- **Non-terminating Repeating (or Recurring) Decimals:** The decimal expansion obtained from $\frac{p}{q}$ repeats periodically, then it is called non-terminating repeating (or recurring) decimal.
- Just divide the numerator by the denominator of a fraction. If you end up with a remainder of 0, you have a terminating decimal otherwise repeating or recurring decimal.
- The sum or difference of a rational and an irrational number is irrational.
- The product and quotient of a non-zero rational and an irrational number is irrational.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which terminates.
- Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form $\frac{p}{q}$, where p and q are co-primes and the prime factorization of q is of the form 2^m5^n, where m and n are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating.

Know the Formulae

For two positive integers a and b, we have

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

and

$$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

❑❑

UNIT II: ALGEBRA

Chapter - 2 : Polynomials

Revision Notes

Zeroes of a Polynomial and Coefficients of Quadratic Polynomials

- **Polynomial:** An algebraic expression in the form of $a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, ..., a_n$ are real numbers) is called a polynomial in one variable x of degree n.
- **Value of a Polynomial at a given point :** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$,
  is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial:** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.
  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-co-ordinates of the points, where the graph of $y = p(x)$ intersects the X-axis.
  (i) A linear polynomial has one and only one zero.
  (ii) A quadratic polynomial has at most two zeroes.
  (iii) A cubic polynomial has at most three zeroes.
  (iv) In general, a polynomial of degree n has at most n zeroes.

- **Graphs of Different types of Polynomials:**
  - **Linear Polynomial:** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.
  - **Quadratic Polynomial: (i)** Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects X-axis at a maximum of two distinct points.
    (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two distinct points.
  - **Cubic polynomial:** Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects X-axis at a maximum of three distinct points.
- **Relationship between the Zeroes and the Coefficients of a Polynomial :**

(i) Zero of a linear polynomial $= \dfrac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\dfrac{-b}{a}$

(ii) In a quadratic polynomial,

$$\text{Sum of zeroes of a quadratic polynomial} = \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes of a quadratic polynomial} = \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

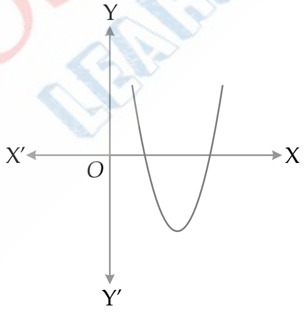
(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

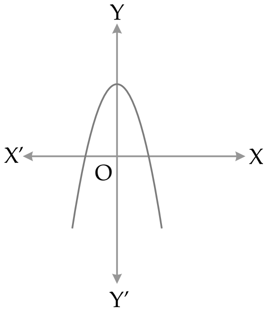
$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma = (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D.

The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

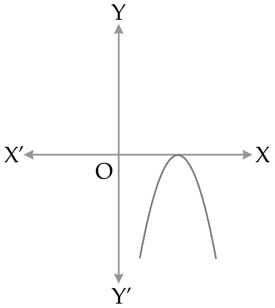
Case I : If $D > 0$, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-co-ordinates of points of intersection with X-axis is known as 'zeroes' of $f(x)$.

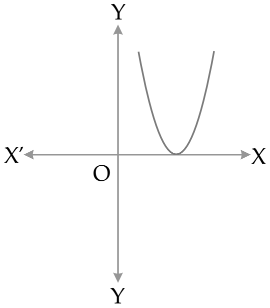




$\therefore f(x)$ will have two zeroes and we can say that roots/zeroes of the two given polynomials are real and unequal.

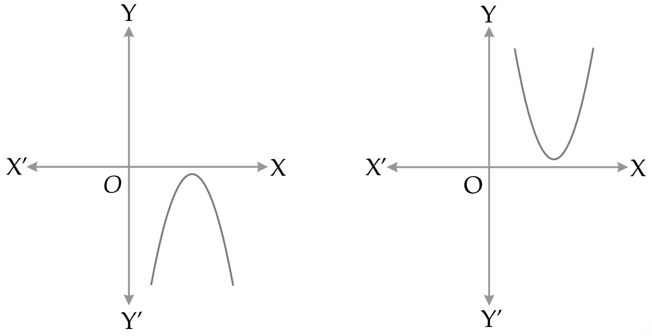
Case II : If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.





$\therefore f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

Case III: If D < 0, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.



$\therefore$ $f(x)$ will not have any real zero.

Know the Formulae

Relationship between the zeroes and the coefficients of a Polynomial :

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b$, where $a \neq 0$	1	$k = -\frac{b}{a}$, *i.e.*, $k = \frac{-\text{Constant term}}{\text{Coefficient of } x}$
2.	Quadratic	$ax^2 + bx + c$, where $a \neq 0$	2	Sum of zeroes, $(\alpha + \beta) = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$ Product of zeroes, $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d$, where $a \neq 0$	3	Sum of zeroes, $(\alpha + \beta + \gamma) = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time, $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$ Product of zeroes, $(\alpha\beta\gamma) = \frac{-\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$

Mnemonics

Concept: Formula $\rightarrow \alpha.\beta = \frac{c}{a}$

Amitabh **B**achchan went **C**anada **by** **a**eroplane.

Interpretation:

Amitabh's A $\Rightarrow$ Alpha (α)
Bachchan's B $\Rightarrow$ Beta (β)
Canada's C $\Rightarrow$ Constant (c)
By for Divide by and aeroplane's a $\Rightarrow$ Variable.

CHAPTER -2 : POLYNOMIALS

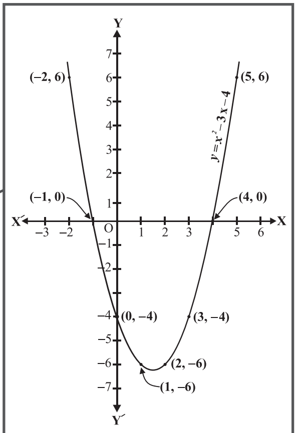
Polynomials

Division Algorithm

If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then –
$p(x) = g(x) \times q(x) + r(x)$
where, $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Graphical Representation- Quadratic Polynomial

Parabola



$y = x^2 - 3x - 4$

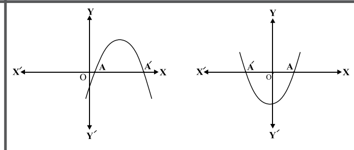
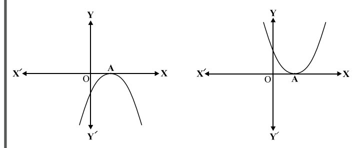
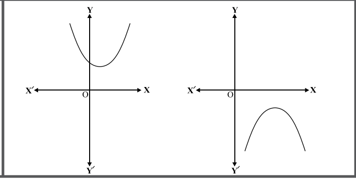
Degree of Polynomial

Highest power of x in Polynomial, p(x)

Types

Polynomial	Degree	General Form
Linear	1	$ax+b$
Quadratic	2	ax^2+bx+c, $a \neq 0$
Cubic	3	ax^3+bx^2+cx+d, $a \neq 0$

Zeroes of polynomial Graphically

Case	Graph	Number of Zeroes
Case1- Graph cuts x-axis at 2 points		2
Case2- Graph cuts x-axis at exactly one point		1
Case3- Graph does not cut x-axis		0

Relationship-Zeroes and Coefficient of Polynomials

Quadratic

α and β are zeroes of Quadratic Polynomial
$ax^2 + bx + c$
Then,
Sum of zeroes,
$\alpha + \beta = -\frac{b}{a}$
Product of zeroes
$\alpha\beta = \frac{c}{a}$

Cubic

α, β and Υ are zeroes of Cubic Polynomial
$ax^3 + bx^2 + cx + d$
Sum of zeroes,
$\alpha + \beta + \Upsilon = -\frac{b}{a}$
Sum of products of the zeroes taken two at a time
$\alpha\beta + \beta\Upsilon + \Upsilon\alpha = \frac{c}{a}$
Product of zeroes
$\alpha\beta\Upsilon = -\frac{d}{a}$