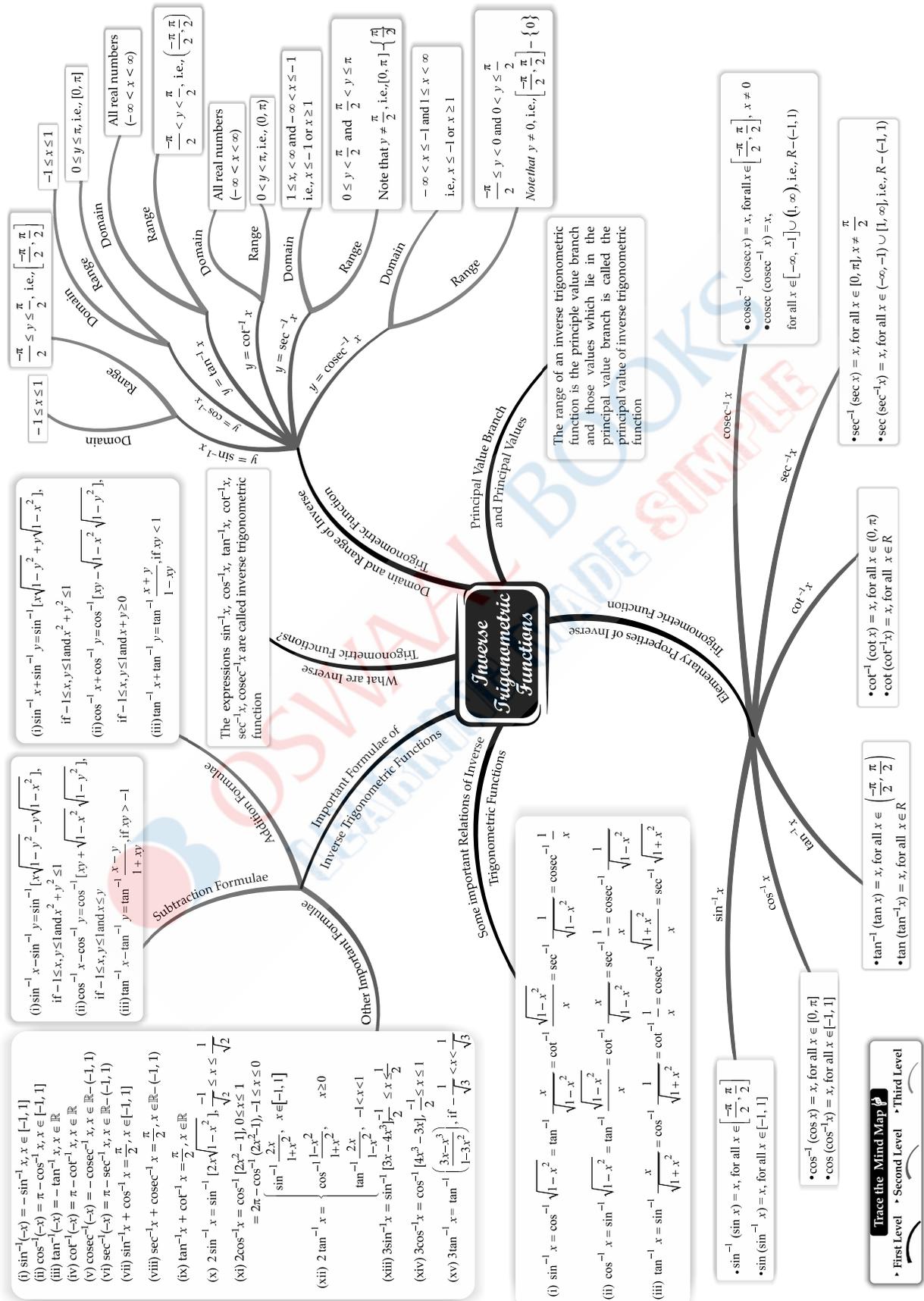
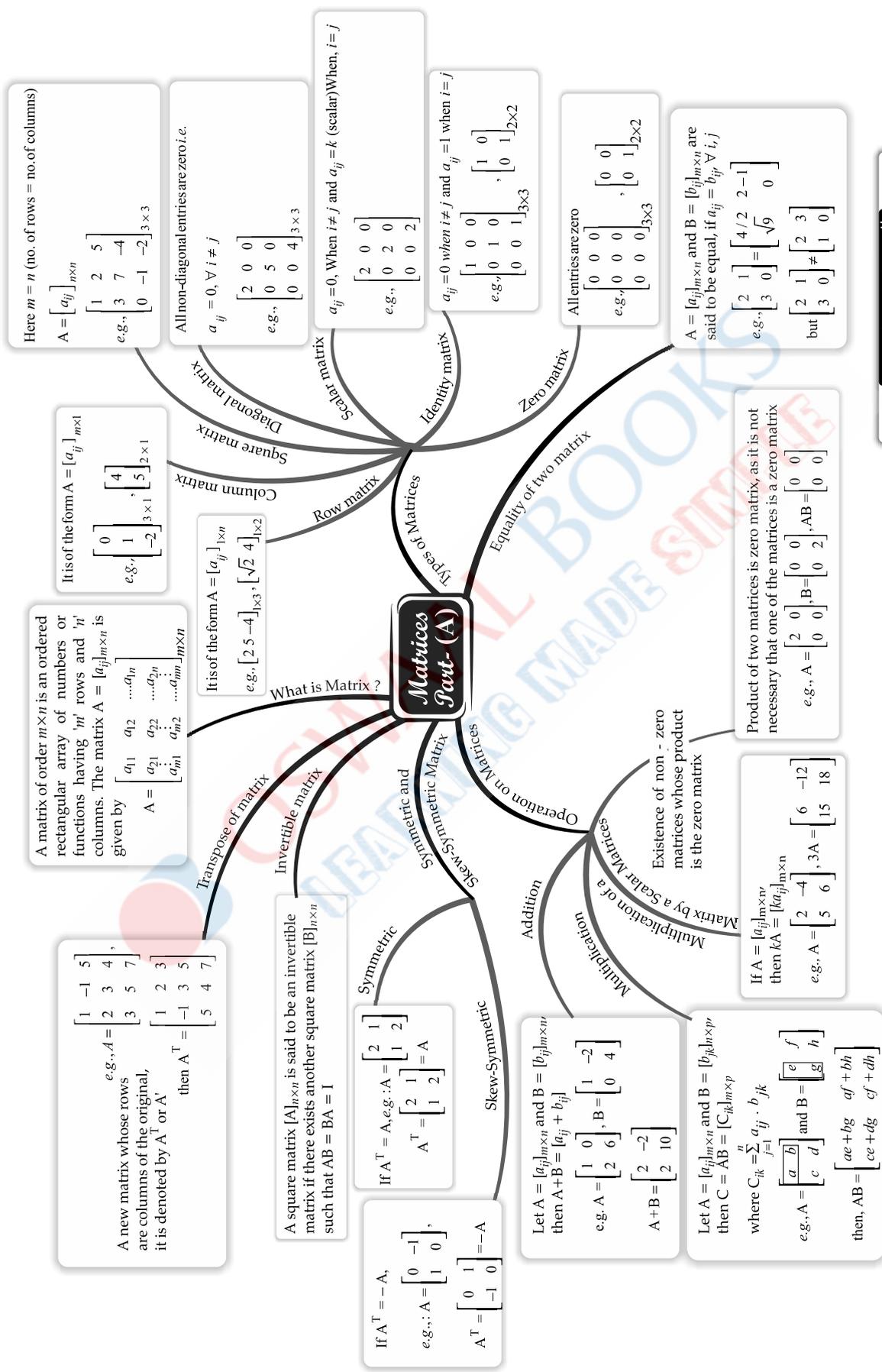


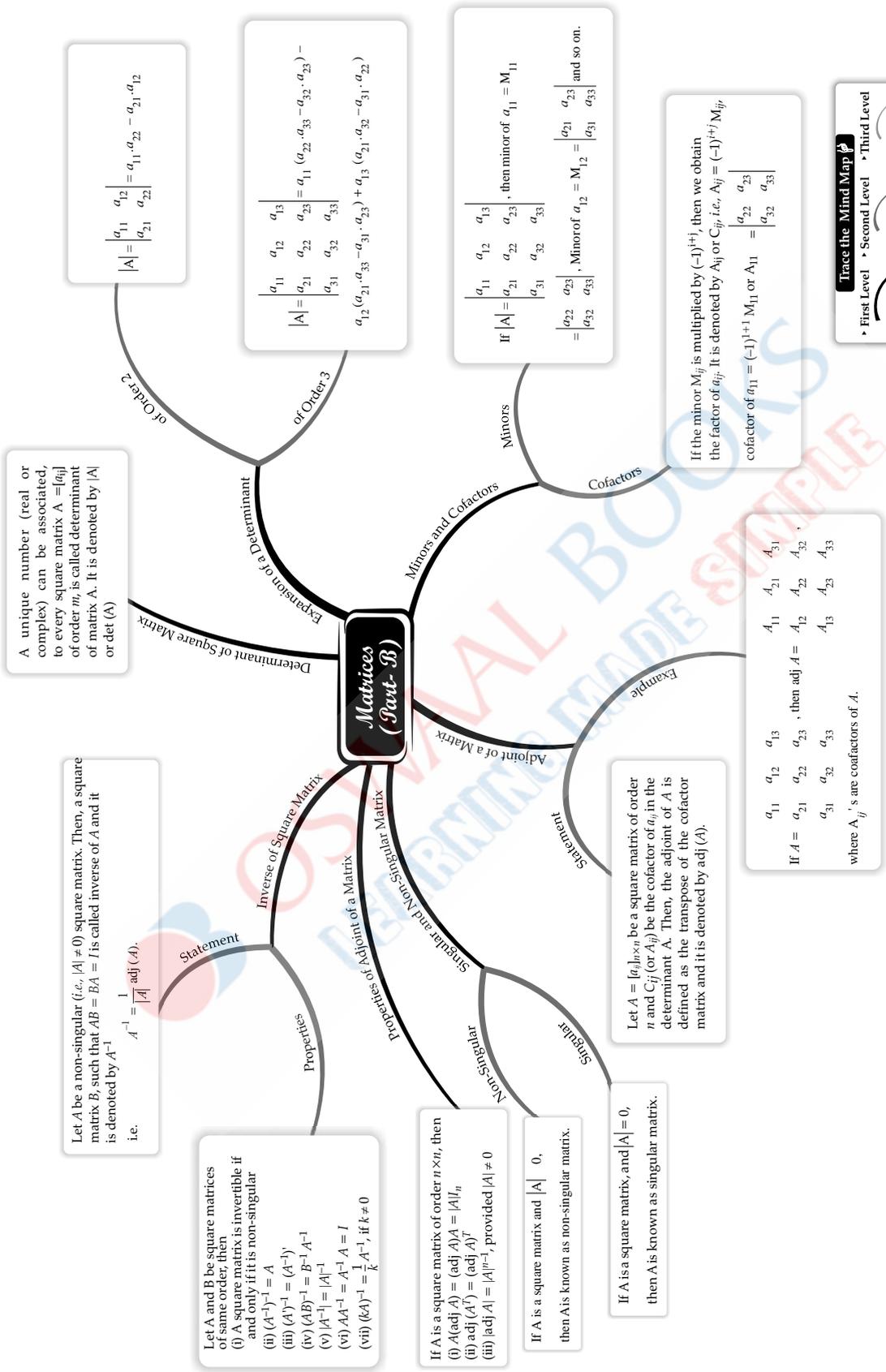
Trace the Mind Map

- First Level
- Second Level
- Third Level

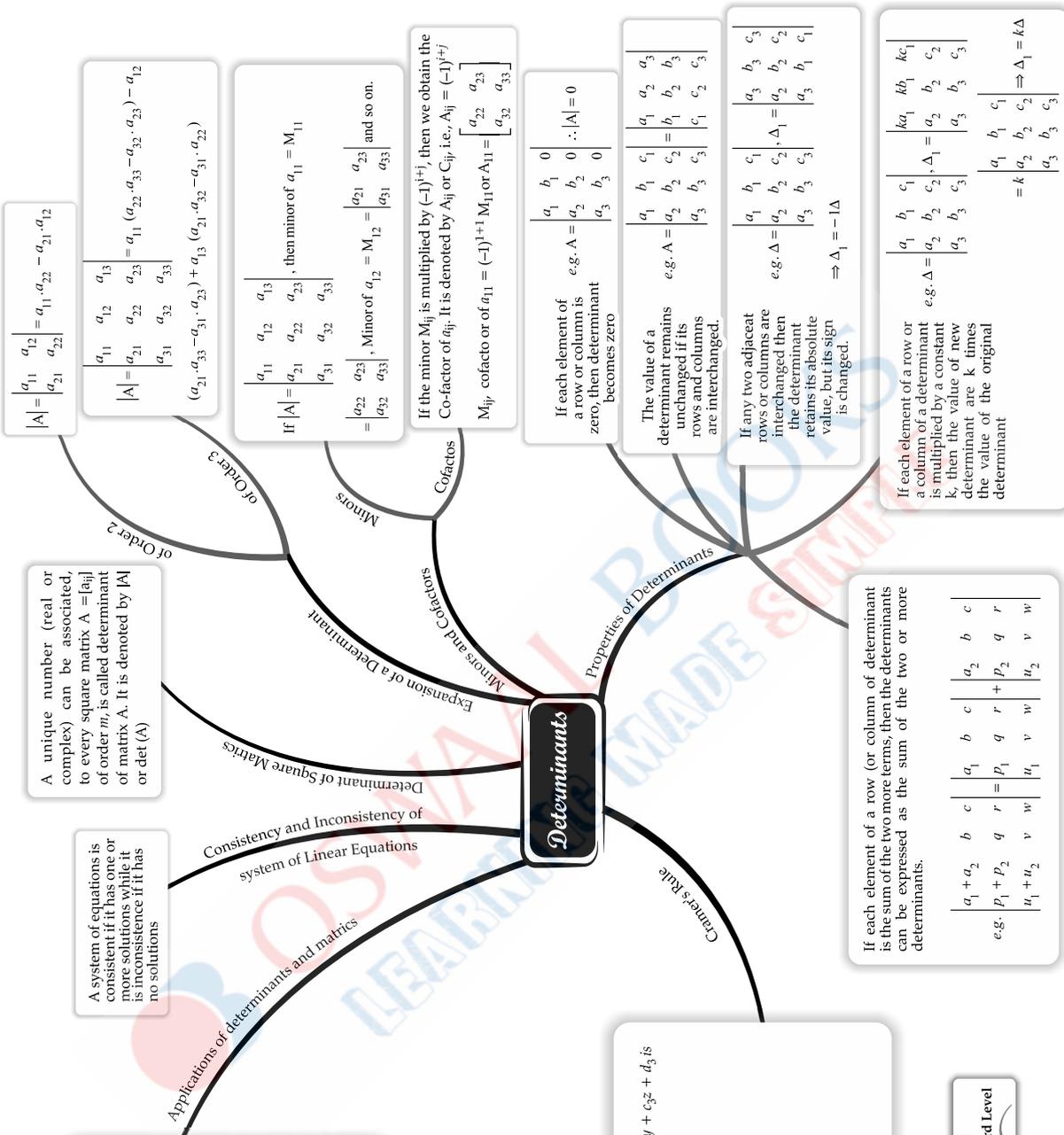




Trace the Mind Map
 → First Level → Second Level → Third Level



Determinants



• If $a_1x + b_1y + c_1z = d_1$
 $a_2x + b_2y + c_2z = d_2$
 $a_3x + b_3y + c_3z = d_3$
 then we can write $AX = B$,
 Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

- Unique solution of $AX = B$ is $X = A^{-1}B$, $|A| \neq 0$ as the solution exists or not.
- For a square matrix A in $AX = B$, if
 - (i) $|A| \neq 0$ then there exists unique solution.
 - (ii) $|A| = 0$ and $(adj. A)B \neq 0$, then no solution.
 - (iii) if $|A| = 0$ and $(adj. A)B = 0$ then system may or may not be consistent.

The solution of the system
 $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$ is
 given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$ and $z = \frac{D_z}{D}$, where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

provided that $D \neq 0$

Trace the Mind Map

- First Level
- Second Level
- Third Level

A unique number (real or complex) can be associated, to every square matrix $A = [a_{ij}]$ of order m , is called determinant of matrix A . It is denoted by $|A|$ or $\det(A)$

A system of equations is consistent if it has one or more solutions while it is inconsistent if it has no solutions

Consistency and Inconsistency of system of Linear Equations

Determinant of Square Matrix

Expansion of Determinant

Minors and Cofactors

Properties of Determinants

Cofactors

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{21} \cdot a_{12}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

If $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

If the minor M_{ij} is multiplied by $(-1)^{i+j}$, then we obtain the Co-factor of a_{ij} . It is denoted by A_{ij} or C_{ij} ; i.e., $A_{ij} = (-1)^{i+j} M_{ij}$; cofactor of $a_{11} = (-1)^{1+1} M_{11}$ or $A_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

If each element of a row or column is zero, then determinant becomes zero

e.g. $A = \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \therefore |A| = 0$

The value of a determinant remains unchanged if its rows and columns are interchanged.

e.g. $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

If any two adjacent rows or columns are interchanged then the determinant retains its absolute value, but its sign is changed.

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = -\Delta$

If each element of a row or a column of a determinant is multiplied by a constant k , then the value of new determinant are k times the value of the original determinant

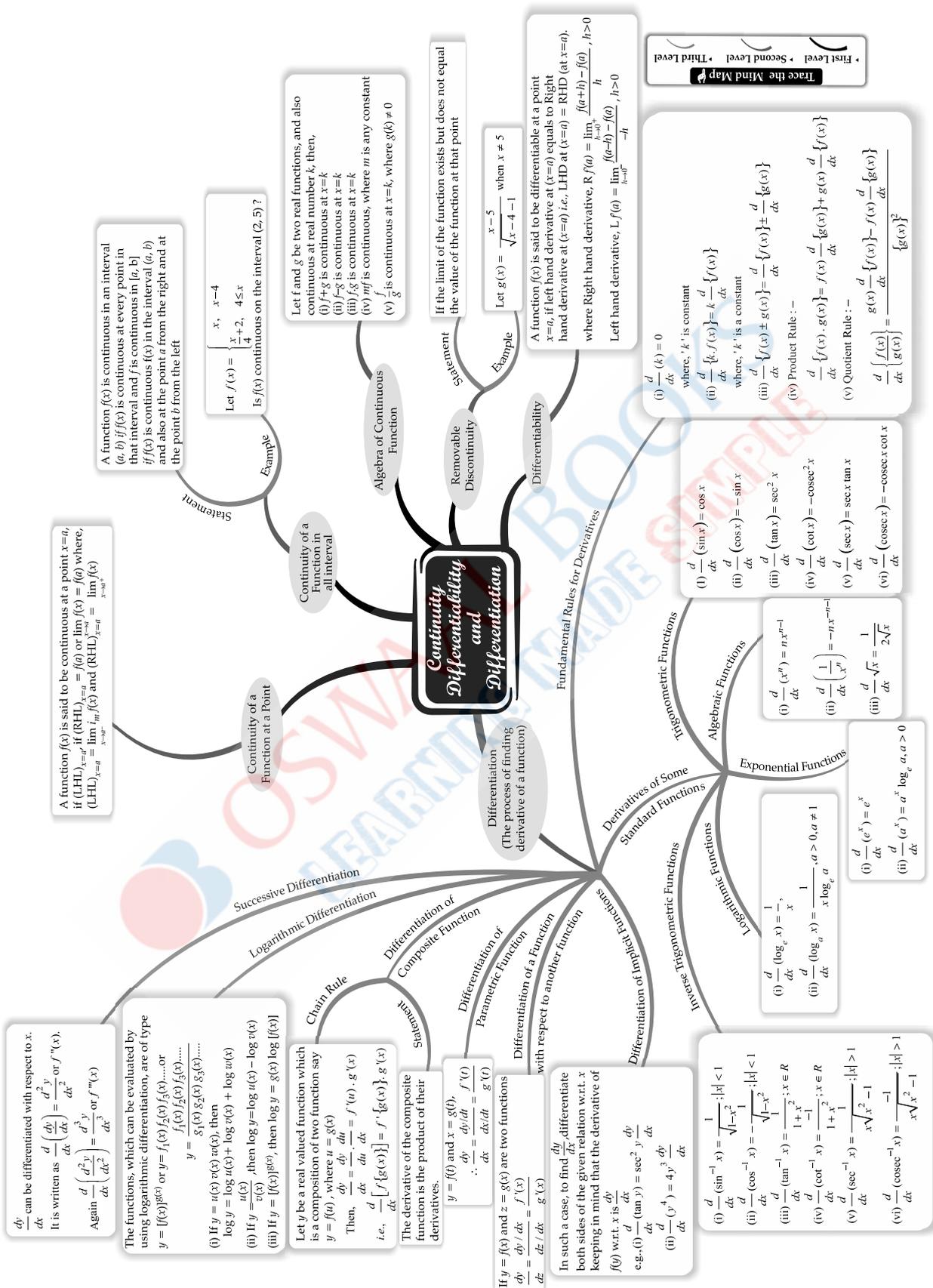
e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Rightarrow \Delta_1 = k\Delta$

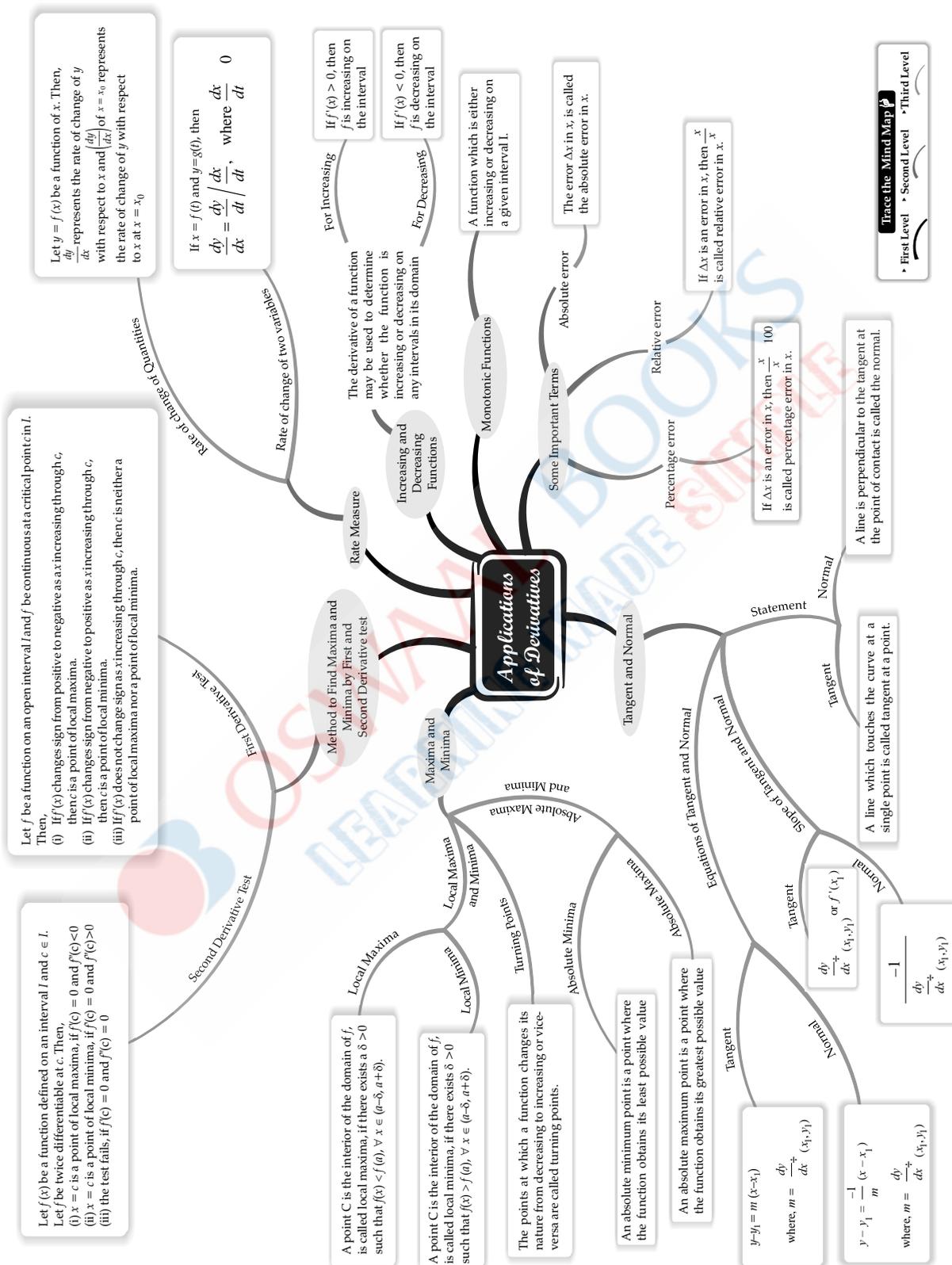
$$= k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

If each element of a row (or column) of determinant is the sum of the two more terms, then the determinant can be expressed as the sum of the two or more determinants.

$$\begin{vmatrix} a_1 + a_2 & b & c \\ a_1 & b & c \\ a_1 & b & c \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ a_2 & b & c \\ a_1 & b & c \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ a_1 & b & c \\ a_1 & b & c \end{vmatrix}$$

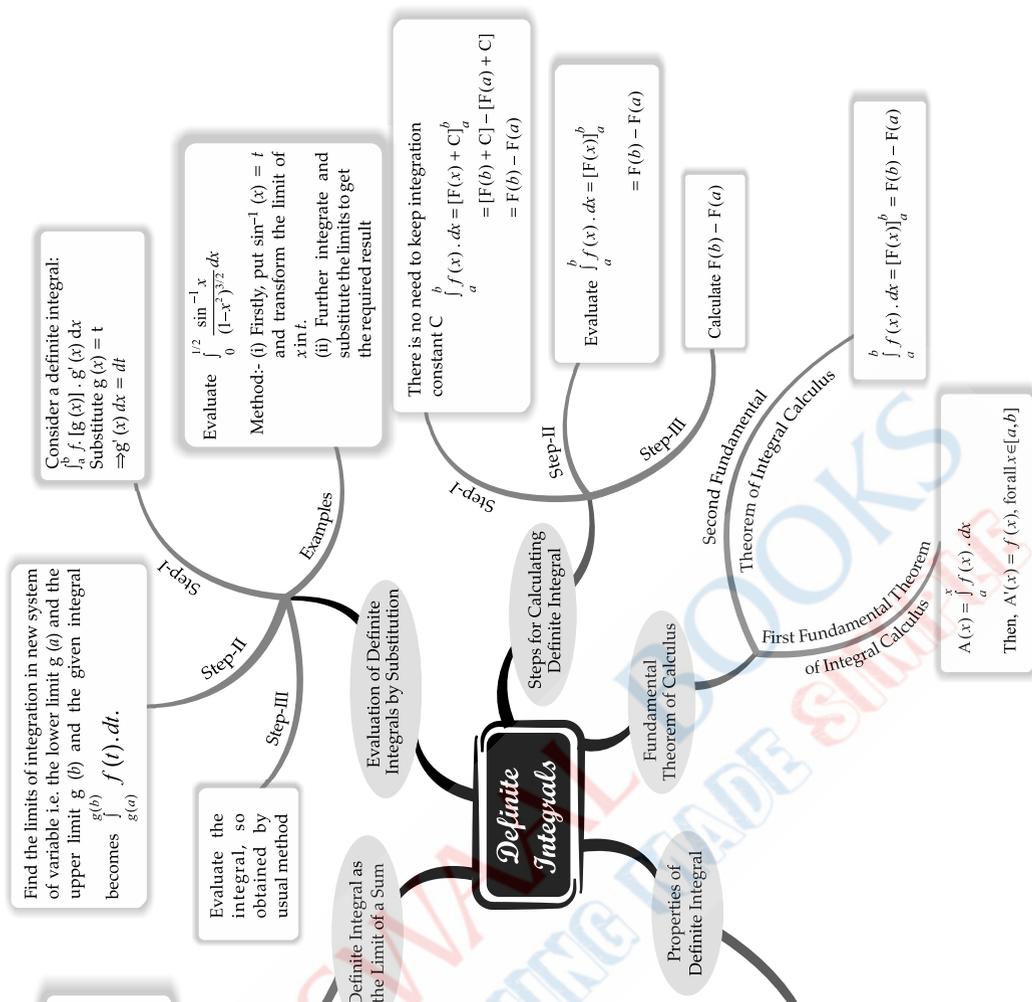
e.g. $\begin{vmatrix} a_1 + a_2 & b & c \\ a_1 & b & c \\ a_1 & b & c \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ a_2 & b & c \\ a_1 & b & c \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ a_1 & b & c \\ a_1 & b & c \end{vmatrix}$





Trace the Mind Map

- First Level
- Second Level
- Third Level



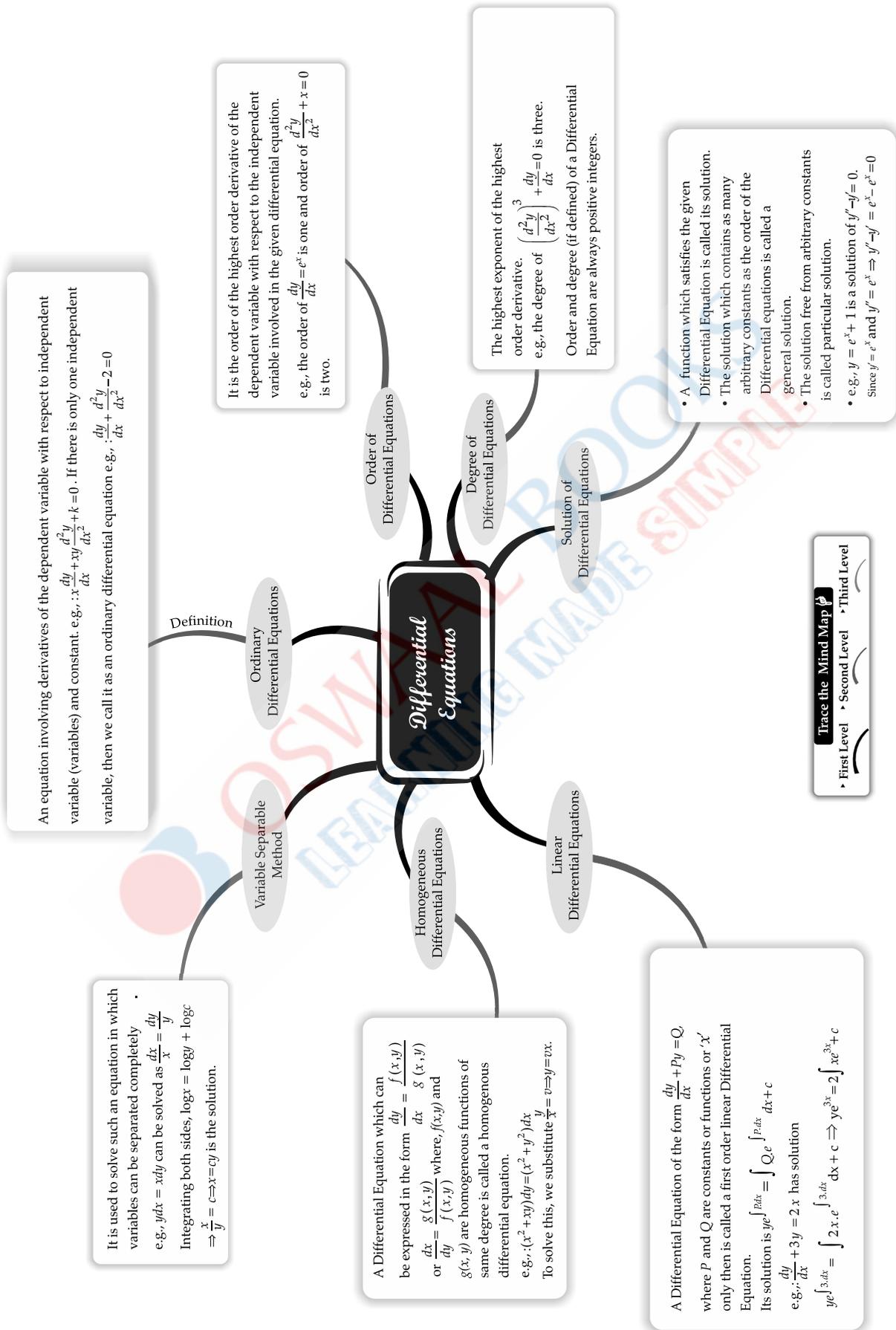
$$\int_a^b f(x) \cdot dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h) + f(a+nh)]$$

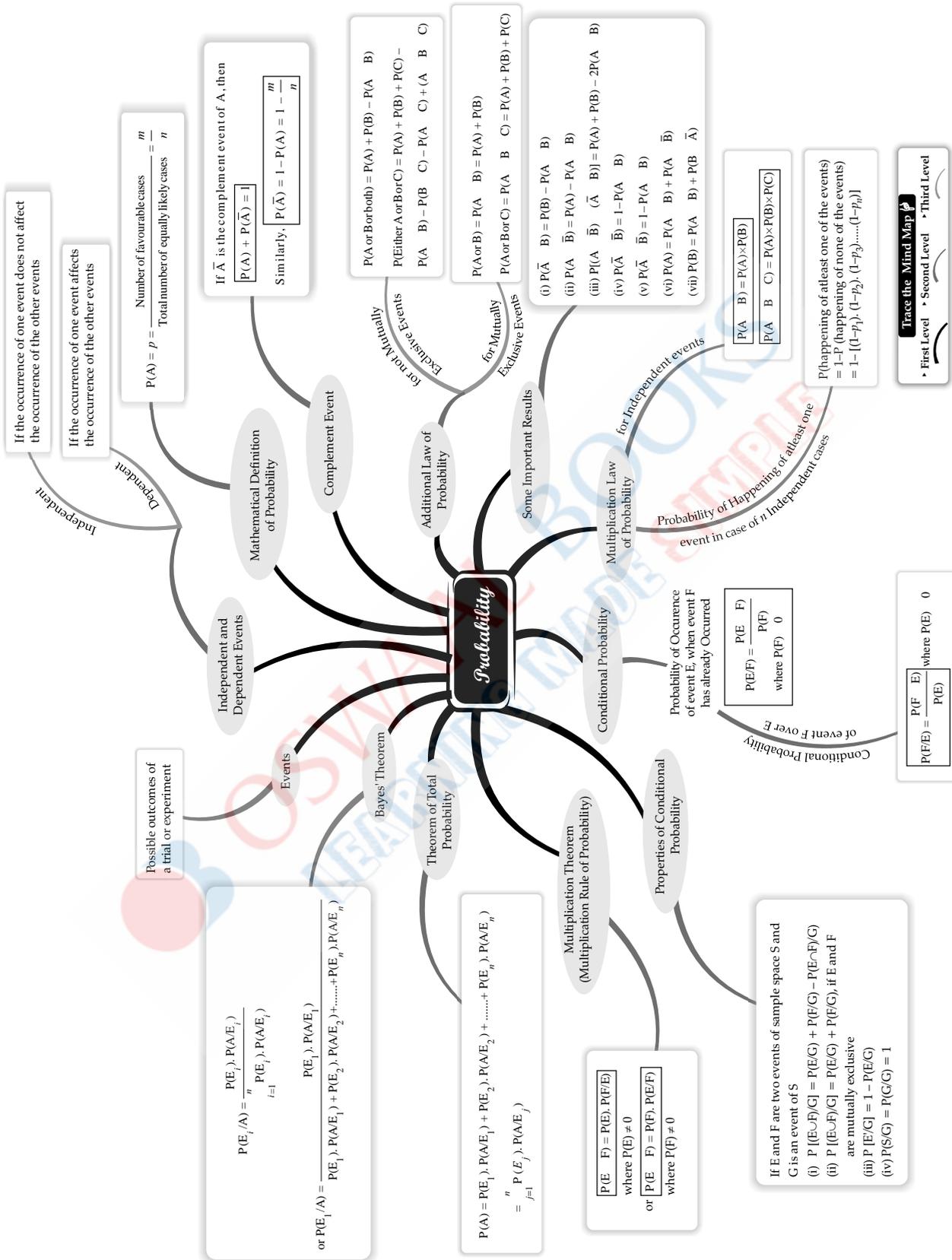
where, $h = \frac{b-a}{n}$ or $nh = b-a$

- (i) $\int_a^b f(x) dx = \int_a^b f(t) dt$
- (ii) $\int_a^b f(x) dx = -\int_b^a f(x) dx$
- (iii) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$, where $a < c < b$
- (iv) $\int_a^a f(x) dx = 0$
- (v) $\int_a^a f(x) dx = \int_a^a f(a-x) dx$
- (vi) $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
- (vii) $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$
- (viii) $\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$
- or $\int_a^b f(x) dx = \begin{cases} 0, & \text{if } f(a+x) = -f(b-x) \\ 2 \int_a^{\frac{a+b}{2}} f(x) dx, & \text{if } f(a+x) = f(b-x) \end{cases}$
- (ix) $\int_{-a}^a f(x) dx = \int_0^a [f(x) + f(-x)] dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is even i.e. } f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is odd i.e. } f(-x) = -f(x) \end{cases}$

Trace the Mind Map

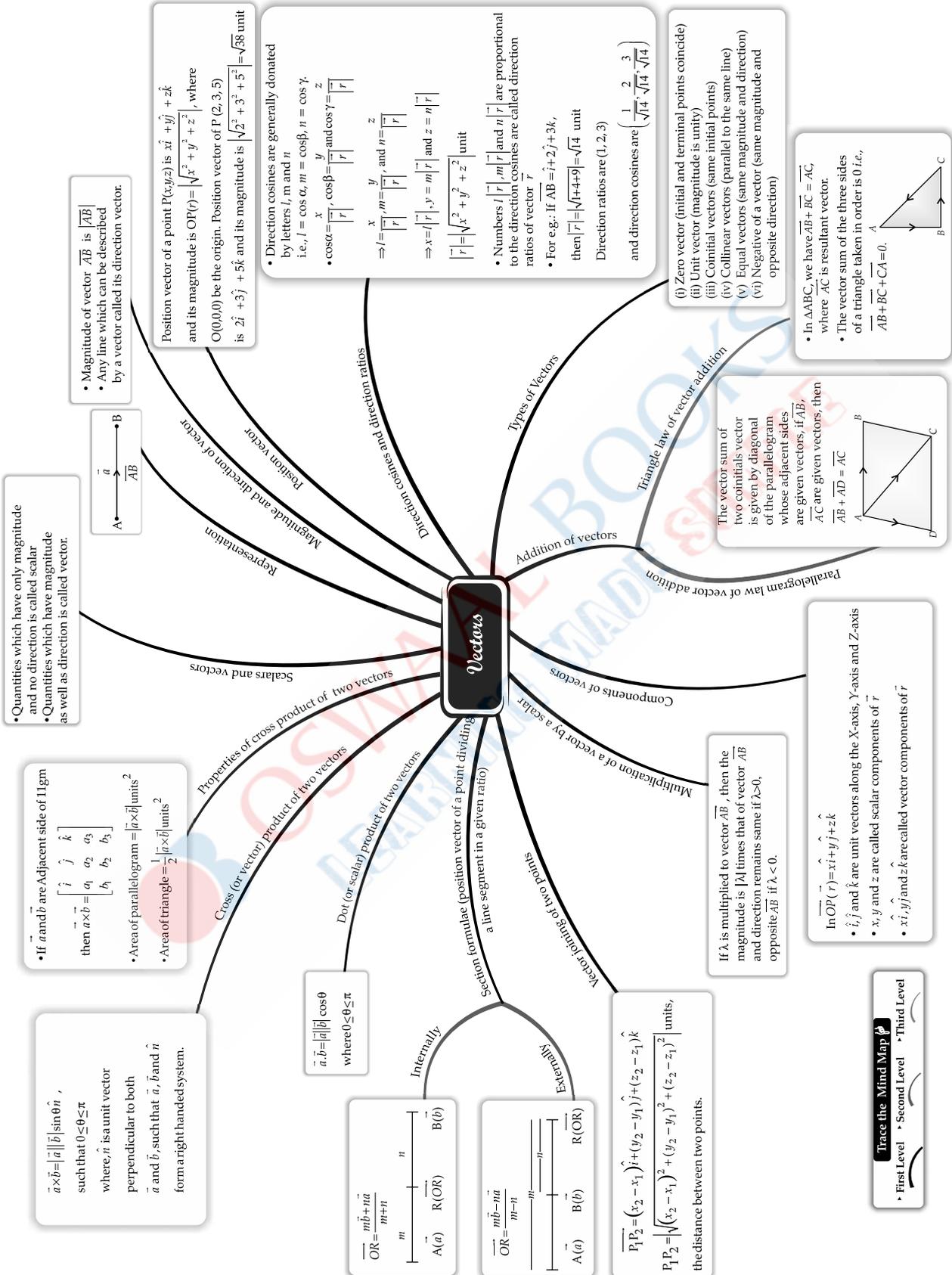
- First Level
- Second Level
- Third Level





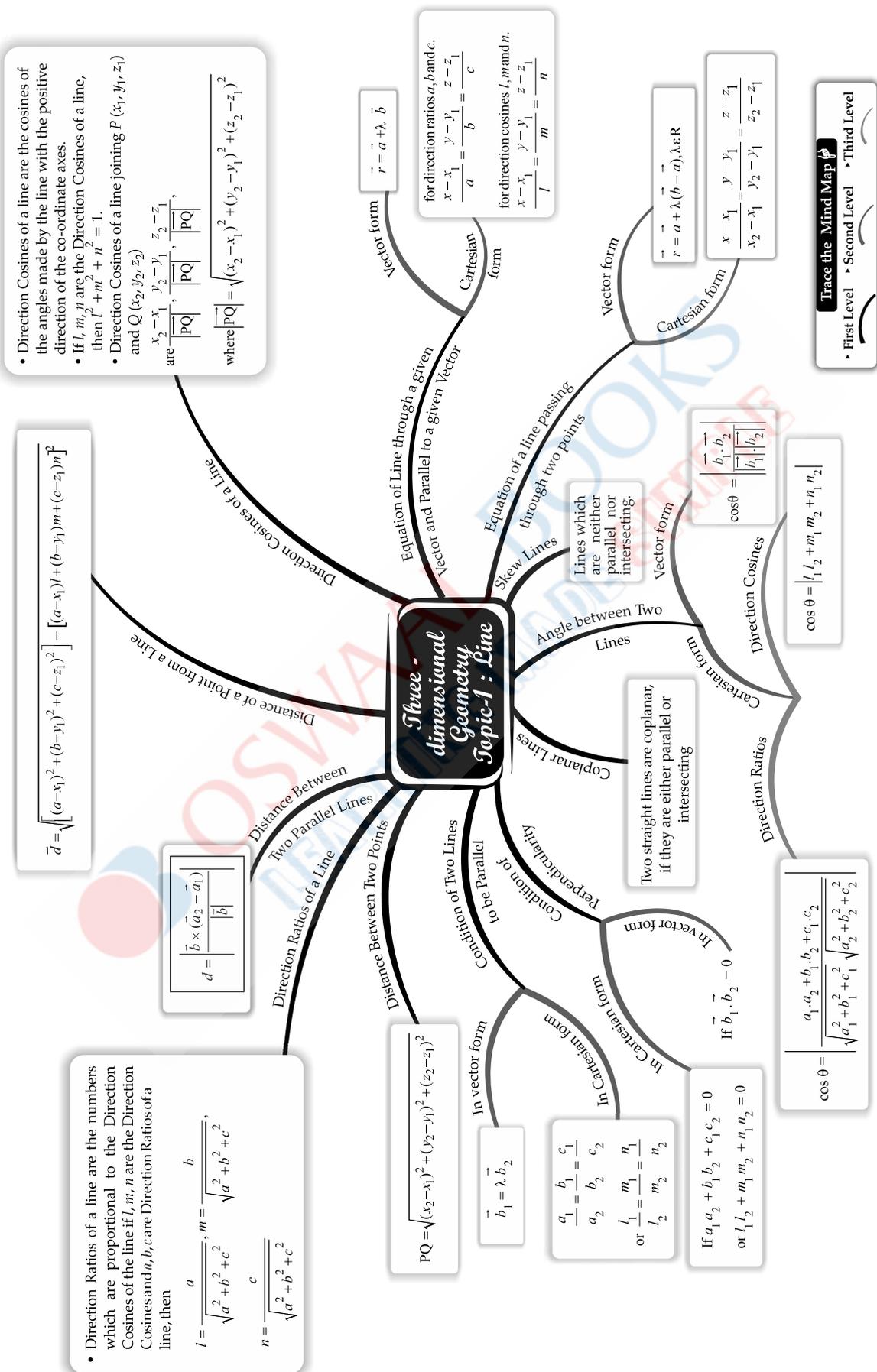
Trace the Mind Map

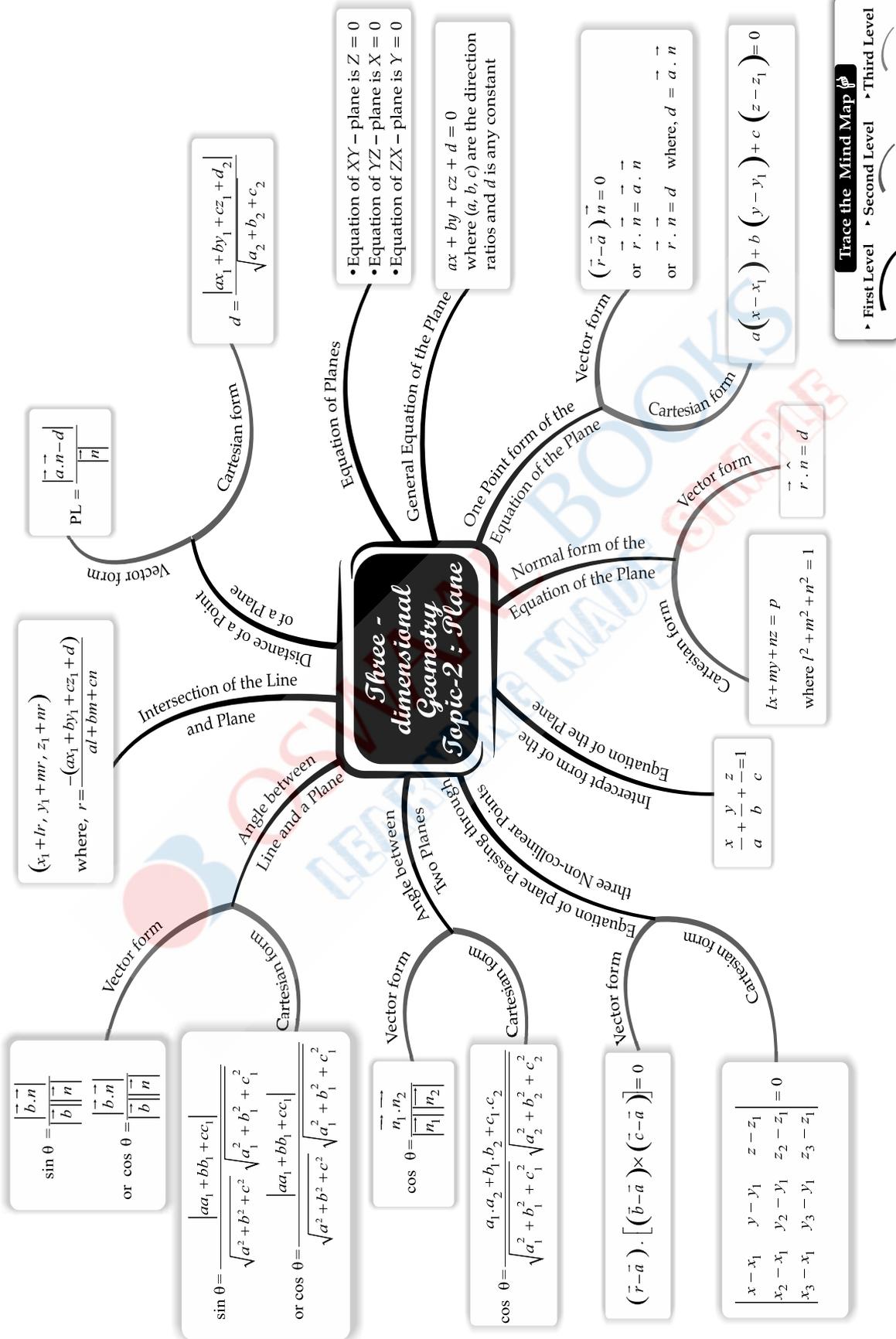
- First Level
- Second Level
- Third Level



Trace the Mind Map

- First Level
- Second Level
- Third Level





Trace the Mind Map

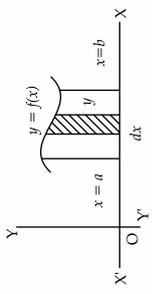
► First Level ► Second Level ► Third Level

Application of Integrals

Area Under Simple Curves and Coordinate Axes

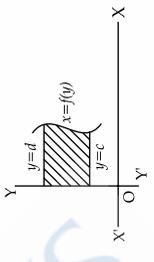
Case I

The area of the region bounded by the curve $y = f(x)$ above X - axis and between the lines $x=a$ and $x=b$ ($b > a$) is given by

$$A = \int_a^b y dx \text{ or } \int_a^b f(x) dx.$$


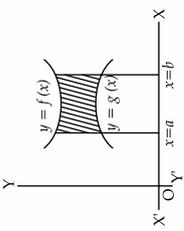
Case II

The area of the region bounded by the curve $x = f(y)$ right to the Y - axis and between the lines $y=c$ and $y=d$ ($d > c$) is given by

$$A = \int_c^d x dy \text{ or } \int_c^d f(y) dy.$$


Area enclosed between two curves

Case I



The areas of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is given by

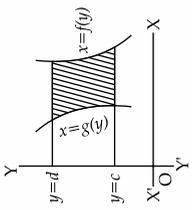
$$A = \int_a^b [f(x) - g(x)] dx, \text{ where } f(x) \geq g(x) \text{ in } [a, b]$$

Case II

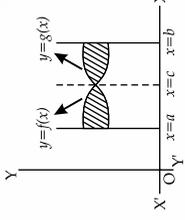
The area of the region enclosed between curves $x = f(y)$, $x = g(y)$ and the lines $y = c$, $y = d$ is given by

$$A = \int_c^d [f(y) - g(y)] dy$$

where $f(y) \geq g(y)$ in $[d, c]$



Case III

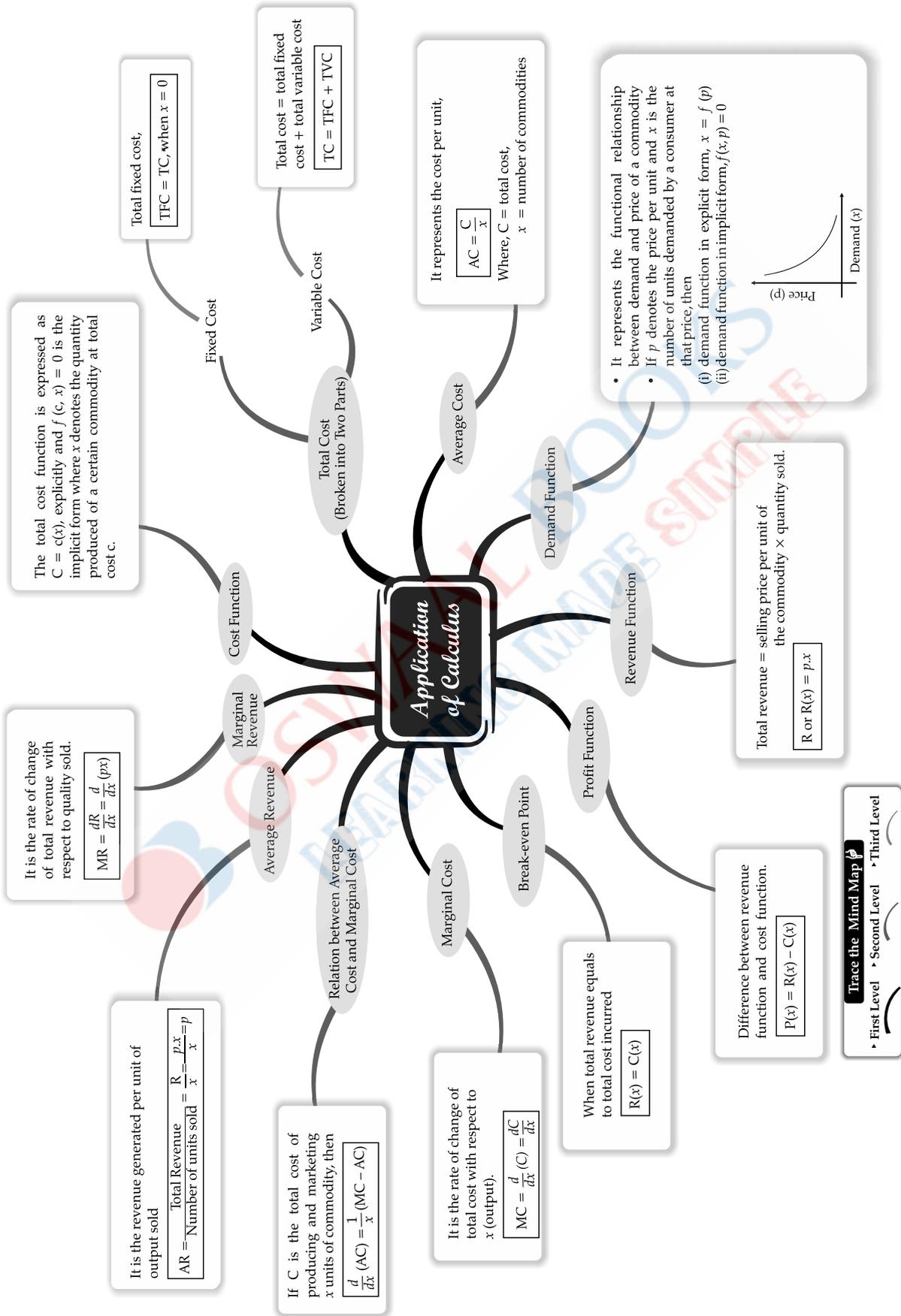


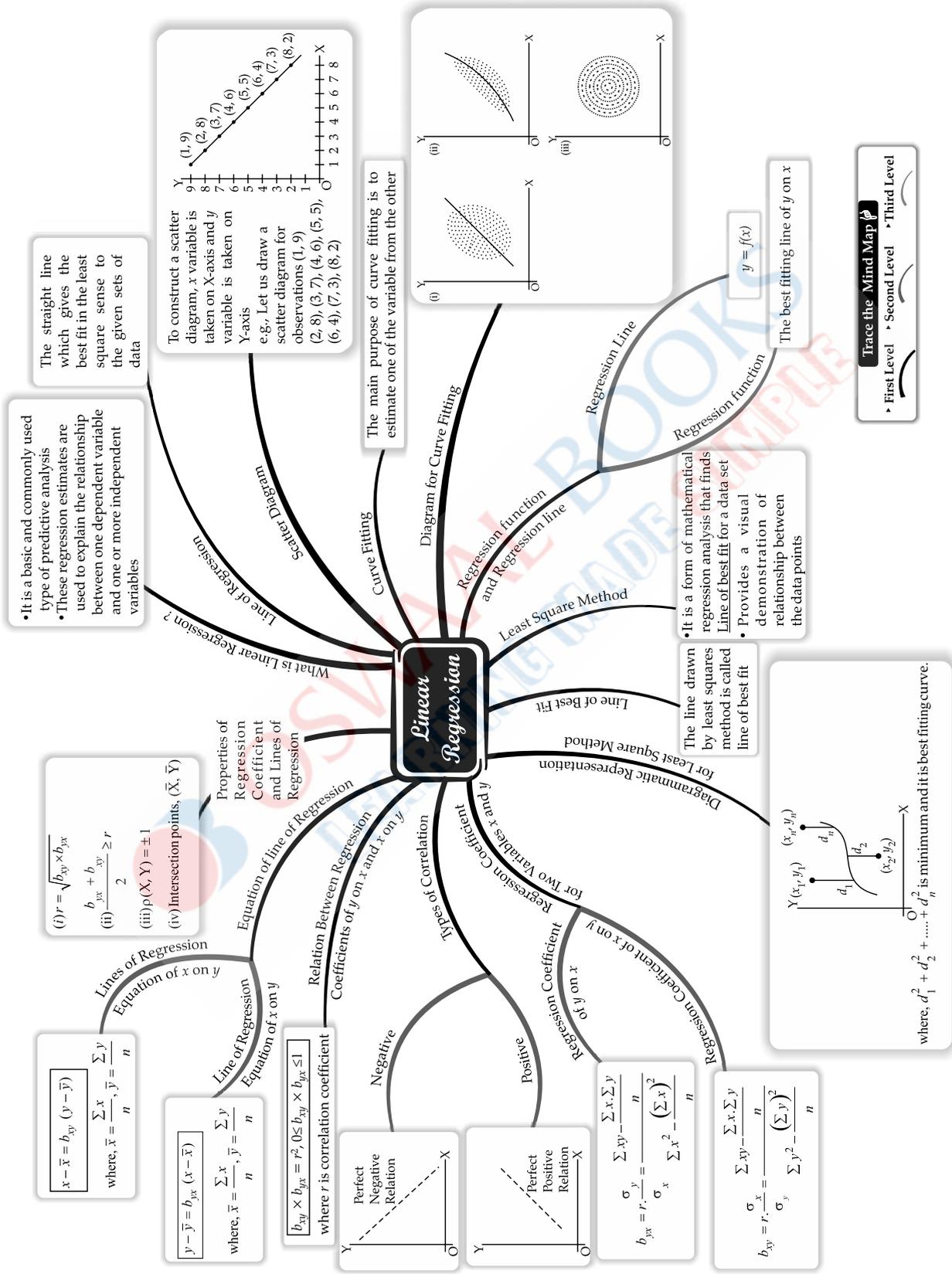
If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$ where $a < c < b$, then the area of the region bounded by the curves is

$$A = \int_a^c [f(x) - g(x)] dx + \int_c^b [g(x) - f(x)] dx$$

Trace the Mind Map

▶ First Level
▶ Second Level
▶ Third Level

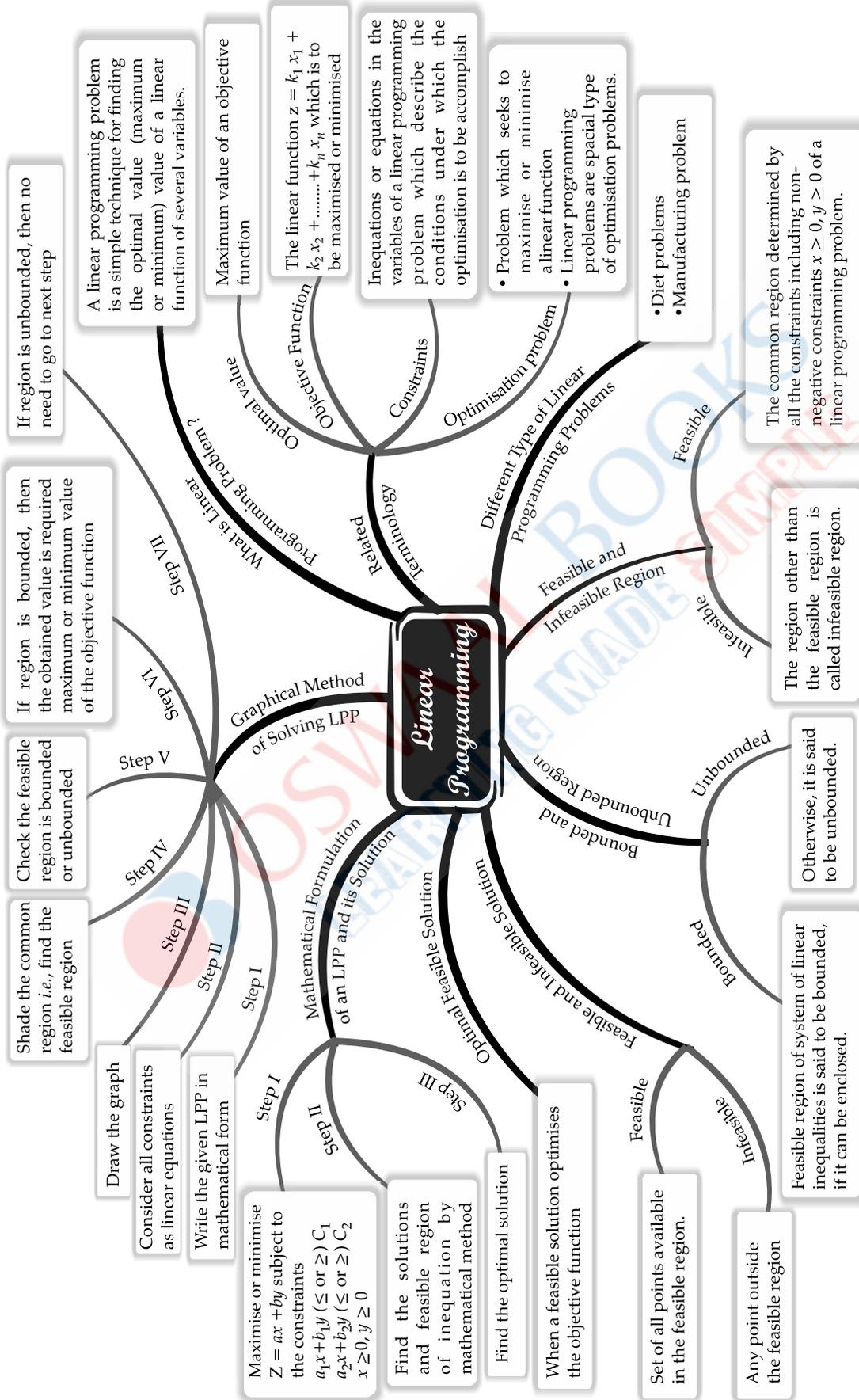




Trace the Mind Map

- First Level
- Second Level
- Third Level

where, $d_1^2 + d_2^2 + \dots + d_n^2$ is minimum and it is best fitting curve.



Trace the Mind Map

→ First Level → Second Level → Third Level