JEE Advanced (2023)

PAPER 1

Physics

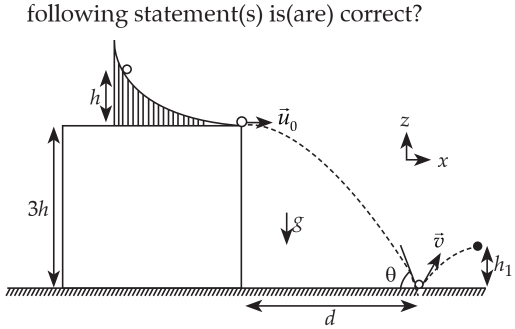
General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Mark : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −2 marks.

Q. 1. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height $3h$ from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct?



(A) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$ **(B)** $\vec{v} = \sqrt{2gh}\left(\hat{x} - \hat{z}\right)$

(C) $\theta = 60°$ **(D)** $d/h_1 = 2\sqrt{3}$

Q. 2. A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

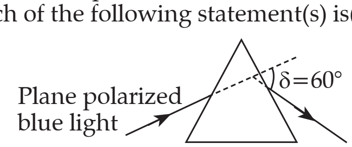


Figure-1

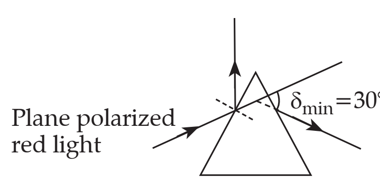


Figure-2

(A) The blue light is polarized in the plane of incidence.

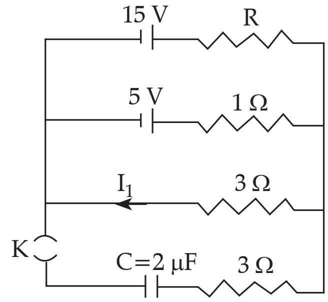
(B) The angle of the prism is 45°.

(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.

(D) The angle of refraction for blue light in air at the exit plane of the prism is 60°.

Q. 3. In a circuit shown in the figure, the capacitor C is initially uncharged and the key K is open. In this condition, a current of 1 A flows through the 1 Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct?

[Given: $e^{-1} = 0.36$]



(A) The value of the resistance R is 3 Ω.

(B For $t < t_0$, the value of current I_1 is 2 A.

(C) At $t = t_0 + 7.2$ μs, the current in the capacitor is 0.6 A.

(D) For $t \to \infty$, the charge on the capacitor is 12 μC.

***General Instructions*:**

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

Q. 4. A bar of mass $M = 1.00$ kg and length $L = 0.20$ m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $m = 0.10$ kg is moving on the same horizontal surface with 5.00 m s^{-1} speed on a path perpendicular to the bar. It hits the bar at a distance L/2 from the pivoted end and returns back on the same path with speed υ. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct?

(A) $\omega = 6.98$ rad s^{-1} and $\upsilon = 4.30$ m s^{-1}

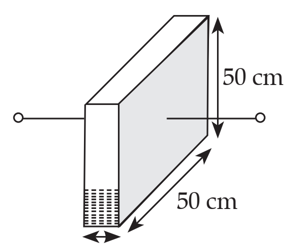
(B) $\omega = 3.75$ rad s^{-1} and $\upsilon = 4.30$ m s^{-1}

(C) $\omega = 3.75$ rad s^{-1} and $\upsilon = 10.0$ m s^{-1}

(D) $\omega = 6.80$ rad s^{-1} and $\upsilon = 4.10$ m s^{-1}

Q. 5. A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 cm^3 s^{-1}. What is the value of the capacitance of the container after 10 seconds?

[Given: Permittivity of free space $\varepsilon_0 = 9 \times 10^{-12}$ C^2 N^{-1} m^{-2}, the effects of the non-conducting walls on the capacitance are negligible]



(A) 27 pF **(B)** 63 pF

(C) 81 pF **(D)** 135 pF

Q. 6. One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state (T_f, $5V_0$). Another mole of the same gas expands isothermally from a different initial state (T_B, V_0) to the same final state (T_f, $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A/T_B?

(A) $5^{\gamma-1}$ **(B)** $5^{1-\gamma}$

(C) 5^{γ} **(D)** $5^{1+\gamma}$

Q. 7. Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are h_P and h_Q, respectively, where $h_P = R/3$. The accelerations of P and Q due to Earth's gravity are g_P and g_Q, respectively. If $g_P/g_Q = 36/25$, what is the value of h_Q?

(A) $3R/5$ **(B)** $R/6$

(C) $6R/5$ **(D)** $5R/6$

General Instructions:

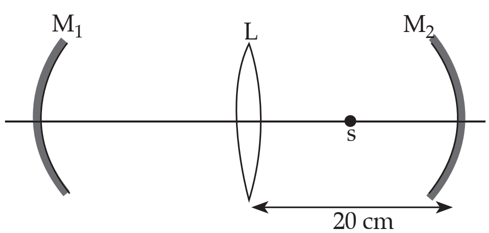
SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Q. 8. A Hydrogen-like atom has atomic number Z. Photons emitted in the electronic transitions from level $n = 4$ to level $n = 3$ in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1.95 e V. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of Z is _______.

[Given: hc = 1240 e V-n m and Rhc = 13.6 e V, where R is the Rydberg constant, h is the Planck's constant and c is the speed of light in vacuum]

Q. 9. An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is $n/7$ cm, one of the images coincides with S. The value of n is _______.

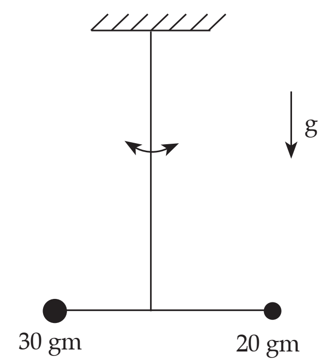


Q. 10. In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is 10 ± 0.1 cm and the distance of its real image from the lens is 20 ± 0.2 cm. The error in the determination of focal length of the lens is n%. The value of n is _______.

Q. 11. A closed container contains a homogeneous mixture of two moles of an ideal monoatomic gas ($\gamma = 5/3$) and one mole of an ideal diatomic gas ($\gamma = 7/5$). Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is _______ Joule.

Q. 12. A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, the speed of the tip of the person's shadow on the ground with respect to the person is _______ cm s^{-1}.

Q. 13. Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is 1.2×10^{-8} N m rad^{-1}. The angular frequency of the oscillations is $n \times 10^{-3}$ rad s^{-1}. The value of n is _______.



General Instructions:

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

Q. 14. List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

List-I	List-II
(P) $^{238}_{92}U \rightarrow ^{234}_{91}Pa$	(1) one α particle and one β^+ particle
(Q) $^{214}_{82}Pb \rightarrow ^{210}_{82}Pb$	(2) three β^- particles and one α particle
(R) $^{210}_{81}Tl \rightarrow ^{206}_{82}Pb$	(3) two β^- particles and one α particle
(S) $^{228}_{91}Pa \rightarrow ^{224}_{88}Ra$	(4) one α particle and one β^- particle
	(5) one α particle and two β^+ particles

(A) P → 4, Q → 3, R → 2, S → 1
(B) P → 4, Q → 1, R → 2, S → 5
(C) P → 5, Q → 3, R → 1, S → 4
(D) P → 5, Q → 1, R → 3, S → 2

Q. 15. Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option.

[Given: Wien's constant as 2.9×10^{-3} m-K and $\frac{hc}{e} = 1.24 \times 10^{-6}$ V-m]

List-I	List-II
(P) 2000 K	(1) The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV.
(Q) 3000 K	(2) The radiation at peak wavelength is visible to human eye.
(R) 5000 K	(3) The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S) 10000 K	(4) The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K.
	(5) The radiation at peak emission wavelength can be used to image human bones.

(A) P → 3, Q → 5, R → 2, S → 3
(B) P → 3, Q → 2, R → 4, S → 1
(C) P → 3, Q → 4, R → 2, S → 1
(D) P → 1, Q → 2, R → 5, S → 3

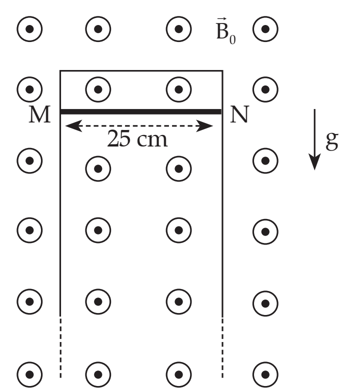
Q. 16. A series LCR circuit is connected to a 45sin (ωt) Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad s^{-1}, the current amplitude in the circuit is 0.05 I_0. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option.

List-I	List-II
(P) I_0 in mA	(1) 44.4
(Q) The quality factor of the circuit	(2) 18
(R) The bandwidth of the circuit in rad s^{-1}	(3) 400
(S) The peak power dissipated at resonance in Watt	(4) 2250
	(5) 500

(A) P → 2, Q → 3, R → 5, S → 1
(B) P → 3, Q → 1, R → 4, S → 2
(C) P → 4, Q → 5, R → 3, S → 1
(D) P → 4, Q → 2, R → 1, S → 5

Q. 17. A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field B_0 = 4 T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.

[Given: The acceleration due to gravity g = 10 m s^{-2} and e^{-1} = 0.4]



List-I	List-II
(P) At t = 0.2 s, the magnitude of the induced emf in Volt	(1) 0.07
(Q) At t = 0.2 s, the magnitude of the magnetic force in Newton	(2) 0.14
(R) At t = 0.2 s, the power dissipated as heat in Watt	(3) 1.20
(S) The magnitude of terminal velocity of the rod in m s^{-1}	(4) 0.12
	(5) 2.00

(A) P → 5, Q → 2, R → 3, S → 1
(B) P → 3, Q → 1, R → 4, S → 5
(C) P → 4, Q → 3, R → 1, S → 2
(D) P → 3, Q → 4, R → 2, S → 5

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
Section -I			
1	(A, C, D)	Collision	Center of Mass
2	(A, C, D)	Deviation and Dispersion from Prism	Geometric Optics
3	(A, B, C, D)	Kirchhoff's law and Combination of batteries	Current Electricity
Section -II			
4	(A)	Collision of Point mass with Rigid bodies	System of Particles and Rotational Motion
5	(B)	Polarisation and Dielectric	Electrostatics and Capacitors
6	(A)	Thermodynamic process	Kinematic Theory of Gases and Thermodynamics
7	(A)	Acceleration due to gravity	Gravitation
Section -III			
8	3	Photoelectric Effect	Dual Nature of Matter and Radiation
9	150	Lens and Mirror combination	Geometric Optics
10	1	Refraction through lens	Geometric Optics
11	121	Thermodynamic process	Kinematic Theory of Gases and Thermodynamics
12	40	Miscellaneous	Geometric Optics
13	10	Torsional Pendulum	Simple Harmonic Motion
Section -IV			
14	(A)	Radioactive Decay	Nuclear Physics
15	(C)	Black body Radiation	Thermal Properties of Matter
16	(B)	LCR Circuit	Electromagnetic Effect and Alternating Current
17	(D)	Motional Electromagnetic Effect	Electromagnetic Effect and Alternating Current

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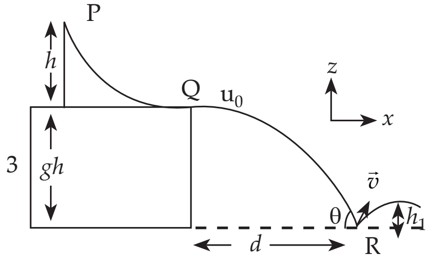
ANSWERS WITH EXPLANATIONS

1. Correct options are (A, C and D).



From energy conservation between P and Q

$$\downarrow PE = \uparrow KE$$

$$mgh = \frac{1}{2}mu_0^2$$

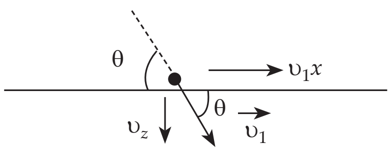
$\Rightarrow$ $$u_0 = \sqrt{2gh}$$

$$\bar{u}_0 = \sqrt{2gh}\,\hat{x}$$

(A) is correct.

At R

Before impact



Between P and R

$$\downarrow \text{PE} = \uparrow \text{KE}$$

$$mg(4h) = \frac{1}{2}mv_1^2$$

$$v_1 = \sqrt{8gh}$$

Also $v_{1x} = u_0$ [$\because$ Horizontal velocity doesn't change in projectile motion.]

Again $$d = u_0 t = u_0 \times \sqrt{\frac{2 \times 3h}{g}} = \sqrt{2gh}\,.\sqrt{\frac{6h}{g}}$$

$$= \sqrt{12}h = 2\sqrt{3}\,h$$

$$v_1^2 = v_{1x}^2 + v_{1z}^2$$

$$= u_0^2 + v_{1z}^2$$

$$8gh = 2gh + v_{1z}{}^2$$

$\Rightarrow$ $$v_{1z} = \sqrt{6gh}$$

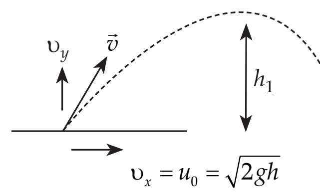
$$\tan\theta = \frac{v_{1z}}{v_{1x}} = \frac{\sqrt{6gh}}{\sqrt{2gh}} = \sqrt{3}$$

$\Rightarrow$ $$\tan\theta = \sqrt{3}$$

$\Rightarrow$ $$\theta = 60°$$

(C) is correct.

After impact



$$v_x = u_0 = \sqrt{2gh}$$

Velocity of separation = $e \times$ velocity of approach.

$$v_y = e \times v_{1y}$$

$$= \frac{1}{\sqrt{3}} \times \sqrt{6gh} = \sqrt{2gh}$$

$$h_1 = \frac{v_y^2}{2g}$$

$$= \frac{2gh}{2g} = h$$

$$\frac{d}{h} = \frac{2\sqrt{3}\,h}{h} = 2\sqrt{3}$$

(D) is correct.

$$\vec{\upsilon}_x = v_x\hat{x} + v_3\hat{z}$$

$$= \sqrt{2gh}\left(\hat{x} + \hat{z}\right)$$

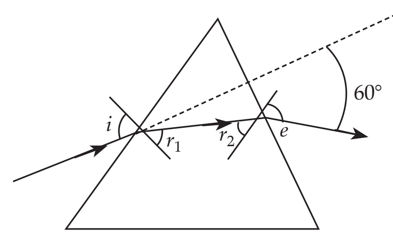
(B) is not correct.

2. Correct options are (A, C and D).

If a light is incident on a surface then for Brewster's angle light is polarised in direction perpendicular to plane of incidence.

$\therefore$ No reflection for blue light is seen.

$\Rightarrow$ It must be polarised in plane of incidence.



$\tan \underline{i} = \mu$

[for incidence at Brewster's angle]

$$\tan i = \sqrt{3}$$
$$i = 60°$$
$$\delta = i + e - A$$
$$60 = 60 + e - A$$
$$\Rightarrow \quad A = e$$
$$\frac{\sin i}{\sin r_1} = \sqrt{3}$$
$$\Rightarrow \quad \frac{\sin 60}{\sin r_1} = \sqrt{3} \quad \Rightarrow \quad r_1 = 30°$$
$$r_2 = A - r_1$$
$$\mu \sin (A - r_1) = \sin e$$
$$\Rightarrow \quad \sqrt{3} \sin (A - 30) = \sin A$$
$$r_3 [\sin A \cos 30° - \cos A \sin 30] = \sin A$$
$$\Rightarrow \quad \frac{\sqrt{3}}{2}(\sqrt{3} - \cot A) = 1$$
$$\Rightarrow \quad \cot A = \sqrt{3} - \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \cot A = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad A = 60°$$

So, $e = 60°$ (for blue)

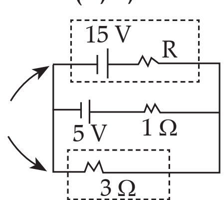
For red,

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin \frac{A}{2}}$$
$$= \frac{\sin\left(\frac{30 + 60}{2}\right)}{\sin \frac{60}{2}} = \frac{\sin 45}{\sin 30} = \frac{1/\sqrt{2}}{1/2}$$
$$\boxed{\mu_{red} = \sqrt{2}}$$

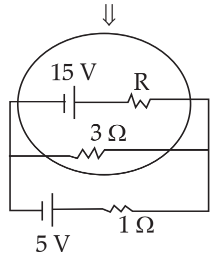
(A), (C), (D) are correct.

3. Correct options are (A, B, C and D).

At t < t_0

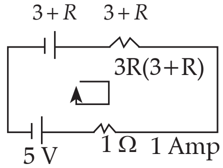


$\Downarrow$

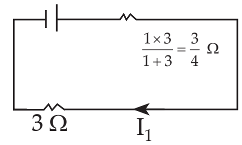


$\Downarrow$

$$\frac{15 \times 3}{3 + R} = \frac{45}{3 + R}$$



$$\frac{15 \times 1 + 5 \times R}{1 + R} = \frac{15}{2} \text{ V}$$



At t < t_0

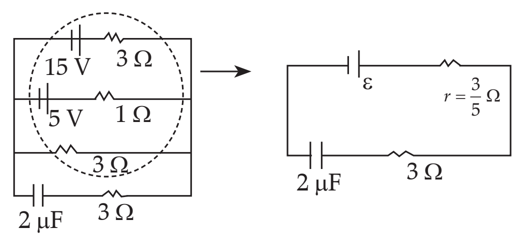
$$i = \frac{\left(\frac{45}{3 + R}\right) - 5}{1 = \frac{3R}{3 + R}}$$
$$1 = \frac{45 - 15 - 5R}{3 + 4R}$$
$$\Rightarrow \quad R = 3\,\Omega$$

(A) is correct.

$$I_1 = \frac{\frac{15}{2}}{3 + \frac{3}{4}} \quad \Rightarrow \quad I_1 = 2\text{ A}$$

(B) is correct.

At t > t_0



$$\frac{\varepsilon}{r} = \frac{15}{3} + \frac{5}{1} + \frac{0}{3}$$
$$\frac{1}{r} = \frac{1}{3} + \frac{1}{1} + \frac{1}{3} = \frac{5}{3}$$
$$\varepsilon = 10 \times \frac{3}{5} \quad \Rightarrow \quad 6\text{ V}$$

At $t \to \infty$

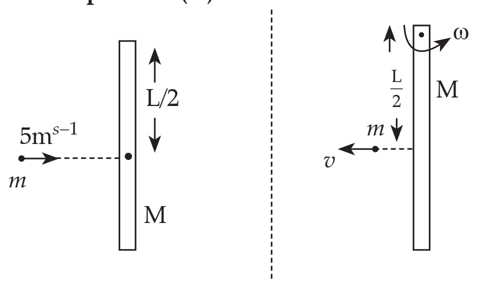
$$q = C\varepsilon$$
$$= 2 \times 10^{-6} \times 6 = 12\,\mu\text{C}$$

Time constant $\tau = RC$

$$= (r + 3)\, 2\,\mu\text{C}$$
$$= \left(\frac{3}{5} + 3\right) \times 12 = 7.2\,\mu\text{s}$$
$$i = i.\ e^{-t/\tau}$$
$$i_0 = \frac{\varepsilon}{r + 3} = \frac{6}{\left(3 + \frac{3}{5}\right)}$$
$$\Rightarrow \quad i = \frac{6}{\frac{18}{5}} \times e^{-7.2/1.2}$$
$$= \frac{5}{3} \times 0.36 = 0.6\text{ A}$$

All options are correct.

4. Correct option is (A).



About hinge → angular momentum is conserved.

$$L_{\text{before}} = L_{\text{after}}$$

$$mv_0\frac{L}{2} = I w - mv\frac{L}{2}$$

$$\Rightarrow 0.1 \times 5 \times \frac{0.2}{2} = \frac{1}{3} \times 1 \times 0.2^2\omega - 0.1 \times v \times \frac{0.2}{2}$$

$$\Rightarrow \frac{4\omega}{3} = 5 + v \qquad ...(1)$$

For Elastic collision

At point of impact,

velocity of separation = velocity of approach

$$\frac{\omega L}{2} - (-v) = 5$$

$$\Rightarrow \frac{\omega \times 0.2}{2} + v = 5$$

$$\frac{\omega}{10} = 5 - v \qquad ...(2)$$

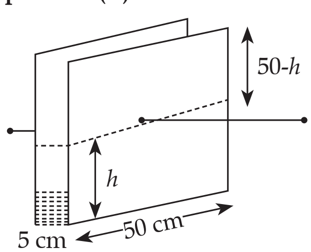
From (1) and (2),

$$\omega = \frac{300}{43} \text{ rad sec}^{-1}$$

$$\Rightarrow \omega = 6.976 \text{ rad sec}^{-1}$$

and $v = 4.3 \text{ m sec}^{-1}$

5. Correct option is (B).

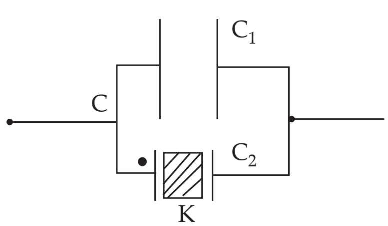


Volume of dielectric filled in 10 sec

$$= 250 \times 10 = 2500 \text{ cm}^3$$

$$h \times 50 \times 5 = 2500$$

$$\Rightarrow h = 10 \text{ cm}$$



$$C_1 = \frac{\varepsilon_0 A_1}{d}, C_2 = \frac{\varepsilon_0 A_2}{d}K$$

$$A_1 = 50 \times (50 - h)$$

$$A_2 = 50 \times h$$

$$C_{eq} = C_1 + C_2$$

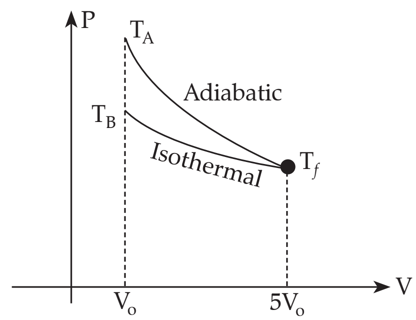
$$= \frac{\varepsilon_0 50(50-h)\text{cm}^2}{5 \text{ cm}} + \frac{3 \times \varepsilon_0 \times 50 \times h}{5 \text{ cm}}$$

$$= \varepsilon_0\left[\frac{50 \times 40}{5} + \frac{3 \times 50 \times 10}{5}\right] \times 10^{-2}$$

$$= 7\varepsilon_0 = 63 \times 10^{-12}$$

$$= 63 \text{ pF}$$

6. Correct option is (A).



$$T_B = T_f \qquad [\because \text{ process is isothermal}]$$

$$T_A V_0^{\gamma-1} = T_f (5V_0)^{\gamma-1} \qquad [\because \text{ process is adiabatic}]$$

$$T_A = T_f 5^{\gamma-1}$$

$$= T_B 5^{\gamma-1}$$

$$\Rightarrow \frac{T_A}{T_B} = 5^{\gamma-1}$$

7. Correct option is (A).

Above the surface of earth

$$g = \frac{g_0}{\left(1 + \frac{h}{R}\right)^2}$$

$$\frac{g_P}{g_Q} = \frac{g_0}{\left(1 + \frac{h_P}{R}\right)^2} \times \frac{\left(1 \times \frac{h_Q}{R}\right)^2}{g_0}$$

$$\Rightarrow \frac{36}{25} = \frac{\left(1 + \frac{h_Q}{R}\right)^2}{\left(1 + \frac{h_P}{R}\right)^2}$$

$$\Rightarrow \frac{6}{5} = \frac{\left(1 + \frac{h_Q}{R}\right)}{\left(1 + \frac{R/3}{R}\right)}$$

$$\Rightarrow \frac{h_Q}{R} + 1 = \frac{6}{5} \times \frac{4}{3}$$

$$= \frac{8}{5}$$

$$\boxed{h_Q = \frac{3}{5}R}$$

8. Correct answer is [3].

Energy of photon (E)

$$E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 13.6Z^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= \frac{13.6 \times 7 \times z^2}{16 \times 9}$$

During photo electric effect

$$KE_{max} = E - \phi,\ \phi = \frac{hc}{\lambda_0} = \frac{1240}{310} = 4\text{eV}$$

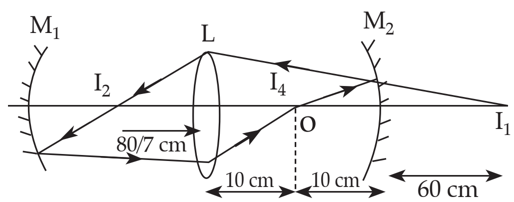
$$\Rightarrow \quad 1.95 = \frac{13.6 \times 7 \times Z^2}{16 \times 9} - 4$$

$$13.6 \times 7 \times Z^2 = 9 \times 95.2$$

$$Z^2 = 9$$

$$\Rightarrow \quad Z = 3$$

9. Correct answer is [150].



For Image I_1

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{-10} = \frac{1}{-12}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{12}$$

$$\Rightarrow \quad v = 60 \text{ cm}$$

For Image I_2,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-80} = \frac{1}{10}$$

$$\Rightarrow \quad v = \frac{80}{7} \text{ cm}$$

For Image I_3

Image I_3 must be such that I_4 is formed at O and O is at focus of L, so I_3 must be at infinity.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\infty} + \frac{1}{u} = \frac{1}{-10}$$

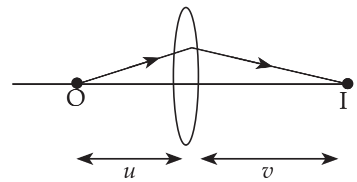
$$\Rightarrow \quad u = -10 \text{ cm}$$

Separation between L and M,

$$10 + \frac{80}{7} = \frac{150}{7}$$

$$\Rightarrow \quad \boxed{n = 150}$$

10. Correct answer is [1].



$$u = -10 \text{ cm}$$

$$\Delta u = 0.1 \text{ cm}$$

$$v = +20 \text{ cm}$$

$$\Delta v = 0.2 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{+20} - \frac{1}{-10} = \frac{1}{f}$$

$$f = \frac{20}{3} \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{dv}{v^2} - \frac{du}{u^2} = \frac{df}{f^2}$$

Error

$$\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f}\left(\frac{1}{f}\right)$$

$$\frac{0.2}{20^2} + \frac{0.1}{10^2} = \frac{\Delta f}{f} \times \frac{3}{20}$$

$$\frac{\Delta f}{f} = \frac{1}{100}$$

$$\Rightarrow \quad \frac{\Delta f}{f} \times 100 = n$$

$$\Rightarrow \quad \boxed{n = 1}$$

11. Correct answer is [121].

$$\Delta U = nLV\,\Delta T$$

$$\Delta W = n\,R\,\Delta T \quad \text{[at constant pressure]}$$

$$\Rightarrow \quad 66 = n\,R\,\Delta T$$

Also $(n_1 + n_2)C_V = n_1 Cv_1 + n_2 Cv_2$

$$= 2 \times \frac{R}{\gamma_1 - 1} + 1 \times \frac{R}{\gamma_2 - 1}$$

$$(2 + 1)C_V = 2 \times \frac{3}{2}R + 1 \times \frac{5R}{2}$$

$$\Rightarrow \quad C_V = \frac{11}{6}R$$

$$\Delta U = \frac{11}{6} nR\Delta T$$

$$\Rightarrow \quad \Delta U = \frac{11}{6} \times 60 = 121 \text{ J}$$

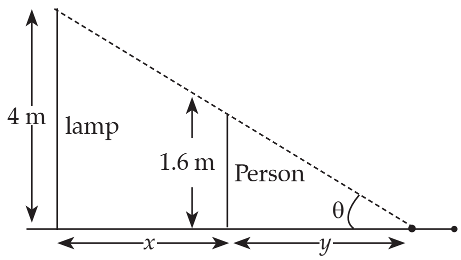
12. Correct answer is [40].

y = distance of tip of shadow from person.

$$\tan\theta = \frac{4}{x+y} = \frac{1.6}{y}$$

$$\Rightarrow \quad 2x = 3y$$

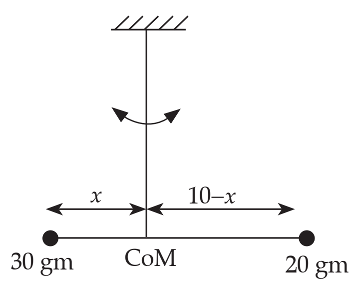
$$2\frac{dx}{dt} = 3\frac{dy}{dt}$$



$$\Rightarrow \quad 2 \times 60 = 3 \times \frac{dy}{dt}$$

$$\Rightarrow \quad \frac{dy}{dt} = 40 \text{ cm s}^{-1}$$

13. Correct answer is [10].



Moment of inertia about CoM

$$I = 30 \times x^2 + 20 \times (10 - x)^2$$
$$= (30 \times 4^2 + 20 \times 6^2) \times 10^{-3} \times (10^{-2})^2$$
$$= 1.2 \times 10^{-4} \text{ kg m}^2$$
$$\tau = I\alpha$$
$$-K\theta = I\alpha$$
$$\Rightarrow \quad \alpha = \frac{-K}{I}\theta = -\omega^2\theta$$
$$\Rightarrow \quad \omega^2 = \frac{1.2\times10^{-8}}{1.2\times10^{-4}}$$
$$\omega^2 = 10^{-4}$$
$$\omega = 10^{-2} \text{ rad/s}$$
$$\omega = n \times 10^{-3}$$
$$\Rightarrow \quad \boxed{n = 10}$$

14. Correct answer is [A].

$${}^{A}_{Z}X \rightarrow {}^{A-4n_1}_{Z-4n_1+n_2}Y$$

η_1 = no. of α decay

η_2 = no. of β decay

(P) ${}^{238}_{92}U \longrightarrow {}^{234}_{91}Pa$

$$238 - 4n_1 = 234 \Rightarrow n_1 = 1$$
$$92 - 2n_1 + n_2 = 91$$
$$\Rightarrow \quad 2n_1 - n_2 = 1 \quad \Rightarrow n_2 = 1$$

$\Rightarrow$ 1 α and 1 β^- decay

$\boxed{P \rightarrow 4}$

(Q) ${}^{214}_{82}Pb \longrightarrow {}^{210}_{82}Pb$

$$214 - 4n_1 = 210$$
$$\Rightarrow \quad n_1 = 1$$
$$82 - 2n_1 + n_2 = 82$$
$$2n_1 = n_2$$
$$n_2 = 2$$

1 α and 2 β^- decay

$\boxed{Q \rightarrow 3}$

(R) ${}^{210}_{81}Tl \longrightarrow {}^{206}_{82}Pb$

$$210 - 4n_1 = 206$$
$$\Rightarrow \quad n_1 = 1$$
$$81 - 2n_1 + n_2 = 82$$
$$\Rightarrow \quad n_2 = 3$$

1 α and 3β^- decay

$\boxed{R \rightarrow 2}$

(S) ${}^{228}_{91}Pa \longrightarrow {}^{224}_{88}Pa$

$$228 - 4n_1 = 224$$
$$\Rightarrow \quad n_1 = 1$$
$$91 - 2n_1 + n_2 = 88$$
$$n_2 = -1$$

$\Rightarrow$ β^+ decay occurs

1 α and 1 β^+ decay

$\boxed{S \rightarrow 1}$

P → 4, Q → 3, R → 2, S → 1.

15. Correct option is (C).

$$\lambda_0 T = b$$

(P) $\lambda_0(2000) = 2.9 \times 10^{-3}$

$$\lambda_0 = 1450 \text{ n m}$$

(Q) $\lambda_0\,(3000) = 2.9 \times 10^{-3}$

$$\lambda_0 = \frac{2900}{3} \text{n m}$$

(R) $\lambda_0 = 580$ n m

(S) $\lambda_0 = 290$ n m

$$\phi = 4\,e\,V$$

$$\Rightarrow \quad \lambda_T = \frac{1240}{4} = 310 \text{ n m}$$

To emit photoelectron

$$\lambda_0 < \lambda_T$$

$\Rightarrow$ $\boxed{S \rightarrow 1}$

$\lambda_0 = 580$ lie in visible range

$\Rightarrow \boxed{R \to 2}$

Power $\alpha\ T^4$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\frac{1}{16} = \left(\frac{T_1}{6000}\right)^4$$

$$\frac{T_1}{6000} = \frac{1}{2}$$

$\Rightarrow \quad T_1 = 3000$ K

$\boxed{Q \to 4}$

λ is maximum for P.

$$\sin\theta = \frac{\lambda}{d}$$

2θ = width of central maximum

$\Rightarrow$ width $\propto \lambda$.

$\Rightarrow$ max^m width of (P)

$\boxed{P \to 3}$

P $\to$ 3, Q $\to$ 4, R $\to$ 2, S $\to$ 1.

16. Correct option is (B).

$\omega_0 = 10^5$ rad s^{-1}, $L = 50 \times 10^{-3}$ H

$\Rightarrow \quad X_L = 5000\ \Omega = X_C$ (at resonance)

$$\frac{1}{\sqrt{LC}} = \omega_0$$

$$\Rightarrow \quad L_C = \frac{1}{\omega_0^2}$$

$\Rightarrow \quad C = 2 \times 10^{-9}\ F$

$$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_0 = \frac{\varepsilon_0}{R} \quad \text{...(1)}$$

$$0.05 I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \text{...(2)}$$

at $\quad \omega = 8 \times 10^4$ rad s^{-1}

$X_L = 8 \times 10^4 \times 50 \times 10^{-3} = 4000\ \Omega$

$$X_C = \frac{1}{\omega_C} = 6250\ \Omega$$

From (1) and (2), we get

$$\sqrt{R^2 + (X_L - X_C)^2} = \frac{R}{0.05}$$

$R^2 + (6250 - 4000)^2 = 400R^2$

$$\Rightarrow R^2 \simeq \frac{(2250)^2}{400}$$

$$R = \frac{2250}{20} = 112.5\ \Omega$$

$$I_0 = \frac{\varepsilon_0}{R} = \frac{45}{112.5}$$

$= 400$ m A $\qquad$ (P $\to$ 3)

$$\to \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\Rightarrow \quad Q = \frac{2}{2250} \times \sqrt{\frac{50 \times 10^{-3}}{2 \times 10^{-7}}} = \frac{10^4}{225}$$

$= 44.4 \qquad$ (Q $\to$ 1)

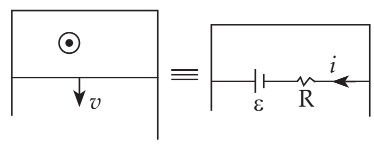
$$Q = \frac{\omega_0}{BW}$$

$$\Rightarrow \quad BW = \frac{\omega_0}{Q} = \frac{10^5}{\frac{10^4}{225}} = 2250 \qquad (R \to 4)$$

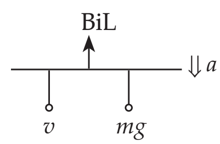
$$P = \frac{\varepsilon_0^2}{R} \qquad \text{(peak power)}$$

$$= \frac{45^2}{112.5} = 18\ W \qquad (S \to 2)$$

17. Correct option is (D).



$$i = \frac{\varepsilon}{R} = \frac{Blv}{R}$$



$$a = \frac{\Sigma F}{m}$$

$$= \frac{mg - Bil}{m}$$

$$= g - \frac{B^2 l^2}{mR} v = 10 - 5v$$

$$\frac{dv}{dt} = 10 - 5v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{10 - 5v} = \int_0^t dt$$

$\Rightarrow \quad v = 2(1 - e^{-5t})$

at $t = 0.2$ sec $\quad v = 1.2$ m s^{-1}

$\varepsilon = Bl\,v$

$= 1.2V$

$$F = \frac{B^2 l^2 v}{R} = 0.12 \text{ N}$$

$$P = \frac{B^2 l^2 v^2}{R} = 0.144 \text{ W}$$

Terminal velocity, $\quad g = \dfrac{B^2 l^2 v_0}{mR}$

$\boxed{v_0 = 2\,ms^{-1}}$

P $\to$ 3, Q $\to$ 4, R $\to$ 2, S $\to$ 5.

JEE Advanced (2023)

PAPER 2

Physics

General Instructions:

SECTION 1 (Maximum Marks: 12)

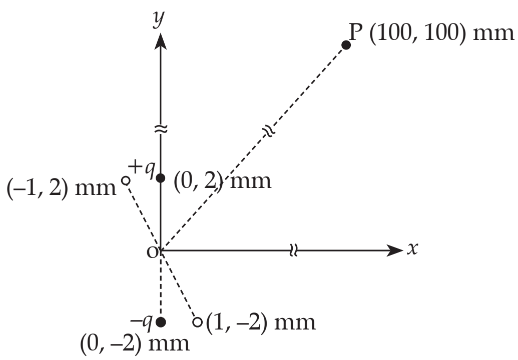
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

Q. 1. An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at (0, 2) mm and (0, −2) mm, respectively, as shown in the figure. The electric potential at point P (100,100) mm due to the dipole is V_0. The charges $+q$ and $-q$ are then moved to the points (−1, 2) mm and (1, −2) mm, respectively. What is the value of electric potential at P due to the new dipole?



(A) $V_0/4$ (B) $V_0/2$

(C) $V_0/\sqrt{2}$ (D) $3V_0/4$

Q. 2. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha}h^{\beta}G^{\gamma}$. Which of the following is the correct option?

(A) $\alpha = 7, \beta = -1, \gamma = -2$

(B) $\alpha = -7, \beta = -1, \gamma = -2$

(C) $\alpha = 7, \beta = -1, \gamma = 2$

(D) $\alpha = -7, \beta = 1, \gamma = -2$

Q. 3. A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle?

(A) $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$

(B) $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$

(C) $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$

(D) $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$

Q. 4. An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas is n. The internal energy of one mole of the gas is U_n and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option?

(A) $v_3 < v_6$ and $U_3 > U_6$

(B) $v_5 > v_3$ and $U_3 > U_5$

(C) $v_5 > v_7$ and $U_5 < U_7$

(D) $v_6 < v_7$ and $U_6 < U_7$

General Instructions:

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

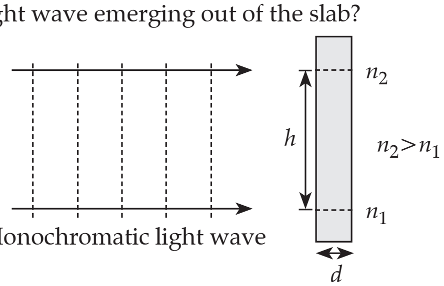
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks

Q. 5. A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is(are) true about the light wave emerging out of the slab?



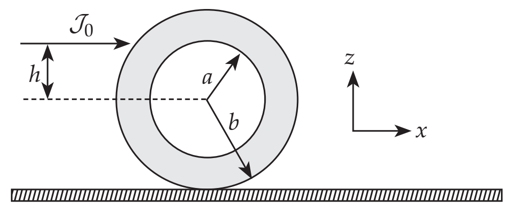
(A) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2-n_1^2)d}{2h}\right]$.

(B) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2-n_1)d}{h}\right]$.

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

Q. 6. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $\mathcal{J}_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



(A) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$.

(B) For $\mu \neq 0$ and $a \to b$, $h_m = b$.

(C) For $h = h_m$, the initial angular velocity does **not** depend on the inner radius a.

(D) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

Q. 7. The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x}+\hat{y})\sin\left[2\pi\left(5\times10^{14}t-\frac{10^7}{3}z\right)\right]$ V m^{-1}. Which of the following option(s) is(are) correct?

[Given: The speed of light in vacuum,
$c = 3 \times 10^8$ m s^{-1}]

(A) $B_x = -2\times10^{-7}\sin\left[2\pi\left(5\times10^{14}t-\frac{10^7}{3}z\right)\right]$ Wb m^{-2}.

(B) $B_y = 2\times10^{-7}\left[2\pi\left(5\times10^{14}t-\frac{10^7}{3}z\right)\right]$ Wb m^{-2}.

(C) The wave is polarized in the xy-plane with polarization angle 30° with respect to the x-axis.

(D) The refractive index of the medium is 2.

General Instructions:

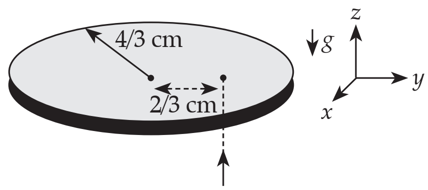
SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases.

Q. 8. A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy-plane. The coin is tossed vertically up ($+z$ direction) by applying an impulse of $\sqrt{\frac{\pi}{2}}\times10^{-2}$ N s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the $+z$ direction. By the time the coin reaches back to its initial position, it completes n rotations. The value of n is ______ .

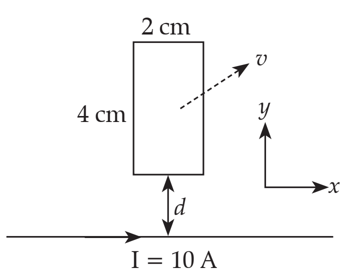
[Given: The acceleration due to gravity g = 10 m s^{-2}]



Q. 9. A rectangular conducting loop of length 4 cm and width 2 cm is in the xy-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x}+\frac{1}{2}\hat{y}$ with a constant speed v. The wire is carrying a steady current I = 10 A in the positive x-direction. A current of 10 μA flows through the loop when it is at a distance d = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of v is _______m s^{-1}.

[Given: The permeability of free space

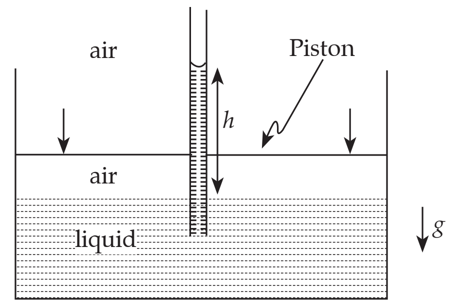
$\mu_0 = 4\pi\times10^{-7}$ N A^{-2}]



Q. 10. A string of length 1 m and mass 2×10^{-5} kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is ______ Newton.

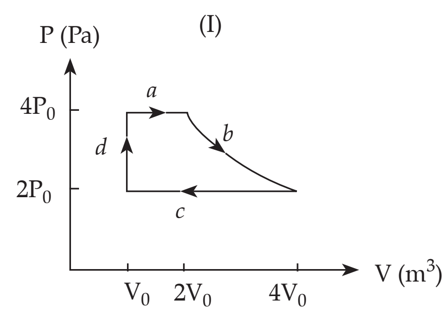
Q. 11. An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is _________ .

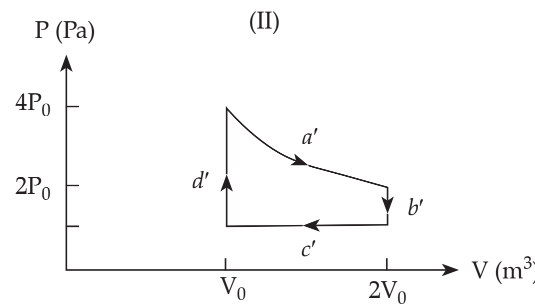
[Given: Surface tension of the liquid is 0.075 N m^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero]



Q. 12. In a radioactive decay process, the activity is defined as $A=-\frac{dN}{dt}$, where N(t) is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time t = 0. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3rd and 7th half-lives, respectively, the ratio A_1/A_2 is ________ .

Q. 13. One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the P-V diagrams below. In cycle I, processes a, b, c and d are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes a', b', c' and d' are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I/W_{II} is ________ .





General Instructions:

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

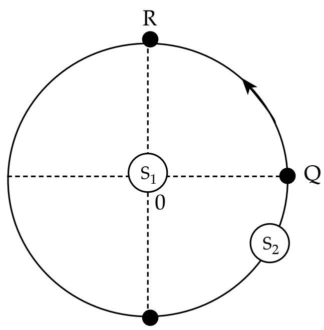
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

PARAGRAPH I

S_1 and S_2 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}$ m s^{-1} on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP.

[Given: The speed of sound in air is 324 m s^{-1}]



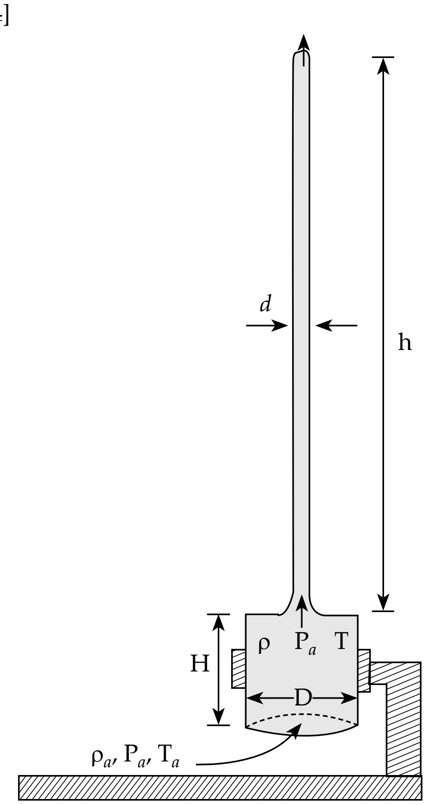
Q. 14. When only S_2 is emitting sound and it is at Q, the frequency of sound measured by the detector in Hz is ________ .

Q. 15. Consider both sources emitting sound. When S_2 is at R and S_1 approaches the detector with a speed 4ms^{-1}, the beat frequency measured by the detector is ________ Hz.

PARAGRAPH II

A cylindrical furnace has height (h) and diameter (d) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_α and its temperature becomes T = 360 K. The hot air with density ρ rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density ρ_α = 1.2 kg m^{-3}, pressure P_α and temperature T_α = 300 K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

[Given: The acceleration due to gravity g = 10 m s^{-2} and π = 3.14]



Q. 16. Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is _______ gs^{-1}.

Q. 17. When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ________ N m^{-2}.

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
Section -I			
1	(B)	Electric Potential due to a dipole	Electrostatic Potential & Capacitance
2	(A)	Dimensional Analysis	Units & Measurement
3	(A)	General Kinematics	Motion in a plane
4	(C)	Degree of freedom, Internal Energy	Kinetic theory of gasses
Section -II			
5	(B and D)	Optical & geometrical path, Wave front	Wave Optics
6	(A, B, C and D)	Impulse, Rolling motion	System of particles & Rotational motion
7	(A and D)	EM waves—Mathematical representation	Electromagnetic waves
Section -III			
8	30	Impulse	System of particles & Rotational Motion
9	4	Motion in Electromagnetic Field	Electromagnetic Induction
10	5	Standing Waves	Waves
11	25	Capillary tube, Thermodynamic processes	Mechanical properties of fluids
12	16	Radioactive decay	Nuclei
13	2	Workdone in cyclic process	Thermodynamics
Section -IV			
14	648	Doppler's Effect	Waves
15	8.2	Doppler's Effect	Waves
16	60.81	Kinetic theory of gasses, Continuity equation	Mechanical properties of fluids
17	30	Kinetic theory of gasses, Continuity equation	Mechanical properties of fluids

JEE Advanced (2023)

PAPER 2

ANSWERS WITH EXPLANATIONS

Physics

1. Correct option is (B).

When the dipole is at (0. 2) mm and (0, –2) mm,

$$\vec{P_1} = q \times 4 \times 10^{-3}\ \hat{j}$$

$V_{\text{at } (100,\,100)\text{ mm}}$,

$$V_0 = \frac{k\vec{P_i}\cdot\vec{r_1}}{|\vec{r_1}|^3}$$

$$\Rightarrow \quad V_0 = \frac{kq \times [4\times10^{-3}\hat{j}]\cdot[0.1\hat{i}+0.1\hat{j}]}{[\sqrt{(0.1)^2+(0.1)^2}]^3}$$

$$\Rightarrow \quad V_0 = \frac{kq[0.4\times10^{-3}]}{[\sqrt{(0.1)^2+(0.1)^2}]^3}$$

Now $+q$ and $-q$ are moved to (–1, 2) mm and (1, – 2) mm

So $\quad \vec{P} = q(-2\hat{i}+4\hat{j})\times10^{-3}$

$\quad \vec{r_1} = 0.1\hat{i}+0.1\hat{j}$

So $\quad V_0 = \dfrac{k\vec{P}\cdot\vec{r_1}}{|\vec{r_1}|^3}$

$$= \frac{k\times q[-2\hat{i}+4\hat{j}]\times10^{-3}\cdot[0.1\hat{i}+0.1\hat{j}]}{[\sqrt{(0.1)^2+(0.1)^2}]^3}$$

$$\Rightarrow \quad V = \frac{kq(-0.2+0.4)\times10^{-3}}{[\sqrt{(0.1)^2+(0.1)^2}]^3}$$

$$= \frac{kq(0.2)\times10^{-3}}{[\sqrt{(0.1)^2+(0.1)^2}]^3}$$

$$\Rightarrow \quad V = \frac{V_0}{2}$$

2. Correct option is (A).

$$Y = C^{\alpha}h^{\beta}G^{\gamma}$$

$\Rightarrow [M^1L^{-1}T^{-2}] = [LT^{-1}]^{\alpha}\ [M^1L^2T^{-1}]^{\beta}\ [M^{-1}L^3T^{-2}]^{\gamma}$

$\Rightarrow [M^1L^{-1}T^{-2}] = M^{\beta-\gamma}\,L^{\alpha+2\beta+3\gamma}\,T^{-\alpha-\beta-2\gamma}$

Comparing both the sides

$$\beta - \gamma = 1 \quad \text{...(i)}$$

$$\alpha + 2\beta + 3\gamma = -1 \quad \text{...(ii)}$$

$$-\alpha - \beta - 2\gamma = -2 \quad \text{...(iii)}$$

Adding eqns (ii) and (iii),

$$\beta + \gamma = -3 \quad \text{...(iv)}$$

Again adding eqn (i) and (iv), $\boxed{\beta = -1}$

Putting the value of β in eqn (i), $\boxed{\gamma = -2}$

Putting all these values in eqn (iii), $\boxed{\alpha = 7}$

3. Correct option is (A).

$$\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = \alpha y \quad \text{and} \quad v_y = \frac{dy}{dt} = 2\alpha x$$

Now $\quad \vec{a} = \dfrac{d\vec{v}}{dt} = \alpha\dfrac{dy}{dt}\hat{x} + 2\alpha\dfrac{dx}{dt}\hat{y}$

$$\vec{a} = \alpha(2\alpha x)\hat{x} + 2\alpha(\alpha y)\hat{y}$$

$$\Rightarrow \quad \vec{a} = (2\alpha^2 x)\hat{x} + (2\alpha^2 y)\hat{y}$$

$$\Rightarrow \quad \vec{a} = 2\alpha^2(x\hat{x} + y\hat{y})$$

So $\quad \vec{F} = m\vec{a} = 2m\alpha^2(x\hat{x} + y\hat{y})$

4. Correct option is (C).

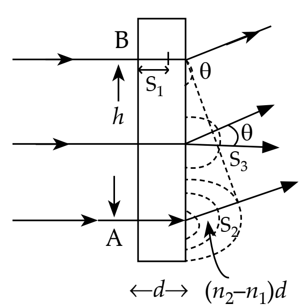
$$v = \sqrt{\frac{\gamma RT}{M}}$$

where $\quad \gamma = 1 + \dfrac{2}{n}$

and $\quad u = \dfrac{nRT}{2}$

If n is more, γ will be less, so velocity of sound will also be less, but the internal energy (U_n) will be more.

5. Correct option is (B and D).



At A, let the wave front reaches at the end of the slab in time t.

So $$t = \frac{\text{distance}}{\text{speed}} = \frac{d}{\frac{c}{n_1}} = \frac{n_1 d}{c}$$

In time t, the wave front at B, let travels s_1.

$$s_1 = \frac{c}{n_2} \times \frac{n_1 d}{c} = \frac{dn_1}{n_2}$$

When wave front of B, reaches at end of the slab, further t_1 time takes place.

So $$t_1 = \left(\frac{d - s_1}{c}\right) n_2 = \left(d - \frac{dn_1}{n_2}\right)\frac{n_2}{c} = \frac{d(n_2 - n_1)}{c}$$

In time t_1, wave front at A will travel s_2,

$$s_2 = ct_1 = d(n_2 - n_1)$$

and $$\tan\theta = \frac{(n_2 - n_1)d}{h}$$

6. Correct options are [A, B, C and D].

Angular impulse about center of mass = ΔL

$$J_0 h = I_{cm} w$$

$$\Rightarrow \quad w = \frac{J_0 h}{I_{cm}}$$

Linear impulse = ΔP

$$J_0 = \Delta P$$

$$\Rightarrow \quad J_0 = MV$$

$$\Rightarrow \quad V = \frac{J_0}{M}$$

As the disc is pure rolling,

$$v = wb$$

$$\Rightarrow \quad w = \frac{v}{b}$$

$$\Rightarrow \quad w = \frac{J_0}{Mb}$$

So w is independent of radius (a)

Therefore, the option (C) is the answer.

If $a \to b$, then the disc is a ring of radius b.

So $$I_{cm} = Mb^2$$

$$\Rightarrow \quad w = \frac{J_0 h}{Mb^2}$$

$$\Rightarrow \quad \frac{J_0}{Mb} = \frac{J_0 h}{Mb^2}$$

$$\Rightarrow \quad h = b \qquad \because \quad h_m = b$$

Therefore, the option (B) is the answer.

If $h = 0$, $w = \frac{J_0 h}{Mb^2} = 0$

but $v = \frac{J_0}{M}$, wheel always slide without rolling

Therefore, the option (D) is the answer.

If $a \to 0$, then disc is a solid disc of radius b.

$$\therefore \quad I_{cm} = \frac{Mb^2}{2}$$

$$\Rightarrow \quad w = \frac{J_0 h \times 2}{Mb^2}$$

$$\Rightarrow \quad \frac{J_0}{Mb} = \frac{J_0 h \times 2}{Mb^2};\ h = \frac{b}{2}$$

So $$h_m = \frac{b}{2}$$

Therefore, the option (A) is the answer.

7. Correct options are [A and D].

Speed of light in the medium

$$v = \frac{5 \times 10^{14}}{\frac{10^7}{3}} = \frac{5 \times 10^{14} \times 3}{10^7} = 1.5 \times 10^8$$

$$\Rightarrow \quad v = \frac{c}{2}$$

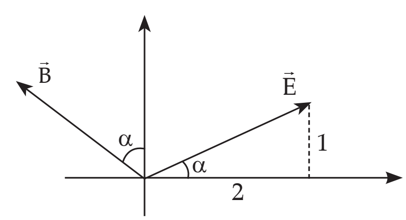
$n = 2$ (refractive index of the medium)

Therefore, the option (D) is the answer.

Now magnitude of

$$B = \frac{E}{v}$$

$$\Rightarrow \quad B = \frac{30\sqrt{5}}{1.5 \times 10^8}$$



$\tan\alpha = \frac{1}{2}$

$\sin\alpha = \frac{1}{\sqrt{5}}$

The direction of $\vec{B}$ is along the direction of $\vec{v} \times \vec{E}$

i.e. $\hat{v} \times \hat{E} = \hat{B}$

Now $B_x = -B\sin\alpha$

$$\Rightarrow \quad B_x = -\frac{30\sqrt{5}}{1.5 \times 10^8} \times \frac{1}{\sqrt{5}} = -2 \times 10^7$$

Therefore, option (A) is the answer.

8. Correct answer is [30].

Impulse = $\Delta P = MV$

$$\Rightarrow \quad \sqrt{\frac{\pi}{2}} \times 10^{-2} = \frac{5}{1000} \times V$$

$$\Rightarrow \quad V = \sqrt{2\pi} \text{ m/s (velocity of CoM)}$$

Now time of flight

$$t = \frac{2\sqrt{2\pi}}{10} = \frac{\sqrt{2\pi}}{5}$$

Angular impulse = impulse × $\perp^r$ distance

$$= I_{\text{diameter}} \times \omega$$

$$= \frac{mr^2}{4} \times \omega$$

$$\Rightarrow \quad \sqrt{\frac{\pi}{2}} \times 10^{-2} \times \frac{2}{3} \times 10^{-2}$$

$$= \frac{5 \times 10^{-3} \times \frac{16}{9} \times 10^{-4} \times \omega}{4}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{\pi}{2}} \times 300$$

Now $\theta = \omega t + \frac{1}{2}\alpha t^2$

$$\Rightarrow \quad \theta = \omega t = \sqrt{\frac{\pi}{2}} \times 300 \times \frac{\sqrt{2\pi}}{5} = 60\pi$$

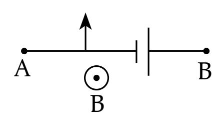
Hence number of revolutions,

$$\Rightarrow \quad n = \frac{60\pi}{2\pi} = 30.$$

9. Correct answer is [4].

Induced emf in AB $(e_1) = (\vec{v} \times \vec{B}) \cdot \vec{l}$

$$V_y = V \times \frac{1}{2}$$

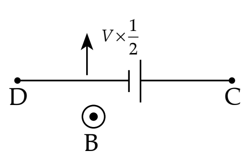


So $B = \frac{\mu_0 I}{2\pi r} = \frac{1}{2} \times 10^{-4}$ T

emf in $AB = e_1 = B \times \left(\frac{1}{2} \times \text{V}\right) \times (2 \times 10^{-2})$

$$= \frac{\text{V}}{2} \times 10^{-6} \text{ volt}$$

Induced emf in CD $(e_2) = (\vec{V} \times \vec{B}) \cdot \vec{l}$



emf in $CD = l_2$

$$= B \times \frac{1}{2} \times e \times 2 \times 10^{-2}$$

$$= \left(\frac{\mu_o I}{2\pi(8 \times 10^{-2})}\right) \times \left(\frac{1}{2}\right) \times 2 \times 10^{-2} \times (\text{V})$$

$$= \text{V} \times \frac{1}{4} \times 10^{-6}$$

The two sides perpendicular to the wire would contribute net zero emf.

So net emf in the loop $= e_1 - e_2$

$$= \left(\frac{V}{2} - \frac{V}{4}\right) \times 10^{-6} \text{ volt}$$

$$= \frac{V}{4} \times 10^{-6} \text{ volt}$$

Resistance of the loop = 0.1 Ω

So current in the loop,

$$I = \frac{10^{-6}}{4 \times 0.1} \times v$$

$$\Rightarrow \quad \frac{10v}{4} = 10$$

$$\Rightarrow \quad v = 4 \text{ m s}^{-1}$$

10. Correct answer is [5].

$$\frac{nv}{2l} = 750$$

$$(n+1)\frac{v}{2l} = 1000$$

$$\Rightarrow \quad \frac{v}{2l} = 1000 - 750 = 250$$

$$\Rightarrow \quad v = 2 \times (1) \times 250 = 500 \text{ ms}^{-1}$$

and $\mu = \frac{m}{l} = 2 \times 10^{-5}$

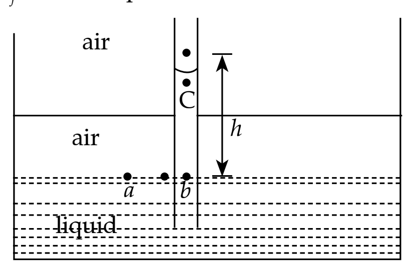
So $\sqrt{\frac{T}{\mu}} = 500$

$$\Rightarrow \quad T = (500)^2 \times \mu$$

$$= 500 \times 500 \times 2 \times 10^{-5} = 5 \text{ N}$$

11. Correct answer is [25].

Let P_f be the air pressure.



$$P_0 V_0 = P_f V_f$$

$$\Rightarrow \quad P_0 V_0 = P_f \left(\frac{100}{101}\right) V_0$$

$$\Rightarrow \quad P_f = P_0 \times \frac{100}{101} \text{ V}_0 = 101 \times 10^3 \text{ Pa}$$

$[\because P_0 = 10^5 \text{ N m}^{-2}]$

Now consider 4 points as shown in the diagram

$$P_d - P_c = \frac{2T}{R}$$

$$\Rightarrow \quad P_c = P_d - \frac{2T}{R}$$

$$\Rightarrow \quad P_c = P_0 - \frac{2T}{R} \qquad [\because \quad P_d = P_0]$$

Now $P_a = P_b$ and also $P_a = P_f$

$$\Rightarrow \quad P_f = fgh + P_c$$

$$\Rightarrow\ 101 \times 10^3 = (10^3 \times 10 \times h) + \left(10^5 - \frac{2 \times 0.075}{0.1 \times 10^{-3}}\right)$$

$$\Rightarrow\ \boxed{h = 25\,\text{cm}}$$

12. Correct answer is [16].

$$n^{\text{th}} \text{ half life} = \frac{A_0}{2^n}$$

So $$\frac{A_1}{A_2} = \frac{\frac{A_0}{2^3}}{\frac{A_0}{2^7}} = 2^4 = 16$$

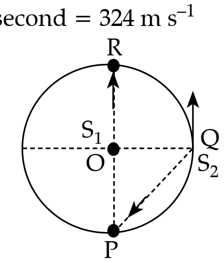
13. Correct answer is [2].

$$W_I = W_a + W_b + W_c + W_d$$
$$= 4P_0(2V_0 - V_0) + 4P_0(2V_0)\ln\frac{4V_0}{2V_0} + 2P_0(V_0 - 4V_0) + 0$$
$$= 8P_0V_0\ln_2 - 2P_0V_0$$
$$W_{II} = W_a' + W_b' + W_c' + W_d'$$
$$= 4P_0V_0\,\ln\left(\frac{2V_0}{V_0}\right) + 0\ P_0(V_0 - 2V_0) + 0$$
$$= 4P_0V_0\ln_2 - P_0V_0$$
$$\frac{W_I}{W_{II}} = \frac{2}{1} = 2$$

14. Correct answer is [648].

$$f_0 = 656 \text{ Hz}$$

Velocity of second $= 324 \text{ m s}^{-1}$



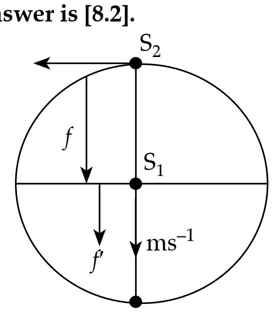
(Detector)

Velocity of the source away from detector,

$$v_s = 4\sqrt{2}\cos 45° = 4 \text{ m s}^{-1}$$

$$\therefore\ f = \left(\frac{v}{v + v_s}\right)f_0$$
$$= \left(\frac{324}{324 + 4}\right) \times 656 = 648 \text{ Hz}$$

15. Correct answer is [8.2].



(Detector)

$$f' = \left(\frac{v}{v - v_s}\right) \times f_0$$
$$= \left(\frac{324}{324 - 4}\right) \times 656 = 664.2$$
$$f_{\text{beats}} = f' - f_0 = 664.2 - 656 = 8.2 \text{ Hz}$$

16. Correct answer is [60.81].

$$PM_0 = \rho RT$$
$$\Rightarrow\ \frac{PM_0}{R} = \rho T$$

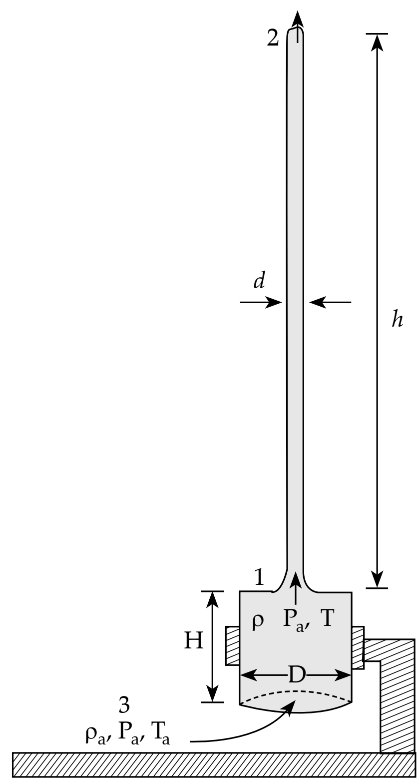
Now $$\rho_a T_a = \rho T$$
$$\Rightarrow\ \rho = 1.2 \times \frac{300}{360}$$
$$\Rightarrow\ \rho = 1 \text{ kg m}^{-3}$$

Applying be Moulli equation between diagram points 1 & 2,

$$P_a + \frac{1}{2}\rho v_1^2 = P + \rho gh + \frac{1}{2}\rho v_2^2 \quad \text{...(i)}$$

Applying be Moulli equation between diagram points 2 & 3,

$$P = P_a + \rho_a(H + h)g \quad \text{...(ii)}$$



$$P_a = P_a - \rho_a(H + h)g + \rho gh + \frac{1}{2}\rho v_2^2$$
$$\Rightarrow\ \frac{1}{2}\rho v_2^2 = \rho_a(H + h)g - \rho gh$$
$$\Rightarrow\ \frac{1}{2} \times 1 \times v_2^2 = 1.2 \times (10)(10) - 1 \times 10 \times 9$$

$$\Rightarrow \quad \frac{v_2^2}{v} = 120 - 90 = 30$$

$$\Rightarrow \quad v_2 = \sqrt{60}\ \text{ms}^{-1}$$

Now $$\frac{dm}{dt} = \rho A_2 v_2 = 1 \times 3.14 \times \left(\frac{0.1}{2}\right)^2 \times \sqrt{60}$$

$$= 60.81\ \text{gs}^{-1}$$

17. Correct answer is [30].

$$P_{top} = P_a - \rho_a g(H + h)$$

$$P_{bottom} = P_a - \rho g h$$

$$\Rightarrow \quad P_{bottom} - P_{top} = \rho_a g(H + h) - \rho g h$$

$$= 30\ \text{Nm}^{-2}$$