MOCK TEST PAPER 1

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).*
2. *Each subject is divided into two sections. Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice for Section A questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. The position of a particle in x–y plane is described by the variables $x = at^3$ and $y = 2at$. Then the acceleration of the particle.......

(1) is $6a$ at $t = 0$ **(2)** is $6a$ at $t = 1$

(3) is $3a$ at $t = 0$ **(4)** is $3a$ at $t = 1$

Q. 2. Which of the following options may be the correct estimate of the mean free path of gas particles ? [n : Number of gas particle per unit volume, d : diameter]

(1) $\lambda = \dfrac{1}{d^2n^2}$ **(2)** $\lambda = \dfrac{n^2d}{\sqrt{2}}$

(3) $\lambda = \dfrac{nd^2}{\sqrt{2}}$ **(4)** $\lambda = \dfrac{1}{\sqrt{2}nd^2}$

Q. 3. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and required 1 s to cover. How long the drunkard takes to fall in a pit 13 m away from the start ?

(1) 9 s **(2)** 21 s

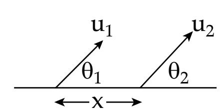
(3) 32 s **(4)** 37 s

Q. 4. If T be the total time of flight of a current of water and H be the maximum height attained by it from the point of projection, then H/T will be : (u = projection velocity, θ = projection angle)

(1) $\left(\dfrac{1}{2}\right)u \sin\theta$ **(2)** $\left(\dfrac{1}{4}\right) u \sin\theta$

(3) $u \sin\theta$ **(4)** $2u \sin\theta$

Q. 5. Two particles are projected simultaneously from the level ground as shown figure. They may collide after a time :



(1) $\dfrac{x\sin\theta_2}{u_1}$ **(2)** $\dfrac{x\sin\theta_2}{u_2}$

(3) $\dfrac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$ **(4)** $\dfrac{x\sin\theta_2}{u_2\sin(\theta_2-\theta_1)}$

Q. 6. If a body of mass m is moving on a rough horizontal surface of coefficient of kinetic friction μ, the net electromagnetic force exerted by surface on the body is :

(1) $mg\sqrt{1+\mu^2}$ **(2)** μmg

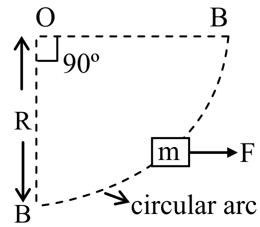
(3) mg **(4)** $mg\sqrt{1-\mu^2}$

Q. 7. An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 r.p.m. The acceleration of a point on the tip of the blade is about :

(1) 1600 m/s^2 **(2)** 4740 m/s^2

(3) 2370 m/s^2 **(4)** 5055 m/s^2

Q. 8. A block of mass m is taken from A to B slowly under the action of a constant force F. Work done by this force is :



(1) FR **(2)** $\frac{\pi}{2}$FR

(3) $\frac{FR}{\sqrt{2}}$ **(4)** $\frac{FR}{4}$

Q. 9. A monkey of mass 20 kg rides on a 40 kg trolley moving with constant speed of 8 m/s along a horizontal track. If the monkey jumps vertically to grab the overhanging branch of a tree, the speed of the trolley after the monkey has jumped off is :

(1) 8 m/s **(2)** 1 m/s

(3) 4 m/s **(4)** 12 m/s

Q. 10. A rod of mass 'm' hinged at one end is free to rotate in a horizontal plane. A small bullet of mass m/4 travelling with speed 'u' hits the rod and attaches to it at its centre. Find the angular speed of rotation of rod just after the bullet hits the rod 3.

[take length of the rod as 'l']

(1) $\frac{6}{19}\frac{u}{l}$ **(2)** $\frac{6}{13}\frac{u}{l}$

(3) $\frac{3}{19}\frac{u}{l}$ **(4)** $\frac{3}{13}\frac{u}{l}$

Q. 11. If R is the radius of the earth and g is the acceleration due to gravity on the earth's surface, the mean density of the earth is :

(1) $\frac{4\pi G}{3gR}$ **(2)** $\frac{3\pi R}{4gG}$

(3) $\frac{3g}{4\pi RG}$ **(4)** $\frac{\pi Rg}{12G}$

Q. 12. A particle is oscillating according to the equation X = 7 cos 0.5 πt, where 't' is in second. The point moves from the position of equilibrium to maximum displacement in time :

(1) 4.0 seconds **(2)** 2.0 seconds

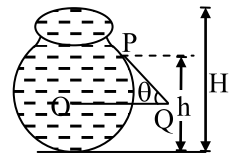
(3) 1.0 seconds **(4)** 0.5 seconds

Q. 13. A metal wire of length L, area of cross section A and Young's modulus Y behaves as a spring of spring constant k given by:

(1) k = YA/L **(2)** k = 2YA/L

(3) k = YA/2L **(4)** k = YL/A

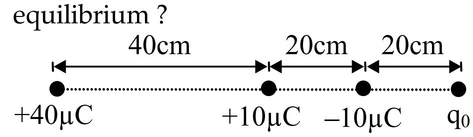
Q. 14. Figure shows the vertical cross-section of a vessel filled with liquid of density ρ. The normal thrust per unit area on the walls of the vessel at point P, as shown will be :



(1) $h \rho g$ **(2)** $(H - h) \rho g$

(3) $(H - h) \rho g \cos\theta$ **(4)** $H \rho g$

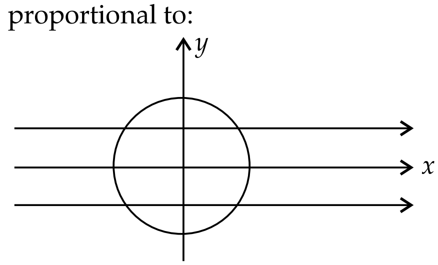
Q. 15. Four point charges are placed in a straight line with magnitude and separation as shown in the diagram. What should be the value of q_0 such that + 10μC charge is in equilibrium ?



(1) – 80 μC **(2)** + 40 μC

(3) + 80 μC **(4)** – 20 μC

Q. 16. A conducting loop of resistance R and radius r has its centre at the origin of the coordinate system in a magnetic field of induction B. When it is rotated about y-axis through 90°, the net charge flown in the loop is directly proportional to:



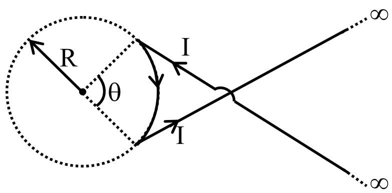
(1) R^{-1} **(2)** R

(3) r^2 **(4)** r

Q. 17. In copper, each copper atom releases one electron. If a current of 1.1 A is flowing in the copper wire of uniform cross-sectional area of diameter 1 mm, then drift velocity of electrons will approximately be : (Density of copper = 9 × 10^3 kg /m^3, Atomic weight of copper = 63)

(1) 10.3 mm/s **(2)** 0.1 mm/s
(3) 0.2 mm/s **(4)** 0.2 cm/s

Q. 18. A wire carrying current i has the configuration shown in figure. For the magnetic field to be zero at the centre of the circle, θ must be :



(1) 1 radian **(2)** 2 radian
(3) π radian **(4)** 2π radian

Q. 19. When a clock is viewed in a mirror, the needles exhibit a time which appears to be 08:20. Then the actual time will be :
(1) 04:40 **(2)** 03:40
(3) 08:20 **(4)** 03:20

Q. 20. The value of angular momentum for He^+ ion in the first Bohr orbit is :
(1) $\frac{h}{2\pi}$ **(2)** $4 \times \frac{h}{2\pi}$
(3) $2 \times \frac{h}{2\pi}$ **(4)** nothing can be said

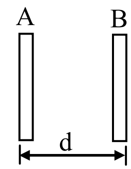
Section B

Q. 21. ^{23}Ne decays to ^{23}Na by negative beta emission. Mass of ^{23}Ne is 22.994465 amu mass of ^{23}Na is 22.989768 amu. The maximum kinetic energy of emitted electrons neglecting the kinetic energy of recoiling product nucleus is ....MeV

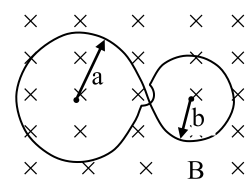
Q. 22. If photons of ultraviolet light of energy 12 eV are incident on a metal surface of work function of 4 eV, then the stopping potential (in eV) will be :

Q. 23. A light is entering from one medium refractive index $\left(RI = \frac{5}{3}\right)$ to another medium at an angle 30°. The angle of refraction for other medium is $\sin^{-1}\left(\frac{5}{6}\right)$. then the increase in angle of incidence is ...., such that the ray of light reflected back into the same medium.

Q. 24. Two plates A and B of a parallel plate capacitor are arranged in such a way, that the area of each plate is $S = 5 \times 10^{-3}$ m^2 and distance between them is d = 8.85 mm. Plate A has a positive charge $q_1 = 10^{-10}$ C and Plate B has charge $q_2 = +2 \times 10^{-10}$ C. Then the charge induced on the plate B due to the plate A be – (....... $\times 10^{-11}$)C



Q. 25. A plane loop is shaped in the form as shown in figure with radii a = 20 cm and b = 10 cm and is placed in a uniform time varying magnetic field $B = B_0 \sin \omega t$, where $B_0 = 10\ m$T and ω = 100 rad/s. The amplitude of the current induced in the loop if its resistance per unit length is equal to 50×10^{-3} Ω/m. The inductance of the loop is negligible is .... A.

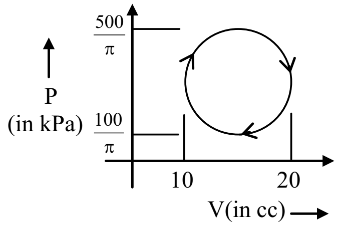


Q. 26. A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltage across resistance and inductance are 60 V and 40 V respectively. At what frequency the current in the circuit lags the voltage by 45°. Give answer in ....$\times 10^5$ rad s^{-1}.

Q. 27. On an X temperature scale, water freezes at –125°X and boils at 375°X. On a Y temperature scale, water freezes at –70°Y and boils at –30°Y. The value of temperature on X scale is....... on which value of temperature on y scale becomes 50° Y

Q. 28. A diatomic molecule can be modelled as two rigid balls connected with spring such that the balls can vibrate with respect to centre of mass of the system (spring + balls). Consider a diatomic gas made of such diatomic molecule. If the gas performs 20 Joule of work under isobaric condition, then heat given to the gas is .... J.

Q. 29. Work done by gas in cyclic process is .... J.



Q. 30. In a Quincke's tube experiment, a tuning fork of frequency 300 Hz is vibrated at one end. It is observed that intensity decreases from maximum to 50 % of its maximum value, as tube is moved by 6.25 cm. Velocity of sound is .... m/s.

Chemistry

Section A

Q. 31. There are two common oxides of Sulphur, one of which contains 50% O_2 by weight, the other almost 60%. The weights of sulphur which combine with 1 g of O_2 (fixed) are in the ratio of :

(1) 1 : 1 **(2)** 2 : 1
(3) 2 : 3 **(4)** 3 : 2

Q. 32. Which of the following options of species have identical shapes?

(1) $BeCl_2$, XeF_2, CO_2
(2) PF_5, IF_5, IF_7
(3) BF_3, NH_3, PCl_3,
(4) CF_4, SF_4, XeF_4

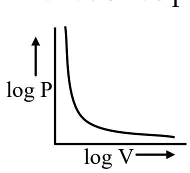
Q. 33. The correct set of four quantum numbers for the valence electron of potassium (atomic number 19) is:

(1) 5, 0, 0, 1/2 **(2)** 4, 1, 1, 1/2
(3) 4, 0, 0, 1/2 **(4)** 4, 1, 0, 1/2

Q. 34. What is the pH of the solution, if the cell potential for the cell Pt / H_2 (g) /H^+(aq) IICu^{2+} (0.01M)/ Cu(s) is 0.576V at 298K.
Given, $E^\circ_{Cu^{2+}/Cu} = 0.3V$.

(1) 4 **(2)** 9
(3) 6 **(4)** 2

Q. 35. Which of the following represents log P vs log V variation as per Boyle's law ?

(1)


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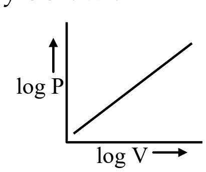

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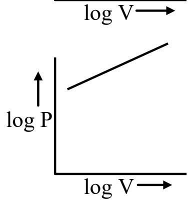
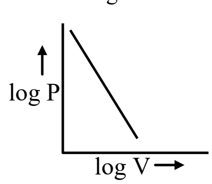


(4)


Q. 36. Find the work done when 2 moles of hydrogen expand isothermally from 15 to 50 litres against a constant pressure of 1 atm at 25°C.

(1) 847.0 cals **(2)** 847 k cal
(3) 84.7 cals **(4)** 84.7 k cal

Q. 37. In which of the following species O.N. per atom of the underlined elements is/are equal to + 1?

(1) $\underline{S}_2O_3^{2-}$, $\underline{P}_3O_9^{3-}$ **(2)** $\underline{P}_3O_9^{3-}$, $\underline{N}_2O$
(3) $H_3\underline{P}O_2$, $\underline{Fe}_2O_3$ **(4)** $\underline{N}_2O$, $H_3\underline{P}O_2$

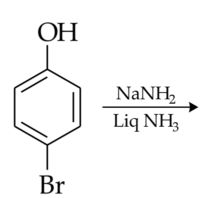
Q. 38. The decomposition of N_2O_4 to NO_2 was carried out in chloroform at 280°C. At equilibrium, 0.2 mol of N_2O_4 and 2 x 10^{-3} mol of NO_2 were present in 2 L of solution. The equilibrium constant for the reaction $N_2O_4 \rightleftharpoons 2NO_2$ is :

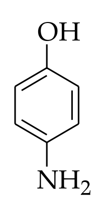
(1) 0.01×10^{-3} **(2)** 2.0×10^{-3}
(3) 2.0×10^{-5} **(4)** 1.0×10^{-5}

Q. 39. Boric acid (H_3BO_3) is :

(1) Monobasic and weak Lewis acid
(2) Tribasic and strong Lewis acid
(3) Monobasic and weak Bronsted acid
(4) Tribasic and weak Bronsted acid

Q. 40. The major product of the following reaction is:



(1)


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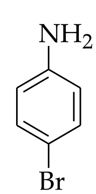
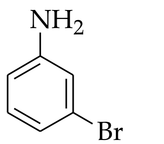
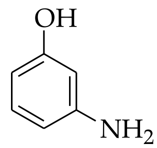


(3) 

(4) 

Q. 41. The major product of the following reaction is:

$$CH_3-\overset{\displaystyle Cl}{\overset{|}{C}H}-CH_3 \xrightarrow[\substack{\text{(ii) HBr, peroxide}\\ \text{(iii) aq. KOH}}]{\text{(i) Alc, KOH}}$$

(1) $CH_3-\overset{\displaystyle OH}{\overset{|}{C}H}-CH_3$

(2) $CH_3-CH_2-CH_2-OH$

(3) $CH_3CH=CH_2$

(4) $CH_3-\overset{\displaystyle Br}{\overset{|}{C}H}-CH_3$

Q. 42. In S_N1 reactions, the correct order of reactivity for the following compounds: CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is:

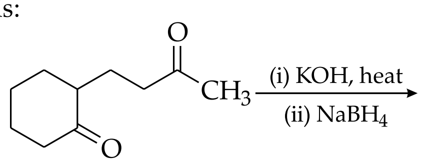
(1) $(CH_3)_3CCl > (CH_3)_2CHCl > CH_3Cl > CH_3CH_2Cl$

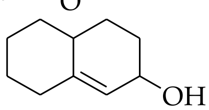
(2) $(CH_3)_3CCl > (CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl$

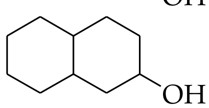
(3) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

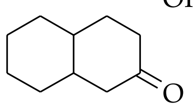
(4) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

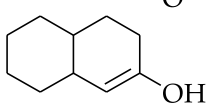
Q. 43. The major product of the following reaction is:



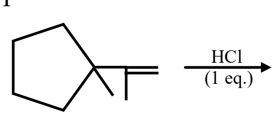
(1) 

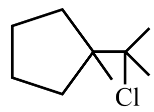
(2) 

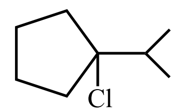
(3) 

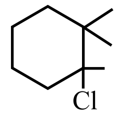
(4) 

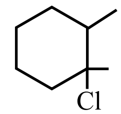
Q. 44. Major product of the reaction is :



(1) 

(2) 

(3) 

(4) 

Q. 45. Among the following statements on the nitration of aromatic compounds, the false one is :

(1) The rate of nitration of benzene is almost the same as that of hexadeuterobenzene

(2) The rate of nitration of toluene is greater than that of benzene

(3) The rate of nitration of benzene is greater than that of hexadeuterobenzene

(4) Nitration is an electrophilic substitution reaction

Q. 46. To prepare 3-ethylpentane-3-ol, the reactants needed are :

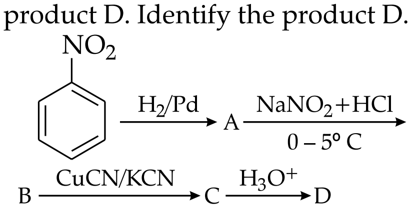
(1) $CH_3CH_2MgBr + CH_3COCH_2CH_3$

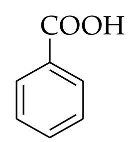
(2) $CH_3MgBr + CH_3CH_2CH_2COCH_2CH_3$

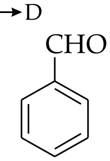
(3) $CH_3CH_2MgBr + CH_3CH_2COCH_2CH_3$

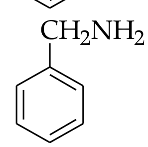
(4) $CH_3CH_2CH_2MgBr + CH_3COCH_2CH_3$

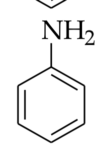
Q. 47. In a set of reactions nitrobenzene gave a product D. Identify the product D.



(1) 

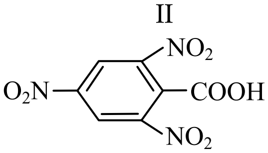
(2) 

(3) 

(4) 

Q. 48. Give the order of decarboxylation of the following acid :

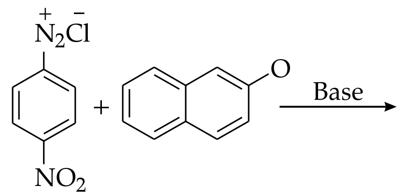
CH_3COOH; (I) $CH_2=CH-CH_2-COOH$; (II)

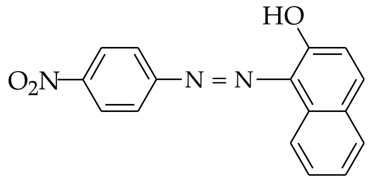
$CH_2(COOH)_2$; (III)  (IV)

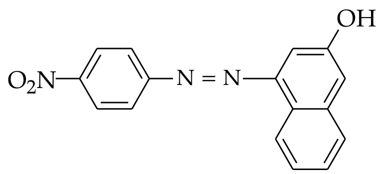
(1) I > II > III > IV **(2)** III > IV > II > I

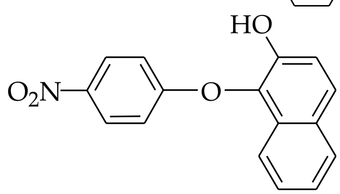
(3) IV > III > II > I **(4)** I > III > II > IV

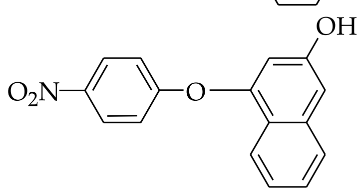
Q. 49. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

Q. 50. The number of asymmetric carbon atom in the glucose molecule in open and cyclic form is:

(1) Four, Five **(2)** Four, Four

(3) Five, Four **(4)** Five, six

Section B

Q. 51. The specific rate constant of the decomposition of N_2O_5 is 0.008 min^{-1}. The volume of O_2 collected after 20 minutes is 16 ml. The volume that would be collected at the end of reaction. NO_2 formed is dissolved in CCl_4 .... mL.

Q. 52. The e.m.f. of cell Zn | $ZnSO_4$ || $CuSO_4$| Cu at 25°C is 0.03 V and the temperature coefficient of e.m.f. is -1.4×10^{-4} V per degree. The heat of reaction for the change taking place inside the cell is .... kJ/mole.

Q. 53. The energy released in joule and MeV in the following nuclear reaction

$${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n$$

Assume that the masses of ${}_1^2H$, ${}_2^3He$ and neutron (n) respectively are 2.0141, 3.0160 and 1.0087 in amu, is .... $\times 10^{-13}$ J.

Q. 54. A unit cell of sodium chloride has four formula units. The edge length of the unit cell is 0.564 nm. The density of sodium chloride is .... g/cm^3.

Q. 55. If weight of the non-volatile solute urea (NH_2—CO—NH_2) to be dissolved in 100 g of water, in order to decrease the vapour-pressure of water by 25%, then the weight of the solute will be .... g.

Q. 56. Volume of N_2 at NTP required to form a monolayer on the surface of iron catalyst is 8.15 ml/g of the adsorbent. The surface area of the 100 g adsorbent if each nitrogen molecule occupies 16×10^{-22} m^2 will be .... m^2.

Q. 57. The number of π-bonds are present in marshall's acid is ....

Q. 58. The effective atomic number (EAN) of a metal carbonyl, $m(Co)_x$ is 36. The atomic number of the metal is 26. The value of 'x' is ....

Q. 59. The magnetic moment of central atom of $[Co(NH_3)_6]^{3+}$ is ……..

Q. 60. $Au + CN^- + H_2O + O_2 \rightarrow [Au(CN)_2]^- + OH^-$. The number of CN^- ions are involved in the balanced equation is ....

Mathematics

Section A

Q. 61. The statement $\sim (p \leftrightarrow \sim q)$ is:

(1) a tautology

(2) a fallacy

(3) equivalent to $p \leftrightarrow q$

(4) equivalent to $\sim p \leftrightarrow q$

Q. 62. If $\dfrac{\tan 2\theta + \tan\theta}{1 - \tan\theta \tan 2\theta} = 0$, then the general value of θ is :

(1) $n\pi; n \in I$ **(2)** $\dfrac{n\pi}{3}; n \in I$

(3) $\dfrac{n\pi}{4}; n \in I$ **(4)** $\dfrac{n\pi}{6}; n \in I$

Q. 63. The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be :

(1) 41 **(2)** 49

(3) 40.5 **(4)** 42.5

Q. 64. In a triangle ABC, in usual notation, $(a + b + c)(b + c - a) = \lambda bc$ will be true if :

(1) $\lambda < 0$ **(2)** $\lambda > 0$

(3) $0 < \lambda < 4$ **(4)** $\lambda > 4$

Q. 65. In an equilateral triangle of side $2\sqrt{3}$ cm, the circum radius is :

(1) 1 cm **(2)** $\sqrt{3}$ cm

(3) 2 cm **(4)** $2\sqrt{3}$ cm

Q. 66. The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\ldots\sqrt[p]{p}}}}}_{n \text{ radical signs}}$

where $p \geq 2, p \in \text{N}\,; n \in \text{N}$ when simplified is :

(1) independent of p

(2) independent of p and of n

(3) dependent on both p and n

(4) positive

Q. 67. The number of real solutions of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is:

(1) 1 **(2)** 2

(3) 0 **(4)** 3

Q. 68. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots$ are an A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots\ldots + a_{23} + a_{24}$ is equal to :

(1) 909 **(2)** 75

(3) 750 **(4)** 900

Q. 69. The middle term in the expansion of $(1-3x + 3x^2 - x^3)^6$ is :

(1) ${}^{18}C_{10}\, x^{10}$ **(2)** ${}^{18}C_9\, (-x)^9$

(3) ${}^{18}C_9\, x^9$ **(4)** ${}^{18}C_9\, x^{10}$

Q. 70. If ${}^{m+n}P_2 = 90$ and ${}^{m-n}P_2 = 30$, then (m, n) is given by :

(1) (7, 3) **(2)** (16, 8)

(3) (9, 2) **(4)** (8, 2)

Q. 71. The domain of function

$$f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}} \text{ is:}$$

(1) (1, 4) **(2)** (–2, 4)

(3) (2, 4) **(4)** (2, ∞)

Q. 72. $\lim\limits_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is:

(1) 1 **(2)** –1

(3) zero **(4)** does not exist

Q. 73. If $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ -1 \text{ if } x \text{ is irrational} \end{cases}$ is continuous on :

(1) R **(2)** ϕ

(3) (–1, 1) **(4)** (–1, 0)

Q. 74. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to :

(1) $\frac{(2^x+2^y)}{(2^x-2^y)}$ **(2)** $\frac{(2^x+2^y)}{(1+2^{x+y})}$

(3) $2^{x-y} \cdot \frac{2^y-1}{1-2^x}$ **(4)** $\frac{(2^{x+y}-2^x)}{2^y}$

Q. 75. If m be the slope of a tangent to the curve $e^{2y} = 1 + 4x^2$, then :

(1) $m < 1$ **(2)** $|m| \leq 1$

(3) $|m| > 1$ **(4)** $|m| \geq 1$

Q. 76. $\int \frac{(a^x - b^x)^2}{a^x b^x}\, dx$ equals :

(1) $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln\left(\frac{b}{a}\right)} + 2x + c$

(2) $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln\left(\frac{a}{b}\right)} + 2x + c$

(3) $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln\left(\frac{b}{a}\right)} - 2x + c$

(4) $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln\left(\frac{a}{b}\right)} - 2x + c$

Q. 77. $\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\, dx$ equals :

(1) $\log_e \frac{2}{3}$ **(2)** $\log_e 3$

(3) $\frac{1}{2}\log_e \frac{4}{3}$ **(4)** $\log_e \frac{4}{3}$

Q. 78. Area bounded by $y = \sec^2 x$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$ and x- axis is :

(1) $\frac{2}{\sqrt{3}}$ **(2)** $\frac{\sqrt{3}}{2}$

(3) $\frac{\sqrt{2}}{3}$ **(4)** $\sqrt{\frac{2}{3}}$

Q. 79. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$, is

(1) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

(2) $(x - 2) = ke^{2\tan^{-1} y}$

(3) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(4) $xe^{\tan^{-1} y} = \tan^{-1} y + k$

Q. 80. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = -1$ is :

(1) 1 **(2)** 2

(3) 3 **(4)** 4

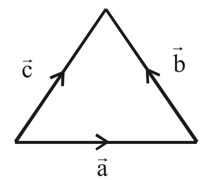
Section B

Q. 81. Two boxes are containing 20 balls each and each ball is either black or white. The total number of black balls in the two boxes is different from the total number of white balls. One ball is drawn at random from each box and the probability that both are white is 0.21 and the probability that both are black is k, then $\frac{100k}{13}$ is equal to ....

Q. 82. If $x \in$ R and $\begin{vmatrix} 8 & 2 & x \\ 2 & x & 8 \\ x & 8 & 2 \end{vmatrix} = 0$, then $\left|\frac{x}{2}\right|$ is equal to ....

Q. 83. If A is a square matrix such that $A(\text{adj}A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $\frac{|\text{adj}(\text{adj}A)|}{|\text{adj}A|}$ is equal to ....

Q. 84. If $\vec{a} = (\lambda x)\hat{i} + (y)\hat{j} + (4z)\hat{k}$, $\vec{b} = y\hat{i} + x\hat{j} + 3y\hat{k}$, $\vec{c} = -z\hat{i} - 2z\hat{j} - ((\lambda+1)x)\hat{k}$ are sides of triangle as shown is figure then value of λ is .... (where x, y, z are not all zero)



Q. 85. Consider a plane $2x + y - 3z = 5$ and the point P(–1, 3, 2). A line L has the equation $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{4}$. The co-ordinates of a point Q of the line L such that $\overrightarrow{PQ}$ is parallel to the given plane are (α, β, γ), then the product $\beta\gamma$ is ....

Q. 86. A rectangle PQRS has sides $\overrightarrow{PQ} = 11$ and QR = 5. A triangle ABC has P as orthocentre, Q as circumcentre, R as mid point of BC and S as the foot of altitude from A. Then length of BC is k, where $k/4$ is equal to ....

Q. 87. If two circles $x^2 + y^2 + 2n_1 x + 2y + \frac{1}{2} = 0$ and $x^2 + y^2 + n_2 x + n_2 y + n_1 = \frac{1}{2}$, intersect each other orthogonally where n_1, $n_2 \in$ I, then number of possible of ordered pairs (n_1, n_2) is ....

Q. 88. Let a variable point A be lying on the directrix of parabola $y^2 = 4ax$ $(a > 0)$. Tangents AB and AC are drawn to the curve where B and C are points of contact of tangents. The locus of centroid of ΔABC is a conic whose length of latus rectum is λ, then $\frac{\lambda}{a}$ is equal to ....

Q. 89. The ratio of the area of the ellipse and the area enclosed by the locus of mid-point of PS where P is any point on the ellipse and S is the focus of the ellipse, is equal to ....

Q. 90. If the radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > b)$ are $2r$ and r respectively, then $\frac{e_2^2}{e_1^2}$ is equal to (where e_1, e_2 are their eccentricities respectively)

Answers

Physics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
1	(2)	Kinematics	16	(3)	Equal potential surface
2	(4)	Kinetic theory of gases	17	(2)	Current Electricity
3	(4)	One Dimension	18	(2)	Magnetic Effect of Current
4	(2)	Projectile Motion	19	(2)	Ray Optics
5	(3)	Newton's Laws of Motion	20	(1)	Atomic Structure & Matter Wave
6	(1)	Friction	21	4.00	Nuclear Physics & Radioactivity
7	(2)	Circular Motion	22	8.00	Photoelectric Effect
8	(1)	Work Energy and Power	23	7.00	Refraction at Plane Surface
9	(4)	Electromagnetic field	24	5.00	Capacitance
10	(1)	Angular momentum	25	1.00	Electromagnetic Induction
11	(3)	Gravitation	26	8.00	Alternative Current
12	(3)	Simple harmonic motion	27	1375	Calorimetry
13	(1)	Properties of Matter	28	140	Kinetic Theory of Gases
14	(2)	Fluid Mechanics	29	1.00	Thermodynamics
15	(3)	Electrostatics	30	150	Sound Wave

Chemistry

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
31	(4)	Mole Concepts	46	(3)	Alcohol, Ether and Phenol
32	(1)	Chemical Bonding	47	(1)	Nitrogen Compound
33	(3)	Atomic Structure	48	(3)	Carboxylic Acid
34	(3)	Electrochemistry	49	(2)	Nitrogen Containing
35	(4)	Gaseous State	50	(1)	Biomolecules
36	(1)	Chemical Energetics	51	17.94	Chemical Kinetics
37	(4)	Redox Reaction	52	13.84	Electrochemistry
38	(4)	Chemical Equilibrium	53	5.22	Nuclear Chemistry
39	(1)	Ionic Equilibrium	54	2.16	Solid State
40	(4)	Haloarens	55	111.1	Solution
41	(2)	Alcohol	56	35.00	Surface Chemistry
42	(2)	Haloalkanes	57	4.00	p-Blocks
43	(4)	Carbonyl Compound	58	5.00	Coordination Compound
44	(3)	Halogen Derivative	59	0.00	Coordination Compound
45	(3)	Aromatic Compound	60	8.00	Metallurgy

Mathematics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
61	(3)	Mathematical Reasoning	76	(4)	Indefinite Integration
62	(2)	Trigonometric Equation	77	(4)	Definite Integration
63	(1)	Statistics	78	(1)	Area under the Curve
64	(3)	Solutions of Triangle	79	(3)	Differential Equations
65	(3)	Properties of Triangles	80	(2)	Complex Number
66	(1)	Logarithms	81	2.00	Probability

67	(3)	Quadratic Equations	82	5.00	Determinants
68	(4)	Progressions	83	9.00	Matrices
69	(2)	Binomial Theorem	84	0.00	Vectors
70	(4)	Permutation and Combination	85	6.00	Three Dimentional Geometry
71	(4)	Function	86	7.00	Straight Lines
72	(4)	Limit	87	2.00	Circles
73	(2)	Continuity	88	3.00	Parabola
74	(3)	Differentiation	89	4.00	Ellipse
75	(2)	Tangent & Normal	90	4.00	Hyperbola

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JEE (Main) MOCK TEST PAPER 1

ANSWERS WITH EXPLANATION

Physics

1. (2) Given that :

$x = at^3 \rightarrow v_x = \frac{dx}{dt} = 3at^2 \rightarrow a_x = \frac{dv_x}{dt} = 6at$

and $y = 2at \rightarrow v_y = \frac{dy}{dt} = 2a \rightarrow a_y = \frac{dv_y}{dt} = 0.$

$\therefore \quad a^2 = a_x^2 + a_y^2$

$\rightarrow \quad a^2 = ax^2 + O$

$\therefore \quad a = ax$

$a = 6at$

$at\ t = 1$

$a = 6a$

2. (4) We define the mean free path as the average distance a gas particle travels before colliding.

$\therefore \quad n = [M^0 L^{-3} T^0]\ d = [M^0 L^1 T^0]$

$$\therefore \frac{1}{\sqrt{2}nd^2} = \frac{1}{[M^0L^{-3}T^0]}[M^0L^1T^0]^2 = [M^0L^1T^0]$$

3. (4) Distance covered with 1 step = 1 m

Time taken = 1 s

Time taken to move 5 m forward = 5 s

Time taken to move 3 m backward = 3 s

Net distance covered = 5 – 3

= 2 m

Net time taken to cover 2 m = 8 s

Drunkard covers 2 m in 8 s

Drunkard covers 4 m is 16 s, 6 m in 24 s, 8 m in 32 s. In next 5s he he will fall into pit.

In next 5s, he will fall into pit

Net time taken by drunkard = 32 + 5 = 37 s

4. (2) Given : T – time period, H – maximum height

$$\frac{H}{T} = \frac{\frac{u^2 \sin^2\theta}{2g}}{\frac{2u\sin\theta}{g}} = \frac{1}{4}\ u \sin\theta.$$

5. (3) The two projected particle will collide if the the particle will be at the same position in the same time.

$$t = \frac{x}{u_1\cos\theta_1 - u_2\cos\theta_2} \quad \text{... (1)}$$

And $u_1\sin\theta_1 \cdot t - \frac{1}{2}gt^2 = u_2\sin\theta_2 \cdot t - \frac{1}{2}gt^2$

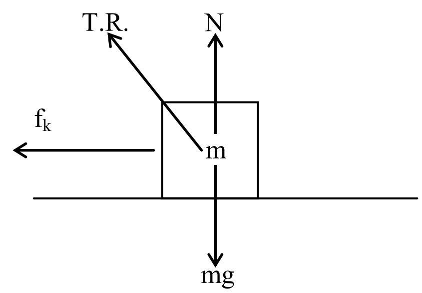
$\therefore \quad u_1\sin\theta_1 = u_2\sin\theta_2$ ...(2)

from equation no. (1) & (2)

$$t = \frac{x}{u_1\cos\theta_1 - \frac{u_1\sin\theta_1\cdot\cos\theta_2}{\sin\theta_2}}$$

$$\therefore \quad t = \frac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$$

6. (1)



The only electromagnetic forces acting on the box are the normal force and the friction.

$f_k = \mu\, mg$

T.R. $= \sqrt{N^2 + f^2}$

$= \sqrt{(mg)^2 + (\mu mg)}$

$= mg\sqrt{1+\mu^2}$

7. (2) Given, f = 1200 rpm

Now, converting in seconds

and $\quad f = \frac{1200}{60} = 20\ \frac{\text{rps}}{\text{sec}}$

Angular velocity of particle.

$\omega = 2\pi f$

$= 20 \times 2\pi = 40\pi$ rod/s

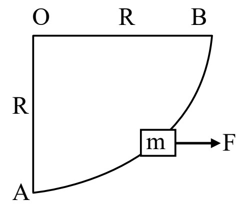
Now, acceleration $a = \omega^2 r$

$= (40\pi)^2 \times \frac{30}{100}$

$= 4737\ m/s^2$

$\sim 4740\ m/s^2$

8. (1)



A block of mass m is taken from A to B under constant force F.

$\therefore$ work done $\quad W = \vec{F}\cdot\vec{d}$

$$W_{AB} = \vec{F}x\cdot\vec{R}x + \vec{F}x\cdot\vec{R}y$$

$$= FR\cos 0^\circ + FR\cos 90^\circ$$

$\Rightarrow \quad W_{AB} = FR$

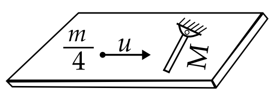
9. (4) Let the final speed of the trolley becomes V
Initial momentum of the system in x direction = $(20 + 40) \times 8$ when the monkey jumped off from the trolley, then momentum along x direction =(40) V
Now, applying the conservation of momentum along the x axis,

$60 \times 8 = 40V$

$V = 12$ m/s

Hence, speed of the trolley will become 12 m/s.

10. (1)



Angular momentum of bullet with respect to the poivot :

$$L_B = \frac{m}{4}u\left(\frac{L}{2}\right) = \frac{muL}{8}$$

Angular momentum of system when the bullet hits rod :

$$L_S = [I_B + I_R]\,\omega = \left[\frac{m}{4}\left(\frac{L}{2}\right)^2 + \frac{1}{3}mL^2\right]\omega$$

$$= \frac{19}{48}mL^2\omega$$

By conservation of angular momentum

$$\frac{mul}{8} = \frac{19}{48}ml^2\omega$$

$$\omega = \frac{6}{19}\frac{u}{l}$$

11. (3)

$$g_e = \frac{Gm_e}{R^2}$$

(Given R $\rightarrow$ Radius of earth,
$\rho \rightarrow$ mean density of earth)

$$g_e = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2}\ ;\ g_e = \frac{4}{3}\pi R\,\rho G$$

$$\rho = \frac{3}{4}\cdot\frac{g_e}{\pi RG}$$

12. (3) $x = 7\cos 0.5\,\pi t$ general form of equation.
$x = A\cos\omega t$
Given equation compare with standard form

$$\omega = \frac{2\pi}{T} = 0.5\,\pi;$$

$$T = 4\text{ s}$$

$$T' = \frac{T}{4} = \frac{4}{4} = 1\text{ s}$$

13. (1)

$$Y = \frac{\frac{F}{A}}{\frac{\ell}{L}} = \frac{FL}{A\ell} \text{ on comparing}$$

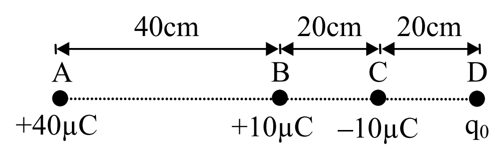
$$\Rightarrow \quad F = \frac{YA}{L}\ell$$

$$\Rightarrow \quad F = k\,x$$

$$k = \frac{YA}{l}$$

14. (2) Liquid pressure is always normal to the surface
Hence thrust per unit area at point
$P = (H - h)\,\rho g$.

15. (3)



Force on B due to A $(+x)$ $f_{AB} = \dfrac{Kq_Aq_B}{r_{AB}^2}$

$$= \frac{9\times10^9\times40\times10^{-6}\times10\times10^{-6}}{\left(40\times10^{-2}\right)^2} = \frac{90}{4}$$

$$= 22.5\text{ N}$$

Force B due to C $(+x)$$f_{CB} = \dfrac{Kq_Cq_B}{r_{BC}^2}$

$$= \frac{9\times10^9\times10\times10^{-6}\times10\times10^{-6}}{\left(20\times10^{-2}\right)^2} = \frac{90}{4} = 22.5\text{ N}$$

Force on B due to D $(-x)$ $f_{BD} = \dfrac{Kq_Bq_D}{r_{BD}^2}$

$$= \frac{9\times10^9\times10\times10^{-6}\times q_0}{\left(40\times10^{-2}\right)^2} = \frac{9}{16}\times10^6\,q_0$$

For the equilibrium,

$$22.5 + 22.5 = \frac{9}{16}\times10^6\times q_0$$

$$= \frac{45 \times 16}{9 \times 10^6} = q_0$$

$$q_0 = +80 \times 10^{-6}$$

$$q_0 = +80 \mu C$$

16. (3) Hence, the correct option is (3).

$$\varepsilon = \frac{d\phi}{dt} \Rightarrow \int_{\phi_1}^{\phi_2} d\phi = \Delta\phi = \int \varepsilon dt$$

The total charge flown in the loop $q = \int I dt$

$$\text{or } q = \int \frac{\varepsilon}{R} dt = \frac{\Delta\phi}{R} = \frac{B\pi r^2}{R}$$

$q \propto B$, $q \propto r^2$, and $q \propto \dfrac{1}{R}$

17. (2) Given $I = 1.1$ A

$$e = 1.6 \times 10^{-19} C$$

$$A = \pi r^2 = 3.14 \times (0.05)^2$$

$$= 78.5 \times 10^{-4} \text{ cm}^2$$

$$n = \frac{6 \times 10^{23}}{7 \text{ cm}^3} = 0.86 \times 10^{23}$$

$$v_d = \frac{I}{neA}$$

$$= 0.86 \times 10^{23} / \text{m}^3$$

$$v_d = \frac{1.1}{0.86 \times 1.6 \times 10^{-19} \times 78.5 \times 10^{-4}}$$

(volume of 63g Cu)

$$v_d = 0.01 \text{ cm/s}$$

$$= 0.1 \text{ mm/s}$$

18. (2) Refer to figure given in question
For magnetic field at centre to be zero

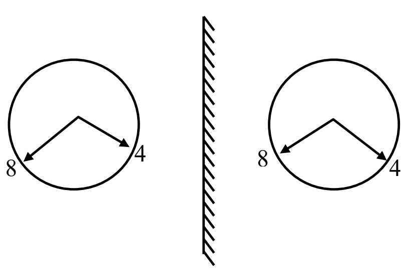
$$\frac{\mu_0 I dl}{4\pi R^2} = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R}$$

$$\frac{\mu_0 I R\theta}{4\pi R^2} = \frac{\mu_0 I}{4\pi}\left[\frac{1}{R} + \frac{1}{R}\right]$$

$\because$ $dl = R\theta$

or $\theta = 2$ rad

19. (2)



Mirror turns left into right and right into left. Hence actual time is 3 : 40

20. (1) $L = \dfrac{nh}{2\pi}$; for $n = 1$, $L = \dfrac{h}{2\pi}$

21. [4.00] ${}^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + e^- + \overline{\nu}$

$$Q = [m(^{23}NF) - m(^{23}\text{Na})] \times 931.5 \text{ MeV}$$

$$Q = 4.375 \text{ MeV} = 4.4 \text{ MeV}$$

$$Q = 4 \text{ MeV}$$

$$Q = KE_y + KE_e + E\overline{\nu}$$

KE_y is very very small

$$Q \approx KE_e + E\overline{\nu}$$

when KE_e is maximum $E\overline{\nu}$ is negligible

$$KE_e = Q = 4 \text{ MeV}$$

22. [8.00] $eV_s = h\nu - W$

$$= 12\text{eV} - 4\text{eV}$$

$$eVs = 8\text{eV or } Vs = 8\text{V}$$

23. [7.00] As per Snell's rule,

$\sin(i)/\sin(r) = n_2/n_1$

Here, $i = 30°$

$r = \sin^{-1}(5/6)$

$\sin r = 5/6$

$n_2 = ?$

$n_1 = 5/3$

$$\frac{5}{3} \sin 30° = n_2 . \frac{5}{6} \Rightarrow n_2 = 1$$

$$\frac{n_2}{n_1} = \sin c$$

$$\frac{1}{5/3} = \sin c$$

$$\frac{3}{5} = \sin c$$

$$c = 37°$$

So, required difference in angle of incidence be $37° - 30° = 7$

24. [5.00] $10^{-10} - x$ | | x $-x$ | | $2 \times 10^{-10} + x$

A B

Let the induced charge be x,
At the steady state, the potential will be equal, we know that potential due to charged plate $V = [\frac{Q}{2}(\varepsilon \times S)] \times d$

$$10^{-10} - x = 2 \times 10^{-10} + x$$

$$2x = -10^{-10}$$

$$x = -5 \times 10^{-11} \text{ C}$$

25. [1.00] Instantaneous flux

$$= \pi a^2 B \cos 0° + \pi b^2 B \cos 180°$$

$$= \pi(a^2 - b^2) B$$

$$\phi = \pi(a^2 - b^2) B_0 \sin \omega t$$

$$l = \frac{d\phi}{dt}$$

$$i = \frac{\ell}{R}$$

$$i = \frac{\pi\left(a^2 - b^2\right)B_0\omega\cos\omega t}{R}$$

$$R = \rho \times 2\pi\,(a + b)$$

$$\therefore \quad i_{max} = \frac{1}{2\ell}(a-b)B_0\omega = 1 \text{ Amp}$$

26. [8.00] At resonance reactance = 0

$$I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} \text{ Amp.}$$

$$V_L = I \times X_L = I \times \omega L$$

$$\therefore \quad L = \frac{V_L}{I\omega} \qquad \dots(1)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{L\omega_0^2} \qquad \dots(2)$$

Calculate L and C from (1) and (2) current will lag the applied voltage by 45°

$$\text{if } \tan 45° = \frac{\omega L - \frac{1}{\omega c}}{R}$$

Solve for ω

$$\omega = 8 \times 10^5 \text{ rad/s}$$

27. [1375] Relation between temperature on the unknown X scale to unknown scale Y,

$(X - T_2)/(T_2 - T_1) = (Y - T_3)/(T_4 - T_3)$

Here, $T_1 = 375X$

$T_2 = -125X$

$T_3 = -70Y$

$T_4 = -30Y$

$Y = 50$

$X = ?$

Now, substituting the values in the equation

$$\frac{X-(-125)}{500} = \frac{Y-(-70)}{40}$$

if $Y = 50$

$X = 1375°X$

28. [140]

$$\frac{\Delta Q}{W} = \frac{nC_p\Delta T}{nR\Delta T}$$

$$\Rightarrow \quad \Delta Q = \frac{C_P}{R}\,.\,W$$

$$= \frac{7}{2} \times 20$$

$$= 140 \text{ J}$$

29. [1.00] Work done by gas

$$= \pi \cdot \frac{\left(\frac{400}{\pi}\times 10^3\right)}{2} \times \frac{\left(20\times 10^{-6}\right)}{2} \text{J} = 1 \text{ J}$$

30. [150] $I = I_m\cos^2\phi$

(ϕ = Phase difference,

I_m = maximum intensity)

$$I = \frac{I_m}{2}$$

$$\Rightarrow \quad I = I_m\cos^2\phi$$

Now, substituting the value of $I = \frac{I_m}{2}$

$$\Rightarrow \quad \cos^2\phi = \frac{1}{2}$$

$$\Rightarrow \quad \cos\phi = \frac{1}{\sqrt{2}}$$

$$\cos\phi = \cos\frac{\pi}{4}$$

We know that the general equation of wave $x = a\sin(\text{wt-}\,k\,\Delta x)$

$$\phi = 2\pi \times \frac{\Delta x}{\lambda}$$

$$\frac{\pi}{4} = 2\pi \times \frac{6.25}{\lambda} \times 100$$

$$\lambda = \frac{1}{2}m$$

$$v = n \times \lambda$$

$$v = 300 \times \frac{1}{2}$$

$$v = 150 \text{ m/s}$$

Chemistry

31. (4)

First oxide	Second oxide
Sulphur 50%	40%
Oxygen 50%	60%

The amount of sulphur that reacts with the oxygen in oxide 1 is $= \frac{50}{50} = 1$

Similarly, for second oxide the amount of sulphur that combines with oxygen $= \frac{40}{60} = 0.67$

Thus, the ratio of sulphur in two oxides are is 1:0.67 or 3 : 2

32. (1) Molecules that have identical hybridisation would have identical shapes.

	Type of hybridization	Geometry
$BeCl_2$	$Cl \overset{s}{—} Be \overset{p}{—} Cl$ $H = \frac{1}{2}(2 + 2 - 0 + 0) = 2$ $H = 2 = sp$	Linear
XeF_2	$F \overset{d}{—} \ddot{Xe} \overset{p}{—} F$ (s, p, p) $H = \frac{1}{2}(8 + 2 - 0 + 0) = 5$ $H = 5 = sp^3d$	Linear
CO_2	$O \overset{s}{=} C \overset{p}{=} O$ $H = \frac{1}{2}(4 + 0 - 0 + 0) = 2$ $H = 2 = sp$	Linear

33. (3) The electron configuration of potassium atom is $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 4s^1$.
Quantum numbers for the valence electron in 4*s* orbital. Principle quantum number (n) is 4
Azimuth quantum number (l) for *s* orbital is 0.
Magnetic quantum numbers (m_l) for *s* orbital is 0.
Spin quantum numbers (s) is 1/2.

34. (3) According to Nernst equation

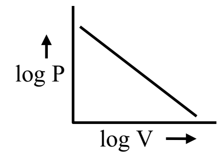
$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log \frac{[P]}{[R]}$$

For the given cell

$$E_{cell} = 0.576V,\ E^\circ_{cell} = 0.34V$$
$$n = 2$$
$$0.576 = 0.34 - \frac{0.0591}{2} \log \frac{[H^+]^2}{[Cu^{2+}]}$$
$$0.236 = \frac{0.0591}{2} \times 2 \log \frac{[H^+]}{0.01}$$
$$3.993 = [\log H^+ - \log 0.01]$$
$$3.993 = \log 10^{-2} - \log H^+$$
$$3.993 = -2 - \log H^+$$
$$5.993 = -\log [H^+]$$

Also $pH = -\log [H^+]$
$$= 5.993 \approx 6.$$

35. (4) $PV = RT = k$ (Constant) (Boyle's law)
Taking log both side
$\log P + \log V = \log k$
$\log P = -\log V + \text{constant}$
comparing the above equation with straight line equation, $y = mx + c$
so



36. (1) Here,
$$n = 2$$
$$V_1 = 15\ l$$
$$V_2 = 50\ l$$
Temperature, T = 25° = 298k
Pressure, P = 1 atm
Work done $= -P(V_2 - V_1)$
$= -1\,(50 - 15)$
$= -1 \times 35$
$= -35\ l$/atm
As $1\ l$/atm = 101.3 J
Therefore, $-35\ l$ atm $= -35 \times 101.3$
$= -3545.5$ J
As 1 calorie = 4.184 J

So, $-3545.5\text{J} = \frac{-3545.5}{4.184}$ cal
$= -848.2$ cals

37. (4) N_2O
$2x - 2 = 0$
$\Rightarrow \quad x = +1$
$H_3\underline{P}O_2$
$3\,(1) + x + 2\,(-2) = 0$
$x = 4 - 3 = +1$

38. (1)
$$N_2O_4 \rightleftharpoons 2NO_2$$

At equilibrium $\frac{0.2}{2}$ $\qquad \frac{2\times10^{-3}}{2}$ (2ℓ)

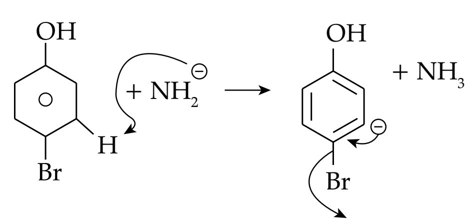
$0.1 \qquad 1 \times 10^{-3}$

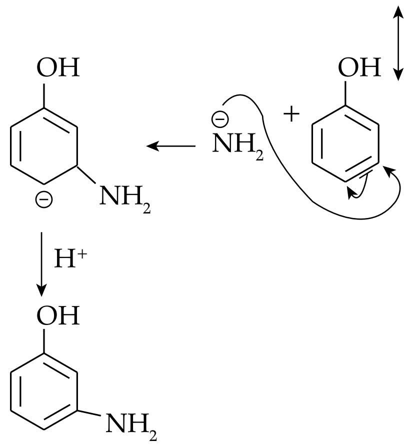
$$Kc = \frac{(1\times10^{-3})^2}{0.1} = 10^{-5}$$

39. (1) Boric acid H_3BO_3, is monobasic and works as Lewis acid according to the following reaction.

$$H_3BO_3 + H_2O \rightarrow B(OH)_4^- + H^-$$

40. (4)





41. (2) $CH_3-\underset{}{\overset{Cl}{\overset{|}{C}}}H-CH_3 \xrightarrow[\text{Alcohol}]{\text{KOH}} CH_3CH=CH_2$

2-chloropropane Heat

$\xrightarrow[\text{Peroxide}]{\text{HBr}} CH_3-CH_2-CH_2-Br \xrightarrow{\text{aq. KOH}}$

$CH_3-CH_2-CH_2-OH$

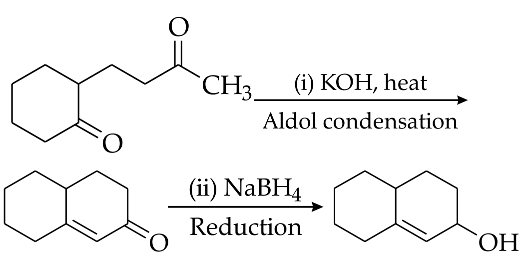
propan-1-ol

42. (2) The order of reactivity in S_N1 reaction is mainly dependant on stability of carbocation, formed thus the order of reactivity of the given compounds are as follows.

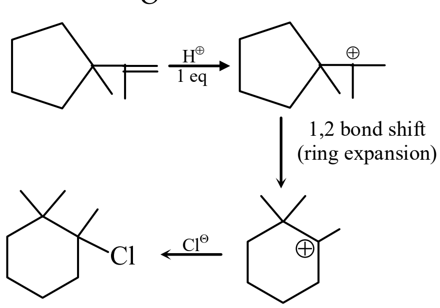
$$CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Cl > CH_3-\overset{Cl}{CH}-CH_3$$

$$> CH_3-CH_2-Cl > CH_3-Cl$$

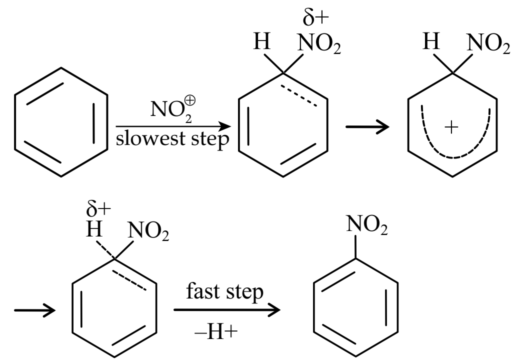
43. (4)



44. (3)



45. (3)



The rate determining step in electrophilic substitution reaction, is the bonding of the electrophile to the aromatic ring without cleavage of C – H or C – D bond.
This bond is broken in the fast step (second step) that restore the stable aromatic system. Also the bond strength of C-H and C-D bonds are equal. Hence the rate of Nitration of benzene is almost the same as that of Hexa deuterobenzene.

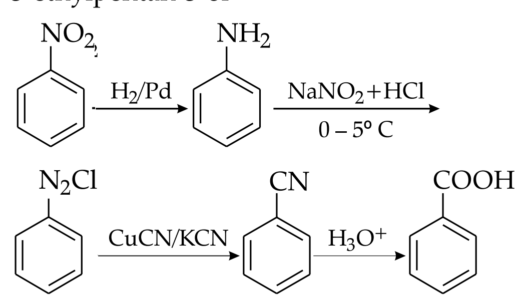
46. (3) $CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_2-CH_3 + CH_3\overset{-}{C}H_2\overset{+}{M}gBr \rightarrow$

$$CH_3-CH_2-\underset{OMgBr}{\overset{CH_2CH_3}{C}}-CH_2-CH_3 \xrightarrow{H^+/HOH}$$

$$CH_3-CH_2-\underset{OH}{\overset{CH_2-CH_3}{C}}-CH_2-CH_3$$

3-ethylpentan-3-ol

47. (1)



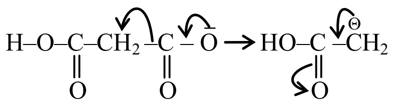
48. (3) The reactivity of decarboxylation depends upon the stability of the conjugate base .The conjugate bases of the given compounds are as follows:

$$CH_3-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O} \longrightarrow \overset{\ominus}{C}H_3 + CO_2$$

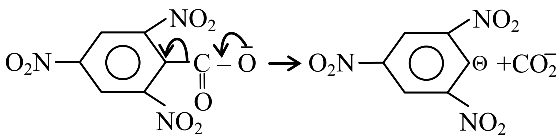
unstable (I)

$$CH_2=CH-CH_2-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O} \rightarrow CH_2=CH-\overset{\ominus}{C}H_2 + CO_2$$

Stable due to conjugation (II)



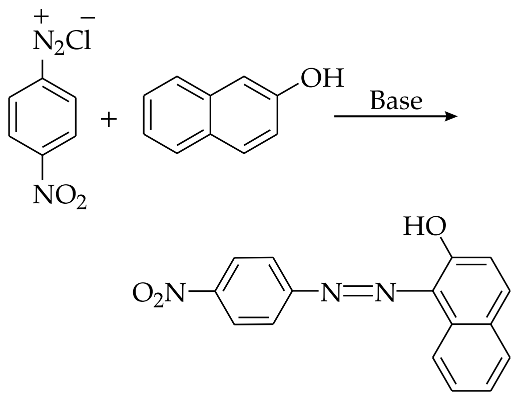
stabilised due to
resonance (III)



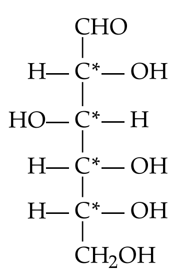
stabilised due
to resonance

(IV) conjugate base is more stable than (III) as it has more resonating structures. Therefore, stability of carbanion α-decarboxylation is IV > III > II > I.

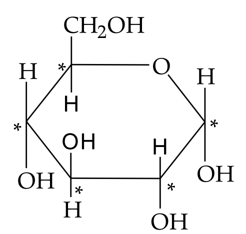
49. (2)



50. (1)



D–(+)–Glucose



α–D–(+)–Glucopyranose
(Haworth structure)

There are four asymmetric carbon in open structure of glucose,whereas there are five asymmetric carbon in cyclic form.

51. [17.49] Given $k = 0.008\ \text{min}^{-1}$

From unit of k, the reaction is a first order reaction.

From $\therefore \quad k = \dfrac{2.303}{t} \log \dfrac{V_\infty}{V_\infty - V}$

$\Rightarrow \quad 0.008 = \dfrac{2.303}{20} \log \dfrac{V_\infty}{V_\infty - 16}$

$\Rightarrow \quad 0.0695 = \log \dfrac{V_\infty}{V_\infty - 16}$

$\Rightarrow \quad V_\infty = 17.49\ \text{mL}$

52. [13.842] According to Gibb's Helmholtz equation,

heat of reaction ΔH, given as

$$\Delta H = nF\left[T\left(\frac{\delta E}{\delta T}\right)_P - E\right]$$

$T = (273 + 25)\ K$

$= 298\ K,\ n = 2,$

$F = 96500\ C,\ E = 0.03\ CV$

$\left(\dfrac{\delta E}{\delta T}\right)_P = -1.4 \times 10^{-4}\ V/K$

$\Delta H = 2 \times 96500\,[298 \times (-1.4 \times 10^{-4})] - 0.03$

$= -13842\ J = -13.842\ kJ/mole$

53. [5.22] $\Delta m = [2 \times 2.0141] - 3.0160 - 1.0087$

$= 3.5 \times 10^{-3}$ amu

$\Delta E = \Delta m \times 931.478$ MeV

$\Delta E = 3.5 \times 10^{-3} \times 931.478$

$= 3.260$ MeV.

$\Delta E = 5.22 \times 10^{-13}$ J

54. [2.16] Density of solid $= \dfrac{n \times \text{At.wt}}{\text{Av.no.} \times a^3}$

For NaCl

n= 4(fcc)

At.wt = 58.5

$= \dfrac{4 \times 58.5}{6.023 \times 10^{23} \times \left(5.64 \times 10^{-7}\right)^3}$

$= 2.16\ g/cm^3$

55. [111.10] Assume the vapoar pressure of water = 100

$\therefore$ Vapour pressure of urea solution = 75

Weight of urea = w_1

Molecular weight of urea = Mw_1

Weight of water = w_2

Molecular weight of water = Mw_2

By Raoult's law

$$= \frac{\dfrac{w_1}{Mw_1}}{\dfrac{w_2}{Mw_2} + \dfrac{w_1}{Mw_1}}$$

So, $\dfrac{100 - 75}{100} = \dfrac{\dfrac{w_1}{60}}{\dfrac{100}{18} + \dfrac{w_1}{60}}$

$\Rightarrow$ Weight of urea $w_1 = 111.1$ g

56. [35.00] Given, P = 1atm

volume = 8.15ml/g=0.00815L/g

T = 273K

$R = 0.0821 L\ atm/mole\ K$

From ideal gas equation

$PV = nRT$

Now substituting these values in the ideal gas equation:

$n = 1\ atm \times 8.15 \times 10 - 3L/gx\ 0.0821\ L\ atm/mole\ K \times 273\ K$

Solving this we will get the value of

$n = 3.64 \times 10^{-4}$ mole/g.

$\therefore$ no of molecules

$= 0.036 \times 6.023 \times 10^{23}$

$= 2.19 \times 10^{22}$

$\therefore$ Area required

$= 2.19 \times 10^{22} \times 16 \times 10^{-22}$

$= 35.00\ m^2$

57. [4.00]

```
         O         O
         ||        ||
HO – S – O – O – S – OH
         ||        ||
         O         O
```

$\Rightarrow$ π-bonds = 4.00

58. [5.00] $EAN = 36 = 26 + 2 \times x$

or $2x = 10$

$x = 5.00$

59. [0.00] $Co = 27 = [Ar]\ 3d^7\ 4s^2$

Co oxidation state : 3^+

$Co^{3+} = 24\ [Ar]\ 3d^6\ 4s^0$

Orbitals of Co^{3+} ion	3d: ↑↓ ↑ ↑ ↑ ↑ ; 4s: ☐ ; 4p: ☐ ☐ ☐
$[Co(NH_3)_6]^{3+}$	3d: ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ; 4s: ↑↓ ; 4p: ↑↓ ↑↓ ↑↓ ; $\underbrace{\qquad}_{d^2sp^3}$
Geometry shape	Octahedral
Type of hybridization	d^2sp^3
No. of unpaired electrons (n)	0
Magnetic nature	Diamagnetic
Nature of complex	Low spin complex, inner orbital complexes
Magnetic moment calculation $\mu = \sqrt{n(n+2)}$	0

60. [8.00] $4Au + 8CN^- + 2H_2O + O_2 \rightarrow 4[Au(CN)_2]^- + 4OH^-$

Mathematics

61. (3)

p	q	$\sim q$	$p \leftrightarrow \sim q$	(i) $\sim (p \leftrightarrow \sim q)$	(ii) $p \leftrightarrow q$
F	F	T	F	T	T
F	T	F	T	F	F
T	F	T	T	F	F
T	T	F	F	T	T

From column (i) and (ii) are equivalent, $\sim (p \leftrightarrow \sim q)$ and $p \leftrightarrow q$

62. (2) $\dfrac{\tan 2\theta + \tan \theta}{1 - \tan\theta \tan 2\theta} = 0$

$\Rightarrow \tan(3\theta) = 0$

$\Rightarrow 3\theta = n\pi$

$\Rightarrow \theta = \dfrac{n\pi}{3}, n \in I$

63. (1) Correct mean $\dfrac{20 \times 40 - 33 + 53}{20} = 41$

Correct option : (1) 41

64. (3) $\because (b+c)^2 - a^2 = \lambda bc$

$\Rightarrow b^2 + c^2 - a^2 + 2bc = \lambda bc$

$\Rightarrow 2bc \cos A + 2bc = \lambda bc$

(from cosine rule)

$\Rightarrow 2(\cos A + 1) = \lambda$

$\Rightarrow \cos A = \dfrac{\lambda}{2} - 1$

But $-1 < \cos A < 1$

$\Rightarrow -1 < \dfrac{\lambda}{2} - 1 < 1$

$\Rightarrow 0 < \dfrac{\lambda}{2} < 2$

$\Rightarrow 0 < \lambda < 4$

65. (3) $\because$ Triangle is equilateral, so

$\Delta = \dfrac{\sqrt{3}}{4}a^2$ and $R = \dfrac{a^3}{4\Delta}$

$\Rightarrow R = \dfrac{a^3}{\frac{4\sqrt{3}}{4}a^2}$

$= \dfrac{a}{\sqrt{3}}$

$\because \quad a = 2\sqrt{3}$

$\Rightarrow \quad R = 2$

Shortcut Method:

We have $a = 2R \sin 60°$

$\Rightarrow \quad 2\sqrt{3} = 2R \cdot \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad R = 2$

66. (1) $\log_p \log_p p^{p^{\frac{1}{n}}} = \log_p \log_p p^{p^{-n}}$

$= \log_p p^{-n} \log_p p$

$= -n \log_p p = -n$

67. (3) $\left(\dfrac{9}{10}\right)^x = -(x^2 - x + 3)$

$\Rightarrow \quad \left(\dfrac{9}{10}\right)^x = -\left\{\left(x - \dfrac{1}{2}\right)^2 + \dfrac{11}{4}\right\}$

LHS is always positive while RHS is always negative. Hence LHS $\neq$ RHS

$\therefore$ No solution

68. (4) We know that in an A.P. $a_1 + a_{24} = a_5 + a_{20}$

$= a_{10} + a_{15} = a_{12} + a_{13}$

So, $3(a_{12} + a_{13}) = 225$

$\Rightarrow \quad a_{12} + a_{13} = 75$

Therefore,

$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_{23} + a_{24}$

$= 12(a_{12} + a_{13})$

$= 12 \times 75 = 900$

69. (2) $(1 - 3x + 3x^2 - x^3)^6 = (1 - x)^{18}$

If in the expansion of $(1 - x)^n$, is even, then the middle term is $\left(\dfrac{n+2}{2}\right)$

So, the middle term is $\dfrac{18+2}{2} = 10^{th}$ term

$T_{10} = {}^{18}C_9(-x)^9$

70. (4) $\Rightarrow (m + n)(m + n - 1) = 90 = 10 \times 9$

$\Rightarrow m + n = 10 \quad \ldots(1)$

and $(m - n)(m - n - 1) = 30 = 6 \times 5$

$\Rightarrow m - n = 6 \quad \ldots(2)$

Solving eq.(1) and (2) we get

$m = 8, \quad n = 2$

71. (4) For $x^2 + 2x + 8 > 0$ here $D = 4 - 8(4) < 0$

$\therefore \quad x^2 + 2x + 8 > 0 \quad \forall\, x \in R$

$-\log_{0.3}(x - 1) \geq 0$

$\Rightarrow \quad \log_{0.3}(x - 1) \leq 0$

$\Rightarrow \quad (x - 1) \geq 1$

$\Rightarrow \quad x \geq 2$

Also, $x - 1 \neq 1$

$\Rightarrow \quad x \neq 2$

$\therefore$ Domain is $R \cap [2, \infty) = [2, \infty)$

72. (4) $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$;

$\lim\limits_{x \to 0} \dfrac{\sqrt{2\sin^2 x}}{\sqrt{2}x} = \lim\limits_{x \to 0} \dfrac{|\sin x|}{x} \left[\because \lim\limits_{x \to 0} \dfrac{\sin\theta}{\theta} = 1\right]$

The above limit does not exist as

LHL $= -1 \neq$ RHL $= 1$

73. (2) $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ -1 \text{ if } x \text{ is irrational} \end{cases}$

Let 'a' is any rational number

$\Rightarrow \quad f(a) = 1$

Then, $\lim\limits_{\substack{x \to a \\ x \in Q}} f(x) = 1 = f(a)$

and $\lim\limits_{\substack{x \to a \\ x \to Q^C}} f(x) = -1 \neq f(a)$

$\Rightarrow f(x)$ is not continuous at any rational number.

Now, Let $a \in Q^C \Rightarrow f(a) = -1$

Then, $\lim\limits_{\substack{x \to a \\ x \in Q}} f(x) = 1 \neq f(a)$

and $\lim\limits_{\substack{x \to a \\ x \to Q^C}} f(x) = -1 = f(a)$

$\Rightarrow f(x)$ is not continuous at any irrational number.

$\therefore$ The set of points of continuity $= \phi$

74. (3) $2^x + 2^y = 2^{x+y}$

Differentiating both the sides of above equation w.r.t. x, we get

$\Rightarrow 2^x \ln 2 + 2^y \ln 2 \dfrac{dy}{dx} = 2^{x+y} \ln 2\left(1 + \dfrac{dy}{dx}\right)$

$\dfrac{dy}{dx} = \dfrac{\left(2^x \ln 2 - 2^y 2^x \ln 2\right)}{\left(2^y \ln 2 - 2^y 2^x \ln 2\right)}$

$= -2^{x-y}\left[\dfrac{1 - 2^y}{1 - 2^x}\right]$

$\Rightarrow \quad \dfrac{dy}{dx} = 2^{x-y}\left[\dfrac{2^y - 1}{1 - 2^x}\right]$

75. (2) $e^{2y} = 1 + 4x^2$

Taking logartihm on both sides of the above equation

$$2y = \log_e(1+4x^2)$$
$$y = \frac{1}{2}\log_e(1+4x^2)$$
$$\frac{dy}{dx} = \frac{1}{2}\times\frac{1}{1+4x^2}\times 4\times 2x = \frac{4x}{1+4x^2}$$
$$\frac{dy}{dx} = \frac{4x}{1+4x^2} = m$$
$$\Rightarrow \quad 4mx^2 - 4x + m = 0$$
for $x \in R$,

Discriminant ≥ 0
$$\Rightarrow \quad 16 - 16m^2 \geq 0$$
$$\Rightarrow \quad |m| \leq 1$$

76. (4) Let $I = \int \frac{a^{2x}+b^{2x}-2a^xb^x}{a^xb^x}\,dx$
$$= \int\left[\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x - 2\right]dx$$
$$= \left(\frac{a}{b}\right)^x/\ln\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)^x/\ln\left(\frac{b}{a}\right) - 2x + c$$
$$= \frac{\left(\frac{a}{b}\right)^x}{\ln\left(\frac{a}{b}\right)} + \frac{\left(\frac{b}{a}\right)^x}{-\ln\left(\frac{a}{b}\right)} - 2x + c$$
$$= \frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\log\left(\frac{a}{b}\right)} - 2x + c$$

77. (4) Let $\tan x = t$
$$\Rightarrow \sec^2 x\,dx = dt$$
$$I = \int_0^1 \frac{dt}{(1+t)(2+t)} = \int_0^1\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt$$
$$= \left[\ln(1+t) - \ln(2+t)\right]_0^1$$
$$= \ln 2 - \ln 3 + \ln 2 = \ln\frac{4}{3}$$
$$= \log_e\frac{4}{3}$$

78. (1) $A = \int_{\pi/6}^{\pi/3} \sec^2 x\;dx = [\tan x]_{\pi/6}^{\pi/3}$
$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

79. (3) $(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$
$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{(1+y^2)}$$

It is form of linear differential equation.
$$I.F = e^{\int \frac{1}{1+y^2}dy = e^{\tan^{-1} y}}$$
$$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1+y^2}\,e^{\tan^{-1} y}\,dy$$
$$x(e^{\tan^{-1} y}) = \frac{e^{\tan^{-1} y}}{2} + c$$
$$\left[\because \int e^{2x}dx = \frac{e^{2x}}{2}\right]$$
$$\therefore\; 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k \qquad [k = 2c]$$

80. (2) $\left(\frac{1+i}{1-i}\right)^n = \left[\frac{(1+i)}{(1-i)}\times\frac{(1+i)}{(1+i)}\right]^n$
$$= \left[\frac{(1+i)^2}{1+1}\right]^n$$
$$= \left[\frac{1-1+2i}{2}\right]^n$$
$$= (i)^n$$
The Smallest positive integer must be 2 so that $\left(\frac{1+i}{1-i}\right)^n = -1$

81. [2.00] Let first box has exactly a and the other has exactly b white balls.

$\Rightarrow$ Probability that both balls are white
$$= \frac{a}{20}\cdot\frac{b}{20} = \frac{21}{100}$$
$$\Rightarrow \quad ab = 84$$
$\Rightarrow$ (a, b) is either (6, 14) or (7, 12), (14, 6), (12, 7)

But (6, 14) & (14, 6) is not possible

$\because a + b = 20$

$\Rightarrow$ (a, b) is (7, 12) or (12, 7)

$\Rightarrow P$(both drawn balls are black)
$$= \frac{13}{20}\times\frac{8}{20}$$
$$= 0.26 = k$$
Now, $\frac{100k}{13} = \frac{100\times 0.26}{13}$
$$= 2.00$$

82. [5.00] $\begin{vmatrix} 8 & 2 & x \\ 2 & x & 8 \\ x & 8 & 2 \end{vmatrix} = 0$

$\Rightarrow$ Applying $R_1 \to R_1 + R_2 + R_3$

or $\begin{vmatrix} (x+10) & (x+10) & (x+10) \\ 2 & x & 8 \\ x & 8 & 2 \end{vmatrix} = 0$

or $(x + 10)(x^2 - 10x + 52) = 0$

$\Rightarrow \quad x + 10 = 0 \qquad [\because x \text{ is real}]$

$\Rightarrow \quad x = -10$

$\therefore \quad \left|\dfrac{x}{2}\right| = \left|\dfrac{-10}{2}\right| = 5$

83. [9.00] $A(\text{adj } A) = |A| I_3 = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad |A| = 3$

$|\text{adj}(\text{adj } A)| = 3^{(3-1)^2} = 3^4$

$|\text{adj } A| = 3^{(3-1)} = 3^2$

$\Rightarrow \quad \dfrac{|\text{adj}(\text{adj } A)|}{|\text{adj } A|} = \dfrac{3^4}{3^2} = 3^2 = 9$

84. [0.00] $\vec{a} + \vec{b} = \vec{c}$

$((\lambda x)\hat{i} + y\hat{j} + 4z\hat{k}) + (y\hat{i} + x\hat{j} + 3y\hat{k})$

$= -z\hat{i} - 2z\hat{j} - (\lambda + 1)x\hat{k}$

$\Rightarrow \lambda x + y + z = 0; \; x + y + 2z = 0$

and $(\lambda + 1)x + 3y + 4z = 0$

$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ 1 & 1 & 2 \\ (\lambda+1) & 3 & 4 \end{vmatrix} = 0$

$\Rightarrow \lambda(4 - 6) - (4 - 2(\lambda + 1)) + (3 - (\lambda + 1)) = 0$

$\Rightarrow \quad -2\lambda - 4 + 2\lambda + 2 + 3 - \lambda - 1 = 0$

$\Rightarrow \quad -\lambda = 0$

$\Rightarrow \quad \lambda = 0$

85. [6.00] Let $Q \equiv (3\lambda + 2, 2\lambda + 1, 4\lambda + 3)$

$\overrightarrow{PQ} = (3\lambda + 3, \; 2\lambda - 2, \; 4\lambda + 1)$

Since $\overrightarrow{PQ}$ is parallel to $2x + y - 3z = 5$

$\Rightarrow \quad 2(3\lambda + 3) + (2\lambda - 2) - 3(4\lambda + 1) = 0$

$\Rightarrow \quad \lambda = \dfrac{1}{4}$

$\therefore \quad Q \equiv \left(\dfrac{11}{4}, \dfrac{6}{4}, 4\right)$

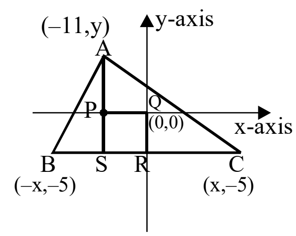
$\therefore \quad \beta\gamma = 6$

86. [7.00] Let Q(0, 0), P(–11, 0), R(0, –5) and S(–11, –5)

Now BP is an altitude, therefore BP. is perpendicular to AC

$\Rightarrow m_{BP} \cdot m_{AC} = -1$

or $\left(\dfrac{5}{x - 11}\right)\left(\dfrac{y + 5}{-11 - x}\right) = -1$



or $5(y + 5) = (x + 11)(x - 11)$ ...(1)

Also Q is equidistant from A and C,

so $y^2 + 121 = x^2 + 25$ ...(2)

From (1) and (2), we get

$5y + 25 = (y^2 + 96) - 121$

or $y^2 - 5y - 50 = 0$ gives $y = 10, -5$

But $y = -5$ is not possible

Hence $y = 10$

$\Rightarrow \quad x = 14$

$\therefore \quad BC = 2x = 28$

87. [2.00] $2(g_1 g_2 + f_1 f_2) = c_1 + c_2$

$\Rightarrow \quad 2\left(n_1\left(\dfrac{n_2}{2}\right) + (1)\left(\dfrac{n_2}{2}\right)\right) = n_1$

$\Rightarrow \quad n_1 n_2 + n_2 = n_1$

$\Rightarrow \quad n_2 = \dfrac{n_1}{(1 + n_1)}$

$\Rightarrow \quad n_2 = 1 - \dfrac{1}{(1 + n_1)}$

$1 + n_1 = 1$ or $1 + n_1 = -1$

$n_1 = 0$ or $n_1 = -2$

$\Rightarrow \quad n_2 = 0$ or $n_2 = 2$

The number of ordered pairs (n_1, n_2) is 2 i.e., (0, 0) and (–2, 2)

88. [3.00] Let

$A\left(-a, a\left(t_1 + t_2\right)\right)$, $B(at_1^2, 2at_1)$, $C(at_2^2, 2at_2)$

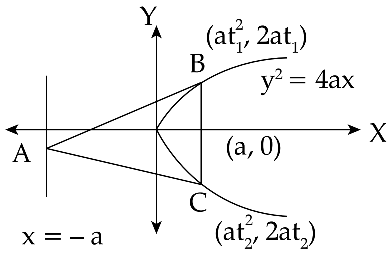
Let (h, k) is centroid of ΔABC

$\Rightarrow \quad h = \dfrac{a(t_1^2 + t_2^2) - 1}{3}$

and $\quad k = a\left(t_1 + t_2\right)$

$\because$ B and C are the end points of the chord of parabola

$\therefore t_1 t_2 = -1$



$\Rightarrow \quad 3h = a((t_1 + t_2)^2 + 2) - a$

$\Rightarrow \quad 3h = a\left(\frac{k^2}{a^2}+1\right)$

$\Rightarrow \quad 3h = \frac{k^2}{a} + a$

$\Rightarrow \quad k^2 = 3a\left(h - \frac{a}{3}\right)$

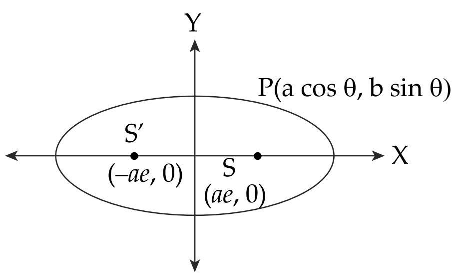
So, the locus of centroid of Δ ABC is

$$y^2 = 3a\left(x - \frac{a}{3}\right)$$

$\Rightarrow$ The length of latus rectum is $\lambda = 3a$

$\Rightarrow \quad \frac{\lambda}{a} = 3$

89. [4.00] Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



be an ellipse

Area of ellipse = $A_1 = \pi ab$

Let (h, k) be the mid-point of PS

$\Rightarrow \; 2h = a\cos\theta + ae$ and $2k = b\sin\theta$

Eliminating θ, we get

$$\frac{\left(x - \frac{ae}{2}\right)^2}{\left(\frac{a}{2}\right)^2} + \frac{y^2}{\left(\frac{b}{2}\right)^2} = 1$$

The area enclosed by the locus of mid-point of PS is $A_2 = \pi \frac{a}{2}\cdot\frac{b}{2} = \frac{\pi ab}{4}$

$\Rightarrow A_1 : A_2 = 4 : 1$

90. [4.00] $a^2 + b^2 = r^2$

$a^2 - b^2 = \frac{r^2}{4}$

$a^2 = \frac{5r^2}{8}$ and $b^2 = \frac{3r^2}{8}$

$b^2 = a^2\left(1 - e_1^2\right)$ if $\frac{b^2}{a^2} = \left(e2^2 - 1\right)$

$\Rightarrow e_2^2 = \frac{8}{5}$ and $e_1^2 = \frac{2}{5}$

Now, $\frac{e_2^2}{e_1^2} = \frac{8}{2} = 4$

Shortcut Method:

$a^2 + b^2 = a^2 e_2^2 = r^2$

$a^2 - b^2 = a^2 e_1^2 = \frac{r^2}{4}$

$\Rightarrow \quad \frac{e_2^2}{e_1^2} = 4$