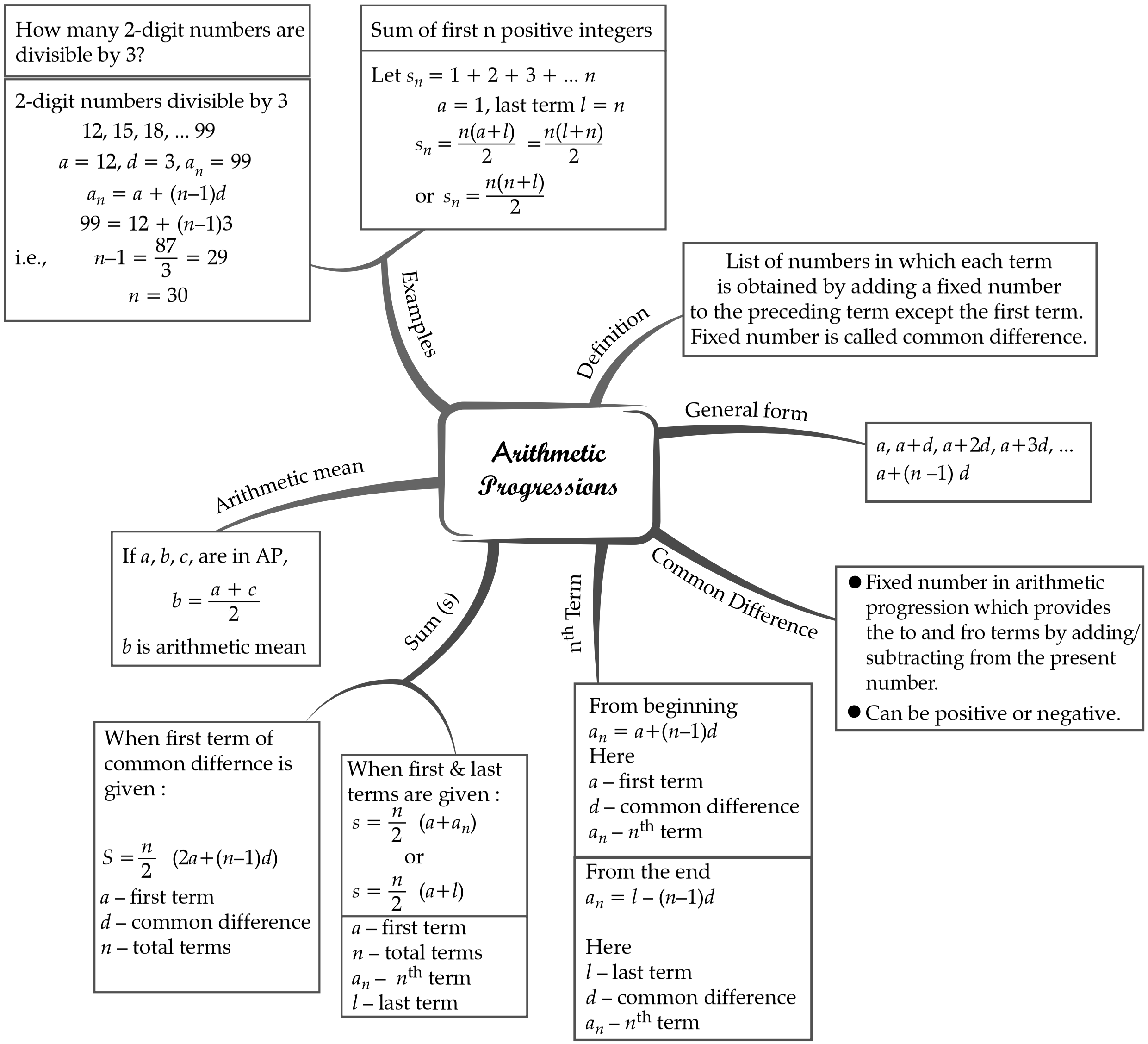
CHAPTER -5 : ARITHMETIC PROGRESSIONS



- The Old-Babylonians (400 *BC*) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid's developed a geometrical approach for finding out roots, which are solutions of quadratic equations.
- In Vedic manuscripts, procedures are described for solving quadratic equations by geometric methods related to completing a square.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.
- Sridharacharya (C.E. 870-930) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.
- Golden ratio ϕ is the root of quadratic equation $x^2 - x - 1 = 0$.

Discriminant and Nature of Roots

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant D $= b^2 - 4ac$.
- Nature of roots of a quadratic equation :
  (i) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  (ii) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii) If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

Know the Terms

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$
- **Quadratic identities :**
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$

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CHAPTER-2
ARITHMETIC PROGRESSION

Revision Notes

To Find nth Term of the Arithmetic Progression

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number *d* to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.

- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots..$
- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, the common difference changes to negative of original sequence common difference.

Know the Formulae

- The general term of an A.P. is expressed as :

  $a_n = a + (n - 1)d$. .... from the starting.

  where, a is the first term d is the common difference and n is the number of terms.
- The general term of an A.P. $l, l - d, l - 2d, \ldots\ldots.., a$ is given by :

  $a_n = l + (n - 1)(- d) = l - (n - 1)d$ .... from the end.

  where, l is the last term, d is the common difference and n is the number of terms.
- Sum of n terms of an A.P. is given by :

  $$S_n = \frac{n}{2}[2a + (n - 1)d]$$

  where, a is the first term, d is the common difference and n is the total number of terms.
- Sum of n terms of an A.P., when first and last term is given, is

  $$S_n = \frac{n}{2}[a + l]$$

  where, a is the first term and l is the last term.
- The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum of first $(n - 1)$ terms of it. *i.e.*,

  $$a_n = S_n - S_{n-1}.$$

Mnemonics

SAND

S : Means Sum of terms
A : Means first term
N : Means nth term of n term
D : Means common difference

Know the Terms

- A sequence is defined as an ordered list of numbers.
  The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.
- If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.
  Example : 2 + 4 + 6 + 8 + ... is a series.
- The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician Leonardo Fibonacci, when he was dealing with the problem of rabbit population.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by a constant, the resulting sequence is also an A.P.
- The selection of three terms in an A.P. are
  (i) $a - d, a, a + d$
  (ii) The selection of four terms in an A.P. are $a - 3d, a - d, a + d, a + 3d$.

- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.
- If the terms are selected at a regular interval, the given sequence is in A.P.
- If three consecutive number a, b and c are in A.P., the sum of first and third number is twice the middle number *i.e.*, $2b = a + c$.

Facts about the Common Difference

If common difference is:

(a) Positive, the A.P. is increasing. E.g. 2,4,6,8,…….

(b) Zero, the A.P. is constant. E.g. 5,5,5,5,5,……..

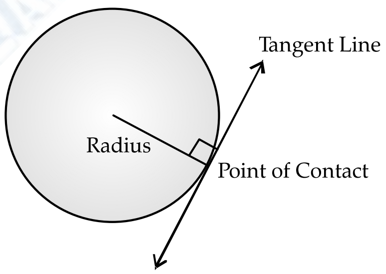
(c) Negative, the A.P. is decreasing. E.g. 24,21,18,15,……

UNIT II: GEOMETRY

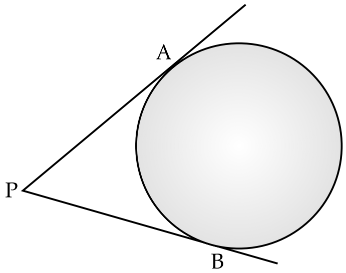
CHAPTER-3
CIRCLES

Revision Notes

- A tangent to a circle is a line that intersects the circle at one point only.
- The common point of the circle and the tangent is called the point of contact.
- The length of the segment of the tangent drawn from the external point P and the point of contact with the circle is called the length of the tangent.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point outside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.



- The lengths of the tangents drawn from an external point to a circle are equal.



In the figure, $PA = PB$.

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- The standard form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots..$
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Sum of n Terms of an Arithmetic Progression

- Sum of n terms of an A.P. is given by:

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where, a is the first term, d is the common difference and n is the total number of terms.
- Sum of n terms of an A.P. when first and last term is given.

$$S_n = \frac{n}{2}[a + l]$$

where, a is the first term and l is the last term.
- The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum to first $(n-1)$ terms of it.
i.e., $a_n = S_n - S_{n-1}$.

Mnemonics

Concept: n^{th} Term of Arithmetic Progression $n = a + (n-1)d$.

Nokia Offers Additional Programmer in English To Attract Positive New One Buyer Daily

Interpretation:

Nokia's 'N' is n^{th} term.
Offer's 'O' is **of**
Additional's 'A' is Arithmetic
Programmer's 'P' is Progression
In's 'I' is **is**.
English's 'E' is **Equal**
To's 'T' is **To**
Attract's 'A' is a
Positive's 'P' is **+**
New's 'N' is n
One buyer is **– 1**
Daily's 'D' is d

Know the Formulae

- The general (n^{th}) term of an A.P. is expressed as:

$$a_n = a + (n-1)d. \text{ .... from the starting.}$$

where, a is the first term and d is the common difference.

- The general (n^{th}) term of an A.P. $l, l-d, l-2d, \ldots\ldots, a$ is given by:

$$a_n = l + (n-1)(-d) = l - (n-1)d \text{ .... from the end.}$$

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UNIT II: GEOMETRY

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- **Secant:** Two common points (A and B) between line PQ and circle.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord coincide.
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