CUET (UG) Exam Paper 2023

National Testing Agency

Held on 26th May 2023

MATHEMATICS/APPLIED MATHEMATICS

Solved

[This includes Questions pertaining to Domain Specific Subject only]

Time Allowed: 45 Mins. **Maximum Marks: 200**

General Instructions :

(i) *Section A will have 15 questions covering both i.e., Mathematics/Applied Mathematics which will be compulsory for all candidates.*
(ii) *Section B1 will have 35 questions from Applied Mathematics out of which 25 questions need to be attempted. Section B2 will have 35 questions purely from Mathematics out of which 25 questions will be attempted.*
(iii) *Correct answer or the most appropriate answer : Five marks (+ 5)*
(iv) *Any incorrect option marked will be given minus one mark (– 1).*
(v) *Unanswered/Marked for Review will be given no mark (0).*
(vi) *If more than one option is found to be correct then Five marks (+5) will be awarded to only those who have marked any of the correct options.*
(vii) *If all options are found to be correct then Five marks (+5) will be awarded to all those who have attempted the question.*
(viii) *If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given five marks (+5).*
(ix) *Calculator / any electronic gadgets are not permitted.*

Section - A

Mathematics/Applied Mathematics

1. The area enclosed by the ellipse $\frac{x^2}{9^2}+\frac{y^2}{6^2}=1$ is:

(1) 15π **(2)** 54π
(3) 18π **(4)** $\frac{3}{2}\pi$

Ans. Option (2) is correct

Explanation: Equation of Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

We know that Area of ellipse = πab

On comparing given ellipse equation $\frac{x^2}{9^2}+\frac{y^2}{6^2}=1$

with above ellipse equation $a=9$ and $b=6$, then

Area enclosed by the ellipse $= \pi ab$
$= \pi\times 9\times 6$
$= 54\pi$

2. If A is a square matrix of order 3, $B=kA$ and $|B|=x|A|$ then,

(1) $x=2k$ **(2)** $x=k^2$
(3) $x=k^3$ **(4)** $x=3k$

Ans. Option (3) is correct

Explanation: From the properties of the determinants, we know that $|kA|=k^n\,|A|$, where n is the order of the determinant. Here, $n = 3$, therefore, on comparing $|B|=x|A|$ with $k^3|A|$ the answer x is k^3

3. The matrix A= $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ is a

(1) Diagonal matrix
(2) Symmetric matrix
(3) Skew-symmetric matrix
(4) Scalar matrix

Ans. Option (3) is correct

Explanation: A matrix can be skew symmetric only if it is square and if the transpose of a matrix is equal to the negative of itself, then the matrix is said to be skew symmetric matrix. This means that for a matrix to be skew symmetric, $A'=-A$

Also, The diagonal elements of a skew symmetric matrix are equal to zero.

Here in question,

$$A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix}$$

$$A' = -\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$A' = -A$$

4. The degree of the differential equation

$$\left(1+\frac{dy}{dx}\right)^4 = \left(\frac{d^2y}{dx^2}\right)^2 \text{ is:}$$

1. 1 **2.** 3
3. 2 **4.** 4

Ans. Option (3) is correct

Explanation: We know that degree of differential equation is the power of the highest derivative in a differential equation.
Therefore, Degree of given differential equation is 2.

5. The differential equation $\frac{dy}{dx}+\frac{x}{y} = 0$, represents the family of curves:

(1) $x^2-y^2=C$ **(2)** $\frac{x}{y}=C$

(3) $xy=C$ **(4)** $x^2+y^2=C$

Ans. Option (4) is correct

Explanation: $\frac{dy}{dx}+\frac{x}{y} = 0$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$ydy = -xdx$$

$$-ydy = xdx$$

on integrating both sides

$$-\int ydy = \int xdx$$

$$\frac{-y^2}{2}+\frac{C}{2} = \frac{x^2}{2}$$

$$\frac{C}{2} = \frac{x^2}{2}+\frac{y^2}{2}$$

$$C=x^2+y^2 \quad (C_1=C)$$

$$x^2+y^2=C$$

6. If $\begin{vmatrix} 2 & 3-x \\ x & 1 \end{vmatrix} = 0$, then the values of x are:

(1) 1 and 3 **(2)** 1 and 2 **(3)** 2 and 3 **(4)** 3 and 0

Ans. Option (2) is correct

Explanation: $\begin{vmatrix} 2 & 3-x \\ x & 1 \end{vmatrix} = 0$

$$(2)(1)-(x)(3-x)=0$$

$$2-3x+x^2=0$$

$$x^2-2x-x+2=0$$

$$x(x-2)-1(x-2)=0$$

$$(x-2)(x-1)=0$$

$$x=2, 1$$

7. $\int_0^{1.5} [x]dx$ where $[x]$ denotes the greatest integer function $\le x$ is equal to:

(1) $\frac{1}{4}$ **(2)** $\frac{1}{2}$ **(3)** 1 **(4)** 0

Ans. Option (2) is correct

Explanation:

$$\int_0^{1.5}[x]dx = \int_0^1 [x]dx + \int_1^{1.5}[x]dx$$

$$= \int_0^1 [0]dx + \int_1^{1.5}[1]dx$$

$$= 0+[x]_1^{1.5}$$

$$= 0 + (1.5-1)$$

$$= 0.5 = 1/2$$

8. If matrix A is of order 2 × 3 and B of order 3 × 2, then

(1) AB, BA both are defined and are equal
(2) AB is defined but BA is not defined
(3) AB is not defined but BA is defined
(4) AB, BA are defined but are not equal

Ans. Option (4) is correct

Explanation: We know that if A, B are, respectively $m\times n, k\times l$ matrices, then both AB and BA are defined if and only if $n = k$ and $l=m$
and If AB and BA are both defined, it is not necessary that $AB = BA$.

9. In a box containing 100 bulbs. 10 are defective. Then the probability, that out of a sample of 5 bulbs none is defective, is:

1(1) 10^{-1} **(2)** $\left(\frac{1}{2}\right)^5$ **(3)** $\left(\frac{9}{10}\right)^5$ **(4)** $\left(\frac{9}{10}\right)$

Ans. Option (3) is correct

Explanation: The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denote the number of defective bulbs out of a sample of 5 bulbs.
Probability of getting a defective bulb,

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{10}$

$\therefore p(X=r) = {}^nC_r q^{n-r} p^r$

$$p(X=r) = 5_{C_r}\left(\frac{9}{10}\right)^{5-r}\left(\frac{1}{10}\right)^r$$

p (none of the bulb is defective) $= p(X=0)$

$$p(X=0) = 5_{C_0}\left(\frac{9}{10}\right)^{5-0}\left(\frac{1}{10}\right)^0$$

$$= (1)\left(\frac{9}{10}\right)^5 (1) \quad = \left(\frac{9}{10}\right)^5$$

10. If $y = \frac{1}{x+1}$, then $\frac{d^2y}{dx^2}$ at $x=2$ is:

(1) $\frac{2}{9}$ (2) $\frac{3}{2}$ (3) $\frac{2}{27}$ (4) $\frac{3}{8}$

Ans. Option (3) is correct

Explanation: $y = \frac{1}{x+1}$

d.w.r to x

$$\frac{dy}{dx} = \frac{-1}{(x+1)^2}$$

again d.w.r to x

$$\frac{d^2y}{dx^2} = -1 \times -2 \times \frac{1}{(x+1)^3}$$

$$\left.\frac{d^2y}{dx^2}\right\}_{x=2} = 2 \times \frac{1}{(2+1)^3}$$

$$= \frac{2}{27}$$

11. Match List I with List II

	LIST I		LIST II
A	Maximum value of $f(x) = -\lvert x+1\rvert + 3$	I	6
B	Minimum value of $f(x) = (2x-1)^2 + 5$	II	5
C	Maximum value of $f(x) = 6 - x^2$	III	no maximum value
D	Maximum value of $f(x) = x^3 + 1$	IV	3

Choose the correct answer from the options given below:

(1) A-IV, B-II, C-I, D-III
(2) A-III, B-IV, C-I, D-II
(3) A-I, B-II, C-III, D-IV
(4) A-II, B-III, C-IV, D-I

Ans. Option (1) is correct

Explanation:

A. Maximum value of $f(x) = -\lvert x+1\rvert + 3$ is 3 when $x = -1$ Such that A is belongs to IV.

B. Minimum value of $f(x) = (2x-1)^2 + 5$ is 5 when $x = \frac{1}{2}$.

Such that option B from list I belongs to option II of list II.

C. Maximum value of $f(x) = 6 - x^2$ is 6 when $x=0$. Such that option C from list I belongs to option I of list II.

D. There is no maximum value of $f(x) = x^3 + 1$ Such that option D from list I belongs to option III of list II.

12. The mean number of heads in two tosses of a coin is:

(1) 2 (2) $\frac{1}{2}$ (3) 1 (4) $\frac{3}{2}$

Ans. Option (3) is correct

Explanation: Total possible outcomes = {TT, TH, HT, HH}

No head P(0) = 1 (TT)
One Head P(1) = 2 (HT, TH
Two Head P(2) = 1 (HH)

So,

X_i	0	1	2
P_i	1/4	2/4	1/4

$$\text{Mean} = \sum X_i P_i = 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$$

$$= 0 + \frac{1}{2} + \frac{1}{2} = 1$$

13. If $f(x) = \frac{1}{1-x}$, then for $x > 1$, $f(x)$ is:

(1) decreasing
(2) constant
(3) increasing
(4) neither decreasing nor increasing

Ans. Option (1) is correct

Explanation: $f(x) = \frac{1}{1-x}$

for $x>1$ $\rightarrow -x < -1$

$1-x < 1-1$

$f(x)$ is decreasing $\quad 1-x < 0$

14. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X = 0$ if he opposed. and $X = 1$ if he is in favour. Then. E (X) is :

(1) $\frac{7}{10}$ (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{7}{11}$

Ans. Option (1) is correct

Explanation:

Case 1: If selected member opposed

$$p(X=0) = \frac{30}{100}$$

Case 2: If selected member favours

$$p(X=1) = \frac{70}{100}$$

Required potability distribution is

X	0	1
$p(X)$	$\frac{30}{100}$	$\frac{70}{100}$

Then

$$E(x) = 0\times\frac{30}{100} + 1\times\frac{70}{100}$$

$$= \frac{70}{100}$$

$$= \frac{7}{10}$$

15. The solution of a LPP with basic feasible solutions (0, 0), (10, 0), (0, 20). (10, 15) and objective function Max $Z=2x+3y$ is:

(1) $x=0, y=20$, Max $Z=60$

(2) $x=10, y=15$, Max $Z=65$

(3) $x=10, y=20$. Max $Z=70$

(4) $x=15, y=10$, Max $Z=60$

Ans. Option (2) is correct

Explanation: At $(x, y)=(10, 15)$

$$Z = 2x+3y$$
$$= 2(10) + 3(15)$$
$$= 20+45$$
$$= 65$$

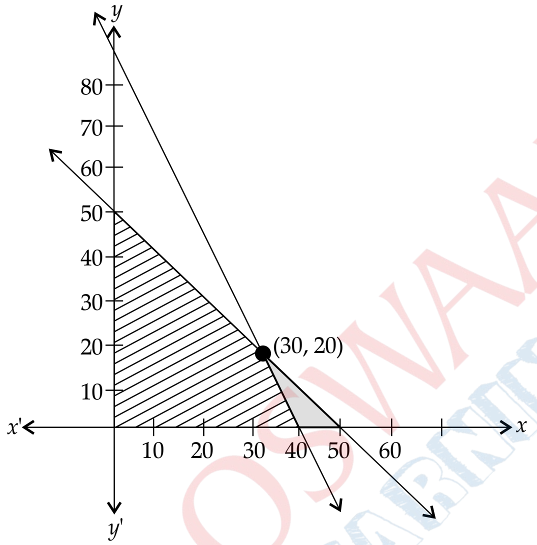
Here, objective function is Max. at $x=10$ and $y=15$.

Note: (1920) is not given solution of feasible solution.

Section - B1

Mathematics

16. The linear constraints, for which the shaded area in the figure is the feasible region of an LPP. are:



(1) $x + y \geq 50$
$2x + y \leq 80$
$x, y \geq 0$

(2) $x + y \leq 50$
$2x + y \geq 80$
$x, y \geq 0$

(3) $x + y \leq 50$
$2x + y \leq 80$
$x, y \geq 0$

(4) $x + y \geq 50$
$2x + y \geq 80$
$x, y \geq 0$

Ans. Option (3) is correct

Explanation: Shaded region is bounded between Four Corner points (0,0), (40,0), (30, 20) and (0,50)

Hence, $x + y \leq 50$
$2x + y \leq 80$
$x; y \geq 0$

are required equations.

17. Two dice are thrown simultaneously. If X denotes the number of sixes, then the variance of X is:

(1) $\frac{5}{18}$ **(2)** $\frac{7}{18}$ **(3)** $\frac{1}{3}$ **(4)** $\frac{2}{3}$

Ans. Option (2) is correct

Explanation: Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1 or 2

$\therefore$ $p(x=0) = p$ (not getting six on any of the dice)

$$= \frac{5}{6} \times \frac{5}{6}$$

$$= \frac{25}{36}$$

$p(X=1) = p$ (six on first dice and number after than six on second dice) $+ p$ (number other than six on first dice and six on second dice)

$$= \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}$$

$$= \frac{5}{36}+\frac{5}{36}$$

$$= \frac{10}{36}$$

$p(X=2)$ = P (six on both the dice)

$$= \frac{1}{6}\times\frac{1}{6}$$

$$= \frac{1}{36}$$

X	0	1	2
$p(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Mean (expected value) $\mu = X_i\, p(X_i)$

$$= 0\times\frac{25}{36} + 1\times\frac{10}{36} + 2\times\frac{1}{36}$$

$$= \frac{10}{36} + \frac{2}{36} = \frac{12}{36} = \frac{1}{3}$$

Variance $(\sigma^2) = \sum p(X_i)\,(X_i)^2 - (\text{Mean})^2$

$$\sum p(X_i)\,(X_i)^2 - \sum(X_i\, p(X_i))^2$$

$$= \left[(0)^2\times\frac{25}{36}+1^2\times\frac{10}{36}+4\times\frac{1}{36}\right]-\left(\frac{1}{3}\right)^2$$

$$= \frac{10}{36} + \frac{1}{9} - \frac{1}{9}$$

$$= \frac{10}{36} = \frac{5}{18}$$

18. If f : R→R is defined by $f(x) = \sin x + x$. then $f(f(x))$ is:

(1) $2\sin x + 2x$

(2) $\sin^2 x + x^2$

(3) $\sin(\sin x + x) + \sin x + x$

(4) $\sin^2 x + 2\sin x + x$

Ans. Option (3) is correct

Explanation: If f: R→R, $f(x) = \sin x + x$

$$f(f(x)) = f(\sin x + x) = \sin(\sin x + x) + \sin x + x$$

19. Which of the following statements is <u>NOT CORRECT</u>.

(1) A row matrix has only one row.

(2) A diagonal matrix has all diagonal elements equal to zero.

(3) A symmetric matrix is a square matrix satisfying certain conditions.

(4) A skew-symmetric matrix has all diagonal elements equal to zero.

Ans. Option (2) is correct

Explanation: Because, we know that a square matrix in which every element except the diagonal elements are zero is called diagonal Matrix.

20. The angle between the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z+5}{6}$ and the plane $2x + 10y - 11z = 5$ is:

(1) $\cos^{-1}\left(\frac{8}{21}\right)$ (2) $\sin^{-1}\left(\frac{-8}{21}\right)$

(3) $\cos^{-1}\left(\frac{21}{82}\right)$ (4) $\sin^{-1}\left(\frac{21}{82}\right)$

Ans. Option (2) is correct

Explanation: We know that the Angle θ between the line

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and}$$

plane $a_2x+b_2y+c_2z + D = 0$ is,

$$\sin\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\frac{x-(-2)}{3} = \frac{y-3}{2} = \frac{z-(-5)}{6}$$

plane $2x+10y-11z-5=0$

when, $a_1=3, b_1=2, c_1=6$ and $a_2=2, b_2=10, c_2=-11$,

Now Angle between the line and plane

$$\sin\theta = \frac{(3)(2)+(2)(10)+(6)(-11)}{\sqrt{(3)^2+(2)^2+(6)^2}\sqrt{(2)^2+(10)^2+(-11)^2}}$$

$$\sin\theta = \frac{6+20-66}{\sqrt{49}\sqrt{225}}$$

$$\sin\theta = \frac{-40}{7\times 15}$$

$$\sin\theta = \frac{-8}{21}$$

$$\theta = \sin^{-1}\left(\frac{-8}{21}\right)$$

21. Match List I with List II

LIST I		LIST II	
A.	$\int \frac{\sin x}{1+\cos x}dx$	I.	$e^{\tan^{-1}x} + C$
B.	$\int \frac{1}{1-\tan x}dx$	II.	$\log(\log x + 1) + C$
C.	$\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$	III.	$-\log\lvert 1+\cos x\rvert + C$
D.	$\int \frac{1}{x+x\log x}dx$	IV.	$\frac{x}{2}-\frac{1}{2}\log\lvert\cos x - \sin x\rvert + C$

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-IV, C-I, D-II

(3) A-I, B-II, C-III, D-IV

(4) A-IV, B-I, C-III, D-II

Ans. Option (2) is correct

Explanation:

A. $$\int \frac{\sin x}{1+\cos x}dx$$

let $1+\cos x = t$

$-\sin x\,dx = dt$

$$= -\int \frac{1}{t}dt$$

$$= -\log t + C$$

$$= -\log\lvert 1+\cos x\rvert + C$$

Here, A Match with III

B. $$\int \frac{1}{1-\tan x}dx$$

$$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$$

$$= \int \frac{\cos x}{\cos x - \sin x}dx$$

$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$

$$= \frac{1}{2}\int \frac{\cos x + \cos x + \sin x - \sin x}{\cos x - \sin x}dx$$

$$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$$

$$= \frac{1}{2}\int \left(\frac{\cos x - \sin x}{\cos x - \sin x} + \frac{\cos x + \sin x}{\cos x - \sin x}\right)dx$$

$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

let $\cos x - \sin x = t$

$(-\sin x - \cos x)dx = dt$

$-(\cos x + \sin x)dx = dt$

$(\cos x + \sin x)dx = -dt$

$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\times -\int \frac{1}{t}dt$$

$$= \frac{1}{2}x - \frac{1}{2}\log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

Here, B Match with IV

C. $\displaystyle\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

let $\tan^{-1}x = t$

$$\frac{1}{1+x^2}dx = dt$$

$= \int e^t\,dt$

$= e^t + C$

$= e^{\tan^{-1}x} + C$

Here, C Match with I

D. $\displaystyle\int \frac{1}{x + x\log x}dx$

$$= \int \frac{1}{x(1+\log x)}dx$$

let $1 + \log x = t$

$$\left(0 + \frac{1}{x}\right)dx = dt$$

$$\frac{1}{x}dx = dt$$

$$= \int \frac{1}{t}dt$$

$= \log|t| + C$

$= \log|1 + \log x| + C$

Here, D Match with II

22. The function $f(x) = \dfrac{x-1}{x(x^2-1)}$, $x \neq 1$, $f(1) = 1$, is discontinuous at:

(1) Exactly one point **(2)** Exactly two points

(3) Exactly three points **(4)** No point

Ans. Option (3) is correct

Explanation: $f(x) = \dfrac{x-1}{x(x^2-1)}$ Thus given function $f(x)$ is discontinuous when $f(x)$ is not defined when denominator is equal to zero.

$\therefore\quad x(x^2-1) = 0$

$x(x-1)(x+1) = 0$

$x = 0, x = 1$ and $x = -1$

Therefore, given function is not defined at Three Values of x

Hence, $f(x)$ is discontinuous at Exactly three points.

23. The area of the region bounded by the lines $x = 2y + 3$, $x = 0$, $y = 1$ and $y = -1$ is:

(1) 4 sq. units **(2)** 6 sq. units

(3) 8 sq. units **(4)** $\frac{3}{2}$ sq. units

Ans. Option (2) is correct

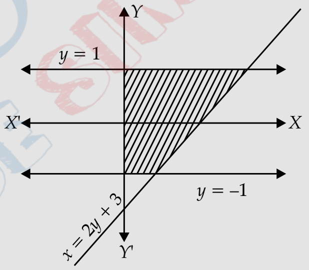
Explanation: Required area, $A = \int_{-1}^{1}(2y+3)dy$

$$= \left[\frac{2y^2}{2} + 3y\right]_{-1}^{1}$$

$$= \left[y^2 + 3y\right]_{-1}^{1}$$

$= [1 + 3 - 1 + 3]$

$= 6$ sq units.



24. Match List I with List II

	LIST I		LIST II
A.	The area of parallelogram determined by vectors $2\hat{i}$ and $3\hat{j}$	**I.**	2
B.	The value of $(\hat{i}\times\hat{j})\cdot\hat{k} + (\hat{j}\times\hat{k})\cdot\hat{i}$	**II.**	4
C.	The value of a for which the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} - 6\hat{j} + 8\hat{k}$ are collinear.	**III.**	0
D.	The value of λ for which the vectors $2\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are perpendicular	**IV.**	6

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV

(2) A-II, B-I, C-III, D-IV

(3) A-III, B-IV, C-II, D-I

(4) A-IV, B-I, C-II, D-III

Ans. Option (4) is correct

Explanation:

A. Area of parallelogram $= |2\hat{i} \times 3\hat{j}|$

$= |6\hat{k}|$

$= 6$

Here, A Match with IV

B. $\left(\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{j}\times\hat{k}\right)\cdot\hat{i} = \hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}$
$= 1+1$
$= 2$

Here, B Match with I

C. Vectors are collinear when

$$\frac{2}{a}=\frac{-3}{-6}=\frac{4}{8}$$

$$a = 4$$

Here, C Match with II

D. Vectors are perpendicular when

$$(2)(2) + (1)(-4)+(1)(\lambda) = 0$$
$$4-4+\lambda = 0$$
$$\lambda = 0$$

Here, D Match with III

25. If the matrix $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then A^2 is equal to:

(1) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

(2) $\begin{bmatrix}\cos^2\theta & \sin^2\theta\\ -\sin^2\theta & \cos^2\theta\end{bmatrix}$

(3) $\begin{bmatrix}\cos\theta^2 & \sin\theta^2\\ -\sin\theta^2 & \cos\theta^2\end{bmatrix}$

(4) $\begin{bmatrix}\cos\theta+\sin\theta & \cos\theta-\sin\theta\\ \sin\theta-\sin\theta & \cos\theta+\sin\theta\end{bmatrix}$

Ans. Option (1) is correct

Explanation:

$$A = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$A^2 = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta-\sin^2\theta & \sin\theta\cos\theta+\sin\theta\cos\theta\\ -\sin\theta\cos\theta-\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta-\sin^2\theta & 2\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$$

26. The maximum slope of the curve

$y = -x^3 + 3x^2 + 9x - 27$ is:

(1) 0 (2) 12 (3) 16 (4) 32

Ans. Option (2) is correct

Explanation: The given equation is

$$y=-x^3+3x^2+9x-27$$

To find the slope we differentiate the function once.

$\Rightarrow$ $y'=-3x^2+6x+9$

To find the extreme points we again differentiate and equate it to zero

$\Rightarrow$ $y''=-6x+6$

Now, $y''=0$

$\Rightarrow$ $-6x+6=0$

$\Rightarrow$ $x=1$

Now,

$\Rightarrow$ $y'''=-6$

$\Rightarrow$ $y'''(1)=-6$

$\Rightarrow$ $y'''(1)<0$

Hence we find a maxima at $x=1$ in the equation of the slope. Hence the maximum value of the slope is

$\Rightarrow$ $y'(1)$ which is,

$\Rightarrow$ $y'(1)=-3(1)^2+6(1)+9$

$\Rightarrow$ $y'(1)=-3+6+9$

$\Rightarrow$ $y'(1)=12$

27. The approximate volume of a cube of side a metres on increasing the side by 4% is:

(1) $1.04a^3$ m^3 (2) $1.004a^3$ m^3
(3) $1.12a^3$ m^3 (4) $1.12a^2$ m^3

Ans. Option (3) is correct

Explanation: Volume (V) of a cube with side a is given by, $V=a^3$

$$\Rightarrow \frac{dV}{da} = 3a^2 = \Delta V = 3a^2\Delta a,$$

Now it is given that $\Delta a = 0.04a$

$\therefore$ $\Delta V = 3a^2\Delta a$

$\Delta V = 3a^2(0.04a)$

$\Delta V = 0.12a^3$

Approximate volume of a cube $= V + \Delta V$

$= a^3 + .12a^3$

$= 1.12a^3$ m^3

28. Match List I with List II

	LIST I		LIST II
A.	$\sin^{-1}x+\cos^{-1}x, x\in[-1,1]$	I.	$-\frac{\pi}{2}$
B.	$\tan^{-1}\sqrt{3}-\cos^{-1}\left(-\sqrt{3}\right)$	II.	$-\frac{\pi}{6}$
C.	$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$	III.	$\frac{\pi}{2}$
D.	$\sin^{-1}\left(-\frac{1}{2}\right)$	IV.	$\frac{\pi}{6}$

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II
(2) A-IV, B-I, C-II, D-III
(3) A-II, B-III, C-IV, D-I
(4) A-I, B-II, C-III, D-IV

Ans. Option (1) is correct

Explanation:

A. $\sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}$

Here, A Match with III

B. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= \frac{-3\pi}{6}$

$= -\frac{\pi}{2}$

Here, B Match with I

C. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$\cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$

$\cos^{-1}\left[\cos\frac{\pi}{6}\right]$

$= \frac{\pi}{6}$

Here, C Match with IV

D. $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Here, D Match with II

29. If the matrix A $= \begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}$ is skew-symmetric, then :

(1) $x = 2,\ y = 1,\ z = 0$
(2) $x = 2,\ y = 2,\ z = 0$
(3) $x = -2,\ y = -1,\ z = 0$
(4) $x = -2,\ y = -1,\ z = -1$

Ans. Option (3) is correct

Explanation:

$A = \begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}$

Given, A is skew-symmetric

$A' = -A$

$$\begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}' = -\begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & x-y \\ x+y & z & -2 \\ 1 & 2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -(x+y) & -1 \\ -3 & -z & -2 \\ -x+y & 2 & 0 \end{bmatrix}$$

On comparing

$x+y = -3$ ... (i)

$x-y = -1$ ... (ii)

on solving eg. (i) and eg. (ii)

$x = -2$ and $y = -1$

and

$z = -z$

$z+z = 0$

$2z = 0$

$z = 0$

$x = -2, y = -1$ and $z=0$

30. If three points A(a_1, b_1), B(a_2, b_2),C(a_3, b_3) and are collinear and D= $\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then:

(1) D=0
(2) D=±1
(3) D^2=0 or 1
(4) D= $(a_1+a_2+a_3)-(b_1+b_2+b_3)$

Ans. Option (1) is correct

Explanation: We know that when three points. are collinear, then area between them is zero
D = 0

31. If a line makes angles 90°. 60° and θ with X, Y and Z axis respectively, where θ is acute, then value of θ is:

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Ans. Option (1) is correct

Explanation: if α, β and γ are the angles made by the line with the x, y and z axes respectively, then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Therefore, $\cos^2 90° + \cos^2 60° + \cos^2\theta = 1$

$0 + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30° = \frac{\pi}{6}$

32. The area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum is:

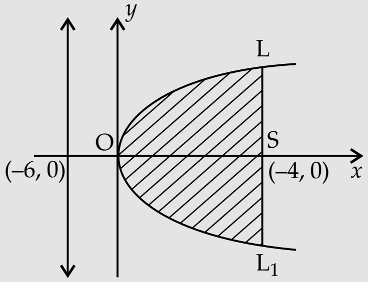
(1) $\frac{4a^2}{3}$ sq. units (2) $\frac{8a^2}{3}$ sq. units

(3) $\frac{2a^2}{3}$ sq. units (4) $\frac{9a^2}{5}$ sq. units

Ans. Option (2) is correct

Explanation: The equation of parabola is

$y^2=4ax$ ... (i)



Let O be the vertex, S be the focus and LL′ be the latus rectum of parabola.

The equation of latus rectum is

$x = a$.

Also, we know that parabola is symmetric about X-axis.

∴ Required area =2(area of OSLO)

∴ Required area

$$=2\int_0^a y\,dx = 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$=2.2\sqrt{a}\int_0^a x^{1/2}\,dx = 4\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^a$$

$$=4\sqrt{a}\,\frac{2}{3}\left[x^{3/2}\right]_0^a = \frac{8\sqrt{a}}{3}\left[a^{3/2}-0\right]$$

$$=\frac{8\sqrt{a}}{3}a^{3/2} = \frac{8}{3}a^2 \text{ sq. units}$$

33. Particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$ given that when $x=0$, $y=0$ is:

(1) $e^x+e^{-y}=2$ **(2)** $e^{-x}+e^y=2$

(3) $e^x+e^y=2$ **(4)** $e^{-x}+e^{-y}=2$

Ans. Option (1) is correct

Explanation: $\log\left(\frac{dy}{dx}\right)=x+y$

$$\left(\frac{dy}{dx}\right)=e^{x+y}$$

$$\int\frac{dy}{e^y}=\int e^x dx$$

$$\frac{e^{-y}}{-1}=e^x+c_1$$

$e^x+e^{-y}=c$ $(c=c_1)$

$e^x+e^{-y}=c$ ... (i)

when $x=0$ and $y=0$

$e^0+e^0=c$

$1+1=c$

$c=2$ Put in eq. (i)

$e^x+e^{-y}=2$

34. Solution of $\frac{dy}{dx}=(1+x^2)(1+y^2)$ is:

(1) $\tan^{-1}y=x+\frac{x^3}{3}+c$

(2) $\tan^{-1}y=x+\frac{x^3}{3}+c$

(3) $\tan^{-1}y=x^2+\frac{x^3}{3}+c$

(4) $\tan^{-1}y=x^2+\frac{x^3}{3}+c$

Ans. Option (1) is correct

Explanation: $\frac{dy}{dx}=(1+x^2)(1+y^2)$

$$dy=(1+x^2)(1+y^2)dx$$

$$\frac{dy}{1+y^2}=(1+x^2)dx$$

Integrating both sides,

$$\because\int\frac{1}{1+y^2}=\int(1+x^2)dx$$

$$\tan^{-1}y=x+\frac{x^3}{3}+C$$

$\left[\because\int\frac{1}{1+y^2}dy=\tan^{-1}y\right]$

35. A manufacturer can sell x items at a price of ₹$3x+5$ each. The cost price of x items is ₹x^2+5x. If x is the number of items she should sell to get no profit and no loss. then:

(1) $x=10$ **(2)** $x=30$ **(3)** $x=0$ **(4)** $x=-10$

Ans. Option (3) is correct

Explanation:

Number of items = x

Selling Price of each item = ₹$3x+5$

Selling Price of all x items $(x)(3x+5)$

= $3x^2+5x$

Cost price of all x items = x^2+5x

For not getting profit or loss

S.P = C.P

$3x^2+5x=x^2+5x$

$x^2=0$

$x=0$

36. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then $\frac{dy}{dx}=$

(1) $\sqrt{\frac{1-x^2}{1-y^2}}$ **(2)** $\sqrt{\frac{1-y^2}{1-x^2}}$

(3) $\sqrt{\frac{1-x^2}{1+y^2}}$ **(4)** $\sqrt{\frac{1+x^2}{1-y^2}}$

Ans. Option (2) is correct

Explanation: $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Put $x = \sin A$ and $y = \sin B$

$\sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B} = a[\sin A-\sin B]$

$\cos A + \cos B = a\,[\sin A-\sin B]$

$$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = a\,2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$$

$$\tan\left(\frac{A-B}{2}\right)=\frac{1}{a}$$

$$\frac{A-B}{2}=\tan^{-1}\left(\frac{1}{a}\right)$$

$$A-B = 2\tan^{-1}\left(\frac{1}{a}\right)$$

$$\sin^{-1}x - \sin^{-1}y = 2\tan^{-1}\left(\frac{1}{a}\right)$$

d.w.r.t x

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}=0$$

$$\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

37. $\int\left(\frac{1+x+x^2}{1+x^2}\right)e^{\tan^{-1}x}dx =$

(1) $x = e^{\tan^{-1}x}+c$ (2) $e^{\tan^{-1}x}-x+c$

(3) $e^{\tan^{-1}x}+c$ (4) $xe^{\tan^{-1}x}+c$

Ans. Option (4) is correct

Explanation:

$$\int\left(\frac{1+x+x^2}{1+x^2}\right)e^{\tan^{-1}x}dx$$

$$= \int\left(\frac{(1+x^2)+(x)}{(1+x^2)}\right)e^{\tan^{-1}x}\,dx$$

let $e^{\tan^{-1}}x = t$

$$\frac{1}{1+x^2}\,dx = dt$$

$= \int(1+x^2+x)\,e^t dt$

$\tan^{-1}x = t$

$x = \tan t$

$= \int(1+\tan^2 t+\tan t)\,e^t dt$

$= \int(\sec^2 t + \tan t)\,e^t dt$

$= \int(\tan t + \sec^2 x)\,e^t dt$

we know that

$\int e^x[f(x)+f'(x)\,dx=e^x f(x) + c$

$= e^t \tan t + c$

$= e^{\tan^{-1}x}\tan(\tan^{-1}x) + c$

$= x\,e^{\tan^{-1}x} + c$

38. **A.** Equation of the line passing through the point (1, 2, 3) and parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ is $\frac{x-1}{3}=\frac{y-2}{2}=\frac{y-3}{-2}$.

B. Equation of line passing through (1, 2, 3) and parallel to the line given by $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$ is $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z+3}{6}$.

C. Equation of line passing through the origin and (5, −2, 3) is $\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$

D. Equation of plane passing through the point (1,2,3) and perpendicular to the line with direction ratio's 2, 3, −1 is $2(x-1)+3(y-2)-1(z-3)=0$

E. Equation of plane with intercepts 2, 3 and 4 on X, Y and Z-axes respectively is $2x+3y+4z=1$.

Choose the correct answer from the options given below:

(1) A, E only (2) A, C, D only

(3) C, D, E only (4) E only

Ans. Option (2) is correct

Explanation:

A. Equation of line Passing through a point (1, 2, 3) and parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ is $\frac{x-1}{3}=\frac{y-2}{2}=\frac{y-3}{-2}$

which is given correct

B. Equation of line passing through (1, 2, 3) and Parallel to the line given by

$$\frac{x-(-3)}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$$

But given $x_1=1, y_1=2, z_1=3$

$a_1=3, b_1=-5, c_1=6$

Parallel line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

$\therefore \quad \frac{x-1}{3}=\frac{y-2}{-5}=\frac{z-3}{6}$

but in question, it is incorrect.

C. Equation of a line, Passing through the origin $(x_1, y_1, z_1) = (0, 0, 0)$ and $(x_2, y_2, z_2) = (5,-2, 3)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-0}{5-0}=\frac{y-0}{-2-0}=\frac{z-0}{3-0}$$

$$\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$$

which) is correct

D. Equation of plane Passing through the point $(x_1, y_1, z_1) = (1, 2, 3)$ and perpendicular to the line with direction ratios $(a, b, c) = (2, 3, -1)$ is

$a(x-x_1) + b(y-y_1)+c(z-z_1)=0$

$2(x-1)+3(y-2)+(-1)(z-3)=0$

$2(x-1)+3(y-2)-(z-3)=0$

which) is correct

E. Equation of plane with intercepts $(a, b, c) = .(2, 3, 4)$ on x, y, z axis is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$$

$$\frac{6x+4y+3z}{12}=1$$

$$6x+4y+3z = 12$$

But in question, it is incorrect

39. If $A=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ and B is a square matrix of order 3. then $|AB|$ is equal to:

(1) $|B|^2$ **(2)** $|B|$

(3) $\sin 2\theta|B|$ **(4)** $\cos 2\theta|B|$

Ans. Option (4) is correct

Explanation:

$A = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ and B is a square matrix of order 3, then $|AB|$ is equal to $|AB| = |B|\,|A|$

$|A| = \begin{vmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{vmatrix}$ is

$|A| = \cos\theta(\cos\theta) + \sin\theta(-\sin\theta) + 0(0)$

$= \cos 2\theta\ |A|\ |B|$

$\cos 2\theta|B|$

40. Value of $\dfrac{e^{\sin\left(\tan^{-1}x+\cot^{-1}x\right)}}{e^{\sin\left(\sin^{-1}x+\cos^{-1}x\right)}}$, $x \in [-1,1]$, is:

(1) 0 **(2)** $\frac{\pi}{2}$ **(3)** 1 **(4)** $-\frac{\pi}{2}$

Ans. Option (3) is correct

Explanation: $= \dfrac{e^{\sin\left(\tan^{-1}x+\cot^{-1}x\right)}}{e^{\sin\left(\sin^{-1}x+\cos^{-1}x\right)}}$

$= \dfrac{e^{\sin\frac{\pi}{2}}}{e^{\sin\frac{\pi}{2}}}$

$= \dfrac{e^1}{e^1}$

$= 1$

41. Probabilities to solve a specific problem by A, B and C are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Probability that at least one will solve the problem is:

(1) $\frac{1}{24}$ **(2)** $\frac{1}{4}$ **(3)** $\frac{23}{24}$ **(4)** $\frac{3}{4}$

Ans. Option (4) is correct

Explanation: P(Probability of solving problem by A),

$P(A) = \frac{1}{2}$

P(problem not solve by A), $P(A)$

$P(\bar{A}) = 1-P(A)$

$= 1-\frac{1}{2}$

$= \frac{1}{2}$

P(Probability of solving problem by B),

$P(B) = \frac{1}{3}$

P(Probability not solve by B),

$P(B) = 1-P(B)$

$= 1-\frac{1}{3}$

$= \frac{2}{3}$

P(Probability of solving problem by C),

$P(C) = \frac{1}{4}$

P(Probability not solve by C) = $P(\bar{C})$

$P(\bar{C}) = 1-P(C)$

$= 1-\frac{1}{4}$

$= \frac{3}{4}$

P(at least one will solve the problem) = 1 − P(Problem not solve by any one among A, B and C).

$= 1 - (PA)P(B)P(C)$

$= 1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$

$= 1-\frac{1}{4}$

$= \frac{3}{4}$

42. The derivative of $\sin(\tan^{-1}e^{2x})$ with respect to x is:

(1) $\dfrac{2e^{2x}\sin(\tan^{-1}e^{2x})}{1+e^{4x}}$

(2) $\dfrac{2e^{2x}\cos(\tan^{-1}e^{2x})}{1+e^{4x}}$

(3) $\dfrac{2e^{2x}\sin(\tan^{-1}e^{2x})}{1+e^{x^2}}$

(4) $\dfrac{2e^{2x}\cos(\tan^{-1}e^{2x})}{1+e^{2x}}$

Ans. **Option (2) is correct**

Explanation: $y = \sin(\tan^{-1} e^{2x})$
d.w.r.t. x

$$\frac{dy}{dx} = \cos(\tan^{-1} e^{2x}) \times \frac{1}{1+e^{4x}} \times e^{2x} \times 2$$

$$= \frac{2e^{2x}\cos(\tan^{-1} e^{2x})}{1+e^{4x}}$$

43. If A is a square matrix of order 3, then $|\text{adj } A|$ is equal to:

(1) $|A|$ **(2)** $|A|^2$ **(3)** $|A|^3$ **(4)** $3|A|$

Ans. **Option (2) is correct**

Explanation: $|\text{adj } A| = |A|^{x-1}$, where x is order of the matrix

$$|\text{adj } A| = |A|^{3-1}$$
$$= |A|^2$$

44. The feasible region of an LPP Max $Z = 3x + 2y$ subject to $x \geq 0, y \geq 0, x-2y \leq 3$ is:

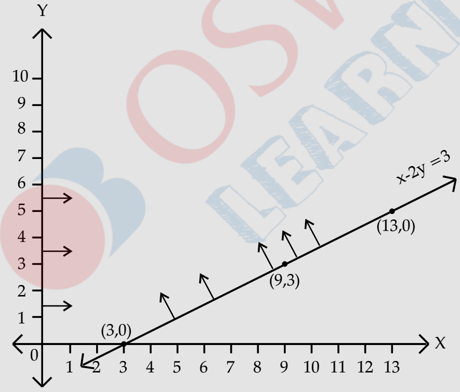
(1) Bounded in first quadrant but has no solution
(2) Unbounded in first quadrant but has a solution
(3) Unbounded in first quadrant and has no solution
(4) Bounded and has a solution $x=0, y=0, Z=0$

Ans. **Option (3) is correct**

Explanation:

$x \geq 0, y \geq 0$
$x-2y \leq 3$
$x-2y = 3$

x	3	9	13
y	0	3	5



Unbounded in first quadrant and has no solution.

45. Let A={1.2.3}. Consider the relation R={(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}. Then R is

(1) reflexive only
(2) reflexive and transitive
(3) symmetric and transitive
(4) neither symmetric nor transitive

Ans. **Option (2) is correct**

Explanation: R={(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}
Given Relation is Reflexive and transitive but not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$

46. The range of the function $f(x) = \frac{1}{3-\sin 4x}$ is

(1) $\left[\frac{1}{4}, \frac{1}{2}\right]$ **(2)** $\left[\frac{1}{2}, 1\right]$

(3) $\left[\frac{1}{4}, \frac{3}{4}\right]$ **(4)** $\left[\frac{1}{2}, \frac{3}{4}\right]$

Ans. **Option (1) is correct**

Explanation: $f(x) = \frac{1}{3-\sin 4x}$ $f(x)$ will maximum when $3-\sin 4x$ is minimum
and $3-\sin 4x$ is minimum when $\sin 4x = 1$

Maximum Range $= \frac{1}{3-1}$

$$= \frac{1}{2}$$

$f(x)$ will minimum when $3-\sin 4x$ is maximum and $3-\sin 4x$ is maximum when $\sin 4x = -1$

Minimum Range $= \frac{1}{3-(-1)}$

$$= \frac{1}{4}$$

$$= \left[\frac{1}{4}, \frac{1}{2}\right]$$

47. The equation of the tangent. to the curve $= x^2-2x-3$ which is perpendicular to the line $x+2y+3=0$, is:

(1) $4x-2y=7$ **(2)** $2x-y=7$
(3) $2x-y=5$ **(4)** $4x-2y=5$

Ans. **Option (2) is correct**

Explanation:

$y = x^2-2x-3$

$\frac{dy}{dx} = 2x-2$

At $(x_1, y_1) m_1 = 2x_1 - 2$ ... (i)

$x+2y+3=0$

$1+2\frac{dy}{dx}+0=0$

$\frac{dy}{dx} = \frac{-1}{2}$

At $(x_1, y_1) m_2 = \frac{-1}{2}$

$m_1 \times m_2 = \frac{-1}{2}$ [as m_1 and m_2 are perpendicular]

$(2x_1-2)\left(\frac{1}{2}\right) = -1$

$2x_1-2=2$
$2x_1=4$
$x_1=2$
$x_1=2$ Put in $y_1 = x_1^2-2x_1-3$
$= (2)^2-2(2)-3$
$= 4-4-3 = -3$

$(x_1, y_1) = (2, -3)$

Equation of tangent

$y - y_1 = m_1(x - x_1)$

$y - (-3) = [2(2) - 2]\,(x - 2)$

$y + 3 = 2\,(x - 2)$

$y + 3 = 2x - 4$

$2x - y = 7$

48. The solution of the differentiable equation $2x\dfrac{dy}{dx} + y = 14x^3,\ x > 0$, is:

(1) $y = 2x^3 + cx^{½}$

(2) $y = x^3 + cx^{½}$

(3) $y = 2x^3 + cx^{-½}$

(4) $y = x^3 + cx^{-½}$

Ans. Option (3) is correct

Explanation: $2x\dfrac{dy}{dx} + y = 14x^3$

$$2x\left(\frac{dy}{dx} + \frac{1}{2x}y\right) = 14x^3$$

$$\frac{dy}{dx} + \frac{1}{2x}y = 7x^2 \quad \text{.... (i)}$$

On comparing with

$\dfrac{dy}{dx} + Py = $ as, $P = \dfrac{1}{2x}$ and

$Q = 7x^2$

I.F $= e^{\int P x}$

$$= e^{\int \frac{1}{2x}dx}$$

$$= e^{1/2 \log x}$$

$$= e^{\log \sqrt{x}}$$

$$= \sqrt{x}$$

On Multiply $\sqrt{x}$ on both sides in eq. (i)

$$\sqrt{x}\frac{dy}{dx} + \frac{\sqrt{x}}{2x}y = 7x^2\sqrt{x}$$

$$\sqrt{x}\frac{dy}{dx} + \frac{1}{2\sqrt{x}} \times y = 7x^{5/2}$$

$$\frac{dy}{dx}(\sqrt{x}\,y) = 7x^{5/2}$$

On Integrating both sides

$$\sqrt{x}\,y = 7\int x^{5/2}dy$$

$$\sqrt{x}\,y = 7\frac{x^{7/2}}{\frac{7}{2}} + c$$

$$y = 2\frac{x^{7/2}}{\sqrt{x}} + \frac{c}{\sqrt{x}}$$

$$y = 2x^3 + c\,x^{-½}$$

49. Let the vectors $\vec{a} = \hat{i} - 3\hat{j} + 2\hat{k},\ \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} + 5\hat{j} - 2\lambda\hat{k}$ be coplanar. Then λ is equal to:

(1) -1 **(2)** 1

(3) -2 **(4)** 2

Ans. Option (4) is correct

Explanation: $\vec{a} = \hat{i} - 3\hat{j} + 2\hat{k},\ \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} + 5\hat{j} - 2\lambda\hat{k}$ are coplanar.

$[abc] = 0$

$$\begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -1 \\ 3 & 5 & -2\lambda \end{vmatrix} = 0$$

$1(-2\lambda + 5) + 3(-4\lambda + 3) + 2(10 - 3) = 0$

$-2\lambda + 5 - 12\lambda + 9 + 14 = 0$

$-2\lambda + 5 - 12\lambda + 23 = 0$

$-14\lambda = 2B$

$\lambda = 2$

50. A coin is tossed 7 times. The probability of getting at least 4 heads is:

(1) $\dfrac{5}{8}$ **(2)** $\dfrac{3}{4}$

(3) $\dfrac{1}{4}$ **(4)** $\dfrac{1}{2}$

Ans. Option (4) is correct

Explanation: Use the binomial distribution directly. Let us assume that the number of heads is represented by x (where a result of heads is regarded at success and in this case $x \geq 4$.

Assuming that the coin is unbiased you have a probability of failure 'q' is 1/2 (where q is considered as failure). The number of trials is represented by the letter n and for this question $n = 7$, you have a probability of success 'p' (where p is considered as success is 1/2.

Now just use the probability function for a binomial distribution

$p(X = x) = {}^nC_x P^x q^{n-x}$

Using the information in the problem we get,

$p(x \geq 4) = p(x = 4) + p(x = 5) + p(x = 6) + p(x = 7)$

$$7_{c_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + 7_{c_5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2$$

$$+ 7_{c_6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + 7_{c_7}\left(\frac{1}{2}\right)^7$$

$$= \left(\frac{1}{2}\right)^7[7_{c_4} + 7_{c_5} + 7_{c_6} + 7_{c_7}]$$

$$= \left(\frac{1}{2}\right)^7[35 + 21 + 7 + 1]$$

$$\frac{1}{128}[64]$$

$$= \frac{1}{2}$$

Section - B2

Applied Mathematics

51. Consider the following data:

Commodity	Price year 2010	Price year 2016	Quantity year 2010	Quantity year 2016
A	1	2	10	13
B	5	10	12	16
C	6	10	15	18

The Laspeyre's price index number for year 2016 with year 2010 as base year is:

(1) 160 **(2)** 200 **(3)** 150 **(4)** 170

Ans. Option (1) is correct

Explanation:

Commodity	P_0	P_1	Q_0	Q_1	P_0Q_1	P_1Q_1	P_1Q_0
A	1	2	10	13	13	26	20
B	5	10	12	16	80	160	120
C	6	10	15	18	108	180	150
					$\sum P_0Q_1=201$	$\sum P_1Q_1=366$	$\sum P_1Q_0=290$

$$P_{01}=\frac{\sum P_1Q_0}{\sum P_0Q_0}\times 100$$

$$=\frac{290}{160}\times 100$$

52. The present value of a perpetuity of ₹1200 payable at the beginning of each year, if money is worth 5% per annum is:

(1) ₹25,500 **(2)** ₹24,000 **(3)** ₹24,200 **(4)** ₹25,200

Ans. Option (4) is correct

Explanation: PV = ₹25,200

$$P = R+\frac{R}{I} = 1200+\frac{1200}{5}$$

$$= 1200 + 24000 = ₹25200$$

53. Pure honey costs ₹300 per litre. A shopkeeper adds water to 10 litres of pure honey and sells the resulting syrup at ₹250 per litre. The quantity of water added by the shopkeeper is :

(1) 2 litres **(2)** 5 litres **(3)** 3 litres **(4)** 1.5 litres

Ans. Option (1) is correct

Explanation: Given, Quantity of pure honey = 10 litre.

Per litre Price of pure honey = ₹300

Total selling price of 10 litre honey

= 10 × 300 = ₹3000

After Adding x ltr of water into 10 ltr of honey, Now new quantity of syrup (mixture of water and honey) = $(10+x)$ ltr

Given, Shopkeeper selling syrup at the rate of ₹250 per litre,

∴ Total sealing price = $250(10+x)$

According to the question.

$$3000 = 250(10+x)$$
$$3000 = 2500+250x$$
$$500 = 250x$$
$$x = 2 \text{ litre}$$

54. Three persons A, B and C enter into a partnership to run a business. They invested their capitals in the ratio $\frac{4}{3}:\frac{5}{2}:\frac{6}{5}$. After 5 months B increases his share by 40%. If the total profit at the end of a year is ₹50,550, then A's share in the profit is:

(1) ₹8,000 **(2)** ₹10,000 **(3)** ₹20,000 **(4)** ₹12,000

Ans. Option (4) is correct

Explanation: One month ratio of investment of three persons A, B and C $=\frac{4}{3}:\frac{5}{2}:\frac{6}{5}$

$= 40 : 75 : 36$

A's Investment for a year $= 40x \times 12$

$= 480x$

B's Investment for a year $= 75x \times 5 + \frac{140}{100}\times 75x \times 7$

$= 375x+735x$

$= 1110x$

C's Investment for a year $= 36x \times 12$

$= 432x$

Yearly ratio of capital investment of three persons A, B and C = $480x : 1110x : 432x$ = 80 : 185 : 72

Given, Total profit at the end of a year = ₹50550

A's profit share $=\frac{80}{80+185+72}\times 50550$

$=\frac{80}{337}\times 50550$

= ₹12000

55. Two positive numbers x and y whose sum is 25 and the product x^3y^2 is maximum are:

(1) $x=10, y=15$ **(2)** $x=15, y=10$

(3) $x=12, y=13$ **(4)** $x=16, y=9$

Ans. Option (2) is correct

Explanation:

Given, $x+y = 25$

$y = 25 - x$ ... (i)

$p = x^3y^2$

$p = x^3(25-x)^2$ [from eq. (i)]

d.w.r.t. x

$$\frac{dp}{dx} = x^3\, 2(25-x)(-1) + (25-x)^2\,(3x)^2$$
$$\frac{dp}{dx} = x^2\,(25-x)[-2x+75-3x]$$
$$\frac{dp}{dx} = x^2\,(25-x)(-5x+75)$$
$$\frac{dp}{dx} = 5x^2\,(25-x)(x-15) \quad \text{... (ii)}$$

On putting $\frac{dp}{dx} = 0$

$$-5x^2\,(25-x)(x-15) = 0$$
$$x = 0, x = 25, x = 15$$

again differentiating eq. (ii) w.r.t. x

$$\frac{d^2p}{dx^2} = -5x^2(25-x)+-5x^2(x-15)(-1) + (25-x)\,(x-15)(-10x)$$
$$\left.\frac{d^2p}{dx^2}\right\}_{x=0} = 0$$
$$\left.\frac{d^2p}{dx^2}\right\}_{x=15} = -12250$$
$$\left.\frac{d^2p}{dn^2}\right\}_{x=25} = +31250$$

from above

$$\left.\frac{d^2p}{dx^2}\right\}_{x=15} = -12250 < 0$$

Hence at $x = 15$ and $y = 10$ the product x^3y^2 is maximum.

56. If the function $f(x)=a\log x+\frac{b}{x}+x$ has extreme values at $x=1$ and $x=3$, then (a, b) is:

(1) $\left(-\frac{1}{2}, -\frac{3}{2}\right)$ **(2)** (4, 3)

(3) (−2, −1) **(4)** (−4, −3)

Ans. Option (4) is correct

Explanation:

$$f(x)=a\log x+\frac{b}{x}+x$$
$$f'(x) = \frac{a}{x}-\frac{b}{x^2}+1$$

As we know, An extremum is calculated from the derivative of the function about a point where the derivative is equal to zero.

$\Rightarrow f'(x)=0$

$$\frac{a}{x}-\frac{b}{x^2}+1 = 0 \quad \text{... (i)}$$

Now it is given that the extremum is at x=1 and x=3

on putting $x=1$ in eq. (i)

$$a-b = -1 \quad \text{... (ii)}$$

on putting $x=3$ in eq. (i)

$$\frac{a}{3}-\frac{b}{9} = -1$$
$$3a - b = -9 \quad \text{... (iii)}$$

On solving eq (ii) and eq (iii)

$a = -4$ and $b = -3$

$(a, b) = (-4, -3)$

57. The maximum value of $Z=3x+y$ subject to the constraints $x+y\le30$, $2x+y\le40$, $x, y\ge0$ is:

(1) 50 **(2)** 30 **(3)** 25 **(4)** 60

Ans. Option (4) is correct

Explanation: when,

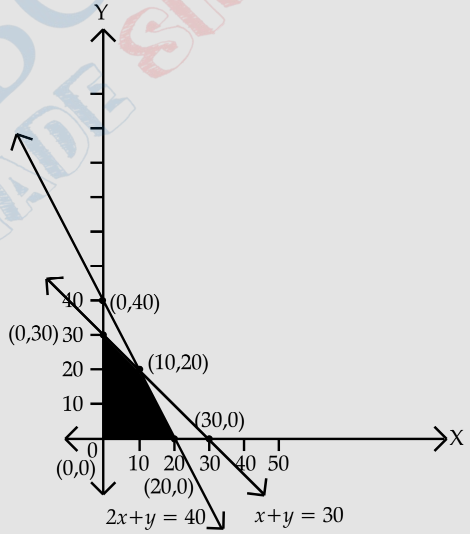
$x+y = 30$

X	0	30	10
Y	30	0	20

$2x+y = 40$

X	0	20	10
Y	40	0	20

For $x+y\le30$, $2x+y\le40$ and $x, y\ge0$



(x, y)	$z=3x+y$
(0, 0)	0
(20, 0)	60
(10, 20)	50
(0, 30)	30

Here, at $x=20$ and $y=0$ Maximum value of $z=3x+y$ is 60.

58. If $57 = x(\text{mod } 5)$. Then the least positive value of x is:

(1) 57 **(2)** 5 **(3)** 4 **(4)** 2

Ans. Option (4) is correct

Explanation: We know that when 57 is divided by 5, we get 2 as reminder,

Hence $x=2$

59. If $x = \log t$ and $y = \dfrac{1}{t^2}$, then $\dfrac{d^2y}{dx^2}$ is:

(1) $\dfrac{2}{t^2}$ **(2)** $\dfrac{4}{t^2}$ **(3)** $-\dfrac{1}{t}$ **(4)** $-\dfrac{4}{t^2}$

Ans. Option (2) is correct

Explanation: Given, $x = \log t$

d.w.r.t. x $\dfrac{dx}{dt} = \dfrac{1}{t}$ ... (i)

and $y = \dfrac{1}{t^2}$

d.w.r.t. t

$$\frac{dy}{dt} = \frac{-2}{t^3} \quad \text{... (ii)}$$

on dividing eq (i) by eq (ii) we get,

$$\frac{dy}{dx} = \frac{-2}{t^3} \times \frac{t}{1}$$

$$\frac{dy}{dx} = \frac{-2}{t^2}$$

d.w.r.t. x

$$\frac{d^2y}{dx^2} = -2 \times \frac{-2}{t^3} \times \frac{dt}{dx}$$

$$= \frac{4}{t^3} \times \frac{t}{1} \quad \text{[from eq (i)]}$$

$$= \frac{4}{t^2}$$

60. The speed of a motor boat in still water is 14.4 times the speed of the current of water. If the motor boat covers a certain distance upstream in 6 hours 25 minutes, then the time taken by the motor boat to come back is:

(1) 5 hours 35 minutes
(2) 5 hours 25 minutes
(3) 5 hours 10 minutes
(4) 5 hours 55 minutes

Ans. Option (1) is correct

Explanation: Let the speed of current of water $= x$ km/hr speed of motor boat in still water $= 14.4x$ km/hr

Speed in upstream $= (14.4x - x)$ km/hr

Speed of motor boat in down stream

$= (14.4x \times x)$ km/hr

time taken by boat in upstream = 6 hr 25 min

$$= 6 + \frac{25}{60}$$

$$= \frac{77}{12} \text{ hr}$$

Speed of motor boat in upstream $= \dfrac{\text{Distance}}{\text{time}}$

$$14.4\,x - x = \frac{\text{Distance}}{\frac{77}{12}}$$

$$13.4\,x = \frac{12D}{77}$$

$$D = 13.4 \times \frac{77}{12} x$$

Now, time taken by motor in Downstream

$$= \frac{\text{Distance}}{\text{Speed of motor boat in Downstream}}$$

$$= \frac{D}{14.4x + x}$$

$$= \frac{13.4x}{15.4x} \times \frac{77}{12}$$

$$= \frac{134}{2} \times \frac{1}{12}$$

= 5 hours 35 minutes.

61. A simple random sample consists of five observations 2, 4, 6, 7, 6. The point estimate of population standard deviation is:

(1) 4 **(2)** 2.5 **(3)** 5 **(4)** 2

Ans. Option (4) is correct

Explanation:

Mean $= \dfrac{2+4+6+7+6}{5}$

$= \dfrac{25}{5}$

$\bar{x} = 5$

Standard deviation $= \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{f}}$

$$= \sqrt{\frac{(2-5)^2 + (4-5)^2 + (6-5)^2 + (7-5)^2 + (6-5)^2}{5}}$$

$$= \sqrt{\frac{9+1+1+4+1}{5}}$$

$$= \sqrt{\frac{16}{5}} = 1.78$$

62. The price relatives and weights of a set of commodities are given as:

Commodity	**A**	**B**	**C**
Price Relative	150	130	180
Weight	x	$3x$	y

If the sum of weights is 30 and the index for the set is 144, then the values of x and y are:

(1) $x=6, y=8$ **(2)** $x=8, y=4$
(3) $x=6, y=6$ **(4)** $x=5, y=10$

Ans. Option (3) is correct

Explanation:

Commodity	**Weight (w)**	**Price relative (p)**	wp
A	x	150	$150x$
B	$3x$	130	$390x$
C	y	180	$180y$
Sum	$4x+y$		$540x+180y$

Sum of weights = 30

$4x + y = 30$ ... (i)

Index Number $= 144$

$$\frac{540x+180y}{30} = 144$$

$9x + 3y = 72$

$3x + y = 24$ ... (ii)

On solving eq. (i) and eq. (ii) to get x and y, we get $x = 6$ and $y = 6$

63. Given the data for the sales of a product in a state is as follows:

Year	2005	2006	2007	2008	2009
Sales(In lakh ₹)	150	130	160	170	200

The equation of the straight-line trend by method of least squares is:

(1) $14 + 162x$ **(2)** $126 + 15x$

(3) $128 + 14x$ **(4)** $162 + 14x$

Ans. Option (4) is correct

Explanation: Calculating for the straight line trend by least square method

Year (x)	Sales (y)	$X = x - 2007$	X^2	XY
2005	150	−2	4	−300
2006	130	−1	1	−130
2007	160	0	0	0
2008	170	1	1	170
2009	200	2	4	400
$n = 5$	$\sum y = 810$	$\sum x=0$	$\sum x^2 = 10$	$\sum xy = 140$

$$a = \frac{\sum y}{n} = \frac{810}{5} = 162$$

$$= \frac{\sum xy}{\sum x^2} = \frac{140}{10} = 14$$

$$Y = a+bX$$

$$= 162+14x$$

64. If $\begin{bmatrix} 2x+3 & 3y & 3 \\ y+1 & 2x-z & -1 \\ 3z+1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & -9 & 3 \\ -2 & -4 & -1 \\ 25 & 2 & 5 \end{bmatrix}$ then the values of x, y and z are:

(1) $x=2$, $y=3$, $z=8$

(2) $x=2$, $y=-3$, $z=8$

(3) $x=-3$, $y=2$, $z=6$

(4) $x=-2$, $y=3$, $z=8$

Ans. Option (2) is correct

Explanation: On, comparing,

$2x+3=7$

$2x=4$

$x=2$

$y+1=-2$

$y=-3$

and $3z+1=25$

$3z = 24$

$z = 8$

$x=2, y = -3, z=8$

65. Consider the following hypothesis test:

$H_0 : \mu \geq 20$

$H_1 : \mu < 20$

A sample of 64 provided a sample mean of 19.5. The population standard deviation is 2. The value of the test statistic is:

(1) −2.5 **(2)** −2 **(3)** 2 **(4)** −1.5

Ans. Option (2) is correct

Explanation:

$$\text{Test statistic } t = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{\pi}}$$

$$= \frac{19.5-20}{2/\sqrt{64}} = \frac{-0.5}{2/8} = -2$$

66. Between 3 p.m. and 5 p.m. the average number of phone calls per minute coming into the help line desk of a bank is 5. The probability that during one particular minute there will be only one phone call is:

(1) $0.5e^{-5}$ **(2)** $5e^{-5}$ **(3)** e^{-5} **(4)** $25e^{-5}$

Ans. Correct Option is (2)

Explanation: Let X be denoting the number of phone calls per minutes. Given that Mean $\lambda = 5$ and we need to Find $P(X=1)$

Now,

$$P(X=1) = \frac{e^{-\lambda}(\lambda)^1}{1!} \left[P(X=r) = \frac{e^{-\lambda}(\lambda)^r}{r!} \right]$$

$$= \frac{e^{-5}(5)^1}{1!} = 5e^{-5}$$

67. If the sum and product of the mean and variance of a binomial distribution are 18 and 72 respectively, then the probability of obtaining at most one success is

(1) $25\left(\frac{1}{2}\right)^{24}$ **(2)** $\left(\frac{1}{2}\right)^{24}$

(3) $24\left(\frac{1}{2}\right)^{24}$ **(4)** $24\left(\frac{1}{2}\right)^{23}$

Ans. Option (1) is correct

Explanation: Let the binomial distribution be $(p+q)^n$ we have,

Mean $= np$

Variance $= npq$

Given, $np+npq = 18$

$np(1+q) = 18$

On squaring both sides

$n^2p^2(1+q)^2 = 324$ ... (i)

and $(np)(npq) = 72$

$n^2p^2q = 72$ ... (ii)

On dividing eq (i) by eq (ii), we get

$$\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{324}{72}$$

$$\frac{1+q^2+2q}{q}=\frac{9}{2}$$
$$2q^2-5q+2=0$$
$$2q^2-4q-q+2=0$$
$$2q(q-2)-1(q-2)=0$$
$$(q-2)(2q-1)=0$$
$$q = 2 \text{ (not possible)}, q=\frac{1}{2}$$

We know $p+q = 1$

$$p+\frac{1}{2} = 1$$
$$p=\frac{1}{2}$$

So by eq (ii), we get

$$n^2\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = 72$$
$$n^2 = 72\times 8$$
$$n = 24$$

Therefore, The binomial distribution is

$$(p+q)^n = \left(\frac{1}{2}+\frac{1}{2}\right)^{24}$$

Now, Probability of obtaining almost one success
$p\,(x\le 1) = p(x=0) + p(x=1)$

$$= {}^{24}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{24} + {}^{24}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{23}$$
$$= 25\left(\frac{1}{2}\right)^{24}$$

68. Match List I with List II

	LIST I		LIST II
A.	The variance of a Poisson distribution with mean λ is	**I.**	$\sqrt{\lambda}$
B.	The standard deviation of a Poisson distribution with mean is	**II.**	4
C.	In a Poisson distribution, if mean is 4, then the standard deviation is	**III.**	λ
D.	In a Poisson distribution, if mean is 4, then the variance is	**IV.**	2

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV
(2) A-III, B-I, C-IV, D-II
(3) A-I, B-III, C-II, D-IV
(4) A-I, B-III, C-IV, D-II

Ans. Option (2) is correct

Explanation:

A. We know, Variance of a poisson distribution with mean λ=λ
Here, A Match with III

B. Standard Deviation of a Poisson distribution with mean λ = $\sqrt{\lambda}$
Here, B Match with I

C. S.D. when λ = 4 = √4 = 2
Here, C Match with IV

D. Variance when λ = 4 = 4
Here, D Match with II

69. Match List I with List II

	LIST I		LIST II
A.	A matrix which is not a square matrix is called	**I.**	Non-singular matrix
B.	If the determinant of any matrix is non-zero, then the matrix is called	**II.**	Null matrix
C.	A diagonal matrix having same diagonal elements is called	**III.**	Rectangular matrix
D.	A matrix which is both symmetric and skew-symmetric matrix is called	**IV.**	Scalar matrix

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II
(2) A-IV, B-III, C-I, D-II
(3) A-IV, B-I, C-II, D-III
(4) A-II, B-I, C-IV, D-III

Ans. Option (1) is correct

Explanation: A → III
B → I
C → IV
D → II

70. The longest side of a triangle is four times the shortest side. The third side of the triangle is 3 cm shorter than the longest side. If the perimeter of the triangle is at least 69 cm, then its:

(1) Shortest-side < 8 cm
(2) Shortest-side > 8 cm
(3) Shortest-side ≤ 8 cm
(4) Shortest-side ≥ 8 cm

Ans. Option (4) is correct

Explanation: Let the
Shortest side $= x$ cm
Longest side $= 4x$ cm
Third side $= (4x-3)$ cm
Given, Perimeter ≥ 69
$x+ 4x + 4x-3\ge 69$
$9x\ge 72$
$x\ge 8$
Shortest side ≥ 8

71. An asset costing ₹2,00,000 is expected to have a useful life of 10 years and a final scrap valve of ₹40,000. The book value of the machine at the end of sixth year is:

(1) ₹1,36,000 **(2)** ₹1,04,000
(3) ₹1,20,000 **(4)** ₹88,000

Ans. Option (2) is correct

Explanation:

Annual Depreciation = $\frac{\text{Cost Price - Scrap Value}}{\text{Estimated life of machine}}$

$= \frac{200000 - 40000}{10}$

$= 16000$

Book value = Cost Price − no. of years after which book value is to be computed × Annual Depreciation

$= 200000 - 6 \times 16000$

$= 200000 - 96000$

$= 1,04,000$

72. The point on the straight line $3x + 4y = 8$, which is closest to the origin is:

(1) $\left(\frac{13}{24}, \frac{17}{24}\right)$ **(2)** $\left(\frac{24}{25}, \frac{32}{25}\right)$

(3) $\left(\frac{5}{24}, \frac{7}{24}\right)$ **(4)** $\left(1, \frac{5}{4}\right)$

Ans. Option (2) is correct

Explanation: Given, $3x + 4y = 8$

at $\left(\frac{13}{24}, \frac{17}{24}\right)$

L.H.S. $= 3\left(\frac{13}{24}\right)+4\left(\frac{17}{24}\right)$

$= \frac{39}{24}+\frac{68}{24}$

$= \frac{107}{24}$

$=$ R.H.S.

$\left(\frac{13}{24}, \frac{17}{24}\right)$ Does not lie on line $3x+4y = 8$

Now, at $\left(\frac{24}{25}, \frac{32}{25}\right)$

L.H.S. $= 3\left(\frac{24}{25}\right)+4\left(\frac{32}{25}\right)$

$= \frac{72}{25}+\frac{128}{25}$

$= \frac{200}{25}$

$= 8$

$=$ R.H.S.

$\left(\frac{24}{25}, \frac{32}{25}\right)$ Does lie on line $3x+4y = 8$

Now, at $\left(\frac{5}{24}, \frac{7}{24}\right)$

L.H.S. $= 3\left(\frac{5}{24}\right)+4\left(\frac{7}{24}\right)$

$= \frac{15}{24}+\frac{28}{24}$

$= \frac{43}{24}$

$\neq 8$

$\left(\frac{5}{24}, \frac{7}{24}\right)$ Does not lie on line $3x+4y=8$

Now at, $\left(1, \frac{5}{4}\right)$

L.H.S. $= 3(1) + 4\left(\frac{5}{4}\right)$

$= 3 + 5$

$= 8$

$=$ R.H.S.

$\left(1, \frac{5}{4}\right)$ Does lie on line $3x+4y = 8$

Now, Distance between $\left(\frac{24}{25}, \frac{32}{25}\right)$ and (0,0)

$d_1 = \sqrt{\left(\frac{24}{25}-0\right)^2+\left(\frac{32}{20}-0\right)^2}$

$= \sqrt{\frac{576+1024}{625}}$

$= \sqrt{\frac{1600}{625}}$

$= \frac{40}{25}$

$= \frac{8}{5}$

$= 1.6$ unit

Now, Distance between $\left(1, \frac{5}{4}\right)$ and (0, 0)

$d_2 = \sqrt{(1-0)^2+\left(\frac{5}{4}-0\right)^2}$

$= \sqrt{1+\frac{25}{16}}$

$= \sqrt{\frac{41}{16}}$

$= \frac{6.403124}{4}$

$$= 1.600781$$

Hence, $\left(\frac{24}{25}, \frac{32}{25}\right)$ is the point on the straight line which is closest to the origin.

73. Match List I with List II

	LIST I		LIST II
A.	A special characteristic of a population is known as a	**I.**	statistic
B.	A special characteristic of a sample is known as a	**II.**	Confidence interval
C.	The uncertainty of a sampling process is expressed by	**III.**	Estimation
D.	The process by which one makes the inferences about a population based on the information obtained from a sample is known as	**IV.**	Parameter

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I
(2) A-I, B-IV, C-II, D-III
(3) A-IV, B-I, C-II, D-III
(4) A-IV, B-I, C-III, D-II

Ans. Option (4) is correct

Explanation:
A → IV
B → I
C → III
D → II

74. Match List I with List II

	LIST I		LIST II
A.	The solution set of the inequality $3x+7>12$	**I.**	$[-1, \infty]$
B.	The solution set of the inequality $\frac{3x+5}{2} \geq 1, x \in R$	**II.**	$\left[\frac{17}{8}, \infty\right]$
C.	The solution set of the inequality $2x+5<7x+9, x \in R$ is	**III.**	$\left[\frac{5}{3}, \infty\right]$
D.	The solution set of the inequality $6x - 5 \geq -2x+12, x \in R$ is	**IV.**	$\left[-\frac{4}{5}, \infty\right]$

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II
(2) A-III, B-I, C-IV, D-II
(3) A-I, B-III, C-IV, D-II
(4) A-III, B-I, C-II, D-IV

Ans. Option (2) is correct

Explanation:

A. $3x+7>12$
$3x>5$
$x>\left[\frac{5}{3}\right]$
$x \in \left[\frac{5}{3}, \infty\right]$

Here, A Match with III

B. $\frac{3x+5}{2} \geq 1$
$3x+5 \geq 2 \Rightarrow \quad 3x \geq -3$
$x \geq -1 \Rightarrow \quad x \in (-1, \infty)$

Here, B Match with I

C. $2x+5<7x+9$
$5-9<7x-2x \Rightarrow -y<5x$
$-\frac{4}{5} < x \Rightarrow \quad x \in \left[-\frac{4}{5}, \infty\right]$

Here, C Match with IV

D. $6x - 5 \geq -2x+12$
$6x+2x \geq 12+5 \Rightarrow \quad 8x \geq 17$
$x \geq \frac{17}{8} \Rightarrow \quad x \in \left[\frac{17}{8}, \infty\right]$

Here D Match with II

75. Let X be a discrete random variable whose probability distribution is defined as:

$$P(X=x) = \begin{cases} 0.5 & \text{, if} \quad x = 0 \\ k(x+1) & \text{, if} \quad x = 1 \text{ or } 2 \\ k(6-x) & \text{, if} \quad x = 3 \text{ or } 4 \\ 0 & , \quad \text{otherwise} \end{cases}$$

The value of k is:

(1) $\frac{1}{10}$ **(2)** $\frac{1}{20}$ **(3)** $\frac{1}{2}$ **(4)** $\frac{1}{4}$

Ans. Option (2) is correct

Explanation: $P(0) + P(1) + P(2) + P(3) + P(4) = 1$
$0.5+2k+3k+3k+2k=1$
$\frac{1}{2} + 10k = 1$
$10k = \frac{1}{2}$
$k = \frac{1}{20}$

76. If the matrix $\begin{bmatrix} a & -2 & 5b \\ 2 & 0 & -15 \\ 15 & 3c & 0 \end{bmatrix}$ is skew-symmetric. then the value of $a^2 + b^2 + c^2$ is:

(1) 15 **(2)** 34 **(3)** 25 **(4)** 16

Ans. Option (2) is correct

Explanation: Given, A is skew-symmetric matrix
$A' = -A$

$$\begin{bmatrix} a & -2 & 5b \\ 2 & 0 & -15 \\ 15 & 3c & 0 \end{bmatrix}' = -\begin{bmatrix} a & -2 & 5b \\ 2 & 0 & -15 \\ 15 & 3c & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & 2 & 15 \\ -2 & 0 & 3c \\ 5b & -15 & 0 \end{bmatrix} = \begin{bmatrix} -a & 2 & -5b \\ -2 & 0 & 15 \\ -15 & -3c & 0 \end{bmatrix}$$

On comparing

$a = -a$

$a = 0$ ... (i)

$5b = -15$

$b = -3$ ... (ii)

$3c = 15$

$c = 5$ ... (iii)

$a^2+b^2+c^2=(0)^2+(-3)^2+(5)^2$

[From eq. (i), (ii) and (iii)]

$= 0 + 9 + 25 \quad = 34$

77. The minimum value of the objective function $Z = 30x + 10y$ subject to the constraints $x+2y\leq 30$, $3x+y\geq 30$, $4x+3y\geq 60$, $x, y\geq 0$ is:

(1) 100 **(2)** 450 **(3)** 300 **(4)** 1200

Ans. Option (3) is correct

Explanation:

For, $x+2y = 30$

x	0	30	6
y	15	0	12

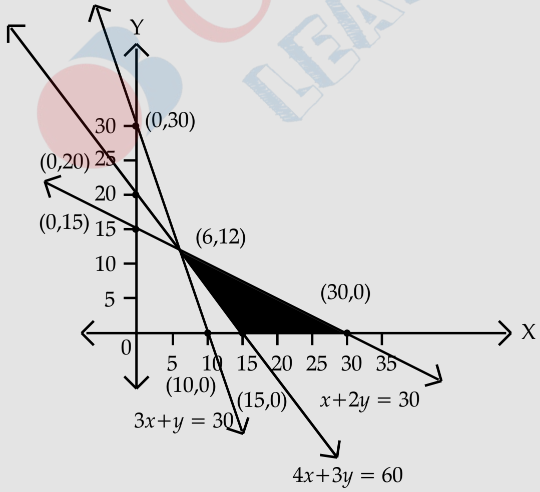
For, $3x+y = 30$

x	0	10	6
y	30	0	12

For, $4x+3y = 60$

x	0	15	6
y	20	0	12

For $x+2y\leq 30$, $3x+y\geq 30$, $4x+3y\geq 60$ and $x, y\geq 0$



(x, y)	$= 30x+10y$
(6, 12)	300
(15, 0)	450
(30, 0)	900

Here, at $x=6$ and $y=12$ minimum value of the objective function $z=30x+10y$ is 300

78. Match List I with List II

	LIST I		LIST II
A.	The set of values of decision variables which do not satisfy all the constrains and non-negativity condition of a LPP is called.	**I.**	Linear
B.	In a LPP, the objective function is always	**II.**	Convex polygon
C.	In a LPP, the linear inequalities on variable are called	**III.**	Infeasible solution
D.	The feasible region for a LPP is always a	**IV.**	Constraints

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-III, B-I, C-II, D-IV

Ans. Option (3) is correct

Explanation: A → III

B → I

C → IV

D → II

79. A person takes a car loan of ₹9,00,000 at the rate of 12% per annum for 5 years from a bank. The EMI under flat rate system is:

(1) ₹24,000 **(2)** ₹20,000 **(3)** ₹16,000 **(4)** ₹28,000

Ans. Option (1) is correct

Explanation: $P = 900000$, $r = 12\%$, $A = 5$ years

Total Interest in 5 years,

$$I = \frac{p\times r\times t}{100}$$

$$= \frac{900000\times 12\times 5}{100}$$

$$= 5{,}40{,}000$$

Total Amount Payable,

$$A = P + I$$

$$= 900000 + 540000$$

$$= 14{,}40{,}000$$

EMI under flat rate system $= \dfrac{14{,}40{,}000}{60}$

$= ₹24000$

80. A company produces bikes at the rate of x bikes per day and its total cost function is $C(x)=x^3-60x^2+13x+50$. The optimal number of bikes produced per day at which the marginal cost is minimum is:

(1) 15 **(2)** 40 **(3)** 20 **(4)** 25

Ans. Option (3) is correct

Explanation: given, $C(x) = x^3-60x^2+13x+50$

Marginal cost, $C'(x) = 3x^2-120x+13$

$C''(x) = 6x-120$

Now, $C''(x)=0$

$\therefore$ $6x-120 = 0$

$\Rightarrow$ $x=20$

Now, $C'''(x)=6$

$C'''(20)=6>0$

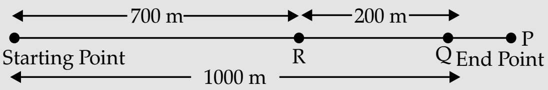
Hence, Marginal Cost is Minimum at $x=20$

81. In 1000 metres race, P, Q, R scored first, second and third positions respectively. If P beats Q by 100 metres and Q beats R by 200 metres, then the gap between P and R is:

(1) 300 m **(2)** 280 m **(3)** 260 m **(4)** 240 m

Ans. Option (1) is correct

Explanation:



Clearly, The gap between P and R is 300 m

82. If $y=\log_e\left(\frac{x^3}{e^3}\right)$, then $\frac{d^2y}{dx^2}$ is equal to:

(1) $\frac{3}{x^2}$ **(2)** $-\frac{2}{x^2}$ **(3)** $-\frac{3}{x^2}$ **(4)** $-\frac{2}{x}$

Ans. Option (3) is correct

Explanation:

$$y=\log_e\left(\frac{x^3}{e^3}\right)$$

d.w.r.t. x

$$\frac{dy}{dx}=\frac{1}{\frac{x^3}{e^3}}\times\frac{1}{e^3}\times 3x^2$$

$$\frac{dy}{dx}=\frac{3}{x}$$

again d.w.r.t. x

$$\frac{d^2y}{dx^2}=\frac{-3}{x^2}$$

83. The probability distribution of a discrete random variable X is given as:

X	0	1	2	3
P(X)	$2k^2$	k^2	$3k^2$	k

The mean of the distribution is:

(1) $\frac{4}{3}$ **(2)** $\frac{5}{3}$ **(3)** $\frac{7}{6}$ **(4)** $\frac{16}{9}$

Ans. Option (4) is correct

Explanation:

$2k^2+k^2+3k^2+k=1$

$6k^2+k-1=0$

$6k^2+3k-2k-1=0$

$3k(2k+1)-1(2k+1) = 0$

$(2k+1)(3k-1) = 0$

$k = \frac{-1}{2}$ (not possible), $k = \frac{1}{3}$

Now,

X	0	1	2	3
P(X)	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{3}$

Mean $= 0\times\frac{2}{9}+1\times\frac{1}{9}+2\times\frac{1}{3}+3\times\frac{1}{3}$

$= 0+\frac{1}{9}+\frac{2}{3}+1$

$= \frac{1+6+9}{9}=\frac{16}{9}$

84. The wholesale price index of sugar in 2018 compared to 2015 is 125. If the cost of sugar was ₹20 per kg in 2015, then the cost of sugar in 2018 is:

(1) ₹25 per kg **(2)** ₹30 per kg

(3) ₹15 per kg **(4)** ₹45 per kg

Ans. Option (1) is correct

Explanation:

$I = \frac{P_1}{P_0}\times 100$

$125 = \frac{P_1}{20}\times 100$

$P_1 = \frac{125\times 20}{100}$

$P_1 =$ ₹25 per kg

The cost of sugar in 2018 is ₹25/kg

85. The corner points of the feasible region for a LPP are (0, 4), (2, 3), (4, 5), (7, 0). If objective function is $Z=px+qy$; $p, q> 0$ then the condition on p and q so that the minimum of Z occurs at (2, 3) and (7, 0) is:

(1) $7p=4q$ **(2)** $5p=3q$

(3) $4p=q$ **(4)** $3p=3q$

Ans. Option (2) is correct

Explanation:

(x, y)	$Z=px+qy$
(0, 4)	$4q$
(2, 3)	$2p+3q$
(4, 5)	$4p+5q$
(7, 0)	$7p$

Given, minimum of Z occurs at (2, 3) and (7, 0)

Therefore, $2p+3q=7p$

$5p = 3q$

❑❑❑

CUET Question Paper 2022

National Testing Agency

MATHEMATICS/APPLIED MATHEMATICS

(*This includes Questions pertaining to Domain Specific Subject only*)

Solved

Max. Marks : 200 | **Time allowed : 45 Min.**

General Instructions :

(i) *Section A will have 15 questions covering both i.e., Mathematics/Applied Mathematics which will be compulsory for all candidates.*

(ii) *Section B1 will have 35 questions from Applied Mathematics out of which 25 questions need to be attempted. Section B2 will have 35 questions purely from Mathematics out of which 25 questions will be attempted.*

(iii) *Correct answer or the most appropriate answer : Five marks (+ 5)*

(iv) *Any incorrect option marked will be given minus one mark (– 1).*

(v) *Unanswered/Marked for Review will be given no mark (0).*

(vi) *If more than one option is found to be correct then Five marks (+5) will be awarded to only those who have marked any of the correct options.*

(vii) *If all options are found to be correct then Five marks (+5) will be awarded to all those who have attempted the question.*

(viii) *If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given five marks (+5).*

(ix) *Calculator / any electronic gadgets are not permitted.*

Section - A

Mathematics/Applied Mathematics

1. Let A and B be two non zero square matrix and AB and BA both are defined. If means

(1) No. of columns of A ≠ No. of rows of B

(2) No. of rows of A ≠ No. of columns of B

(3) Both matrices (A) and (B) have same order

(4) Both matrices (A) and (B) does not have same order

Ans. **Option (3) is correct.**

> ***Explanation:*** Option A is incorrect as given AB is defined then columns of A = rows of B
> Options B is incorrect as given BA is defined then columns of B = rows of A
> Options C is correct as:
> Let A be $m \times n$. Since AB and BA both exist hence B must be $n \times m$. Thus, AB is $m \times m$ and BA is $n \times n$. So, for, all we have established is both AB and BA are square matrices. If $m \neq n$ then AB and BA can be compared but in general AB ≠ BA.

2. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, then which of the following statements are correct?

(A) A is a square matrix

(B) A^{-1} exists

(C) A is a symmetric matrix

(D) $|A| = 19$

(E) A is a null matrix

Choose the correct answer from the options given below:

(1) A, B, C only **(2)** A, D, E only

(3) A, B, D only **(4)** C, D, E only

Ans. **Option (3) is correct.**

> ***Explanation:*** Given $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$
> Order of A is 2 × 2. (A square matrix)
> $$|A| = 10 + 9 = 19$$
> Since $|A| \neq 0$ therefore A^{-1} exists
> $$A' = \begin{bmatrix} 2 & 3 \\ -3 & 5 \end{bmatrix} \neq A$$
> Hence, A is not symmetric
> Thus, statements A, B and D is correct

3. The number of all possible matrices of order 2 × 2 with each entry 0 or 1 is:

(1) 27 **(2)** 18

(3) 16 **(4)** 81

Ans. **Option (3) is correct.**

> ***Explanation:*** In a 2 × 2 matrix, the total no. of elements present are 4. Each element can be replaced with either 1 or 0. Thus, there are two ways of filling each of the four spaces.

Thus the total no. of number of such matrices will be $2 \times 2 \times 2 \times 2 = 2^4$ matrices.

Therefore, the no. of possible matrix of 2×2 order with each entry as 0 or 1 is equal to 16.

4. If $y = \left(\frac{1}{x}\right)^x$, then value of $e^e\left(\frac{d^2y}{dx^2}\right)_{x=e}$ is:

(1) $2-\frac{1}{e}$ **(2)** $4-\frac{1}{e}$

(3) $\frac{1}{e}$ **(4)** $1-\frac{1}{e}$

Ans. **Option (2) is correct.**

Explanation: Given

$$y = \left(\frac{1}{x}\right)^x$$

$$\log_2 y = x\log_e\frac{1}{x}$$

(Taking log both sides)

Differentiating w.r.t., x both sides.

$$\frac{1}{y}\frac{dy}{dx} = 1.\log_e\frac{1}{x}+x\frac{1}{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = y\left[\log_e\left(\frac{1}{x}\right)-1\right]$$

$$= -y(\log_e x + 1)$$

$$\frac{d^2y}{dx^2} = -y'(\log_e x+1)-y\left(\frac{1}{x}+0\right)$$

$$= -y'(\log_e x+1)-\frac{y}{x}$$

$$= x^{-x}(\log_e x+x)^2-\frac{x^{-x}}{x}$$

$$= x^{-x}\left[(\log_e x+1)^2-\frac{1}{x}\right]$$

$$\frac{d^2y}{dx^2}\Big]_{\text{at } x=e} = \left[(\log_e e+1)^2-\frac{1}{e}\right]x^{-x}$$

$$= \left(4-\frac{1}{e}\right)e^{-e}$$

Now, $e^e\left(\frac{d^2y}{dx}\right)_{dx=e} = e^e\left(4-\frac{1}{e}\right)e^{-e}$

$$= 4-\frac{1}{e}$$

5. The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval

(1) $(-\infty, -1)$ **(2)** $(-1, \infty)$

(3) $(-\infty, 1)$ **(4)** $(-1, \infty)$

Ans. **Option (3) is correct.**

Explanation: **Given** $f(x) = x^2 - 2x$

For strictly decreasing function,

$$f'(x) < 0$$

i.e., $2x - 2 < 0$

or, $2(x-1) < 0$

or, $x < 1$

i.e., $x \in (-\infty, 1)$

Hence, f is strictly decreasing in $(-\infty, 1)$

6. $\int\frac{dx}{x(x^5+3)}$ is equal to

(1) $\frac{1}{3}\log\left|\frac{x^5}{x^5+3}\right|+C$ **(2)** $\frac{1}{15}\log\left|\frac{x^5}{x^5+3}\right|+C$

(3) $\frac{1}{5}\log\left|\frac{x^5}{x^5+3}\right|+C$ **(4)** $\frac{1}{25}\log\left|\frac{x^5}{x^5+3}\right|+C$

Ans. **Option (2) is correct.**

Explanation: Let $I = \int\frac{dx}{x(x^5+3)}$

$$= \int\frac{x^4}{x^5(x^5+3)}dx$$

Let $x^5 = t \Rightarrow 5x^4dx = dt \Rightarrow x^4dx = \frac{dt}{5}$

$$\therefore \quad I = \int\frac{dt}{5t(t+3)}$$

$$= \frac{1}{5}\int\frac{1}{3}\left[\frac{1}{t}-\frac{1}{t+3}\right]dt$$

[By partial fraction]

$$= \frac{1}{15}\left[\int\frac{1}{t}dt-\int\frac{1}{t+3}dt\right]$$

$$= \frac{1}{15}[\log t-\log(t+3)]+C$$

$$= \frac{1}{15}\log\left(\frac{t}{t+3}\right)+C$$

$$= \frac{1}{15}\log\left(\frac{x^5}{x^5+3}\right)+C$$

[Substituting $t = x^5$]

7. If $\int\frac{x^3}{x+1}dx = q(x)-\log|x+1|+C$ then $q(x)$ is equal to:

(1) $q(x)=\frac{x^3}{3}+x$ **(2)** $q(x)=\frac{x^3}{2}-x$

(3) $q(x) = x^2 - x + 1$ **(4)** $q(x)=\frac{x^3}{3}-\frac{x^2}{2}+x$

Ans. **Option (4) is correct.**

Explanation: **Given,**

$$\int\frac{x^3}{x+1}dx = q(x)-\log|x+1|+C \quad ..(i)$$

Let, $I = \int\frac{x^3}{x+1}dx$

$$= \int \frac{(x^3+1-1)}{x+1}dx$$
$$= \int \frac{x^3+1}{x+1}dx - \int \frac{1}{x+1}dx$$
$$= \int \frac{(x+1)(x^2-x+1)}{(x+1)}dx - \int \frac{dx}{x+1}$$
$$= \int (x^2-x+1)dx - \int \frac{dx}{x+1}$$
$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$$

Thus, $\int \frac{x^3}{x+1}dx$

$$= \left(\frac{x^3}{3} - \frac{x^2}{2} + x\right) - \log|x+1| + C \quad \text{...(ii)}$$

Comparing RHS of eps. (i) and (ii), we get

$$q(x) = \frac{x^3}{3} - \frac{x^2}{2} + x$$

8. $\int_{-1}^{1} (|x-2| + |x|)dx =$

(1) 7 **(2)** 5
(3) 4 **(4)** 6

Ans. **Option (2) is correct.**

Explanation:

Let $I = \int_{-1}^{1}(|x-2|+|x|)dx$

$$= \int_{-1}^{1}|x-2| + \int_{-1}^{1}|x|\,dx$$
$$= -\int_{-1}^{1}(x-2)dx + \int_{-1}^{0}(-x)dx + \int_{0}^{1} x\,dx$$
$$= -\left[\frac{x^2}{2} - 2x\right]_{-1}^{1} + \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$
$$= -\left[\left(\frac{1}{2}-2\right) - \left(\frac{1}{2}+2\right)\right] - \left(\frac{0}{2}-\frac{1}{2}\right) + \left(\frac{1}{2}+\frac{0}{2}\right)$$
$$= -\left[\frac{-3}{2} - \frac{5}{2}\right] + 1$$
$$= -(-4) + 1$$
$$= 5$$

9. If a and b are order and degree of differential equation $y'' + (y')^2 + 2y = 0$, then value of $2a + 6b$, is:

(1) 3 **(2)** 4
(3) 6 **(4)** 10

Ans. **Option (4) is correct.**

Explanation: Given differential equation is:

$$y'' + (y')^2 + 2y = 0$$

Order of differential equation is 2. Thus, $a = 2$
Degree of differential equation is 1. Thus, $b = 1$
Therefore, $2a + 6b = 2 \times 2 + 6 \times 1 = 4 + 6 = 10$

10. The solution of the differential equation $xdy - ydx = 0$ represent family of

(1) Circles passing through origin.
(2) Straight line parsing through (–1, 6).
(3) Straight line passing through the origin.
(4) Circle whose centre is at the origin.

Ans. **Option (3) is correct.**

Explanation:

$$x\,dy - y\,dx = 0$$
$$x\,dy = y\,dx$$
$$\frac{dy}{y} = \frac{dx}{x}$$
$$\log y = \log x + \log c$$
$$\log y = \log (x\,c)$$
$$y = cx,$$

which represent the straight line passing through origin.

11. For differential equation $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy$, $y(0) = 1$, the value of $x(e)$ is equal to:

(1) 0 **(2)** 1
(3) 2 **(4)** e

Ans. **Option (4) is correct.**

Explanation: We have.

$$y\,e^{x/y}\,dx = (xe^{x/y} + y^2)\,dy$$
$$\frac{dx}{dy} = \frac{x\,e^{x/y} + y^2}{y\,e^{x/y}}$$
$$\frac{dy}{dx} = \frac{xe^{x/y}}{ye^{x/y}} + \frac{y}{e^{x/y}}$$

Put $x = vy$

$\Rightarrow$ $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore$ $v + y\frac{dv}{dy} = vde^v + \frac{y}{e^v}$

$$y\frac{dv}{dy} = v - v + \frac{y}{e^v}$$
$$y\frac{dv}{dx} = \frac{y}{e^v}$$
$$\int e^v dv = \int dy$$
$$e^v + c = y$$

or, $y = e^{x/y} + c$
Since, $y(0) = 1$
Therefore, $1 = e^0 + c \Rightarrow c = 0$
$\therefore$ $y = e^{x/y}$
or, $\log y = \frac{x}{y}\log e$
or, $x = y\frac{\log y}{\log e}$
or, $(x)(e) = \frac{e\log e}{\log e} = e$

12. $\int_{-1}^{1} e^{|x|} dx =$

(1) $2(e^{-1} - 1)$ **(2)** $2(e + 1)$

(3) $e - 1$ **(4)** $2(e - 1)$

Ans. **Option (4) is correct.**

Explanation: Let

$$I = \int_{-1}^{1} e^{|x|} dx$$

$$= \int_{-1}^{1} e^{-x} dx + \int_{0}^{1} e^{+x}$$

$$= -\left[e^{-x}\right]_{-1}^{0} + \left[x^{e}\right]_{0}^{1}$$

$$= -(1 - e) + (e - 1)$$

$$= 2e - 2$$

$$= 2(e - 1)$$

13. For two events A, B

$P(A \cup B) = \frac{7}{12}, P(A) = \frac{5}{12}, P(B) = \frac{3}{12}$ Then $P(A \cap B) =$

(1) $\frac{1}{2}$ **(2)** $\frac{1}{12}$

(3) $\frac{1}{6}$ **(4)** $\frac{1}{3}$

Ans. **Option (2) is correct.**

Explanation: We have,

$$P(A \cup B) = \frac{7}{12},\ P(A) = \frac{5}{12},\ P(B) = \frac{3}{12}$$

$\because$ $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = \frac{5}{12} + \frac{3}{12} - \frac{7}{12}$$

$$= \frac{8}{12} - \frac{7}{12}$$

$$= \frac{1}{2}$$

14. The probability distribution of X is:

x	0	1	2	3	4
P(X = x)	0.1	$2k$	k	k	$2k$

Then var(X) =

(1) $\frac{3}{20}$ **(2)** $\frac{9}{4}$

(3) $\frac{141}{20}$ **(4)** $\frac{159}{80}$

Ans. **Option (4) is correct.**

Explanation: Since,

$$\Sigma P(X) = 1$$

$$\therefore 0.1 + 2k + k + k + 2k = 1$$

$$6k = 1 - 0.1$$

$$k = \frac{0.9}{6} = 0.15$$

So, distribution is:

x	0	1	2	3	4	Total
$P(X = x)$	0.1	0.30	0.15	0.15	0.30	
$x_i P(X)$	0	0.30	0.30	0.45	1.20	2.25
$x_i^2 P(X)$	0	0.30	0.60	1.35	4.80	7.05

$$\textbf{Var}(X) = \Sigma x_i^2 P(X) - (\Sigma x_i P(X))^2$$

$$= 7.05 - (2.25)^2$$

$$= 7.05 - 5.0625$$

$$= 1.9875$$

$$= \frac{19875}{1000} = \frac{159}{80}$$

15. The maximum value of $z = 4x + 2y$ subject to constraints $2x + 3y \leq 28$, $x + y \leq 10$, $x, y \geq 0$ is:

(1) 36 **(2)** 40

(3) $\frac{100}{3}$ **(4)** 32

Ans. **Option (2) is correct.**

Explanation: Given LPP is

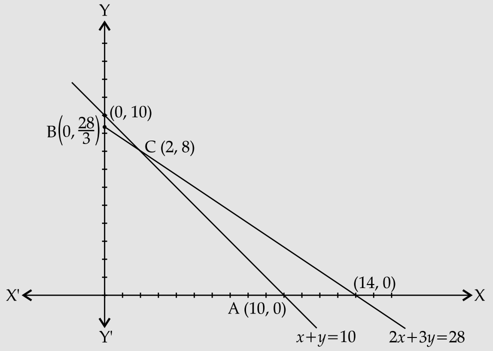
$$\text{Max } z = 4x + 2y$$

Subject to constraints:

$$2x + 3y \leq 28$$

$$x + y \leq 10$$

$$x, y \geq 0$$



Corner Points	Value of $z = 4x + 2y$
$A(10, 0)$	$z = 40 \rightarrow$ Maximum
$B\left(0, \frac{28}{3}\right)$	$z = \frac{56}{3}$
C(2, 8)	$z = 24$

Section - B1

Applied Mathematics

16. The number at unit place of number 17^{123} is:

(1) 1 **(2)** 3
(3) 7 **(4)** 9

Ans. **Option (4) is correct.**

Explanation: $17^{123} = 17^{17\times7+4}$

Unit's digit of 17^{123} = unit digit of 17^4
= unit digit of 17^{2+2}
= unit digit of 17^2
= unit digit of 289 = 9

17. Match List I with List II

LIST I		LIST II	
A.	$3^3 \equiv b(\text{mod } 9)$	I.	4
B.	$2^5 \equiv b(\text{mod } 15)$	II.	0
C.	$4^3 \equiv b(\text{mod } 10)$	III.	2
D.	$5^3 \equiv b(\text{mod } 12)$	IV.	5

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I **(2)** A-II, B-III, C-I, D-IV
(3) A-I, B-II, C-III, D-IV **(4)** A-III, B-I, C-IV, D-II

Ans. **Option (2) is correct.**

Explanation:
A. $3^3 \equiv b(\text{mod } 9)$
Here, $3^3 \equiv 0(\text{mod } 0)$
B. $2^5 \equiv b(\text{mod } 15)$
Here, $2^5 \equiv 2(\text{mod } 15)$
C. $4^3 \equiv b(\text{mod } 10)$
Here, $4^3 \equiv 4(\text{mod } 10)$
D. $5^3 \equiv b(\text{mod } 12)$
Here, $5^3 \equiv 5(\text{mod } 12)$
There, A-II, B-III, C-I, D-IV

18. A mixture contains milk and water in the ratio 8 : x. If 3 litres of water is added in 33 lites of mixture, the ratio of milk and water becomes 2 : 1, then value of x is:

(1) 3 Litres **(2)** 4 Litres
(3) 2 Litres **(4)** 11 Litres

Ans. **Option (1) is correct.**

Explanation: Ratio of milk and water in original mixture

i.e., M_1 Milk : Water
8 : x

After adding 3 litres of water in 33 litres of mixture the ratio of mixture is

M_2 Milk : Water
2 : 1
or 8 : 4

Since the ratio of milk does not change as we are adding water only. Thus, the ratio of water increases by 3(= 4 – 1).

19. A motorboat can travel in still water at the speed 15 km/h, while the speed of the current is 3 km/h. Time taken by boat to go 36 km upstream is:

(1) 2 hr **(2)** 3 hr
(3) 12 hr **(4)** 18 hr

Ans. **Option (2) is correct.**

Explanation: Speed of motor boat in still water = 15 km/h
Speed of current = 3 km/h
Speed of upstream = speed of motor boat – speed of current
= 15 – 3 km/h
= 12 km/h
Distance = speed × time

$\therefore$ Time $= \frac{36}{12} = 3$ hours

20. Hari covers 100 m distance in 36 seconds. Ram covers the same distance in 45 seconds. In a 100 m race, Hari ahead from Ram is

(1) 20 m **(2)** 30 m
(3) 25 m **(4)** 40 m

Ans. **Option (1) is correct.**

Explanation: Hari beats Ram by
= (45 – 36) seconds
= 9 seconds

Speed of Ram $= \frac{100}{45}$

Distance covered by Ram in 9 seconds

$= \frac{100}{45} \times 9$

= 20 m

So, Hari is 20 m ahead from Ram.

21. A pipe can empty $\left(\frac{5}{6}\right)^{\text{th}}$ part of a cistern in 20 minutes. The part of cistern which will be empty in 9 minutes is:

(1) $\frac{3}{5}$ **(2)** $\frac{3}{8}$
(3) $\frac{4}{5}$ **(4)** $\frac{5}{9}$

Ans. **Option (2) is correct.**

Explanation:

Emptied part in 20 minutes $= \frac{5}{6}$

Emptied part in 1 minutes $= \frac{5}{6\times20}$

Emptied part in 9 minutes $= \frac{5\times9}{6\times20} = \frac{3}{8}$

22. The system of linear inequalities $2x - 1 \geq 3$ and $x - 3 > 5$ has solution:

(1) $(2, \infty)$ **(2)** $(2, 8)$

(3) $(8, \infty)$ **(4)** $(-\infty, 8)$

Ans. **Option (3) is correct.**

Explanation:

$$2x - 1 \geq 3 \text{ and } x - 3 > 5$$
$$2x \geq 4 \text{ and } x > 8$$
$$x \geq 2 \text{ and } x > 8$$
$$x \in [2, \infty) \geq 2 \text{ and } x \in (8, \infty)$$

Therefore, $x \in (8, \infty)$

23. The value of x which satisfied $|3x| \geq |6 - 3x|$

(A) $(0, 1]$ **(B)** $[1, 4]$

(C) $(4, \infty)$ **(D)** $(-1, 0)$

(E) $(-\infty, 0)$

Choose the correct answer form the options given below:

(1) A and B only **(2)** C and E only

(3) B and C only **(4)** D and E only

Ans. **Option (3) is correct.**

Explanation:

$$|3x| \geq |6 - 3x|$$
$$(|3x|)^2 \geq (|6 - 3x|)^2$$
$$9x^2 \geq (6 - 3x)^2$$
$$9x^2 \geq 36 - 36x + 9x^2$$
$$0 \geq 36 - 36x$$
$$0 \geq 36(1 - x)$$
$$0 \geq 1 - x$$
$$x \geq 1$$
$$x \in [1, \infty)$$

Only options B and C lies in range of x.

24. If $\begin{bmatrix} x & y & z \\ 2 & u & v \\ -1 & 6 & w \end{bmatrix}$ is skew symmetric matrix, then value of $x^2 + y^2 + z^2 + u^2 + v^2 + w^2$ is:

(1) 1 **(2)** 4

(3) 36 **(4)** 41

Ans. **Option (4) is correct.**

Explanation:

Let, $A = \begin{bmatrix} x & y & z \\ 2 & u & v \\ -1 & 6 & w \end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix} x & 2 & -1 \\ y & u & 6 \\ z & v & w \end{bmatrix}$

Since, A is skew-symmetric i.e., $A = -A'$

$$\therefore \begin{bmatrix} x & y & z \\ 2 & u & v \\ -1 & 6 & w \end{bmatrix} = -\begin{bmatrix} x & 2 & -1 \\ y & u & 6 \\ z & v & w \end{bmatrix}$$

On comparing matrices, we get

$$y = -2, z = +1, v = -6$$
$$y = -x \Rightarrow 2x = 0 \Rightarrow x = 0$$
$$u = -u \Rightarrow 2u = 0 \Rightarrow u = 0$$
$$w = -w \Rightarrow 2w = 0 \Rightarrow w = 0$$
$$\therefore x^2 + y^2 + z^2 + u^2 + v^2 + w^2$$
$$= 0^2 + (-2)^2 + (1)^2 + 0^2 + (-6)^2 + (0)^2$$
$$= 4 + 1 + 36 = 41$$

25. If $y = e^{nx}$, then n^{th} derivative of y is:

(1) e^{nx} **(2)** n^2e^{nx}

(3) ny **(4)** n^ny

Ans. **Option (4) is correct.**

Explanation: Given,

$$y = e^{nx}$$
$$y' = ne^{nx} = ny$$
$$y'' = n^2e^{nx} = n^2y$$
$$y'' = n^3e^{nx} = n^3y$$

n^{th} derivative of $y(y_n) = n^ne^{nx} = n^ny$

26. The total revenue (in Rs.) received by selling 'x' units of a certain products is given by: $R(x) = 4x^2 + 10x + 3$.

What is the marginal revenue on selling 20 such units?

(1) ₹130 **(2)** ₹170

(3) ₹173 **(4)** ₹360

Ans. **Option (2) is correct.**

Explanation: Given, $R(x) = 4x^2 + 10x + 3$

Marginal Revenue, $\dfrac{dR(x)}{dx} = 8x + 10$

Marginal Revenue, $\left.\dfrac{dR(x)}{dx}\right|_{at\, x=20}$

$$= 8 \times 20 + 10$$
$$= 160 + 10$$
$$= ₹170$$

27. If x is a real, then minimum value of $x^2 - 8x + 17$ is:

(1) -1 **(2)** 0

(3) 1 **(4)** 2

Ans. **Option (3) is correct.**

Explanation: Let,

$$f(x) = x^2 - 8x + 17$$

$\therefore$ $f'(x) = 2x - 8$

For minimum value, put $f'(x) = 0$

$\therefore$ $2x - 8 = 0$

$\Rightarrow$ $x = 4$

Now, $f''(x) = 2 > 0$

So, $f(x)$ is minium at $x = 4$

Hence, value of $f(x)$ at $x = 4$ is

$$f(4) = (4)^2 - 8(4) + 17$$
$$= 16 - 32 + 17$$
$$= 33 - 32$$
$$= 1$$

28. If μ is mean of random variable X, with probability distribution

x	0	1	2
$P(X = x)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

then value of $9\mu + 4$ is:

(1) 4 **(2)** 9

(3) 10 **(4)** 17

Ans. Option (3) is correct.

Explanation:

x	$P(X = x)$	$x.P(X = x)$
0	$\frac{4}{9}$	0
1	$\frac{4}{9}$	$\frac{4}{9}$
2	$\frac{1}{9}$	$\frac{2}{9}$
Total		$\Sigma x.P(X = x) = \frac{6}{9}$

Since, **Mean(μ)** $= \Sigma x.P(X = x)$

$\therefore$ $\mu = \frac{6}{9}$

Now, $9\mu + 4 = 9.\frac{6}{9} + 4 = 6 + 4 = 10$

29. In a game, a child will win ₹5 if he gets all heads or all tails when three coins are tossed simultaneously and he will lose ₹3 for all other cases. The expected amount to lose in the game is

(1) ₹0 **(2)** ₹0.8

(3) ₹1 **(4)** ₹2

Ans. Option (3) is correct.

Explanation: Let X be the amount received by the person. Then X can take values 5 and –3 such that

$P(X = 5)$: Probability of getting all heads or all tails when three coins are tossed

$$P(X = 5) = \frac{2}{8} = \frac{1}{4}$$

$P(X = -3)$: Probability of getting one or two heads

$$\therefore \quad P(X = -3) = \frac{6}{8} = \frac{3}{4}$$

Therefore, expected amount to win, on the average, per game

$$= \overline{X} = \Sigma|p_i x_i$$

$$= 5 \times \frac{1}{4} + (-3) \times \frac{3}{4}$$

$$= \frac{5}{4} - \frac{9}{4}$$

$$= -1$$

Thus, the person will, on average lose ₹1 per toss of the coin.

30. The Probability mass functions of Random variable X is:

$P(X = x) = (0.6)^x(0.4)^{1-x}$, $x = 0, 1$ The variance of X is:

(1) 0.60 **(2)** 0.124

(3) 0.244 **(4)** 0.240

Ans. Option (4) is correct.

Explanation: Given,

$$P(X = x) = (0.6)^x(0.4)^{1-x}, x = 0, 1$$

Here, $p = 0.6, q = 0.4, n = 1$

Variance $= npq$

$= 1 \times 0.6 \times 0.4$

$= 0.24$

31. Match List I with List II

LIST I		LIST II	
A.	Quantity index	I.	Measures relative price change over a period of time.
B.	Time series	II.	Measures change in quantity of consumption of goods over a specific period of time.
C.	Price index	III.	Measures average value of goods for specific time period.
D.	Value index	IV.	Statistical observation taken at different points of time for specific period of time.

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV **(2)** A-II, B-III, C-I, D-IV

(3) A-III, B-IV, C-I, D-II **(4)** A-II, B-IV, C-I, D-III

Ans. Option (4) is correct.

32. Given that, $\Sigma p_0 q_0 = 700$, $\Sigma p_0 q_1 = 1450$, $\Sigma p_1 q_0 = 855$ and $\Sigma p_1 q_1 = 1300$.

Where subscripts 0 and 1 are used for base year and current year respectively. The Laspeyer's price index number is:

(1) 118.46 **(2)** 119.35

(3) 120.23 **(4)** 122.14

Ans. Option (4) is correct.

Explanation: Laspeyre's price index

$$= \frac{\Sigma p_i q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{855}{700} \times 100$$

$$= 122.14$$

33. If $y = a + b(x - 2005)$ fits the time series date

x(year) :	2003	2004	2005	2006	2007
y (yield in tons) :	6	13	17	20	14

Then the value of $a + b$ is:

(1) 16 **(2)** 20.3

(3) 43 **(4)** 80.3

Ans. **Option (2) is correct.**

Explanation:

x(year)	y(yield in tons)	$X = x_i - 2005$	X^2	Xy
2003	6	–2	4	–12
2004	13	–1	1	–13
2005	17	0	0	0
2006	20	1	1	20
2007	24	2	4	48
	$\Sigma y = 80$	$\Sigma X = 0$	$\Sigma X^2 = 10$	$\Sigma Xy = 23$

Now, $a = \frac{\Sigma y}{n} = \frac{80}{5} = 18 \quad (n = 5)$

$b = \frac{\Sigma Xy}{\Sigma X^2} = \frac{23}{10} = 2.3$

$\therefore \quad a + b = 18 + 2.3 = 20.3$

34. Which of the following statements are correct?

(A) If discount rate > coupon rate, then present value of a bond > face value

(B) An annuity in which the periodic payment begins on a fixed date and continues forever is called perpetuity

(C) The issuer of bond pays interest at fixed interval at fixed rate of interest to investor is called coupon payment

(D) A sinking fund is a fixed payment made by a borrower to a lender at a specific date every month to clear off the loan

(E) The issues of bond repays the principle i.e., face value of the bond to the investor at a later date termed as maturity date

Choose the correct answer from the options given below:

(1) (A), (C), (E) only **(2)** (A), (B), (D) only
(3) (B), (C), (E) only **(4)** (A), (B), (C) only

Ans. **Option (3) is correct.**

35. Which of the following statements is true?

(A) EMI in flat rate method,

$$\text{EMI} = \frac{\text{Principle + Interest}}{\text{Number of Payment}}$$

(B) EMI in reducing balance method,

$$\text{EMI} = P \times \frac{i}{1 + (1+i)^n}$$

where P = Principle, i = interest rate, n = no. of payments

(C) In sinking fund, a fixed amount at regular intervals is deposited.

(D) Approximate Yield to Maturity

$$= \frac{\text{Coupen Payment} + \frac{\text{Face Value + Present Value}}{\text{Number of Payment}}}{\frac{\text{Face Value + Present Value}}{2}}$$

Choose the correct answer from the options given below:

(1) (A) and (B) only **(2)** (B) and (C) only
(3) (A) and (C) only **(4)** (C) and (D) only

Ans. **Option (3) is correct.**

Explanation: Option B, EMI $= \frac{P \times i}{1-(1+i)^{-n}}$

where, P = Principle, i = interest rate
n = no. of payments

Option D, Approx. YTM $= \frac{c + \{(F - P.V.)/N\}}{\frac{(F + P.V.)}{2}}$

Where F is face value
c is coupon rate
$P.V.$ is present value
N is no. of payments

36. Mr. Dev wishes to purchase an AC for ₹45,000 with a down payment of ₹5000 and balance in EMI for 5 years. If Bank charges 6% per annum compounded monthly then monthly EMI is:

$\left(\text{use } \frac{0.005}{1-(1.005)^{-60}} = 0.0194\right)$

(1) ₹776 **(2)** ₹700
(3) ₹737 **(4)** ₹673

Ans. **Option (1) is correct.**

Explanation: Given,

Cost of AC = ₹45,000
Down payment = ₹5000
$\therefore$ Balance = ₹40,000
So, P = ₹40,000,

$i = \frac{6}{12 \times 100} = 0.005$,

$n = 5 \times 12 = 60$

$$\text{EMI} = \frac{P \times i}{1-(1+i)^{-n}}$$

$$= \frac{40{,}000 \times 0.005}{1-(1+0.005)^{-60}}$$

$$= 40{,}000 \times \frac{0.005}{1-(1.005)^{-60}}$$

$$= 40{,}000 \times 0.0194$$

$\left[\text{Given, } \frac{0.005}{1-(1.005)^{-60}} = 0.0194\right]$

= ₹776

37. The cost of a machine is ₹20,000 and its estimated useful life is 10 years. The scrap value of the machine, when its value depreciates at 10% p.a, is: use $(0.9)^{10} = 0.35$

(1) ₹9672 **(2)** ₹7000
(3) ₹6982 **(4)** ₹3500

Ans. **Option (2) is correct.**

> *Explanation:* Amount on depreciation
>
> $= \text{Original amount}\left(1-\frac{r}{100}\right)^n$
>
> $= 20000 \times \left(1-\frac{10}{100}\right)^{10}$
>
> $= 20000 \times (0.9)^{10}$
>
> $= 20000 \times 0.35$ [Given $(0.9)^{10} = 0.35$]
>
> $= ₹70000$

38. One of the following is true of relation between sample mean ($\bar{x}$) and population mean (μ).

(1) $|\bar{x} - \mu|$ increases when increases the size of sample

(2) $\bar{x} = \mu$, for all sample sizes

(3) $|\bar{x} - \mu|$ do not change with size of sample

(4) $|\bar{x} - \mu|$ decreases when increases the size of sample

Ans. **Option (2) is correct.**

39. Below are the stages for Drawing statistical inferences.

(A) Sample **(B)** Population

(C) Making Inference **(D)** Data tabulation

(E) Data Analysis

Choose the correct answer from the options given below:

(1) (B), (D), (A), (E), (C) **(2)** (A), (B), (D), (C), (E)

(3) (B), (A), (D), (E), (C) **(4)** (D), (B), (A), (C), (E)

Ans. **Option (3) is correct.**

40. Corner points of the feasible region for an LPP, are (0, 2), (3, 0), (6, 0) and (6, 8). If $z = 2x + 3y$ is the objective function of LPP then max.(z) – min.(z) is equal to:

(1) 30 **(2)** 24

(3) 21 **(4)** 9

Ans. **Option (1) is correct.**

Explanation:

Corner point	Value of $Z = 2x + 5y$
(0, 2)	$Z = 6 \rightarrow$ Minimum
(3, 0)	$Z = 6$
(6, 0)	$Z = 12$
(6, 8)	$Z = 36 \rightarrow$ Maximum

$\therefore$ **max(≥) – min(≥) = 36 – 6 = 30**

Read the below passage and solve the questions from Q.No. 41-Q.No.45.

Passage

Sitaram, a money lender lent a part of ₹200000 to Shyam at simple interest 6% p.a. and the remaining to Sushil at 10% p.a. at simple interest. Sitaram earned an annual interest income of ₹18000. Based on the given information answer the following question:

41. What is the mean rate of interest?

(1) 6% p.a. **(2)** 8% p.a.

(3) 9% p.a. **(4)** 16% p.a.

Ans. **Option (3) is correct.**

> *Explanation:* Let part of money given as a loan to shyam be ₹x.
>
> $x \times \frac{6}{100} = A_1$ where A_1 is income earned on Shyam's money
>
> Similarly
>
> $(2{,}00{,}000 - x) \times \frac{10}{100} = A_2$
>
> where A_2 is income earned on Sushil's money
>
> Given, $A_1 + A_2 = 18{,}000$
>
> $\therefore 0.06x + (2{,}00{,}000 - x) \times 0.1 = 18{,}000$
>
> $0.06x - 0.1x + 20000 = 18000$
>
> $0.04x = 2000$
>
> $x = ₹50{,}000$
>
> (part of money given to Shyam)
>
> Therefore, part of money given to Shushil
>
> = ₹1,50,000
>
> Mean interest rate
>
> $= \frac{50{,}000 \times 6 + 1{,}50{,}000 \times 10}{2{,}00{,}000}$
>
> $= \frac{3{,}00{,}000 + 1{,}5{,}00{,}000}{2{,}00{,}000} = 9\%$

42. In what ratio did Sitaram lent the money at 6% p.a. and 10% p.a. respectively?

(1) 1 : 3 **(2)** 3 : 1

(3) 2 : 3 **(4)** 3 : 5

Ans. **Option (1) is correct.**

> *Explanation:*
>
> Money lent to Shyam = ₹50,000
>
> Money lent to Shushil = ₹1,50,000
>
> [from Q. 41]
>
> Ratio = 50,000 : 1,50,000 = 1:3

43. How much money did Shyam borrow?

(1) ₹150000 **(2)** ₹75000

(3) ₹50000 **(4)** ₹12000

Ans. **Option (3) is correct.**

> *Explanation:* See Q. 41

44. What amount of money is lent at 10% p.a. simple interest?

(1) ₹20,000 **(2)** ₹50,000

(3) ₹75,000 **(4)** ₹1,50,000

Ans. **Option (4) is correct.**

> *Explanation:* Money is lent at 10% p.a. Money lent to Shushil
>
> = 1,50,000
>
> [from Q.41]

45. What is the ratio of the interest paid by Shyam and Sushil respectively

(1) 1 : 3 **(2)** 1 : 5
(3) 3 : 5 **(4)** 2 : 3

Ans. **Option (2) is correct.**

Explanation: Amount of interest paid by Shyam $= 50,000 \times \frac{6}{100} = ₹3000$

Amount of interest paid by Shushil

$= 1,50,000 \times \frac{10}{100} = ₹15,000$

So, ratio of interest paid = 3000 : 15,000
= 1:5

Read the below passage and solve the questions from Q.No. 46–Q.No.50.

Passage

Item are based on the information below:

A cable network provider in a small town has 500 subscribes and he used to collect ₹300 per month from each subscribes. He proposes to increase the monthly charges and it is believed from the past experiences that for every increase of ₹1, one subscribes will discontinue the service. Based on the above in formation, answer the following question:

46. If ₹x is the monthly increase in subscription amount, then the number of subscribes are

(1) x **(2)** $500 - x$
(3) $x - 500$ **(4)** 500

Ans. **Option (2) is correct.**

Explanation: If ₹x is monthly increase in subscription amount, then x subscribers will discontinue the service.

So, no. of subscriber = $500 - x$

47. Total revenue 'R' is given by (in Rs.)

(1) $R = 300x + 300(500 - x)$
(2) $R = (300 + x)(500 + x)$
(3) $R = (300 + x)(500 - x)$
(4) $R = 300x + 500(x + 1)$

Ans. **Option (3) is correct.**

Explanation: Total revenue of cable network provider after the increment is given by

$$R(x) = (500 - x)(300 + x)$$

48. The number of subscribes which gives the maximum revenue is

(1) 100 **(2)** 200
(3) 300 **(4)** 400

Ans. **Option (4) is correct.**

Explanation: Since,

$$R(x) = (500 - x)(300 - x)$$
$$= 1,50,000 + 500x - 300x - x^2$$
$$= -x^2 + 200x + 1,50,000$$

$\therefore$ $R'(x) = -2x + 200$

Put $R'(x) = 0$, we get

$-2x + 200 = 0$

$x = 100$

Now, $R''(x) = -2 < 0$ maximum

So, $R(x)$ is maximum when $x = 100$.

Hence, no. of subscribes which given the maximum revenue = 500 – 100 = 400

49. What is increase in changes per subscribes that yields maximum revenue?

(1) 100 **(2)** 200
(3) 300 **(4)** 400

Ans. **Option (1) is correct.**

Explanation: Increase in change per subscriber $= x = 100$

50. The maximum revenue generated is

(1) ₹200000 **(2)** ₹180000
(3) ₹160000 **(4)** ₹150000

Ans. **Option (3) is correct.**

Explanation: $R(100) = (500 - 100)(300 + 100)$
$= 400 \times 400$
$= ₹1,60,000$

Section - B2

Mathematics

51. Match List I with List II

LIST I		LIST II	
A.	$R = \{(x, y): x$ and y are student of the same school$\}$	I.	Symmetric
B.	$R = \{(L_1, L_2) : L_1 \perp L_2, L_1, L_2 \in L$, where L is a set of all lines$\}$	II.	one-one
C.	A function $f : R \to R$ defined $f(x) = 2 - 3x$ is	III.	bijective
D.	A function $f[0, 1] \to R$ defined by $f(x) = 1 + x^2$ is $f : [0, 1] \to R$	IV.	Equivalence

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-II, D-III **(2)** A-IV, B-I, C-III, D-II
(3) A-I, B-IV, C-III, D-II **(4)** A-IV, B-I, C-II, D-III

Ans. **Option (2) is correct.**

Explanation:

A. $R = \{(x, y): x$ and y are students of the same school$\}$ is an equivalence relation.

B. $R\{(L_1, L_2): L_1 \perp L_2, L_1, L_2 \perp L$ where L is a set of all lines$\}$ only follows symmetric relation.

C. A function $f : R \to R$ such that $f(x) = 2 - 3x$ is one-one onto relation. Hence, bijective

D. A function $f : [0, 1] \to R$ such that $f(x) = 1 + x^2$ is one-one.

52. The value of $\tan^{-1}\left[2\sin\left\{2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$, is:

(1) $\frac{\pi}{3}$ **(2)** $\frac{2\pi}{3}$

(3) $-\frac{\pi}{3}$ **(4)** $\frac{\pi}{6}$

Ans. Option (1) is correct.

Explanation: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$= \tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right]$

$= \tan^{-1}\left[2\sin\left(2\left(\frac{\pi}{6}\right)\right)\right]$

$= \tan^{-1}\left[\left(2\times\sin\frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left[2\times\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\sqrt{3}$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

53. Match List I with List II

LIST I		LIST II	
A.	The range of $\sin^{-1}x$ is	I.	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$
B.	The range of $\tan^{-1}x$ is	II.	$\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
C.	The range of $\text{cosec}^{-1}x$ is	III.	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
D.	The range of $\sec^{-1}x$ is	IV.	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I **(2)** A-IV, B-I, C-III, D-II

(3) A-I, B-IV, C-II, D-III **(4)** A-IV, B-II, C-I, D-III

Ans. Option (4) is correct.

54. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then:

(1) A'A = I **(2)** A'A = 0

(3) A'A = 2I **(4)** A'A = –I

Ans. Option (1) is correct.

Explanation: $A = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$AA' = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}\cos^2\alpha+\sin^2\alpha & -\cos\alpha\sin\alpha+\cos\alpha\sin\alpha\\ -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$

55. If $A = \begin{bmatrix}2 & 1 & 0\\ 3 & 1 & 2\\ 0 & 4 & -1\end{bmatrix}$ then $|\text{adj}(A)|$ is equal to

(1) 11 **(2)** 12

(3) 225 **(4)** –225

Ans. Option (3) is correct.

Explanation: **Given**

$A = \begin{bmatrix}2 & 1 & 0\\ 3 & 1 & 2\\ 0 & 4 & -1\end{bmatrix}$

$|A| = 2(-1-8)-1(-3)$
$= -18+3$
$= -15 \neq 0$

Since, $|\text{adj } A| = |A|^{n-1}$

where n = order of matrix

Therefore, $|\text{adj } A| = |-15|^{3-1}$
$= (-15)^2$
$= 225$

56. Identify the correct option (s)

(A) A modulus function is continuous at every point in its domain.

(B) A modulus function may or may not be continuous at every point in its domain.

(C) Every rational function is continuous in its domain.

(D) If a function f is differentiable at a point then it is also continuous at that point.

(E) If a function f is continuous at a point then it is also differentiable at that point.

Choose the correct answer from the options given below:

(1) (A) and (C) only **(2)** (B) and (E) only

(3) (A), (C) and (D) only **(4)** (C) and (E) only

Ans. Option (3) is correct.

Explanation: E is incorrect as Differentiability $\Rightarrow$ continuity, But converse is not true.
B is incorrect as modules function is continuous at every point in its domain.

57. If $f(x) = \begin{cases} \dfrac{\sqrt{1-\cos 2x}}{x\sqrt{2}}, & x \neq 0 \\ k, & x = 0 \end{cases}$, then the value of k will make function f continuous at $x = 0$ is:

(1) 1 **(2)** –1
(3) 0 **(4)** No value

Ans. **Option (1) is correct.**

Explanation: **Since, $f(x)$ is continuous at $x = 0$**

$$\Rightarrow \quad k = \lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{x\sqrt{2}}$$
$$= \lim_{x\to 0} \frac{\sqrt{1-(1-2\sin^2 x)}}{x\sqrt{2}}$$
$$= \lim_{x\to 0} \frac{\sqrt{2}\sin x}{x\sqrt{2}}$$
$$= \lim_{x\to 0} \frac{\sin x}{x}$$
$$= 1$$

58. If $y = \log(\sec e^{x^2})$, then $\dfrac{dy}{dx} =$

(1) $x^2e^{x^2}\tan e^{x^2}$ **(2)** $e^{x^2}\tan e^{x^2}$
(3) $2xe^{x^2}\tan e^{x^2}$ **(4)** $xe^{x^2}\tan e^{x^2}$

Ans. **Option (3) is correct.**

Explanation: **Given,**

$$y = \log(\sec e^{x^2})$$
$$\frac{dy}{dx} = \frac{1}{\sec(e^{x^2})}(\sec e^{x^2}\tan e^{x^2}).(e^{x^2}).2x$$

[Using chain rule of differentiation]

$$= 2xe^{x^2}.\tan e^{x^2}$$

59. If $y = e^{\log\sin^{-1}x} + e^{\log\cos^{-1}x}, 0 < x < 1$, then

(1) $\dfrac{dy}{dx} = 0$ **(2)** $\dfrac{dy}{dx} = \dfrac{\pi}{2}$

(3) $\dfrac{dy}{dx} = \dfrac{\pi}{3}$ **(4)** does not exist

Ans. **Option (1) is correct.**

Explanation:

$y = e^{\log\sin^{-1}x} + e^{\log\cos^{-1}x}, 0 < x < 1$

$y = \sin^{-1}x + \cos^{-1}x, 0 < x < 1$

$y = \dfrac{\pi}{2}$ $\left[\because \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}\right]$

$\therefore \dfrac{dy}{dx} = 0$

60. Match List I with List II

LIST I		LIST II	
A.	$\displaystyle\int \frac{1}{x+\sqrt{x}}dx$	I.	$2\sqrt{x}+C$
B.	$\displaystyle\int \frac{e^{\log\sqrt{x}}}{x}dx$	II.	$2(\sqrt{x}-1)e^{\sqrt{x}}+C$
C.	$\displaystyle\int \frac{dx}{4x^2-9}$	III.	$2\log(\sqrt{x}+1)+C$
D.	$\displaystyle\int e^{\sqrt{x}}dx$	IV.	$\dfrac{1}{12}\log\left\|\dfrac{2x-3}{2x+3}\right\|+C$

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III **(2)** A-III, B-II, C-IV, D-I
(3) A-III, B-I, C-IV, D-II **(4)** A-I, B-II, C-III, D-IV

Ans. **Option (3) is correct.**

Explanation: **(A)** $I_1 = \displaystyle\int \frac{1}{x+\sqrt{x}}dx$

put $\sqrt{x} = t$

$\Rightarrow \dfrac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow dx = 2tdt$

$\therefore I_1 = \displaystyle\int \frac{2tdt}{t^2+t}$

$= 2\displaystyle\int \frac{1}{t+1}dt$

$= 2\log(t + 1) + c$

$\therefore I_1 = 2\log|\sqrt{x}+1|+c$

(B) Let $I_2 = \displaystyle\int \frac{e^{\log\sqrt{x}}}{x}dx$

$= \displaystyle\int \frac{\sqrt{x}}{x}dx$

$= \displaystyle\int x^{-\frac{1}{2}}dx$

$= \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$

$= \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$

$= 2\sqrt{x}+c$

(C) Let $I_3 = \displaystyle\int \frac{dx}{4x^2-9}$

$= \dfrac{1}{4}\displaystyle\int \frac{dx}{x^2-\frac{9}{4}}$

$= \dfrac{1}{4}\displaystyle\int \frac{dx}{x^2-\left(\frac{3}{2}\right)^2}$

$= \dfrac{1}{4}\dfrac{1}{2.\left(\frac{3}{2}\right)}\log\left|\dfrac{x-\frac{3}{2}}{x+\frac{3}{2}}\right|+c$

$$\left[\text{Using } \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$= \frac{1}{12}\log\left|\frac{2x-3}{2x+3}\right| + c$$

(D) Let $I_4 = \int e^{\sqrt{x}}dx$

Put $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow dx = 2tdt$

$\therefore I_4 = \int e^t(2t)dt$

$= 2\int te^t dt$

$= 2\left[te^t - \int e^t dt\right]$

[Integration by parts]

$= 2[te^t - e^t] + c$

$= 2e^t(t-1) + c$

$= 2e^{\sqrt{x}}(\sqrt{x}-1) + c$

61. The order of the differential equation whose general solution is $y = e^x(a\cos x + b\sin x)$, where a and b are arbitrary constants is:

(1) 1 **(2)** 3

(3) 2 **(4)** 6

Ans. **Option (3) is correct.**

Explanation: General solution is

$y = e^x(a\cos x + b\sin x)$ ...(i)

$y' = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$

or, $y' = y - ae^x\sin x + be^x\cos x$

or, $y' = y + e^x(-a\sin x + b\cos x)$ ...(ii)

Differentiating again w.r.t. x

$\frac{d^2y}{dx^2}$

$= \frac{dy}{dx} + e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x)$

$= \frac{dy}{dx} + e^x(-a\sin x + b\cos x) - y$ ...(iii)

Subtracting eq. (ii) from eq. (iii), we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = \frac{dy}{dx} - 2y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Thus, order of differential equation is 2.

62. $\frac{d}{dx}\left[\int_0^{2a} f(\sin 2x)dx\right] =$

(1) $2a$ **(2)** $f(\sin 2a)$

(3) $f(\cos 2a)$ **(4)** 0

Ans. **Option (4) is correct.**

63. $\int \tan x(\sec x - \tan x)dx =$

(1) $\sec x - \tan x + x - C$

(2) $\sec x - \tan^2 x + C$

(3) $\sec x + \tan x + x + C$

(4) $\sec x - \tan x + C$

Ans. **Option (1) is correct.**

Explanation: Let

$I = \int \tan x(\sec x - \tan x)dx$

$= \int \tan x \sec x dx - \int \tan^2 x dx$

$= \sec x - \left\{\int (\sec^2 - 1)dx\right\}$

$= \sec x - \left\{\int (\sec^2 x dx - \int dx\right\}$

$= \sec x - [\tan x - x + c]$

$= \sec x - \tan x + x - c$

64. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of vector $\vec{a}$, then value of $\cos 2\alpha^+ \cos 2\beta^+ \cos 2\gamma$ is equal to:

(1) 3 **(2)** 0

(3) 2 **(4)** –1

Ans. **Option (4) is correct.**

Explanation: Since, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are direction cosines of vector $\vec{a}$, then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Now, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$= (2\cos^2\alpha - 1) + (2\cos^2\beta - 1) + (2\cos^2\gamma - 1)$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2 \times 1 - 3$

$= 2 - 3$

$= -1$

65. The value of $i.(\hat{j}\times\hat{k}) + j.(\hat{i}\times\hat{k}) + k.(\hat{i}\times\hat{j})$ is

(1) 0 **(2)** –1

(3) 1 **(4)** 3

Ans. **Option (4) is correct.**

Explanation: We have,

$\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$

$= \hat{i}.\hat{i} + \hat{j}.\hat{j} + \hat{k}.\hat{k}$

$= 1 + 1 + 1$

$= 3$

66. The corner points of the feasible region for an L.P.P are (2, 0), (7, 0), (4, 5) and (0, 3) and $z = 2x + 3y$ is the objective function. The difference of the maximum and minimum value of z is:

(1) 19 **(2)** 4

(3) 23 **(4)** 14

Ans. **Option (1) is correct.**

Explanation:

Corner Points	Value of $z = 2x + 3y$
(2, 0)	$z = 4 \rightarrow$ min
(7, 0)	$z = 14$
(4, 5)	$z = 23 \rightarrow$ max
(0, 3)	$z = 9$

$$\text{Difference} = z_{max} - z_{min} = 23 - 14 = 19$$

67. The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is

(1) 3 **(2)** $\sqrt{2}$

(3) 4 **(4)** $\sqrt{3}$

Ans. **Option (4) is correct.**

Explanation: Area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is given by $|\vec{a}\times\vec{b}|$

Here, $\vec{a} = \hat{i}+\hat{j}$ and $\vec{b} = 2\hat{i}+\hat{j}+\hat{k}$

So, $$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(1-0) - \hat{j}(1-0) + \hat{k}(1-2)$$

$$= \hat{i} - \hat{j} - \hat{k}$$

Thus, $$|\vec{a}\times\vec{b}| = \sqrt{1^2+(-1)^2+(-1)^2} = \sqrt{3}$$

68. If $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector then value of x is:

(1) $\pm\sqrt{3}$ **(2)** $\pm\frac{1}{3}$

(3) ± 3 **(4)** $\pm\frac{1}{\sqrt{3}}$

Ans. **Option (4) is correct.**

Explanation: Given

$$|x(\hat{i}+\hat{j}+\hat{k})| = 1$$

$$\Rightarrow |x||\hat{i}+\hat{j}+\hat{k}| = 1$$

$$\Rightarrow |x|\sqrt{1^2+1^2+1^2} = 1$$

$$\Rightarrow |x|\sqrt{3} = 1$$

$$\Rightarrow |x| = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

69. The point of intersection the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$, is:

(1) (1, 1, 1) **(2)** (1, –1, –1)

(3) (–1, 1, –1) **(4)** (–1, –1, –1)

Ans. **Option (4) is correct.**

Explanation: Let

Line 1 $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$$

$$\Rightarrow x = 2r + 1,$$
$$y = 3r + 2, z = 4r + 3 \quad ...(i)$$

Line 2 $$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$$

Substitute value of x, y and z in line 2, we get

$$\frac{2r+1-4}{5} = \frac{2r+2-1}{2} = 4r + 3$$

Equation first and last term,

$$\frac{2r+1-4}{5} = 4r + 3$$

$$\Rightarrow 2r - 3 = 20\,r + 15r$$
$$\Rightarrow 18r = -18$$
$$\Rightarrow r = -1$$

Required point of intersection, from eq. (i)

$$x = -1, y = -1 \text{ and } z = -1$$

70. The distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x + y + z = 17$, is

(1) 3 **(2)** 2

(3) 1 **(4)** 0

Ans. **Option (1) is correct.**

Explanation: Let the point of intersection of line

$$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$$

and the plane $x + y + z = 17$ be (x_0, y_0, z_0)

As (x_0, y_0, z_0) is the point of intersection of the line and plane, So it must satisfy both of the equations of line and plane

i.e., $$\frac{x_0-3}{1} = \frac{y_0-4}{2} = \frac{z_0-5}{2} = k \text{ (let)}$$

$$\frac{x_0-3}{1} = k, \frac{y_0-4}{2} = k, \frac{z_0-5}{2} = k$$

or, $$x_0 = k + 3, y_0 = 2k + 4,$$
$$z_0 = 2k + 5$$

Put these values in equation off plane, we get

$$(k + 3) + (2k + 4) + (2k + 5) = 17$$
$$5k + 12 = 17$$
$$5k = 5$$
$$k = 1$$

Thus, required point of intersection is

$$x_0 = 4, y_0 = 6, z_0 = 7$$

Now, distance between (3, 4, 5) and (4, 6, 7) is

$$d = \sqrt{(3-4)^2+(4-6)^2+(5-7)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3 \text{ units}$$

71. If events A and B are independent, then identify the correct statements

(A) A and B must be mutually exclusive

(B) The sum of their probabilities must be equal to 1

(C) P(A).P(B) = P(A ∩ B)

(D) A' and B' are also independent

Choose the correct answer from the options given below:

(1) (A) and (B) only **(2)** (B) and (C) only

(3) (C) and (D) only **(4)** (A) and (D) only

Ans. **Option (3) is correct.**

Explanation: **(A)** If two events are independent, they cannot be mutually exclusive.

(B) To test if probability of independent events is 1 or not.

Let A be the event of getting head

$$\therefore \quad P(A) = \frac{1}{2}$$

Let B be the event of getting 5 on a die

$$\therefore \quad P(B) = \frac{1}{6}$$

Here, A and B are independent events

Therefore, $P(A) + P(B) = \frac{1}{2}+\frac{1}{6} = \frac{4}{12}$

$$= \frac{1}{3} \neq 1$$

Thus, statement (B) is also incorrect.

72. The equation of plane passing through the point (0, 7, –7) and containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$, is:

(1) $x - y - z = 0$ **(2)** $4x + y + z = 0$

(3) $x + y + z = 0$ **(4)** $3x + 2y + 2z = 0$

Ans. **Option (3) is correct.**

Explanation: Any plane passing through (0, 7, –7) is

$a(x - 0) + b(y - 7) + c(z + 7) = 0$ ...(i)

If plane (i) contains thee given line, then it must passes through the point (–1, 3, –2) and must be parallel to the given line.

If (i) passes through (–1, 3, –2), then

$a(-1 - 0) + b(3 - 7) + c(-2 + 7) = 0$

$a + 4b - 5c = 0$ ...(ii)

If (i) is parallel to thee given line, then

$(-3)a + 2.b + 1.c = 0$

$-3a + 2b + c = 0$ ...(iii)

Solving eqs. (ii) and (iii), we get

$$\frac{a}{4+10} = \frac{b}{15-1} = \frac{c}{2+12}$$
$$\frac{a}{14} = \frac{b}{14} = \frac{c}{14}$$
$$\frac{a}{1} = \frac{b}{1} = \frac{c}{1} = k$$

$\therefore \quad a = k, b = k$ and $c = k$

Substituting these values in eq. (i), we get

$k(x - 0) + k(y - 7) + k(z + 7) = 0$

$kx + by + kz - 7k + 7k = 0$

$kx + by + kz = 0$

$k(x + y + z) = 0$

or $(x + y + z) = 0$

73. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ is equal to

(1) $\frac{4}{15}$ **(2)** $\frac{8}{45}$

(3) $\frac{1}{3}$ **(4)** $\frac{2}{9}$

Ans. **Option (4) is correct.**

Explanation:

$$P(A' \cap B') = P(A \cup B)'$$
$$= 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - [P(A) + P(B) - P(A).P(B)]$$

[∵ A and B are independent $P(A \cap B) = P(A).P(B)$]

$$= 1-\left[\frac{3}{5}+\frac{4}{9}-\frac{3}{5}\times\frac{4}{9}\right]$$
$$= 1-\left[\frac{27+20}{45}-\frac{12}{45}\right]$$
$$= 1-\left[\frac{47}{45}-\frac{12}{45}\right]$$
$$= 1-\frac{35}{45}$$
$$= \frac{10}{45}$$
$$= \frac{2}{9}$$

74. A line L: $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-1}{-1}$ is perpendicular to a plane (P), which passing through the point (4, 3, 9). If the mirror image of point 'S' on the line (L) in the given plane (P) is (2, 3, 1), then co-ordinates of point S, is:

(1) (1, 0, 3) **(2)** (0, –1, 3)

(3) (–2, –3, –1) **(4)** (4, 7, –1)

Ans. **Option (2) is correct.**

Explanation: The direction ratios of given line are $(\hat{i}, 2\hat{j}, -\hat{k})$

ATQ, Given line is perpendicular to the plane.

$\Rightarrow$ The direction ratios of plane are also $(\hat{i}, 2\hat{j}, -\hat{k})$

$\therefore$ The equation of plane passing through (4, 3, 9) is $a(x-4) + b(y-3) + c(z-9) = 0$

$1.(x-4) + 2(y-3) - 1(z-9) = 0$

$x + 2y - z - 4 - 6 + 9 = 0$

$x + 2y - z = 1$

75. A biased dice is thrown once. If X denotes the number appearing on it and have probability distribution:

x	1	2	3	4	5	6
P(X = x)	k	$\frac{k}{2}$	$2k$	$8k^2$	$1-5k$	$\frac{k}{2}$

where $k > 0$. Then consider the following statements:

(A) P(X = 3) **(B)** P(X ≤ 2)

(B) P(X ≥ 5) **(D)** P(X = 4)

(E) P(X = 1) + P(X = 5)

Choose the correct answer from the options given below:

(1) C > D > B > A > E **(2)** E > C > D > A > B

(3) E > C > A > B > D **(4)** C > E > A > B > D

Ans. Option (3) is correct.

Explanation: $\Sigma[P(X = x)] = 1$

$\Rightarrow k + \frac{k}{2} + 2k + 8k^2 + (1-5k) + \frac{k}{2} = 1$

$\Rightarrow 2k + k + 4k + 16k^2 + 2 - 10k + k = 2$

$\Rightarrow 16k^2 - 2k = 0$

$\Rightarrow 2k(8k-1) = 0$

$\Rightarrow k = 0 \text{ or } k = \frac{1}{8}$

$\Rightarrow k = \frac{1}{8}$ (Rejecting $k = 0$)

Probability Distribution is:

x	1	2	3	4	5	6
P(X = x)	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{16}$

(A) P(X = 3) = $\frac{1}{4}$

(B) P(X ≤ 2) = P(X = 1) + P(X = 2)

$= \frac{1}{8} + \frac{1}{16} = \frac{2+1}{16} = \frac{3}{16}$

(C) P(X ≥ 5) = P(X = 5) + P(X = 6)

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

(D) P(X = 4) = $\frac{1}{8}$

(E) P(X = 1) + P(X = 5) = $\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

Thus, E > C > A > B > D

Read the below passage and solve the questions from Q.No.76- Q.No.80.

Passage

In a school, a auditorium was used for its cultural activities. The shape of the floor of the auditorium is rectangular with dimensions x and $y(x > y)$. has fixed perimeter p.

76. If x and y represent the length and breadth of the rectangular region, then:

(1) $p = x + y$ **(2)** $p^2 = x^2 + y^2$

(3) $p = 2(x + y)$ **(4)** $p = x + 2y$

Ans. Option (3) is correct.

Explanation: Perimeter of rectangular region $= 2(x + y)$

$\therefore \quad p = 2(x + y) \quad$ ...(i)

77. The area (A) of the floor, as a function of x can be expressed as:

(1) $A(x) = px + \frac{x}{2}$ **(2)** $A(x) = \frac{px + x^2}{2}$

(3) $A(x) = \frac{px - 2x^2}{2}$ **(4)** $A(x) = \frac{x^2}{2} + px^2$

Ans. Option (3) is correct.

Explanation: Area off region (floor) is given by

$A = xy \quad$ ...(ii)

Since, $p = 2(x + y)$

$\Rightarrow \quad y = \frac{p-2x}{2} \quad$ ...(iii)

[from eq. (i) of Q. 76]

$\therefore \quad A(x) = x\left(\frac{p-2x}{2}\right)$

$\Rightarrow \quad A(x) = \frac{px - 2x^2}{2} \quad$ ...(iv)

78. The value of x, for which area of floor of auditorium is maximum is:

(1) $\frac{p}{4}$ **(2)** $\frac{p}{2}$

(3) p **(4)** $\frac{p}{3}$

Ans. Option (1) is correct.

Explanation: From eq. (iv) of Q. 77, we have

$A = \frac{px - 2x^2}{2}$

$$\therefore \quad \frac{dA}{dx} = \frac{1}{2}(p-4x)$$

Put $\quad \frac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{1}{2}(p-4x) = 0$$

$$\Rightarrow \quad x = \frac{p}{4}$$

Also, $\quad \frac{dA}{dx^2} = \frac{1}{2}(0-4) = -2 < 0$

Thus, area is maximum at $x = \frac{p}{4}$

79. The value of y, for which area of floor of auditorium is maximum is:

(1) $\frac{p}{2}$ (2) $\frac{p}{3}$

(3) $\frac{p}{4}$ (4) $\frac{p}{16}$

Ans. Option (3) is correct.

Explanation: From Q. 78, we have $x = \frac{p}{4}$

Substituting $x = \frac{p}{4}$ in eq. (iii) of Q. 77

We get

$$y = \frac{p-2x}{2} = \frac{p-2\times\frac{p}{4}}{2}$$

$$= \frac{p-\frac{p}{2}}{2} = \frac{p}{4}$$

$$\therefore \quad y = \frac{p}{4}$$

80. Maximum area of floor is:

(1) $\frac{p^2}{4}$ (2) $\frac{p^2}{16}$

(3) $\frac{p^2}{28}$ (4) $\frac{p^2}{64}$

Ans. Option (2) is correct.

Explanation: From Q. 78 and Q. 79, we have

$$x = \frac{p}{4} \text{ and } y = \frac{p}{4}$$

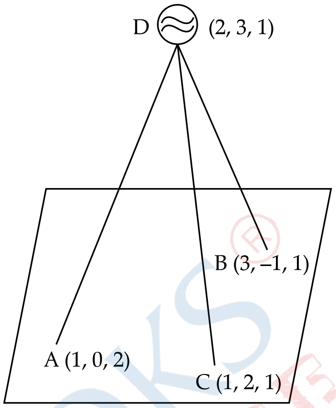
from eq. (ii) of Q. 77, we get

$$A = xy = \frac{p}{4}\times\frac{p}{4} = \frac{p^2}{16}$$

Read the below passage and solve the questions from Q.No. 81-Q.No.85.

Passage

A ball is thrown upwards from the plane surface of the ground. Suppose the plane surface from which the ball is thrown also consists of the points A(1, 0, 2), B(3, –1, 1) and C(1, 2, 1) on it. The highest point of the ball takes, is D(2, 3, 1) as shown in the figure, Using this information answer the questions.



81. The equation of the plane passing through the points A, B and C is:

(1) $3x - 2y + 4z = -11$ (2) $3x + 2y + 4z = 11$

(3) $3x - 2y - 4z = 11$ (4) $-3x + 2y + 4z = -11$

Ans. Option (2) is correct.

Explanation: The equation off plane passing through three non-collinear points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-0 & z-2 \\ 3-1 & -1-0 & 1-2 \\ 1-1 & 2-0 & 1-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y & z-2 \\ 2 & -1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(1+2) - y(-2+0) + (z-2)(4+0) = 0$$

$$\Rightarrow 3(x-1) + 2y + 4(z-2) = 11$$

82. The maximum heights of the ball from the ground is:

(1) $\frac{5}{\sqrt{29}}$ units (2) $\frac{7}{\sqrt{29}}$ units

(3) $\frac{6}{\sqrt{29}}$ units (4) $\frac{8}{\sqrt{29}}$ units

Ans. Option (1) is correct.

Explanation: Height of the ball = Perpendicular distance from point (2, 3, 1) to the plane $3x + 2y + 4z = 11$

So, $\left|\frac{6+6+4-11}{\sqrt{3^2+2^2+4^2}}\right| = \left|\frac{5}{\sqrt{29}}\right| = \frac{5}{\sqrt{29}}$ units

83. The equation of the perpendicular line drawn from the maximum height of the ball to the ground, is:

(1) $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-5}{-2}$

(2) $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z-1}{-4}$

(3) $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-1}{4}$

(4) $\frac{x+1}{-2}=\frac{y+3}{-1}=\frac{z-5}{2}$

Ans. **Option (3) is correct.**

Explanation: D.R.s of perpendicular are <3, 2, 4>

[Since, perpendicular is parallel to the normal to the plane]

Since, perpendicular is passing through the point (2, 3, 1), therefore its equation is

$$\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-1}{4}$$

84. The co-ordinates of the foot of the perpendicular drawn from the maximum height of the ball to the ground are

(1) $\left(\frac{43}{29},\frac{-77}{29},\frac{-9}{29}\right)$

(2) $\left(\frac{9}{7},\frac{-1}{7},\frac{-10}{7}\right)$

(3) $\left(\frac{43}{29},\frac{77}{29},\frac{9}{29}\right)$

(4) $\left(-\frac{13}{29},-\frac{7}{29},-\frac{19}{29}\right)$

Ans. **Option (3) is correct.**

Explanation: Let the coordinate of foot of perpendicular are $(3\lambda + 2, 2\lambda + 3, 4\lambda + 1)$

Since, this point lie on the plane $3x + 2y + 4z = 11$, therefore we get

$3(3\lambda + 2) + 2(2\lambda + 3) + 4(4\lambda + 1) = 11$

$9\lambda + 6 + 4\lambda + 6 + 16\lambda + 4 = 11$

$29\lambda + 16 = 11$

$29\lambda = 11 - 16$

$$\lambda = \frac{-5}{29}$$

Thus, coordinates of foot of perpendicular are

$$\left[\left(3\left(-\frac{5}{29}\right)+2\right),\left(2\left(-\frac{5}{29}\right)+3\right),\left(4\left(-\frac{5}{29}\right)+1\right)\right]$$

$$=\left[\left(\frac{-15}{29}+2\right),\left(\frac{-10}{29}+3\right),\left(\frac{-20}{29}+1\right)\right]$$

$$=\left(\frac{43}{29},\frac{77}{29},\frac{9}{29}\right)$$

85. The area of ΔABC is:

(1) $\sqrt{29}$ sq. units

(2) $\frac{1}{4}\sqrt{29}$ sq. units

(3) $\frac{1}{16}\sqrt{29}$ sq. units

(4) $\frac{1}{2}\sqrt{29}$ sq. units

Ans. **Option (4) is correct.**

Explanation: Clearly area of ΔABC

$$=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$$

$$=\frac{1}{2}|(2\hat{i}-\hat{j}-\hat{k})\times(0\hat{i}+2\hat{j}-\hat{k})|$$

$$=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & -1\\ 0 & 2 & -1\end{matrix}\right\|$$

$$=\frac{1}{2}|\hat{i}(1+2)-\hat{j}(-2+0)+\hat{k}(4+0)|$$

$$=\frac{1}{2}|3\hat{i}+2\hat{j}+4\hat{k}|$$

$$=\frac{1}{2}\sqrt{9^2+2^2+4^2}$$

$$=\frac{1}{2}\sqrt{29}\text{ sq. units}$$

●●●

CUET Question Paper 2021

National Testing Agency

UIQP02 23rd SEP 2021—SHIFT 2

MATHEMATICS/APPLIED MATHEMATICS

Solved

(*This includes Questions pertaining to Domain Specific Subject only*)

Max. Marks : 80 **Time allowed : 30 Min.**

General Instructions:

(i) *This paper consists of 25 MCQs, attempt any 20 out of 25.*
(ii) *Correct answer or the most appropriate answer: Four marks (+4) .*
(iii) *Any incorrect option marked will be given minus One mark (–1) .*
(iv) *Unanswered/Marked for Review will be given No mark (0) .*
(v) *If more than one option is found to be correct then Four marks (+4) will be awarded to only those who have marked any of the correct options .*
(vi) *If all options are found to be correct then Four marks (+4) will be awarded to all those who have attempted the question .*
(vii) *If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given Four marks (+4).*
(viii) *Calculator / any electronic gadgets are not permitted .*

1. Let P be any non-empty set containing p elements. Then, what is the number of relations on P?

(1) $2p$ **(2)** 2^{p^2}
(3) p^2 **(4)** p^p

Ans. Option (2) is correct.

Explanation: Any subset of PXP will be a relation, as we know that with p elements $2p$ subsets are possible, so these are number of relations on a set with p elements will be $2^{p\times p} = 2^{p^2}$

2. The domain of the function defined by $f(x) = \log_x 10$ is

(1) $x < 10$ excluding $x = -10$
(2) $x > 10$
(3) $x \geq 10$
(4) $x > 10$ excluding $x = 1$

Ans. Option (4) is correct.

3. Let det M denotes the determinant of the matrix M. Let A and B be 3 × 3 matrices with det A = 3 and det B = 4. Then the det (2AB) is

(1) 24 **(2)** 42
(3) 96 **(4)** 48

Ans. Option (3) is correct.

Explanation: A and B are square matrices of order 3

$\therefore$ AB is a square materix of order 3

We know that if A is a square materix or order n then $|kA| = k^n|A|$

Hence $|2AB| = 2^3|A||B| = 2^3 (3) (4) = 96$

4. If 19th term of a non-zero arithmetic progression (AP) is zero, then its (49th term) : (29th term) is

(1) 2 : 1 **(2)** 4 : 1
(3) 1 : 3 **(4)** 3 : 1

Ans. Option (4) is correct.

Explanation: Let the first term and common difference of A.P. be a and a respectively

Given $a_{19} = 0 \therefore a + 18d = 0$

$a = -18d \; [a_n = a + (n-1)d]$

$$\frac{a_{49}}{a_{29}} = \frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d} = \frac{30d}{10d} = 3:1$$

5. The value of k for which the system of equations $x + ky + 3z = 0, 4x + 3y + kz = 0, 2x + y + 2z = 0$ has non-trivial solution is

(1) k = 0 or $\frac{9}{2}$ **(2)** $k = 10$
(3) $k < 9$ **(4)** $k > 0$

Ans. Option (1) is correct.

Explanation: $\left.\begin{aligned} x + ky + 3z &= 0 \\ 4x + 3y + kz &= 0 \\ 2x + y + 2z &= 0 \end{aligned}\right]$ has non-trival

$$\begin{vmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 3 & k \\ 1 & 2 \end{vmatrix} - k\begin{vmatrix} 4 & k \\ 2 & 2 \end{vmatrix} + 3\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = 0$$

$$6 - k - 8k + 2k - 6 = 0$$
$$2k^2 - 9k = 0$$
$$k(2k-9) = 0$$
$$\therefore \quad k = 0 \text{ or } k = \frac{9}{2}$$

6. A card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is either Ace or a King?

(1) $\frac{4}{13}$ **(2)** $\frac{1}{13}$

(3) $\frac{2}{13}$ **(4)** None of these

Ans. Option (3) is correct.

Explanation: No. of kings in a pack = 4

No of ace in a pack = 4

$\therefore$ favourable events $n(E) = 4 + 4 = 8$

Total events $n(S) = 52$

Probability of (Ace or King)

$$= \frac{\text{Favourable events}}{\text{Total events}}$$

$$= \frac{8}{52} = \frac{2}{13}$$

7. Let E_1 and E_2 be two independent events. Let P(E) denotes the probability of the occurrence of the event E. Further, let E_1 and E_2 denote the complements of E_1 and E_2, respectively. If $P(E_1' \cap E_2) = \frac{2}{15}$ and $P(E_1 \cap E_2') = \frac{1}{6}$, then $P(E_1)$ is

(1) $\frac{2}{15}$ **(2)** $\frac{13}{15}$

(3) $\frac{2}{13}$ **(4)** $\frac{1}{5}$

Ans. Option (4) is correct.

Explanation: $P(E^1 \cap E_2) = \frac{2}{15}$ (Given)

$$P(E_1) . P(E_2) = \frac{2}{15} \text{ (Independent events)}$$

$$[1 - P(E_1)]. P(E_2) = \frac{2}{15}$$

$$P(E_2) - P(E_1) . P(E_2) = \frac{2}{15} \quad ...(i)$$

$$P(E_1 \cap E_1) = \frac{1}{6}$$

$$P(E_1) . P(E_2) = \frac{1}{6}$$

$$P(E_1) (1 - P(E_2)) = \frac{1}{6}$$

$$P(E_1) - P(E_1) P(E_2) = \frac{1}{6} \quad ...(ii)$$

$$P(E_1) - P(E_2) = \frac{1}{6} - \frac{2}{15} = \frac{5-4}{30} = \frac{1}{30}$$

$$P(E_2) = P(E_1) - \frac{1}{30} \quad ...(iii)$$

Now, from equations (i) and (iii),

$$P(E_1) - \frac{1}{30} - P(E_1)\left(P(E_1) - \frac{1}{30}\right) = \frac{2}{15}$$

Let $P(E_1) = x$

$$x - \frac{1}{30} - x\left(x - \frac{1}{30}\right) = \frac{2}{15}$$

$$x - \frac{1}{30} - x^2 + \frac{x}{30} = \frac{2}{15}$$

$$-x^2 + \frac{31x}{30} - \frac{5}{30} = 0$$

or $30x^2 - 31x + 5 = 0$

$$30x^2 - 25x - 6x + 5 = 0$$

$$(6x - 5)(5x - 1) = 0$$

if $6x - 5 = 0$ then $x = \frac{5}{6}$

if $5x - 1 = 0$ then $x = \frac{1}{5}$

8. If $a = \frac{2\sin\theta}{1+\cos\theta+\sin\theta}$, then $\frac{1+\sin\theta-\cos\theta}{1+\sin\theta}$ is

(1) $\frac{1}{a}$ **(2)** $1 - a$

(3) a **(4)** $1 + a$

Ans. Option (3) is correct.

Explanation:

$$a = \frac{2\sin\theta}{1+\cos\theta+\sin\theta} \times \frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}$$

$$= \frac{2\sin\theta(1+\sin\theta-\cos\theta)}{(1+\sin\theta)^2-\cos^2\theta}$$

$$= \frac{2\sin\theta(1+\sin\theta-\cos\theta)}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}$$

$$= \frac{2\sin\theta(1+\sin\theta-\cos\theta)}{2\sin\theta(1+\sin\theta)}$$

$$= \frac{1+\sin\theta-\cos\theta}{1+\sin\theta}$$

$$= a$$

9. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then

(1) $x + y + z - xyz = 0$ **(2)** $xy + yz + zx - 1 = 0$

(3) $x + y + z + xyz = 0$ **(4)** $xy + yz + zx + 1 = 0$

Ans. Option (2) is correct.

Explanation: $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

$$\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$$

$$\frac{x+y}{1-xy} = \frac{1}{z}$$

$$xz + yz = 1 - xy$$

$$xy + yz + zx - 1 = 0$$

10. The equation $ax + by + c = 0$ represents a straight line

(1) for all real numbers a, b and c
(2) only when $b \neq 0$
(3) only when $a \neq 0$
(4) only when at least one of a and b is non-zero

Ans. Option (4) is correct.

Explanation: Any equation of the form $ax + by + c = 0$ will represent a straight line on the xy-plane at least one of a and b is non-zero.

11. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1.$

(1) $x^2 + y^2 - 6y - 5 = 0$ **(2)** $x^2 + y^2 - 6y + 7 = 0$
(3) $x^2 + y^2 - 6y - 7 = 0$ **(4)** $x^2 + y^2 - 6y + 5 = 0$

Ans. Option (3) is correct.

Explanation: Eqn. of ellipse $\frac{x^2}{16}+\frac{y^2}{9} = 1$

$$\therefore \quad a = 4 \text{ and } b = 3$$
$$b^2 = a^2(1 - e^2)$$
$$9 = 16(1 - e^2)$$
$$e^2 = 1-\frac{9}{16}$$
$$= \frac{7}{16}$$
$$\therefore \quad e = \frac{\sqrt{7}}{4}$$

Focus $(ae, 0) \Leftrightarrow \left(\sqrt{7},0\right)$

Circle's centre $= (0, 3)$

$\therefore$ Radius of circle $= \left|\sqrt{(\sqrt{7})^2+3^2}\right| = 4$ units

Equation of circle
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - 3)^2 = (4)^2$$
$$x^2 + y^2 - 6y + 9 = 16$$
$$x^2 + y^2 - 6y - 7 = 0$$

12. Let $[x^r]$ denotes the greatest integer of x^r and $|x|$ denotes the modulus of x.

Then $\lim\limits_{x\to 0}\frac{\sum_{r=1}^{100}[x^r]}{1+|x|}$

(1) does not exist **(2)** is –1
(3) is 1 **(4)** is 100

Ans. Option (1) is correct.

Explanation: $\lim\limits_{x\to 0}\frac{\sum_{r=1}^{100}[x^r]}{1+|x|}$

$$\text{L.H.L.} = \lim_{x\to 0^-}\frac{[x]+[x^2]+[x^3]+\ldots\ldots\ldots+[x^{100}]}{1-x}$$
$$= \lim_{x\to 0^-}\frac{[x]+[x^2]+[x^3]+\ldots\ldots\ldots+[x^{100}]}{1-x}$$
$$= \frac{-1+0-1+0-1\ldots\ldots\ldots 0}{1-0} = -50$$
$$\text{R.H.L.} = \lim_{x\to 0^+}\frac{[x]+[x^2]+[x^3]+\ldots\ldots\ldots+[x^{100}]}{1+x}$$
$$= \frac{0+0+0+0\ldots\ldots\ldots+0}{1+0} = 0$$

L.H.L. $\neq$ R.H.S.
So, Limit does not exist

13. If $f(x) = ax^2 + 6x + 5$ attains its maximum value at $x = 1$. then the value of a is

(1) 0 **(2)** 5
(3) 3 **(4)** –3

Ans. Option (4) is correct.

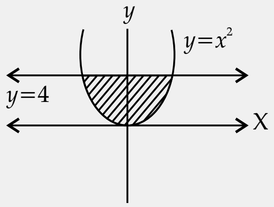
Explanation: For maximum/minimum value
$$f'(x) = 0$$
$$2ax + 6 + 0 = 0$$
at $\quad x = 1 \Rightarrow 2a + b = 0 \Rightarrow a = -3$

14. The area of the region bounded by the line $y = 4$ and the curve $y = x^2$ is

(1) $\frac{32}{3}$ square units **(2)** 0 square unit
(3) 1 square unit **(4)** 32 square units

Ans. Option (1) is correct.

Explanation: Required area $= 2\int_0^4 xdy$



$$= 2\int_0^4 \sqrt{y}dy$$
$$= 2(y)^{\frac{3}{2}}\times\frac{2}{3}$$
$$= 2(4)^{\frac{3}{2}}\times\frac{2}{3}$$
$$= \frac{4}{3}\times 8$$
$$= \frac{32}{3}\text{units}^2$$

15. The equation of the tangent to the curve given by $x = a\sin^3 t$, $y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$ is

(1) $y = 1$ **(2)** $y = 0$
(3) $x = 0$ **(4)** $x = 1$

Ans. Option (2) is correct.

Explanation: at $t = \frac{\pi}{2}$
$$x = a\sin^3\frac{\pi}{2} = a$$
$$y = b\cos^3\frac{\pi}{2} = 0$$

$x = a\sin^3 t$ | $y = b\cos^3 t$

$\frac{dx}{dt} = 3a\sin^2 t\cos t$ | $\frac{dy}{dt} = -3b\cos^2 t\sin t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{-3b\cos^2 t\sin t}{3a\sin^2 t\cos t}$$

$$\frac{dy}{dx} = -\frac{b}{a}\cot t$$

$$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{2}} = -\frac{b}{a}\cot\frac{\pi}{2}$$

$$= 0$$

Equ. of tangent

$$y - y_1 = \left(\frac{dy}{dx}\right)_{x_1 y_1}(x - x_1)$$

$$y - 0 = 0\,(x - a)$$

$$\therefore \quad y = 0$$

16. The solution of the differential equation $\frac{dx}{dy} + Px = Q$, where P and Q are constants or functions of y, is given by

(1) $xe^{\int Pdx} = \int Qe^{\int Pdx}dx + c$

(2) $ye^{\int Pdy} = \int Qe^{\int Pdy}dy + c$

(3) $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$

(4) $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + c$

Ans. Option (4) is correct.

Explanation: Solutions of $\frac{dx}{dy} + Px = Q$ is given by

$$xe^{\int pdy} = \int Qe^{\int pdy}\,dy + c$$

17. Let $\vec{\alpha} = \hat{i} + 2\hat{i} - \hat{k}, \vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$. If $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to a vector $\vec{\delta}$ and $\vec{\delta}.\vec{\gamma} = 10$, then the magnitude of $\vec{\delta}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $2\sqrt{3}$

(3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Ans. Option (2) is correct.

Explanation: Let $\vec{\delta} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{\delta} \perp \vec{\alpha}$ and $\vec{\delta} \perp \vec{\beta}$

$$\therefore \quad a + 2b - c = 0$$

$$2a - b + 3c = 0$$

$$\frac{a}{6-1} = \frac{b}{-2-3} = \frac{c}{-1-4}$$

$$\frac{a}{5} = \frac{b}{-5} = \frac{c}{-5}$$

$$\frac{a}{1} = \frac{b}{-1} = \frac{c}{-1}$$

$$\therefore \quad \vec{\delta} = k\left(\hat{i} - \hat{j} - \hat{k}\right)$$

$$\vec{\delta}.\vec{\gamma} = 10$$

$$k\left(\hat{i} - \hat{j} - \hat{k}\right).\left(2\hat{i} + \hat{j} + 6\hat{k}\right) = 10$$

$$k(2 - 1 - 6) = 10$$

$$k = \frac{10}{-5} = -2$$

$$\vec{\delta} = -2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\left|\vec{\delta}\right| = \sqrt{2^2 + 2^2 + 2^2}$$

$$= 2\sqrt{3} \text{ units}$$

18. Let $\hat{a}, \hat{b}$ and $\hat{c}$ be three unit vectors such that $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{\sqrt{3}}{2}(\hat{b} + \hat{c})$. If $\hat{b}$ is not parallel to $\hat{c}$, then the angle between $\hat{a}$ and $\hat{c}$ is

(1) 0 **(2)** 2π

(3) $\frac{5\pi}{6}$ **(4)** $\frac{\pi}{6}$

Ans. Option (4) is correct.

Explanation: $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} \times \vec{c}\right)$

$$\left(\vec{a}.\vec{c}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

on comparing we get

$$\left(\vec{a}.\vec{c}\right) = \frac{\sqrt{3}}{2}$$

$$\left|\vec{a}\right|\left|\vec{c}\right|\cos\theta = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$

19. Which one of the following statements is correct for a moving body?

(1) If its velocity changes, its speed must change and it must have some acceleration

(2) If its speeds changes, its velocity must change and it must have some acceleration

(3) If its speed changes but direction of motion does not change, its velocity will remain constant

(4) None of the above

Ans. Option (1) is correct.

> ***Explanation:*** When velocity changes then its magnitude (speed) and direction may change. Hence, the speed must change and must have acceleration.

20. a ball is thrown upward at a speed of 28 metre per second. What is the speed of ball one second before reaching maximum height? (Given that g = 10 metre per second^2)

(1) 10 metre per second **(2)** 1 metre per second

(3) 2 metre per second **(4)** 18 metre per second

Ans. Option (1) is correct.

> ***Explanation:*** $v = u + at$
>
> where v = final velocity
>
> t = time taken
>
> u = initial velocity
>
> a = acceleration
>
> $\therefore$ V $= 4 - gt$ (upwards)
>
> $0 = 28 - 10t$
>
> $\therefore$ $t = 2.8$
>
> Mass height reach at in 2.8 s
>
> $\therefore$ Velocity at 2.8 – 1 = 1.8 s
>
> V $= u - gt$
>
> $= 28 - 10 \times 1.8$
>
> $= 28 - 18 = 10$ m/s

21. A bullet of mass m and velocity a is fired into a large block of wood of mass M. Then final velocity of the system is

(1) $\frac{m}{m+M}a$ **(2)** $\frac{M}{m+M}a$

(3) $\frac{m+M}{m}a$ **(4)** $\frac{m+M}{M}a$

Ans. Option (1) is correct.

> ***Explanation:*** Since there is no extra force other than the action and reaction force, so the linear momentum should be conserved.
>
> Momentum before collision
>
> $= m \times a + m \times o$
>
> $= ma$
>
> Momentum after collision = $(M+m)v$
>
> (where v is velocity of the system)
>
> By law of conservation of momentum
>
> $ma = (M + m)v$
>
> $\therefore \quad v = \frac{m}{M+m}a$

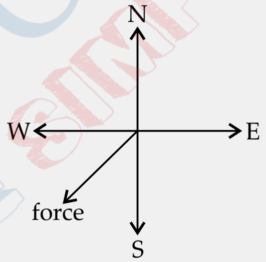
22. If the horizontal and vertical components of a force are negative, then that force is acting in between

(1) North and East **(2)** North and West

(3) South and West **(4)** South and East

Ans. Option (3) is correct.

> ***Explanation:*** Force is acting between south and west.



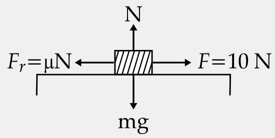
23. Suppose we have block of 4 kilogram kept on a horizontal surface and we are applying a horizontal force of 10 newton. Let the coefficient of friction is 0.2. Find the force of friction. Assume that $g = 10$.

(1) 4 newton **(2)** 8 newton

(3) 1 newton **(4)** None of above

Ans. Option (2) is correct.

> ***Explanation:***
>
> From the figure
>
> If the block is in motion, applying the force 10 N then



> Force of friction (F_r) = μN
>
> = μ mg
>
> = 0.2 × 4 × 10
>
> = 8.0 N
>
> Here μ is coefficient of friction

24. The general solution of the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$ is

(1) $x^2 + y^2 = cxy$ **(2)** $x^2 + y^2 = c$

(3) $x^2 - y^2 = c$ **(4)** $x + y = c$

Ans. Option (2) is correct.

Explanation: $\frac{dy}{dx} + \frac{x}{y} = 0$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$ydy = -xdx$$

$$\int ydy = \int -xdx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + c_1$$

$$x^2 + y^2 = 2c_1$$

$$x^2 + y^2 = c \qquad (c = 2c_1)$$

25. Let A = $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix}$ and B = $\begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$.

Then the determinant of the matrix A + B is

(1) 1 **(2)** 10

(3) 0 **(4)** 2

Ans. Option (3) is correct.

Explanation:

$$A + B = \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

◆◆◆