BEST SELLER

For **2024** EXAM

OSWAAL BOOKS®

LEARNING MADE SIMPLE

ISSUED BY CBSE
SAMPLE QUESTION PAPER
2023-24 • Fully Solved
Released on 31st March, 2023

CBSE QUESTION BANK

Chapterwise & Topicwise

SOLVED PAPERS

CLASS 10

MATHEMATICS

STANDARD

Strictly as per the latest CBSE Syllabus & Circulars released on 6th April & 31st March, 2023 (CBSE Cir No. Acad-39/2023; Acad-45/2023)

Get the #OswaalEdge

1. **100% Updated for 2023-24**
   with **Latest Syllabus** 2024 & Fully Solved (All Sets) **Board Paper 2023**
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   with Crisp Revision Notes, Smart Mind Maps & Mnemonics
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5. **100% as per NEP 2020**
   with **Art Integration, Artificial Intelligence** & **Competency-based Questions**

**with competency focused practice questions (CFPQ)*

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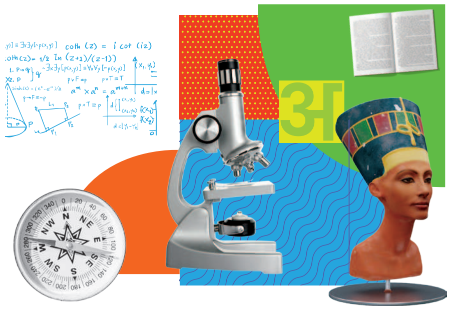

- Latest CBSE Circulars & Syllabus released on 6th April & 31st March 2023 for Academic Year 2023-2024 (CBSE Cir. No. Acad 45/2023; Acad 39/2023) 6 - 10
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In each chapter, for better understanding, questions have been classified the typology issued by CBSE as :

R - Remembering, U - Understanding, A - Analysing, Ap - Applying C - Creating E - Evaluating

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NEP Derived Learning Resources Prescribed by CBSE for Year 2023-24

Artificial Intelligence (AI)

Mode of Assessment

Elective Subject (Classes 8, 9 & 10)
+
Integrated with other subjects like: Mathematics, Science, Physics, Economics, Sociology, Languages from classes 9-12

Skills to be Developed

- Analysis & Critical Thinking
- Interaction and cooperation
- Innovativeness and initiative
- Vocational ethics
- Communication skills

Implications For Students

Students should opt for this curriculum to:
- Become future ready & become at par with their counterparts at a global level.
- Visualise jobs of the future and prepare for them.

Art Integration Learning (AIL)

Mode of Assessment

Art-Integrated Projects as Subject Enrichment Activity

Skills to be Developed

- Art-based enquiry
- Investigation & Exploration
- Critical Thinking
- Creativity

Implications For Students

- At least one Art-Integrated Project in each subject shall be taken up by all students of classes I to XII.
- In a minimum one project work of students from classes I – XII, it is mandatory to integrate any art form of the paired State/UT, as defined under Ek Bharat Shrestha Bharat Programme.

Competency Based Assessment (CBA)

Mode of Assessment

Competency based Questions makes the students understand and apply what they learn in real life situations

Skills to be Developed

- Analysis
- Critical Thinking
- Conceptual Clarity

Implications For Students

40% of the Questions will be asked in the form of:
- MCQs, True/False, Matching Types & Cloze/Gap filling Questions
- Stand Alone Short & Long Answer Questions
- Linked Questions (Source & Case-based Questions)

UNIT II ALGEBRA

Study Time
Maximum time: 4:00 Hrs
Maximum questions: 56

CHAPTER 2

POLYNOMIALS

Syllabus

Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials.

Revision Notes

- **Polynomial:** An algebraic expression in the form of $a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, ..., a_n$ are real numbers) is called a polynomial in one variable x of degree n.

- **Value of a Polynomial at a given point:** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$.

Scan to know more about this topic

Zeroes of polynomials

- **Zero of a Polynomial:** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.

  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-co-ordinates of the points, where the **graph** of $y = p(x)$ intersects the X-axis.

  (i) A linear polynomial has one and only one zero.

  (ii) A quadratic polynomial has at most two zeroes.

  (iii) A cubic polynomial has at most three zeroes.

  (iv) In general, a polynomial of degree n has at most n zeroes.

- **Graphs of Different types of Polynomials:**

  - **Linear Polynomial:** The **graph** of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.

  - **Quadratic Polynomial: (i) Graph** of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects X-axis at a maximum of two distinct points.

    (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two **distinct** points.

Scan to know more about this topic

Relationship between zeroes and coefficients of quadratic polynomials

  - **Cubic polynomial:** Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects X-axis at a maximum of three **distinct** points.

- **Relationship between the Zeroes and the Coefficients of a Polynomial:**

  (i) Zero of a linear **polynomial**

  $$= \frac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$$

  If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\frac{-b}{a}$.

  (ii) In a quadratic polynomial,

  Sum of zeroes of a quadratic polynomial

Polynomials

Degree of Polynomial

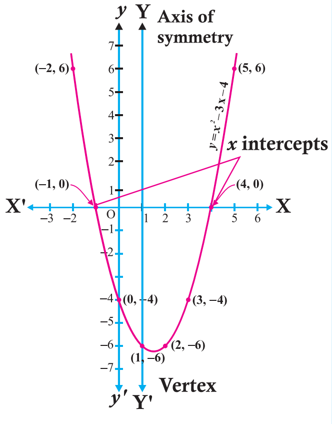
Highest power of x in Polynomial, $p(x)$

Types

Polynomial	Degree	General Form
Linear	1	$f(x) = ax + b$ $a \neq 0$
Quadratic	2	$f(x) = ax^2 + bx + c$ $a \neq 0$
Cubic	3	$f(x) = ax^3+bx^2+cx+d$ $a \neq 0$

Graphical Representation of Quadratic Polynomial

Parabola



$y = x^2 - 3x - 4$

Trace the Mind Map

- First Level
- Second Level
- Third Level

$$= \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes of a quadratic polynomial

$$= \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

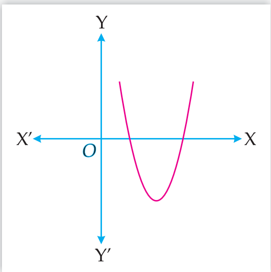
$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha$$

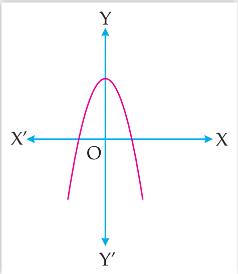
$$= (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma$$

$$= (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D. The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

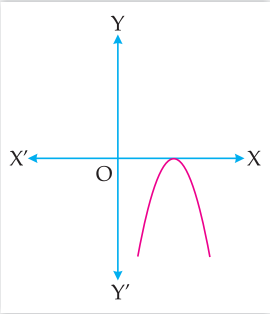
Case I: If D > 0, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-co-ordinates of points of intersection with X-axis are known as 'zeroes' of $f(x)$.

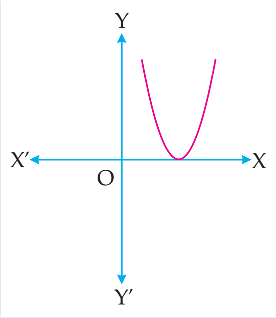




$\therefore f(x)$ will have two zeroes and we can say that roots/zeroes of the two given polynomials are real and unequal.

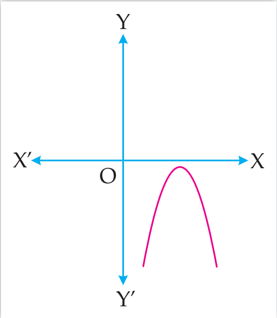
Case II: If D = 0, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.

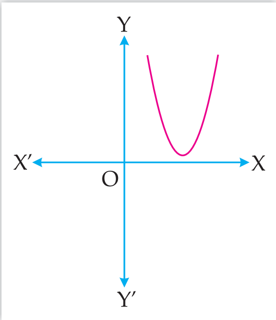




$\therefore f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

Case III: If D < 0, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.





$\therefore f(x)$ will not have any real zero.

Fundamental Facts

(1) Polynomials are also an essential tool in describing and predicting traffic patterns so appropriate traffic control measures, such as traffic lights, can be implemented.

Mnemonics

Concept: $\alpha.\beta = \frac{c}{a}$

Mnemonics: Amitabh Bachchan went Canada by aeroplane.

Interpretation:

Amitabh's A $\Rightarrow$ **Alpha** (α)

Bachchan's B $\Rightarrow$ **Beta** (β)

Canada's C $\Rightarrow$ **Constant** (c)

By for Divide by

Aeroplane's a $\Rightarrow$ **Variable** (a).

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

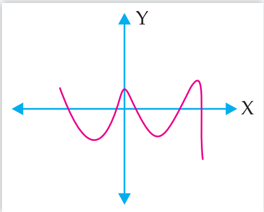
Q. 1. Form a quadratic polynomial, the sum and product of whose zeroes are (– 3) and 2 respectively.
A [CBSE Delhi Board, 2020]

Sol. Given, sum of zeroes $= -3$
and Product of zeroes $= 2$
$\therefore$ Required Polynomial

$$\begin{aligned} p(x) &= k[x^2 - (\text{sum of zeroes})x \\ &\quad + (\text{product of zeroes})] \\ &= k[x^2 - (-3)x + 2] \\ &= k[x^2 + 3x + 2] \end{aligned}$$

Q. 2. If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.
C+U [CBSE SQP, 2020-21]

Q. 3. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows:



Find the number of zeroes of $p(x)$.
AI R [CBSE SQP, 2020]

Short Answer Type Questions-I (2 marks each)

Q. 1. Find a quadratic polynomial where zeroes are $5-3\sqrt{2}$ and $5+3\sqrt{2}$ AI A [CBSE SQP, 2020-21]

Sol. Sum of zeroes $= 5-3\sqrt{2}+5+3\sqrt{2} = 10$ ½

Product of zeroes $= (5-3\sqrt{2})(5+3\sqrt{2})$ 1

$= 25 - 18 = 7$

Polynomial is given by

$x^2 - (\text{sum of zeroes})x + (\text{product of zeroes}) = 0$

$p(x) = x^2 - 10x + 7$ ½

[CBSE Marking Scheme, 2020-21]

Q. 2. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students:
$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

Answer the following questions:
(i) How many of the above ten, are not polynomials ?
(ii) How many of the above ten, are quadratic polynomials ?
A + C [CBSE OD Set-I II III, 2020]

Q. 3. Find the value of p, for which one root of the quadratic polynomial $px^2 - 14x + 8 = 0$ is 6 times the other. AI C + A [CBSE OD Set-III, 2017]

Topper Answer, 2017

Sol. Let α and β be the roots of given quadratic equation.

$\beta = 6\alpha$

Here, $a = p$ $b = -14$, $c = 8$.

$\alpha + \beta = \frac{-(-14)}{p} = \frac{-b}{a}$

$\cancel{7}\alpha = \frac{+\cancel{14}\,2}{p}$

$\alpha = \frac{2}{p}$ —(1)

Also, $\alpha\beta = \frac{8}{p} = \frac{c}{a}$

$\alpha \times 6\alpha = \frac{8}{p}$

$6\alpha^2 = \frac{8}{p}$

from (1),

$6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$

$6 \times \frac{\cancel{4}}{p^2} = \frac{\cancel{8}\,2}{p}$

$\frac{6}{p^2} = \frac{2}{p}$

$\frac{\cancel{6}\,3}{2} = \frac{p^2}{\cancel{p}}$

$\therefore p = 3$ 2

Q. 4. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$, then find the value of a.
C + U [Board Term, 2016]

Q. 5. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.
AI A [Board Term-1, 2016]

Sol. Given,

Sum of zeroes $= \frac{21}{8}$

and Product of zeroes $= \frac{5}{16}$ 1

So, quadratic polynomial

$= x^2 - (\text{Sum of zeroes})x$
$+ \text{Product of zeroes}$

$$= x^2 - \left(\frac{21}{8}\right)x + \frac{5}{16}$$

$$= \frac{1}{16}(16x^2 - 42x + 5) \quad \mathbf{1}$$

Q. 6. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

C + A [Board Term-1, 2015]

Sol. Let, $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$

Then, $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow \quad \alpha + \beta = -\frac{(-4\sqrt{3})}{1}$

$\Rightarrow \quad \alpha + \beta = 4\sqrt{3}$ **1**

and $\quad \alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

$\Rightarrow \quad \alpha\beta = 3$

$\therefore \quad \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3.$ **1**

Short Answer Type Questions-II (3 marks each)

Q. 1. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation. AI C + A [CBSE SQP, 2020-21]

Sol. $3x^2 + px + 4 = 0$

$\because \frac{2}{3}$ is a root so it must satisfy the given equation

$$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$$

$$\frac{4}{3} + \frac{2p}{3} + 4 = 0 \quad \frac{1}{2}$$

On solving, we get

$p = -8$ ½

$3x^2 - 8x + 4 = 0$

$3x^2 - 6x - 2x + 4 = 0$ ½

$3x(x - 2) - 2(x - 2) = 0$ ½

$x = \frac{2}{3}$ or $x = 2$ ½

Hence, $\quad x = 2$ ½

So the other root is 2.

[CBSE Marking Scheme, 2020-21]

Q. 2. The roots α and β of the quadratic equation $x^2 - 5x + 3(k - 1) = 0$ are such that $\alpha - \beta = 1$. Find the value of k. AI U [CBSE SQP, 2020-21]

Sol. We have $\quad \alpha + \beta = 5$ ...(i) ½

$\alpha - \beta = 1$ ...(ii) ½

Solving (i) and (ii), we get

$\alpha = 3$ and $\beta = 2$ ½

also $\quad \alpha\beta = 6$ ½

or $\quad 3(k - 1) = 6$ ½

$k - 1 = 2$

$k = 3$ ½

[CBSE Marking Scheme, 2020-21]

Q. 3. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

A [CBSE Delhi Set-I, 2020]

Q. 4. If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

A + E [CBSE Delhi Set-I, 2020]

Sol. $x^3 - 3x - 10x + 24$

Let α, β and γ be the zeroes of given polynomial

$\therefore \quad \alpha + \beta + \gamma = 3$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = -10$ ...(ii)

$\alpha\beta\gamma = -24$ ...(iii) **1**

Given, $\quad \alpha = 4$

from eqn. (i) $\beta + \gamma = -1$

from eqn (ii) $\beta\gamma = -6$

$(\beta - \gamma)^2 = (\beta + \gamma)^2 - 4\beta\gamma$

$= (-1)^2 - 4(-6)$

$= 25$ ½

$\therefore \quad \beta - \gamma = \pm 5$

$\beta - \gamma = 5$ ...(iv) ½

$\beta + \gamma = -1$

$2\beta = 4 \Rightarrow \beta = 2$

and $\gamma = -3$

Hence zeroes are -3, 2 and 4. **1**

[CBSE Marking Scheme, 2020]

Q. 5. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product. R [CBSE Delhi Set-I, 2019]

Sol. Sum of zeroes $= k + 6$ ½

Product of zeroes $= 2(2k - 1)$ ½

Hence $\quad k + 6 = \frac{1}{2} \times 2\,(2k - 1)$ **1**

$\Rightarrow \quad k = 7$ **1**

[CBSE Marking Scheme, 2019]

Q. 6. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

A [CBSE OD Set-III, 2019]

Sol. Given, polynomial is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$\Rightarrow \quad \frac{1}{3}(21y^2 - 11y - 2)$ ...(i) ½

$\Rightarrow \quad \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$\Rightarrow \quad \frac{1}{3}[(7y(3y-2)+1(3y-2)]$

$\Rightarrow \quad \frac{1}{3}(3y-2)(7y+1)$

$\Rightarrow \quad y = \frac{2}{3}$ or $y = \frac{-1}{7}$

Hence, zeroes of given polynomial are

$y = \frac{2}{3}$ and $y = \frac{-1}{7}$ **1**

On comparing eq (i) with $ax^2 + bx + c = 0$, we get $a = 21, b = -11$ and $c = -2$

Now, sum of zeroes $= \frac{2}{3} + \left(-\frac{1}{7}\right)$ ½

$= \frac{2}{3} - \frac{1}{7}$

$= \frac{14-3}{21}$

$= \frac{11}{21}$

$= -\frac{b}{a}$ **Hence verified.** ½

and product of roots $= \frac{2}{3} \times \left(-\frac{1}{7}\right)$

$= \frac{-2}{21}$

$= \frac{c}{a}$ **Hence verified.** ½

[Based on CBSE Marking Scheme, 2019]

Q. 7. Find the zeroes of the following polynomial:

$5\sqrt{5}x^2 + 30x + 8\sqrt{5}$ U [CBSE SQP, 2018]

Sol. $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$

$= 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$ ½

$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}(\sqrt{5}x + 4)$ ½

$= (\sqrt{5}x + 4)(5x + 2\sqrt{5})$ **1**

Thus, zeroes are $\frac{-4}{\sqrt{5}} = \frac{-4\sqrt{5}}{5}$ and $\frac{-2\sqrt{5}}{5}$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Candidates commit error in simplifying the equation $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$.

Answering Tip

- Adequate Practice is necessary for factorization problems.

Q. 8. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q.

[CBSE SQP, 2022-23]

Sol. Let α and β be the zeros of the polynomial $2x^2 - 5x - 3$

Then $\alpha + \beta = \frac{5}{2}$ ½

And $\alpha\beta = -\frac{3}{2}$ ½

Let 2α and 2β be the zeros $x^2 + px + q$

Then $2\alpha + 2\beta = -p$ ½

$2(\alpha + \beta) = -p$

$2 \times \frac{5}{2} = -p$

So $p = -5$ ½

And $2\alpha \times 2\beta = q$ ½

$4\alpha\beta = q$

So $q = 4 \times -\frac{3}{2}$

$= -6$ ½

Long Answer Type Questions (5 marks each)

Q. 1. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q. A [Board Term-1, 2015]

Sol. Factors of $x^2 + 7x + 12$:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x+4) + 3(x+4) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow x = -4$ or -3 ...(i) **1**

Since, $p'(x) = x^4 + 7x^3 + 7x^2 + px + q$

If $p'(x)$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ are its zeroes. So, putting $x = -4$ and $x = -3$.

$p'(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$

but $p'(-4) = 0$

$\therefore \quad 0 = 256 - 448 + 112 - 4p + q$

$0 = -4p + q - 80$

$\Rightarrow 4p - q = -80$ ...(i) **1**

and $p'(-3) = (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q$

but $p'(-3) = 0$

$\therefore \quad 0 = 81 - 189 + 63 - 3p + q$

$0 = -3p + q - 45$ **1**

$\Rightarrow 3p - q = -45$ ...(ii)

Subtracting equation (ii) from equation (i),

$4p - q = -80$

$3p - q = -45$

$- \quad + \quad +$

$p = -35$

On putting the value of p in eq. (i), **1**

$4(-35) - q = -80$

$-140 - q = -80$

$-q = 140 - 80$

$\Rightarrow \quad -q = 60$

$\therefore \quad q = -60$

Hence, $\quad p = -35$ and $q = -60$ **1**

Q. 2. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

C + A

Sol. Given, $\quad p(x) = 2x^2 + 5x + k$

Then, $\quad$ sum of zeroes $= -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$ ½

$$\Rightarrow \quad \alpha + \beta = \frac{-5}{2} \quad \textbf{1}$$

and product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$ ½

$$\Rightarrow \quad \alpha\beta = \frac{k}{2} \quad \textbf{1}$$

According to equation,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\text{or, } (\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4} \quad \textbf{1}$$

$$\Rightarrow \quad \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \quad \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$$

Hence, $\quad k = 2$. **1**

Q. 3. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$; find the value of 'a' if $3\alpha + 2\beta = 20$.

COMPETENCY AND CRITICAL THINKING BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020.

LEVEL-1: Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions

Q. 1. The graph of a polynomial $p(x)$ cuts the X-axis at 3 points and touches it at 2 other points. The number of zeroes of $p(x)$ is

(A) 1 (B) 2

(C) 3 (D) 5

R [CBSE Board Term-I, 2021]

Ans. Option (D) is correct.

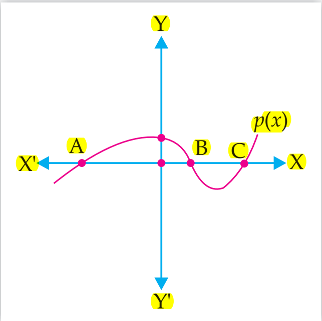
Explanation: According to the property of the polynomials,

Number of zeroes = Number of points at which graph intersects the X-axis.

According to given condition, the graph intersects the X-axis at 5 different points.

Therefore, number of zeroes = 5.

Q. 2. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



(A) 1 (B) 2

(C) 3 (D) 4

R [CBSE Board Term-I, 2021]

Q. 3. A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively is

(A) $k[x^2 - 8x + 5]$ (B) $k[x^2 + 8x + 5]$

(C) $k[x^2 - 5x + 8]$ (D) $k[x^2 + 5x + 8]$

A [CBSE Board Term-I, 2021]

Ans. Option (A) is correct.

Explanation: For any quadratic polynomial,

$$ax^2 + bx + c$$

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$8 = \frac{-b}{a}$$

$$\frac{8}{1} = \frac{-b}{a}$$

or $b = -8k$, $a = 1k$

$$\text{Also, product of zeroes} = \frac{c}{a}$$

$$5 = \frac{c}{a}$$

$$\frac{5}{1} = \frac{c}{a}$$

or $c = 5k$, $a = 1k$

Polynomial whose sum of zeroes or product of zeroes are given,

Required Polynomial $= ax^2 + bx + c$

$= kx^2 - 8kx + 5k$

$= k[x^2 - 8x + 5]$

Q. 4. If $x - 1$ is a factor of the polynomial $p(x) = x^3 + ax^2 + 2b$ and $a + b = 4$, then

(A) $a = 5, b = -1$ **(B)** $a = 9, b = -5$

(C) $a = 7, b = -3$ **(D)** $a = 3, b = 1$

A [CBSE Board Term-I, 2021]

Ans. Option (B) is correct.

Explanation: Given,

$$p(x) = x^3 + ax^2 + 2b$$

$$a + b = 4 \quad \text{...(i)}$$

$x - 1$ is a factor of the polynomial $p(x)$,

it means $x = 1$ is a zero of the polynomial $p(x)$.

$\therefore \quad p(1) = 0$

or $\quad (1)^3 + a(1)^2 + 2b = 0$

or $\quad 1 + a + 2b = 0$

or $\quad a + 2b = -1 \quad$ ...(ii)

Subtracting (i) from (ii), we get

$$b = -5$$

Substituting the value of b in (i), we get $a = 9$

$\therefore \quad a = 9 \ \& \ b = -5$

Q. 5. If α, β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$, then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$, is

(A) -7 **(B)** 7

(C) -3 **(D)** 3

A [CBSE Board Term-I, 2021]

Ans. Option (B) is correct.

Explanation: $p(x) = x^2 - (k + 6)x + 2(2k - 1)$ is the given polynomial

Here, $a = 1, b = -(k + 6) \ \& \ c = 2(2k - 1)$

Sum of zeroes $= \alpha + \beta$

$$= \frac{-b}{a}$$

$$= k + 6$$

Product of zeroes $= \alpha\beta$

$$= \frac{c}{a}$$

$$= \frac{2(2k-1)}{1} = 2(2k - 1)$$

It is given that,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$\Rightarrow \quad k + 6 = \frac{1}{2}2(2k - 1)$

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad -k = -7$

or $\quad k = 7.$

Q. 6. How many zero(es) does $(x - 2)(x + 3)$ have ?

U [CBSE Q.B., 2021-22]

(A) Zero **(B)** One

(C) Two **(D)** Three

Q. 7. Which of these is the polynomial whose zeroes are $\frac{1}{3}$ and $\left(-\frac{3}{4}\right)$?

R [CBSE Q.B., 2021-22]

(A) $12x^2 + 5x - 3$ **(B)** $12x^2 - 5x - 3$

(C) $12x^2 + 13x + 3$ **(D)** $12x^2 - 13x - 3$

Ans. Option (A) is correct.

Explanation: Sum of zeroes $= \frac{1}{3} + \left(-\frac{3}{4}\right)$

$$= \frac{4-9}{12} = \frac{-5}{12}$$

and product of zeroes $= \left(\frac{1}{3}\right)\left(-\frac{3}{4}\right)$

$$= -\frac{1}{4}$$

$\therefore$ Required polynomial $= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$$= x^2 - \left(\frac{-5}{12}\right)x + \left(-\frac{1}{4}\right)$$

$$= \frac{12x^2 + 5x - 3}{12}$$

$$= \frac{1}{12}[12x^2 + 5x - 3]$$

Hence, $12x^2 + 5x - 3$ is a polynomial and $\frac{1}{12}$ is a constant.

Q. 8. How many zero(es) does the polynomial $293x^2 - 293x$ have ? A [CBSE Q.B., 2021-22]

(A) 0 **(B)** 1

(C) 2 **(D)** 3

Ans. Option (C) is correct.

Explanation: Given polynomial is $293x^2 - 293x$

$\Rightarrow 293x(x - 1)$

For the property of zeroes,

$$293x(x - 1) = 0$$

Either, $\quad 293x = 0 \Rightarrow x = 0$

or, $\quad x - 1 = 0 \Rightarrow x = 1$

Hence, it has two zeroes.

Q. 9. p and q are the zeroes of the polynomial $4y^2 - 4y + 1$. What is the value of $\frac{1}{p} + \frac{1}{q} + pq$?

A+E [CBSE Q.B., 2021-22]

(A) $-\frac{15}{4}$ **(B)** $-\frac{3}{4}$

(C) $\frac{5}{4}$ **(D)** $\frac{17}{4}$

Ans. Option (D) is correct.

Explanation: Given polynomial is $4y^2 - 4y + 1$

$\therefore \quad$ Sum of zeroes $= -\left(\frac{-4}{4}\right) = 1$

and product of zeroes $= \frac{1}{4}$

Now, $\quad \frac{1}{p} + \frac{1}{q} + pq = \frac{q+p}{pq} + pq$

We have proved above, $p + q = 1$ and $pq = \frac{1}{4}$

So, $\quad \frac{p+q}{pq} + pq = \frac{1}{1/4} + \frac{1}{4}$

$$= 4 + \frac{1}{4} = \frac{17}{4}.$$

Q. 10. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
C+U [CBSE Delhi Set-I, 2020]

(A) 10 **(B)** – 10
(C) – 7 **(D)** – 2

Ans. Option (B) is correct.

Explanation: Let $p(x) = x^2 + 3x + k$

$\because$ 2 is a zero of $p(x)$, then

$$p(2) = 0$$
$$\therefore \quad (2)^2 + 3(2) + k = 0$$
$$\Rightarrow \quad 4 + 6 + k = 0$$
$$\Rightarrow \quad 10 + k = 0$$
$$\Rightarrow \quad k = -10$$

Q. 11. The quadratic polynomial, the sum of whose zeroes is – 5 and their product is 6, is
A [CBSE Delhi Set-I, 2020]

(A) $x^2 + 5x + 6$ **(B)** $x^2 - 5x + 6$
(C) $x^2 - 5x - 6$ **(D)** $-x^2 + 5x + 6$

Ans. Option (A) is correct.

Explanation: Let α and β be the zeroes of the quadratic polynomial, then

$$\alpha + \beta = -5$$
and $$\alpha\beta = 6$$

So, required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6.$$

Q. 12. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

(A) $m, m + 3$ **(B)** $-m, m + 3$
(C) $m, -(m + 3)$ **(D)** $-m, -(m + 3)$

U [CBSE OD Set-I, 2020]

Ans. Option (B) is correct.

Explanation: Given, $x^2 - 3x - m(m + 3)$

putting $x = -m$, we get

$$= (-m)^2 - 3(-m) - m(m + 3)$$
$$= m^2 + 3m - m^2 - 3m = 0,$$

putting $x = m + 3$, we get

$$= (m + 3)^2 - 3(m + 3) - m(m + 3)$$
$$= (m + 3)[m + 3 - 3 - m]$$
$$= (m + 3)[0] = 0.$$

Hence, $-m$ and $m + 3$ are the zeroes of given polynomial.

Commonly Made Error

- Students often make mistakes in analyzing the zeroes as they get confused with different terms.

Answering Tip

- Understand different cases for zeroes.

Q. 13. The degree of polynomial having zeroes – 3 and 4 only is
A [CBSE Board, 2020]

(A) 2 **(B)** 1
(C) more than 3 **(D)** 3

Topper Answer, 2020

Sol. (A) 2

Detailed Solution:

By the definition of the polynomial,

A polynomial of degree n has at most n zeroes.

Hence, the degree of polynomial zeroes having – 3 and 4 only is 2.

Q. 14. If α and β are the zeros of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is

(A) $-\frac{2}{3}$ **(B)** $\frac{2}{3}$
(C) $\frac{1}{3}$ **(D)** $-\frac{1}{3}$

[CBSE SQP, 2022-23]

Ans. Option (B) is correct.

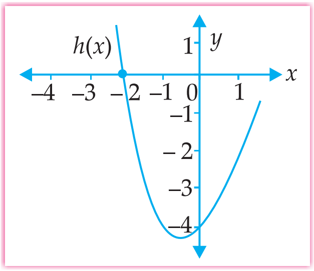
Explanation:

Given, $f(x) = px^2 - 2x + 3p$

Since α and β are the zeroes of given polynomial.

$$\therefore \quad \alpha + \beta = -\frac{(-2)}{p} = \frac{2}{p}$$
and $$\alpha\beta = \frac{3p}{p} = 3$$
$$\because \quad \alpha + \beta = \alpha\beta \quad \text{(given)}$$
$$\therefore \quad \frac{2}{p} = 3$$
$$\Rightarrow \quad p = \frac{2}{3}$$

Q. 15. Shown below is a part of the graph of a polynomial $h(x)$.



On dividing $h(x)$ by which of the following will the remainder be zero?

(i) $(x - 2)$ (ii) $(x + 2)$
(iii) $(x - 4)$ (iv) $(x + 4)$

(A) Only (ii)
(B) Only (i) and (iii)
(C) Only (ii) and (iv)
(D) Cannot be determined without knowing the polynomial $h(x)$

[Q. 2, CBSE CFPQ]

Sol. Option (A) is correct.

Explanation: From graph zero of polynomial

$$h(x) = (-2)$$
therefore $$h(x) = (x + 2)$$

So, divisor of $h(x)$ will be

$$x + 2)\overline{x + 2}(1$$
$$x + 2$$
$$- \quad -$$
$$\overline{\quad 0 \quad}$$

[B] Assertion and Reason Based Questions

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

Reason (R): If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

Ans. Option (D) is correct.

Explanation: In case of assertion:

Let α and β be the roots of the quadratic polynomial. If α and β are positive then

$\alpha + \beta = \dfrac{-b}{a}$ it shows that $\dfrac{-b}{a}$ is negative but sum of two positive numbers (α, β) must be +ve *i.e.*, either b or a must be negative. So, a, b and c will have different signs.

$\therefore$ Given statement is incorrect.

In case of reason:

Let $\beta = 0, \gamma = 0$

$$f(x) = (x-\alpha)(x-\beta)(x-\gamma)$$
$$= (x-\alpha)\, x \cdot x$$
$$\Rightarrow \quad f(x) = x^3 - \alpha x^2$$

which has no linear (coefficient of x) and constant terms.

$\therefore$ Given statement is correct.

Thus, assertion is false but reason is true.

Q. 2. Assertion (A): The value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes are $\pm\dfrac{1}{2}$.

Reason (R): If all the three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

Ans. Option (C) is correct.

Explanation: In case of assertion:

$$f(x) = kx^2 + x + k$$
$$[\text{here, } a = k, b = 1, c = k]$$

For equal roots $b^2 - 4ac = 0$

$$\Rightarrow \quad (1)^2 - 4(k)(k) = 0$$
$$\Rightarrow \quad 4k^2 = 1$$
$$\Rightarrow \quad k^2 = \frac{1}{4}$$
$$\Rightarrow \quad k = \pm\frac{1}{2}$$

So, there are $+\dfrac{1}{2}$ and $-\dfrac{1}{2}$ values of k so that the given equation has equal roots.

$\therefore$ Assertion is true.

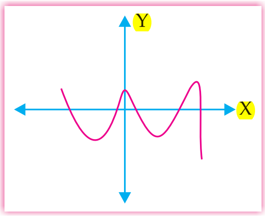
In case of reason:

All the zeroes of cubic polynomial are positive only when all the constants a, b, and c are negative.

$\therefore$ Reason is false.

Thus, assertion is true but reason is false.

Q. 3. Assertion (A): The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows:



The number of zeroes of $p(x)$ is 5.

Reason (R): If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

Q. 4. Assertion (A): If the zeroes of the quadratic polynomial $(k-1)\,x^2 + kx + 1$ is -3, then the value of k is $\dfrac{4}{3}$.

Reason (R): If -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$, then the value of k is -2.

Ans. Option (B) is correct.

Explanation: In case of assertion:

Let $p(x) = (k-1)x^2 + kx + 1$

As -3 is a zero of $p(x)$, then

$$p(-3) = 0$$
$$\Rightarrow \quad (k-1)(-3)^2 + k(-3) + 1 = 0$$
$$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$$
$$\Rightarrow \quad 9k - 3k = +9 - 1$$
$$\Rightarrow \quad 6k = 8$$
$$\Rightarrow \quad k = \frac{4}{3}$$

$\therefore$ Assertion is true.

In case of reason:

Since, -1 is a zero of the polynomial

and $\quad p(x) = kx^2 - 4x + k,$

then $\quad p(-1) = 0$

$$\therefore \quad k(-1)^2 - 4(-1) + k = 0$$
$$\Rightarrow \quad k + 4 + k = 0$$
$$\Rightarrow \quad 2k + 4 = 0$$
$$\Rightarrow \quad 2k = -4$$
$$\text{Hence,} \quad k = -2$$

$\therefore$ Reason is also true.

Thus, both assertion and reason are true but reason is not the correct explanation for assertion.

This question is for practice and its solution is available at the end of the chapter

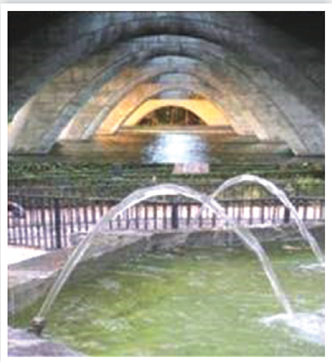
LEVEL-2: Case Based Questions (4 marks each)

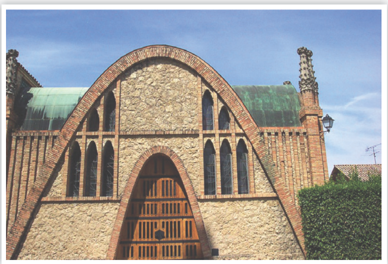
[A] Case Based MCQs

I. Read the following and answer any four questions from Q.1. to Q.5.

The below pictures are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.

[CBSE QB, 2021]





Q. 1. In the standard form of quadratic polynomial, $ax^2 + bx + c$, a, b and c are

(A) All are real numbers.

(B) All are rational numbers.

(C) 'a' is a non zero real number and b and c are any real numbers.

(D) All are integers.

Ans. Option (C) is correct.

Explanation: According to the standard form of quadratic polynomial, 'a' is a non-zero real number and b and c are real numbers.

Q. 2. If the roots of the quadratic polynomial are equal, where the discriminant $D = b^2 - 4ac$, then

(A) $D > 0$ (B) $D < 0$

(C) $D \neq 0$ (D) $D = 0$

Ans. Option (D) is correct.

Explanation: If the roots of the quadratic polynomial are equal, then discriminant is equal to zero

$$D = b^2 - 4ac = 0$$

Q. 3. If α and $\dfrac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is

(A) 4 (B) $\dfrac{1}{4}$

(C) $\dfrac{-1}{4}$ (D) 2

Ans. Option (B) is correct.

Explanation: Given equation, $2x^2 - x + 8k$

$$\text{Sum of zeroes} = \alpha + \frac{1}{\alpha}$$

$$\text{Product of zeroes} = \alpha . \frac{1}{\alpha} = 1$$

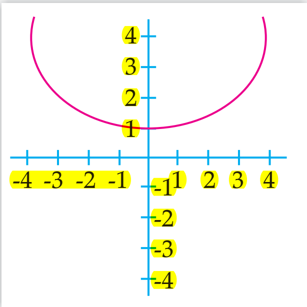
$$\text{Product of zeroes} = \frac{c}{a} = \frac{8k}{2}$$

So, $$\frac{8k}{2} = 1$$

$$k = \frac{2}{8}$$

$$k = \frac{1}{4}$$

Q. 4. The graph of $x^2 + 1 = 0$



(A) Intersects X-axis at two distinct points.

(B) Touches X-axis at a point.

(C) Neither touches nor intersects X-axis.

(D) Either touches or intersects X-axis.

Ans. Option (C) is correct.

Explanation: From the graph of the polynomial, we get to know that it neither touches nor intersects X-axis.

Q. 5. If the sum of the roots is $-p$ and product of the roots is $-\dfrac{1}{p}$, then the quadratic polynomial is

(A) $k\left(-px^2 + \dfrac{x}{p} + 1\right)$ (B) $k\left(px^2 - \dfrac{x}{p} - 1\right)$

(C) $k\left(x^2 + px - \dfrac{1}{p}\right)$ (D) $k\left(x^2 + px + \dfrac{1}{p}\right)$

Ans. Option (C) is correct.

Explanation: Required polynomial,

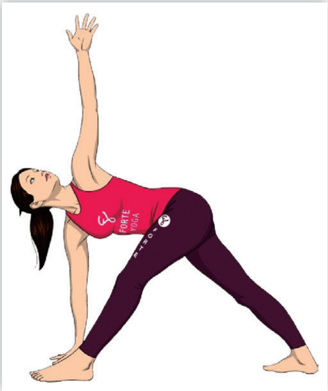
$$P(x) = k[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$$

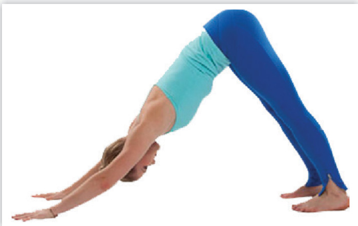
$$= k\left[x^2 - (-p)x + \left(-\frac{1}{p}\right)\right]$$

$$= k\left(x^2 + px - \frac{1}{p}\right)$$

II. Read the following text and answer any four questions from Q1 to Q5.

An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial. **[CBSE QB, 2021]**





Q. 1. The shape of the poses shown is

(A) Spiral (B) Ellipse

(C) Linear (D) Parabola

Ans. Option (D) is correct.

Explanation: All the shapes resembles the Parabola.

Q. 2. The graph of parabola opens downwards, if ______

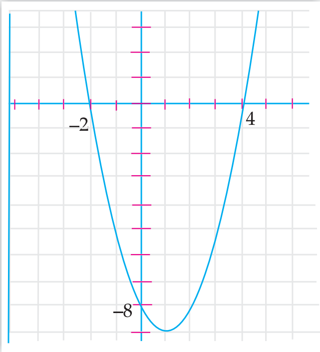
(A) $a \geq 0$ (B) $a = 0$

(C) $a < 0$ (D) $a > 0$

Ans. Option (C) is correct.

Explanation: According to the standard form of the quadratic polynomial the graph open downwards if $a<0$.

Q. 3. In the graph, how many zeroes are there for the polynomial?



(A) 0 (B) 1

(C) 2 (D) 3

Ans. Option (C) is correct.

Explanation: Since, graph, cuts X-axis at two points. Hence, zeroes of polynomial are 2.

Q. 4. The two zeroes in the above shown graph are

(A) 2, 4 (B) – 2, 4

(C) – 8, 4 (D) 2, – 8

Ans. Option (B) is correct.

Explanation: From the graph we get the zeroes of the quadratic polynomial as –2 and 4.

Q. 5. The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are

(A) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (B) $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

(C) $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ (D) $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

Ans. Option (B) is correct.

Explanation: $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ (given)

$$= 4\sqrt{3}x^2 + (8-3)x - 2\sqrt{3}$$

$$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

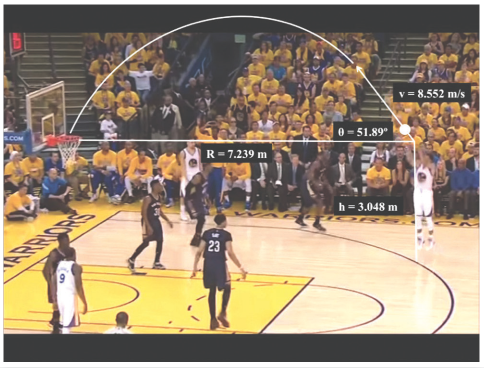
$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

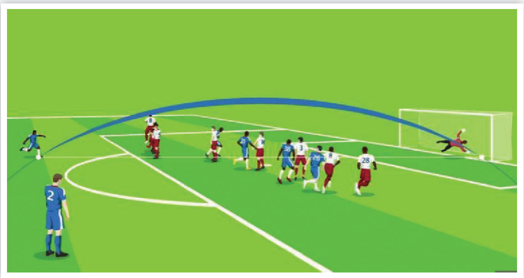
$$= (\sqrt{3}x + 2)(4x - \sqrt{3})$$

Hence, zeroes of polynomial $= \frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

III. Read the following text and answer the following questions on the basis of the same:

Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial. **[CBSE QB, 2021]**





Q. 1. The shape of the path traced shown is

(A) Spiral (B) Ellipse
(C) Linear (D) Parabola

Ans. Option (D) is correct.

Explanation: The shape of the path traced by the balls is a parabola.

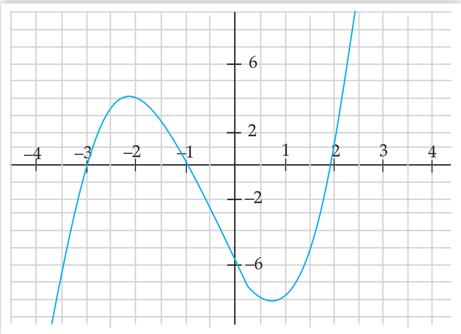
Q. 2. The graph of parabola opens upwards, if ________

(A) $a = 0$ (B) $a < 0$
(C) $a > 0$ (D) $a \geq 0$

Ans. Option (B) is correct.

Explanation: According to the standard form of the quadratic polynomial the graph of a parabola opens downwards if $a < 0$.

Q. 3. Observe the following graph and answer:



In the above graph, how many zeroes are there for the polynomial?

(A) 0 (B) 1
(C) 2 (D) 3

Ans. Option (D) is correct.

Explanation: The number of zeroes of polynomial is the number of times the curve intersects the X-axis, *i.e.*, attains the value 0.

Here, the polynomial meets the X-axis at 3 points.

So, number of zeroes = 3.

Q. 4. The three zeroes in the above shown graph are

(A) 2, 3, –1 (B) –2, 3, 1
(C) –3, –1, 2 (D) –2, –3, –1

Ans. Option (C) is correct.

Explanation: From the graph we get the zeroes of the polynomial as –3, –1 and 2.

Q. 5. What will be the expression of the polynomial?

(A) $x^3 + 2x^2 - 5x - 6$ (B) $x^3 + 2x^2 - 5x - 6$
(C) $x^3 + 2x^2 + 5x - 6$ (D) $x^3 + 2x^2 + 5x + 6$

Ans. Option (A) is correct.

Explanation: Since, the three zeroes $= -3, -1, 2$

Hence, the expression is $(x + 3)(x + 1)(x - 2)$

$= [x^2 + x + 3x + 3](x - 2)$

$= x^3 + 4x^2 + 3x - 2x^2 - 8x - 6$

$= x^3 + 2x^2 - 5x - 6$

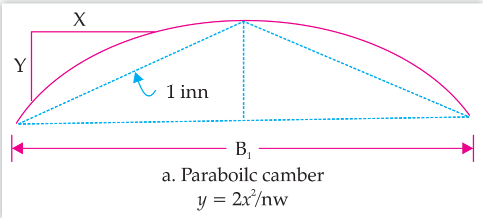
[B] Subjective Case Based Questions

I. Read the following text and answer the following questions:

Applications of Parabolas: Highway Overpasses/ Underpasses

A highway underpass is parabolic in shape.

Shape of cross slope:



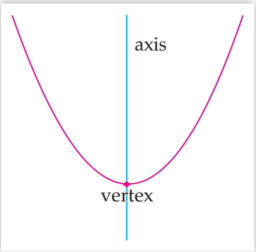
Parabola

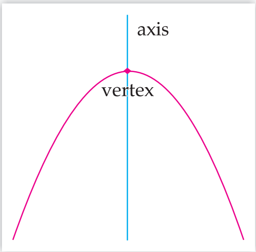
A parabola is the graph that results from

$$p(x) = ax^2 + bx + c.$$

Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the vertex.





A I C + AE [CBSE SQP, 2020-21]

Q. 1. If the highway overpass is represented by $x^2 - 2x - 8$. Find its zeroes.

Sol.
$$x^2 - 2x - 8 = 0$$
or, $x^2 - 4x + 2x - 8 = 0$

or, $x(x-4) + 2(x-4) = 0$

or, $(x-4)(x+2) = 0$

or, $x = 4, x = -2.$ **1**

Q. 2. Find the product of zeroes of given polynomial in question 1.

OR

Find the number of zeroes that polynomial $f(x) = (x-2)^2 + 4$ can have.

Sol. Given polynomial is $x^2 - 2x - 8 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -2$ and $c = -8$

$$\therefore \quad \text{product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$
$$= \frac{c}{a} = \frac{-8}{1} = -8.$$ **2**

OR

We have,
$$f(x) = (x-2)^2 + 4$$
$$= x^2 + 4 - 4x + 4$$
$$= x^2 - 4x + 8.$$

i.e., It has $D = b^2 - 4ac = 16 - 32 < 0$

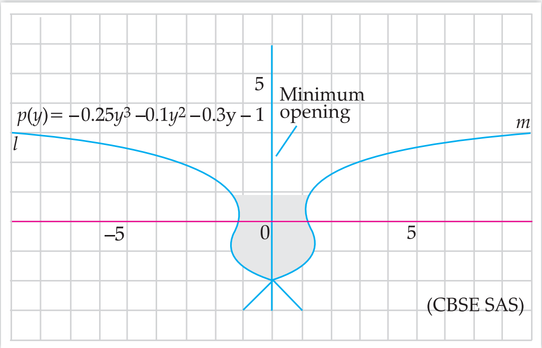
Hence no real value of x is possible, i.e., no zero.

Q. 3. Write the name of graph, which represent above case.

Sol. Here, the given graph of a quadratic polynomial is a parabola. **1**

II. Read the following text and answer the following questions:

Amit designs a flower vase using a graph of polynomial equations. Equation of the curve is given in the graph



Q. 1. Sara looks at the graphical model and makes an observation, "The zero of the polynomial is at the origin." Is the correct? If not, what are the coordinates of the zero of the polynomial. **1**

Sol. Zero of the polynomial $p(y)$ is not at origin point. The coordinates of the zero are (0, –2)

Q. 2. The curve m is a mirror image of $p(y)$ on the y-axis. Which polynomial represents curve m? **1**

Sol. Since, the curve m is a mirror image of the curve l, so all the signs are reversed i.e., polynomial represents the curve m is

$p(y) = 0.25y^3 + 0.1y^2 + 0.3y + 1.$ **1**

Q. 3. Sara changes the coefficient of y^3 in the polynomials for the curves l and m. How does if affect the shape of the flowerpot?

OR

Amit wants to decrease the minimum opening of the flower pot. Which term of the polynomials for the curves l and m should he change?

Sol. The curves of the flower vase changed when the value of the coefficient of y^3 changes. The curves of the flower pots can be different.

OR

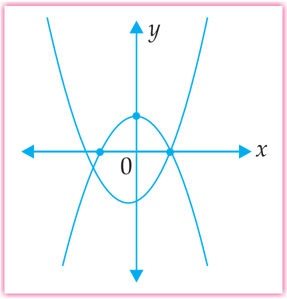
(i) Change the points where polynomial cut the x-axis

(ii) Change points on x-axis where curve l and m intersect.

LEVEL-3:

Very Short Answer Type Questions [1 mark each]

Q. 1. Two polynomials are shown in the graph below.



Find the number of zeroes that are common to both the polynomials. Explain your answer.

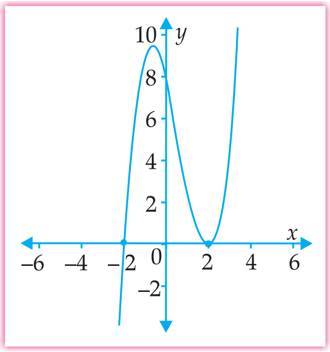
[Q. 5, CBSE CFPQ]

Sol. From graph, number of zeroes for each polynomial is 2.

But both polynomial intersect at only one point.

$\therefore$ Number of zero that is common to both the polynomials = 1. **1**

Q. 2. Students of a class were shown the graph below.



Based on their answers, they were divided into two groups. Group 1 said the graph represented a quadratic polynomial whereas group 2 said the graph represented a cubic polynomial.

(i) Which group was correct?

(ii) Write the polynomial represented by the graph.

[Q. 15, CBSE CFPQ]

Sol. (i) Group 2 was correct. ½

(ii) Polynomial represented by the graph is

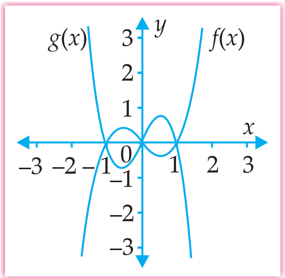
$= (x-2)^2(x+2)$

$= (x^2+4-4x)(x+2)$

$= x^3+2x^2+4x+8-4x^2-8x$

$= x^3-2x^2-4x+8$ ½

Q. 3. Shown below are the graphs of two cubic polynomials, $f(x)$ and $g(x)$. Both polynomials have the zeroes (–1), 0 and 1.



Anya said, "Both the graphs represent the same polynomial, $f(x) = g(x) = (x+1)(x-0)(x-1)$ as they have the same zeroes."

Pranit said," Both the graphs represent two different polynomials, $f(x) = (x+1)(x-0)(x-1)$ and $g(x) = -(x+1)(x-0)(x-1)$ and only two such polynomials exist that can have the zeroes (–1), 0 and 1."

Aadar said," Both the graphs represent two different polynomials and infinitely many such polynomials exists that have the zeroes (–1), 0 and 1."

Who is right? Justify your answer.

[Q. 16, CBSE CFPQ]

Sol. Aadar is right ½

$\because$ Polynomial in factored form with zeroes (–1), 0 and 1 can be written as; $K(x+1)(x-0)(x-1)$

Where K is an integer ½

Short Answer Type Questions-I

[2 marks each]

Q. 1. $\dfrac{x^2-3\sqrt{2}x+4}{x-\sqrt{2}}$; $x \neq \sqrt{2}$

At how many points does the graph of the above expression intersect the x-axis? [Q. 4, CBSE CFPQ]

Sol. $\dfrac{x^2-3\sqrt{2}x+4}{x-\sqrt{2}}$; $x \neq \sqrt{2}$

On factorising we get

$$= \frac{x^2-2\sqrt{2}x-\sqrt{2}x+4}{x-\sqrt{2}}$$

$$= \frac{x(x-2\sqrt{2})-\sqrt{2}(x-2\sqrt{2})}{x-\sqrt{2}}$$

$$= \frac{(x-\sqrt{2})(x-2\sqrt{2})}{x-\sqrt{2}} \quad 1$$

$$= x-2\sqrt{2}$$

So, $x = 2\sqrt{2}$

Since, there is only one zero

$\therefore$ $(x-2\sqrt{2})$ intersects the x-axis at exactly one point. 1

Q. 2. p and q are zeroes of the polynomial $2x^2+5x-4$. Without finding the actual values of p and q, evaluate $(1-p)(1-q)$. Show your steps.

[Q. 5, CBSE CFPQ]

Sol. $2x^2+5x-4$

Here, p and q are zeroes of polynomial

$\therefore$ Sum of roots $(p+q) = \dfrac{-b}{a} = \dfrac{-5}{2}$ ...(i) ½

Product of roots $(p \times q) = \dfrac{c}{a} = \dfrac{-4}{2} = -2$ ...(ii) ½

On expanding $(1-p)(1-q)$ we get

$= 1-q-p+pq$

$= 1-(p+q)+pq$ ½

$$= 1-\left(\frac{-5}{2}\right)+(-2)$$

$\left(\because p+q = \dfrac{-5}{2} \text{ and } pq = -2\right)$

$$= 1+\frac{5}{2}-2$$

$$= \frac{2+5-4}{2} = \frac{3}{2} \quad ½$$

Q. 3. $p(x) = 2x^2-6x-3$. The two zeroes are of the form: $\dfrac{3\pm\sqrt{k}}{2}$; Where k is a real number

Use the relationship between the zeroes and coefficients of a polynomial to find the value of k. Show your steps. [Q. 10, CBSE CFPQ]

Sol. $p(x) = 2x^2-6x-3$

Zeroes of polynomial $\Rightarrow \dfrac{3+\sqrt{k}}{2}$ and $\dfrac{3-\sqrt{k}}{2}$

And product of zeroes $= \dfrac{c}{a}$

$$\therefore \left(\frac{3+\sqrt{K}}{2}\right)\left(\frac{3-\sqrt{K}}{2}\right) = \frac{-3}{2} \quad \text{...(i) } \mathbf{1}$$

$$\Rightarrow \frac{9-K}{4} = \frac{-3}{2} \quad \tfrac{1}{2}$$

$$18 - 2K = -12$$
$$-2K = -30$$
$$K = 15 \quad \tfrac{1}{2}$$

Short Answer Type Questions-II

[3 marks each]

Q. 1. $f(x) = x^3 - ax^2 + (a-3)x + 6$**, where** a **is a non-zero real number. When** $f(x)$ **is divided by** $(x+1)$**, there is no remainder.**

If $f(x)$ **is completely factorable, find the zeroes of** $f(x)$**. Show your steps.** [Q. 13, CBSE CFPQ]

Sol. Since, $f(x)$ is divisible by $(x+1)$

$\therefore$ $f(-1) = 0$

Thus $f(-1) = (-1)^3 - a(-1)^2 + (a-3) - 1 + 6 = 0$

$$-1 - a - a + 3 + 6 = 0 \quad \tfrac{1}{2}$$
$$8 - 2a = 0$$
$$8 = 2a$$
$$a = 4$$

After substituting value of 'a' we get

$$f(x) = x^3 - 4x^2 + x + 6 \quad \tfrac{1}{2}$$

On dividing $f(x)$ by $(x+1)$,

$$
\begin{array}{r|l}
x+1 & x^3 - 4x^2 + x + 6 \;(x^2 - 5x + 6 \\
 & x^3 + x^2 \\
 & -\quad - \\
\hline
 & -5x^2 + x + 6 \\
 & -5x^2 - 5x \\
 & +\quad + \\
\hline
 & 6x + 6 \\
 & 6x + 6 \\
 & -\quad - \\
\hline
 & 0
\end{array}
\quad \mathbf{1}
$$

$\therefore$ quotient $= x^2 - 5x + 6$

On factorising $x^2 - 5x + 6$, we get

$$= x^2 - 3x - 2x + 6$$
$$= x(x-3) - 2(x-3)$$
$$= (x-3)(x-2)$$

$x = 3$ and $x = 2$ ½

Thus, zeroes of $f(x)$ are (-1), 2 and 3. ½

Q. 2. A polynomial is given by $q(x) = x^3 - 2x^2 - 9x + k$**, where** k **is a constant.**

The sum of two zeroes of $q(x)$ **is zero.**

Using the relationship between the zeroes and coefficients of a polynomial, find the:

(i) zeroes of $q(x)$**.**

(ii) value of k**.**

Show your steps. [Q. 7, CBSE CFPQ]

Sol. (i) $q(x) = x^3 - 2x^2 - 9x + k$

Let zeroes of cubic polynomial be $-\alpha$, α, β

As sum of zeroes $= \dfrac{-b}{a}$ ½

$$\therefore \quad -\alpha + \alpha + \beta = \frac{-(-2)}{1}$$
$$\beta = 2 \quad \tfrac{1}{2}$$

$$\text{Sum of products} = \frac{c}{a}$$

$$\therefore (-\alpha \times \alpha) + (\alpha \times \beta) + (-\alpha \times \beta) = \frac{-9}{1}$$
$$-\alpha^2 + \alpha\beta - \alpha\beta = -9$$
$$-\alpha^2 = -9$$
$$\alpha = (\pm 3) \quad \mathbf{1}$$

Thus, 3 zeroes of $q(x)$ are (-3), 3 and 2. ½

(ii) $$\text{Product of zeroes} = \frac{-d}{a}$$

$$\therefore \quad -\alpha \times \alpha \times \beta = -k$$
$$-\alpha^2\beta = -k$$
$$9 \times 2 = k$$
$$k = 18 \quad \tfrac{1}{2}$$

Q. 3. $p(x) = ax^2 - 8x + 3$**, where a is** a **non-zero real number. One zero of** $p(x)$ **is 3 times the other zero.**

(i) Find the value of a**. Show your work.**

(ii) What is the shape of the graph of $p(x)$**? Give a reason for your answer.** [Q. 8, CBSE CFPQ]

Sol. $p(x) = ax^2 - 8x + 3$

Let zeroes of $p(x) = \alpha$ and β

$\therefore$ $\alpha = 3\beta$ (given) ½

$$\text{Sum of roots } (\alpha + \beta) = \frac{-b}{a}$$

$$(3\beta + \beta) = \frac{-(-8)}{a}$$

$$4\beta = \frac{8}{a}$$

$$\beta = \frac{2}{a} \quad \text{...(i) } \tfrac{1}{2}$$

$$\text{Product of roots } (\alpha\beta) = \frac{c}{a}$$

$$(3\beta \times \beta) = \frac{3}{a}$$

$$3\beta^2 = \frac{3}{a}$$

$$\beta^2 = \frac{1}{a} \quad \text{...(ii)}$$

On equating (i) and (ii), we get

$$\left(\frac{2}{a}\right)^2 = \frac{1}{a}$$

$$\frac{4}{a^2} = \frac{1}{a}$$

$$a = 4 \quad \mathbf{1}$$

(ii) Since, 'a' is positive, therefore the graph of $p(x)$ is an open upward parabola. **1**

Solutions for Practice Questions

Very Short Answer Type Questions

Sol. 2: Let the roots of the given quadratic equation be α and β.
So we have,

$$\alpha + \beta = \frac{k}{3} \quad \tfrac{1}{2}$$

$$3 = \frac{k}{3}$$

$$k = 9 \quad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2020-21]

Sol. 3: Since the graph intersects the X-axis 5 times.
Hence, the number of zeroes of $p(x)$ is 5. **1**

[CBSE SQP Marking Scheme, 2020]

Short Answer Type Questions-I

Sol. 2: (i) $x^3 + \sqrt{3x} + 7,\ \ 2x^2 + 3 - \frac{5}{x}$ and $x + \frac{1}{x}$ are not polynomials. **1**

(ii) $3x^2 + 7x + 2$ is only one quadratic polynomial. **1**

[CBSE Marking Scheme, 2020]

Sol. 4: Product of (zeroes) roots $= \frac{c}{a}$

$$= \frac{2a}{1} = \alpha \cdot \frac{2}{\alpha} \quad \mathbf{1}$$

or, $2a = 2$

$\therefore$ $a = 1$ **1**

[CBSE Marking Scheme, 2016]

Short Answer Type Questions-II

Sol. 3: Let α and β be zeroes of the given polynomial $ax^2 + bx + c$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a} \quad \mathbf{1}$$

$$s = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

$$p = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c} \quad \mathbf{1}$$

Quadratic polynomial

$$= x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}$$

$$p(x) = \frac{1}{c}(cx^2 + bx + a) \quad \mathbf{1}$$

[CBSE Marking Scheme, 2020]

Long Answer Type Questions

Sol. 3: We have quadratic polynomial $= x^2 - 6x + a$

$$\therefore \quad \alpha + \beta = \frac{-(-6)}{1} = 6 \quad \mathbf{1}$$

and $\alpha\beta = \frac{a}{1} = a$ **1**

It is given that; $3\alpha + 2\beta = 20$ ...(i)

and $\alpha + \beta = 6$ ...(ii)

Multiplying by 2 in eq. (ii), we get

$2\alpha + 2\beta = 12$ ...(iii) ½

Subtracting eq. (iii) from eq. (i), we get

$\alpha = 8$ ½

Substituting $\alpha = 8$ in eq. (ii), we get

$8 + \beta = 6$

$\Rightarrow$ $\beta = 6 - 8 = -2$ **1**

$\because$ $\alpha\beta = a$

Then, $a = 8(-2) = -16.$ **1**

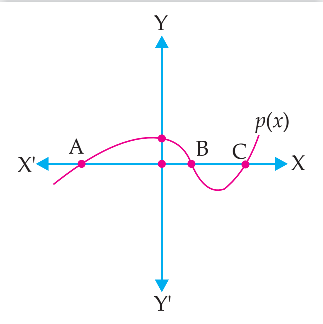
LEVEL-1 OBJECTIVE TYPE QUESTIONS

[A] Multiple Choice Questions

Sol. 2: Option (C) is correct.

Explanation: According to the property of the polynomials,

Number of zeroes = Number of points at which graph intersects the x-axis.



From the figure it is clear that the graph intersects X-axis at three different points. Therefore, the polynomial has 3 zeroes.

Sol. 6: Option (C) is correct.

Explanation: For zeroes

$$(x - 2)(x + 3) = 0$$

Either, $x - 2 = 0 \Rightarrow x = 2$

or, $x + 3 = 0 \Rightarrow x = -3$

So, we get two values of x *i.e.*,

$$x = 2 \text{ or } -3.$$

Hence, quadratic polynomial intersects x-axis at two points.

[B] Assertion and Reason Based Questions

Ans. 3. Option (B) is correct.

Explanation: In case of assertion:

Since the graph intersects the x-axis 5 times,

So, the number of zeroes of $p(x)$ is 5.

$\therefore$ Assertion is true.

In case of reason:

If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, and then graph of the polynomial will intersect at two points on x-axis.

$\therefore$ Reason is also true.

Thus, both assertion and reason are true but reason is not the correct explanation for assertion.

REFLECTIONS

- Did you understand the relationship between the zeroes and the co-efficients of a polynomial?
- Will you be able to evaluate the sum and product of zeroes in given polynomial $\sqrt{3}x^2 - 10x + \frac{3}{5}$

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