Sample Question Paper-1

(Specimen Paper issued by CISCE dated 12th July, 2022)

MATHEMATICS

Class-10

SOLVED

Time Allowed : 2½ hours **Maximum Marks : 80**

Answers to this Paper must be written on the paper provided separately.

*You will **not** be allowed to write during the first **15** minutes.*

This time is to be spent in reading the Question Paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from **Section A** and any four questions from **Section B**.*

All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of question are given in brackets [ ]

Mathematical tables are provided.

Section-A

(*Attempt **all** questions from this **Section***)

Question 1.

Choose the Correct answer to the questions from the given options: **[15 Marks]**

(i) The SGST paid by a customer to the shopkeeper for an article which is priced at ₹500 is ₹15. The rate of GST charged is:

(a) 1.5% **(b)** 3%

(c) 5% **(d)** 6%

(ii) When the roots of a quadratic equation are real and equal then the discriminant of the quadratic equation is:

(a) Infinite **(b)** Positive

(c) Zero **(d)** Negative

(iii) If $(x - 1)$ is a factor of $2x^2 - ax - 1$, then the value of 'a' is:

(a) -1 **(b)** 1

(c) 3 **(d)** -3

(iv) Given $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times X = \begin{bmatrix} p \\ q \end{bmatrix}$. The order of matrix X is:

(a) 2×2 **(b)** 1×2

(c) 2×1 **(d)** 1×1

(v) 57, 54, 51, 48, ... are in Arithmetic Progression. The value of the 8^{th} term is:

(a) 36 **(b)** 78

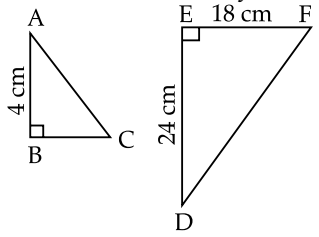
(c) -36 **(d)** -78

(vi) The point A (p, q) is invariant about $x = p$ under reflection. The coordinates of it's image A' is:

(a) $A'(p, -q)$ **(b)** $A'(-p, q)$

(c) $A'(p, q)$ **(d)** $A'(-p, -q)$

(vii) In the given diagram the ΔABC is similar to ΔDEF by the axiom:



(a) SSS **(b)** SAS
(c) AAA **(d)** RHS

(viii) The volume of a right circular cone with same base radius and height as that of a right circular cylinder, is 120 cm^3. The volume of the cylinder is:

(a) 240 cm^3 **(b)** 60 cm^3
(c) 360 cm^3 **(d)** 480 cm^3

(ix) The solution set for the given in equation is: $-8 \le 2x < 8, x \in W$

(a) {–4, –3, –2, –1, 0, 1,2, 3, 4} **(b)** {–4,–3,–2,–1}
(c) {0, 1, 2, 3} **(d)** {–8, –7, –6, –5, –4, –3, –2, –1, 0, 1,2, 3, 4, 5,6 7, 8}

(x) The probability of the Sun rising from the east is P(S). The value of P(S) is:

(a) P(S) = 0 **(b)** P(S) < 0
(c) P(S) = 1 **(d)** P(S) > 1

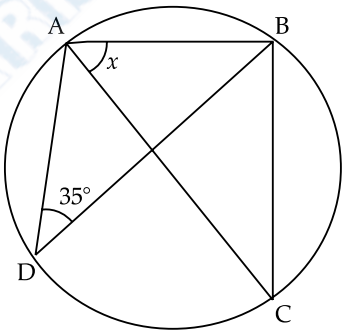
(xi) If $\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$. The value of x is:

(a) 2 **(b)** 3
(c) 4 **(d)** 5

(xii) The centroid of a ΔABC is G (6, 7). If the coordinates of the vertices A, Band C are (a, 5), (7, 9) and (5, 7) respectively. The value of a is:

(a) 9 **(b)** 6
(c) 3 **(d)** 7

(xiii) In the given diagram AC is a diameter of the circle and ∠ADB=35°. The degree measure of x is:



(a) 55° **(b)** 35°
(c) 45° **(d)** 70°

(xiv) If the n^{th} term of an Arithmetic Progression (A.P.) is (n + 3), then the first three terms of the (A.P.) are:

(a) 1, 2, 3 **(b)** 2, 4, 6
(c) 4, 5, 6 **(d)** 7, 8, 9

(xv) The median of a grouped frequency distribution is found graphically by drawing:

(a) a linear graph **(b)** a histogram
(c) a frequency polygon **(d)** a cumulative frequency curve

Question 2

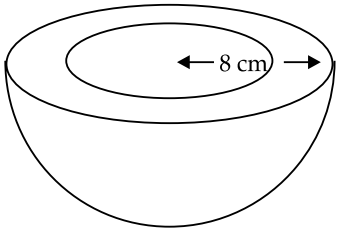
(i) Salman deposits ₹ 1200 every month in a recurring deposit account for 2 ½ years. If the rate of interest is 6% per annum, find the amount he will receive on maturity. **[4]**

(ii) 3, 9, *m*, 81 and *n* are in continued proportion. Find the values *of m* and *n*. **[4]**

(iii) Prove that: $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$ **[4]**

Question 3

(i) The inner circumference of the rim of a circular metal tub is 44 cm. [4]



Find:

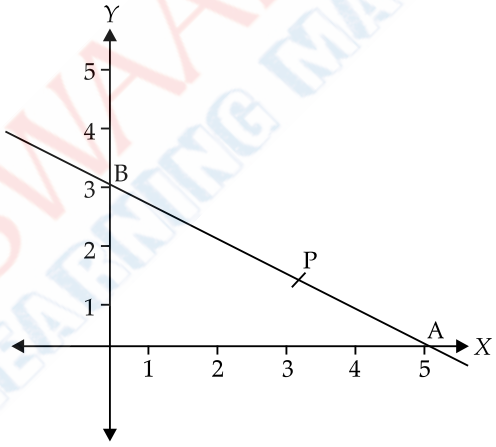
(a) The inner radius of the tub

(b) The volume of the material of the tub if it's outer radius is 8 cm.

Use $\pi = \frac{22}{7}$

Give your answer correct to three significant figures.

(ii) From the given figure: **[4]**



(a) Write down the coordinates of A and B.

(b) If P divides AB in the ratio 2:3, find the coordinates of point P.

(c) Find the equation of a line parallel to line AB and passing through origin.

(iii) Use graph set for this question. Take 2 cm = 1 unit along the axes. **[5]**

Plot the ΔOAB, where O (0, 0), A (3, –2), B (2, –3).

(a) Reflect the ΔOAB through the origin and name it as ΔOA'B'.

(b) Reflect the ΔOA'B' on the y – axis and name it as ΔOA''B''.

(c) Reflect the ΔOA'B' on the x – axis and name it as ΔOA'''B'''.

(d) Join the points AA''B''B'A'A'''B'''B and give the geometrical name of the closed figure so formed.

Section-B

(*Attempt **any four** questions from this Section.*)

Question 4

(i) The following bill shows the GST rates and the marked price of articles: **[3]**

BILL: COMPUTERS		
Articles	**Marked price**	**Rate of GST**
Graphic Card	Rs 15500.00	18%
Laptop adapter	Rs 1900.00	28%

Find the total amount to be paid for the above bill.

(ii) Solve the following quadratic equation, **[3]**

$$7x^2 + 2x - 2 = 0$$

Give your answer correct to two places of decimal

(iii) Use graph sheet for this question. Draw a histogram for the daily earnings of 54 medical stores in the following table and hence estimate the mode for the following distribution. Take 2 cm = ₹500 units along the x-axis and 2 cm = 5 stores along the y-axis. **[4]**

Daily earnings (₹)	4500 – 5000	5000 – 5500	5500 – 6000	6000 – 6500	6500 – 7000
No. of medical stores	20	14	12	5	3

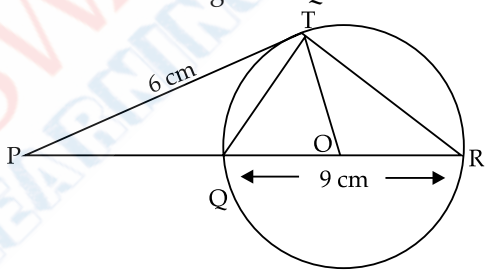
Question 5

(i) $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ *and* $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$, Evaluate AB − 5C **[3]**

(ii) In the given figure, O is the centre of circle. The tangent PT meets the diameter RQ produced at P. **[3]**

(a) Prove $\Delta PQT \sim \Delta PTR$

(b) If PT = 6 cm, QR = 9 cm. Find the length of PQ



(iii) Factorise the given polynomial completely, using Remainder Theorem: **[4]**

$6x^3 + 25x^2 + 31x + 10$

Question 6

(i) ABCD is a square where B (1, 3), D (3, 2) are the end points of the diagonal BD. **[3]**

Find:

(a) the coordinates of point of intersection of the diagonals AC and BD

(b) the equation of the diagonal AC

(ii) Prove that: $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sec\theta.\operatorname{cosec}\theta$ **[3]**

(iii) The first, the last term and the common difference of an Arithmetic Progression are 98, 1001 and 7 respectively. Find the following for the given Arithmetic Progression: **[4]**

(a) number of terms 'n'.

(b) Sum of the 'n' terms.

Question 7

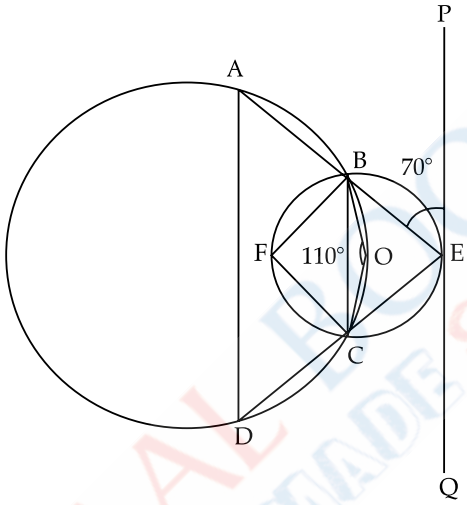
(i) A box contains some green, yellow and white tennis balls. The probability of selecting a green ball is $\frac{1}{4}$ and yellow ball is $\frac{1}{3}$. If the box contains 10 white balls, then find: **[3]**

(a) total number of balls in the box.

(b) probability of selecting a white ball.

(ii) A cone and a sphere having the same radius are melted and recast into a cylinder. The radius and height of the cone are 3 cm and 12 cm respectively. If the radius of the cylinder so formed is 2 cm, find the height of the cylinder. **[3]**

(iii) In the given diagram, ABCD is a cyclic quadrilateral and PQ is a tangent to the smaller circle at E. Given $\angle AEP = 70°$, $\angle BOC = 110°$. Find: **[4]**



(a) $\angle ECB$, **(b)** $\angle BEC$,

(c) $\angle BFC$, **(d)** $\angle DAB$,

Question 8

(i) Solve the following equation: **[3]**

$$-\frac{x}{3} - 4 \leq \frac{x}{2} - \frac{7}{3} < -\frac{7}{6},\ x \in R$$

Represent the solution set on a number line.

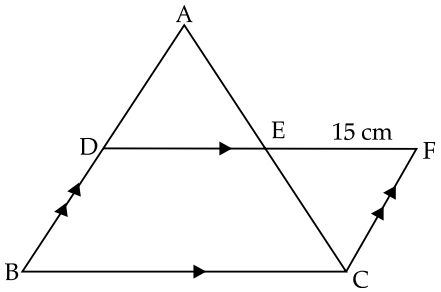
(ii) The following table gives the petrol prices per litre for a period of 50 days. **[3]**

Price (₹)	85 – 90	90 – 95	95 – 100	100 – 105	105 – 110
No. of days	12	10	8	15	5

Find the mean price of petrol per litre to the nearest rupee using step-deviation method.

(iii) In the given diagram, ABC is a triangle and BCFD is a parallelogram. **[4]**

AD : DB = 4 : 5 and EF = 15 cm.



Find:

(a) $AE : EC$ **(b)** DE

(c) BC

Question 9

(i) Amit takes 12 days less than the days taken by Bijoy to complete a certain work. If both, working together, takes 8 days to complete the work, find the number of days taken by Bijoy to complete the work, working alone. **[4]**

(ii) Use a graph sheet for this question. The daily wages of 120 workers working at a site are given below: **[6]**

Wages (₹)	250 – 300	300 – 350	350 – 400	400 – 450	450 – 500	500 – 550	550 – 600
No. of workers	8	15	20	30	25	15	7

Use 2 cm = ₹50 and 2 cm = 20 workers along x – axis and y – axis respectively to draw an ogive and hence estimate

(a) the median wages

(b) the inter – quartile range of wages

(c) percentage of workers whose daily wage is above ₹475.

Question 10

(i) Solve for x, using the properties of proportion. **[3]**

$$\frac{\sqrt{2+x}+\sqrt{3-x}}{\sqrt{2+x}-\sqrt{3-x}}=3$$

(ii) Using ruler and compasses, construct a regular hexagon of side 4.5 cm. Hence construct a circle circumscribing the hexagon. Measure and write down the length of the circum-radius. **[3]**

(iii) An observer standing on the top of a lighthouse 150 m above the sea level watches a ship sailing away. As he observes, the angle of depression of the ship changes from 50° to 30°. Determine the distance travelled by the ship during the period of observation. Give your answer correct to the nearest meter. (Use Mathematical Table for this question.) **[4]**

■■

SOLUTIONS

Sample Question Paper-1

MATHEMATICS

SECTION-A

1. (i) Option (d) is correct.

Explanation: SGST paid = ₹15

Purchase price = ₹500

Total GST = SGST + CGST

= 15 + 15

(∵ SGST = CGST)

= 30

$$\text{Rate} = \frac{\text{Amount of GST}}{\text{Total Price}} \times 100$$

$$= \frac{30}{500} \times 100 = 6\%$$

(ii) Option (c) is correct.

Explanation: We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since the roots are real and equal

$$\Rightarrow \quad x = \frac{-b}{2a}, \frac{-b}{2a}$$

$$\Rightarrow \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad D = 0$$

(iii) Option (b) is correct.

Explanation: Since $(x - 1)$ is a factor of $2x^2 - ax - 1$

$$\therefore \quad x = 1$$

$$\Rightarrow \quad 2(1)^2 - a(1) - 1 = 0$$

$$\Rightarrow \quad 2 - a - 1 = 0$$

$$\Rightarrow \quad 1 - a = 0$$

$$\Rightarrow \quad 1 = a$$

$$\Rightarrow \quad a = 1$$

(iv) Option (c) is correct.

Explanation: Here $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2 × 2 matrix

and $\begin{pmatrix} p \\ q \end{pmatrix}$ is a 2 × 1 matrix

So, to obtain a 2 × 1 matrix from a 2 × 2 matrix, we multiply a 2 × 2 matrix by 2 × 1 matrix.

A $m \times n$ matrix is multiplied by $n \times p$ matrix to obtain a $m \times p$ matrix.

(v) Option (a) is correct.

Explanation: AP = 57, 54, 51, 48, ...

Here, $a = 57$

$d = 54 - 57$

$d = -3$

Here, $a_n = a + (n - 1)d$

$\Rightarrow \quad a_8 = 57 + (8 - 1)(-3)$

$\Rightarrow \quad a_8 = 57 + 7(-3)$

$\Rightarrow \quad a_8 = 57 - 21$

$\Rightarrow \quad a_8 = 36$

(vi) Option (a) is correct.

Explanation: We know that only those points which lie on the line are invariant points when reflected in the line. So, only those points are invariant which are on the line $x = p$.

Hence, line passing through $x = p$ will be considered x– axis and thus the coordinates of image are A′ $(p, -q)$

(vii) Option (b) is correct.

Explanation: Here $\frac{4}{24} = \frac{3}{18}$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF}$$

and $\angle ABC = \angle DEF$

So, by SAS similarity criterion, ΔABC ~ ΔDEF

(viii) Option (c) is correct.

Explanation: Volume of cone = 120 cm^3

We know that,

$$\Rightarrow \quad \text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 120$$

$$\Rightarrow \quad \pi r^2 h = 120 \times 3$$

$\Rightarrow$ Volume of cylinder = 360 cm^3

(ix) Option (c) is correct.

Explanation: Here $-8 \le 2x < 8$

$$\Rightarrow \quad -8 \div 2 \le 2x \div 2 < 8 \div 2$$

$$\Rightarrow \quad -4 \le x < 4$$

$\Rightarrow$ $x = -4, -3, -2, -1, 0, 1, 2, 3$

But since $x \in W$,

$x = \{0, 1, 2, 3\}$

(x) Option (c) is correct.

Explanation: Since the sun always rises from east, it is a certain event. Therefore, the probability $P(S) = 1$

(xi) Option (d) is correct.

Explanation: $\begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 3 \\ 12 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 12 & 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2+6 & x+3 \\ 0+12 & 1+0 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 12 & 1 \end{pmatrix}$

On comparison

$x + 3 = 8$

$\Rightarrow$ $x = 5$

(xii) Option (b) is correct.

Explanation: The centroid of triangle

$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

$\Rightarrow$ $(6, 7) = \left(\frac{a+7+5}{3}, \frac{5+9+7}{3} \right)$

$\Rightarrow$ $(6, 7) = \left(\frac{a+12}{3}, \frac{21}{3} \right)$

On comparison, $6 = \frac{a+12}{3}$

$\Rightarrow$ $18 = a + 12$

$\Rightarrow$ $a = 6$

(xiii) Option (a) is correct.

Explanation: Join CD

Since AC is diameter

$\Rightarrow$ $\angle ADC = 90°$

(Angle in a semicircle are equal to 90°)

$\Rightarrow$ $\angle CDB + \angle ADB = 90°$

$\Rightarrow$ $\angle CDB + 35° = 90°$

$\Rightarrow$ $\angle CDB = 55°$

Now $\angle CAB = \angle CDB$

(Angles in the same segment)

$\Rightarrow$ $\angle CAB = 55°$

(xiv) Option (c) is correct.

Explanation: Since n^{th} term $= (n + 3)$

$\Rightarrow$ $a_n = (n + 3)$

$\Rightarrow$ $a_1 = (1 + 3)$

$\Rightarrow$ $a_1 = 4$

$\Rightarrow$ $a_2 = (2 + 3) = 5$

and $a_3 = (3 + 3) = 6$

So, the first three terms of the AP are 4, 5 and 6

(xv) Option (d) is correct.

Explanation: The median for a grouped frequency distribution is found graphically by drawing a cumulative frequency curve.

2. (i) Monthly deposit = ₹ 1200,

$n = 30$ months, $r = 6\%$

$$\text{Interest} = \frac{n(n+1) \times P \times r}{2400}$$

$$= \frac{30(30+1) \times 1200 \times 6}{2400}$$

$$= \frac{30 \times 31 \times 1200 \times 6}{2400}$$

$$= 30 \times 31 \times 3$$

$$= 2790$$

So, Maturity Value $= P \times n + I$

$= 1200 \times 30 + 2790$

$= 36000 + 2790$

$=$ ₹ 38790

(ii) Since, 3, 9, m, 81 and n are in continued proportion

$\Rightarrow$ $\frac{3}{9} = \frac{9}{m} = \frac{m}{81} = \frac{81}{n}$

So, $3m = 9 \times 9$

$\Rightarrow$ $m = 81 \div 3$

$\Rightarrow$ $m = 27$

Now, $27n = 81 \times 81$

$\Rightarrow$ $n = 81 \times 3$

$\Rightarrow$ $n = 243$

(iii) $LHS = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$\Rightarrow \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)(\cos A)}$

$\Rightarrow \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)}$

$\Rightarrow \frac{1 + 1 + 2 \sin A}{(1 + \sin A)(\cos A)}$

$(\cos^2 A + \sin^2 A = 1)$

$\Rightarrow \frac{2 + 2 \sin A}{(1 + \sin A)(\cos A)}$

$\Rightarrow \frac{2\,(1 + \sin A)}{(1 + \sin A)(\cos A)}$

$\Rightarrow \frac{2}{\cos A}$ $\left(\because \frac{1}{\cos A} = \sec A \right)$

$\Rightarrow 2 \sec A = \text{RHS}$ **Hence Proved**

3. (i) Inner circumference of circular metal tub
$= 44$ cm

$\Rightarrow \quad 2\pi r = 44$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow \quad r = 44 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow \quad r = 7$ cm

Now, Volume of material of tub

$= \frac{2}{3}\pi (R^3 - r^3)$

$= \frac{2}{3} \times \frac{22}{7} \times (8^3 - 7^3)$

$= \frac{2}{3} \times \frac{22}{7} \times (512 - 343)$

$= \frac{2}{3} \times \frac{22}{7} \times 169$

$= 354.095 \text{ cm}^3$

$= 354 \text{ cm}^3$

(ii) (a) Coordinates of $A = (5, 0)$,
Coordinates of $B = (0, 3)$

(b) $P = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$

$= \left(\frac{2(0) + 3(5)}{2+3}, \frac{2(3) + 3(0)}{2+3} \right)$

$= \left(\frac{15}{5}, \frac{6}{5} \right) = \left(3, \frac{6}{5} \right)$

Therefore, the coordinates of point P, are $\left(3, \frac{6}{5} \right)$.

(c) For parallel lines, $m_1 = m_2$

Slope of line $AB = \frac{3-0}{0-5} = \frac{3}{-5} = \frac{-3}{5}$

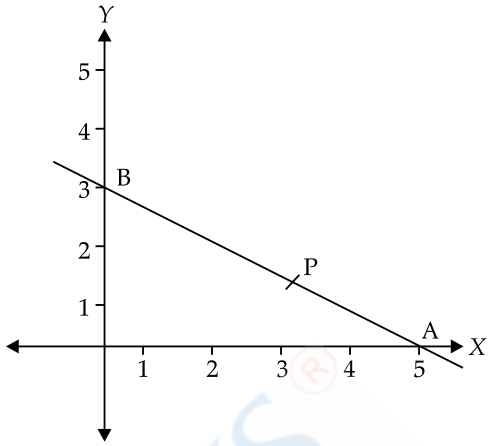
Now, $\frac{y - y_1}{x - x_1} = \frac{-3}{5}$

$\Rightarrow \quad \frac{y-0}{x-0} = \frac{-3}{5}$

$\Rightarrow \quad \frac{y}{x} = \frac{-3}{5}$

$\Rightarrow \quad -3x = 5y$

$\Rightarrow \quad 3x + 5y = 0$



SECTION-B

4. (i) MP of Graphic card = ₹15,500

GST = 18%

$\Rightarrow$ GST = 0.18×15500

= ₹2790

So, SP = 15500 + 2790

$\Rightarrow$ SP = ₹18,290

Now, MP of Laptop Adapter = ₹1900

GST = 28%

$\Rightarrow$ GST = 0.28×1900

$\Rightarrow$ GST = ₹532

So, SP = 1900 + 532

$\Rightarrow$ SP = ₹2432

Total amount to be paid = 18,290 + 2432

= ₹20,722

(ii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(7)(-2)}}{2(7)}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 + 56}}{14}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{60}}{14}$

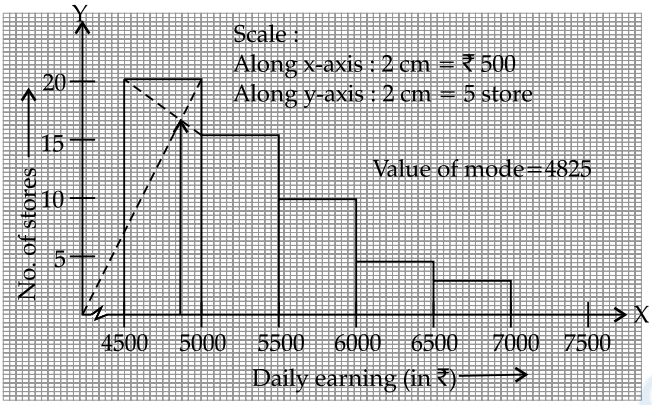
$\Rightarrow \quad x = \frac{-2 \pm 7.746}{14}$

$\Rightarrow \quad x = \frac{-2 + 7.746}{14}$ and $x = \frac{-2 - 7.746}{14}$

$\Rightarrow \quad x = \frac{5.746}{14}$ and $x = \frac{-9.746}{14}$

$\Rightarrow \quad x = 0.41$ and $x = 0.696 \sim 0.70$

(iii)



5. (i) $A = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

So, $AB = \begin{pmatrix} 18 & -2 \\ -6 & +4 \end{pmatrix} = \begin{pmatrix} 16 \\ -2 \end{pmatrix}$

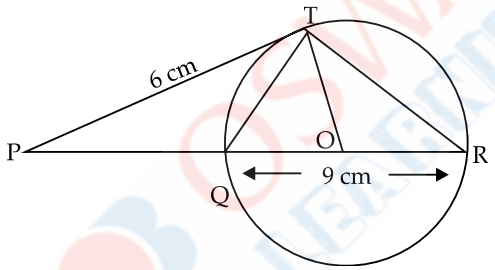
$C = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

So, $5C = \begin{pmatrix} -20 \\ 25 \end{pmatrix}$

Now,

$$AB - 5C = \begin{pmatrix} 16 \\ -2 \end{pmatrix} - \begin{pmatrix} -20 \\ 25 \end{pmatrix} = \begin{pmatrix} 16 & +20 \\ -2 & -25 \end{pmatrix} = \begin{pmatrix} 36 \\ -27 \end{pmatrix}$$

(ii) (a)



Here, in ΔPQT and ΔPTR

$\angle PQT = \angle PRT$

(Alternate Segment Theorem; For any circle, the angle formed between the tangent and the chord through the point of contact of the tangent is equal to the angle formed by the chord in the alternate segment)

Also, $\angle QPT = \angle TPR$ (common)

So, by AA similarity criterion,

$\Delta PQT \sim \Delta PTR$

(b) Now, If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

So, $PT^2 = PR \times PQ$

Let PQ be x cm

$\Rightarrow 6 \times 6 = (x + 9)(x)$

$\Rightarrow 36 = x^2 + 9x$

$\Rightarrow x^2 + 9x - 36 = 0$

$\Rightarrow x^2 + 12x - 3x - 36 = 0$

$\Rightarrow x(x + 12) - 3(x + 12) = 0$

$\Rightarrow (x - 3)(x + 12) = 0$

$\Rightarrow x = 3$ and $x = -12$

Since, x represents length of a segment, it can't be negative. Thus $PQ = 3$ cm

(iii) $p(x) = 6x^3 + 25x^2 + 31x + 10$

Here,

$p(-2) = 6(-2)^3 + 25(-2)^2 + 31(-2) + 10$

$= 6 \times -8 + 25 \times 4 + 31 \times -2 + 10$

$= -48 + 100 - 62 + 10$

$= 0$

So, $x = -2$ is the zero of the given polynomial.

$\Rightarrow x + 2$ is a factor of the given polynomial.

Now, by long division:

$$\begin{array}{l} x + 2 \overline{)\, 6x^3 + 25x^2 + 31x + 10 \,(6x^2 + 13x + 5} \\ \quad 6x^3 + 12x^2 \\ \quad (-)\quad (-) \\ \quad \overline{\quad 13x^2 + 31x + 10} \\ \quad\quad 13x^2 + 26x \\ \quad\quad (-)\quad\ (-) \\ \quad\quad \overline{\quad\quad + 5x + 10} \\ \quad\quad\quad\ 5x + 10 \\ \quad\quad\quad (-)\ (-) \\ \quad\quad\quad \overline{\quad 0} \end{array}$$

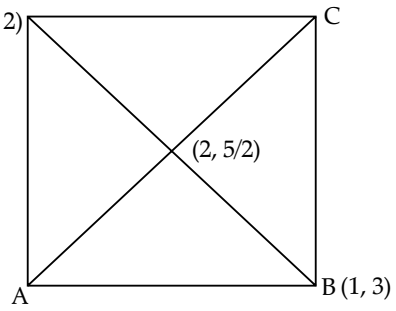
$p(x) = (x + 2)(6x^2 + 13x + 5)$

$= (x + 2)(6x^2 + 10x + 3x + 5)$

$= (x + 2)[2x(3x + 5) + 1(3x + 5)]$

$p(x) = (x + 2)(2x + 1)(3x + 5)$

6. (i)



Since ABCD is a square, diagonals bisect each other

$\Rightarrow$ diagonals AC and BD intersect each other at the midpoint of BD

$$\Rightarrow \text{Midpoint of } BD = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+3}{2}, \frac{3+2}{2}\right)$$

$$= \left(\frac{4}{2}, \frac{5}{2}\right)$$

$$= \left(2, \frac{5}{2}\right)$$

$\because$ diagonals of square are at right angles with each other. So $m_1 m_2 = -1$

$$\left(\frac{3-2}{1-3}\right)\left(\frac{y-\frac{5}{2}}{x-2}\right) = -1$$

$$\left(\frac{1}{-2}\right)\left(\frac{2y-5}{2x-4}\right) = -1$$

$$\frac{2y-5}{2x-4} = 2$$

$$\Rightarrow \quad 2y - 5 = 4x - 8$$

$$\Rightarrow \quad 4x - 2y - 3 = 0$$

(ii) $LHS = \sqrt{\sec^2\theta + \text{cosec}^2\theta}$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \ \sin^2\theta}}$$

$$= \sqrt{\frac{1}{\cos^2\theta \ \sin^2\theta}}$$

$$= \sqrt{\sec^2 q \ \text{cosec}^2 q}$$

$= \sec\theta \, \text{cosec}\theta = \text{RHS}$

Hence proved

(iii) $a = 98, a_n = 1001, d = 7$

(a) We know that, $a_n = a + (n-1)d$

$$\Rightarrow \quad 1001 = 98 + (n-1)(7)$$

$$\Rightarrow \quad 903 = (n-1)(7)$$

$$\Rightarrow \quad 129 = n - 1$$

$$\Rightarrow \quad n = 130$$

So, the number of terms is 130.

(b) Now, $S_n = \frac{n}{2}(a+l)$

$$= \frac{130}{2}(98 + 1001)$$

$$= 65 \times 1099$$

$$= 71{,}435$$

Therefore, sum of n terms is 71,435.

7. (i) (a) P (selecting green ball) $= \frac{1}{4}$

P (selecting yellow ball) $= \frac{1}{3}$

Now, P (selecting green ball) + P (selecting yellow ball) + P (selecting white ball) = 1

$$\Rightarrow \quad \frac{1}{4} + \frac{1}{3} + P \text{ (selecting white ball)} = 1$$

$$\Rightarrow \quad \frac{7}{12} + P \text{ (Selecting white ball)} = 1$$

$$\Rightarrow \quad \text{P (selecting white ball)} = 1 - \frac{7}{12}$$

$$\Rightarrow \text{P (selecting white ball)} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{10}{\text{Total number of balls}} = \frac{5}{12}$$

$\Rightarrow$ Total number of balls = 24

Therefore, total number of balls = 24

(b) P (selecting white ball) $= \frac{5}{12}$

(ii) Since, a cone and sphere are melted and re-casted into a cylinder

$\Rightarrow$ Volume of cone + Volume of sphere = Volume of cylinder

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h + \frac{4}{3}\pi r^3 = \pi R^2 H$$

$$\Rightarrow \quad \frac{4}{3}(3)^2(12) + \frac{4}{3}(3)^3 = (2)^2 H$$

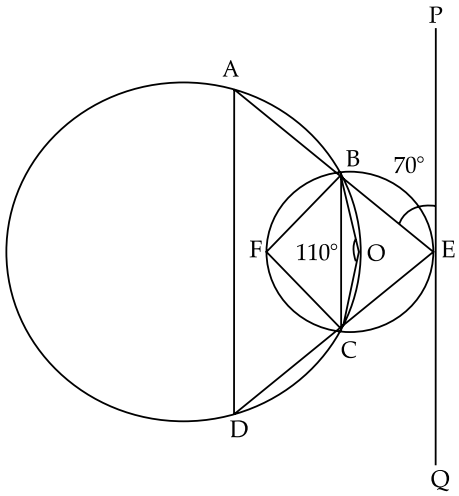
$$\Rightarrow \quad 36 + 36 = 4H$$

$$\Rightarrow \quad 72 = 4H$$

$$\Rightarrow \quad H = 18$$

Height of cylinder = 18 cm

(iii)



(a) Since $\angle AEP = 70°$

$\Rightarrow \quad \angle BEP = 70°$

$\Rightarrow \quad \angle BCE = \angle ECB = 70°$

(Alternate segments theorem)

(b) Since, $\angle BOC = 110°$

So, $\angle BEC = 110 \div 2 = 55°$

(Degree measure theorem)

(c) Here, reflex

$\angle BOC = 360° - 110° = 250°$

$\Rightarrow \quad \angle BFC = 250 \div 2 = 125°$

(Degree measure theorem)

(d) Since, $\angle BCE = 70°$

$\Rightarrow \quad \angle BCD = 180° - 70°$ (linearpair)

$\Rightarrow \quad \angle BCD = 110°$

Now, $\angle DAB + \angle BCD = 180°$ (Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow \quad \angle DAB = 180° - 110°$

$\Rightarrow \quad \angle DAB = 70°$

8. (i) $\frac{-x}{3} - 4 \le \frac{x}{2} - \frac{7}{3} < \frac{-7}{6}$

$\Rightarrow \quad \frac{-x}{3} - 4 + \frac{7}{3} \le \frac{x}{2} - \frac{7}{3} + \frac{7}{3} < \frac{-7}{6} + \frac{7}{3}$

$\Rightarrow \quad \frac{-x - 12 + 7}{3} \le \frac{x}{2} < \frac{7}{6}$

$\Rightarrow \quad \frac{-x-5}{3} \le \frac{x}{2} < \frac{7}{6}$

$\Rightarrow \quad \frac{-x}{3} - \frac{5}{3} \le \frac{x}{2} < \frac{7}{6}$

So, $\quad \frac{-x}{3} - \frac{5}{3} \le \frac{x}{2}$

$\Rightarrow \quad \frac{-x}{3} - \frac{x}{2} \le \frac{5}{3}$

$\Rightarrow \quad \frac{-2x-3x}{6} \le \frac{5}{3}$

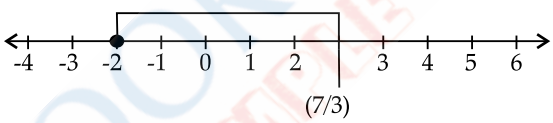
$\Rightarrow \quad \frac{-5x}{6} \le \frac{5}{3}$

$\Rightarrow \quad -x \le 2$

$\Rightarrow \quad x \ge -2$

And $\quad \frac{x}{2} < \frac{7}{6}$

$\Rightarrow \quad x < \frac{7}{3}$



(ii)

Price (₹)	No. of days	Class mark	$d_i = x_i - A$	$u_i = \frac{d_i}{h}$	$f_i u_i$
85–90	12	87.5	–10	–2	–24
90–95	10	92.5	–5	–1	–10
95–100	8	97.5=A	0	0	0
100–105	15	102.5	5	1	15
105–110	5	107.5	10	2	10
	$\Sigma f_i = 50$				$\Sigma f_i u_i = -9$

$$\overline{X} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 97.5 + \left(\frac{-9}{50}\right) \times 5$$

$$= 97.5 + \left(\frac{-9}{10}\right)$$

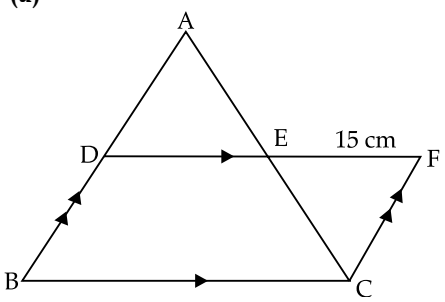
$$= 97.5 - 0.9$$

$\bar{X} \quad = 96.6$

So, price of petrol per litre (to nearest rupee)

= ₹ 97

(iii) (a)



Here, $DF \parallel BC$

(Opposite sides of parallelogram are parallel)

$\Rightarrow \quad DE \parallel BC$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$

(Basic Proportionality Theorem)

$\Rightarrow \quad \frac{4}{5} = \frac{AE}{EC}$

(b) $\angle ADE = \angle ABC$ (Corresponding angles)

And $\angle AED = \angle ACB$ (Corresponding angles)

By AA Similarity criterion, $\Delta ADE \sim \Delta ABC$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{AC} = \frac{DE}{BC}$

(By CPST)

Now, Since $\frac{AD}{DB} = \frac{4}{5}$

$\Rightarrow \quad \frac{DB}{AD} = \frac{5}{4}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{5}{4} + 1$

$\Rightarrow \quad \frac{DB + AD}{AD} = \frac{5+4}{4}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{9}{4}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{4}{9} = \frac{AE}{AC} = \frac{DE}{BC}$

$\Rightarrow \quad \frac{DE}{DF} = \frac{4}{9}$

(BC = DF; opposite sides of parallelogram are equal)

$\Rightarrow \quad \frac{DE}{DE+15} = \frac{4}{9}$

$\Rightarrow \quad 9DE = 4DE + 60$

$\Rightarrow \quad 5DE = 60$

$\Rightarrow \quad DE = 60 \div 5$

$\Rightarrow \quad DE = 12$ cm

(c) Also, since $\frac{DE}{BC} = \frac{4}{9}$

$\Rightarrow \quad \frac{12}{BC} = \frac{4}{9}$

$\Rightarrow \quad BC = 27$ cm

9. (i) Let number of days taken by Bijoy to complete the work alone = x

$\Rightarrow$ Time taken by Amit to finish the work alone = $(x - 12)$ days

$\Rightarrow$ Work done by Bijoy in one day = $\frac{1}{x}$

And Work done by Amit in one day = $\frac{1}{x-12}$

Time taken when both work together = 8 days.

According to question,

$$\frac{1}{x} + \frac{1}{x-12} = \frac{1}{8}$$

$$\Rightarrow \quad \frac{x - 12 + x}{(x)(x-12)} = \frac{1}{8}$$

$$\Rightarrow \quad \frac{2x-12}{x^2-12x} = \frac{1}{8}$$

$$\Rightarrow \quad 16x - 96 = x^2 - 12x$$

$$\Rightarrow \quad x^2 - 12x - 16x + 96 = 0$$

$$\Rightarrow \quad x^2 - 28x + 96 = 0$$

$$\Rightarrow \quad x^2 - 24x - 4x + 96 = 0$$

$$\Rightarrow \quad x(x - 24) - 4(x - 24) = 0$$

$$\Rightarrow \quad (x - 24)(x - 4) = 0$$

$$\Rightarrow \quad x - 24 = 0 \text{ and } x - 4 = 0$$

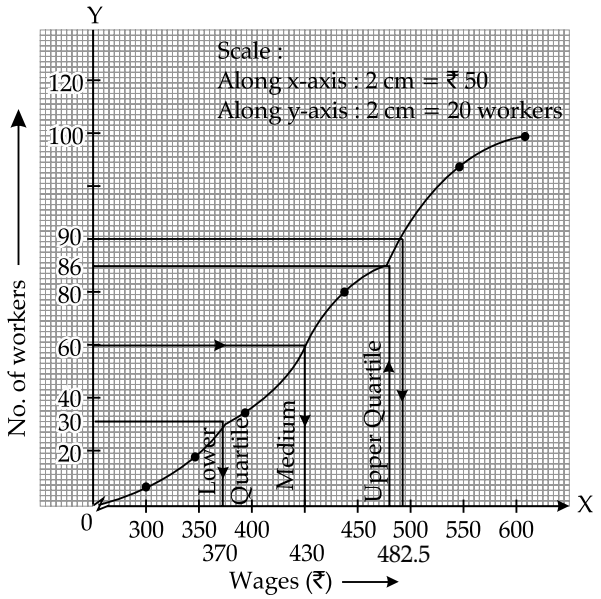
$$\Rightarrow \quad x = 24 \text{ and } x = 4$$

but if $x = 4$, then time taken by Amit will be $x - 12 = 4 - 12 = -8$ days; which is not possible.

Therefore, time taken by Bijoy to complete the work alone is 24 days.

(ii)

Wages (in ₹)	No. of workers (f)	Wages (in ₹)	No. of workers ($c.f.$)
250-300	8	Less than 300	8
300-350	15	Less than 350	23
350-400	20	Less than 400	43
400-450	30	Less than 450	73
450-500	25	Less than 500	98
500-550	15	Less than 550	113
550-600	7	Less than 600	120



(a) $N = 120$, which is even

$$\text{Median} = \left(\frac{120}{2}\right)^{\text{th}} \text{term}$$

$$= 60^{\text{th}} \text{ term.}$$

From the graph,

60^{th} term is: 430

Therefore, median = 430.

(b) $$\text{Upper quartile} = \left(\frac{3N}{4}\right)^{\text{th}} \text{term}$$

$$= \left(\frac{3\times 120}{4}\right)^{\text{th}} \text{term}$$

$$= 90^{\text{th}} \text{ term}$$

From the graph,

90^{th} term is: 482.5

Now, Lower quartile $= \left(\frac{N}{4}\right)^{\text{th}}$ term

$= 30^{\text{th}}$ term

from the graph,

30^{th} term is: 370

Inter – quartile range

$= Q_3 - Q_1$

$= 482.5 - 370$

$= 112.5$

(c) Number of workers above wage rate of ₹ 475

$= 120 - 86 = 34$

$$\text{Percentage} = \frac{34}{120} \times 100$$

$$= 28.33\%$$

10. (i) Here $\dfrac{\sqrt{2+x}+\sqrt{3-x}}{\sqrt{2+x}-\sqrt{3-x}} = 3$

On applying componendo and dividendo:

$$\frac{3+1}{3-1} = \frac{\sqrt{2+x}+\sqrt{3-x}+\sqrt{2+x}-\sqrt{3-x}}{\sqrt{2+x}+\sqrt{3-x}-\sqrt{2+x}+\sqrt{3-x}}$$

$$\Rightarrow \quad \frac{4}{2} = \frac{2\sqrt{2+x}}{2\sqrt{3-x}}$$

On squaring both sides

$$(2)^2 = \left(\frac{\sqrt{2+x}}{\sqrt{3-x}}\right)^2$$

$$4 = \frac{2+x}{3-x}$$

$$12 - 4x = 2 + x$$

$$10 = 5x$$

$$\Rightarrow \quad x = 10 \div 5$$

$$\Rightarrow \quad x = 2$$

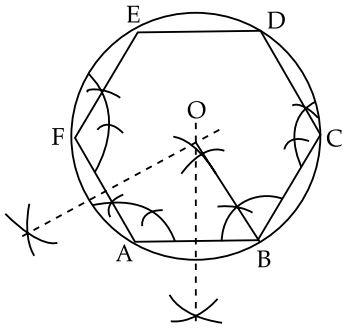
(ii) Step of construction:

(a) Draw a line segment AB = 4.5 cm.

(b) Taking A and B as centres draw lines AF, BC each of angle 120° and each of length 4.5 cm. Similarly, we draw other segment CD, DE and EF. Hence, we get the regular hexagon ABCDEF.

(c) Draw perpendicular bisector of AB and BC which meets at point O.

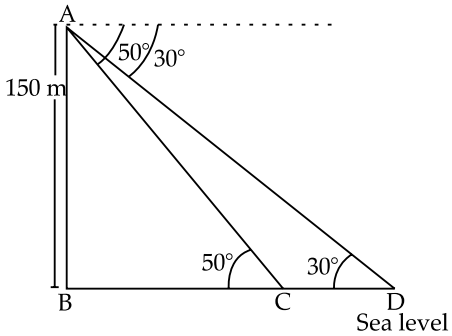
(d) Taking O as centre and OA as radius draw a circle passes through point A, B, C, D, E and F.



Circum-radius = 4.5 cm

For a hexagon OA = Radius

(iii)



Find CD

Here $\tan 50° = \frac{AB}{BC}$

$\Rightarrow \quad 1.1918 = \frac{150}{BC}$

$\Rightarrow \quad BC = \frac{150}{1.1918}$

$\Rightarrow \quad BC = 125.86$ m

Now, $\tan 30° = \frac{AB}{BD}$

$\Rightarrow \quad 0.57735 = \frac{150}{BD}$

$\Rightarrow \quad BD = \frac{150}{0.57735}$

$\Rightarrow \quad BD = 259.80$ m

So, distance travelled during observation

$= BD - BC$

$= 259.80 - 125.86$

$= 133.94$ m

~ 134 m

■■