

# UNIT – I: ELECTROSTATICS

## CHAPTER-1

### ELECTRIC CHARGES AND FIELDS

#### Topic-1

#### Electric Field and Dipole

**Concepts Covered** • Electric charge, • Electrostatic charge, • Properties of electric charge, • Coulomb's law, • Principle of superposition, • Electric field, • Electric field lines, • Electric dipole, • Torque on a dipole, • Electric dipole moment, • Electric field due to dipole



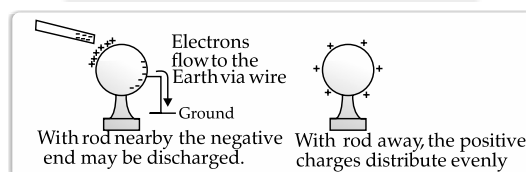
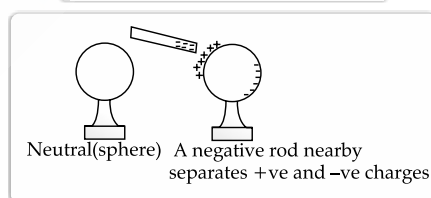
#### Revision Notes

##### Electric Charge

- Electric charge is the property of a matter due to which, it experiences a force when placed in an electromagnetic field.
- Point charge is an accumulation of the electric charges at a point, without spatial extent.
- Electrons are the smallest and lightest fundamental particles in an atom having negative charge as these are surrounded by invisible field known as electrostatic field.
- Protons are comparatively larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons or two protons will tend to repel each other as they carry like charges, negative and positive respectively.
- The electron and proton will get attracted towards each other due to their unlike charges.
- The charge present on the electron is equal and opposite to charge on the proton.  
Charge on a proton =  $+ 1.6 \times 10^{-19} \text{ C}$   
and, charge on an electron =  $- 1.6 \times 10^{-19} \text{ C}$

##### Electrostatic Charge

- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental physical quantity like length, mass and time.



- Charge on a body is expressed as  $q = \pm ne$
- The magnitude of charge is independent of the speed of the particle.
- Based on the flow of charge across them, materials are classified as:
  - **Conductors:** Allow **electric charge** to flow freely, e.g., metals.
  - **Semi-conductors:** Behave as the conductor or insulator depending on the number of free electrons and holes availability. e.g., silicon.
  - **Insulators:** Do not allow electric charge to flow, e.g., rubber, wood, plastic, etc.

► **Net charge on a body is given by:**

- **Charging by friction:** Charging insulators
- **Charging by conduction:** Charging metals and other conductors
- **Charging by induction:** Wireless charging

**Charging by Rubbing**

- On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged.
- If a plastic rod is rubbed with wool, it becomes negatively charged.

**Charging by Induction**

- Charging by induction means charging without contact.
- If a negatively charged rod is brought near **neutral metal** with insulator mounting, it repels free electrons and attracts positive charges on metal.



## Key Words

**Electric Charge:** Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

**Friction:** It is the opposing force.

**Neutral metal:** Metal having no net charge.

- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.
- When the rod is removed, the captive positive charge is distributed evenly.

**Properties of Electric Charge**

**Addition of charges**

- If a system contains three point charges  $q_1$ ,  $q_2$ , and  $q_3$ , then the total charge of the system will be the algebraic addition of  $q_1$ ,  $q_2$  and  $q_3$ , i.e., charges will add up.

$$q = q_1 + q_2 + q_3$$

**Conservation of charges**

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge can neither be created nor destroyed in the process, but only exists in positive-negative pairs.

**Quantization of charges**

- Electric charge is always quantized i.e., electric charge is always an integral multiple of charge 'e'.

- Net charge  $q_{net}$  of an object having  $N_e$  electrons,  $N_p$  protons and  $N_n$  neutrons is:

$$q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = \pm ne$$

- Neutron ( $n$ ):  $m = 1.675 \times 10^{-27}$  kg;  $q = 0$
- Proton ( $p$ ):  $m = 1.673 \times 10^{-27}$  kg;  $q = +1.6 \times 10^{-19}$  C
- Electron ( $e$ ):  $m = 9.11 \times 10^{-31}$  kg;  $q = -1.6 \times 10^{-19}$  C

**Coulomb's Law**

- The force of attraction or repulsion between two point charges  $q_1$  and  $q_2$  separated by a distance  $r$  is directly proportional to product of magnitude of charges and inversely proportional to square of the distance between charges, written as:

$$F = k \frac{|q_1| |q_2|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2}$$

where,  $F$  = Force of attraction/repulsion between charge  $q_1$  and  $q_2$ .

$q_1, q_2$  = Magnitudes of charge 1 and charge 2 respectively

$r$  = Distance between charges  $q_1, q_2$

$k$  = Constant whose value depends on medium where charges are kept,  $k = \frac{1}{4\pi\epsilon_0}$

As  $\epsilon = K'\epsilon_0$ ,  $k = \frac{1}{4\pi K'\epsilon_0}$

$\epsilon_0$  = Permittivity of vacuum or free space =  $8.854 \times 10^{-12}$  F/m

$K'$  = Relative permittivity of medium or dielectric constant.

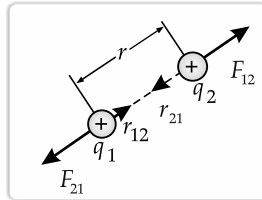
- For vacuum, relative permittivity,  $K' = 1$ ,
- As  $\epsilon = K'\epsilon_0$ , therefore the force of attraction/repulsion between two electric charges  $q_1, q_2$  placed in the vacuum and medium will be:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \text{ (vacuum) and } F = \frac{1}{4\pi\epsilon_0 \epsilon_r} \cdot \frac{q_1 q_2}{r^2} \text{ (medium)}$$

- ▶ The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- ▶ Current is the rate at which charge moves past a point or through a region,  $i = \frac{dq}{dt}$ , hence  $1 \text{ C} = (1 \text{ A}) \times (1 \text{ s})$ .
- ▶ The vector form of Coulomb force with  $\hat{r}_{12}$  = unit vector from  $q_1$  to  $q_2$  is given as:

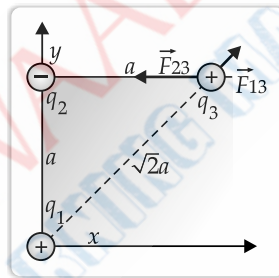
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \text{and} \quad \vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$\Rightarrow \vec{F}_{21} = -\vec{F}_{12}$$



### Principle of Superposition

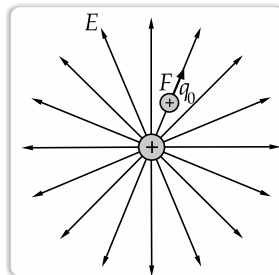
- ▶ The force on any charge due to other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- ▶ The individual forces are unaffected due to presence of other charges.
- ▶ Force exerted by  $q_1$  on  $q_3 = \vec{F}_{13}$
- ▶ Force exerted by  $q_2$  on  $q_3 = \vec{F}_{23}$



- ▶ Net force exerted on  $q_3$  is vector sum of  $\vec{F}_{13}$  and  $\vec{F}_{23}$

### Electric field

- ▶ The space around a charge up to which its electric force can be experienced is called electric field.

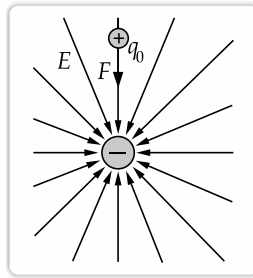


- ▶ If a test charge  $q_0$  is placed at a point where electric field is  $E$ , then force on the test charge is  $F = q_0 E$
- ▶ The electric field strength due to a point source charge ' $q$ ' at an observation point ' $A$ ' at a distance ' $r$ ' from the source charge is given by:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \vec{r} \quad \text{or} \quad E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

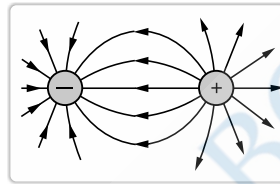
- ▶ The unit of electric field is N/C.

- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in an external field, the field in the conductor is zero.
- In the case of a charged conductor, electric field is independent of the shape of the conductor.



### Electric field lines

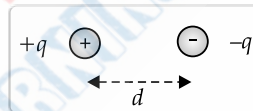
- Electric field lines are imaginary lines that originate from the positive charge and terminate at negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge  $q_0$  located around source charge.



- Electric field has the same direction as force on the positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge  $q_0$ .
- The electric field lines never intersect each other.
- Strength of electric field is encoded in density of field lines.

### Electric Dipole

- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.
- The electric field exists due to a dipole.



- The force on a dipole in a uniform electric field is zero in both stable as well as unstable equilibrium.
- The potential energy of a dipole in an uniform electric field is minimum for a stable equilibrium and maximum for an unstable equilibrium.

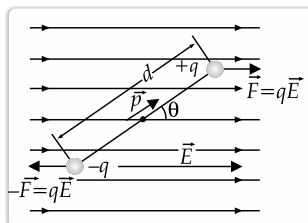
### Torque on a dipole

- In a dipole, when the net force on dipole due to electric field is zero and center of mass of dipole remains fixed, the forces on charged ends produce net torque  $\tau$  about its center of mass.

$$\tau = F d \sin \theta = qE d \sin \theta = pE \sin \theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

- $d = 2a$



- If  $\theta = 0^\circ$  or  $360^\circ$ , dipole exists in stable equilibrium state.
- If  $\theta = 180^\circ$ , dipole exists in an unstable equilibrium state.
- In the uniform electric field, the dipole experiences torque, the net force on dipole is zero.
- In the uniform electric field, the dipole experiences a rotatory motion.
- In the non-uniform electric field, dipole experiences torque and net force.

- In the non-uniform electric field, dipole experiences rotatory and translatory motion.
  - The torque aligns the dipole with the electric field and it becomes zero.
  - The direction of the torque is normal to the plane going inward.

### Electric Dipole Moment

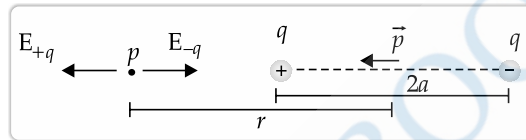
- Dipole moment is a vector quantity whose unit is coulomb-metre (Cm).
- Dipole moment vector of electric dipole is  $\vec{p} = \vec{q} \times 2a$  between pair of charges  $q, -q$ , along the line, separated by distance  $2a$ .

### Electric field due to a dipole

- For point  $P$  at distance  $r$  from the centre of the dipole on charge  $q$ , for  $r \gg a$ , total field at point  $P$  is

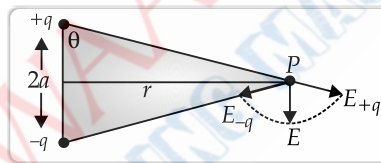
$$E = \frac{4qa}{4\pi\epsilon_0 r^3}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \quad (\text{if } a \ll r)$$



- For point  $P$  on the equatorial plane due to charges  $+q$  and  $-q$ , electric field of dipole at a large distance,

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$



## Topic-2

### Gauss' Theorem and its Applications

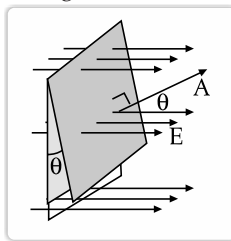
**Concepts Covered** • Electric flux, • Continuous charge distribution, • Gauss' theorem



### Revision Notes

#### Electric Flux

- Electric flux is proportional to algebraic number of electric field lines passing through the surface, outgoing lines with positive sign, incoming lines with negative sign.



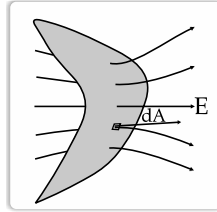
- Due to arbitrary arrangement of electric field lines, electric flux can be quantified as  $\phi_E = EA$
- If vector  $A$  is perpendicular to the surface, magnitude of vector  $A$  parallel to electric field is  $A \cos \theta$ 

$$A_{\parallel} = A \cos \theta$$

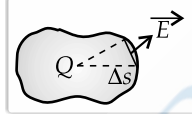
$$\phi_E = EA_{\parallel} = EA \cos \theta$$
- In non-uniform electric field, the flux will be  $\phi_E = \int E \cdot dA$

### Continuous Charge Distribution

- It is a system in which the charge is uniformly distributed over the material. In this system, infinite number of charges are closely packed and have minor space among them. Unlike the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the material. There are three types of continuous charge distribution system.



- For linear charge distribution ( $\lambda$ ),  $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{\lambda}{r^2} d\hat{r}$  (Where,  $\lambda$  = linear charge density)
- For surface charge distribution ( $\sigma$ ),  $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{\sigma}{r^2} dS\hat{r}$  (Where,  $\sigma$  = surface charge density)



- For volume charge distribution ( $\rho$ ),  $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{\rho}{r^2} dV\hat{r}$  (Where,  $\rho$  = volume charge density)

### Gauss' theorem

- The net outward normal electric flux through any closed surface of any shape is equal to  $1/\epsilon_0$  times to net charge enclosed by the surface.
- The electric field flux at all points on Gaussian surface is  $\phi = E \oint dA = \frac{q}{\epsilon_0}$ .

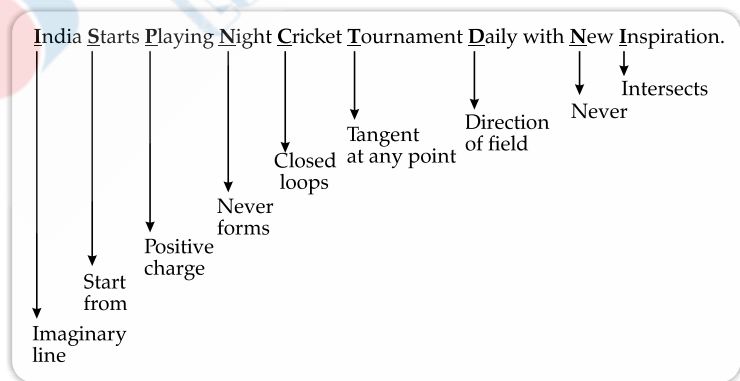


### Mnemonics

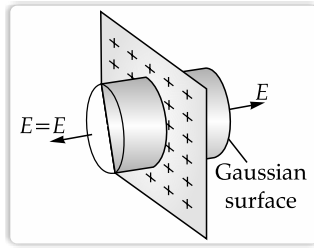
**Concept:** Characteristics of Electric field lines

**Mnemonic:** India Starts Playing Night Cricket Tournament Daily with New Inspiration.

**Interpretation:**



- If there is a positive flux, net positive charge is enclosed.
- If there is a negative flux, net negative charge is enclosed.
- If there is zero flux, no net charge is enclosed.
- The expression for electric field due to a point charge on Gaussian surface is  $E = \frac{q}{4\pi\epsilon_0 r^2}$



- ▶ In an insulating sheet, the charge remains in the sheet, so electric field,  $E = \frac{\sigma}{2\epsilon_0}$
- ▶ Gauss theorem works in cases of cylindrical, spherical and rectangular symmetries.
- ▶ The field outside the wire points radially outward which depends on distance from wire,  $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$ , where,  $\lambda$  is linear charge density.
- ▶ **Closed surface:** It is a surface which divides the space inside and outside region, where one can't move from one region to another without crossing the surface.
- ▶ **Gaussian surface:** It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- ▶ **Electrostatic Shielding:** It is the phenomenon of protecting a certain region of space from external electric field.
- ▶ **Dielectric:** The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. e.g., air,  $H_2$  gas, glass, mica, paraffin wax, transformer oil, etc.

### Key Formulae

▶ Coulomb's force:  $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ ;

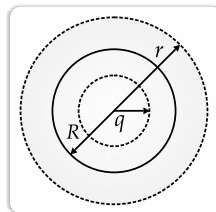
where all alphabets have their usual meanings.

- ▶ Electric field due to point charge  $q$ :

$$E = \frac{k|q|}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

- ▶ Electric field due to a dipole at a point on the dipole axis:  $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$  ( $r \gg \gg a$ )

- ▶ Electric field due to a dipole at a point on an equatorial plane:  $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$  ( $r \gg \gg a$ )



- ▶ Torque on an electric dipole placed in an electric field,  $\tau = pE \sin \theta$
- ▶ Electric flux through an area  $A$ :  $\phi = E \cdot A = EA \cos \theta$
- ▶ Electric flux through a Gaussian surface:  $\phi = \oint E \cdot dS$
- ▶ Gauss's Law:  $\phi = \frac{q_{enc}}{\epsilon_0}$

- ▶ Electric Field due to an infinite line of charge:  $E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$

where,  $E$  = electric field [N/C],

$\lambda$  = charge per unit length [C/m]

$\epsilon_0 =$  permittivity of free space  $= 8.85 \times 10^{-12} [\text{C}^2/\text{N m}^2]$ ,  $r =$  distance (m),  $k = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$

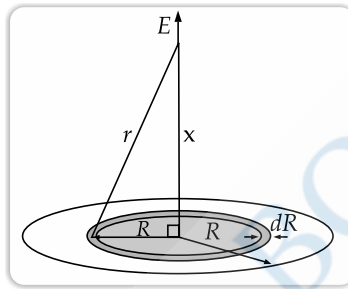
- Electric field due to a ring at a distance  $x$  is:

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{(r^2 + x^2)^{3/2}}$$

- When,  $x \gg r$ :  $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$

- When  $x \ll r$ :  $E = 0$

- Electric field due to a charged disc:  $E = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{x}{\sqrt{R^2 + x^2}} \right]$



where,

$E =$  electric field [N/C]

$\sigma =$  charge per unit area [ $\text{C}/\text{m}^2$ ]

$\epsilon_0 = 8.85 \times 10^{-12} [\text{C}^2/\text{Nm}^2]$

$x =$  distance from charge [m]

$R =$  radius of the disc [m]

- Electric field due to a thin infinite sheet:  $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$

## CHAPTER-2

# ELECTROSTATIC POTENTIAL AND CAPACITANCE

### Topic-1

### Electric Potential

**Concepts Covered** *Electric potential, potential difference, equipotential surfaces. Electrical potential energy of system of two point charges and of electric dipole.*



### Revision Notes

#### Electric potential

- Electric potential is the amount of work done by an external force in moving a unit positive charge from infinity to a point in an electrostatic field without producing an acceleration.

- It is written as  $V = \frac{W}{q}$

where,  $W =$  work done in moving charge  $q$  through the field,  $q =$  charge being moved through the field.

- The SI units of electric potential are  $\frac{\text{J}}{\text{C}}$ , Volt,  $\frac{\text{Nm}}{\text{C}}$ .



**Potential difference**

- Electric potential difference is defined as the amount of work done in carrying a unit charge from one point to another in an electric field.

Electric potential difference

$$= \frac{\text{Work}}{\text{Charge}} = \frac{\Delta PE}{\text{Charge}} = \frac{W}{q}$$

Between two points  $A$  and  $B$ ,  $W_{AB} = -V_{AB} \times q$

where,  $V_{AB} = V_B - V_A$  is potential difference between  $A$  and  $B$ .

- In a region of space having an electric field, the work done by electric field  $dW$ , when test charge  $q$ , is displaced by a distance  $dI$ , then,

$$dW = q \vec{E} \cdot d\vec{l}$$

$$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B q \vec{E} \cdot d\vec{l}}{q} = -\int_A^B \vec{E} \cdot d\vec{l}$$

**Electric potential due to point charge**

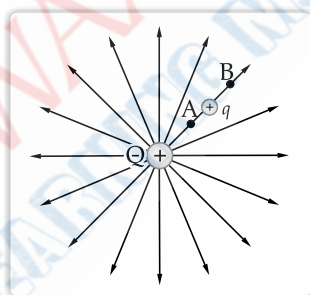
- The electric potential by point charge  $q$ , at a distance  $r$  from the charge, can be written as,

$$V_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

where,  $\epsilon_0$  is absolute electrical permittivity of vacuum (free space).

- **Electric potential** is a scalar quantity.
- Dimension of Electric potential is  $[ML^2T^{-3}A^{-1}]$ .
- For a single point charge  $q$ , the potential difference between  $A$  and  $B$  is given by,

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} = -\int_A^B E dl \cos 0^\circ = -E \int_A^B dl$$



where,  $E$  is the field due to a point charge,  $dI = dr$ , so that,

$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q}{4\pi\epsilon_0} \cdot \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} \right]_{r_A}^{r_B} = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_B} - \frac{1}{r_A} \right]$$

- If  $r_B = \infty$ , then  $V_B = 0$  so,

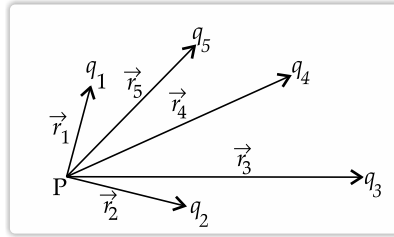
$$V_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_A} = \frac{kq}{r_A}$$

**Dipole and system of charges**

- Electric **dipole** consists of two equal but opposite electric charges which are separated by a certain or least distance.
- The net potential due to a dipole at any point on its equatorial line is always zero. So, work done in moving a charge on an equatorial line is always zero.
- Electric potential due to dipole at a point at distance  $r$  and making an angle  $\theta$  with the dipole moment  $p$  is given by,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{r}}{r^2} (r \gg a)$$

- Potential at a point due to system of charges is the sum of potentials due to individual charges.
- In a system of charges  $q_1, q_2, q_3, \dots, q_n$  having positive vectors  $r_1, r_2, r_3, \dots, r_n$  relative to point  $P$ , the potential at point  $P$  due to total charge configuration is algebraic sum of potentials due to individual charges, so,



$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots + \frac{q_n}{r_n} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

- It is known that in a uniformly charged spherical shell, electric potential outside the shell is given as:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad (r \geq R)$$

where,  $q$  is the total charge on shell and  $R$  is the shell radius.

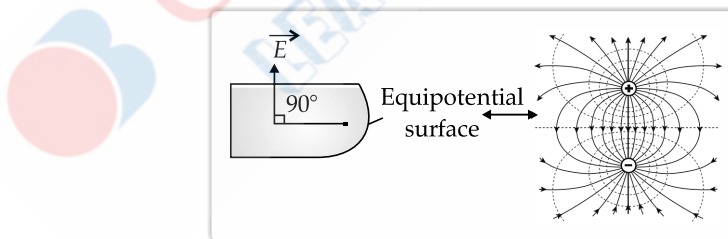
### Key Words

**Electric potential:** The amount of work needed to move a unit charge from a reference point to a specific point against an electric field.

**Dipole:** A pair of equal and oppositely charged or magnetized poles separated by a certain distance.

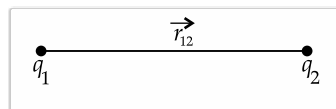
### Equipotential surfaces

- Equipotential surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface, hence the surface will have no electric field, so  $\vec{E}$  will be at right angle to the surface.
- Work done in moving a charge over equipotential surface is always zero.
- Electric field is always perpendicular to the equipotential surface.
- Spacing among equipotential surfaces allows to locate regions of strong and weak electric field.
- Equipotential surfaces never intersect each other. If they intersect then the intersecting point of two equipotential surfaces results in two values of electric potential at that point, which is impossible.



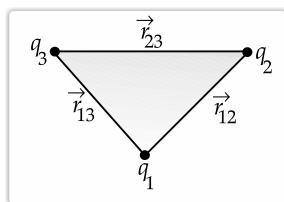
- Potential energy of a system of two charges,

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$



- Potential energy of a system of three charges,

$$U = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



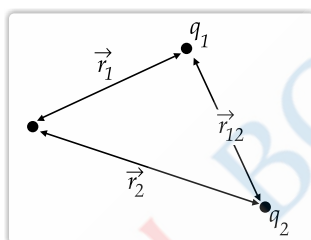
- Potential energy due to single charge in an external field:  
Potential energy of a charge  $q$  at a distance  $r$  in an external field,

$$U = qV(\vec{r})$$

Here,  $V(\vec{r})$  is the external potential at point  $r$ .

- Potential energy due to two charges in an external field,

$$U = q_1V(\vec{r}_1) + q_2V(\vec{r}_2) + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r_{12}}$$



- Potential energy of a dipole in an external field:

When a dipole of charge  $q_1 = +q$  and  $q_2 = -q$  having separation ' $2a$ ' is placed in an external field ( $\vec{E}$ ).

$$U(\theta) = -pE \cos \theta$$

Here,  $p = 2aq$  and  $\theta$  is the angle between electric field and dipole.

## Topic-2

### Capacitance

**Concepts Covered** Dielectrics, electric polarization, capacitor and capacitance, combination of capacitor, energy stored in capacitor.



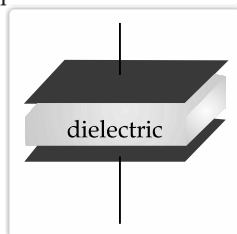
## Revision Notes

### Conductors and insulators

- Conductors are the materials through which charge can move freely. Examples: Metals, semi-metals as carbon, graphite, antimony and arsenic.
- Insulators are materials in which the electrical current does not flow easily. **Examples:** Plastics and glass.

### Dielectrics

- These are the materials in which induced dipole moment is linearly proportional to the applied electric field.



- Electrical displacement or electrical flux density,  $D = \epsilon_r \epsilon_0 E$ .  
where,  $\epsilon_r$  = Electrical relative permittivity,  $\epsilon_0$  = Electrical permittivity of free space and  $E$  is electric field.
- If a dielectric is kept in between the plates of capacitor, capacitance increases by factor ' $\kappa$ ' (kappa) known as dielectric constant, so  $C = \kappa \epsilon_0 \frac{A}{d}$   
where,  $A$  = area of plates

$\kappa$  = dielectric constant of material is also called relative permittivity  $\kappa = \epsilon_r = \frac{\epsilon}{\epsilon_0}$

| Material    | Dielectric Constant ( $\kappa$ ) | Dielectric strength ( $10^6$ V/m) |
|-------------|----------------------------------|-----------------------------------|
| Air         | 1.00059                          | 3                                 |
| Paper       | 3.7                              | 16                                |
| Pyrex Glass | 5.6                              | 14                                |
| Water       | 80                               | -                                 |

- In dielectric, polarization and production of induced charge takes place when dielectric is kept in an external electric field.

### Electric polarization

- **Electric polarization**  $P$  is the difference between electric fields  $D$  (induced) and  $E$  (imposed) in dielectric due to bound and free charges written as  $P = \frac{D-E}{4\pi}$
- In term of electric susceptibility:  $P = \chi_e E$
- In MKS:  $P = \epsilon_0 \chi_e E$ ,
- The dielectric constant  $\kappa$  is always greater than 1 as  $\chi_e > 0$



### Key Word

**Electric polarization:** It is the separation of center of positive charge and the center of negative charge in a material. The separation can be caused by a sufficiently high electric field.

### Capacitor

- A capacitor is a device which is used to store charge.
- Amount of charge ' $Q$ ' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate is connected to negative terminal.
- Electric charge stored between plates of capacitor is directly proportional to potential difference between its plates, i.e.,

$$Q = CV$$

where,  $C$  = Capacitance of capacitor,  $V$  = potential difference between the plates

- In capacitor, energy is stored in the form of electrical energy, in the space between the plates.

### Capacitance

- Capacitance of a capacitor is ratio of magnitude of charge stored on the plate to potential difference between the plates, written as  $C = \frac{Q}{\Delta V}$ .

where,  $C$  = capacitance in Farads ( $F$ ),  $Q$  = charge in Coulombs ( $C$ ),  $\Delta V$  = electric potential difference in Volts ( $V$ ),

- SI unit of capacitance is Farad ( $F$ )

$$1 F = \frac{1 C}{1 V} = 9 \times 10^{11} \text{ stat Farad,}$$

Where, stat-Farad is electrostatic unit of capacitance in C.G.S. system

- Capacitance of a conductor depends on size, shape, medium and other conductors in surrounding.
- Parallel plate capacitor with dielectric among its plates has capacitance which is given as:

$$C = \kappa \epsilon_0 \frac{A}{d},$$

where,  $\epsilon_0 = 8.85 \times 10^{-12}$  F/m

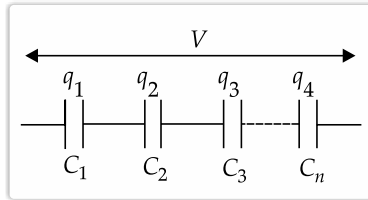
- Capacitor having capacitance of 1 Farad is too large for electronics applications, so components with lesser values of capacitance such as  $\mu$  (micro),  $n$  (nano) and  $p$  (pico) are applied such as:

| PREFIX | MULTIPLIER                     |                       |
|--------|--------------------------------|-----------------------|
| $\mu$  | $10^{-6}$ (millionth)          | $1 \mu F = 10^{-6} F$ |
| $n$    | $10^{-9}$ (thousand-millionth) | $1 nF = 10^{-9} F$    |
| $p$    | $10^{-12}$ (million-millionth) | $1 pF = 10^{-12} F$   |

**Combination of capacitors in series and parallel****Capacitors in series**

- (i) If a number of capacitors of capacitances  $C_1, C_2, C_3, \dots, C_n$  are connected in series, then their equivalent capacitance is given by:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$



In series combination, the charge on each capacitor is same, but the potential difference on each capacitor depends on their respective capacitance, i.e.,

$$q_1 = q_2 = q_3 \dots q_n = q$$

- If  $V_1, V_2, V_3, \dots, V_n$  be the potential differences across the capacitors and  $V$  be the emf of the charging battery, then

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

- As charge on each capacitor is same, therefore

$$q = V_1 C_1 = V_2 C_2 = V_3 C_3 \dots$$

the potential difference is inversely proportional to the capacitance, i.e.,

$$V \propto \frac{1}{C}$$

- In series, potential difference across largest capacitance is minimum.  
 ► The equivalent capacitance in series combination is less than the smallest capacitance in combination.

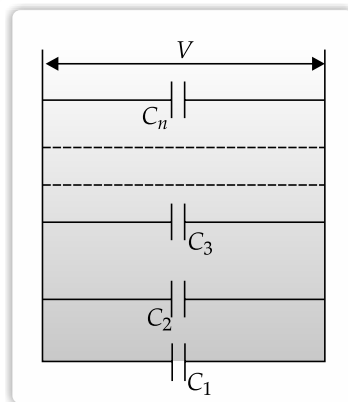
**Capacitors in parallel**

- (i) If a number of capacitors of capacitances  $C_1, C_2, C_3, \dots, C_n$  are connected in parallel, then their equivalent capacitance is given by,

$$C_p = C_1 + C_2 + C_3 + \dots + C_n$$

- In parallel combination, the potential difference across each capacitor is same and equal to the emf of the charging battery, i.e.,

$$V_1 = V_2 = V_3 = \dots = V_n = V$$



while the charge on different capacitors may be different.

- If  $q_1, q_2, q_3, \dots, q_n$  be the charges on the different capacitors, then  $q_1 + q_2 + q_3 + \dots + q_n = VC_p$   
 ► As potential drop across each capacitor is same, so

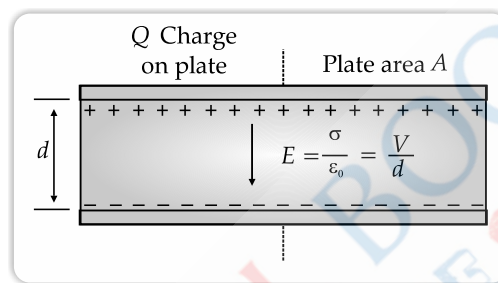
$$\Rightarrow V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \dots = \frac{q_n}{C_n}$$

- ▶ The charges on capacitors are directly proportional to capacitances, i.e.,  $q \propto C$
- ▶ Parallel combination is useful when large capacitance with large charge gets accumulated on combination.
- ▶ Force of attraction between parallel plate capacitor will be  $F = \frac{1}{2} \left[ \frac{QV}{d} \right] = \frac{1}{2} QE$  where  $Q$  is charge on capacitor.

#### Capacitance of parallel plate capacitor with and without dielectric medium between the plates

- ▶ Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium.
- ▶ The electric field between two large parallel plates is given as:

$$E = \frac{\sigma}{\epsilon_0},$$



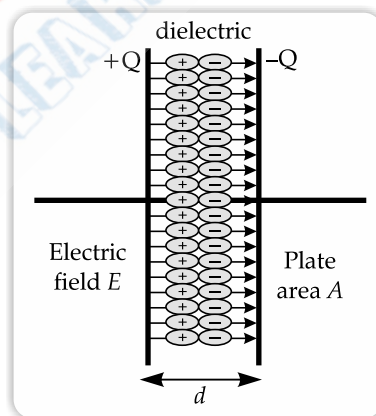
Where,  $\sigma$  = charge density and  $\epsilon_0$  = permittivity of free space  
Surface charge density,

$$\sigma = \frac{Q}{A},$$

where,  $Q$  = charge on plate and  $A$  = plate area

- ▶ Capacitance of parallel-plate capacitor with area  $A$  separated by a distance  $d$  is written as

$$C = \epsilon_r \epsilon_0 \frac{A}{d}$$



- ▶ If a dielectric slab is placed in between the plates of a capacitor, then its capacitance will increase by certain amount.
- ▶ Capacitance of parallel plate capacitor depends on plate area  $A$ , distance  $d$  between the plates, medium between the plates ( $\kappa$ ) and not on charge on the plates or potential difference between the plates.
- ▶ If we have number of dielectric slabs of same area as the plates of the capacitor and thicknesses  $t_1, t_2, t_3, \dots$  and dielectric constant  $\kappa_1, \kappa_2, \kappa_3, \dots$  between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\epsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \frac{t_3}{\kappa_3} + \dots}$$

Where,  $d = t_1 + t_2 + t_3 + \dots$

- If slab of conductor of thickness  $t$  is introduced between the plates, then

$$C = \frac{\epsilon_0 A}{\frac{t}{\kappa} + \frac{(d-t)}{1}} = \frac{\epsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}}$$

$$C = \frac{\epsilon_0 A}{d-t} \quad (\because \kappa = \infty \text{ for a conductor})$$

- When the medium between the plates consists of slabs of same thickness but areas  $A_1, A_2, A_3, \dots$  and dielectric constants  $\kappa_1, \kappa_2, \kappa_3, \dots$ , then capacitance is given by

$$C = \frac{\epsilon_0(\kappa_1 A_1 + \kappa_2 A_2 + \kappa_3 A_3 \dots)}{d}$$

$$\therefore \kappa = \frac{C_m}{C_0} = \frac{\text{Capacitance in medium}}{\text{Capacitance in vacuum}}$$

- When space between the plates is partly filled with medium of thickness  $t$  and dielectric constant  $\kappa$ , then capacitance will be:

$$C = \frac{\epsilon_0 A}{d-t + \frac{t}{\kappa}} = \frac{\epsilon_0 A}{d-t \left(1 - \frac{1}{\kappa}\right)}$$

When there is no medium between the plates, then  $\kappa = 1$ , so  $C_{\text{vacuum}} = \frac{\epsilon_0 A}{d}$

- Capacitance of spherical conductor of radius  $R$  in a medium of dielectric constant  $\kappa$  is given by,

$$C = 4\pi\epsilon_0\kappa R$$

### Energy stored in capacitor

- In capacitor, energy gets stored when a work is done on moving a positive charge from negative conductor to positive conductor against the repulsive forces.

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

- **Polar atom:** Atom in which positive and negative charges possess asymmetric charge distribution about its centre.
- **Polarisation:** The stretching of atoms of a dielectric slab under an applied electric field.
- **Dielectric strength:** The maximum value of electric field that can be applied to dielectric without its electric breakdown.
- **Dielectric:** It is an electrically insulated or non-conducting material considered for its electric susceptibility.
- **Permittivity:** It is a property of a dielectric medium that shows the forces which electric charges placed in medium exerts on each other.

OR

It is the measure of resistance that is encountered when forming an electric field in a particular medium. More specifically, permittivity describes the amount of charge needed to generate one unit of electric flux in a particular medium.



### Mnemonics

**Concept:** Characteristics of equi-potential surface

**Mnemonics:** Exclusive peace and No war; Noble India is super power

**Interpretations:** Exclusive peace: Electric field is perpendicular to the surface

No war: No Work is done on moving a charge on the surface

Noble India: Never Intersects

Super Power: Same potential everywhere on the surface

## Key Formulae

► Electric Potential,  $V = \frac{W}{q}$ , measured in volt; 1 volt = 1 Joule / coulomb.

► Electric potential difference or "voltage" ( $\Delta V$ ) =  $V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$ .

► Electric potential due to a point charge  $q$  at a distance  $r$  away:  $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

► Finding  $V$  from  $E$ :  $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{S}$

► Potential energy of two point charges in absence of external electric field:  $U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{r_{12}} \right]$

► Potential energy of two point charges in presence of external electric field:  $q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$

► Capacitance,  $C = \frac{Q}{V}$ , measured in Farad; 1 F = 1 coulomb/volt

► **Parallel plate capacitor:**

$$C = \kappa\epsilon_0 \frac{A}{d}$$

► **Cylindrical capacitor:**

$$C = 2\pi\kappa\epsilon_0 \frac{L}{\ln(b/a)}$$

where,  $L$  = length,  $b$  = radius of the outer conductor,  $a$  = radius of the inner conductor.

► **Spherical capacitor:**

$$C = 4\pi\kappa\epsilon_0 \left( \frac{ab}{b-a} \right)$$

where,  $b$  = radius of the outer conductor,  $a$  = radius of the inner conductor

► Maximum charge on a capacitor:

$$Q = VC$$

► For capacitors connected in series, the charge  $Q$  is equal for each capacitor as well as for the total equivalent. If the **dielectric constant**  $\kappa$  is changed, the capacitance is multiplied by  $\kappa$ , the voltage is divided by  $\kappa$  and  $Q$  is unchanged. In vacuum,  $\kappa = 1$  and when dielectrics are used, replace  $\epsilon_0$  with  $\kappa\epsilon_0$ .

► Electrical energy stored in a capacitor: [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

► Surface charge density or Charge per unit area: [ $C/m^2$ ]

$$\sigma = \frac{q}{A}$$

► **Energy density:**

- Electric energy density is also called Electrostatic pressure.
- Electric force between plates of capacitor,

$$F = \frac{1}{2} \epsilon_0 E^2 \cdot A$$

- Energy stored in terms of Energy density,

$$\frac{E}{A \times d} = \frac{1}{2} \epsilon_0 E^2$$

$$U = \frac{1}{2} \epsilon_0 E^2$$

where,  $U$  = energy per unit volume [ $J/m^3$ ],  $\epsilon_0$  = permittivity of free space, =  $8.854 \times 10^{-12} C^2/Nm^2$ ,  $E$  = energy [J]

- **Capacitors in series:**

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \dots$$

- **Capacitors in parallel:**

$$C_{eff} = C_1 + C_2 \dots$$



**UNIT – II: CURRENT ELECTRICITY****CHAPTER-3****CURRENT ELECTRICITY****Topic-1****Electric Current & Cells**

**Concepts Covered** • Electric current, • drift velocity, • Ohm's law, • V-I characteristics, Resistivity and conductivity, • Temperature dependence of resistance, • Cells and their combinations.

**Revision Notes****Electric current**

- ▶ Electric current is defined as the rate of flow of charge, across the cross section of conductor i.e.,  $I = \frac{dq}{dt}$
- ▶ When charge flows at a constant rate, the corresponding electric current can be written as :  $I = \frac{q}{t}$
- ▶ Conventional current in an external circuit flows from positive terminal to negative terminal.
- ▶ Free electrons flow from the negative terminal to the positive terminal in the external circuit.
- ▶ 1 Ampere current =  $6.25 \times 10^{18}$  electrons flowing per second.
- ▶ Direct current is unidirectional flow of electric charge.

**Flow of electric charges in metallic conductor**

- ▶ When an electric field is applied to a metal at certain points, free electrons experience force and start moving.
- ▶ Without external applied electric field, free electrons will move randomly through metal from one point to other giving zero net current.
- ▶ Motion of conducting electrons in electric field is a combination of motion due to random collisions.
- ▶ Drift velocity, mobility and their relation with electric current
- ▶ Drift velocity is an average velocity which is obtained by certain particles like electrons due to the presence of electric field.

- ▶ Drift velocity is written as :

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

where, relaxation time,  $\tau = \frac{\lambda}{v}$ ,

here  $e$  = charge,  $m$  = mass,

$\lambda$  = mean free path

- ▶ When electric current is set up in a conductor, electrons drift through the conductor with velocity  $v_d$ , is given as

$$v_d = \frac{I}{neA} \text{ or } I = neAv_d$$

where,  $I$  = Electric current through conductor,

$n$  = Number of free electrons per unit volume,

$A$  = Area of cross-section,  $e$  = Charge of electron

- ▶ Drift velocity of electrons under ordinary conditions is of the order of 0.1 mm/s.
- ▶ Mobility is the drift velocity of an electron when applied electric field is unity.

$$\text{Mobility, } \mu = \frac{v_d}{E}$$

$$\text{or } \mu = \frac{e\tau E / m}{E} = \frac{e\tau}{m}$$

**Electrical resistivity and conductivity**

- ▶ Resistivity is the specific resistance that is given by the conductor having unit length and unit area of cross-section.

$$\rho = \frac{m}{ne^2\tau}$$

- Conductivity is the reciprocal of resistivity shown as :

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

### Ohm's law

- The flow of current through conductor is directly proportional to the potential difference established across the conductor, provided other physical conditions remain constant.

$$\text{or, } I \propto V$$

$$\text{or, } I = GV$$

$$\text{Here, } G = \frac{1}{R}$$

$$\text{or } I = \frac{1}{R}V$$

$$\text{or } V = IR$$

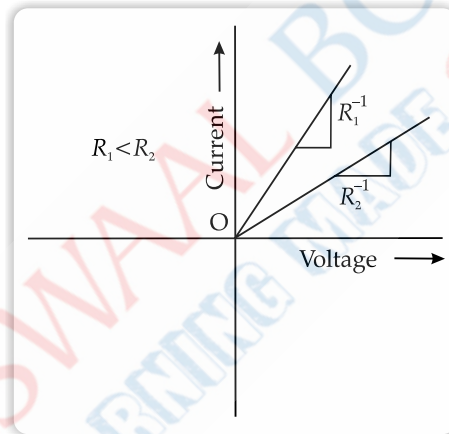
where,  $R$  = resistance of conductor

### V-I characteristics (linear and non-linear)

- V-I characteristic curves show the relationship between the current flowing through an electronic device and applied voltage across its terminals.

#### Linear V-I Characteristics

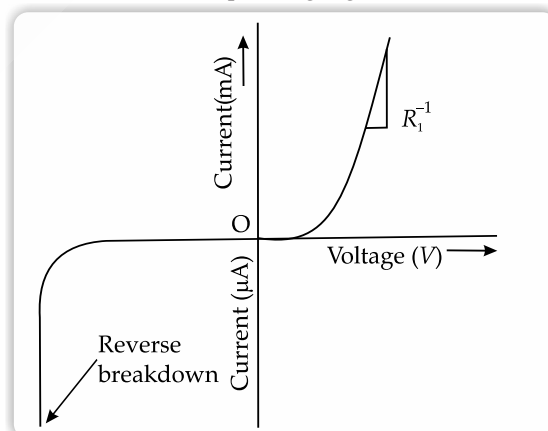
- A linear V-I curve has a constant slope and hence a constant resistance. Carbon resistors and metals obey the Ohm's law and have a constant resistance. This means that the V-I curve is a straight line passing through the origin.



- An electronic component may exhibit linear characteristic only in a particular region. For example, a resistance shows linear behaviour mostly in its operating region.

#### Non-linear V-I Characteristics

- A circuit component has a non-linear characteristic if the resistance is not constant throughout and is some function of voltage or current. The diode, for example, has varying resistance for different values of voltage.  
 ► However, it has linear characteristic for a narrow operating region.



Note that in the graph above, we can also see the maximum forward and reverse voltage in which the diode can be operated without causing breakdown and burning up of the diode.

**Electrical energy and power**

- Electrical energy due to conduction of charged particles in a conductor causing electric current ( $i$ ) is given as

$$E = V \times i \times t = i^2 \times R \times t = \frac{V^2}{R} \times t$$

where,  $E$  = Electrical energy,  $V$  = Potential difference,  $t$  = Time taken,  $i$  = Current,  $R$  = Resistance  
The SI unit of energy is Joule (J).

- Power is the work done per unit time which is the rate of energy consumed in a circuit.

$$P = \frac{W}{t}$$

Since, Voltage  $V = \frac{W}{q}$ ,

$$\text{So } P = V \frac{q}{t} = VI$$

$$\text{or } P = I^2 R \text{ or } \frac{V^2}{R} \quad \left[ \text{Here, } I = \frac{q}{t} \right]$$

The unit of power is J/s or W (Watt).

**Temperature dependence of resistivity**

- With small change in temperature, resistivity varies with temperature as :

$$\rho = \rho_0(1 + \alpha \Delta T)$$

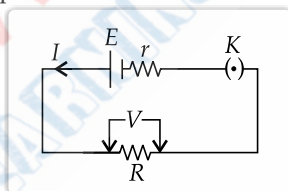
where,  $\alpha$  = temperature coefficient of resistivity.

**Internal resistance of cell**

- Cell is a device that maintains the potential difference that is present between the two electrodes as a result of chemical reaction.  
► Internal resistance is the resistance of electrolyte that is present in a battery which resists the flow of current when connected to a circuit.  
► Emf  $E$  is the potential difference between the electrodes of cell, when no current flows through it.

**Potential difference and emf of a cell**

- **The emf and terminal potential difference of a cell :** Let emf of a cell be  $E$  and its internal resistance,  $r$ . If an external resistance  $R$  be connected across the cell through a key, then  $IR = V =$  potential difference across the external resistance  $R$ . This is equal to the terminal potential difference across the cell.



$$E = V + Ir$$

$$\Rightarrow I = \frac{E - V}{r}$$

$$\text{So } V = E - Ir$$

$$\therefore V < E. \quad (\text{if there is flow of current})$$

When current is drawn from a cell, its terminal potential difference is less than the emf.

**Combination of cells in series and parallel**

- **(i) Series combination of cells :** This combination is used when an external resistance ( $R$ ) of the circuit is much larger as compared to the internal resistance ( $r$ ) of the cell i.e.,

$$R \gg r$$

Let  $n$  cells, each of emf  $E$  and internal resistance  $r$  be connected in series across an external resistance  $R$ , then the current in the circuit will be

$$I_s = \frac{nE}{R + nr}$$

- **(ii) Parallel combination of cells :** This combination is used when the external resistance  $R$  is much smaller as compared to the internal resistance ( $r$ ) of the cell, i.e.,

$$R \ll r$$

When  $m$  cells are connected in parallel across a resistance  $R$ , then current through the resistance is given by

$$I_p = \frac{E}{R + r/m} = \frac{mE}{mR + r}$$

If  $m$  cells of emfs  $E_1, E_2, E_3, \dots, E_m$  and of internal resistances  $r_1, r_2, r_3, \dots, r_m$  are connected in parallel across an external resistance  $R$ , then the current through the external resistance is given by

$$I_p = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \dots + \frac{E_m}{r_m}}{R + \left( \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots + \frac{1}{r_m} \right)}$$

► **Conductance** : The reciprocal of resistance with unit as Siemen, "S".

## Topic-2

### Kirchhoff's Rules & Wheatstone Bridge

**Concepts Covered** • Kirchhoff's rules, • Wheatstone bridge.



### Revision Notes

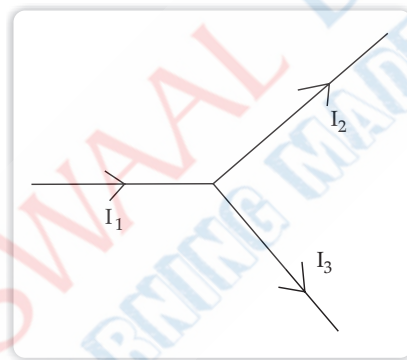
Kirchhoff's rules

► Kirchhoff's rules tell us about the relationship between voltages and currents in circuits.

First rule

► Kirchhoff's first rule is also known as junction rule which states that for a given junction or node in a circuit, sum of the currents entering in a junction is equal to sum of currents leaving that junction.

$$I_1 = I_2 + I_3$$



OR

► The algebraic sum of all currents meeting at a junction in a closed circuit is zero. i.e.,  $\Sigma I = 0$

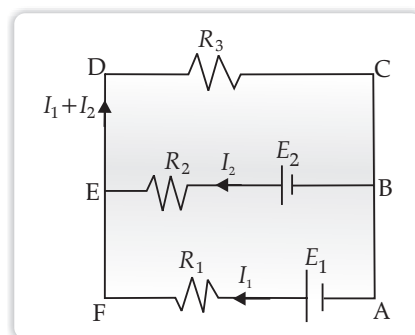
► This is also called as the law of conservation of charge.

Second rule

► Kirchhoff's second rule is also known as loop rule which shows that around any closed loop in a circuit, sum of the emfs and the potential differences across all elements is zero.

$$\text{i.e., } \Sigma V = 0 \text{ or } \Sigma V = \Sigma IR$$

► This is also called as the law of conservation of energy.



**For example** : Applying voltage law in loop AFEBA,

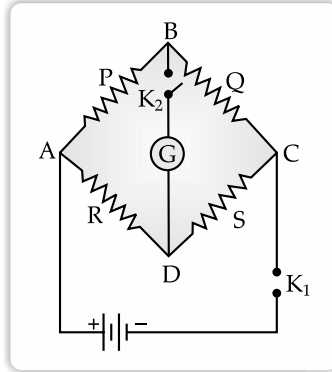
$$E_1 - E_2 = I_1 R_1 - I_2 R_2$$

**Wheatstone Bridge**

- It is a circuit having four resistances  $P$ ,  $Q$ ,  $R$  and  $S$ , a galvanometer ( $G$ ) and a battery connected as shown.

**Wheatstone Bridge**

- Balanced condition:**  $P/Q = R/S$
- Node :** An end point to any branch of a network or a junction common to two or more branches.

**UNIT – III: MAGNETIC EFFECTS OF CURRENT AND MAGNETISM****CHAPTER-4****MOVING CHARGES AND MAGNETISM****Topic-1****Magnetic Field and Biot-Savart law**

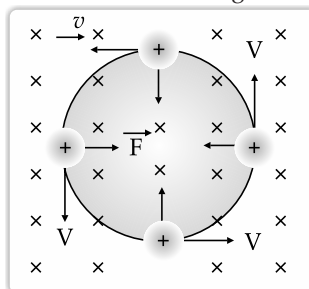
**Concepts Covered** • Magnetic field, • Oersted's experiment, • Biot-Savart law and its application to current carrying circular loop.

**Revision Notes****Concept of Magnetic field**

- Magnetic field** is a region around a magnet where force of magnetism acts which affects other magnets and magnetic materials.
- Magnetic field also known as  $B$ -field can be pictorially represented by magnetic field lines.
- Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.
- Lorentz Force:** When a charge  $q$  moving with velocity  $v$  enters a region where both magnetic fields and electric fields exist, both fields exert a force on it.

$$\text{Lorentz Force, } \vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$$

where,  $\vec{F}$  = magnetic force,  $q$  = charge,  $\vec{v}$  = velocity,  $\vec{B}$  = magnetic field,  $\vec{E}$  = electric field,  $q\vec{E}$  = electric force on the charge,  $q(\vec{v} \times \vec{B})$  = magnetic force on the charge



- ▶ SI unit of magnetic field is Tesla, while smaller magnetic fields are measured in terms of Gauss.

$$1 \text{ Tesla} = 10^4 \text{ G}$$

- ▶ When a test charge  $q_0$  enters a magnetic field  $\vec{B}$  directed along negative z-axis with a velocity  $\vec{v}$  making an angle  $\theta$  with the z-axis, then,

$$\vec{F}_m = q_0(\vec{v} \times \vec{B}) = q_0 v B \sin \theta \hat{n}$$

Characteristics of motion of particle in magnetic field

- ▶ Velocity and kinetic energy of particle do not change, as force is always perpendicular to velocity.
- ▶ Direction of velocity will continuously change, if  $\theta = 0$ .

When  $\theta = 0$ , no force will act on the particle, hence there will be no change in velocity.

- ▶ When  $\theta = 90^\circ$ , test charge describes a circle of radius

$$\frac{mv}{q_0 B},$$

where,  $m$  is mass of the particle; larger the momentum, bigger the circle described.

- ▶ In case of  $\theta$  being any other angle than  $0^\circ$  and  $90^\circ$ , test charge will show circular path of radius  $\frac{mv \sin \theta}{q_0 B}$ ,

which moves along the direction of magnetic field with speed of  $v \cos \theta$ .

- ▶ Momentum along the direction of magnetic field will remain same.

- ▶ **Angular speed** of test charge  $\frac{q_0 B}{m}$  is independent of initial speed of particle.

- ▶ Centripetal force on test charge  $q_0 v B \sin \theta$  is independent of the mass of particle.

- ▶ When the particle enters the magnetic field with the same momentum, then radius of path will be,

$$r = \frac{mv}{q_0 B}$$

where,  $r \propto \frac{1}{q_0}$

Oersted's experiment

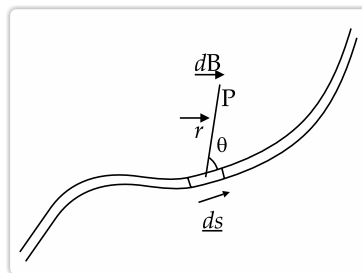
Oersted observed that:

- ▶ When there is no current, compass needle below a wire shows no deflection.
- ▶ When the flow of current is in single direction, then the compass needle deflects in a particular direction.
- ▶ When the flow of current is reversed, deflection in compass needle occurs in the opposite direction.
- ▶ From an experiment, it is concluded that an electrical current produces a magnetic field which surrounds the wire.

Biot-Savart's law

- ▶ The magnetic field due to a current element at a nearby point is given by:

$$\text{where, } \vec{dB} = \left[ \frac{\mu_0}{4\pi} \right] I \frac{d\vec{S} \times \vec{r}}{r^3}$$



$\vec{dB}$  = Magnetic field produced by current element

$d\vec{S}$  = Vector length of small section of wire in direction of current

$\vec{r}$  = Positional vector from section of wire to where magnetic field is measured

$I$  = Current in the wire

$\theta$  = Angle between  $\vec{ds}$  and  $\vec{r}$

$\mu_0$  = **Permeability** of free space and  
The magnitude of magnetic field,

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb/Am}$$

$$|d\vec{B}| = \left(\frac{\mu_0}{4\pi}\right) \frac{Idl \sin\theta}{r^2}$$

### Key Words

**Angular Speed:** It is defined as the rate of change of angular displacement, and it is expressed as follows:  $\omega = \theta t$ .

**Permeability:** It is also called magnetic permeability, is a constant of proportionality that exists between magnetic induction and magnetic field intensity.

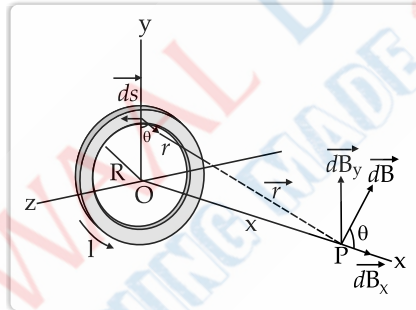
Applications of Biot-Savart's Law

- Magnetic field at a point in circular loop will be:

$$\vec{B} = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} \vec{r} \quad [\text{Here, } r^2 = R^2 + x^2]$$

- Magnetic field at the centre of the coil

$$\vec{B} = \frac{\mu_0 NI}{2R} \vec{r} \quad [x = 0]$$

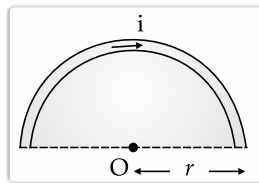


- Magnetic field at very large distance from the centre:  $B = \frac{2\mu_0 NiA}{4\pi x^3}$  [Here,  $R^2 \ll r^2$  or  $R^2 + x^2 \approx x^2$ ]

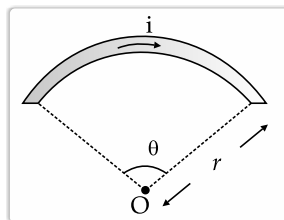
where,  $A$  = Area of circular loop  
 $= \pi R^2$

Magnetic field due to current carrying circular arc with centre O will be:

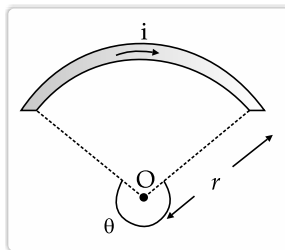
$$(i) B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$$



$$(ii) B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$$

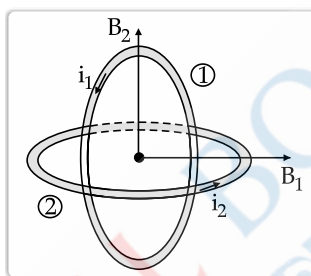


$$(iii) B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta)i}{r}$$



- Magnetic field at common centre of non-coplanar and concentric coils, where both coils are perpendicular to each other will be:

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r} \sqrt{i_1^2 + i_2^2}$$



### Key Formulae

- Lorentz force,  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- In uniform magnetic field B, frequency of circular motion of charged particle,
- $$f = \frac{qB}{2\pi m}$$
- and  $KE_m = \frac{q^2 r^2 B^2}{2m}$
- Biot-Savart's law,  $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \vec{r})}{r^3}$
- Magnetic field at a point due to circular loop,
- $$\vec{B} = \frac{\mu_0}{2} \cdot I \frac{R^2}{(R^2 + x^2)^{3/2}}$$



### Mnemonics

**Concept:** Path of a charged particle in a uniform magnetic field depending on the angle between the magnetic field and the velocity of the particle:

**Mnemonics:** Circle-circle ninety (90°) angle, go straight if it is zero (0°), go for helical, all other angle magnetic field is zero.

Circle circle ninety (90°) angle ⇒

Path is a circle if angle between magnetic field and velocity of charged particle is 90°.

Go straight if it zero (0°) ⇒

Path is a straight line if angle between magnetic field and velocity of charged particles is 90°

Go for helical, all other angle ⇒

Path is helix if any other angle between magnetic field and velocity of charged particle.



**Topic-2****Ampere's Circuital Law and its Applications**

**Concepts Covered** • Ampere's law and its applications to infinitely long straight wire,  
• straight solenoid

**Revision Notes**

- Ampere's circuital law states that the line integral of magnetic field around a closed path is  $\mu_0$  times of total current enclosed by the path,  $\oint B \cdot dl = \mu_0 I$

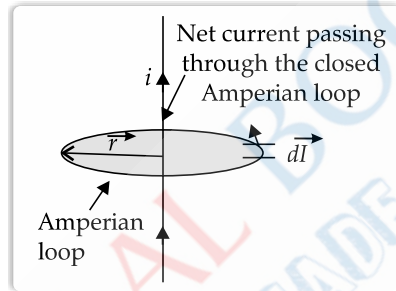
where,

$B$  = Magnetic field

$dl$  = Infinitesimal segment of the path

$\mu_0$  = Magnetic permeability of free space

$I$  = Enclosed electric current by the path



- Magnetic field at a point will not depend on the shape of Amperian loop and will remain same at every point on the loop.

Forces between two parallel currents

- Two parallel wires separated by distance  $r$  having currents  $I_1$  and  $I_2$  where magnetic field strength at second wire due to current flowing in first wire is given as:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

- In this, the field is orientated at right-angles to second wire where force per unit length on the second wire will be:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

- Magnetic field-strength at first wire due to the current flowing in second wire will be:

$$B = \frac{\mu_0 I_2}{2\pi r}$$

- One ampere is the magnitude of current which, when flowing in each parallel wire one metre apart, results in a force between the wires as  $2 \times 10^{-7}$  N per metre of length.

Applications of Ampere's law to infinitely long straight wire, straight solenoids

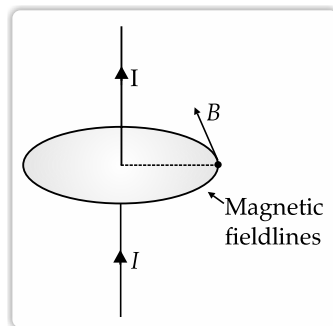
**(i) Magnetic Field due to long straight wire**

Ampere's law describes the magnitude of magnetic field of a straight wire as:

$$B = \frac{\mu_0 I}{2\pi r}$$

where,

- Field  $B$  is tangential to a circle of radius  $r$  centered on the wire.
- Magnetic field  $B$  and path length  $L$  will remain parallel where magnetic field travels.



(ii) Magnetic Field due to Solenoid

### Solenoid:

Solenoid is a tightly wound helical coil of wire whose diameter is small compared to its length.

Magnetic field generated in the centre, or core of a current carrying solenoid is uniform and is directed along the axis of solenoid.

Magnetic field due to a straight solenoid:

- at any point in the solenoid,  $B = \mu_0 n I$
- at the ends of solenoid,  $B_{\text{end}} = \frac{\mu_0 n I}{2}$

where,  $n$  = number of turns per unit length,  $I$  = current in the coil.



## Key Formulae

➤ Ampere's circuital law:  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$$r = \frac{\mu_0 N i}{2\pi r}$$

➤ Magnetic field at the surface of a solid cylinder:

$$B = \frac{\mu_0 I}{2\pi R}$$

➤ Force between two parallel wires,

$$F = \frac{\mu_0}{4\pi} \times \frac{2i_1 i_2}{a} \times l$$

➤ Magnetic field inside the solenoid:

$$B = \mu_0 n I$$

➤ Force between two moving charge particle,

$$F_m = \frac{\mu_0}{4\pi} \times \frac{q_1 q_2 v_1 v_2}{r^2}$$

➤ Magnetic field in a toroid with mean radius  $r$ :

## Topic-3

### Torque and Galvanometer

**Concepts Covered** • Torque experienced by a current loop in uniform magnetic field, • moving coil galvanometer and its current sensitivity • conversion to ammeter and voltmeter.



## Revision Notes

➤ **Torque:** It is a force that produces or tends to produce rotation or torsion.

➤ **Galvanometer:** A galvanometer is an electromechanical measuring instrument for electric current.

Torque experienced by a current loop in uniform magnetic field

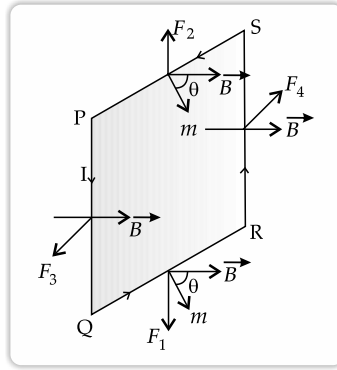
➤ If a rectangular loop of length  $l$ , breadth  $b$  with current  $I$  flowing through it is in a uniform magnetic field of induction,  $B$  where angle  $\theta$  is between the normal and in direction of magnetic field, then the torque experienced will be:

$$\tau = n B I A \sin \theta$$

where,  $n$  = number of turns in the coil

$$\therefore n I A = M$$

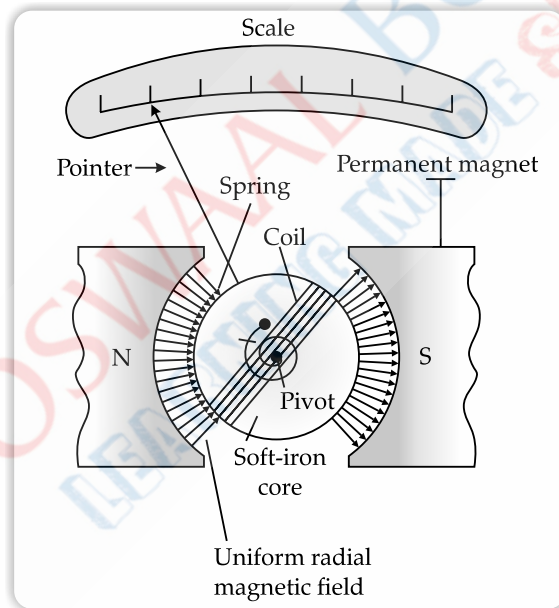
Further,  $\tau = M B \sin \theta$



- Torque will be maximum when the coil is parallel to magnetic field and will be zero when coil is perpendicular to magnetic field.
- In vector notation, torque  $\vec{\tau}$  experienced will be  $\vec{\tau} = \vec{M} \times \vec{B}$

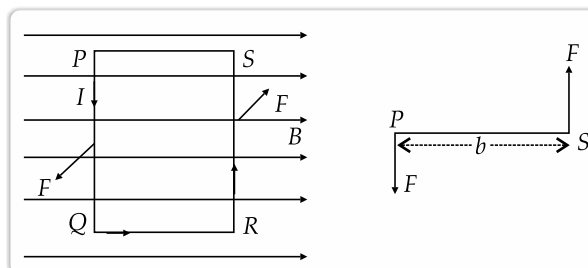
**Moving coil galvanometer**

- It is an instrument used for detection and measurement of small electric currents.
- In this, when a current carrying coil is suspended in uniform magnetic field, it experiences a torque which rotates the coil.
- The force experienced by each side of the galvanometer will be  $F = BIl$  which are opposite in direction.



- Opposite and equal forces form the couple which generates deflecting torque on the coil having number of turns  $n$  is given as:

$$\begin{aligned} \tau &= F \times b \\ &= nBIl \times b \\ &= nBIA \end{aligned}$$



- In moving coil galvanometer, current in the coil will be directly proportional to the angle of the deflection of the coil,  $I \propto \theta$  *i.e.*, where,  $\theta$  is the angle of deflection.

### Current sensitivity of galvanometer

- ▶ Current sensitivity of galvanometer is the deflection produced when unit current passes through the galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

$$I = \frac{C}{nBA} \theta$$

$$\text{Current Sensitivity, } \frac{\theta}{I} = \frac{nBA}{C}$$

- ▶ Voltage sensitivity of galvanometer is the deflection per unit voltage given as

$$\text{Voltage Sensitivity, } \frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$$

where,  $G$  = galvanometer resistance,  $C$  = torsional constant.

- ▶ Increase in sensitivity of moving coil galvanometer depends on:

- number of turns  $n$
- magnetic field  $B$
- area of coil  $A$  and
- torsional constant.

### Conversion of galvanometer into ammeter

- ▶ Galvanometer can be converted into ammeter by connecting a low resistance known as shunt in parallel with the galvanometer coil.
- ▶ If  $I_g$  being the maximum current with full scale deflection passes through galvanometer, then current through shunt resistance will be

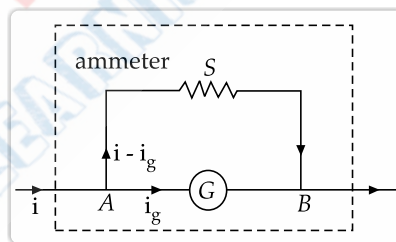
$$i_s = (i - i_g)$$

where,  $G$  = Galvanometer resistance,  $S$  = Shunt resistance and  $i$  = Current in circuit

- ▶ Now, effective resistance of ammeter will be:

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

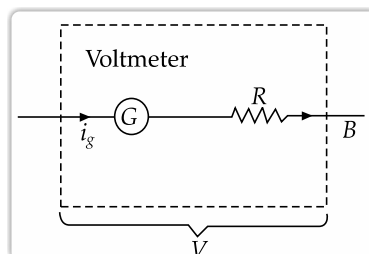
$$R_a = \frac{GS}{G+S}$$



### Conversion of galvanometer into voltmeter

- ▶ Voltmeter measures the potential difference between the two ends of a current carrying conductor.
- ▶ Galvanometer can be converted to voltmeter by connecting high resistance in series with galvanometer coil.
- ▶ As resistance  $R$  is connected in series with galvanometer, current through the galvanometer will be,

$$i_g = \frac{V}{R+G} \quad \text{or, } R = \frac{V}{i_g} - G$$



- ▶ Effective resistance of voltmeter is  $R_v = G + R$ ,

where,  $R_v$  is very large making the voltmeter to connect in parallel since it can draw less current from the circuit.

**Key Formulae**

$$\tau_{\max} = NBiA$$

$$\text{Current Sensitivity} = \frac{\theta}{I} = \frac{nAB}{C}$$

$$\text{Voltage Sensitivity} = \frac{\theta}{V} = \frac{nAB}{CG}$$

**CHAPTER-5****MAGNETISM AND MATTER****Revision Notes**

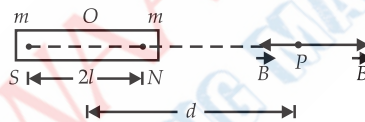
**Magnetic field intensity due to magnetic dipole (bar magnet) along its axis and perpendicular to its axis**

- Bar magnet has two magnetic poles known as north pole and south pole each having strength  $m$ . If the separation between the poles known as **magnetic length** is  $2l$ , then bar magnet is said to have **magnetic dipole moment** which is
 
$$M = m \times 2l = 2ml$$
- Direction of magnetic dipole moment is from South pole to North pole of bar magnet.
- Lines of magnetic force run in closed loops and form continuous closed loops.

**Point lies on axial line of bar magnet:**

- If  $2l$  is length of bar magnet,  $m$  is **magnetic strength** or **intensity** of each pole,  $M$  is magnetic dipole moment, then magnetic field at point  $P$  due to:

North pole of magnet (N)



$$B = \frac{\mu_0(+m)}{4\pi(d-l)^2}$$

**Key Words**

- Magnetic length:** It is the distance between the two poles of a magnetic dipole.
- Magnetic dipole moment:** It is the product of pole strength and separation between two poles. It is denoted by  $M$ .
- Magnetic intensity:** It is the magnetic moment per unit volume.

South pole of magnet (S)

$$B = \frac{\mu_0(-m)}{4\pi(d+l)^2}$$

Hence, resultant at point P when  $2l \ll d$ :

$$B = \frac{\mu_0 2M}{4\pi d^3}$$

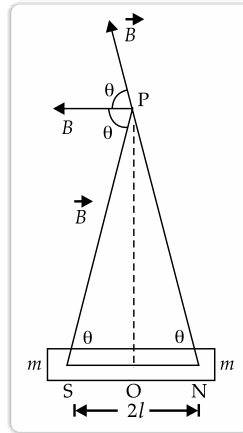
where,  $M = m \times 2l$ .

**Point lies on equatorial line of bar magnet**

- If  $2l$  is magnet length of bar magnet,  $m$  is magnetic intensity of each pole,  $\mu_0$  is permeability of free space, then magnetic field at point P due to:

North pole of magnet (direction N-P)

$$B = \frac{+\mu_0(m)}{4\pi(d^2 - l^2)}$$



- South pole of magnet (direction P-S)

$$B = \frac{-\mu_0 m}{4\pi(d^2 + l^2)}$$

- Hence resultant at point P when  $2l \ll d$ :

$$B = \frac{\mu_0 M}{4\pi d^3}$$

where,  $M = m \times 2l$

#### Torque on a magnetic dipole (bar magnet) in uniform magnetic field

- A bar magnet with length  $2l$  and pole strength  $m$  in uniform magnetic field induction  $B$  at angle  $\theta$  with force  $mB$  acting on North and South pole along the direction opposite to magnetic field results as a couple where torque  $\tau$  due to couple will be:

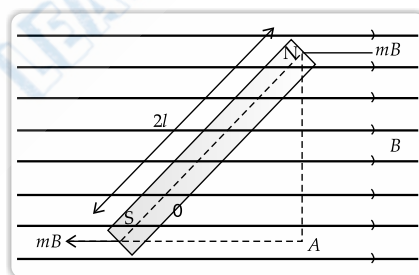
### Key Words

- **Magnetic dipole:** A system of two equal and opposite magnetic poles separated by small distance.

$\tau = \text{force} \times \text{perpendicular distance}$

$$\begin{aligned} \tau &= F \times NA \\ &= mB \times NA \\ &= mB \times 2l \sin \theta \\ &= MB \sin \theta \end{aligned}$$

where,  $M = m \times 2l$



In vector form:

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of  $\tau$  is perpendicular to plane containing, so when  $B = 1$  and  $\theta = 90^\circ$ ,

$$\tau = M$$

- Magnetic moment  $M$  of the magnet will be equal to the torque required to keep the magnet at right angles to a magnetic field of unit magnetic induction.

Magnetic field lines

- They are imaginary closed loops which continuously represent the direction of magnetic field at any point. Tangent at any point of these loops give the direction of magnetic field at that point.
- Concentration of field lines gives strength of magnetic field.
- Two field lines can never intersect each other. If they would, there will be two tangents at the point of intersection, which mean two directions of magnetic lines, which is impossible.
- In a 'uniform' magnetic field, the field lines are parallel and equidistant.

- If a magnetic needle is placed in a magnetic field and left, it will experience a torque and start oscillating in simple harmonic motion.
- Torque on a magnetic needle,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

where,  $\vec{B}$  is external magnetic field in which magnetic needle is kept.

- Time period of oscillations of magnetic needle,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where,  $I$  is moment of inertia of magnetic needle.

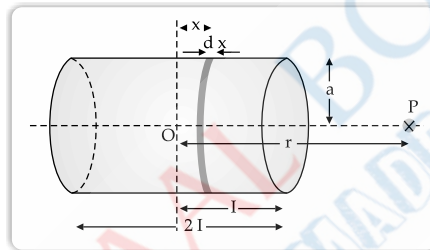
- Potential energy of a magnet in a magnetic field,

$$U = -\vec{M} \cdot \vec{B}$$

### Bar magnet as an equivalent solenoid

- If a solenoid of length  $2l$ , radius  $a$  with current  $I$  having  $n$  number of turns per unit length, then the magnetic moment of solenoid,

$$M (= NIA), B = \frac{\mu_0 2M}{4\pi d^3}$$



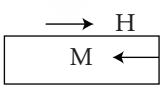
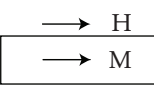
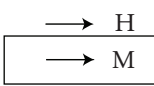
- Magnetic moment of a bar magnet is equal to magnetic moment of an equivalent solenoid that produces same magnetic field.

### Gauss' Law for Magnetic Fields

- Gauss' Law for magnetism applies to the magnetic flux through a closed surface.
- It shows that no magnetic monopoles exist and total flux through closed surface will be zero.
- The Gauss's law for magnetic fields in integral form is given by

$$\phi = \int \vec{B} \cdot d\vec{A} = 0$$

- **Properties of Dia, para and Ferromagnetic Substances:**

| S.No. | Diamagnetic Substances  | Paramagnetic Substances  | Ferromagnetic Substances   |
|-------|---|--|--|
| 1.    | These substances acquire feeble magnetism opposite to the direction of magnetic field when placed in magnetic field.<br> | These substances acquire feeble magnetism in the direction of magnetic field when placed in magnetic field.<br> | These substances acquire strong magnetism in the direction of magnetic field when placed in magnetic field.<br> |
| 2.    | Permeability is less than one ( $\mu < 1$ ).  | Permeability is slightly greater than one ( $\mu > 1$ ).   | Permeability is much greater than one ( $\mu \gg 1$ ).   |
| 3.    | Susceptibility is small, negative, and independent of temperature.  | Susceptibility is small and positive and inversely proportional to the absolute temperature, $\chi \propto \frac{1}{T}$ (Curie's Law)  | Susceptibility is large and positive and follow Curie's Law, $\chi \propto \frac{1}{T}$ (Curie's Law)<br>At Curie temperature, ferromagnetic substance changes to paramagnetic substance.            |



## Mnemonics

**Concept:** Four characteristics of magnetic field lines

**Mnemonics:** I love new stories Tina found new Cook-ies.

**Interpretations:**

(i) Imaginary Lines

(ii) Extended from North to South pole

(iii) Tangent gives (magnetic) field direction

(iv) Never Cross each other



## Key Formulæ

- Magnetic field due to short dipole at distance 'd' on axial line:

$$B_{\text{axial}} = \frac{\mu_0 2M}{4\pi d^3}$$

- Magnetic field due to short dipole at distance 'd' on equatorial line:

$$B_{\text{equi}} = \frac{\mu_0 M}{4\pi d^3}$$

- Torque on a magnetic dipole in uniform magnetic field:  $\tau = MB \sin \theta$

# UNIT – IV: ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

## CHAPTER-6

## ELECTROMAGNETIC INDUCTION

### Topic-1

### Magnetic Flux and Faraday's Laws

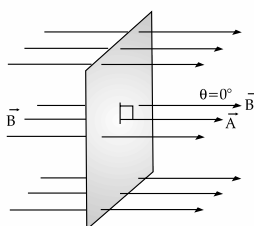
**Concepts Covered** • Electromagnetic induction, Faraday's laws & Lenz's Law.



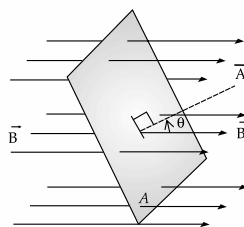
## Revision Notes

### Electromagnetic induction

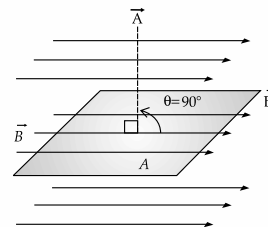
- Electromagnetic induction is the process of generating the electric current with a changing magnetic field.
- It takes place whenever a magnetic field is changing or electric conductors move relative to one another when they are in fluctuating magnetic field.
- The current produced by electromagnetic induction is more when the magnet or coil moves faster. When magnet or coil moves back and forth repeatedly, then alternating current is produced.
- If magnetic field is changing, the changing magnetic flux will be  $\phi_B = NBA \cos \theta$ , where,  $\theta$  is the angle between magnetic field and normal to the plane.



$\vec{B}$  parallel to  $A$  ( $\theta = 0^\circ$ )  
magnetic flux  $\phi_B = BA$ .



$\vec{B}$  at an angle  $\theta$  with the perpendicular to  $A$ :  
magnetic flux  $\phi_B = BA \cos \theta$



$\vec{B}$  perpendicular to  $A$  ( $\theta = 90^\circ$ ):  
magnetic flux  $\phi_B = 0$ .



**Magnetic flux density**

- The change in **magnetic flux** per unit change in area is called magnetic flux density.

**Key Word**

**Magnetic flux:** Magnetic flux is a measurement of the total magnetic field which passes through a given area. It is the number of magnetic field lines passing through a given closed surface. It is the product of the average magnetic field and the perpendicular area that it penetrates.

- Magnetic flux is given by:

$$d\phi = \vec{B} \cdot \vec{dA}$$

- For  $\vec{B}$  parallel to  $\vec{dA}$ , we have

$$d\phi = B(dA)\cos 0^\circ = B(dA)$$

$$\text{Therefore, } B = \frac{d\phi}{dA} \quad \dots(i)$$

i.e., **magnetic induction** is equal to the magnetic flux density. In other words, the magnetic field may be measured in terms of magnetic flux density. From equation (i), we find:

$$\text{Unit of } B = \frac{\text{Unit of } d\phi}{\text{Unit of } dA}$$

$$\text{Or, } T = \frac{Wb}{m^2}$$

i.e., Tesla = Weber per square metre.

**Faraday's Laws of Electromagnetic Induction**

- The **induced emf** in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law.
- **Faraday's First Law** of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field, an emf is induced which is known as induced emf and if the conductor circuit is closed, current is also induced which is called induced current.
- **Faraday's Second Law** of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is the product of number of turns in the coil and flux associated with the coil.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

- $\phi_B$  is magnetic flux through the circuit and is represented as  $\phi_B = \int \vec{B} \cdot \vec{dA}$

- With  $N$  loops of similar area in a circuit and  $\phi_B$  being the flux through each loop, emf is induced in every loop. Writing the formula for Faraday's law as

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

- where,  $\varepsilon$  = Induced emf [V],  $N$  = Number of turns in the coil
- $\Delta\phi$  = Change in the magnetic flux [Wb],
- $\Delta t$  = Change in time [s]
- The negative sign indicates that  $\varepsilon$  opposes its cause.
- If there is no change in magnetic flux, no emf is induced.

**Induced emf and current**

- A changing magnetic flux induces an electric field which induces a current in the circuit.
- A wire moving in the field induces a current which acts same as current provided by a battery.
- Changing magnetic flux and **induced electric field** are related to induced emf as per Faraday's law.
- The induced EMF in a conductor moving is related to the magnetic field as

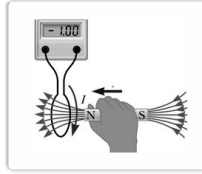
$$E = B.l.v\sin\theta$$

**Key Words**

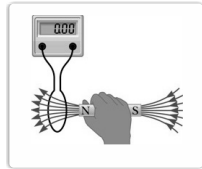
- **Induced emf:** A short-lived voltage generated by a conductor or coil, moving in a magnetic field.
- **Induced electric field:** Field generated due to changing magnetic flux with time.

**Induced current**

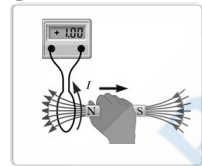
- ▶ When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.
- ▶ When a magnet is moved towards a loop of wire connected to an ammeter, ammeter shows current induced in the loop.



- ▶ When a magnet is held stationary, there will be no induced current in the loop, even though the magnet is inside the loop.

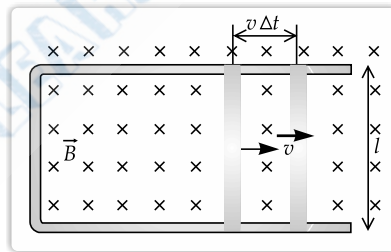


- ▶ When a magnet is moved away from the loop, the ammeter shows opposite current induced in the loop.

**Motional emf**

- ▶ The relationship between an induced emf  $\epsilon$  in a wire or a conductor moving at a constant speed  $v$  through a magnetic field  $B$  is given by:

$$\begin{aligned}\phi_B &= Blx \\ \epsilon &= \frac{-d\phi_B}{dt} = \frac{-d}{dt}(Blx) \\ &= -Bl \frac{dx}{dt} \\ &= Blv \left( \frac{dx}{dt} = -v \right)\end{aligned}$$



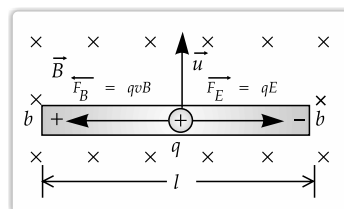
- ▶ An induced emf from Faraday's law is generated from a **motional emf** that opposes the change in flux.
- ▶ Magnetic and electric forces on charges in a rod moving perpendicular to magnetic field is given as:

At equilibrium,

$$\begin{aligned}F_E &= F_B \\ qE &= qvB \\ E &= vB\end{aligned}$$

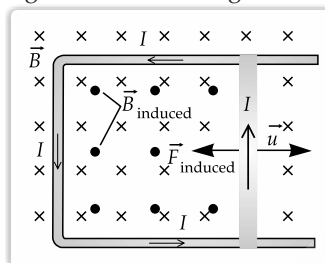
$$\frac{V}{l} = vB \quad \left[ \text{Here, } E = \frac{V}{l} \right]$$

$$V = Bvl$$



**Lenz's law**

- ▶ Lenz's law is used to determine the direction of induced magnetic fields, currents and emfs.
- ▶ The direction of an induced emf always opposes the change in magnetic flux which causes the emf.
- ▶ It explains the negative sign in Faraday's rule,  $\varepsilon = -\frac{d\phi_B}{dt}$  showing that the polarity of induced emf tends to produce a current that opposes the cause i.e., change in magnetic flux.
- ▶ As per conservation of energy, induced emf opposes its cause, making mechanical work to continue with the process which gets converted into electrical energy.
- ▶ Slide wire containing induced current, magnetic field and magnetic force:

**Electric Generators and Back Emf**

- ▶ Electric generator rotates a coil in a magnetic field inducing an emf which is given as a function of time  $\varepsilon = NBA\omega \sin(\omega t)$ .

where, A = Area of N-turn coil rotated at constant angular velocity  $\omega$  in uniform magnetic field  $\vec{B}$ .

- ▶ The **peak emf** of a generator is,  $\varepsilon_0 = NBA\omega$
- ▶ Any rotating coil produces an induced emf. In motors, it is known as **back emf** as it opposes the emf input to the motor.

**Key Words**

- ▶ **Electric generator:** Device for converting mechanical work into electrical energy that induces an emf by rotating a coil in magnetic field.
- ▶ **Motional emf:** Voltage produced by the movement of conducting wire or a conductor in a magnetic field.
- ▶ **Peak emf:** The maximum emf produced by a generator.
- ▶ **Back emf:** The emf generated by a running motor due to coil that turns in a magnetic field which opposes the voltage that powers the motor.

**Topic-2****Self and Mutual Induction**

**Concepts Covered** • Self Inductance • Mutual Inductance.

**Revision Notes****Mutual Induction**

- ▶ The production of induced emf in a circuit, when the current in the neighbouring circuit changes is called **mutual induction**.  
When the circuit of the primary coil is closed or opened, deflection is produced in the galvanometer of the secondary coil. This is due to the mutual induction.
- ▶ The mutual induction between two coils depends on the following factors:
  - The number of turns of primary and secondary coils.
  - The shape, size or geometry of the two coils. i.e., the area of cross-section and the length of the coils.

**Coefficient of mutual induction:**

- ▶ Suppose, the instantaneous current in the primary coil is I. Let the magnetic flux linked with the secondary coil be  $\phi$ . It is found that the magnetic flux is proportional to the current. i.e.,  $\phi \propto I$  or  $\phi = MI$  ... (i)

where, M is the constant of proportionality. It is called coefficient of mutual induction.

The induced emf  $\varepsilon$  in the secondary coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \quad \dots \text{(ii)}$$

The negative sign is in accordance with the Lenz's law i.e., the induced emf in the secondary coil opposes the variation of current in the primary coil.

Taking magnitude of induced emf from the equation (ii), we find:

$$M = \frac{\varepsilon}{(dI/dt)}$$

Therefore, Unit of  $M = \frac{V}{As^{-1}} = VA^{-1}s$

If  $n_1, n_2$  be the number of turns per unit length in primary and secondary coils per unit length and  $r$  be their radius, then coefficient of mutual **inductance** is given as

$$M = \mu_0 n_1 n_2 \pi r^2 l$$



## Key Word

**Inductance:** Inductance is the tendency of an electrical conductor to oppose a change in electric current flowing through it.

Inductance is defined as the ratio of the induced voltage to the rate of change of current causing it. .

The unit of inductance is the henry (H),

### Self-Induction:

- ▶ The production of induced emf in a circuit, when the current in the same circuit changes is known as **self-induction**.

Suppose the instantaneous current in the circuit is  $I$  and if the magnetic flux linked with the solenoid is  $\phi$ , then it is found that:

$$\phi \propto I \text{ or } \phi = LI \quad \dots(i)$$

where,  $L$  is the constant of proportionality. It is called **coefficient of self-induction**.

The induced emf  $\varepsilon$  in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad \dots(ii)$$

The negative sign is in accordance with the Lenz's law i.e., the induced emf opposes the variation of current in the coil.

Taking the magnitude of the induced emf from the equation (ii), we find:

$$L = \varepsilon / (dI/dt) \quad \dots(iii)$$

**Then, the coefficient of self-induction** is the ratio of induced emf in the circuit to the rate of change of the current in the circuit.

**Unit of L:** The unit of self-induction is called henry (symbol H).

From equation (ii), we find that if

$$dI/dt = 1 \text{ As}^{-1} \text{ and } \varepsilon = 1 \text{ V,}$$

then  $L = 1 \text{ H} \Rightarrow 1 \text{ VA}^{-1}\text{s}$

- ▶ If a rod of length  $l$  moves perpendicular to a magnetic field  $B$  with a velocity  $v$ , then the induced emf produced across it, is given by

$$\varepsilon = vBl$$

In general, we have,  $\varepsilon = Blv \sin \theta$

- ▶ If a metallic rod of length  $l$  rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across its ends is given by

$$\varepsilon = \frac{B\omega l^2}{2} = \frac{B2\pi fl^2}{2} = BAf$$

Here,  $\omega$  = angular velocity of rotation,

$A = \pi l^2$  = area of circle and  $f$  = frequency of rotation.

- ▶ Inductance in the electrical circuit is equivalent to the inertia (mass) in mechanics.
- ▶ When a bar magnet is dropped into a coil, the electromagnetic induction in the coil opposes its motion, so the magnet falls with acceleration less than that due to gravity.
- ▶ The inductance of a coil depends on the following factors:
  - area of cross-section,
  - number of turns
  - permeability of the core.

► Unit of induction,  $H = \frac{Wb}{A} = \frac{Vs}{A} = \Omega.s$

► The self inductance of a circular coil is given by:

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi NI)}{rI} \times AN \quad \left[ \because B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi NI}{r} \right]$$

$$L = \frac{\mu_0 N^2}{2r} A = \frac{\mu_0 N^2}{2r} \times \pi r^2$$

or  $L = \frac{\mu_0 N^2 \pi r}{2}$

► The self inductance of a solenoid of length  $l$  is given by

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left( \frac{\mu_0 NI}{l} \right) \frac{AN}{I} \quad \left[ \because B = \frac{\mu_0 NI}{l} \right]$$

or  $L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al = \mu_0 n^2 V$   $\left[ \because n = \frac{N}{l} \right]$

Here,  $n = N/l =$  Number of turns per unit length and  $V = Al =$  Volume of the solenoid.

► If two coils of inductance  $L_1$  and  $L_2$  are coupled together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

where,  $k$  is called the coupling constant.

► The value of  $k$  lies between 0 and 1.

For perfectly coupled coils,  $k = 1$ , it means that the magnetic flux of primary coil is completely linked with the secondary coil.

► If a current  $I$  is set up in a coil of inductance  $L$ , then the magnetic field energy stored in it is given by

$$U_m = \frac{1}{2} LI^2$$



## Mnemonics

**Concept:** Induced emf in a conductor moving in a magnetic field:

**Mnemonics:** I eat Plain Loaf and Boiled Vegetables

**Interpretations:**

**I:** Induce

eat: emf

Plain: product of

Loaf: Length of Conductor

Boiled: B (magnetic field)

Vegetables: V (Velocity)

Induced emf =  $Blv$



## Key Formulae

► Magnetic flux:  $\phi_m = \int \vec{B} \cdot d\vec{A}$

► Faraday's law:  $\epsilon = -N \frac{d\phi_m}{dt}$

► Motional induced emf:  $\epsilon = Blv$

► Motional emf around a circuit:

$$\epsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_m}{dt}$$

► emf produced by an electric generator

$$\epsilon = NBA \sin \omega t$$

► For Self Induction

$$\epsilon = -\frac{d\phi}{dt} = -L \frac{dI}{dt}$$

► For Mutual Induction

$$\epsilon = -\frac{d\phi}{dt} = -M \frac{dI}{dt}$$

► The inductance in series is given by

$$L_s = L_1 + L_2 + L_3 + \dots$$

► The inductance in parallel is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

► Mutual Inductance of two coils is given by

$$M = \frac{\mu_0 \epsilon_r N_p N_s A_p}{l_p} = \frac{n_0 n_r N_p N_s A_s}{l_p}$$

where,  $\mu_0$  is the permeability of free space ( $4\pi \times 10^{-7}$ ).

$\mu_r$  is the relative permeability of the soft iron core.

$N_s$  is number of turns in secondary coil.

$N_p$  is number of turns in primary coil.

$A_p$  is the cross-sectional area of primary coil in  $m^2$ .

$A_s$  is the cross-sectional area of secondary coil in  $m^2$ .

$I$  is the coil current.

► For a.c. Generator  $\epsilon = \epsilon_0 \sin \omega t$  or  $\epsilon = \epsilon_0 \sin 2\pi \nu t$

# CHAPTER-7

## ALTERNATING CURRENT

### Topic-1

### Alternating Current

#### Concepts Covered

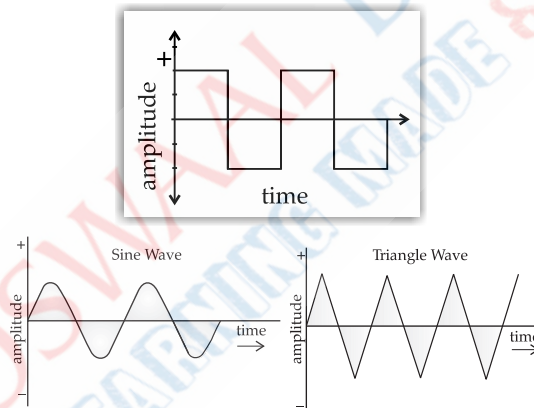
• Alternating current, its peak and rms value, reactance and impedance,



### Revision Notes

#### Alternating current

- Alternating current changes continuously in magnitude and periodically in direction.
- It is represented by sine curve or cosine curve as  $I = I_0 \sin \omega t$  or  $I = I_0 \cos \omega t$  where,  $I_0$  is peak value of current and  $I$  is instantaneous value of current.
- Frequency of an alternating current supply  $f$ , is defined as the number of cycles completed per second. It is measured in Hertz (Hz). In India, the frequency of ac current is 50 Hz.
- The time period  $T$ , of an alternating supply, is time taken to complete one cycle.
- The behaviour of ohmic resistance  $R$  in ac circuit is the same as in dc circuit.
- Alternating current can be produced by using a device called as an alternator.
- AC waveforms are:



#### Peak and rms value of alternating current/voltage:

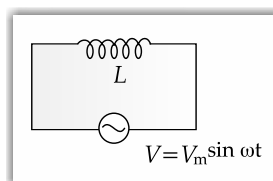
- Root mean square or rms is the root mean square of voltage or current in an ac circuit for one complete cycle denoted by  $V_{\text{rms}}$  or  $I_{\text{rms}}$ .
- rms value is the standard way of measuring alternating current and voltage as it gives the dc equivalent values.
- rms value of ac is also called effective value or virtual value of ac represented as  $I_{\text{rms}}$ ,  $I_{\text{eff}}$  or  $I_v$  shown as

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707I_0$$

- rms voltage value is the square root of averages of the squares of instantaneous voltages in a time varying waveform.

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707 V_0$$

- AC voltage applied to pure **inductive circuits**:



$$V = V_m \sin \omega t$$

$$I = I_m \left( \sin \omega t - \frac{\pi}{2} \right)$$

[which shows current lags the voltage by  $\frac{\pi}{2}$ ]

$$\text{Average } P_L = \frac{I_m V_m}{2} [\sin(2\omega t)] = 0$$

[Since average of  $\sin 2\omega t$  over a complete cycle is zero]



## Key Words

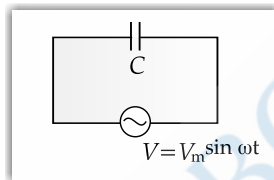
**Inductive circuits:** A Pure inductive circuit is one in which the only quantity in the circuit is inductance (L), with no other components such as resistance or capacitance.

**Capacitive circuit:** A Pure capacitor circuit is a circuit that contains a pure capacitor with capacitance C farads.

**Impedance:** For A.C. circuits, Impedance is the measure of the total opposition that a circuit presents to electric current. Impedance includes both resistance and reactance.

Thus the average power supplied to an inductor over one complete cycle is zero.

- AC applied to pure **capacitive circuit:**



$$V = V_m \sin \omega t$$

$$I = I_m \sin \left( \omega t + \frac{\pi}{2} \right)$$

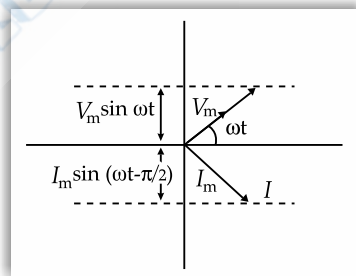
[which shows current leads the voltage by  $\frac{\pi}{2}$ ]

$$\text{Average } P_C = \frac{I_m V_m}{2} \sin(2\omega t) = 0 \quad [\text{Since average of}$$

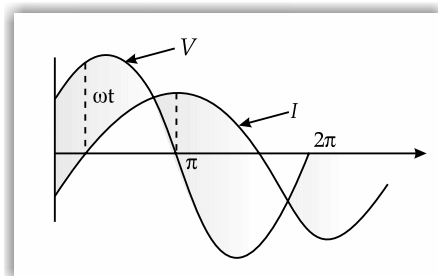
$\sin 2\omega t$  over a complete cycle is zero]

Thus, the average power supplied to a capacitor over one complete cycle is zero.

- **Phasor-diagram:** A phasor diagram represents sinusoidal ac current and sinusoidal voltage in a circuit along with the phase difference between current and voltage. The length of phasor is proportional to the instantaneous values of  $V, I$  and the maximum length is proportional to  $V_0$  and  $I_0$ .



Phasor diagram of purely Inductive circuit



Graphical representation of  $V$  and  $i$  versus  $\omega t$ .

### Reactance and Impedance

- ▶ When an ac current is passed through a resistor, a voltage drop is produced which is in phase with the current and is measured in ohms ( $\Omega$ ).
- ▶ Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is  $90^\circ$  out of phase with the current.
- ▶ Reactance is shown by "X" and is measured in ohms ( $\Omega$ ).
- ▶ Reactance is of two types: inductive and capacitive.
- ▶ Inductive reactance is linked with varying magnetic field that surrounds a wire or a coil carrying a current.
- ▶ Inductive reactance ( $X_L$ ) is the resistance offered by an inductor and is given by  $X_L = \omega L = 2\pi fL$ .
- ▶ Through a pure inductor, alternating current lags behind the alternating emf by phase angle of  $90^\circ$ .
- ▶ Capacitive reactance is linked with changing electric field between two conducting surfaces separated from each other by an insulating medium.
- ▶ Capacitive reactance ( $X_C$ ) is the resistance offered by a capacitor and is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

- ▶ Through a pure capacitor, alternating current leads the alternating emf by a phase angle of  $90^\circ$ .  
Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between  $0^\circ$  and  $90^\circ$  which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X_C^2}$$

where, Z = Impedance of circuit, R = Resistance, X = Reactance

- ▶ **Impedance:** In an ac, the impedance is analogous to resistance in a dc circuit that measures the combined effect of resistance, capacitive reactance and inductive reactance.



### Key Formulae

- ▶ rms value for current

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

- ▶ rms value for voltage

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

- ▶ Power  $P = V_{\text{rms}} I_{\text{rms}}$

- ▶ In a purely inductive circuit if,

$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right),$$

where  $I_m = \frac{V_m}{X_L}$  and  $X_L = \omega L$

$$(P_{\text{avg}})_L = 0$$

- ▶ In a purely capacitive circuit if,

$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where,}$$

$$I_m = \frac{V_m}{X_C} \text{ and } X_C = \frac{1}{\omega C}$$

- ▶ Average Power =  $\frac{1}{2} V_0 I_0 \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi$

(where,  $\cos \phi = \frac{R}{Z}$  is power factor)

- ▶  $Z = \sqrt{R^2 + (X_L - X_C)^2}$

- ▶ Induced emf =  $e = -L \frac{dI}{dt}$

## Topic-2

### LCR Series Circuit

**Concepts Covered** • LCR series circuit (phasors only), resonance, power in AC circuits, power factor, wattless current.

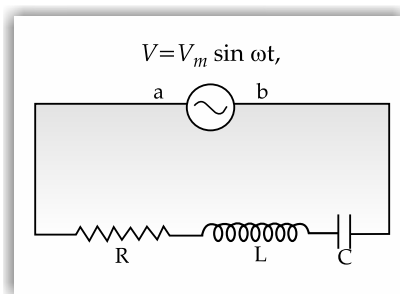


### Revision Notes

#### LCR series circuit

- ▶ In an LCR series circuit with resistor, inductor and capacitor, the expression for the instantaneous potential difference between the terminals a and b is given as





- ▶ The potential difference in this will be equal to the sum of the magnitudes of potential differences across R, L and C elements as

$$V = V_m \sin \omega t = RI + L \frac{dI}{dt} + \frac{1}{C}q$$

where,  $q$  is the charge on capacitor.

- ▶ The **steady state** situation will be

$$i = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \text{ and}$$

$$i_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

where,  $\phi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$

- ▶ From the equation, steady-state current varies sinusoidal with time, so steady-state current can be written as  $I = I_m \sin(\omega t - \phi)$

- ▶ In an LCR circuit:

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here,  $Z$  = Impedance of the circuit,  $X$  = Reactance of the circuit,  $X_L$  and  $X_C$  = Inductive and Capacitive reactance.

- ▶ For steady-state currents, maximum current  $I_m$  is related to maximum potential difference  $V_m$  by

$$I_m = \frac{V_m}{Z}$$

- ▶ Total effective resistance of LCR circuit is called Impedance ( $Z$ ) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

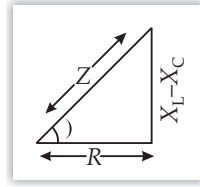
## Key Word

**Steady state:** As in the case of forced oscillations of a spring-mass system with damping, we call  $Q_p$  the steady state charge on the capacitor of the RLC circuit. Since  $I = Q' = Q'_c + Q'_p$  and  $Q'_c$  also tends to zero exponentially as  $t' \rightarrow \infty$ , we say that  $I_c = Q'_c$  is the transient current and  $I_p = Q'_p$  is the steady state current.

- ▶ The angle by which alternating voltage leads the alternating current in LCR circuit is given by

$$\tan \phi = \frac{X_L - X_C}{R}$$

- ▶ In an LCR circuit, impedance triangle is a right-angled triangle in which base is ohmic resistance  $R$ , perpendicular is reactance  $(X_L - X_C)$  and hypotenuse is impedance  $(Z)$



- ▶ When a condenser of capacity  $C$  charged to certain potential is connected to inductor  $L$ , energy stored in  $C$  oscillates between  $L$  and  $C$  where frequency of energy oscillations is given by

$$X_L = X_C \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

- ▶ In LCR circuit, if there is no loss of energy, then total energy in  $L$  and  $C$  at every instant will remain constant.
- ▶ Sign for phase difference ( $\phi$ ) between  $I$  and  $V$  for a series LCR circuit:
  - $\phi$  is positive, when  $X_L > X_C$ .
  - $\phi$  is negative, when  $X_L < X_C$ .
  - $\phi$  is zero, when  $X_L = X_C$ .
  - $\phi = \pi/2$ , when  $\omega = \phi$ .
  - $\phi = -\pi/2$ , when  $\omega = 0$ .

#### Resonance

- ▶ Circuit in which inductance  $L$ , capacitance  $C$  and resistance  $R$  are connected in series and the circuit admits maximum current, such circuit is called as series resonant circuit.
- ▶ The necessary condition for resonance in LCR series circuit is:  $V_C = V_L$

$$X_L = X_C \quad \text{which gives} \quad \omega^2 = \frac{1}{LC} \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

- ▶ In this, frequency of ac fed to circuit will be equal to natural frequency of energy oscillations in the circuit under conditions,

$$Z = R$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$$

- ▶ The sharpness of tuning at resonance is measured by  $Q$  factor or quality factor of the circuit given as

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

- ▶ At series LCR resonance or acceptor circuit, current is maximum.

$$I_{\max} = \frac{V}{R}$$

#### Power in AC circuits

- ▶ When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives apparent power.
- ▶ If the instantaneous values of the voltage and current in an ac circuit are given by

$$V = V_0 \sin \omega t$$

$$i = i_0 \sin (\omega t - \phi)$$

where  $\phi$  is the phase difference between voltage and the current. Then, the **instantaneous power**

$$P_{in} = V \times i = V_0 i_0 \sin \omega t \cdot \sin (\omega t - \phi)$$

or average power  $P_{avg} = \frac{1}{2} V_0 i_0 \cos \phi$

$$= \frac{V_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos \phi$$

$$= V_{rms} \times I_{rms} \times \cos \phi$$

where,  $\cos \phi$  is known as power factor.

- Power factor ( $\cos \phi$ ) is important in power systems as it shows how closely the effective power equals the apparent power which is given as:

$$\cos \phi = \frac{\text{Effective power}}{\text{Apparent power}}$$

- The value of power factor varies from 0 to 1.
- The instantaneous rate at which energy is supplied to an electrical device by ac circuit is  $P = VI$
- Average power in LCR where,  $X_L = X_C$  over a complete cycle in a non-inductive circuit or pure resistive circuit is given as  $P = V_0 I_0$  or  $I_0^2 R$
- The current in an AC circuit is said to be Wattless Current when the average power consumed in such circuit corresponds to Zero. This happens in case of pure inductive and capacitive circuits.

### Key Word

**Instantaneous power:** The amount of power in a circuit at any instant of time is called the instantaneous power and is given by the well-known relationship of power equals volts times amps ( $P = V \times I$ )

### Key Formulae

- Impedance for a series LCR circuit,

$$Z = \sqrt{R^2 + X^2} = \left[ R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}$$

- Average power,

$$P = \frac{E_0 I_0}{2} \cos \phi = V_{rms} I_{rms} \cos \phi$$

- Quality factor

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$



### Mnemonics

**Concept:** Current leads in pure capacitive circuit, voltage leads in pure inductive circuit.

**Mnemonics:** Chocolate Cookies are Very Interesting !

**Interpretations:**

**C:** Current leads

**C:** Capacitive circuit

**V:** Voltage leads

**I:** Inductive circuit

## Topic-3

### AC Generator and Transformer

**Concepts Covered** • AC generator and transformer.



### Revision Notes

#### AC generator

- An AC generator is an electrical machine which converts mechanical energy into alternating electrical energy.
- AC generator or a synchronous generator has a stator and rotor.
- It is similar to the basic working principle of a dc generator.
- It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced emf given as:

$$e = e_0 \sin \omega t = NBA\omega \sin \omega t$$

**Transformer**

- ▶ Transformer is an electrical device used for changing the alternating voltages. It is based on the phenomenon of mutual induction.
- ▶ The main use of transformer is in transmission of ac over long distances at extremely high voltages which reduces the energy losses in transmission.
- ▶ It comprises of two sets of coils which are insulated from each other and are wound on **soft-iron** core.
- ▶ In this, one of the coil is called as primary (input coil) having  $N_p$  turns while other coil is secondary (output coil) having  $N_s$  turns, so we have

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- ▶ Transformation Ratio:

$$E_s = \left(\frac{N_s}{N_p}\right)E_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$  is defined as the transformation ratio.

The value of turns ratio of a transformer

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = n$$

- ▶ **Step-up transformer:** If secondary coil has more number of turns than primary ( $N_s > N_p$ ), voltage gets stepped up ( $V_s > V_p$ ).

In this, there is less current in secondary as compared to primary ( $\frac{N_s}{N_p} > 1$  and  $I_s < I_p$ ).

The value of transformation ratio  $k > 1$

- ▶ **Step-down transformer:** In this, the secondary coil has less number of turns than primary ( $N_s < N_p$ ). In this,  $V_s < V_p$  and  $I_s > I_p$  as voltage gets stepped down or reduced with increase in current.

In this, value of **transformation ratio**  $k < 1$

- ▶ The main use of transformers is in stepping up voltage for power transmission.

**Key Words**

**Transformation Ratio:** The transformation ratio is defined as the ratio of output voltage to the input voltage of a transformer. This gives the information about the change in voltage level by the transformer.

$$\text{Transformation ratio} = k = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

If  $k > 1$ , then it is a step-up transformer.

If  $k < 1$ , then it is a step-down transformer.

**Soft iron:** Iron that has a low carbon content and is easily magnetized and demagnetized, used to make the cores of solenoids and other electrical equipment.

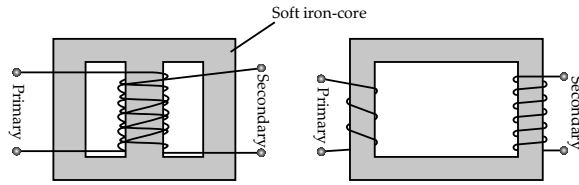
**Electrical power:** Electric power is the rate, per unit time, at which electrical energy is transferred by an electric circuit.

- ▶ **Electric power** can be transmitted efficiently at high voltages than at low voltages due to less ( $I^2R$ ) heat loss in a high voltage / low current transmission.

- ▶ **Efficiency of transformer:**

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$



- ▶ The efficiency in a transformer is usually above 90%.
- ▶ An ideal transformer is 100% efficient as it delivers all energy it receives.
- ▶ Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- ▶ A transformer operating with constant voltage and frequency with very high capacity, efficiency results as 98%.
- ▶ Energy losses in transformers:
  1. Flux Leakage
  2. Resistance of windings
  3. Eddy currents
  4. Hysteresis

### Key Word

**Hysteresis loss:** The hysteresis loss instead is the energy entrapped in magnetic materials exposed to a magnetic field in the form of residual magnetization—a typical behavior of FM materials—which is then lost as heat during the demagnetization step.

### Key Formulae

▶ For transformer:

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

$$V_s = \left(\frac{N_s}{N_p}\right) V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right) I_p$$

▶ The value of transformer ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.

$$\eta = \frac{E_s I_s}{E_p I_p}$$

$$\% \text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100\%$$

▶ For generator:

$$e = e_0 \sin \omega t = NBA \omega \sin \omega t$$

$$I = \frac{e}{r} = \frac{NBA \omega \sin \omega t}{R}$$

## UNIT – V : ELECTROMAGNETIC WAVES

### CHAPTER-8

## ELECTROMAGNETIC WAVES

### Topic-1

### Electromagnetic Waves and Maxwell's Equations

**Concepts Covered** • Basic idea of displacement current, Electromagnetic waves and their characteristics, their transverse nature.

### Revision Notes

Basic idea of displacement current:

- ▶ Displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of *electric flux*.
- ▶ Displacement current has the units of electric current and has associated magnetic field similar as actual currents.
- ▶ Displacement current plays an important role in the propagation of electromagnetic radiation, such as light and radio waves, through empty space.

- Displacement current is defined as:

$$I_d = \epsilon_0 \frac{d\phi_E}{dt},$$

$$\left( I_d = \epsilon_0 A \frac{dE}{dt} \right)$$

### Key Word

**Electric flux:** It is the rate of flow of electric field of lines passing through a given area.

where,  $\phi_E = \int \vec{E} \cdot d\vec{A}$  is the electric flux and  $\epsilon_0$  is the permittivity of free space.

- As per Ampere Maxwell law, line integral of magnetic field around any closed path is equal to  $\mu_0 \times$  (sum of conduction current and displacement current through that path.)

$$\text{i.e., } \oint \vec{B} \cdot d\vec{l} = \mu_0 [I_c + I_d]$$

$$\begin{aligned} \oint \vec{B} \cdot d\vec{l} &= \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \\ &= \mu_0 I_c + \mu_0 \epsilon_0 \frac{d}{dt}(E.A) \end{aligned}$$

This proves that change in electric field  $E$ , is responsible for the induction of magnetic field.

### Electromagnetic waves

- Waves that can travel through vacuum of outer space and do not need the presence of material medium for transporting energy from one location to another.
- EM waves are produced by accelerated charged particles.
- The electric and magnetic fields produced by accelerated charge change with time, which radiate electromagnetic waves.

### Example:

- Electron jumping from its outer to inner orbits radiates EM waves.
- Electrical oscillations in **LC circuit** produce EM waves.
- Electric sparking generates EM waves. Characteristics of EM waves:
- EM waves are propagated as electric and magnetic fields oscillating in mutually perpendicular directions.
- EM waves travel in vacuum along a straight line with the velocity  $2.997924591 \times 10^8$  m/s which is often assumed as  $3 \times 10^8$  m/s.
- EM waves are not affected by electric and magnetic fields.
- Relation between electric and magnetic field components is:

$$B_0 = E_0/c$$

where,  $c \cong 3 \times 10^8$  m/s. and  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

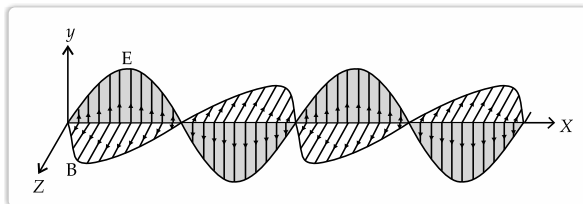
The  $\lambda$  and  $f$  are related as

$$c = f\lambda.$$

where  $\lambda$  is the wavelength and  $f$  is the frequency.

### Transverse nature of electromagnetic waves

- In electromagnetic wave, electric and magnetic field vectors are perpendicular to each other in the direction of propagation of wave which shows its transverse nature.



- A plane EM wave travelling in the  $x$ -direction is of the form:

$$E(x, t) = E_{max} \cos(kx - \omega t + \phi)$$

$$B(x, t) = B_{max} \cos(kx - \omega t + \phi)$$

where,  $E$  = electric field vector,  $B$  = magnetic field vector

- In this, wave propagates along z-axis, the electric and magnetic field propagation will be:

$$E = E_0 \sin(kz - \omega t)$$

$$B = B_0 \sin(kz - \omega t)$$

### Key Word

**LC circuit:** It is a resonating circuit that consists of an inductor and a capacitor.

- **Gauss Law:** For electricity, electric flux of closed surface equals to the charge enclosed divided by permittivity.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

For magnetism, total magnetic flux of the closed surface is zero

$$\oint \vec{B} \cdot d\vec{A} = 0$$

### Key Formulae

- Displacement current between the plates of a capacitor

$$I_D = \epsilon_0 \frac{d(EA)}{dt} = \epsilon_0 A \frac{dE}{dt}$$

$$I_D = \epsilon_0 A \frac{d\left(\frac{V}{d}\right)}{dt} = \frac{\epsilon_0 A}{d} \frac{dV}{dt} = C \frac{dV}{dt}$$

Here,  $E$  = Electric field between the plates of the capacitor,  $V$  = Potential difference,  $d$  = Separation between the plates,  $C$  = Capacitance of the capacitor,  $A$  = Area of plates.

- For the EM waves, the energy density is given by

$$U_E = \frac{1}{2} \epsilon_0 E^2 \quad (\text{Due to electric field})$$

$$U_B = \frac{1}{2} \frac{B^2}{\mu_0} \quad (\text{Due to magnetic field})$$

- The energy transported by EM waves per unit area per second is called Poynting vector ( $\vec{S}$ ).

It is given by  $\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0}$ .

Since,  $\vec{E} \perp \vec{B}$ , hence  $S = \frac{EB}{\mu_0}$ .

- In EM waves, the total energy density of EM waves is

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$$

$$U = \epsilon_0 E^2 = \frac{B^2}{\mu_0} \quad \left[ \text{As, } E = \frac{B}{\sqrt{\mu_0 \epsilon_0}} \right]$$

- In the EM waves:  $E = E_0 \sin(\omega t - kx)$ ,  $B = B_0 \sin(\omega t - kx)$ .

Maxwell's Equations:

1.  $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$  (Gauss's law for electricity).
2.  $\oint \vec{B} \cdot d\vec{A} = 0$  (Gauss's law for magnetism).
3.  $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$  (Faraday's law).
4.  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$  (Ampere-Maxwell law).  $E = eV$



## Mnemonics

**Concept: Electromagnetic spectrum:** Arrangement of em waves with increasing frequency (decreasing wavelength) :

**Mnemonics:** Russian magicians introduced and very unusual X-ray eye Game.

**Mnemonic:** **R**ussian **m**agicians **i**ntroduced a **V**ery **u**nusual **X**-ray eye **G**ame.



## Topic-2

### Electromagnetic Spectrum

**Concepts Covered** • Electromagnetic spectrum (radio waves, microwaves, infrared, visible, UV, X-rays, gamma rays) including elementary facts about their uses



## Revision Notes

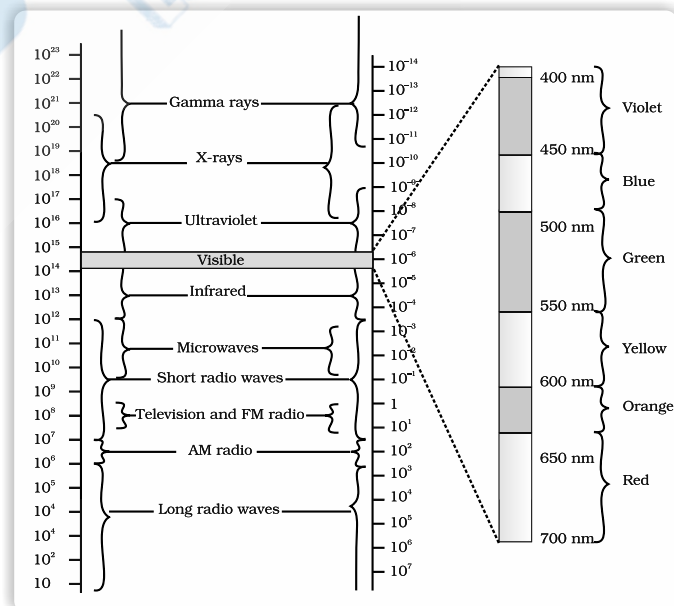
- **Gamma rays:** Rays with smallest wavelengths and highest frequencies having high energy capable of travelling long distances through air and these are most penetrating.
- **X-rays:** These are the rays with long and small wavelengths having higher energy as compared to ultraviolet radiation.
- **Ultraviolet (UV) radiation:** It is a part of electromagnetic spectrum that lies between X-rays and visible light.
- **Visible light:** It is a visible spectrum which is part of electromagnetic spectrum which can be seen by human eyes.
- **Infrared (IR) radiation:** These are thermal radiations which is the part of electromagnetic spectrum that lie between visible light and microwaves.
- **Radio waves:** Waves with long wavelengths used in television, cell phone and radio communications.

Electromagnetic spectrum

- Classification of EM-waves is based on their frequency or wavelength range.
- EM radiations are classified as per the frequency and wavelength of wave such as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

Frequency (in Hz)

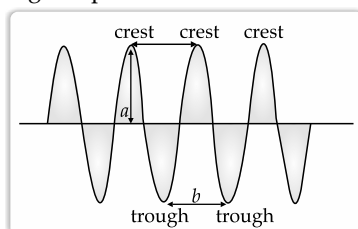
Wavelength (in m)





General properties of electromagnetic waves (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays)

- ▶ Electromagnetic waves require no medium to travel or propagate.
- ▶ Varying electric and magnetic fields are the sources of electromagnetic waves.
- ▶ Electromagnetic waves are transverse waves which are characterized by their amplitude, wavelength, or distance between highest/lowest points.
- ▶ In electromagnetic waves, a crest is the highest point of the wave and trough the lowest point of wave in a cycle.



$a$  = Amplitude  
 $b$  = wavelength

#### Electromagnetic spectrum is divided into following regions:

The electromagnetic spectrum is the distribution of electromagnetic radiation in terms of energy, frequency or wavelength. The electromagnetic radiation can be described as a stream of photons travelling in a wave like pattern, at the speed of light.

| Type of radiation     | Frequency range                         | Wavelength range                     |
|-----------------------|---|--------------------------------------|
| Gamma rays            | $> 3 \times 10^{20}$                    | $< 1$ fm                             |
| X-rays                | $3 \times 10^{17} - 3 \times 10^{20}$   | 1 fm – 1 nm                          |
| Ultraviolet           | $7.5 \times 10^{14} - 3 \times 10^{17}$ | 1 nm – 400 nm                        |
| Visible               | $4 \times 10^{14} - 7.5 \times 10^{14}$ | $0.4 \mu\text{m} - 0.75 \mu\text{m}$ |
| Near-infrared         | $10^{14} - 7.5 \times 10^{14}$          | $0.75 \mu\text{m} - 3.0 \mu\text{m}$ |
| Midwave infrared      | $5 \times 10^{13} - 10^{14}$            | $3.0 \mu\text{m} - 6 \mu\text{m}$    |
| Long wave infrared    | $2 \times 10^3 - 5 \times 10^{13}$      | $6.0 \mu\text{m} - 15 \mu\text{m}$   |
| Extreme infrared      | $3 \times 10^{13} - 2 \times 10^{13}$   | $15 \mu\text{m} - 15 \mu\text{m}$    |
| Micro and radio waves | $< 3 \times 10^{11}$                    | $> 1$ mm                             |

Uses of Electromagnetic waves:

| Band designation                     | Applications   |
|--------------------------------------|--|
| Audible                              | Acoustics  |
| Extremely Low Frequency (ELF) Radio  | Electronics, Submarine Communications  |
| Infra Low Frequency (ILF)            | Not applicable   |
| Very Low Frequency (VLF) Radio       | Navigation, Weather System Forecasting   |
| Low Frequency (LF) Radio             | Navigation, Maritime Communications, Information and Weather Systems, Time Systems                         |
| Medium Frequency (MF) Radio          | Navigation, AM Radio, Mobile Radio   |
| High Frequency (HF) Radio            | Citizens Band Radio, Mobile Radio, Maritime Radio  |
| Very High Frequency (VHF) Radio      | Amateur (Ham) Radio, VHF TV, FM Radio, Mobile Satellite, Mobile Radio, Fixed Radio                         |
| Ultra High Frequency (UHF) Radio     | Microwave, Satellite, UHF TV, Paging, Cordless Telephone, Cellular and PCS Telephony, Wireless LAN (Wi-Fi) |
| Super High Frequency (SHF) Radio     | Microwave, Satellite, Wireless LAN (Wi-Fi)   |
| Extremely High Frequency (EHF) Radio | Microwave, Satellite, Radiolocation  |
| Infrared Light (IR)                  | Wireless LAN Bridges, Wireless LANs, Fiber Optics Remote control   |

|                       |   |
|-----------------------|---|
| Visible Light         | Photographic plate, photocells.               |
| Ultraviolet (UV)      | Photocells, kill bacteria and germs.          |
| X-Rays                | In medical, Geiger tubes, ionization chamber. |
| Gamma and Cosmic Rays | In medical (cancer cell killing)              |

Types of Electromagnetic waves, wavelength range, Production and Detection:

| Type of radiation | Wavelength range   | Production   | Detection   |
|-------------------|--|--|---|
| Radio             | $> 1.0 \times 10^{-1} \text{ m}$                               | Rapid acceleration and decelerations of electrons in aerials                               | Receiver's aerials                                  |
| Microwave         | $0.1 \text{ m} - 1.0 \times 10^{-3} \text{ m}$                 | Klystron valve or magnetron valve  | Point contact diodes                                |
| Infra-red         | $1.0 \times 10^{-3} \text{ m} - 700 \times 10^{-9} \text{ m}$  | Vibration of atoms and molecules   | Thermopiles Bolometer, Infrared photographic film   |
| Light             | $700 \times 10^{-9} \text{ m} - 400 \times 10^{-9} \text{ m}$  | Electrons in atoms emit light when they move from one energy level to a lower energy level | The eyes, Photocells Photographic film              |
| Ultraviolet       | $400 \times 10^{-9} \text{ m} - 1.0 \times 10^{-9} \text{ m}$  | Inner shell electrons in atoms moving from one energy level to a lower level               | Photocells Photographic film                        |
| X-rays            | $1.0 \times 10^{-9} \text{ m} - 1.0 \times 10^{-12} \text{ m}$ | X-ray tubes or inner shell electrons   | Photographic film, Geiger tubes, Ionization chamber |
| Gamma rays        | $< 1.0 \times 10^{-12} \text{ m}$                              | Radioactive decay of the nucleus   | Photographic film, Geiger tubes, Ionization chamber |

## UNIT – VI : OPTICS

### CHAPTER-9

## RAY OPTICS AND OPTICAL INSTRUMENTS

### Topic-1

#### Reflection by Spherical Mirrors

**Concepts Covered** Reflection of light, spherical mirrors, image formation by concave and convex mirror, mirror formula, magnification by mirror



#### Revision Notes

- Light is a form of energy. Ray of light represents the direction of propagation of light energy.
- The speed of light in vacuum is the highest speed attainable in nature. Its approximate value is  $3.0 \times 10^8 \text{ m/s}$ .
- When light falls on any object/surface, there are three possible optical phenomenon *i.e.*, reflection, refraction and absorption of light by the object/surface.  
By law of conservation of energy, sum of reflected, absorbed and transmitted light is always equal to the incident light.
- Depending upon the amount of light it reflects, transmits or absorbs, the object is classified into reflector, transmitter or absorber of light.

#### Spherical Mirror

- Curved shaped mirrors are known as spherical mirrors. On the basis of curve of reflecting surface, spherical mirrors are categorised as:
  - **Concave mirror:** A spherical mirror, whose reflecting surface is curved inwards is called a concave mirror. It means reflecting surface faces the centre of the sphere using which it is made.
  - **Convex mirror:** A spherical mirror whose reflecting surface is curved outwards is called a convex mirror.

#### Important terms related to spherical mirrors

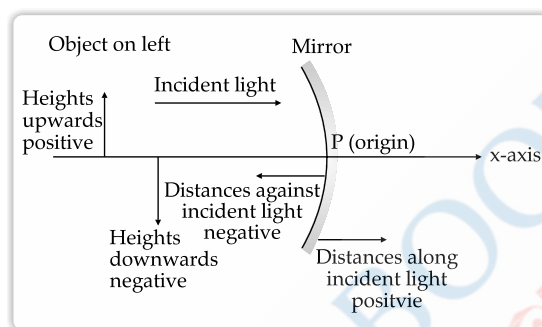
- The mid point or the centre of the reflecting surface of the mirror is known as **pole** of the mirror. It is represented by *P*.

- The centre of the hollow sphere from which the mirror is made, is known as **centre of curvature**. It is represented by  $C$ . Centre of curvature in concave mirror, it is in front of the mirror and in convex mirror, it is behind the mirror.
- An imaginary straight line which joins the pole and centre of curvature of the mirror is known as **principal axis** and the distance between the centre of curvature and pole of the mirror is called the **radius of curvature**. It is represented by  $R$ .
- Mirrors in which radius of curvature is much larger than aperture, there will be relation between  $R$  and  $f$  such that

$$f = \frac{R}{2}$$

- The image is virtual replica of an object. If rays emanating from a point actually meet at another point, then the point is real image of the object; The image will be virtual if the rays do not actually meet but appear to meet at the point when produced backward.

### The Cartesian sign conventions



#### ► Image formation in concave mirror for different positions of object

| Position of the object | Position of the image | Size of the image              | Nature of the image |
|------------------------|-----------------------|--------------------------------|---------------------|
| At infinity            | At the focus $F$      | Highly diminished, point sized | Real and inverted   |
| Beyond $C$             | Between $F$ and $C$   | Diminished                     | Real and inverted   |
| At $C$                 | At $C$                | Same size                      | Real and inverted   |
| Between $C$ and $F$    | Beyond $C$            | Enlarged                       | Real and inverted   |
| At $F$                 | At infinity           | Highly enlarged                | Real and inverted   |
| Between $P$ and $F$    | Behind the mirror     | Enlarged                       | Virtual and erect   |

#### ► Image formation in convex mirror for different positions of object

| Position of the object                          | Position of the image                   | Size of the image              | Nature of the image |
|---|---|--------------------------------|---------------------|
| At infinity                                     | At the focus $F$ , behind the mirror    | Highly diminished, point sized | Virtual and erect   |
| Between infinity and the pole $P$ of the mirror | Between $P$ and $F$ , behind the mirror | Diminished                     | Virtual and erect   |

### Reflection of Light by Spherical Mirror

- When light falls on an object, it bounces the light in the same medium from where the light comes. This is called the reflection of light.
- Mirrors are good reflectors. A mirror can be made by silvering a metal surface or with glass in front and paint at its back.
- **Laws of reflection:** It is observed that light obeys the following laws while reflecting from any type of surface.
  - The angle of incidence is equal to the angle of reflection, and
  - The incident ray, the normal to the surface at the point of incidence and the reflected ray, all lie in the same plane.
- **Mirror formula:** In a spherical mirror, there is a relation between object's distance  $u$ , image distance  $v$  and principal focus of the mirror  $f$ .

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

- **Magnification by Mirror:** The extent by which mirror extends or reduces the size of image with respect to object is called the magnification factor of mirror. It is represented by  $m$ . If size of an object is  $h$  and its image by spherical mirror is  $h'$ . Then magnification factor of mirror is

$$m = -\frac{v}{u} = \frac{h_i}{h_o} = \frac{h'}{h}$$

## Topic-2

# Refraction through Glass Slab, Prism, Lenses and Total Internal Reflection

**Concepts Covered** Total internal reflection, refraction at spherical surfaces, lenses, thin lens formula, lens maker's formula, magnification, power of lenses, refraction of light through prism.



## Revision Notes

- **Refraction of light:** Refraction is deviation of light when it obliquely travels from one medium to another medium. Snell experimentally found the following laws of refraction.

### Laws of Refraction of Light

- The incident ray, the refracted ray and the normal to the interface of two transparent media, at the point of incidence, all lie in the same plane.
- The ratio of sine of angle of incidence to the sine of angle of refraction is a constant, for the light of a given colour and for the given pair of media. This constant value is called the refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r} = \text{constant } (n_{21})$$

This is known as Snell's law.

- From Snell's law

$$\sin i = \sin r \times n_{21}$$

It shows that if  $\angle i = 0$ , then  $\angle r$  is also zero. This proves that the light rays do not deviate when they travel normally from one medium to another.

- If the first medium is air, then the refractive index is known as the absolute refractive index of the second medium. The absolute refractive index of a medium is expressed by

$$n_2 = \frac{\text{Velocity of light in free space}}{\text{Velocity of light in medium}} = \frac{c}{v}$$

since,  $c > v \Rightarrow n_2 > 1$

- If a ray of light enters from one medium to another medium in such a way that bending of light happens away from normal, then second medium is optically rarer with respect to the first medium. If bending of light is towards normal, then second medium is optically denser with respect to the first medium.

### Principle of Reversibility

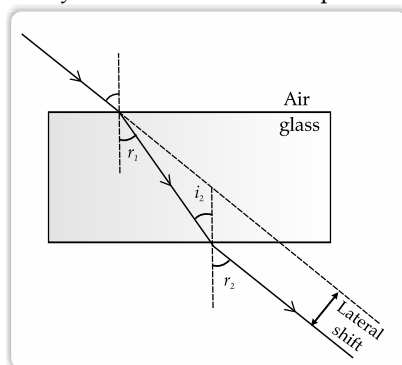
- According to the principle of reversibility, the path of light is reversible even if it is going through several media. It means light follows exactly the same path when its direction is reversed.
- Applying this rule, we may find that if light travels through several media say medium 1 to medium 2 and then to medium 3, then to medium 1.

$$n_{21} \times n_{32} \times n_{13} = 1$$

- Though refraction rules are universal but direction of emergent ray depends upon the shape of the medium or in other words, on the shape and angle between incident and emergent interfaces (refracting surfaces).

### Refraction through Glass Slab

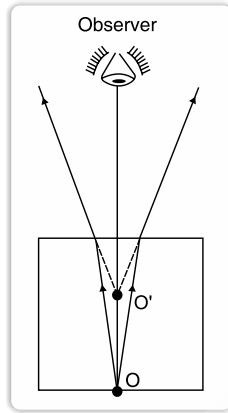
- In a glass slab, refracting surfaces are plane and parallel to each other.
- Emergent ray is parallel to the incident ray but it suffers lateral displacement.



- The apparent depth of the object is always less than actual depth when looking through glass or water.

$$\text{Rise of image} = \text{Real depth} \left( 1 - \frac{1}{n_{21}} \right)$$

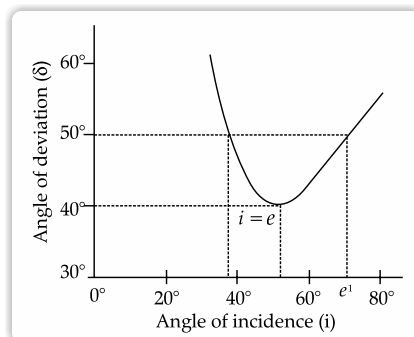
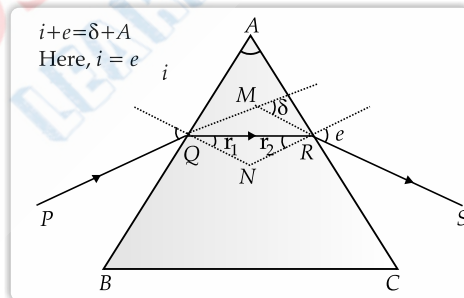
Here,  $n_{21} = \frac{\text{Real depth}}{\text{Apparent depth}}$



- The following phenomena occur due to the refraction of light:-
  - Bottom surface of water pool seems to be raised.
  - The letter appears to be raised when we observe it through a glass slab.
  - Object looks bigger than its actual size and raised when we dip it into liquid.
  - Twinkling of stars.
  - Delayed sunset and early sunrise.

**Refraction through Prism**

- In prism, refracting surfaces are planes but inclined to each other.
- Refracted ray always bends towards the base.
- Angle of deviation,  $\delta = (i - r_1) + (e - r_2)$
- **Angle of minimum deviation:** When incident angle is gradually increased, the angle of deviation initially decreases and after obtaining a minimum value, it starts increasing again. This angle obtained at the lowermost point is called angle of minimum deviation  $\delta_m$ .



- (i)  $i + e = \delta + A$ . Here,  $i = e$
- (ii)  $r_1 + r_2 = A$

- ...(i)
- ...(ii)

- At minimum deviation stage, it is observed that angle of  $i_1 = i_2 (= i)$  and  $r_1 = r_2 (= r)$ , then

$$r = \frac{A}{2} \quad \dots \text{using (ii)}$$

$$i = \frac{\delta_m + A}{2} \quad \dots \text{using (i)}$$

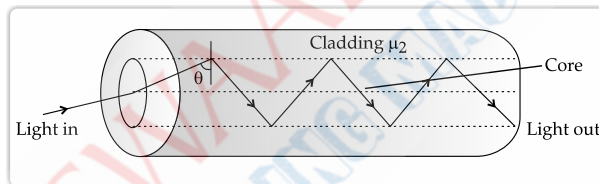
$$n_{21} = \frac{n_2}{n_1} = \frac{\sin[(A + \delta_m) / 2]}{\sin[A / 2]}$$

As angle of prism and deviation can be found experimentally, this equation is used to determine the refractive index of the material of prism.

- For thin prism,  $\delta_m = (n_{21} - 1)A$ . This equation implies that thin prisms do not cause much deviation of light.
- Total internal reflection:** When light travels from an optically denser medium to a rarer medium at the interface, it is reflected into the same medium at a certain angle. This reflection is called total internal reflection.
- Critical angle is that value of incident angle for which angle of refraction is  $90^\circ$ . The refracted ray just brushes the surface. The critical angle for water-air, glass-air and diamond-air are  $45^\circ$ ,  $42^\circ$  and  $24^\circ$  respectively.

$$n_{12} = \frac{1}{\sin C} \quad (\text{where, } C \text{ is critical angle})$$

- If the angle of incidence is more than the critical angle, refraction is not possible and incident ray reflects in denser medium. This process is known as total internal reflection.
- Hence, the conditions for total internal reflection are:
  - The light should travel from denser medium to the rarer medium.
  - Angle of incidence should be larger than the critical angle.
- Optical fibre:** Optical fibre is the technology associated with data transmission using light pulses travelling along with a long fibre which is usually made of plastic or glass.
- Optical fibre works on the principle of total internal reflection. When light travelling in an optically dense medium hits a boundary at a steep angle (larger than the critical angle for the boundary), the light is completely reflected. This is called total internal reflection.



- This effect is used in optical fibres to confine light in the core. Light travels through the fibre core, bouncing back and forth off the boundary between the core and cladding. Because the light must strike the boundary with an angle greater than the critical angle, the only light that enters the fibre within a certain range of angles can travel down the fibre without leaking out.
- Refraction at spherical surface:** If the rays are incident from a medium of refractive index  $n_1$ , to another medium of refractive index  $n_2$ , the formula comes out to be

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Where,  $R$  = Radius of curvature of spherical surface and object is placed at rarer medium.

$u$  = Object distance from spherical surface

$v$  = Image distance from spherical surface

- Lens:** A lens is a piece of transparent glass which is bounded by two surfaces out of which at least one surface is spherical.

There are two types of lenses:

- Convex lens:** A convex lens is one which is thinner at sides and thick at centre.
- Concave lens:** A concave lens is one which is thicker at sides and thin at centre.

- Relation between object distance, image distance with focal length of lens:**

The relation can be expressed as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- Magnification by lens:**

$$m = \frac{\text{Height of the image } (h')}{\text{Height of the object } (h)} = \frac{v}{u}$$

► **Power of a lens:**

The power of a lens is defined as the reciprocal of its focal length. It is represented by the letter  $P$ . The power  $P$  of a lens of focal length  $f$  is given by

$$P = \frac{1}{f}$$

The SI unit of power is dioptre when focal length is in metre. It is denoted by  $D$ . Hence, 1 dioptre is a power of lens whose focal length is 1 metre.

- When two or more lenses are combined, then the power of combined lens is sum of individual power of lenses.

$$P = P_1 + P_2 + \dots$$

► **Lens maker's Formula:**

$$\frac{1}{f} = (n_{21} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \left( \text{where, } n_{21} = \frac{n_2}{n_1} \right)$$

Power of a lens,  $P = \frac{1}{f(m)}$ .

So, the above formula is used to make lenses of required power. Hence, this formula is known as lens maker's formula.

- **Image formation in convex lens for different positions of object**

| Position of the object                 | Position of the image                  | Relative size of the image     | Nature of the image |
|--|--|--------------------------------|---------------------|
| At infinity                            | at focus $F_2$                         | Highly diminished, point sized | Real and inverted   |
| Beyond $2F_1$                          | Between $F_2$ and $2F_2$               | Diminished                     | Real and inverted   |
| At $2F_1$                              | at $2F_2$                              | Same sized                     | Real and inverted   |
| Between $2F_1$ and $2F_2$              | Beyond $2F_2$                          | Enlarged                       | Real and inverted   |
| At Focus $F_1$                         | At infinity                            | Infinitely enlarged            | Real and inverted   |
| Between focus $F_1$ and optical centre | On the same side of the lens as object | Enlarged                       | Virtual and erect   |

- **Image formation in concave lens for different positions of object**

| Position of the object                                  | Position of the image                      | Relative size of the image    | Nature of the image |
|---|--|-------------------------------|---------------------|
| At infinity   | At focus $F_1$                             | Highly diminished point sized | Virtual and erect   |
| Between infinity and the optical centre $O$ of the lens | Between focus $F_1$ and optical centre $O$ | Diminished                    | Virtual and erect   |



## Mnemonics

**Concept name:** Mirror Formula and lens formula:

**Go M** means **More**



**Mirror**

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

**Go L** means **Less**



**Lens**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

**Magnification will have opposite sign:**

$$m = -\frac{v}{u} \quad (\text{For mirror})$$

$$m = \frac{v}{u} \quad (\text{For lens})$$

## Key Formulae

► Snell's law of refraction,  $\frac{\sin i}{\sin r} = \text{constant } (n_{21})$

►  $n_{21} = \frac{c}{v}$

►  $n_{21} \times n_{32} \times n_{13} = 1$

► Rise of image = Real depth  $\left(1 - \frac{1}{n_{21}}\right)$

► Deviation through prism,  $\delta = (i - r_1) + (e - r_2)$

► For thin prism,  $\delta_m = (n_{21} - 1)A$

► Relation between refractive index, angle of prism and minimum deviation

$$n_{21} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

►  $n_{21} = \frac{1}{\sin C}$  (for TIR)

► For lens  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$  and  $m = \frac{h'}{h} = \frac{v}{u}$

► Power of lens,  $P = \frac{1}{f}$

► When two or more lenses are combined, then the power of combined lens is sum of individual power of lenses.

$$P = P_1 + P_2 + \dots$$

► Lens maker's Formula,

$$\frac{1}{f} = (n_{21} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \left( \text{where, } n_{21} = \frac{n_2}{n_1} \right)$$

## Topic-3 Optical Instruments

**Concepts Covered** Simple and compound microscope, telescope, astronomical telescope and their magnifying powers.



### Revision Notes

- Based upon phenomenon of reflecting and refracting properties of mirrors, lenses and prisms, a number of optical devices and instruments have been designed.
- **Microscope** is an optical instrument which helps us to see and study micro objects or organisms. It forms magnified image of the object.
- **Telescope** is an optical instrument which helps us to see and study far off objects magnified and resolved (with clarity).
- We generally set these instruments at two different image vision positions and they are as follows:
  - Image at least distance of distinct vision: This is the least distance from eye where we are able to see objects distinctly. For normal human eye, the distance is 25 cm from our eye.
  - Image at relaxed vision: This is the distance from eye where we are able to see objects distinctly in relaxed vision i.e., no strain to eye. For normal human eye, the distance is infinity from our eye.
  - Magnification at distinct vision is always greater than magnification at relaxed vision.

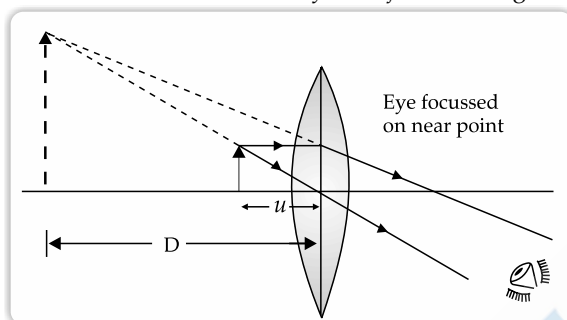


**Simple Microscope:** Convex lens behaves as simple microscope.

The magnifying power of the simple microscope

$$(i) \text{ For least distance of distinct vision, } m = 1 + \frac{D}{f}$$

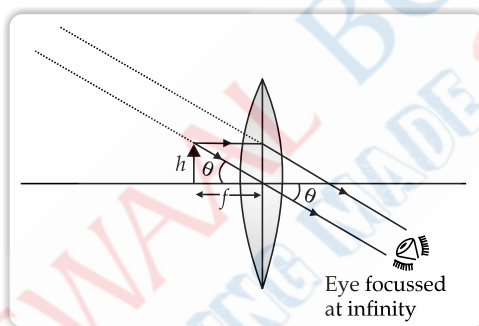
where,  $D$  is the least distance of distinct vision of the eye and  $f$  is focal length of the lens.



(ii) For relaxed eye,

$$m = \frac{D}{f}$$

From above formulae, it is clear that for larger magnifying power, the focal length of the convex lens should be small.



The angular magnification by optical instruments is the linear magnification by lenses only. So, magnification of an instrument means how many times it enlarges the image of an object. It can be written as

$$m = \frac{h'}{h}$$

where,  $h$  is size of object (in one dimension) and  $h'$  is the size of image.

**Compound Microscope:** For much large magnification, compound microscope is used. It is a combination of two convex lenses when the magnification of each lens is compounded.

- The two lenses are placed co-axially and the distance between them is adjustable.
- The lens towards the object is called objective and that towards the eye is called eyepiece.
- The final image formed by the compound microscope is magnified and inverted.
- Total magnification by compound lens,

$$m = m_o \times m_e$$

where,  $m_o$  is magnification by objective lens and  $m_e$  is magnification by eyepiece.

- For least distance of distinct vision, magnification by objective lens is

$$m_o = \frac{v_o}{u_o} \approx \frac{L}{f_o}$$

where,  $L$  is the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length  $f_e$ ). It is called the tube length of the compound microscope.

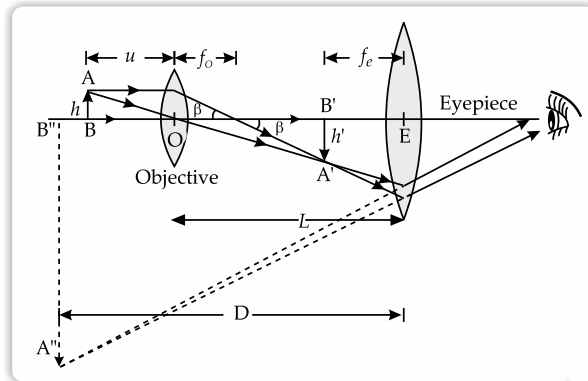
Eyepiece lens will act as simple microscope.

Magnification by eyepiece lens is

$$m_e = 1 + \frac{D}{f_e}$$

Hence, Magnification by compound lens

$$= \frac{L}{f_o} \left( 1 + \frac{D}{f_e} \right)$$



► For Relaxed Eye (normal adjustment)

For relaxed eye, the magnification by objective lens remain same, the magnification by eyepiece will be  $+\frac{D}{f_e}$

Hence, the total magnification of compound microscope in relaxed eye condition is

$$m = \frac{L}{f_o} \times \frac{D}{f_e}$$

► **Properties of Compound Microscope**

- For large magnification of a compound microscope, both  $f_o$  and  $f_e$  should be small.
- If the length of the microscope tube increases, then its magnifying power increases.
- Generally  $f_o$  is much smaller. So, the objective is placed very near to principal focus.
- The aperture of the eyepiece is generally small so that whole of the light may enter the eye.
- The aperture of the objective is also small, so the field of view may be restricted.

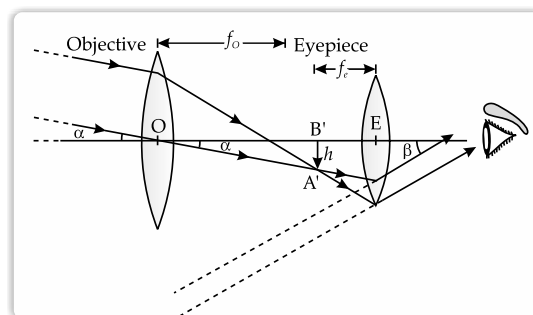
► **Magnification by Telescope**

- Telescope is an instrument to magnify and resolve far off objects.
- Far off objects make much smaller angle at our eye. Telescope makes that angle larger without much intensity loss.
- To maximise the intensity, aperture size of objective lens is quite large. It focuses a bright point size image at its focal plane.
- Now with eyepiece, we will observe the point size image to final inverted magnified image. This type of telescope is known as astronomical telescope.
- For least distance of distinct vision,

$$m = -\frac{f_o}{f_e} \left( 1 + \frac{f_e}{D} \right)$$

► For Relaxed Eye (normal adjustment)

$$m = \frac{\alpha}{\beta} = -\frac{f_o}{f_e}$$



► **Properties of astronomical telescope**

- For larger magnifying power,  $f_o$  should be large and  $f_e$  should be small.
- The length of the tube of an astronomical telescope is  $L = f_o + f_e$  for relaxed vision adjustment.
- When the length of the tube of the telescope increases,  $f_o$  increases and magnifying power also increases.

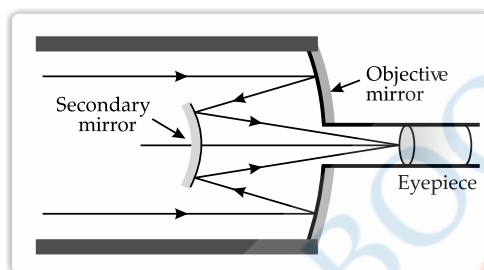
► **Limitations of refractive telescope**

- Large objective lens makes the telescope very heavy. So, it is difficult to handle it by hand.
- It has spherical and chromatic aberrations.

► **Modern Telescope (Reflective Telescope)**

- Reflecting telescope consists of a concave mirror of large radius of curvature in place of objective lens
- A secondary convex mirror is used to focus the incident light, which passes through a hole in the objective primary mirror .

- The magnifying power of the reflecting telescope is  $m = \frac{f_o}{f_e}$



► **Advantages of reflective telescope**

- Very sharp point image by objective mirror removes spherical aberrations.
- As it is very light, large aperture of parabolic mirror can be used for desired magnification.
- This is based on the principle of reflection and there will be no chromatic aberrations.



## Key Formulae

► **Magnification by simple microscope**

$$m = 1 + \frac{D}{f} \text{ (for distinct vision)}$$

$$m = \frac{D}{f} \text{ (For relaxed eye)}$$

► **Magnification by compound microscope**

$$\frac{L}{f_o} \left( 1 + \frac{D}{f_e} \right) \text{ or } \frac{v_o}{u_o} \left( 1 + \frac{D}{f_e} \right) \text{ (for distinct vision)}$$

$$\frac{L}{f_o} \times \frac{D}{f_e} \text{ or } \frac{v_o}{u_o} \times \frac{D}{f_e} \text{ (for relaxed eye)}$$

► **Magnification by telescope**

$$m = - \frac{f_o}{f_e} \left( 1 + \frac{f_e}{D} \right) \text{ (for distinct vision)}$$

$$m = - \frac{f_o}{f_e} \text{ (for relaxed eye)}$$

# CHAPTER-10

## WAVE OPTICS

### Topic-1

### Wave Theory and Huygens' Principle

**Concepts Covered** • Wave front, Huygens' principle, reflection and refraction of plane wave at a plane surface using wave fronts, proof of laws of reflection and refraction using Huygens' principle.



### Revision Notes

- Newton supported 'Descartes' corpuscular theory' of light and developed it further.
- According to the corpuscular theory, "sources of light emit large number of tiny massless particles known as corpuscles in a medium surrounding the source. They are perfectly elastic, rigid and have high speed. This theory could explain reflection and refraction of light but could not explain many other optical phenomenon like interference and diffraction of light. It was unable to explain the concept of partial reflection and refraction through a transparent surface.
- **Huygens' proposed wave theory of light;** According to the theory, light travels in the form of longitudinal waves with uniform speed in a homogenous medium. Different wavelengths of light represent different colours of light.
- As longitudinal and mechanical waves need medium to travel, he assumed a hypothetical medium known as 'ether'. He also proved that speed of light is slower in optically denser medium.
- Initially, Huygens' wave theory of light didn't get much success. Its main point of rejection was, that it was considered as longitudinal wave which need medium, but experimentally found that it could also travel in vacuum and there is no medium like ether.

But later Maxwell's theory of electromagnetic waves and Young's famous double slit experiment firmly established this theory. Maxwell explained that light is an electromagnetic wave which does not need medium and its speed in vacuum is  $3 \times 10^8$  m/s. Phenomenon of optical interference, diffraction and polarisation can be explained with wave nature of light.

- It had some points of failure. It could not explain Photoelectric effect and Compton effect. With polarisation, it is established that light is not a longitudinal wave but a transverse wave.
- Huygens' principle brings concept of formation of new wave fronts and its propagation in forward direction.
- Wavefront is locus of all points in which light waves are in same phase. Propagation of wave energy is perpendicular to the wavefront.

#### Huygens' Principle:

- Every point of a wavefront becomes secondary source of light.
- These secondary sources give their own light waves. Within small time, they produce their own wave called secondary wavelets. These secondary waves have same speed and wavelengths as waves by primary sources.
- At any instant, a common tangential surface on all these wavelets give new wavefronts in forward direction.

#### Shapes of wavefronts:

| Source                     | Wavefronts            |
|----------------------------|-----------------------|
| Point source               | Spherical wavefront   |
| Line source                | Cylindrical wavefront |
| Plane source               | Plane wavefront       |
| Point source very far away | Plane wavefront       |

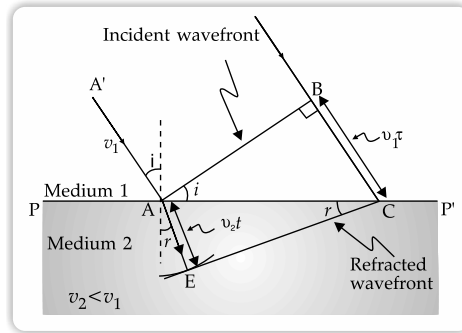
Concave lens converts plane wavefront to convex wavefront and convex lens convert plane wavefront to concave wavefront.

#### Refraction of light by Huygens' Principle:

- Snell's law can be proved by Huygens' principle.

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant}$$

- It has also been proved that the velocity of light in denser medium is less than velocity of light in rarer medium.



$AB$  = incident wavefront

$EC$  = refracted wavefront

$\angle i$  = angle between incident wavefront  $AB$  and interface  $PP'$

$\angle r$  = angle between refracted wavefront  $EC$  and interface  $PP'$

If medium 2 is optically denser than medium 1 and  $\tau$  is the time in which disturbance from  $B$  reaches  $C$ . This is the same time  $\tau$  in which disturbance from  $A$  reaches  $E$  where distance  $AE < BC$ .

$$\triangle AEC \cong \triangle ABC$$

$$\sin i = \frac{BC}{AC}$$

$$\sin r = \frac{AE}{AC}$$

$$\frac{\sin i}{\sin r} = \frac{BC}{AE}$$

$BC$  = Distance travelled by wave at  $B$  in time  $\tau$  in medium 1

$AE$  = Distance travelled by wave at  $A$  in time  $\tau$  in medium 2

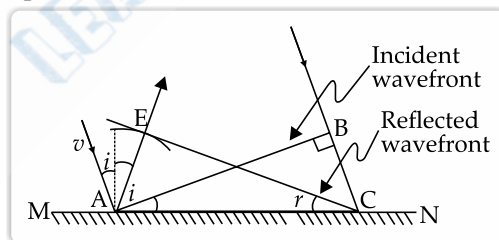
$$\frac{\sin i}{\sin r} = \frac{v_1 \tau}{v_2 \tau}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant}$$

Hence

This is law of refraction (Snell's law).

**Reflection of light by Huygens' Principle:**



$AB$  = incident wavefront

$EC$  = reflected wavefront

$\angle i$  = angle between incident wavefront  $AB$  with the interface  $AC$

$\angle r$  = angle between reflected wavefront  $EC$  with the interface  $AC$

If disturbance at  $A$  is reflected from the interface  $AC$ , then disturbance at  $B$  and disturbance at  $A$  both travel in same medium. Thus, they will have travelled equal distance in time  $\tau$ , where  $\tau$  is the time in which disturbance from  $B$  reaches at  $C$ .

Now  $AE = BC = v\tau$  (distance travelled in same medium in same time)

$$\triangle AEC \cong \triangle ABC$$

$$\angle i = \angle r$$

This is law of reflection.

## Topic-2

# Superposition of Light Waves (Interference and Diffraction)

**Concepts Covered** • Interference, Young's double slit experiment, Coherent sources, Diffraction due to a single slit.



## Revision Notes

- According to superposition principle, "At a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each wave".

It means that if individual displacement produced at a point by two coherent waves at any instant is given by

$$y_1 = a \cos \omega t \text{ and } y_2 = a \cos \omega t.$$

Then, resultant displacement at that point will be

$$y = y_1 + y_2 = 2a \cos \omega t.$$

Hence, the total intensity at that point will be:

$$I = 4I_0$$

where,  $I_0 \propto a^2$ ; maximum intensity due to one wave.

### Interference

- **Constructive Interference:** If two waves are propagating such that crest and trough of both waves would reach at a point in the same instant, then we say there is constructive interference of two waves at that point. The resultant amplitude of the wave is the sum of individual amplitudes. (We can generalize this to superposition of more than two waves)  $a = a_1 + a_2$
- **Destructive Interference:** If two waves are propagating such that crest of one wave and trough of other wave reaching at a point in same instant, then we say that there is destructive interference of two waves at that point. The resultant amplitude of the wave is the difference of individual amplitudes. (We can generalize this to superposition of more than two waves)  $a = a_1 - a_2$
- Two independent sources can never be coherent. We may create two coherent sources by deriving them from one source.

### Condition for constructive Interference:

- Waves would be coherent in nature. Coherent wave means that they should have equal frequency and constant phase difference ( $0, 2\pi, \dots, 2n\pi$ ) with each other at any time interval  $t$ .

Path difference between waves at this phase difference =  $0, \lambda, \dots, n\lambda$ , Here,  $n = 0, 1, 2, 3, \dots$

$$a_r = a_1 + a_2$$

if

$$a_1 = a_2 = a$$

then

$$a_r = 2a$$

∴

$$I \propto a^2$$

$$I_r = 4a^2$$

### Condition for destructive interference:

- Waves would be coherent in nature. The phase between the waves should be odd multiples of  $\pi$ , i.e.,  $0, \pi, \dots, (2n-1)\pi$

Path difference between waves at this phase difference =  $\frac{\lambda}{2}, \frac{3\lambda}{2}, (2n-1)\frac{\lambda}{2}$ , Here,  $n = 1, 2, 3, 4, \dots$

$$a_r = a_1 - a_2$$

if

$$a_1 = a_2$$

then

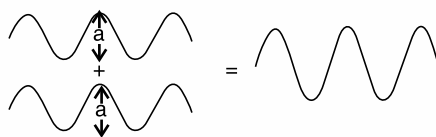
$$a_r = 0$$

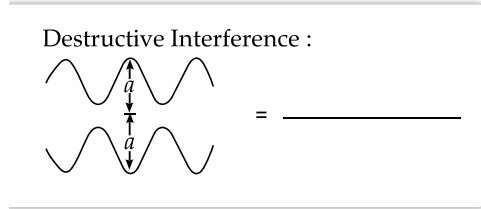
∴

$$I \propto a^2$$

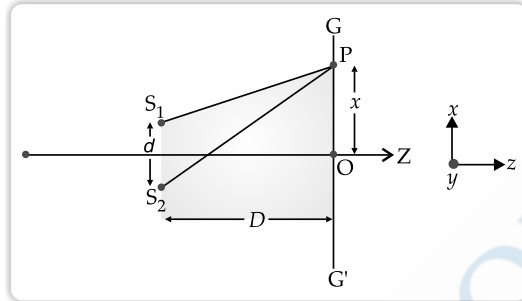
$$I_r = 0$$

### Constructive Interference :





**Young's double slit Experiment:**



- At "O" we get central maxima. Here, path difference  $(S_2P - S_1P) = 0$
- At "P", which is at "x" height from "O" path difference

$$(S_2P - S_1P) = \frac{xd}{D}$$

- **Condition for P to be a bright spot:**

$$\frac{xd}{D} = 0, \lambda, 2\lambda, \dots, n\lambda$$

$$x_{n^{\text{th}} \text{ bright}} = \frac{nD}{d} \lambda$$

where,  $n$  is number of bright fringes after central fringe.

- **Condition for P to be a dark spot:**

$$\frac{xd}{D} = 0, \frac{3\lambda}{2}, \dots, (2n+1) \frac{\lambda}{2}$$

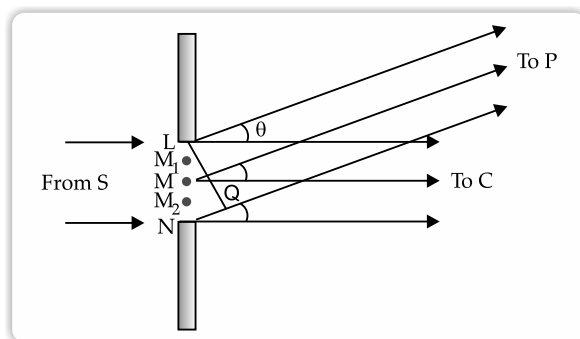
$$x_{n^{\text{th}} \text{ dark}} = \frac{(2n+1)D}{2d} \lambda$$

Here,  $n$  is the number of dark fringes after central fringe.

- Width of the bright fringe  $(\beta_B) = x_{nB} - x_{(n-1)B} = \frac{D\lambda}{d}$
- Width of the dark fringe  $(\beta_D) = x_{nD} - x_{(n-1)D} = \frac{D\lambda}{d}$
- Width of the central fringe  $(\beta_C) = \frac{D\lambda}{d}$
- Hence  $\beta_B = \beta_D = \beta_C$

**Diffraction**

It is defined as the bending of light around the corners of an obstacle or aperture into the region where we expect shadow of the obstacle.



If width of the opening =  $a$

$\theta$  is the angle of elevation of point P from principal axis.

Path difference between ray from L and ray from N =  $NQ = a \sin \theta$

$$a \sin \theta = \lambda$$

$$\theta = \frac{\lambda}{a}$$

$$(\therefore \sin \theta \cong \theta) \sin \theta \ll \ll 1$$

$\therefore$  for first maxima

It is observed that when path difference =  $\lambda, 2\lambda, \dots, (2n-1)\lambda$ , P is a dark point.

When  $a \sin \theta = \frac{3\lambda}{2}, \dots, (2n+1)\frac{\lambda}{2}$ , P is a bright point.

Elevation angle for first bright fringe,  $\theta_{1D} = \frac{3\lambda}{2a}$

- ▶ Height of first dark fringe,  $x_{1D} = \frac{3\lambda D}{2a}$
- ▶ Elevation angle for first dark fringe,  $\theta_{1D} = \frac{\lambda}{a}$
- ▶ Width of the bright fringe =  $\frac{D\lambda}{a}$
- ▶ Width of the dark fringe =  $\frac{D\lambda}{a}$
- ▶ Width of the central fringe =  $\frac{2D\lambda}{a}$

There is no gain or loss of energy in interference or diffraction, which is consistent with the principle of conservation of energy. Energy only redistributes in these phenomena.



## Key Formulae

### Condition for constructive interference for coherent waves:

- Constant phase difference ( $0, 2\pi, \dots, 2n\pi$ )
- Path difference =  $0, \lambda, \dots, n\lambda$

### Condition for destructive interference for coherent waves:

- Phase difference ( $0, \pi, \dots, (2n-1)\pi$ ) with each other at any time interval  $t$ .
- Path difference =  $\frac{\lambda}{2}, \dots, (2n-1)\frac{\lambda}{2}$

### In Interference Pattern

- Width of the bright fringe =  $\frac{D\lambda}{d}$
- Width of the dark fringe =  $\frac{D\lambda}{d}$
- Width of the central fringe =  $\frac{D\lambda}{d}$
- All fringes have equal fringe width.

### In Diffraction Pattern:

- Angle of elevation of any point P on screen =  $\frac{\lambda}{a}$
- Condition that P would be dark point when path difference =  $\lambda, 2\lambda, \dots, (2n-1)\lambda$
- Condition that P would be bright point when path difference =  $\frac{3\lambda}{2}, \dots, (2n+1)\frac{\lambda}{2}$
- Width of the bright fringe =  $\frac{D\lambda}{a}$
- Width of the dark fringe =  $\frac{D\lambda}{a}$
- Width of the central fringe =  $\frac{2D\lambda}{a}$
- Height of first bright fringe  $x_{1B} = \frac{3\lambda D}{2a}$



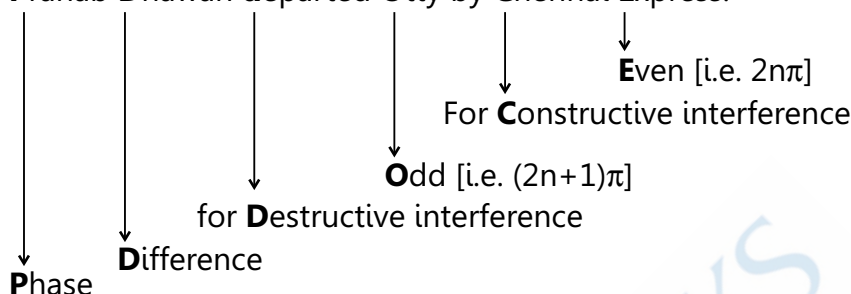


## Mnemonics

**Concept:** Phase difference for constructive and destructive interference:

**Mnemonics:** **P**ranab **D**hawan **d**eported **O**ddy by **C**hennai **E**xpress.

**Interpretations:**



## UNIT – VII : DUAL NATURE OF RADIATION AND MATTER

### CHAPTER-11

## DUAL NATURE OF RADIATION AND MATTER

### Topic-1

#### Photoelectric Effect

**Concepts Covered** • Photoelectric effect, • Hertz and Lenard's observations, • Einstein's photoelectric equation, • Particle nature of light, • Experimental study of photoelectric effect



### Revision Notes

- ▶ In an attempt towards unification of study of Physics, Photoelectric effect was established in the 19<sup>th</sup> century which stated that everything in nature can be classified into either matter or radiation.
- ▶ Several important experiments were carried out independently on matter and radiations during that time. In 1897, Maxwell established electromagnetic theory which unified all radiations like light and heat. Maxwell established the wave theory of light. X-ray radiation was also discovered during that time in 1895.
- ▶ Simultaneously, in study of matter, a milestone discovery of electron was done by J.J. Thomson in 1897. It established that atoms of different matters constitute same particles and one of them is the electron.
- ▶ **Electron Emission:** Electron has two types of motion, i.e. orbital and zig-zag motion in free state depending upon its energy. i.e., free electrons have higher energy than orbital electrons.
  - Free electrons in metals cannot come out from the surface due to force by positive ions present in metals. Electrons can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull.
  - **Work Function of a metal:** Work function of a metal is the minimum amount of work done (energy given) to its electron so that it can escape from the metal surface. Work function is different for different metals. It is measured in electron volt (eV).
  - One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt.

$$V = \frac{W}{e} \quad (q = e, \text{ for an electron})$$

When  $V = 1 \text{ V}$ ;  $W = 1 \text{ eV}$ ; putting these values in equation

Hence,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Three types of electron emissions are as follows:-

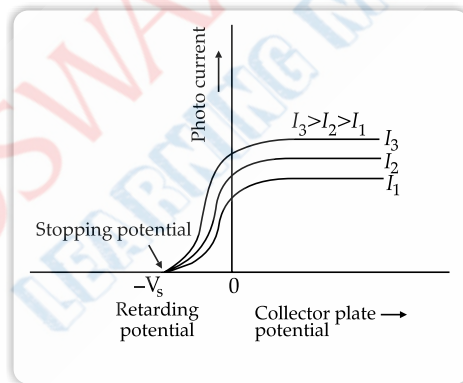
- ▶ **Thermionic emission:** When electron emission occurs by heating the metal, it is known as thermionic emission. Emitted electrons are called thermionic electrons.

- ▶ **Field emission:** When electron emission occurs by applying a strong electric field, it is known as field emission and emitted electrons are called field electrons.
- ▶ **Photoelectric effect:** When electron emission is occurred by illumination of metal by the light of suitable frequency, it is known as photoelectric emission. Here, emitted electrons are called photo electrons.
  - When light falls on the metal surface, free electrons absorb energy from light and if this energy is more than the work function of metal, the electron escapes from the surface. This phenomenon is known as photoelectric emission. This was first observed by Hertz.
- ▶ **Hallwachs' and Lenard's detailed study of Photoelectric effect:**
  - In 1888, Lenard observed that when ultraviolet light falls on zinc metal, metal becomes positively charged. With the discovery of electrons, it was established that this is due to emission of electrons. The current produced by these photoelectrons is called photoelectric current.
  - The frequency of the incident light below which emission of electrons do not take place, is called its threshold frequency.
- ▶ **Hertz and Lenard's experiment:** This experiment led the formation of quantum theory of light as wave theory could not explain photoelectric effect. Experiment was carried out to study the following two properties of light:-
  - **Intensity of light:** Power of light is directly proportional to the intensity of light. A higher power bulb (say 100 watts) has more intensity than the lower power bulb (say 50 watts).
  - **Frequency of light:** Colour of light is due to its characteristic property of frequency.

$$v = \frac{c}{\lambda}$$

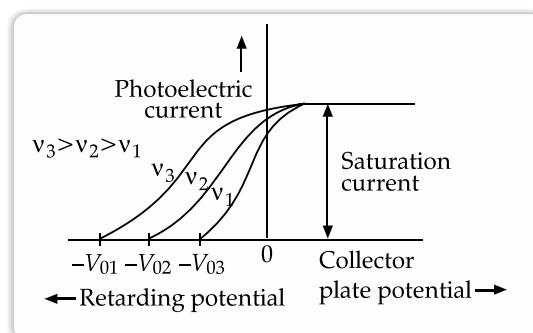
where,  $v$  is frequency of light.

- ▶ **Experimental outcome:** It showed that intensity of light has linear relationship with photoelectric current at a potential higher than the stopping potential.
- ▶ **Effect of Potential on photoelectric current:** For a given frequency of the incident radiation, the stopping potential is independent of its intensity.



- ▶ **Effect of frequency of incident radiation on stopping potential:**

- It was observed that photoemission current starts only at certain minimum frequency of light known as **threshold frequency** of that metal. Below this frequency, photoemission does not take place inspite of intensity of light of that frequency, falling on a photosensitive plate.



- Wave theory was inadequate or failed to explain the photoelectric effect due to the following reasons:
- According to wave theory, higher amplitude means higher energy but experiments show that even larger amplitude (higher intensity) of light below threshold frequency can not give photoelectric effect.
  - According to wave theory, same intensity of different colour should have same energy but experiment shows that energy depends upon frequency, not on amplitude.
  - According to wave theory, wavefront should take some time to give energy to electron but experimentally, it was found that ejection of electron is instantaneous.
- In 1900, Max Planck stated that electromagnetic energy can be emitted only in quantized form.

$$E = h\nu$$

where,  $h$  is Planck's constant.

- Based upon this postulate, Einstein established quantum theory of radiation and was able to explain photoelectric phenomenon by this theory. It states that light energy packets are known as photons (Particle nature of light).
- In photoelectric effect, an electron absorbs a quantum of energy ( $E = h\nu$ ) of radiation. If this absorbed quantum of energy exceeds, the minimum energy needed for the electron to escape from the metal surface (work function  $\phi_0$ ), the electron is emitted with maximum kinetic energy.

$$K.E. = h\nu - \phi_0$$

where,  $\phi_0$  is the work function of the metal.

- At stopping potential, kinetic energy of the ejected electron is zero. Below this potential, the electrons can not be ejected. Hence, maximum kinetic energy of an electron is calculated by

$$K.E._{max} = eV_0$$

where,  $V_0$  is stopping potential.

Work function of metal,  $\phi_0 = \nu_0 h$

where,  $\nu_0$  is the cutoff frequency or threshold frequency.

- Maximum speed of emitted photoelectrons can be calculated as

$$v_{max} = \sqrt{\frac{2K.E._{max}}{m}}$$

- According to quantum theory, all photons of specific light frequency have equal energy. Intensity of light only increases the number of photons per unit area and not the energy of photons.
- Photons are electrically neutral and are not deflected by electric and magnetic field.
- Photon has energy to propagate, hence it has momentum.

$$\text{Momentum of photon } p = \frac{h\nu}{c}$$

- In photon- electron collision, the number of electrons or photons are not conserved but energy and momentum are conserved.

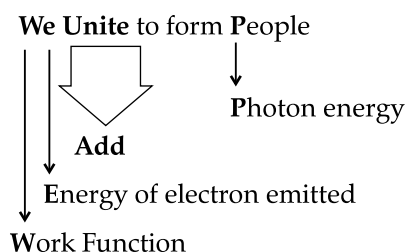
- As interference, diffraction and polarization cannot be explained by quantum theory of light, hence it was said that light has dual nature. When it travels in a medium, it travels as wave and while interacting with other medium, it acts like particles (photons).



## Mnemonics

**Concept:** Einstein's equation of photoelectric effect:

**Mnemonics:**



**Interpretations:** So, Work function + Energy of electron emitted = Energy of incident photon.

## Key Formulae

### ▶ Einstein equation of photoelectric effect

$$K.E. = h\nu - \phi_0$$

### ▶ Work function $\phi_0 = h\nu_0$

where,  $\nu_0$  is the cutoff frequency.

### ▶ Maximum speed of emitted photoelectrons can be calculated as

$$v_{\max} = \sqrt{\frac{2KE_{\max}}{m}}$$

### ▶ Momentum of photon, $p = \frac{h\nu}{c} = \frac{h}{\lambda}$ as $c = \lambda\nu$

### ▶ 1eV (one electron volt) = $1.6 \times 10^{-19}$ J

### ▶ Energy of light $E = \frac{hc}{\lambda} = \frac{12375}{\lambda(\text{in } \text{Å})}$ (eV)

### ▶ If applied potential is $V$ , the emitted energy $E = eV$

### ▶ 1eV = $1.6 \times 10^{-19}$ J

## Topic-2

## Dual Nature of Matter

**Concepts Covered** • Matter waves, • de-Broglie wavelength.



## Revision Notes

- ▶ de-Broglie's postulate is based upon the symmetry of nature. If radiation has dual nature, then matter should also have dual nature.
- ▶ According to his hypothesis, moving particles of matter should display wave nature under suitable conditions. He named the wave as **matter wave**. It is the third type of wave. It is different from mechanical wave and electromagnetic wave.
- ▶ **Properties of matter wave:** Whenever a particle moves, the matter wave envelops it and controls its motion.
  - de-Broglie proposed that the wavelength  $\lambda$  known as de-Broglie wavelength; associated with momentum of particle  $p$  is given as

$$\lambda = \frac{h}{p}$$

Hence, de-Broglie's wavelength of particle,

$$\lambda = \frac{h}{mv}$$

- **Calculation of wavelength of electron-wave:**

In photoelectric equation, kinetic energy of electron at potential  $V$  is  $K = eV$ . Putting this value of kinetic energy in de-Broglie wavelength equation,

$$\lambda_e = \frac{h}{\sqrt{2meV}}$$

By putting the value of mass of electron, its charge and Planck's constant, it becomes  $\lambda_e = \frac{1.227}{\sqrt{V}}$  nm. This is theoretical calculation of de-Broglie wavelength of electron, where,  $V$  is the magnitude of accelerating potential in volts.

### Key Word

#### Matter waves:

According to de-Broglie, a wave is associated with each moving particle which is called matter waves.

- Lighter the particle, larger the de-Broglie wavelength.
- Faster the particle moves, the smaller is its de-Broglie wavelength.
- de-Broglie's wavelength is independent of the charge.

- From this formula, wavelength of particle is inversely proportional to the mass of particle and its velocity. Hence, heavier particles have shorter wavelengths.

### Key Formulae

- de Broglie wavelength associated with momentum of particle  $p$  as

$$\lambda = \frac{h}{p} \text{ or } \lambda = \frac{h}{mv}$$

- $\lambda_e = \frac{h}{\sqrt{2meV}}$  or  $\lambda_e = \frac{1.227}{\sqrt{V}}$  where  $V$  is the magnitude of accelerating potential in Volts.

## UNIT – VIII : ATOMS & NUCLEI

### CHAPTER-12

### ATOMS

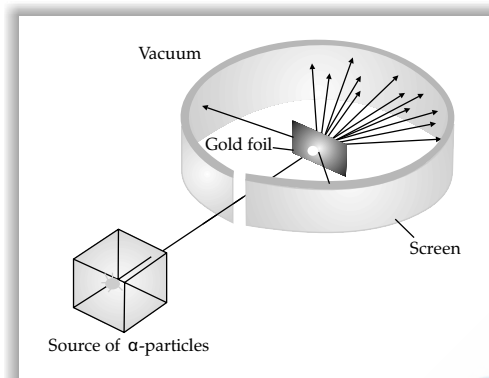
### Revision Notes

- There are roughly a hundred types of atoms. (An atom is the identity of an element. 118 types of elements are known to us till date.)
- All atoms radiate different line spectra which shows these atoms are different and may be the smallest particles.
- With the discovery of electron by J. J. Thomson, it was evident that atoms have identical sub atomic particles and different line spectra of different atoms exists due to the motion of these particles.
- **Atomic models**
  - As atom is electrically neutral, the discovery of electron led by J. J. Thomson established that it should also have positive charge. Hence, he proposed first model of atom- Plum-Pudding model.
  - **Plum-Pudding model:** According to plum pudding model, "the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon."
  - But subsequent studies on atom showed the results very different from this atomic model.
- **Rutherford's atomic model:**
  - With the discovery of Avogadro number, the atomic size was understood to be quite big as compared to the sizes of atomic sub-particles.
  - This led Rutherford to establish the second theoretical atomic model known as "nuclear model of the atom". It was inspired by planetary position around the Sun.
  - According to this model "The entire positive charge and most of the mass of the atom is concentrated in a small volume called the nucleus, with electrons revolving around the nucleus just as planets revolve around the Sun."

- Though, it was initially a theoretical model but it was a major step towards the modern atomic model.
- **Geiger and Marsden** experimentally proved Rutherford's atomic model.

### Geiger and Marsden scattering experiment or Alpha particle scattering experiment:

#### Experimental setup:



- Radioactive element  ${}_{83}^{214}\text{Bi}$  was taken as  $\alpha$ -particles generating source.
- Gold was taken as the target metal. The selection of gold was based upon its two important characteristics:
  - Gold has the highest malleability. Gold foil that was used in the experiment was almost transparent.
  - Gold is a heavy metal, hence it helped in the discovery of the nucleus.
- Lead bricks absorbed the  $\alpha$ -particles which were not in the direction of gold foil. They worked as collimator.
- The detector was made from ZnS.

#### Experimental observations:

- When  $\alpha$ -particles hit ZnS screen, it absorbs and glows. Hence, the number of  $\alpha$ -particles can be counted by intensity variation.
- Most of the  $\alpha$ -particles passed roughly in a straight line (within  $1^\circ$ ) without deviation. This showed that no force was acting upon most of  $\alpha$ -particles.
- A very small number of  $\alpha$ -particles were deflected. (1 out of 8000)

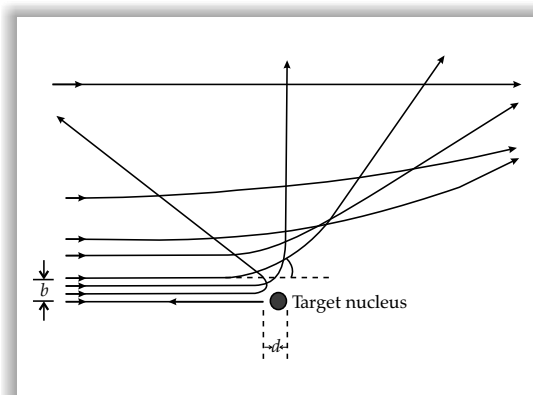
#### Conclusions:

- Most of the space in the atom is mostly empty (only 0.14% scatters more than  $1^\circ$ ).
- Experiment suggests that all positively charged particles are together at one location at centre. It was called nucleus. So, nucleus has all the positive charges and the mass. Therefore, it has the capability to reflect heavy positive  $\alpha$ -particles.
- Size of the nucleus is calculated to be about  $10^{-15}$  m to  $10^{-14}$  m. According to kinetic theory, size of one atom is of the order of  $10^{-10}$  m.
- Force between  $\alpha$ -particles and gold nucleus

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r^2}$$

#### Alpha-particle trajectory:

- **Impact parameter:** It is the perpendicular distance between the direction of the given  $\alpha$ -particle and centre of the nucleus. It is represented by ' $b$ '.
- **Distance of closest approach:** It is the distance between centre of nucleus and the  $\alpha$ -particle where it stops and reflects back. It is represented by ' $d$ '. This distance gives an approximation of nucleus size.



**Electron Orbits**

- We can calculate the energy of an electron and the radius of its orbit based upon Rutherford model.
- The electrostatic force of attraction,  $F_e$  between the revolving electrons and the nucleus provides the requisite centripetal force ( $F_c$ ) to keep them in their orbits.

$$F_e = F_c$$

$$\text{For hydrogen atom, } \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$\text{or, } r = \frac{e^2}{4\pi\epsilon_0 mv^2}$$

Electron has kinetic energy,  $K = \frac{1}{2}mv^2$ . Putting the value of  $mv^2$  in the above equation

$$K = \frac{e^2}{8\pi\epsilon_0 r}$$

$$\text{And } v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

P.E. of an electron,  $U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$  (negative sign shows that it is due to attractive force)

$$\begin{aligned} \text{Total energy, } E &= K + U \\ E &= \frac{e^2}{\pi 8\epsilon_0 r} + \left( -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r} \right) \\ &= -\frac{e^2}{8\pi\epsilon_0 r} \end{aligned}$$

- Due to this negative energy, the electron is bound to the nucleus and revolves around it. This energy is known as the binding energy of an electron.
- From the equation, it is clear that if energy is zero, then radius is infinity. Practically, if we provide this amount of energy to this electron, it gets free.

**Atomic Spectra:**

- Each element has a characteristic spectrum of radiation, which it emits. There are two types of **atomic spectra**: Emission atomic spectra and absorption atomic spectra.
- **Emission atomic spectra**: Due to excitation of atom usually by electricity, light of particular wavelength is emitted. This atomic spectra is known as emission spectra.
- **Absorption atomic Spectra**: If atoms are excited in presence of white light, it absorbs its emission spectral colours and black lines appear in the same places of that atoms' emission spectra. This type of spectra is known as absorption spectra.

**Spectral series:**

- The atom shows range of spectral lines. Hydrogen is the simplest atom and has the simplest spectrum.
- The spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way. Each of these sets is called a spectral series.
- **Balmer Series**: Balmer observed the first hydrogen spectral series in the visible range of the hydrogen spectrum. It is known as Balmer Series.

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

Longest wavelength = 6566.4 Å

Shortest wavelength = 3648 Å

where,  $R$  is Rydberg's constant. The value of  $R$  is  $1.097 \times 10^7 \text{ m}^{-1}$ ;  $n = 3, 4, 5, \dots$

$$\frac{1}{\lambda} = \frac{\nu}{c}$$

$$\text{Hence, } \nu = Rc \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

Other series of spectra for hydrogen were as follows:

- ▶ **Lyman Series:**  $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$ ;  $n = 2, 3, 4, 5, \dots$  This is in UV range.  
Longest wavelength = 1216 Å  
Shortest wavelength = 912 Å
- ▶ **Paschen Series:**  $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ ;  $n = 4, 5, 6, \dots$   
Longest wavelength = 18761.14 Å  
Shortest wavelength = 8208 Å
- ▶ **Brackett Series:**  $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ ;  $n = 5, 6, \dots$   
Longest wavelength = 40533.33 Å  
Shortest wavelength = 14592 Å
- ▶ **Pfund Series:**  $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$ ;  $n = 6, 7, 8, \dots$   
Longest wavelength = 74618.1 Å  
Shortest wavelength = 22800 Å
- ▶ The Lyman series is in the ultraviolet region while the Paschen, Brackett and Pfund series are in the infrared region.

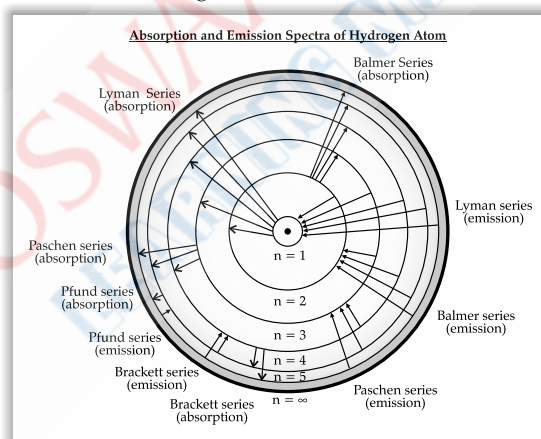


## Key Word

### Atomic Spectrum

When electrons are excited, they jump from lower to higher energy level by absorbing energy. In this spectrum, dark lines are observed on bright background.

When excited electrons are de-excited, they jump from higher to lower energy level by emitting energy. In this spectrum, bright lines are observed on dark background.



| Spectral series | Emission        | Absorption      | Frequency     |
|-----------------|-----------------|-----------------|---------------|
| Lyman series    | Down to $n = 1$ | Up from $n = 1$ | Ultraviolet   |
| Balmer series   | Down to $n = 2$ | Up from $n = 2$ | Visible light |
| Paschen series  | Down to $n = 3$ | Up from $n = 3$ | Near infrared |
| Brackett series | Down to $n = 4$ | Up from $n = 4$ | Far infrared  |
| Pfund series    | Down to $n = 5$ | Up from $n = 5$ | Far infrared  |

### Limitation of Rutherford model:

- ▶ **It could not explain the stability of the atom:** The electron orbiting around the nucleus radiates energy. As a result, the radius of the electron orbit should continuously decrease and ultimately the electron should fall into the nucleus.
- ▶ **It could not explain the nature of energy spectrum:** According to the Rutherford's model, the electrons can revolve around the nucleus in all possible orbits. Hence, the atom should emit radiations of all possible wavelengths or in other words, it should have a continuous spectrum. However, in practice, the atoms are found to have a line spectrum or discrete spectrum.



**Bohr's Model and Postulates:**

- ▶ An electron can revolve in certain stable orbits without emission of radiant energy. These orbits are called stationary states of the atom.
- ▶ Electron revolves around nucleus only in those orbits for which the angular momentum is the integral multiple of  $\frac{h}{2\pi}$ , where,  $h$  is Planck's constant.
- ▶ Hence, angular momentum,  $L = \frac{nh}{2\pi}$
- ▶ An electron may make a transition from one of its specified non-radiating orbit to another of lower energy. When it does so, a photon is radiated having energy equal to energy difference between initial and final state.

$$h\nu = E_i - E_f \quad (\text{where, } \nu \text{ is frequency})$$

$$\text{Angular momentum,} \quad L = mv_n r_n$$

$$\text{According to Bohr's postulate,} \quad L = \frac{nh}{2\pi}$$

$$\text{Hence,} \quad mv_n r_n = \frac{nh}{2\pi}$$

$$mr_n = \frac{nh}{2\pi v_n}$$

$$\text{For hydrogen atom,} \quad v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

Combining these two equations, we get

$$v_n = \frac{1}{n} \cdot \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{(h/2\pi)}$$

This equation depicts that electron speed in  $n^{\text{th}}$  orbit falls by a  $n$  factor.

$$r_n = \left(\frac{n^2}{m}\right) \left(\frac{h}{2\pi}\right)^2 \frac{4\pi\epsilon_0}{e^2}$$

For innermost orbit  $n = 1$ ; the value of  $r_1$  is known as Bohr's radius  $a_0$ .

$$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

If we put values of all constants, we get

$$a_0 = 5.29 \times 10^{-11} \text{ m} \approx 0.53 \text{ \AA}$$

It can also be observed that radii of  $n^{\text{th}}$  orbit increases by  $n^2$  times.

By putting this value in total energy of an electron and convert the unit in eV, we get

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

Negative value shows that electron is bound to nucleus.

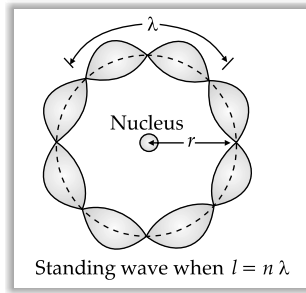
- ▶ The explanation of the hydrogen atom spectrum provided by Bohr's model was a brilliant achievement.

**de-Broglie's explanation of Bohr's second postulate by quantization theory:**

- ▶ According to Bohr's postulate, an electron in a hydrogen atom can revolve in a certain orbit only in which its angular momentum,  $L = n \frac{h}{2\pi}$ . In these stationary orbits, an electron does not radiate energy.
- ▶ de-Broglie proved it with the help of the wave nature of electron.
- ▶ Travelling wave propagates energy but stationary wave does not propagates energy. In analogy to waves travelling on a string, particle waves can lead to standing waves under resonant conditions. Resonant condition is

$$l = 2\pi r$$

where,  $l$  = perimeter of orbit.



For a hydrogen atom, length of the innermost orbit is its perimeter. Hence,

$$2\pi a_0 = n\lambda \quad \dots(i)$$

According to deBroglie's wavelength of electron,

$$\lambda = \frac{h}{p}$$

Now equation (i) can be written as (taking  $a_0 = r$ )

$$2\pi r = n \frac{h}{p} \quad \dots(ii)$$

But  $p = mv$

Hence, equation (ii) can be reduced as,

$$2\pi r = n \frac{h}{mv}$$

$$mvr = \frac{nh}{2\pi}$$

$$L = \frac{nh}{2\pi}$$

This is Bohr's second postulate.

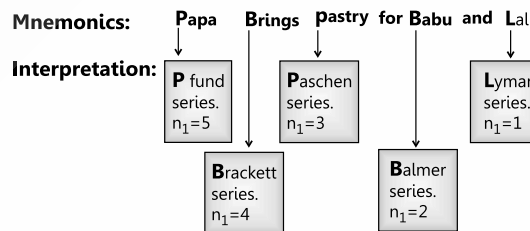
Limitation of Bohr's atomic model:

▶ Bohr's model is for hydrogenic atoms. It does not hold true for a multi-electron model.

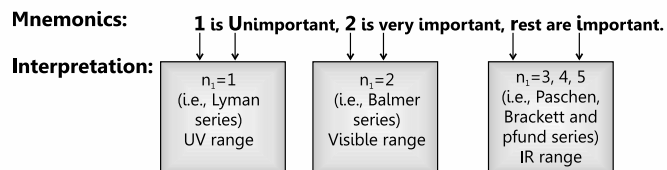


## Mnemonics

**Concept: Hydrogen Spectra:**



**Concept: Range of each series of Hydrogen Spectra:**





## Key Formula

- Radius of orbit,

$$r = \frac{e^2}{4\pi\epsilon_0 m v^2}$$

- Kinetic energy of electron in its orbit,

$$K = \frac{e^2}{4\pi\epsilon_0 r}$$

- PE of an electron  $U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$

- Velocity of electron in its orbit,  $v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}}$

- Total energy of an electron in an orbit,  $E = -\frac{e^2}{8\pi\epsilon_0 r}$

### Spectral series

- **Balmer Series:**  $\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$ ,  $n = 3, 4, 5, \dots$ . This is in Visible range.

- **Lyman Series:**  $\frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$ ;  $n = 2, 3, 4, 5, \dots$ . This is in UV range.

- **Paschen Series:**

$$\frac{1}{\lambda} = R \left( \frac{1}{3^2} - \frac{1}{n^2} \right); n = 4, 5, 6$$

- **Brackett Series:**

$$\frac{1}{\lambda} = R \left( \frac{1}{4^2} - \frac{1}{n^2} \right); n = 5, 6, 7, \dots$$

- **Pfund Series:**

$$\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right); n = 6, 7, 8, \dots$$

These series are in infrared region.

- **Relation between speed, total energy of an electron and its radius with respect to orbital number  $n$ :**

$$v_n = \frac{1}{n} \frac{e^2}{4\pi\epsilon_0} \frac{1}{(h/2\pi)}$$

$$r_n = \left( \frac{n^2}{m} \right) \left( \frac{h}{2\pi} \right) \frac{4\pi\epsilon_0}{e^2}$$

Bohr radius,  $a_0 = \frac{h^2 \epsilon_0}{\pi m e^2} = 0.53 \text{ \AA}$

Energy for  $n^{\text{th}}$  orbiting electron,  $E_n = -\frac{13.6}{n^2} \text{ eV}$

# CHAPTER-13

## NUCLEI



### Revision Notes

- ▶ As per Rutherford scattering experiment, it is established that radius of atom is  $10^4$  times of its nucleus. Hence, volume of nucleus is  $10^{-12}$  times smaller than atom. This concludes that atom is almost empty.
- ▶ For measuring atomic mass and its sub-particles, new unit of mass is introduced as atomic mass unit 'u'.

$$1 \text{ u} = \frac{\text{Mass of one } {}_6^{12}\text{C atom}}{12}$$

$$1 \text{ u} = 1.660539 \times 10^{-27} \text{ kg}$$

- ▶ Atomic mass unit is not an integral multiple of u due to presence of isotopes (atoms of same element with different atomic masses).
- ▶ All mass and positive charge of an atom is concentrated in its centre known as nucleus.
- ▶ Chadwick discovered a new sub-particle in nucleus known as neutron. It is electrically neutral in nature.

$$\text{Mass of neutron, } m_n = 1.00866 \text{ u} = 1.6749 \times 10^{-27} \text{ kg}$$

- ▶ The composition of a nucleus can now be described using the following terms and symbols :

- $Z$  = Atomic number = Number of protons (equal to the number of electrons)
- $N$  = Neutron number = Number of neutrons
- $A$  = Atomic mass number =  $(Z + N)$  = Total number of protons and neutrons

- ▶ An atom is represented by  ${}_Z^A X$ , where

$X$  = Symbol of element

$A$  = Mass number

$Z$  = Atomic number

- ▶ **Isotopes** : Two atoms of an element having the same atomic number ( $Z$  is same) but different atomic mass number (due to the different number of neutrons) are said to be isotopes.
- ▶ **Isobars** : Two atoms of different elements having the same mass number but different atomic numbers are said to be isobars.
- ▶ **Isotones**: Two atoms of different elements having different mass numbers and atomic numbers such that their difference is same are said to be isotones. It means they have same number of neutrons.
- ▶ **Size of the nucleus** : A nucleus of mass number  $A$  has a radius

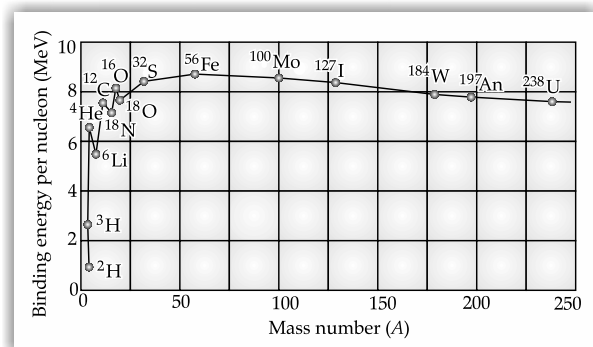
$$R = R_0 A^{1/3} \qquad \text{where, } R_0 = 1.2 \times 10^{-15} \text{ m}$$

- ▶ Nuclear matter density =  $2.3 \times 10^{17} \text{ kgm}^{-3}$
- ▶ Earlier, it was believed that anything in the universe can be classified into matter or radiation. Einstein proposed that there are two forms of energy which are interconvertible.  
 $E = mc^2$ ; where,  $c$  is speed of light.
- ▶ With this relation, we may calculate  $1 \text{ u} = 931.5 \text{ MeV}$
- ▶ **Mass defect** : The difference in mass of a nucleus and its constituents,  $\Delta M$ , is called the mass defect, and is given by,  
 $\Delta M = [Zm_p + (A - Z)m_n] - M$ .
- ▶ **Binding energy** : Binding energy **Nuclear binding energy** of a nucleus is that quantity of energy which when given to nucleus, its nucleons will become free and will leave the nucleus.  
 $E_b = \Delta Mc^2$  where,  $E_b$  is binding energy.

- ▶ **Binding energy per nucleons ( $E_b/A$ )**: A measure of the binding between the constituents of the nucleus is the binding energy per nucleon,  $E_{bn}$  or  $E_b/A$ , which is the ratio of the binding energy  $E_b$  of a nucleus to the number of the nucleons,  $A$ , in that nucleus.

$$E_{bn} = \frac{E_b}{A}$$

► **Relation between  $E_b/A$  and Stability of elements**



- Higher the Binding Energy per Nucleons, more stable is the element. Higher binding energy per nucleon means we have to supply more energy to free nucleons or it is difficult to break the nucleus.



### Key Word

#### Nuclear Binding Energy:

Nuclear binding energy is the minimum amount of energy required to separate the nucleons of a nucleus, or, equivalently, the energy that would be liberated by combining individual nucleons into a single nucleus.

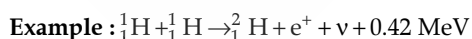
Nucleus is made up of protons and neutrons. But the mass of a nucleus is always less than the sum of the individual masses of the protons and neutrons which constitute it. The difference is a measure of the nuclear binding energy which holds the nucleus together. This binding energy can be calculated from the Einstein relationship,  $E = \Delta mc^2$

- Most of the atoms where atomic mass number are in the range  $30 < A < 200$ , the binding energy per nucleon is fairly constant and quite high. It is maximum for  $A = 56$  about 8.75 MeV.
  - For  $A < 30$  and  $A > 170$ ; Binding energy per nucleon is quite low and remain constant.
- If a nucleus of lower binding energy is converted into higher binding energy, then energy is released.
- There are two methods of converting lower binding energy nucleons into higher binding energy nucleons.
- **Fission** : A heavy nucleus (low binding energy per nucleon) is broken into two lighter nuclei (higher binding energy per nucleon) with the release of energy. This process is known as fission.

#### Example:



- **Fusion** : Two light nucleus (low binding energy per nucleon) are joined to form one nucleus of higher binding energy per nucleon and energy is released. This process is known as fusion.



► **Nuclear force :**

- The binding energy per nucleon is approximately 8 MeV, which is much larger than the binding energy in atoms.
- This high binding energy per nucleon counters the repulsive force between protons and bind both protons and neutrons into the tiny nuclear volume.
- The nuclear force is much stronger than the Coulomb force acting between charges or the Gravitational forces between masses but it's a short range force  $\left( \propto \frac{1}{r^7} \right)$ .



## Mnemonics

**Concept:** Isotope, Isotone and Isobar

**Mnemonics:** Zakir with a topee, found Ansari in a bar singing in nasal tone.



**Interpretations:** Same Z (i.e. same number of protons)    **ISOTOPE**    Same A (i.e. same number of nucleons)    **ISOBAR**    Same number of neutrons    **ISOTONE**

[Isotopes have the same number of protons. Isobars have the same number of nucleons. Isotones have the same number of neutrons.]

## Key Formula

➤ Size of the nucleus:

$$R = R_0 A^{1/3}, \text{ where } R_0 = 1.2 \times 10^{-15} \text{ m}$$

➤ Mass defect :  $\Delta M = [Zm_p + (A - Z)m_n] - M$

➤ Binding energy :  $E_b = \Delta M c^2$

➤ Binding energy per nucleon (BE/N):  $E_{bn} = \frac{E_b}{A}$

## UNIT – IX : ELECTRONIC DEVICES

### CHAPTER-14

## SEMICONDUCTOR ELECTRONICS: MATERIALS, DEVICES AND SIMPLE CIRCUITS

### Topic-1

### Energy Bands

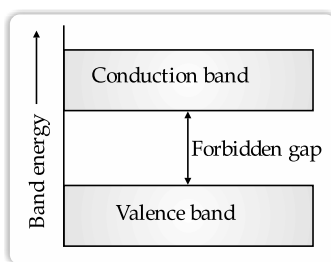
**Concepts Covered** Energy bands in conductors, semiconductors and insulators, intrinsic and extrinsic semiconductors.



## Revision Notes

**Energy bands:**

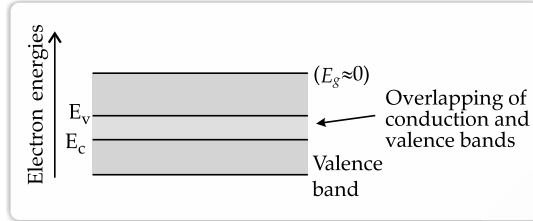
- In crystal, each electron has a different energy level with continuous energy variation.
- **Energy bands** consist of large number of closely spaced energy levels that exist in crystalline materials.
- In solids, there are three important energy bands such as Valence band, Conduction band, forbidden band or forbidden gap.



- The collection of energy levels of free electrons which move freely around the material are called conduction band.
- There is an extra energy required for valence electrons to move to conduction band which is known as forbidden energy.
- The energy associated with forbidden band is known as energy gap which is measured in electron volt (eV) where,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ .
- The collection of energy levels which are partially or wholly filled are known as valence band.
- Materials may be classified as conductors, insulators or semiconductors on the basis of energy band theory.

#### Energy bands in Conductors:

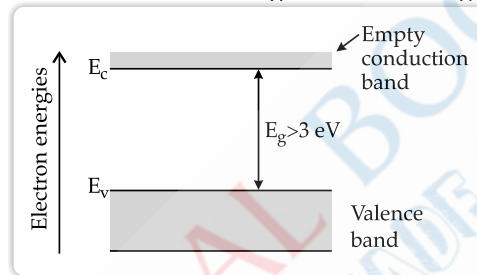
- In conductors, the overlapping of conduction and valence bands without energy gap forms a conduction band.



In this, an electron that receives any acceptable low energy is able to move freely among the bands.

#### Energy bands in Insulators:

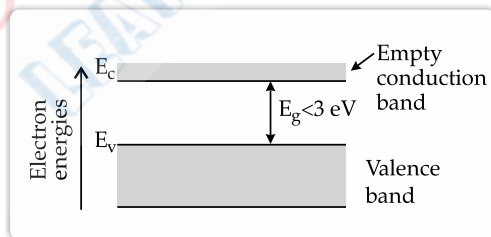
- In insulators, conduction band and valence band have large forbidden energy gap.



- The gap between conduction band and valence band exceeds by 3 eV as electrons that transfer from valence band to conduction band need more energy.
- Due to requirement of more energy, insulators do not conduct any electric current.
- Example of an insulator is diamond with energy gap of around 5.4 eV.

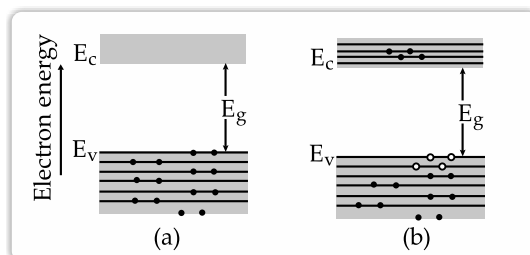
#### Energy bands in Semiconductors:

- Semiconductors, are materials in which, conduction band and valence band are neither overlapped nor have wide gap.
- In such materials, the energy provided by the heat at room temperature is sufficient to lift the electrons from the valence band to the conduction band.



- Semiconductors behave as insulators at 0 K as no electron exist in conduction band.
- Examples of semiconductors are Silicon and Germanium having energy gaps as 1.12 eV and 0.75 eV respectively.
- Intrinsic semiconductors: Pure semiconductors are called intrinsic semiconductors. They can not be used in electronic circuits as their conductivity is low.
- For intrinsic semiconductors, the number of free electrons is equal to the number of holes.

$$n_e = n_h = n_i$$



- ▶ An intrinsic semiconductor (a) at  $T = 0\text{ K}$  behaves like insulators. (b) At  $T > 0\text{ K}$  forms thermally generated electron hole pairs. The filled circles ( $\bullet$ ) represent electrons and empty circles ( $\circ$ ) represent holes.

### Key Word

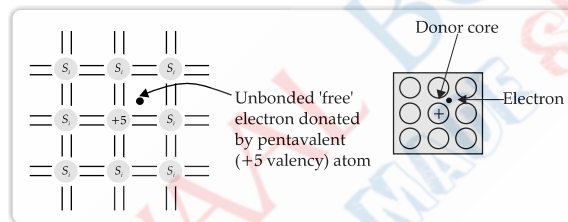
**Energy base gap** is the distance between the valence band and the conduction band. Essentially, the band gap represents the minimum energy required to excite an electron to shift it to conduction band where it can participate in conduction.

This band gap is also known as forbidden band gap.

- ▶ Extrinsic semiconductors: When a small fixed amount of charged impurity is mixed with intrinsic semiconductors, they become extrinsic.
- ▶ In extrinsic semiconductors:
  - (i) Conductivity increases
  - (ii) Conductivity is controlled by doping carriers
- ▶ In extrinsic semiconductors, the number of free electrons is not equal to number of holes.
 
$$n_e \neq n_h$$
- ▶ Doping is adding impurities to intrinsic semiconductors crystal lattice so as to increase the number of carriers.
- ▶ For raising electrical conductivity, semiconductors are mixed with either pentavalent impurity such as Antimony (Sb), Arsenic (As) and Phosphorus (P) or trivalent impurity such as Indium (In), Gallium (Ga) or Boron (B).

#### ***n*-type Semiconductors:**

- ▶ If Phosphorous with 5 valence-band electrons is added, it will give an extra  $e^-$  which will freely move around and leave a positively charged nucleus.



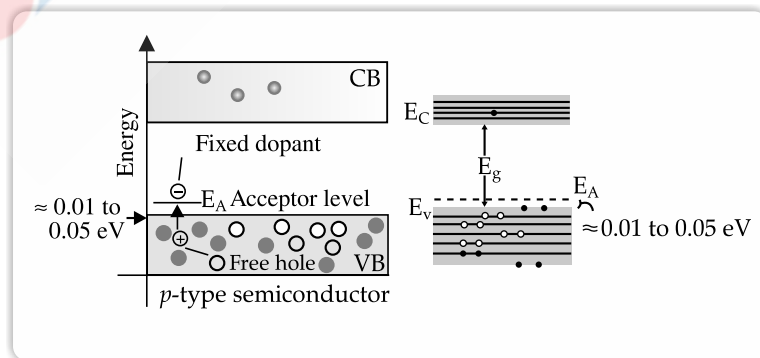
- ▶ The crystal is electrically neutral, known as "*n*-type" material with negative carriers where concentration of donor atoms is  $10^{15}\text{ cm}^{-3} \sim 10^{20}\text{ cm}^{-3}$  having mobility  $\mu_n \approx 1350\text{ m}^2/\text{Vs}$ .

Energy Diagram of *n*-Type Semiconductor:

- ▶ On doping a semiconductor with pentavalent impurity like Antimony (Sb) or Arsenic (As), extrinsic semiconductor so obtained is known as *n*-type.
- ▶ *n*-type semiconductor has large number of free electrons known as majority (charge) carriers and small number of holes known as minority (charge) carriers.

$$n_e \gg n_h$$

- ▶ Impurity atom in *n*-type semiconductor is called donor as it generates new energy level below the conduction band, known as  $E_D$ .

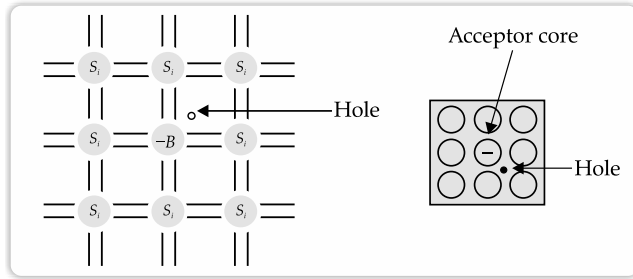


**Energy bands for *p*-type semiconductor at  $T > 0\text{ K}$**

#### ***p*-type Semiconductors:**

- ▶ If Boron atom with 3 valence band electrons is added, it accepts  $e^-$  and give extra holes ( $h^+$ ) to move freely which leaves behind negatively charged nucleus.





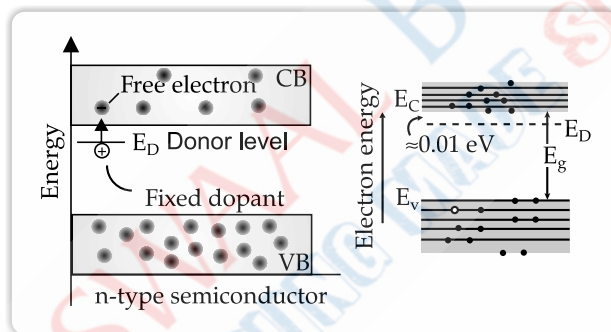
- ▶ The crystal is electrically neutral known as “p-type” silicon in which concentration of acceptor atoms  $\sim 10^{28} \text{ cm}^{-3}$  where hole movement needs breaking of bond thereby giving low mobility, where,  $\mu_p \approx 500 \text{ m}^2/\text{Vs}$ .

**Energy Band Diagram of p-Type Semiconductor**

- ▶ On doping a semiconductor with trivalent impurity like Indium (In) or Gallium (Ga), extrinsic semiconductor so obtained is known as p-type.
- ▶ p-type semiconductor has large number of holes known as majority (charge) carriers where number of free electrons is less known as minority (charge) carriers.

$$n_h \gg n_e$$

- ▶ Impurity atom in p-type semiconductor is known as acceptor atom.
- ▶ In p-type, extra holes in band gap allow excitation of valence band electrons which leaves mobile holes in valence band.
- ▶ Large number of holes in covalent bond is created in crystal with trivalent impurity.
- ▶ In extrinsic semiconductors  $n_e \neq n_h$  but  $n_e \cdot n_h = n_i^2$



**Energy bands for n-type semiconductor at  $T > 0 \text{ K}$**

- ▶ **Energy band:** Range of energies which an electron may possess in an atom.
- ▶ **Valence Band:** Range of energy levels possessed by valence electrons.
- ▶ **Conduction Band:** Range of energy levels possessed by conductive (free) electrons.
- ▶ **Forbidden Band:** Energy band in between the conduction band and valence band.

**Topic-2**

**Semiconductor Diodes and Applications**

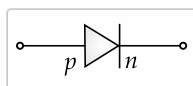
**Concepts Covered** Semiconductor diode and its I-V characteristics in forward and reverse bias, Diode as a rectifier.



**Revision Notes**

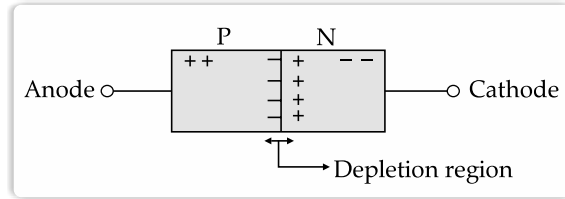
**Diode**

- ▶ Diode is an electronic device consisting of a junction of semiconductors p-type and n-type. It is represented as:



**Semiconductor diode**

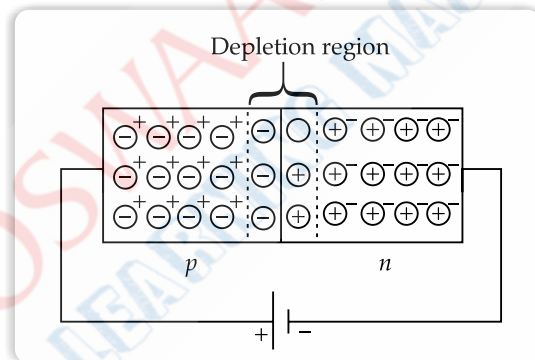
- ▶ Semiconductor diodes were first semiconductor electronic devices which is basically a p-n junction.



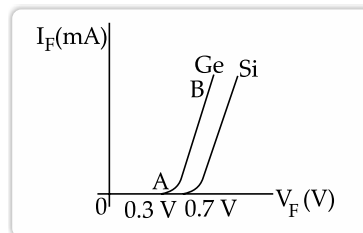
- ▶ When a  $p$ -type semiconductor material is suitably joined to  $n$ -type semiconductor, the contact surface is called a  $p$ - $n$  junction.
- ▶ It is an electrical device that allows current only in one direction. The direction of arrow is the direction of current when it is forward biased.
- ▶ At junction, electrons and holes diffuse to form the diffusion current.
- ▶ A  $p$ - $n$  junction layer is also called the depletion layer. Potential barrier is created at junction due to diffusion current. It acts as a barrier for majority charge carriers.
- ▶ The potential barrier helps the minority charge carriers to flow. A drift current is formed which is opposite in direction of the diffusion current.
- ▶ Under equilibrium condition, diffusion current is equal to the drift current and net current is zero as both are in opposite direction.
- ▶ There are many types of semiconductor diodes such as: Avalanche diodes, Gunn diodes, Light Emitting Diodes (LED), Photodiodes, etc.
- ▶ Semiconductor diode can be made either from Silicon or Germanium and each differs in size and properties.

#### Forward Bias

- ▶ When an external voltage is applied, where negative terminal of battery is connected to  $n$ -side while positive terminal of battery is connected to  $p$ -side, the barrier potential and the width of the depletion layer get reduced and more current can flow across the junction. Thus the resistance is reduced.



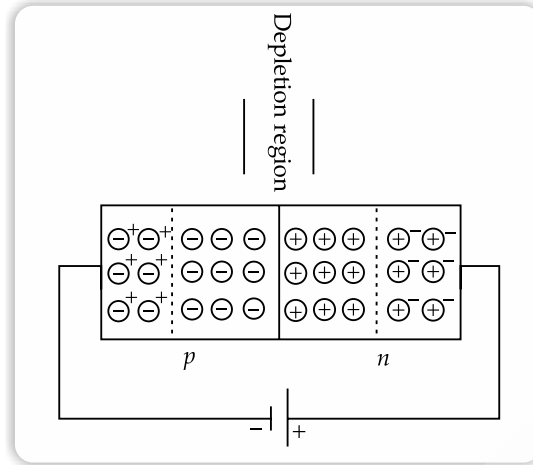
- ▶ The positive terminal of battery repels majority carriers, holes in  $p$ -region while negative terminal repels electrons in  $n$ -region which pushes them towards the junction.
- ▶ Here, an increase in concentration of charge carriers near the junction is observed, where recombination takes place thereby reducing width of depletion region.



- ▶ Due to rise in forward bias voltage, depletion region continue to reduce its width, which results in more and more recombination process.

#### Reverse Bias

- ▶ If an external voltage is applied in reverse direction where positive terminal of battery is connected to  $n$ -side while negative terminal of battery is connected to  $p$ -side, then barrier potential and width of depletion region will increase and minority charge carriers will flow across junction.



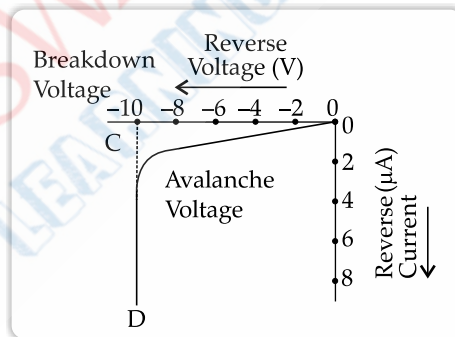
- In this, the current will be quite small and is independent of external voltage.
- Beyond certain voltage, diode will break down with avalanche breakdown mechanism or zener breakdown mechanism.
- Here, negative terminal of battery will attract majority charge carriers, holes in  $p$ -region and positive terminal attracts electrons in  $n$ -region which pulls them away from the junction.
- As a result of this, there will be decrease in concentration of charge carriers near junction which increases the width of depletion region.

**Key Word**

**Diode:**

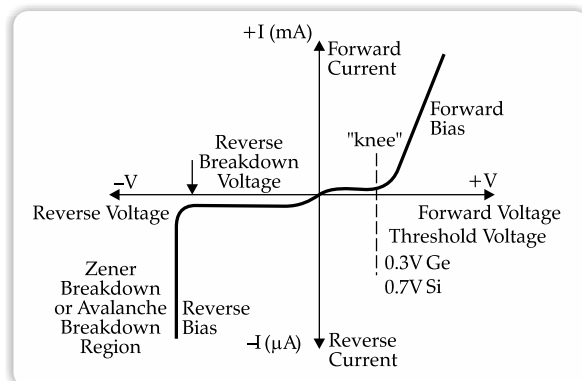
Diode is an electrical component which allows current to flow only in one direction.

The most common type of diode is a  $p$ - $n$  junction. An intrinsic semiconductor is doped with acceptor impurity from one side and  $n$ -type impurity from other side. Thus, a  $p$ - $n$  junction is formed.



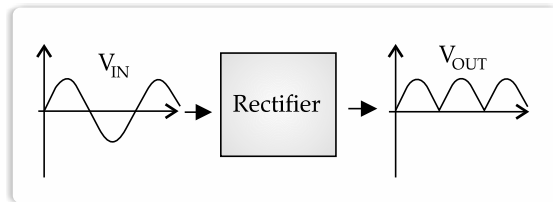
- A small amount of current flows due to minority carriers known as reverse saturation current or leakage current and with rise in reverse bias voltage, depletion region continues to increase in width without any increase in flow of current. After a certain reverse voltage, known as break down voltage, reverse current will increase drastically.

**$V$ - $I$  Characteristics of Diode**

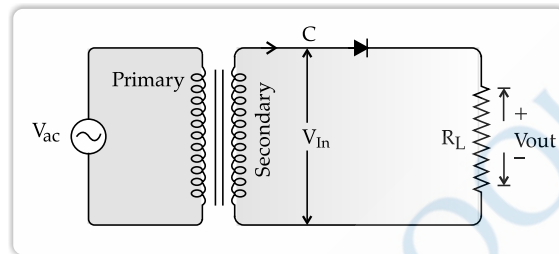


**Diode as rectifier**

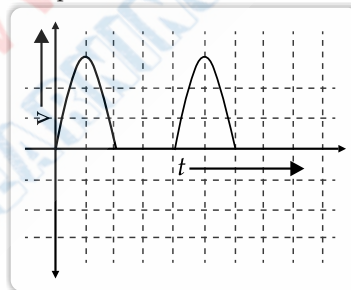
- Rectifier is a circuit which converts AC supply into unidirectional DC supply.



- With rectification, alternating current (AC) gets converted to direct current (DC).
- There are two types of rectifier: half-wave and full-wave.

**Half-wave rectifier**

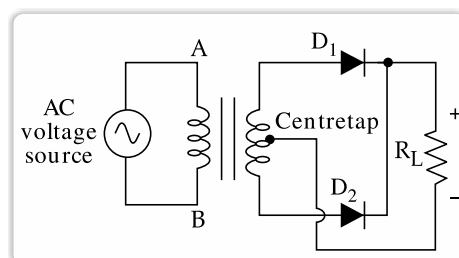
- The half-wave rectifier with single diode, allows current to flow in one direction.
- Here, AC power source  $V_{ac}$  is connected to primary side of transformer, while secondary terminals of transformer are connected to diode and a load resistor in series.
- If  $V_{ac}$  is in positive cycle, a positive voltage is produced on secondary side of transformer.
- The positive voltage forward biases the diode and diode starts passing the current. As a result of which the voltage becomes available across the load.
- If  $V_{ac}$  is in negative cycle, then secondary side have negative voltage where diode is reverse biased and does not pass any current.
- Voltage waveform across load resistor is shown below, where positive side of sinusoidal cycle is present while negative side of sinusoidal cycle has been clamped off.

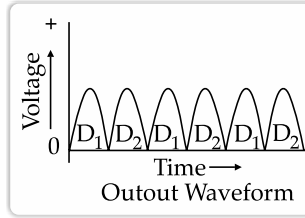


- The output voltage  $V_{dc}$  is not exactly similar to the output of a battery.
- The output of half-wave rectifier is bumpy as only one half of input AC is rectified.

**Full-wave rectifier**

- For rectifying AC input for both half cycles full wave rectifier is used.





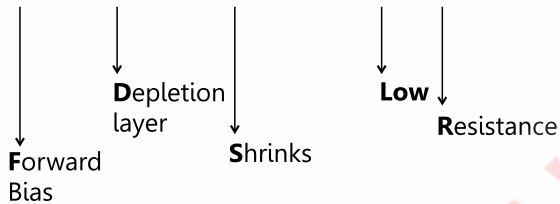
- A simple kind of full-wave rectifier uses centre tap transformers with two diodes.
- In full wave rectifier, for positive half cycle, A is +ve, B is -ve, then only top diode conducts, while bottom diode blocks the current. For negative half cycle, A is -ve and B is +ve, only bottom diode conducts while the top diode blocks the current.



## Mnemonics

Concept : Forward biased  $p-n$  Junction.

**Football deserves Stamina and low risk.**

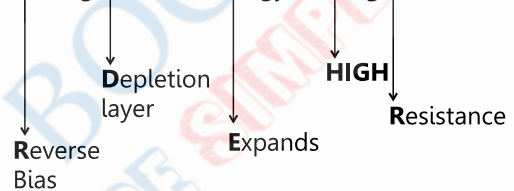


In Forward Biased  $p-n$  Junction:

- (i) Depletion layer shrinks
- (ii) Resistance is low

Concepts : Reverse biased  $p-n$  Junction.

**Rafting deserves Energy and high risk.**



In Reverse Biased  $p-n$  Junction:

- (i) Depletion layer expands
- (ii) Resistance is high