

CBSE

Solved Paper 2023

Mathematics

Class-12th

(Delhi & Outside Delhi Sets)

Time : 3 Hours

Max. Marks : 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

- (i) This question paper contains 38 questions. All questions are compulsory.
- (ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.
- (iii) In Section A, Questions Number 1 to 18 are Multiple Choice Questions (MCQs) type and Questions Number 19 and 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B, Questions Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.
- (v) In Section C, Questions Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
- (vi) In Section D, Questions Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
- (vii) In Section E, Questions Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is NOT allowed.

Delhi Set-I

65/5/1

SECTION — A

(Multiple Choice Questions)

Each question carries 1 mark.

Section the correct option out of the four given options:

1. Let $A = \{3, 5\}$. Then number of reflexive relations of A is:
(a) 2 (b) 4
(c) 0 (d) 8
2. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to:
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$
3. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals :
(a) A (b) $A + I$
(c) $I - A$ (d) $A - I$
4. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals:
(a) ± 1 (b) -1
(c) 1 (d) 2
5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is:
(a) 1 (b) 2
(c) 3 (d) 4

6. The derivative of x^{2x} w.r.t. x is:
 (a) x^{2x-1} (b) $2x^{2x} \log x$
 (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$
7. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x , is continuous at:
 (a) $x = 1$ (b) $x = 1.5$
 (c) $x = -2$ (d) $x = 4$
8. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to:
 (a) x (b) $-x$
 (c) $16x$ (d) $-16x$
9. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is:
 (a) $(-1, \infty)$ (b) $(-2, -1)$
 (c) $(-\infty, -2)$ (d) $(-1, 1)$
10. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals:
 (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$
 (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$
11. $\int_{-1}^1 \frac{|x-2|}{x-2} dx$, $x \neq 2$ is equal to:
 (a) 1 (b) -1
 (c) 2 (d) -2
12. The sum of the order and the degree of the differential equation $\frac{d}{dx} \left(\left(\frac{dy}{dx} \right)^3 \right)$ is:
 (a) 2 (b) 3
 (c) 5 (d) 0
13. Two vector $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ are collinear if:
 (a) $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
 (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$
14. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is:
 (a) 1 (b) 5
 (c) 7 (d) 23
15. If a line makes angles of 90° , 135° and 45° with the x , y and z axes respectively, then its direction cosines are:
 (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$
 (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is:
 (a) 0° (b) 30°
 (c) 45° (d) 90°
17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equals to:
 (a) $\frac{1}{10}$ (b) $\frac{1}{8}$
 (c) $\frac{7}{8}$ (d) $\frac{17}{20}$
18. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is:
 (a) $\frac{27}{32}$ (b) $\frac{5}{32}$
 (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

Assertion-Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
 (c) (A) is true and (R) is false.
 (d) (A) is false, but (R) is true.

19. **Assertion (A):** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R): Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

20. **Assertion (A):** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx = 3$

Reason (R): $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

SECTION — B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Write the domain and range (principle value branch) of the following functions:

$$f(x) = \tan^{-1} x$$

22. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

- (b) Find the value(s) of ' λ ', if the function

$$f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0.$$

23. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y -axis. Hence, obtain its area using integration.

24. (a) If the vectors \vec{a} and \vec{b} are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between \vec{a} and \vec{b} .

OR

- (b) Find the area of a parallelogram whose adjacent side are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

25. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, -1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

SECTION — C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

27. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$.

28. (a) Evaluate: $\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$

OR

(b) Find: $\int \frac{x^4}{(x-1)(x^2+1)} dx$

29. Find the area of the following region using integration:

$$\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$$

30. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

OR

(b) Three vectors \vec{a} , \vec{b} and \vec{c} satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

31. Find the distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

SECTION — D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

33. Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

34. Solve the following Linear Programming Problem graphically :

Maximize: $P = 70x + 40y$

Subject to: $3x + 2y \leq 9$,

$3x + y \leq 9$

$x \geq 0, y \geq 0$

35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the

probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who

guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the

answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

SECTION — E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub-parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories-Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions:

- (I) How many relations are possible from B to G ?
 (II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
 (III) Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation.

OR

- (III) A function $f : B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.
 Check if f is bijective. Justify your answer.

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions:

- (I) Convert the given above situation into a matrix equation of the form $AX = B$.
 (II) Find $|A|$.
 (III) Find A^{-1} .

OR

- (III) Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions:

- (I) Show that $(x^2 - y^2) dx + 2xy dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.
 (II) Solve the above equation to find its general solution.

Delhi Set-II

65/5/2

Note: Except these, all other questions are from Delhi Set-I

SECTION — A

4. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then A^2 is:

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is:

- (a) 10
(c) 7

- (b) 8
(d) -7

9. The function $f(x) = x|x|$, $x \in \mathbb{R}$ is differentiable

- (a) only at $x = 0$
(c) in \mathbb{R}

- (b) only at $x = 1$
(d) in $\mathbb{R} - \{0\}$

11. The value of $\int_0^{\pi/4} (\sin 2x) dx$ is:

- (a) 0
(c) $\frac{1}{2}$

- (b) 1
(d) $-\frac{1}{2}$

14. A unit vector \hat{a} makes equal but acute angles on the co-ordinate axes. The projection of the vector \hat{a} on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is:

- (a) $\frac{11}{15}$
(c) $\frac{4}{5}$

- (b) $\frac{11}{5\sqrt{3}}$
(d) $\frac{3}{5\sqrt{3}}$

18. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P\left(\frac{B'}{A}\right)$ is:

- (a) $\frac{1}{4}$
(c) $\frac{3}{4}$

- (b) $\frac{1}{8}$
(d) 1

21. Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$.

25. Find the angle between the following two lines :

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k});$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

SECTION — C

26. Using determinants, find the area of ΔPQR with vertices $P(3, 1)$, $Q(9, 3)$ and $R(5, 7)$. Also, find the equation of line PQ using determinants.

28. (a) Evaluate: $\int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1 + \cos 2x} dx$

OR

(b) Find: $\int e^{x^2} (x^5 + 2x^3) dx$

29. Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

SECTION — D

32. Evaluate: $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

SECTION — A

1. Let R be a relation in the set N given by

$$R = \{(a, b) : a = b - 2, b > 6\}$$

Then

(a) $(8, 7) \in R$

(b) $(6, 8) \in R$

(c) $(3, 8) \in R$

(d) $(2, 4) \in R$

2. If
- $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$
- and
- $A = A^T$
- , where
- A^T
- is the transpose of the matrix A, then

(a) $x = 0, y = 5$

(b) $x = y$

(c) $x + y = 5$

(d) $x = 5, y = 0$

6. If
- $f(x) = |\cos x|$
- , then
- $f\left(\frac{3\pi}{4}\right)$
- is:

(a) 1

(b) -1

(c) $\frac{-1}{\sqrt{2}}$

(d) $\frac{1}{\sqrt{2}}$

9. The function
- $f(x) = x^3 + 3x$
- is increasing in interval

(a) $(-\infty, 0)$

(b) $(0, \infty)$

(c) \mathbb{R}

(d) $(0, 1)$

12. The order and the degree of the differential equation
- $\left(1 + 3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$
- respectively are:

(a) $1, \frac{2}{3}$

(b) 3, 1

(c) 3, 3

(d) 1, 2

13. If
- $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$
- , then
- \vec{a}
- is:

(a) \hat{k}

(b) \hat{i}

(c) \hat{j}

(d) $\hat{i} + \hat{j} + \hat{k}$

SECTION — B

21. (a) Find the value of k for which the function f given as

$$f(x) = \begin{cases} \frac{1 - \cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0$$

OR

- (b) If
- $x = a \cos t$
- and
- $y = b \sin t$
- , then find
- $\frac{d^2y}{dx^2}$
- .

22. Find the value of
- $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1}1$
- .

SECTION — C

26. Show that the determinant
- $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$
- is independent of
- θ
- .

27. Using integration, find the area of the region bounded by
- $y = mx (m > 0)$
- ,
- $x = 1$
- ,
- $x = 2$
- and the x-axis.

28. (a) Find the coordinates of the foot of the perpendicular drawn from point
- $(5, 7, 3)$
- to the line
- $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
- .

OR

- (b) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

SECTION — D

32. Solve the following Linear Programming Problem graphically:

$$\text{Minimise : } Z = 60x + 80y$$

Subject to constraints:

$$3x + 4y \geq 8$$

$$5x + 2y \geq 11$$

$$x, y \geq 0$$

Outside Delhi Set-I

65/2/1

SECTION — A

(Multiple Choice Questions)

Each question carries 1 mark.

1. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to:

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

2. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to:

(a) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

3. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is:

(a) \mathbb{R}

(b) $\{0\}$

(c) $\{4\}$

(d) $\mathbb{R} - \{4\}$

4. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is:

(a) 1

(b) -1

(c) 2

(d) 0

5. If $\frac{d}{dx} [f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to:

(a) $a + b$

(b) $\frac{ax^2}{2} + bx$

(c) $\frac{ax^2}{2} + bx + c$

(d) b

6. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is:
- (a) 2 (b) 1
(c) not defined (d) 0
7. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay$, $(-1 < y < 1)$ is:
- (a) $\frac{1}{y^2 - 1}$ (b) $\frac{1}{\sqrt{y^2 - 1}}$
(c) $\frac{1}{1 - y^2}$ (d) $\frac{1}{\sqrt{1 - y^2}}$
8. Unit vector along \vec{PQ} , where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$ is:
- (a) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (b) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
(c) $\frac{-2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$ (d) $\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$
9. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is:
- (a) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (b) $4\hat{i} + \hat{j} - 2\hat{k}$
(c) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (d) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$
10. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is:
- (a) 0 (b) 12
(c) $\frac{12}{\sqrt{13}}$ (d) $\frac{-12}{\sqrt{13}}$
11. Equation of a line passing through point $(1, 1, 1)$ and parallel to z-axis is:
- (a) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$
(c) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (d) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$
12. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is:
- (a) $\frac{1}{9}$ (b) $\frac{4}{9}$
(c) $\frac{1}{18}$ (d) $\frac{1}{2}$
13. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is:
- (a) $\sec^2\left(\frac{\pi}{4} - x\right) + c$ (b) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$
(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$
14. If (a, b) , (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC , then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to:
- (a) $2\Delta^2$ (b) $4\Delta^2$
(c) 2Δ (d) 4Δ

15. The function $f(x) = x|x|$ is:
- (a) continuous and differentiable at $x = 0$. (b) continuous but not differentiable at $x = 0$.
 (c) differentiable but not continuous at $x = 0$. (d) neither differentiable nor continuous at $x = 0$.
16. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to:
- (a) $\frac{-y}{x}$ (b) $\frac{y}{x}$
 (c) $\sec^2\left(\frac{y}{x}\right)$ (d) $-\sec^2\left(\frac{y}{x}\right)$
17. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ?
- (a) $a = 9, b = 1$ (b) $a = 5, b = 2$
 (c) $a = 3, b = 5$ (d) $a = 5, b = 3$
18. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to:
- (a) 40 (b) 96
 (c) 120 (d) 136

Assertion-Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
 (c) (A) is true and (R) is false.
 (d) (A) is false, but (R) is true.
19. **Assertion (A):** Maximum value of $(\cos^{-1} x)^2$ is π^2 .
- Reason (R):** Range of the principal value branch of $\cos^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.
20. **Assertion (A):** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.
- Reason (R):** The sum of squares of the direction cosines of a line is 1.

SECTION — B

21. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$.

OR

- (b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also write its range.
22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x -coordinate 1, y -coordinate is changing twice as fast as x -coordinate. Find the value of a .
23. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between \vec{a} and $\vec{b} - \vec{c}$.
24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.
25. (a) If $y = \sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$.
- (b) If $f(x) = \begin{cases} ax+b & ; 0 < x \leq 1 \\ 2x^2-x & ; 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b .

SECTION — C

26. (a) Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$.

OR

(b) Find $\int \frac{dx}{\sqrt{\sin^3 \cos(x - \alpha)}}$

27. Find $\int e^{\cot^{-1} x} \left(\frac{1 - x + x^2}{1 + x^2} \right) dx$.

28. Evaluate $\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$

29. (a) Find the general solution of the differential equation:
 $(xy - x^2) dy = y^2 dx$.

OR

(b) Find the general solution of the differential equation:
 $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$

30. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

31. Solve the following linear programming problem graphically :

Minimize : $Z = 5x + 10y$

subject to constraints : $x + 2y \leq 120$
 $x + y \geq 60$
 $x - 2y \geq 0$
 $x \geq 0, y \geq 0$.

SECTION — D

32. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations:

$$\begin{aligned} x - 2y &= 3 \\ 2x - y - z &= 2 \\ -2y + z &= 3 \end{aligned}$$

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$

33. (a) Find the equation of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q(5, -3, 1), R(12, 4, 5) and S(11, 9, -2) Use these equation to find the point of intersection of diagonals.

OR

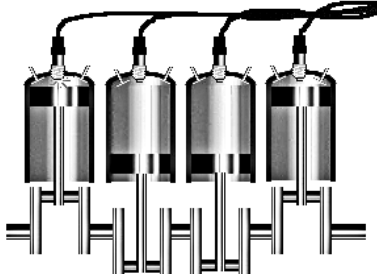
(b) A line l passes through point $(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l . Hence, obtain its distance from origin.

34. Using integration, find the area of region bounded by line $y = \sqrt{3x}$, the curve $y = \sqrt{4 - x^2}$ and y -axis in first quadrant.

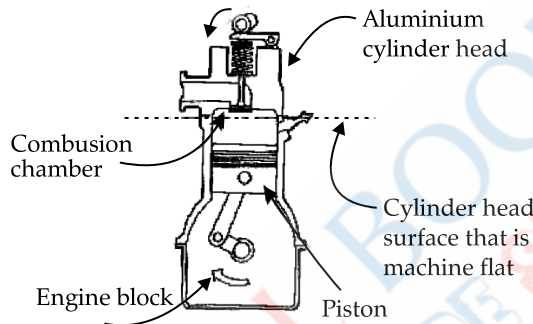
35. A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$

SECTION — E

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



One complete cycle of a four-cylinder four stroke engine. The volume displaced is marked



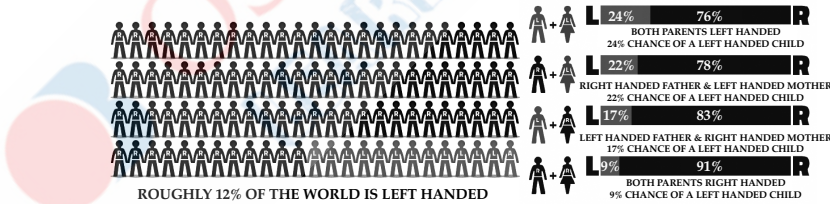
The cylinder bore in the form of circular cylinder open at the top is made from a metal sheet of area $75\pi \text{ cm}^2$. Based on the above information, answer the following questions:

- (i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r .
- (ii) Find $\frac{dV}{dr}$
- (iii) (a) Find the radius of cylinder when its volume is maximum.

OR

- (b) For maximum volume, $h > r$. State true or false and justify.

37. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

- A : When both father and mother are left handed:
Chances of left handed child is 24%.
- B : When father is right handed and mother is left handed:
Chances of left handed child is 22%.
- C : When father is left handed and mother is right handed:
Chances of left handed child is 17%.
- D : When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following question:

- (i) Find $P\left(\frac{L}{C}\right)$
- (ii) Find $P\left(\frac{\bar{L}}{A}\right)$

(iii) (a) Find $P\left(\frac{A}{L}\right)$

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

38. The use of electric vehicles will curb air pollution in the long run.



The use electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and $t = 1, 2, 3, \dots$ corresponds to year 2001, 2002, 2003, respectively.

Based on the above information, answer the following questions:

- (i) Can the above function be used to estimate number of vehicles in the year 2000? Justify.
 (ii) Prove that the function $V(t)$ is an increasing function.

Outside Delhi Set-II

65/2/2

Note: Except these, all other questions are from Outside Delhi Set - I

SECTION — A

1. If $\frac{d}{dx}f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is:
- (a) $x^2 + 3 \log |x| + 1$ (b) $x^2 + 3 \log |x|$
 (c) $2 - \frac{3}{x^2}$ (d) $x^2 + 3 \log |x| - 4$
5. If in $\triangle ABC$, $\vec{BA} = 2\vec{a}$ and $\vec{BC} = 3\vec{b}$, then \vec{AC} is:
- (a) $2\vec{a} + 3\vec{b}$ (b) $2\vec{a} - 3\vec{b}$
 (c) $3\vec{b} - 2\vec{a}$ (d) $-2\vec{a} - 3\vec{b}$
6. If $|\vec{a} \times \vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = -3$, then angle between \vec{a} and \vec{b} is:
- (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$
 (c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{6}$
7. Equation of line passing through origin and making 30° , 60° and 90° with x , y , z axes respectively is
- (a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$ (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$
 (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$ (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$
8. If A and B are two events such that $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$ and $P(A) + P(B) = \frac{2}{3}$, then $P(B)$ is equal to
- (a) $\frac{2}{9}$ (b) $\frac{7}{9}$
 (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

15. If A is a 2×3 matrix such that AB and AB' both are defined, then order of the matrix B is
- (a) 2×2 (b) 2×1
(c) 3×2 (d) 3×3

SECTION — B

23. If the equation of a line is $x = ay + b, z = cy + d$, then find the direction ratios of the line and a point on the line.
25. If the circumference of circle is increasing at the constant rate, prove that rate of change of area is directly proportional to its radius.

SECTION — C

29. Solve the following linear programming problem graphically:

$$\text{Maximize : } Z = x + 2y$$

$$\text{subject to constraints : } x + 2y \geq 100,$$

$$2x - y \leq 0,$$

$$2x + y \leq 200,$$

$$x \geq 0, y \geq 0.$$

30. (a) Evaluate $\int_{-1}^1 |x^4 - x| dx$.

OR

(b) Find $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$.

31. Find $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$

33. Using Integration, find the area of triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$.

Outside Delhi Set-III

65/2/3

Note: Except these, all other questions are from Outside Delhi Set - I & Set - II

SECTION — A

1. If the angle between \vec{a} and \vec{b} is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is:
- (a) 9 (b) 3
(c) $\frac{1}{9}$ (d) $\frac{1}{3}$
2. The position vector of three consecutive vertices of a parallelogram ABCD are $A(4\hat{i} + 2\hat{j} - 6\hat{k})$, $B(5\hat{i} - 3\hat{j} + \hat{k})$ and $C(12\hat{i} + 4\hat{j} + 5\hat{k})$. The position vector of D is given by
- (a) $-3\hat{i} - 5\hat{j} - 10\hat{k}$ (b) $21\hat{i} + 3\hat{j}$
(c) $11\hat{i} + 9\hat{j} - 2\hat{k}$ (d) $-11\hat{i} - 9\hat{j} + 2\hat{k}$
3. If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to
- (a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) $\frac{2}{5}$ (d) $\frac{2}{3}$
4. If $\int_0^{2\pi} \cos^2 x dx = k \int_0^{\pi/2} \cos^2 x dx$, then the value of k is
- (a) 4 (b) 2
(c) 1 (d) 0
10. Number of symmetric matrices of order 3×3 with each entry 1 or -1 is
- (a) 512 (b) 64
(c) 8 (d) 4

18. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is

1

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

(b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

(c) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$

(d) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

SECTION — B

21. If points A, B and C have position vectors $2\hat{i}$, \hat{j} and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.

23. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/s, how fast is the area of triangle increasing at an instant when all sides become equal ?

SECTION — C

26. Solve the following Linear Programming problem graphically:

Maximize: $Z = 3x + 3.5y$

subject to constraints: $x + 2y \geq 240$,

$3x + 1.5y \geq 270$,

$1.5x + 2y \leq 310$,

$x \geq 0, y \geq 0$.

27. (a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx$

OR

(b) Evaluate $\int_{-a}^a f(x) dx$, where $f(x) = \frac{9^x}{1+9^x}$

31. (a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X.

OR

(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not.

SECTION — D

35. Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration.

■ ■

ANSWERS

Delhi Set-I

65/5/1

SECTION — A

1. Option (b) is correct

Explanation: The number of reflexive relations is $2^{n(n-1)}$

$$\Rightarrow 2^{2(2-1)} = 4$$

2. Option (a) is correct

Explanation: $\sin \left[\frac{\pi}{3} + \sin^{-1} \left(\frac{1}{2} \right) \right]$

$$\sin \left(\frac{\pi}{3} + \frac{\pi}{6} \right) = \sin \frac{\pi}{2} = 1$$

3. Option (c) is correct

Explanation:

$$A^2 - A + I = 0$$

$$A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$$

$$IA - I + A^{-1} = 0$$

$$A^{-1} = I - A$$

4. Option (c) is correct

Explanation:

$$A = B^2$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

$$x^2 = 1 \text{ and } x + 1 = 2$$

$$\therefore x = \pm 1$$

$$\therefore x = 1$$

Hence $x = 1$

5. Option (d) is correct

Explanation:

$$\begin{bmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \alpha(2-4) - 3(1-1) + 4(4-2) = 0$$

$$-2\alpha + 8 = 0$$

$$\alpha = 4$$

6. Option (c) is correct

Explanation:

$$y = x^{2x}$$

$$\log y = 2x \log x$$

$$\frac{d}{dx} \log y = \frac{d}{dx} 2x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \left[x \frac{d}{dx} \log x + \log x \frac{d}{dx} x \right]$$

$$\frac{dy}{dx} = 2y \left[x \times \frac{1}{x} + \log x \right]$$

$$\frac{dy}{dx} = 2x^{2x} [1 + \log x]$$

7. Option (b) is correct

Explanation: The function $f(x) = [x]$ is continuous for all except all integral values of x .

8. Option (d) is correct

Explanation:

$$x = A \cos 4t + B \sin 4t$$

$$\frac{dx}{dt} = -4A \sin 4t + 4B \cos 4t$$

$$\frac{d^2x}{dt^2} = -16A \cos 4t - 16B \sin 4t$$

$$= -16(A \cos 4t + B \sin 4t)$$

$$\therefore \frac{d^2x}{dt^2} = -16x$$

9. Option (b) is correct

Explanation:

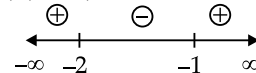
$$f(x) = 2x^3 + 9x^2 + 12x - 1$$

$$f'(x) = 6x^2 + 18x + 12 - 0$$

for decreasing function $f'(x) < 0$

$$6(x^2 + 3x + 2) < 0$$

$$6(x+2)(x+1) < 0$$



$f(x)$ is decreasing in interval $(-2, -1)$

10. Option (b) is correct

Explanation:

$$\int \frac{\sec x}{\sec x - \tan x} dx$$

$$\int \frac{\sec x (\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)} dx$$

$$\int \sec^2 x dx + \int \sec x \tan x dx \quad [\sec^2 x - \tan^2 x = 1]$$

$$\tan x + \sec x + c$$

11. Option (d) is correct

Explanation:

$$\int_{-1}^1 \frac{|x-2|}{x-2} dx$$

$$\int_{-1}^1 \frac{-(x-2)}{x-2} dx = [-x]_{-1}^1 = -2$$

12. Option (b) is correct

Explanation:

$$\frac{d}{dx} \left[\left(\frac{dy}{dx} \right)^3 \right] = 3 \left(\frac{dy}{dx} \right)^2 \frac{d^2y}{dx^2}$$

order is 2 and degree is 1

\therefore required answer $2 + 1 = 3$

13. Option (b) is correct*Explanation:*

$$\text{Two vectors } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\text{are collinear if } \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$$

14. Option (c) is correct*Explanation:*

$$\vec{a} = 6\hat{i} - 2\hat{j} + 3\hat{k}$$

$$|\vec{a}| = |\sqrt{6^2 + 2^2 + 3^2}| = 7 \text{ units}$$

15. Option (a) is correct*Explanation:* Direction cosines are $\cos 90^\circ$, $\cos 135^\circ$ and $\cos 45^\circ$

$$\therefore \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

16. Option (d) is correct*Explanation:*

$$2x = 3y = -z \quad 6x = -y = -4z$$

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \quad \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

$$\cos \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos \theta = \frac{3 \times 2 + 2(-12) + (-6)(-3)}{\sqrt{3^2 + 2^2 + 6^2} \sqrt{2^2 + (-12)^2 + (-3)^2}}$$

$$\cos \theta = \frac{6 - 24 + 18}{7 \cdot \sqrt{157}} = 0$$

$$\therefore \theta = \frac{\pi}{2} = 90^\circ$$

17. Option (c) is correct*Explanation:*

$$P(A) = \frac{4}{5}, P(A \cap B) = \frac{7}{10}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}}$$

$$= \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

18. Option (c) is correct*Explanation:*

Probability of the event that at least one head comes up

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$

19. Option (a) is correct*Explanation:*

$$S = \{(HH), (H, T), (T, H), (T, T)\}$$

Probability of getting two heads; $\{H, H\}$

$$n(F) = \frac{1}{4}$$

Probability of getting at least one head:

$$\{(H, H), (H, T), (T, H)\}, n(E) = \frac{3}{4}$$

Required probability $P\left(\frac{F}{E}\right)$

$$= \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

20. Option (a) is correct*Explanation:*

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx$$

$$= \int_2^8 \frac{\sqrt{10-10+x}}{\sqrt{10-x} + \sqrt{10-10+x}} dx$$

$$= \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x} + \sqrt{x}} dx$$

$$2I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x} + \sqrt{x}} dx + \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x} + \sqrt{x}} dx$$

$$2I = \int_2^8 dx = [x]_2^8 = 6$$

$$I = 3$$

SECTION — B**21.**

$$f(x) = \tan^{-1} x$$

Domain = Real number

$$\text{Range} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

22. (a)

$$f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$$

 $f(x)$ is defined at $x = 1$ and $f(1) = 1$

$$f'_-(1) = \lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$$

$$= \frac{1+h-1}{h} = 1$$

$$f'_+(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2+h)}{h}$$

$$f'_-(1) \neq f'_+(1)$$

Hence $f(x)$ is not differentiable at $x = 1$

OR

$$(b) f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0.$$

$$f(0) = 1$$

$$f(x) = \lim_{x \rightarrow 0^-} \frac{\sin^2 \lambda(0-h)}{(0-h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{\sin^2 \lambda h}{h^2}$$

$$= \lim_{h \rightarrow 0} \lambda^2 \left(\frac{\sin \lambda h}{\lambda h} \right)^2$$

$$= \lambda^2 \times 1 = \lambda^2$$

$$f(0) = \lim_{x \rightarrow 0^-} f(x)$$

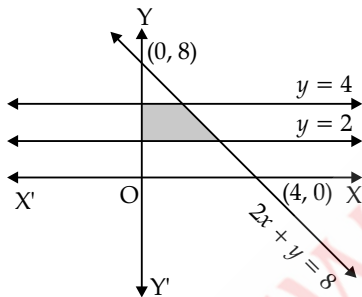
$$1 = \lambda^2 \therefore \lambda = \pm 1$$

23.

$$2x + y = 8$$

$$y = 2$$

$$y = 4$$



$$\text{Required Area} = \int_2^4 x dy = \int_2^4 \frac{8-y}{2} dy$$

$$= \left[4y - \frac{y^2}{4} \right]_2^4$$

$$= [16 - 4 - 8 + 1]$$

$$\text{Required area} = 5 \text{ unit}^2$$

$$24. (a) \text{ Given } |\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$$

$$\text{Since, } \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$\text{Now, } |\vec{a} \times \vec{b}| = \frac{|\vec{a} \times \vec{b}|}{\hat{n}} = \frac{|\vec{a}| |\vec{b}| \sin \theta \hat{n}}{\hat{n}}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$1 = 3 \times \frac{2}{3} \times \sin \theta$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

Angle between \vec{a} and \vec{b} is 30°

OR

$$(b) \vec{a} = \hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

Required area of $||^{\text{gm}}$

$$= |\vec{a} \times \vec{b}|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$$

$$= |20\hat{i} + 5\hat{j} - 5\hat{k}|$$

$$= |\sqrt{400 + 25 + 25}|$$

$$= 15\sqrt{2} \text{ unit}^2$$

25. Given point A (1, 2, -1)

$$\text{Given line } 5x - 25 = 14 - 7y = 35z,$$

$$= 5(x-5) = -7(y-2) = 35z,$$

$$= \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1} \text{ Divide by 35}$$

Direction ratio of the line (7, -5, 1)

∴ Direction ratio of the parallel line (7, -5, 1)

Equation of the line passing through the point A (1, 2, -1) and parallel to the given line

$$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$

vector form of the line

$$\hat{i} + 2\hat{j} - \hat{k} + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$

SECTION — C

26.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^2 \times A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

L.H.S.

$$A^3 - 23A - 40I$$

$$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 - 23A - 40I$$

$$= \begin{bmatrix} 63-23-40 & 46-46-0 & 69-69-0 \\ 69-69-0 & -6+46-40 & 23-23-0 \\ 92-92-0 & 46-46-0 & 63-23-40 \end{bmatrix}$$

$$A^3 - 23A - 40I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 - 23A - 40I = 0$$

Hence proved.

27. (a) $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ and $v = \sin^{-1}(2x\sqrt{1-x^2})$

Let $x = \sin \theta$

$$u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \sec^{-1}\frac{1}{\cos\theta} = \theta \quad 28.$$

 \therefore

$$u = \sin^{-1}x$$

$$v = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$v = \sin^{-1}\sin 2\theta = 2\theta$$

$$v = 2\sin^{-1}x$$

$$\frac{du}{dx} = \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dv}{dx} = \frac{d}{dx}(2\sin^{-1}x) = \frac{2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{\frac{dx}{dx}}{\frac{dv}{dx}} = \frac{1}{2}$$

OR

(b)

$$y = \tan x + \sec x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\tan x) + \frac{d}{dx}(\sec x)$$

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$\frac{dy}{dx} = \sec x (\sec x + \tan x)$$

$$\frac{d^2y}{dx^2} = (\sec x + \tan x) \frac{d}{dx} \sec x$$

$$+ \sec x \frac{d}{dx} (\sec x + \tan x)$$

$$\frac{d^2y}{dx^2} = (\sec x + \tan x) \sec x \tan x$$

$$+ \sec x (\sec x \tan x + \sec^2 x)$$

$$= (\sec x + \tan x) \sec x \tan x + \sec^2 x (\sec x + \tan x)$$

$$= \sec x (\sec x + \tan x) (\tan x + \sec x)$$

$$= \sec x (\sec x + \tan x)^2$$

$$= \frac{1}{\cos x} \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right)^2$$

$$= \frac{1}{\cos x} \left(\frac{1+\sin x}{\cos x} \right)^2$$

$$= \frac{1}{\cos x} \frac{(1+\sin x)^2}{(1-\sin^2 x)}$$

$$= \frac{1+\sin x}{\cos x(1-\sin x)}$$

$$\frac{d^2y}{dx^2} = \frac{(1+\sin x)}{\cos x(1-\sin x)} \times \frac{1-\sin x}{1-\sin x}$$

$$= \frac{1-\sin^2 x}{\cos x(1-\sin x)^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2} \quad \text{Hence proved.}$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}} dx$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}} dx$$

$$= \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx$$

$$2I = \int_0^{2\pi} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx$$

$$2I = \int_0^{2\pi} 1 dx$$

$$= [x]_0^{2\pi}$$

$$2I = 2\pi - 0$$

 \therefore

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} = \pi$$

OR

$$\int \frac{x^4}{(x-1)(x^2+1)} dx$$

$$\int \frac{x^4}{x^3-x^2+x-1} dx$$

	$x + 1$
$x^3 - x^2 + x - 1$	x^4
	$x^4 - x^3 + x^2 - x$
	(-) (+) (-) (+)
	$x^3 - x^2 + x$
	$x^3 - x^2 + x - 1$
	(-) (+) (-) (+)
	$\times 1$

$$\int \frac{x^4}{(x-1)(x^2+1)} dx = \int \left[x+1 + \frac{1}{(x-1)(x^2+1)} \right] dx$$

$$= \frac{x^2}{2} + x + \int \frac{1}{(x-1)(x^2+1)} dx$$

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)(x-1)$$

$$1 = (A+B)x^2 + (A-C) + x(C-B)$$

$$\therefore A+B=0 \Rightarrow A=-B$$

$$C-B=0 \Rightarrow C=B$$

$$A-C=1$$

$$-2C=1 \Rightarrow C=-\frac{1}{2}$$

$$B=-\frac{1}{2} \text{ and } A=\frac{1}{2}$$

$$\int \frac{1}{(x-1)(x^2+1)} dx = \frac{1}{2} \int \left[\frac{+1}{x-1} + \frac{-x-1}{x^2+1} \right] dx$$

$$= \frac{1}{2} \left[\int \frac{dx}{x-1} - \int \frac{x}{x^2+1} dx - \int \frac{dx}{x^2+1} \right]$$

$$= \frac{1}{2} \left[\log|x-1| - \frac{1}{2} \log|x^2+1| - \tan^{-1}x \right] + c_1$$

$$= \frac{1}{2} \log \left| \frac{x-1}{\sqrt{x^2+1}} \right| - \frac{1}{2} \tan^{-1}x + c$$

$$\int \frac{x^4}{(x-1)(x^2+1)} dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2} \log \left| \frac{x-1}{\sqrt{x^2+1}} \right| - \frac{1}{2} \tan^{-1}x + c$$

29. (a) Given: $y^2 = 2x$... (1)

$y = x - 4$... (2)

Required area is OABCO

from (1) and (2)

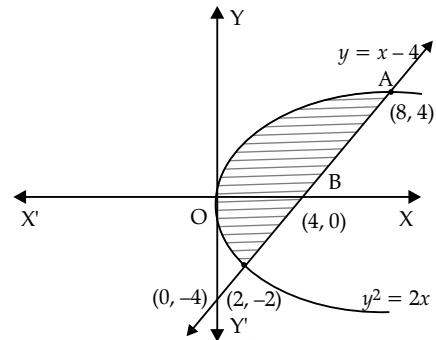
$$(x-4)^2 = 2x$$

$$x^2 - 10x + 16 = 0$$

$$(x-8)(x-2) = 0$$

$$x = 8 \text{ and } x = 2$$

\(\therefore\) Intersection points (2, -2) and (8, 4)



$$\text{Required Area} = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^4$$

$$= \left(8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3} \right)$$

$$= 30 - 12$$

$$= 18 \text{ unit}^2$$

30. P (0, 2, 3)

$$\text{line } \frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$$

General point on the line is

$$[(5\lambda - 3), (2\lambda + 1), (3\lambda - 4)]$$

Direction ratio of the perpendicular line

$$[(5\lambda - 3), (2\lambda - 1), (3\lambda - 7)]$$

$$\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$$

$$25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$$

$$38\lambda - 38 = 1$$

$$\lambda = 1$$

\(\therefore\) foot of perpendicular line is

$$[(5 - 3), (2 + 1), (3 - 4)]$$

$$(2, 3, -1)$$

OR

(b) $\vec{a} + \vec{b} + \vec{c} = 0$

$$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$$

$$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$3^2 + 4^2 + 2^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -29$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{29}{2}$$

31. Given:

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

These lines are parallel

∴ Distance between two parallel lines

$$= \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

$$a_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$a_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$$

and $|\vec{b}| = \sqrt{4+9+36} = 7$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & +6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-3-6) - \hat{j}(-2-12) + \hat{k}(2-6)$$

$$= -9\hat{i} + 14\hat{j} - 4\hat{k}$$

$$|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = |\sqrt{9^2 + 14^2 + 4^2}|$$

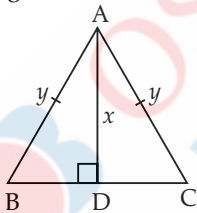
$$= |\sqrt{81 + 196 + 16}|$$

$$= \sqrt{293} \text{ units}$$

Shortest distance = $\frac{\sqrt{293}}{7}$ units

SECTION — D

32. (a) Let the length of the median be x cm



and side of equilateral triangle be y cm

In $\triangle ABD$ $AB^2 = AD^2 + BD^2$ ($\angle D = 90^\circ$)

$$y^2 = x^2 + \left(\frac{y}{2}\right)^2$$

$$\frac{3}{4}y^2 = x^2$$

$$y^2 = \frac{4}{3}x^2$$

$$y = \frac{2}{\sqrt{3}}x$$

$$\frac{d}{dt}(y) = \frac{2}{\sqrt{3}} \frac{d}{dt}(x)$$

$$\frac{dy}{dt} = \frac{2}{\sqrt{3}} \times \frac{dx}{dt}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{2}{\sqrt{3}} \times 2\sqrt{3} \\ &= 4 \text{ cm/s} \end{aligned}$$

Hence side of equilateral triangle be increase at 4cm/s

OR

(b) Let the number be x and y

$$\therefore x + y = 5 \quad \dots(1)$$

$$S = x^3 + y^3 \quad \dots(2)$$

from (1) & (2)

$$S = x^3 + (5-x)^3$$

$$\frac{dS}{dx} = 3x^2 - 3(5-x)^2$$

$$\frac{dS}{dx} = 3x^2 - 75 + 30x - 3x^2$$

$$= -75 + 30x$$

for maximum and minimum

$$\frac{dS}{dx} = 0$$

$$-75 + 30x = 0$$

$$x = \frac{75}{30} = \frac{5}{2}$$

$$\frac{d^2S}{dx^2} = -75 + 30x$$

$$\frac{d^2S}{dx^2} = 30$$

$$\frac{d^2S}{dx^2} > 0$$

Hence S is minimum at $x = \frac{5}{2}$

Minimum value of $x^2 + y^2$

$$= \frac{25}{4} + \frac{25}{4} = \frac{25}{2}$$

33. $I = \int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$

$$I = \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x) dx$$

Let $\sin x = t$

$$\cos x dx = dt$$

when $x = 0, t = 0$

$$x = \frac{\pi}{2}, t = 1$$

$$\therefore I = \int_0^1 2t \tan^{-1} t dt$$

$$= 2 \left[\tan^{-1} t \int t dt - \int \left[\frac{d}{dx} \tan^{-1} t \right] \int t dt \right] dt$$

$$= 2 \left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2(1+t^2)} dt \right]_0^1$$

$$= \left[t^2 \tan^{-1} t \right]_0^1 - \int_0^1 \frac{t^2}{(1+t^2)} dt$$

$$= \tan^{-1} 1 - \left[\int_0^1 1 dt - \int_0^1 \frac{1}{1+t^2} dt \right]$$

$$= \frac{\pi}{4} - \left[t - \tan^{-1} t \right]_0^1$$

$$= \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx = \frac{\pi}{2} - 1$$

34. Maximize: $P = 70x + 40y$

$$3x + 2y \leq 9, 3x + y \leq 9$$

$$x \geq 0, y \geq 0$$

$$3x + 2y = 9$$

$$3x + y = 9$$

x	0	3
y	9/2	0

x	0	3
y	9	0

Feasible area is OABO

$$P = 70x + 40y$$

At (0, 0) $P = 70 \times 0 + 40 \times 0 = 0$

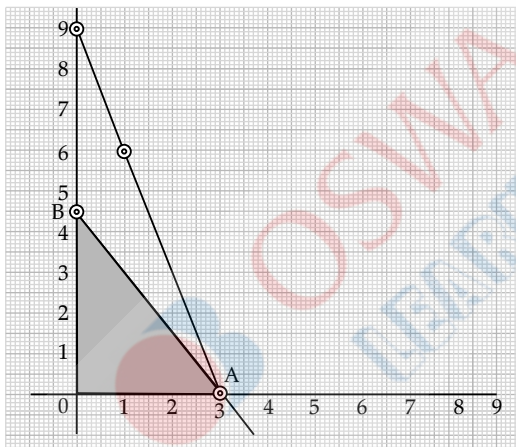
At (3, 0) $P = 70 \times 3 + 40 \times 0 = 210$

At (0, $\frac{9}{2}$) $P = 70 \times 0 + 40 \times \frac{9}{2} = 180$

Maximise at (3, 0)

$$x = 3 \text{ and } y = 0$$

Maximum value = 210



35. (a) Let E_1 = Student knows the answer
 E_2 = Student guesses the answer
 A = Student has answered the question correctly

$$\therefore P(E_1) = \frac{3}{5}, P(E_2) = 1 - \frac{3}{5} = \frac{2}{5}$$

$P\left(\frac{A}{E_1}\right)$ = Probability of student answered the question correctly given that he knows the answer
 $= 1$

$P\left(\frac{A}{E_2}\right)$ = Probability of student answered the question correctly given that he guesses

the answer

$$= \frac{1}{3}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$$

$$= \frac{1 \times \frac{3}{5}}{1 \times \frac{3}{5} + \frac{2}{5} \times \frac{1}{3}}$$

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{3}{5}}{\frac{11}{15}} = \frac{9}{11}$$

$$\text{Required probability} = \frac{9}{11}$$

OR

(b) Probability of prize ₹ 8 each = $\frac{2}{10} = \frac{1}{5}$

Probability of a prize ₹ 4 each = $\frac{5}{10} = \frac{1}{2}$

Probability of a prize ₹ 2 each = $\frac{3}{10}$

Mean value of the prize

$$= 8 \times \frac{1}{5} + 4 \times \frac{1}{2} + 2 \times \frac{3}{10}$$

$$= \frac{8}{5} + 2 + \frac{3}{5}$$

$$= \frac{21}{5}$$

$$\therefore \text{Mean value of the prize} = ₹ 4.20$$

SECTION — E

Case Study I

36. (i) Number of relations = 2^{mn}
 $= 2^{2 \times 3} = 2^6 = 64$

(ii) Number of functions from B to G
 $= 2^3 = 8$

(iii) $R = \{(x, y) : x \text{ and } y \text{ are students of same sex.}\}$

Since x and x are of the same sex

So $(x, x) \in R$ for all x

$\therefore R$ is reflexive

If x and y are of the same sex then y and x are also of the same sex

$\therefore R$ is symmetric

If $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

Then x and z will be of the same sex

$\therefore R$ is transitive

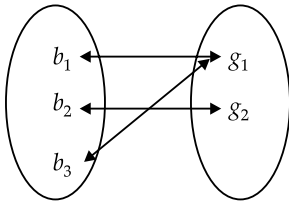
Since R is reflexive, symmetric and transitive

$\therefore R$ is an equivalence relation.

OR

(iii) Given

$$R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$$



Since b_1 and b_3 have the same image g_1

$\therefore R$ is not injective

Since all elements of G has a pre-image

$\therefore R$ is bijective

Case Study II

37.	Pen	Bags	Instrument
Gautam	5	3	1
Vikram	2	1	3
Ankur	1	2	4

$$(i) \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

where

x = cost of Pen

y = cost of Bag

z = cost of Instrument

$$(ii) A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} \\ = 5(4-6) - 3(8-3) + 1(4-1) \\ = -10 - 15 + 3 \\ = -22$$

$$(iii) C_{11} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2$$

$$C_{12} = - \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = -5$$

$$C_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$$C_{21} = - \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -10$$

$$C_{22} = \begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} = 19$$

$$C_{23} = - \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -7$$

$$C_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$$

$$C_{32} = - \begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = -13$$

$$C_{33} = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = -1$$

$$\text{Adj } A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= -\frac{1}{22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$= \frac{1}{22} \begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

OR

$$(iii) P = A^2 - 5A$$

$$= \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} - 5 \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 25+6+1 & 15+3+2 & 5+9+4 \\ 10+2+3 & 6+1+6 & 2+3+12 \\ 5+4+4 & 3+2+8 & 1+6+16 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

Case study III

$$38. (i) (x^2 - y^2) dx + 2xy dy = 0$$

$$\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy}$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2 \left(\frac{y^2}{x^2} - 1 \right)}{2xy}$$

$$= \frac{\left(\frac{y^2}{x^2} - 1 \right)}{\frac{2y}{x}}$$

$$\therefore \frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

$$(ii) \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$= \frac{v^2 - 1 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{-(1 + v^2)}{2v}$$

$$\frac{2v}{1+v^2} dv = -\frac{dx}{x}$$

$$\int \frac{2v}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\log |1 + v^2| = -\log |x| + \log |c|$$

$$\log |x(1 + v^2)| = \log |c|$$

$$x(1 + v^2) = c$$

$$x \left(1 + \frac{y^2}{x^2} \right) = c$$

$$x^2 + y^2 = cx$$

Delhi Set-II

65/5/2

SECTION — A

4. Option (d) is correct

Explanation:

$$a_{ij} = \begin{cases} 1 & \text{when } i \neq j \\ 0 & \text{when } i = j \end{cases}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. Option (d) is correct

$$\text{Explanation: } \begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 6(3-4) - 0(6-4) - 1(2-1)$$

$$= -6 - 1$$

$$= -7$$

9. Option (d) is correct

Explanation: $f(x) = x|x|$ is not differentiable at $x = 0$

11. Option (c) is correct

$$\text{Explanation: } \int_0^{\pi/4} (\sin 2x) dx$$

$$= \int_0^{\pi/4} 2 \sin x \cos x dx$$

$$\text{Let } \sin x = t$$

$$\cos x dx = dt$$

$$\text{when } x = 0 \text{ then } t = 0$$

$$\text{when } x = \frac{\pi}{4} \text{ then } t = \frac{1}{\sqrt{2}}$$

$$= 2 \int_0^{1/\sqrt{2}} t dt$$

$$= 2 \left[\frac{t^2}{2} \right]_0^{1/\sqrt{2}}$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

14. Option (a) is correct

Explanation: \vec{a} makes equal acute angles from axis

$$\therefore \cos \alpha = \cos \beta = \cos \gamma$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\therefore \vec{a} = \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}$$

$$\hat{a} = \frac{\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}}{|\vec{a}|}$$

$$= \frac{\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}}{\sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}}$$

$$\hat{a} = \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}$$

$$\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$$

Projection of vector \vec{a} on \vec{b}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{\left(\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k} \right) \cdot (5\hat{i} + 7\hat{j} - \hat{k})}{\sqrt{5^2 + 7^2 + 1^2}}$$

$$\begin{aligned} &= \frac{\frac{5}{\sqrt{3}} + \frac{7}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{\sqrt{75}} \\ &= \frac{11}{\sqrt{3}} \times \frac{1}{5\sqrt{3}} = \frac{11}{15} \end{aligned}$$

18. Option (c) is correct

Explanation: $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

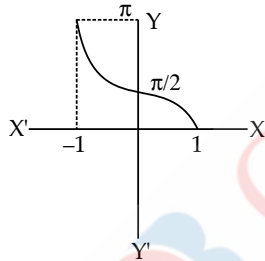
A and B are two independent events

$$\begin{aligned} \therefore P(A) \cdot P(B) &= P(A \cap B) \\ P(B') &= 1 - P(B) \\ &= 1 - \frac{1}{4} = \frac{3}{4} \\ P(A \cap B') &= P(A) \cdot P(B') \\ &= \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4} \\ P\left(\frac{B'}{A}\right) &= \frac{P(B' \cap A)}{P(A)} \\ &= \frac{1}{\frac{1}{3}} = \frac{3}{1} \end{aligned}$$

$$\therefore [P(B' \cap A) = P(B' \cap A)]$$

$$\begin{aligned} &= \frac{1}{\frac{1}{3}} = \frac{3}{1} \end{aligned}$$

21. $f(x) = \cos^{-1}x$



25.

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore \cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| |\vec{c}|} = \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{3^2 + 2^2 + 6^2} \sqrt{1^2 + 2^2 + 2^2}}$$

$$\cos \theta = \frac{3 + 4 + 12}{7 \times 3}$$

$$\cos \theta = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

SECTION — C

26.

$$\text{Ar.}\Delta \text{ PQR} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [3(3-7) - 1(9-5) + 1(63-1)]$$

$$= \frac{1}{2} |-12 - 4 + 48|$$

$$= \frac{32}{2} = 16 \text{ unit}^2$$

Equation of the line PQ

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$x(1-3) - y(3-9) + 1(9-9) = 0$$

$$-2x + 6y = 0$$

$$x - 3y = 0$$

28. $\int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1 + \cos 2x} dx$

$$f(x) = \frac{\cos 2x}{1 + \cos 2x}$$

$$\therefore f(-x) = \frac{\cos 2x}{1 + \cos 2x}$$

Hence $f(x)$ is even function

$$I = \int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1 + \cos 2x} dx$$

$$= 2 \int_0^{\pi/4} \frac{\cos 2x}{1 + \cos 2x} dx$$

$$= 2 \int_0^{\pi/4} \left[1 - \frac{1}{1 + \cos 2x} \right] dx$$

$$= 2 \left[\int_0^{\pi/4} 1 dx - \int_0^{\pi/4} \frac{1}{2} \sec^2 x dx \right]$$

$$= 2 \left[x - \frac{1}{2} \tan x \right]_0^{\pi/4}$$

$$I = 2 \left[\frac{\pi}{4} - \frac{1}{2} - 0 - 0 \right]$$

$$= \frac{\pi}{2} - 1$$

OR

(b) $\int e^{x^2} (x^5 + 2x^3) dx$

$$= \int x e^{x^2} (x^4 + 2x^2) dx$$

Let $x^2 = t$
 $2x dx = dt$

$$= \frac{1}{2} \int e^t (t^2 + 2t) dt$$

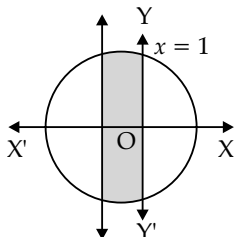
$$\begin{aligned} f(t) &= t^2 \\ \therefore f'(t) &= 2t \\ \int e^t(f(t) + f'(t))dt &= e^t f(t) \end{aligned}$$

$$\therefore \frac{1}{2} e^t \cdot t^2 + c = \frac{1}{2} x^4 e^{x^2} + c$$

29.

$$x^2 + y^2 = 4$$

$$\therefore y = \sqrt{4 - x^2}$$



$$\text{Required Area} = 2 \int_0^1 y dx$$

$$= 2 \int_0^1 \sqrt{4 - x^2} dx$$

$$= 2 \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{x}{2} \right]_0^1$$

$$= 2 \left[\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{6} - 0 - 0 \right]$$

$$= \left(\sqrt{3} + \frac{2\pi}{3} \right) \text{ units}^2$$

SECTION — D

32.

$$I = \int_0^\pi \frac{x}{1 + \sin x} dx$$

$$I = \int_0^\pi \frac{\pi - x}{1 + \sin(\pi - x)} dx$$

$$= \int_0^\pi \frac{\pi - x}{1 + \sin x} dx$$

$$2I = \int_0^\pi \frac{\pi}{1 + \sin x} dx$$

$$= \pi \int_0^\pi \frac{1 - \sin x}{1 - \sin^2 x} dx$$

$$= \pi \left[\int_0^\pi \frac{1}{\cos^2 x} dx - \int_0^\pi \tan x \sec x dx \right]$$

$$= \pi [\tan x - \sec x]_0^\pi$$

$$= \pi [0 + 1 - 0 + 1] = 2\pi$$

$$\therefore I = \int_0^\pi \frac{x}{1 + \sin x} dx = \pi$$

Delhi Set-III

65/5/3

SECTION — A

1. Option (b) is correct

Explanation: $a = b - 2, b > 6$

$$\therefore 6 = 8 - 2$$

2. Option (b) is correct

Explanation: $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$

$$A = A^T$$

$$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$

$$\therefore x = y$$

6. Option (d) is correct

Explanation: $f(x) = |\cos x|$

$$f\left(\frac{3\pi}{4}\right) = \left| \cos \frac{3\pi}{4} \right|$$

$$= \left| -\frac{1}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

9. Option (c) is correct

Explanation: $f(x) = x^3 + 3x$

$$f'(x) = 3x^2 + 3$$

for increasing $f'(x) > 0$

$$3x^2 + 3 > 0 \therefore x \in R \quad (x \in R \therefore x^2 > 0)$$

12. Option (c) is correct

Explanation:

$$\left(1 + 3 \frac{dy}{dx} \right)^2 = 4 \frac{d^3 y}{dx^3}$$

$$\therefore \text{order} = 3 \text{ and degree} = 1$$

13. Option (b) is correct

Explanation: Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \cdot \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i} = x$$

$$\vec{a} \cdot (\hat{i} + \hat{j}) = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j})$$

$$= x + y$$

$$\vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= x + y + z$$

Given, $x = x + y = x + y + z = 1$

$$\therefore x = 1, y = 0 \text{ and } z = 0$$

$$\vec{a} = \hat{i}$$

SECTION — B

$$21. (a) f(x) = \begin{cases} \frac{1 - \cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0$$

$$\text{L.H.L.} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \frac{1 - \cos(0 - h)}{2(0 - h)^2}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{1 - \cosh}{2h^2} \\
 &= \lim_{h \rightarrow 0} \frac{2 \sin^2 h / 2}{2h^2} \\
 &= \lim_{h \rightarrow 0} \left(\frac{\sin h / 2}{2h / 2} \right)^2 = \frac{1}{4}
 \end{aligned}$$

$f(x)$ is continuous at $x = 0$

$$\therefore k = \frac{1}{4} \quad \therefore (f(0) = \lim_{x \rightarrow 0} f(x))$$

OR

(b) $x = a \cos t, y = b \sin t$

$$\frac{dx}{dt} = \frac{d}{dt}(a \cos t) = -a \sin t$$

$$\frac{dy}{dt} = \frac{d}{dt}(b \sin t) = b \cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{b \cos t}{-a \sin t}$$

$$= -\frac{b}{a} \cot t$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a} \frac{d}{dx} \cot t$$

$$= +\frac{b}{a} \operatorname{cosec}^2 t \cdot \frac{dt}{dx}$$

$$= \frac{b}{a} \operatorname{cosec}^2 t \times \frac{1}{-a \sin t}$$

$$= -\frac{b}{a^2} \operatorname{cosec}^3 t$$

$$22. \tan^{-1} \left[2 \cos \left(2 \sin^{-1} \frac{1}{2} \right) \right] + \tan^{-1} 1$$

$$= \tan^{-1} \left[2 \cos \left(2 \times \frac{\pi}{6} \right) \right] + \frac{\pi}{4}$$

$$= \tan^{-1} \left(2 \times \frac{\sqrt{3}}{2} \right) + \frac{\pi}{4}$$

$$= \tan^{-1}(\sqrt{3}) + \frac{\pi}{4}$$

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

SECTION — C

$$26. \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

$$= x \begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin \theta \begin{vmatrix} -\sin \theta & 1 \\ \cos \theta & x \end{vmatrix} + \cos \theta \begin{vmatrix} -\sin \theta & -x \\ \cos \theta & 1 \end{vmatrix}$$

$$= x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta)$$

$$+ \cos \theta(-\sin \theta + x \cos \theta)$$

$$= -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta$$

$$- \sin \theta \cos \theta + x \cos^2 \theta$$

$$= -x^3 - x + x(\sin^2 \theta + \cos^2 \theta)$$

$$= -x^3 - x + x$$

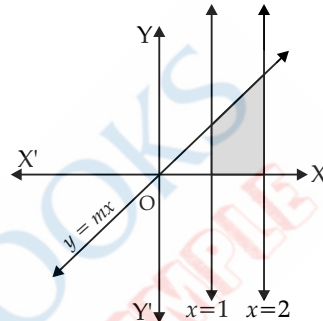
$$= -x^3$$

It is independent of θ .

Hence Proved.

$$27. \text{ Required Area} = \int_1^2 y dx$$

$$= \int_1^2 mx dx$$



$$= \left[\frac{mx^2}{2} \right]_1^2$$

$$= 2m - \frac{m}{2}$$

$$= \frac{3m}{2} \text{ unit}^2$$

$$28. \text{ Given line } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

General point of the line $(3\lambda + 15, 8\lambda + 29, -5\lambda + 5)$

Direction ratio of the perpendicular line which is passes through $(5, 7, 3)$ is

$$(3\lambda + 10, 8\lambda + 22, -5\lambda + 2)$$

lines are perpendicular: $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$$\therefore 3(3\lambda + 10) + 8(8\lambda + 22) - 5(-5\lambda + 2) = 0$$

$$9\lambda + 30 + 64\lambda + 176 + 25\lambda - 10 = 0$$

$$98\lambda = -196$$

$$\lambda = -2$$

foot of perpendicular $(-6 + 15, -16 + 29, 10 + 5)$

$$= (9, 13, 15)$$

OR

$$(b) \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$$

Perpendicular vector to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = \hat{i}(-6+4) - \hat{j}(-4-0) + \hat{k}(-2-0)$$

$$= -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\text{Required unit vector} = \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{|\sqrt{2^2 + 4^2 + 2^2}|}$$

$$= \frac{1}{\sqrt{6}}(-\hat{i} + 2\hat{j} - \hat{k})$$

32. $Z = 60x + 80y$

$$3x + 4y \geq 8$$

$$5x + 2y \geq 11$$

$$3x + 4y = 8$$

$$5x + 2y = 11$$

x	2	0	8/3
y	0.5	2	0

x	0	1	2	11/5
y	5.5	3	0.5	0

Point (x,y)	$Z = 60x + 80y$
At $(0, 11/2)$	$Z = 60 \times 0 + 80 \times \frac{11}{2}$ $= 440$

At $(2, 0.5)$	$Z = 60 \times 2 + 80 \times 0.5 = 160$
$(8/3, 0)$	$Z = 60 \times \frac{8}{3} + 80 \times 0 = 160$

$$60x + 80y \leq 160$$

$$60x + 80y = 160$$

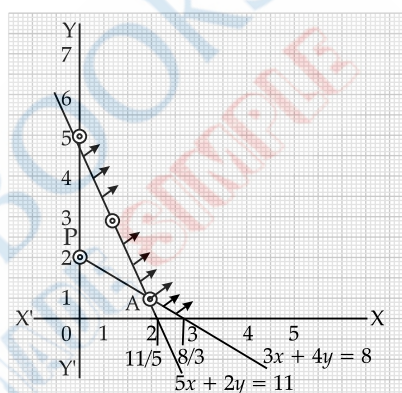
x	0	2	8/3
y	2	0.5	0

feasible region is unbounded so we consider

$$60x + 80y \leq 160$$

and feasible region.

Hence minimum value of Z is 160 which is each point of A and P.



Outside Delhi Set-I

65/2/1

SECTION — A

1. Option (c) is correct

$$\text{Explanation: } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A^{2023} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2. Option (b) is correct

$$\text{Explanation: } \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$$

$$= (A + A^t) + (A - A^t)$$

$$2A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$$

$$2A^t = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$$

$$Q = A - A^t$$

$$= \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$$

3. Option (d) is correct

$$\text{Explanation: } \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix} \text{ is non-singular matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix} \neq 0$$

\therefore

$$1(3-a) - 2(2-3) + 1(2a-9) \neq 0$$

$$3-a+2+2a-9 \neq 0$$

$$a \neq 4$$

\therefore

$$A = R - \{4\}$$

4. Option (d) is correct

$$\text{Explanation: } |A| = |kA|$$

$$|A| = k^n |A|$$

where n is the order of matrix

$$1 = k^n$$

$$k^2 = 1$$

\Rightarrow

$$k = \pm 1$$

Sum of all values of $k = +1 - 1 = 0$

5. **Option (b) is correct**

Explanation: $\frac{d}{dx}[f(x)] = ax + b$

$$\int \frac{d}{dx}[f(x)]dx = \int (ax + b)dx$$

$$= \frac{ax^2}{2} + bx + c$$

$$f(0) = 0$$

$$\therefore c = 0$$

Hence $f(x) = \frac{ax^2}{2} + bx$

6. **Option (b) is correct**

Explanation:

$$\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$$

$$\cos\left(\frac{dy}{dx}\right) = y^2 - \sin x$$

$$\frac{dy}{dx} = \cos^{-1}(y^2 - \sin x)$$

Hence degree of the differential equation is 1

7. **Option (d) is correct**

Explanation:

$$(1 - y^2)\frac{dx}{dy} + yx = ay$$

$$\frac{dx}{dy} + \frac{y}{1 - y^2}x = \frac{ay}{1 - y^2}$$

$$\text{I.F. is } e^{\int \frac{y}{1 - y^2} dy} = e^{-\frac{1}{2} \log|1 - y^2|}$$

$$\text{I.F.} = e^{\log(1 - y^2)^{-1/2}}$$

$$= \frac{1}{\sqrt{1 - y^2}}$$

8. **Option (d) is correct**

Explanation: P(2, 1, -1) and Q(4, 4, -7)

$$\vec{PQ} = (4 - 2)\hat{i} + (4 - 1)\hat{j} + (-7 + 1)\hat{k}$$

$$= 2\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\hat{PQ} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

9. **Option (b) is correct**

Explanation: Position vector of A = $2\hat{i} + 3\hat{j} - 4\hat{k}$

Position vector of midpoint AB = $3\hat{i} + 2\hat{j} - 3\hat{k}$

Let Position vector of B = $x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \frac{x+2}{2} = 3, \frac{y+3}{2} = 2, \frac{z-4}{2} = -3$$

$$\therefore x = 4, y = 1 \text{ and } z = -2$$

Hence position vector of B = $4\hat{i} + \hat{j} - 2\hat{k}$

10. **Option (a) is correct**

Explanation:

$$\vec{a} = 2\hat{i} + 3\hat{j}, \vec{b} = 3\hat{i} - 2\hat{j}$$

Projection of \vec{a} on \vec{b} is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 2\hat{j})}{\sqrt{3^2 + 2^2}}$$

$$= \frac{6 - 6}{\sqrt{13}} = 0$$

11. **Option (d) is correct**

Explanation: Direction ratio of z-axis is (0, 0, 1)

Line passing through the point (1, 1, 1) and parallel to z-axis

$$\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$$

12. **Option (d) is correct**

Explanation:

A = Sum of numbers obtained on the pair of dice is 9

$$= \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

B = Number obtained on 1 dice 1 and 4

$$= \{(4, 5), (5, 4)\}$$

$$P\left(\frac{B}{A}\right) = \text{Required probability}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{2}{\frac{36}{4}} = \frac{1}{2}$$

13. **Option (d) is correct**

Explanation:

Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ w.r.t. x is

$$\int \frac{\tan x - 1}{\tan x + 1} dx = \int -\tan\left(\frac{\pi}{4} - x\right) dx$$

$$= \frac{-\log \sec\left(\frac{\pi}{4} - x\right)}{-1}$$

$$= \log \left| \sec\left(\frac{\pi}{4} - x\right) \right| + c$$

14. Option (b) is correct

$$\text{Explanation: } \Delta = \frac{1}{2} \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$2\Delta = \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$4\Delta^2 = \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$$

15. Option (a) is correct

16. Option (b) is correct

$$\text{Explanation: } \tan\left(\frac{x+y}{x-y}\right) = k$$

$$\frac{x+y}{x-y} = \tan^{-1} k$$

$$\frac{d}{dx} \frac{x+y}{x-y} = \frac{d}{dx} \tan^{-1} k$$

$$\frac{(x-y)\left(1 + \frac{dy}{dx}\right) - (x+y)\left(1 - \frac{dy}{dx}\right)}{(x-y)^2} = 0$$

$$(x-y+x+y)\frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} = \frac{2y}{2x} = \frac{y}{x}$$

17. Option (c) is correct

$$\text{Explanation: } Z = ax + by$$

$$\therefore 4a + 6b = 42 \quad \dots(i)$$

$$3a + 2b = 19 \quad \dots(ii)$$

from (i) & (ii) $a = 3, b = 5$

18. Option (c) is correct

$$\text{Explanation: } Z = 30x + 24y$$

$$\text{At } (8, 0) \quad Z = 30 \times 8 + 24 \times 0 = 240$$

$$\text{At } (0, 4) \quad Z = 30 \times 0 + 24 \times 4 = 96 \text{ Minimum}$$

$$\text{At } \left(\frac{20}{3}, \frac{4}{3}\right) \quad Z = 30 \times \frac{20}{3} + 24 \times \frac{4}{3} = 232 \text{ Maximum}$$

$$= \text{Maximum} - \text{Minimum}$$

$$= 232 - 96 = 136$$

19. Option (c) is correct

Explanation: Range of the principal value of $\cos^{-1} x$ is $[0, \pi]$

20. Option (a) is correct

Explanation:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1$$

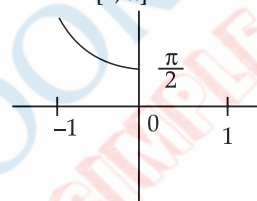
$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

SECTION — B

$$\begin{aligned} 21. \text{ (a) } & \sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1) \\ &= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{4}\right)\right) + \pi + \tan^{-1}\left(\tan \frac{\pi}{4}\right) \\ &= \sin^{-1}\sin\left(\frac{\pi}{4}\right) + \pi + \frac{\pi}{4} \\ &= \frac{\pi}{4} + \pi + \frac{\pi}{4} \\ &= \frac{3\pi}{2} \end{aligned}$$

OR

(b) Range of $\cos^{-1} x$ is $[0, \pi]$



22.

$$3y = ax^3 + 1$$

$$\text{Given: } \frac{dy}{dt} = 2\left(\frac{dx}{dt}\right) \text{ at } x = 1$$

$$3y = ax^3 + 1$$

$$\frac{3dy}{dt} = 3x^2 a \frac{dx}{dt} + 0$$

$$\frac{dy}{dt} = ax^2 \frac{dx}{dt}$$

$$2\left(\frac{dx}{dt}\right) = a(1)^2 \frac{dx}{dt}$$

$$\therefore a = 2$$

$$23. \quad \vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$$

$$= 0$$

$$(a \cdot b = a \cdot c)$$

\therefore Angle between \vec{a} and $(\vec{b} - \vec{c})$ is right angle i.e., 90°

$$24. \text{ Given line } \frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$$

Gen. point on the line $(\lambda, 2\lambda + 1, 2\lambda - 1)$

$$\text{Distance from origin } \left| \sqrt{x_1^2 + y_1^2 + z_1^2} \right|$$

$$\therefore \left| \sqrt{\lambda^2 + (2\lambda + 1)^2 + (2\lambda - 1)^2} \right| = \sqrt{11} \quad (\text{Given})$$

$$\lambda^2 + (2\lambda + 1)^2 + (2\lambda - 1)^2 = 11$$

$$\lambda^2 + 4\lambda^2 + 4\lambda + 1 + 4\lambda^2 - 4\lambda + 1 = 11$$

$$9\lambda^2 = 9$$

$$\Rightarrow \lambda = \pm 1$$

if $\lambda = 1$ point on the line $(1, 3, 1)$

if $\lambda = -1$ point on the line $(-1, -1, -3)$

25. (a) $y = \sqrt{ax+b}$
 $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{ax+b})$
 $\frac{dy}{dx} = \frac{1}{2\sqrt{ax+b}}(a+0)$
 $y \frac{dy}{dx} = \frac{a}{2}$
 $\frac{d}{dx}\left(y \frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{a}{2}\right)$
 $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ **Hence Proved**

OR
 (b) $f(x) = ax + b \quad 0 < x \leq 1$
 $\quad \quad \quad = 2x^2 - x \quad 1 < x < 2$
 $\therefore f(1) = a + b \quad \dots(i)$
 $\therefore f'_-(1) = f'_+(1)$
 $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$
 $\lim_{h \rightarrow 0^+} \frac{2(1+h)^2 - (a+b) - (1+h)}{h}$
 $\quad \quad \quad = \lim_{h \rightarrow 0^-} \frac{a(1+h) + b - (a+b)}{h}$

$$\lim_{h \rightarrow 0} \frac{2h^2 + 3h + 1 - a - b}{h} = \lim_{h \rightarrow 0^-} \frac{ah}{h} = a$$

$$\lim_{h \rightarrow 0^+} \frac{2h^2 + 3h + 1 - a - b}{h} = a$$

$f(x)$ is also continuous at $x = 1$

$$\therefore \lim_{x \rightarrow 1^+} f(x) = f(1) = \lim_{x \rightarrow 1^-} f(x)$$

$$\therefore a + b = 1 \quad \dots(ii)$$

$$\lim_{h \rightarrow 0^+} \frac{2h^2 + 3h + 1 - 1}{h} = a$$

$$\lim_{h \rightarrow 0} \frac{h(2h+3)}{h} = a$$

$$\lim_{h \rightarrow 0} (2h+3) = a$$

from $a = 3$
 $b = -2$
 $\therefore a = 3$ and $b = -2$

SECTION — C

26. (a) $I = \int_0^{\pi/4} \log(1 + \tan x) dx$
 $I = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$
 $I = \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$
 $I = \int_0^{\pi/4} \log \frac{2}{1 + \tan x} dx$

$$2I = \int_0^{\pi/4} \log(1 + \tan x) dx + \int_0^{\pi/4} \log \frac{2}{1 + \tan x} dx$$

$$2I = \int_0^{\pi/4} \log(1 + \tan x) \left(\frac{2}{1 + \tan x}\right) dx$$

$$2I = \int_0^{\pi/4} \log 2 dx$$

$$2I = [x \log 2]_0^{\pi/4}$$

$$2I = \frac{\pi}{4} \log 2$$

$$\therefore I = \int_0^{\pi/4} \log(1 + \tan x) = \frac{\pi}{8} \log 2$$

OR

(b) $\int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$
 $= \int \frac{dx}{\sqrt{\sin^3 x (\cos x \cos \alpha + \sin x \sin \alpha)}}$
 $= \int \frac{dx}{\sin^2 x \sqrt{\frac{\cos x}{\sin x} \cos \alpha + \sin \alpha}}$

$$= \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cot x \cos \alpha + \sin \alpha}}$$

$$= \frac{1}{\sqrt{\cos \alpha}} \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cot x + \tan \alpha}}$$

Let $\cot x + \tan \alpha = t$

$$-\operatorname{cosec}^2 x dx = dt$$

$$= \frac{1}{\sqrt{\cos \alpha}} \int \frac{-dt}{\sqrt{t}}$$

$$= -\frac{2\sqrt{t}}{\sqrt{\cos \alpha}} + c$$

$$= -2 \frac{\sqrt{\cot x + \tan \alpha}}{\sqrt{\cos \alpha}} + c$$

27. $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2}\right) dx$

Let $\cot^{-1} x = t$

$$-\frac{1}{1+x^2} dx = dt$$

$$= \int -e^t (1 - \cot t + \cot^2 t) dt$$

$$= \int -e^t (\operatorname{cosec}^2 t - \cot t) dt$$

$$= \int e^t (\cot t - \operatorname{cosec}^2 t) dt$$

$$f(t) = \cot t$$

$$f'(t) = -\operatorname{cosec}^2 t$$

$$\int e^t (f(t) + f'(t)) dt = e^t f(t) + c$$

$$\therefore = e^t \cot t + c$$

$$= e^{\cot^{-1} x} \cot \cot^{-1} x + c$$

$$= x e^{\cot^{-1} x} + c$$

$$28. \quad I = \int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$$

$$I = \int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{\left(e^x + \frac{1}{e^x}\right)\left(e^x - \frac{1}{e^x}\right)} dx$$

$$I = \int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{e^{2x}}{(e^{2x} + 1)(e^{2x} - 1)} dx$$

$$\text{Let } e^{2x} = t$$

$$2e^{2x} dx = dt$$

$$x = \log \sqrt{3} \text{ then } t = 3$$

$$x = \log \sqrt{2} \text{ then } t = 2$$

$$\therefore I = \frac{1}{2} \int_2^3 \frac{dt}{(t+1)(t-1)}$$

$$I = \frac{1}{2} \int_2^3 \frac{dt}{t^2 - (1)^2}$$

$$= \frac{1}{2} \cdot \frac{1}{2 \times 1} \log \left| \frac{t-1}{t+1} \right|_2^3$$

$$I = \frac{1}{4} \left[\log \frac{2}{4} - \log \frac{1}{3} \right] = \frac{1}{4} \log \frac{3}{2}$$

$$29. \quad (xy - x^2)dy = y^2 dx$$

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

It is a homogenous equation

$$\therefore y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{v^2}{v-1}$$

$$x \frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$$

$$\frac{v-1}{v} dv = \frac{dx}{x}$$

$$\int \left(1 - \frac{1}{v}\right) dv = \int \frac{dx}{x}$$

$$v - \log|v| = \log|x| + \log|c|$$

$$v = \log vcx$$

$$\frac{y}{x} = \log cy$$

$$\Rightarrow cy = e^{y/x} \text{ or } y = c_1 e^{y/x} \quad \left(c_1 = \frac{1}{c}\right)$$

OR

$$(b) \quad (x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

$$\frac{dy}{dx} + \frac{2x}{x^2 + 1} y = \frac{\sqrt{x^2 + 4}}{x^2 + 1}$$

Equation is the linear form

\(\therefore\) Integrating factor

$$\text{I.F.} = e^{\int \frac{2x}{x^2+1} dx}$$

$$\text{I.F.} = e^{\log|x^2+1|} = x^2 + 1$$

\(\therefore\) Solution of differential equation

$$y \times \text{I.F.} = \int \text{I.F.} \times \frac{\sqrt{x^2 + 4}}{x^2 + 1} dx$$

$$(x^2 + 1)y = \int \sqrt{x^2 + 4} dx$$

$$(x^2 + 1)y = \frac{x}{2} \sqrt{x^2 + 4}$$

$$+ 2 \log|x + \sqrt{x^2 + 4}| + c$$

30. Let the no. of Red balls = no of Green balls be x

$$\text{Total balls} = 2x$$

Probability of no Red balls

$$\frac{x}{2x} \times \frac{x}{2x} = \frac{1}{4}$$

Probability of 1 Red balls

$$2 \times \frac{x}{2x} \times \frac{x}{2x} = \frac{1}{2}$$

Probability of 2 Red balls

$$\frac{x}{2x} \times \frac{x}{2x} = \frac{1}{4}$$

$$\text{Required mean} = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 1$$

OR

Let W and F be the probabilities for win and fail respectively when single die is thrown alternatively.

$$\text{Since, } p(\text{getting } 6) = \frac{1}{6}$$

$$q(\text{not getting } 6) = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Starting with A,

$$P(\text{A win } 1^{\text{st}} \text{ throw}) = W$$

$$P(\text{A Win } 3^{\text{rd}} \text{ throw}) = F_A F_B W$$

$$P(\text{A Win } 5^{\text{th}} \text{ throw}) = F_A F_B F_A F_B W$$

So the probabilities for A to Win the game

$$P(\text{A Win}) = P(1^{\text{st}}) + P(3^{\text{rd}}) + P(5^{\text{th}}) \dots$$

$$= W + F_A F_B W + F_A F_B F_A F_B W \dots$$

$$= p + q \cdot q \cdot p + q \cdot q \cdot q \cdot q \cdot p \dots$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \dots$$

$$= \frac{1}{6} \left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 \dots \right] \because \left(S_{\infty} = \frac{a}{1-r}\right) r < 1$$

$$= \frac{1}{6} \times \frac{1}{1 - \frac{25}{36}}$$

$$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

$$P(\text{B Win}) = 1 - P(\text{A Win})$$

$$= 1 - \frac{6}{11}$$

$$= \frac{5}{11}$$

31.

$$Z = 5x + 10y$$

$$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0$$

$$x \geq 0, y \geq 0$$

$$x + 2y = 120$$

x	0	120	60
y	60	0	30

$$x + y = 60$$

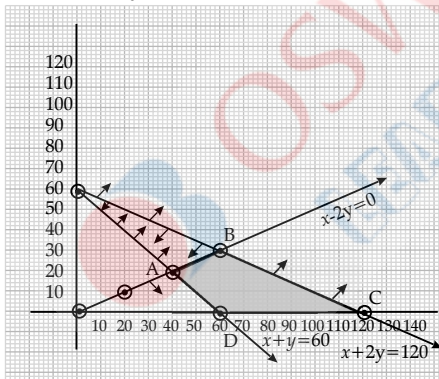
x	0	60	30
y	60	0	30

$$x - 2y = 0$$

x	0	10	20
y	0	5	10

Point (x, y)	$Z = 5x + 10y$
At $(40, 20)$	$Z = 5 \times 40 + 10 \times 20 = 400$
At $(60, 0)$	$Z = 5 \times 60 + 10 \times 0 = 300$ (Minimum)
At $(120, 0)$	$Z = 5 \times 120 + 10 \times 0 = 600$
At $(60, 30)$	$Z = 5 \times 60 + 10 \times 30 = 600$

Minimum value = 300

at $x = 60$ and $y = 0$ 

SECTION — D

$$32. \text{ (a) } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4+0 & -6+2+4 & 0+4-4 \\ 2-2+0 & 4-1-2 & 0-2+2 \\ 2-2+0 & 4-1-3 & 0-2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore B^{-1} = A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$x - 2y = 3$$

$$2x - y - 1 = 2$$

$$\text{and } -2y + z = 3$$

Equation can be written in matrix form

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\therefore (B^t)^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, y = -1 \text{ and } z = 1$$

OR

$$(b) \quad f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(-\beta) = \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(\alpha) \cdot f(-\beta) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cos \beta + \sin \alpha \sin \beta & \cos \alpha \sin \beta - \sin \alpha \cos \beta & 0 \\ \sin \alpha \cos \beta - \sin \beta \cos \alpha & \sin \alpha \sin \beta + \cos \alpha \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha - \beta) & -\sin(\alpha - \beta) & 0 \\ \sin(\alpha - \beta) & \cos(\alpha - \beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$33. \quad f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$$

Equation of the diagonal PR

Hence Proved

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$$

Equation of the diagonal QS

$$\frac{x-5}{6} = \frac{y+3}{+12} = \frac{z-1}{-3}$$

or
$$\frac{x-5}{2} = \frac{y+3}{4} = \frac{z-1}{-1}$$

General point on the diagonal PR

$$= (8\lambda + 4, 2\lambda + 2, 11\lambda - 6)$$

General point on the diagonal QS

$$= 2\mu + 5, 4\mu - 3, -\mu + 1$$

Intersection point of PR and QS

$$8\lambda + 4 = 2\mu + 5, 2\lambda + 2 = 4\mu - 3, 11\lambda - 6 = -\mu + 1$$

$$8\lambda - 2\mu = 1 \qquad 11\lambda + \mu = 7$$

$$2\lambda - 4\mu = -5 \qquad 11\lambda + \mu = 7 \text{ is}$$

$$14\lambda = 7 \qquad \text{Satisfies these values}$$

$$\lambda = \frac{1}{2} \text{ and } \mu = \frac{3}{2}$$

So intersection point of diagonals or mid point of diagonals is

$$= \left(4+4, 1+2, \frac{11}{2}-6 \right)$$

$$= \left(8, 3, -\frac{1}{2} \right)$$

OR

(b) Let the direction ratio of the line be (a, b, c)

Equation of the line passes through $(-1, 3, -2)$

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$$

Line is perpendicular to the given line

$$\therefore a + 2b + 3c = 0$$

$$-3a + 2b + 5c = 0$$

$$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

or
$$\frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$$

Required line

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

Vector form

$$= -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

General point on the line

$$= (2\lambda - 1, -7\lambda + 3, 4\lambda - 2)$$

Direction ratio of the line passing through origin and $(2\lambda - 1, -7\lambda + 3, 4\lambda - 2)$

$$\therefore (2\lambda - 1) + (-7)(-7\lambda + 3) + 4(4\lambda - 2) = 0$$

$$4\lambda - 2 + 49\lambda - 21 + 16\lambda - 8 = 0$$

$$69\lambda - 31 = 0$$

$$\lambda = \frac{31}{69}$$

Foot of perpendicular from origin on the line is $\frac{-7}{69}, \frac{-10}{69}, \frac{-14}{69}$

$$\text{Distance from origin} = \sqrt{\left(\frac{-7}{69}\right)^2 + \left(\frac{-10}{69}\right)^2 + \left(\frac{-14}{69}\right)^2}$$

$$= \sqrt{\frac{49+100+196}{69^2}}$$

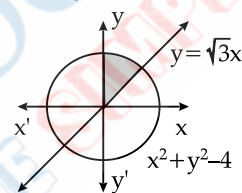
$$= \sqrt{\frac{345}{69}} = \sqrt{\frac{5}{9}} \text{ units}$$

34.

$$y = \sqrt{4-x^2}$$

$$y = \sqrt{3}x$$

$$y = \sqrt{4-x^2}$$



Squaring both side

$$\therefore x^2 + y^2 = 4$$

Intersection point of $x^2 + y^2 = 4$ and $y = \sqrt{3}x$

$$x^2 + 3x^2 = 4$$

$$\Rightarrow x = \pm 1$$

Intersection point in I Quadrant is $(1, \sqrt{3})$

$$\text{Required area} = \int_0^{\sqrt{3}} x_{\text{line}} dy + \int_{\sqrt{3}}^2 x_{\text{circle}} dy$$

$$= \int_0^{\sqrt{3}} \frac{y}{\sqrt{3}} dy + \int_{\sqrt{3}}^2 \sqrt{4-y^2} dy$$

$$= \left[\frac{y^2}{2\sqrt{3}} \right]_0^{\sqrt{3}} + \left[\frac{y}{2} \sqrt{4-y^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{y}{2} \right]_{\sqrt{3}}^2$$

$$= \left(\frac{\sqrt{3}}{2} - 0 \right) + \left[0 + 2 \sin^{-1} 1 - \frac{\sqrt{3}}{2} - 2 \sin^{-1} \frac{\sqrt{3}}{2} \right]$$

$$= \frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{3}$$

$$= \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3} \text{ unit}^2$$

35.

$$f(x) = \sqrt{16-x^2}$$

for $x = 0$

$$(0 \in (-4, 4))$$

$$f(0) = \pm 4$$

So it is not one-one and for each value of y these exists x

$$f(a) = \sqrt{7}$$

$\therefore f$ is an onto function

$$\sqrt{16-a^2} = \sqrt{7}$$

Squaring both sides

$$\begin{aligned} 16-a^2 &= 7 \\ a^2 &= 16-7=9 \\ a &= \pm 3 \\ a &= 3 \text{ and } -3 \end{aligned}$$

SECTION — E

36. Area of bore = $75\pi \text{ cm}^2$

$$2\pi rh + \pi r^2 = 75\pi$$

$$2rh + r^2 = 75$$

$$h = \frac{75-r^2}{2r}$$

(i) Vol. of cylinder = $\pi r^2 h$

$$\begin{aligned} &= \pi r^2 \left(\frac{75-r^2}{2r} \right) \\ &= \frac{\pi}{2} (75r - r^3) \text{ cm}^3 \end{aligned}$$

(ii) $\frac{dV}{dr} = \frac{\pi}{2} [75 - 3r^2]$

$$\frac{dV}{dr} = 0 \text{ for maxima}$$

$$\frac{\pi}{2} (75 - 3r^2) = 0$$

$$\Rightarrow \begin{aligned} r^2 &= 25 \\ r &= \pm 5 \end{aligned}$$

$$\frac{d^2V}{dr^2} = -6r$$

(iii) (a) $\left(\frac{d^2V}{dr^2} \right)_{(r=5)} = -30 < 0$

Hence volume is maximum at $r = 5 \text{ cm}$

(b) $h = \frac{75-25}{2 \times 5} = 5 \text{ cm}$

So $h = r$

at maximum volume

Hence statement is false.

37. $P(A) = 24\%$ $P(C) = 17\%$

$P(B) = 22\%$ $P(D) = 9\%$

(i) $P\left(\frac{L}{C}\right) = \frac{1}{4} \times \frac{12}{100}$

$$= \frac{17}{400}$$

$$= 0.0425$$

(ii) $P\left(\frac{L}{A}\right) = 1 - 0.0425$

$$= 0.9575$$

(iii) (a) $P\left(\frac{A}{L}\right) = \frac{P(A) \cdot P\left(\frac{L}{A}\right)}{P(A) \cdot P\left(\frac{L}{A}\right) + P(B) \cdot P\left(\frac{L}{B}\right) + P(C) \cdot P\left(\frac{L}{C}\right) + P(D) \cdot P\left(\frac{L}{D}\right)}$

$$\begin{aligned} &= \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} \\ &= \frac{24}{24 + 22 + 17 + 9} \\ &= \frac{24}{72} = \frac{1}{3} \end{aligned}$$

OR

(b) $= \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100}$

$$= \frac{39}{400} = 0.0475$$

38. $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

(i) $V(0) = -2 < 0$

So the above function can not be used to estimate number of vehicles in the year 2000.

(ii) $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$$V'(t) = \frac{3}{5}t^2 - 5t + 25$$

$$= \frac{3}{5} \left[t^2 - \frac{25}{3}t + \frac{125}{3} \right]$$

$$= \frac{3}{5} \left[\left(t - \frac{25}{6} \right)^2 - \frac{625}{36} + \frac{125}{3} \right]$$

$$= \frac{3}{5} \left[\left(t - \frac{25}{6} \right)^2 + \frac{875}{36} \right]$$

$$V'(t) > 0 \text{ for any value of } t$$

Hence $V(t)$ is increasing function.

SECTION — A

1. Option (a) is correct.

Explanation:

$$\frac{d}{dx} f(x) = 2x + \frac{3}{x}$$

$$f(x) = \int \left(2x + \frac{3}{x} \right) dx$$

$$f(x) = \frac{2x^2}{2} + 3 \log |x| + C$$

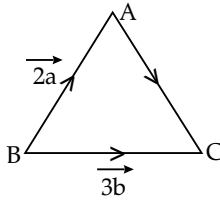
$$f(1) = (1)^2 + 3 \log |1| + C$$

$$1 = 1 + 0 + C \Rightarrow C = 0$$

$$f(x) = x^2 + 3 \log |x|$$

5. Option (c) is correct.

Explanation:



$$\vec{BA} + \vec{AC} = \vec{BC}$$

$$\vec{AC} = \vec{BC} - \vec{BA}$$

$$\vec{AC} = 3\vec{b} - 2\vec{a}$$

6. Option (d) is correct.

Explanation:

$$|\vec{a} \times \vec{b}| = \sqrt{3}$$

$$\vec{a} \cdot \vec{b} = -3$$

$$|\vec{a} \times \vec{b}| = \frac{|\vec{a} \times \vec{b}|}{\hat{n}} = \frac{|\vec{a}||\vec{b}|\sin \theta \hat{n}}{\hat{n}}$$

$$|\vec{a}||\vec{b}|\sin \theta = \sqrt{3} \quad \dots(i)$$

$$|\vec{a}||\vec{b}|\cos \theta = -3 \quad \dots(ii)$$

From eq. (i) and eq. (ii)

$$\tan \theta = \frac{\sqrt{3}}{-3} = -\frac{1}{\sqrt{3}}$$

$$\theta = \frac{5\pi}{6}$$

7. Option (b) is correct.

Explanation: $\cos \alpha = \cos 30 = \frac{\sqrt{3}}{2}$

$$\cos \beta = \cos 60 = \frac{1}{2}$$

$$\cos \gamma = \cos 90 = 0$$

Equation of required line

$$\frac{x-0}{\sqrt{3}/2} = \frac{y-0}{1/2} = \frac{z-0}{0}$$

$$\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$$

8. Option (a) is correct.

Explanation:

$$P\left(\frac{A}{B}\right) = 2 \times P\left(\frac{B}{A}\right)$$

$$\frac{P(A \cap B)}{P(B)} = \frac{2 \times P(A \cap B)}{P(A)}$$

$$P(A) = 2P(B)$$

$$P(A) + P(B) = \frac{2}{3}$$

$$2P(B) + P(B) = \frac{2}{3}$$

$$P(B) = \frac{2}{9}$$

$$P(A) = \frac{2}{3} - \frac{2}{9} = \frac{4}{9}$$

15. Option (d) is correct.

Explanation: $A = 2 \times 3$ $B = m \times n$

For defined $AB = m = 3$

For defined $AB' = n = 3$

$\therefore B = 3 \times 3$

SECTION — B

23.

$$x = ay + b$$

$$z = cy + d$$

$$\frac{x-b}{a} = y$$

$$\frac{z-d}{c} = y$$

\therefore Equation of the line

$$\frac{x-b}{a} = y = \frac{z-d}{c}$$

Direction ratio of the line $(a, 1, c)$

Point on the line $(b, 0, d)$

$$\frac{dC}{dt} = k \text{ (given)}$$

25.

$$\frac{d}{dt}(2\pi r) = k$$

(where C is the circumference of the circle)

$$2\pi \frac{dr}{dt} = k$$

$$\therefore \frac{dr}{dt} = \frac{k}{2\pi} = \text{Constant}$$

$A = \pi r^2$ (A and r is the area and radius of circle respectively)

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = \left(2\pi \frac{dr}{dt} \right) r$$

$$\frac{dA}{dt} \propto r$$

Hence Proved.

29.

$$z = x + 2y$$

$$x + 2y \geq 100$$

$$2x - y \leq 0$$

$$2x + y \leq 200$$

$$x \geq 0, y \geq 0$$

$$x + 2y = 100$$

x	0	100	60
y	50	0	20

$$2x - y = 0$$

x	10	0	20
y	20	0	40

$$2x + y = 200$$

x	0	100	50
y	200	0	100

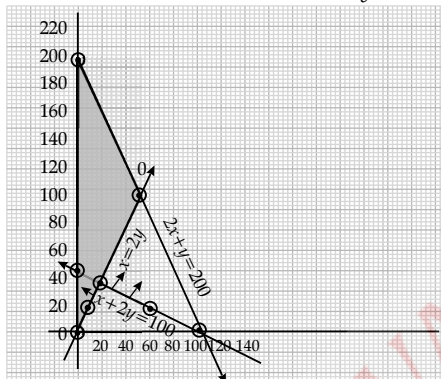
$$z = x + 2y$$

$$At (0, 200) = 0 + 2 \times 200 = 400 \text{ (Maximum)}$$

$$At (50, 100) = 50 + 2 \times 100 = 250$$

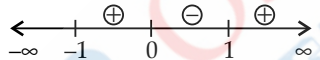
$$At (20, 40) = 20 + 2 \times 40 = 80$$

Maximum value is 400 at $x = 0$ and $y = 200$



30. (a) $\int_{-1}^1 |x^4 - x| dx$

$$\int_{-1}^1 |x(x-1)(x^2+x+1)| dx$$



$$\therefore I = \int_{-1}^0 (x^4 - x) dx + \int_0^1 -(x^4 - x) dx$$

$$= \left[\frac{x^5}{5} - \frac{x^2}{2} \right]_{-1}^0 - \left[\frac{x^5}{5} - \frac{x^2}{2} \right]_0^1$$

$$= \left(\frac{+1}{5} + \frac{1}{2} \right) - \left(\frac{1}{5} - \frac{1}{2} \right)$$

$$I = \frac{1}{2} + \frac{1}{2} = 1$$

OR

(b) $I = \int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

Let $\sin^{-1} x = t$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{t}{(1-\sin^2 t)} dt = \int t \sec^2 t dt$$

$$I = t \int \sec^2 t dt - \int \left[\frac{d}{dt} t \int \sec^2 t dt \right]$$

$$= t \tan t - \int \tan t dt$$

$$= t \tan t + \log |\cos t| + C$$

$$= \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \log |\sqrt{1-x^2}| + C$$

31. $I = \int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

$$I = \int e^x \left(\frac{1-2\sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} \right)$$

$$I = \int e^x \left(\frac{1}{2\sin^2 \frac{x}{2}} - \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} \right)$$

$$I = \int e^x \left(\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right) dx$$

$$I = \int e^x \left(-\cot \frac{x}{2} + \frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} \right) dx$$

Let $f(x) = -\cot \frac{x}{2}$

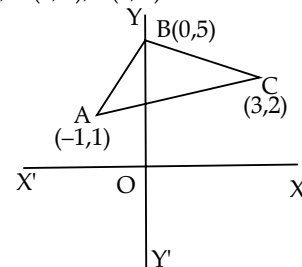
$$f'(x) = +\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2}$$

$$\int e^x ((f(x) + f'(x)) dx = e^x f(x) + C$$

$$\therefore I = \int e^x \left(-\cot \frac{x}{2} + \frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} \right) dx$$

$$= -e^x \cot \frac{x}{2} + c$$

33. $A(-1, 1), B(0, 5), C(3, 2)$



Equ. of the side AB

$$y - 1 = \frac{5-1}{0+1}(x+1)$$

$$y = 4x + 5$$

Equ. of the side BC

$$y - 5 = \frac{2-5}{3-0}(x-0)$$

$$y - 5 = -x$$

$$y = 5 - x$$

Equ. of the side AC

$$y - 1 = \frac{2-1}{3+1}(x-1)$$

$$4y - 4 = x + 1$$

$$y = \frac{x+5}{4}$$

Required Area of ΔABC

$$= \int_{-1}^0 (4x+5) dx + \int_0^3 (5-x) dx - \int_{-1}^3 \left(\frac{x+5}{4}\right) dx$$

$$= \left[\frac{x^2}{2} + 5x \right]_{-1}^0 + \left[5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4} \left[\frac{x^2}{2} + 5x \right]_{-1}^3$$

$$= \left(0 + \frac{9}{2} \right) + \left(\frac{21}{2} - 0 \right) - \frac{1}{4} \left(\frac{39}{2} + \frac{9}{2} \right)$$

$$= \frac{9}{2} + \frac{21}{2} - 6$$

$$= 15 - 6 = 9 \text{ unit}^2.$$

SECTION — A

1. Option (b) is correct.

Explanation:

$$|\vec{a} \times \vec{b}| = 3\sqrt{3}$$

$$|\vec{a} \times \vec{b}| = \frac{\vec{a} \times \vec{b}}{\hat{n}} = \frac{|a||b|\sin\theta\hat{n}}{\hat{n}}$$

$$|\vec{a}| |\vec{b}| \sin\theta = 3\sqrt{3}$$

$$|\vec{a}| |\vec{b}| \sin 60^\circ = 3\sqrt{3}$$

$$|\vec{a}| |\vec{b}| = 3\sqrt{3} \times \frac{2}{\sqrt{3}} = 6$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta$$

$$= 6 \cos 60^\circ$$

$$= 6 \times \frac{1}{2} = 3$$

2. Option (c) is correct.

Explanation:

$$A(4\hat{i} + 2\hat{j} - 6\hat{k}) \quad C(12\hat{i} + 4\hat{j} + 5\hat{k})$$

$$B(5\hat{i} - 3\hat{j} + \hat{k}) \quad D(x\hat{i} + y\hat{j} + z\hat{k})$$

Diagonal of \square bisect each other

$$\therefore \text{Position vector of mid point of AC } \left(8\hat{i} + 3\hat{j} - \frac{1}{2}\hat{k} \right)$$

Position vector of midpoint of \vec{BD}

$$\left(\frac{5+x}{2}\hat{i} + \frac{y-3}{2}\hat{j} + \frac{z+1}{2}\hat{k} \right)$$

$$\frac{5+x}{2} = 8 \quad \therefore x = 11$$

$$\frac{y-3}{2} = y = 9$$

$$\frac{z+1}{2} = -\frac{1}{2} \quad z = -2 \quad \therefore D\left(11\hat{i} + 9\hat{j} - 2\hat{k} \right)$$

3. Option (d) is correct.

Explanation:

$$P(A - B) = \frac{1}{5}$$

$$P(A) - P(A \cap B) = \frac{1}{5}$$

$$P(A \cap B) = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

4. Option (a) is correct.

Explanation:

$$\int_0^{2\pi} \cos^2 x dx = k \int_0^{\pi/2} \cos^2 x dx$$

$$f(x) = \cos^2 x$$

$$f(2\pi - x) = \cos^2(2\pi - x) = \cos^2 x$$

$$\therefore \int_0^{2\pi} \cos^2 x dx = 2 \int_0^{\pi} \cos^2 x dx$$

Similarly

$$\int_0^{\pi} \cos^2 x dx = 2 \int_0^{\pi/2} \cos^2 x dx$$

$$\therefore \int_0^{2\pi} \cos^2 x dx = 4 \int_0^{\pi/2} \cos^2 x dx$$

$$\therefore k = 4$$

10. Option (b) is correct.

Explanation: Number of Symmetric matrices of order $3 \times 3 = 2^6 = 64$

18. Option (d) is correct

Explanation: Equation of line passing through Point (x_1, y_1, z_1) .

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$x_1 = 1$$

$$y_1 = 2$$

$$z_1 = 3$$

$$a = 1$$

$$b = 1$$

$$c = 1$$

Required Eq.

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

SECTION — B

21. $A(2\hat{i})$, $B(\hat{j})$ and $C(2\hat{k})$

$$\vec{AB} = \hat{j} - 2\hat{i} \quad |\vec{AB}| = \sqrt{1^2 + 2^2} = |\sqrt{5}| \text{ unit}$$

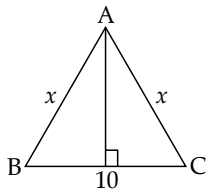
$$\vec{BC} = 2\hat{k} - \hat{j} \quad |\vec{BC}| = \sqrt{2^2 + 1^2} = |\sqrt{5}| \text{ unit}$$

$$\vec{AC} = 2\hat{k} - 2\hat{i} \quad |\vec{AC}| = \sqrt{2^2 + 2^2} = |2\sqrt{2}| \text{ unit}$$

$$\therefore AB = BC = \sqrt{5} \text{ unit}$$

Hence $\triangle ABC$ is an isosceles triangle

23. $\text{Ar } \triangle ABC = \frac{1}{2} BC \times AD$
 $= \frac{1}{2} \times 10 \times \sqrt{x^2 - 25}$



$$A = 5\sqrt{x^2 - 25} \text{ units}$$

$$A = 5\sqrt{x^2 - 25}$$

$$\frac{dA}{dt} = 5 \frac{d}{dt} (\sqrt{x^2 - 25})$$

$$= 5 \cdot \frac{1}{2} (x^2 - 25)^{-1/2} (2x) \frac{dx}{dt}$$

$$= \frac{5x}{\sqrt{x^2 - 25}} \frac{dx}{dt}$$

$$\left(\frac{dA}{dt} \right)_{x=10} = \frac{50}{\sqrt{25}} \cdot 4 = 40 \text{ cm}^2 / \text{s}$$

SECTION — C

26. $x + 2y \geq 240$
 $3x + 1.5y \geq 270 \quad x \geq 0, y \geq 0$
 $1.5x + 2y \leq 310$
 $x + 2y = 240$

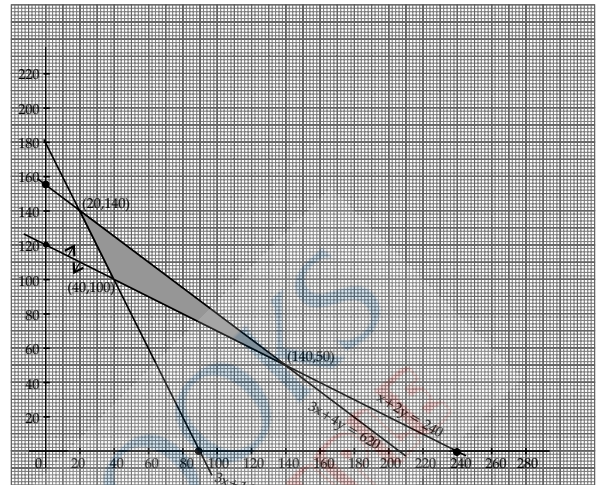
x	0	240	80
y	120	0	80

$$3x + 1.5y = 270$$

x	0	90	80
y	180	0	20

$$1.5x + 2y = 310$$

x	$\frac{620}{3}$	0	100	60
y	0	155	80	110



Point (x, y)	$Z = 3x + 3.5y$
at $(40, 100)$	$Z = 3 \times 40 + 3.5 \times 100 = 470$
at $(20, 140)$	$Z = 3 \times 20 + 140 \times 3.5 = 550$
at $(140, 50)$	$Z = 3 \times 140 + 3.5 \times 50 = 594$ (Maximum)

Maximum value 595

$$\text{at } x = 140 \text{ and } y = 50$$

27. (a) $\int \frac{x+2}{\sqrt{x^2-4x-5}} dx$

$$I = \int \frac{x+2}{\sqrt{x^2-4x-5}} dx$$

$$= \int \frac{\frac{1}{2}(2x-4)+4}{\sqrt{x^2-4x-5}} dx$$

$$= \frac{1}{2} \int \frac{2x-4}{\sqrt{x^2-4x-5}} dx + 4 \int \frac{dx}{\sqrt{x^2-4x-5}}$$

$$= \frac{1}{2} \cdot 2 \sqrt{x^2-4x-5} + 4 \int \frac{dx}{\sqrt{(x-2)^2-3^2}}$$

$$= \sqrt{x^2-4x-5} + 4 \log |x-2 + \sqrt{(x-2)^2-3^2}| + C$$

$$= \sqrt{x^2-4x-5} + 4 \log |x-2 + \sqrt{x^2-4x-5}| + C$$

OR

$$I = \int_{-a}^a f(x) = \int_{-a}^a \frac{9^x}{1+9^x} dx$$

$$I = \int_{-a}^a \frac{9^{(a-a-x)}}{1+9^{a-a-x}} dx$$

$$I = \int_{-a}^a \frac{9^{-x}}{1+9^{-x}} dx$$

$$= \int_{-a}^a \frac{1}{1+9^x} dx$$

$$2I = \int_{-a}^a \frac{9^x}{1+9^x} dx + \int_{-a}^a \frac{1}{1+9^x} dx$$

$$2I = \int_{-a}^a \frac{1+9^x}{1+9^x} dx = \int_{-a}^a dx$$

$$2I = [x]_{-a}^a$$

$$2I = 2a$$

$$\therefore I = a$$

31. (a) Given first six positive even integers

(2, 4, 6, 8, 10, 12)

Two numbers can be selected from the first six even integers = $6 \times 5 = 30$ ways

X denote the large of the two numbers

Hence x can take any value of 4, 6, 8, 10, 12

For x = 4 i.e., (2, 4) and (4, 2)

$$P(x) = \frac{2}{30} = \frac{1}{15}$$

For x = 6 i.e., (2, 6), (4, 6), (6, 2), (6, 4)

$$P(x) = \frac{4}{30} = \frac{2}{15}$$

For x = 8 i.e., (2, 8), (8, 2), (4, 8), (8, 4), (6, 8), (8, 6)

$$P(x) = \frac{6}{30} = \frac{3}{15}$$

For x = 10 i.e., (2, 10), (10, 2), (4, 10), (10, 4), (6, 10), (10, 6), (8, 10), (10, 8)

$$P(x) = \frac{8}{30} = \frac{4}{15}$$

For x = 12 i.e., (2, 12), (12, 2), (4, 12), (12, 4), (6, 12), (12, 6), (12, 8), (8, 12), (10, 12), (12, 10)

$$P(x) = \frac{10}{30} = \frac{1}{3}$$

x	4	6	8	10	12
y	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

OR

- (b) A fair coin and unbiased dice is tossed

$$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

$$n(S) = 12$$

$$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

$$B = \{(H, 3), (3, T)\}$$

$$n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

$$A \cap B = \{(H, 3)\}$$

$$\Rightarrow n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{12}$$

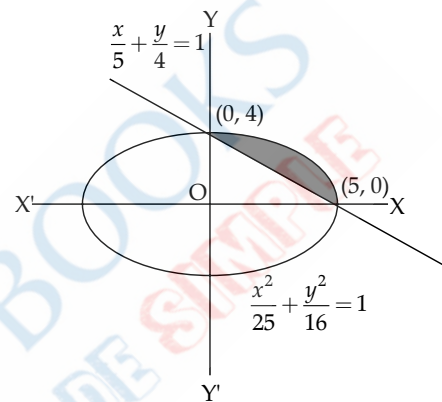
$$P(A).P(B) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

$$P(A \cap B) = \frac{1}{12}$$

$$\therefore P(A \cap B) = P(A).P(B)$$

These are independent events

35.



Eqn. the line $\frac{x}{5} + \frac{y}{4} = 1$

$$y = 4 \left(1 - \frac{x}{5} \right) = 4 - \frac{4x}{5}$$

Eqn. of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$y = 4 \sqrt{1 - \frac{x^2}{25}} = \frac{4}{5} \sqrt{25 - x^2}$$

$$\text{Required area} = \int_0^5 (y_{\text{ellipse}} - y_{\text{line}}) dx$$

$$= \int_0^5 \left\{ \frac{4}{5} \sqrt{25 - x^2} - \left(4 - \frac{4x}{5} \right) \right\} dx$$

$$= \int_0^5 \frac{4}{5} \sqrt{25 - x^2} dx - \int_0^5 \left(4 - \frac{4x}{5} \right) dx$$

$$= \frac{4}{5} \left[\frac{x}{2} \sqrt{25 - x^2} + \frac{1}{2} 25 \sin^{-1} \frac{x}{5} \right]_0^5 - \left[4x - \frac{4x^2}{10} \right]$$

$$= \frac{4}{5} \left[\frac{1}{2} \sin^{-1} 1 \right] - [20 - 10]$$

$$= \frac{4}{5} \times \frac{\pi}{4} - 10$$

$$= (5\pi - 10)$$

Required Area $(5\pi - 10)$ unit²