Solved Paper 2023
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This question paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections-Section **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions Number **1** to **18** are Multiple Choice Questions (MCQs) type and Questions Number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C,** Questions Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1** SA type question is of **2** marks. Internal choice is provided in **2** marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B, 3** questions in Section **C, 2** questions in Section **D** and **2** questions in Section **E.***

(ix) *Use of calculators is **NOT** allowed.*

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SECTION - A

1. Let $A = \{3, 5\}$. Then number of reflexive relations of A is:

(a) 2 **(b)** 4

(c) 0 **(d)** 8

Sol. Option (b) is correct

Explanation: The number of reflexive relations is $2^{n(n-1)}$

$\Rightarrow \quad 2^{2(2-1)} = 4$

2. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to:

(a) 1 **(b)** $\frac{1}{2}$

(c) $\frac{1}{3}$ **(d)** $\frac{1}{4}$

Sol. Option (a) is correct

Explanation: $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$

$$\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right) = \sin\frac{\pi}{2} = 1$$

3. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals:

(a) A **(b)** A + I

(c) I – A **(d)** A – I

Sol. Option (c) is correct

Explanation:

$$A^2 - A + I = 0$$
$$A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$$
$$IA - I + A^{-1} = 0$$
$$A^{-1} = I - A$$

4. If $A = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B = \begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A = B^2$, then x equals:

(a) ± 1 **(b)** -1

(c) 1 **(d)** 2

Sol. Option (c) is correct

Explanation:

$$A = B^2$$
$$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} = \begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$$
$$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} = \begin{bmatrix}x^2 & 0\\ x+1 & 1\end{bmatrix}$$
$$x^2 = 1 \text{ and } x + 1 = 2$$
$$\therefore \quad x = \pm 1$$
$$\therefore \quad x = 1$$

Hence $x = 1$

5. **If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is:**

(a) 1 (b) 2
(c) 3 (d) 4

Sol. Option (d) is correct

Explanation:

$$\begin{bmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \alpha(2-4) - 3(1-1) + 4(4-2) = 0$$
$$-2\alpha + 8 = 0$$
$$\alpha = 4$$

6. **The derivative of x^{2x} w.r.t. x is:**

(a) x^{2x-1} (b) $2x^{2x}\log x$
(c) $2x^{2x}(1+\log x)$ (d) $2x^{2x}(1-\log x)$

Sol. Option (c) is correct

Explanation:

$$y = x^{2x}$$
$$\log y = 2x\log x$$
$$\frac{d}{dx}\log y = \frac{d}{dx}2x\log x$$
$$\frac{1}{y}\frac{dy}{dx} = 2\left[x\frac{d}{dx}\log x + \log x\frac{d}{dx}x\right]$$
$$\frac{dy}{dx} = 2y\left[x\times\frac{1}{x} + \log x\right]$$
$$\frac{dy}{dx} = 2x^{2x}[1+\log x]$$

7. **The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at:**

(a) $x = 1$ (b) $x = 1.5$
(c) $x = -2$ (d) $x = 4$

Sol. Option (b) is correct

Explanation: The function $f(x) = [x]$ is continuous for all except all integral values of x.

8. **If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to:**

(a) x (b) $-x$
(c) $16x$ (d) $-16x$

Sol. Option (d) is correct

Explanation:

$$x = A\cos 4t + B\sin 4t$$
$$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$$
$$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$$
$$= -16(A\cos 4t + B\sin 4t)$$
$$\therefore \quad \frac{d^2x}{dt^2} = -16x$$

9. **The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is:**

(a) $(-1, \infty)$ (b) $(-2, -1)$
(c) $(-\infty, -2)$ (d) $(-1, 1)$

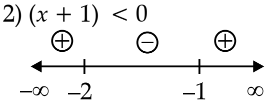
Sol. Option (b) is correct

Explanation:

$$f(x) = 2x^3 + 9x^2 + 12x - 1$$
$$f'(x) = 6x^2 + 18x + 12 - 0$$

for decreasing function $f'(x) < 0$

$$6(x^2 + 3x + 2) < 0$$
$$6(x+2)(x+1) < 0$$



$f(x)$ is decreasing in interval $(-2, -1)$

10. **$\int \frac{\sec x}{\sec x - \tan x}dx$ equals:**

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$
(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

Sol. Option (b) is correct

Explanation:

$$\int \frac{\sec x}{\sec x - \tan x}dx$$
$$\int \frac{\sec x(\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}dx$$
$$\int \sec^2 x\,dx + \int \sec x\tan x\,dx \qquad [\sec^2 x - \tan^2 x = 1]$$

$\tan x + \sec x + c$

11. **$\int_{-1}^{1} \frac{|x-2|}{x-2}dx,\ x \neq 2$ is equal to:**

(a) 1 (b) -1
(c) 2 (d) -2

Sol. Option (d) is correct

Explanation:

$$\int_{-1}^{1} \frac{|x-2|}{x-2}dx$$
$$\int_{-1}^{1} \frac{-(x-2)}{x-2}dx = [-x]_{-1}^{1} = -2$$

12. **The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is:**

(a) 2 (b) 3
(c) 5 (d) 0

Sol. Option (b) is correct

Explanation:

$$\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^3\right] = 3\left(\frac{dy}{dx}\right)^2\frac{d^2y}{dx^2}$$

order is 2 and degree is 1

$\therefore$ required answer $2 + 1 = 3$

13. **Two vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if:**

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$

(d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

Sol. Option (b) is correct

Explanation:

Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

are collinear if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$

14. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is:

(a) 1 **(b)** 5

(c) 7 **(d)** 12

Sol. Option (c) is correct

Explanation:

$$\vec{a} = 6\hat{i} - 2\hat{j} + 3\hat{k}$$

$$|\vec{a}| = |\sqrt{6^2 + 2^2 + 3^2}| = 7 \text{ units}$$

15. If a line makes angles of 90°, 135° and 45° with the *x*, *y* and *z* axes respectively, then its direction cosines are:

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **(b)** $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ **(d)** $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Sol. Option (a) is correct

Explanation: Direction cosines are cos 90°, cos 135° and cos 45°

$$\therefore \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

16. The angle between the lines 2*x* = 3*y* = – *z* and 6*x* = – *y* = – 4*z* is:

(a) 0° **(b)** 30°

(c) 45° **(d)** 90°

Sol. Option (d) is correct

Explanation:

$2x = 3y = -z \qquad 6x = -y = -4z$

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \qquad \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\cos\theta = \left|\frac{3\times 2 + 2(-12) + (-6)(-3)}{\sqrt{3^2 + 2^2 + 6^2}\sqrt{2^2 + (-12)^2 + (-3)^2}}\right|$$

$$\cos\theta = \left|\frac{6 - 24 + 18}{7.\sqrt{157}}\right| = 0$$

$$\therefore \qquad \theta = \frac{\pi}{2} = 90°$$

17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equals to:

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

Sol. Option (c) is correct

Explanation:

$$P(A) = \frac{4}{5}, P(A \cap B) = \frac{7}{10}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}}$$

$$= \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

18. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is:

(a) $\frac{27}{32}$ **(b)** $\frac{5}{32}$

(c) $\frac{31}{32}$ **(d)** $\frac{1}{32}$

Sol. Option (c) is correct

Explanation:

Probability of the event that at least one head comes up

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$

Assertion-Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. Assertion (A): Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R): Let E and F be two events with a random experiment, then P(F/E) = $\frac{P(E \cap F)}{P(E)}$.

Sol. Option (a) is correct

Explanation:

$$S = \{(HH), (H, T), (T, H) (T, T)\}$$

Probability of getting two heeds; {H, H}

$$n(F) = \frac{1}{4}$$

Probability of getting at least one head:

$\{(H, H), (H, T) (T, H)\}, n(E) = \frac{3}{4}$

Required probability $P\left(\frac{F}{E}\right)$

$$= P\frac{(E\cap F)}{P(E)} = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

20. Assertion (A): $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx = 3$

Reason (R): $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Sol. Option (a) is correct

Explanation:

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$= \int_2^8 \frac{\sqrt{10-10+x}}{\sqrt{10-x}+\sqrt{10-10+x}}dx$$

$$= \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}dx$$

$$2I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}dx + \int_2^8 \frac{\sqrt{x}}{\sqrt{(10-x}+\sqrt{x}}dx$$

$$2I = \int_2^8 dx = [x]_2^8 = 6$$

$$I = 3$$

SECTION - B

21. Write the domain and range (principle value branch) of the following functions:

$f(x) = \tan^{-1} x$

Sol.

$$f(x) = \tan^{-1} x$$

Domain = Real number

$$\text{Range} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

22. (a) If $f(x) = \begin{cases} x^2, \text{ if } x \geq 1 \\ x, \text{ if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(b) Find the value(s) of 'λ', if the function

$f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, \text{ if } x \neq 0 \\ 1, \text{ if } x = 0 \end{cases}$ is continuous at $x = 0$.

Sol. (a)

$$f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$$

$f(x)$ is defined at $x = 1$ and $f(1) = 1$

$$f'_-(1) = \lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h}$$

$$= \frac{1+h-1}{h} = 1$$

$$f'_+(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1+h)^2-1}{h}$$

$$= \lim_{h\to 0} \frac{h(2+h)}{h}$$

$$f'_-(1) \neq f'_+(1)$$

Hence $f(x)$ is not differentiable at $x = 1$

OR

(b) $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, \text{ if } x \neq 0 \\ 1, \text{ if } x = 0 \end{cases}$ is continuous at $x = 0$.

$$f(0) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0^-} \frac{\sin^2 \lambda(0-h)}{(0-h)^2}$$

$$= \lim_{h\to 0} \frac{\sin^2 \lambda h}{h^2}$$

$$= \lim_{h\to 0} \lambda^2 \left(\frac{\sin \lambda h}{\lambda h}\right)^2$$

$$= \lambda^2 \times 1 = \lambda^2$$

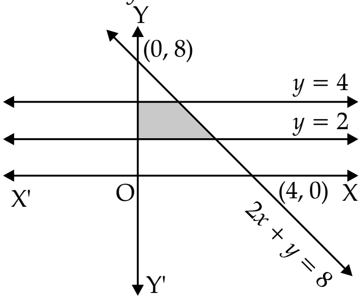
$$f(0) = \lim_{x\to 0^-} f(x)$$

$$1 = \lambda^2 \therefore \lambda = \pm 1$$

23. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

Sol.

$$2x + y = 8$$
$$y = 2$$
$$y = 4$$



$$\text{Required Area} = \int_2^4 x\,dy = \int_2^4 \frac{8-y}{2}dy$$

$$= \left[4y - \frac{y^2}{4}\right]_2^4$$

$$= [16 - 4 - 8 + 1]$$

Required area = 5 unit^2

24. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent

side are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Sol. **(a)** Given $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$

Since, $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$

Now, $|\vec{a}\times\vec{b}| = \frac{\vec{a}\times\vec{b}}{\hat{n}} = \frac{|\vec{a}||\vec{b}|\sin\theta\, \hat{n}}{\hat{n}}$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$1 = 3\times\frac{2}{3}\times\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30°$$

Angle between $\vec{a}$ and $\vec{b}$ is 30°

OR

(b) $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

Required area of $||^{gm}$

$$= |\vec{a}\times\vec{b}|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= i(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$$

$$= |20\hat{i} + 5\hat{j} - 5\hat{k}|$$

$$= |\sqrt{400+25+25}|$$

$$= 15\sqrt{2}\ \text{unit}^2$$

25. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, – 1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

Sol. Given point A (1, 2, – 1)

Given line $5x - 25 = 14 - 7y, = 35z,$

$= 5(x-5), = -7(y-2), = 35z,$

$= \frac{x-5}{7} = \frac{y-7}{-5}, = \frac{z}{1}$ Divide by 35

Direction ration of the line (7, –5, 1)

∴ Direction ratio of the parallel line (7, –5, 1)

Equation of the line passing through the point A (1, 2, –1) and parallel to the given line

$$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$

vector form of the line

$\hat{i} + 2\hat{j} - \hat{k} + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

SECTION - C

26. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

Sol.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^2 \times A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

L.H.S.

$A^3 - 23A - 40\,I$

$$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^3 - 23\,A - 40I$

$$= \begin{bmatrix} 63-23-40 & 46-46-0 & 69-69-0 \\ 69-69-0 & -6+46-40 & 23-23-0 \\ 92-92-0 & 46-46-0 & 63-23-40 \end{bmatrix}$$

$$A^3 - 23\,A - 40\,I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$A^3 - 23A - 40I = 0$ **Hence proved.**

27. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ **w.r.t.** $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, **then prove that** $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

Sol. (a) $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ and $v = \sin^{-1}(2x\sqrt{1-x^2})$

Let $x = \sin\theta$

$$u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\frac{1}{\cos\theta} = \theta$$
$$\therefore \quad u = \sin^{-1}x$$
$$v = \sin^{-1}(2\sin\theta(\sqrt{1-\sin^2\theta})$$
$$= \sin^{-1}(2\sin\theta\cos\theta)$$
$$v = \sin^{-1}\sin 2\theta = 2\theta$$
$$v = 2\sin^{-1}x$$
$$\frac{du}{dx} = \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{dv}{dx} = \frac{d}{dx}(2\sin^{-1}x) = \frac{2}{\sqrt{1-x^2}}$$
$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2}$$

OR

(b)
$$y = \tan x + \sec x$$
$$\frac{dy}{dx} = \frac{d}{dx}(\tan x) + \frac{d}{dx}(\sec x)$$
$$\frac{dy}{dx} = \sec^2 x + \sec x\tan x$$
$$\frac{dy}{dx} = \sec x(\sec x + \tan x)$$
$$\frac{d^2y}{dx^2} = (\sec x + \tan x)\frac{d}{dx}\sec x + \sec x\frac{d}{dx}(\sec x + \tan x)$$
$$\frac{d^2y}{dx^2} = (\sec x + \tan x)\sec x\tan x + \sec x(\sec x\tan x + \sec^2 x)$$
$$= (\sec x + \tan x)\sec x\tan x + \sec^2 x(\sec x + \tan x)$$
$$= \sec x(\sec x + \tan x)(\tan x + \sec x)$$
$$= \sec x(\sec x + \tan x)^2$$
$$= \frac{1}{\cos x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)^2$$
$$= \frac{1}{\cos x}\left(\frac{1+\sin x}{\cos x}\right)^2$$
$$= \frac{1}{\cos x}\frac{(1+\sin x)^2}{(1-\sin^2 x)}$$
$$= \frac{1+\sin x}{\cos x(1-\sin x)}$$
$$\frac{d^2y}{dx^2} = \frac{(1+\sin x)}{\cos x(1-\sin x)} \times \frac{1-\sin x}{1-\sin x}$$
$$= \frac{1-\sin^2 x}{\cos x(1-\sin x)^2}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$$

Hence proved.

28. (a) Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}dx$

OR

(b) Find: $\int \frac{x^4}{(x-1)(x^2+1)}dx$

Sol.
$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}dx$$
$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}}dx$$
$$I = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}dx$$
$$= \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}dx$$
$$2I = \int_0^{2\pi} \frac{1+e^{\sin x}}{1+e^{\sin \pi}}dx$$
$$2I = \int_0^{2\pi} 1dx$$
$$= [x]_0^{2\pi}$$
$$2I = 2\pi - 0$$
$$\therefore \quad I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} = \pi$$

OR

$$\int \frac{x^4}{(x-1)(x^2+1)}dx$$
$$\int \frac{x^4}{x^3-x^2+x-1}dx$$

	$x + 1$
$x^3 - x^2 + x - 1$	x^4
	$x^4 - x^3 + x^2 - x$
	(–) (+) (–) (+)
	$x^3 - x^2 + x$
	$x^3 - x^2 + x - 1$
	(–) (+) (–) (+)
	$+ 1$

$$\int \frac{x^4}{(x-1)(x^2+1)}dx = \int\left[x+1+\frac{1}{(x-1)(x^2+1)}\right]dx$$

$$= \frac{x^2}{2} + x + \int \frac{1}{(x-1)(x^2+1)} dx$$

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$1 = A(x^2 + 1) + (Bx + C)(x - 1)$

$1 = (A + B)x^2 + (A - C) + x(C - B)$

$\therefore \quad A + B = 0 \Rightarrow A = -B$

$C - B = 0 \Rightarrow C = B$

$A - C = 1$

$-2C = 1 \Rightarrow C = -\frac{1}{2}$

$B = -\frac{1}{2}$ and $A = \frac{1}{2}$

$$\int \frac{1}{(x-1)(x^2+1)} dx = \frac{1}{2}\int\left[\frac{+1}{x-1} + \frac{-x-1}{x^2+1}\right]dx$$

$$= \frac{1}{2}\left[\int \frac{dx}{x-1} - \int \frac{x}{x^2+1}dx - \int \frac{dx}{x^2+1}\right]$$

$$= \frac{1}{2}\left[\log|x-1| - \frac{1}{2}\log|x^2+1| - \tan^{-1}x\right] + c_1$$

$$= \frac{1}{2}\log\left|\frac{x-1}{\sqrt{x^2+1}}\right| - \frac{1}{2}\tan^{-1}x + c$$

$$\int \frac{x^4}{(x-1)(x^2+1)} dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log\left|\frac{x-1}{\sqrt{x^2+1}}\right| - \frac{1}{2}\tan^{-1}x + c$$

29. Find the area of the following region using integration:

$\{(x, y) : y^2 \le 2x$ and $y \ge x - 4\}$

Sol. Given: $y^2 = 2x$ ...(1)

$y = x - 4$ ...(2)

Required area is OABCO

from (1) and (2)

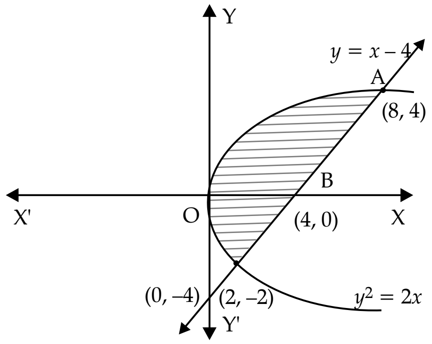
$(x - 4)^2 = 2x$

$x^2 - 10x + 16 = 0$

$(x - 8)(x - 2) = 0$

$x = 8$ and $x = 2$

$\therefore$ Intersection points $(2, -2)$ and $(8, 4)$



$$\text{Required Area} = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$$

$$= \left(8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3}\right)$$

$= 30 - 12$

$= 18$ unit^2

30. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

OR

(b) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

Sol. (a) P (0, 2, 3)

line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

General point on the line is

$[(5\lambda - 3), (2\lambda + 1), (3\lambda - 4)]$

Direction ratio of the perpendicular line

$[(5\lambda - 3), (2\lambda - 1), (3\lambda - 7)]$

$\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$38\lambda - 38 = 1$

$\lambda = 1$

$\therefore$ foot of perpendicular line is

$[(5 - 3), (2 + 1), (3 - 4)]$

$(2, 3, -1)$

OR

(b) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{c} + \vec{b}.\vec{b}$

$+ \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c} = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$3^2 + 4^2 + 2^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -29$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{29}{2}$

31. Find the distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

Sol. Given:

$$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

$$\vec{r} = (3\hat{i}+3\hat{j}-5\hat{k})+\mu(4\hat{i}+6\hat{j}+12\hat{k})$$

These lines are parallel

∴ Distance between two parallel lines

$$= \frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$$

$$a_1 = \hat{i}+2\hat{j}-4\hat{k}$$

$$a_2 = 3\hat{i}+3\hat{j}-5\hat{k}$$

$$\vec{a}_2-\vec{a}_1 = 2\hat{i}+\hat{j}-\hat{k}$$

and $|\vec{b}| = \sqrt{4+9+36} = 7$

$$\vec{b}\times(\vec{a}_2-\vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & +6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-3-6)-\hat{j}(-2-12)+\hat{k}(2-6)$$

$$= -9\hat{i}+14\hat{j}-4\hat{k}$$

$$|\vec{b}\times(\vec{a}_2-\vec{a}_1)| = |\sqrt{9^2+14^2+4^2}|$$

$$= |\sqrt{81+196+16}|$$

$$= \sqrt{293} \text{ units}$$

Shortest distance $= \dfrac{\sqrt{293}}{7}$ units

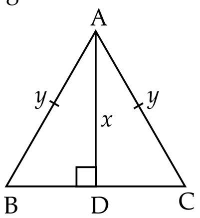
SECTION - D

32. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

Sol. (a) Let the length of the median be x cm



and side of equilateral triangle be y cm

In ΔABD $AB^2 = AD^2 + BD^2$ $(\angle D = 90°)$

$$y^2 = x^2 + \left(\frac{y}{2}\right)^2$$

$$\frac{3}{4}y^2 = x^2$$

$$y^2 = \frac{4}{3}x^2$$

$$y = \frac{2}{\sqrt{3}}x$$

$$\frac{d}{dt}(y) = \frac{2}{\sqrt{3}}\frac{d}{dt}(x)$$

$$\frac{dy}{dt} = \frac{2}{\sqrt{3}}\times\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{2}{\sqrt{3}}\times 2\sqrt{3}$$

$$= 4 \text{ cm/s}$$

Hence side of equilateral triangle be increase at 4cm/s

OR

(b) Let the number be x and y

∴ $x + y = 5$ ...(1)

$S = x^3 + y^3$ ...(2)

from (1) & (2)

$$S = x^3 + (5-x)^3$$

$$\frac{dS}{dx} = 3x^2 - 3(5-x)^2$$

$$\frac{dS}{dx} = 3x^2 - 75 + 30x - 3x^2$$

$$= -75 + 30x$$

for maximum and minimum

$$\frac{dS}{dx} = 0$$

$$-75 + 30x = 0$$

$$x = \frac{75}{30} = \frac{5}{2}$$

$$\frac{dS}{dx} = -75 + 30x$$

$$\frac{d^2S}{dx^2} = 30$$

$$\frac{d^2S}{dx^2} > 0$$

Hence S is minimum at $x = \dfrac{5}{2}$

Minimum value of $x^2 + y^2$

$$= \frac{25}{4}+\frac{25}{4} = \frac{25}{2}$$

33. Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\,dx$

Sol.

$$I = \int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\,dx$$

$$I = \int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x)\,dx$$

Let $\sin x = t$

$\cos x\,dx = dt$

when $x = 0, t = 0$

$x = \dfrac{\pi}{2}, t = 1$

$$\therefore \quad I = \int_0^1 2t\tan^{-1}t\,dt$$

$$= 2\left[\tan^{-1}t\int t\,dt - \int\left[\frac{d}{dx}\tan^{-1}t\right]\int t\,dt\right].dt$$

$$= 2\left[\frac{t^2}{2}\tan^{-1}t - \int\frac{t^2}{2(1+t^2)}dt\right]_0^1$$

$$= \left[t^2\tan^{-1}t\right]_0^1 - \int_0^1\frac{t^2}{(1+t^2)}dt$$

$$= \tan^{-1}1 - \left[\int_0^1 1dt - \int_0^1\frac{1}{1+t^2}dt\right]$$

$$= \frac{\pi}{4} - \left[t - \tan^{-1}t\right]_0^1$$

$$= \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

34. Solve the following Linear Programming Problem graphically :

Maximize: $P = 70x + 40y$

Subject to: $3x + 2y \le 9$,

$3x + y \le 9$

$x \ge 0, y \ge 0$

Sol. Maximize: $P = 70x + 40y$

$3x + 2y \le 9, 3x + y \le 9$

$x \ge 0, y \ge 0$

$3x + 2y = 9$

x	0	3
y	9/2	0

$3x + y = 9$

x	0	3
y	9	0

Feasible area is OABO

$P = 70x + 40y$

At (0, 0) $P = 70 \times 0 + 40 \times 0 = 0$

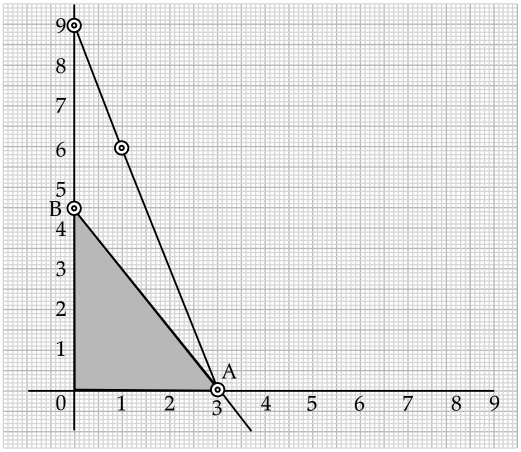
At (3, 0) $P = 70 \times 3 + 40 \times 0 = 210$

At $(0, \frac{9}{2})$ $P = 70 \times 0 + 40 \times \frac{9}{2} = 180$

Maximise at (3, 0)

$x = 3$ and $y = 0$

Maximum value = 210



35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

Sol. (a) Let E_1 = Student knows the answer

E_2 = Student guesses the answer

A = Student has answered the question correctly

$$\therefore P(E_1) = \frac{3}{5}, P(E_2) = 1 - \frac{3}{5} = \frac{2}{3}$$

$P\left(\frac{A}{E_1}\right)$ = Probability of student answered the question correctly given that he knows the answer

= 1

$P\left(\frac{A}{E_2}\right)$ = Probability of student answered the question correctly given that he guesses the answer

$= \frac{1}{3}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{1\times\frac{3}{5}}{1\times\frac{3}{5} + \frac{2}{5}\times\frac{1}{3}}$$

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{3}{5}}{\frac{11}{15}} = \frac{9}{11}$$

Required probability $= \frac{9}{11}$

OR

(b) Probability of prize ₹ 8 each $= \frac{2}{10} = \frac{1}{5}$

Probability of a prize ₹ 4 each $= \frac{5}{10} = \frac{1}{2}$

Probability of a prize ₹ 2 each $= \frac{3}{10}$

Mean value of the prize

$$= 8\times\frac{1}{5}+4\times\frac{1}{2}+2\times\frac{3}{10}$$

$$= \frac{8}{5}+2+\frac{3}{5}$$

$$= \frac{21}{5}$$

$\therefore$ Mean value of the prize = ₹ 4.20

SECTION - E

Case Study I

36. An organization conducted bike race under two different categories-Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions:

(i) How many relations are possible from B to G ?

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(iii) Let R : B → B be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(iii) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.
Check if f is bijective. Justify your answer.

Sol. (i) Number of relations $= 2^{mn}$

$$= 2^{2\times3} = 2^6 = 64$$

(ii) Number of functions from B to G

$$= 2^3 = 8$$

(iii) $R = \{(x, y) : x$ and y are students of same sex.$\}$

Since x and x are of the same sex

So $(x, x) \in$ R for all x

$\therefore$ R is reflexive

If x and y are of the same sex then y and x are also of the same sex

$\therefore$ R is symmetric

If $(x, y) \in$ R and $(y, z) \in$ R then $(x, z) \in$ R

Then x and z will be of the same sex

$\therefore$ R is transitive

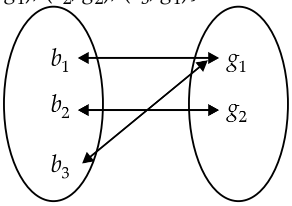
Sine R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation.

OR

(iii) Given

R = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$



Since b_1 and b_3 have the same image g_1

$\therefore$ R is not injective

Since all elements of G has a pre-image

$\therefore$ R is bijective

Case Study II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop. Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions:

(i) Convert the given above situation into a matrix equation of the form $AX = B$.

(ii) Find |A|.

(iii) Find A^{-1}.

OR

(iii) Determine $P = A^2 - 5A$.

Sol.

	Pen	Bags	Instrument
Gautam	5	3	1
Vikram	2	1	3
Ankur	1	2	4

(i)

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

where x = cost of Pen
y = cost of Bag
z = cost of Instrument

(ii)

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix}$$

$$= 5(4-6) - 3(8-3) + 1(4-1)$$

$$= -10 - 15 + 3$$

$$= -22$$

(iii)

$$C_{11} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2$$

$$C_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = -5$$

$$C_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$$C_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -10$$

$$C_{22} = \begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} = 19$$

$$C_{23} = -\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -7$$

$$C_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$$

$$C_{32} = -\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = -13$$

$$C_{33} = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = -1$$

$$\text{Adj } A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$= -\frac{1}{22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

OR

(iii) $P = A^2 - 5A$

$$= \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} - 5\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 25+6+1 & 15+3+2 & 5+9+4 \\ 10+2+3 & 6+1+6 & 2+3+12 \\ 5+4+4 & 3+2+8 & 1+6+16 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

Case study III

38. **An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is a homogenous function of degree n if F(λx, λy) = $\lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx}$ = F(x, y) = $g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.**

Based on the above, answer the following questions:

(i) **Show that $(x^2 - y^2)\,dx + 2xy\,dx = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.**

(ii) **Solve the above equation to find its general solution.**

Sol. **(i)** $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$$\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy}$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2\left(\frac{y^2}{x^2} - 1\right)}{2xy}$$

$$= \frac{\left(\frac{y^2}{x^2} - 1\right)}{\frac{2y}{x}}$$

$$\therefore \quad \frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

(ii)

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put $y = vx$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$= \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-(1 + v^2)}{2v}$$

$$\frac{2v}{1+v^2}dv = -\frac{dx}{x}$$

$$\int\frac{2v}{1+v^2}dv = -\int\frac{dx}{x}$$

$$\log|1+v^2| = -\log|x| + \log|c|$$

$$\log|x(1+v^2)| = \log|c|$$

$$x(1+v^2) = c$$

$$x\left(1+\frac{y^2}{x^2}\right) = c$$

$$x^2+y^2 = cx$$

Delhi Set-II 65/5/2

Note: *Except these, all other questions are from Delhi Set-1*

SECTION - A

4. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases}1, & \text{when } i \neq j\\ 0, & \text{when } i = j\end{cases}$, then A^2 is:

(a) $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$ **(b)** $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ **(d)** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Sol. Option (d) is correct

Explanation:

$$a_{ij} = \begin{cases}1 & \text{when } i \neq j\\ 0 & \text{when } i = j\end{cases}$$

$$A = \begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$$

$$A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

5. The value of the determinant $\begin{vmatrix}6 & 0 & -1\\2 & 1 & 4\\1 & 1 & 3\end{vmatrix}$ is:

(a) 10 **(b)** 8

(c) 7 **(d)** –7

Sol. Option (d) is correct

Explanation: $\begin{vmatrix}6 & 0 & -1\\2 & 1 & 4\\1 & 1 & 3\end{vmatrix}$

$$= 6(3-4) - 0(6-4) - 1(2-1)$$
$$= -6-1$$
$$= -7$$

9. The function $f(x) = x\,|x|, x \in$ R is differentiable

(a) only at $x = 0$ **(b)** only at $x = 1$

(c) in R **(d)** in R – {0}

Sol. Option (d) is correct

Explanation: $f(x) = x|x|$ is not differentiable at $x = 0$

11. The value of $\int_0^{\pi/4}(\sin 2x)dx$ is:

(a) 0 **(b)** 1

(c) $\frac{1}{2}$ **(d)** $-\frac{1}{2}$

Sol. Option (c) is correct

Explanation: $\int_0^{\pi/4}(\sin 2x)dx$

$$= \int_0^{\pi/4} 2\sin x\cos x\,dx$$

Let $\sin x = t$

$\cos x\,dx = dt$

when $x = 0$ then $t = 0$

when $x = \frac{\pi}{4}$ then $t = \frac{1}{\sqrt{2}}$

$$= 2\int_0^{1/\sqrt{2}} t\,dt$$

$$= 2\left[\frac{t^2}{2}\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

14. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is:

(a) $\frac{11}{15}$ **(b)** $\frac{11}{5\sqrt{3}}$

(c) $\frac{4}{5}$ **(d)** $\frac{3}{5\sqrt{3}}$

Sol. Option (a) is correct

Explanation: $\vec{a}$ makes equal acute angles from axis

$\therefore \cos\alpha = \cos\beta = \cos\gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

$$\therefore \vec{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{3}\hat{k}$$

$$\hat{a} = \frac{\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{3}\hat{k}}{|\vec{a}|}$$

$$= \frac{\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}}{\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}}$$

$$\hat{a} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$$

$$\vec{b} = 5\hat{i}+7\hat{j}-\hat{k}$$

Projection of vector $\vec{a}$ on $\vec{b}$

$$= \frac{\vec{a}.\vec{b}}{|\vec{b}|}$$

$$= \frac{\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right).(5\hat{i}+7\hat{j}-\hat{k})}{\left|\sqrt{5^2+7^2+1^2}\right|}$$

$$= \frac{\frac{5}{\sqrt{3}}+\frac{7}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt{75}}$$

$$= \frac{11}{\sqrt{3}}\times\frac{1}{5\sqrt{3}} = \frac{11}{15}$$

18. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P\left(\frac{B'}{A}\right)$ is:

(a) $\frac{1}{4}$ **(b)** $\frac{1}{8}$

(c) $\frac{3}{4}$ **(d)** 1

Sol. Option (c) is correct

Explanation: $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

A and B are two independent events

$\therefore$ $P(A).P(B) = P(A \cap B)$

$$P(B') = 1 - P(B) = 1-\frac{1}{4} = \frac{3}{4}$$

$$P(A \cap B') = P(A).P(B') = \frac{1}{3}.\frac{3}{4} = \frac{1}{4}$$

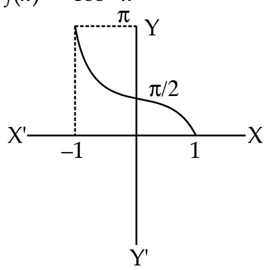
$$P\left(\frac{B'}{A}\right) = \frac{P(B'\cap A)}{P(A)}$$

$\therefore [P(B'\cap A) = P(B'\cap A)]$

$$= \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

21. Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$.

Sol. $f(x) = \cos^{-1}x$



25. Find the angle between the following two lines:

$$\vec{r} = 2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k});$$

$$\vec{r} = 7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$$

Sol.

$$\vec{r} = 2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$$

$$\vec{r} = 7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$$

$$\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}$$

$$\vec{c} = \hat{i}+2\hat{j}+2\hat{k}$$

$$\therefore \cos\theta = \frac{\vec{b}.\vec{c}}{|b||c|} = \frac{(3\hat{i}+2\hat{j}+6\hat{k})(\hat{i}+2\hat{j}+2\hat{k})}{\left|\sqrt{3^2+2^2+6^2}\right|\left|\sqrt{1^2+2^2+2^2}\right|}$$

$$\cos\theta = \frac{3+4+12}{7\times 3}$$

$$\cos\theta = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

SECTION - C

26. Using determinants, find the area of ΔPQR with vertices P(3, 1), Q(9, 3) and R(5, 7). Also, find the equation of line PQ using determinants.

26. $\text{Ar. } \Delta PQR = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(3-7)-1(9-5)+1(63-1)]$$

$$= \frac{1}{2}|-12-4+48|$$

$$= \frac{32}{2} = 16\text{unit}^2$$

Equation of the line PQ

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$x(1-3) - y(3-9) + 1(9-9) = 0$

$-2x+6y = 0$

$x - 3y = 0$

28. (a) Evaluate: $\int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1+\cos 2x} dx$

OR

(b) Find: $\int e^{x^2}(x^5 + 2x^3)dx$

Sol. (a) $\int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1+\cos 2x} dx$

$$f(x) = \frac{\cos 2x}{1+\cos 2x}$$

$$\therefore \quad f(-x) = \frac{\cos 2x}{1+\cos 2x}$$

Hence $f(x)$ is even function

$$I = \int_{-\pi/4}^{\pi/4} \frac{\cos 2x}{1+\cos 2x} dx$$

$$= 2\int_{0}^{\pi/4} \frac{\cos 2x}{1+\cos 2x} dx$$

$$= 2\int_{0}^{\pi/4} \left[1 - \frac{1}{1+\cos 2x}\right] dx$$

$$= 2\left[\int_{0}^{\pi/4} 1 dx - \int_{0}^{\pi/4} \frac{1}{2}\sec^2 x \, dx\right]$$

$$= 2\left[x - \frac{1}{2}\tan x\right]_0^{\pi/4}$$

$$I = 2\left[\frac{\pi}{4} - \frac{1}{2} - 0 - 0\right]$$

$$= \frac{\pi}{2} - 1$$

OR

(b) $\int e^{x^2}(x^5 + 2x^3)dx$

$$= \int x e^{x^2}(x^4 + 2x^2)dx$$

Let $x^2 = t$

$2x\,dx = dt$

$$= \frac{1}{2}\int e^t (t^2 + 2t)dt$$

$f(t) = t^2$

$\therefore \quad f'(t) = 2t$

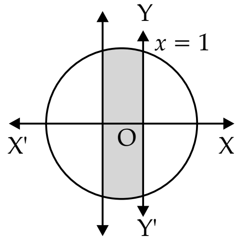
$\int e^t (f(t) + f'(t))dt = e^t f(t)$

$$\therefore \quad \frac{1}{2}e^t . t^2 + c = \frac{1}{2}x^4 e^{x^2} + c$$

29. Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

Sol. $x^2 + y^2 = 4$

$\therefore \quad y = \sqrt{4 - x^2}$



$$\text{Required Area} = 2\int_0^1 y\,dx$$

$$= 2\int_0^1 \sqrt{4-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{4-x^2} + \frac{1}{2} \times 4\sin^{-1}\frac{x}{2}\right]_0^1$$

$$= 2\left[\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{6} - 0 - 0\right]$$

$$= \left(\sqrt{3} + \frac{2\pi}{3}\right) \text{units}^2$$

SECTION - D

32. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x} dx$

Sol.

$$I = \int_0^{\pi} \frac{x}{1+\sin x} dx$$

$$I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)} dx$$

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx$$

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x} dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \pi\left[\int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \tan x \sec x \, dx\right]$$

$$= \pi[\tan x - \sec x]_0^{\pi}$$

$$= \pi[0 + 1 - 0 + 1] = 2\pi$$

$$\therefore \quad I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \pi$$

Delhi Set-III — 65/5/3

Note: *Except these, all other questions are from Delhi Set-1 & Set-2*

SECTION - A

1. Let R be a relation in the set N given by

$$R = \{(a, b) : a = b - 2, b > 6\}$$

Then

(a) $(8, 7) \in R$ **(b)** $(6, 8) \in R$

(c) $(3, 8) \in R$ **(d)** $(2, 4) \in R$

Sol. Option (b) is correct

Explanation: $a = b - 2, b > 6$

$\therefore \quad 6 = 8 - 2$

2. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ **and** $A = A^T$, **where** A^T **is the transpose of the matrix A, then**

(a) $x = 0, y = 5$ **(b)** $x = y$

(c) $x + y = 5$ **(d)** $x = 5, y = 0$

Sol. Option (b) is correct

Explanation:
$$A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$$
$$A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$
$$A = A^T$$
$$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$
$$\therefore \quad x = y$$

6. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is:

(a) 1 **(b)** –1

(c) $\frac{-1}{\sqrt{2}}$ **(d)** $\frac{1}{\sqrt{2}}$

Sol. Option (d) is correct

Explanation:
$$f(x) = |\cos x|$$
$$f\left(\frac{3\pi}{4}\right) = \left|\cos\frac{3\pi}{4}\right|$$
$$= \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

9. The function $f(x) = x^3 + 3x$ is increasing in interval

(a) $(-\infty, 0)$ **(b)** $(0, \infty)$

(c) $\mathbb{R}$ **(d)** $(0, 1)$

Sol. Option (c) is correct

Explanation:
$$f(x) = x^3 + 3x$$
$$f'(x) = 3x^2 + 3$$
for increasing $f'(x) > 0$
$$3x^2 + 3 > 0 \therefore x \in R \qquad (x \in R \therefore x^2 > 0)$$

12. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are:

(a) $1, \frac{2}{3}$ **(b)** 3, 1

(c) 3, 3 **(d)** 1, 2

Sol. Option (c) is correct

Explanation:
$$\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$$
$\therefore$ order = 3 and degree = 1

13. If $\vec{a}.\hat{i} = \vec{a}.(\hat{i}+\hat{j}) = \vec{a}.(\hat{i}+\hat{j}+\hat{k}) = 1$, then $\vec{a}$ is:

(a) $\hat{k}$ **(b)** $\hat{i}$

(c) $\hat{j}$ **(d)** $\hat{i}+\hat{j}+\hat{k}$

Sol. Option (b) is correct

Explanation: Let $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$
$$\vec{a}.\hat{i} = (x\hat{i}+y\hat{j}+z\hat{k})i = x$$
$$\vec{a}.(\hat{i}+\hat{j}) = (x\hat{i}+y\hat{j}+z\hat{k})(\hat{i}+\hat{j})$$
$$= x + y$$
$$\vec{a}.(\hat{i}+\hat{j}+\hat{k}) = (x\hat{i}+y\hat{j}+z\hat{k})(\hat{i}+\hat{j}+\hat{k})$$
$$= x + y + z$$
Given, $x = x + y = x + y + z = 1$

$\therefore$ $x = 1, y = 0$ and $z = 0$
$$\vec{a} = \hat{i}$$

SECTION - B

21. (a) Find the value of k for which the function f given as
$$f(x) = \begin{cases} \frac{1-\cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$
is continuous at $x = 0$

OR

(b) If $x = a \cos t$ and $y = b \sin t$, then find $\frac{d^2y}{dx^2}$.

Sol. (a) $f(x) = \begin{cases} \frac{1-\cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$
$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{1-\cos(0-h)}{2(0-h)^2}$$
$$= \lim_{h \to 0} \frac{1-\cos h}{2h^2}$$
$$= \lim_{h \to 0} \frac{2\sin^2 h/2}{2h^2}$$
$$= \lim_{h \to 0} \left(\frac{\sin h/2}{2h/2}\right)^2 = \frac{1}{4}$$
$f(x)$ is continuous at $x = 0$
$$\therefore \quad k = \frac{1}{4} \qquad \therefore \left(f(0) = \lim_{x \to 0^-} f(x)\right)$$

OR

(b) $x = a \cos t, y = b \sin t$
$$\frac{dx}{dt} = \frac{d}{dt}(a\cos t) = -a\sin t$$
$$\frac{dy}{dt} = \frac{d}{dt}(b\sin t) = b\cos t$$
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{b\cos t}{-a\sin t}$$
$$= \frac{-b}{a}\cot t$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a}\frac{d}{dx}\cot t$$

$$= +\frac{b}{a}\text{cosec}^2 t.\frac{dt}{dx}$$

$$= \frac{b}{a}\text{cosec}^2 \times \frac{1}{-a\sin t}$$

$$= -\frac{b}{a^2}\text{cosec}^3\, t$$

22. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1}1$.

Sol. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1}1$

$$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] + \frac{\pi}{4}$$

$$= \tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) + \frac{\pi}{4}$$

$$= \tan^{-1}\left(\sqrt{3}\right) + \frac{\pi}{4}$$

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

SECTION - C

26. Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Sol. $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$$= x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$

$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= -x^3 - x + x$

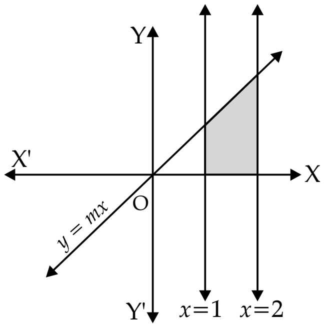
$= -x^3$

It is independent of 0. **Hence Proved.**

27. Using integration, find the area of the region bounded by $y = mx (m > 0)$, $x = 1$, $x = 2$ and the x-axis.

Sol. Required Area $= \int_1^2 y\,dx$

$$= \int_1^2 mx\,dx$$



$$= \left[\frac{mx^2}{2}\right]_1^2$$

$$= 2m - \frac{m}{2}$$

$$= \frac{3m}{2}\text{unit}^2$$

28. (a) Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

(b) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Sol. (a) Given line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

General point of the line $(3\lambda + 15, 8\lambda + 29, -5\lambda + 5)$

Direction ratio of the perpendicular line which is passes through (5, 7, 3) is

$(3\lambda + 10, 8\lambda + 22, -5\lambda + 2)$

lines are perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore 3(3\lambda + 10) + 8(8\lambda + 22) - 5(-5\lambda + 2) = 0$

$9\lambda + 30 + 64\lambda + 176 + 25\lambda - 10 = 0$

$98\lambda = -196$

$\lambda = -2$

foot of perpendicular $(-6 + 15, -16 + 29, 10 + 5)$

$= (9, 13, 15)$

OR

(b)

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$$

Perpendicular vector to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = \hat{i}(-6+4)-\hat{j}(-4-0)+\hat{k}(-2-0)$$

$$= -2\hat{i}+4\hat{j}-2\hat{k}$$

$$\text{Required unit vector} = \frac{-2\hat{i}+4\hat{j}-2\hat{k}}{|\sqrt{2^2+4^2+2^2}|}$$

$$= \frac{1}{\sqrt{6}}(-\hat{i}+2\hat{j}-\hat{k})$$

32. Solve the following Linear Programming Problem graphically:

Minimise : Z = 60x + 80y

Subject to constraints:

$3x + 4y \geq 8$

$5x + 2y \geq 11$

$x, y \geq 0$

Sol. $Z = 60x + 80y$

$3x + 4y \geq 8$ $\quad$ $5x + 2y \geq 11$

$3x + 4y = 8$ $\quad$ $5x + 2y = 11$

x	2	0	8/3
y	0.5	2	0

x	0	1	2	11/5
y	5.5	3	0.5	0

Point (x,y)	$Z = 60x + 80y$
At (0, 11/2)	$Z = 60 \times 0 + 80 \times \frac{11}{2}$ = 440
At (2, 0.5)	$Z = 60 \times 2 + 80 \times 0.5 = 160$
(8/3, 0)	$Z = 60 \times \frac{8}{3} + 80 \times 0 = 160$

$60x + 80y \leq 160$

$60x + 80y = 160$

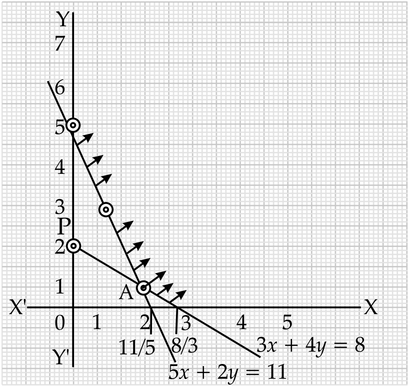
x	0	2	8/3
y	2	0.5	0

feasible region is unbounded so we consider

$60x + 80y \leq 160$

and feasible region.

Hence minimum value of Z is 160 which is each point of A and P.



Outside Delhi Set-I **65/2/1**

SECTION - A

1. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to:

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

Sol. Option (c) is correct

Explanation: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \quad A^{2023} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to:

(a) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

Sol. Option (b) is correct

Explanation: $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$

$$= (A + A^t) + (A - A^t)$$

$$2A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$$

$$2A^t = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$$

$$Q = A - A^t$$
$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

3. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a ∈ A, then the set A is:

(a) IR **(b)** {0}
(c) {4} **(d)** IR – {4}

Sol. Option (d) is correct

Explanation: $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix

$$\therefore \quad \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{vmatrix} \neq 0$$
$$1(3 - a) - 2(2 - 3) + 1(2a - 9) \neq 0$$
$$3 - a + 2 + 2a - 9 \neq 0$$
$$a \neq 4$$
$$\therefore \quad A = R - \{4\}$$

4. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is:

(a) 1 **(b)** –1
(c) 2 **(d)** 0

Sol. Option (d) is correct

Explanation: $|A| = |kA|$
$$|A| = k^n|A|$$
where n is the order of matrix
$$1 = k^n$$
$$k^2 = 1$$
$$\Rightarrow \quad k = \pm 1$$
Sum of all values of $k = +1 - 1 = 0$

5. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to:

(a) $a + b$ **(b)** $\frac{ax^2}{2} + bx$
(c) $\frac{ax^2}{2} + bx + c$ **(d)** b

Sol. Option (b) is correct

Explanation: $\frac{d}{dx}[f(x)] = ax + b$
$$\int \frac{d}{dx}[f(x)]dx = \int (ax + b)dx$$
$$= \frac{ax^2}{2} + bx + c$$
$$f(0) = 0$$
$$\therefore \quad c = 0$$
Hence $f(x) = \frac{ax^2}{2} + bx$

6. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is:

(a) 2 **(b)** 1
(c) not defined **(d)** 0

Sol. Option (b) is correct

Explanation:
$$\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$$
$$\cos\left(\frac{dy}{dx}\right) = y^2 - \sin x$$
$$\frac{dy}{dx} = \cos^{-1}(y^2 - \sin x)$$
Hence degree of the differential equation is 1

7. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is:

(a) $\frac{1}{y^2 - 1}$ **(b)** $\frac{1}{\sqrt{y^2 - 1}}$
(c) $\frac{1}{1 - y^2}$ **(d)** $\frac{1}{\sqrt{1 - y^2}}$

Sol. Option (d) is correct

Explanation:
$$(1 - y^2)\frac{dx}{dy} + yx = ay$$
$$\frac{dx}{dy} + \frac{y}{1 - y^2}x = \frac{ay}{1 - y^2}$$
$$\text{I.F. is } e^{\int \frac{y}{1-y^2}dy} = e^{-\frac{1}{2}\log|1-y^2|}$$
$$\text{I.F.} = e^{\log(1-y^2)^{-1/2}}$$
$$= \frac{1}{\sqrt{1 - y^2}}$$

8. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7) is:

(a) $2\hat{i} + 3\hat{j} - 6\hat{k}$ **(b)** $-2\hat{i} - 3\hat{j} + 6\hat{k}$
(c) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ **(d)** $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

Sol. Option (d) is correct

Explanation: P(2, 1, –1) and Q(4, 4, –7)
$$\overrightarrow{PQ} = (4 - 2)\hat{i} + (4 - 1)\hat{j} + (-7 + 1)\hat{k}$$

$$= 2\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\hat{PQ} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

9. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is:

(a) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ **(b)** $4\hat{i} + \hat{j} - 2\hat{k}$

(c) $5\hat{i} + 5\hat{j} - 7\hat{k}$ **(d)** $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

Sol. Option (b) is correct

Explanation: Position vector of $A = 2\hat{i} + 3\hat{j} - 4\hat{k}$

Position vector of midpoint $AB = 3\hat{i} + 2\hat{j} - 3\hat{k}$

Let Position vector of $B = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \frac{x+2}{2} = 3, \frac{y+3}{2} = 2, \frac{z-4}{2} = -3$

$\therefore x = 4, y = 1$ and $z = -2$

Hence position vector of $B = 4\hat{i} + \hat{j} - 2\hat{k}$

10. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is:

(a) 0 **(b)** 12

(c) $\frac{12}{\sqrt{13}}$ **(d)** $\frac{-12}{\sqrt{13}}$

Sol. Option (a) is correct

Explanation:

$\vec{a} = 2\hat{i} + 3\hat{j}, \ \vec{b} = 3\hat{i} - 2\hat{j}$

Projection of $\vec{a}$ on $\vec{b}$ is $\left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$

$$= \left|\frac{(2\hat{i} + 3\hat{j}).(3\hat{i} - 2\hat{j})}{\sqrt{3^2 + 2^2}}\right|$$

$$= \left|\frac{6-6}{\sqrt{13}}\right| = 0$$

11. Equation of a line passing through point (1, 1, 1) and parallel to *z*-axis is:

(a) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ **(b)** $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

(c) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ **(d)** $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

Sol. Option (d) is correct

Explanation: Direction ratio of *z*-axis is (0, 0, 1)

Line passing through the point (1, 1, 1) and parallel to *z*-axis

$$\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$$

12. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is:

(a) $\frac{1}{9}$ **(b)** $\frac{4}{9}$

(c) $\frac{1}{18}$ **(d)** $\frac{1}{2}$

Sol. Option (d) is correct

Explanation:

A = Sum of numbers obtained on the pair of dice is 9

= {(3, 6), (4, 5), (5, 4), (6, 3)}

B = Number obtained on 1 dice 1 and 4

= {(4, 5), (5, 4)}

$P\left(\frac{B}{A}\right)$ = Required probability

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{1}{2}$$

13. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to *x* is:

(a) $\sec^2\left(\frac{\pi}{4} - x\right) + c$ **(b)** $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$ **(d)** $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

Sol. Option (c) is correct

Explanation:

Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ w.r.t. x is

$$\int \frac{\tan x - 1}{\tan x + 1} dx = \int -\tan\left(\frac{\pi}{4} - x\right) dx$$

$$\frac{-\log\sec\left(\frac{\pi}{4} - x\right)}{-1}$$

$$= \log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$$

14. If (*a*, *b*), (*c*, *d*) and (*e*, *f*) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to:

(a) $2\Delta^2$ **(b)** $4\Delta^2$

(c) 2Δ **(d)** 4Δ

Sol. Option (b) is correct

Explanation:
$$\Delta = \frac{1}{2}\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$
$$2\Delta = \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$
$$4\Delta^2 = \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$$

15. The function $f(x) = x|x|$ is:

(a) continuous and differentiable at $x = 0$.

(b) continuous but not differentiable at $x = 0$.

(c) differentiable but not continuous at $x = 0$.

(d) neither differentiable nor continuous at $x = 0$.

Sol. Option (a) is correct

16. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{-y}{x}$ **(b)** $\frac{y}{x}$

(c) $\sec^2\left(\frac{y}{x}\right)$ **(d)** $-\sec^2\left(\frac{y}{x}\right)$

Sol. Option (b) is correct

Explanation:
$$\tan\left(\frac{x+y}{x-y}\right) = k$$
$$\frac{x+y}{x-y} = \tan^{-1} k$$
$$\frac{d}{dx}\frac{x+y}{x-y} = \frac{d}{dx}\tan^{-1} k$$
$$\frac{(x-y)\left(1+\frac{dy}{dx}\right)-(x+y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2} = 0$$
$$(x-y+x+y)\frac{dy}{dx} - 2y = 0$$
$$\frac{dy}{dx} = \frac{2y}{2x} = \frac{y}{x}$$

17. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ?

(a) $a = 9, b = 1$ (b) $a = 5, b = 2$

(c) $a = 3, b = 5$ (d) $a = 5, b = 3$

Sol. Option (c) is correct

Explanation: $Z = ax + by$

$\therefore$ $4a + 6b = 42$ ...(i)

$3a + 2b = 19$ ...(ii)

from (i) & (ii) $a = 3, b = 5$

18. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to:

(a) 40 **(b)** 96

(c) 120 **(d)** 136

Sol. Option (c) is correct

Explanation: $Z = 30x + 24y$

At (8, 0) $Z = 30 \times 8 + 24 \times 0 = 240$

At (0, 4) $Z = 30 \times 0 + 24 \times 4 = 96$ Minimum

At $\left(\frac{20}{3}, \frac{4}{3}\right)$ $Z = 30\times\frac{20}{3} + 24\times\frac{4}{3} = 232$ Maximum

= Maximum – Minimum

= 232 – 96 = 136

Assertion-Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. Assertion (A): Maximum value of $(\cos^{-1} x)^2$ is π^2.

Reason (R): Range of the principal value branch of $\cos^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

Sol. Option (c) is correct

Explanation: Range of the principal value of $\cos^{-1} x$ is $[0, \pi]$

20. Assertion (A): If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

Reason (R): The sum of squares of the direction cosines of a line is 1.

Sol. Option (a) is correct

Explanation:
$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$
$$1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1$$
$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

SECTION - B

21. (a) Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$.

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also write its range.

Sol. **(a)** $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$

$= \sin^{-1}\left(\sin\left(\pi-\frac{\pi}{4}\right)\right)+\pi+\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

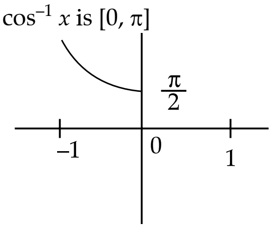
$= \sin^{-1}\sin\left(\frac{\pi}{4}\right)+\pi+\frac{\pi}{4}$

$= \frac{\pi}{4}+\pi+\frac{\pi}{4}$

$= \frac{3\pi}{2}$

OR

(b) Range of $\cos^{-1} x$ is $[0, \pi]$



22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate 1, y-coordinate is changing twice as fast at x-coordinate. Find the value of a.

Sol. $3y = ax^3 + 1$

Given: $\frac{dy}{dt} = 2\left(\frac{dx}{dt}\right)$ at $x = 1$

$3y = ax^3 + 1$

$\frac{3dy}{dt} = 3x^2a\frac{dx}{dt}+0$

$\frac{dy}{dt} = ax^2\frac{dx}{dt}$

$2\left(\frac{dx}{dt}\right) = a(1)^2\frac{dx}{dt}$

$\therefore$ $a = 2$

23. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b}-\vec{c}$.

Sol. $\vec{a}.(\vec{b}-\vec{c}) = \vec{a}.\vec{b}-\vec{a}.\vec{c}$

$= 0$ $(a.b = a.c)$

$\therefore$ Angle between $\vec{a}$ and $(\vec{b}-\vec{c})$ is right angle i.e., 90°

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.

Sol. Given line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$

Gen. point on the line $(\lambda, 2\lambda + 1, 2\lambda - 1)$

Distance from origin $\left|\sqrt{x_1^2+y_1^2+z_1^2}\right|$

$\therefore$ $\left|\sqrt{\lambda^2+(2\lambda+1)^2+(2\lambda-1)^2}\right| = \sqrt{11}$ (Given)

$\lambda^2 + (2\lambda + 1)^2 + (2\lambda - 1)^2 = 11$

$\lambda^2 + 4\lambda^2 + 4\lambda + 1 + 4\lambda^2 - 4\lambda + 1 = 11$

$9\lambda^2 = 9$

$\Rightarrow$ $\lambda = \pm 1$

if $\lambda = 1$ point on the line $(1, 3, 1)$

if $\lambda = -1$ point on the line $(-1, -1, -3)$

25. (a) If $y = \sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2 = 0$.

(b) If $f(x) = \begin{cases} ax+b\ ; & 0<x\le 1 \\ 2x^2-x; & 1<x<2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.

Sol. **(a)** $y = \sqrt{ax+b}$

$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{ax+b})$

$\frac{dy}{dx} = \frac{1}{2\sqrt{ax+b}}(a+0)$

$y\frac{dy}{dx} = \frac{a}{2}$

$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{a}{2}\right)$

$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0$ **Hence Proved**

OR

(b) $f(x) = ax + b$ $\quad 0 < x \le 1$

$= 2x^2 - x$ $\quad 1 < x < 2$

$\therefore$ $f(1) = a + b$ ...(i)

$\therefore$ $f'_-(1) = f'_+(1)$

$\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}$

$\lim_{h\to 0^+}\frac{2(1+h)^2-(a+b)-(1+h)}{h}$

$= \lim_{h\to 0^-}\frac{a(1+h)+b-(a+b)}{h}$

$\lim_{h\to 0}\frac{2h^2+3h+1-a-b}{h} = \lim_{h\to 0^-}\frac{ah}{h} = a$

$\lim_{h\to 0^+}\frac{2h^2+3h+1-a-b}{h} = a$

$f(x)$ is also continuous at $x = 1$

$\therefore$ $\lim_{x\to 1^+} f(x) = f(1) = \lim_{x\to 1^-} f(1)$

$\therefore$ $a + b = 1$ ...(ii)

$\lim_{h\to 0^+}\frac{2h^2+3h+1-1}{h} = a$

$\lim_{h\to 0}\frac{h(2h+3)}{h} = a$

$$\lim_{h\to 0}(2h+3) = a$$

$a = 3$

from $b = -2$

$\therefore$ $a = 3$ and $b = -2$

SECTION - C

26. (a) Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$ ·

OR

(b) Find $\int \frac{dx}{\sqrt{\sin^3 \cos(x-\alpha)}}$

Sol. (a)

$$I = \int_0^{\pi/4}\log(1+\tan x)dx$$

$$I = \int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$I = \int_0^{\pi/4}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$I = \int_0^{\pi/4}\log\frac{2}{1+\tan x}dx$$

$$2I = \int_0^{\pi/4}\log(1+\tan x)dx+\int_0^{\pi/4}\log\frac{2}{1+\tan x}dx$$

$$2I = \int_0^{\pi/4}\log(1+\tan x)\left(\frac{2}{1+\tan x}\right)dx$$

$$2I = \int_0^{\pi/4}\log 2dx$$

$$2I = [x\log 2]_0^{\pi/4}$$

$$2I = \frac{\pi}{4}\log 2$$

$$\therefore \quad I = \int_0^{\pi/4}\log(1+\tan x) = \frac{\pi}{8}\log 2$$

OR

(b) $\int\frac{dx}{\sqrt{\sin^3 x\cos(x-\alpha)}}$

$$= \int\frac{dx}{\sqrt{\sin^3 x(\cos x\cos\alpha+\sin x\sin\alpha)}}$$

$$= \int\frac{dx}{\sin^2 x\sqrt{\frac{\cos x}{\sin x}\cos\alpha+\sin\alpha}}$$

$$= \int\frac{\operatorname{cosec}^2 x\,dx}{\sqrt{\cot x\cos\alpha+\sin\alpha}}$$

$$= \frac{1}{\sqrt{\cos\alpha}}\int\frac{\operatorname{cosec}^2 x\,dx}{\sqrt{\cot x+\tan\alpha}}$$

Let $\cot x + \tan\alpha = t$

$-\operatorname{cosec}^2 x\,dx = dt$

$$= \frac{1}{\sqrt{\cos\alpha}}\int\frac{-dt}{\sqrt{t}}$$

$$= -\frac{2\sqrt{t}}{\sqrt{\cos\alpha}}+c$$

$$= -2\frac{\sqrt{\cot x+\tan\alpha}}{\sqrt{\cos\alpha}}+c$$

27. Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx.$

Sol. $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$

Let $\cot^{-1}x = t$

$$-\frac{1}{1+x^2}dx = dt$$

$$= \int -e^t(1-\cot t+\cot^2 t)dx$$

$$= \int -e^t(\operatorname{cosec}^2 t-\cot t)dt$$

$$= \int e^t(\cot t-\operatorname{cosec}^2 t)dt$$

$f(t) = \cot t$

$f'(t) = -\operatorname{cosec}^2 t$

$\int e^t(f(t)+f'(t))dt = e^t f(t) + c$

$\therefore \quad = e^t\cot t + c$

$= e^{\cot^{-1}x}\cot\cot^{-1}x + c$

$= xe^{\cot^{-1}x}+c$

28. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$

Sol.

$$I = \int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$$

$$I = \int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{\left(e^x+\frac{1}{e^x}\right)\left(e^x-\frac{1}{e^x}\right)}dx$$

$$I = \int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{e^{2x}}{(e^{2x}+1)(e^{2x}-1)}dx$$

Let $e^{2x} = t$

$2e^{2x}\,dx = dt$

$x = \log\sqrt{3}$ then $t = 3$

$x = \log\sqrt{2}$ then $t = 2$

$$\therefore \quad I = \frac{1}{2}\int_2^3\frac{dt}{(t+1)(t-1)}$$

$$I = \frac{1}{2}\int_2^3\frac{dt}{t^2-(1)^2}$$

$$= \frac{1}{2}\cdot\frac{1}{2\times 1}\log\left|\frac{t-1}{t+1}\right|_2^3$$

$$I = \frac{1}{4}\left[\log\frac{2}{4}-\log\frac{1}{3}\right] = \frac{1}{4}\log\frac{3}{2}$$

29. (a) Find the general solution of the differential equation:

$(xy - x^2)\, dy = y^2\, dx$.

OR

(b) Find the general solution of the differential equation:

$$(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$$

Sol. (a)

$$(xy - x^2)dy = y^2 dx$$

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

It is a homogenous equation

$\therefore$ $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$$

$$\frac{xdv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$$

$$\frac{v-1}{v}dv = \frac{dx}{x}$$

$$\int\left(1 - \frac{1}{v}\right)dv = \int\frac{dx}{x}$$

$$v - \log|v| = \log|x| + \log|c|$$

$$v = \log vcx$$

$$\frac{y}{x} = \log cy$$

$$\Rightarrow \quad cy = e^{y/x} \text{ or } y = c_1 e^{y/x} \qquad \left(c_1 = \frac{1}{c}\right)$$

OR

(b)

$$(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$$

$$\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{\sqrt{x^2+4}}{x^2+1}$$

Equation is the linear form

$\therefore$ Integrating factor

$$\text{I.F.} = e^{\int \frac{2x}{x^2+1}dx}$$

$$\text{I.F.} = e^{\log|x^2+1|} = x^2 + 1$$

$\therefore$ Solution of differential equation

$$y \times \text{I.F.} = \int \text{I.F.} \times \frac{\sqrt{x^2+4}}{x^2+1}dx$$

$$(x^2+1)y = \int\sqrt{x^2+4}\,dx$$

$$(x^2+1)y = \frac{x}{2}\sqrt{x^2+4} + 2\log|x + \sqrt{x^2+4}| + c$$

30. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number or red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first.

Sol. (a) Let the no. of Red balls = no of Green balls be x

Total balls $= 2x$

Probability of no Red balls

$$\frac{x}{2x} \times \frac{x}{2x} = \frac{1}{4}$$

Probability of 1 Red balls

$$2 \times \frac{x}{2x} \times \frac{x}{2x} = \frac{1}{2}$$

Probability of 2 Red balls

$$\frac{x}{2x} \times \frac{x}{2x} = \frac{1}{4}$$

$$\text{Required mean} = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 1$$

OR

(b) Let W and F be the probabilities for win and fail respectively when single die is thrown alternatively.

Since, p**(getting 6)** $= \frac{1}{6}$

q**(not getting 6)** $= 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Starting with A,

P(A win 1st throw) = W

P(A Win 3rd throw) = $F_A F_B W$

P(A Win 5th throw) = $F_A F_B F_A F_B W$

So the probabilities for A to Win the game

P(A Win) = P(1st) + P(3rd) + p(5th)..........

$= W + F_A F_B W + F_A F_B F_A F_B W$.................

$= p + q.q\,p + q.q.q.q.p...$

$$= \frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}...$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 ...\right] \because \left(S_\infty = \frac{a}{1-r}\right) r < 1$$

$$= \frac{1}{6} \times \frac{1}{1 - \frac{25}{36}}$$

$$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

P(B Win) = 1 – P(A Win)

$$= 1 - \frac{6}{11}$$

$$= \frac{5}{11}$$

31. Solve the following linear programming problem graphically :

Minimize : Z = 5x + 10y

subject to constraints :

$$x + 2y \le 120$$
$$x + y \ge 60$$
$$x - 2y \ge 0$$
$$x \ge 0, y \ge 0.$$

Sol. $Z = 5x + 10y$

$x + 2y \le 120, x + y \ge 60, x - 2y \ge 0$

$x \ge 0, y \ge 0$

$x + 2y = 120$

x	0	120	60
y	60	0	30

$x + y = 60$

x	0	60	30
y	60	0	30

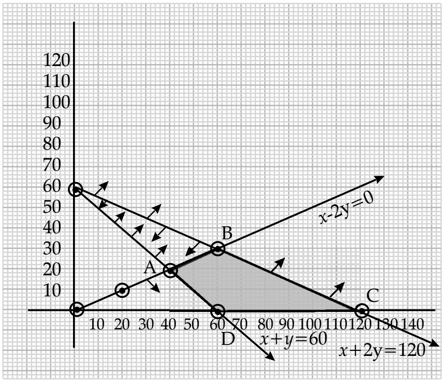
$x - 2y = 0$

x	0	10	20
y	0	5	10

Point (x, y)	$Z = 5x + 10y$
At (40, 20)	$Z = 5 \times 40 + 10 + 20 = 400$
At (60, 0)	$Z = 5 \times 60 + 10 \times 0 = 300$ (Minimum)
At (120, 0)	$Z = 5 \times 120 + 10 \times 0 = 600$
At (60, 30)	$Z = 5 \times 60 + 10 \times 30 = 600$

Minimum value = 300

at x = 60 and y = 0



SECTION - D

32. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, **then find AB and use it to solve the following system of equations:**

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, **prove that** $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$

Sol. (a) $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4+0 & -6+2+4 & 0+4-4 \\ 2-2+0 & 4-1-2 & 0-2+2 \\ 2-2+0 & 4-1-3 & 0-2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad B^{-1} = A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$x - 2y = 3$$
$$2x - y - 1 = 2$$
and $-2y + z = 3$

Equation can be written in matrix form

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\therefore \quad (B^t)^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = -1$ and $z = 1$

OR

(b) $$f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(-\beta) = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(\alpha).f(-\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos\alpha\sin\beta - \sin\alpha\cos\beta & 0 \\ \sin\alpha\cos\beta - \sin\beta\cos\alpha & \sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

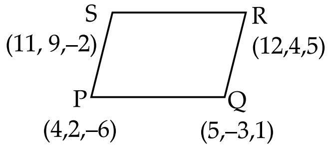
$f(\alpha).f(-\beta) = f(\alpha - \beta)$ **Hence Proved**

33. (a) Find the equation of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2) Use these equation to find the point of intersection of diagonals.

OR

(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

Sol. (a)



Equation of the diagonal PR

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$$

Equation of the diagonal QS

$$\frac{x-5}{6} = \frac{y+3}{+12} = \frac{z-1}{-3}$$

or $$\frac{x-5}{2} = \frac{y+3}{4} = \frac{z-1}{-1}$$

General point on the diagonal PR

$= (8\lambda + 4, 2\lambda + 2, 11\lambda - 6)$

General point on the diagonal QS

$= 2\mu + 5, 4\mu - 3, -\mu + 1$

Intersection point of PR and QS

$8\lambda + 4 = 2\mu + 5, 2\lambda + 2 = 4\mu - 3, \ 11\lambda - 6 = -\mu + 1$

$8\lambda - 2\mu = 1$ $\qquad 11\lambda + \mu = 7$

$2\lambda - 4\mu = -5$ $\qquad 11\lambda + \mu = 7$ is

$14\lambda = 7$ $\qquad$ Satisfies these values

$\lambda = \frac{1}{2}$ and $\mu = \frac{3}{2}$

So intersection point of diagonals or mid point of diagonals is

$$= \left(4+4, 1+2, \frac{11}{2} - 6\right)$$

$$= \left(8, 3, -\frac{1}{2}\right)$$

OR

(b) Let the direction ratio of the line be (a, b, c)

Equation of the line passes through (–1, 3, –2)

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$$

Line is perpendicular to the given line

$\therefore \quad a + 2b + 3c = 0$

$-3a + 2b + 5c = 0$

$$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

or $$\frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$$

Required line

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

Vector form

$$= -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

General point on the line

$= (2\lambda - 1, -7\lambda + 3, 4\lambda - 2)$

Direction ratio of the line passing through origin and $(2\lambda - 1, -7\lambda + 3, 4\lambda - 2)$

$\therefore \quad (2\lambda - 1) + (-7)(-7\lambda + 3) + 4(4\lambda - 2) = 0$

$4\lambda - 2 + 49\lambda - 21 + 16\lambda - 8 = 0$

$69\lambda - 31 = 0$

$$\lambda = \frac{31}{69}$$

Foot of perpendicular from origin on the line is $\frac{-7}{69}, \frac{-10}{69}, \frac{-14}{69}$

Distance from origin $= \left|\sqrt{\left(\frac{-7}{69}\right)^2 + \left(\frac{-10}{69}\right)^2 + \left(\frac{-14}{69}\right)^2}\right|$

$$= \left|\sqrt{\frac{49+100+196}{69^2}}\right|$$

$$= \sqrt{\frac{345}{69}} = \sqrt{\frac{5}{69}} \text{ units}$$

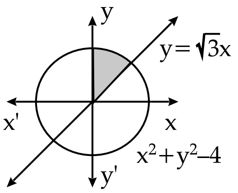
34. Using integration, find the area of region bounded by line $y = \sqrt{3x}$, the curve $y = \sqrt{4-x^2}$ and y-axis in first quadrant.

Sol.

$$y = \sqrt{4-x^2}$$

$$y = \sqrt{3}x$$

$$y = \sqrt{4-x^2}$$



Squaring both side

$\therefore \quad x^2 + y^2 = 4$

Intersection point of $x^2 + y^2 = 4$ and $y = \sqrt{3}x$

$$x^2 + 3x^2 = 4$$

$$\Rightarrow \quad x = \pm 1$$

Intersection point in I Quadrant is $(1, \sqrt{3})$

$$\text{Required area} = \int_0^{\sqrt{3}} x_{\text{line}}\, dy + \int_{\sqrt{3}}^{2} x_{\text{circle}}\, dy$$

$$= \int_0^{\sqrt{3}} \frac{y}{\sqrt{3}} dy + \int_{\sqrt{3}}^{2} \sqrt{4-y^2}\, dy$$

$$= \left[\frac{y^2}{2\sqrt{3}}\right]_0^{\sqrt{3}} + \left[\frac{y}{2}\sqrt{4-y^2} + \frac{1}{2}\times 4\sin^{-1}\frac{y}{2}\right]_{\sqrt{3}}^{2}$$

$$= \left(\frac{\sqrt{3}}{2} - 0\right) + \left[0 + 2\sin^{-1}1 - \frac{\sqrt{3}}{2} - 2\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$= \frac{\sqrt{3}}{2} + 2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{3}$$

$$= \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3} \text{ unit}^2$$

35. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$

Sol. $f(x) = \sqrt{16 - x^2}$

for $x = 0$ $\quad (0 \in (-4, 4))$

$$f(0) = \pm 4$$

So it is not one-one and for each value of y these exists x

$$f(a) = \sqrt{7}$$

$\therefore f$ is an onto function

$$\sqrt{16 - a^2} = \sqrt{7}$$

Squaring both sides

$$16 - a^2 = 7$$

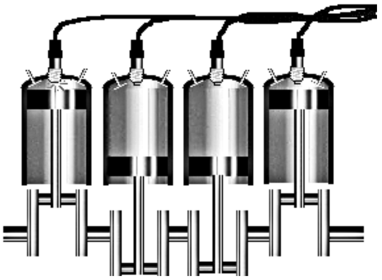
$$a^2 = 16 - 7 = 9$$

$$a = \pm 3$$

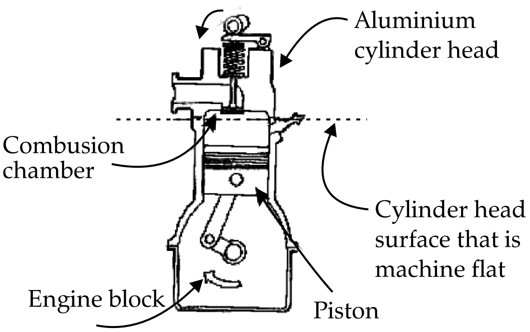
$$a = 3 \text{ and } -3$$

SECTION - E

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



One complete cycle of a four-cylinder four stroke engine. The volume displaced is marked



The cylinder bore in the form of circular cylinder open at the top is be made from a metal sheet of area 75π cm².

Based on the above information, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

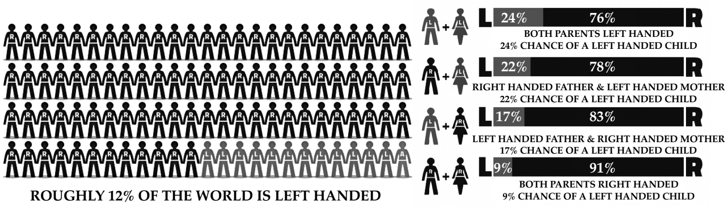
(ii) Find $\frac{dV}{dr}$

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(b) For maximum volume, $h > r$. State true or false and justify.

37. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A : When both father and mother are left handed:
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D : When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following question:

(i) Find $P\left(\frac{L}{C}\right)$ **(ii) Find $P\left(\frac{\bar{L}}{A}\right)$**

(iii) (a) Find $P\left(\frac{A}{L}\right)$

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

Sol. Area of bore $= 75\pi \text{ cm}^2$

$$2\pi rh + \pi r^2 = 75\pi$$

$$2rh + r^2 = 75$$

$$h = \frac{75 - r^2}{2r}$$

(i) Vol. of cylinder $= \pi r^2 h$

$$= \pi r^2\left(\frac{75 - r^2}{2r}\right)$$

$$= \frac{\pi}{2}(75r - r^3) \text{ cm}^3$$

(ii) $$\frac{dV}{dr} = \frac{\pi}{2}[75 - 3r^2]$$

$$\frac{dV}{dr} = 0 \text{ for maxima}$$

$$\frac{\pi}{2}(75 - 3r^2) = 0$$

$$r^2 = 25$$

$$\Rightarrow \quad r = \pm 5$$

$$\frac{d^2V}{dr^2} = -6r$$

(iii) (a) $$\left(\frac{d^2V}{dr^2}\right)_{(r=5)} = -30 < 0$$

Hence volume is maximum at $r = 5$ cm

OR

(b) $$h = \frac{75 - 25}{2 \times 5} = 5 \text{ cm}$$

So $h = r$
at maximum volume
Hence statement is false.

38. The use of electric vehicles will curb air pollution in the long run.

The use electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions:

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(ii) Prove that the function V(t) is an increasing function.

37. $P(A) = 24\%$ $P(C) = 17\%$
$P(B) = 22\%$ $P(D) = 9\%$

(i) $$P\left(\frac{L}{C}\right) = \frac{1}{4} \times \frac{12}{100}$$

$$= \frac{17}{400}$$

$$= 0.0425$$

(ii) $$P\left(\frac{\bar{L}}{A}\right) = 1 - 0.0425$$

$$= 0.9575$$

(iii) (a)

$$P\left(\frac{A}{L}\right) = \frac{P(A).P\left(\frac{L}{A}\right)}{P(A).P\left(\frac{L}{A}\right)+P(B).P\left(\frac{L}{B}\right)+P(C).P\left(\frac{L}{C}\right)+P(D).P\left(\frac{L}{D}\right)}$$

$$= \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}}$$

$$= \frac{24}{24+22+17+9}$$

$$= \frac{24}{72} = \frac{1}{3}$$

OR

(b)

$$= \frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}$$

$$= \frac{39}{400} = 0.0475$$

38.

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

(i) $V(0) = -2 < 0$

So the above function can not be used to estimate number of vehicles in the year 2000.

(ii)

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

$$V'(t) = \frac{3}{5}t^2 - 5t + 25$$

$$= \frac{3}{5}\left[t^2 - \frac{25}{3}t + \frac{125}{3}\right]$$

$$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 - \frac{625}{36} + \frac{125}{3}\right]$$

$$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right]$$

$V'(t) > 0$ for any value of t

Hence $V(t)$ is increasing function.

Outside Delhi Set-II **65/2/2**

Note: *Except these, all other questions are from Outside Delhi Set-1*

SECTION - A

1. If $\frac{d}{dx}f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is:

(a) $x^2 + 3\log|x| + 1$ **(b)** $x^2 + 3\log|x|$

(c) $2 - \frac{3}{x^2}$ **(d)** $x^2 + 3\log|x| - 4$

Sol. Option (a) is correct.

Explanation:

$$\frac{d}{dx}f(x) = 2x + \frac{3}{x}$$

$$f(x) = \int\left(2x + \frac{3}{x}\right)dx$$

$$f(x) = \frac{2x^2}{2} + 3\log|x| + C$$

$$f(1) = (1)^2 + 3\log|1| + C$$

$$1 = 1 + 0 + C \Rightarrow C = 0$$

$$f(x) = x^2 + 3\log|x|$$

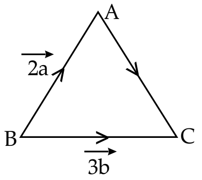
5. If in ΔABC, $\overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is:

(a) $2\vec{a} + 3\vec{b}$ **(b)** $2\vec{a} - 3\vec{b}$

(c) $3\vec{b} - 2\vec{a}$ **(d)** $-2\vec{a} - 3\vec{b}$

Sol. Option (c) is correct.

Explanation:



$$\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$$

$$\overrightarrow{AC} = \overrightarrow{BC} - \overrightarrow{BA}$$

$$\overrightarrow{AC} = 3\vec{b} - 2\vec{a}$$

6. If $|\vec{a}\times\vec{b}| = \sqrt{3}$ and $\vec{a}.\vec{b} = -3$, then angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\frac{2\pi}{3}$ **(b)** $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{5\pi}{6}$

Sol. Option (d) is correct.

Explanation:

$$|\vec{a}\times\vec{b}| = \sqrt{3} \qquad \vec{a}.\vec{b} = -3$$

$$|\vec{a}\times\vec{b}| = \frac{\vec{a}\times\vec{b}}{\hat{n}} = \frac{|a||b|\sin\theta\hat{n}}{\hat{n}}$$

$$|a||b|\sin\theta = \sqrt{3} \quad ...(i)$$

$$|a||b|\cos\theta = -3 \quad ...(ii)$$

From eq. (i) and eq. (ii)

$$\tan\theta = \frac{\sqrt{3}}{-3} = -\frac{1}{\sqrt{3}}$$

$$\theta = \frac{5\pi}{6}$$

7. Equation of line passing through origin and making 30°, 60° and 90° wih x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$ **(b)** $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

(c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$ **(d)** $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

Sol. Option (b) is correct.

Explanation: $\cos\alpha = \cos 30 = \frac{\sqrt{3}}{2}$

$$\cos\beta = \cos 60 = \frac{1}{2}$$

$$\cos\gamma = \cos 90 = 0$$

Equation of required line

$$\frac{x-0}{\sqrt{3}/2} = \frac{y-0}{1/2} = \frac{z-0}{0}$$

$$\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$$

8. If A and B are two events such that $P\left(\frac{A}{B}\right) = P\left(\frac{A}{B}\right)$ and $P(A) + P(B) = \frac{2}{3}$, then P(B) is equal to

(a) $\frac{2}{9}$ **(b)** $\frac{7}{9}$

(c) $\frac{4}{9}$ **(d)** $\frac{5}{9}$

Sol. Option (a) is correct.

Explanation:

$$P\left(\frac{A}{B}\right) = 2\times P\left(\frac{B}{A}\right)$$

$$\frac{P(A\cap B)}{P(B)} = \frac{2\times P(A\cap B)}{P(A)}$$

$$P(A) = 2P(B)$$

$$P(A) + P(B) = \frac{2}{3}$$

$$2P(B) + P(B) = \frac{2}{3}$$

$$P(B) = \frac{2}{9}$$

$$P(A) = \frac{2}{3} - \frac{2}{9} = \frac{4}{9}$$

15. If A is a 2 × 3 matrix such that AB and AB′ both are defined, then order of the matrix B is

(a) 2 × 2 **(b)** 2 × 1

(c) 3 × 2 **(d)** 3 × 3

Sol. Option (d) is correct.

Explanation: $A = 2\times 3 \quad B = m\times n$

For defined $AB = m = 3$

For defined $AB' = n = 3$

$\therefore$ $B = 3\times 3$

SECTION - B

23. If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line.

Sol.

$$x = ay + b$$

$$z = cy + d$$

$$\frac{x-b}{a} = y$$

$$\frac{z-d}{c} = y$$

$\therefore$ Equation of the line

$$\frac{x-b}{a} = y = \frac{z-d}{C}$$

Direction ratio of the line $(a, 1, c)$

Point on the line (b, 0, d)

25. If the circumference of circle is increasing at the constant rate, prove that rate of change of area is directly proportional to its radius.

Sol.

$$\frac{dC}{dt} = k \text{ (given)}$$

$$\frac{d}{dt}(2\pi r) = k$$

(where C is the circumference of the circle)

$$2\pi\frac{dr}{dt} = k$$

$$\therefore \quad \frac{dr}{dt} = \frac{k}{2\pi} = \text{Constant}$$

$A = \pi r^2$ (A and r is the area and radius of circle respectively)

$$\frac{dA}{dt} = \pi 2r\frac{dr}{dt}$$

$$\frac{dA}{dt} = \left(2\pi\frac{dr}{dt}\right)r$$

$$\frac{dA}{dt} \propto r$$

Hence Proved.

SECTION - C

29. Solve the following linear programming problem graphically:

Maximize : Z = $x + 2y$

subject to constraints :

$$x + 2y \geq 100,$$
$$2x - y \leq 0,$$
$$2x + y \leq 200,$$
$$x \geq 0, y \geq 0.$$

Sol.

$$z = x + 2y$$
$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x \geq 0,\ y \geq 0$$

$x + 2y = 100$

x	0	100	60
y	50	0	20

$2x - y = 0$

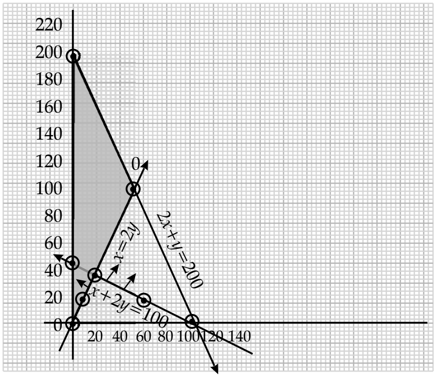
x	10	0	20
y	20	0	40

$2x + y = 200$

x	0	100	50
y	200	0	100

$$z = x + 2y$$
$$At\ (0, 200) = 0 + 2 \times 200 = 400 \text{ (Maximum)}$$
$$At\ (50, 100) = 50 + 2 \times 100 = 250$$
$$At\ (20, 40) = 20 + 2 \times 40 = 80$$

Maximum value is 400 at $x = 0$ and $y = 200$



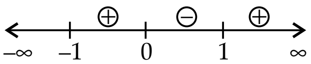
30. (a) Evaluate $\int_{-1}^{1} |x^4 - x|\, dx$.

OR

(b) Find $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$.

Sol. (a) $\int_{-1}^{1} |x^4 - x|\, dx$

$\int_{-1}^{1} |x(x-1)(x^2+x+1)|\, dx$



$$\therefore \quad I = \int_{-1}^{0}(x^4 - x)dx + \int_{0}^{1} -(x^4 - x)\, dx$$
$$= \left[\frac{x^5}{5} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^5}{5} - \frac{x^2}{2}\right]_{0}^{1}$$
$$= \left(\frac{+1}{5} + \frac{1}{2}\right) - \left(\frac{1}{5} - \frac{1}{2}\right)$$
$$I = \frac{1}{2} + \frac{1}{2} = 1$$

OR

(b) $$I = \int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\, dx$$

Let $\sin^{-1}x = t$

$$\frac{1}{\sqrt{t - x^2}} dx = dt$$
$$I = \int \frac{t}{(1 - \sin^2 t)} dt = \int t \sec^2 t\, dt$$
$$I = t\int \sec^2 t dt - \int\left[\frac{d}{dt} t \int \sec^2 t\, dt\right]$$
$$= t \tan t - \int \tan dt$$
$$= t \tan t + \log|\cos t| + C$$
$$= \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \log|\sqrt{1-x^2}| + C$$

Sol. Find $\int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

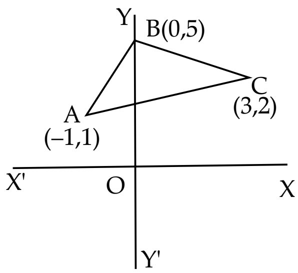
31. $$I = \int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$$
$$I = \int e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)$$
$$I = \int e^x \left(\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)$$
$$I = \int e^x \left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$$
$$I = \int e^x \left(-\cot\frac{x}{2} + \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right) dx$$

Let $f(x) = -\cot\frac{x}{2}$

$$f'(x) = +\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$$
$$\int e^x((fx) + f'(x))dx = e^x f(x) + C$$
$$\therefore \quad I = \int e^x \left(-\cot\frac{x}{2} + \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right) dx$$
$$= -e^x \cot\frac{x}{2} + c$$

33. Using Integration, find the area of triangle whose vertices are (–1, 1), (0, 5) and (3, 2).

Sol. $A(-1, 1)$, $B(0, 5)$, $C(3, 2)$



Equ. of the side AB

$$y - 1 = \frac{5-1}{0+1}(x+1)$$
$$y = 4x + 5$$

Equ. of the side BC

$$y-5 = \frac{2-5}{3-0}(x-0)$$

$$y-5 = -x$$

$$y = 5-x$$

Equ. of the side AC

$$y-1 = \frac{2-1}{3+1}(x-1)$$

$$4y-4 = x+1$$

$$y = \frac{x+5}{4}$$

Required Area of ΔABC

$$= \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \int_{-1}^{3}\left(\frac{x+5}{4}\right)dx$$

$$= \left|\left[\frac{x^2}{2}+5x\right]_{-1}^{0}\right| + \left[5x-\frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$$

$$= \left(0+\frac{9}{2}\right)+\left(\frac{21}{2}-0\right)-\frac{1}{4}\left(\frac{39}{2}+\frac{9}{2}\right)$$

$$= \frac{9}{2}+\frac{21}{2}-6$$

$= 15 - 6 = 9$ unit^2.

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SECTION - A

1. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a}\times\vec{b}| = 3\sqrt{3}$, then the value of $\vec{a}.\vec{b}$ is:

(a) 9 (b) 3

(c) $\frac{1}{9}$ (d) $\frac{1}{3}$

Sol. Option (b) is correct.

Explanation:

$$|\vec{a}\times\vec{b}| = 3\sqrt{3}$$

$$|\vec{a}\times\vec{b}| = \frac{\vec{a}\times\vec{b}}{\hat{n}} = \frac{|a||b|\sin\theta\hat{n}}{\hat{n}}$$

$$|\vec{a}||\vec{b}|\sin\theta = 3\sqrt{3}$$

$$|\vec{a}||\vec{b}|\sin 60^\circ = 3\sqrt{3}$$

$$|\vec{a}||\vec{b}| = 3\sqrt{3}\times\frac{2}{\sqrt{3}} = 6$$

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$= 6\cos 60^\circ$$

$$= 6\times\frac{1}{2} = 3$$

2. The position vector of three consecutive vertices of a parallelogram ABCD are $A(4\hat{i}+2\hat{j}-6\hat{k})$, $B(5\hat{i}-3\hat{j}+\hat{k})$ and $C(12\hat{i}+4\hat{j}+5\hat{k})$. The position vector of D is given by

(a) $-3\hat{i}-5\hat{j}-10\hat{k}$ (b) $21\hat{i}+3\hat{j}$

(c) $11\hat{i}+9\hat{j}-2\hat{k}$ (d) $-11\hat{i}-9\hat{j}+2\hat{k}$

Sol. Option (c) is correct.

Explanation:

$A(4\hat{i}+2\hat{j}-6\hat{k})$ $C(12\hat{i}+4\hat{j}+5\hat{k})$

$B(5\hat{i}-3\hat{j}+\hat{k})$ $D(x\hat{i}+y\hat{j}+z\hat{k})$

Diagonal of $||^{gm}$ bisect each other

∴ Position vector of mid point of AC $(8\hat{i}+3\hat{j}-\frac{1}{2}\hat{k})$

Position vector of midpoint of $\vec{BD}$

$$\left(\frac{5+x}{2}\hat{i}+\frac{y-3}{2}\hat{j}+\frac{z+1}{2}\hat{k}\right)$$

$$\frac{5+x}{2} = 8 \qquad \therefore x = 11$$

$$\frac{y-3}{2} = y = 9$$

$$\frac{z+1}{2} = -\frac{1}{2} \quad z = -2 \quad \therefore D\left(11\hat{i}+9\hat{j}-2\hat{k}\right)$$

3. If for two events A and B, $P(A-B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{2}{3}$

Sol. Option (d) is correct.

Explanation:

$$P(A-B) = \frac{1}{5}$$

$$P(A)-P(A\cap B) = \frac{1}{5}$$

$$P(A\cap B) = \frac{3}{5}-\frac{1}{5} = \frac{2}{5}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$$

$$= \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

4. If $\int_0^{2\pi}\cos^2 x\,dx = k\int_0^{\pi/2}\cos^2 x\,dx$, then the value of k is

(a) 4 **(b)** 2

(c) 1 **(d)** 0

Sol. Option (a) is correct.

Explanation:

$$\int_0^{2\pi}\cos^2 x\,dx = k\int_0^{\pi/2}\cos^2 x\,dx$$

$$f(x) = \cos^2 x$$

$$f(2\pi - x) = \cos^2(2\pi - x) = \cos^2 x$$

$$\therefore \int_0^{2\pi}\cos^2 x\,dx = 2\int_0^{\pi}\cos^2 x\,dx$$

Similarly

$$\int_0^{\pi}\cos^2 x = 2\int_0^{\pi/2}\cos^2 x\,dx$$

$$\therefore \int_0^{2\pi}\cos^2 x\,dx = 4\int_0^{\pi/2}\cos^2 x\,dx$$

$$\therefore k = 4$$

10. Number of symmetric matrices of order 3 × 3 with each entry 1 or –1 is

(a) 512 **(b)** 64

(c) 8 **(d)** 4

Sol. Option (b) is correct.

Explanation: Number of Symmetric matrices of order 3× 3 = 2^6 = 64

18. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is 1

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ **(b)** $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(c) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ **(d)** $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$

Sol. Option (d) is correct

Explanation: Equation of line passing though Point (x_1, y_1, z_1).

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$x_1 = 1$

$y_1 = 2$

$z_1 = 3$

$a = 1$

$b = 1$

$c = 1$

Required Eq.

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

SECTION - B

21. If points A, B and C have position vectors $2\hat{i}$, $\hat{j}$ and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.

Sol. $A(2\hat{i})$. $B(\hat{j})$ and $\vec{C}(2\hat{k})$

$\vec{AB} = \hat{j} - 2\hat{i}$ $\quad |\vec{AB}| = \sqrt{1^2+2^2} = |\sqrt{5}|$ unit

$\vec{BC} = 2\hat{k} - \hat{j}$ $\quad |\vec{BC}| = \sqrt{2^2+1^2} = |\sqrt{5}|$ unit

$\vec{AC} = 2\hat{k} - 2\hat{i}$ $\quad |\vec{AC}| = \sqrt{2^2+2^2} = |2\sqrt{2}|$ unit

$\therefore AB = BC = \sqrt{5}$ unit

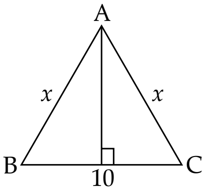
Hence ΔABC is an isosceles triangle

23. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/s, how fast is the area of triangle increasing at an instant when all sides become equal ?

Sol.

$$\text{Ar } \Delta ABC = \frac{1}{2}BC \times AD$$

$$= \frac{1}{2}\times 10 \times \sqrt{x^2-25}$$



$$A = 5\sqrt{x^2-25} \text{ units}$$

$$A = 5\sqrt{x^2-25}$$

$$\frac{dA}{dt} = 5\frac{d}{dt}(\sqrt{x^2-25})$$

$$= 5.\frac{1}{2}(x^2-25)^{-1/2}(2x)\frac{dx}{dt}$$

$$= \frac{5x}{\sqrt{x^2-25}}\frac{dx}{dt}$$

$$\left(\frac{dA}{dt}\right)_{x=10} = \frac{50}{\sqrt{25}}.4 = 40 \text{ cm}^2/\text{s}$$

SECTION - C

26. Solve the following Linear Programming problem graphically:

Maximize: Z = 3x + 3.5y

subject to constraints:

$x + 2y \ge 240$,

$3x + 1.5y \ge 270$,

$1.5x + 2y \le 310$,

$x \ge 0, y \ge 0$.

Sol. $x + 2y \ge 240$

$3x + 1.5y \ge 270$ $\qquad x \ge 0, y \ge 0$

$1.5x + 2y \le 310$

$x + 2y = 240$

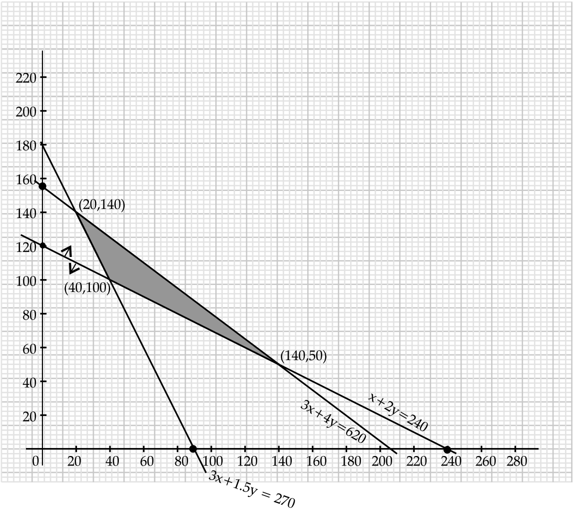
x	0	240	80
y	120	0	80

$3x + 1.5y = 270$

x	0	90	80
y	180	0	20

$1.5x + 2y = 310$

x	$\frac{620}{3}$	0	100	60
y	0	155	80	110



Point (x, y)	$Z= 3x + 3.5y$
at(40,100)	$Z= 3\times 40 + 3.5 \times 100 = 470$
at(20,140)	$Z= 3 \times 20 + 140 \times 3.5 = 550$
at(140,50)	$Z= 3 \times 140 + 3.5 \times 50 = 594$ (Maximum)

Maximum value 595

at $x = 140$ and $y = 50$

27. (a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}}dx$

OR

(b) Evaluate $\int_{-a}^{a} f(x)dx$, **where** $f(x) = \frac{9^x}{1+9^x}$

Sol. (a) $\int \frac{x+2}{\sqrt{x^2-4x-5}}dx$

$$I = \int \frac{x+2}{\sqrt{x^2-4x-5}}dx$$

$$= \int \frac{\frac{1}{2}(2x-4)+4}{\sqrt{x^2-4x-5}}dx$$

$$= \frac{1}{2}\int \frac{2x-4}{\sqrt{x^2-4x-5}}dx + 4\int \frac{dx}{\sqrt{x^2-4x-5}}$$

$$= \frac{1}{2}.2\sqrt{x^2-4x-5} + 4\int \frac{dx}{\sqrt{(x-2)^2-3^2}}$$

$$= \sqrt{x^2-4x-5} + 4\log|x-2+\sqrt{(x-2)^2-3^2}| + C$$

$$= \sqrt{x^2-4x-5} + 4\log|x-2+\sqrt{x^2-4x-5}| + C$$

OR

(b) $I = \int_{-a}^{a} f(x) = \int_{-a}^{a} \frac{9^x}{1+9^x}$

$$I = \int_{-a}^{a} \frac{9^{(a-a-x)}}{1+9^{a-a-x}}dx$$

$$I = \int_{-a}^{a} \frac{9^{-x}}{1+9^{-x}}dx$$

$$= \int_{-a}^{a} \frac{1}{1+9^x}dx$$

$$2I = \int_{-a}^{a} \frac{9^x}{1+9^x}dx + \int_{-a}^{a} \frac{1}{1+9^x}dx$$

$$2I = \int_{-a}^{a} \frac{1+9^x}{1+9^x}dx = \int_{-a}^{a} dx$$

$$2I = [x]_{-a}^{a}$$

$$2I = 2a$$

$$\therefore \quad I = a$$

31. (a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X.

OR

(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not.

Sol. (a) Given first sin positive even integer

(2, 4, 6, 8, 10, 12)

Two numbers can be selected from the first six even integer = $6 \times 5 = 30$ ways

X denote the large of the two numbers

Hence x can take any value of 4, 6, 8, 10, 12

For $x = 4$ i.e., (2, 4) and (4, 2)

$$P(x) = \frac{2}{30} = \frac{1}{15}$$

For $x = 6$ i.e., (2, 6), (4, 6), (6, 2), (6, 4)

$$P(x) = \frac{4}{30} = \frac{2}{15}$$

For $x = 8$ i.e., (2, 8), (8, 2), (4, 8), (8, 4), (6, 8), (8, 6)

$$P(x) = \frac{6}{30} = \frac{3}{15}$$

For $x = 10$ i.e., (2, 10), (10, 2), (4, 10), (10, 4), (6, 10), (10, 6) (8,10), (10,8)

$$P(x) = \frac{8}{30} = \frac{4}{15}$$

For $x = 12$ i.e., (2, 12), (12, 2), (4, 12), (12, 4), (6, 12), (12, 6), (12, 8), (8, 12), (10, 12), (12, 10),

$$P(x) = \frac{10}{30} = \frac{1}{3}$$

x	4	6	8	10	12
y	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

OR

(b) A fair coin and unbiased dice is tossed

$$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

$$n(S) = 12$$

$$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} - \frac{1}{2}$$

$$B = \{(H, 3), (3, T)\}$$

$$n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

$$A \cap B = \{(H, 3)\}$$

$$\Rightarrow \quad n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{A(S)} = \frac{1}{12}$$

$$P(A).P(B) = \frac{1}{2}.\frac{1}{6} = \frac{1}{12}$$

$$P(A \cap B) = \frac{1}{12}$$

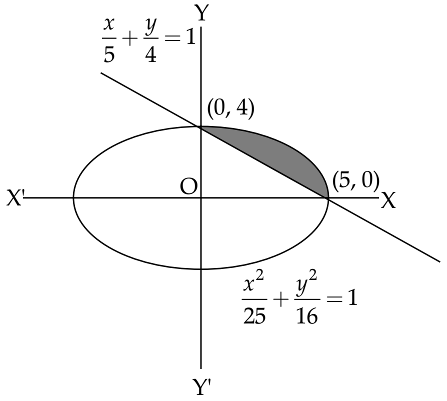
$$\therefore \quad P(A \cap B) = P(A).P(B)$$

These are independent events

SECTION - D

35. Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration.

Sol.



Eqn. the line $\frac{x}{5} + \frac{y}{4} = 1$

$$y = 4\left(1 - \frac{x}{5}\right) = 4 - \frac{4x}{5}$$

Eqn. of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$y = 4\sqrt{1 - \frac{x^2}{25}} = \frac{4}{5}\sqrt{25 - x^2}$$

$$\text{Required area} = \int_0^5 (y_{\text{ellipse}} - y_{\text{line}})dx$$

$$= \int_0^5 \left\{\frac{4}{5}\sqrt{25 - x^2} - \left(4 - \frac{4x}{5}\right)\right\}dx$$

$$= \int_0^5 \frac{4}{5}\sqrt{25 - x^2}dx - \int_0^5 \left(4 - \frac{4x}{5}\right)dx$$

$$= \frac{4}{5}\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{1}{2}25\sin^{-1}\frac{x}{5}\right]_0^5 - \left[4x - \frac{4x^2}{10}\right]$$

$$= \frac{4}{5}\left[\frac{1}{2}\sin^{-1}1\right] - [20 - 10]$$

$$= \frac{4}{5} \times \frac{\pi}{4} - 10$$

$$= (5\pi - 10)$$

Required Area $(5\pi - 10)$ unit^2

❑❑