarrow I(1) = \log\left|\frac{e}{1+e}\right| + \frac{1}{1+e} = 1 - \log(1+e) + \frac{1}{1+e}$$

$$= \frac{2+e}{1+e} - \log|1+e|$$

HINT:

(1) $\lim_{x\to\infty} \frac{1}{x} = 0$

(2) $\log 1 = 0$

21. The correct answer is (19).

We have, A = {0, 3, 4, 6, 7, 8, 9, 10}

Case I: $x - y$ is odd, if one is odd and one is even and $x > y$.

$\therefore$ Possibilites are {(3, 0), (4, 3), (6, 3), (7, 6), (7, 4), (7, 0), (8, 7), (8, 3), (9, 8), (9, 6), (9, 4), (9, 0), (10, 9), (10, 7), (10, 3)}

No. of cases = 15

Case II: $x - y = 2$

$\therefore$ Possibilities are {(6, 4), (8, 6), (9, 7), (10, 8)}

$\therefore$ No. of cases = 4

So, minimum ordered pair to be added = 15 + 4 = 19

HINT:

Any relations said to be symmetric if $(a, b) \in R$ $\Rightarrow (b, a) \in R$

22. The correct answer is (1275).

Let T_{r+1} be the constant term.

$$T_{r+1} = {}^7C_r (3x^2)^{7-r}\left(\frac{-1}{2x^5}\right)^r$$

For constant term, power of x should be zero.

i.e., $14 - 2r - 5r = 0$

$\Rightarrow 14 = 7r \Rightarrow r = 2$

Now, constant term = α

$$\Rightarrow {}^7C_2 (3)^5 \left(\frac{-1}{2}\right)^2 = \alpha$$

$$\Rightarrow 21 \times 243 \times \frac{1}{4} = \alpha$$

$\Rightarrow [\alpha] = [1275.75] = 1275$

HINT:

Let $(a + b)^n$, then $T_{r+1} = {}^nC_r\, a^{n-r} . b^r$

23. The correct answer is (9).

Since $\left(\frac{5}{2}, 1, \lambda\right)$ and (–2, 0, 1) are equidistant from plane $2x + 3y - 6z + 7 = 0$

$$\therefore \left|\frac{2\left(\frac{5}{2}\right) + 3(1) - 6(\lambda) + 7}{\sqrt{2^2+3^2+6^2}}\right| = \left|\frac{2(-2)+3(0)-6(1)+7}{\sqrt{2^2+3^2+6^2}}\right|$$

$\Rightarrow |5 + 3 - 6\lambda + 7| = |-4 -6 + 7|$

$\Rightarrow |15 - 6\lambda| = |-3|$

$\Rightarrow 15 - 6\lambda = \pm 3$

$\Rightarrow 15 - 6\lambda = 3$ or $15 - 6\lambda = -3$

$\Rightarrow 6\lambda = 12$ or $6\lambda = 18$

$\Rightarrow \lambda = 2$ or $\lambda = 3$

$\because \lambda_1 > \lambda_2$

$\therefore \lambda_1 = 3$ and $\lambda_2 = 2$

So, point will be (1, 2, 3)

Let $M_0 = (1, 2, 3)$

M_1 is the point through which line passes i.e, (5, 1, –7)

and $\vec{s} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \overrightarrow{M_0M_1} = 4\hat{i} - \hat{j} - 10\hat{k}$

Now, required distance $= \left|\frac{\overrightarrow{M_0M_1} \times \vec{s}}{|\vec{s}|}\right|$

$$= \frac{|(4\hat{i} - \hat{j} - 10\hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k})|}{\sqrt{1+4+4}}$$

$$= \frac{|18\hat{i} - 18\hat{j} + 9\hat{k}|}{3} = 9$$

HINT:

Distance of a point (a, b, c) from a plane $px + qy + rz + s = 0$ is $\frac{|ap + bq + cr + s|}{\sqrt{p^2+q^2+r^2}}$

24. The correct answer is (3).

The given differential equation is,

$(y - 2\log x)\,dx + (x \log x^2)\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{(2\log x - y)}{2x\log x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{2x\log x} = \frac{1}{x}$$

It is a linear differential equation.

$$\therefore \text{I.F.} = e^{\int \frac{1}{2x\log x}dx}$$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore \text{I.F.} = e^{\int \frac{1}{2t}dt} = e^{\log(t)^{\frac{1}{2}}} = \sqrt{t} = \sqrt{\log x}$$

So, required solution is,

$$y\sqrt{\log x} = \int \frac{\sqrt{\log x}}{x} dx$$

$$\log x = v \Rightarrow \frac{1}{x} dx = dv$$

$$\Rightarrow y\sqrt{\log x} = \int \sqrt{v}\, dv + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2v^{3/2}}{3} + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2}{3}(\log x)^{3/2} + C$$

Now, this curve passes through $\left(e, \frac{4}{3}\right)$ and (e^4, α)

$$\therefore \frac{4}{3}\sqrt{\log e} = \frac{2}{3}(\log e)^{3/2} + C$$

$$\Rightarrow C = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

Also, $\alpha\sqrt{\log e^4} = \frac{2}{3}(\log e^4)^{3/2} + \frac{2}{3}$

$$\Rightarrow 2\alpha = \frac{2}{3} \times (4)^{3/2} + \frac{2}{3} = \frac{16}{3} + \frac{2}{3} = \frac{18}{3}$$

$\Rightarrow \alpha = 3$

HINT:

Reduce the given differential equation to linear differential equation and find its solution.

25. The correct answer is (11).

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Now, $\vec{a}.\vec{c} = -12$

$\Rightarrow 6C_1 + 9C_2 + 12C_3 = -12$ ...(i)

Also, $\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$

$\Rightarrow C_1 - 2C_2 + C_3 = 5$ ...(ii)

Now, $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$

$\Rightarrow \vec{a} \times (\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a}$ is parallel to $(\vec{c} - \vec{b})$

$\Rightarrow \vec{a} = \lambda(\vec{c} - \vec{b})$

$$\Rightarrow 6\hat{i} + 9\hat{j} + 12\hat{k} = \lambda(c_1 - \alpha)\hat{i} + \lambda(c_2 - 11)\hat{j} + \lambda(c_3 + 2)\hat{k}$$

On comparing, we get

$$c_1 = \frac{6}{\lambda} + \alpha, c_2 = \frac{9}{\lambda} + 11, c_3 = \frac{12}{\lambda} - 2$$

Put there values in (ii), we get

$$\frac{6}{\lambda} + \alpha - \frac{18}{\lambda} - 22 + \frac{12}{\lambda} - 2 = 5$$

$\Rightarrow \alpha = 29$

From (i) and values of C_1, C_2, C_3, and α we have

$$6\left(\frac{6}{\lambda} + 29\right) + 9\left(\frac{9}{\lambda} + 11\right) + 12\left(\frac{12}{\lambda} - 2\right) = -12$$

$$\Rightarrow \frac{261}{\lambda} = -261 \Rightarrow \lambda = -1$$

So, $C_1 = 23$, $C_2 = 2$, $C_3 = -14$

$$\therefore \vec{c}.(\hat{i} + \hat{j} + \hat{k}) = (23\hat{i} + 2\hat{j} + -14\hat{k}).(\hat{i} + \hat{j} + \hat{k})$$

$= 23 + 2 - 14 = 11$

HINT:

$$\vec{a} \times \vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{a} \,||\, (\vec{c} - \vec{b}) \Rightarrow a = \lambda(\vec{c} - \vec{b})$$

26. The correct answer is (31).

We have,

$$\left[\frac{66}{3}\right] = 22$$

$$\left[\frac{66}{3^2}\right] = 7$$

$$\left[\frac{66}{3^3}\right] = 2$$

Highest powers of 3 is greater than 66. So, their g.i.f. is always 0

$\therefore$ Required natural number = 22 + 7 + 2 = 31

27. The correct answer is (5).

Let $y = \dfrac{x^3}{x^4 + 147}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^4 + 147) \times 3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

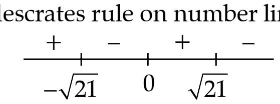
$$= \frac{3x^6 + 441x^2 - 4x^6}{(x^4 + 147)^2} = \frac{441x^2 - x^6}{(x^4 + 147)^2}$$

For maxima/minima, put $\frac{dy}{dx} = 0$

$\Rightarrow 441\, x^2 - x^6 = 0 \Rightarrow x^4 = 441$

$\Rightarrow x = \pm\sqrt{21}, \ \pm\sqrt{21}i$

Now, by descrates rule on number line we have



Since sign changes from negative to positive at 0.

$\therefore$ Maximum value of is at $x = \sqrt{21} = 4.58$

Now, 4 < 4.5 < 5

$$\therefore y \text{ at } x = 4 = \frac{64}{403} = 0.159$$

$$y \text{ at } x = 5 = \frac{125}{772} = 0.162$$

So, y is maximum at $x = 5$

$\therefore \alpha = 5$

HINT:

For maximum value, find $\frac{dy}{dx}$ and then observe the change in signs using decrates rule.

28. The correct answer is (25).

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$

x	$x-9$	$(x-9)^2$
y	$y-9$	$(y-9)^2$
10	1	1
12	3	9
6	–3	9
12	3	9
4	–5	25
8	–1	1
$x+y+92$		$(x-9)^2+(y-9)^2+54$

Now, mean $(\bar{x}) = 9$

$$\Rightarrow \frac{x+y+52}{8} = 9$$

$\Rightarrow x + y = 20$ ...(i)

Also, variance = 9.25

$$\Rightarrow \frac{(x-9)^2+(y-9)^2+54}{8} = 9.25$$

$\Rightarrow x^2 + y^2 + 81 + 81 - 2 \times 9(x + y) = 20$

$\Rightarrow x^2 + y^2 - 18 \times 20 = -142$

$\Rightarrow x^2 + y^2 = 218$

$\Rightarrow x^2 + (20 - x)^2 = 218$

$\Rightarrow x^2 + 400 + x^2 - 40x = 218$

$\Rightarrow 2x^2 - 40x + 182 = 0$

$$\Rightarrow x = \frac{40 \pm 12}{4}$$

$\Rightarrow x = 13$ or $x = 7 \Rightarrow y = 7$ or $y = 13$

But $x > y$

$\therefore x = 13$ and $y = 7$

So, $3x - 2y = 39 - 14 = 25$

HINT:

(1) Mean $= \dfrac{\Sigma x_i}{n}$

(2) Variance $= \dfrac{\Sigma(x_i - \bar{x})^2}{n}$

29. The correct answer is (2).

We have,

$C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$

$C_1 : (x-2)^2 + (y-1)^5 - 5 = \alpha - 5$

$C_1 : (x-2)^2 + (y-1)^2 = (\sqrt{\alpha})^2$

So, centre and radius of C_1 are (2, 1) and $\sqrt{\alpha}$ respectively

Now, image of (2, 1) along the line $y = 2x + 1$ is,

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{2^2+(-1)^2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$

$$\Rightarrow x = \frac{-6}{5} \text{ and } y = \frac{13}{5}$$

Now, $\left(\frac{-6}{5}, \frac{13}{5}\right)$ will be the centre of C_2

$$\therefore f = \frac{6}{5} \text{ and } g = \frac{-13}{5}$$

Now, radius of $C_2 = r = \sqrt{f^2 + g^2 - \frac{36}{5}}$

$$\Rightarrow r = \sqrt{\frac{36}{25} + \frac{169}{25} - \frac{36}{5}} = 1$$

$\because$ $r = 1$ so, $\alpha = 1$

$\therefore \alpha + r = 1 + 1 = 2$

HINT:

Image of a point (x_1, y_1) w.r.t. $ax + by + c = 0$ is (x, y), then

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{(a^2 + b^2)}$$

30. The correct answer is (14).

$$\text{Let I} = \frac{2}{\pi}\int_{\frac{\pi}{6}}^{5\frac{\pi}{6}} \{8[\text{cosec } x] - 5[\cot x]\}dx$$

$$= \frac{2}{\pi}\left[8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\text{cosec } x]dx - 5\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x]dx\right]$$

$$= \frac{2}{\pi}\left[8\int_{\pi/6}^{5\pi/6} dx - 5\left\{\int_{\pi/6}^{\pi/4} dx + \int_{\pi/4}^{\pi/2} 0.dx + \int_{\pi/2}^{3\pi/4} (-1)dx + \int_{3\pi/4}^{5\pi/6} (-2)dx\right\}\right]$$

$$= \frac{2}{\pi}\left[8 \times \left(\frac{5\pi}{6} - \frac{\pi}{6}\right) - 5\left\{\left(\frac{\pi}{4} - \frac{\pi}{6}\right) - \left(\frac{3\pi}{4} - \frac{\pi}{2}\right)\right\} - 2\left(\frac{5\pi}{6} - \frac{3\pi}{4}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{16\pi}{3} + \frac{5\pi}{3}\right] = 14$$

HINT:

Check the graph of [cosec x] and [cot x].

JEE (Main) MATHEMATICS SOLVED PAPER

2023
08th April Shift 2

General Instructions :

(i) *There are 30 questions in this section.*
(ii) *Section A consists of 20 Multiple choice questions and Section B consists of 10 Numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
(iii) *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted questions.*
(iv) *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
(v) *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
(vi) *All calculations/ written work should be done in the rough sheet which is provided with Question Paper.*

Section A

Q. 1. Let $A=\left\{\theta\in(0,2\pi):\frac{1+2i\sin\theta}{1-i\sin\theta}\text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is

(A) π **(B)** 3π
(C) 4π **(D)** 2π

Q. 2. Let P be the plane passing through the line $\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, γ) then $\alpha+\beta+\gamma$ is equal to

(A) 12 **(B)** 9
(C) 10 **(D)** 11

Q. 3. If $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$. $A^{-1}=\alpha A+BI$ and $\alpha+\beta=-2$, then $4\alpha^2+\beta^2+\lambda^2$ is equal to :

(A) 14 **(B)** 12
(C) 19 **(D)** 10

Q. 4. The area of the quadrilateral ABCD with vertices A(2,1,1), B (1,2, 5), C(–2,–3, 5) and D (1, –6, –7) is equal to

(A) 54 **(B)** $9\sqrt{38}$
(C) 48 **(D)** $8\sqrt{38}$

Q. 5. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by

(A) 34 but not by 14 **(B)** 14 but not by 34
(C) Both 14 and 34 **(D)** Neither 14 nor 34

Q. 6. Let O be the origin and OP and OQ be the tangents to the circle $x^2+y^2-6x+4y+8=0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha,\frac{1}{2}\right)$, then a value of α is.

(A) $-\frac{1}{2}$ **(B)** $\frac{5}{2}$
(C) 1 **(D)** $\frac{3}{2}$

Q. 7. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 +.... and $S_n=\sum_{k=1}^{n}a_k$. Then $S_{30}-a_{40}$ is equal to

(A) 11260 **(B)** 11280
(C) 11290 **(D)** 11310

Q. 8. If $\alpha>\beta>0$ are the roots of the equation $ax^2+bx+1=0$, and $\lim_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{\frac{1}{2}}=\frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$. then k is equal to

(A) β **(B)** 2α
(C) 2β **(D)** α

Q. 9. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)k$, is equal to

(A) 1890 **(B)** 945
(C) 2835 **(D)** 5670

Q. 10. Let S be the set of all values of $\theta\in[-\pi,\pi]$ for which the system of linear equations

$$x+y+\sqrt{3}z=0$$
$$-x+(\tan\theta)y+\sqrt{7}z=0$$
$$x+y+(\tan\theta)z=0$$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in S}\theta$ is equal to

(A) 20 **(B)** 40
(C) 30 **(D)** 10

Q. 11. For $a, b\in Z$ and $|a-b|\le 10$, let the angle between the plane P : $ax+y-z=b$ and the line l : $x-1=a-y=z+1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then a^4+b^2 is equal to

(A) 85 **(B)** 48
(C) 25 **(D)** 32

Q. 12. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}, \vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a + b + c)$ is equal to
(A) 4 **(B)** 12
(C) 6 **(D)** 0

Q. 13. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to
(A) $10^3 - 10$ **(B)** $11^3 - 11$
(C) $12^3 - 12$ **(D)** $13^3 - 13$

Q. 14. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = $\{(x, y) \in A \times A : x + y = 7\}$ is
(A) Symmetric but neither reflexive nor transitive
(B) Transitive but neither symmetric nor reflexive
(C) An equivalence relation
(D) Reflexive but neither symmetric nor transitive

Q. 15. If the probability that the random variable X takes values x is given by $P(X=x) = k(x+1)3^{-x}$, $x = 0, 1, 2, 3, \ldots$, where k is a constant, then $P(X \geq 2)$ is equal to
(A) $\frac{7}{27}$ **(B)** $\frac{11}{18}$
(C) $\frac{7}{18}$ **(D)** $\frac{20}{27}$

Q. 16. The integral $\int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to
(A) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$ **(B)** $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$
(C) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$ **(D)** $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

Q. 17. The value of $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is
(A) 27 **(B)** 54
(C) 18 **(D)** 36

Q. 18. Let A (0, 1), B(1, 1) and C (1, 0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{3}, 0)$, , where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to
(A) 6 **(B)** 8
(C) $\frac{9}{2}$ **(D)** 12

Q. 19. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to
(A) $p \wedge (\sim q)$ **(B)** $p \wedge (q \wedge (\sim p))$
(C) $p \vee (q \vee (\sim p))$ **(D)** $p \wedge q$

Q. 20. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $m + n$ is equal to
(A) 316 **(B)** 317
(C) 315 **(D)** 314

Section B

Q. 21. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions f : R → S such that $f(a) \neq 1$ is equal to _____.

Q. 22. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5[x + 2] - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then $m^2 + mn + n^2$ is equal to ________.

Q. 23. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1) on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

Q. 24. If domain of the function $\log_e\left(\frac{6x^2 + 5x + 1}{2x - 1}\right) + \cos^{-1}\left(\frac{2x^2 - 3x + 4}{3x - 5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then, $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to

Q. 25. Let the area enclosed by the lines $x + y = 2$, $y = 0, x = 0$ and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where $[x]$ deontes the greatest integer $\leq x$, be A. Then the value of 12A is ______.

Q. 26. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _________.

Q. 27. Let the solution curve $x = x(y), 0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\,dx - (1 + 3x\log_e(\cos y))\sin y\,dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$.

If $x\left(\frac{\pi}{6}\right)=\frac{1}{\log_e m-\log_e n}$, where m and n are co-prime, then mn is equal to

Q. 28. Let $[t]$ denote the greatest integer function. If $\int_0^{24}[x^2]dx=\alpha+\beta\sqrt{2}+\gamma\sqrt{3}+\delta\sqrt{5}$, then $\alpha+\beta+\gamma+\delta$ is equal to ________.

Q. 29. The ordinates of the points P and Q on the parabola with focus (3.0) and directrix $x=-3$ are in the ratio 3 : 1. If R (α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to ________.

Q. 30. Let k and m be positive real numbers such that the function $f(x)=\begin{cases}3x^2+k\sqrt{x+1}, & 0<x<1\\ mx^2+k^2 & x\geq 1\end{cases}$ is differentiable for all $x>0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to _____.

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	C	General form	Complex Numbers
2	C	Equation of plane	Three Dimensional Geometry
3	A	Charactersictic equation	Matrices and Determinants
4	D	Area of quadrilateral	Vector Algebra
5	A	Remainder theorem	Binomial Theorem
6	B	Circumcircle	Circle
7	C	Special series	Sequences and Series
8	B	Limits of trigonometry	Limits
9	D	Number of words	Permutation and Combination
10	A	System of linear equations	Matrices and Determinants
11	D	Distance of a point from a plane	Three Dimensional Geometry
12	B	Scalar triple product	Vector Algebra
13	C	General term	Binomial Theorem
14	A	Equivalence relation	Relation and Function
15	A	Probaility distribuution	Probability
16	D	Indefinite Integral	Integral Calculus
17	D	Trigonometric relations	Trigonometry
18	B	Incentre of triangle	Parabola
19	D	Equivalent statement	Mathematical Reasoning
20	B	Mean, Variance	Statistics
21	[180]	Number of onto fuctions	Relation and Function
22	[9]	Roots of equation	Quadratic equations
23	[11]	Equation of plane	Three dimensional geometry
24	[20]	Domain of a function	Function
25	[17]	Area between the curves	Integral Calculus
26	[150]	A.P., G.P.	Sequences and series
27	[12]	Linear differential equation	Differential equations
28	[6]	Definite Integral	Integral Calculus
29	[16]	Parabola	Conic Section
30	[309]	First derivative	Differentiability

JEE (Main) MATHEMATICS SOLVED PAPER

2023
08^{th} April Shift 2

Solutions

Section A

1. Option (C) is correct.

Here, $z = \dfrac{1+2i\sin\theta}{1-i\sin\theta} \times \dfrac{1+i\sin\theta}{1+i\sin\theta}$

$\dfrac{1+i\sin\theta+2i\sin\theta-2\sin^2\theta}{1-i^2\sin^2\theta}$

$= \dfrac{(1-2\sin^2\theta)+i(3\sin\theta)}{1+\sin^2\theta}$

$\because$ z is purely imaginary, so Re $z = 0$

$\Rightarrow \dfrac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$

$\Rightarrow 2\sin^2\theta = 1 \Rightarrow \sin^2\theta = \dfrac{1}{2}$

$\Rightarrow \sin\theta = \pm\dfrac{1}{\sqrt{2}}$

$\therefore A = \left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right] \qquad \because \theta \in (0, 2\pi)$

$\therefore$ Sum $= \dfrac{\pi+3\pi+5\pi+7\pi}{4} = \dfrac{16\pi}{4} = 4\pi$

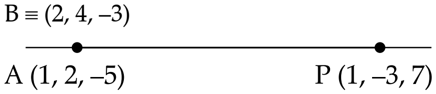
HINT:

For a complex number, $z = a + ib$, if z is purely imaginary, then Re $z = 0 \Rightarrow a = 0$

2. Option (C) is correct.

Equation of line : $\dfrac{x-1}{1} = \dfrac{y-2}{-3} = \dfrac{z+5}{7}$

Let B ≡ (2, 4, –3)



So, $\overrightarrow{AB} = (2-1)\hat{i} + (4-2)\hat{j} + (-3+5)\hat{k}$

$= \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 7 \\ 1 & 2 & 2 \end{vmatrix} = (-6-14)\hat{i} - (2-7)\hat{j} + (2+3)\hat{k}$

$= -20\hat{i} + 5\hat{j} + 5\hat{k}$

$= -5(4\hat{i} - \hat{j} - \hat{k})$

$\therefore$ Eqn. of plane is :

$4(x-1) + (-1)(y-2) - 1(z+5) = 0$

$\Rightarrow 4x - 4 - y + 2 - z - 5 = 0$

$\Rightarrow 4x - y - z - 7 = 0$

$\because$ Image of point (–1, 3, 4) is (α, β, γ)

So, $\dfrac{\alpha+1}{4} = \dfrac{\beta-3}{-1} = \dfrac{\gamma-4}{-1} = \dfrac{-2(-4-3-4-7)}{16+1+1} = 2$

$\Rightarrow \alpha = 7, \beta = 1, \gamma = 2$

So, $\alpha + \beta + \gamma = 10$

HINT:

Equation of plane passing through the line and a point can be find by using the normal vector.

3. Option (A) is correct.

$A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$

$\Rightarrow |A - xI| = 0$

$\Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0$

$\Rightarrow (1-x)(10-x) - 5\lambda = 0$

$\Rightarrow 10 - 11x + x^2 - 5\lambda = 0$

Also, $\Rightarrow A^{-1} = \alpha A + \beta I$

$\Rightarrow \alpha A^2 + \beta A - I = 0$

and $A^2 - 11A + (10 - 5\lambda) I = 0$

On solving, we get

$\alpha = \dfrac{1}{5}, \beta = -\dfrac{11}{5}$

So, $5\lambda - 10 = 5 \Rightarrow \lambda = 3$

$\therefore 4\alpha^2 + \beta^2 + \lambda^2$

$= 4\left(\dfrac{1}{25}\right) + \left(\dfrac{121}{25}\right) + 9$

$= \dfrac{125}{25} + 9 = 14$

HINT:

The characteristic equation is :

$|A - xI| = 0$

4. Option (D) is correct.

Here $\overrightarrow{AC} = (-2-2)\hat{i} + (-3-1)\hat{j} + (5-1)\hat{k}$

$= -4\hat{i} - 4\hat{j} + 4\hat{k}$

$\overrightarrow{BD} = (1-1)\hat{i} + (-6-2)\hat{j} + (-7-5)\hat{k}$

$= -8\hat{j} - 12\hat{k}$

So, area of quadrilateral $= \dfrac{1}{2} | |\overrightarrow{AC} \times \overrightarrow{BD}|$

$$= \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -4 & 4 \\ 0 & -8 & -12 \end{matrix} \right\|$$

$$= \frac{1}{2} |(48+32)\hat{i} - (48-0)\hat{j} + (32-0)\hat{k}|$$

$$= \frac{1}{2} |80\hat{i} - 48\hat{j} + 32\hat{k}|$$

$$= \frac{1}{2} 16 |15\hat{i} - 3\hat{j} + 2\hat{k}|$$

$$= 8\sqrt{25+9+4} = 8\sqrt{38} \text{ sq units.}$$

HINT:

Area of quadrilateral = Half of product of diagonal vectors.

5. **Option (A) is correct.**

The given expression is divisible by 6 and 17.

Also, $25^{190} - 8^{190}$ is not divisible by 7

but $19^{190} - 2^{190}$ is divisible by 7,

So, $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by 34 but not by 14.

6. **Option (B) is correct.**

Centre (3, –2)

Equation of circumcircle is

$x(x-3) + y(y+2) = 0$

$\Rightarrow x^2 - 3x + y^2 + 2y = 0$

Since $\left(\alpha, \frac{1}{2}\right)$ is on the circle

So $\alpha^2 - 3\alpha + \frac{1}{4} + 1 = 0$

$\Rightarrow 4\alpha^2 - 12\alpha + 5 = 0$

$$\Rightarrow \alpha = \frac{12 \pm \sqrt{144-80}}{8}$$

$$= \frac{12 \pm \sqrt{64}}{8} = \frac{12 \pm 8}{8}$$

$$\alpha = \frac{20}{8}, \frac{4}{8} = \frac{5}{2}, \frac{1}{2}$$

HINT:

Equation of circumcircle whose diametric points are (a, b) & (c, d) is $(x-a)(x-c) + (y-b)(y-d) = 0$

7. **Option (C) is correct.**

Let $S_n = 5 + 8 + 14 + 23 + \dots + a_n$

and $S_n = 0 + 5 + 8 + 14 + \dots + a_n$

On subtracting, we get

$0 = 5 + 3 + 6 \dots - a_n$

$\Rightarrow a_n = 5 + 3 + 6 + 9 + \dots (n-1)$ terms

$$= 5 + \left[\frac{(n-1)}{2}(6+(n-2)3)\right]$$

$$= 5 + \left[\frac{(n-1)}{2}(6+3n-6)\right]$$

$$= 5 + \frac{(n-1)(3n)}{2}$$

$$= \frac{10+3n^2-3n}{2}$$

So, $a_{40} = \frac{3(40)^2 - 3(40) + 10}{2}$

$$= \frac{4800-120+10}{2} = 2345$$

Now, $S_n = \sum_{k=1}^{n} a_k$

$$\Rightarrow S_{30} = \frac{3\sum_{n=1}^{30} n^2 - 3\sum_{n=1}^{30} n + 10\sum_{n=1}^{30} 1}{2}$$

$$= \frac{3 \times (30)(30+1)(60+1)}{12} - \frac{3 \times 30 \times 31}{4} + \frac{10 \times 30}{2}$$

$$= \frac{28365 - 1395 + 300}{2} = \frac{27270}{2}$$

$= 13635$

$\therefore S_{30} - a_{40} = 13635 - 2345 = 11290$

HINT:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{k(k+1)(2k+1)}{6}$$

8. **Option (B) is correct.**

Since, α, β are roots of $ax^2 + bx + 1 = 0$

Replace $x \to \frac{1}{x}$

$$\frac{a}{x^2} + \frac{b}{x} + 1 = 0 \Rightarrow x^2 + bx + a = 0$$

So, $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots

Now, $\lim_{x \to \frac{1}{\alpha}} \left[\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right]^{\frac{1}{2}}$

$$= \lim_{x \to \frac{1}{\alpha}} \left[\frac{2\sin^2\left(\frac{x^2+bx+a}{2}\right)}{2(1-\alpha x)^2}\right]^{\frac{1}{2}}$$

$$= \lim_{x \to \frac{1}{\alpha}} \left[\frac{2\sin^2 \frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}}{4 \times 2\alpha^2 \frac{\left(x-\frac{1}{\alpha}\right)^2\left(x-\frac{1}{\beta}\right)^2}{4}} \left(x-\frac{1}{\beta}\right)^2 \right]^{\frac{1}{2}}$$

$$= \lim_{x \to \frac{1}{\alpha}} \left[\pm\frac{1}{2} \frac{\sin \frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}}{\alpha \frac{\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)}{2}} \left(x-\frac{1}{\beta}\right) \right]$$

$$= \frac{1}{2\alpha}\left(\frac{-1}{\alpha}+\frac{1}{\beta}\right)$$

$$\Rightarrow \frac{1}{k}\left[\frac{1}{\beta}-\frac{1}{\alpha}\right] = \frac{1}{2\alpha}\left[\frac{1}{\beta}-\frac{1}{\alpha}\right]$$

$\Rightarrow k = 2\alpha$

HINT:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

9. Option (D) is correct.

Total number of words = $\frac{11!}{2!2!2!}$

Number of words in which C and S are together

$$= \frac{10!}{2!2!2!} \times 2!$$

So, required number of words

$$= \frac{11!}{2!2!2!} - \frac{10!}{2!2!}$$

$$= \frac{11 \times 10!}{2!2!2!} - \frac{10!}{2!2!}$$

$$= \frac{10!}{2!2!}\left[\frac{11}{2}-1\right] = \frac{10!}{2!2!} \times \frac{9}{2}$$

$= 5670 \times 6!$

$\Rightarrow k\,(6!) = 5670 \times 6!$

$\Rightarrow k = 5670$

HINT:

Out of n objects if r things are same, then number of ways = $\frac{n!}{r!}$

10. Option (A) is correct.

Since, the given system has a non trivial solution,

So, $\Delta = 0$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$$\Rightarrow 1(\tan^2\theta - \sqrt{7}) - 1(-\tan\theta - \sqrt{7}) + \sqrt{3}(-1-\tan\theta) = 0$$

$$\Rightarrow \tan^2\theta - \sqrt{7} + \tan\theta + \sqrt{7} - \sqrt{3} - \sqrt{3}\tan\theta = 0$$

$$\Rightarrow \tan\theta(\tan\theta - \sqrt{3}) + 1(\tan\theta - \sqrt{3}) = 0$$

$\Rightarrow \tan\theta = \sqrt{3}$ or $\tan\theta = -1$

$$\therefore \theta = \left\{\frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4}\right\}$$

So, $\frac{120}{\pi}\sum_{\theta \in S} \theta = \frac{120}{\pi}\left\{\frac{4\pi - 8\pi - 3\pi + 9\pi}{12}\right\}$

$$= \frac{120}{\pi}\left[\frac{2\pi}{12}\right] = 20$$

HINT:

For a system of linear equation having non trivial solution, $\Delta = 0$

11. Option (D) is correct.

We have, $\theta = \cos^{-1}\frac{1}{3}$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

$$\therefore \sin\theta = \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

The given plane line and are

$ax + y - z = b$ & $x - 1 = a - y = z + 1$

$$\therefore \sin\theta = \frac{a.1 + (1)(-1) + (-1)(1)}{\sqrt{a^2+1^2+1^2}\sqrt{1^2+1^2+1^2}}$$

$$\Rightarrow \frac{a-1-1}{\sqrt{a^2+2}\sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow 3(a-2) = 2\sqrt{6}\sqrt{a^2+2}$$

$\Rightarrow 9\,(a^2 + 4 - 4a) = 24\,(a^2 + 2)$

$\Rightarrow 9a^2 + 36 - 36a = 24a^2 + 48$

$\Rightarrow 15a^2 + 36a + 12 = 0$

$\Rightarrow 5a^2 + 12a + 4 = 0$

$\Rightarrow 5a^2 + 10a + 2a + 4 = 0$

$\Rightarrow 5a\,(a + 2) + 2\,(a + 2) = 0$

$$\Rightarrow a = \frac{-2}{5}, -2$$

So, $a = -2$ $\quad \because a \in Z$

Hence, the eqn. of plane is $-2x + y - z - b = 0$

Now, $d = \left|\frac{-12-6-4-b}{\sqrt{4+1+1}}\right| = 3\sqrt{6}$

$\Rightarrow |-(b + 22)| = 18$

$\Rightarrow b = 18 - 22 = -4$

$\therefore a^4 + b^2 = (-2)^4 + (-4)^2$

$= 16 + 16 = 32$

HINT:

Distance of a point (a_1, b_1, c_1) from the plane $ax + by + cz + d = 0$ is $d = \left|\dfrac{aa_1 + bb_1 + cc_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

12. Option (B) is correct.

Since, $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are coplanar.

So, $[\vec{u}_1\ \vec{u}_2\ \vec{u}_3] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(b-1) - 1(1-c) + a(1-bc) = 0$
$\Rightarrow b - 1 - 1 + c + a - abc = 0$
$\Rightarrow a + b + c - 2 = abc$ ...(i)

Also, $[\vec{v}_1\ \vec{v}_2\ \vec{v}_3] = 0$

$$\Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$\Rightarrow (a + b)[bc + ba + c^2 + ca - ab] - c[ac + a^2 - ab] + c[ab - b^2 - bc] = 0$

$\Rightarrow abc + ac^2 + a^2c + b^2c + bc^2 + abc - ac^2 - a^2c + abc + abc - b^2c - bc^2 = 0$

$\Rightarrow 4abc = 0 \Rightarrow abc = 0$ ...(ii)

So, $a + b + c - 2 = 0$ [from (i)]

$\Rightarrow a + b + c = 2$

$\Rightarrow 6(a + b + c) = 12$

HINT:

If three non-zero vectors are coplanar, then their scalar triple product is zero.

13. Option (C) is correct.

General term of $\left(2x^2 + \dfrac{1}{2x}\right)^{11}$ is:

$$T_{r+1} = {}^{11}C_r (2x^2)^{11-r}\left(\frac{1}{2x}\right)^r$$

$= {}^{11}C_r\, 2^{11-r}\, x^{22-2r}\, 2^{-r}\, x^{-r}$
$= {}^{11}C_r\, 2^{11-r}\, x^{22-3r}$

Now, $22 - 2r = 10$ and $22 - 3r = 7$

$\Rightarrow 3r = 12$ $\Rightarrow 3r = 15$

$\Rightarrow r = 4$ $\Rightarrow r = 5$

$\therefore$ Coeff. of $x^{10} = {}^{11}C_4 . 2^{11-8} = {}^{11}C_4 \times 8$

Coeff. of $x^7 = {}^{11}C_5 . 2^{11-10} = {}^{11}C_4 \times 2$

Now, required difference

$= {}^{11}C_4 \times 8 - {}^{11}C_5 \times 2$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7!}{4! \times 7!} \times 8 - \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6! \times 2}{5!\,6!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 8}{24} - \frac{11 \times 10 \times 9 \times 8 \times 7 \times 2}{120}$$

$= 11 \times 10 \times 8 \times 3 - 11 \times 3 \times 4 \times 7$
$= 11 \times 3 \times 4\,[20 - 7]$
$= 11 \times 12 \times 13 = (12 - 1) \times 12 \times (12 + 1)$
$= 12\,(12^2 - 1) = 12^3 - 12$

HINT:

General term of $(a + b)^n$ is
$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

14. Option (A) is correct.

Here, A = {1, 2, 3, 4, 5, 6, 7}
Since, $x + y = 7 \Rightarrow y = 7 - x$
So, R = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
$\because (a, b) \in R \Rightarrow (b, a) \in R$
$\therefore$ R is symmetric only.

HINT:

For a relation,
if $(a, a) \in R \Rightarrow$ R is reflexive
if $(a, b) \in R \Rightarrow (b, a) \in R$ So, R is symmetric
if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So, R is transitive

15. Option (A) is correct.

As, we know that sum of all the probabilities = 1

So, $\sum_{x=1}^{\infty} P(X = x) = 1$

$\Rightarrow k[1 + 2.3^{-1} + 3.3^{-2} + ...\infty] = 1$

Let $S = 1 + \dfrac{2}{3} + \dfrac{3}{3^2} + ... + \infty$

$\Rightarrow \dfrac{S}{3} = 0 + \dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{3}{3^3} + ... + \infty$

On subtracting, we get

$\dfrac{2S}{3} = 1 + \dfrac{1}{3} + \dfrac{1}{3^2} + ... + \infty$

$\Rightarrow \dfrac{2S}{3} = \dfrac{1}{1 - \frac{1}{3}} = \dfrac{1}{\frac{2}{3}}$

$\Rightarrow \dfrac{2S}{3} = \dfrac{3}{2}$

$\Rightarrow S = \dfrac{9}{4}$

So, $k \times \dfrac{9}{4} = 1 \Rightarrow k = \dfrac{4}{9}$

Now, $P(X \geq 2) = 1 - P(X < 2)$
$= 1 - P(X = 0) - P(X = 1)$

$= 1 - \dfrac{4}{9}(1) - \dfrac{4}{9} \times \dfrac{2}{3}$

$= 1 - \dfrac{4}{9} - \dfrac{8}{27} = \dfrac{27 - 12 - 8}{27} = \dfrac{7}{27}$

Sum of probabilities = 1

$$\sum_{x=0}^{\infty} P(X = x) = 1$$

16. Option (D) is correct.

Note: Given integral is wrong it may be

$$\int\left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right]\ln\left(\frac{ex}{2}\right)dx$$

Let I = $\int\left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right]\ln\left(\frac{ex}{2}\right)dx$

$$= \int\left[e^{x\ln x - x\ln 2} + e^{x\ln 2 - x\ln x}\right]dx$$

Let $x \ln x - x \ln 2 = t$

$(\ln x + 1 - \ln 2)dx = dt$

$$\Rightarrow \ln\left(\frac{ex}{2}\right)dx = dt$$

$$\therefore I = \int\left[e^t - e^{-t}\right]dt$$

$= e^t + e^{-t} + c$

$$= \left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + c$$

17. Option (D) is correct.

$4\cos^2\theta - 1 = 4(1 - \sin^2\theta) - 1$

$= 3 - 4\sin^2\theta$

$$= \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}$$

$$= \frac{\sin 3\theta}{\sin\theta}$$

So, $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$

$$= 36\left[\frac{\sin 27°}{\sin 9°}\times\frac{\sin 81°}{\sin 27°}\times\frac{\sin 243°}{\sin 81°}\times\frac{\sin 729°}{\sin 243°}\right]$$

$$= 36\left[\frac{\sin 729°}{\sin 9°}\right] = 36 \times 1 = 36$$

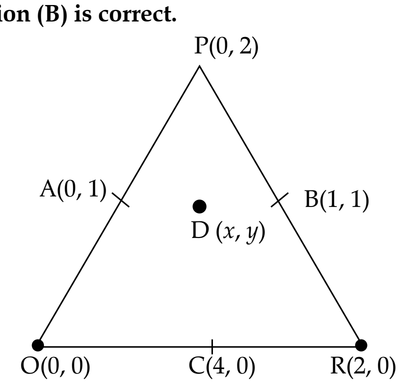
HINT:

Use the formula:

$\sin^2\theta + \cos^2\theta = 1$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

18. Option (B) is correct.



So, $D \equiv \left(\frac{4}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right)$

$$\equiv \left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$$

$$= \left(\frac{2}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}, \frac{2}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}\right)$$

$\equiv (2-\sqrt{2}, 2-\sqrt{2})$

$\because y^2 = 4ax$

$(2-\sqrt{2})^2 = 4a(2-\sqrt{2})$

$$\Rightarrow 4a = 2-\sqrt{2} \Rightarrow a = \frac{2-\sqrt{2}}{4}$$

$$\Rightarrow \frac{1}{2} - \frac{\sqrt{2}}{4} = \alpha + \beta\sqrt{2}$$

$$\Rightarrow \alpha = \frac{1}{2}, \beta = \frac{-1}{4}$$

So, $\frac{\alpha}{\beta^2} = \frac{\frac{1}{2}}{\frac{1}{16}} = 8$

HINT:

The incentre of a triangle is the intersection point of all the three interior angle bisectors of the triangle.

19. Option (D) is correct.

$(p \wedge (\sim q)) \vee (\sim p)$

$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$\equiv T \wedge (\sim q \vee \sim p)$

$\equiv \sim q \vee \sim p$ negation $p \wedge q$

HINT:

$a \vee \sim a \equiv T$

$\sim a \vee \sim b \equiv b \wedge a$

20. Option (B) is correct.

Since, Mean = $\frac{9}{2}$

$$\Rightarrow \Sigma x = \frac{9}{2}\times 12 = 54$$

Also, variance = 4

$$\Rightarrow \frac{\sum x^2}{12} = \left[\frac{\sum x_i}{12}\right]^2 = 4$$

$$\Rightarrow \frac{\sum x^2}{12} = 4 + \frac{81}{4} = \frac{97}{4}$$

$$\Rightarrow \sum x^2 = 291$$

$\sum x' = 54 - (9 + 10) + 7 + 14$

$= 54 - 19 + 21 = 56$

and $\sum x^2 = 291 - (81 + 100) + 49 + 196$

$= 291 - 181 + 49 + 196 = 355$

So, $\sigma^2_{new} = \frac{\sum x^2_{new}}{12} - \left(\frac{\sum x_{new}}{12}\right)^2$

$= \frac{355}{12} - \left(\frac{56}{12}\right)^2$

$= \frac{4260 - 3136}{144} = \frac{1124}{144} = \frac{281}{36}$

$= \frac{m}{n}$

$\Rightarrow m = 281$ & $n = 36$

$\Rightarrow m + n = 281 + 36 = 317$

HINT:

Mean $= \frac{\sum x}{n}$

Variance $(\sigma^2) = \frac{\sum x^2}{n} - \left[\frac{\sum x}{n}\right]^2$

Section B

21. The correct answer is (180).

Total number of onto functions

$= \frac{5!}{3!2!} \times 4!$

$= \frac{5 \times 4}{2} \times 24 = 240$

When $f(a) = 1$, number of onto functions

$= 4! + \frac{4!}{2!2!} \times 3!$

$= 24 + 36 = 60$

So, required number of onto functions

$= 240 - 60 = 180$

22. The correct answer is (9).

The givne eqn is : $x^2 - 12x + [x] + 31 = 0$

$\Rightarrow \{x\} - x = x^2 - 12x + 31$

$\Rightarrow \{x\} = x^2 - 11x + 31$

So, $0 \le x^2 - 11x + 31 < 1$

$\Rightarrow x^2 - 11x + 30 \le 0$

$\Rightarrow (x - 5)(x - 6) < 0$

$\Rightarrow x \in (5, 6)$

$\therefore [x] = 5$

$\therefore x^2 - 12x + 5 + 31 = 0$

$\Rightarrow x^2 - 12x + 36 = 0$

$\Rightarrow (x - 6)^2 = 0 \Rightarrow x = 6$

Hence, $x \in \phi$ $(\because x \in (5, 6))$

$\therefore m = 0$

Another equation is $x^2 - 5[x + 2] - 4 = 0$

Case I: $x \ge -2$

$x^2 - 5x - 14 = 0 \Rightarrow x = 7, -2$

Case II: $x < -2$

$x^2 + 5x + 6 = 0 \Rightarrow x = -3 - 2$

$\therefore x \in \{-3, -2, 7\}$

$\therefore n = 3$

Hence, $m^2 + mx + n^2 = 0 + 0 + 9 = 9$

HINT:

The relation between the greatest integer function and fractional part is :

$[x] = x - \{x\}$

23. The correct answer is (11).

Equation of plane P_2 passing through (2, –1, 0), (2, 0, –1) and (5, 1, 1) is

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow (x - 5)(4 - 1) - (y - 1)(6 - 3) + (z - 1)(3 - 6) = 0$

$\Rightarrow 3x - 15 - 3y + 3 - 3z + 3 = 0$

$\Rightarrow 3x - 3y - 3z - 9 = 0$

$\Rightarrow x - y - z = 3$ ...(i)

Now, direction ratios of line of intersection of P_1 and P_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 3 & -1 & -7 \end{vmatrix}$$

$= \hat{i}(7 - 1) - \hat{j}(-7 + 3) + \hat{k}(-1 + 3)$

$= 6\hat{i} + 4\hat{j} + 2\hat{k}$

At $z = 0$, $x - y = 3$ [from (i)]

$3x - y = 11$

on solving, we get

$x = 4$ and $y = 1$

So, equation of line is

$\frac{x-4}{6} = \frac{y-1}{4} = \frac{z-2}{6} = k$

$\therefore (\alpha, \beta, \gamma) = (6k + 4, 4k + 1, 2k)$

$\Rightarrow (6)(\alpha - 7) + 4(\beta - 4) + 2(\gamma + 1) = 0$

$\Rightarrow 6(6k + 4 - 7) + 4(4k + 1 - 4) + 2(2k + 1) = 0$

$\Rightarrow 36k - 18 + 16k - 12 + 4k + 4 = 0$

$\Rightarrow 56k = 26 \Rightarrow k = \frac{1}{2}$

So, $\alpha = 7$, $\beta = 3$ and $\gamma = 1$

$\therefore \alpha + \beta + \gamma = 7 + 3 + 1 = 11$

HINT:

Equation of plane passing through (a, b, c), (d, c, f) and (g, h, i) is

$$\begin{vmatrix} x-h & y-h & z-i \\ g-a & h-b & i-e \\ g-d & h-e & i-f \end{vmatrix} = 0$$

24. The correct answer is (20).

Domain of $\log_e\left(\frac{6x^2 + 5x + 1}{2x - 1}\right)$

So, $\frac{6x^2 + 5x + 1}{2x - 1} > 0$

$$\Rightarrow \frac{(3x+1)(2x+1)}{2x-1} > 0$$

$$\Rightarrow x \in \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$$

For domain of $\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ domain of $\cos^{-1}x \rightarrow [-1, 1]$

$$-1 \leq \frac{2x^2-3x+4}{3x-5} \leq 1$$

$$\frac{2x^2-1}{3x-5} \geq 0 \text{ and } \frac{2x^2-6x+9}{3x-5} \leq 0$$

$$\Rightarrow x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \cup \left(\frac{5}{3}, \infty\right)$$

So, common domain is $\left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$

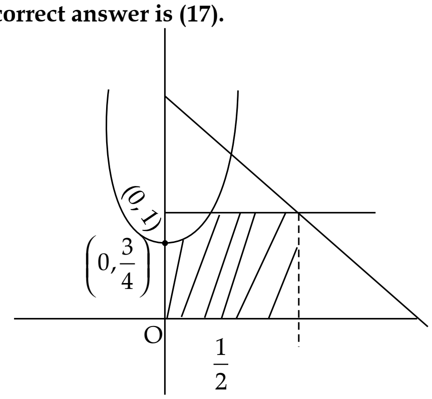
$$\therefore 18(\alpha^2+\beta^2+\gamma^2+\delta^2) = 18\left(\frac{1}{4}+\frac{1}{9}+\frac{1}{4}+\frac{1}{2}\right)$$

$$= 18\left(\frac{9+4+9+18}{36}\right) = \frac{1}{2}(40) = 20$$

HINT:

For $\log_e x$, $x > 0$ and $-1 \leq \cos^{-1} x \leq 1$

25. The correct answer is (17).



$$\text{Required area} = \left[\int_0^{\frac{1}{2}}\left(x^2+\frac{3}{4}\right)dx\right] + \left[\frac{1}{2}\left(\frac{3}{2}+\frac{1}{2}\right)\times 1\right]$$

$$= \left[\frac{x^3}{3}+\frac{3x}{4}\right]_0^{\frac{1}{2}} + 1$$

$$= \frac{1}{24}+\frac{3}{8}-0+1 = \frac{1+9+24}{24} = \frac{34}{24} = \frac{17}{12}$$

So, $12A = 12 \times \frac{17}{12} = 17$

HINT:

Find the common region bounded by all the given curves and then using integration, find the required area.

26. The correct answer is (150).

$\because \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

$$\Rightarrow \frac{1}{x}+\frac{1}{z} = \frac{2}{y} \quad \text{...(i)}$$

and $x, \sqrt{2}y, z$ are in G.P.

$$\Rightarrow 2y^2 = xz \quad \text{...(ii)}$$

from (i), $\frac{2}{y} = \frac{x+z}{xz} = \frac{x+z}{2y^2}$

$\Rightarrow 4y = x + z$

Also, $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$

$$y(4y) + xz = \frac{3}{\sqrt{2}}(2y^2)y$$

$$\Rightarrow 4y^2 + 2y^2 = 3\sqrt{2}y^3$$

$$\Rightarrow 6y^2 = 3\sqrt{2}y^3 \Rightarrow y = \sqrt{2}$$

$$\therefore 3(x+y+z)^2 = 3(5y)^2 = 3(5\sqrt{2})^2$$

$= 150$

HINT:

$a, b, c \rightarrow$ A.P. $\Rightarrow a + c = 2b$

$a, b, c \rightarrow$ G.P. $\Rightarrow b^2 = ac$

27. The correct answer is (12).

Given:

$(\cos y), (\ln(\cos y))^2\, dx = (1 + 3x \ln \cos y) \sin y\, dy$

$$\Rightarrow \frac{dx}{dy} = \frac{(1+3x\ln\cos y)\sin y}{(\ln\cos y)^2\cos y}$$

$$= \tan y\left[\frac{1}{(\ln\cos y)^2} + \frac{3x}{\ln\cos y}\right]$$

$$\Rightarrow \frac{dx}{dy} - \left(\frac{3\tan y}{\ln\cos y}\right)x = \frac{\tan y}{(\ln\cos y)^2}$$

which is a linear differential equation.

$$\text{I.F.} = e^{-\int\frac{3\tan y}{\ln\cos y}dy} = (\ln\cos y)^3 \qquad \text{I.F.} = e^{\int P.dx}$$

So, the solution is :

$$x \times (\ln\cos y)^3 = \int\left((\ln\cos y)^3 \times \frac{\tan y}{(\ln\cos y)^2}\right)dy$$

$$x \times (\ln\cos y)^3 = \frac{-(\ln\cos y)^2}{2} + C$$

At $y = \frac{\pi}{3}$,

$$\frac{1}{2\ln 2}\times\left(\ln\left(\frac{1}{2}\right)\right)^3=-\frac{\left(\ln\left(\frac{1}{2}\right)\right)^2}{2}+C$$

$\Rightarrow C = 0$

So, $x \times \ln^3 \cos y = \dfrac{-\ln^2\cos y}{2}$

At $y=\frac{\pi}{6}, x\times\left(\ln\left(\frac{\sqrt{3}}{2}\right)\right)^3=-\frac{1}{2}\left(\ln\left(\frac{\sqrt{3}}{2}\right)\right)^2$

$$\Rightarrow x=-\frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$$

$$=-\frac{1}{2[\ln\sqrt{3}-\ln 2]}=\frac{-1}{2\left[\frac{1}{2}\ln 3-\ln 2\right]}$$

$$=\frac{-1}{2\left[\frac{\ln 3-\ln 4}{2}\right]}=\frac{1}{\ln 4-\ln 3}$$

$\Rightarrow m = 4, n = 3$

$\Rightarrow mn = 12$

HINT:

For a linear differential equation, $\frac{dx}{dy}+P(y)x=Q(y)$,

the solution is $x \times$ I.F. $= \int \text{I.F.}\times Q(y)\,dy$

where I.P. $= e^{\int P(y)dy}$

28. The correct answer is (6).

$$\int_0^{2.4}[x^2]dx=\int_0^1[x^2]dx+\int_1^{\sqrt{2}}[x^2]dx+\int_{\sqrt{2}}^{\sqrt{3}}[x^2]dx+\int_{\sqrt{3}}^{2}[x^2]dx+\int_{2}^{\sqrt{5}}[x^2]dx+\int_{\sqrt{5}}^{2.4}[x^2]dx$$

$$=\int_0^1 0.dx+\int_1^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx+\int_{\sqrt{3}}^{2}3dx+\int_{2}^{\sqrt{5}}4dx+\int_{\sqrt{5}}^{2.4}5dx$$

$$0+[x]_1^{\sqrt{2}}+2[x]_{\sqrt{2}}^{\sqrt{3}}+3[x]_{\sqrt{3}}^{2}+4[x]_2^{\sqrt{5}}+5[x]_{\sqrt{5}}^{2.4}$$

$$=\sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}+4\sqrt{5}-8+12-5\sqrt{5}$$

$$=-\sqrt{2}-\sqrt{3}-\sqrt{5}+9$$

$\therefore \alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

So, $\alpha + \beta + \gamma + \delta = 9 - 1 - 1 - 1 = 6$

HINT:

The greater integer value is that integeral value which is less than or equal to that number.

29. The correct answer is (16).

Give parabola is : $y^2 = 12x$ $(\because a = 3)$

So, $P \equiv (at_1^2, 2at_1)$

$Q \equiv (at_2^2, 2at_2)$

So, point R $(\alpha, \beta) \equiv (at_1t_2, a(t_1 + t_2))$

$\equiv ((3t)(3t), 3(t + 3t)) = (9t^2, 12t)$

$$\therefore \frac{\beta^2}{\alpha}=\frac{144t^2}{9t^2}=16$$

HINT:

For equation of parabola $y^2 = 4ax$, focus is $(a, 0)$

30. The correct answer is (309).

Here, $f(x) = \begin{cases} 3x^2+k\sqrt{x+1}, & 0<x<1 \\ mx^2+k^2, & x\geq 1\end{cases}$

$\because f(x)$ is differentiable at $x > 0$

So, $f(x)$ is differentiable at $x = 1$

$f(1^-) = f(1) = f(1^+)$

$3+k\sqrt{2}=m+k^2$ ...(i)

$f'(1^-) = f'(1^+)$

$$6(1)+\frac{k}{2\sqrt{1+1}}=2m(1)$$

$$\Rightarrow 6+\frac{k}{2\sqrt{2}}=2m \quad ...(ii)$$

Using (i) and (ii),

$$3+k\sqrt{2}=3+\frac{k}{4\sqrt{2}}+k^2$$

$$\Rightarrow k^2+k\left[\frac{1}{4\sqrt{2}}-\sqrt{2}\right]=0$$

$$\Rightarrow k\left[k+\frac{1-8}{4\sqrt{2}}\right]=0\Rightarrow k=0,\frac{7}{4\sqrt{2}}$$

$$\text{for } k=\frac{7}{4\sqrt{2}}, m=3+\frac{\frac{7}{4\sqrt{2}}}{4\sqrt{2}}$$

$$=3+\frac{7}{32}=\frac{96+7}{32}=\frac{103}{32}$$

$$\text{So, } \frac{8f'(8)}{f'\left(\frac{1}{8}\right)}=\frac{8\times\left[2\times\frac{103}{32}\times 8\right]}{6\times\frac{1}{8}+\frac{7}{4\sqrt{2}}\times 2\sqrt{918}}$$

$$=\frac{412}{\frac{3}{4}+\frac{7}{12}}=\frac{412}{\frac{9+7}{12}}=\frac{412\times 12}{16}=309$$

HINT:

$f(x)$ is differentiable at $x = a$, if $f'(a^-) = f'(a^+)$