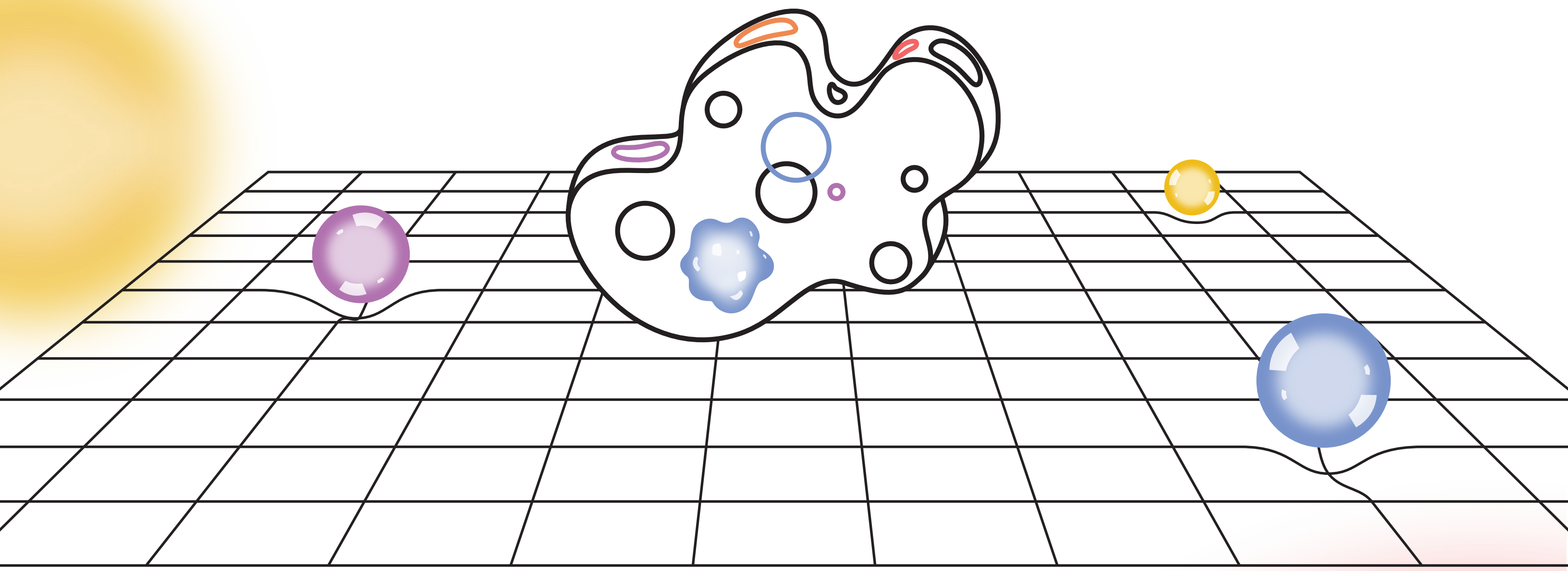
THE IDEA SPACE:

A NEW METRIC FOR GENERAL RELATIVITY

BY

CLÉMENT DECROP



"In order to understand physical laws, you must understand that they are all some kind of approximation."

- Richard Feynman

Abstract

This paper proposes a new metric for general relativity: an idea space. The elements of your idea space are your thoughts, emotions, sensations, perceptions, and the empty set, Ø (i.e. nothing). An idea space is a topological space with a pseudo metric, where the distance between any number of ideas is zero. Overall, an idea space is an object that sits at the center of your own observable universe; and, it is uncountably deep and has zero measure, like The Cantor Set. Since it has zero measure, it looks like nothing to an outside observer and sits outside of space and time. All in all, an idea space provides an approximate mathematical model for the mind that passes testable observations. In turn, your idea space can be used as your object of meditation.

Introduction

The idea space addresses the following questions: How does consciousness, or the mind, relate to the universe at large? Is there a way to structure the mind in a mathematical format based on observations that are congruent with modern physics?

The idea space provides a way to look at thoughts, emotions, sensations, and perceptions as objectively as one would view objects in spacetime—devoid of "I". In other words, the idea space allows you to look at the world from who you are at zero measure: your Non-Self. And, as Alan Watts says, "The true Self is Non-Self."

This paper serves as a technical complement to the book, THE IDEA SPACE: AWAKENING YOUR NON-SELF, and The Idea Space Overview. While this paper dives into the more scientific aspects of the idea space and how they relate to general relativity, the book serves as a digestible tool to help the reader better understand the repercussions of this new metric. To be clear, the book is made to be non-technical, so it can be read and understood by a mass audience.

Before starting, it is worth noting that this is an approximation to what we call consciousness, or the mind. As discussed in the book & overview, it is not a definitive proof, or statement. The only place where definitive proofs are allowed is mathematics. Then, when one maps mathematical properties onto testable observations, a scientific theory is born.

Specifically, science is based on falsification. For instance, in elementary statistics, you start with a hypothesis, or assumption. Then, you conduct a test to see how that hypothesis holds up. This is the familiar scientific method. At the end of the test, you can either (a) fail to reject the hypothesis (i.e. not enough information) or (b) reject the hypothesis. At no point do you "confirm" a hypothesis. If you fail to reject a hypothesis through multiple tests, then the hypothesis is said to be "true", or a "natural law".

In this spirit, the idea space is an approximation for the mind based on mathematical properties. Physicist Richard Feynman captures the sentiment behind how physics is a tool of approximations and idealizations well:

> *What is a chair? The atoms are evaporating from it from time to time—not many atoms, but a few—dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible.*
>
> *To define … a single object is impossible, because there are not any single, left-alone objects in the world—every object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations…*
>
> *In order to understand physical laws, you must understand that they are all some kind of approximation.*

With this understanding, the paper starts by detailing the elements of an idea space. Then, it highlights general mathematical traits of metric spaces and topological spaces. This information is then used to demonstrate the properties of an idea space and show how an idea space fits within the scheme of general relativity.

Although a complete understanding of general relativity, especially Einstein's field equation, is not necessary, this paper does not derive the equation in full. More information on the equation, including derivation, is well established in literature, like in *Spacetime and Geometry* by Sean Carroll, *The Large Scale Structure of Spacetime* by Stephen Hawking, and other references listed at the end of this paper. If the reader is interested in this field, then these are great sources to pull from.

Lastly, the paper concludes with The Sunset Conjecture which states: (1) everyone lives at the center of their own observable universe and thus experiences their own Singularity Sunset, and (2) at the center lies their idea space of zero measure and uncountable depth.

Elements of An Idea Space

Let *A* be an **idea space** (Figure 1). Drawing inspiration from Zen, the elements of *A* are thoughts, emotions, sensations, perceptions, and the empty set, Ø.

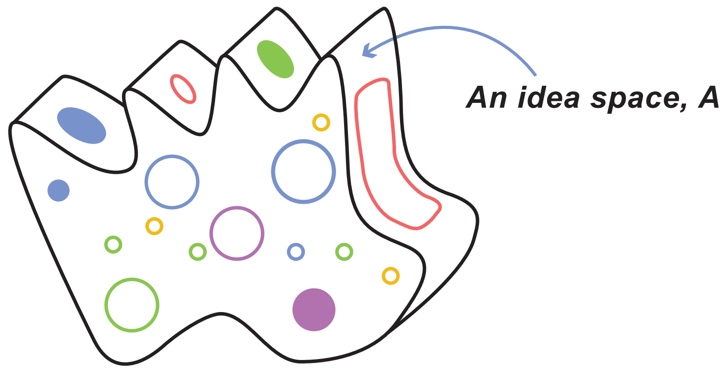


Figure 1. *An idea space.*

Thoughts consists of mental formations, like words, music, images, daydreams, and memories. **Emotions** include any feeling between pleasant and unpleasant, including neutral, or:

$$pleasant \leq emotions \leq unpleasant.$$

Sensations include the classical five: sights, sounds, touch, smell, and taste. **Perception** is the ability to recognize an object. For instance, a pen is a pen. A computer a computer.

Then, ***consciousness*** is the light one casts onto an idea space, as all forms of experience are captured by thoughts, emotions, sensations, perceptions, and emptiness. Can you think of an experience which is not captured by, or does not fall under, this grouping?

In reality, there is no "I". Your name, or identity, is an amalgamation of all your thoughts, emotions, sensations, perceptions, and consciousness at a particular point in spacetime. It is always changing, or impermanent. As Shunryu Suzuki says, *"What we call 'I' is just a swinging door which moves when we inhale and when we exhale."* In other words, "I" is an approximation others, and sometimes even yourself, use to capture the complexity of who you really are. Realizing there is no "I", or unveiling *The Illusion of Self,* is the first glimpse in understanding your **Non-Self**. More information on Non-Self can be found in the book.

The Empty Set, ∅

The empty set is nothing, Ø. An easy way to understand it is as follows: Imagine you have three apples. Each apple has distinct characteristics that can be used to describe it (Table 1).

***Table 1**. Apple Characteristics.*

Trait	Apple 1	Apple 2	Apple 3
Color	Red	Green	
Weight	80 grams	100 grams	70 grams
Stem? (Y/N)		N	Y

Clearly, this table is missing entries. That's the empty set, Ø! Your mind may quickly try to fill in those gaps by thinking something like, *the third apple is yellow*! However, you and I will never know what that information is, because it is missing. There's simply nothing there.

Now extrapolate the same concept of the empty set, Ø, to the idea space. It is simply a place where no ideas exist, or the space between breaths. For instance, by focusing on the breath for a short period of time we can experience emptiness of thoughts.

Zero Measure

The regular, or Euclidean, distance between any two points is simply $x - y$, or $d(x, y)$.

An object has zero measure if the distance between any two points is less than ε for all $\varepsilon > 0$ (Figure 2). In other words:

$$d(x, y) < \varepsilon \text{ for all } \varepsilon > 0.$$

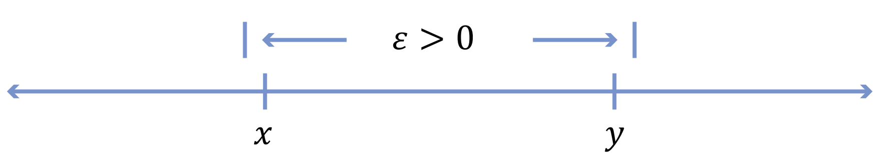


***Figure 2**. Zero measure: the distance between two points, $d(x, y)$, is less than ε for all $\varepsilon > 0$.*

The outer measure of a zero set is 0. The outer measure of the empty set, Ø, is also 0[1]. Thus, the two look the same. For instance, we can make a pen smaller, and smaller, until it has zero measure (Figure 3). At which point, it looks like the empty set, Ø: *nothing.*

1 *The outer measure of a set relates to Lebesgue theory, where m* denotes outer measure. So, m*0 = 0 = m*Ø.*

(a) A pen. (b) A smaller pen. (c) An even smaller pen. (d) A pen with zero measure.

Figure 3. *A pen with zero measure looks like nothing.*

To be clear, $0 \neq \emptyset$! Although the two look the same, behind an object of zero measure can lie uncountable depth, like your idea space (Figure 4).

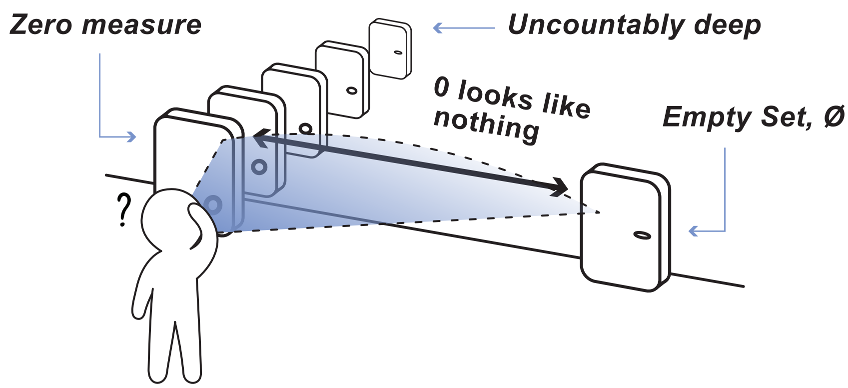


Figure 4. *Although 0 looks like the empty set, Ø, they are not the same.*

Metric Space

A metric space is a set *M*, whose elements are called points, together with a metric, d. The metric, d, is a way to define distances between any two points in *M*. The three main properties of a metric space are:

(1) Positive definiteness: The distance between any two points is positive. The distance between two points is zero if and only if the two points are the same point, or at the same location:

$$d(x, y) \geq 0 \text{ and } d(x, y) = 0 \text{ if and only if } x = y.$$

(2) Symmetry: The distance between x and y is the same as the distance between y and x:

$$d(x, y) = d(y, x).$$

(3) Triangle *inequality*: The distance between x and y, plus the distance between y and z, must be greater than or equal to the distance between x and z:

$$d(x, y) + d(y, z) \geq d(x, z) \text{ for all } x, y, z \in M.$$

A **pseudo metric space** is one where the first condition allows for the distance between any two, unique points to be 0 for any x and y:

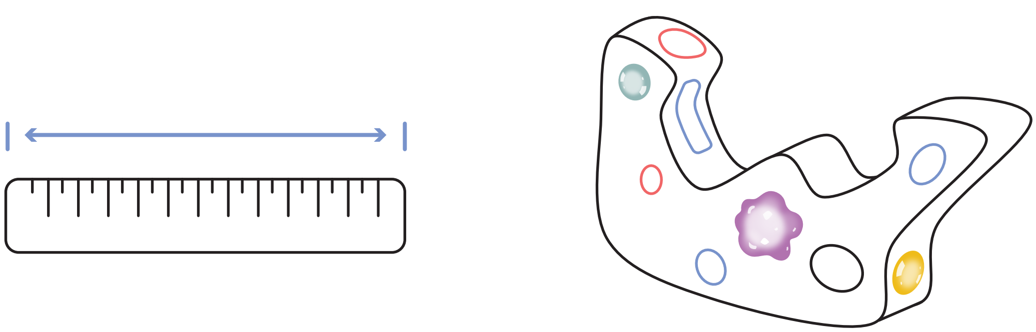
$$d(x, y) = 0 \text{ for any } x \text{ and } y.$$

The Idea Space Metric

An idea space is a pseudo metric space. The distance between any two elements of an idea space (e.g. thoughts, emotions, sensations, and perception) is zero. To test this, perform the following experiment:

Hold something in your hand, like a pencil, a book, or a computer. Clearly, you can measure it. You can feel it. Others can see it. Now, close your eyes and bring a mental image of what you're holding into your mind. Can anyone else see the image in your head? No—it has zero measure and therefore looks like nothing to an outside observer. Is there nothing there? No—clearly there is something there. Something only you can see. This holds true for all your thoughts, emotions, sensations, and perceptions. No one else can see them, but you.

Thus, a dividing line occurs (Figure 5). **The science of objects** are approximations used for observations that can be measured, or anything in spacetime, like physics, biology, chemistry, and the other natural sciences. Conversely, **the science of the first person** are approximations used for observations that have zero measure, like your idea space. The former can be seen by all. The latter is unique to you.



(b) The science of the objects deals with things with measure.

(b) The science of the person deals with things with zero measure.

***Figure 5**. (a) Physics, biology, and chemistry deal with the science of objects, while (b) the idea space falls under the science of the first person.*

General Relativity and Metric Spaces

General relativity dictates how spacetime curves in relation to matter, or energy. Specifically, according to general relativity, the universe is made up of different types of metric spaces. A metric, $g_{\mu v}$, is simply how you define the distance between two events, or two points in spacetime.

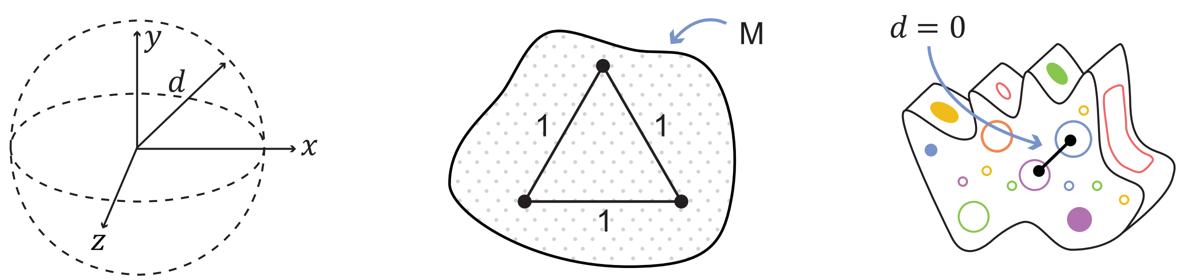
Each metric works well in its respective range. For instance, in day to day life, we may use Euclidean metric to measure the distance between any two points. Near stationary stars, we may switch to the Schwarzschild Metric. For spinning stars, we turn to the Kerr Metric. To look at the observable universe, we use the Robertson-Walker Metric. This paper proposes using the pseudo metric as the metric of your idea space, or your mind.

A couple examples of metric spaces are listed below with their corresponding figures.

(a) *Euclidean distance:* $d(x,y,z)^2 = x^2 + y^2 + z^2$
- This is your typical way to measure spatial distances. Think Pythagorean Theorem.

(b) *Discrete metric:* $d(x,y) = 1$
- The distance between any two points is always 1.
- Think of an equilateral triangle with equal sides of 1.

(c) *Pseudo metric space:* $d(x,y) = 0$ *for all points* x *and* y
- The distance between every point is zero. Think of an idea space.



(a) Euclidean metric space *(b) Discrete metric space* *(c) Pseudo metric*

Figure 6*. Examples of metric spaces.*

(d) *Minkowski Metric* (flat spacetime, or special relativity):

$$d(x,y,z,t)^2 = dx^2 + dy^2 + dz^2 - (c \cdot dt)^2$$

- *(x, y, z)* represent spatial coordinates, *(t)* is time. Think Pythagorean Theorem in four dimensions (three for space and one for time).
- *c* is the speed of light: 186,000 miles per second
- Einstein Equivalence Principle: in small enough regions of spacetime, the laws of physics reduce to those of special relativity (i.e. spacetime is locally flat with global curvature)

(e) *Robertson-Walker Metric* (the observable universe):

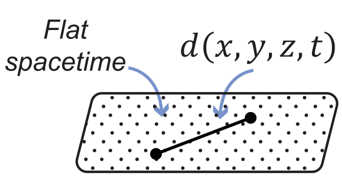
$$d(t,r,\theta,\phi)^2 = (c \cdot dt)^2 - a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right]$$

- Describes an expanding, homogenous, and isotropic universe in polar coordinates.
- You are at the center of your own observable universe (a giant sphere centered on you).
- Everything you see is in the past, as it takes time for light to go from point A to point B. So, in theory, the edge of your observable universe is its creation: The Big Bang.

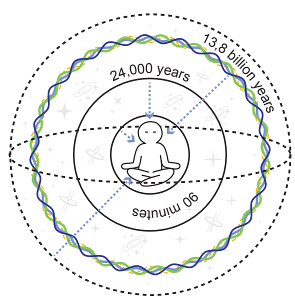
(f) *Schwarzschild Metric* (the space around Earth, the sun, or a stationary black hole):

$$d(t,r,\theta,\phi)^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$$

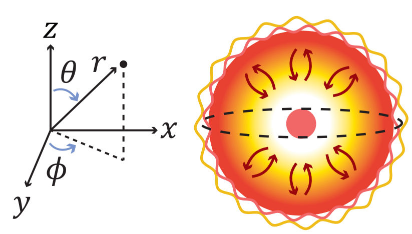
- G is Newton's gravitational constant: $6.7 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$
- M is the mass of the gravitating object (the origin is at the center of the object)



(d) Minkowski Metric, or flat spacetime, used for special relativity



(e) Robertson-Walker Metric used for the observable universe (a giant sphere)



(f) Schwarzschild Metric, for spherically non-spinning, symmetrical objects

Figure 7*. Examples of metric spaces in physics.*

These are examples of metrics. Once you have the metric, $g_{\mu\nu}$, you take its derivative to obtain the Christoffel Symbols, $\Gamma^{\sigma}_{\mu\nu}$. From there, you take the derivative again to obtain the Riemann tensor, $R^{\rho}{}_{\sigma\mu\nu}$. The Riemann tensor contracts to the Ricci Tensor, $R_{\mu\nu}$. Finally, taking the trace of the Ricci Tensor produces the Ricci Scalar, *R*. These relations are stated more explicitly through the below equations:

$$\Gamma^{\sigma}_{\mu\nu} = 1/2 \cdot g^{\sigma\rho}\left(\partial_{\mu} g_{\nu\rho} + \partial_{\nu} g_{\rho\mu} - \partial_{\rho} g_{\mu\nu}\right);$$

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma};$$

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu};$$

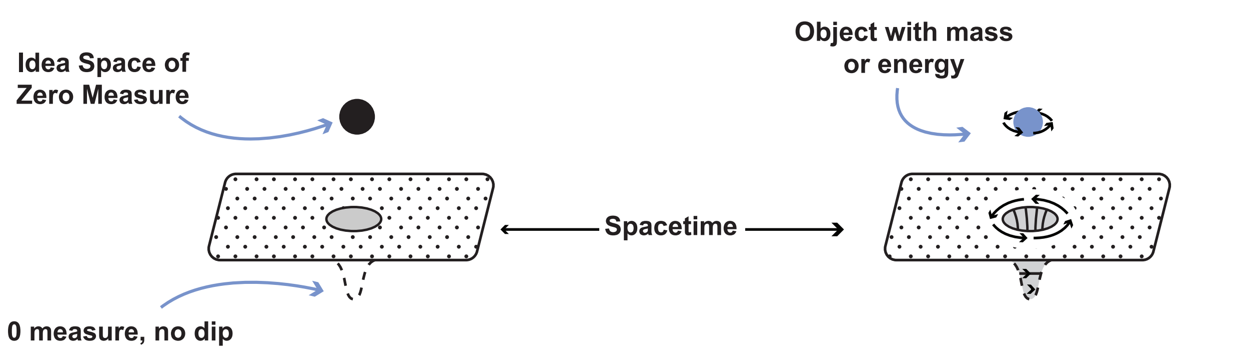
$$R = R^{\mu}{}_{\mu} = g^{\mu\nu}R_{\mu\nu}.$$

From there, you plug in your values of the Ricci Tensor, Ricci Scalar, and metric into Einstein's Field Equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}.$$

The equation defines how spacetime curves (left hand portion) in relation to matter, or energy (right hand portion). More details on this derivation can be found in the texts mentioned in the references, or online.

Since the idea space relies on the pseudo-metric, $g_{\mu\nu} = R_{\mu\nu} = R = 0$. In other words, an idea space has no direct effect on spacetime: **an idea does not curve spacetime at all** *(Figure 8).*



(a) *An object with zero measure, like your idea space, has no direct effect on spacetime.*

(b) *An object with measure has a direct effect on spacetime.*

Figure 8*. Your idea space has no direct effect on spacetime.*

Sets with zero measure are measurable. However, since an idea space does not cause a dip in spacetime and has no direct effect on spacetime, *then your idea space sits outside of spacetime*.

Specifically, space and time describe a framework for things we can measure. For instance, you can measure 20 feet here or 30 seconds there. If you were to place an object with zero measure in the way of your measurements, then you'd still get the same 20 feet here and 30 seconds there. An object with zero measure has no effect on the total measurement.

Put another away, imagine a car traveling 100ft in three seconds. If I place an object of zero measure directly in the way of the car, the car would still travel 100ft in three seconds. The car wouldn't be affected at all. Therefore, objects of zero measure must sit outside the realm of space and time, because they have no direct effect on neither space nor time.

In mathematical terms, let *Z* be a zero set and *E* be a set with measure, like 10 meters. Then:

$$E = Z \cup E.$$

Here, the union (U) of a set with zero measure, *Z*, and a set with measure, *E*, is no different than a set with measure, *E*, by itself. In other words, although the zero set is measurable, we see the inclusion or exclusion of a zero set has no effect on outer measure, or measurability.

Limits

In English, a **limit** occurs when two points are so close together they are essentially the same point.

In math, a sequence is a list of points, $p_1, p_2, \dots, p_n$, in a metric space *M*, where the points p_n belong to *M*. This is often written as (p_n). The sequence (p_n) **converges to the limit p** in *M* if

For all $\varepsilon > 0$, there exists an $N \in \mathbb{N}$ such that
$n \in \mathbb{N}$ and $n \geq N$ implies $d(p_n, p) < \varepsilon$.

Similarly, a Cauchy sequence is a sequence (p_n) in *M* that satisfies the Cauchy condition (Figure 9). Namely,

For all $\varepsilon > 0$, there exists an $N \in \mathbb{N}$ such that
$n, k \in \mathbb{N}$ and $n, k \geq N$ implies $d(p_n, p_k) < \varepsilon$.

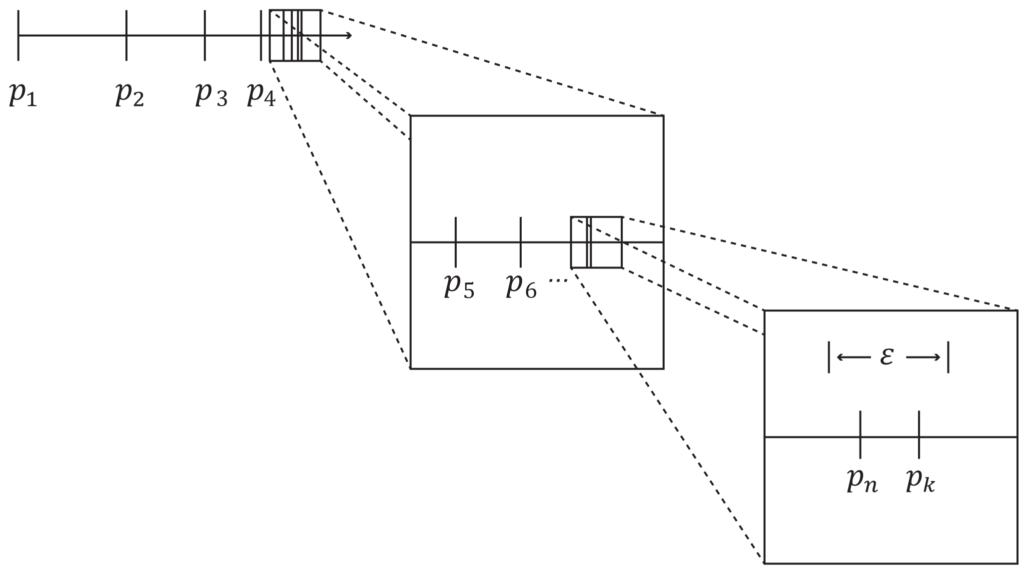


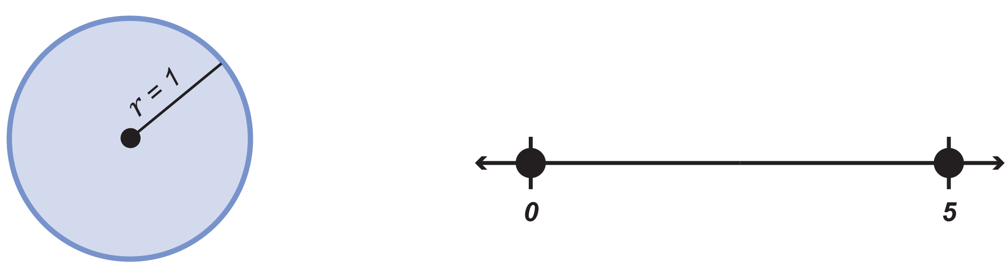
***Figure 9**. Cauchy Condition: The distance between p_n and p_k is less than ε for all $\varepsilon > 0$.*

Closed and Open

Let *M* be a metric space and *S* be a subset of *M*. A point $p \in M$ is a limit of *S* if there exists a sequence (p_n) in S that converges to p.

Closed

S is a **closed set** if it contains all its limits. For instance, the below disk with radius 1 (Figure 10-a) and the line interval [0,5] (Figure 10-b) are both closed. They contain all their points, including those on the boundary.



***(a)** Closed Disc.*

***(b)** The interval [0,5] is closed.*

***Figure 10**. Two different representations of closed.*

The empty set, Ø, is by definition closed since it contains all its limits (since it has none!). So too is the metric space, *M*: it certainly contains the limit of any sequence that converges in *M*.

An idea space is considered closed if it has all of its limits. This is similar to a closed mindset: every thought, sensation, emotion, and perception in a closed idea space is self-contained and can be reached (Figure 11). This is not necessarily a "bad" thing. Closed idea spaces are achieved through focused awareness.

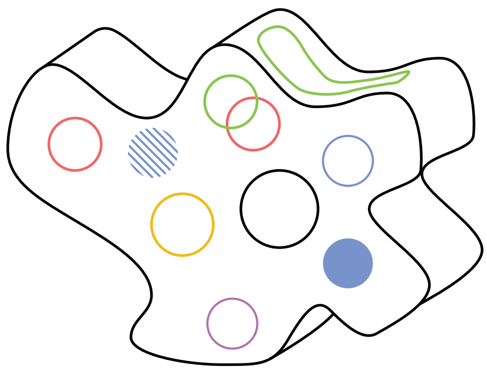
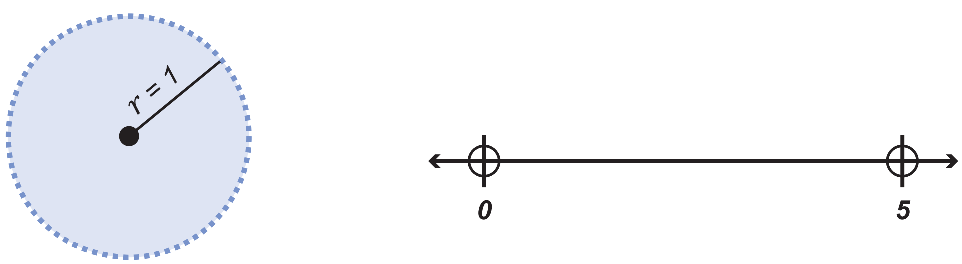


Figure 11. *A closed idea space: focused awareness.*

<u>Open</u>

In English, **open** means: given a point, p, there will always be another point, q, within a certain positive radius of the original point, p. For instance, the below disk with radius 1 (Figure 12-a) and the line interval (0,5) (Figure 12-b) are both open. They contain all the point in the interior, but do not include the points on the boundary.



(a) *Open Disc.*

(b) *The interval (0,5) is open.*

Figure 12. *Two different representations of open.*

In other words, open means I can draw an infinite amount of smaller and smaller circles around a point and there will always be a new point within the vicinity of the original point.

In math, S is an open set if for each $p \in S$ there exists an $r > 0$ such that $d(p,q) < r$ implies that $q \in S$.

This is made more evident through the concepts of neighborhoods. A **neighborhood** of a point p in M is any open set S that contains p. This is usually written as $S = M_r p$ and is illustrated below. Note how q is within r distance away from p. This holds true for any $r > 0$ (Figure 13).

Furthermore, one can draw another open neighborhood, $W = M_l q$, at the point q which shows that there exists an $l > 0$ such that $d(q,x) < l$ implies that $x \in W$.

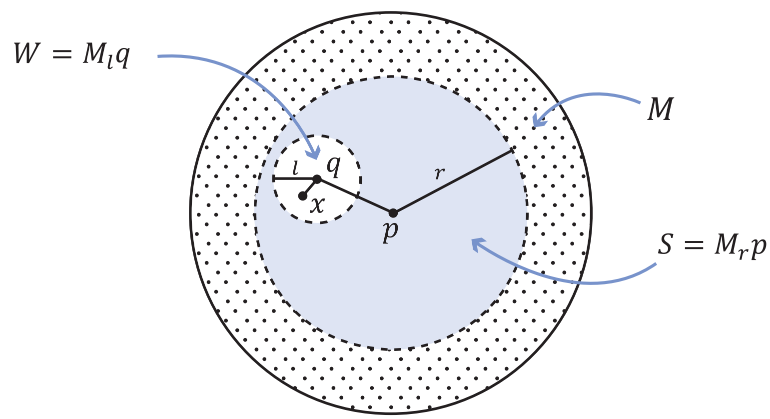


Figure 13. *The r-neighborhood of p is an open set, because q is within r. Hence, S is open. Similarly, the l-neighborhood of q is an open set, because x is within q. Hence, W is open.*

An idea space is considered open if it does not contain all its limits (Figure 14). This is similar to an open mindset: you can have many thoughts, emotions, sensations, and perceptions present within your idea space, but there are some ideas that are unreachable. As you approach a specific idea, there is always a new idea that will populate the idea space. This is not a “good” thing. Open idea spaces are achieved through choiceless awareness.

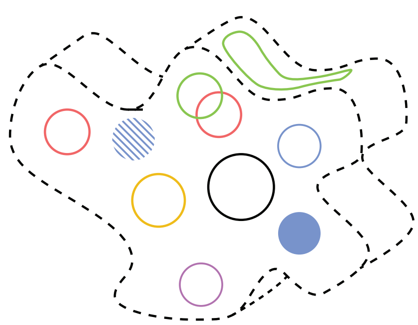


Figure 14*. An open idea space: choiceless awareness.*

A good way to differentiate open and closed is through video games. If you've ever played one, then you may be familiar with the concept of lag, or a glitch. On one hand, imagine watching a character continuously walk towards a wall, but never reach it. That's open. On the other hand, if the character goes—SPLAT—against the wall and reach it, then that's closed.

Topological Space

A topological space is a set, M, together with a collection of all the open subsets, $\mathcal{U}$, of M that satisfy the following three properties:

(a) $\emptyset \in \mathcal{U}, M \in \mathcal{U}$
- The empty set and the set M are open sets. Both Ø and M are covered by the collection of all open subsets, $\mathcal{U}$

(b) Every union of any members of $\mathcal{U}$ is in $\mathcal{U}$
- The union of any open subset is within the collection of all open subsets.

(c) The intersection of finitely many members of $\mathcal{U}$ is in $\mathcal{U}$
- The intersection of finitely many open subsets is within the collection of all open subsets

A subset C of a topological space M is said to be closed if and only if its complement, M-C, is open. In other words, openness is dual to closedness: the complement of an open set is closed and the complement of a closed set is open.

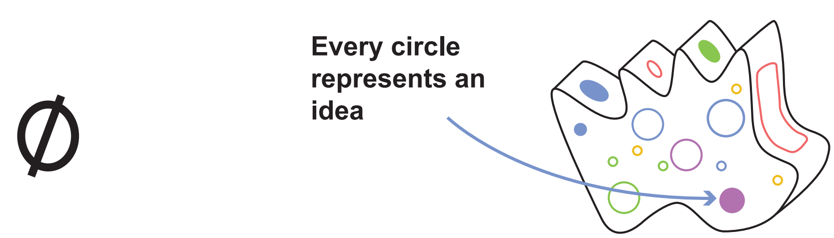
From this we can define a topological space through its closed sets:

(a) Ø and M are closed

(b) the finite union of any pair of closed sets is closed

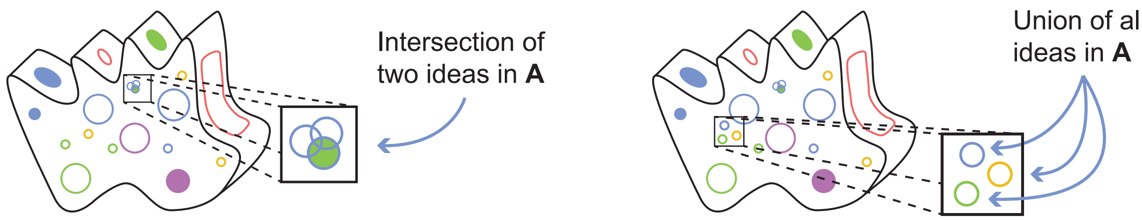
(c) the intersection of any number of closed sets is closed.

We see Ø and M are both open and closed sets; hence, **clopen**. Furthermore, an idea space is therefore a topological space, because it fulfills the three properties listed above. (Figure 15).



***(a)** The idea space contains the null set - intuitively it contains nothing.*

***(b)** The idea space contains all of its elements (e.g. thoughts, emotions, sensations, and perceptions.)*



(c) *The intersection of any two ideas is within The Idea Space,* **A.**

(d) *The union of any all ideas is within The Idea Space,* **A.**

Figure 15*. Your idea space is a topological space.*

A topological space needs no metric! In a way, groupings precede the need to count or define distances. It is one level of abstraction higher. In other words, all metric spaces are topological spaces, but not all topological spaces are metric spaces.

Furthermore, most of the metric spaces in general relativity, including the idea space, are both open and closed, or clopen.

The Idea Space is Uncountable

Numbers

There are four main types of numbers:

Natural Numbers, $\mathbb{N} = 1, 2, 3, 4, 5, 6, \ldots$ | Whole numbers whose value is greater than 1.

Integers, $\mathbb{Z} = \cdots, -2, -1, 0, 1, 2, \ldots$ | Positive whole numbers, zero, and negative whole numbers.

Rational Numbers, $\mathbb{Q} = \frac{2}{3}, \frac{5}{4}, \frac{8}{3}, \frac{9}{10}$, etc. | $\mathbb{Q} = \frac{p}{q}$ for $p, q \in \mathbb{Z}, q \neq 0$ | The fraction between any two integers, such that the bottom integer is not 0.

Real Numbers, $\mathbb{R} = 2.3432\ldots$, or $231.12312\ldots$, or $2.97631\ldots$, etc. | $\mathbb{R} = Z.x_1x_2x_3x_4x_5\ldots$ for $Z \in \mathbb{Z}$ and $x \in \{0,1,2,3,4,5,6,7,8,9\}$ | Any decimal expansion that never terminates. It helps to picture a number line.

Function

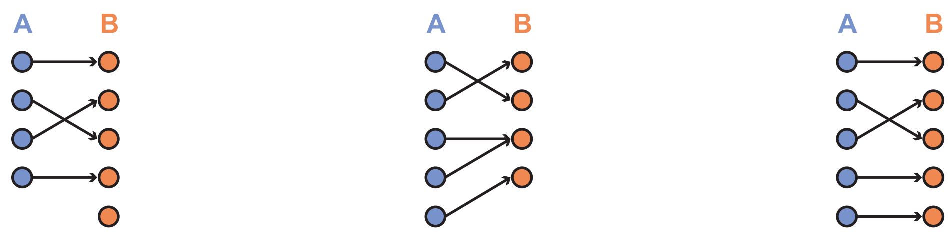
A **function**, f, takes a point from a space *A* to another space *B*. Functions are also called maps or transformations.

A mapping $f: A \to B$ is an injection (i.e. one-to-one) if for each pair of distinct elements $a, a' \in A$ the elements $f(a)$ and $f(a')$ are distinct in *B*:

$$a \neq a' \text{ implies } f(a) \neq f(a').$$

The mapping f is a surjection (i.e. onto) if for each $b \in B$ there is at least one $a \in A$ such that $f(a) = b$.

A mapping is a **bijection** if it is both injective and surjective.



***(a)** Injective (one-to-one)* ***(b)** Surjective (onto)* ***(c)** Bijective (one-to-one & onto)*

***Figure 16**. Basic types of functions.*

Counting

If there is a bijection between the natural numbers, $\mathbb{N}$, and a set, then that set is **countable**. If there is no bijection between the natural numbers, $\mathbb{N}$, and a set, then that set is **uncountable**. For instance, how many oranges are there in the set S (Figure 17)?

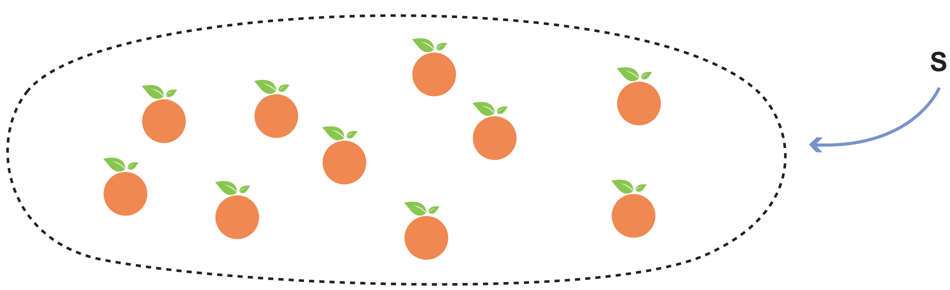


***Figure 17**. How many oranges are in this arbitrary set, S?* [2]

In your head, you probably did something like the following (Figure 18). You mapped the natural numbers, one by one, onto our oranges. One orange, two oranges, three oranges… You'll agree, I could add as many oranges as I want to—an infinite amount even. Then, you would continue the pattern and count all the way until infinity.

2. 10, in case you forgot how to count.

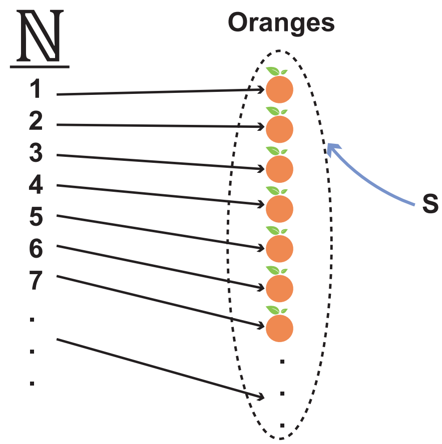


Figure 18. *To count something means that there is a bijection, or one to one and onto function between the natural numbers, ℕ, and a set.*

So, the natural numbers are countable, even though there are infinite amount of them. In fact, contrary to what one may believe, the natural numbers, ℕ (i.e. 1, 2, 3, …), has the same **cardinality**, or the same number of elements, as the integers, ℤ (i.e. …, -2, -1, 0, 1, 2, …), and the rational numbers, ℚ (i.e. $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$, etc.).

Countable means that there is a bijection between the natural numbers and a specific set. So, if we prove that there exists a bijection between the natural numbers and [ℕ × ℕ, 0], then that will show that ℕ, ℤ, and ℚ have equal cardinality.

1 Theorem. ℕ×ℕ and zero ([ℕ×ℕ, 0]) is countable.

Proof: Think of ℕ×ℕ and zero as an $\infty \times \infty$ matrix and walk along the counter-diagonals (Figure 19). Clearly, we can list every integer and rational number:

(0), (1, 1), (2,1), (1,2), (3,1), (2,2), (1,3), (4,1), (3,2), (2,3), (1,4), etc.

There are no integers nor rational numbers that were left off the list. Therefore, there exists a bijection between the natural numbers and the integers and rational numbers. Thus, by definition, the integers and rational numbers are countable.

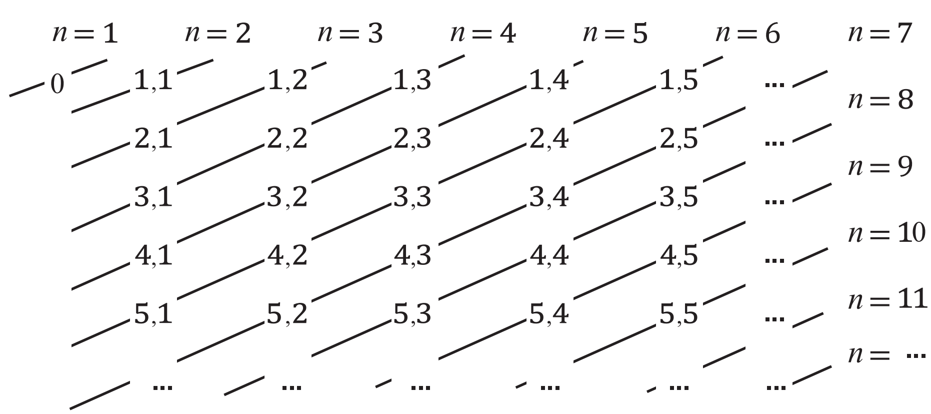


Figure 19. *The integers and rational numbers are countable.* ■

On the other hand, the real numbers are not countable, because there is no bijection between the natural numbers and the real numbers. In other words, **there is more infinity in the real numbers, $\mathbb{R}$, than in the natural numbers, $\mathbb{N}$**. The proof is known as Cantor's Diagonal argument and boils down to: if you list any real numbers—even an infinite amount—then I'll always be able to pick a real number that you did not state.

2 Theorem. The real numbers, $\mathbb{R}$, are uncountable.

Proof: A real number is: $Z.x_1x_2x_3x_4x_5\ldots$ where Z is any integer (…, -2, -1, 0, 1, 2, …) and x_i is any number between 0-9. This number can be uniquely determined if one agrees never to terminate its decimal expansion. Now, our goal is to prove that the real numbers are uncountable. To do so, let's assume it is countable and force a contradiction.

If the real numbers are countable, then, per our definition, there must exist a bijective map between the Natural Numbers, $\mathbb{N}$, and the Real Numbers, $\mathbb{R}$ (Figure 20). So, let's count the real numbers, $\mathbb{R}$, like the above oranges. In order to list the real numbers, we introduce elements x_{ij}, where *i* and *j* represent the row and column index, respectively. Again, x_{ij} is simply a number between 0-9.

$$
\begin{array}{ccl}
\mathbb{N} & & \qquad\mathbb{R} \\
1 & \longrightarrow & Z.x_{11}x_{12}x_{13}x_{14}x_{15}x_{16}x_{17}\ldots \\
2 & \longrightarrow & Z.x_{21}x_{22}x_{23}x_{24}x_{25}x_{26}x_{27}\ldots \\
3 & \longrightarrow & Z.x_{31}x_{32}x_{33}x_{34}x_{35}x_{36}x_{37}\ldots \\
4 & \longrightarrow & Z.x_{41}x_{42}x_{43}x_{44}x_{45}x_{46}x_{47}\ldots \\
5 & \longrightarrow & Z.x_{51}x_{52}x_{53}x_{54}x_{55}x_{56}x_{57}\ldots \\
6 & \longrightarrow & Z.x_{61}x_{62}x_{63}x_{64}x_{65}x_{66}x_{67}\ldots \\
7 & \longrightarrow & Z.x_{71}x_{72}x_{73}x_{74}x_{75}x_{76}x_{77}\ldots \\
\vdots & \longrightarrow & \vdots
\end{array}
$$

Figure 20. *A bijection between the natural numbers, $\mathbb{N}$, and the real numbers, $\mathbb{R}$.*

By looking at the above figure, you'll agree that we listed all the possible real numbers. Now, can you think of a number that is not represented in the Real Numbers table above?

Let's arbitrarily introduce a new number:

$$a = Z.a_1a_2a_3a_4\ldots,$$

such that $a_i \neq x_{ii}$ Put plainly, the first decimal point (a_1) cannot equal the first decimal point in our first row (x_{11}); the second decimal point (a_2) cannot equal the first decimal point in our first row (x_{22}); etc. In mathematical terms, $a_1 \neq x_{11}, a_2 \neq x_{22}$, $a_3 \neq x_{33}$, etc…

So, where is this new number that we introduced? Is it in the first row? Nope—for x_{11} cannot be the digit at that place. How about the second row? No, again—x_{22} now cannot be the digit. Keep this going for k digits and you'll note that every single diagonal in the Real Numbers table is not part of our new number, $a = Z.a_1a_2a_3a_4...$ (Figure 21).

$$
\begin{array}{ccl}
\mathbb{N} & & \mathbb{R} \\
1 & \longrightarrow & Z_1.x_{11}x_{12}x_{13}x_{14}x_{15}x_{16}x_{17}\cdots \\
2 & \longrightarrow & Z_2.x_{21}x_{22}x_{23}x_{24}x_{25}x_{26}x_{27}\cdots \\
3 & \longrightarrow & Z_3.x_{31}x_{32}x_{33}x_{34}x_{35}x_{36}x_{37}\cdots \\
4 & \longrightarrow & Z_4.x_{41}x_{42}x_{43}x_{44}x_{45}x_{46}x_{47}\cdots \\
5 & \longrightarrow & Z_5.x_{51}x_{52}x_{53}x_{54}x_{55}x_{56}x_{57}\cdots \\
6 & \longrightarrow & Z_6.x_{61}x_{62}x_{63}x_{64}x_{65}x_{66}x_{67}\cdots \\
7 & \longrightarrow & Z_7.x_{71}x_{72}x_{73}x_{74}x_{75}x_{76}x_{77}\cdots \\
\vdots & \longrightarrow & \vdots
\end{array}
$$

? ? ?

? $\boxed{a = Z.a_1a_2a_3a_4\ldots}$?

? ? ?

$$
\boxed{\begin{array}{c} a_1 \neq x_{11} \\ a_2 \neq x_{22} \\ a_3 \neq x_{33} \\ \cdots \end{array}}
$$

***Figure 21**. Where is the number $a = Z.a_1a_2a_3a_4...$, such that $a_i \neq x_{ii}$? Nowhere! Thus Cantor's Diagonal proves that the Real Numbers, $\mathbb{R}$, are uncountable, because the bijection no longer holds.*

We have created a contradiction! There is a number that is not part of this table and therefore the bijection no longer holds! We must have made a wrong assumption somewhere; and, the only assumption we made was that we could map the Natural Numbers, $\mathbb{N}$, bijectively to the Real Numbers, $\mathbb{R}$. Why? Because we assumed that the Real Numbers, $\mathbb{R}$, were countable. Therefore, by contradiction, the Real Numbers, $\mathbb{R}$, are uncountable. ■

Space and time are uncountable, because according to general relativity they rely on the real numbers. For every measurement you make, I can always make a measurement you didn't make.

An idea space is uncountable, because for every idea you pick, I could always pick an idea you did not have. This is based on the proposition: **every solution to a problem raises new unsolved problems; whenever an idea is discovered, a new idea appears; a new discovery creates more questions than answers.**

For an illustrative proof, picture yourself living in the year 1680. You are tasked with counting all the ideas in the world. You go out and you count an infinite amount of them. Then, in 1687, Isaac Newton publishes *Philosophiæ Naturalis Principia Mathematica* and *the idea of gravity* came into the picture. Where was the idea of gravity in all the idea spaces before? Was it in the first one? No. The second? No. It is nowhere found in the infinite list of ideas (Figure 22). In other words, gravity didn't "exist" until Isaac Newton brought forth the *idea of gravity*. Similarly, the idea of the idea space didn't "exist" until this book. Where was it in the infinite list of idea spaces beforehand?

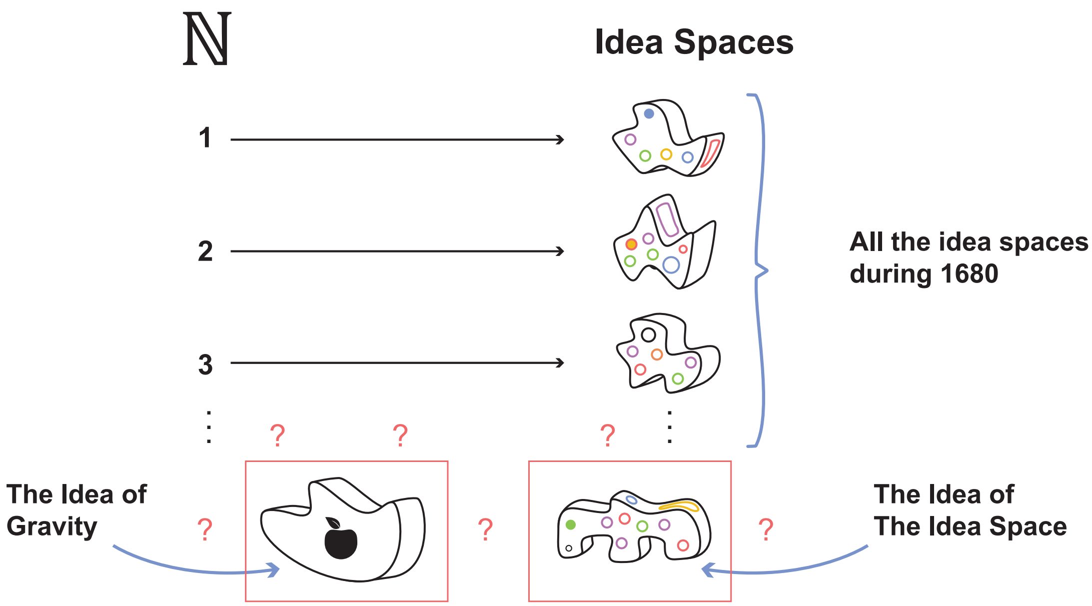


Figure 22*. Where was the idea space of gravity in 1680?*

To test the uncountability of your idea space for yourself, simply sit for a couple minutes and try to count all your ideas. Do not be fooled, some are very soft, like "It's quiet in here." Others come from behind and say, "There haven't been many thoughts lately." What you'll find when you do this is that for every thought, sensation, emotion, or perception you count, another one appears.

Uncountability directly translates to impermanence. Both the world and your mind are impermanent, or in a constant state of flux. In the universe, this shows up as the mysterious **dark energy**, which is constant at every point in space and time and is responsible for the universal expansion of space. In your mind, this shows up as an ever changing landscape of thoughts, emotions, sensations, and perceptions. All in all, "No man steps into the same river twice, for it is not the same river and he is not the same man" (Heraclitus).

The Cantor Set

The Cantor Set provides an example of a mathematical phenomenon that is uncountably deep and has zero measure, like your idea space (Figure 23).

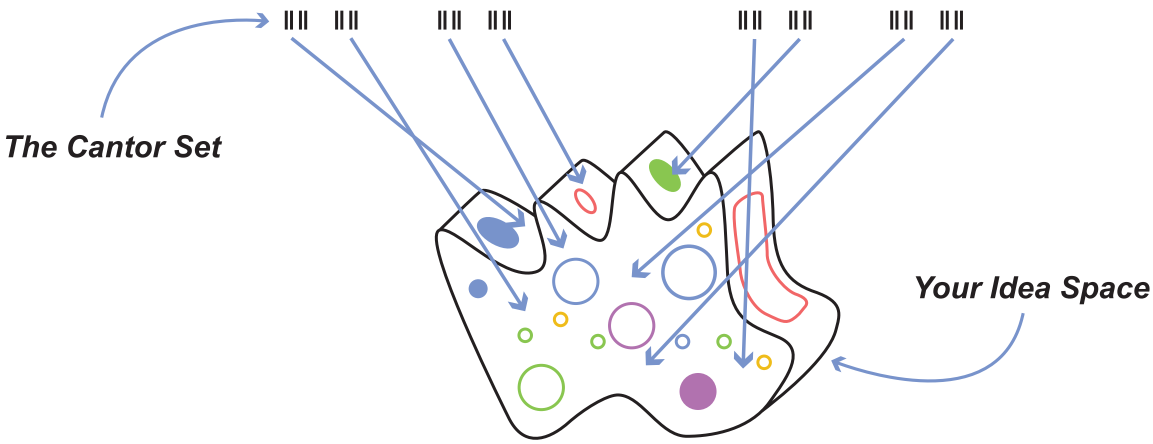


***Figure 23**. Your idea space is uncountable and has zero measure, like The Cantor Set.*

To build The Cantor Set start with the unit interval [0,1]. Then, remove its open middle thirds, (1/3, 2/3). This leaves us with the closed intervals [0, 1/3] and [2/3, 3/3]. Then, remove the open middle thirds of the remaining two intervals. Repeat this ad nauseam.

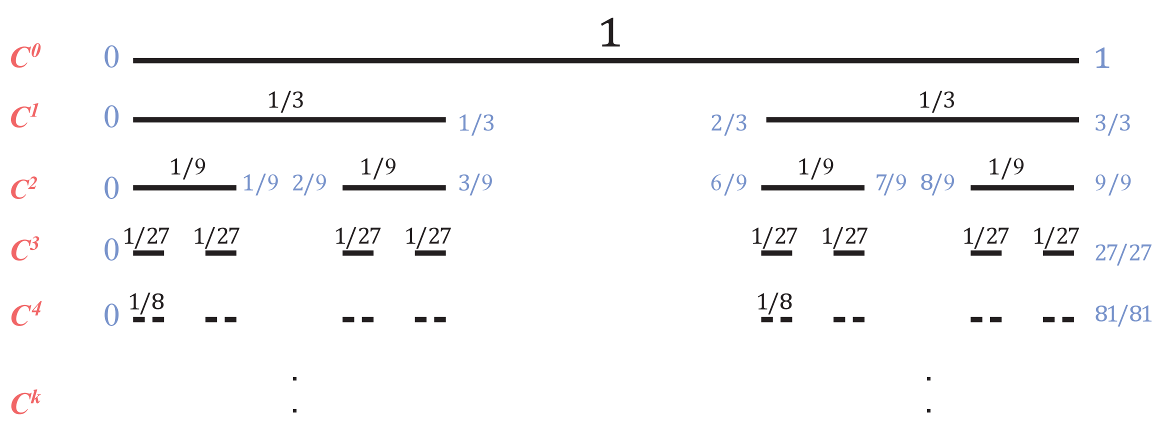


***Figure 24**. Constructing the Cantor Set. In **blue**, we have the start and end points of each iteration, C^k. Iterations are noted in **red**. The total measure of each iteration is in **black**.*

Let's break this down for a couple iterations:

C^0. We start with C^0 which constitutes the closed interval $[0,1]$. This has measure of **1**.

C^1. Remove the open middle thirds of the line to obtain C^1. This becomes the union of two intervals $[0,1/3]$ and $[2/3,1]$. This has a total measure of $1/3+1/3 = \mathbf{2/3}$.

C^2. Take away the middle thirds of the two intervals in C^1 and you're left with C^2. This consists of four intervals $[0,1/9]$, $[2/9,3/9]$, $[6/9,7/9]$, and $[8/9,1]$. The total measure of these intervals is $1/9+1/9+1/9+1/9 = \mathbf{4/9}$.

C^3. You can continue this for C^3 and you get eight intervals:

$$[0/27, 1/27],\ [2/27, 3/27],\ [6/27, 7/27],\ [8/27, 9/27],$$
$$[18/27, 19/27], [20/27, 21/27],\ [24/27, 25/27], [26/27, 27/27]$$

Each interval has a measure of $1/27$ for a total measure of $\mathbf{8/27}$.

C^k. Continue for each iteration, k. The number of intervals goes up by a factor of 2^k and the measure of each interval goes down by a factor of $1/3$. Therefore, the total measure for a particular iteration is simply: $\mathbf{2^k \cdot (1/3)^k}$. Let's do an example for an arbitrary $k = 27$.

C^{27}. At C^{27}, one would have 2^{27} intervals, with a total measure of $\mathbf{2^{27} \cdot (\frac{1}{3})^{27}}$.

If one keeps iterating towards infinity, all that's left are points (Figure 25). Clearly, the distance between any point and itself is zero. If you add together an infinite amount of points together, each with zero measure, then your total measure is still zero. Thus, The Cantor Set is a set of zero measure.

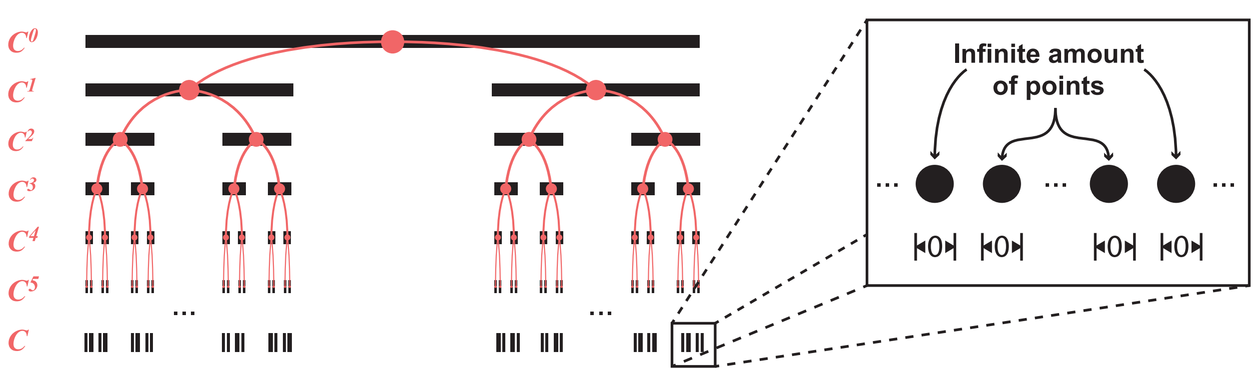


Figure 25. *At the end of the Cantor Set, all we're left with are an infinite amount of points, each with zero measure.*

The question now becomes: Does The Cantor Set have a countable or uncountable amount of end points? The next theorem sheds light on this question.

3 Theorem. The Standard Middle Thirds Cantor Set is uncountable.

Proof: In order to prove the Cantor Set is uncountable, we need to introduce a geometric coding. *Seeing the geometric coding is easier than trying to understand the words behind it*. This coding will in turn allow us to use Cantor's Diagonal argument, thus proving that the set is uncountable. Ok, so what are we coding? For each interval, 0 = left and 2 = right. So, the left interval of C^1 would be denoted as C_0. The right interval in C^1 would be denoted as C_2. Therefore, C^1 becomes the union of C_0 and C_2. If we look at the next iteration, C^2 would then be the union of the four segments: C_{00}, C_{02}, C_{20}, and C_{22}. The continuation of this geometric coding is made explicit in the below figure.

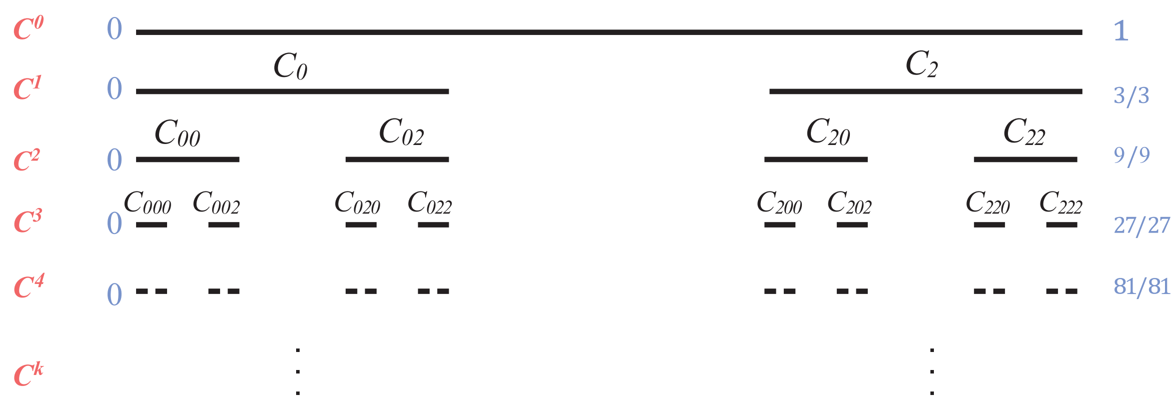


Figure 26. *The geometric coding of the Cantor Set.*

As one can see, we can continue this pattern indefinitely. For example, the left most interval for C^{27} can be represented as $C_{00...0}$ with 27 zeros! Now, what happens when we reach infinity? You will note that each interval can be represented as an infinite string consisting of either 0 or 2. Think of each of these intervals as a sort of address, or zip code.

We are trying to prove that the number of intervals is uncountable. Therefore, let us assume that they are countable and force a contradiction—just like our second theorem! If there are a countable amount of intervals, then there exists a bijection (i.e. one-to-one and onto mapping) between the Natural Numbers, $\mathbb{N}$, to all the addresses in the Cantor Set. This is demonstrated in our figure below where we have an infinite string of coordinates, $C_{\omega_{ij}}$. Here, $\omega_{ij} = 0$ or $\omega_{ij} = 2$. Once again, all the i and j denote are the indices of the row and column.

$$\begin{array}{ccl}
\mathbb{N} & & \textbf{Cantor Set} \\
1 & \longrightarrow & C\omega_{11}\,\omega_{12}\,\omega_{13}\,\omega_{14}\,\omega_{15}\,\omega_{16}\,\omega_{17}\cdots \\
2 & \longrightarrow & C\omega_{21}\,\omega_{22}\,\omega_{23}\,\omega_{24}\,\omega_{25}\,\omega_{26}\,\omega_{27}\cdots \\
3 & \longrightarrow & C\omega_{31}\,\omega_{32}\,\omega_{33}\,\omega_{34}\,\omega_{35}\,\omega_{36}\,\omega_{37}\cdots \\
4 & \longrightarrow & C\omega_{41}\,\omega_{42}\,\omega_{43}\,\omega_{44}\,\omega_{45}\,\omega_{46}\,\omega_{47}\cdots \\
5 & \longrightarrow & C\omega_{51}\,\omega_{52}\,\omega_{53}\,\omega_{54}\,\omega_{55}\,\omega_{56}\,\omega_{57}\cdots \\
6 & \longrightarrow & C\omega_{61}\,\omega_{62}\,\omega_{63}\,\omega_{64}\,\omega_{65}\,\omega_{66}\,\omega_{67}\cdots \\
7 & \longrightarrow & C\omega_{71}\,\omega_{72}\,\omega_{73}\,\omega_{74}\,\omega_{75}\,\omega_{76}\,\omega_{77}\cdots \\
\vdots & \longrightarrow & \vdots
\end{array}$$

Figure 27. *The Cantor Set represented as an infinite string of coordinates, C, where $\omega_{ij} = 0$ or $\omega_{ij} = 2$.*

Now, we'll do exactly what we did in theorem 2. You'll agree that this lists all the possible coordinates in our Cantor Set. However, can we think of a coordinate that is not in our above table?

Let's pick an arbitrary coordinate:

$$a = C_{a_1 a_2 a_3 a_4} \cdots,$$

such that $a_i \neq \omega_{ii}$. Namely, if $\omega_{ii} = 0$, then we replace it with $\omega_{ii} = 2$; and, vice versa. Where is this coordinate in our list? Is it in the first row? Nope, because $a_1 \neq \omega_{11}$. Is it in the second row? Nope, because $a_2 \neq \omega_{22}$. Continue this forever and you'll see that our coordinate, a, is nowhere to be found! These words and the below figure concludes our proof.

$$\begin{array}{ccl}
\mathbb{N} & & \textbf{Cantor Set} \\
1 & \longrightarrow & C\omega_{11}\,\omega_{12}\,\omega_{13}\,\omega_{14}\,\omega_{15}\,\omega_{16}\,\omega_{17}\cdots \\
2 & \longrightarrow & C\omega_{21}\,\omega_{22}\,\omega_{23}\,\omega_{24}\,\omega_{25}\,\omega_{26}\,\omega_{27}\cdots \\
3 & \longrightarrow & C\omega_{31}\,\omega_{32}\,\omega_{33}\,\omega_{34}\,\omega_{35}\,\omega_{36}\,\omega_{37}\cdots \\
4 & \longrightarrow & C\omega_{41}\,\omega_{42}\,\omega_{43}\,\omega_{44}\,\omega_{45}\,\omega_{46}\,\omega_{47}\cdots \\
5 & \longrightarrow & C\omega_{51}\,\omega_{52}\,\omega_{53}\,\omega_{54}\,\omega_{55}\,\omega_{56}\,\omega_{57}\cdots \\
6 & \longrightarrow & C\omega_{61}\,\omega_{62}\,\omega_{63}\,\omega_{64}\,\omega_{65}\,\omega_{66}\,\omega_{67}\cdots \\
7 & \longrightarrow & C\omega_{71}\,\omega_{72}\,\omega_{73}\,\omega_{74}\,\omega_{75}\,\omega_{76}\,\omega_{77}\cdots \\
\vdots & \longrightarrow & \vdots
\end{array}$$

$$\begin{array}{ccc}
 & ?\quad ?\quad ? & \\
? & \boxed{a = C\,a_1 a_2 a_3 \ldots} & ? \\
 & ?\quad ?\quad ? &
\end{array}
\qquad
\boxed{\begin{array}{c} a_1 \neq \omega_{11} \\ a_2 \neq \omega_{22} \\ a_3 \neq \omega_{33} \\ \cdots \end{array}}$$

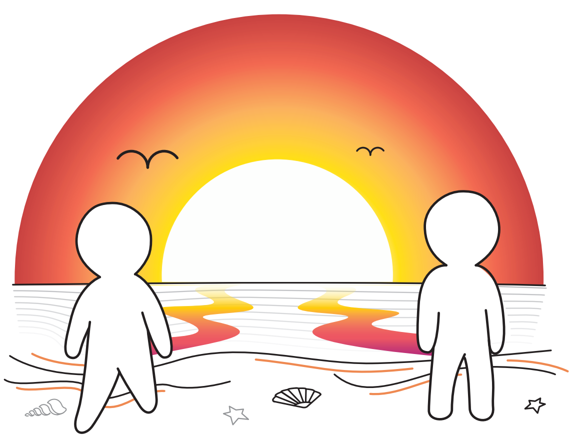
Figure 28. *Where is the coordinate $a = C_{a_1 a_2 a_3 a_4} \cdots$, such that $a_i \neq \omega_{ii}$ in the above table? Nowhere, thus proving that the Cantor Set is uncountable.* ■

You will quickly note there are an uncountable number of Cantor Sets. Instead of taking out the middle thirds, you can take out the middle fourth, middle 1/5.4323, middle 1/pi, etc. In other words, I can always create a new Cantor Set you did not, by taking out a different open middle set.

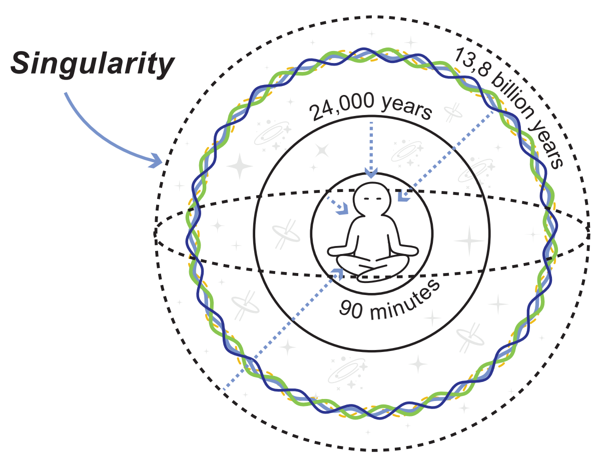
The Sunset Conjecture

The Sunset Conjecture is two fold: (a) everyone is at the center of their own observable universe, and (b) at the center lies your idea space of uncountable depth and zero measure. This conjecture explains why, during a sunset, it appears as if the sun's golden rays are unique for both people (figure 29-a).

Your observable universe is a giant sphere centered on you. Everything you see in your observable universe is in the past. It takes time for light to go from point A to point B, even when it travels at 186,000 miles per second. Thus, we can *theoretically* see The Big Bang. Of course, the Cosmic Microwave Background (CMB) prevents light from that era from making it past, but the gravitational effects of The Big Bang are able to make it past the CMB as the hypothetical **graviton** decoupled at Planck Time (10^{-44} seconds post the start of the universe). Thus, extrapolating the sunset idea, everyone experiences their own Singularity Sunset, or everyone experiences the gravitational effects of the start of our universe, The Big Bang, differently (Figure 29-b). Your perspective of the universe is unique to you.



(a) During a sunset, the sun's golden rays appear unique for both people.



(b) Everyone lives at the center of their own observable universe. Everyone experiences their own Singularity Sunset.

Figure 29. *The Sunset Conjecture.*

Plus, at the center of it all lies your idea space—hidden from the outside world.

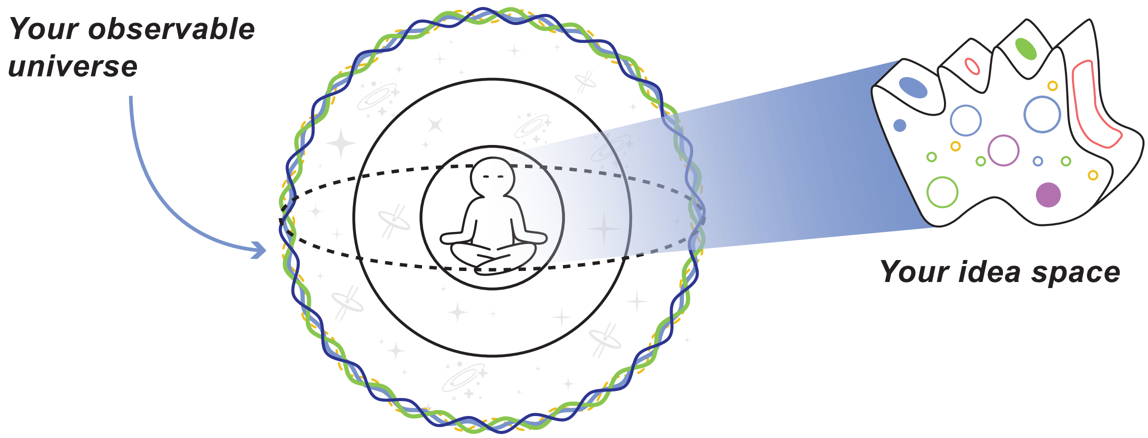


Figure 30. *At the center of your observable universe lies your idea space, hidden from the outside world.*

Conclusion

The goal of physics is to provide mathematical models we can overlay on top of the world to make better sense of reality. The idea space expands on the guidelines set forth by general relativity to introduce a new metric, the pseudo-metric, to describe the mind. Since your mind is hidden from the outside world, because it has zero measure, a new science is needed to study the consciousness: the science of the first person, or mindfulness.

Thankfully, many mindfulness practices already exist, like The Idea Space, which is a union of Zen and physics. Other practices include Buddhism, Transcendental Meditation, Effortless Mindfulness, Breathing Techniques, Metta, The Headless Way, Stoicism, Yoga, etc. To build a well-rounded idea space, explore each of these practices during your own time to see which resonates with you the most.

Remember, do not to take any parts of the practice, including this whitepaper and the book, at face value. Test to see whether what others say lines up with your view of reality. Build your own practice.

Thank you for taking the time to read this and be well.

References

The following have consciously and subconsciously helped produce this paper. A more complete bibliography can be found in The Idea Space book.

Carroll, Sean. *From Eternity to Here: The Quest for the Ultimate Theory of Time.* Dutton, 2010.

Carroll, Sean. *Something Deeply Hidden: Quantum Worlds and the Emergence of Spacetime.* Dutton, 2019.

Carroll, Sean. *Spacetime and Geometry: An Introduction to General Relativity.* Cambridge University Press, 2019.

Carroll, Sean. *The Big Picture: On the Origins of Life, Meaning, and the Universe Itself*. Dutton, 2016.

Einstein, Albert. *Relativity: The Special & General Theory.* Henry Holt and Company, 1920.

Falconer, K.J. *The Geometry of Fractal Sets.* Cambridge University Press, 1985.

Feynman, Richard. *The Feynman Lectures on Physics Volumes I-III*. Basic Books, 1965.

Harding, Douglas. *On Having No Head: Zen and the Rediscovery of the Obvious*. The Buddhist Society, 1961.

Harris, Sam. *Waking Up*. Waking Up LLC, Version 2.2. https://www.wakingup.com/, https://apps.apple.com/us/app/waking-up-guided-meditation/id1307736395.

Hawking Stephen. *The Illustrated A Brief History of Time*. Bantam, 1988.

Hawking, Stephen and George Ellis. *The Large Scale Structure of Space-Time.* Cambridge University Press, 1973.

Kolb, Edward and Michael Turner. *The Early Universe*. Westview Press, 1990.

Kosniowki, Czes*. A First Course in Algebraic Topology*. Cambridge University Press, 1980.

Mack, Katie. *The End of Everything (Astrophysically Speaking).* Scribner, 2020.

Mandelbrot, Benoit. *The Fractal Geometry of Nature.* Freeman, 1977.

Misner, Charles, Kip Thorne, and John Wheeler. *Gravitation*. W. H. Freeman and Company, 1973.

Popper, Karl. *Conjectures and Refutations*. Routledge & Kegan Paul, 1963.

Pugh, Charles. *Real Mathematical Analysis*. Springer, 2002.

Thorne, Kip. *Black Holes and Time Warps: Einstein's Outrageous Legacy*. Norton, 1994.

Tou, Hseuh. *The Blue Cliff Record.* Translated by Thomas Cleary & J.C. Cleary. ~1000.

Watts, Alan. *The Way of Zen.* Vintage Books. 1957.

Weinberg, Steven. *Gravitation and Cosmology: Principles and Applications of The General Theory of Relativity.* John Wiley & Sons, 1972.

THANK YOU
FOR READING!