

SECTION-I

General Instructions:

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

Q. 1. Consider a triangle Δ whose two sides lie on the X-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is

- (A) $x^2 + y^2 - 3x + y = 0$
(B) $x^2 + y^2 + x + 3y = 0$
(C) $x^2 + y^2 + 2y - 1 = 0$
(D) $x^2 + y^2 + x + y = 0$

Q. 2. The area of the region

$$\left\{ (x, y) : 0 \leq x \leq \frac{9}{4}, 0 \leq y \leq 1, x \leq 3y, x + y \geq 3 \right\}$$

is

- (A) $\frac{11}{32}$ (B) $\frac{35}{96}$
(C) $\frac{37}{96}$ (D) $\frac{13}{32}$

Q. 3. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1 , and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

- (A) $\frac{1}{5}$ (B) $\frac{3}{5}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Q. 4. Let $\theta_1, \theta_2, \dots, \theta_{10}$ be positive valued angles (in radian) such that

$\theta_1 + \theta_2 + \dots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1} e^{i\theta_k}$ for $k = 2, 3, \dots, 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below:

$$P : |z_2 - z_1| + |z_3 - z_2| + \dots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$$

$$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$$

Then,

- (A) P is TRUE and Q is FALSE
(B) Q is TRUE and P is FALSE
(C) both P and Q are TRUE
(D) both P and Q are FALSE

SECTION-II

General Instructions:

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.

Question Stem for Question Nos. 5 and 6**Question Stem**

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \dots, 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

Q. 5. The value of $\frac{625}{4} p_1$ is _____.

Q. 6. The value of $\frac{125}{4} p_2$ is _____.

Question Stem for Question Nos. 7 and 8**Question Stem**

Let α , β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P .

Q. 7. The value of $|M|$ is _____.

Q. 8. The value of D is _____.

Question Stem for Question Nos. 9 and 10**Question Stem**

Consider the lines L_1 and L_2 defined by

$$L_1: x\sqrt{2} + y - 1 = 0 \text{ and } L_2: x\sqrt{2} - y + 1 = 0$$

For a fixed constant λ , let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2 . The line $y = 2x + 1$ meets C at two points R and S , where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S' . Let D be the square of the distance between R' and S' .

Q. 9. The value of λ^2 is _____.

Q. 10. The value of D is _____.

SECTION-III**General Instructions:**

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

Q. 11. For any 3×3 matrix M , let $|M|$ denote the determinant of M . Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and}$$

$$F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3 , then which of the following statements is (are) TRUE ?

(A) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(C) $|(EF)^3| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Q. 12. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) TRUE ?

(A) f is decreasing in the interval $(-2, -1)$

(B) f is increasing in the interval $(1, 2)$

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

Q. 13. Let E, F and G be three events having probabilities

$$P(E) = \frac{1}{8}, P(F) = \frac{1}{6} \text{ and } P(G) = \frac{1}{4}, \text{ and let}$$

$$P(E \cap F \cap G) = \frac{1}{10}.$$

For any event H, if H^c denotes its complement, then which of the following statements is (are) TRUE ?

(A) $P(E \cap F \cap G^c) \leq \frac{1}{40}$

(B) $P(E^c \cap F \cap G) \leq \frac{1}{15}$

(C) $P(E \cup F \cup G) \leq \frac{13}{24}$

(D) $P(E^c \cap F^c \cap G^c) \leq \frac{5}{12}$

Q. 14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE ?

(A) $|FE| = |I - FE| |FGE|$

(B) $(I - FE)(I + FGE) = I$

(C) $EFG = GEF$

(D) $(I - FE)(I - FGE) = I$

Q. 15. For any positive integer n , let $S_n : (0, \infty) \rightarrow \mathbb{R}$ be defined by

$$S_n(x) = \sum_{k=1}^n \cot^{-1} \left(\frac{1+k(k+1)x^2}{x} \right),$$

where for any $x \in \mathbb{R}$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE ?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1} \left(\frac{1+11x^2}{10x} \right)$ for all $x > 0$

(B) $\lim_{x \rightarrow \infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

(D) $\tan(S_n(x)) \leq \frac{1}{2}$, for all $n \geq 1$ and $x > 0$

Q. 16. For any complex number $\omega = c + id$, let $\arg(\omega) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$

Then which of the following statements is (are) TRUE ?

(A) $\alpha = -1$ (B) $\alpha\beta = 4$

(C) $\alpha\beta = -4$ (D) $\beta = 4$

SECTION-IV

General Instructions:

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.

Q. 17. For $x \in \mathbb{R}$, the number of real roots of the equation

$$3x^2 - 4|x^2 - 1| + x - 1 = 0 \text{ is } \underline{\hspace{2cm}}.$$

Q. 18. In a triangle ABC, let $AB = \sqrt{23}$, $BC = 3$ and

$$CA = 4. \text{ Then the value of } \frac{\cot A + \cot C}{\cot B} \text{ is}$$

Q. 19. Let \vec{u} , \vec{v} and \vec{w} be vectors in three-dimensional space, where \vec{u} and \vec{v} are unit

vectors which are not perpendicular to each other and

$$\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1 \text{ and } \vec{w} \cdot \vec{w} = 4$$

If the volume of the parallelepiped, whose adjacent sides are represented by the vectors \vec{u} , \vec{v} and \vec{w} , is $\sqrt{2}$, then the value of

$$|3\vec{u} + 5\vec{v}| \text{ is } \underline{\hspace{2cm}}.$$

Answers

Q. No.	Answer	Topic Name	Chapter Name
1	(B)	Special Points in Triangles	Conic Section
2	(A)	Straight Line and a Point	Conic Section
3	(A)	Conditional Probability	Probability
4	(C)	Geometry of Complex Numbers	Complex Number
5	[76.35]	Algebra of Probabilities	Probability
6	[24.50]	Algebra of Probabilities	Probability
7	[1]	Systems of Linear Equations	Matrices and Determinants
8	[1.5]	Plane and a Point	Three Dimensional Geometry
9	[9]	Distance of a Point from a Line	Point and Straight Line
10	[77.14]	Interaction between two Lines	Point and Straight Line
11	(A, B, D)	Inverse of a Matrix	Matrices and Determinants
12	(A, B)	Maxima and Minima	Application of Derivatives
13	(A, B, C)	Algebra of Probabilities	Probability
14	(A, B, C)	Properties of Matrix Operations	Matrices and Determinants
15	(A, B)	Properties of Inverse Trigonometric Functions	Inverse Trigonometric Functions
16	(B, D)	Geometry of Complex Numbers	Complex Number
17	[4]	Types of Functions	Functions
18	[2]	Relations between Sides and Angles of a Triangle	Properties of Triangle
19	[7]	Triple Products	Vector Algebra

ANSWERS WITH EXPLANATION

1. Option (B) is correct.

Given: A triangle Δ with one side on X-axis and other on $x + y + 1 = 0$

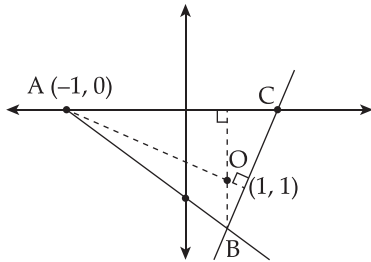
Orthocenter of Δ is $(1, 1)$

Let the triangle be ABC

Point A is the intersection point of X-axis and $x + y + 1 = 0$

\Rightarrow coordinates of A $\equiv (-1, 0)$

Let orthocenter be O,



Let B $\equiv (x_2, y_2) \rightarrow c = (x_2, y_2)$

Since, the orthocenter is the intersection point of the altitudes.

$\Rightarrow BO \perp AC$ and $AO \perp BC$

Slope of AC = slope of X-axis = 0

As we know, if slope of two perpendicular lines be m_1 and m_2 , then $m_1 \cdot m_2 = -1$

$\therefore BO \perp AC \Rightarrow m_{BO} \cdot m_{AC} = -1$

$\Rightarrow m_{BO} = \infty \quad \{\because m_{AC} = 0\}$

Also, slope of a line passing through (α_1, β_1) and (α_2, β_2) is

$$m = \frac{\beta_2 - \beta_1}{\alpha_2 - \alpha_1}$$

$$\Rightarrow m_{BO} = \frac{y_1 - 1}{x_1 - 1} = \infty$$

$$\Rightarrow x_1 - 1 = 0$$

$$\Rightarrow x_1 = 1$$

\therefore B passes through $x + y + 1 = 0$

$$\Rightarrow y_1 = -1 - x_1$$

$$\Rightarrow y_1 = -2$$

So, coordinates of B are $(1, -2)$

Similarly, $AO \perp BC$

$$\Rightarrow m_{AO} \cdot m_{BC} = -1$$

$$m_{BC} = \frac{-2 - y_2}{1 - x_2}$$

$$\text{and } m_{AO} = \frac{(1-0)}{(1-(-1))} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} \left(\frac{y_2 + 2}{x_2 - 1} \right) = -1$$

$$\Rightarrow y_2 + 2 = -2x_2 + 2$$

$$\Rightarrow y_2 = -2x_2$$

\therefore C passes through X-axis

$$\Rightarrow y_2 = 0$$

$$\therefore x_2 = 0 \text{ and } y_2 = 0$$

So, coordinates of C are $(0, 0)$

Since, the centroid of a triangle ABC with vertices $(a_1, b_1), (a_2, b_2)$ is given by

$$\left(\frac{a_1 + a_2 + a_3}{3}, \frac{b_1 + b_2 + b_3}{3} \right)$$

$$\begin{aligned} \therefore \text{Centroid of } \Delta ABC &= \left(\frac{-1+1+0}{3}, \frac{0-2+0}{3} \right) \\ &= \left(0, \frac{-2}{3} \right) \end{aligned}$$

Let centroid be M

As we know, M $\left(0, \frac{-2}{3} \right)$ divides the line segment joining circumcenter N(h, k) and orthocenter O(1, 1) in the ratio 1 : 2.

$$N(h, k) \frac{1:2}{M\left(0, \frac{-2}{3} \right)} O(1, 1)$$

As we know, by section formula the coordinates of point dividing a line passing through (x_1, y_1) and (x_2, y_2) in the ratio $m : n$, is given by.

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\Rightarrow \left(0, \frac{-2}{3} \right) = \left(\frac{2h+1}{1+2}, \frac{2k+1}{1+2} \right)$$

$$\Rightarrow \left(0, \frac{-2}{3} \right) = \left(\frac{2h+1}{3}, \frac{2k+1}{3} \right)$$

$$\Rightarrow 2h + 1 = 0 \text{ and } 2k + 1 = -2$$

$$\Rightarrow h = \frac{-1}{2} \text{ and } k = \frac{-3}{2}$$

So, circumcenter $N(h, k) = \left(\frac{-1}{2}, \frac{-3}{2} \right)$

\Rightarrow center of the required circle is $N \left(\frac{-1}{2}, \frac{-3}{2} \right)$
and radius is CN

$$\Rightarrow \text{radius CN} = \sqrt{\left(\frac{-1}{2} - 0 \right)^2 + \left(\frac{-3}{2} - 0 \right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{9}{4}}$$

$$= \sqrt{\frac{10}{2}} \text{ units}$$

Equation of circle with center (a, b) and radius r of is

$$(x-a)^2 + (y-b)^2 = r^2$$

So, required equation of the circle

$$= \left(x - \left(\frac{-1}{2} \right) \right)^2 + \left(y - \left(\frac{-3}{2} \right) \right)^2 = \left(\frac{\sqrt{10}}{2} \right)^2$$

$$= \left(x + \frac{1}{2} \right)^2 + \left(y + \frac{3}{2} \right)^2 = \frac{10}{4}$$

$$\Rightarrow x^2 + x + \frac{1}{4} + y^2 + 3y + \frac{9}{4} = \frac{10}{4}$$

$$\Rightarrow x^2 + y^2 + x + 3y = 0$$

Hints:

- Find the coordinates of vertices of triangle using condition of slope for perpendicular lines.
- Use the concept that a centroid divided the line segment joining circumcenter and orthocenter of a triangle in the ratio 1 : 2.
- The equation of circle having center (a, b) and radius r is given by $(x-a)^2 + (y-b)^2 = r^2$

Shortcut Method:

Given: orthocenter of Δ is $(1, 1)$

As we know, the mirror image of orthocenter in side of a triangle lies on the circumcircle.

Image of $(1, 1)$ in X-axis is $(1, -1)$

And image of $(1, 1)$ in $x + y + 1 = 0$ is

$$\frac{x-1}{1} = \frac{y-1}{1} = -\frac{2(1+1+1)}{1+1}$$

$$\Rightarrow x-1 = y-1 = -3$$

$$\Rightarrow x = -2 \text{ and } y = -2$$

\Rightarrow image of $(1, 1)$ in $x + y + 1 = 0$ is $(-2, -2)$

Now, the required circle will pass through $(1, -1)$ and $(-2, -2)$

According to option only (B) $x^2 + y^2 + x + 3y = 0$ satisfies the condition

So, required equation is $x^2 + y^2 + x + 3y = 0$

2. Option (A) is correct.

Given: The region is:

$$\left\{ (x, y) : 0 \leq x \leq \frac{9}{4}, 0 \leq y \leq 1, x \geq 3y, x + y \geq 2 \right\}$$

The intersection point of $x + y = 2$ and $x = 3y$ be J.

$$\Rightarrow 3y + y = 2$$

$$\Rightarrow y = \frac{2}{4} = \frac{1}{2}$$

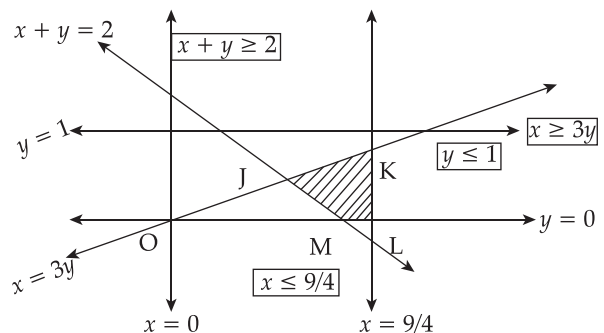
$$\Rightarrow x = \frac{3}{2}$$

\therefore Coordinates of point J are $\left(\frac{3}{2}, \frac{1}{2} \right)$

Similarly, intersection point of $x = \frac{9}{4}$ and

$x - 3y = 0$ be K

\Rightarrow Coordinates of point K are $\left(\frac{9}{4}, \frac{3}{4} \right)$



Coordinates of point M is intersection of $x + y = 2$ and $y = 0$

\therefore Coordinates of M are (2, 0)

and coordinates of L is intersection of

$$y = 0 \text{ and } x = \frac{9}{4}$$

\therefore coordinates of L are $\left(\frac{9}{4}, 0\right)$

So, the required region is JKLM with

$$J\left(\frac{3}{2}, \frac{1}{2}\right), K\left(\frac{9}{4}, \frac{3}{4}\right), L\left(\frac{9}{4}, 0\right), M(2, 0)$$

Required area = area of ΔJKM + area of ΔMKL

$$\text{Area of } \Delta JKM = \frac{1}{2} \begin{vmatrix} \frac{3}{2} & \frac{1}{2} & 1 \\ \frac{9}{4} & \frac{3}{4} & 1 \\ 2 & 0 & 1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2} \left\{ 2 \left(\frac{1}{2} - \frac{3}{4} \right) + 1 \left(\frac{9}{8} - \frac{9}{8} \right) \right\} \text{ sq. units}$$

$$= \frac{1}{2} \left\{ -\frac{1}{2} + 0 \right\} \text{ sq. units}$$

$$= \frac{1}{4} \text{ sq. units}$$

$$\text{Similarly area of } \Delta MKL = \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{9}{4} & \frac{3}{4} & 1 \\ \frac{9}{4} & 0 & 1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2} \left\{ 3 \left(2 - \frac{9}{4} \right) \right\} \text{ sq. units}$$

$$= \frac{3}{32} \text{ sq. units}$$

$$\therefore \text{ Required area} = \left(\frac{1}{4} + \frac{3}{32} \right) \text{ sq. units}$$

$$= \frac{11}{32} \text{ sq. units}$$

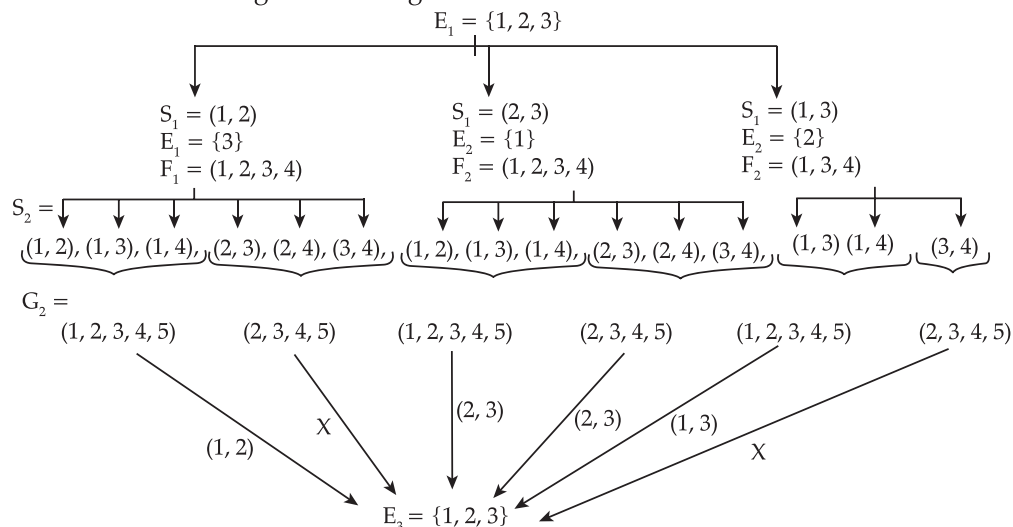
Hints:

- (i) Draw the graph showing the region bounded by the curves, finding the intersection points.
- (ii) Use area of triangle choose vertices are (x_1, y_1) (x_2, y_2)

And (x_3, y_3) is given by. $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ sq. units}$

3. Option (A) is correct.

Let us check the cases using the tree diagram:



$$P(E_1 = E_2) = P(S_1 = (1, 2)) P(S_1 = (2, 3)) + P(S_1 = (1, 3))$$

$$= \left(\frac{1}{3} \times \frac{3}{6} \times \frac{1}{5C_2} + 0 \right) + \left(\frac{1}{3} \times \frac{3}{6} \times \frac{1}{5C_2} + \frac{1}{3} \times \frac{3}{6} \times \frac{1}{4C_2} \right) + \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{5C_2} + 0 \right)$$

$$= \frac{1}{12}$$

Required probability

$$= \frac{P(S_1 = (1, 2)) \cap (E_1 = E_3)}{P(E_1 = E_3)}$$

$$= \frac{1}{3} \times \frac{3}{6} \times \frac{1}{5} C_2$$

$$= \frac{1}{12}$$

$$= \frac{1}{5}$$

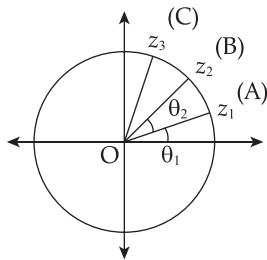
4. Option (C) is correct.

Given: $\theta_1 + \theta_2 + \dots + \theta_{10} = 2\pi$

$$z_1 = e^{i\theta}, z_k = z_{k-1} e^{i\theta_k}$$

$$|z_1| = |z_2| \dots |z_{10}| = 1$$

$$|z_2 - z_1| = \text{Length of line AB}$$



As we know, length of line AB is less than or equal to length of arc AB

$$\Rightarrow |z_2 - z_1| \leq \text{Length of arc AB}$$

Similarly, $|z_3 - z_2| \leq \text{Length of arc BC}$

$$\therefore Q_1 + Q_2 + \dots + Q_{10} = 2\pi$$

$$\Rightarrow \text{Sum of length of all arcs} = 2\pi$$

$$\Rightarrow |z_2 - z_1| + |z_3 - z_2| + \dots + |z_1 - z_{10}| \leq 2\pi$$

As we know, $|z_k^2 - z_{k-1}^2| = |z_k - z_{k-1}| |z_k + z_{k-1}|$

Also, $|z_k + z_{k-1}| \leq |z_k| + |z_{k-1}| \leq 2$

$$\Rightarrow |z_k + z_{k-1}| |z_k - z_{k-1}| \leq 2|z_k - z_{k-1}|$$

$$\Rightarrow |z_k^2 - z_{k-1}^2| \leq 2|z_k - z_{k-1}|$$

$$\Rightarrow |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_1^2 - z_{10}^2| \leq 2$$

$$\left[|z_2 - z_1| + |z_3 - z_2| + \dots + |z_1 - z_{10}| \right]$$

$$\Rightarrow |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_1^2 - z_{10}^2| \leq 2(2\pi)$$

$$\Rightarrow |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_1^2 - z_{10}^2| \leq 4\pi$$

Hints:

(i) Use $|z_1| = |z_2| \dots |z_{10}| = 1$

(ii) $|z_2 - z_1| = \text{length of line A B}$

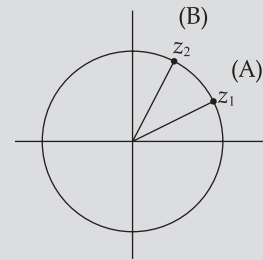
Hints:

(i) Use the conditional probability

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

(ii) Use the number of ways of selecting r object out of n objects is ${}^n C_r$

(iii) Use the multiplication rule of probability.



(iii) length of arcs \geq length of chord.

(iv) $|z_a^2 - z_b^2| = |z_a - z_b| |z_a + z_b|$

5. Correct answer is [76.25].

Given: $S = \{1, 2, 3, \dots, 100\}$

p_1 = probability that the maximum of chosen numbers is at least 81.

$\Rightarrow p_1 = 1 - \text{probability that the maximum of chosen numbers is at most 80}$

$$\Rightarrow p_1 = 1 - \frac{80 \times 80 \times 80}{100 \times 100 \times 100}$$

$$\Rightarrow p_1 = 1 - \frac{64}{125}$$

$$\Rightarrow p_1 = \frac{61}{125}$$

$$\therefore \frac{625}{4} p_1 = \frac{625}{4} \times \frac{61}{125} = \frac{305}{4} = 76.25$$

Hints:

(i) Probability that the maximum of chosen numbers is at least 81 = $1 - \text{probability that the maximum of chosen number is at most 80.}$

6. Correct answer is [24.50].

Given: $S = \{1, 2, 3, \dots, 100\}$

p_2 = the probability that the minimum of chosen at most 40.

$\Rightarrow p_2 = 1 - \text{probability that the minimum of chosen number is at least 41.}$

$$\Rightarrow p_2 = 1 - \frac{60 \times 60 \times 60}{100 \times 100 \times 100}$$

$$\Rightarrow p_2 = 1 - \frac{27}{125}$$

$$\Rightarrow p_2 = \frac{98}{125}$$

$$\Rightarrow \frac{125}{4} p_2 = \frac{125}{4} \times \frac{98}{125} = 24.50$$

Hints:

- (i) The probability that the minimum of chosen number S is at most 40 = 1 - probability that the minimum of chosen numbers is at least 41.

7. Correct answer is [1].

Given: The system of linear equation is

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1$$

$$D_1 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$\Rightarrow D_1 = -3 - 2(-6) + 3(-3)$$

$$\Rightarrow D_1 = 0$$

$$D_1 = \begin{vmatrix} \alpha & 2 & 3 \\ \beta & 5 & 6 \\ \gamma - 1 & 8 & 9 \end{vmatrix}$$

$$= \alpha(45 - 48) - 2(9\beta - 6\gamma + 6) + 3(8\beta - 5\gamma + 5)$$

$$\Rightarrow D_1 = -3\alpha - 18\beta + 12\gamma - 12 + 24\beta - 15\gamma + 15$$

$$\Rightarrow D_1 = -3\alpha + 6\beta - 3\gamma + 3$$

$$\Rightarrow D_1 = -3(\alpha - 2\beta + \gamma - 1)$$

Similarly, $D_2 = \begin{vmatrix} 1 & \alpha & 3 \\ 4 & \beta & 6 \\ 7 & \gamma - 1 & 9 \end{vmatrix}$

$$\Rightarrow D_2 = 1(9\beta - 6\gamma + 6) - \alpha(36 - 42) + 3(4\gamma - 4 - 7\beta)$$

$$\Rightarrow D_2 = 6\alpha - 12\beta + 6\gamma - 6$$

And, $D_3 = \begin{vmatrix} 1 & 2 & \alpha \\ 4 & 5 & \beta \\ 7 & 8 & \gamma - 1 \end{vmatrix}$

$$= 1(5\gamma - 5 - 8\beta) - 2(4\gamma - 4 - 7\beta)$$

$$+ \alpha(32 - 35)$$

$$\Rightarrow D_3 = -3\alpha + 6\beta - 3\gamma + 3$$

$$\Rightarrow D_3 = -3(\alpha - 2\beta + \gamma - 1)$$

For system of linear equation to be consistent,

$$D_1 = D_2 = D_3 = 0$$

Now,

$$M = \begin{vmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow M = \alpha(1) - 2(\beta) + \gamma(1)$$

$$\Rightarrow M = \alpha - 2\beta + \gamma$$

$$\therefore M = 1 \quad \{ \because \alpha - 2\beta + \gamma - 1 = 0 \}$$

Hints:

- (i) For a system of linear equations $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$

And $a_3x + b_3y + c_3z = d_3$ where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

To be a consistent system, if $\Delta = 0$ then

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

8. Correct answer is [1.5].

As from the above question we get,

$$\alpha - 2\beta + \gamma - 1 = 0$$

- \therefore P is the plane containing all those (α, β, γ) for which the given system of linear equation is consistent.

\Rightarrow The equation of the plane P is

$$x - 2y + z - 1 = 0$$

Now, distance of the point $(0, 1, 0)$ from the plane $x - 2y + z - 1 = 0$ is

$$\frac{|0 - 2(1) + 0 - 1|}{\sqrt{(1)^2 + (-2)^2 + (1)^2}}$$

$$\Rightarrow \sqrt{D} = \left| \frac{-2 - 1}{\sqrt{6}} \right| \text{ units}$$

$$\Rightarrow \sqrt{D} = \frac{3}{\sqrt{6}}$$

$$\Rightarrow (\sqrt{D})^2 = \left(\frac{3}{\sqrt{6}} \right)^2$$

$$\Rightarrow D = \frac{9}{6} = \frac{3}{2} = 1.5$$

Hints:

- (i) Use the distance of a point (x_1, y_1, z_1) from the plane

$ax + by + cz + d = 0$ is given by

$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right| \text{ units}$$

9. Correct answer is [9].

Given: Line $L_1 : \sqrt{2}x + y - 1 = 0$

Line $L_2 : \sqrt{2}x - y + 1 = 0$

Let point P be (h, k)

$$\begin{aligned} \text{Distance of P } (h, k) \text{ from } L_1 &= \left| \frac{\sqrt{2}h + k - 1}{\sqrt{(\sqrt{2})^2 + (1)^2}} \right| \\ &= \left| \frac{\sqrt{2}h + k - 1}{\sqrt{3}} \right| \end{aligned}$$

Similarly, distance of P (h, k) from

$$L_2 = \left| \frac{\sqrt{2}h - k + 1}{\sqrt{3}} \right|$$

$$\text{Now, } \left| \frac{\sqrt{2}h + k - 1}{\sqrt{3}} \right| \left| \frac{\sqrt{2}h - k + 1}{\sqrt{3}} \right| = \lambda^2$$

$$\Rightarrow \left| (\sqrt{2}h)^2 - (k-1)^2 \right| = 3\lambda^2$$

$$\Rightarrow \text{Locus is } \left| 2x^2 - (y-1)^2 \right| = 3\lambda^2$$

Now, $y = 2x + 1$ cuts C at R and S,

$$\Rightarrow \left| 2x^2 - (2x+1-1)^2 \right| = 3\lambda^2$$

$$\Rightarrow \left| 2x^2 - 4x^2 \right| = 3\lambda^2$$

$$\Rightarrow \left| -2x^2 \right| = 3\lambda^2$$

$$\Rightarrow x = \pm \sqrt{\frac{3}{2}} |\lambda|$$

Let R be (x_1, y_1) and S be (x_2, y_2)

$$\Rightarrow |x_1 - x_2| = \left| \sqrt{\frac{3}{2}} |\lambda| + \sqrt{\frac{3}{2}} |\lambda| \right| = \sqrt{6} |\lambda|$$

$$\begin{aligned} \text{And } |y_1 - y_2| &= |2x_1 + 1 - 2x_2 - 1| = 2|x_1 - x_2| \\ &= 2\sqrt{6} |\lambda| \end{aligned}$$

Now, distance between R (x_1, y_1) and S (x_2, y_2) is $\sqrt{270}$

$$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{270}$$

$$\Rightarrow \sqrt{(\sqrt{6} |\lambda|)^2 + (2\sqrt{6} |\lambda|)^2} = \sqrt{270}$$

$$\Rightarrow 6\lambda^2 + 4(6\lambda^2) = 270$$

$$\Rightarrow \lambda^2 = \frac{270}{30}$$

$$\Rightarrow \lambda^2 = 9$$

Hints:

- (i) Use distance of a point (x_1, y_1) from the line

$$ax + by + c = 0 \text{ is given by } \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| \text{ units}$$

- (ii) Use distance formula to find the distance between two points (x_1, y_1) and

$$(x_2, y_2) \text{ by } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \text{ units}$$

10. Correct answer is [77.14].

Let R be (x_1, y_1) and S be (x_2, y_2)

We have $|x_1 - x_2| = \sqrt{6}\lambda$ and

$$|y_1 - y_2| = 2\sqrt{6}\lambda$$

$$\text{Slope of line RS} = \frac{y_1 - y_2}{x_1 - x_2} = 2$$

Now, R'S' \perp RS

$$\Rightarrow (\text{slope of RS}) \cdot (\text{slope of R'S'}) = -1$$

$$\Rightarrow \text{slope of R'S'} = -\frac{1}{2}$$

Using mid-point formula, mid-point of RS will be T

$$T = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

As we had, x_1 and $x_2 = \pm \sqrt{\frac{3}{2}} |\lambda|$ and

$$y_1 = 2x_1 + 1, y_2 = 2x_2 + 1$$

$$\Rightarrow T = \left(\frac{\sqrt{\frac{3}{2}} |\lambda| - \sqrt{\frac{3}{2}} |\lambda|}{2}, \frac{2x_1 + 1 + 2x_2 + 1}{2} \right)$$

$$\Rightarrow T = (0, x_1 + x_2 + 1)$$

$$\Rightarrow T = (0, 1)$$

So, slope of R'S' = $-\frac{1}{2}$ and R'S' passes through $(0, 1)$

So, using point slope form of line, equation of R'S' will be,

$$(y - 1) = -\frac{1}{2} (x - 0)$$

$$\Rightarrow y - 1 = -\frac{1}{2} x$$

Let R' be (x_3, y_3) and S' be (x_4, y_4)

Now $y - 1 = -\frac{1}{2}x$ meets curve

$$|2x^2 - (y-1)^2| = 3\lambda^2 \text{ at R' and S'}$$

$$\Rightarrow \left| 2x^2 - \left(-\frac{1}{2}x\right)^2 \right| = 3\lambda^2$$

$$\Rightarrow \left| \frac{7}{4}x^2 \right| = 3\lambda^2$$

$$\Rightarrow x^2 = \frac{12}{7}\lambda^2$$

$$\Rightarrow x = \pm\sqrt{\frac{12}{7}}|\lambda|$$

$$\Rightarrow |x_1 - x_2| = \left| \sqrt{\frac{12}{7}}|\lambda| + \sqrt{\frac{12}{7}}|\lambda| \right| = 2\sqrt{\frac{12}{7}}|\lambda|$$

and

$$|y_1 - y_2| = \left| 1 - \frac{1}{2}x_1 - 1 + \frac{1}{2}x_2 \right| = \frac{1}{2}|x_2 - x_1| = \sqrt{\frac{12}{7}}|\lambda|$$

Using distance formula,

$$R'S' = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$R'S' = \sqrt{\left(2\sqrt{\frac{12}{7}}|\lambda|\right)^2 + \left(\sqrt{\frac{12}{7}}|\lambda|\right)^2}$$

$$(R'S')^2 = 5 \times \frac{12}{7}(\lambda^2)$$

$$\Rightarrow D = (R'S')^2 = 5 \times \frac{12}{7} \times 9 \quad \{\therefore \lambda^2 = 9\}$$

$$\Rightarrow D = 77.14$$

Hints:

(i) Use if slope of two perpendicular lines be m_1 and m_2 then $m_1 \cdot m_2 = -1$

(ii) Slope of line passing through (x_1, y_1) and

$$(x_2, y_2) \text{ is } \frac{y_2 - y_1}{x_2 - x_1}$$

(iii) Equation of straight line have slope m and passing through

$$(x_1, y_1) \text{ is } (y - y_1) = m(x - x_1)$$

(iv) Distance between two points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

11. Options (A), (B) and (D) are correct.

Given: $|Q| \neq 0$

$$E = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix}, P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}, F = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$PEP = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\Rightarrow PEP = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\Rightarrow PEP = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$\Rightarrow PEP = F$$

$$\text{Also, } P^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow P^2 = I$$

$$\text{Now, } |E| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix}$$

$$= 1(54 - 52) - 2(36 - 32) + 3(26 - 24)$$

$$\Rightarrow |E| = 0$$

$$\text{And } |F| = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= 1(54 - 52) - 3(24 - 26) + 2(32 - 36)$$

$$\Rightarrow |F| = 0$$

$$\text{Now, } |EQ| = |E||Q| = 0 \quad \{\therefore |E| = 0\}$$

$$\text{and } |PFQ^{-1}| = \frac{|P||F|}{|Q|} = 0 \quad \{\therefore |F| = 0\}$$

$$\text{let } A = EQ + PFQ^{-1}$$

post multiplying the above equation by Q we get,

$$AQ = EQ^2 + PF$$

$$\Rightarrow AQ = EQ^2 + P(PEP) \quad \{\therefore PEP = F\}$$

$$\Rightarrow AQ = EQ^2 + P^2EP$$

$$\Rightarrow AQ = EQ^2 + EP \quad \{\therefore P^2 = I\}$$

$$\Rightarrow AQ = E(Q^2 + P)$$

$$\Rightarrow |AQ| = |E||Q^2 + P|$$

$$\Rightarrow |AQ| = 0 \quad \{\therefore |E| = 0\}$$

$$\Rightarrow |A| = 0 \quad \{\therefore |Q| \neq 0\}$$

$$\Rightarrow |EQ + PFQ^{-1}| = 0$$

$$\Rightarrow |EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$$

$$\text{Now, as } |E| = 0, |F| = 0$$

$$\Rightarrow |(EF)^3| = 0, \text{ and } |(EF)^2| = 0$$

$$\therefore 0 > 0 \text{ is not true}$$

$$\Rightarrow |(EF)^3| > |EF|^2 \text{ is also not true}$$

$$\text{Now, } P^2 = I$$

$$\Rightarrow P^{-1} = P$$

$$\Rightarrow P^{-1}FP = PFP$$

$$\Rightarrow P^{-1}FP = P(PEP)P \quad \{\therefore PEP = F\}$$

$$\Rightarrow P^{-1}FP = IEI$$

$$\Rightarrow P^{-1}FP = E$$

$$\therefore E + P^{-1}FP = E + E = 2E$$

$$\text{And } P^{-1}EP + F = PEP + PEP$$

$$\{\therefore P^{-1} = P \text{ and } PEP = F\}$$

$$\Rightarrow P^{-1}EP + F = 2PEP$$

$$\Rightarrow P^{-1}EP + F = 2F$$

Now diagonal elements of E are 1, 3, 18 and diagonal elements of F are 1, 18, 3

$$\Rightarrow \text{Sum of diagonal elements of E} = \text{sum of diagonal elements of F}$$

$$\Rightarrow \text{Sum of diagonal elements of } 2E = \text{sum of diagonal elements of } 2F$$

$$\Rightarrow \text{Sum of diagonal elements of } E + P^{-1}FP = \text{sum of diagonal elements of } P^{-1}EP + F$$

Hints:

(i) If A is a non singular matrix then $|A| \neq 0$

(ii) Use properties of determinants $|AB| = |A||B|$

$$\text{and } |AB^{-1}| = \frac{|A|}{|B|}$$

(iii) Use multiplication of two matrices.

12. Options (A) and (B) are correct.

$$\text{Given: } f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Differentiating both the sides of the above equation we get,

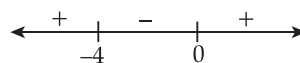
$$f'(x) = \frac{(x^2 + 2x + 4)(2x - 3) - (x^2 - 3x - 6)(2x + 2)}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow f'(x) = \frac{2x^3 - 3x^2 + 4x^2 - 6x + 8x - 12 - 2x^3 - 2x^2 + 6x^2 + 6x + 12x + 12}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow f'(x) = \frac{5x^2 + 20x}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow f'(x) = \frac{5x(x+4)}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow f'(x) = 0 \text{ at } x = 0, -4$$



$$\therefore f' < 0 \text{ in } (-2, -1)$$

$$\Rightarrow f(x) \text{ is decreasing in } (-2, -1)$$

$$\therefore f'(x) > 0 \text{ in } (1, 2)$$

$$\Rightarrow f(x) \text{ is increasing in } (1, 2)$$

$\therefore f'(x)$ changes from $+ve$ to $-ve$ at -4 so $x = -4$ is the point of maxima and $f'(x)$ changes from $-ve$ to $+ve$ at $x = 0$ so $x = 0$ is the point of minima.

$$f(-4) = \frac{(-4)^2 - 3(-4) - 6}{(-4)^2 + 2(-4) + 4} = \frac{11}{6}$$

$$\text{And } f(0) = \frac{-6}{4} = \frac{-3}{2}$$

$$\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{x^2 - 3x - 6}{x^2 + 2x + 4} = \lim_{x \rightarrow \pm\infty} \frac{1 - \frac{3}{x} - \frac{6}{x^2}}{1 + \frac{2}{x} + \frac{4}{x^2}}$$

$$\Rightarrow \lim_{x \rightarrow \pm\infty} f(x) = 1$$

$$\Rightarrow \text{maximum value of } f(x) = \frac{11}{6}$$

$$\text{And minimum value of } f(x) = \frac{-3}{2}$$

Now, Co-domain of $f(x)$ is R

$$\Rightarrow \text{Range of } f(x) = \left[\frac{-3}{2}, \frac{11}{6} \right]$$

$$\Rightarrow \text{Range of } f(x) \neq \text{codomain of } f(x)$$

$\therefore f$ is into function

Hints:

(i) Use quotient rule of differentiation

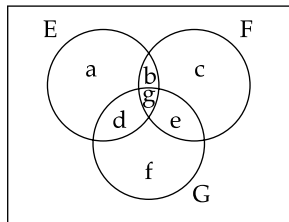
$$\left(\frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

(ii) If $f'(x) < 0$ in an interval then $f(x)$ is decreasing in that interval and if $f'(x) > 0$, then $f(x)$ is increasing.
 (iii) A function is onto function if Range = codomain.

13. Options (A), (B) are C) are correct.

Given: $P(E) = \frac{1}{8}, P(F) = \frac{1}{6}, P(G) = \frac{1}{4},$

$P(E \cap F \cap G) = \frac{1}{10}$



Here, $P(E \cap F \cap G) = g = \frac{1}{10}$

$P(E) = a + b + d + g = \frac{1}{8}$

$\Rightarrow a + b + d = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$... (i)

$P(F) = c + b + e + g = \frac{1}{6}$

$\Rightarrow c + b + e = \frac{1}{6} - \frac{1}{10} = \frac{1}{15}$... (ii)

$P(G) = f + d + e + g = \frac{1}{4}$

$\Rightarrow f + d + e = \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$

Now, $P(E \cap F \cap G^c) = b$

From eq. (i) $b \leq \frac{1}{40}$

$\Rightarrow P(E \cap F \cap G^c) \leq \frac{1}{40}$

Similarly, $P(E^c \cap F \cap G) = e$

From eq. (ii) $e \leq \frac{1}{15}$

$\Rightarrow P(E^c \cap F \cap G) \leq \frac{1}{15}$

Also, $P(E \cup F \cup G) \leq P(E) + P(F) + P(G)$

$\Rightarrow P(E \cup F \cup G) \leq \frac{1}{8} + \frac{1}{6} + \frac{1}{4}$

$\Rightarrow P(E \cup F \cup G) \leq \frac{13}{24}$

By De Morgan's Law, $P(E^c \cap F^c \cap G^c)$

$= P(E \cup F \cup G)^c$
 $\Rightarrow P(E^c \cap F^c \cap G^c) \geq 1 - \frac{13}{24}$

$\Rightarrow P(E^c \cap F^c \cap G^c) \geq \frac{11}{24}$

Hints:

- (i) Use venn diagram for three variables and solve it.
- (ii) Use De Morgan's Law $P(A \cup B \cup C) = P(A^c \cap B^c \cap C^c)$
- (iii) Use complement rule of probability, $1 - P(A) = P(A^c)$

14. Options (A), (B) and C) are correct.

Given: $I - EF$ is invertible matrix

$G = (I - EF)^{-1}$

$\Rightarrow G^{-1} = I - EF$... (i)

Post multiplying by G , we get,

$G^{-1}G = IG - EFG$

$\Rightarrow I = G - EFG$... (ii)

Pre multiplying equation (i) by G we get,

$GG^{-1} = GI - GEF$

$\Rightarrow I = G - GEF$... (iii)

From equation (ii) and (iii), we get

$EFG = GEF$

Now, $(I - FE)(I + FGE)$

$= I + FGE - FE - FEFGE$

$= I + FGE - FE - F(G - I)E$

$= I + FGE - FE - FGE + FE$

$= I$

$\Rightarrow (I - FE)(I + FGE) = I$

And $(I - FE)(I - FGE)$

$= I - FE - FGE + FEFGE$

$= I - FE - FGE + F(G - I)E$

$= I - FE - FGE + FGE - FE$

$= I - 2FE$

$\Rightarrow (I - FE)(I - FGE) \neq I$

Now, $(I - FE)(FGE) = FGE - FEFGE$

$\Rightarrow (I - FE)(FGE) = FGE - F(G - I)E$

$\Rightarrow (I - FE)(FGE) = FGE - FGE + FE$

$\Rightarrow (I - FE)(FGE) = FE$

$\Rightarrow |I - FE||FGE| = |FE|$

Hints:

- (i) Similarly using identities of multiplication of matrices.
- (ii) Use properties of inverse of matrix $AA^{-1} = I = A^{-1}A$

15. Options (A) and (B) are correct.

$$\text{Given: } S_n(x) = \sum_{k=1}^n \cot^{-1} \left(\frac{1+k(k+1)x^2}{x} \right) \forall x \in \mathbb{R}$$

$$\Rightarrow S_n(x) = \sum_{k=1}^n \tan^{-1} \left(\frac{x}{1+k(k+1)x^2} \right)$$

$$\left\{ \because \cot^{-1} x = \tan^{-1} \frac{1}{x} \right\}$$

$$\Rightarrow S_n(x) = \sum_{k=1}^n \tan^{-1} \left(\frac{kx+x-kx}{1+(kx+x)kx} \right)$$

$$\Rightarrow S_n(x) = \sum_{k=1}^n [\tan^{-1}(kx+x) - \tan^{-1}(kx)]$$

$$\left\{ \because \tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A-B}{1+AB} \right) \right\}$$

$$\Rightarrow S_n(x) = \sum_{k=1}^n \{ \tan^{-1}((k+1)x) - \tan^{-1}(kx) \}$$

$$\Rightarrow S_n(x) = \tan^{-1} 2x - \tan^{-1} x + \tan^{-1} 3x - \tan^{-1} 2x$$

$$+ \dots + \tan^{-1}(n+1)x - \tan^{-1} nx$$

$$\Rightarrow S_n(x) = \tan^{-1}(nx+x) - \tan^{-1} x$$

$$\Rightarrow S_n(x) = \tan^{-1} \left(\frac{nx}{1+(n+1)x^2} \right)$$

$$\Rightarrow S_{10}(x) = \tan^{-1} \left(\frac{10x}{1+11x^2} \right)$$

$$\Rightarrow S_{10}(x) = \frac{\pi}{2} - \cot^{-1} \left(\frac{10x}{1+11x^2} \right)$$

$$\left\{ \because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \right\}$$

$$\Rightarrow S_{10}(x) = \frac{\pi}{2} - \tan^{-1} \left(\frac{1+11x^2}{10x} \right) \forall x > 0$$

$$\Rightarrow \text{Now, } \lim_{n \rightarrow \infty} \cot(S_n(x)) = \lim_{n \rightarrow \infty} \cot$$

$$\left[\tan^{-1} \left(\frac{nx}{1+(n+1)x^2} \right) \right]$$

$$\Rightarrow \lim_{n \rightarrow \infty} \cot(S_n(x)) = \lim_{n \rightarrow \infty} \cot$$

$$\left[\cot^{-1} \left(\frac{1+(n+1)x^2}{nx} \right) \right]$$

$$\Rightarrow \lim_{n \rightarrow \infty} \cot(S_n(x)) = \lim_{n \rightarrow \infty} \left(\frac{1+(n+1)x^2}{nx} \right)$$

$$\left\{ \because \cot(\cot^{-1} x) = x \right\}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \cot(S_n(x)) = \lim_{n \rightarrow \infty} \left(\frac{\frac{1}{n} + \left(1 + \frac{1}{n}\right)x^2}{x} \right)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \cot(S_n(x)) = \frac{x^2}{x} = x \forall x > 0$$

$$\Rightarrow \text{Now, } S_3(x) = \tan^{-1} \left(\frac{3x}{1+4x^2} \right)$$

$$\Rightarrow \text{So, for equation } S_3(x) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{3x}{1+4x^2} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{3x}{1+4x^2} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{3x}{1+4x^2} = 1$$

$$\Rightarrow 4x^2 - 3x + 1 = 0$$

$$\Rightarrow x = \frac{3x \pm \sqrt{9-16}}{8}$$

{Using Shri dharacharya's rule}

$$\Rightarrow x \in \mathbb{R}$$

$$\text{So, equation } S_3(x) = \frac{\pi}{4} \text{ has no root in } (0, \infty)$$

$$\text{Now, } \tan(S_n(x)) = \tan \left(\tan^{-1} \left(\frac{nx}{1+(n+1)x^2} \right) \right)$$

$$\Rightarrow \tan(S_n(x)) = \frac{nx}{1+(n+1)x^2}$$

$$\forall n \geq 1; x > 0; n \in \mathbb{I}^+$$

$$\Rightarrow \text{Now for } \tan(S_n(x)) \leq \frac{1}{2}$$

$$\Rightarrow \frac{nx}{1+(n+1)x^2} \leq \frac{1}{2}$$

$$\Rightarrow 2nx \leq 1 + (n+1)x^2$$

$$\Rightarrow (n+1)x^2 - 2nx + 1 \geq 0$$

Discriminant of $(n+1)x^2 - 2nx + 1 = 0$ is

$$D = (-2n)^2 - 4(n+1)(1)$$

$$D = 4n^2 - 4n - 4$$

$$D = 4n^2 - 4(n+1)$$

$$\text{For } n = 1, D = -4 \Rightarrow D < 0$$

$$\Rightarrow (n+1)x^2 - 2nx + 1 > 0$$

$$\text{For } n \geq 2, D > 0$$

$$\text{For some } x > 0, (n+1)x^2 - 2nx + 1 < 0$$

$\therefore \tan (S_n(x)) \leq \frac{1}{2} \forall n \geq 1$ and $x > 0$ is not true.

Hints:

(i) use identities of inverse trigonometric functions

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}, \tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A-B}{1+AB} \right)$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \cot(\cot^{-1} x) = x$$

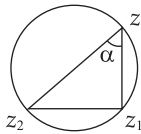
(ii) find roots of quadratic equation using Sridharacharya rule i.e. roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(iii) use nature of roots of quadratic equation using discriminant for $ax^2 + bx + c = 0$, if $D = b^2 - 4ac < 0$, then $ax^2 + bx + c > 0$ and if $D > 0$ then x has real values so for some x $ax^2 + bx + c < 0$.

16. Options (B) and D) are correct.

Given: $\arg \left(\frac{z-\alpha}{z+\beta} \right) = \frac{\pi}{4}$

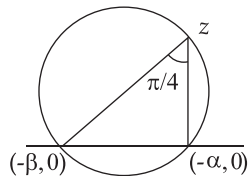
As we know, if $\arg \left(\frac{z+z_1}{z+z_2} \right) = \alpha$ where $\alpha \in (0, \pi)$



$\Rightarrow z$ would lie on an arc of segment of a circle on $z_1 z_2$ containing angle α .

So, $\arg \left(\frac{z+\alpha}{z+\beta} \right) = \frac{\pi}{4}$

$\Rightarrow z$ lie on an arc of segment of a circle with $(-\alpha, 0)$ and $(-\beta, 0)$ and subtend an angle $\frac{\pi}{4}$ on z



Also, z lie on circle $x^2 + y^2 + 5x - 3y + 4 = 0$
 $\Rightarrow (-\alpha, 0)$ and $(-\beta, 0)$ also lie on the circle
 So, $y = 0, x^2 + 5x + 4 = 0$
 $\Rightarrow (x + 1)(x + 4) = 0$

$\Rightarrow x = -1, -4$
 $\Rightarrow \alpha = 1$ and $\beta = 4$
 $\therefore \alpha\beta = 4$ and $\beta = 4$

Hints:

(i) If $\arg \left(\frac{z-z_1}{z-z_2} \right) = \alpha$ where α whose $\alpha \in (0, \pi)$ then z would lie on an arc of segment of a circle on $z_1 z_2$ containing angle α

17. Correct answer is [4].

Given: $3x^2 - 4|x^2 - 1| + x - 1 = 0$

For $x \in [-1, 1] \quad |x^2 - 1| = -(x^2 - 1)$

$\therefore 3x^2 + 4(x^2 - 1) + x - 1 = 0$
 $\Rightarrow 7x^2 + x - 5 = 0$

$\Rightarrow x = \frac{(-1) \pm \sqrt{1 + 4(5)(7)}}{2(7)}$

(Using Sridharacharya's rule)

$\Rightarrow x = \frac{-1 \pm \sqrt{141}}{14}$

\Rightarrow Both values of $x \in [-1, 1]$

Now, for $x \in (-\infty, -1) \cup (1, \infty)$

$|x^2 - 1| = (x^2 - 1)$

$\therefore 3x^2 - 4(x^2 - 1) + x - 1 = 0$
 $\Rightarrow x^2 - x - 3 = 0$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{1 - 4(-3)}}{2}$

$\Rightarrow x = \frac{1 \pm \sqrt{13}}{2}$

\Rightarrow Both values $x \in (-\infty, -1) \cup (1, \infty)$

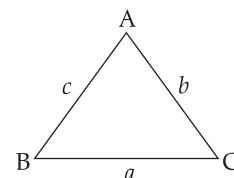
\therefore given equation has 4 real roots.

Hints:

(i) Use definition of modulus function.

(ii) Use Sridharacharya's rule for $ax^2 + bx + c = 0$ roots are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

18. Correct answer is [2].



Given: ΔABC such that

$AB = \sqrt{23}, BC = 3, CA = 4$

$$\Rightarrow c = \sqrt{23}, a = 3, b = 4$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{b^2 + c^2 - a^2}{2bc \sin A}$$

$$\therefore \text{Area of triangle } \Delta ABC = \Delta = \frac{1}{2} bc \sin A$$

$$\Rightarrow \cot A = \frac{b^2 + c^2 - a^2}{4\Delta}$$

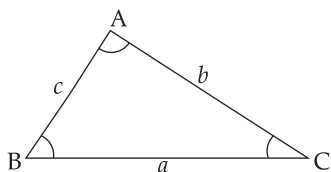
$$\text{Similarly, } \cot B = \frac{a^2 + c^2 - b^2}{4\Delta} \text{ and}$$

$$\cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$$

$$\therefore \frac{\cot A + \cot C}{\cot B} = \frac{\frac{b^2 + c^2 - a^2}{4\Delta} + \frac{a^2 + b^2 - c^2}{4\Delta}}{\frac{a^2 + c^2 - b^2}{4\Delta}}$$

$$\Rightarrow \frac{\cot A + \cot C}{\cot B} = \frac{2b^2}{a^2 + c^2 - b^2} = \frac{2(4)^2}{(3)^2 + (\sqrt{23})^2 - (4)^2}$$

$$\Rightarrow \frac{\cot A + \cot C}{\cot B} = 2$$

**Hints:**

(i) Use cosine formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(ii) Use area of triangle $= \frac{1}{2} bc \sin A = \frac{1}{2}$

$$ab \sin c = \frac{1}{2} ac \sin B$$

19. Correct answer is [7].

$$\text{Given: } |\vec{u}| = 1, |\vec{v}| = 1$$

\vec{u} is not perpendicular to \vec{v}

$$\vec{u} \cdot \vec{v} \neq 0$$

$$\text{And } \vec{u} \cdot \vec{w} = 1 \quad \vec{v} \cdot \vec{w} = 1 \quad \vec{w} \cdot \vec{w} = 4$$

$$\Rightarrow \vec{w} \cdot \vec{w} = |\vec{w}|^2 = 4$$

$$\Rightarrow |\vec{w}| = 2$$

Volume of parallelepiped with \vec{u}, \vec{v} and \vec{w} as its sides,

$$= [\vec{u} \vec{v} \vec{w}] = \sqrt{2}$$

$$\text{Now, } [\vec{u} \vec{v} \vec{w}]^2 = \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{v} & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & \vec{w} \cdot \vec{w} \end{vmatrix} = 2$$

$$\Rightarrow \begin{vmatrix} 1 & \vec{u} \cdot \vec{v} & 1 \\ \vec{v} \cdot \vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$\Rightarrow 1(4-1) - \vec{u} \cdot \vec{v}(4\vec{u} \cdot \vec{v} - 1) + 1(\vec{u} \cdot \vec{v} - 1) = 2$$

$$\Rightarrow 3 - 4(\vec{u} \cdot \vec{v})^2 + \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{v} - 1 = 2$$

$$\Rightarrow -4(\vec{u} \cdot \vec{v})^2 + 2\vec{u} \cdot \vec{v} + 2 = 2$$

$$\Rightarrow -4(\vec{u} \cdot \vec{v})^2 + 2\vec{u} \cdot \vec{v} = 0$$

$$\Rightarrow 2\vec{u} \cdot \vec{v}(-2\vec{u} \cdot \vec{v} + 1) = 0$$

$$\Rightarrow \vec{u} \cdot \vec{v} = \frac{1}{2}, \vec{u} \cdot \vec{v} \neq 0$$

$$\text{Now, } |3\vec{u} + 5\vec{v}| = \sqrt{9 + 25 + 30\left(\frac{1}{2}\right)}$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = \sqrt{49}$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = 7$$

Hints:

(i) If two vectors \vec{a} and \vec{b} are not perpendicular then $\vec{a} \cdot \vec{b} \neq 0$

(ii) Volume of parallelepiped with $\vec{a}, \vec{b}, \vec{c}$ as its sides is $[\vec{a} \vec{b} \vec{c}]$

$$\text{(iii) } [\vec{a} \vec{b} \vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$