2023

COMPUTER SCIENCE (CS)

Q. 1. The Lucas sequence L_n is defined by the recurrence relation:

$L_n = L_{n-1} + L_{n-2}$, for $n \geq 3$, with $L_1 = 1$ and $L_2 = 3$.

Which one of the options given is TRUE?

(a) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

(b) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$

(c) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$

(d) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

Q. 2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

Let det(A) and det(B) denote the determinants of the matrices A and B, respectively. Which one of the options given below is TRUE?

(a) det(A) = det(B)

(b) det(B) = –det(A)

(c) det(A) = 0

(d) det(AB) = det(A) + det(B)

Q. 3. Let $f(x) = x^3 + 15x^2 - 33x - 36$ be a real-valued function.

Which of the following statements is/are TRUE?

(a) $f(x)$ does not have a local maximum.

(b) $f(x)$ has a local maximum.

(c) $f(x)$ does not have a local minimum.

(d) $f(x)$ has a local minimum.

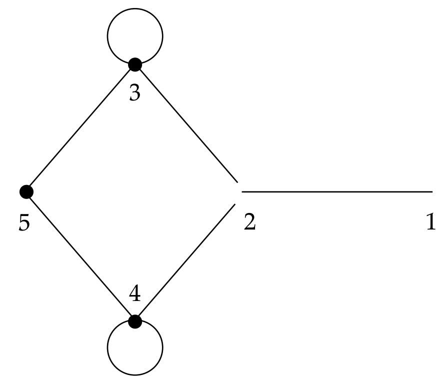
Q. 4. Let f and g be functions of natural numbers given by $f(n) = n$ and $g(n) = n^2$.

Which of the following statements is/are TRUE?

(a) $f \in O(g)$ **(b)** $f \in \Omega(g)$

(c) $f \in o(g)$ **(d)** $f \in \theta(g)$

Q. 5. Let A be the adjacency matrix of the graph with vertices {1, 2, 3, 4, 5}.



Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 be the five eigen values of A. Note that these eigen values need not be distinct.

The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ ______.

Q. 6. The value of the definite integral $\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1} (4x^2y - z^3)\, dzdydx$ is ____________.

(Rounded off to the nearest integer)

Q. 7. Let $U = \{1, 2, ..., n\}$, where n is a large positive integer greater than 1000. Let k be a positive integer less than n. Let A, B be subsets of U with $|A| = |B| = k$ and $A \cap B = \phi$.

We say that a permutation of U separates A from B if one of the following is true.

(a) $n!$

(b) $\binom{n}{2k}(n-2k)!$

(c) $\binom{n}{2k}(n-2k)!\,(k!)^2$

(d) $2\binom{n}{2k}(n-2k)!\,(k!)^2$

Q. 8. Let $f : A \to B$ be an onto (or surjective) function, where A and B are non-empty sets.

Define an equivalence relation - on the set A as

$$a_1 \sim a_2 \text{ if } f(a_1) = f(a_2),$$

where $a_1, a_2 \in A$. Let $\in = \{[x]: x \in A\}$ be the set of all the equivalence classes under $\sim$. Define a new mapping $F : \in \to B$ as

$F([x]) = f(x)$, for all the equivalence classes $[x]$ in $\in$.

Which of the following statements is/are TRUE?

(a) F is NOT well-defined.
(b) F is an onto (or surjective) function.
(c) F is a one-to-one (or injective) function.
(d) F is a bijective function.

Q. 9. Let X be a set and 2^X denote the power set of X.

Define a binary operation Δ on 2^X as follows:

$$A \,\Delta\, B = (A - B) \cup (B - A)$$

Let $H = (2^X, \Delta)$. Which of the following statements about H is/are correct?

(a) H is a group.
(b) Every element in H has an inverse, but H is NOT a group.
(c) For every $A \in 2^X$, the inverse of A is the complement of A.
(d) For every $A \in 2^X$, the inverse of A is A.

Q. 10. Consider a random experiment where two fair coins are tossed. Let A be the event that denotes HEAD on both the throws, B be the event that denotes HEAD on the first throw, and C be the event that denotes HEAD on the second throw. Which of the following statements is/are TRUE?

(a) A and B are independent.
(b) A and C are independent.
(c) B and C are independent.
(d) Prob $(B|C)$ = Prob(B)

Q. 11. Let G be a simple, finite, undirected graph with vertex set $\{v_1, ..., v_n\}$. Let $\Delta(G)$ denote the maximum degree of G and let $N = \{1, 2, ...\}$ denote the set of all possible colors. Color the vertices of G using the following greedy strategy:

for $i = 1, ..., n$

color$(v_i) \leftarrow$ min $\{j \in N$: no neighbour of v_i is colored $j\}$

Which of the following statements is/are TRUE?

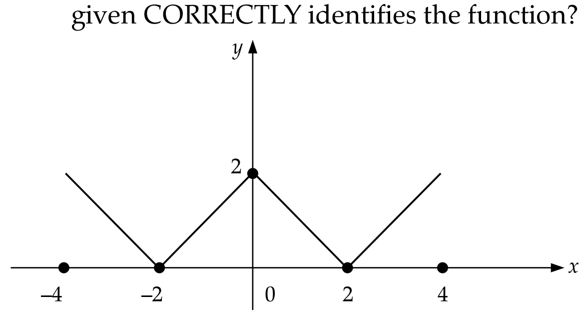
(a) This procedure results in a proper vertex coloring of G.
(b) The number of colors used is at most $\Delta(G) + 1$.
(c) The number of colors used is at most $\Delta(G)$.
(d) The number of colors used is equal to the chromatic number of G.

Q. 12. Let $U = \{1, 2, 3\}$. Let 2^U denote the power set of U. Consider an undirected graph G whose vertex set is 2^U. For any $A, B \in 2^U$, (A, B) is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subseteq B$ or $B \subseteq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by $B(A)$.

If ϕ denotes the empty set, then the cardinality of $B(\phi)$ is ____________ .

MECHANICAL ENGINEERING (ME)

Q. 13. The figure shows the plot of a function over the interval [–4, 4]. Which one of the options given CORRECTLY identifies the function?

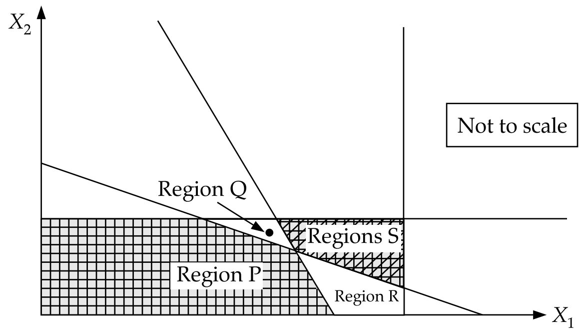


(a) $|2 - x|$ **(b)** $|2 - |x||$
(c) $|2 + |x||$ **(d)** $2 - |x|$

Q. 14. Which one of the options given represents the feasible region of the linear programming model:

Maximize $45x_1 + 60X_2$

$$x_1 \le 45$$
$$x_2 \le 50$$
$$10x_1 + 10x_2 \ge 600$$
$$25x_1 + 5x_2 \ge 750$$



(a) Region P **(b)** Region Q
(c) Region R **(d)** Region S

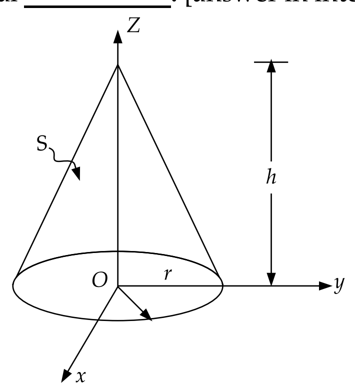
Q. 15. A vector field

$$B(x, y, z) = x\hat{i} + y\hat{j} - 2z\hat{k}$$

is defined over a conical region having height $h = 2$, base radius $r = 3$ and axis along z, as shown in the figure. The base of the cone lies in the x-y plane and is centered at the origin. If n denotes the unit outward normal to the curved surface S of the cone, the value of the integral.

$$\int_S B \cdot n dS$$

equal ___________. [answer in integer]



Q. 16. A linear transformation maps a point (x, y) in the plane to the point $(\hat{x}, \hat{y})$ according to the rule

$$\hat{x} = 3y, \ \hat{y} = 2x$$

Then, the disc $x^2 + y^2 \leq 1$ gets transformed to a region with an are equal to _____. (Rounded off to two decimals) Use $\pi = 3.14$.

Q. 17. The value of k that makes the complex-valued function

$$f(z) = e^{-kx}(\cos 2y - i \sin 2y)$$

analytic, where $z = x + iy$, is _______. [Answer in integer]

Q. 18. Which one of the options given is the inverse Laplace transform of $\dfrac{1}{S^3 - S}$?

u(t) denotes the unit-step function.

(a) $\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}\right)u(t)$

(b) $\left(\frac{1}{3}e^{-t} - e^{t}\right)u(t)$

(c) $\left(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{(t-1)}\right)u(t-1)$

(d) $\left(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{(t-1)}\right)u(t-1)$

Q. 19. The smallest perimeter that a rectangle with area of 4 square units can have is _________ units. (Answer in integer)

Q. 20. Consider the second-order linear ordinary differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0, \ x \geq 1$$

with the initial conditions

$$y(x=1) = 6, \left.\frac{dy}{dx}\right|_{x=1} = 2$$

the value of y at $x = 2$ equals _________. [Answer in integer]

Q. 21. The initial value problem

$$\frac{dy}{dt} + 2y = 0, \ y(0) = 1$$

is solved numerically using the forward Euler's method with a constant and positive time step of Δt.

Let Y_n represent the numerical solution obtained after n steps. The condition $|y_{n+1}| \leq |y_n|$ is satisfied if and only if .tit does not exceed _________. (Answer in integer)

ELECTRICAL ENGINEERING (EE)

Q. 22. For a given vector w = $[1 \ 2 \ 3]^T$, the vector normal to the plane defined by $w^T x = 1$ is

(a) $[-2 \ \ -2 \ \ 2]^T$ **(b)** $[3 \ \ 0 \ \ -1]^T$
(c) $[3 \ \ 2 \ \ 1]^T$ **(d)** $[1 \ \ 2 \ \ 3]^T$

Q. 23. The Fourier transform $X(\omega)$ of the signal $x(t)$ is given by

$$X(\omega) = 1, \ \text{for } |\omega| < W_0$$
$$= 0, \ \text{for } |\omega| > W_0$$

Which one of the following statements is true?

(a) $x(t)$ tends to be an impulse as $W_0 \to \infty$.

(b) $x(0)$ decreases as W_0 increases.

(c) At $t = \dfrac{\pi}{2W_0}$, $x(t) = -\dfrac{1}{\pi}$

(d) At $t = \dfrac{\pi}{2W_0}$, $x(t) = \dfrac{1}{\pi}$

Q. 24. The Z-transform of a discrete signal $x[n]$ is

$$X(z) = \frac{4z}{\left(z - \frac{1}{5}\right)\left(z - \frac{2}{3}\right)(z-3)} \text{ with ROC} = R$$

Which one of the following statements is true?

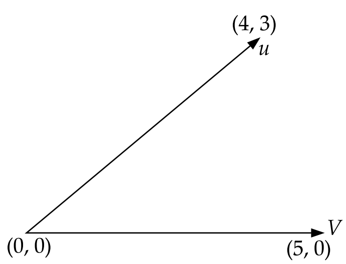
(a) Discrete-time Fourier transform of $x[n]$ converges if R is $|z| > 3$.

(b) Discrete-time Fourier transform of $x[n]$ converges if R is $\dfrac{2}{3} < |z| < 3$.

(c) Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a left-sided sequence.

(d) Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a right sided sequence.

Q. 25. In the figure, the vectors **u** and **v** are related as: **Au** = **v** by a transformation matrix **A**. The correct choice of **A** is



(a) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$ **(d)** $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

Q. 26. Three points in the x-y plane are (–1, 0.8), (0, 2.2) and (1, 2.8). The value of the slope of the best fit straight line in the least square sense is ________ (Round off to 2 decimal places).

Q. 27. Consider the following equation in a 2-D real-space.

$$|x_1|^p + |x_2|^p = 1 \text{ for } p > 0$$

Which of the following statement(s) is/are true.

(a) When $p = 2$, the area enclosed by the curve is π.

(b) When p tends to ∞, the area enclosed by the curve tends to 4.

(c) When p tends to 0, the area enclosed by the curve is 1.

(d) When $p = 1$, the area enclosed by the curve is 2.

Q. 28. Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = [3 \quad -2],\ D = 1$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _______ (Round off to the nearest integer).

Q. 29. The discrete-time Fourier transform of a signal $x[n]$ is $X(\Omega) = (1 + \cos \Omega)\, e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

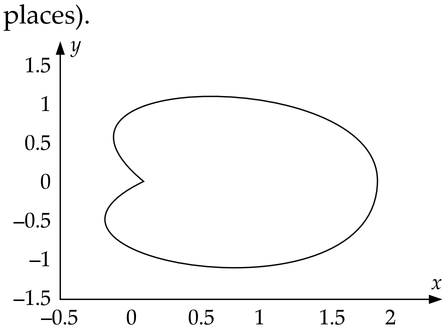
$$x_p[n] = x[n], \text{ for } n = 0, 1, 2$$
$$= 0, \text{ for } n = 3, 4$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is ________ (Round off to 3 decimal places).

Q. 30. The closed curve shown in the figure is described by $r = 1 + \cos\theta$,

where $r = \sqrt{x^2 + y^2}$; $x = r\cos\theta$, $y = r\sin\theta$.

The magnitude of the line integral of the vector field $F = -y\hat{i} + x\hat{j}$ around the closed curve is ___________ (Round off to 2 decimal places).



Q. 31. A quadratic function of two variables is given as

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$$

The magnitude of the maximum rate of change of the function at the point (1,1) is ________ (Round off to the nearest integer).

ELECTRONICS COMMUNICATION (EC)

Q. 32. Let $V_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$ and $V_2 = \begin{bmatrix}2\\1\\3\end{bmatrix}$ be two vectors.

The value of the coefficient α in the expression $V_1 = \alpha V_2 + e$, which minimizes the length of the error vector e, is

(a) $\frac{7}{2}$ **(b)** $-\frac{2}{7}$

(c) $\frac{2}{7}$ **(d)** $-\frac{7}{2}$

Q. 33. The rate of increase, of a scalar field $f(x, y, z) = xyz$ in the direction $v = (2, 1, 2)$ at a point $(0, 2, 1)$ is

(a) $\frac{2}{3}$ **(b)** $\frac{4}{3}$

(c) 2 **(d)** 4

Q. 34. Let $w^4 = 16j$. Which of the following cannot be a value of w?

(a) $2e^{\frac{j2\pi}{8}}$ **(b)** $2e^{\frac{j\pi}{8}}$

(c) $2e^{\frac{j5\pi}{8}}$ **(d)** $2e^{\frac{j9\pi}{8}}$

Q. 35. The value of the contour integral, $\oint_c \left(\frac{z+2}{z^2+2z+2}\right) dz$ where the contour C is $\left\{z : \left|z + 1 - \frac{3}{2}j\right| = 1\right\}$ taken in the counter clockwise direction, is

(a) $-\pi(1 + j)$ **(b)** $\pi(1 + j)$

(c) $\pi(1 - j)$ **(d)** $-\pi(1 - j)$

Q. 36. Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{v_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B = P^{-1}AP$, respectively, are

(a) $\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{Pv_k \mid 1 \le k \le n\}$

(b) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{v_k \mid 1 \le k \le n\}$

(c) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{Pv_k \mid 1 \le k \le n\}$

(d) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}v_k \mid 1 \le k \le n\}$

Q. 37. The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

(a) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$ **(b)** $\frac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

(c) $\sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$ **(d)** $\sqrt{\pi}\, e^{-\frac{\omega^2}{2}}$

Q. 38. The value of the line integral $\int_P^Q (z^2\, dx + 3y^2\, dy + 2xz\, dz)$ along the straight line joining the points $P(1, 1, 2)$ and $Q(2, 3, 1)$ is:

(a) 20 **(b)** 24

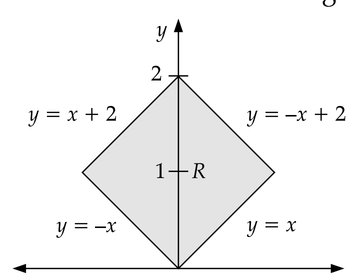
(c) 29 **(d)** –5

Q. 39. Let x be an $n \times 1$ real column vector with length $l = \sqrt{x^T x}$. The trace of the matrix $P = xx^T$ is

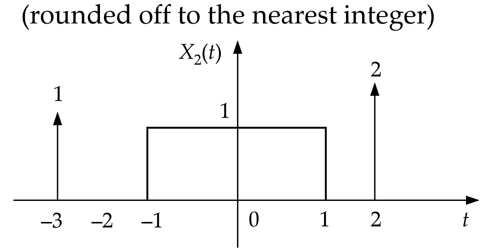
(a) l^2 **(b)** $\frac{l^4}{4}$

(c) l **(d)** $\frac{l^2}{2}$

Q. 40. The value of the integral $\iint_R xy\, dx\, dy$ over the region R, given in the figure, is _______. (rounded off to the nearest integer).



Q. 41. Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) \bullet x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\, dt$ is __________. (rounded off to the nearest integer)



INSTRUMENTATION ENGINEERING (IN)

Q. 42. Choose solution set S corresponding to the systems of two equations

$$x - 2y + z = 0$$
$$x - z = 0$$

Note: R denotes the set of real numbers

(a) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

(b) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(c) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(d) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

Q. 43. $F(z) = \dfrac{1}{1-z}$ when expanded as a power series around $z = 2$, would result in $F(z) = \sum_{k=0}^{\infty} a_k (z-2)^k$ with the region of convergence (ROC)$|z - 2| < 1$. The coefficients $a_k \geq 0$, are given by the expression ________.

(a) $(-1)^k$ **(b)** $(-1)^{k+1}$

(c) $\left(\dfrac{1}{2}\right)^k$ **(d)** $\left(\dfrac{-1}{2}\right)^{k+1}$

Q. 44. The solution $x(t)$, $t \geq 0$, to the differential equation $\ddot{x} = -k\dot{x}, k > 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ is

(a) $x(t) = 2e^{-kt} + 2kt - 1$

(b) $x(t) = 2e^{-kt} + 1$

(c) $x(t) = 1$

(d) $x(t) = 2e^{-kt} - kt - 1$

Q. 45. A system has the transfer-function $\dfrac{Y(s)}{X(s)} = \dfrac{s-\pi}{s+\pi}$. Let $u(t)$ be the unit-step function. The input x(that results in a steady-state output $y(t) = \sin \pi t$ is ________.

(a) $x(t) = \sin(pt)u(t)$

(b) $x(t) = \sin\left(\pi t + \dfrac{\pi}{2}\right)u(t)$

(c) $x(t) = \sin\left(\pi t - \dfrac{\pi}{2}\right)u(t)$

(d) $x(t) = \cos\left(\pi t + \dfrac{\pi}{2}\right)u(t)$

Q. 46. What is $\lim_{x \to \infty} f(x)$, where $f(x) = x \sin \dfrac{1}{x}$?

(a) o **(b)** 1

(c) ∞ **(d)** Limit does not exist

Q. 47. The number of zeros of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right-half plane is ________.

Q. 48. Let $y(t) = x(4t)$, where $x(t)$ is a continuous-time periodic signal with fundamental period of 100s. The fundamental period of $y(t)$ is ________s (rounded off to the nearest integer).

Q. 49. X is a discrete random variable which takes values 0, 1 and 2. The probabilities are $P(X = 0) = 0.25$ and $P(X = 1) = 0.5$. With $E[\,.\,]$ denoting the expectation operator, the value of $E[X] - [X^2]$ is ________(rounded off to one decimal place).

Q. 50. The Laplace transform of the continuous-time signal $x(t) = e^{-3t}\, u(t-5)$ is________, where $u(t)$ denotes the continuous-time unit step signal.

(a) $\dfrac{e^{-5s}}{s+3}$, Real $\{s\} > -3$

(b) $\dfrac{e^{-5(s-3)}}{s-3}$, Real $\{s\} > -3$

(c) $\dfrac{e^{-5(s+3)}}{s+3}$, Real $\{s\} > -3$

(d) $\dfrac{e^{-5(s-3)}}{s+3}$, Real $\{s\} > -3$

Q. 51. Let $f(z) = j\dfrac{1-z}{1+z}$, where z denotes a complex number and j denotes $\sqrt{-1}$. The inverse function $f^{-1}(z)$ maps the real axis to the________.

(a) unit circle with centre at the origin

(b) unit circle with centre not at the origin

(c) imaginary axis

(d) real axis

Q. 52. How many five-digit numbers can be formed using the integers 3, 4, 5 and 6 with exactly one digit appearing twice?

Q. 53. Five measurements are made using a weighing machine, and the readings are 80 kg, 79 kg, 81 kg, 79 kg and 81 kg. The sample standard deviation of the measurement is ________ kg (rounded off to two decimal places).

Q. 54. Consider the real-valued function $g(x) = \max\{(x-2)^2, -2x+7\}$, where $x \in (-\infty, \infty)$. The minimum value attained by $g(x)$ is ________ (rounded off to one decimal place).

Q. 55. The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is ________ (rounded off to the nearest integer). $A = \begin{bmatrix} 1 & A \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$

CHEMICAL ENGINEERING (CH)

Q. 56. Which one of the following is the CORRECT value of y, as defined by the expression given below?

$$\hat{v} = \lim_{x \to 0} \frac{2x}{e^x - 1}$$

(a) 1 **(b)** 2
(c) 0 **(d)** ∞

Q. 57. The vector $\vec{v}$ is defined as

$$\vec{v} = zx\hat{i} + 2xy\hat{j} + 3yz\hat{k}$$

Which one of the following is the CORRECT value of divergence of $\vec{v}$, evaluated at the point $(x, y, 0) = (3, 2, 1)$?

(a) 0 **(b)** 3
(c) 14 **(d)** 13

Q. 58. Given that

$$F = \frac{|z_1 + z_2|}{|z_1| + |z_2|}$$

where $z_1 = 2 + 3i$ and $z_2 = -2 + 3i$ with $i = \sqrt{-1}$ which one of the following options is CORRECT?

(a) $F < 0$ **(b)** $F < 1$
(c) $F > 0$ **(d)** $F = 1$

Q. 59. For a two-dimensional plane, the unit vectors, $(\hat{e}_r, \hat{e}_\theta)$ of the polar coordinate system and $(\hat{i}, \hat{j})$ of the cartesian coordinate system, are related by the following two equations.

$$\hat{e}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$$
$$\hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

Which one of the following is the CORRECT value of $\frac{\partial(\hat{e}_r + \hat{e}_\theta)}{\partial\theta}$?

(a) 1 **(b)** 8a
(c) $\hat{e}_r + \hat{e}_\theta$ **(d)** $-\hat{e}_r + \hat{e}_\theta$

Q. 60. The position $x(t)$ of a particle, at constant ω, is described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The initial conditions are $x(t = 0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$ then position of particle at $t = \left(\frac{3\pi}{\omega}\right)$ is ________ (in integer).

Q. 61 An exhibition was held in a hall on 15 August 2022 between 3 PM and 4 PM during which any person was allowed to enter only once. Visitors who entered before 3:40 PM exited the hall exactly after 20 minutes from their time of entry. Visitors who entered at or after 3:40 PM, exited exactly at 4 PM. The probability distribution of the arrival time of any visitor is uniform between 3 PM and 4 PM. Two persons X and Y entered the exhibition hall independent of each other. Which one of the following values is the probability that their visits to the exhibition overlapped with each other?

(a) $\frac{5}{9}$ **(b)** $\frac{4}{9}$
(c) $\frac{2}{9}$ **(d)** $\frac{7}{9}$

Q. 62. Simpson's one-third rule is used to estimate the definite integral

$$I = \int_{-1}^{1} \sqrt{(1 - x^2)}\, dx$$

with an interval length of 0.5. Which one of the following is the CORRECT estimate of I obtained using this rule?

(a) $\frac{1}{3}-\frac{1}{\sqrt{3}}$ **(b)** $\frac{1}{3}+\frac{2}{\sqrt{3}}$

(c) $\frac{1}{3}+\frac{1}{\sqrt{3}}$ **(d)** $\frac{1}{3}-\frac{2}{\sqrt{3}}$

Q. 63. If a matrix M is defined as $M = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$, the sum of all the eigenvalues of M^3 is equal to ________ (in integer).

Q. 64. The first derivative of the function

$$U(r) = 4\left[\left(\frac{1}{r}\right)^{12} - \left(\frac{1}{r}\right)^{6}\right]$$

evaluated at $r = 1$ is ________ (in integer).

CIVIL ENGINEERING (CE) P1

Q. 65. For the integral

$$I = \int_{-1}^{1} \frac{1}{2}\,dx$$

which of the following statements is TRUE?

(a) $I = 0$

(b) $I = 2$

(c) $I = -2$

(d) The integral does not converge

Q. 66. The following function is defined over the interval [– L, L]:

$$F(x) = px^4 + qx^5$$

If it is expressed as a Fourier series, which options amongst the following are true?

(a) a_n, $n = 1, 2, ..., \infty$ depend on p

(b) a_n, $n = 1, 2, ..., \infty$ depend on q

(c) b_n, $n = 1, 2, ..., \infty$ depend on p

(d) b_n, $n = 1, 2, ..., \infty$ depend on q

Q. 67. The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)? ____________.

Q. 68. In the differential equation $\frac{dy}{dx} + \alpha xy = 0$, is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is ______________ (rounded off to three decimal places).

Q. 69. For the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Which of the following statements is/are TRUE?

(a) The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$

(b) The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$

(c) The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$

(d) The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$

Q. 70. For the function $f(x) = e^x|\sin x|$, $x \in R$ which of the following statements is/are TRUE?

(a) The function is continuous at all x

(b) The function is differentiable at all x

(c) The function is periodic

(d) The function is bounded

Q. 71. The differential equation,

$$\frac{du}{dt} + 2tu^2 = 1,$$

is solved by employing a backward difference scheme within the finite difference framework. The value of u at the $(n-1)^{th}$ time-step, for some n, is 1.75. The corresponding time (t) is 3.14 s. Each time step is 0.01 s long. Then, the value of $(u_n - u_{n-1})$ is ________. (round off to three decimal places).

CIVIL ENGINEERING (CE) P2

Q. 72. For the matrix

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

which of the following statements is/are TRUE?

(a) $[A]\{x\} = \{b\}$ has a unique solution

(b) $[A]\{x\} = \{b\}$ does not have a unique solution

(c) $[A]$ has three linearly independent eigenvectors

(d) $[A]$ is a positive definite matrix

Q. 73. The solution of the differential equation

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

is expressed as $y = C_1e^{2.5x} + C_2e^{\alpha x}, C_3e^{\beta x}$ where C_1, C_2, C_3, α, and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α and β?

(a) 1 and 2
(b) –1 and –2
(c) 2 and 3
(d) –2 and –3

Q. 74. Two vectors $[2\ 1\ 0\ 3]^T$ and $[1\ 0\ 1\ 2]^T$ belong to the null space of a 4 × 4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

(a) $[1\ 1\ –1\ 1]^T$ **(b)** $[2\ 0\ 1\ 2]^T$
(c) $[0\ –2\ 1\ –1]^T$ **(d)** $[3\ 1\ 1\ 2]^T$

Q. 75. Cholesky decomposition is carried out on the following square matrix [A].

$$[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

Let L_{ij} and A_{ij} be the $(i, j)^{th}$ elements of matrices [L] and [A], respectively. If the element l_{22} of the decomposed lower triangular matrix $[L]$ is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?

(a) 5 **(b)** 7
(c) 9 **(d)** 11

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(a)	Finding CI and PI	Linear Higher Order Differential Equations
2	(a)	Determinants	Linear Algebra
3	(b) & (d)	Maxima nad Minima	Advanced Calculus
4	(a) & (c)	Functions	Functions
5	2	Eigen values and Eigen vectors	Eigen values and Eigen vectors
6	0	Multiple Integrals	Triple Integrals
7	(d)	Permutations and Combinations	Combinations
8	(b, c, d)	Types of Functions	Functions
9	(a, d)	Laws of Indices	Group Theory
10	(d)	Conditional Probability	Probability
11	(a) & (b)	Graph theory	Graph Theory
12	5040	Power Set	Set Theory
13	(b)	Graphs of Two Functions	Graphs
14	(d)	Linear Programming Problem	Operation Research
15	0	Gauss Divergence Theorem	Vector Integration
16	18.84	Minor and Major Axis	Geometry

17	2	C-R Equations	Complex Analysis
18	(a)	Partial Fractions Method	Inverse Laplace Transforms
19	8	Perimeters	Areas
20	9	CI and PI	Higher Order Linear Differen-tial Equations
21	1	Eulers Method	Numerical Methods
22	(d)	Normal Vectors	Vector Calculus
23	(b)	Signals and Systems	PDE
24	(b)	Region of Convergence	Z Transform
25	(a)	Vector Space	Vector Space
26	1.93	Straight line	Curve fitting
27	(a, b, d)	Vector Space	Vector Space
28	2	Laplace Transform	Laplace Transform
29	0.038	Periodic Signal	Fourier Transform
30	9.42	Green' theorem	Vector Calculus
31	10	Gradient	Vector Differentiation
32	(c)	Maxima and Minima	Vector Space
33	(b)	Gradient	Vector Differentiation
34	(a)	De Moiver's	De Moiver's
35	(b)	Region of Convergence	Complex Analysis
36	(c)	Diagonalization of a Matrix	Linear Algebra
37	(c)	Fourier Transform	Fourier Transform
38	(b)	Line Integral	Vector Integration
39	(a)	Orthogonalization of a Matrix	Linear Algebra
40	0	Areas of Double Integration	Multiple Integrals
41	15	Unit Laplace Transform	Laplace Transform
42	(a)	Solving Equations	Theory of Equations
43	(b)	Region of Convergence	Z Transform
44	(c)	CI and PI	Higher order Differential Equa-tions
45	(c)	Unit Step Function	Laplace Transform
46	(b)	Limits of a Function	Calculus
47	2	Polynomials	Polynomials
48	25	Time Period	Periodic Function
49	–0.5	Expectations	Random Variables
50	(c)	Second Shifting Theorem	Laplace Transform

51	(a)	Complex Number	Complex Analysis
52	240	Eulers Formula	Numerical Methods
53	1	Standard Deviation	Measures of Dispersion
54	1	Maxima and Minima	Graphs of Functions
55	–8	Minor of a Matrix	Matrices
56	(b)	Limits	Calculus
57	(d)	Divergence	Vector Differentiation
58	(b)	Complex function	Complex Analysis
59	(d)	Demoveries	Complex Analysis
60	–1	CI and PI	Higher Order Linear Differential Functions
61	(b)	Probability	Probability
62	(b)	Simpsons 1/3rd rule	Numerical Integration
63	41160	Eigen Values	Liner Algebra
64	–24	Differentiation	Differentiation
65	(d)	Convergence	Integration
66	(b, c)	Change of interval	Fourier Series
67	0.9	Independent Events	Probability
68	0.446	Solving the DE	Differential Equations
69	(a, b, c, d)	Eigen Values and Eigen vectors	Linear Algebra
70	(a)	Countinuous and Bounded	Real Analysis
71	–0.1823	Eulers Formula	Numerical methods
72	(b, c)	Linear Solutions	Simultaneous solutions
73	(a)	Complementary functions	Higher Order Lde
74	(a)	Nullity Theorem	Vector Spaces
75	(b)	LU Decomposition	Linear Algebra

ENGINEERING MATHEMATICS

GATE Solved Papers 2023

ANSWERS WITH EXPLANATIONS

COMPUTER SCIENCE (CS)

1. Option (a) is correct.

Given $L_n = L_{n-1} + L_{n-2}$, for $n \geq 3$ (1)

Here, $L_1 = 1$ and $L_2 = 3$

Replacing n by $n + 2$ in equation (1),

$L_{n+2} = L_{n+1} + L_n$

$\Rightarrow L_{n+2} - L_{n+1} - L_n = 0$ (2)

A.E. because $E^2 - E - 1 = 0$

So, $E = \dfrac{1 \pm \sqrt{1-4\times1\times-1}}{2\times1} = \dfrac{1\pm\sqrt{5}}{2}$

$\therefore$ C.F. $= C_1 (E_1)^n + C_2 (E_2)^n$

$$= C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

P. I = 0, $\because$ R.H.S. = 0

Complete solution

$$L_n = \text{C.F.} + \text{P.I.} = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{n}\right)^n$$

When $n = 1$, $L_1 = 1 = C_1\left(\dfrac{1+\sqrt{5}}{2}\right) + C_2\left(\dfrac{1-\sqrt{5}}{2}\right)$ (3)

and $n = 2$ $L_2 = 3 = C_1\left(\dfrac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\dfrac{1-\sqrt{5}}{2}\right)^2$ (4)

By solving equations (3) and (4), we get

$$C_1\left(\frac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$$

$$C_1\left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2 = \frac{1-\sqrt{5}}{2}$$

$$-\qquad\qquad -\qquad\qquad -$$

$$\left(\frac{1+\sqrt{5}}{2}\right)C_1\left[\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right] = 3 - \left(\frac{1-\sqrt{5}}{2}\right)$$

$$\Rightarrow \left(\frac{1+\sqrt{5}}{2}\right)C_1\left[\frac{1+\sqrt{5}-1+\sqrt{5}}{2}\right] = \frac{6-1+\sqrt{5}}{2}$$

$$\Rightarrow \left(\frac{1+\sqrt{5}}{2}\right)\times\frac{2\sqrt{5}}{2}C_1 = \frac{5+\sqrt{5}}{2}$$

$$\Rightarrow \frac{\sqrt{5}+5}{2}C_1 = \frac{5+\sqrt{5}}{2}$$

$\Rightarrow C_1 = 1$

From (3), $\left(\dfrac{1+\sqrt{5}}{2}\right)\times 1 + C_2\left(\dfrac{1-\sqrt{5}}{2}\right) = 1$

$$\Rightarrow \left(\frac{1-\sqrt{5}}{2}\right)C_2 = 1 - \left(\frac{1+\sqrt{5}}{2}\right) = \frac{2-1-\sqrt{5}}{2}$$

$$\Rightarrow \left(\frac{1-\sqrt{5}}{2}\right)C_2 = \frac{1-\sqrt{5}}{2} \qquad C_2 = 1$$

Hence, $L_n = \left(\dfrac{1+\sqrt{5}}{2}\right)^n + \left(\dfrac{1-\sqrt{5}}{2}\right)^n$

2. Option (a) is correct.

Given

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

Option (a):

det (A) = det (B)

Since, interchanging the rows of a matrix, the determinant does not change

So, det (A) = –192 and det (B) = –192

3. Options (b & d) are correct.

Given $f(x) = x^3 + 15x^2 - 33x - 36$ be a real valued function.

Differentiate with respect to x both sides, we get

Initial coordination

$$\frac{dy}{dx} = f'(x) = 3x^2 + 30x - 33 = 0$$

$\Rightarrow x^2 + 10x - 11 = 0$

$\Rightarrow (x + 11)(x - 1) = 0$

$\Rightarrow x = 1, x = -11$

$$\frac{d^2y}{dx^2} = f''(x) = 6x + 30$$

When, we check at $x = 1$, $f''(1) = 6 + 30 = 36$ +ve

$f(x)$ has local Minima at $x = 1$.

Now, we check at $x = -11$

$f''(-11) = -66 + 30 = -36 < 0$

$\therefore$ $f(x)$ has local Maxima at $x = -11$.

4. Options (a) & (c) are correct.

Here, f and g be two functions of natural numbers given by

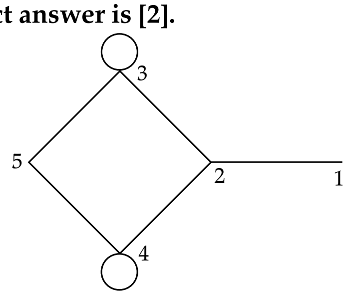
$f(x) = n$ and $g(n) = n^2$

$f(n) \in O(g(n))$ TH $f(n)$ asymptotically smaller or equal to $g(n)$.

$f(n) \in O(g(n))$ TH $f(n)$ asympotically smaller than $g(n)$

$$n \in O(n^2)$$
$$n \in \Omega(n^2)$$
$$n \in O(n^2)$$
$$n \in \theta(n^2)$$

5. Correct answer is [2].



A be the adjacency matrix of the graph with vertices $\{1, 2, 3, 4, 5\}$

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 0 \end{bmatrix} \end{array} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 0 \end{bmatrix} \end{array}$$

Let $\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4$ and λ_5 be the five five eigen values of A.

We have sum of eigen values of A = Trace of A

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 0 + 0 + 01 + 01 + 0 = 2$

6. Correct answer is (0).

$$I = \int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)\,dz\,dy\,dx$$

$$= \int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1} 4x^2y\,dx\,dy\,dz - \int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1} z^3\,dx\,dy\,dz$$

$$= \int_{-3}^{3}\int_{-2}^{2} 4x^2y\,[z]_{-1}^{1}\,dx\,dy - \int_{-3}^{3}\int_{-2}^{2}\left[\frac{z^4}{4}\right]_{-1}^{1} dx\,dy$$

$$= 8\int_{-3}^{3}\int_{-2}^{2} x^2y\,dxdy - \frac{1}{4}\int_{-3}^{3}\int_{-2}^{2}[1-1]\,dx\,dy$$

$$= 8\int_{-3}^{3} x^2\left[\frac{4^2}{2}\right]^2 dx - \frac{1}{4}\times 0 \quad = \quad 8\int_{-3}^{3}\frac{x^2}{2}[4-4]\,dx$$

$$= 4\int_{-3}^{3} x^2 \times 0\,dx = 0 \qquad \text{Hence, } I = 0$$

7. Option (d) is correct.

given $U = \{1, 2, 3, 4, \ldots, n\}$ where $n > 1000$

$A \subseteq U$, $B \subseteq U$ and $|A| = |B| = K$

and $A \cap B = \varnothing$ $(k < n)$

Case I: It all element of A appear before the elements of B then the number of permutations.

$$= {}^nC_{2k} \cdot (n-2k)! \cdot k! \cdot (k!)$$
$$= {}^nC_{2k} \cdot (n-2k)!\,(k!)^2$$

Case II: If all element of B appear before the elements of 'A' then number of permutations

$$= {}^nC_{2k}\,(n-2k)! \cdot (k!)^2$$

$\therefore$ Total number of permutations

= Case I + Case II

$$= 2\,{}^nC_{2k}\,(n-2k)! \cdot (k!)^2$$

8. Options (b), (c) & (d) are correct.

Given $f : A \to B$ be an auto (or subjective) functions.

We have $f(x_1) = f(x_2)$

$$x_1 = x_2 \ \forall\ x_1, x_2 \in A$$

Every equivalence class of 'x' under F is uniquely mapped with some element 'x', hence F is a function and every function is well defined so option (a) is not correct.

Distinct equivalence $[x]$, $[y]$ are having distinct images every element of co-domain 'B' is associated with same element of $\in$ under F so F is auto hence F is bijective.

9. Options (a) & (d) are correct.

Let X be any set and 2^X is power set of X

$A \Delta B = (A - B) \cup (B - A)$ for $A, B \in 2^X$

$H = (2^X, \Delta)$

H has satisfies the following properties

(a) H satisfies Closure law under the operation 'Δ'

(b) H satisfies Associative law under the operation 'Δ'

(c) H satisfies Identity law under '$\varnothing$' is identity.

$a \Delta \varnothing = \varnothing \Delta a = a \ \forall \ a \in Z^X$

(d) H is satisfies Inverse law for $A \in 2^X$ we have Inverse of $A = A$.

(e) H is satisfying commutative law

$$A \Delta B = B \Delta A \text{ for all } A, B \in 2^X$$

$$H = (2^X, \Delta) \text{ is abelian group.}$$

10. Option (d) is correct.

Let S be the sample space of tossing two coins

$$S = \{HH, HT, TH, TT\}$$

$P(A)$ = Probability of getting heads in both toss

$$= \frac{2}{4} = \frac{1}{2}$$

$P(B)$ = Probability of getting heads on first toss

$$= \frac{2}{4} = \frac{1}{2}$$

$P(C)$ = Probability of getting heads on second toss

$$= \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4} = P(A) \cdot P(B)$$

$\therefore$ A and B are not Independent.

$$P(B \cap C) = \frac{1}{4} = P(B) \cdot P(C)$$

$\therefore$ B and C are not independent

$$P(A \cap C) = \frac{1}{4} = P(A) \cdot P(C)$$

$\therefore$ A and C are not Independent

$$P\left(\frac{B}{C}\right) = P(B) \text{ is True.}$$

$$P\left(\frac{B}{C}\right) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{1}{2} = P(B)$$

11. Option (b) is correct.

Let $V = \{V_1 \ V_2, \ldots \ V_n\}$ be the set of vertices of graph G.

$\Delta(G)$ = Maximum degree of G

$N = \{11, 2, 3, \ldots\}$ set of all possible colours.

Greedy strategy:

Colour $(V_i) \leftarrow$ Min. $\{j \in N$: no neighbour of V_i is coloured $j\}$

By using the above strategy. no, two adjacent vertex have same colour so it is proper vertex colouring hence option (a) is True.

By using Leonard Brooke's Theorem, we chromatic number of G is almost $\Delta + 1$ hence option (b) is True.

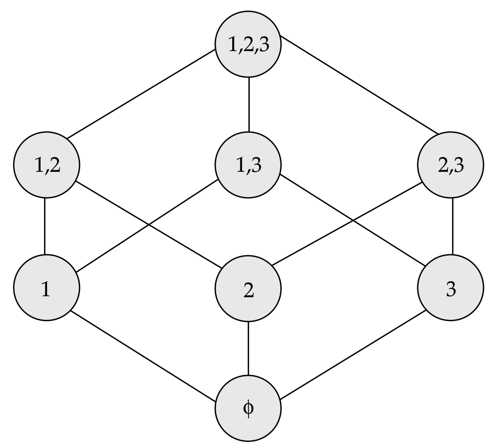
12. Correct answer is [5040].

given $U = \{1, 2, 3\}$ graph according to description U has 3 element hence if power set 2^U consist 8 elements

let 2^U denote the power set of U.

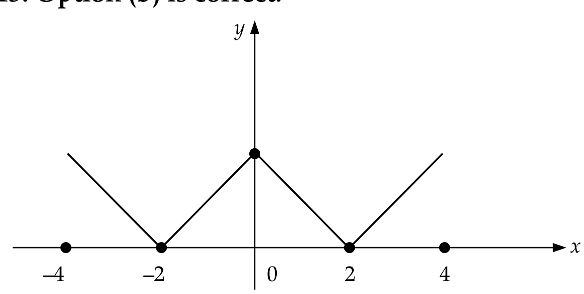
$2^U = \{\varnothing, \{1\}, \{2\}, \{3\} \ \{1, 2\} \ \{1, 3\} \ \{2, 3\} \ \{1, 2, 3\}$

Number of BFS sequences from $\varnothing$ is B($\varnothing$) are $7! = 5040$

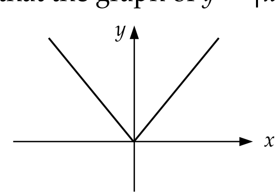


MECHANICAL ENGINEERING (ME)

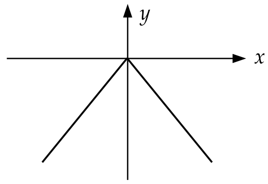
13. Option (b) is correct.



We know that the graph of $y = |x|$ is



and the graph of $y = -|x|$ is



it can be observed that the given graphs can be obtained by first shifting the graph of $y = -|x|$ up by 2 units and then taking the modulus of resultant function.

Shifting up by 2 units transforms the equations to

$$y = 2 - |x|$$

and taking modulus gives the resultant eqn. as

$$y = |2 - |x||$$

14. Option (d) is correct.

Given LPP is Maximize

$$z = 45x_1 + 60x_2$$

Given constraint are

$x_1 \le 45$ ...(A)

$x_2 \le 50$ ...(B)

$10x_1 + 10x_2 \ge 600$ ...(C)

$25x_1 + 5x_2 = 750$ ...(D)

Step 1: Writing the given inequalities as equalities

$x_1 = 45$ ...(1)

$x_2 = 50$ ...(2)

$10x_1 + 10x_2 = 600$ ...(3)

$25x_1 + 5x_2 = 750$ ...(4)

Step 2: Putting the above line on the graph

from (3) $10x_1 + 10x_2 = 600$

if $x_1 = 0 \Rightarrow x_2 = 60$

$\therefore (x_1, x_2) = (10, 60)$

$x_2 = 0 \Rightarrow x_1 = 60$

$\therefore (x_1, x_2) = (60,0)$

From (4) $25x_1 + 5x_2 = 750$

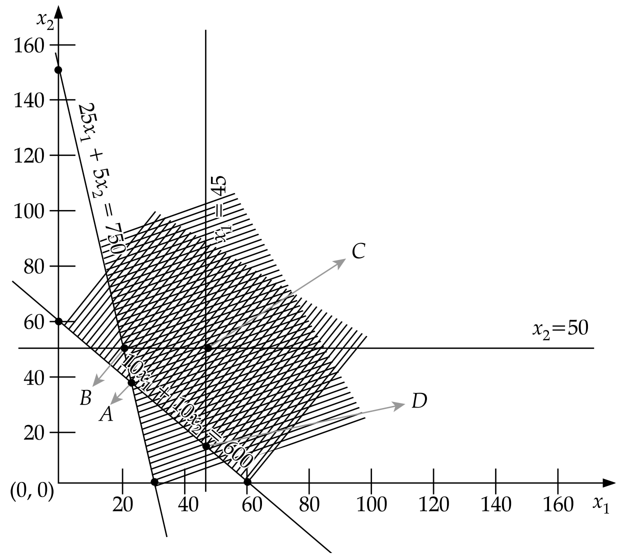
Put $x_1 = 0 \Rightarrow x_2 = 150$

$\Rightarrow (x_1, x_2) = (0, 150)$

$x_2 = 0 \Rightarrow x_1 = 30$

$\Rightarrow (x_1, x_2) = (30, 0)$

Since (c), (d) are the constraints with '≥' sign, shade the region above the line.



ABCD is the feasible region.

Option (d) gives the feasible region.

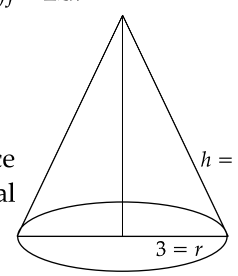
15. Correct answer is [0].

Given vector point function

$$B(x, y, z) = x\hat{i} + y\hat{j} - 2z\hat{k}$$

then

$$\int_s \overline{B} \cdot \hat{n}\, ds$$



Relation between surface integral and volume integral

$$\iint_s \overline{F} \cdot \hat{n}\, ds = \iiint_v \text{div } \overline{F}\, dv \quad \text{...(1)}$$

$$\int_s \overline{B} \cdot \hat{n}\, ds = \iiint_c \text{div } \overline{B}\, dv$$

$$\text{div } \overline{B} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(x\hat{i} + y\hat{j} + 2z\hat{k}\right)$$

$$= 1 + 1 - 2 = 0$$

$$\therefore \int_s \overline{B} \cdot \hat{n}\, ds = 0$$

16. Correct answer is [18.84].

Given $\hat{x} = 3y,\ \hat{y} = 2x$

$$\therefore \quad y = \frac{\hat{x}}{3} \text{ and } x = \frac{\hat{y}}{2}$$

The given region

$$x^2 + y^2 \le 1$$

or $$\left(\frac{\hat{y}}{2}\right)^2 + \left(\frac{\hat{x}}{3}\right)^2 \le 1$$

$$\frac{\hat{y}^2}{4}+\frac{\hat{x}^2}{9}\le 1$$

This is equation of an ellipse with semi major axis a = 3 and semi minor axis b = 2

Then area of transformation region is

A = πab = π × 3 × 2 = 6π = 18.84 units

17. Correct answer is [2].

Given

$$f(z) = e^{-kx}(\cos 2y - i\sin 2y) \text{ is the} \quad \text{...(1)}$$

Where function of a complex variable.

$$z = x + iy$$

We have

$$\omega = f(z) = u + iv \quad \text{...(2)}$$

Then

$$u(x\cdot y) = e^{-kx}\cos 2y$$
$$v(x\cdot y) = -e^{-kx}\sin 2y$$

Given function is analytic if its satisfy cauchy–Riemann equation.

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = -ke^{-kx}\cos 2y \quad \text{...(3)}$$

$$\frac{\partial u}{\partial y} = 2e^{-kx}\sin 2y \quad \text{...(4)}$$

$$\frac{\partial v}{\partial x} = ke^{-kx}\sin 2y \quad \text{...(5)}$$

$$\frac{\partial v}{\partial y} = -2e^{-kx}\cos 2y \quad \text{...(6)}$$

Since $f(z)$ is analytic

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$$

$\Rightarrow -ke^{-kx}\cos 2y = -2e^{-kx}\cos 2y$

Hence, k = 2

18. Option (a) is correct.

Given

$$f(s)=\frac{1}{s^3-s}=\frac{1}{s(s^2-1)}$$

$$f(s)=\frac{1}{s(s-1)(s+1)}$$

We have by Partial Fraction Method

$$\frac{1}{s(s-1)(s+1)} = \frac{A}{s}+\frac{B}{s-1}+\frac{C}{s+1}$$

$1 = A(s-1)(s+1) + B(s+1)s + C(s)(s-1)$

Put s = 1 both sides

1 = 2B ∴ B = $\frac{1}{2}$ again put s = –1

1 = 2C ∴ C = $\frac{1}{2}$ again put s = 0

1 = A(–1) ∴ A = –1

$$\frac{1}{s(s-1)(s+1)} = \frac{-1}{s}+\frac{1}{2(s-1)}+\frac{1}{2(s+1)}$$

Taking L^{-1} both sides

$L^{-1}\{f(s)\}$ =

$$-L^{-1}\left\{\frac{1}{s}\right\}+\frac{1}{2}L^{-1}\left\{\frac{1}{s-1}\right\}+\frac{1}{2}L^{-1}\left\{\frac{1}{s+1}\right\}$$

$$f(t) = -1 + \frac{1}{2}e^{t} + \frac{1}{2}e^{-t}$$

19. Correct answer is [8]

Let x and y be length and breadth of the rectangle

Given

Area $x \times y = 4$ ...(1)

Let perimeter minimum $B = 2(x + y)$ ...(2)

From (1)

$$B = 2\left(x+\frac{4}{x}\right) \text{ is minimum}$$

$$\text{or} \quad \frac{dB}{dx} = 2\left(1+\left(\frac{4}{x^2}\right)\right)=0$$

$$\Rightarrow 2-\frac{8}{x^2} = 0$$

$$\Rightarrow 2 = \frac{8}{x^2}$$

$$\therefore \quad x^2 = \frac{8}{2}=4$$

∴ $x = \pm 2$

$x = 2$ (+ve)

∴ $y = 2$

Then the smallest perimeter is

$B = 2(x + y) = 2(2 + 2) = 8$ units

20. Correct answer is [9].

Given second order Differential equation.

$$x^2\frac{dy^2}{dx^2}+x\frac{dy}{dx}-y=0,\ x\ge 1$$

with initial condition $y(1) = 6$, $y'(1) = 2$

Find y at x = 2.

Which is Homogeneous Linear Deferential Equation of higher order with variable coefficients.

$\boxed{x = e^z}$ $\boxed{z = \log x}$ $\boxed{D' = \frac{d}{dz}}$

Put $x\frac{dy}{dx} = D'(y)$

$x^2\frac{d^2y}{dx^2} = D'(D'-1)y$

$\Rightarrow D'(D'-1)y + D'y - y = 0$

$\Rightarrow (D'^2 - D' + D' - 1)\,y = 0$

$\Rightarrow f(D') = 0, \quad D' = m$

$\Rightarrow m^2 - 1 = 0 \quad \therefore\ m = \pm 1,$ Real and distinct.

C.F. $= C_1\dot{e}^z + C_2\bar{e}^z$ P. Z. = 0

$\because$ R.H.S. = 0

Complete solutions y = C.F. + P.Z.

$y = C_1e^z + C_2e^{-z}$ Put $\boxed{Z = \log x}$

$y = C_1x + \frac{C_2}{x}$ ...(1)

Apply initial condition

$y(1) = 6, \quad \therefore \quad 6 = C_1 + C_2$ ...(2)

$\frac{dy}{dx} = C_1 + C_2\left(\frac{-1}{x^2}\right)$

$y'(1) = 2 \quad 2 = C_1 - C_2$ ...(3)

From (2) and (3)

$C_1 + C_2 = 6$

$C_1 - C_2 = 2$

$2C_1 = 8 \quad \Rightarrow \quad C_1 = 4$

or $C_2 = 6 - 4 = 2$

$\therefore \quad C_2 = 2$

Hence $y = 4x + \frac{2}{x}$ ...(4)

Put $x = 2 \quad y = 8 + \frac{2}{2} = 8 + 1 = 9$

Since, $\boxed{y = 9}$ at $x = 2$

21. Correct answer is [1].

Given initial value problem,

$\frac{dy}{dt} + 2y = 0, \quad y(0) = 0, \quad t_0 = 0, \quad y_0 = 0$

We have by Euler's method

$y_{n+1} = y_n + hf(x_n, y_n)$ ...(1)

$\frac{dy}{dt} = -2y = f(t, y), \quad h = \Delta t =$ Positive.

$\therefore \quad y_{n+1} = y_n + \Delta t \times f(t_n, y_n)$

$\Rightarrow y_{n+1} = y_n + \Delta t \times (-2y_n)\,y_n\,(1 - 2\Delta t)$

$\Rightarrow \frac{y_{n+1}}{y_n} = 1 - 2\Delta t$

$\Rightarrow \left|\frac{y_{n+1}}{y_n}\right| \le 1 \quad \Rightarrow \quad |1 - 2\Delta t| \le 1$

$\Rightarrow \quad -1 \le 1 - 2\Delta t \le 1$

$\Rightarrow \quad 0 \le 2\Delta t \le 2$

$\Rightarrow \quad 0 \le \Delta t \le 1$

ELECTRICAL ENGINEERING (EE)

22. Option (d) is correct.

We have normal vector to the surface $\varnothing(x, y, z)$ is $\overline{N}$

$\overline{N}$ = grad $\varnothing$ = $\overline{\nabla}\varnothing$

Given a vector $W = [1, 2, 3]^T$

Plane defined by $W^TX = 1$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$W^TX = 1 \quad \Rightarrow \quad \begin{bmatrix} 1, & 2, & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$

$\therefore \quad x + 2y + 3z = 1$

$\varnothing = x + 2y + 3z - 1 = 0$

$\frac{\partial\varnothing}{\partial x} = 1, \quad \frac{\partial\varnothing}{\partial y} = 2 \quad \frac{\partial\varnothing}{\partial z} = 3$

$\overline{N} = \text{grad}\,\varnothing\,\overline{\nabla}\varnothing = \frac{\partial\varnothing}{\partial x}\hat{i} + \frac{\partial\varnothing}{\partial y}\hat{j} + \frac{\partial\varnothing}{\partial z}\hat{k}$

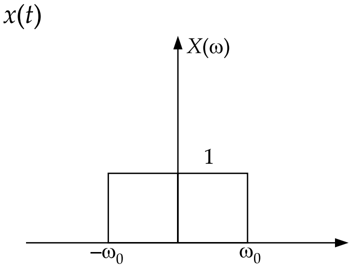
$= \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Normal vector to the surface

$\overline{N} = \hat{i} + 2\hat{j} + 3\hat{k} = [1, 2, 3]^T$

23. Option (b) is correct.

Given $x(t)$



$x(w) = 1$ for $|w| < w_0$

$= 0$ for $|w| > w_0$

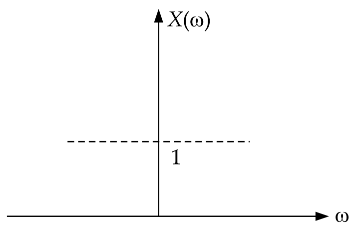
$\therefore \quad y(t) = \dfrac{\sin w_0 t}{\pi t}$

At $t = \dfrac{\pi}{2w_0} \Rightarrow x\left(\dfrac{\pi}{2w_0}\right) = \dfrac{\sin \pi/2}{\pi \times \dfrac{\pi}{2w_0}}$

$= \dfrac{2w_0}{\pi^2}$

$\therefore$ Option (c) and (d) are wrong.

If $w_0 \to \infty$ then



if $X(w)$ = DC – signal = 1

$\therefore \quad X(t) = \delta(t)$

So option (B) is correct.

Now $X[0] = \dfrac{\text{Area of } X(w)}{2\pi} = \dfrac{2w_0}{2\pi} = \dfrac{w_0}{\pi}$

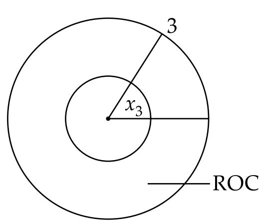
$\therefore$ $X(0)$ will increase if w_0 increase

So option (*a*) is wrong.

24. Option (b) is correct.

Given

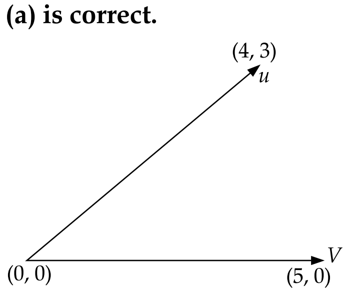
$$X(z) = \frac{4z}{\left(\frac{z-1}{5}\right)\left(\frac{z-2}{3}\right)(z-3)} \text{ with ROC} = R$$



$\dfrac{2}{3} < z < 3$

If ROC is $\dfrac{2}{3} < z < 3$ then it is including z = 1 circle or unit circle so, DTFT will convergence.

25. Option (a) is correct.



Then vectors u and v are related as

$Au = v$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$4a_{11} + 3a_{12} = 5$ ...(1)

$4a_{21} + 3a_{22} = 0$ ...(2)

$A = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$ satisfy the above two equations.

Now, $4 \times \dfrac{4}{5} + 3 \times \dfrac{3}{5} = 5$

And $\dfrac{4 \times (-3)}{5} + \dfrac{4 \times 3}{5} = 0$

26. Correct answer is [1.93].

Given

X	– 1	0	1
Y	0.8	2.2	2.8

Let $y = a + bx$ ...(1)

be the equation of one degree curve (straight line)

We have the normal euation of one degree curve,

No. of data $n = 3$

$\Sigma y = na + b\Sigma x$ ...(2)

$\Sigma xy = a\Sigma x + b\Sigma x^2$ ...(3)

X	Y	X^2	XY
–1	0.8	1	– 0.8
0	2.2	0	0
1	2.8	1	2.8
$\Sigma X = 0$	$\Sigma Y = 5.8$	$\Sigma X^2 = 2$	$\Sigma XY = 2.0$

Then $5.8 = 3a + b \times 0$ ...(1)

$a = \dfrac{5.8}{3} = 1.93$

$2 = a \times 0 + b \times 2$ $(\therefore \; b = 1)$

$\therefore$ Equation of curve,

$Y = a + bx = 1.93 + x$

27. Options (a), (b) & (d) are correct.

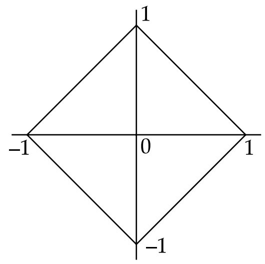
Given space $|x_1|\ P + |x_2|^P = 1$ for $P > 0$

(a) For $P = 2$

Eqn. $|x_1|^2 + |x_2|^2 = 1$ is a unit circle.

Area $= \pi(1)^2 = \pi$ it is True.

(b) For $P = 1$



Equation $|x_1| + |x_2| = 1$ is square.

$$\text{Area} = \frac{dz}{Z} = \frac{2^2}{2} = 2$$

28. Correct answer is [2].

Given

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [3, -2] \quad D = 1$$

The transfer function

$TF = C[SI - A]^{-1} B + D$

$$[SI - A]^{-1} = \begin{bmatrix} S & -1 \\ 1 & S+2 \end{bmatrix}^{-1}$$

$$= \frac{1}{(S^2 + 2S + 1)} \begin{bmatrix} S+2 & 1 \\ -1 & S \end{bmatrix}$$

$$TF = \frac{1}{(S^2 + 2S + 1)} [3, -2] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$= \frac{3 - 2S}{S^2 + 2S + 1} + 1 = \frac{S^2 + 4}{S^2 + 2S + 1}$$

$$H(Jw) = \frac{4 - \omega^2}{1 - \omega^2 + 2J\omega}$$

The output will be zero, for $\omega^2 - 4 = 0$

$\Rightarrow \omega = 2$ Rad/sec.

29. Correct answer is [0.038].

Given $x(n) \rightleftarrows X(e^{i\Omega}) = (1 + \cos \Omega)\, e^{-j\Omega}$

and $X_p(n)$ = Periodic signal $\rightleftarrows a_k$ = DFS-coefficient with $N = 5$

$$\text{Where } X_P(n) = \begin{cases} X(n) & \text{for } n = 0, 1, 2 \\ 0 & \text{for } n = 3, 4 \end{cases}$$

$$a_k = \frac{X(e^{iK\Omega_0})}{N}, \text{ where } \Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{5}$$

$$= \frac{1}{5}(1 + \cos K\Omega_0) \cdot e^{-jK\Omega_0}$$

$$= \frac{1}{5}\left[1 + \cos \frac{2\pi}{5} K\right] e^{-j\frac{2\pi}{5}K}$$

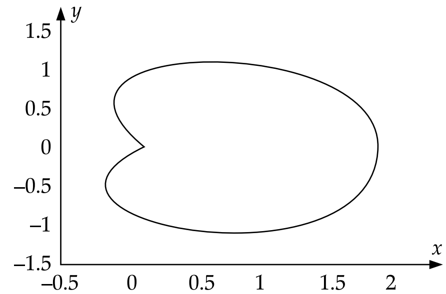
$$\text{Now} \quad |a_k| = \frac{1}{5}\left[1 + \cos \frac{2\pi}{5} K\right]$$

$$\text{Put} \quad K = 3 \; |a_3| = \frac{1}{5}\left[1 + \cos \frac{6\pi}{5}\right]$$

$$= \frac{1}{5}(1 - 0.809) = 0.038$$

30. Correct answer is [9.42].

given $r = 1 + \cos\theta$ $\quad r = \sqrt{x^2 + y^2}$ $\quad \overline{F} = -y\hat{i} + x\hat{j}$



C: $r = 1 + \cos\theta$

$$\oint_C \overline{F} \cdot d\overline{r} = \oint_C -4dx + x\,dy$$

By Green's Theorem

Relation beween line integral and surface integral

$$\int p dx + Q dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx dy$$

$$\int_C -y dx + x dy = \iint_R 2\, dx dy$$

$= 2$ Area of region bounded by C

$$= 2 \times \frac{1}{2}\int_{\theta=0}^{2\pi} r^2 d\theta = \int_0^{2\pi} (1 + \cos\theta)^2 \, d\theta$$

$$= \int_0^{2\pi} (1 + \cos^2\theta + 2\cos\theta)\, d\theta$$

$$= [\theta]_0^{2\pi} + 4\int_0^{\pi/2} \cos^2\theta\, d\theta + 4\int_0^{\pi} \cos\theta\, d\theta$$

$$= 2\pi + 4 \times \frac{1}{2} \times \frac{\pi}{2} + 0 = 2\pi + \pi = 3\pi$$

$= 3 \times 3.14 = 9.42$

31. Correct answer is [10].

Let

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1 x_2 + 1$$

$$= \overline{\nabla} f = \frac{\partial f}{\partial x_1}\hat{i} + \frac{\partial f}{\partial x_2}\hat{j}$$

$$= (2x_1 + 3 + x_2)\,\hat{i} + (4x_2 + 3 + x_1)\,\hat{j}$$

$$(\overline{\nabla} f) = 6\hat{i} + 8\hat{j}$$

$$|\overline{\nabla} f| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

ELECTRONICS COMMUNICATION (EC)

32. Option (c) is correct.

given

$$V_1 = \begin{bmatrix}1\\2\\0\end{bmatrix} \quad V_2 = \begin{bmatrix}2\\1\\3\end{bmatrix} \text{ be two vectors}$$

$$V_1 = \alpha V_2 + e$$

$$\therefore \quad e = V_1 - \alpha V_2$$

$$e = \begin{bmatrix}1 & 2 & 0\end{bmatrix} - \alpha\begin{bmatrix}2, & 1, & 3,\end{bmatrix}$$

$$= (\hat{i} + 2\hat{j} + 0\hat{k}) - \alpha(2\hat{i} + \hat{j} + 3\hat{k})$$

$$= (1 - 2\alpha)\,\hat{i} + (2 - \alpha)\,\hat{j} - 3\alpha\hat{k}$$

$$\hat{e} = \sqrt{(1-2\alpha)^2 + (2-\alpha)^2 + (-3\alpha)^2}$$

$$A = |e|^2 = (1-2\alpha)^2 + (2-\alpha)^2 + 9\alpha^2$$

$$= 1 + 4\alpha^2 - 4\alpha + 4 + \alpha^2 - 4\alpha + 9\alpha^2$$

$$= 5 + 14\alpha^2 - 8\alpha \text{ is minimum}$$

$$\text{If} \quad \frac{\partial e^2}{\partial \alpha} = \frac{\partial A}{\partial \alpha} = 28\alpha - 8 = 0$$

$$\therefore \quad \alpha = \frac{8}{28} = \frac{2}{7}$$

$$\boxed{\alpha = \frac{2}{7}}$$

33. Option (b) is correct.

Given $f(x, y, z) = xyz$ be any scalar point functions. We have the directional derivative of f in the direction of the vector $\bar{v} = 2\hat{i} + \hat{j} + 2\hat{k}$ at the point $P(0, 2, 1)$.

$$\frac{df}{ds} = \text{grad } f \cdot \hat{a} = \text{grad } f \cdot \hat{v} \qquad ...(1)$$

$$\text{grad} f = \vec{\nabla} \cdot f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$$

$$\vec{v} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\text{at } P(0, 2, 1)\ (\text{grad} f)_{\text{at } p} = 2\hat{i} \qquad ...(1)$$

$$\frac{df}{ds} = \text{grad } f \cdot \hat{v} = 2\hat{i} \times \frac{(2\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{4+1+4}} = \frac{4}{\sqrt{9}}$$

$$\text{or } \frac{df}{ds} = \frac{4}{3}$$

34. Option (a) is correct.

given $\quad \omega^4 = 16j$

$$\omega = (16j)^{\frac{1}{4}} = (2^4 \cdot j)^{\frac{1}{4}} = 2(j)^{\frac{1}{4}}$$

$$\omega = 2(0 + j)^{\frac{1}{4}} = 2[e^{j(2n+1)\pi/2}]^{\frac{1}{4}}$$

$$= 2[e^{j(2n+1)\pi/8}]$$

For $n = 0$, $\quad \omega = 2e^{j\pi/8}$

For $n = 2$, $\quad \omega = 2e^{5\pi j/8} = 2e^{5\pi j/8}$

For $n = 4$, $\quad \omega = 2e^{9\pi J/8}$

35. Option (b) is correct.

Given contour integral

$$\oint_C \left(\frac{z+2}{z^2 + 2z + z}\right) dz \text{, where } C \text{ is}$$

$$\left\{z : \left|z + 1 - \frac{3}{2}i\right| = 1\right\}$$

Taken in the counter clockwise direction.

Pole: equating denominator to zero

$z^2 + 2z + 2 = 0 \quad \therefore\ z^2 + 2z + 1 + 1 = 0$

$(z + 1)^2 + 1 = 0 \quad \therefore\ (z + 1)^2 = -1$

$\therefore\ z + 1 = \pm\sqrt{-1} = \pm i$

$\therefore\ z = -1 \pm i$

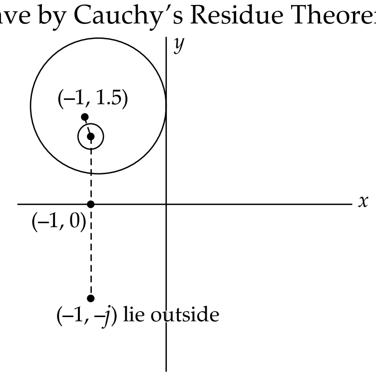
$z = -1 + i, -1 - i$, There are two poles.

$$\text{C: } \left|z + 1 - \frac{3}{2}\right| = 1; \ \sqrt{(x+1)^2 + \left(y - \frac{3}{2}\right)^2} = 1,$$

$$\Rightarrow (x+1)^2 + \left(y - \frac{3}{2}\right)^2 = 1$$

is a circle of Radius 1 and centre (–1, 3/2) only he pole $z = -1 + i$ lie inside the circle.

We have by Cauchy's Residue Theorem



$$\oint_C f(z)\,dz = 2\pi j \text{ Res } [f(z)]_{z=-1+i}$$

$$= 2\pi j\left(\frac{z+2}{2(z+1)}\right)_{z=-1+j}$$

$$= 2\pi j\left(\frac{-1+j+2}{2(-1+j+1)}\right) = \pi(1+j)$$

36. Option (c) is correct.

We have diagonalization of a Matrix.

$$D = P^{-1}AP \qquad ...(1)$$

where D is the diagonal Matrix, whose diagonal elements are the eigen values of A.

$$B = P^{-1}\text{AP} \Rightarrow A = PBP^{-1}$$

$\Rightarrow$ A and B are called similar Matrices.

$\Rightarrow$ Both A, B have same set eigen values but eigen vectors of A, B are different.

Let $BX = \lambda X \quad \Rightarrow \quad (P^{-1}\text{AP})X = \lambda X$

$$(P^{-1}\text{AP})\,X = \lambda X$$

$$\Rightarrow A(PX) = \lambda(PX)$$

$\therefore$ Eigen vectors of A are PX.

37. Option (c) is correct.

We have by Fourier transform

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{isx} f(x)\,dx = f(s)$$

we know $e^{-at^2};\ a > 0 \xleftrightarrow{FT} \sqrt{\dfrac{\pi}{a}}\, e^{-\omega^2/4a}$

Here; $a = 1$

$$\therefore \quad X(\omega) = \sqrt{\pi} e^{-\omega^2/4}$$

38. Option (b) is correct.

We have

$$\text{L.I.} = \int_C \vec{F}\cdot dr, \qquad d\overline{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

Given $\text{L.I.} = \int_P^Q (z^2dx + 3y^2dy + 2xzdz)$

$$= \int_P^Q 3y^2dy + \int_P^Q d(xz^2)$$

$$= \int_P^Q 3y^2dy + \left[xz^2\right]_{(1,1,2)}^{(2,3,1)}$$

$$\text{L.I.} = \left[\frac{3y^3}{3}\right]_1^3 + [2-4]$$

$$= 27 - 1 - 2 = 27 - 3 = 24$$

39. Option (a) is correct.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}, \quad l = \sqrt{x^T x}$$

The trace of the matrix $P = x\,x^{\text{T}}$

Given $\quad l = \sqrt{x^T x},\ P = (xx^T)_{m\times n}$

$$l = \sqrt{x^T \cdot x} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \cdots x_4^2}$$

$$P = xx^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}\begin{bmatrix} x_1 & x_2 & - & x_n \end{bmatrix}$$

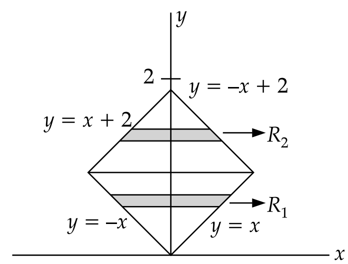
$$= x_1^2 + x_2^2 + \cdots + x_n^2$$

$$= \begin{bmatrix} x_1^2 & \cdots & \cdots & \cdots \\ \vdots & x_2^2 & & \\ \vdots & & \ddots & \\ \vdots & & & x_n^2 \end{bmatrix}$$

$$= x_1^2 + x_2^2 + \cdots + x_n^2 = l^2$$

40. Correct answer is [0].

$I = \iint_R xy\,dxdy$ over the Region



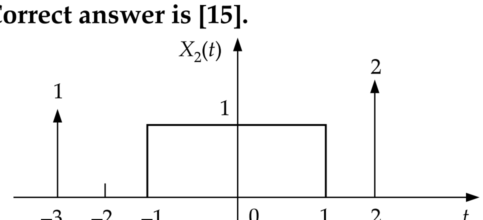
$$I = \iint_{R_1} xy\,dx\,dy + \iint_{R_2} xy\,dx\,dy$$

$$I = \int_0^1\int_{-y}^{y} xy\,dx\,dy + \int_1^2\int_{y-2}^{2-y} xy\,dx\,dy$$

$\because \int_{-a}^{0} f(x)\,dx = 0$. If $f(x)$ is an odd function.

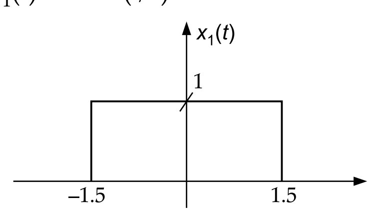
$$I = 0 + 0 = 0$$

41. Correct answer is [15].



given $x_1(t) = u(t+1.5) - u(t-1.5)$

$\Rightarrow \quad x_1(t) = \text{rect}\,(t/3)$



$$x_1(t) = \text{rect}\,(t/3) \xrightarrow{FT} 3Sa\,(1.5\,w)$$

Now $x_2(t) = \delta(t+3) + \text{rect}\,(t/2) + 2\delta(t-2)$

We have by Fourier Transform

$$x_2(w) = e^{3jw} + 2sa(w) + 2e^{-2jw}$$

$$y(t) = x_1(t) \cdot x_2\,(t)$$

We know $y(w) = \int_{-\infty}^{\infty} y(t)\, e^{-jwt}\, dt$

$\therefore \quad \int_{-\infty}^{\infty} y(t) = y(0)$

$\therefore \quad y(0) = x_1(0) \cdot x_2(0) = 3\,[1 + 2 + 2] = 15$

INSTRUMENTATION ENGINEERING (IN)

42. Option (a) is correct.

$x - 2y + z = 0,\ x + 0y - z = 0$ solving

$$\frac{x}{(-2)\,(-1) - 0} = \frac{y}{(1)\,(0) - (-1)\,(1)}$$

$$= \frac{z}{1(0) - (-2)\,(1)} = \propto$$

$$\Rightarrow \quad \frac{x}{2-0} = \frac{y}{1+1} = \frac{z}{0+2} = \propto$$

$$\Rightarrow \frac{x}{2} = \frac{y}{2} = \frac{z}{2} = \propto$$

or $\quad \frac{x}{1} \cdot \frac{y}{1} = \frac{z}{1} = \propto$

$\Rightarrow \quad x = \propto,\ y = \propto,\ z = \propto$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \propto \\ \propto \\ \propto \end{bmatrix} = \propto \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \text{ when } \propto \in R$$

$$\text{S} = \left[\propto \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \propto \in R \right]$$

43. Option (b) is correct.

$$F(Z) = \frac{1}{1-Z} = \frac{-1}{Z-1}$$

$$= -\left[\frac{1}{Z-2+1}\right] = -\left[\frac{1}{1+(Z-2)}\right]$$

$\therefore \quad$ Region of convergence is $|Z-2| < 1$

$= -[1 + (Z-2)]^{-1}\ [\because\ (1+x)^{-1} = 1 - x + x^2 - x^3]$

$= -[1 - (Z-2) + (Z-2)^2 - (Z-3)^3 + \ldots]$

$$= -\left[\sum_{K=0}^{\infty} (-1)^K (Z-2)^K\right] \text{for } |Z-2| < 1$$

$$= \sum_{K=0}^{\infty} (-1)^{K+1} (Z-2)^K \text{ which is of the form}$$

$$\sum_{K=0}^{\infty} a_k (Z-2)^K \text{, where } a_k = (-1)^{K+1}$$

44. Option (c) is correct.

The given equation can be written as

$$\frac{d^2x}{dt^2} = -K\frac{dx}{dt} \quad \Rightarrow \quad \frac{d^2x}{dt^2} + K\frac{dx}{dt} = 0$$

Operator form is $(D^2 + KD)\,x = 0$

Auxillary equation $f(m) = 0,\ m^2 + km = 0$

$\Rightarrow \quad m(m+k) = 0$

$\Rightarrow \quad m = 0, -k$ are the roots.

$\therefore \quad x = C_1 e^{0(t)} + C_2 e^{-kt} \quad \Rightarrow \quad x = C_1 + C_2 e^{-kt}$ ...(A)

Given Initial Conditions are

$x(0) = 1$ i.e., when $t = 0,\ x = 1$ ...(1)

$x(0) = 0$ i.e., when $t = 0,\ x' = 0$ ...(2)

From (A) $x' = 0 + C_2 \cdot e^{-kt} \cdot (-k)$

$\Rightarrow \quad x' = -C_2 k e^{-kt}$ (B)

Using (1) in (A) we get $C_1 + C_2 = 1$ ...(C)

Using (2) in (B) we get $0 = -C_2 k.e^0 \Rightarrow \boxed{C_2 = 0}$

$\boxed{\therefore\ C_1 = 1}$

Substituting C_1 and C_2 values in A, we get

$x = 1$ or $x(t) = 1$

45. Option (c) is correct.

Let $u(t)$ be the unit step function. We know that by the definition of unit step function

$$X(t) = \longleftarrow \boxed{\begin{matrix} V(s) \\ LTI\ System \end{matrix}} \longrightarrow Y(t)$$

$X(t) = A\sin(pt + q)$ and $Y(t) = A'\sin(\pi t + \varnothing)$

Here $A' = A \cdot |U(w)|_{w = w_0}$, $\varnothing = Q +$ Angle

$u(\omega)|_{\omega=\omega_0}$

we have $\omega_0 = \pi$, A′ = 1, $\phi = 0$

By using the unit step function formula:

$$|u(\omega)|_{\omega=p} = \frac{\sqrt{\omega^2 + \pi^2}}{\sqrt{\omega^2 + \pi^2}} = 1$$

$$\frac{u(\omega)}{\omega^2\pi} = 180 - \tan^{-1}\left(\frac{\omega}{\pi}\right) - \tan^{-1}\left(\frac{\omega}{\pi}\right)$$

$$= 180 - 90 = 90$$

$\therefore \quad A = 1 \Rightarrow \theta = -90°$

Now $x(t) = \sin(\pi t - 90°) = \sin\left[\pi t - \frac{\pi}{2}\right]$

$\therefore \quad x(t) = \sin\left[\pi t - \frac{\pi}{2}\right] u(t)$

46. Option (b) is correct.

$$\underset{x\to\infty}{\text{Lt}} f(x) = \underset{x\to\infty}{\text{Lt}}\, x \sin\frac{1}{x} = \underset{\frac{1}{x}\to 0}{\text{Lt}}\, x\sin\frac{1}{x}$$

$$= \underset{\frac{1}{x}\to 0}{\text{Lt}} \frac{\sin 1/x}{1/x} = 1 \qquad \left[\because \underset{x\to 0}{\text{Lt}} \frac{\sin x}{x} = 1\right]$$

47. Correct answer is [2]

$P(S) = S^3 + 2S^2 + 5S + 80$

The roots are –4.64, 1.32, 1.32

$\therefore$ Number of zeros of $P(S)$ in the right half plane is 2

48. Correct answer is [25]

$y(t) = x(4t)$ T = 100 sec $x(t) = \sin(\omega_0 t)$

we know

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{100} \text{ rad/sec}$$

$$\therefore \quad y(t) = x(4t) = \sin(4\omega_0 t) = \sin\left[4\left(\frac{2\pi}{100}\right)t\right]$$

$$= \sin\left[\frac{8\pi}{100}t\right]$$

$$\therefore \quad \frac{2p}{T} = \frac{8p}{100} \Rightarrow \frac{1}{T^1} = \frac{1}{25} \Rightarrow \boxed{T^1 = 25 \text{ sec}}$$

49. Correct answer is [–0.5].

From the given data probability distribution is

X	0	1	2
$P(X)$	0.25	0.5	k

we know that for discrete random variable $\Sigma P_i = 1$

$\Rightarrow \quad 0.25 + 0.5 + k = 1$

$\Rightarrow \quad k = 0.25$

$\therefore P(X = 0) = 0.25, P(x = 1) = 0.5, P(x = 2) = 0.25$

$E(X) = \Sigma x_i P_i = 0(0.25) + 1(0.5) + 2(0.25)$
$= 1$

$E(X^2) = \Sigma x_i^2 P_i = 0^2(0.25) + 1^2(0.5) + 2^2(0.25)$
$= 1.5$

$\therefore \quad E(X) - E(X^2) = 1 - 1.5 = -0.5$

50. Option (c) is correct.

$x(t) = e^{-3t} u(t-5) \Rightarrow L[x(t)] = [Le^{-3t} u(t-5)]$

From second shifting theorem we have

If $L[f(t)] = F(s)$ and $g(t) = \begin{bmatrix} f(t-a) & t > a \\ 0 & t < 0 \end{bmatrix}$

then $L[g(t)] = e^{-as} F(s)$

where $u(t-5)$ is unit step function

$$\therefore \quad L[u(t-5)] = \frac{1}{S} e^{-5s} \Big|_{S\to S+3}$$

By using first shifting theorem

$$= \frac{1}{S+3} e^{-5(S+3)} = \frac{e^{-5(S+3)}}{S+3}, \text{ Real } \{S\} > -3$$

51. Option (a) is correct.

Let

$f(z) = \frac{i(1-z)}{1+z}$, where z is a complex number, $i = \sqrt{-1}$

Let

$$\omega = u + iv = u + \frac{i(1-z)}{1+z}$$

$$= \frac{i[1-(x+iy)]}{1+(z+iy)} = \frac{i - ix + y}{(1+x)+iy}$$

$$= \frac{i(1-x)+y}{(1+z)+iy} \times \frac{i(1+x)-iy}{(1+z)-iy}$$

$$= \frac{[i(1-x)+y][(1+x)-iy]}{(1+x)^2+y^2}$$

$$= \frac{i(1-x^2-y^2)+(1-x)y+y(1+x)}{(1+x)^2+y^2}$$

$$= \frac{2y+i(1-x^2-y^2)}{(1+x)^2+y^2}$$

$$= \frac{2y}{(1+x)^2+y^2} + \frac{i(1-x^2-y^2)}{(1+x)^2+y^2}$$

To find the image of real axis in w-plane, $V = 0$

Then $\frac{1-x^2-y^2}{(1+x)^2+y^2} = 0 \Rightarrow 1 - x^2 - y^2 = 0$

$\Rightarrow \quad x^2 + y^2 = 1$

$\therefore f^{-1}(z)$ maps the real axis in w-maps into a unit circle in z-plane.

52 .Correct answer is [240].

Given 5 digits out of which 2 are identical.

Number of ways $= \frac{5!}{2!} = \frac{120}{2} = 60$

Out of four digits exactly one digits appearing twice.

Number of ways $= {}^4C_1 = 4$

Total number of ways for five digit number $= 4 \times 60$
$= 240$

53. Correct answer is [1].

The sample measurement are 80 kg, 79 kg, 81 kg, 79 kg and 81 kg

No. of Samples $n = 5$

$$\bar{X} = \frac{\text{Sum of observations}}{\text{No. of observations}}$$

$$= \frac{80 + 79\ 81 + 79 + 81}{5} = 80$$

Sample standard deviation =

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(80-80)^2 + (79-80)^2 + (81-80)^2 + (79-80)^2 + (81-80)^2}{5-1}}$$

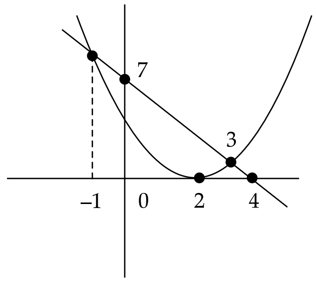
$$= \sqrt{\frac{4}{4}} = 1$$

$\therefore$ S.D. = σ = 1 kg

54. Correct answer is [1].

$g(x) = \max\{(x-2)^2, -2x + 7\}$, where $x \in (-\infty, \infty)$

$g(x) = \max\{(x-2)^2, -2x + 7\}$ graph



From the figure

Minimum value of g(x) is at $x = 3$

$\Rightarrow g(3) = (3-2)^2$ or $[-2(3) + 7]$

$= 1$

$\therefore$ The minimum value of $g(x)$ is 1.

55. Correct answer is [–8].

The matrix A = $\begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$, $(A) = 1$

Minor of order 2 = 0 $\because P(A) = 1$

$$\begin{vmatrix} 1 & 4 \\ -3 & \alpha \end{vmatrix} = 0 \quad \Rightarrow \quad \alpha + 12 = 0 \quad \Rightarrow \alpha = -12$$

and also

$$\begin{bmatrix} -3 & \alpha \\ \beta & 6 \end{bmatrix} = 0 \quad \Rightarrow \quad -18 - \alpha\beta = 0 \quad \Rightarrow \alpha\beta = -18$$

$$\beta = \frac{-18}{12} = \frac{3}{2}$$

or

$$\begin{bmatrix} 1 & 4 \\ \beta & 6 \end{bmatrix} = 0 \quad \Rightarrow \quad 6 - 4\beta = 0 \quad \Rightarrow 6 = 4\beta$$

$$\beta = \frac{3}{2}$$

$$\therefore \quad \frac{\alpha}{\beta} = \frac{-12}{3/2} = -8$$

CHEMICAL ENGINEERING (CH)

56. Option (b) is correct.

As $x \to 0$ we get $\frac{0}{0}$ (undefined form)

By L Hospital Rule $\left[Lt \frac{f}{g} = Lt \frac{f'}{g'} \right]$

$$y = \underset{x\to 0}{Lt} \frac{2x}{e^x - 1} = \underset{x\to 0}{Lt} \frac{2}{e^x - 0}$$

$$= \underset{x\to 0}{Lt} \frac{2}{e^x} = \frac{2}{e^0} = 2$$

57. Option (d) is correct.

Given that $\bar{v} = zx\bar{i} + 2xy\bar{j} + 3yz\bar{k}$

We have if $\bar{v} = V_1\bar{i} + V_2\bar{j} + V_3\bar{k}$

$$\text{div } \bar{v} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

$$y = \frac{\partial}{\partial x}(zx) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(syz) = z + 2x + 3y$$

div $\bar{v}$ at the point (3, 2, 1) $= 1 + 2(3) + 3(2)$

$= 13$

58. Option (b) is correct.

$$F = \frac{|z_1 + z_2|}{|z_1| + |z_2|}$$

$$= \frac{|(2+3i) + (-2+3i)|}{|2+3i| + |-2+3i|}$$

$$= \frac{|0+6i|}{\sqrt{2^2+3^2} + \sqrt{(-2)^2+3^2}}$$

$$= \frac{\sqrt{(0)^2+6^2}}{\sqrt{4+9}+\sqrt{4+9}} = \frac{\sqrt{36}}{\sqrt{13}\sqrt{13}}$$

$$= \frac{6}{13} = 0.4615 < 1$$

F < 1

59. Option (d) is correct.

Given $\hat{e}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$, $\hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$

$$\hat{e}_r + \hat{e}_\theta = (\cos\theta\hat{i} + \sin\theta\hat{j}) + (-\sin\theta\hat{i} + \cos\theta\hat{j})$$
$$= (\cos\theta - \sin\theta)\hat{i} + (\sin\theta + \cos\theta)\hat{j}$$
$$\frac{\partial}{\partial\theta}(\hat{e}_r + \hat{e}_\theta) = (-\sin\theta - \cos\theta)\hat{i} + (\cos\theta - \sin\theta)\hat{j}$$
$$= -\sin\theta\hat{i} - \cos\theta\hat{i} + \cos\theta\hat{j} - \sin\theta\hat{j}$$
$$= (-\sin\theta\hat{i} + \cos\theta\hat{j}) - (\cos\theta\hat{i} + \sin\theta\hat{j})$$
$$= \hat{e}_\theta - \hat{e}_r$$
$$= -\hat{e}_r + \hat{e}_\theta$$

60. Correct answer is [–1].

Given that $\frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$

In operator form $(D^2 + w^2)x = 0$

Auxiliary equation is

$f(m) = 0 \Rightarrow m^2 + \omega^2 = 0 \Rightarrow m = \pm\omega i$

$\therefore x(t) = C_1 \cos\omega t + C_2 \sin\omega t$ (A)

The initial conditions are

$x(t = 0) = 1$ when $t = 0, x = 1$ (1)

$\left.\frac{dx}{dt}\right|_{t=0}$ i.e., $t = 0, x' = 0$ (2)

From (A)

$x = C_1 \cos\omega t + C_2 \sin\omega t$

$\Rightarrow x' = -C_1 \omega \sin\omega t + C_2 \omega \cos\omega t$ (B)

Using (1) in (A) $t = 0, x = 1 \Rightarrow C_1 = 1$

Using (2) in (B) $0 = -C_1(0) + C_2\omega(1) \Rightarrow C_2 = 0$

From (A)

$x = \cos\omega t$ $[\because \cos x\pi = (-1)^n]$

At $t = \frac{3\pi}{\omega}$

$x = \cos\omega\left(\frac{3\pi}{\omega}\right)$

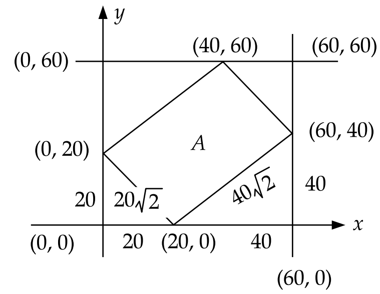
$= \cos 3\pi$ $[\because \cos n\pi = (-1)^n]$

$x = -1$

61. Option (b) is correct.

$$\text{Probability} = \frac{\text{Area of rectangle } A}{\text{Area of square}}$$

Based on the data given in the problem, we will get the rectangle and square shape.



Area of rectangle $= |x - y| \le 20$

$$\text{Probability} = \frac{40\sqrt{2} \times 20\sqrt{2}}{60 \times 60} = \frac{4}{9}$$

62. Option (b) is correct.

Given $I = \int_{-1}^{1} \sqrt{1 - x^2}\,dx$ with length of the interval is 0.5 i.e., $\lambda = 0.5$; $n = \frac{b - a}{h} = \frac{2}{0.5} = 4$

By Sampson's 1/3rd Rule

$y = \frac{h}{3}$ [(sum of first and last ordinates) + 4 (sum of odd) + 2 (sum of even)]

x	–1	–0.5	0	0.5	1
$I = f(x) = \sqrt{1 - x^2}$	0	0.8660	1	0.8660	0

$$I = \frac{h}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$$
$$= \frac{0.5}{3}[(0 + 0) + 4(0.8660 + 0.8660) + 2(1)]$$
$$= \frac{0.5}{3}(8.928) = 1.488$$

i.e., $\frac{1}{3} + \frac{2}{\sqrt{3}}$

63. Correct answer is [41160].

Given that

$$M = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$
$$|M - \lambda I| = \begin{vmatrix} 10 - \lambda & 6 \\ 6 & 10 - \lambda \end{vmatrix}$$
$$= (10 - \lambda)^2 - 36 = 0$$

Expanding we get

$$\lambda^2 - 20\lambda + 64 = 0$$
$$\Rightarrow \lambda^2 - 16\lambda - 4\lambda + 64 = 0$$
$$\Rightarrow \lambda(\lambda - 16) - 4(\lambda - 16) = 0$$
$$\lambda = 4, 16$$

$\therefore$ Eigen values of M are 4, 16

Eigen values of M^3 are 4^3, 16^3

Sum of M^3 Eigen values = $4^3 + 16^3 = 41160$

64. Correct answer is [–24].

Given that

$$u(r) = 4\left[\left(\frac{1}{r}\right)^{12} - \left(\frac{1}{r}\right)^{6}\right] = 4\,[r^{-12} - r^{-6}]$$

$$u^1(r) = 4\,[-12r^{-13} - (-6)r^{-7}] \qquad [\because d(x^n) = nx^{n-1}]$$

At r = 1

$$u^1(1) = 4\,[-12(1)^{-13} + 6(1)^{-7}]$$

$$= 4\,[-12 + 6] = 4[-6] = -24$$

CIVIL ENGINEERING (CE) P1

65. Option (d) is correct.

The integral lies in between –1 to +1. But at $x = 0$

$f(x) = \dfrac{1}{x^2}$ does not exist. Because $\dfrac{1}{0} = \infty$ which is not defined.

$\therefore$ The integral does not converge.

66. Options (b) & (c) are correct.

Given $F(x) = px^4 + qx^5$ on (–L, L)

The fourier series expansion is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin\left(\frac{\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \cos\left(\frac{\pi x}{L}\right)$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

Consider

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} (Px^4 + qx^5)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \left\{\frac{1}{L}\int_{-L}^{L} Px^4 \sin\left(\frac{n\pi x}{L}\right)dx + \frac{1}{L}\int_{-L}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx\right\}$$

$$= \frac{1}{L}\left\{0 + q\int_{-L}^{L} x^5 \sin\left(\frac{n\pi x}{L}\right)dx\right\}$$, Since I integral is an odd function.

$$= \frac{1}{L}\left\{2\int_{0}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx\right\}$$

i.e. an depends on q

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\left\{\int_{-L}^{L} Px^4 \cos\left(\frac{n\pi x}{L}\right)dx + \int_{-L}^{L} qx^5 \cos\left(\frac{n\pi x}{L}\right)dx\right\}$$

$$= \frac{1}{L}\left[2\int_{0}^{L} Px^4 \cos\left(\frac{n\pi x}{L}\right)dx\right]$$, Since II integral is an even function.

i.e. b_n depends on p.

67. Correct answer is [0.9].

Given A, B are independent events $P(A) = 0.5$, $P(B) = 0.8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since A & B are independent events.

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cup B) = 0.5 + 0.8 - (0.5)\,(0.8)$

$= 1.3 - 0.4 = 0.9$

68. Correct answer is [0.446].

Given differential equation $\dfrac{dy}{dx} + \alpha xy = 0$

$\dfrac{dy}{dx} = -\alpha xy \Rightarrow \dfrac{dy}{y} = -\alpha x\,dx$. Integrating on both sides, we get

$$\int\frac{dy}{y} = -\alpha\int x\,dx \Rightarrow \log y = -\alpha\frac{x^2}{2} + \log c$$

$$\Rightarrow \log\frac{y}{c} = -\alpha\frac{x^2}{2}$$

$$\Rightarrow \frac{y}{c} = e^{-\alpha\frac{x^2}{2}} \Rightarrow y = ce^{-\alpha\frac{x^2}{2}} \qquad \text{(A)}$$

The given conditions are $y = 1$ at $x = 0$ (1)

$y = 0.8$ at $x = 1$ (2)

Substituting (1) in A, we get $c = 1$

Put the value of (2) in A $\Rightarrow 0.8 = ce^{-\frac{\alpha}{2}} \Rightarrow e^{-\frac{\alpha}{2}} = 0.8$

$$\Rightarrow e^{\frac{\alpha}{2}} = \frac{1}{0.8} \Rightarrow \frac{\alpha}{2} = \log\left(\frac{1}{0.8}\right) \Rightarrow \frac{\alpha}{2} = 0.22314$$

$\Rightarrow \alpha = 0.44628 \cong 0.446$

69. Options (a, b, c, d) are correct.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad |A - \lambda I| = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 2-\lambda & 1 \\ 3 & 1 & 2-\lambda \end{bmatrix}$$

Expanding we get 6, 1, –2

From the properties of Eigen values satisfies

(i) Eigen values of A^T and A are same.

(ii) Eigen vectors of A and A^T are same

(iii) Eigen values of A^{-1} is reciprocal of Eigen values of A.

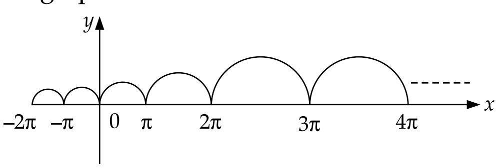
(iv) Eigen vectors of A^{-1} are same as the eigen vectors of A.

So answer is a, b, c, d

70. Option (a) is correct.

$f(x) = e^x|\sin x|$ given function is continuous at all x. But it is not bounded and not differentiable and not periodic.

The graph is



71. Correct answer is [–0.1823].

Given D.E. is $\frac{du}{dt} + 2tu^2 = 1$

$$\frac{du}{dt} = 1 - 2tu^2 = f(t, u)$$

where $f(t, u) = 1 - 2tu^2$

From Eulers iterative formula

$$u_n = u_{n-1} + hf(t_n, u_n)$$

Given that $h = 0.01$, $u_n = 1.75$, $t_n = 3.14$

$u_n - u_{n-1} = h\,[1 - 2t_n u_n^2]$

$= 0.01\,[1 - 2(3.14)(1.75)^2]$

$= -0.1823$

CIVIL ENGINEERING (CE) P2

72. Options (b, c) are correct.

Given that

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1 \quad R_3 \to R_3 + R_2$

which is the Echelon form rank of $A = 2$ but $n = 3$ (no of unknown)

If $r < n$, No. of Lineraly independent solutions $= n - r = 3 - 2 = 1$

$\therefore$ $[A][x]$ has infinite solutions.

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = 0$$

By expanding we get $(1 - \lambda)(\lambda^2 - 3\lambda + 1 - \lambda) = 0$

$\Rightarrow (1 - \lambda)\,\lambda\,(\lambda - 3) = 0 \Rightarrow \lambda = 0, 1, 3$

$\therefore$ Eigen values of A are 0, 1, 3

It has 3 linearly independent eigen vectors and is positive semi definite.

73. Option (a) is correct.

Given that

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

Operator form is $m^3 - 5.5\,m^2 + 9.5\,m - 5 = 0$

$\therefore m = 2.5, 1, 2$

$\therefore y = C_1 e^{2.5x} + C_2 e^x + C_3 e^{2x}$

$\therefore \alpha = 1, \beta = 2$

74. Option (a) is correct.

Given that rank of $A_{4\times4} = 2$ i.e. $P(A) = 2$, $n = 4$

By using Rank and Nullity theorem

Rank of A + Nullity of A = number of columns in A.

$\Rightarrow$ Nullity of $A = 4 - 2 = 2 \Rightarrow N(A) = 2$

$\therefore$ $N(A)$ consists only 2 linearly independent vectors.

The given vectors are $x = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

These are linearly independents, remaining are Lineraly dependent

$$\therefore \quad x - y = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

75. Option (b) is correct.

Given that $A = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$

We know that $LL^T = A$

$$\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} l_{11} & l_{21} \\ 0 & l_{22} \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} l_{11}^2 + 0 & l_{11}l_{21} + 0 \\ l_{21}l_{11} + 0 & l_{21}^2 + l_{22}^2 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} l_{11}^2 & l_{11}l_{21} \\ l_{21}l_{11} & l_{21}^2 + l_{22}^2 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

$$\Rightarrow l_{11}^2 = 8 \Rightarrow l_{11} = \sqrt{8};$$

$$l_{11}l_{21} = -5 \Rightarrow l_{21} = \frac{-5}{\sqrt{8}}$$

$$l_{21}^2 + l_{22}^2 = a_{22}$$

Given that $l_{22} = 1.968$

$$\therefore \quad \left(\frac{-5}{\sqrt{8}}\right)^2 + (1.968)^2 = a_{22}$$

$\Rightarrow a_{22} = 6.998 \simeq 7$