Sample Question Paper

(Issued by Board on 31st March, 2023)

MATHEMATICS BASIC (041)

Class- X

Session-2023-24

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

General Instructions :

(i) *This Question Paper has 5 Sections A, B, C, D, and E.*
(ii) *Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.*
(iii) *Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.*
(iv) *Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.*
(v) *Section D has 4 Long Answer (LA) type questions carrying 5 marks each.*
(vi) *Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.*
(viii) *Draw neat figures wherever required. Take π =22/7 wherever required if not stated.*

Section – A

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is

(A) xy **(B)** xy^2
(C) x^3y^3 **(D)** x^2y^2

2. The LCM of smallest two-digit composite number and smallest composite number is

(A) 12 **(B)** 4
(C) 20 **(D)** 44

3. If $x = 3$ is one of the roots of the quadratic equation $x^2 - 2kx - 6 = 0$, then the value of k is

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$
(C) 3 **(D)** 2

4. The pair of equations $y = 0$ and $y = -7$ has

(A) One solution **(B)** Two solutions
(C) Infinitely many solutions **(D)** No solution

5. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(A) 0 only **(B)** 4
(C) 8 only **(D)** 0,8

6. The distance of the point(3, 5) from x-axis(in units) is

(A) 3 **(B)** -3
(C) 5 **(D)** -5

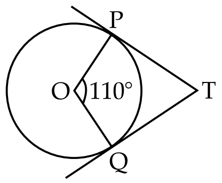
7. If in ΔABC and ΔPQR, we have $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

(A) $\Delta PQR \sim \Delta CAB$ **(B)** $\Delta PQR \sim \Delta ABC$
(C) $\Delta CBA \sim \Delta PQR$ **(D)** $\Delta BCA \sim \Delta PQR$

8. Which of the following is NOT a similarity criterion?

(A) AA **(B)** SAS
(C) AAA **(D)** RHS

9. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to



(A) 60° **(B)** 70°
(C) 80° **(D)** 90°

10. If $\cos A = \frac{4}{5}$ then the value of tan A is

(A) $\frac{3}{5}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) $\frac{1}{8}$

11. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is

(A) 30°

(B) 45°

(C) 60°

(D) 90°

12. $1 - \cos^2 A$ is equal to

(A) $\sin^2 A$

(B) $\tan^2 A$

(C) $1 - \sin^2 A$

(D) $\sec^2 A$

13. The radius of a circle is same as the side of a square. Their perimeters are in the ratio

(A) 1 : 1

(B) $2 : \pi$

(C) $\pi : 2$

(D) $\sqrt{\pi} : 2$

14. The area of the circle is 154 cm^2. The radius of the circle is

(A) 7 cm

(B) 14 cm

(C) 3.5 cm

(D) 17.5 cm

15. When a die is thrown, the probability of getting an even number less than 4 is

(A) 1/4

(B) 0

(C) 1/2

(D) 1/6

16. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The lower limit of modal class is

(A) 15

(B) 25

(C) 30

(D) 35

17. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is

(A) 3.5

(B) 7

(C) $\frac{80}{7}$

(D) 5

18. Consider the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	12	10	15	8	11

The median class is

(A) 6-12

(B) 12-18

(C) 18-24

(D) 24-30

19. **Assertion (A):** The point (0, 4) lies on y-axis:

Reason(R): The x coordinate of the point on y-axis is zero

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(B) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(C) Assertions (A) is true but reason (R) is false.

(D) Assertions (A) is false but reason (R) is true.

20. **Assertion (A):** The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

Reason(R): For any two positive integers a and b, HCF(a, (B) × LCM (a, (B) = $a \times b$.

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(B) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(C) Assertions (A) is true but reason (R) is false.

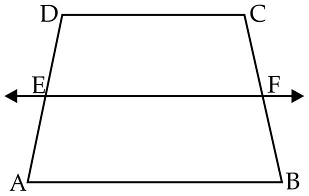
(D) Assertions (A) is false but reason (R) is true.

Section – B

21. Find whether the following pair of linear equations is consistent or inconsistent:

$$3x + 2y = 8$$
$$6x - 4y = 9$$

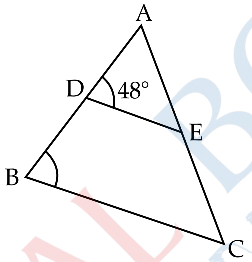
22. In the given figure, if ABCD is a trapezium in which AB $\|$ CD $\|$ EF,



then prove that $\frac{AE}{ED} = \frac{BF}{FC}$

OR

In figure, if AD = 6 cm, DB = 9 cm, AE = 8 cm and EC = 12 cm and $\angle ADE = 48°$. Find $\angle ABC$.



23. The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm. Find the radius of the circle.

24. Evaluate: $\sin^2 60° + 2\tan 45° - \cos^2 30°$.

25. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40cm and 9cm?

OR

A chord of a circle of radius 10cm subtends a right angle at the centre. Find area of minor segment. (Use $\pi = 3.14$)

Section – C

26. Prove that $\sqrt{3}$ is an irrational number.

27. Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

28. The coach of a cricket team buys 4 bats and 1 ball for ₹ 2050. Later, she buys 3 bats and 2 balls for ₹ 1600. Find the cost of each bat and each ball.

OR

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

29. A circle touches all the four sides of quadrilateral ABCD. Prove that $AB + CD = AD + BC$.

30. **Prove that:**

$$(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

OR

Prove that sec A (1 – sin A) (sec A + tan A) = 1.

31. A bag contains 6 red, 4 black and some white balls:

(i) Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{3}$.

(ii) How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$?

Section – D

32. A train travels 360 km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

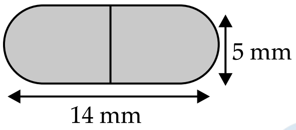
OR

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

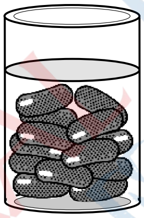
In ΔPQR, S and T are points on PQ and PR respectively. $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle.

34. A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



OR

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.



35. The following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life time of a lamp.

Section – E

36. **CASE STUDY 1**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(1) In which year, the production is ₹ 29,200.

(2) Find the production during 8^{th} year.

OR

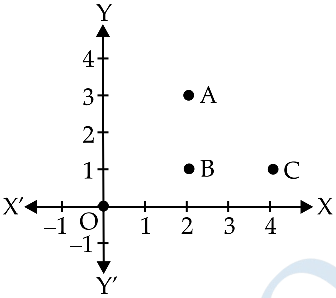
Find the production during first 3 years.

(3) Find the difference of the production during 7^{th} year and 4^{th} year.

37. CASE STUDY 2

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C.

Based on the above information, answer the following questions.



(i) How far is Alia's house from Shagun's house?

(ii) How far is the library from Shagun's house?

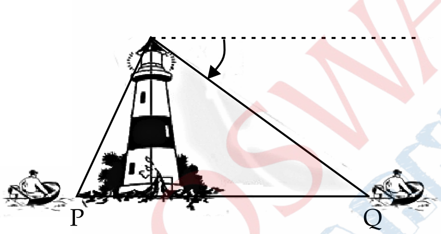
(iii) Show that for Shagun, school is farther compared to Alia's house and library.

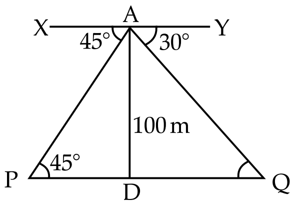
OR

Show that Alia's house, Shagun's house and library for an isosceles right triangle.

38. CASE STUDY 3

A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.





Based on the above information, answer the following questions.

(i) What is the measure of $\angle APD$?

(ii) If $\angle YAQ = 30°$, then $\angle AQD$ is also 30°, Why?

(iii) How far is the boat P from the light house?

OR

How far is the boat Q from the light house?

SOLUTIONS

All solutions are as per the CBSE Board Marking Scheme 2023-24

Section - A

1. Option (B) is correct.
xy^2 **1**

Detailed Answer:
Given that,
$\Rightarrow \quad a = x^3y^2$
$\Rightarrow \quad b = xy^3$
$HCF\ (a, b) = xy^2$

2. Option (C) is correct.
20 **1**

Detailed Answer:
We know that,
Smallest two-digit composite number = 10
Smallest composite number = 4
We have, $LCM\ (4, 10) = 2^2 \times 5 = 20$

3. Option (B) is correct.
$\frac{1}{2}$ **1**

Detailed Answer:
Given that,
$\Rightarrow x = 3$ is root of quadratic equation
$x^2 - 2kx - 6 = 0$
On putting $x = 3$ in given equation,
$\Rightarrow \quad (3)^2 - 2(k)(3) - 6 = 0$
$\Rightarrow \quad 9 - 6k - 6 = 0$
$\Rightarrow \quad 6k = 3$
$\Rightarrow \quad k = 3/6 = \frac{1}{2}$

4. Option (D) is correct.
No Solution **1**

Detailed Answer:
Given that,
Pair of linear equations $y = 0$ and $y = -7$
We know that, on making the graph of these equations the two lines are parallel to each other and never meet each other.
So, given pair of linear equations does not have any Solution.

5. Option (D) is correct.
0, 8 **1**

Detailed Answer:
Given that,
$\Rightarrow \quad 2x^2 - kx + k = 0$
For equal roots, Discriminant = 0
$\Rightarrow \quad b^2 - 4ac = 0$
$\Rightarrow \quad (-k)^2 - 4(2)(k) = 0$
$\Rightarrow \quad k^2 - 8k = 0$
$\Rightarrow \quad k(k - 8) = 0$
$\Rightarrow \quad k = 0, 8$

6. Option (C) is correct.
5 units **1**

Detailed Answer:
The distance of point (3, 5) from the x-axis is equal to the ordinate of the given coordinates. So, the distance from x-axis is 5 units.

7. Option (A) is correct.
$\Delta PQR \sim \Delta CAB$ **1**

Detailed Answer:
Given that,
In triangle ABC and PQR,
$\Rightarrow \quad AB/QR = BC/PR = CA/PQ$
From the given relation we get,
$\Rightarrow \quad PQ/CA = PR/BC = QR/AB$
So, triangle $PQR \sim CAB.$

8. Option (D) is correct.
RHS **1**

Detailed Answer:
RHS is not a similarity criterion, it is a congruence criterion.

9. Option (B) is correct.
70° **1**

Detailed Answer:
We know that,
In given figure,
$\Rightarrow \quad \angle POQ + \angle PTQ = 180°$
$\Rightarrow \quad 110° + \angle PTQ = 180°$
$\Rightarrow \quad \angle PTQ = 180° - 110° = 70°$

10. Option (B) is correct.
¾ **1**

Detailed Answer:
Given that,
$\Rightarrow \quad \text{Cos A} = 4/5$
$\Rightarrow \quad \text{Sec A} = 5/4$

$\Rightarrow \quad \tan^2 A = \sec^2 A - 1$

$= (5/4)^2 - 1 = (3/4)^2$

$\Rightarrow \quad \tan A = ¾$

11. Option (B) is correct.

45° **1**

Detailed Answer:

Given that,

Height of tower = Length of shadow

Let the angle of elevation be x

$$\Rightarrow \quad \tan x = \frac{\text{Height of tower}}{\text{Length of shadow}}$$

$\Rightarrow \quad \tan x = 1 \quad (\tan 45° = 1)$

$\Rightarrow \quad x = 45°$

12. Option (A) is correct.

$\sin^2 A$ **1**

Detailed Answer:

We know that,

$\Rightarrow \quad \sin^2 x + \cos^2 x = 1$

So, we have,

$\Rightarrow \quad 1 - \cos^2 A = \sin^2 A$

13. Option (C) is correct.

$\pi : 2$ **1**

Detailed Answer:

Given that,

Radius of circle = Side of square

Let radius of circle be x

$\Rightarrow$ Radius of circle = Side of square = x

$$\text{Ratio of perimeters} = \frac{\text{Perimeter of circle}}{\text{Perimeter of square}}$$

$= 2\pi x / 4x = \pi/2$

14. Option (A) is correct.

7 cm **1**

Detailed Answer:

Given that,

Area of circle = 154 cm^2

Let the radius of circle be x

$\pi x^2 = 154$

$\Rightarrow \quad x^2 = 49$

$\Rightarrow \quad x = 7$ cm

15. Option (D) is correct.

1/6 **1**

Detailed Answer:

We have,

Even number less than 4 = {2}

Probability of getting even number less than 4 = 1/6

16. Option (A) is correct.

15 **1**

Detailed Answer:

The class having maximum frequency is 15-20

So, Lower limit of modal class is 15

17. Option (A) is correct.

3.5 **1**

Detailed Answer:

Given that,

Dimensions of rectangular sheet = 40 cm × 22 cm

Height of cylinder = 40 cm

Let radius of cylinder be r

$\Rightarrow \quad 2\pi r = 22$

$\Rightarrow \quad r = 3.5$ cm

18. Option (B) is correct.

12-18 **1**

Detailed Answer:

We have,

$\Rightarrow \quad N = 56$

$\Rightarrow \quad N/2 = 28$

Cumulative frequency just greater than or equal to 28 lies in interval 12-18

So, Median class is 12-18

19. Option (A) is correct.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). **1**

Detailed Answer:

Assertion: The point (0, 4) lies on y-axis

Reason: The x-coordinate of the point on y-axis is zero

Hence, both assertion and reason are true and reason is the correct explanation for assertion.

20. Option (D) is correct.

Assertions (A) is true but reason (R) is false. **1**

Detailed Answer:

Assertion: Given that, $HCF = 5$

$LCM = 40$

Product of numbers = 150

We know that, $HCF \times LCM$ = Product of numbers

$HCF \times LCM = 5 \times 40 = 200$ is not equal to product

So, Assertion is False

Reason: for any two positive integers a and b, $HCF(a, b) \times LCM(a, b) = a \times b$.

Hence, Assertion is false but reason is true.

Section - B

21. $3x + 2y = 8$

$6x - 4y = 9$

$a_1 = 3, b_1 = 2, c_1 = 8$

$a_2 = 6, b_2 = -4, c_2 = 9$ **1**

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{8}{9}$$

½

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The given pair of linear equations has unique solution.

So, the given pair of lines is consistent. ½

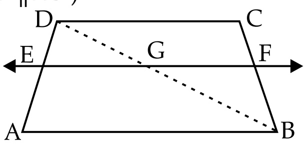
22. **Given:** $AB \parallel CD \parallel EF$

To prove:- $\dfrac{AB}{ED} = \dfrac{BF}{FC}$

Constant:- Join BD which intersect EF at G.

Proof: in Δ ABD

$EG \parallel AB$ ($EF \parallel AB$) ½



$$\frac{AE}{ED} = \frac{BG}{GD} \text{ (by BPT)} \qquad \text{...(1) ½}$$

In ΔDBC

$GF \parallel CD$ ($EF \parallel CD$) ½

$$\frac{BF}{FC} = \frac{BG}{GD} \text{ (by BPT)} \qquad \text{...(2)}$$

From (1) & (2)

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence Proved ½

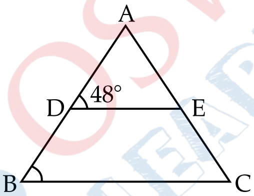
OR

Given: $AD=6$ cm, $DB=9$ cm

$AE=8$ cm, $EC=12$ cm, $\angle ADE=48°$

To find: $\Delta ABC=?$

Proof: In ΔABC



Consider, $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$$\frac{6}{9} = \frac{8}{12}$$

$$\frac{2}{3} = \frac{2}{3} \qquad 1$$

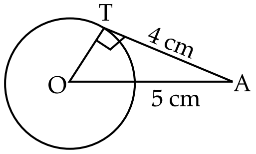
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$DE \parallel BC$ (Converse of BPT)

$\angle ADE = \angle ABC$

(Corresponding angles) So, $\angle ABC=48°$ **1**

23. In ΔOTA, $\angle OTA = 90°$



By Pythagoras theorem

$$OA^2 = OT^2 + AT^2 \qquad ½$$

$$(5)^2 = OT^2 + (4)^2$$

$$25 - 16 = OT^2$$

$$9 = OT^2 \qquad ½$$

$$OT = 3 \text{ cm}$$

radius of circle = 3 cm. **1**

24. $\sin^2 60° + 2 \tan 45° - \cos^2 30°$

On substituting the values we have,

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2 \qquad \mathbf{1}$$

$$= \frac{3}{4} + 2 - \frac{3}{4} = 2 \qquad \mathbf{1}$$

25. Area of the circle = sum of areas of 2 circles

$$\pi R^2 = \pi(40)^2 + \pi(9)^2 \qquad ½$$

$$\pi R^2 = \pi \times (40^2 + 9)^2 \qquad ½$$

$$R^2 = 1600 + 81$$

$$R^2 = 1681$$

$$R = 41 \text{ cm.} \qquad ½$$

Diameter of required circle = 41 × 2 = 82 cm ½

OR

Radius of circle = 10 cm, θ = 90°

Area of minor segment $= \dfrac{\theta}{360°}\pi r^2$ – Area of right angled triangle

$$= \frac{\theta}{360°}\pi r^2 - \frac{1}{2} \times b \times h \qquad ½$$

$$= \frac{90°}{360°} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10 \qquad ½$$

$$= \frac{314}{4} - 50$$

$$= 78.5 - 50 = 28.5 \text{ cm}^2 \qquad ½$$

Area of minor segment = 28.5 cm² ½

Section - C

26. Let √3 be a rational number

$$\sqrt{3} = \frac{a}{b} \text{ where } a \text{ and } b \text{ are co-prime.} \qquad 1$$

squaring on both the sides

$$(\sqrt{3})^2 = \left(\frac{a}{b}\right)^2 \qquad ½$$

$$3 = \frac{a^2}{b^2}$$

$$a^2 = 3a^2$$

a^2 is divisible by 3 so a is also divisible by 3 ...(1)

Let $a = 3c$ for any integer c.

$$(3c)^2 = 3b^2 \qquad ½$$

$$9c^2 = 3b^2$$

$$b^2 = 3c^2$$

since b^2 is divisible by 3 so, b is also divisible by 3 ...(2)

From (1) & (2) we can say that 3 is a factor of a and b ½

which is contradicting the fact that a and b are co-primes.

Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number. ½

27.
$$P(S) = 4S^2 - 4S + 1$$
$$4S^2 - 2S - 2S + 1 = 0$$
$$2S(2S-1) - 1(2S-1) = 0$$
$$(2S-1)(2S-1) = 0$$
$$S = \frac{1}{2}, S = \frac{1}{2} \quad \textbf{1}$$

Zeroes of quadratic polynomial are $\frac{1}{2}, \frac{1}{2}$

From the quadratic equation, we have

$a = 4, b = -4\ c = 1\ \alpha = \frac{1}{2}\ \alpha\beta = \frac{1}{2}$

$$\alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$$
$$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4} \quad \textbf{1}$$
$$\frac{1+1}{2} = \frac{+4}{4}$$
$$\text{L.H.S.} = \text{R.H.S.}$$

$$\alpha.\beta = \frac{c}{a}$$
$$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} = \frac{1}{4}$$
$$\frac{1}{4} = \frac{1}{4}$$
$$\frac{2}{2} = 1 \quad \textbf{1}$$
$$1 = 1$$

Hence, the relationship between zeroes and coefficient is verified

28. Let cost of one bat be ₹ x

Let cost of one ball be ₹ y ½

ATQ

$$4x + 1y = 2050 \quad ...(1)$$
$$3x + 2y = 1600 \quad ...(2)\ ½$$

from (1) $4x + 1y = 2050$

$$y = 2050 - 4x \quad ½$$

Substitute value of y in (2)

$[3x + 2(2050 - 4x) = 1600]$

$$3x + 4100 - 8x = 1600$$
$$-5x = -2500$$
$$x = 500 \quad ½$$

Substitute value of x in (1)

$$4x + 1y = 2050$$
$$4(500) + y = 2050$$
$$2000 + y = 2050$$
$$y = 50 \quad ½$$

Hence

Cost of one bat = ₹ 500

Cost of one ball = ₹ 50

OR

Let the fixed charge for first 3 days = ₹ x

and additional charge after 3 days = ₹ y ½

According to given conditions,

$$x + 4y = 27 \quad ...(1)$$
$$x + 2y = 21 \quad ...(2)$$

Subtract eqn. (2) from (1)

$$x + 4y = 27$$
$$x + 2y = 21$$
$$2y = 6$$
$$y = 3 \quad \textbf{1}$$

Substitute value of y in (2)

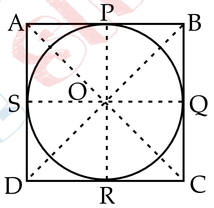
$$x + 2y = 21$$
$$x + 2(3) = 21$$
$$x = 21 - 6$$
$$x = 15 \quad \textbf{1}$$

Fixed charge = ₹ 15

Additional charge = ₹ 3

29. Given circle touching sides of quadrilateral ABCD at P, Q, R and S

To prove: $AB + CD = AD + BC$



1

Proof:

$AP = AS$ ...(1) tangents from same point

$PB = BQ$ ...(2) to a circle are equal in length

$DR = DS$ ... (3)

$CR = CQ$ ...(4) **1**

Adding eqn (1),(2),(3) & (4)

$AP + BP + DR + CR = AS + DS + BQ + CQ$

$AB + DC = AD + BC$ **Hence Proved 1**

30. $(\text{cosec}\ \theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\text{cosec}\ \theta - \cot\theta)^2$$
$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \quad ½$$
$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 \quad ½$$
$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \quad \textbf{1}$$
$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{1-\cos\theta}{1+\cos\theta} \quad \textbf{1}$$

L.H.S. = R.H.S. **Hence Proved 1**

OR

Sec A (1 – sin A) (sec A + tan A) = 1

$$\text{LHS} = \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right) \quad \mathbf{1}$$

$$= \frac{(1-\sin A)}{\cos A}\frac{(1+\sin A)}{\cos A}$$

$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} \qquad (1-\sin^2 A = \cos^2 A) \quad \mathbf{1}$$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1 \quad \mathbf{1}$$

L.H.S. = R.H.S. **Hence Proved**

31. (i) Red balls = 6, Black balls = 4, White balls = x

$$\text{P (white ball)} = \frac{x}{10+x} = \frac{1}{3} \quad \mathbf{1}$$

$\Rightarrow 3x = 10 + x \Rightarrow x = 5$ white balls ½

(ii) Let y red balls be removed, black balls = 4, white balls = 5

$$\text{P(white balls)} = \frac{5}{(6-y)+4+5} = \frac{1}{2} \quad \mathbf{1}$$

$$\Rightarrow \frac{5}{15-y} = \frac{1}{2}$$

$\Rightarrow 10 = 15 - y \Rightarrow y = 5$ white balls ½

So 5 balls should be removed.

Section - D

32. Let the speed of train be x km/hr

distance = 360 km ½

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Time} = \frac{360}{x} \quad ½$$

New speed = $(x + 5)$ km/hr

$$\text{Time} = \frac{D}{5}$$

$$(x+5) = \frac{360}{\left(\frac{360}{x}-1\right)}$$

$$(x+5)\left(\frac{360}{x}-1\right) = 360 \quad \mathbf{1}$$

$$(x+5)(360-x) = 360x$$

$$-x^2 - 5x + 1800 = 0$$

$$x^2 + 5x - 1800 = 0 \quad \mathbf{1}$$

$$x^2 + 45x - 40x - 1800 = 0$$

$$x(x+45) - 40(x+45) = 0$$

$$(x+45)(x-40) = 0 \quad \mathbf{1}$$

$$x + 45 = 0,\ x - 40 = 0$$

$$x = -45 \text{ and } x = 40$$

Speed cannot be negative

Speed of train = 40 km/hr **1**

OR

Let the speed of the stream = x km/hr ½

Speed of boat = 18 km/hr

Upstream speed = $(18-x)$ km/hr

Downstream speed = $(18 + x)$ km/hr ½

$$\text{Time taken (upstream)} = \frac{24}{(18-x)}$$

$$\text{Time taken (downstream)} = \frac{24}{(18+x)}$$

According to the given conditions,

$$\frac{24}{(18-x)} = \frac{24}{(18+x)} + 1 \quad \mathbf{1}$$

$$\frac{24}{(18-x)} - \frac{24}{(18+x)} = 1$$

$$24(18+x) - 24(18-x) = (18-x)(18+x)$$

$$24(18+x-18+x) = (18)^2 - (x)^2$$

$$24(2x) = 324 - x^2$$

$$48x - 324 + x^2 = 0$$

$$x^2 + 48x - 324 = 0 \quad \mathbf{1}$$

$$x^2 - 6x + 54x - 324 = 0$$

$$x(x-6) + 54(x-6) = 0$$

$$(x-6)(x+54) = 0 \quad \mathbf{1}$$

$$x - 6 = 0,\ x + 54 = 0$$

$$x = 6 \text{ and } x = -54$$

Speed cannot be negative **1**

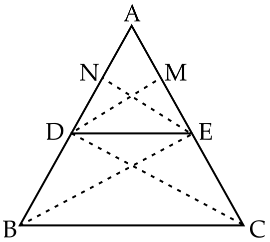
Speed of stream = 6 km/hr

33. **Given:** Δ ABC $= DE || BC$

To prove $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction: Join BE and CD ½

Draw $DM \perp AC$ and $EN \perp AB$



Proof: $\text{Ar. } \Delta ADE = \dfrac{1}{2} \times b \times h$

$$= \frac{1}{2} \times AD \times EN \quad ...(1)$$

$$\text{Ar. } \Delta DBE = \frac{1}{2} \times DB \times EN \quad ...(2)$$

Divide eqn (1) by (2)

$$\frac{\text{Ar.}\Delta ADE}{\text{Ar.}\Delta BDE} = \frac{\frac{1}{2}\times AD \times EN}{\frac{1}{2}\times DB \times EN} = \frac{AD}{DB} \quad ...(3)\ \mathbf{1}$$

$$\text{Ar. } \Delta ADE = \frac{1}{2} \times AE \times DM \quad ...(4)$$

$$\text{Ar. } \Delta DEC = \frac{1}{2} \times EC \times DM \quad ...(5)$$

Divide eqn (3) by (4)

$$\frac{\text{Ar.}\Delta ADE}{\text{Ar.}\Delta DEC} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} = \frac{AE}{EC} \quad ...(6) \quad \mathbf{1}$$

ΔBDE and ΔDEC are on the same base DE and between same parallel lines BC and DE

$$\text{Ar. }(BDE) = \text{Ar. }(DEC)$$

hence

$$\frac{\text{Ar.}\Delta ADE}{\text{Ar.}\Delta BDE} = \frac{\text{Ar.}\Delta ADE}{\text{Ar.}\Delta DEC}$$

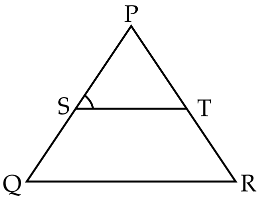
$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{[from (3) and (6)] } \tfrac{1}{2}$$

Hence Proved

Given $\quad \dfrac{PS}{SQ} = \dfrac{PT}{TR}$

$\angle$ PST $= \angle$ PRQ

To prove : PQR is an isosceles triangle



Proof : $\quad \dfrac{PS}{SQ} = \dfrac{PT}{TR}$

$\angle PST = \angle PQR$

(Corresponding angles) **1**

But $\quad \angle PST = \angle PRQ$

$\angle PQR = \angle PRQ$

$PR = PQ$ (sides opposite to equal angles are equal

– ΔPQR is isosceles triangle. **1**

Hence Proved

34. Diameter of cylinder and hemisphere = 5 mm

radius (r) = $\dfrac{5}{2}$

Total length = 14 mm

Height of cylinder = 14 – 5 = 9 mm **1**

CSA of cylinder = $2\pi rh$

$$= 2\times\frac{22}{7}\times\frac{5}{2}\times 9$$

$$= \frac{990}{7}\text{ mm}^2 \quad \mathbf{1}$$

CSA of hemispheres $= 2\pi r^2$

$$= 2\times\frac{22}{7}\times\left(\frac{5}{2}\right)^2$$

$$= \frac{275}{7}\text{ mm}^2 \quad \mathbf{1}$$

CSA of 2 hemispheres $= 2\times\dfrac{275}{7}$

$$= \frac{550}{7}\text{ mm}^2 \quad \mathbf{1}$$

Total area of capsule $= \dfrac{990}{7}+\dfrac{550}{7}$

$$= \frac{1540}{7}$$

$$= 220\text{ mm}^2 \quad \mathbf{1}$$

OR

Diameter of cylinder = 2.8 cm

Radius of cylinder = $\dfrac{2.8}{2}$ = 1.4 cm

Radius of cylinder = Radius of hemisphere = 1.4 cm

Height of cylinder = 5 – 2.8 **1**

= 2.2 cm

Volume of 1 gulab jamun

= vol. of cylinder + 2 × vol. of hemisphere

$= \pi r^2 h + 2\times\dfrac{2}{3}\pi r^3$ **1**

$= \dfrac{22}{7}\times(1.4)^2\times 2.2 + 2\times\dfrac{2}{3}\times\dfrac{22}{7}\times(1.4)^3$ **1**

=13.55+11.50

=25.05 cm^3 **1**

volume of 45 gulab jamuns = 45 × 25.05

syrup in 45 jamuns = 30% × 45 × 25.05

$= \dfrac{30}{100}\times 45\times 25.05$ **1**

= 338.175 cm^3

= 338 cm^3 **1**

35.

Life time (in hours)	Number of lamps	Mid x	d	fd
1500-2000	14	1750	–1500	–21000
2000-2500	56	2250	–1000	–56000
2500-3000	60	2750	–500	–30000
3000-3500	86	3250	0	0
3500-4000	74	3750	500	37000
4000-4500	62	4250	1000	62000
4500-5000	48	4750	1500	72000
	400			64000

2

Using Assumed Mean Method,

Mean $= a + \dfrac{\Sigma fd}{\Sigma f}$ ½

$a = 3250$ ½

Mean $= 3250 + \dfrac{64000}{400}$ **1**

= 3250 + 160

= 3410

Average life of lamp is 3410 hr **1**

Section - E

36. $a_6 = 16000$, $a_9 = 22600$

$$a + 5d = 16000 \quad ...(1)$$
$$a = 16000 - 5d$$
$$a + 8d = 22600 \quad ...(2)$$

substitute in (2)

$$16000 - 5d + 8d = 22600$$
$$3d = 22600 - 16000$$
$$3d = 6600$$
$$d = \frac{6600}{3} = 2200$$
$$a = 16000 - 5(2200)$$
$$a = 16000 - 11000$$
$$a = 5000$$

(i) $a_n = 29200$, $a = 5000$, $d = 2200$

$$a_n = a + (n-1)d$$
$$29200 = 5000 + (n-1)2200 \quad ½$$
$$29200 - 5000 = 2200n - 2200$$
$$24200 + 2200 = 2200n$$
$$26400 = 2200n$$
$$n = \frac{264}{22}$$
$$n = 12 \quad ½$$

in 12th year the production was ₹ 29200

(ii) $n=8$, $a=5000$, $d=2200$

$$a_n = a + (n-1)d \quad ½$$
$$= 5000 + (8-1)2200 \quad ½$$
$$= 5000 + 7 \times 2200$$
$$= 5000 + 15400 \quad ½$$
$$= 20400$$

The production during 8th year is ₹ 20400 ½

OR

$n = 3$, $a = 5000$, $d = 2200$

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad ½$$
$$= \frac{3}{2}[2(5000) + (3-1)2200]$$
$$S_3 = \frac{3}{2}(10000 + 2 \times 2200) \quad ½$$
$$= \frac{3}{2}(10000 + 4400) \quad ½$$
$$= 3 \times 7200$$
$$= 21600 \quad ½$$

The production during first 3 year is ₹ 21600

(iii)
$$a_4 = a + 3d$$
$$= 5000 + 3(2200)$$
$$= 5000 + 6600$$
$$= 11600$$
$$a_7 = a + 6d$$
$$= 5000 + 6 \times 2200$$
$$= 5000 + 13200$$
$$= 18200$$
$$a_7 - a_4 = 18200 - 11600 = 6600 \quad ½$$

37. Coordinates of A (2,3) – Alia's house
coordinates of B (2,1) – Shagun's house
coordinates of C (4,1) – library

(i)
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2-2)^2 + (1-3)^2} \quad ½$$
$$= \sqrt{(0)^2 + (-2)^2}$$
$$AB = \sqrt{0+4}$$
$$= \sqrt{4} \text{ unit} = 2 \text{ units} \quad ½$$

Distance of Alia's house from shagun's house is 2 unit

(ii) C(4, 1), B (2, 1)

$$CB = \sqrt{(2-4)^2 + (1-1)^2} \quad ½$$
$$= \sqrt{(-2)^2 + (0)^2}$$
$$= \sqrt{4+0}$$
$$= \sqrt{4} = 2 \text{ unit} \quad ½$$

Distance of shagun's house from library is 2 unit

(iii) O (0, 0), B(2, 1)

$$OB = \sqrt{(2-0)^2 + (1-0)^2}$$
$$= \sqrt{2^2 + 1^2}$$
$$= \sqrt{4+1} = \sqrt{5} \text{ units} \quad 1$$

Distance between Alia's house and Shagun's house AB = 2 units

Distance between Library and Shagun's house CB = 2 units ½

OB is greater than AB and CB, ½

For shagun, school [O] is farther than Alia's house [A] and Library [C]

OR

C (4,1) A(2,3)

$$CA = \sqrt{(2-4)^2 + (3-1)^2}$$
$$= \sqrt{(-2)^2 + 2^2}$$
$$= \sqrt{4+4} = \sqrt{8}$$
$$= 2\sqrt{2} \text{ units}, AC^2 = 8 \quad 1$$

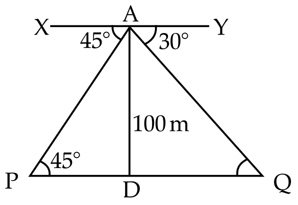
Distance between Alia's house and Shagun's house $AB = 2$ units

Distance between Library and Shagun's house CB = 2 units ½

$AB^2 + BC^2 = 2^2 + 2^2 = 4 + 4 = 8 = AC^2$ ½

Therefore A,B and C form a right triangle.

38. **(i)** $XY \parallel CD$ and AP is transversal.



$\angle APD = \angle PAX$ (alt.int $\angle S$) ½

$\angle APD = 45^\circ$ ½

(ii) $\angle YAQ = 30^\circ$

$\angle AQD = 30^\circ$ ½

Because $XY \,||\, PQ$ and AQ is a transversal so alternate interior angles are equal

$\angle YAQ = \angle AQD$ ½

(iii) In $\Delta\, ADC$

$\tan 45^\circ = \frac{100}{PD}$ ½

$1 = \frac{100}{PD}$ ½

PD = 100 m

Boat P is 100 m from the light house **1**

OR

In ΔADQ

$\tan 30^\circ = \frac{100}{DQ}$ ½

$\frac{1}{\sqrt{3}} = \frac{100}{DQ}$ ½

DQ = $100\sqrt{3}$ m

Boat Q is $100\sqrt{3}$ m from the light house **1**

❑❑