Sample Question Paper-1

(Issued by CBSE on 31st March, 2023)

MATHEMATICS STANDARD

Class-X, Session: 2023-24

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) This Question Paper has 5 Sections A, B, C, D and E.

(ii) Section A has 20 MCQs carrying 1 mark each

(iii) Section B has 5 questions carrying 02 marks each.

(iv) Section C has 6 questions carrying 03 marks each.

(v) Section D has 4 questions carrying 05 marks each.

(vi) Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.

(vii) All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E

(viii) Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section-A

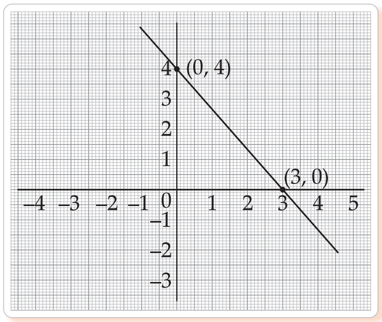
Section A Consists of Multiple Choice Type questions of 1 mark each

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is

(A) xy **(B)** xy^2

(C) x^3y^3 **(D)** x^2y^2 **1**

2. The given linear polynomial $y = f(x)$ has



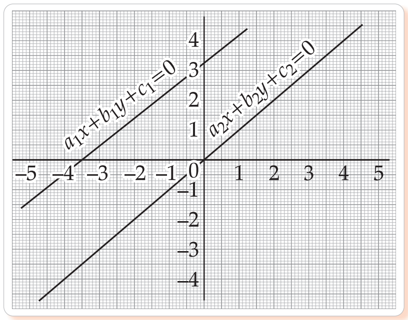
(A) 2 zeros

(B) 1 zero and the zero is '3'

(C) 1 zero and the zero is '4'

(D) No zero **1**

3. The lines representing the given pair of linear equations are non-intersecting. Which of the following statements is true?



(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(C) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(D) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ **1**

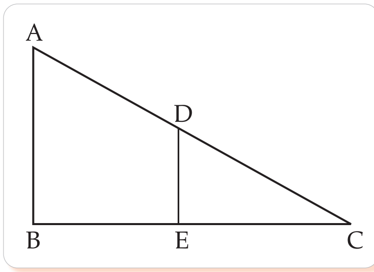
4. The nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$ is:
(A) No real roots (B) 2 equal real roots (C) 2 distinct real roots (D) More than 2 real roots **1**

5. Two APs have the same common difference. The first term of one of these is – 1 and that of the other is – 8. The difference between their 4th terms is:
(A) 1 (B) – 7 (C) 7 (D) 9 **1**

6. What is the ratio in which the line segment joining (2, –3) and (5, 6) is divided by x-axis?
(A) 1 : 2 (B) 2 : 1 (C) 2 : 5 (D) 5 : 2 **1**

7. A point (x, y) is at a distance of 5 units from the origin. How many such points lie in the third quadrant?
(A) 0 (B) 1 (C) 2 (D) infinitely many **1**

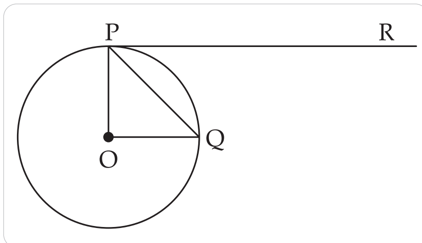
8. In ΔABC, DE || AB. If $AB = a$, $DE = x$, $BE = b$ and $EC = c$. Then x expressed in terms of a, b and c is:



(A) $\frac{ac}{b}$ (B) $\frac{ac}{b+c}$

(C) $\frac{ab}{c}$ (D) $\frac{ab}{b+c}$ **1**

9. If O is centre of a circle and chord PQ makes an angle 50° with the tangent PR at the point of contact P, then the angle subtended by the chord at the centre is:



(A) 130° (B) 100°
(C) 50° (D) 30° **1**

10. A quadrilateral PQRS is drawn to circumscribe a circle.
If $PQ = 12$ cm, $QR = 15$ cm and $RS = 14$ cm, find the length of SP is:
(A) 15 cm (B) 14 cm (C) 12 cm (D) 11 cm **1**

11. Given that $\sin\theta = \frac{a}{b}$, find $\cos\theta$ is:

(A) $\frac{b}{\sqrt{b^2-a^2}}$ (B) $\frac{b}{a}$ (C) $\frac{\sqrt{b^2-a^2}}{b}$ (D) $\frac{a}{\sqrt{b^2-a^2}}$ **1**

12. $(\sec A + \tan A)(1 - \sin A)$ equals:
(A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$ **1**

13. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is:
(A) 60° (B) 45° (C) 30° (D) 90° **1**

14. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is:
(A) 2 units (B) π units (C) 4 units (D) 7 units **1**

15. It is proposed to build a new circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is:
(A) 10 m (B) 15 m (C) 20 m (D) 24 m

16. There is a complete shaded square board of side '$2a$' units circumscribing a shaded circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the complete shaded region.
(A) $\frac{\pi}{4}$ (B) $\frac{4-\pi}{4}$ (C) $\frac{\pi-4}{4}$ (D) $\frac{4}{\pi}$ **1**

17. 2 cards of hearts and 4 cards of spades are missing from a pack of 52 cards. A card is drawn at random from the remaining pack. What is the probability of getting a black card?
(A) $\frac{22}{52}$ (B) $\frac{22}{46}$ (C) $\frac{24}{52}$ (D) $\frac{24}{46}$ **1**

18. The upper limit of the modal class of the given distribution is:

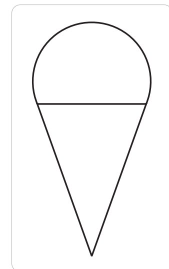
Height [in cm]	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
Number of girls	4	11	29	40	46	51

(A) 165 **(B)** 160 **(C)** 155 **(D)** 150 **1**

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

19. **Statement A (Assertion):** Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.

Statement R (Reason): Top is obtained by joining the plane surfaces of the hemisphere and cone together.



(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true. **1**

20. **Statement A (Assertion):** $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$ is in Arithmetic Progression.

Statement R (Reason): The terms of an Arithmetic Progression cannot have both positive and negative rational numbers.

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true. **1**

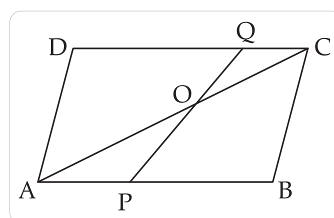
Section-B

Section B consists of 5 questions of 2 marks each.

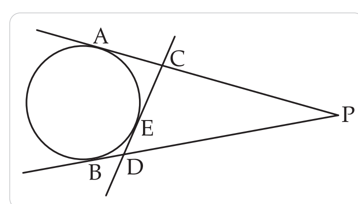
21. Prove that $\sqrt{2}$ is an irrational number. **2**

22. ABCD is a parallelogram. Point P divides AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1.

Prove that OC is half of OA. **2**



23. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If $PA = 10$ cm, find the perimeter of ΔPCD. **2**



24. **(A)** If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B < 90°$; $A > B$, find A and B. **2**

OR

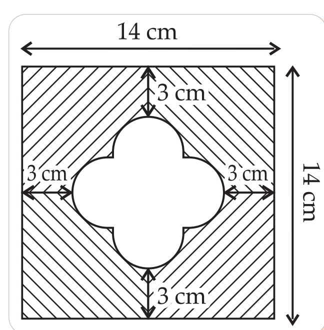
(B) Find the value of x if

$$2 \operatorname{cosec}^2 30° + x \sin^2 60° - \frac{3}{4} \tan^2 30° = 10$$

25. **(A)** With vertices A, B and C of Δ ABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle. **2**

OR

(B) Find the area of the unshaded region shown in the given figure.



Section-C

Section C Consists of 6 questions of 3 marks each

26. National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same art form and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room? **3**

27. If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of

(1) $\alpha^2 + \beta^2$

(2) $\alpha^{-1} + \beta^{-1}$ **3**

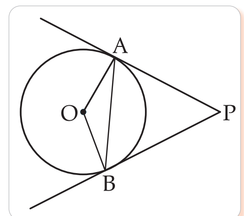
28. **(A)** The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? **3**

OR

(B) Solve: $\dfrac{2}{\sqrt{x}} + \dfrac{3}{\sqrt{y}} = 2;\ \dfrac{4}{\sqrt{x}} - \dfrac{9}{\sqrt{y}} = -1\ \ x, y > 0$

29. **(A)** PA and PB are tangents drawn to a circle of centre O from an external point P. Chord AB makes an angle of 30° with the radius at the point of contact.

If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA. **3**



OR

(B) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

30. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\dfrac{1}{2}$. **3**

31. The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table.

Length [in mm]	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the mean length of the leaves. **3**

Section-D

Section D consists of 4 questions of 5 marks each.

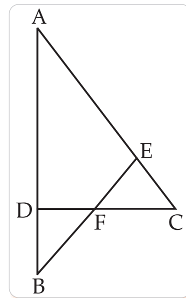
32. **(A)** A motor boat whose speed is 18 km/h in still water takes 1 h. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream. **5**

OR

(B) Two water taps together can fill a tank in $9\dfrac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

33. **(a)** State and prove Basic Proportionality theorem.

(b) In the given figure ∠CEF = ∠CFE. F is the midpoint of DC.



Prove that $\frac{AB}{BD} = \frac{AE}{FD}$ **5**

34. **(A)** Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

What should be the speed of water if the rise in water level is to be attained in 1 hour? **5**

OR

(B) A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.

35. The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data. **5**

Marks obtained	Number of students
20 – 30	p
30 – 40	15
40 – 50	25
50 – 60	20
60 – 70	q
70 – 80	8
80 - 90	10

Section-E

Case study based questions are compulsory.

36. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete.

Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day.

Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week.

During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

(i) How many throws Sanjitha practiced on 11th day of the camp? **1**

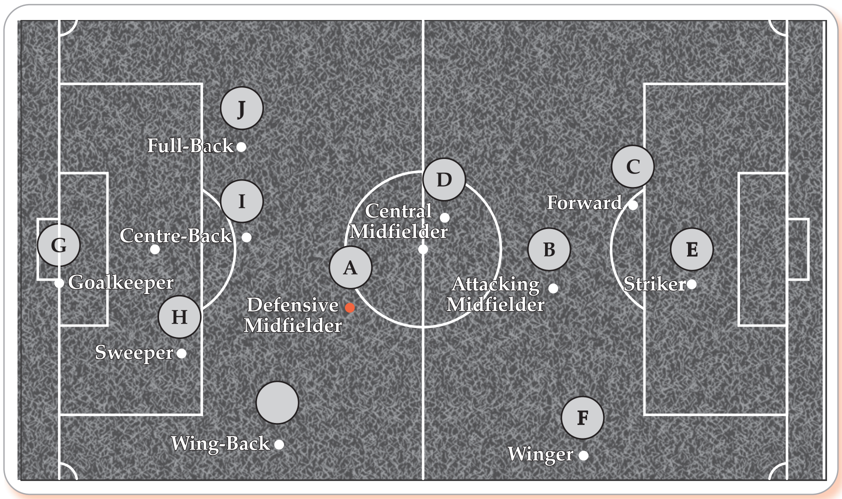
(ii) What would be Sanjitha's throw distance at the end of 6 weeks?

OR

When will she be able to achieve a throw of 11.16 m? **2**

(iii) How many throws did she do during the entire camp of 15 days ? **1**

37. Tharunya was thrilled to know that the football tournament is fixed with a monthly time frame from 20th July to 20th August 2023 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.



(i) At an instance, the mid-fielders and forward formed a parallelogram. Find the position of the central mid-fielder (D) if the position of other players who formed the parallelogram are: A(1, 2), B(4, 3) and C(6, 6) **1**

(ii) Check if the Goal keeper G(–3, 5), Sweeper H(3, 1) and Wing-back K(0, 3) fall on a same straight line.

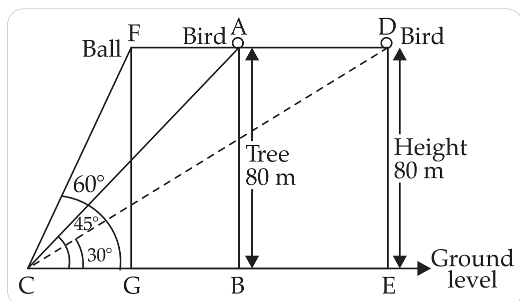
OR

Check if the Full-back J(5, –3) and centre-back I(–4, 6) are equidistant from forward C(0, 1) and if C is the mid-point of IJ. **2**

(iii) If Defensive mid-fielder A(1, 4), Attacking mid-fielder B(2, –3) and Striker E(a, b) lie on the same straight line and B is equidistant from A and E, find the position of E. **1**

38. One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80 m. He observed a bird on the tree at an angle of elevation of 45°.

When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.



(i) At what distance from the foot of the tree was he observing the bird sitting on the tree? **1**

(ii) How far did the bird fly in the mentioned time?

OR

After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball? **2**

(iii) What is the speed of the bird in m/min if it had flown $20(\sqrt{3}+1)$ m? **1**

SOLUTIONS
Sample Question Paper-1
MATHEMATICS STANDARD

All solutions are as per the CBSE Board Marking Scheme 2023-24

Section-A

1. Option (B) is correct. 1

(B) xy^2

Detailed Answer:

Given, $a = x^3y^2$ and $b = xy^3$

$$\text{LCM } (a, b) = x^3y^3$$

$$\text{Product } (A) \text{ and } (B) = x^3y^2 \times xy^3$$

Dividing product (a, b) by LCM (a, b)

$$\frac{\text{Product } a \text{ and } b}{\text{LCM}} = \frac{x^3y^2 \times xy^3}{x^3y^3} = xy^2$$

2. Option (B) is correct. 1

(B) 1 zero and the zero is '3'

Detailed Answer:

Polynomial $y = f(x)$ is intersect x-axis at only one point, so the zero is one and intersect x-axis at point (3), so the zero of the polynomial $y = f(x)$ is 3.

3. Option (B) is correct. 1

(B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Detailed Answer:

Two pair of linear equations is non intersecting so there line is parallel lines, there are no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

4. Option (C) is correct. 1

(C) 2 distinct real roots

Detailed Answer:

Given quadratic equation is $9x^2 - 6x - 2 = 0$

$$9x^2 - 6x - 2 = 0$$
$$a = 9$$
$$b = -6$$
$$c = -2$$

Now, $b^2 - 4ac$

$\Rightarrow (-6)^2 - 4 \times 9 \times (-2)$

$\Rightarrow 36 + 72$

$b^2 - 4ac > 0$

So the roots is distinct real. Here, the given equation is of order 2, so there are two roots.

5. Option (C) is correct. 1

(C) 7

Detailed Answer:

Given two APs and their first term are –1 and –8 and have same common difference.

Since, n^{th} term is given as $T_n = a + (n - 1)\, d$

Now, 4^{th} term of first AP

$$T_4 = (-1) + (4 - 1)d$$
$$T_4 = -1 + 3d \quad \text{...(i)}$$

4^{th} term of second AP

$$T_4' = -8 + 3d \quad \text{...(iii)}$$

Subtract eq (ii) from eq (i)

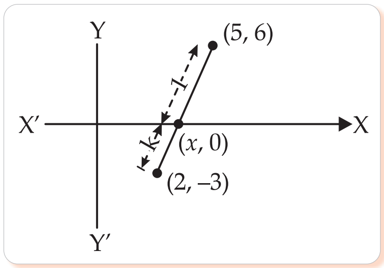
$$T_4 - T_4' = (-1 + 3d) - (-8 + 3d)$$
$$T_4 - T_4' = 7$$

Hence the difference between their 4th term is 7.

6. Option (A) is correct. 1

(A) 1 : 2

Detailed Answer:



When the line is divided by x-axis, the intersect point be $(x, 0)$, Let the ratio is $k : 1$

$$x = \frac{x_2 \times m + x_1 \times n}{m + n}$$

$$y = \frac{y_2 \times m + y_1 \times n}{m + n}$$

$$0 = \frac{k \times 6 - 3 \times 1}{k + 1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

7. Option (D) is correct. **1**

(D) infinitely many

Detailed Answer:

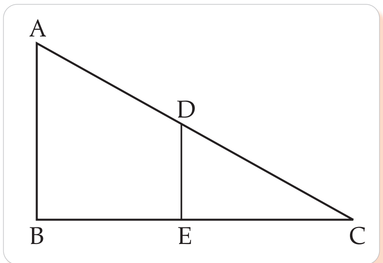
(x, y) is 5 units from the origin. There are infinity many point in from origin between point (x, y).

8. Option (B) is correct. **1**

(B) $\frac{ac}{b+c}$

Detailed Answer:

Using basic proportional theorem.



$$\frac{CE}{BC} = \frac{CD}{AC} = \frac{DE}{AB}$$

$CE = c, BE = b, DE = x, AB = a$

Now $$\frac{DE}{AB} = \frac{CE}{BC}$$

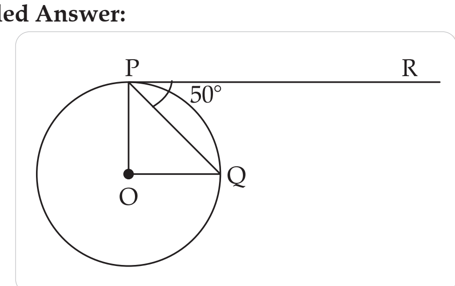
$$\Rightarrow \quad \frac{x}{a} = \frac{c}{b+c}$$

$$\Rightarrow \quad x = \frac{ac}{b+c}$$

9. Option (B) is correct. **1**

(B) 100°

Detailed Answer:



Given $\angle QPR = 50°$

PQ is a tangent, so $\angle OPR$ is 90°.

Then, $\angle OPR = \angle OPQ + \angle QPR$

$90° = \angle OPQ + 50°$

$40° = \angle OPQ = \angle OQP$

(radii of the circle)

In ΔOQP

$\angle POQ + \angle OQP + \angle OPQ = 180°$

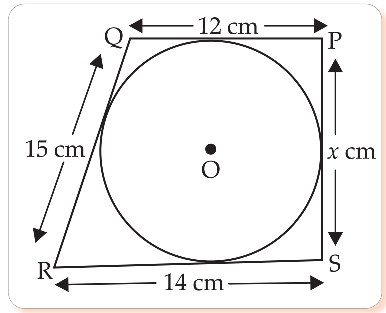
$\angle POQ + 40° + 40° = 180°$

$\angle POQ = 100°$

10. Option (D) is correct. **1**

(D) 11 cm

Detailed Answer:



Given, a quadrilateral PQRS is drawn to circumscribe a circle PQ, QR, RS and SP is the tangent to circle then, using the tangent property

$$PQ + RS = PS + QR$$
$$12 + 14 = x + 15$$
$$26 - 15 = x$$
$$x = 11 \text{ cm}$$

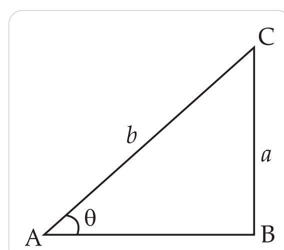
11. Option (C) is correct. **1**

(C) $\frac{\sqrt{b^2 - a^2}}{b}$

Detailed Answer:

$$\sin\theta = \frac{a}{b}$$

$$\sin\theta = \frac{BC}{AC} = \frac{a}{b}$$



Using Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$
$$b^2 = (AB)^2 + (a)^2$$
$$AB^2 = b^2 - a^2$$
$$AB = \sqrt{b^2 - a^2}$$

Now, $$\cos\theta = \frac{AB}{AC}$$

$$= \frac{\sqrt{b^2 - a^2}}{b}$$

12. Option (D) is correct. **1**

(D) cos A

Detailed Answer:

Since, (sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1+\sin A)(1-\sin A)}{\cos A}$$

$$= \frac{1-\sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

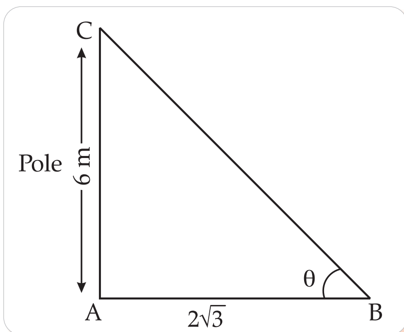
$$= \cos A$$

13. Option (A) is correct. 1

(A) 60°

Detailed Answer:

Let the pole AC is 6 m and shadow AB is $2\sqrt{3}$ m.



Now, $\tan\theta = \frac{AC}{AB}$

$$\tan\theta = \frac{6}{2\sqrt{3}}$$

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = \tan 60°$$

$$\theta = 60°$$

14. Option (A) is correct. 1

(A) 2 units

Detailed Answer:

Let radius of the circle be r

Given, perimeter of circle = Area of circle

$$2\pi r = \pi r^2$$

$$2 = r$$

$$r = 2 \text{ units}$$

15. Option (A) is correct. 1

(A) 10 m

Detailed Answer:

Let radius of new park be R.

$$r_1 = 8 \text{ cm}$$

$$r_2 = 6 \text{ m}$$

Now, Area of new park = Sum of area of two circular parks

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi R^2 = \pi(r_1^2 + r_2^2)$$

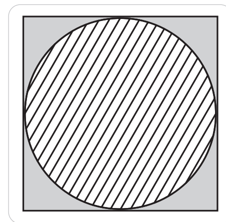
$$R^2 = 64 + 36$$

$$R = 10 \text{ m}$$

16. Option (B) is correct. 1

(B) $\frac{4-\pi}{4}$

Detailed Answer:



Since, Total area of board = Area of complete shaded + Area of shaded region = Area of square board $(2a)^2 = 4a^2$ unit^2

Radius of shaded circle $= \frac{2a}{2} = a$

Now Area of complete shaded region= Area of square board – Area of shaded circle

$$= 4a^2 - \pi a^2$$

$$= a^2(4-\pi)$$

$$\text{Required probability} = \frac{\text{Area of complete shaded region}}{\text{Total area}}$$

$$= \frac{a^2(4-\pi)}{4a^2} = \frac{4-\pi}{4}$$

17. Option (B) is correct. 1

(B) $\frac{22}{46}$

Detailed Answer:

Given, 2 cards of hearts and 4 cards are missing then,

Remaining total cards = 52 – 6

$n(S) = 46$

Total black cards = 26

remaining black cards $n(E) = 26 - 4$

$= 22$

Probability (getting black cards)

$$= \frac{n(E)}{n(S)}$$

$$= \frac{22}{46}$$

18. Option (D) is correct. 1

(D) 150

Detailed Answer:

Class	Frequency
135-140	4
140-145	7
145-150	18
150-155	11
155-160	6
160-165	5

Higher frequency is 18 then modal class is 145 – 150.
The upper limit at modal class is 150.

19. **Option (A) is correct.** 1

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Detailed Answer:

Total surface area of top

= Curved surface of cone

+ Curved surface area of hemisphere

Assertion is true.

Reason: Top is obtained by fixing the plane surface of hemisphere and cone together. The flat surface of the cone and hemisphere will be contact with each other when they are fixed together to form the top.

Reason is true.

Both assertion and reason are true and reason (R) is correct explanation of assertion (A).

20. **Option (C) is correct.** 1

(C) Assertion (A) is true but reason (R) is false.

Detailed Answer:

Assertion: In given series $-5, \frac{-5}{2}, 0, \frac{5}{2}.....$

$$\text{Common difference} = T_2 - T_1 = \frac{-5}{2} - (-5) = \frac{-5}{2} + 5 = \frac{5}{2}$$

$$\text{Common difference} = T_3 - T_2 = 0 - \left(\frac{-5}{2}\right) = \frac{5}{2}$$

Hence, common difference is equal each term so this series is A.P. So, assertion is true.

Reason: The term of A.P. have also negative and positive rational number, so reason is false.

Section-B

21. Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers a and b such that $\sqrt{2} = \frac{a}{b}$

where a and b are coprime.

So, $b\sqrt{2} = a$

Squaring both sides, we get $2b^2 = a^2$

Therefore, 2 divides a^2 and so 2 divides a.

So, we can write $a = 2c$ for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b

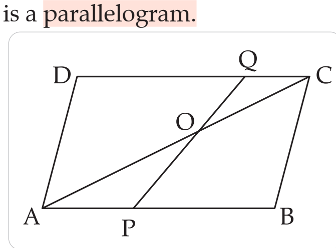
Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational. 2

22. ABCD is a parallelogram.



$$AB = DC = a$$

Point P divides AB in the ratio 2 : 3

$$AP = \frac{2}{5}a, \; BP = \frac{3}{5}a$$

Point Q divides DC in the ratio 4 : 1.

$$DQ = \frac{4}{5}a, \; CQ = \frac{1}{5}a$$

$$\Delta APO \sim \Delta CQO$$

$$\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$$

[AA similarity]

$$\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1}$$

$$\Rightarrow \quad OC = \frac{1}{2}OA$$

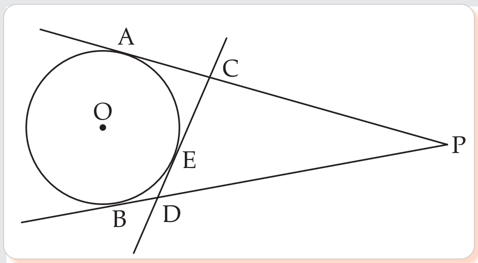
2

23. $PA = PB; CA = CE; DE = DB$

[Tangents to a circle]

$$\begin{aligned}\text{Perimeter of } \Delta PCD &= PC + CD + PD \\ &= PC + CE + ED + PD \\ &= PC + CA + BD + PD \\ &= PA + PB\end{aligned}$$

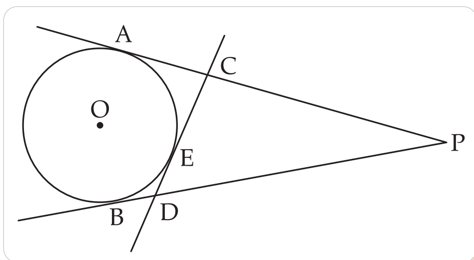
$$\begin{aligned}\text{Perimeter of } \Delta PCD &= PA + PA = 2PA \\ &= 2(10) = 20 \text{ cm}\end{aligned}$$



2

Detailed Answer:

P is a external point and PA and PB are drawn then $PA = PB = 10$ cm



C, D is also external point and drawn CA, CE and ED, BD

Then
$$ED = BD$$
$$AC = CE$$

Now, Perimeter of $\Delta PCD = PC + CD + PD$
$$= PC + CE + ED + PD$$
$$= (PC + AC) + (BD + PD)$$
$$= PA + PB$$
$$= PA + PA$$
$$= 2PA$$
$$= 2 \times 10 \text{ cm}$$
$$= 20 \text{ cm}$$

24. **(A)** $\because$ $\tan(A + B) = \sqrt{3}$

$\therefore$ $A + B = 60°$ ...(i)

$\because$ $\tan(A - B) = \dfrac{1}{\sqrt{3}}$

$\therefore$ $A - B = 30°$ ...(ii)

Adding (i) & (ii), we get $2A = 90°$

$\Rightarrow$ $A = 45°$

Also (i) – (ii), we get $2B = 30°$

$\Rightarrow$ $B = 15°$ **2**

OR

(B) $2\,\text{cosec}^2\,30° + x\sin^2 60° - \dfrac{3}{4}\tan^2 30° = 10$

$$\Rightarrow \quad 2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

$$\Rightarrow \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$$

$$\Rightarrow \quad 8 + x\left(\frac{3}{4}\right) - \frac{1}{4} = 10$$

$$\Rightarrow \quad 32 + x(3) - 1 = 40$$

$$\Rightarrow \quad 3x = 9$$

$$\Rightarrow \quad x = 3$$ **2**

25. **(A)** Total area removed

$$= \frac{\angle A}{360°}\pi r^2 + \frac{\angle B}{360°}\pi r^2 + \frac{\angle C}{360°}\pi r^2$$

$$= \frac{\angle A + \angle B + \angle C}{360°}\pi r^2$$

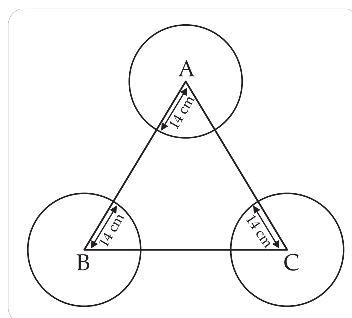
$$= \frac{180°}{360°}\pi r^2$$

$$= \frac{180°}{360°} \times \frac{22}{7} \times (14)^2$$

$= 308 \text{ cm}^2$ **2**

[CBSE Marking Scheme, 2023]

Detailed Answer:



Let there are three circles be C_A, C_B and C_C

Now Area of sector $A = \dfrac{\angle A}{360°}\pi r^2$

Area of sector $B = \dfrac{\angle B}{360°}\pi r^2$

Area of sector $C = \dfrac{\angle C}{360°}\pi r^2$

Total removed area from the triangle

= ar. of sector A + ar. of sector B + ar. of sector C

$$= \frac{\angle A}{360°}\pi r^2 + \frac{\angle B}{360°}\pi r^2 + \frac{\angle C}{360°}\pi r^2$$

$$= \frac{\pi r^2}{360°}(\angle A + \angle B + \angle C)$$

$\therefore (\angle A + \angle B + \angle C = 180°)$

$$= \pi r^2 \times \frac{180°}{360°}$$

$$= \frac{\pi \times 14 \times 14}{2}$$

$$= \frac{\frac{22}{7} \times 14 \times 14}{2} = 308 \text{ cm}^2$$

OR

(B) The side of a square

= Diameter of the semi-circle = a

Area of the unshaded region

= Area of a square of side 'a'

+ 4(Area of a semi-circle of diameter 'a')

The horizontal/vertical extent of the white region

= 14 – 3 – 3 = 8 cm

Radius of the semi-circle + Side of a square + Radius of the semi-circle = 8 cm

2 (radius of the semi-circle) + side of a square

= 8 cm

$2a$ = 8 cm

$\Rightarrow$ a = 4 cm

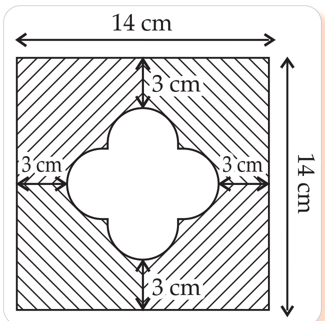
Area of the unshaded region

= Area of a square of side 4 cm

+ 4 (Area of a semi-circle of diameter 4 cm)

$$= (4)^2 + 4\times\frac{1}{2}\pi(2)^2$$

$= 16 + 8\pi$ cm^2



2

Section-C

26. Number of students in each group subject to the given condition = HCF (60, 84, 108)

HCF (60, 84, 108) = 12

Number of groups in Music = $\frac{60}{12} = 5$

Number of groups in Dance = $\frac{84}{12} = 7$

Number of groups in Handicrafts = $\frac{108}{12} = 9$

Total number of rooms required = 21 **3**

27.

$$P(x) = 5x^2 + 5x + 1$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-5}{5} = -1$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{5}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-1)^2 - 2\left(\frac{1}{5}\right)$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

$$\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$$ **3**

28. **(A)** Let the ten's and the unit's digits in the first number be x and y, respectively.

So, the original number = $10x + y$

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit.

So the obtain by reversing the digits = $10y + x$

According to the given condition.

$(10x + y) + (10y + x) = 66$

i.e., $11(x + y) = 66$

i.e., $x + y = 6$ ...(i)

We are also given that the digits differ by 2,

therefore, either $x - y = 2$ ...(ii)

or $y - x = 2$ ...(iii)

If $x - y = 2$, then solving (i) and (ii) by elimination, we get $x = 4$ and $y = 2$.

In this case, we get the number 42.

If $y - x = 2$, then solving (i) and (iii) by elimination, we get $x = 2$ and $y = 4$.

In this case, we get the number 24.

Thus, there are two such numbers 42 and 24. **3**

OR

(B) Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n',

Then the given equations become

$2m + 3n = 2$

$4m - 9n = -1$

$(2m + 3n = 2) \times -2$

$\Rightarrow$ $-4m - 6n = -4$ ...(i)

$4m - 9n = -1$ ...(ii)

Adding (i) and (ii)

We get, $-15n = -5$

$\Rightarrow$ $n = \frac{1}{3}$

Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get

$2m + 1 = 2$

$2m = 1$

$m = \frac{1}{2}$

$\Rightarrow$ $\sqrt{x} = 2$

$\Rightarrow$ $x = 4$ and $n = \frac{1}{3}$

$\Rightarrow$ $\sqrt{y} = 3$

$\Rightarrow$ $y = 9$ **3**

29. **(A)** $\angle OAB = 30°$

$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle PAB = 90° - 30° = 60°$

$AP = BP$

[Tangents to a circle from an external point]

$\angle PAB = \angle PBA$

[Angles opposite to equal sides of a triangle]

In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$

[Angle Sum Property]

$60° + 60° + \angle APB = 180°$

$\angle APB = 60°$

$\therefore$ ΔABP is an equilateral triangle, where $AP = BP = AB$.

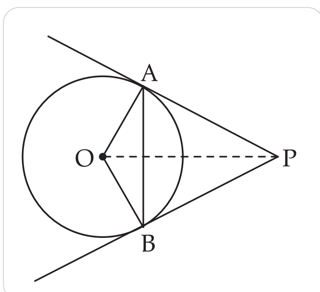
$PA = 6$ cm

In Right ΔOAP, $\angle OPA = 30°$

$$\tan 30° = \frac{OA}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{OA}{6}$$

$$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$ **3**



OR

(B) Let $\angle TPQ = \theta$

$\angle TPO = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle OPQ = 90° - \theta$ **1**

$TP = TQ$

[Tangents to a circle from an external point]

$\angle TPQ = \angle TQP = \theta$

[Angles opposite to equal sides of a triangle]

In ΔPQT, $\angle PQT + \angle QPT + \angle PTQ = 180°$

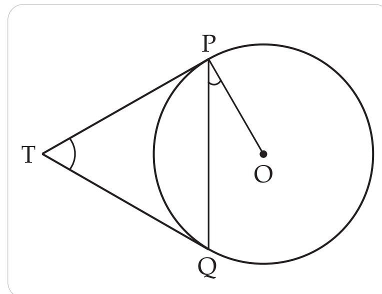
[Angle Sum Property]

$\theta + \theta + \angle PTQ = 180°$

$\angle PTQ = 180° - 2\theta$

$\angle PTQ = 2\,(90° - \theta)$

$\angle PTQ = 2\,\angle OPQ$ [using (1)]



3

30. Given, $1 + \sin^2\theta = 3 \sin\theta \cos\theta$

Dividing both sides by $\cos^2\theta$,

$$\frac{1}{\cos^2\theta} + \tan^2\theta = 3\tan\theta$$

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$$

$$1 + 2\tan^2\theta = 3\tan\theta$$

$$2\tan^2\theta - 3\tan\theta + 1 = 0$$

If $\tan\theta = x$, then the equation becomes

$$2x^2 - 3x + 1 = 0$$

$$\Rightarrow \quad (x-1)(2x-1) = 0$$

$$x = 1 \text{ or } \frac{1}{2}$$

$$\tan\theta = 1 \text{ or } \frac{1}{2}$$ **3**

31.

Length [in mm]	Number of leaves (f)	CI	Mid x	$d = \|x_i - a\|$	fd
118 – 126	3	117.5 – 126.5	122	– 27	– 81
127 – 135	5	126.5 – 135.5	131	– 18	– 90
136 – 144	9	135.5 – 144.5	140	– 9	– 81
145 – 153	12	144.5 – 153.5	$a = 149$	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

$$\text{Mean} = a + \frac{\Sigma fd}{\Sigma f} = 149 + \frac{-81}{40}$$

$$= 149 - 2.025 = 146.975$$

Average length of the leaves = 146.975

Section-D

32. **(A)** Let the speed of the stream be x km/h.

The speed of the boat upstream = $(18 - x)$ km/h and
The speed of the boat downstream = $(18 + x)$ km/h.

The time taken to go upstream

$$= \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{24}{18-x} \text{ hours}$$

The time taken to go downstream

$$= \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{24}{18+x} \text{ hours}$$

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24(18+x) - 24(18-x) = (18-x)(18+x)$$

$$x^2 + 48x - 324 = 0$$

$$x = 6 \text{ or } -54$$

Since x is the speed of the stream, it cannot be negative.

Therefore, $x = 6$ gives the speed of the stream

= 6 km/h **5**

OR

(B) Let the time taken by the smaller pipe to fill the tank = x h

Time taken by the larger pipe = $(x - 10)$ h

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour

$$= \frac{1}{x-10}$$

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Part of the tank filled by both the pipes in 1 hour

$$= \frac{8}{75}$$

Therefore, $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$$8x^2 - 230x + 750 = 0$$

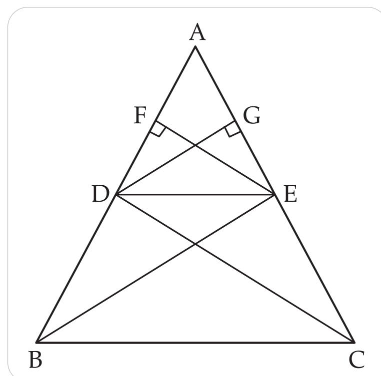
$$x = 25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible.

Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 = 15 hours, respectively. **5**

33. **(a) Statement:** If a line is drawn parallel to one side of a triangle intersecting the other two side of distinct point, then the other two sides are divided in the same ratio.

Let ΔABC in which a line DE parallel to BC intersects AB at D and AC at E



To proof: DE divides the two sides in the same ratio

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Join BE and CD. Draw EF ⊥ AB and DG ⊥ AC

Proof: Since, Area of triangle = $\frac{1}{2}$ × base × height

then $$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$$

$$= \frac{AD}{DB} \quad \text{...(i)}$$

and $$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta DEC} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD}$$

$$= \frac{AE}{EC} \quad \text{...(ii)}$$

Since, ΔBDE and ΔDEC lie between the same parallel DE and BE and are on the same base DE.

We have area of ΔBDE = area of ΔDEC ...(iii)

From eqs. (i), (ii) and (iii) we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved**

(b) Draw DG || BE

In ΔABE, $\frac{AB}{BD} = \frac{AE}{GE}$ [BPT]

$CF = FD$

[F is the midpoint of DC] ...(i)

In ΔCDG, $\frac{DF}{CF} = \frac{GE}{CE} = 1$

[Mid point theorem]

$GE = CE$ ...(ii)

$\angle CEF = \angle CFE$ (Given)

$CF = CE$ ...(iii)

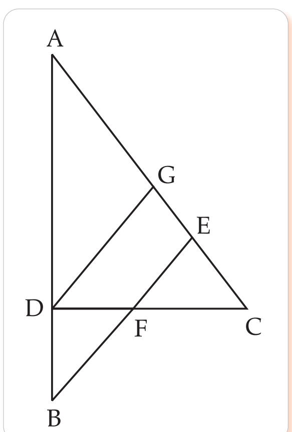
[Sides opposite to equal angles]

From (ii) & (iii) $CF = GE$ ...(iv)

From (i) & (iv) $GE = FD$

$\therefore$ $\frac{AB}{BD} = \frac{AE}{GE}$

$\Rightarrow$ $\frac{AB}{BD} = \frac{AE}{FD}$



5

34. **(A)** Length of the pond, $l = 50$ m, width of the pond, $b = 44$ m

Water level is to rise by, $h = 21 \text{ cm} = \frac{21}{100}$ m

Volume of water in the pond

$$= lbh = 50 \times 44 \times \frac{21}{100} \text{ m}^3$$

$$= 462 \text{ m}^3$$

Diameter of the pipe = 14 cm

Radius of the pipe, $r = 7 \text{ cm} = \frac{7}{100}$ m

Area of cross-section of pipe $= \pi r^2$

$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100}$$

$$= \frac{154}{10000} \text{ m}^2$$

Rate at which the water is flowing through the pipe, $h = 15$ km/h = 15000 m/h

Volume of water flowing in 1 hour = Area of cross-section of pipe × height of water coming out of pipe

$$= \left(\frac{154}{10000} \times 15000\right) \text{ m}^3$$

Time required to fill the pond

$$= \frac{\text{Volume of the pond}}{\text{Volume of water flowing in 1 hour}}$$

$$= \frac{462 \times 10000}{154 \times 15000} = 2 \text{ hours}$$

Speed of water if the rise in water level is to be attained in 1 hour = 30 km/h **5**

OR

(B) Radius of the cylindrical tent (r) = 14 m

Total height of the tent = 13.5 m

Height of the cylinder = 3 m

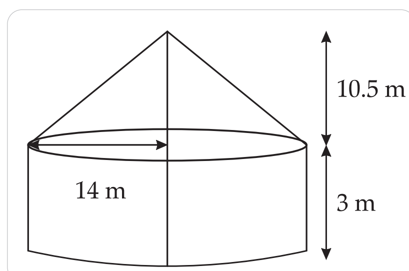
Height of the Conical part = 10.5 m

Slant height of the cone (l) $= \sqrt{h^2 + r^2}$

$$= \sqrt{(10.5)^2 + (14)^2}$$

$$= \sqrt{110.25 + 196}$$

$$= \sqrt{306.25} = 17.5 \text{ m}$$



Curved surface area of cylindrical portion

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 14 \times 3$$

$$= 264 \text{ m}^2$$

Curved surface area of conical portion

$$= \pi rl$$

$$= \frac{22}{7} \times 14 \times 17.5$$

$$= 770 \text{ m}^2$$

Total curved surface area $= 264\text{ m}^2 + 770\text{ m}^2$

$= 1034\text{ m}^2$

Provision for stitching and wastage $= 26\text{ m}^2$

Area of canvas to be purchased $= 1060\text{ m}^2$

Cost of canvas = Rate × Surface area

$= 500 \times 1060$

$= ₹\,5,30,000$ **5**

35.

Marks obtained	Number of students	Cumulative frequency
20 –30	p	p
30 – 40	15	$p + 15$
40 – 50	25	$p + 40$
50 – 60	20	$p + 60$
60 – 40	q	$p + q + 60$
70 – 80	8	$p + q + 68$
80 – 90	10	$p + q + 78$
	90	

$p + q + 78 = 90$ **1**

$p + q = 12$

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$ **1**

$$50 = 50 + \frac{45-(p+40)}{20} \times 10$$

$$\frac{45-(p+40)}{20} \times 10 = 0$$

$45-(p + 40) = 0$

$p = 5$

$5 + q = 12$

$q = 7$ **1**

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}.h$$ **1**

$$= 40 + \frac{25-15}{2(25)-15-20} \times 10$$

$$= 40 + \frac{100}{15}$$

$= 40 + 6.67$

$= 46.67$ **1**

Section-E

36. **(i)** Number of throws during camp, $a = 40$; $d = 12$

$t_{11} = a + 10d$

$= 40 + 10 \times 12$

$= 160$ throws **1**

(ii) $a = 7.56$ m; $d = 9$ cm $= 0.09$ m

$n = 6$ weeks

$t_n = a + (n-1)d$

$t_{11} = 7.56 + 6(0.09)$

$= 7.56 + 0.54$

Sanjitha's throw distance at the end of 6 weeks = 8.1 m **2**

OR

$a = 7.56$ m; $d = 9$ cm $= 0.09$ m

$t_n = 11.16$ m

$t_n = a + (n-1)d$

$11.16 = 7.56 + (n-1)(0.09)$

$3.6 = (n-1)(0.09)$

$$n - 1 = \frac{3.6}{0.09} = 40$$

$n = 41$ **2**

Sanjitha's will be able to throw 11.16 m in 41 weeks.

(iii) $a = 40$; $d = 12$; $n = 15$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{15}{2}[2(40) + (15-1)(12)]$$

$$= \frac{15}{2}[80 + 168]$$

$$= \frac{15}{2}[248]$$

$= 1860$ throws **1**

37. **(i)** Let D be (a, b), then

Mid point of AC = Midpoint of BD

$$\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$$

$4 + a = 7, \quad 3 + b = 8$

$a = 3, \quad b = 5$

Central mid-fielder is at (3, 5). **1**

(ii) $GH = \sqrt{(-3-3)^2 + (5-1)^2}$

$= \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

$GK = \sqrt{(0+3)^2 + (3-5)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

$HK = \sqrt{(3-0)^2 + (1-3)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

$GK + HK = GH$ **2**

⇒ G, H & K lie on a same straight line

OR

$CJ = \sqrt{(0-5)^2 + (1+3)^2}$

$= \sqrt{25+16}$

$= \sqrt{41}$

$$CI = \sqrt{(0+4)^2+(1-6)^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

Full-back J(5, –3) and centre-back I(–4, 6) are equidistant from forward C(0, 1).

$$\text{Mid-point of } IJ = \left(\frac{5-4}{2}, \frac{-3+6}{2}\right)$$
$$= \left(\frac{1}{2}, \frac{3}{2}\right)$$

C is NOT the mid-point of IJ. **2**

(iii) A, B and E lie on the same straight line and B is equidistant from A and E

$\Rightarrow$ B is the mid-point of AE

$$\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2,-3)$$

1 + a = 4; a = 3; 4 + b = – 6; b = – 10; E is (3, – 10). **1**

38. **(i)** $\tan 45° = \dfrac{80}{CB}$

$\Rightarrow$ $CB = 80$ m **1**

(ii) $\tan 30° = \dfrac{80}{CE}$

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{80}{CE}$

$\Rightarrow$ $CE = 80\sqrt{3}$ **1**

Distance the bird flew $= AD = BE$

$= CE - CB = 80\sqrt{3} - 80$

$= 80(\sqrt{3}-1)$ m **1**

OR

$\tan 60° = \dfrac{80}{CG}$

$\Rightarrow$ $\sqrt{3} = \dfrac{80}{CG}$

$\Rightarrow$ $CG = \dfrac{80}{\sqrt{3}}$ **1**

Distance the ball travelled after hitting the tree

$= FA = GB = CB - CG$

$GB = 80 - \dfrac{80}{\sqrt{3}}$

$= 80\left(1 - \dfrac{1}{\sqrt{3}}\right)$ m **1**

(iii) Speed of the bird $= \dfrac{\text{Distance}}{\text{Time taken}}$

$= \dfrac{20(\sqrt{3}+1)}{2}$ m/s

$= \dfrac{20(\sqrt{3}+1)}{2} \times 60$ m/min

$= 600(\sqrt{3}+1)$ m/min **1**

■■■