INVICTA EDUCATION

GEO STRIPS

MATHEMATICS
UNDERSTANDING GEOMETRY

GEOMETRIC EXERCISES IP014159

THIS PRODUCT GUIDE MAY BE PHOTOCOPIED FOR CLASS USE ONLY

DYNAMIC GEOMETRY **H. Shaw, D.C.P.**

The Geo Strips are in four colours, red, blue, yellow and white. There are different lengths in each colour and the difference in length between any two strips is measured by the distance between holes.

In these Working Guides, the strips will be referred to by letters and numbers as follows:-

RED STRIPS

R1
R2
R3

R3 is twice as long as R2 and R2 is twice as long as R1. R3 is four times as long as R1 (the holes in R3 divide the strip into four equal parts).

BLUE STRIPS

B1
B2

B2 is twice as long as B1.

YELLOW STRIPS

Y1
Y2

Y2 is twice as long as Y1 (the holes on the side of the centre hole divide the strips into three equal parts).

WHITE STRIPS

W1
W2

The white strips are special lengths. W2 is longer than W1. The strips fix together by means of brass fasteners.

NOTES FOR TEACHERS

B1 is R1 x √2, B2 is R2 x √2, Y1 is R1 x √3, Y2 is R2 x √3, W1 is R1 x √5.

Geometry is usually studied as a system of rigid non-flexible elements. Many geometric properties are elucidated more clearly by the use of flexible models rather than by similar static ones. Geo Strips make Geometry a dynamic subject.

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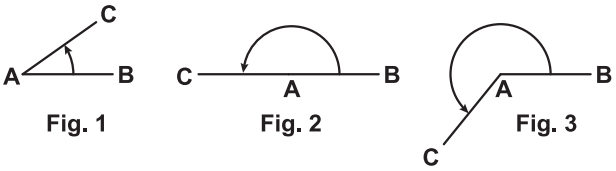
Geometric Exercises IP 014159a (0310)

SECTION 1 - ANGLES (1)

Join two R3 strips. Turn one strip from AB to AC (Fig. 1). The amount of TURNING from AB to AC is called an ANGLE. Continue turning until CAB is a straight line (Fig. 2).



Continue turning until AB and AC are together again. AC has turned through a circle. The unit for measuring an angle is called a DEGREE, written °. There are 360 degrees (360°) in a circle. Angles can be measured by means of a protractor. Examine a protractor. Why are there two sets of figures on it?


Turn AC until AC and AB are a quarter of a circle, 90°. This angle is called a RIGHT ANGLE.


An angle less than a right angle is called an ACUTE ANGLE. Acute means "sharp".


An angle greater than a right angle is called an OBTUSE ANGLE. Obtuse means "blunt".


When AC turns through two right angles, 180° a STRAIGHT ANGLE is made by the strips.


An angle greater than two right angles (Fig. 3), but less than four right angles is called a REFLEX ANGLE.

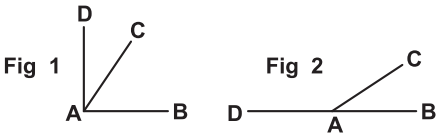
Fasten two R1 strips together and place on to the R3 strips so that points A are together. Turn the two sets of strips together. Does the length of the strip make any difference to the size of the angle?

Join two R1 and one B1 strips to make a right-angle. Use this angle to measure right angles in the classroom, e.g., the corners of books, desks, boxes, etc.

SECTION 2 - ANGLES (2)

1. How many degrees are there in a right angle?
2. How many degrees are there in a straight angle? Your teacher will show you how to use a protractor.
3. Draw these angles: 30°, 15°, 45°, 20°, 60°, 72°, 90°, 108°, 120°, 135°, 150°, 165°.
4. Draw two lines which make an angle and measure the angle.

Make Fig. 1 with three R3 strips. Set angle DAB at 90°. Move AC to various positions. There are two angles, CAD and CAB. When angle DAB is a right angle, angle CAB is called the COMPLEMENT of angle CAD, and CAD is the complement of angle CAB. The sum of the two angles is 90°.



Make Fig. 2 with one R3 and one B2 strips. When angle DAB is a straight angle, angle CAB is the SUPPLEMENT of angle CAD and angle CAD is the supplement of angle CAB. The sum of the angles is 180°.

5. Draw these columns in your book and find the complement and supplement of the following angles: 30°, 45°, 60°, 72°, 90°, 108°, 120°, 135°.

Complement	Given Angle	Supplement
	30°	

6. Draw right angles as in Fig. 1. Draw AC in any position and measure the angles. Write the measurements on your drawings. Is the sum of the two angles 90°?
7. Draw a straight line and a line, AC, in any position as in Fig. 2. Measure the angles and write the measurement on your drawings. Is the sum of the two angles 180°?

WARNING: NOT SUITABLE FOR CHILDREN UNDER 36 MONTHS BECAUSE SMALL PARTS MAY CAUSE A CHOKING HAZARD. TO BE USED ONLY UNDER ADULT SUPERVISION. Please retain the information from this pack for future reference. We reserve the right to alter designs and specifications (including colours and materials) when such changes are unavoidable. This product conforms to the safety requirements of EN71, ASTM, 16 CFR and The Canadian Hazardous Products (Toys) Regulations. MADE IN CHINA

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GEO STRIPS 2

SECTION 3 - **TRIANGLES (1)**

"Tri" means three. What is a tricycle? A triangle has three sides. Join any three strips to form a triangle. Take another set of the same strips and try to make a shape different from the first. Are the two shapes exactly the same? A triangle is a rigid figure. You cannot alter its shape, that is why triangles are used to build bridges, towers, pylons, etc. Look out for the uses of triangles in structure and make a record of where you see them.

1. Make triangles with (a) two R2 and one B1, (b) three Y1, (c) one R1, one R2 and one Y1, (d) one R1, one R2 and one B1, (e) one R2, one W1 and one Y1. Here are some important things to learn about these triangles:

   Triangle (a) has ..... equal sides. It is called an ISOSCELES triangle ("Isosceles" means "equal legs"). Make another triangle the same shape and place one over the other. Turn one triangle over on to the other and you will see that an isosceles triangle also has two ..... angles. These angles are ..... the equal sides.

   Triangle (b) has three ..... sides and three ..... angles. You can prove this by placing a similar triangle over the first. It is called an EQUILATERAL triangle.

   Triangle (c) has one ..... angle. It is called a ..... angled triangle.

   Triangle (d) has one angle ..... than a right angle. It is called an ..... angled triangle.

   Triangle (e) has three angles each ..... than a right angle. It is called an ..... angled triangle. It also has three ..... sides. It is sometimes called a SCALENE triangle.

2. Use different strips to make these triangles; (a) an obtuse angled triangle, (b) an acute angled triangle, (c) an equilateral triangle, (d) a scalene triangle, (e) an isosceles triangle, (f) a right-angled triangle.

3. Complete the following sentences:

   (a) A scalene triangle has .....
   (b) An acute-angle triangle has .....
   (c) An equilateral triangle has .....
   (d) An obtuse-angled triangle has .....
   (e) A right-angled triangle has .....
   (f) An isosceles triangle has .....

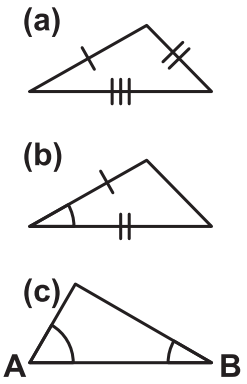
SECTION 4 - **TRIANGLES (2)**

Some things to remember about triangles.

If one triangle can be made to fit exactly on another triangle, the two triangles are said to be CONGRUENT, i.e., they are exactly alike. If we wish to copy a triangle exactly we must have one of these three sets of DATA (Latin, "things given").

(a) Three sides (S.S.S.).
Make any kind of triangle and, using the same kind of strips, make another. Place one triangle over the other. Are the two triangles exactly the alike?

(b) Two sides and the angle between them (S.A.S.).
Make a 30° angle with Y2 and R3 strips and a protractor. Which strip do you need to complete the triangle? Make another triangle using the same strips and angle.
Place one over the other. Are the two triangles exactly alike?



(c) One side and two angles (S.A.A.).
Fasten an R2 strip at 60° at A and Y2 strip at 30° at B at each end of an R3 strip. Repeat with another set of strips. Do you get the same result? Repeat with three Y2 strips. Set the angles at A and B at 60°. Repeat using R3 for AB and two B2 strips. Set the angles at A and B at 45°. In all these exercises, measure the other angle in the triangle and add the three angles together. Complete this: The sum of the angles of a triangle equal ..... degrees.

Draw any triangle on paper. Tear the triangle into three parts (Fig. 1) and place the angles together (Fig. 2). This shows that the angles of a triangle equal two right angles, or 180°.

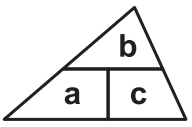


Fig. 1

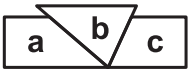


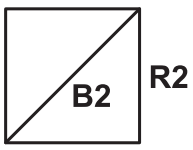
Fig. 2

Your teacher will show you how to draw triangles with a ruler, protractor and compasses. Draw these triangles.

S.S.S.	(a) 4cm, 3cm, 5cm,	(b) 7cm, 4cm, 6cm,	(c) 5cm, 2cm, 5cm.
S.A.S.	(a) 4cm, 60°, 4cm,	(b) 4cm, 90°, 3cm,	(c) 5cm, 120°, 6cm.
S.A.A.	(a) 6cm, 60°, 30°,	(b) 8cm, 45°, 45°,	(c) 6cm, 50°, 70°.

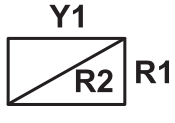
SECTION 5 - **QUADRILATERALS (1)**

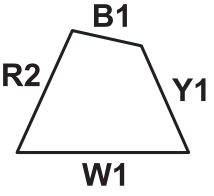
A QUADRILATERAL is a four sided figure. Join four R2 strips. The shape can be altered. It is not rigid. It can be made rigid by fastening a B2 strip from one corner to the opposite corner. This is called a DIAGONAL. Farm gates have a diagonal strip of wood across them to make them strong. The four R2 strips are equal and the angles are right angles. It is called a SQUARE.



Take off the B2 strip. The square can now be pushed out of shape. The angles are not right-angles but the sides are still the same length. They are equal. The opposite angles are equal. This shape is called a RHOMBUS.

Join two R1 strips and two Y1 strips as shown. The shape is not rigid but it can be made rigid using R2 as a diagonal. Check the angles when the figure is rigid. They are all ..... angles. The opposite sides are ..... . The shape is called a RECTANGLE. Remove the diagonal strip. When a rectangle is pushed out of shape, the angles are not right angles. The opposite sides are still ..... . They are always the same distance apart, like railway lines. They are PARALLEL. The shape is called a PARALLELOGRAM. Use protractors to prove that the opposite angles are equal. Make a quadrilateral, as shown, using R2, B1, Y1 and W1 strips. The sides are not ..... . They are not regular in length. It is called an IRREGULAR QUADRILATERAL. It can be deformed into two triangles. When B1 and W1 are parallel, the shape is called a TRAPEZIUM.





Complete the following sentences in your exercise book.

1. A square has four ..... sides and four ..... angles.
2. A rhombus has four ..... sides. The opposite angles are ..... .
3. A rectangle has ..... sides, and four ..... angles. The ..... sides are equal.
4. A parallelogram has ..... sides. Its ..... sides are ..... and its ..... angles are ..... .
5. An irregular quadrilateral has four ..... sides and four ..... angles.

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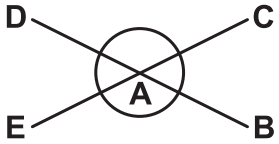
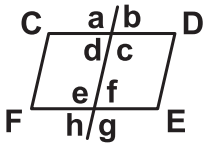
GEO STRIPS 3

SECTION 6 - **ANGLES (3)**



VERTICALLY OPPOSITE ANGLES
Join two R3 strips at the middle hole A. Slowly open out the strips. The same movement which opens angle CAB also opens angle DAE. These are called VERTICALLY OPPOSITE ANGLES and they are equal. Measure them with protractors. Is angle CAD equal to angle BAE?



Make parallelogram CDEF with two R3 and two R2. Join two R3 strips together by placing the end hole of one over the centre hole of the other to make a strip 1 1/2 times as long as R3. Fasten this strip across the centre of the parallelogram as shown.

When the parallelogram CDEF is altered in shape, what do you notice about angles "a, b, c, and d", and angles "e, f, g and h"? You have already learned that angle "a" = angle ....., angle "b" = angle ....., angle "e" = angle ....., and angle "f" = angle ..... ? These are all ..... angles. Does angle "b" = angle "f"? Does angle "g" = angle "c"? Does angle "e" = angle "a"? Angle "h" is equal to angle ..... ?

Angle "c" and angle "g" are called CORRESPONDING ANGLES.
Angle "d" and angle "f" are called ALTERNATE ANGLES.

	a	b	c	d	e	f	g	h
1	30°							
2		45°						
3			60°					

Draw this whole table in your book. Use the model to complete it. 1. (a) 30°, 2. (b) 45°, 3. (c) 60°, 4. (d) 108°, 5. (e) 135°, 6. (f) 72°, 7. (g) 120°, 8. (h) 90°.

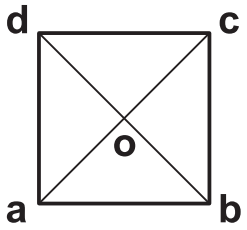
Look at parallelogram CDEF. What do you notice about lines CD and FE? Take off R2 strips DE and CF. When CD and FE are not parallel, are the corresponding and alternate angles equal?

Draw two straight lines that are not parallel and draw another line across them. Measure the corresponding and alternate angles. Are they equal? If the corresponding and alternate angles are equal, what do you know about line CD and line FE?

Can you prove that c+f and e+d above are both equal to 180°?

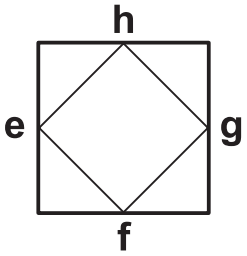
SECTION 7 - **QUADRILATERALS (2)**

Make a square with four R2 strips. Pass a thin coloured elastic thread up through (a), down through (c), up through (b) and down through (d). Tighten the thread and fasten. When the strips are a square, the diagonals are ..... . They meet in the ..... at (o). They bisect each other. "Bisect" means "cut into two equal parts". ao = oc, do = ob, ao = od. When the strips are deformed into a rhombus the diagonals are not ..... . They ..... each other, ao = oc, ob = od, but ao does not equal od.



Use a different coloured thread to join the midpoints of the strips. Pass the thread up through (e), down (f), up (g), down (h), tighten the threads and fasten. When the strips are square, the elastic thread joining the midpoints is a ..... .When the shape is a rhombus, the elastic thread joining the midpoints becomes a ..... .

Make a parallelogram with two R2 and two B2 strips. Put in elastic diagonals and join the midpoints with another coloured thread. The diagonals ..... each other. When the shape is a ..... the diagonals are the same size. When the shape is a rectangle, the threads joining the midpoints is a ..... . When the shape is a parallelogram the threads joining the midpoints is a ..... .



Make a quadrilateral with B1, R2, W1 and Y1. Put in diagonals and join the midpoints with elastic thread. What do you discover?

SECTION 8 - **TRIANGLES (3)**

1. Make a triangle with Y2, B2, and R3. Join the midpoints of B2 and R3 with Y1, R3 and Y2 with B1 and Y2 and B1 with R2. Y1 is parallel to Y2 and is equal to half of Y2. R2 is ..... to R3 and is equal to half of ..... . B1 is ..... to B2 and is equal to ..... of B2.

   LEARN: The straight line joining the middle points of two sides of a triangle is parallel to the third side, and equal to half of it.

2. Make an equilateral triangle with three Y2. Join the midpoints with three Y1. Are the Y1 strips equal to half of the Y2 strips? Are they parallel to them? How many equilateral triangles can you count?

3. SIMILAR TRIANGLES. make a triangle with R1, B1 and Y1. Make a triangle with R2, B2 and Y2. These two triangles are not congruent (exactly alike). Why not? The angles are ..... but the three sides are ..... . Two EQUIANGULAR triangles are called SIMILAR triangles.

   $\frac{R1}{R2} = \frac{B1}{B2} = \frac{Y1}{Y2} = \frac{1}{2}$

   $\frac{R1}{R2}$ may be written R1 : R2. This is read as "R1 is to R2".

   The fraction, $\frac{R1}{R2}$ is called the RATIO of R1 to R2.

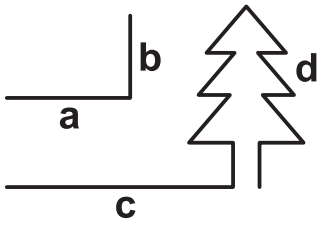
   $\frac{R1}{R2} = \frac{B1}{B2}$ may be written R1 : R2 = B1 : B2, and read "R1 is to R2 as B1 is to B2".

   R1, R2, B1 and B2 are said to be in PROPORTION. Use strips to make other similar triangles.

4. Place a shadow pole (b) in the sun. Measure its shadow (a). Measure the shadow (c) cast by a tree or a building.

   $\frac{a}{b} = \frac{c}{d}$

   If a is to b as c is to d then their cross products are equal:
   a : b = c : d
   ad = bc
   $d = \frac{bc}{a}$



Use this formula for finding the height of buildings, trees etc., with your shadow pole.

GEO STRIPS 4

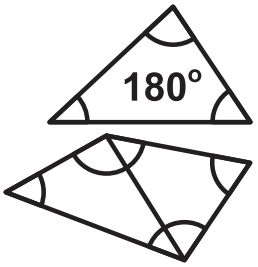
SECTION 9 - **POLYGONS (1)**

A number of strips joined together to form a closed figure is called a POLYGON (poly-gon "many sides"). When the sides are not equal in length it is called an IRREGULAR POLYGON.

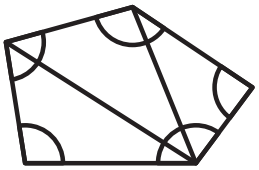
A polygon with three sides is called a ..... .
A polygon with four sides is called a ..... .
A polygon with five sides is called a PENTAGON.
A polygon with six sides is called a HEXAGON.
A polygon with eight sides is called an OCTAGON.
A polygon with ten sides is called a DECAGON.
A polygon with twelve sides is called a DUODECAGON or a DODECAGON.

The sum of the angles of a triangle is 180°.

Make any quadrilateral. Join the opposite corners of the quadrilateral. Two triangles are formed. The sum of the angles is ..... .



Make a pentagon. Divide the figures into Triangles. Find the sum of the angles.

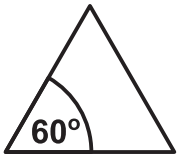


Make irregular polygons.
Divide them into triangles.
Draw this table in your book and complete it.

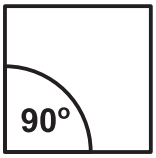
Name of shape	Number of sides	Sum of angles in degrees
Triangle	3	180°
Quadrilateral		
Pentagon		

SECTION 10 - **POLYGONS (2)**

Polygons with equal angles and sides are called REGULAR POLYGONS.



Equilateral triangle



Square

Pentagon

Hexagon

Octagon

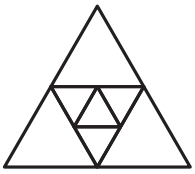
Decagon

The polygons can be built by setting strips at the angles shown.

A pentagon can be deformed into a five-pointed star.
Make a hexagon with Y1 strips. Y2 strips will fit across the opposite angles.
A hexagon can be deformed into a three-pointed star.
Make an octagon with R1. W2 strips will fit across the opposite angles.
An octagon can be deformed into a four-pointed star.
A decagon can be deformed into a five-pointed star and a dodecagon into a six-pointed star.

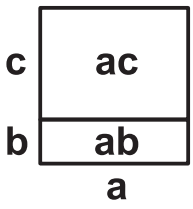
Use the Geo Strips to make geometrical patterns. Here are some ideas:


Y2

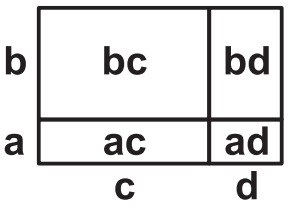

R Strips


R and B Strips

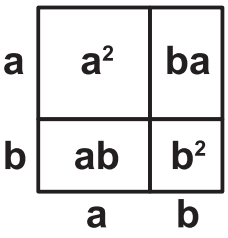
SECTION 11 - **SYMBOLIC REPRESENTATION**



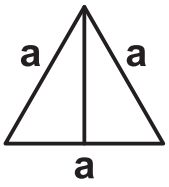
a (b + c)



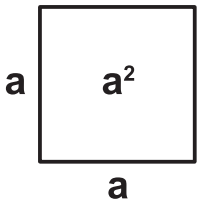
(a + b) (c + d)



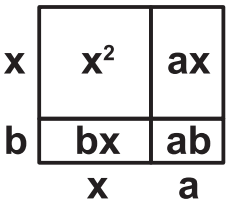
$(a + b)^2 = a^2 + b^2 + 2ab$



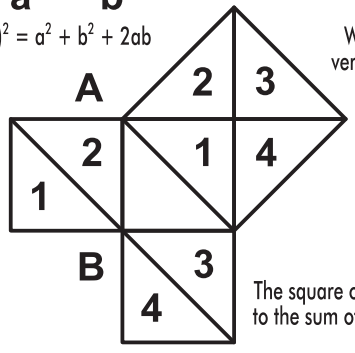
Write the formula for the vertical height in terms of a.



a^2



$(x + a)(x + b) = x^2 + ax + bx + ab$



The square of the hypotenuse is equal to the sum of the squares on the other two sides.

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