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A Personnel Allocation Problem in a Japanese Electric Manufacturer: Examining Algorithmic Solutions

Questions

1. Consider whether a personnel allocation planned by the researcher-proposing DA algorithm can be an effective way to solve a personnel problem for this manufacturer, referring to the theory explained in the body (and Appendix 4).

2. Compare the personnel allocations planned by the researcher-proposing DA algorithm, the division-proposing DA algorithm, and the Boston mechanism, referring to the numerical data shown in Appendices 1, 2 and 3.

3. As of October 2018, a medical trainee matching is the only example of the application of the (trainee-proposing) DA algorithm to real practice in Japan. Infer what happens when a Japanese company introduce DA algorithms in its personnel allocation, with reference to brief notes in Appendix 5 on what happened in the medical system in Japan.

4. What kind of things do we have to care about, in introducing DA algorithms to personnel allocation problems? Consider this question taking employment practices that are commonly observed in Japanese companies into account.

5. In effect, what kind of companies or industries can obtain good results when they apply DA algorithms to their personnel allocation problems?

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The aim of this case

This case describes a personnel allocation problem recognized in a Japanese electric manufacturer and provides some numerical data obtained by examining algorithmic solutions in matching theory at that company. The introduction of matching theory into real practices has steadily advanced in companies and local governments in Europe and the United States, whereas medical trainee matching is the only example of the application in Japan as of October 2018. The aim of this case is to show a precedent and some hints with which Japanese companies can consider how applicable an algorithmic personnel allocation is.

1 A situation

1.1 Allocation of in-house researchers to R&D teams

A Japanese electric manufacturer has some departments which are covered with some inter-departmental divisions. The R&D division is one of those divisions, and it functions as a control office of technical things for each department. In late 2016, there were 113 in-house researchers in total in the R&D division, each belonging to one of 13 R&D teams. The configuration of the R&D teams and the allocation of researchers placed in each team may be fine-tuned as necessary, but in most cases, those are reviewed in the budget formulation period (in January or February) for the next fiscal year. There are a few group managers (**GMs**) who are responsible for senior management in the R&D division to examine the performance of researchers and R&D teams; For each team, the GMs one-sidedly determine a team manager (**TM**) and assign researchers to the team. Researchers' requests are rarely reflected on their assignment.

In assigning researchers to R&D teams, GMs refer to their placement history, past evaluation, the in-house qualifications for positions, and their future career development planned by the company based on long-term employment which is prevalent as an employment practice among many Japanese companies. There is, however, a rigid tendency that the assignment of researchers to each R&D team is fixed. Because each researcher has a technical expertise area, it is difficult to replace researchers even when it is desired according to the amounts or types of various tasks required by the company.

Problem: Further innovation in R&D is constantly being sought within this Japanese electric manufacturer, but there is concern that the rigid tendency of the fixed assignment of researchers may be a factor that hampers this.

In addition, this manufacturer well recognizes that, in order to promote R&D activities, it is **necessary to motivate researchers and induce them to invest for further skill acquisition**. This manufacturer has several R&D themes and priority differs according to each theme. In addition, complementarity among themes should also be considered. There are many evaluation criteria on personnel assignment of inhouse researchers to R&D teams; the company must take into consideration various factors, as noted above.

1.2 Researcher-proposing DA algorithm

Matching theory is a branch of an academic research field called mechanism design and is also considered as an applied game theory.[1] The personnel allocation problem of this manufacturer is formulated as a two-sided matching problem which consists of the following components.

1. Each in-house researcher has his or her preference over R&D teams to which he or she is to be assigned, and each R&D team has its evaluation for researchers. The preferences and evaluation are represented by ranking without ties.
2. Quota: the maximal number of workers that can be accepted by each division, i.e., capacity.
3. Every in-house researcher is assigned to some R&D team.

Example 1: There are six researchers denoted by a, b, c, d, e, and f and three R&D teams denoted by X, Y , and Z. The preferences and evaluation are listed in Tables 1 and 2, respectively, where, e.g., b prefers Z most but b ranks 5th in Z's evaluation, while f prefers Z most and Z gives f the highest rating. The quota of X is 2, that of Y is 1, and that of Z is 3, which are represented by the numerical values in the parentheses in Table 2. An assignment of researchers to R&D teams is called a matching or (personnel) allocation and denoted by μ, such as

$$\mu = \begin{pmatrix} a & b & c & d & e & f \\ X & Z & Z & X & Y & Z \end{pmatrix},$$

where, e.g., researchers a and d are assigned to X.

[1] In 2012, Alvin Roth and Lloyd Shapley were awarded the Nobel Prize in Economic Science for "the development of matching theory and its application in practice".

A solution criterion for this two-sided matching problem is stated easily as follows. A matching is deemed unstable if it sends a researcher to a R&D team when there is another team that is preferred by the researcher, and either has room for him or her or could make room by rejecting someone else it prefers less. For each two-sided matching problem, in general, there are some stable matchings.

Table 1: researchers' preferences over R&D teams

a:	*X*	*Y*	*Z*	*d:*	*Y*	*Z*	*X*
b:	*Z*	*Y*	*X*	*e:*	*Y*	*Z*	*X*
c:	*Z*	*Y*	*X*	*f :*	*Z*	*X*	*Y*

Table 2: R&D teams' evaluation for researchers

(2)	*X :*	*b*	*c*	*a*	*d*	*f*	*e*
(1)	*Y :*	*a*	*d*	*c*	*e*	*f*	*b*
(3)	*Z :*	*f*	*e*	*c*	*d*	*b*	*a*

In late 2016, prior to the budget formulation period for the next fiscal year, this manufacturer computed an allocation plan of researchers by an algorithm which is called researcher-proposing DA algorithm and compared the allocation plan with the one which was determined by the traditional way within that company. In Example 1, a stable matching

$$\mu^1 = \begin{pmatrix} a & b & c & d & e & f \\ X & X & Z & Y & Z & Z \end{pmatrix}$$

was computed by the researcher-proposing DA algorithm as follows.

Table 3: Computation process of μ_1 by the researcher-proposing DA algorithm

		round 1	round 2	round 3	round 4
(2)	*X*	*a*	*a*	*a*	*a, b*
(1)	*Y*	*d,* ~~*e*~~	*d*	*d,* ~~*b*~~	*d*
(3)	*Z*	*b, c, f*	~~*b*~~*, c, f e*	*c , f , e*	*c, f , e*

round 1
- Researchers first apply to the R&D teams they prefer most, respectively.
- Each R&D team accepts temporarily those applicants with higher evaluation as far as they do not exceed its quota.

R&D team *Y* cannot accept as many as two applicants, and thus it rejects the application from researcher e, which is indicated by $\not{e}$ in Table 3, and it keeps the one from *d*.

round 2
- Researcher e next applies to R&D team Z he or she prefers next to *Y* which rejected his or her application in round 1.
- R&D team *Z* temporarily accepts applicants with higher evaluation as far as they do not exceed its quota. At that time, researchers whose applications are kept may be replaced with new applicants.

Table 3 depicts that R&D team *Z* rejects the application from researcher *b* and keeps the current ones from *c*, *f* , and the new one from *e* in this round.

round 3
- Researcher *b* next applies to R&D team *Y* he or she prefers next to *Z* which rejected his or her application in round 2.
- R&D team *Y* temporarily accepts applicants with higher evaluation as far as they do not exceed its quota.

Table 3 depicts that R&D team *Y* rejects the new application from researcher *b* and keeps the current one from *d* in this round.

round 4
- Researcher b next applies to R&D team *X* he or she prefers next to *Y* which rejected his or her application in round 3.
- R&D team *X* temporarily accepts applicants with higher evaluation as far as they do not exceed its quota.

Table 3 depicts that R&D team *X* accepts the new application from researcher *b*, because it has one vacant seat. Now, every researcher is assigned to an R&D team, and thus the computation process ends here.

The researcher-proposing DA algorithm has the following nice properties. Stability is stated more formally here.

- **Stability:** When a matching is proposed, the matching is said to be stable if the following two conditions are satisfied for any other alternatives that are counterproposed by any groups consisting of some researchers and a R&D team; (1) at those alternatives, no researcher in the group is assigned to a higher ranked team in his or her preference, and (2) at those alternatives, the R&D team in the group is not matched with any researchers who are higher ranked in its evaluation.
- **Optimality for researchers:** Every researcher is matched with the R&D team he or she most prefers among those with which he or she can be matched in al stable matchings.

- **Strategy-proofness for researchers:** For each researcher, whatever other researchers report their preferences over R&D teams, it is never assigned to the more preferable R&D team for him or her by misrepresenting their true preference.

Any counter-proposals to stable matchings do not make sense for any groups of researchers and an R&D team. Namely, when the current assignment of researchers is considered as a stable matching, even if researchers and an R&D team have some dissatisfaction at the current assignment, there is no such a reassignment of researchers that improves their welfare in terms of their own individual rankings of preferences and evaluation within the group itself, unless there is no change in the company's R&D policy and there is no unexpected shock in the external environment. Further, the researcher-proposing DA algorithm assigns every researcher to his or her best possible R&D team in stable matchings, and there is no incentive for him or her to misrepresent his or her true preference over R&D teams.

The next property is, unfortunately, not guaranteed at the stable matching computed by the researcher-proposing DA algorithm.

- **Pareto efficiency among researchers:** In order to reassign a researcher to a higher ranked R&D team, there is at least one researcher who must be transferred to a lower ranked team for him or her.

Note that stability implies **Pareto efficiency for both sides** of researchers and R&D teams; at any stable matchings, there is no such a reassignment of researchers that improves their welfare in terms of their individual rankings of preferences and evaluation in both sides. Pareto efficiency among researchers is applied only to the welfare for one side of researchers, not to the welfare for both sides. In general, there are many Pareto efficient allocations of researchers for each two-sided matching problem. We cannot, however, compare any pairs of Pareto efficient assignments in terms of welfare, because they are not measured as numerical values

2 A personnel allocation plan

In late 2016, this Japanese electric manufacturer designated an employee who was in a responsible position as the examiner for its allocation of researchers. The examiner asked researchers to represent their preferences over R&D teams and solicited TMs to state their R&D teams' evaluation for researchers via emails. GMs were asked to determine the quotas of R&D teams for the next fiscal year. All replies were directly sent to the examiner via emails and all data were put IDs in order to secure researchers' privacy. Neither researchers nor TMs were not informed of how an assignment of researchers would be determined, but the purpose of this questionnaire was well explained to them. GMs were explained in advance by the examiner that the researcher-proposing DA algorithm would be applied.

Two questions were given to researchers according to the following order within the same questionnaire.

(1) Researchers were simply asked to represent their preferences over R&D teams

(2) Researchers were asked to state their preferences, after receiving a simple future vision of the R&D division to make further innovation in this electric manufacturer.

There were 113 researchers in the R&D division, but 10 researchers could not reply to the questionnaire by the due date, because they were on overseas business trips. Those researchers were deduced from the quotas and evaluation of R&D teams. Thus, the number of researchers in this examination is 103. Tables 4 and 5 show the excerpts from researchers' preferences over R&D teams for Questions (1) and (2), respectively. In Table 4, e.g., a researcher with ID = 2 prefers R&D team 1 most and prefers team 11 to team 12.

Table 4: Researchers' preferences over R&D teams: Question (1)

researcher ID	rank 1	rank 2	rank 3	· · ·	rank 12	rank 13
1	1	4	2	· · ·	12	13
2	1	11	12	· · ·	3	2
3	3	13	4	· · ·	1	2
⋮	⋮	⋮	⋮		⋮	⋮
102	2	12	9	· · ·	1	8
103	2	11	7	· · ·	1	3

Table 5: Researchers' preferences over R&D teams: Question (2)

researcher ID	rank 1	rank 2	rank 3	· · ·	rank 12	rank 13
1	3	6	5	· · ·	2	1
2	2	11	12	· · ·	3	1
3	3	7	5	· · ·	2	1
⋮	⋮	⋮	⋮		⋮	⋮
102	2	11	6	· · ·	1	5
103	2	11	7	· · ·	1	3

As compared with researchers' preferences, it was relatively difficult for TMs to rank researchers accurately, because they did not necessarily know all 103 researchers. The examiner thus asked TMs to rank researchers referring to evaluation items accumulated at the department of human resources such as skills, knowledge, job qualifications and carriers that each division emphasizes in the execution of tasks. Tables 6 shows a part of R&D teams' evaluation for researchers, where, e.g., an R&D team with ID = 2 places researcher 75 at the highest rank and researcher 98 at the lowest rank in its evaluation.

Table 6: R&D teams' evaluation for researchers

team ID	rank 1	rank 2	rank 3	· · ·	rank 102	rank 103
1	11	4	12	· · ·	13	14
2	75	102	92	· · ·	97	98
3	75	102	92	· · ·	97	98
4	75	102	99	· · ·	98	100
5	74	102	27	· · ·	97	101
6	74	102	24	· · ·	97	101
7	102	103	81	· · ·	100	101
⋮	⋮	⋮	⋮		⋮	⋮
13	9	15	14	· · ·	1	4

Tables 7 lists the quotas of 13 R&D teams, where quota A was the current one for the fiscal year of 2016 and quota B was the one planned by GMs for the next fiscal year, respectively.

Table 7: Quotas of R&D teams

team ID	1	2	3	4	5	6	7	8	9	10	11	12	13	total
quota A	8	12	8	10	11	6	13	4	8	10	5	3	5	103
quota B	8	12	8	9	10	6	12	5	8	10	5	3	7	103

3 Results

As noted at the beginning of Section 2, neither researchers nor TMs were not informed of how an assignment of researchers would be determined. As far as preferences represented by researchers and evaluation stated by TMs are concerned, however, the matching computed by the researcher-proposing DA algorithm is stable and it is thus Pareto efficient for both sides. The purpose of this questionnaire was well explained to both researchers and TMs, and thus we may assume that researchers represented their true preferences and TMs stated their true evaluation.

Tables 8 and 9 are the excerpts from matching results.[2] In Tables 8 and 9, (1) and (2) correspond to Questions (1) and (2), respectively, res i is the abbreviation of researcher with ID = i, and c2016 stands for the current assignment of researchers to R&D teams as of late 2016. For instance, Table 8 shows that a researcher with ID = 103 (res 103) was assigned to an R&D team with ID = 12 (team 12) as of late 2016, team 11 for quota A and team 6 for quota B, respectively, according to preferences represented by researchers who corresponded to Question 1. Table 9 shows that team 12 is ranked 6th, team 11 is ranked 2nd, and team 6 is ranked 4th in res 103's preference represented by him or her corresponding to Question 1.

As noted at the end of Section 1, we cannot compare any pairs of Pareto efficient allocations by numerical values. We thus introduce here indices for measuring the efficiency rates. The efficiency index for researchers is defined by the average rank of R&D teams with which they are matched. The efficiency index for R&D teams is, similarly, defined by the total sum of ranks of researchers with whom they are matched divided by the number of researchers, 103. The efficiency rates increase as the efficiency indices take lower values. Table 10 lists the efficiency rates, separately and jointly, where the values are obtained by rounding off to one decimal digit.

[2] *Excel for Two-Sided Matching version* 3 (sample3.xlsm) was used for computing the matching by the researcher-proposing DA algorithm. As of October 2018, Excel for Two-Sided Matching was upgraded to version 3.2 (sample3 v2.xlsm), which can be freely downloaded from the following URL.
http://labs.kbs.keio.ac.jp/naoki50lab/sample3_v2.xlsm

Table 8: R&D teams with which researchers are matched

	res 1	res 2	res 3	· · ·	res 102	res 103
c2016	1	1	1	· · ·	12	12
(1)+quota A	1	1	3	· · ·	2	11
(1)+quota B	1	1	3	· · ·	2	6
(2)+quota A	3	2	3	· · ·	2	11
(2)+quota B	3	2	3	· · ·	2	11

Table 9: Ranks of R&D teams to which researchers are assigned

	res 1	res 2	res 3	· · ·	res 102	res 103
c2016	1	1	12	· · ·	2	6
(1)+quota A	1	1	1	· · ·	1	2
(1)+quota B	1	1	1	· · ·	1	4
(2)+quota A	1	1	1	· · ·	1	2
(2)+quota B	1	1	1	· · ·	1	2

Table 10: Efficiency rates: researcher-proposing DA algorithm

	c2016	(1)+quota A	(1)+quota B	(2)+quota A	(2)+quota B
total	12.9	39.4	40.0	33.7	33.5
R&D teams	10.4	37.6	38.2	31.4	31.2
researchers	2.5	1.8	1.8	2.3	2.3

As is seen in Table 10, for preferences represented by researchers corresponding to Question (1), the researcher-proposing DA algorithm increased the efficiency rate for researchers but decreased the efficiency rate for R&D teams, as compared with the rates in the allocation for c2016, in both cases of quotas A and B. In total, the joint efficiency rates decreased in both cases of quotas A and B. For preferences represented by researchers corresponding to Question (2), the researcher-proposing DA algorithm decreased the efficiency rate for the researchers but increased the efficiency rate for the R&D teams, as compared with the rates which corresponds to Question (1), in each case of quota A and quota B. In total, the joint efficiency rate increased in each case of quota A and quota B. Appendix 1 lists the efficiency rates defined for each R&D team.

Tables 11 and 12 summarizes the efficiency rates in allocations computed by the R&D team-proposing DA algorithm and the Boston mechanism, respectively. Appendices 2 demonstrates the computation process of the R&D team-proposing DA algorithm and shows the efficiency rates defined for each R&D team. Appendices 3 depicts the difference between researcher-proposing DA algorithm and the Boston mechanism and shows the efficiency rates defined for each R&D team. It is notable that in the case of quota A, the joint efficiency rate in the allocation which corresponds to Question (2) took the nearest value to the rate in the allocation for c2016 among the rates in the allocations for (1)+quota A and (2)+quota A computed by the DA algorithms and the Boston mechanism.

Table 11: Efficiency rates: team-proposing DA algorithm

	c2016	(1)+quota A	(1)+quota B	(2)+quota A	(2)+quota B
total	12.9	38.8	39.9	21.2	19.8
R&D teams	10.4	37.0	38.1	17.8	16.0
researchers	2.5	1.8	1.8	3.4	3.8

Table 12: Efficiency rates: Boston mechanism

	c2016	(1)+quota A	(1)+quota B	(2)+quota A	(2)+quota B
total	12.9	46.6	46.5	47.2	48.2
R&D teams	10.4	44.5	44.8	45.1	45.3
researchers	2.5	2.1	1.7	2.1	2.1

Appendix 1: researcher-proposing DA algorithm

The general computation process of the researcher-proposing DA algorithm is described as follows.

1.
   - Each researcher first applies to the R&D team he or she prefers most.
   - Each R&D team accepts "temporarily" those applicants with higher evaluation as far as they do not exceed the capacity.

2.
   - In round k+1 (k = 1, 2, . . .), the researcher who was rejected in round k applies to the R&D team he or she prefers next to the R&D team which rejected him or her in round k.
   - Each R&D team "temporarily" accepts researchers among those who were temporarily accepted in round k and new applicants with higher evaluation, as far as they do not exceed the capacity. At that time, the R&D team can replace researchers it temporarily accepted with new applicants.

3.
   - Continue this process until every researcher is accepted by some R&D team or rejected by all R&D teams listed in his or her preference.

Tables 13, 14, and 15 show the efficiency rates defined for each R&D team in the allocations for c2016, (1)+A, (1)+B, (2)+A, and (2)+B. Each value indicated by (a), (c), (e), (g), or (i) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103, and each value indicated by (b), (d), (f), (h), or (j) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103. (A) and (B) stand for quotas A and B, respectively.

Table 13: Effciency rates in the current allocation (c2016)

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	45	115	184	78	200	45	91	17	130	115	16	9	24	1069	(a)+(b)
	avg	5.6	9.6	23.0	7.8	18.2	7.5	7.0	4.3	16.3	11.5	3.2	3.0	4.8	10.4 (a)	12.9
researchers (A)	sum	10	20	13	38	36	17	22	4	17	50	6	3	17	253	
	avg	1.3	1.7	1.6	3.8	3.3	2.8	1.7	1.0	2.1	5.0	1.2	1.0	3.4	2.5 (b)	

Table 14: Effciency rates in the allocation corresponding to Q1: researcher-proposing DA algorithm

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	106	567	263	507	571	263	723	136	163	374	61	37	98	3869	(c)+(d)
	avg	13.3	47.3	32.9	50.7	51.9	43.8	55.6	34.0	20.4	33.4	12.2	12.3	19.6	37.6 (c)	39.4
researchers (A)	sum	10	24	14	17	24	8	26	6	18	20	12	3	5	187	
	avg	1.3	2.0	1.8	1.7	2.2	1.3	2.0	1.5	2.3	2.0	2.4	1.0	1.0	1.8 (d)	
R&D teams (B)	sum	95	591	263	429	564	263	673	298	163	334	56	37	164	3930	(e)+(f)
	avg	11.9	49.3	32.9	47.7	56.4	43.8	56.1	59.6	20.4	33.4	11.2	12.3	23.4	38.2 (e)	
researches (B)	sum	10	23	14	16	22	8	23	12	18	20	6	3	7	182	40.0
	avg	1.3	1.9	1.8	1.8	2.2	1.3	3.3	1.5	2.0	2.0	1.2	1.0	1.0	1.8 (f)	

Table 15: Effciency rates in the allocation corresponding to Q2: researcher-proposing DA algorithm

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	127	292	169	298	628	123	624	75	201	530	35	39	91	3232	(g)+(h)
	avg	15.9	24.3	21.1	29.8	57.1	20.5	48.0	18.8	25.1	53.0	7.0	13.0	18.2	31.4 (g)	
researchers (A)	sum	9	22	13	16	29	12	42	7	26	31	18	5	8	238	33.7
	avg	1.1	1.8	1.6	1.6	2.6	2.0	3.2	1.8	3.3	3.1	3.6	1.7	1.6	2.3 (h)	
R&D teams (B)	sum	127	292	145	228	546	123	538	115	297	538	35	39	187	3210	(i)+(j)
	avg	15.9	24.3	18.1	25.3	54.6	20.5	44.0	23.0	37.1	53.8	7.0	13.0	26.7	31.2 (i)	
researchers (B)	sum	9	22	14	15	26	12	37	12	18	32	18	5	15	235	33.4
	avg	1.1	1.8	1.8	1.7	2.6	2.0	3.1	2.4	2.3	3.2	3.6	1.7	2.5	2.3 (j)	

Appendix 2: R&D team-proposing DA algorithm

In Example 1 given in the body of this case, a stable matching

$$\mu^2 = \begin{pmatrix} a & b & c & d & e & f \\ X & X & Z & Y & Z & Z \end{pmatrix}$$

is computed by the R&D team-proposing DA algorithm. Note that $\mu^1 = \mu^2$.

In Example 1, the quota of X is 2, that of Y is 1, and that of Z is 3, which are represented by the numerical values in the parentheses in Table 2. In the R&D teamproposing DA algorithm, each R&D team first makes its offers to researchers with higher evaluation so as to fill its vacant seats. Then, each researcher accepts temporarily an offer which is best among all offers he or she receives. The subsequent steps are similar to the ones in the researcher-proposing DA algorithm. The computation stops when every R&D team fills its vacant seats or when all possible offers are rejected.

Table 16 depicts the computation process of μ_2, where in round 2, researcher *a* rejects the offer from R&D team *Y* which he or she temporarily accepted in round 1 and in round 3.

Table 16: Computation process of μ_2 by the R&D team-proposing DA algorithm

	round 1	round 2	round 3
a	*Y*	~~*Y*~~, *X*	*X*
b	*X*	*X*	*X*
c	~~*X*~~, *Z*	*Z*	*Z*
d			*Y*
e	*Z*	*Z*	*Z*
f	*Z*	*Z*	*Z*

Tables 17, 18, and 19 show the efficiency rates defined for each R&D team in the allocations for c2016, (1)+A, (1)+B, (2)+A, and (2)+B. Table 21 is the same as Table 13, but it is put there for easier comparison with Tables 22 and 23. Each value indicated by (a), (c), (e), (g), or (i) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103, and each value indicated by (b), (d), (f), (h), or (j) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103. (A) and (B) stand for quotas A and B, respectively.

Table 17: Effciency rates in the current allocation (c2016)

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	45	115	184	78	200	45	91	17	130	115	16	9	24	1069	(a)+(b)
	avg	5.6	9.6	23.0	7.8	18.2	7.5	7.0	4.3	16.3	11.5	3.2	3.0	4.8	10.4 (a)	12.9
researchers (A)	sum	10	20	13	38	36	17	22	4	17	50	6	3	17	253	
	avg	1.3	1.7	1.6	3.8	3.3	2.8	1.7	1.0	2.1	5.0	1.2	1.0	3.4	2.5 (b)	

Table 18: Effciency rates in the allocation corresponding to Q1: R&D team-proposing DA algorithm

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	106	530	263	507	571	263	723	137	204	334	38	37	98	3811	(k)+(l)
	avg	13.3	44.2	32.9	50.7	51.9	43.8	55.6	4.0	20.4	33.4	7.6	12.3	19.6	37.0 (k)	38.8
researchers (A)	sum	10	24	14	17	24	8	28	6	18	20	12	3	5	187	
	avg	1.3	2.0	1.8	1.7	2.2	1.3	2.0	1.5	2.3	2.0	2.4	1.0	1.0	1.8 (l)	
R&D teams (B)	sum	95	591	263	429	564	263	673	298	163	334	56	46	164	3929	(m)+(n)
	avg	11.9	49.3	32.9	47.7	56.4	43.8	56.1	59.6	20.4	33.4	11.2	12.0	23.4	38.1 (m)	
researches (B)	sum	10	23	14	16	22	8	23	12	18	20	6	3	7	182	39.9
	avg	1.3	1.9	1.8	1.8	2.2	1.3	1.9	2.4	2.3	2.1	2.1	1.0	1.0	1.8 (n)	

Table 19: Effciency rates in the allocation corresponding to Q2: R&D team-proposing DA algorithm

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	99	219	145	127	330	104	311	33	84	283	23	19	60	1837	(o)+(p)
	avg	12.4	18.3	18.1	12.7	30.0	17.3	23.9	8.3	12.0	28.3	4.6	6.0	12.0	17.8 (o)	
researchers (A)	sum	12	41	14	19	48	13	39	14	40	37	27	20	25	349	21.2
	avg	1.5	3.4	1.8	1.9	4.4	2.2	3.0	3.5	5.0	3.7	2.5	6.7	5.0	3.4 (p)	
R&D teams (B)	sum	85	219	127	101	273	104	190	56	73	270	23	22	109	1652	(q)+(r)
	avg	10.6	18.3	15.9	11.2	27.3	17.3	15.8	11.2	9.1	27.0	4.6	7.3	15.6	16.0 (q)	
researchers (B)	sum	13	41	20	17	43	13	41	15	37	56	27	19	50	392	19.8
	avg	1.6	3.4	2.5	1.9	4.3	2.2	3.4	3.0	4.6	5.6	5.4	6.3	7.1	3.8 (r)	

Appendix 3: Boston mechanism

In Example 1 given in the body of this case, a matching

$$\mu^3 = \begin{pmatrix} a & b & c & d & e & f \\ X & Z & Z & Y & X & Z \end{pmatrix},$$

is computed by the Boston mechanism. Note that $\mu^1 = \mu^2$ but that $\mu^1 \neq \mu^3$. In general, the R&D team-proposing DA mechanism computes a stable matching which is different from the one the researcher-proposing DA algorithm computes. According to a well-known structure of the set of stable matchings, the coincidence of those two stable matchings, i.e., $\mu^1 = \mu^2$, suggests that it be the unique stable matching in Example 1. Thus, μ^3 is not a stable matching.

Unlike DA algorithms, the Boston mechanism does not allow R&D teams to replace researchers they accepted once with others. Table 20 depicts the computation process of μ^3, where researcher *e* cannot apply to R&D team *Z* which is ranked 2nd in his or her preference because there is no vacancy there.[3]

Table 20: Computation process of μ^3 by the Boston mechanism

		round 1	round 2
(2)	*X*	*a*	*a, e*
(1)	*Y*	*d*, ~~*e*~~	*d*
(3)	*Z*	*b, c, f*	*b, c, f*

Tables 21, 22, and 23 show the efficiency rates defined for each R&D team in the allocations for c2016, (1)+A, (1)+B, (2)+A, and (2)+B. Table 21 is the same as Table 13, but it is put there for easier comparison with Tables 22 and 23. Each value indicated by (a), (c), (e), (g), or (i) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103, and each value indicated by (b), (d), (f), (h), or (j) is not the total sum of average values listed in avg for teams, but the total sum of ranks divided by 103. (A) and (B) stand for quotas A and B, respectively.

[3] This mechanism was actually used for assigning students to public schools in municipalities in many countries. In Boston, Massachusetts in the United States, however, this mechanism was replaced by the student-proposing DA algorithm in 2005. Since then, this mechanism has come to be called the Boston mechanism in matching theory. As noted, the Boston mechanism does not necessarily generate a stable matching.

Table 21: Effciency rates in the current allocation (c2016)

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	45	115	184	78	200	45	91	17	130	115	16	9	24	1069	(a)+(b)
	avg	5.6	9.6	23.0	7.8	18.2	7.5	7.0	4.3	16.3	11.5	3.2	3.0	4.8	10.4 (a)	12.9
researchers (A)	sum	10	20	13	38	36	17	22	4	17	50	6	3	17	253	
	avg	1.3	1.7	1.6	3.8	3.3	2.8	1.7	1.0	2.1	5.0	1.2	1.0	3.4	2.5 (b)	

Table 22: Effciency rates in the allocation corresponding to Q1: Boston mechanism

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R& Dteams (A)	sum	145	728	476	506	491	354	689	208	384	383	83	37	98	4582	(s)+(t)
	avg	18.1	60.7	59.5	50.6	44.6	59.0	53.0	52.0	48.0	38.3	16.6	12.3	19.6	44.5 (s)	46.6
researchers (A)	sum	8	19	24	40	21	6	42	15	12	15	5	3	5	215	
	avg	1.0	1.6	3.0	4.0	1.9	1.0	3.2	3.8	1.5	1.5	1.0	1.0	1.0	2.1 (t)	
R&D teams (B)	sum	145	713	476	469	476	354	629	300	384	383	83	37	164	4613	(u)+(v)
	avg	18.1	59.4	59.5	52.1	47.6	59.0	52.4	60.0	48.0	38.3	16.6	12.3	23.4	44.8 (u)	46.5
researches (B)	sum	8	20	8	14	19	6	34	20	12	15	5	3	7	171	
	avg	1.0	1.7	1.0	1.6	1.9	1.0	2.8	4.0	1.5	1.5	1.0	1.0	1.0	1.7 (v)	

Table 23: Effciency rates in the allocation corresponding to Q2: Boston mechanism

		team ID														
		1	2	3	4	5	6	7	8	9	10	11	12	13		
R&D teams (A)	sum	128	601	280	471	737	244	690	82	382	582	276	52	125	4650	(w)+(x)
	avg	16.0	50.1	35.0	47.1	67.0	40.7	53.1	20.5	47.8	58.2	55.2	17.3	25.0	45.1 (w)	
researchers (A)	sum	8	12	8	13	27	8	50	5	14	56	5	3	5	214	47.2
	avg	1.0	1.0	1.0	1.4	2.7	1.3	4.2	1.0	1.8	5.6	1.0	1.0	1.0	2.1 (x)	
R&D teams (B)	sum	128	601	280	381	641	244	694	150	384	567	276	52	269	4667	(y)+(z)
	avg	16.0	50.1	35.0	42.3	64.1	40.7	57.8	30.0	48.0	56.7	55.2	17.3	38.4	45.3 (y)	
researchers (B)	sum	8	12	8	11	23	8	48	7	14	51	5	3	7	205	48.2
	avg	1.0	1.0	1.0	1.2	2.3	1.3	4.0	1.4	1.8	5.1	1.0	1.0	1.0	2.9 (z)	

Appendix 4: DA Algorithm and Strategy-Proofness

The allocation problem of in-house researchers to R&D teams considered in the body of this case is here simplified in such a way that everyone has one's capacity of 1. This situation is named as a marriage problem. The strategy-proofness is explained in this simple situation. There are men and women. Each man has his preference over women and each woman has her preference over men. For simplicity, preferences are represented by ranking without ties. Assume monogamy and heterosexual marriage. Then, this situation is formalized as a two-sided matching problem.

Example 2: There are four men denoted by *A*, *B*, *C*, and *D* and four women denoted by *a*, *b*, *c*, and *d*. Their preferences are listed in Table 24, where, e.g., woman a prefers man *B* most and prefers *A* to *D* and so on.

Table 24: Lists of preferences.

women	a:	B	A	D	C	men	A:	a	b	c	d
	b:	A	B	C	D		B:	d	a	b	c
	c:	B	D	C	A		C:	a	c	d	b
	d:	A	B	D	C		D:	d	c	b	a

- Denote by μ a list of couples (pair). We call μ a matching.

- What matching is socially desirable? Pareto efficiency is a criterion to choose a socially desirable matching. Then, what matching do you choose when there are plural Pareto efficient matchings?

- A matching μ is **Pareto efficient** if there is no matching μ' such that for every man or woman, his or her mate matched at μ' is preferred to his or her current mate at μ.

- We say that a matching μ is **stable** if every man or woman prefers being matched to being single and there is no pair of a man m_i and a woman w_j which makes a joint objection against μ by $w_j >_{m_i} w_j'$ and $m^i >_{w_j} m_i'$, where w_j' is the current mate of m_i and m_i' is the current mate of w_j at μ.

- By the definitions, stable matchings are Pareto efficient.

In Example 2, is a matching $\mu =< Aa, Bb, Cc, Dd >$ stable? The answer is no. A pair $\{B, a\}$ will elope together. In other words, If there is no pair of a man and a woman who elope from a matching given to them, the matching is stable.

Theorem 1 (Gale and Shapley, 1962). *Every marriage game has at least one stable matching.*

- Gale and Shapley developed a simple algorithm to show the existence of stable matchings in marriage games, which is currently called the deferred acceptance (DA) algorithm or Gale-Shapley (GS) algorithm. There are men-proposing procedure and women-proposing procedure. Below is a men-proposing one.

1. Men make proposals to women and women await men's proposals.
2. Each man makes his proposal to the most preferable woman in his preference.
3. Each women who received proposals chooses the best one as a tentative mate among those proposals in reference to her preference. (She keeps the man as her boy friend.)
4. All men who were rejected go to the next round, and make their proposals to the second most preferable women in their preferences. It is prohibited for each man to make a proposal to the woman by whom he was rejected once.
5. Each woman who received one or more proposals choose the best one among her current mate, if any, and those proposals in reference to her preference. (She can reject her current mate and accept another one as her new boy friend.)
6. All men who were rejected go to the next round. The same process as above lasts until no man is rejected or all men are rejected.

In Example 2, the stable matching under the men-proposing procedure is

$$\mu =< Aa, Bd, Cb, Dc > .$$

See Table 2 to understand how the DA algorithm generate μ.
Under the women-proposing procedure, the stable matching is also

$$\mu' =< Aa, Bd, Cb, Dc >,$$

and thus, $\mu' = \mu$, which implies that in Example 1, there exists a unique stable matching $\mu =< Aa, Bd, Cb, Dc >$. (exercise: Why is it unique?) Later, Knuth developed an algorithm to find all stable matchings. (Knuth also developed plain $\TeX$.)

Table 25: DA algorithm (men-proposing).

	1st round	2nd round	3rd round	4th round
a	A, ~~C~~	A	A	A
b				C
c		~~C~~, D	D	D
d	B, ~~D~~	B	B, ~~C~~	B

Theorem 2. *In the stable matching obtained under the men-proposing (women-proposing) procedure, every man (women) is matched with the most preferred woman (man) with whom he (she) can be matched in all stable matchings.*

- As a corollary of Theorem 6, we can show that given lists of men's and women's preferences, if the DA algorithm generates different stable matchings under menproposing and women-proposing procedures, then players on the passive side (players who only await proposals) are not matched with the most preferred persons with whom they can be matched in all stable matchings.

- Question: Is it true that the players on the passive side in the DA algorithm are matched with the worst persons with whom they can be matched in all stable matchings? If your answer is yes, prove it. Otherwise, give a counter example.

Question: Can passive players improve their outcomes?

Theorem 3. *Suppose that given lists of preferences, the DA algorithm generates different stable matchings under men-proposing and women-proposing procedures. Then, players on the active side tell their true preferences, whereas players on the passive side may not do so.*

- Theorem 3 states that every player on the positive side has a dominant strategy, i.e., truth-telling, whereas players on the passive side have an incentive to misrepresent their preferences.

Example 3: Let $M = \{m_1, m_2, m_3\}$ and $W = \{w_1, w_2, w_3\}$. The preferences of players are given in Table 26. Confirm that $\mu =< m_1w_2, m_2w_3, m_3w_1 >$ is a stable matching obtained under the menproposing procedure and that $\mu' = < m_1w_1, m_2w_3, m_3w_2 >$ is the stable matching obtained under the women-proposing procedure. Thus, $\mu \neq \mu'$. Under the men-proposing

Table 26: Lists of preferences.

mens'	m_1:	w_2	w_1	w_3	womens'	w_1:	m_1	m_3	m_2
	m_2:	w_1	w_3	w_2		w_2:	m_3	m_1	m_2
	m_3:	w_1	w_2	w_3		w_3:	m_1	m_3	m_2

procedure, if women w1 misrepresents her true preference as

$$w_1 : m_1\, m_2\, m_3,$$

then the DA algorithm generates another stable matching

$$\hat{\mu} =< m_1w_1,\, m_2w_3,\, m_3w_2 >,$$

as shown in Table 27, which implies that both women w_1 and w_2 are better off and that men m_1 and m_3 are worse off.

Table 27: Woman w_1 misrepresents her true preference.

	1st round	2nd round	3rd round	4th round
w_1	m_2, ~~m_3~~	m_2	m_1, ~~m_2~~	m_1
w_2	m_1	~~m_1~~, m_3	m_3	m_3
w_3				m_2

The interpretation is as follows. In the first round, m_1 whom w_1 prefers most did not come to make his proposal to her, and she could know that her close friend w_2 received his proposal. Note that w_1 prefers m_3 to m_2 in her true preference, but rejected m_3 in the first round; w_1 might foresee the subsequent events that in the second round, m_3 makes his proposal to w_2 (because truth-telling is his dominant strategy), but w_2 rejects his proposal (Probably, w1 know the true preference of w_2, because they are close friends.) In the third round, m_1 whom w1 prefers most will come to make his proposal to her, because she knows that she is preferred to w_3 by him.

Note that In the world of the standard theory of matching game, every player knows the others' preferences; Namely, the game has complete and perfect information. Only the "mechanism designer" does not know their preferences.

Appendix 5: Matching in the medical education system

In 2004, as part of reform in the medical education system, the Japan ministry of health, labor and welfare introduced the medical trainee matching actually used in the United States, where the assignment of trainee doctors to training hospitals with different capacities is determined by a trainee-proposing DA algorithm.[4] Thus, the medical trainee matching has in its standard form the same structure as the one we considered for a Japanese electric manufacturer in the body of this case. At that time, the medical trainee matching was a precedent of the application of DA algorithms to real practice in Japan, and it is still the only example, as of October 2018.[5]

Matching theory has developed exponentially since the late 1990s, expanding its sphere of application to various areas such as organ transplant matching between patients and donors, students' school choice, and so on. The matching theory, however, has not taken into sufficient consideration how the introduction of mechanisms designed for target situations affects people's other social and economic activities. A number of issues were reported after the introduction of medical trainee matching.

Medical training in the United States is merely a preparatory stage for subsequent specialist training programs called fellowships. After the specialist training, many doctors work in private practice and often engage in specialist medicine by renting operating rooms and beds from hospitals. The assignment to training hospitals is thus not directly connected to long-term career choice for young doctors. In contrast, Japan had its own medical system, whereby doctors' associations known as medical offices (centered on university hospitals) intensively managed the provision of doctors to each hospital, including training hospitals. It is not a common practice for doctors to rent operating rooms and beds from hospitals. Moreover, from 1968 to 2003, even a preparatory medical training was not mandatory for doctors who passed the national examination.[6] Specialist training programs began to be set up at hospitals after 2006. Medical offices still function in the medical system, although the participation rate of doctors is decreasing.

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[4] In a matching computed with the trainee-proposing DA algorithm, some trainee doctors are not matched with hospitals, when they represent their preferences only for a few hospitals. At that time, a scramble matching is conducted with hospitals which have vacant seats..

[5] In the United States, a hospital-proposing DA algorithm had been used for a long time, but it was changed to a trainee-proposing DA algorithm in the late 1990s, under the supervision of Alvin Roth who was awarded the Nobel prize in Economic Science in 2012 jointly with Lloyd Shapley.

[6] Medical activities are not permitted for any one even in medical training, unless he or she passes the national examination.

The following major changes to the medical system in Japan have occurred since the introduction of medical trainee matching.

(1) Too many applications by trainee doctors to urban hospitals, and too few applications to university hospitals.

(2) In private small and medium-sized hospitals that relied partly on trainee doctors for medical activities but can no longer accept them, there has been no option but to reduce the number of medical departments, leading to the consolidation and abolition of medical departments among hospitals. As a result, some cases were reported about elderly or serious patients who have to go to distant hospitals with appropriate medical departments.

(3) The only hospitals that can participate in matching are the hospitals that can provide training programs, and other hospitals are becoming unable to secure enough young doctors.

(4) The number of agencies mediating between doctors and hospitals is said to have increased since the introduction of mandatory medical training, although it may not yet be considered a direct cause.

(5) The latest medical techniques developed at university hospitals and graduate schools of medicine may no longer be adequately transferred to small and medium-sized hospitals.

(6) Young doctors currently have broader choices regarding their own careers.

(7) As trainee doctors can obtain cross-disciplinary knowledge and skills across several medical departments, possibilities for patients to receive more appropriate medical treatment have widened.

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