JEE Advanced
(2023)
PAPER 1

Mathematics

General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

Q. 1. Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is (are) true?

(A) There are infinitely many functions from S to T.

(B) There are infinitely many strictly increasing functions from S to T.

(C) The number of continuous functions from S to T is at most 120.

(D) Every continuous function from S to T is differentiable.

Q. 2. Let T_1 and T_2 be two distinct common tangents to the ellipse $E : \frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola $P : y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is (are) true?

(A) The area of the quadrilateral $A_1\ A_2\ A_3\ A_4$ is 35 square units.

(B) The area of the quadrilateral $A_1\ A_2\ A_3\ A_4$ is 36 square units.

(C) The tangents T_1 and T_2 meet the x-axis at the points $(–3, 0)$.

(D) The tangents T_1 and T_2 meet the x-axis at the points $(–6, 0)$.

Q. 3. Let $f : [0, 1] \to [0, 1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region $S = [0, 1] \times [0, 1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0, 1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is (are) true?

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h.

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h.

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h.

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h.

***General Instructions*:**

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
  Negative Marks : −1 In all other cases.

Q. 4. Let $f : (0, 1) \to \mathbb{R}$ be the function defined as $f(x) = \sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in \mathbb{N}$. Let $g : (0, 1) \to \mathbb{R}$ be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}$ for all $x \in (0, 1)$. Then $\lim_{x \to 0} f(x)g(x)$

(A) does NOT exists **(B)** is equal to 1
(C) is equal to 2 **(D)** is equal to 3

Q. 5. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines l_1 and l_2, let $d(l_1, l_2)$ denote the shortest distance between them. Then the maximum value of $d(l_1, l_2)$, as l_1 varies over F and l_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ **(B)** $\frac{1}{\sqrt{8}}$

(C) $\frac{1}{\sqrt{3}}$ **(D)** $\frac{1}{\sqrt{12}}$

Q. 6. Let $X = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R from a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ **(B)** $\frac{73}{220}$

(C) $\frac{79}{220}$ **(D)** $\frac{83}{220}$

Q. 7. Let P be a point on the parabola $y^2=4ax$, where $a>0$. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is

(A) (2, 3) **(B)** (1, 3)
(C) (2, 4) **(D)** (3, 4)

***General Instructions*:**

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Q. 8. Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$. Then the number of real solutions of the equation

$\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

Q. 9. Let $n \geq 2$ be a natural number and $f : [0, 1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function f is

Q. 10. Let $\overbrace{75\cdots57}^{r}$ denote the $(r + 2)$ digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \cdots + \overbrace{75\cdots57}^{98}$. If $S = \dfrac{\overbrace{75\cdots57}^{99} + m}{n}$, where m and n are natural numbers less than 3000, then the value of $m + n$ is

Q. 11. Let $A = \left\{\dfrac{1967 + 1686i\sin\theta}{7 - 3i\cos\theta} : \theta \in \mathbb{R}\right\}$. If A contains exactly one positive integer n, then the value of n is

Q. 12. Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let $S = \{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1$ and the distance of (α, β, γ) from the plane P is $\frac{7}{2}\}$.

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\dfrac{80}{\sqrt{3}}V$ is

Q. 13. Let a and b be two non-zero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2 + \dfrac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax - \dfrac{1}{bx^2}\right)^7$, then the value of $2b$ is

General Instructions:

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none none of the options is chosen (i.e., the question is unanswered);
Negative Marks : –1 In all other cases.

Q. 14. Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$
$x + \alpha z = 11$
$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solution
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11$, $y = -2$ and $z = 0$ as a solution
	(5) $x = -15$, $y = 4$ and $z = 0$ as a solution

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

Q. 15. Consider the given data with frequency distribution

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (4) (S) → (5)
(B) (P) → (3) (Q) → (2) (R) → (1) (S) → (5)
(C) (P) → (2) (Q) → (3) (R) → (4) (S) → (1)
(D) (P) → (3) (Q) → (3) (R) → (5) (S) → (5)

Q. 16. Let l_1 and l_2 be the lines $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\vec{i} + \hat{k})$, respectively. Let X be the set of all the planes H that contain the line l_1. For a plane H, let $d(H)$ denote the smallest possible distance between the points of l_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of $d(H)$ as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes $y = z$, $x = 1$ and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct option is :

(A) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)
(B) (P) → (5) (Q) → (4) (R) → (3) (S) → (1)
(C) (P) → (2) (Q) → (1) (R) → (3) (S) → (2)
(D) (P) → (5) (Q) → (1) (R) → (4) (S) → (2)

Q. 17. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non-zero.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z - \bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2 + |z + \bar{z}|^2$ is equal to	(3) 8
(S) $|z + 1|^2$ is equal to	(4) 10
	(5) 7

The correct option is:

(A) (P) → (1) (Q) → (3) (R) → (5) (S) → (4)
(B) (P) → (2) (Q) → (1) (R) → (3) (S) → (5)
(C) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)
(D) (P) → (2) (Q) → (3) (R) → (5) (S) → (4)

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
Section-I			
1	(A, C, D)	Number of Functions	Function, Continuity and Differentiability
2	(A, C)	Parabola and Ellipse	Ellipse
3	(B, C, D)	Area under two curves	Area under curves
4	(C)	Sandwich Theorem	Limits and Definite Integral
Section-II			
5	(A)	Shortest distance between two line	Three Dimensional
6	(B)	Probability based on geometrical problem	Probability, Parabola, Ellipse
7	(A)	Normal of parabola	Parabola
Section-III			
8	3	Number of solution of equation	Inverse Trigonometric Functions
9	8	Area under simple curves	Area under the curves
10	1219	Geometric Progression	Sequence and Series
11	281	Components of a complex number	Complex Number
12	45	Volume of Parallelpipped	Vector, Three Dimensional
13	3	General term	Binomial Theorem
Section-IV			
14	(A)	System of Linear Equations	Determinants
15	(A)	Mean, Median, Mean Deviation, Variance	Statistics
16	(B)	Point, Line and Plane	Three Dimensional
17	(B)	Modulus of complex number	Complex Number

JEE Advanced

(2023)

PAPER 1

ANSWERS WITH EXPLANATIONS

Mathematics

1. **Correct options are (A, C and D).**

Given $S = (0, 1) \cup (1, 2) \cup (3, 4)$

$T = \{0, 1, 2, 3\}$

$\because$ For function $S \to T$, set S (domain) has infinite elements but set T (codomain) has only 4 elements.

$\therefore$ There are infinite functions from S to T and it is impossible to make a function which is strictly increasing from S to T.

$\therefore$ All functions must be many one.

$\therefore$ Option (A) is correct.

and option (B) is not correct.

According to domain it is possible to make a continuous function from S to T.

Total no of such functions are $= 4^3 = 64$.

$\therefore$ Option (C) is correct.

Also every continuous function is differentiable.

$\therefore$ Option (D) is correct.

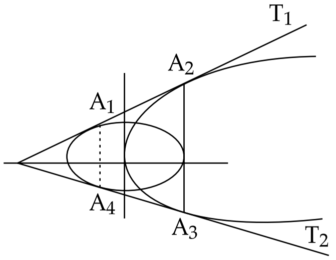
2. **Correct options are (A and C).**

E : $\frac{x^2}{6} + \frac{y^2}{3} = 1$

$a^2 = 6$

$b^2 = 3$

P : $y^2 = 12x$



Equation of tangent for ellipse

$$y = mx \pm \sqrt{6m^2 + 3} \quad ...(1)$$

Equation of tangent for parabola

$$y = mx + \frac{a}{m}$$

$$\Rightarrow \quad y = mx + \frac{3}{m} \quad ...(2)$$

By (1) and (2), we get

$$\frac{3}{m} = \pm \sqrt{6m^2 + 3}$$

and $\frac{9}{m^2} = 6m^2 + 3$

and $9 = 6m^4 + 3m^2$

$2m^4 - m^2 + 3 = 0$

$(m^2 - 1)(2m^2 + 3) = 0$

Or $m^2 = 1$

and $m = \pm 1$

$2m^2 + 3 = 0$ (which is not possible)

$\therefore$ **Equation of tangents are**

$$y = x + 3 \text{ and } y = -x - 3$$

Now their point of intersection is $(-3, 0)$.

Equation of $A_1 A_4$

$T = 0$ (Chord of contact for ellipse)

$$\frac{x(-3)}{6} + \frac{y(0)}{4} = 1$$

$x = -2$

$\Rightarrow$ $A_1 = (-2, 1)$

and $A_4 = (-2, -1)$

Equation of $A_2 A_3$

$T = 0$ (Chord of contact for parabola)

$$y(0) = 12\left(\frac{x - 3}{2}\right)$$

$\Rightarrow$ $x = 3$

$\Rightarrow$ $A_2 = (3, 6)$ and $A_3 = (3, -6)$

$\therefore$ Area of quadrilateral $A_1 A_2 A_3 A_4$

$$= \frac{1}{2} \times (2 + 12) \times 5 = 35 \text{ sq. units}$$

3. **Correct options are (B, C and D).**

Given $f : [0, 1] \to [0, 1]$

$$f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$$

$$f'(x) = \frac{3x^2}{3} - 2x + \frac{5}{9}$$

$$f'(x) = 0$$

$$9x^2 - 18x + 5 = 0$$

$$\Rightarrow 9x^2 - 15x - 3x + 5 = 0$$

$$\Rightarrow 3x(3x - 5) - 1(3x - 5) = 0$$

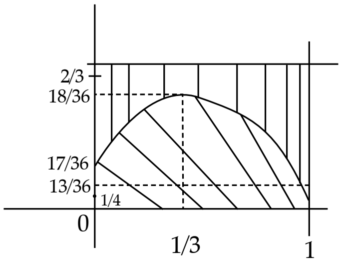
$$\Rightarrow (3x - 5)(3x - 1) = 0$$

$$\Rightarrow \quad x = \frac{1}{3} \text{ or } \frac{5}{3}$$

$$f''(x) = 2x - 2$$

$$f''\left(\frac{1}{3}\right) = \frac{2}{3} - 2 < 0 \text{ point of maxima}$$

Graph of $f(x)$



$$\text{Area}_{\text{red}} = \int_0^1 f(x)dx$$

$$= \left[\frac{x^4}{12} - \frac{x^3}{3} + \frac{5x^2}{18} + \frac{17x}{36}\right]_0^1$$

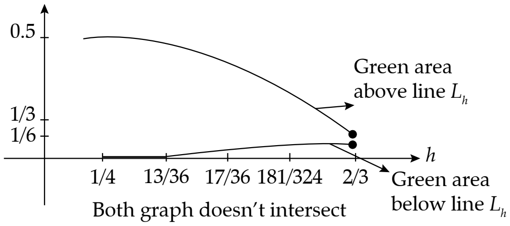
$$= \frac{1}{12} - \frac{1}{3} + \frac{5}{18} + \frac{17}{36}$$

$$= \frac{3 - 12 + 10 + 17}{36}$$

$$= \frac{18}{36} = \frac{1}{2} = 0.5$$

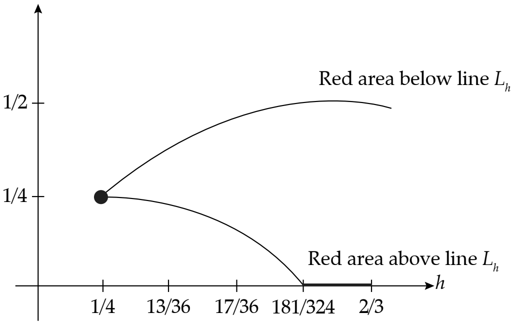
$$\therefore \quad (\text{Area})_{\text{green}} = 1 - \frac{1}{2} = 0.5$$

(A)



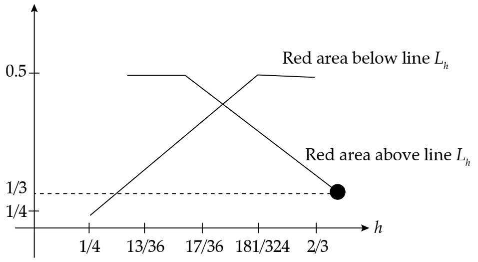
Both graph doesn't intersect

(B)



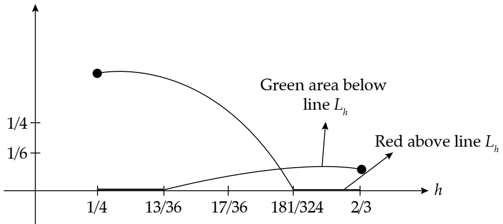
Both Graph intersect at $h = \frac{1}{4}$

(C)



Both Graph intersect.

(D)



Both Graph intersect.

4. **Correct option is (C).**

$f : (0, 1) \to R$,

$$f(x) = \sqrt{n}\ ,\ x \in \left[\frac{1}{n+1}, \frac{1}{n}\right),\ n \in N$$

$g : (0, 1) \to R$ where

$$\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}dt < g(x) < 2\sqrt{x}\ ,\ x \in (0, 1)$$

Now (According to the question)

$$\lim_{x\to\infty} f(x) \cdot g(x)$$

$$\Rightarrow \quad \text{Put} \quad x = \frac{1}{n}$$

$$\lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \sqrt{n-1}\int_{\frac{1}{n^2}}^{\frac{1}{n}} \sqrt{\frac{1-t}{t}}dt \le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right)$$

$$\le \quad \lim_{n\to\infty} \sqrt{n} - 1\frac{2}{\sqrt{n}}$$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{\int_{\frac{1}{n^2}}^{\frac{1}{n}} \sqrt{\frac{1-t}{t}}\,dt}{\frac{1}{\sqrt{n-1}}} \le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) \le 2$$

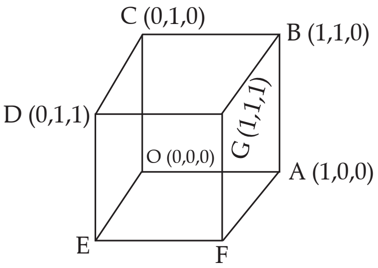
$$\Rightarrow \quad \frac{\lim\limits_{n\to\infty} \frac{-1}{n^2}\sqrt{n-1} + \frac{2}{n^3}\sqrt{n^2-1}}{\frac{1}{2(n-1)^{\frac{3}{2}}}}$$

$$\le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) \le 2$$

$$\therefore \quad \lim_{n\to\infty} \frac{2(n-1)^2}{n^2} - \frac{4(n-1)^{\frac{3}{2}}\sqrt{n^2-1}}{n^3} = 2$$

$$\therefore \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) = 2 \text{ (Using Sandwich Theorem)}$$

5. Correct option is (A).



$$\overrightarrow{OG} = \hat{i} + \hat{j} + \hat{k} = \hat{b}_1$$

$$\overrightarrow{AC} = -\hat{i} + \hat{j} = \hat{b}_2$$

Equation of line OG

$$\Rightarrow \quad \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$$

Equation of line AC

$$\Rightarrow \quad \frac{x-1}{-1} = \frac{y}{1} = \frac{z}{0}$$

$$\text{S.D.} = \frac{|(\bar{a}_2 - \bar{a}_1)\cdot(\bar{b}_1 \times \bar{b}_2)|}{|\bar{b}_1 \times \bar{b}_2|}$$

$$\bar{a}_2 - \bar{a}_1 = -\hat{i}$$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(+1) + \hat{k}(1+1)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$\text{S.D.} = \frac{|(-\hat{i}).(-\hat{i} - \hat{j} + 2\hat{k})|}{|-\hat{i}||-\hat{i} - \hat{j} + 2\hat{k}|}$$

$$= \frac{1}{1\sqrt{1+1+4}} = \frac{1}{\sqrt{6}}$$

6. Correct option is (B).

$$\frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x$$

Let $\frac{x^2}{8} + \frac{y^2}{20} = 1$ ...(1)

and $y^2 = 5x$ ...(2)

On solving (1) and (2), we get

$$\frac{x^2}{8} + \frac{5x}{20} = 1$$

$$\frac{x^2}{8} + \frac{x}{4} = 1$$

$$x^2 + 2x = 8$$

$$\Rightarrow \quad x^2 + 2x - 8 = 0$$

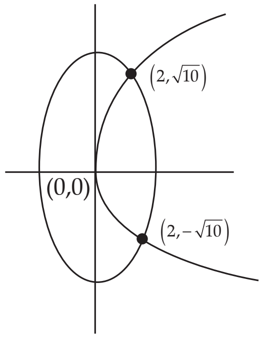
$$\Rightarrow \quad (x+4)(x-2) = 0$$

$$\Rightarrow \quad x = -4, 2$$

$$\Rightarrow \quad x = 2 \text{ (}-4\text{ is not possible)}$$

$$\Rightarrow \quad y^2 = 10$$

$$y = \pm\sqrt{10}$$



X = {(1, 1), (1, 0), (1, –1), (1, 2), (1, –2), (2, 1), (2 –1), (2, 3), (2, 3), (2, –3), (2, –2), (2, 2), (2, 0)}

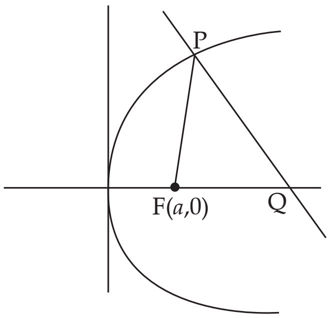
$n(s) = {}^{12}C_3$

A is even of selecting 3 points for which area of Δ is positive integer.

$$n(A) = 4 \times 7 + 9 \times 5 = 73$$

$$P(A) = \frac{73}{{}^{12}C_3} = \frac{73}{220}$$

7. Correct option is (A).



$$y^2 = 4ax$$

Equation of normal

$y = mx - 2am - am^3$

Point of contact

$P(am^2, -2am)$

and Point Q $(2a + am^2, 0)$

$$\text{Area of } \Delta\, PFQ = \frac{1}{2} \times |a + am^2|\; |-2am|$$

$$120 = a^2(1 + m^2)m \qquad \text{...(1)}$$

$$a = 2, m = 3$$

Satisfies the equation (1), hence (2, 3) will be the correct answer.

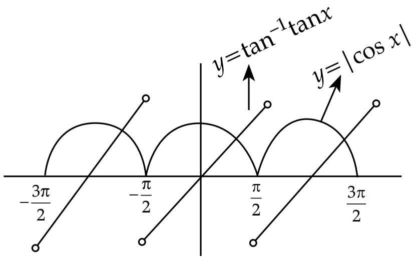
8. Correct answer is [3].

$$\sqrt{1 + \cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$$

$\Rightarrow \sqrt{2\cos^2 x} = \sqrt{2}\ \tan^{-1}\tan x$

$\Rightarrow \sqrt{2}\ |\cos x| = \sqrt{2}\tan^{-1}\tan x$

$\Rightarrow \quad |\cos x| = \tan^{-1}\tan x$

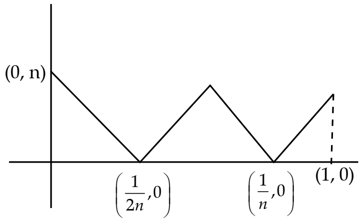


Number of solution = 3.

9. Correct option is [8].

$f : [0, 1] \to R$

$$f(x) = \begin{cases} n(1 - 2nx) & 0 \le x \le \frac{1}{2n} \\ 2n(2nx - 1) & \frac{1}{2n} \le x \le \frac{3}{4n} \\ 4n(1 - nx) & \frac{3}{4n} \le x \le \frac{1}{n} \\ \frac{n}{n-1}(nx - 1) & \frac{1}{n} \le x \le 1 \end{cases}$$



$$\text{Area} = \frac{1}{2} \times \frac{1}{2n} \times n + \frac{1}{2} \times \frac{1}{2n} \times n + \frac{1}{2} \times \left(1 - \frac{1}{n}\right) \times n$$

$$4 = \frac{1}{4} + \frac{1}{4} + \frac{n-1}{2}$$

$$4 = \frac{1}{2} + \frac{n-1}{2}$$

$$4 = \frac{n}{2}$$

$n = 8$

10. Correct answer is [1219].

$$S = 77 + 757 + 7557 + \ldots \overbrace{755 \ldots 57}^{98 \text{ times}}$$

$$S = 70 + 700 + 7000 + \ldots \overbrace{70000 \ldots 00}^{99 \text{ times}}$$

$$+ \underbrace{(50 + 550 + 5550 + \ldots.)}_{98\text{times}}$$

Let T_r be the general term.

$$T_r = 7 \times 10^{r-1} + 5(10 + 100 + \ldots 10^{r-2}) + 7\ r \ge 2$$

$$= 7 \times 10^{r-1} + 5\left[\frac{10(1 - 10^{r-2})}{1 - 10}\right] + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}(10^{r-2} - 1) + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}(10^{r-2}) - \frac{50}{9} + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}10^{r-2} + \frac{13}{9}$$

$$S = \sum_{r=2}^{100} T_r = \sum_{r=2}^{100} 7 \times 10^{r-1} + \frac{50}{90} \times 10^{r-2} + \frac{13}{9}$$

$$= \frac{70}{9}(10^{99} - 1) + \frac{50}{81}(10^{99} - 1) \times 13 \times 11$$

$$\text{RHS} = \frac{\overbrace{7555 \ldots 57}^{99\text{times}} + m}{n}$$

$$\frac{7 \times 10^{100} + \frac{50}{9}(10^{99}) + \frac{13}{9} + m}{n}$$

Now,

$$\frac{70}{9}(10^{99} - 1) + \frac{50}{81}(10^{99} - 1)\ 13 \times 11$$

$$= \frac{\frac{70}{9}10^{100} + \frac{50}{9} \times 10^{99} + \frac{13}{9} + m}{n}$$

$$= \frac{7}{n} + 10100 + \frac{50}{9n}1099 + \frac{13}{9n} + \frac{m}{n}$$

By Comparison,

$9 = n$ or $81 = 9n \Rightarrow n = 9$

$\therefore$ Put $n = 9$

$13 \times 11 \times 9^2 - 50 = 13 + 9m$

$m = 1210$

$\therefore \quad m + n = 1219$

11. Correct answer is [281].

$$A = \left\{\frac{1967 + 1686 i \sin\theta}{7 - 3i\cos\theta}, \theta \in R\right\}$$

$\because$ A contains exactly one positive integer n.

Now simplifying

$$Z = \frac{1967 + 4686 i \cos\theta}{7 - 3i\cos\theta}$$

$$= 281\frac{(7 + 6i\sin\theta)}{7 - 3i\cos\theta} \times \frac{7 + 3i\cos\theta}{7 + 3i\cos\theta}$$

$$= 281\frac{(49 - 9\sin 2\theta)}{49 + 9\cos^2\theta} + \frac{281\,(3)(2\sin\theta + \cos\theta)}{49 + 9\cos^2\theta}i$$

$$= 281\left(\frac{49 - 9\sin 2\theta}{49 + 9\cos^2\theta}\right) + 562\left(\frac{2\sin\theta + \cos\theta}{49 + 9\cos^2\theta}\right)i$$

For positive integer $Im(z) = 0$

We get, $2\sin\theta + \cos\theta = 0$

$$\tan\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \cos^2\theta = \frac{4}{5}$$

$$\Rightarrow \quad \sin 2\theta = \frac{2+\tan\theta}{1+\tan^2\theta}$$

$$= \frac{-1}{1+\frac{1}{4}} = \frac{-4}{5}$$

$$\therefore \quad Z = 281\frac{\left(49-9\left(\frac{-4}{5}\right)\right)}{49+9\left(\frac{4}{5}\right)}$$

$$= 281$$

$$\therefore \quad n = 281$$

12. Correct answer is [45].

$P: \sqrt{3}x + 2y + 3z = 16$

$$S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1, \; d_p = \frac{7}{2}\right\}$$

$\because |\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$ ...(1)

$\vec{u}, \vec{v}, \vec{w}$ are elements of set S and in set S magnitude of vector is 1

$\therefore$ $\vec{u}, \vec{v}, \vec{w}$ are unit vectors and by equation (1) we can system $\vec{u}, \vec{v}, \vec{w}$ are equally inclined and vertices of equilateral triangle also lying on a circle which is intersection of sphere $|\vec{r}| = 1$

Distance from origin to *P*,

$$d = \frac{|-16|}{\sqrt{3+4+9}} = \frac{16}{4} = 4$$

$\therefore$ Plane containing $\hat{u}, \hat{v}, \hat{w}$ are at a distance $4 - \frac{7}{2} = \frac{1}{2}$ from origin and Parallel to $\sqrt{3}x + 2y + 3z = 16$.

$\therefore$ Equation of the plane is

$$\sqrt{3}x + 2y + 3z = \gamma$$

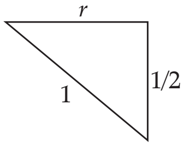
$$\therefore \quad \frac{1}{2} = \left|\frac{\gamma}{4}\right|$$

$$\Rightarrow \quad \gamma = \pm 2$$

$$\sqrt{3}x + 2y + 3z = 2$$

Equation of sphere $x^2 + y^2 + z^2 = 1$

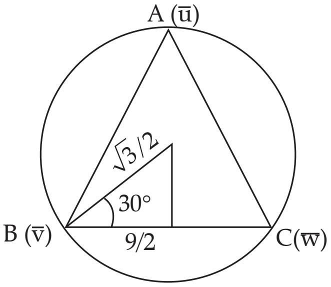
$\therefore$ Radius or circle



$$r = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

then $$\frac{a}{2} = \frac{\sqrt{3}}{2}\cos 30°$$

$$a = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$$



$\therefore$ Area or triangle

$$= \frac{\sqrt{3}}{2}a^2 = \frac{\sqrt{3}}{2} \times \frac{9}{4} = \frac{9\sqrt{3}}{16}$$

$\therefore$ Velocity of Parallelepiped

$$= 2 \times \frac{1}{2} \times \frac{9\sqrt{3}}{16}$$

$$V = \frac{9\sqrt{3}}{16}$$

$$\therefore \quad \frac{80V}{\sqrt{3}} = \frac{80}{\sqrt{3}} \times \frac{9\sqrt{3}}{16} = 45$$

13. Correct answer is [3].

General term of $\left(ax^2 + \frac{70}{27bx}\right)^4$

$$T_{r+1} = {}^4C_r\,(ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r$$

$$= {}^4C_r\,a^{4-r}\,\frac{70^r}{(27b)^r}\,(x^{8-3r})$$

For Coefficient of x^5

$$8 - 3r = 5$$

$$r = 1$$

$$\therefore \quad \text{Coefficient} = {}^4C_1\,a^3 \,.\, \frac{70}{27b}$$

$$= \frac{280}{27}\,\frac{a^3}{2}$$

General term of $\left(ax - \frac{1}{bx^2}\right)^7$ is

$$\mathrm{T}_{r+1} = {}^7C_r\,(ax)^{7-r}\left(\frac{-1}{bx^2}\right)^r$$

$$= {}^7C_r\,a^{7-r}\left(-\frac{1}{b}\right)^r x^{7-3r}$$

For Coefficient of x^{-5}

$$7 - 3r = -5$$

$$r = 4$$

$$\therefore \text{ Coefficient} = {}^7C_4\, a^3 \times \frac{1}{b^4}$$

∴ According to the question,

$$\frac{280}{27}\,\frac{a^3}{b} = \frac{35 \times a^3}{b^4}$$

$$\Rightarrow \quad b^3 = \frac{27}{8}$$

$$\Rightarrow \quad b = \frac{3}{2}$$

$$\therefore \quad 2b = 3$$

14. Correct option is (A).

Given $x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Using Cramer's rule

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix}$$

$$= 1(3\alpha) - 2(\beta - 2\alpha) + 1(-3)$$

$$= 3\alpha - 2\beta + 4\alpha - 3$$

$$= 7\alpha - 2\beta - 3$$

$$\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$$

$$= 7(3\alpha) - 2(11\beta - \gamma\alpha) + 1(-33)$$

$$= 21\alpha - 22\beta + 22\gamma - 33$$

$$\Delta y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$$

$$= 1(11\beta - \alpha\gamma) - 7(\beta - 2\alpha) + 1(\gamma - 22)$$

$$= 11\beta - \alpha\gamma - 7\beta + 14\alpha + \gamma - 22$$

$$= 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix}$$

$$= 1(33) - 2(\gamma - 22) + 7(-3)$$

$$= 33 - 2\gamma + 44 - 21$$

$$= -2\gamma + 56$$

For unique solution $\Delta \neq 0$

For infinite solution

$$\Delta = \Delta x = \Delta y = \Delta z = 0$$

For no solution $\Delta = 0$ and atleast one in $\Delta x, \Delta y, \Delta z$ is non zero.

$$\Delta = 0$$

$$\Rightarrow \quad \beta = \frac{1}{2}(7\alpha - 3)$$

(P) $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$

then $\Delta = 0, \Delta x = \Delta y = \Delta z = 0$

∴ Infinite solution

(Q) $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$

∴ $\Delta = 0$ and $\Delta_2 \neq 0$

⇒ No solution

(R) $\beta \neq \frac{1}{2}(7\alpha - 3),\ \alpha = 1, \gamma \neq 28$

⇒ $\Delta \neq 0$ = unique solution

(S) $\beta \neq \frac{1}{2}(7\alpha - 3),\ \alpha = 1, \gamma = 28$

∴ $\Delta \neq 0,\ \ \Delta = 4 - 2\beta$

$\Delta x = 44 - 22\beta$

$\Delta y = 4\beta - 8$

$\Delta z = 0$

∴ $x = 11, y = -2, z = 0$ is the solution.

15. Correct option is (A).

x_1	f_i	f_ix_i	$f_i\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - N\rvert$
3	5	15	15	10
4	4	16	8	4
5	4	20	4	0
8	2	16	4	6
10	2	20	8	10
11	3	33	15	18
	$\Sigma f_i = 20$	$\Sigma f_ix_i = 120$	sum = 54	sum = 48

(P) $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{20} = 6$

(Q) $\text{Median} = \frac{(10^{\text{th}} + 11^{\text{th}})\text{observation}}{2}$

$$= \frac{5+5}{2} = 5$$

(both observation are same)

(R) Mean deviation

$$= \frac{\Sigma f_i \mid x_i - \bar{x} \mid}{\Sigma f_i} = \frac{54}{20}$$

$$= 2.7$$

(S) Mean deviation about median

$$= \frac{\Sigma f_i \mid x_i - \text{M} \mid}{\Sigma f_i} = \frac{48}{20}$$

$$= 2.4$$

16. Correct option is (B).

$l_1 : \quad \vec{r} = \lambda(\hat{i} + \hat{j} + \hat{k})$

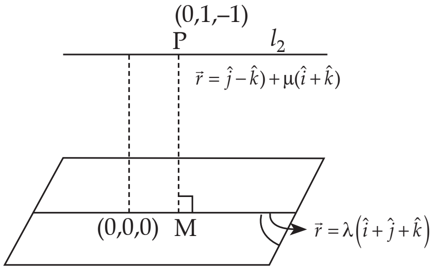
$l_2 : \quad \vec{r} = \hat{j} - \hat{k} + \mu(\hat{i} + \hat{k})$

For plane

$d(H)$ = Smallest possible distance between the points of l_2 and Plane.

$d(H_0)$ = Maximum value of $d(H)$

For $d(H_0)$



l_2 is Parallel to plane containing l_1

Equation of plane

$a(x) + by + cz = 0$

$$a(x) + by + cz = 0 \begin{array}{l} \nearrow \vec{n} \perp l_1 \\ \searrow \vec{n} \perp l_2 \end{array}$$

$\therefore \quad a + b + c = 0$ ...(1)

$a + c = 0$ ...(2)

By (1) and (2) $a = -c, b = 0$

$\therefore$ Equation of plane $x - z = 0$

(P) $d(H_0) = PM = \left|\frac{0-(-1)}{\sqrt{1+1}}\right| = \frac{1}{\sqrt{2}}$

(Q) Distance from (0, 1, 2)

$$= \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$$

(R) Distance from origin (0, 0, 0)

$$= \left|\frac{0}{\sqrt{2}}\right| = 0$$

(S) Point of Intersection,

$x - z = 0$ ...(1)

and $x = 1, y = z$ ...(2)

$\therefore \quad x = 1 = z = y$

$\therefore$ Point of intersection (1, 1, 1)

Distance from origin

$$= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

17. Correct option is (B).

$|Z|^3 + 2Z^2 + 4\bar{Z} - 8 = 0$

let $Z = x + iy$

$|Z| = \sqrt{x^2 + y^2}$

$\bar{Z} = x - iy$

$Z^2 = x^2 - y^2 + 2ixy$

$\therefore \quad |Z|^3 + 2Z^2 + 4Z - 8 = 0$

$(x^2 + y^2)^{3/2} + 2(x^2 - y^2) + 4ixy + 4x - 4iy - 8 = 0$

$\therefore \quad (x^2 + y^2)^{3/2} + 2(x^2 - y^2) + 4x - 8 = 0$ ...(1)

and $2xy - 4y = 0$

$\Rightarrow \quad y = 0$ or $x = 1$

At $x = 1$

$(1 + y^2)^{3/2} + 2 - 2y^2 + 4 - 8 = 0$

$\Rightarrow (1 + y^2)^{3/2} - 2y^2 - 2 = 0$

$\Rightarrow (1 + y^2)^{3/2} - 2(1 + y^2) = 0$

$(1 + y^2)\left(\sqrt{1 + y^2} - 2\right) = 0$

then $1 + y^2 = 0$ (wich is not possible)

or $1 + y^2 = 4$

$\Rightarrow \quad y^2 = 3$

$\therefore \quad x = 1$ and $y^2 = 3$

(P) $|Z|^2 = x^2 + y^2 = 1 + 3 = 4$

(Q) $|Z - \bar{Z}|^2 = |2Im(z)|^2$

$= (2y)^2 = 4y^2 = 12$

(R) $|Z|^2 + |Z + \bar{Z}|^2 = 4 + |2x|^2$

$= 4 + 4(1) = 8$

(S) $|Z + 1|^2 = |x + iy + 1|^2$

$= (x + 1)^2 + y^2$

$= 4 + 3 = 7.$

JEE Advanced (2023)

PAPER 2

Mathematics

General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
  Negative Mark : –1 In all other cases.

Q. 1. Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function such that $f(1) = \frac{1}{3}$ and $3\int_1^x f(t)dt = x\,f(x) - \frac{x^3}{3}, x \in [1, \infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is

(A) $\frac{e^2+4}{3}$ **(B)** $\frac{\log_e 4 + e}{3}$

(C) $\frac{4e^2}{3}$ **(D)** $\frac{e^2-4}{3}$

Q. 2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in heads is $\frac{1}{3}$, then the probability that the experiment stops with head is

(A) $\frac{1}{3}$ **(B)** $\frac{5}{21}$

(C) $\frac{4}{21}$ **(D)** $\frac{2}{7}$

Q. 3. For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3} - 3$ **(B)** $3 - 2\sqrt{3}$

(C) $4\sqrt{3} - 6$ **(D)** $6 - 4\sqrt{3}$

Q. 4. Let the position vectors of the point P, Q, R and S be $\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}, \vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$ and $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$, respectively. Then which of the following statements is true?

(A) The points P, Q, R and S are NOT coplanar

(B) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 95

General Instructions:

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE** (03) questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : –2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e., the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

Q. 5. Let $M = (a_{ij})$, $i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true?

(A) M is invertible

(B) There exists a non-zero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$, where $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3×3 identify matrix

Q. 6. Let $f : (0, 1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to x. Then which of the following statements is (are) true?

(A) The function f is discontinuous exactly at one point in (0, 1)

(B) There is exactly one point in (0, 1) at which the function f is continuous but NOT differentiable

(C) The function f is NOT differentiable at more than three points in (0, 1)

(D) The minimum value of the function f is $-\frac{1}{512}$

Q. 7. Let S be the set of all twice differentiable function f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2 f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is (are) true?

(A) There exists a function $f \in S$ such that $X_f = 0$

(B) For every function $f \in S$, we have $X_f \leq 2$

(C) There exists a function $f \in S$ such that $X_f = 2$

(D) There does NOT exist any function f in S such that $X_f = 1$

***General Instructions*:**

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Q. 8. For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}}\,dt$ is

Q. 9. For $x \in \mathbb{R}$, let $y(x)$ be a solution of the differential equation $(x^2 - 5)\frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2$ such that $y(2) = 7$.

Then the maximum value of the function $y(x)$ is

Q. 10. Let X be the set of all five digit numbers formed using 1, 2, 2, 2, 4, 4, 0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let P be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of $38p$ is equal to

Q. 11. Let A_1, A_2, A_3, ..., A_8 be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for $i = 1, 2, ..., 8$. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \cdots PA_8$, is

Q. 12. Let

$$R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a,b,c,d \in \{0,3,5,7,11,13,17,19\} \right\}.$$

Then the number of invertible matrices in R is

Q. 13. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with centre at the point $A = (4, 1)$, where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If $AB = \sqrt{5}$, then the value of r^2 is

***General Instructions*:**

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

PARAGRAPH "I"

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

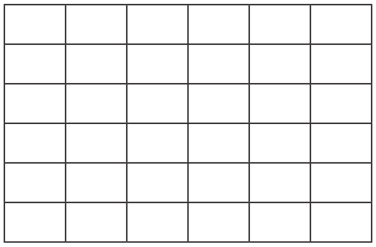
(There are two question based on PARAGRAPH "I", the question given below is one of them)

Q. 14 Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

Q. 15. Then the inradius of the triangle ABC is

PARAGRAPH "II"

Consider the 6 × 6 square in the figure. Let $A_1, A_2, ..., A_{49}$ be the points of intersection (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each points A_i has an equal chance of being chosen.



(There are two question based on PARAGRAPH "II", the question given below is one of them)

Q. 16. Let p_i be the probability that a randomly chosen point has i many friends, $i = 0, 1, 2, 3, 4$. Let X be a random variable such that for $i = 0, 1, 2, 3, 4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

Q. 17. Two distinct points are chosen randomly out of the points $A_1, A_2, ..., A_{49}$. Let p be the probability that they are friends. Then the value of $7p$ is

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
Section-I			
1	(C)	Linear differential equation	Differential equation
2	(B)	Conditional probability	Probability
3	(C)	Solution of Equation	Inverse Trigonometric function
4	(B)	Product of vectors and its Application	Vector
Section-II			
5	(B, C)	Solution of system of linear equations	Matrix and determinants
6	(A, B)	Maxima and Minima	Application of derivatives
7	(A, B, C)	Concavity of curve	Application of derivatives
Section-III			
8	0	Leibnitz theorem & Maxima, Minima	Application of derivatives
9	16	Linear differential equation	Differential equation
10	31	Probability based on permutation & combination	Probability
11	512	Demovire’s theorem and triangular inequality	Complex number
12	3780	Permutation involving in matrix	Matrix
13	2	Radical axis and its properties	Circle
Section-IV			
14	1008	Area of triangle	Properties of triangle
15	0.25	Inradius	Properties of triangle
16	24	Binomial distribution	Probability
17	0.5	Conditional Probability	Probability

JEE Advanced (2023)

PAPER 2

ANSWERS WITH EXPLANATIONS

Mathematics

1. Correct option is (C).

$$3\int_1^x f(t)dt = xf(x) - \frac{x^2}{3} \quad x\in(1,\infty)$$

Using Leibnitz rule,

$$3f(x) = x f'(x) + f(x) - x^2$$

$$\Rightarrow xf'(x) - 2f(x) - x^2 = 0$$

$$\Rightarrow f'(x) - \frac{2}{x}f(x) - x = 0$$

$$\Rightarrow \frac{dy}{dx} - \frac{2}{x}y = x$$

Linear Differential Equation in x

Integrating Factor = $e^{-\int \frac{2}{x}dx} = e^{-2\ln x}$

$$= \frac{1}{x^2}$$

Now $y.\frac{1}{x^2} = \int x.\frac{1}{x^2}dx$

$$= \ln x + C$$

$$\Rightarrow \quad \frac{1}{3} = 0 = C \qquad \left[\because \ f(1) = \frac{1}{3}\right]$$

$$\Rightarrow \quad C = 3$$

$$\Rightarrow \quad y = x^2 \ln x + \frac{x^2}{3}$$

$$f(e) = e^2 + \frac{e^2}{3}$$

$$f(e) = \frac{4e^2}{3}$$

2. Correct option is (B).

$$P(H) = \frac{1}{3} \quad P(T) = \frac{2}{3}$$

Tossing coin is repeatedly this process end with last two head in out come.

⇒ Lets Experiment end with trial : (Two trial) or (Three trial) or (Four trial) or (Five trial) or (Six trial) so, on ....

i.e., (HH) or (THH), (HTHH), (THTHH) (HTHTHH) .....

So, the required probability is given by:

P = (HH) + (THH) + (HTHH) + (THTHH) (HTHTHH) + ... ∞

$$P = \left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + \ldots \infty$$

$$= \left(\left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + \ldots \infty\right) + \left(\frac{2}{3}\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + \ldots \infty\right)$$

$$= \frac{\left(\frac{1}{3}\right)^2}{1-\frac{2}{9}} + \frac{\frac{2}{3}\times\frac{1}{9}}{1-\frac{2}{3}\times\frac{1}{3}}$$

$$= \frac{1}{7} + \frac{2}{21} = \frac{5}{21}$$

3. Correct option is (C).

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3} \qquad \ldots(i)$$

where $0 < |y| < 3$

$$\cot^{-1}\left(\frac{1}{x}\right) = \begin{cases} \tan^{-1} x & x > 0 \\ \pi + \tan^{-1} x & x < 0 \end{cases}$$

Case-I When $0 < y < 3$

$$\tan^{-1}\frac{6y}{9-y^2} + \tan^{-1}\frac{6y}{9-y^2} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\frac{6y}{9-y^2} = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{6y}{9-y^2} = \sqrt{3}$$

$$\Rightarrow \quad 6y = 9\sqrt{3} - \sqrt{3}y^2$$

$$\Rightarrow \quad \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$$\Rightarrow \quad \sqrt{3}y^2 + 9y - 3y - 9\sqrt{3} = 0$$

$$\Rightarrow \quad \sqrt{3}y(y + 3\sqrt{3}) - 3(y + 3\sqrt{3}) = 0$$

$$(\sqrt{3}y - 3)(y + 3\sqrt{3}) = 0$$

So, the value satisfied is $y = \sqrt{3}$

Case II: When $-3 < y < 0$

$$\tan^{-1}\left(\frac{6y}{9 - y^2}\right) + \pi + \tan^{-1}\frac{6y}{9 - y^2} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{6y}{9 - y^2}\right) = \frac{-\pi}{6}$$

$$\frac{6y}{9 - y^2} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 6\sqrt{3}y = y^2 - 9$$

$$\Rightarrow \quad y^2 - 6\sqrt{3}y - 9 = 0$$

$$\Rightarrow \quad y = \frac{6\sqrt{3} \pm \sqrt{108 + 36}}{2}$$

$$= \frac{6\sqrt{3} \pm 12}{2} = 3\sqrt{3} \pm 6$$

So, the value satisfied is $y = 3\sqrt{3} - 6$

Hence, the sum of solutions

$$3\sqrt{3} - 6 + \sqrt{3} = 4\sqrt{3} - 6$$

4. **Correct option is (B).**

$$P(\vec{a}) = \hat{i} + 2\hat{j} - 5\hat{k}$$

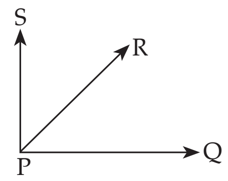
$$Q(\vec{b}) = 3\hat{i} + 6\hat{j} + 3\hat{k}$$

$$R(\vec{c}) = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$$

$$S(\vec{d}) = 2\hat{i} + \hat{j} + \hat{k}$$

From option

(A)

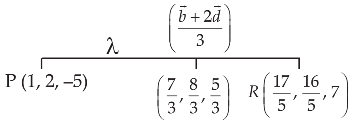


$[\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS}] \rightarrow$ S.T.P

$$\begin{vmatrix} 2 & 4 & 6 \\ \frac{12}{5} & \frac{6}{5} & 12 \\ 1 & -1 & 6 \end{vmatrix} = 0$$

Hence P, Q, R, S are coplanar.

(B)



$$\Rightarrow \quad \frac{\frac{17\lambda}{5} + 1}{1 + \lambda} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{(17\lambda + 5)}{1 + \lambda} = \frac{35}{3}$$

$$\Rightarrow \quad 15\lambda + 15 = 35 + 35\lambda$$

$$\Rightarrow \quad 16\lambda = 20$$

$$\Rightarrow \quad \lambda = \frac{5}{4}$$

Hence option (B) is correct.

(D) $|\vec{b} \times \vec{d}|^2 = |\vec{b}|^2 |\vec{d}|^2 - (\vec{b} \,.\, \vec{d})^2$

$$= 54 \times 6 - 225$$

$$= 324 - 225$$

$$= 99$$

5. **Correct options are (B and C).**

$$M = [a_{ij}] \; i, j \in \{1, 2, 3\}$$

$$a_{ij} = \begin{cases} 1 & \text{if } j + 1 \text{ divisible by } i \\ 0 & \text{other wise} \end{cases}$$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|M| = 0$$

$\Rightarrow$ M^{-1} not exist

$$M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$$

$$\Rightarrow \quad a_1 + a_2 + a_3 = -a_1 \qquad \text{...(i)}$$

$$a_1 + a_3 = -a_2 \qquad \text{...(ii)}$$

$$\Rightarrow \quad a_2 + a_3 = 0 \qquad \text{...(iii)}$$

From (i) and (iii),

$$a_1 = 0$$

From (ii)

$$a_2 + a_3 = 0$$

Hence, there exist infinite many solution for a_2, and a_2

$$MX = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x + y + z = 0 \qquad \text{...(iv)}$$

$$\Rightarrow \quad x + z = 0 \qquad \text{...(v)}$$

$$\Rightarrow \quad y = 0 \qquad \text{...(vi)}$$

From (iv) and (v)

and $M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$|M - 2I| = 0$

Hence, $(M - 2I)^{-1}$ does not exist

6. **Correct options are (A and B).**

Given $f : (0, 1) \to R$

$$f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) \qquad ...(i)$$

when $x \in (0, 1) \Rightarrow 4x \in (0, 4)$

$x : 0 - 1$

$4x : 0 - 1 - 2 - 3 - 4$

$x : 0 - \frac{1}{4} - \frac{1}{2} - \frac{3}{4} - 1$

From (*i*)

$$f(x) = \begin{cases} 0 & 0 < x < \frac{1}{4} \\ \left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{1}{4} \le x < \frac{1}{2} \\ 2\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{3}{4} \le x < 1 \end{cases}$$

Check continuity and differentiability at $x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

Clearly $f(x)$ is discontinuous at $x = \frac{3}{4}$ and continuous at $x = \frac{1}{4}, \frac{1}{2}$

$$\text{also} \quad f'(x) = \begin{cases} 0 & 0 < x < \frac{1}{4} \\ \left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{1}{4} < x < \frac{1}{2} \\ 2\left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{1}{2} < x < \frac{3}{4} \\ 3\left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{3}{4} < x < 1 \end{cases}$$

at $x = \frac{1}{4}$ function is continuous and differentiable

at $x = \frac{1}{2}$ function is continuous but not differentiable

For maxima and minima

Put $f'(x) = 0$

$$x = \frac{1}{4}, \frac{5}{12}$$

Clearly $f(x)$ give minimum value

at $x = \frac{5}{12}$

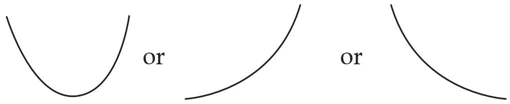
$$f_{\min} = f\left(\frac{5}{12}\right) = \frac{-1}{432}$$

7. **Correct options are (A, B and C).**

$$\frac{d^2 f}{dx^2} > 0$$

$\Rightarrow$ $y = f(x)$ concave upward in (1, 1)

Graph: $y = f(x)$ in $(-1, 1)$



The line $y = x$ cut above goopn either in 0, 1 or 2 point

So, the options A, B, C are correct.

8. **Correct answer is [0].**

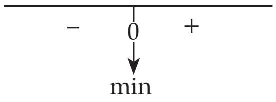
$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t - \cos t}}{1 + t^{2023}}\,dt$$

$$f'(x) = \frac{e^{x\tan^{-1}x - \cos(x\tan^{-1}x)}}{1 + (x\tan^{-1}x)^{2023}}\left(\frac{x}{1 - x^2} + \tan^{-1}x\right)$$

For max/min put $f'(x) = 0$

$$\Rightarrow \quad \frac{x}{1 + x^2} + \tan^{-1}x = 0$$

$$\Rightarrow \quad \boxed{x = 0}$$



$$\boxed{f(0) = 0}$$

9. **Correct answer is [16].**

$$(x^2 - 5)\frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2x}{(x^2 - 5)}y = -2x(x^2 - 5)$$

$$\text{I.F.} = e^{-\int \frac{2x}{x^2 - 5}dx} = \frac{1}{(x^2 - 5)}$$

$$\text{Now} \quad y \cdot \frac{1}{(x^2 - 5)} = -\int 2x\,dx$$

$$= -x^2 + c$$

$$\Rightarrow \quad y = c(x^2 - 5) - x^2(x^2 - 5)$$

$$y(2) = 7$$

$$\Rightarrow \quad 7 = -c + 4$$

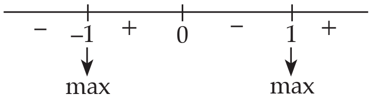
$$\Rightarrow \quad \boxed{c = -3}$$

So, $y = (x^2 - 5)(-x^2 - 3)$ ...(1)

$$\frac{dy}{dx} = (x^2 - 5)(-2x) + (-x^2 - 3)(2x)$$
$$= 2x(-x + 5 - x^2 - 3)$$
$$= 2x(-2x^2 + 2)$$

For maxima and minima, put $\frac{dy}{dx} = 0$

$\Rightarrow$ $x = 0, \pm 1$



From (1)

$y_{\max} = 16$

10. Correct answer is [31].

A = Number of elements in x which is multiple of 5

$$\begin{array}{ccccc} 1 & 2 & 2 & 2 & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{4!}{3!} = 4$$

$$\begin{array}{ccccc} 1 & 4 & 2 & 2 & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{4!}{2!} = 12$$

$$\begin{array}{ccccc} 1 & 2 & 4 & 4 & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{4!}{2!} = 12$$

$$\begin{array}{ccccc} 4 & 2 & 2 & 2 & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{4!}{3!} = 4$$

$$\begin{array}{ccccc} 2 & 2 & 4 & 4 & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{4!}{2!2!} = 6$$

$n(A) = 4 + 12 + 12 + 4 + 6 = 38$

B = Number of elements in x which is multiple of 20

$$\begin{array}{ccccc} 2 & 2 & 2 & & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to 1$$

$$\begin{array}{ccccc} 4 & 2 & 2 & & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{3!}{2!} = 3$$

$$\begin{array}{ccccc} 2 & 4 & 4 & & \\ \hline \multicolumn{1}{|c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{\ } & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{0} \\ \hline \end{array} \to \frac{3!}{2!} = 3$$

So, number of element in x which is multiple of $20 = n(B)$

$$= (4 - 1) + (12 - 3) + (12 - 3) + 4 + 6$$
$$= 31$$

$$\Rightarrow \quad P\left(\frac{B}{A}\right) = \frac{n(A \cup B)}{n(A)} = \frac{31}{38} = P$$

$$\Rightarrow \quad \boxed{38P = 31}$$

11. Correct answer is [512].

Let $z = 2(1)^{1/8}$ $[\because \ |z| = 2]$

$\Rightarrow$ $z = 2, 2x, 2x^2, 2x^3, \ldots 2x^7$ are root.

$\Rightarrow$ $(z^8 - 2^8) = (z - 2)(z - 2x)(z - 2x^2) \ldots (z - 2x^7)$

Using triangular in equalities

$$|z^8 - 2^8| = |z - 2|\,|z - 2x^2|\,|z - 2x^3| \ldots |z - 2x^7|$$
$$\leq |z^8| + |-2^8|$$
$$\leq 2^8 + 2^8$$
$$\leq 2^9$$

Max $PA_1 \cdot PA_2 \cdot PA_3 \ldots PA_8 = 2^9$

12. Correct answer is [3780].

$$R = \begin{bmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{bmatrix}$$

$a, b, c, d, \in \{0, 3, 5, 7, 11, 13, 17, 19\}$

Number of invertible matrices = (Total matrices) – (Non Invertible matrices)

$$\text{Total matrices} = \begin{array}{cccc} a, & b, & c, & d \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 8 & 8 & 8 & 8 \end{array}$$

$$= 8 \times 8 \times 8 \times 8 = 8^4 = 4096$$

For Non-invertible matrices,

$$|R| = 0$$
$$|R| = -5(ad - bc) = 0$$

$$\Rightarrow \quad \boxed{ad = bc = 0}$$

Both side zero

Both side same but not zero

Cases when both side are zero.

(i) All four a, b, c, d are zero.

$ad = bc = 0$ 1 ways

(ii) Three zero and one different digit used for a, b, c, d.

$\Rightarrow$ $ad = bc$

Select three from four a, b, c, d & assign them zero.

i.e., ${}^4C_3 \times 1 \times 7 = 28$ ways

(iii) Two zero and two different digits

$$\text{i.e.,} \quad \begin{array}{ccc} ad & = & bc \\ \Downarrow & & \Downarrow \\ {}^2C_1 \times 1 \times 7 & & {}^2C_1 \times 1 \times 7 \end{array}$$

Hence $2 \times 7 \times 2 \times 7 = 196$ ways

Case II: When both side are same but non zero number.

$$ad = bc \neq 0$$

(i) All four a, b, c, d are same.

i.e., $ad = bc$ (7 ways)

(ii) Two alike & two alike of another.

$ad = bc$

$^7C_1 \times {}^6C_1 \times 2! = 84$ ways

Total number of non invertible matrices are

$= 1 + 28 + 196 + 7 + 84$

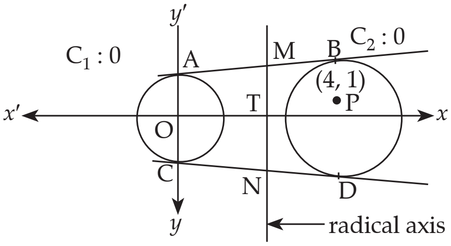
$= 316$

Hence number of invertible matric

$= 8^4 - 316$

$= 3780$

13. Correct answer is [2].



Equation of radical axis : $C_1 - C_2 = 0$

$$\Rightarrow \quad 8x + 2y - 18 + r^2 = 0$$

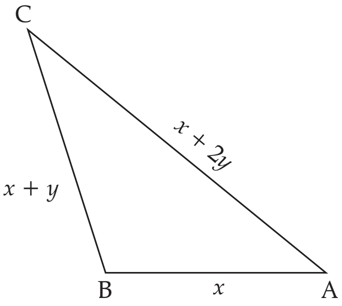
$$T\left(\frac{18 - r^2}{8}, 0\right)$$

$$AT = \sqrt{5} \text{ [given]}$$

$$\Rightarrow \quad \left(\frac{18 - r^2}{8} - 4\right) + (0 - 1)^2 = 5$$

$$r^2 = 2$$

Paragraph I



Let B be greatest angle and C be small angle. Each side of triangle is mention in figure.

Given $\quad B - C = \frac{\pi}{2}$

$$\Rightarrow \quad B = \frac{\pi}{2} + C$$

$$A + B + C = \pi$$

$$\Rightarrow \quad A = \frac{\pi}{2} - 2C$$

Again AB, BC, CA are in AP

$$2BC = AB + AC$$

$$\Rightarrow \quad 4R \sin A = 2R \sin B + 2R \sin C$$

$$\Rightarrow \quad 2 \sin A = \sin B + \sin C$$

$$\Rightarrow \quad 2 \sin\left(\frac{\pi}{2} - 2C\right) = \sin\left(\frac{\pi}{2} + 2C\right) + \sin C$$

$$\Rightarrow \quad 2 \cos 2C = \cos C + \sin C$$

$$\Rightarrow \quad \cos C - \sin C = \frac{1}{2}$$

Squaring both side we get

$$\Rightarrow \quad 1 - \sin 2C = \frac{1}{4}$$

$$\Rightarrow \quad \sin 2C = \frac{3}{4}$$

14. Correct answer is [1008].

$$\text{Area of } \Delta ABC = \frac{AB \cdot BC \cdot AC}{4R}$$

$$\Rightarrow \quad a = \frac{8 \sin A \cdot \sin B \sin C}{4}$$

$$= 2 \sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) \sin C$$

$$= 2 \cos 2\text{C} \cdot \cos \text{C} \cdot \sin \text{C}$$

$$= \cos 2\text{C} \cdot \sin 2\text{C}$$

$$= \sqrt{1 - \sin^2 2C} \cdot \sin 2C$$

$$= \sqrt{1 - \frac{9}{6}} \cdot \times \frac{3}{4}$$

$$\Rightarrow \quad a = \frac{3\sqrt{7}}{16}$$

$$(64\,a)^2 = 1008$$

15. Correct answer is [0.25].

$$\text{In radius} \quad r = \frac{\Delta}{S} = \left[\frac{a}{2R(\sin A + \sin B + \sin C)}\right)$$

$$r = \frac{a}{\sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) + \sin C}$$

$$= \frac{a}{\cos 2C + \cos C + \sin C}$$

$$= \frac{a}{\cos 2C + \sqrt{1 + \sin 2C}}$$

$$= \frac{\frac{3\sqrt{7}}{16}}{\sqrt{\frac{7}{4}} + \sqrt{\frac{7}{2}}} = \frac{1}{4}$$

$$\Rightarrow \quad r = \frac{1}{4} = 0.25$$

$$\Rightarrow \quad r = 0.25$$

16. Correct answer is [24].

$$P(x = 0) = 0$$

$$P(x = 3) = \frac{20}{49}$$

$$P(x = 1) = 0$$

$$P(x = 4) = 1 - \frac{24}{49}$$

$$P(x = 2) = \frac{4}{49}$$

$$= \frac{25}{49}$$

We have

$$E(X_i) = \sum_{i=0}^{4} i\,P(x = i)$$

$$= 0\,.\,P(x = 0) + 1\,P(x = 1) + 2(x = 2) + 3P(x = 3) + 4P(x = 4)$$

$$= 0 + 0 + 2\,\frac{4}{49} + 3\cdot\frac{20}{49} + 4\cdot\frac{25}{49}$$

$$= \frac{8 + 60 + 100}{49} = \frac{168}{49} = \frac{24}{7}$$

$$7E(X_i) = 24$$

17. Correct answer is [0.5].

$$P = \frac{6 \times 7 + 6 \times 7}{{}^{49}C_2} = \frac{2 \times 6 \times 7}{\frac{49 \times 48}{2}}$$

$$P = \frac{1}{14}$$

$$7P = \frac{1}{2} = 0.5$$

$$7P = 0.5$$