

# CBSE

## Solved Paper 2023

### Applied Mathematics

#### Class-XII

(Delhi/Outside Delhi Set)

Time : 3 Hours

Max. Marks : 80

#### General Instructions:

Read the following instructions very carefully and strictly follow them:

- This question paper contains 38 questions. All questions are compulsory.
- This question paper is divided into five Sections - A, B, C, D and E.
- In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
- In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
- In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
- In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
- In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
- There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- Use of calculators is **not** allowed.

Delhi & Outside Delhi Set

Code-465

#### SECTION — A

This section comprises multiple choice questions (MCQs) of 1 mark each

- The last (unit) digit of  $(22)^{12}$  is :  
(a) 2 (b) 4  
(c) 6 (d) 8
- The least non-negative remainder, when  $3^{15}$  is divided by 7 is:  
(a) 1 (b) 5  
(c) 6 (d) 7
- If  $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix}$ , then  $AB$  is:  
(a)  $\begin{bmatrix} -5 & 10 \\ 0 & -5 \end{bmatrix}$  (b)  $\begin{bmatrix} 0 & -5 \\ 25 & 10 \end{bmatrix}$   
(c)  $\begin{bmatrix} 10 & -25 \\ -5 & 0 \end{bmatrix}$  (d)  $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$
- If  $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$ , then the values of  $x$  and  $y$  are:  
(a)  $x = 3, y = 5$  (b)  $x = 5, y = 3$   
(c)  $x = 2, y = 7$  (d)  $x = 7, y = 2$
- The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is:  
(a) 7 : 8 (b) 8 : 7  
(c) 5 : 7 (d) 7 : 5
- If  $\frac{|x+1|}{x+1} > 0$ ,  $x \in \mathbb{R}$ , then:  
(a)  $x \in [-1, \infty)$  (b)  $x \in (-1, \infty)$   
(c)  $x \in (-\infty, -1)$  (d)  $x \in (-\infty, -1]$

7. A and B are square matrices each of order 3 such that  $|A| = -1$  and  $|B| = 3$ . What is the value of  $|3AB|$ ?
- (a)  $-9$  (b)  $-18$   
(c)  $-27$  (d)  $-81$
8. If  $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$ , then the value of  $x$  is:
- (a)  $-1$  (b)  $0$   
(c)  $1$  (d)  $3$
9. The relation between 'Marginal cost' and 'Average cost' of producing ' $x$ ' units of a product is:
- (a)  $\frac{d(AC)}{dx} = x(MC - AC)$  (b)  $\frac{d(AC)}{dx} = x(AC - MC)$   
(c)  $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$  (d)  $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$
10.  $\int (x-1)e^{-x} dx$  is equal to:
- (a)  $(x-2)e^{-x} + C$  (b)  $xe^{-x} + C$   
(c)  $-xe^{-x} + C$  (d)  $(x+1)e^{-x} + C$
11. The solution of the differential equation  $\frac{dx}{x} + \frac{dy}{y} = 0$  is:
- (a)  $\frac{1}{x} + \frac{1}{y} = C$  (b)  $xy = C$   
(c)  $\log x \log y = C$  (d)  $x + y = C$
12. If  $X$  is a Poisson variable such that  $P(X = 1) = 2P(X = 2)$ , then  $P(X = 0)$  is:
- (a)  $e$  (b)  $\frac{1}{e}$   
(c)  $1$  (d)  $e^2$
13. If the calculated value of  $|t| < t_v(\alpha)$ , then the null hypothesis is:
- (a) rejected (b) accepted  
(c) cannot be determined (d) neither accepted nor rejected
14. For testing the significance of difference between the means of two independent samples, the degree of freedom ( $v$ ) is taken as:
- (a)  $n_1 - n_2 + 2$  (b)  $n_1 - n_2 - 2$   
(c)  $n_1 + n_2 - 2$  (d)  $n_1 + n_2 - 1$
15. The straight line trend is represented by the equation:
- (a)  $y_c = a + bx$  (b)  $y_c = a - bx$   
(c)  $y_c = na + b\Sigma x$  (d)  $y_c = na - b\Sigma x$
16. The present value of a perpetuity of ₹  $R$  payable at the end of each payment period, when the money is worth  $i$  per period, is given by:
- (a)  $Ri$  (b)  $R + \frac{R}{i}$   
(c)  $\frac{R}{i}$  (d)  $R - Ri$
17. The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is:
- (a) 10.25% (b) 10.38%  
(c) 10.47% (d) 10.53%
18. Region represented by  $x \geq 0, y \geq 0$  lies in:
- (a) I quadrant (b) II quadrant  
(c) III quadrant (d) IV quadrant

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(c) Assertion (A) is true and Reason (R) is false.  
(d) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A):** The function  $f(x) = (x + 2)e^{-x}$  is increasing in the interval  $(-1, \infty)$ .

**Reason (R):** A function  $f(x)$  is increasing, if  $f'(x) > 0$ .

20. **Assertion (A):** The differential equation representing the family of parabolas  $y^2 = 4ax$ , where 'a' is a parameter, is  $x \frac{dy}{dx} - 2y = 0$ .

**Reason (R):** If the given family of curves has  $n$  parameters, then it is to be differentiated  $n$  times to eliminate the parameter and obtain the  $n^{\text{th}}$  order differential equation.

### SECTION — B

*This section comprises very short answer (VSA) type questions of 2 marks each.*

21. (a) Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes?

OR

(b) In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.

22. Solve for  $x$ :  $\frac{x+3}{x-2} \leq 2$ .

23. (a) Solve the following system of equations by Cramer's rule:

$$2x - y = 17, 3x + 5y = 6$$

OR

(b) Determine the integral value(s) of  $x$  for which the matrix A is singular:

$$A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$$

24. A particle moves along the curve  $6y = x^3 + 2$ . Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.

25. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given  $e^{-2} = 0.135$ )

### SECTION — C

*This section comprises short answer (SA) type questions of 3 marks each.*

26. (a) A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.

OR

(b) A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. If all the three pipes are opened simultaneously, how long will it take to fill the tank?

27. Find all the points of local maxima and local minima for the function  $f(x) = x^3 - 6x^2 + 9x - 8$ .

28. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.

29. The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.

(Use  $t_{0.005} = 1.729$  for 19 d.f.)

30. (a) An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

OR

(b) Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

{Given  $(1.0125)^{-48} = 0.5508565$ }

31. Maximise  $z = 300x + 190y$

subject to constraints:

$$x + y \leq 24,$$

$$2x + y \leq 32,$$

$$x \geq 0, y \geq 0.$$

**SECTION — D**

*This section comprises long answer (LA) type questions of 5 marks each.*

32. (a) Find the inverse of the matrix:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that  $AA^{-1} = I$ .

**OR**

- (b) Using matrix method, solve the following system of equations for  $x$ ,  $y$  and  $z$ :

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

33. (a) Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.

**OR**

- (b) Find a point on the curve  $y^2 = 2x$  which is nearest to the point  $(1, 4)$ .

34. Fit a straight line trend by method of least squares to the following data and find the trend values:

Year:	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹):	65	68	70	72	75	67	73

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below:

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use  $(1.4)^{1/6} = 1.058$ ]

**SECTION — E**

*This section comprises 3 case study based questions of 4 marks each.*

**Case Study-1**

36. A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.

Based on the above information, answer the following questions:

- (i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use:  $e^{-6} = 0.0024$ ) 1
- (ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. 1
- (iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. 2

**OR**

- (iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs. 2

**Case Study-2**

37. A factory manufactures tennis rackets and cricket bats. A tennis racket takes  $1\frac{1}{2}$  hours of machine time and 3 hours

of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions:

- (i) If  $x$  and  $y$  are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. 1
- (ii) Write the constraint that relates the number of craftsmanship hours. 1
- (iii) (a) Determine the maximum profit (in ₹) earned by the factory. 2

**OR**

- (iii) (b) How many bats and rackets respectively, are manufactured to earn maximum profit? 2

**Case Study-3**

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions:

- (i) Determine the EMI. 1
- (ii) Find the principal paid by Mr. Aggarwal in the 150<sup>th</sup> instalment. 1
- (iii) (a) Find the total interest paid by Mr. Aggarwal. 2

**OR**

- (iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ? 2
- [Use  $(1.00625)^{240} = 4.4608$ ;  $(1.00625)^{91} = 1.7629$ ]

□□



# ANSWERS

## SECTION — A

1. **Option (c) is correct.**

*Explanation:* We have  $(22)^{12}$ ,

Here, base number is 22 and its units digit is 2.

Also, exponent is 12, which is divisible by 2 (units digit).

i.e.,  $12 \div 2 = 6$

So, last digit of  $(22)^{12}$  is 6.

2. **Option (c) is correct.**

*Explanation:* Since,  $3^3 = 27 \pmod{7}$

$$3^3 = -1 \pmod{7}$$

$$(3^3)^5 = (-1)^5 \pmod{7}$$

$$3^{15} = 6 \pmod{7}$$

3. **Option (d) is correct.**

$$\begin{aligned} \text{Explanation: } AB &= \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix} \\ &= \begin{bmatrix} -5-0 & 10-0 \\ 10-10 & -20-5 \end{bmatrix} \\ &= \begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix} \end{aligned}$$

4. **Option (a) is correct.**

$$\text{Explanation: } \begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$$

Equating corresponding elements of equal matrices, we get

$$x + y = 8 \text{ and } x + 2 = 5$$

$$\therefore x + 2 = 5 \Rightarrow x = 3$$

$$\text{Also, } x + y = 8 \Rightarrow 3 + y = 8 \Rightarrow y = 5$$

$$\text{Thus, } x = 3 \text{ and } y = 5$$

5. **Option (b) is correct.**

*Explanation:* Let the amount of pulse of cost ₹ 85 =  $x$

Let the amount of pulse of cost ₹ 100 =  $y$

Then the total amount of mixture =  $85x + 100y$

But the cost of per kg of mixture = ₹ 92

ATQ,

$$92(x + y) = 85x + 100y$$

$$(92 - 85)x = (100 - 92)y$$

$$7x = 8y$$

$$\frac{x}{y} = \frac{8}{7}$$

$$x : y = 8 : 7$$

6. **Option (b) is correct.**

*Explanation:* If  $\frac{|x+1|}{x+1} > 0$ , then to make the given

fraction, greater than zero denominator should always be greater than zero.

$$\text{So, } x + 1 > 0 \Rightarrow x > -1$$

$$\text{or, } x \in (-1, \infty)$$

7. **Option (d) is correct.**

$$\begin{aligned} \text{Explanation: } |3AB| &= (3)^3 |A| |B| \\ &= 27 (-1) (3) \\ &= -81 \end{aligned}$$

$[\because |mAB| = m^n |A| |B|$  where  $n$  is the order of square matrices A and B]

8. **Option (a) is correct.**

$$\text{Explanation: } \begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

$$x \begin{vmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0 \quad [\text{Taking } x \text{ common}]$$

$$x \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0 \quad [R_1 \rightarrow R_1 - 2R_2]$$

$$x [0(1-9) - 1(1-4) + 0(9-4)] + 3 = 0 \quad [\text{Expanding along } R_1]$$

$$3x + 3 = 0$$

$$\Rightarrow x = -1$$

9. **Option (d) is correct.**

10. **Option (c) is correct.**

*Explanation:* Let  $I = \int (x-1)e^{-x} dx$

Let  $-x = t \Rightarrow -dx = dt$

$$\therefore I = -\int e^t (-t-1) dt$$

$$= \int e^t (t+1) dt$$

$$[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c]$$

$$= te^t + c$$

$$I = -xe^{-x} + c$$

11. **Option (b) is correct.**

*Explanation:* Given differential equation is

$$\frac{dx}{x} + \frac{dy}{y} = 0$$

Integrating both sides, we get

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\log x = -\log y + \log c$$

$$\log x + \log y = \log c$$

$$\log(xy) = \log c$$

$$xy = c$$

12. **Option (a) is correct.**

*Explanation:* Given,

$$P(x=1) = 2P(x=2)$$

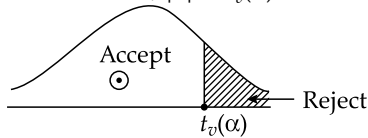
$$\Rightarrow \frac{e^{-m} m^1}{1!} = 2 \frac{e^{-m} m^2}{2!}$$

$$\Rightarrow m = 1$$

$$\text{Now, } P(x=0) = \frac{e^{-m} m^0}{0!} = \frac{e^{-1} 1}{1} = e^{-1}$$

13. Option (b) is correct.

Explanation: Given,  $|t| < t_v(\alpha)$



14. Option (c) is correct.

15. Option (a) is correct.

16. Option (c) is correct.

Explanation: Present value at the end of each payment period is  $\frac{R}{i}$ .

17. Option (b) is correct.

Explanation: Amount of ₹ 100 for 1 year when compounded quarterly

$$\begin{aligned} &= ₹ \left[ 100 \times \left( 1 + \frac{2.5}{100} \right)^4 \right] \\ &= ₹ [100 \times (1 + 0.025)^4] \\ &= ₹ (100 \times 1.1038) \\ &= ₹ 110.38 \end{aligned}$$

$\therefore$  Effective rate =  $(110.38 - 100)\% = 10.38\%$

[Note : Effective rate which is equivalent to nominal rate of 10%. p. a.

When compounded semi-annually = 10.250 %

When compounded Quarterly = 10.381 %

When compounded monthly = 10.471 %

When compounded daily = 10.516 %

18. Option (a) is correct.

Explanation: Since  $x$  and  $y$  both take positive values, so region lies in I quadrant.

19. Option (d) is correct.

Explanation: We have assertion,

$$f(x) = (x + 2)e^{-x}$$

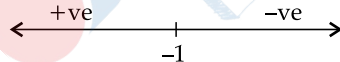
$\therefore f'(x) = e^{-x} - (x + 2)e^{-x}$

$$= e^{-x}(1 - x - 2)$$

$$= -e^{-x}(x + 1)$$

Here,  $e^{-x}$  will be always positive, so the function will increasing when  $x + 1 > 0$

or  $x > -1$



So,  $f'(x) > 0$  when  $x \in (-\infty, -1)$ .

Thus, Assertion is incorrect.

Here, Reason is the base correct condition for a function be an increasing function.

20. Option (d) is correct.

Explanation: For assertion,

$$y^2 = 4ax \quad \dots(i)$$

$\therefore 2yy' = 4a$

or  $yy' = 2a$

or  $a = \frac{yy'}{2}$

Substituting value of  $a$  in eq (i), we get

$$y^2 = 4 \left( \frac{yy'}{2} \right) x$$

$$\text{or, } y^2 = 2yy'x$$

$$\text{or, } y = 2y'x$$

or,  $2x \frac{dy}{dx} - y = 0$  is the differential equation

representing the family of parabolas  $y^2 = 4ax$  where  $a$  is the parameter.

Thus, assertion is incorrect.

Reason is correct.

## SECTION — B

21. (a) Here capacity of the tank be 96 units (LCM of 24 & 32)

$$\text{Pipe A fills} = \frac{96}{24} = 4 \text{ units/minute}$$

$$\text{Pipe B fills} = \frac{96}{32} = 3 \text{ units/minute} \quad 1$$

Let the time for which B is opened is  $x$  minutes.

ATQ,

$$4 \times 18 + 3 \times x = 96$$

$$\Rightarrow 3x = 96 - 72$$

$$\Rightarrow x = 8 \text{ minutes} \quad 1$$

The pipe B should be closed after 8 minutes

OR

- (b) Given,

A beats B by 30 seconds

B beats C by 15 seconds

So, A beats C by  $(30 + 15)$  seconds = 45 seconds

Now, time taken by C to travel 180 m = 45 seconds

$\therefore$  Time taken by C to cover the distance of 1 km

$$(1000 \text{ m}) = \frac{45}{180} \times 1000 = 250 \text{ seconds} \quad 1$$

Thus, time taken by A to cover the distance of 1 km =  $(250 - 45) = 205$  seconds.  $1$

22. We have,

$$\frac{x+3}{x-2} \leq 2 \quad 1$$

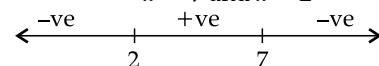
$$\frac{x+3}{x-2} - 2 \leq 0$$

$$\frac{x+3-2x+4}{x-2} \leq 0$$

$$\frac{-x+7}{x-2} \leq 0$$

$\therefore -x + 7 = 0$  and  $x - 2 = 0$

$$x = 7 \text{ and } x = 2$$



Thus, solution set is  $(-\infty, 2) \cup [7, \infty)$ .  $1$

23. (a) Given,

$$2x - y = 17$$

$$3x + 5y = 6$$

$$\text{Here, } \Delta = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}$$

$$= 10 + 3 = 13$$

$$\Delta_x = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 85 + 6 = 91$$

$$\Delta_y = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = 12 - 51 = -39 \quad 1$$

Using Cramer's rule,

$$x = \frac{\Delta_x}{\Delta} = \frac{91}{13} = 7$$

and  $y = \frac{\Delta_y}{\Delta} = \frac{-39}{13} = -3 \quad 1$

**OR**

(b) We have,

$$A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix} \text{ is singular}$$

$$\therefore |A| = 0$$

$$\Rightarrow \begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$$

$$\text{So, } [(x+1)\{(x+2)(x-6)-2\} + 3\{-5(x-6)-8\} + 4\{-5-4(x+2)\}] = 0 \quad 1$$

$$[(x+1)\{(x+2)(x-6)-2\} + 3\{-5(x-6)-8\} + 4\{-5-4(x+2)\}] = 0$$

$$[(x+1)(x^2-4x-14) + 3(-5x+22) + 4(-13-4x)] = 0$$

$$x^3 - 4x^2 - 14x + x^2 - 4x - 14 - 15x + 66 - 52 - 16x = 0$$

$$x^3 - 3x^2 - 49x = 0$$

$$x(x^2 - 3x - 49) = 0$$

$$\therefore x = 0 \text{ or } x^2 - 3x - 49 = 0$$

$$x = 0 \text{ or } x = \frac{3 \pm \sqrt{9+4(49)}}{2}$$

$$x = 0 \text{ or } x = \frac{1}{2}(3 \pm \sqrt{205}) \quad 1$$

24. Let  $(x_1, y_1)$  be the point on the curve  $6y = x^3 + 2$  whose  $y$ -coordinate (ordinate) is changing 8 times as fast as the  $x$ -coordinate (abscissa).

Then,

$$\left(\frac{dy}{dt}\right)_{\text{at}(x_1, y_1)} = 8 \left(\frac{dx}{dt}\right)_{\text{at}(x_1, y_1)} \quad \dots(i) \quad \frac{1}{2}$$

Differentiating  $6y = x^3 + 2$  w.r.t. ' $t$ ', we get

$$6 \frac{dy}{dt} = 3x^2 \frac{dx}{dt} + 0$$

$$2 \frac{dy}{dt} = x^2 \frac{dx}{dt} \quad \frac{1}{2}$$

$$2 \times 8 \left(\frac{dx}{dt}\right)_{\text{at}(x_1, y_1)} = x_1^2 \left(\frac{dx}{dt}\right)_{\text{at}(x_1, y_1)} \quad [\text{Using eq (i)}]$$

$$\therefore x_1^2 = 16 \Rightarrow x_1 = \pm 4 \quad \frac{1}{2}$$

Now,  $(x_1, y_1)$  lies on the curve  $6y = x^3 + 2$

$$\therefore 6y_1 = x_1^3 + 2$$

$$6y_1 = (-4)^3 + 2, \text{ when } x_1 = -4$$

$$y_1 = \frac{-31}{3}$$

and  $6y_1 = (4)^3 + 2, \text{ when } x_1 = 4$

$$y_1 = 11 \quad \frac{1}{2}$$

Hence, the required points on the curve are  $(4, 11)$

and  $\left(-4, \frac{-31}{3}\right)$ .

25. Given,  $n = 100$

$$\text{Mean } (\lambda) = 2\% \text{ of } 100 = 2$$

$$\therefore P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad 1$$

$$P(X = 3) = \frac{e^{-2} 2^3}{3!}$$

$$= \frac{0.135 \times 8}{6}$$

$$= \frac{1.08}{6} = 0.18 \quad 1$$

## SECTION — C

26. (a) Let original quantity of dettol in the bottle be  $x$  litres.

Then initially quantity of water in bottle = 0 litres

After the 1<sup>st</sup> operation,

Quantity of dettol in bottle

$$= \left(x - \frac{x}{3}\right) \text{ litres} = \frac{2x}{3} \text{ litres} \quad \frac{1}{2}$$

After the 2<sup>nd</sup> operation,

$$\text{Quantity of dettol in bottle} = \left(\frac{2x}{3} - \frac{1}{3} \times \frac{2x}{3}\right) \text{ litres}$$

$$= \frac{4x}{9} \text{ litres} \quad \frac{1}{2}$$

After the 3<sup>rd</sup> operation,

$$\text{Quantity of dettol in bottle} = \left(\frac{4x}{9} - \frac{1}{3} \times \frac{4x}{9}\right) \text{ litres}$$

$$= \frac{8x}{27} \text{ litres} \quad \frac{1}{2}$$

Now, Quantity of water in the bottle after 1<sup>st</sup>

$$\text{operation} = \frac{2x}{3} \text{ litres}$$

Quantity of water in the bottle after 2<sup>nd</sup> operation

$$= \left(\frac{x}{3} + \frac{2x}{9}\right) \text{ litres}$$

$$= \frac{5x}{9} \text{ litres}$$

Quantity of water in the bottle offer 3<sup>rd</sup>

$$= \left(\frac{x}{3} + \frac{2x}{9} + \frac{4x}{27}\right) \text{ litres}$$

$$= \frac{19x}{27} \text{ litres} \quad 1$$

$$\therefore \text{Required ratio} = \frac{8x}{27} : \frac{19x}{27}$$

$$= 8 : 19 \quad \frac{1}{2}$$



OR

(b) Pipe A can fill the part of tank in 1 hour =  $\frac{1}{3}$  part  $\frac{1}{2}$ Pipe B can empty the part of tank in 1 hour =  $\frac{1}{7}$  part $\frac{1}{2}$ Pipe C can empty the part of tank in 1 hour =  $\frac{1}{10}$  part  $\frac{1}{2}$ Net part filled in 1 hour =  $\frac{1}{3} - \left(\frac{1}{7} + \frac{1}{10}\right)$ 

$$= \frac{1}{3} - \frac{17}{70}$$

$$= \frac{19}{210}$$

1

Thus, tank to will be filled in  $\frac{210}{19}$  hour i.e.  $11\frac{1}{9}$ hours or 11 hours.  $\frac{1}{2}$ 27. Given,  $y = f(x) = x^3 - 6x^2 + 9x - 8$ 

$$\therefore y' = 3x^2 - 12x + 9$$

Now, put  $y' = 0$ 

$$\therefore 3x^2 - 12x + 9 = 0$$

$$\Rightarrow x = \frac{12 \pm \sqrt{144 - 4 \times 3 \times 9}}{6}$$

$$\Rightarrow x = \frac{12 \pm \sqrt{36}}{6}$$

$$\Rightarrow x = \frac{12 \pm 6}{6}$$

$$\Rightarrow x = \frac{12+6}{6} \text{ and } x = \frac{12-6}{6}$$

$$\Rightarrow x = 3 \text{ and } x = 1$$

Now,  $y'' = 6x - 12$ 

$$\text{At } x = 3, y'' = 6.3 - 12 = 6 > 0$$

$$\text{At } x = 1, y'' = 6.1 - 12 = -6 < 0$$

Thus, at  $x = 3$ , the function is at its local minimum and at  $x = 1$ , the function is at its local maximum. 128. Let  $p$  be the probability of obtaining a 'six' in a single throw of the die.

$$\text{Then, } p = \frac{1}{6} \text{ and } q = 1 - \frac{1}{6} = \frac{5}{6}$$

Obtaining 3<sup>rd</sup> six in the 6<sup>th</sup> throw of the die means that in first five throws there are 2 sixes and the 3<sup>rd</sup> six is obtained in 6<sup>th</sup> throw.Therefore, required probability =  ${}^5C_2 p^2 q^5 \times p$  1

$$= {}^5C_2 \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 \times \frac{1}{6}$$

$$= \frac{5!}{3!2!} \times \frac{1}{36} \times \frac{125}{216} \times \frac{1}{6}$$

$$= \frac{5 \times 4 \times 3! \times 125}{3! \times 2 \times 216 \times 216}$$

$$= \frac{20 \times 125}{2 \times 216 \times 216}$$

$$= \frac{10 \times 125}{216 \times 216}$$

$$= \frac{5 \times 125}{108 \times 216}$$

$$= \frac{625}{23328}$$

2

29. Sample size,  $n = 20$  agenciesSample mean  $\bar{x} = 55$  unitsSample SD,  $\sigma = 10$  unitsPopulation mean,  $\mu = 50$  units

Null Hypothesis : The advertising campaign is not successful

i.e.,  $H_0 : \mu = 50$  ( There is no significant difference between the mean weekly Sales of units in agencies before and after advertising campaign)Alternate Hypothesis,  $H_1 : \mu > 50$ . The advertising campaign was successful. 1Test statistic,  $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}$ 

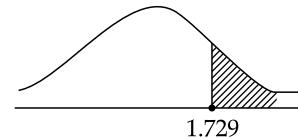
$$z = \frac{55 - 50}{\frac{10}{\sqrt{20-1}}} = \frac{5 \times \sqrt{20-1}}{10}$$

$$= \frac{5 \times \sqrt{19}}{10}$$

$$= \frac{5 \times 4.358}{10}$$

$$= \frac{21.794}{10} = 2.179$$

1



$$z > t_{0.05} = 1.729.$$

Thus, the advertising campaign was successful. 1

30. (a) We know that,

Amount of annual depreciation

$$= \frac{\text{Cast of asset} - \text{scrap value of asset}}{\text{Useful life in years}} \quad \frac{1}{2}$$

$$= \frac{4,50,000 - 1,00,000}{5}$$

$$= \frac{3,50,000}{5} = ₹ 70,000 \quad 1$$

Depreciation Schedule

Year	Book Value (Beginning of the Year)	Depreciation	Book Value (End of the of Year)
1.	₹ 4,50,000	₹ 70,000	₹ 3,80,000
2.	₹ 3,80,000	₹ 70,000	₹ 3,10,000
3.	₹ 3,10,000	₹ 70,000	₹ 2,40,000
4.	₹ 2,40,000	₹ 70,000	₹ 1,70,000
5.	₹ 1,70,000	₹ 70,000	₹ 1,00,000

1½

OR

- (b) We have,  
 Cost of car purchased by Amrita = ₹ 12,50,000  
 Amount of down payment made by Amrita  
 = ₹ 3,00,000  
 So, balanced amount = 12,50,000 – 3,00,000  
 = ₹ 9,50,000  
 Here,  $P = ₹ 9,50,000$   
 $i = \frac{15}{1200} = 0.0125$

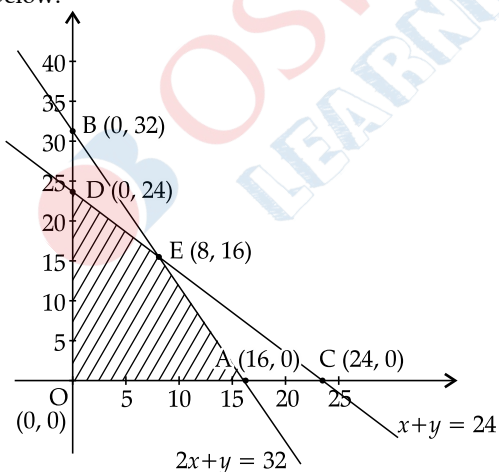
and  $n = 4$  years =  $4 \times 12$  months = 48 months    1  
 Let E be the monthly installment Amrita has to pay.

$$\begin{aligned} \text{Then } E &= \frac{Pi}{1 - (1+i)^{-n}} && 1 \\ &= \frac{9,50,000 \times 0.0125}{1 - (1 + 0.0125)^{-48}} \\ &= \frac{11,875}{1 - (1.0125)^{-48}} \\ &= \frac{11,875}{1 - 0.5508565} \\ &[\because \text{Given } (1.0125)^{-48} = 0.5508565] \\ &= \frac{11,875}{0.4491435} \\ &= ₹ 26,439.2115 \sim ₹ 26,439. && 1 \end{aligned}$$

31. Given, LPP is

$$\begin{aligned} \text{Max } Z &= 300x + 190y \\ \text{Subject to constraints : } &x + y \leq 24 \\ &2x + y \leq 32 \\ &x \geq 0, y \geq 0 \end{aligned}$$

The feasible region of the system of inequation given in constraints is shown in the figure given below.



Corner points of feasible region are O(0, 0), A(16, 0), D(0, 24) and E(8, 16).    2

The values of the objective function  $z$  at the corner points are given in the following table:

Comer point $(x, y)$	Value of $z = 300x + 190y$
A(16, 0)	$Z = 300 \times 16 + 190 \times 0$ = 4800

D(0, 24)	$Z = 300 \times 0 + 190 \times 24$ = 4560
E(8, 16)	$Z = 300 \times 8 + 190 \times 16$ = 2400 + 3040 = 5440 (Maximum)

Thus, maximum value of  $z$  is 5440 at E (8, 16).    1

## SECTION — D

32. Given matrix is

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

$A^{-1}$  exist if  $|A| \neq 0$

$$\begin{aligned} \text{Now, } |A| &= \begin{vmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{vmatrix} \\ &= -1(-4-3) - 1(12+1) + 2(9-1) \\ &= -1(-7) - 1(13) + 2(8) \\ &= 7 - 13 + 16 \\ &= 10 \neq 0 \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{\text{adj}A}{|A|}$$

$$\text{adj} A = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}'$$

where, cofactor  $c_{ij} = (-1)^{i+j} M_{ij}$

$$c_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} = -7$$

$$c_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -1 \\ -1 & 4 \end{vmatrix} = -13$$

$$c_{13} = (-1)^{1+3} \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} = 8$$

$$c_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 2$$

$$c_{22} = (-1)^{2+2} \begin{vmatrix} -1 & 2 \\ -1 & 4 \end{vmatrix} = -2$$

$$c_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 1 \\ -1 & 3 \end{vmatrix} = 2$$

$$c_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 3$$

$$c_{32} = (-1)^{3+2} \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} = 7$$

$$c_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix} = -2$$

$$\text{adj } A = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix} \quad 2$$

$$= \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} \quad 1$$

$$\begin{aligned} \text{Now, } AA^{-1} &= \frac{1}{10} \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 7-13+16 & -2-2+4 & -3+7-4 \\ -21+13+8 & 6+2+2 & 9-7-2 \\ +7-39+32 & -2-6+8 & -3+21-8 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 1 \\ &= I \end{aligned}$$

OR

Given system of equations;

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

This can be written as  $AX = B$ 

$$\text{where } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \text{Here, } |A| &= 1(1+3) + 1(2+3) + 1(2-1) \\ &= 4 + 5 + 1 \\ &= 10 \end{aligned}$$

Since  $|A| \neq 0$ Hence, the system of equations is consistent and has a unique solution given by  $X = A^{-1}B$ 

$$\text{Now, } A^{-1} = \frac{\text{adj } A}{|A|} \text{ and } \text{adj } A = C^T$$

$$c_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 1 + 3 = 4$$

$$c_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$$

$$c_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2-1) = 1$$

$$c_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = 2$$

$$c_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1-1) = 0$$

$$c_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$c_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3-1 = 2$$

$$c_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -(-3-2) = 5$$

$$c_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = (1+2) = 3$$

$$\text{Hence, the co-factor matrix } c = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$\text{adj } A = c^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

 $\therefore$ 

$$\begin{aligned} \text{Thus, } A^{-1} &= \frac{\text{adj } A}{|A|} \\ &= \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \end{aligned}$$

Solution is given by

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Thus, solution is  $x = 2, y = -1$  and  $z = 1$ .33. (a) Let the two numbers be  $x$  and  $y$ .

$$\text{Then, } x + y = 15 \quad \dots(i)$$

ATQ,

$$P = x^2y^3$$

or  $P = x^2(15-x)^3 \dots(ii)$  [from eq (i)]

$$\text{Now, } \frac{dP}{dx} = 2x(15-x)^3 + 3x^2(15-x)^2(-1)$$

$$\text{or, } \frac{dP}{dx} = 2x(15-x)^3 - 3x^2(15-x)^2$$

For maximum or minimum value of  $P$ , put  $\frac{dP}{dx} = 0$ 

$$\text{i.e., } 2x(15-x)^3 - 3x^2(15-x)^2 = 0$$

$$\Rightarrow 2x(15-x) = 3x^2$$

$$\Rightarrow 30x - 2x^2 = 3x^2$$

$$\Rightarrow x = 6$$

Form eq (i), we get

$$y = 9$$

Thus,  $x = 6$  and  $y = 9$

$$\text{Again, } \frac{d^2P}{dx^2} = 2(15-x)^3 - 6x(15-x)^2$$

$$= 2(15-x)^3 - 6x(15-x)^2 + 6x^2(15-x)$$

$$= 2(15-x)^3 - 12x(15-x)^2 + 6x^2(15-x)$$

At  $x = 6$ ,

$$\frac{d^2P}{dx^2} = 2(9)^3 - 12 \times 6(9)^2 + 6 \times (6)^2(9)$$

$$= 1458 - 5832 + 1944$$

$$= 3402 - 5832$$

$$= -2430 < 0$$

Thus, P is maximum when  $x = 6$  and  $y = 9$ .

So, the required two parts into which 15 should be divided are 6 and 9.

**OR**

(b) Given the equation of curve (parabola) is  $y^2 = 2x$ .

Let  $(x, y)$  be the nearest to the point  $(1, 4)$ .

We have,

$$x = \frac{y^2}{2} \quad \dots(i) \quad \frac{1}{2}$$

Using distance formula, we get

$$d = \sqrt{(x-1)^2 + (y-4)^2}$$

$$z = d^2 = (x-1)^2 + (y-4)^2 \quad 1$$

$$\text{or } z = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$$

[from (i)]

$$\text{or } z = \frac{y^4}{4} - 8y + 17$$

$$\therefore \frac{dz}{dy} = y^3 - 8 \quad 1$$

$$\text{Put } \frac{dz}{dy} = 0 \Rightarrow y^3 - 8 = 0 \Rightarrow y = 2 \quad \frac{1}{2}$$

Now, from eq (i), we get

$$x = \frac{(2)^2}{2} = 2 \quad \frac{1}{2}$$

Again differentiating w.r.t.  $y$ , we get

$$\frac{d^2z}{dy^2} = 3y^2$$

$$\left. \frac{d^2z}{dy^2} \right|_{\text{at } y=2} = 3(2)^2 = 12 > 0 \text{ Minima} \quad 1$$

So, the nearest point is  $(2, 2)$ .  $\frac{1}{2}$

34. Let  $y_t = a + bx$  be the trend line.

Let  $X = x - 2014$

Year (x)	Value (y)	X	X <sup>2</sup>	XY
2010	65	-4	16	-260
2012	68	-2	4	-136
2013	70	-1	1	-70
2014	72	0	0	0
2015	75	1	1	75
2016	67	2	4	134
2019	73	5	25	365
	$\Sigma Y = 490$	$\Sigma X = 1$	$\Sigma X^2 = 51$	$\Sigma XY = 108$

2

By the method of least squares, the values of 'a' and 'b' are obtained by solving the normal equations

$$\Sigma y = na + b\Sigma X$$

$$\text{and } \Sigma xy = a\Sigma X + b\Sigma X^2$$

$$\therefore 490 = 7a + b(1)$$

$$\text{and } 108 = a(1) + b(51)$$

$$\text{or, } 7a + b = 490 \quad \dots(i)$$

$$\text{and } a + 51b = 108 \quad \dots(ii)$$

On solving eqs. (i) & (ii), we get

$$a = 69.8933 \text{ and } b = 0.7472 \quad 2$$

$$\text{Thus, } y_t = 69.8933 + (0.7472)x$$

$$\text{or } y_t = 69.89 + 0.75x$$

Trend Values,

$$\text{for 2010, } y_t = 69.89 + 0.75(-4) = 66.89$$

$$\text{for 2012, } y_t = 69.89 + 0.75(-2) = 68.39$$

$$\text{for 2013, } y_t = 69.89 + 0.75(-1) = 69.14$$

$$\text{for 2014, } y_t = 69.89 + 0.75(0) = 69.89$$

$$\text{for 2015, } y_t = 69.89 + 0.75(1) = 70.64$$

$$\text{for 2016, } y_t = 69.89 + 0.75(2) = 71.39$$

$$\text{for 2019, } y_t = 69.89 + 0.75(5) = 73.64 \quad 1$$

35. **Compound Annual Growth Rate (CAGR):**

The compound annual growth rate (CAGR) is annualized average rate of revenue growth between two given years, assuming growth take place at an exponentially compounded rate.

$$\text{CAGR} = \left( \frac{\text{Ending Value}}{\text{Beginning Value}} \right)^{\frac{1}{n}} - 1$$

$$\text{or } \text{CAGR} = \left( \frac{\text{EV}}{\text{BV}} \right)^{\frac{1}{n}} - 1 \quad 1$$

Here, n is investment period

$$\text{CAGR of Vikas's investment} = \left( \frac{\text{EV}}{\text{BV}} \right)^{\frac{1}{n}} - 1$$

$$= \left( \frac{14,000}{10,000} \right)^{\frac{1}{6}} - 1$$

$$= (1.4)^{\frac{1}{6}} - 1$$

$$= 1.058 - 1$$

$$= 0.058$$

$$= 5.8\%$$

3

## SECTION — E

## 36. Case Study-1

Given,  $n = 100$ ,  $p = 6\% = 0.06$

$\therefore$  Mean,  $np = 100 \times 0.06 = 6 (\lambda)$

$$(i) \quad P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

$$P(X = 0) = \frac{e^{-6} 6^0}{0!} \\ = e^{-6} \\ = 0.0024$$

(Given) 1

$$(ii) \quad P(X = 2) = \frac{e^{-6} 6^2}{2!} \\ = \frac{e^{-6} \cdot 36}{2}$$

$$= e^{-6} \times 18 = 0.0024 \times 18 \\ = 0.0432$$

$$(iii) (a) \quad P(X = 1) = \frac{e^{-6} 6^1}{1!} \\ = e^{-6} \times 6 \\ = 0.0024 \times 6 \\ = 0.0144$$

1

Now, Probability that sample of 100 bulbs will include not more than one defective bulb

$$P(X \leq 1) = P(X = 0) + P(X = 1) \\ = [0.0024 + 0.0144] \\ = 0.016$$

1

OR

(iii) (b) We know that mean and variance of the Poisson distribution are same.

1

Thus, mean = 6 and variance = 6.

1

## 37. Case Study-2

(i) Here, given profit on tennis racket is ₹ 20 and profit on cricket bat is ₹ 10

Thus, total profit Max.  $Z = 10x + 20y$  1

(ii) Since, tennis racket requires 1.5 hours and cricket bat requires is 3 hours of machine time. Also, there is maximum 42 hours of machine time available.

$$1.5y + 3x \leq 42 \quad \dots(i) \\ y + 2x \leq 28$$

Since, tennis racket requires 3 hours and cricket bat requires 1 hour of craftsmanship. Also, there is maximum 24 hours of craftsmanship time available.

$$\therefore 3y + x \leq 24 \quad \dots(ii)$$

Thus, required constraints are :

$$2x + y \leq 28 \\ x + 3y \leq 24$$

1

(iii) (a) The L.P.P for given problem can be formulated as

$$\text{Max. } Z = 10x + 20y$$

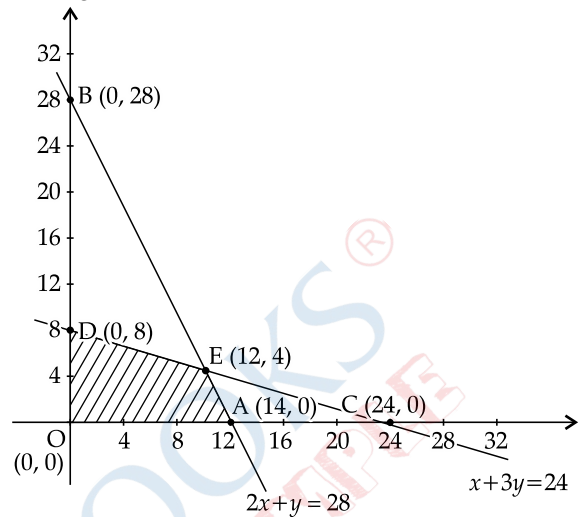
Subject to constraints:

$$2x + y \leq 28$$

$$x + 3y \leq 24$$

$$x \geq 0, y \geq 0$$

Plotting all constraints, we get the following feasible region.



1

Corner Points	Value of $Z = 10x + 20y$
A (14, 0)	140
E (12, 4)	200 (Maximum)
D (0, 8)	160

Thus, maximum profit earned by factory is ₹ 200. 1

OR

(b) From the above solution (iii - a), we get maximum profit at E (12, 4).

So, 12 cricket bats and 4 tennis rackets must be made by factory in order to maximize its profit.

## 38. Case Study-3

(i) Given,  $P = ₹ 30,00,000$ ,  $i = 7.5\% p.a.$   
 $= \frac{7.5}{1200} = 0.00625$

and  $n = 20 \text{ years} = 240 \text{ months}$

$$\text{Since, } EMI = \frac{Pi(1+i)^n}{(1+i)^n - 1} \\ = \frac{30,00,000 \times 0.00625(1+0.00625)^{240}}{(1+0.00625)^{240} - 1} \\ = \frac{30,00,000 \times 0.00625(1+0.00625)^{240}}{(1.00625)^{240} - 1} \\ = \frac{18750 \times 4.4608}{4.4608 - 1} \\ = \frac{83640}{3.4608} \\ = ₹ 24167.82 \\ = ₹ 24168$$

1

(ii) Interest paid by Mr. Aggarwal in 150<sup>th</sup> installment

$$= \frac{EMI[(1+i)^n - 1]}{(1+i)^{n-150+1}}$$

$$= \frac{24167.82[(1.00625)^{240-150+1} - 1]}{(1.00625)^{240-150+1}}$$

$$= \frac{24167.82[(1.00625)^{91} - 1]}{(1.00625)^{91}}$$

$$= \frac{24167.82[1.7629 - 1]}{1.7629}$$

$$= \frac{18437.629}{1.7629}$$

$$= ₹ 10,458.6929$$

$$\begin{aligned} \text{Principal paid} &= 24167.82 - 10,458.69 \\ &= ₹ 13,709.13 \sim ₹ 13709 \end{aligned}$$

$$\begin{aligned} \text{(iii) (a) Total interest paid} &= n(\text{EMI}) - P && \mathbf{1} \\ &= 240 \times 24167.82 - 30,00,000 \\ &= 58,00,276.8 - 30,00,000 \\ &= 28,00,276.8 \\ &= 28,00,277 && \mathbf{2} \end{aligned}$$

**OR**

$$\begin{aligned} \text{(b) Total amount paid} &= n \cdot \text{EMI} \\ &= 240 \times 24,167.82 \\ &= ₹ 58,00,276.8 \\ &= ₹ 58,00,277 \end{aligned}$$

$\square \square$