Duncan Palamourdas specializes in the mathematics of poker and poker education. His poker classes at UCLA always fill up early and have long waiting lists. He is also an author at *Upswing Poker*, *Poker News*, *Card Player* and *Winning Network*. Despite his impressive academic credentials, Duncan's popularity is a product of his love for simple language and metaphors. Profitable poker play essentially revolves around correct risk-reward optimization. This is a complex topic and Duncan's great strength is that he explains it in simple terms without resorting to technical jargon.

Contents

Introduction

Poker is a deceptively complex game. Between the luck of the draw and the unknown cards, it seems that the game is almost too "random" to beat. Worse yet, even if one discovers a long-term winning strategy, it is still quite challenging to apply that knowledge in a convincing and educational way. Albert Einstein has been famously quoted as saying: "*If you cannot explain it to a six year old, you do not understand it yourself*." This book is my attempt to communicate the ins and outs of poker with anyone who – like that kid – has the overwhelming desire to learn but possibly is lacking in experience.

I remember beating my dad at chess for the first time around that age too. I remember it like it was yesterday; it made me feel like a million bucks. Perhaps this is what triggered my obsession with puzzles and games. From math Olympiads and chess competitions early on, to math research and cash games, this passion has not faded in the slightest.

Professor Yiannis Mochovakis[1], one of UCLA's most influential math researchers and educators, once told me that: "*You only know you know a subject when you can teach it*." He was right! After teaching a number of poker classes, I found that not only has my own skill improved, but more importantly I found myself exposed to a wide variety of ideas that I had

[1] http://www.math.ucla.edu/~ynm/

never explored before. Being constantly challenged by my students to articulate in the most efficient and elementary way possible the intricate concepts of poker was a huge catalyst in that direction. This book is an attempt to catalog these concepts in a "user friendly" way.

The book is divided into 13 chapters. Each chapter concentrates on a single concept and – for the most part – can be read independently of the others. Reading this book in order is recommended, but the more experienced reader/player is welcome to read the chapters in any way they see fit. I also included a poker glossary in the appendix section, with all the poker lingo and abbreviations one is likely to encounter either here, in other books and at the tables. Last but not least, to help the reader better visualize the concepts and ideas of the game, I use two main antagonists, representing the two polar opposites found at a poker table: Alex, the professional poker player and successful risk-taker who takes calculated risks for a living, and Bob, the recreational poker player who loves the game and wants to have a good time. My ultimate goal through the narrative of this work is to give a rigorous but comprehensible justification as to why it is always the case that *money "flows" from players such as Bob to players like Alex in the long run*.

Below are brief descriptions of each chapter:

1) The Basics
As the title suggests, this is a collection of the basics of NL (No Limit)-holdem, including the rules, important definitions as well as the different types of games. The experienced player can safely skip this chapter.

2) The Amateur, the Nerd and the Gambler
A short story followed by some important takeaways and general principles that any successful poker player should follow.

3) The Poker Trifecta
In this chapter, the reader will find the three most important innate properties of winning NLHE: Position, Initiative and Card Advantage.

4) We are Only Human
This is a collection of instinctive but unprofitable tendencies of inexperienced players and how to avoid them.

5) Poker is an Honest Game
It is a common misconception that the majority of poker revolves around bluffing. This short essay aims to falsify that line of thinking and present a more realistic view of the game.

6) Money Saved is Money Earned
The proper way of preserving chips at the poker table and how to later use them as weapons.

7) The Cat is Dead and Alive! Sort of...
A chapter on randomness, the great impact it has on poker and how to properly account for it.

8) Give a Man a Fish and He May Turn into One
Alex's full winning strategy, from before she sits at the table all the way to the river.

9) Life is Not an Efficient Process
Here I attempt to give a useful definition of what poker mistakes are and to develop ways to evaluate them quantitatively.

10) The Earth is Not Flat...
A large collection of both common and not so common misconceptions about the game and why they do not work in the long run.

11) We are More Than the Sum of Our Parts
This is a technical chapter that tries to fully dissect a bet in order to understand how players can increase their profitability. (No prior math knowledge is required.)

12) The World is Not Bluff and White
Next, we explore a different decomposition of a bet (bluff vs value-bet), with the same goal of increasing profitability.

13) K.I.S.S. The Power of Fundamentals
In closing, we re-iterate some of the most key and fundamental ideas while emphasizing their importance for winning poker.

Footnotes
As much I like simplicity, there is quite a lot of nuance in poker. Moreover, the academic code of ethics encourages me strongly to use an extensive references backing my claims. This inevitably led to a considerable number of footnotes, many of which may seem pedantic and even repetitive at times (I repeated several of them on purpose to save the reader the trouble of backtracking). In any case, to avoid distractions, the reader may ignore all footnotes safely, without any loss of continuity.

Chapter Eleven

We are More Than the Sum of Our Parts

Bob loses money by calling. Alex makes money by betting.

Alex has wondered many times how much a specific poker hand is actually worth. This is not a mere exercise in futility. If she wants to get the most money out of her hand, it is natural to first understand what is the "true" monetary value associated with it. This is, of course, an extremely vague statement that needs a rigorous definition if we want to make any sense of it. The only obvious part of this phrase is "her hand" which simply means her two hole-cards in connection with the community cards (if any). The other two words (*true* and *value*) are anything but obvious in this context. On the one hand, we have "truth" which has been one of the most investigated and debated terms among philosophers and logicians since the beginning of time. On the other hand, we have "value" which is also a very controversial and often misinterpreted term. This is not only by the non-specialist but also by those involved with finance and economics.

Let's start with the latter as it is easier to conceptualize. What exactly is "value" in general? The term can be interpreted in many ways. To narrow the discussion, let's concentrate on monetary value, the type of value relevant in NLHE. Many economists agree that the (monetary) value of an object is the price that a willing buyer and a willing seller agree on for an

exchange. Quite simple, is it not? However, how does one decide what to pay? The decision is usually based on two components: The intrinsic and the extrinsic value of the object. Intrinsic value is generally "objective," primarily involving the costs of producing the item. Extrinsic value is more "subjective," involving concepts such as speculation (for future price), beauty, innovation, sentiment, etc. For example, the intrinsic value of a typical sized painting (say the Mona Lisa) is probably worth only a few hundred dollars (or less if cheaper materials were used), while its extrinsic value can be anything[308] up to hundreds of millions of dollars. In a similar manner, the raw materials needed to make a standard t-shirt result in a fairly small intrinsic value (a few dollars, if that), while things like brand name, popular demand, style and marketing can skyrocket its extrinsic value to hundreds of dollars or more.

The following question can act as a quick heuristic hinting at the presence of extrinsic value: Is the price of an object as a whole more than the sum of the parts? If yes, extrinsic value is present; if not, then it is not. Incidentally, it is also possible for extrinsic value to be negative. A classic example is a clearance sale where the owner is often willing to sell below intrinsic value (at a loss) to get rid of certain items as quickly as possible.

As can be seen from the examples above, due to the highly subjective nature of extrinsic value, it is the only part of the value equation that fluctuates based on factors such as personal opinion or preference. Extrinsic value is the primary driver of the final price (value).

What does this have to do with poker? Well, everything! Every hand Alex is involved in has a certain intrinsic and extrinsic value even if these values are unknown to her at that moment. First, let's concentrate on the notion of *intrinsic value* in poker, by looking at a simplified example:

- ♠ Alex has A♠-J♣ on the button

[308] In theory extrinsic value can also be negative (up to minus the intrinsic value) if no one is willing to purchase the painting, not even for the value of its raw materials.

- ♠ The flop is J♥-9♦-9♣
- ♠ The pot is $200
- ♠ Bob holds 7♦-7♣

If we assume *no more betting on the flop or any subsequent street*, what are Alex's chances to win this hand? If we use poker software that evaluates all possible combinations of turns and rivers, we see that she is expected to win this pot 90% of the time[309]. That means that we expect her to lose this pot 10% of the time (when Bob spikes a 7, either on the turn and/or the river and Alex does not improve further).

As we have seen before, this means that Alex has 90% pot equity. Since the pot is already $200, this also means that she deserves 90% of the $200. That is, she should make on average:

0.90 x $200 = $180

worth of equity in real dollars. Thus, the intrinsic value of her hand equals the pot equity in dollars which is $180. Similarly, the intrinsic value of Bob's hand is

0.10 x $200 = $20

(which, not surprisingly, is just the remainder of the pot). In layman's terms this means that if we repeat this exact scenario for – say – a million times, on average Alex will make $180 per hand, while Bob will make on average $20 per hand. This is again a version of "the law of large numbers (LLN)"[310] or, as I like to call it, the law of long-run fairness.

[309] We can also estimate our chances by using the 2/4 rule. Bob has 2 outs twice (namely 7♥ and 7♠), for a total of approximately 2x4 = 8% equity. He also has a tiny chance of making a straight, if both a 10 and an 8 show up.

[310] Simply put the LLN states that the more times we run an experiment the closer the results approach the expected outcome. If we toss a fair coin 1 million times we ex-

We therefore arrive at the following simple equation:

Intrinsic Value = Pot Equity

In other words, pot equity represents one's "fair" share of the pot, based on the strength of one's hand.

Of course, if we always received only our fair share in the end, poker would not be a very interesting game. Especially if everyone played approximately the same starting poker hands we do. To understand why this is, let's consider the following hypothetical example. Say that both Alex and Bob have the same starting hand requirements before they get involved in a pot pre-flop. What this means is: If Alex can have A♠-J♣ on the button and Bob can have 7♦-7♣ in the big blind, the reverse situation should also be true. Namely, Bob can have A♠-J♣ on the button and Alex can have 7♦-7♣ in the big blind. More importantly, given enough time, both symmetrical situations *will happen* approximately the same number of times. This is again due to the "fairness" of the LLN.

Putting it numerically, if Alex is involved in several pots against Bob where she has – say – $180 equity and he has $20 equity, in the long run the law of large numbers guarantees that Bob will also be involved in approximately as many pots where he has $180 equity and she has only $20 equity. The LLN guarantees that – on average – they will make approximately the same amount of money from each other. Then when we consider that poker is a negative sum game (due to the rake) this also means that they will both lose money over the long term.

Does this mean that Alex cannot make any money based on intrinsic value (pot equity) alone? Well, not necessarily. If she has a stricter starting hand requirement than Bob, by playing more quality hands than he does, she could in theory profit based on pot equity alone. This is simply because she would flop on average better equity than Bob just because

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pect almost 50% Heads and almost 50% Tails. (Any other distribution is extremely unlikely, indeed almost impossible.)

she plays higher quality hands via a tighter range. This is essentially the concept of Card/Range Advantage from the Trifecta and another reason why hand selection is so important before the flop.

But what if Bob played similar hands with Alex pre-flop? How could Alex profit in that case? The answer is by using the *extrinsic value* of the hand, which is what I call bet equity.

Intrinsic Value = Bet Equity

Let's revisit the previous scenario with Alex holding A♠-J♣, Bob holding 7♦-7♣ and a J♥-9♦-9♣ flop. The only difference is that this time we no longer assume no post-flop betting. Also, for simplicity's sake, let's assume that both players have $200 behind. The action is as follows:

- ♠ Bob checks to Alex from the big blind
- ♠ Alex goes ALL-IN for $200 into a $200 pot
- ♠ Bob has a decision to make (either call or fold)

As we saw in our analysis of Alex's aggression, there is something fundamentally unique about the action of betting and raising: it gives her an opportunity to win the pot then and there by making her opponents fold. Checking and calling cannot do this.

Bob has two choices, neither of which is optimal:

- ♠ He can fold his hand
- ♠ He can call the $200

Let's analyze both choices in terms of profitability for Alex:

1) If Bob folds, Alex wins the full pot for a total of $200. Since Alex already had $180 in pot equity, by making that bet she makes an extra $20 which is her bet equity.

2) If Bob calls, Alex will "deserve" 90% of the now $600 pot or

0.90 x $600 = $540

If we subtract the cost of her bet ($200), her final profit should be (on average) $340. Again, since Alex already had $180 in pot equity, by making that bet she makes an extra

$340-$180=$160

which is her bet equity.

What is interesting in both cases, Alex increases her profitability by betting *regardless of what Bob does*.

In other words:

Value = Pot Equity + Bet Equity

Moreover, we can further dissect Bet Equity into two parts:

- ♠ Fold equity (FE) which is the extra equity from Bob's folds
- ♠ Value-bet equity (VBE) which is the extra equity generated from Bob's calls.

Bet Equity = Fold Equity + VBet Equity

To see how this translates into concrete numbers, let's say that Bob folds/calls at a 50/50 ratio (depending on his mood, how much he has lost that day, etc). This means that 50% of the time Alex would make an extra $20 due to his fold, and 50% of the time she would make (on average) an extra $160 due to his calls. This means that:

Value = Pot Equity + Fold Equity + VBet Equity
Value = 180 + (0.50 x 20) +(0.5 x 160) = $270

Notice that $270 is one and a half times Alex's pot equity (fair

price). Said another way, by exploiting Bob's weaknesses through the action of betting, Alex makes far *more money* she initially deserves *based on the distribution of the cards!* This isexactly how she makes a profit in poker!

Of course, Alex rarely knows her opponent's exact cards, so she needs to find a way to use this formula in a more general and realistic setting. Here is one way:

Let's assume that if Bob calls Alex's (A♠-J♣) button raise from the BB and then checks in the dark, his pre-flop range is:

{10-10 to 2-2, A-Qs to A-2s, K-9s+, Q-9s+, J-9s+, 10-9s, 9-8s, 8-7s, 7-6s, 6-5s, 5-4s, A-Qo to A-8o, K-10o+, Q-10o+, J-10o}

Inputting these hands into Flopzilla we get the following picture:

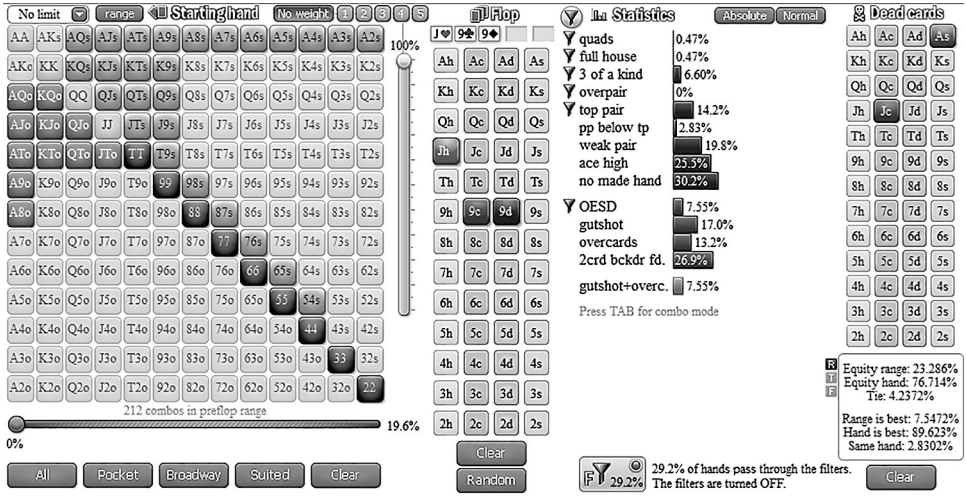

Figure 15: Bob's Preflop range

As we see, accounting for card removal[311], this is approximately 20%

[311] These are the cards that are either in Alex's hand or on the board and thus cannot be in Bob's hand.

of the total possible holdings. After Alex goes all-in, she expects him to call her with something like:

{10-10 to 7-7, A-Js, A-9s, K-9s+, Q-9s+, J-9s+, 10-9s, A-Jo, A-9o, K-10o+, Q-10o+, J-10o}

Once again, it is better to visualize this using Flopzilla:

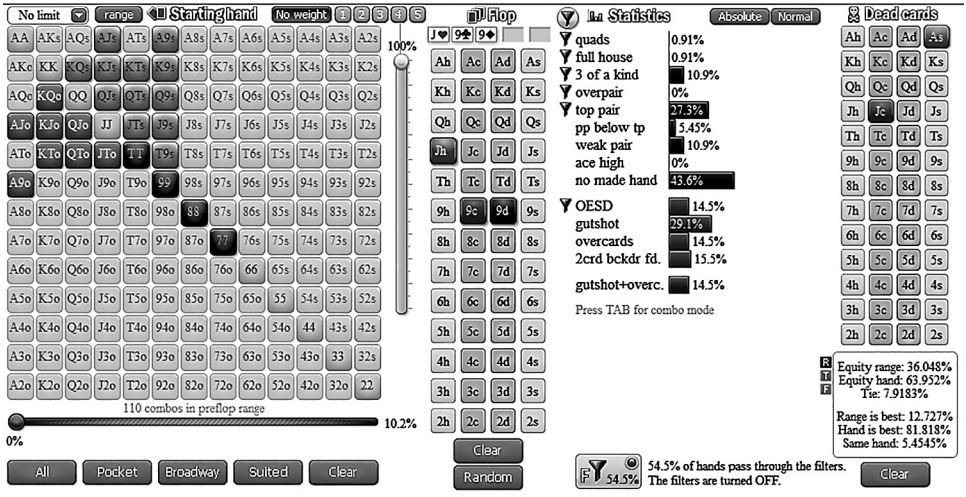

Figure 16: Bob's Flop Calling Range

That is Bob's expected range after he calls Alex's bet. This is roughly 10% of the possible holdings or 50% of his initial pre-flop calling range

10/20 = 50%

Now, a quick Equilab computation shows that A♠-J♣ has approximately a 64% chance of winning against that second range. To sum up:

- ♠ Bob folds about 50% of the time when Alex bets and thus Alex wins a smaller pot ($200)
- ♠ When Bob calls, Alex wins about 64% of a bigger pot ($600)

Alex's total EV (expected value) should be:

Value = EV = 0.5($200) + 0.5[(0.64 x $600) – $200] = 192

Thus the total value of Alex's hand is $192.

How much of this comes from Pot Equity and how much is Bet Equity? To determine this, we need to compute Alex's chances against Bob's entire pre-flop calling range before Alex's flop bet, including hands he would fold to a flop bet. A quick Equilab computation shows that Alex's hand (A♠-J♣) will win about 77% of the time against Bob's pre-flop range. That means that her pot equity before her flop bet is:

Pot Equity = 0.77($200) = $154

Therefore her total bet equity should be:

Bet Equity = Value – Pot Equity = $192 - $154 = $38

In other words, she makes an extra $38 by making that bet! A more accurate way of saying this is that she makes $38 more than she would make had she decided to "check it down" all the way to the river.

Dissecting further, $23=0.5*(200-154) of these $38 dollars come from Fold Equity and thus the remaining $15 come from VBet Equity.

Ok, let's state the important equation one more time:

Value = Pot Equity + Bet Equity

But wait a minute… Does that mean we should always bet in order to increase the value of our hand? I mean, since we always have a non-negative dollar amount from the "pot equity," shouldn't we always bet to add something extra to that amount? Unfortunately, it is not that simple! That is because the Bet Equity could also be negative! Consider the following scenario:

Instead of A♠-J♣ let's say that Alex holds 10♣-10♠. An Equilab calculations shows that Alex's chances of winning vs Bob's 20% range of hands (assuming no more betting[312]) should be approximately 64%[313]. Therefore, her Pot Equity should be exactly 0.64*200 = $128.

Alex's Pot Equity = $128

However, if Alex bets $200, Bob – as before[314] – will call with just half of his hands (10%) and fold the rest. Against those calling hands, Alex now *only* has about a 40% chance of winning. Therefore her total Expected Value should be:

Alex's Value = EV = 0.5($200) + 0.5[(0.40 x $600) - $200] = $120

By making that bet (and using that sizing) something amazing happened! Alex makes less money than her actual Pot Equity! We conclude that her Bet Equity is negative. More precisely:

Bet Equity = Value – Pot Equity = $120 - $128 = -$8

More precisely still, her Fold Equity is $36 (still positive) while her VBet Equity is -$44 (negative). This would be the equivalent of Alex doing a garage sale for a painting or a T-shirt for less than the cost of its raw materials. A large discount indeed!

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[312] That is, we assume that Bob will always check his hand down if Alex doesn't bet the flop.

[313] Not to be confused with the 64% found above, which was the equity of A-J against Bob's flop calling range (10% of staring hands). Here the 64% is the equity of 10-10 against Bob's much wider pre-flop range (20% of starting hands). Not surprisingly, 10-10 is weaker than A-J on the flop, so it performs worse than A-J.

[314] Bob cannot possibly know whether Alex has a different hand this time or not, thus his actions should be roughly the same as before. Another way to say this, is that Bob's decisions can only depend on Alex's betting actions and not her exact hand.

Let's consider one last scenario. With all else being equal, Alex now holds 5♠-4♠. Not surprisingly, this bad hand has only about 13.5% equity versus Bob's pre-flop range (20% of starting holdings). Thus her pot equity should be:

Alex's Pot Equity = 13.5 x $200 = $27

Once again, if Alex bets $200, Bob will call with half of his hands (10%) and fold the rest. Against those calling hands, Alex has only a 7% chance of winning. Therefore her total Expected Value should be:

Alex's Value = EV = 0.5($200) + 0.5[(0.07 x $600) - $200] = $21

This means that Alex's Bet Equity is again negative:

Bet Equity = Value – Pot Equity = $21 - $27 = -$6

Fold Equity here may be massive ($86.5) but VBet Equity is even larger (-$92.5) and negative! The takeaway here is that big bluffs rarely work.

Betting big amounts with low equity hands is almost never a good idea.

By this point, you may have guessed that Bet Equity is the true "game changer" in poker. Sure, Pot Equity is important too. This is why Alex tries to have a higher average Pot Equity than her competition by employing better hand selection. Unfortunately, this is not always possible, given how easily players can learn to play a "reasonable" range of hands. Alternatively, learning when and how much to bet is a far more complex task, one that requires a deep understanding of ranges, something Alex constantly tries to perfect to increase her bottom line.

How does Alex profit by using Bet Equity? There are two main ways:

- ♠ She can identify spots where a bet will add value to her existing Pot Equity. This is usually true when both a fold and a call are desirable outcomes (when she has a big hand – especially one that could be vulnerable to draws). It could also be true if one of the two components (Fold Equity and VBet Equity) adds significant value, even if the other is negative. The classic example here is c-betting a dry board with air. The value from all the folds she will get far outweighs the money she will lose when her opponents finally call her c-bet.
- ♠ She can also combat her opponents' bets – either by calling or raising at appropriate frequencies – in order to minimize their Bet Equity. Ideally, she wants their Bet Equity to be negative. One approach is to raise in spots where her opponents would expect her to fold a good portion of a time (thus they are more likely to bluff). That way she would be denying them the extra equity, not only by not folding, but also by forcing them to fold. This is equivalent of *fighting fire with fire!*[315]

Recall that we began by asking what is "True Value". This is where the word "True" comes into play. True Value refers to the maximum dollar amount a player can generate by applying the best sequence of moves against opponents' given strategy. For example, say Alex makes the following assumptions:

- ♠ Bob's range consists of {10-10 to 2-2, A-Qs to A-2s, K-9s+, Q-9s+, J-9s+, 10-9s, 9-8s, 8-7s, 7-6s, 6-5s, 5-4s, A-Qo to A-8o, K-10o+, Q-10o+, J-10o} (20% of starting hands).
- ♠ The flop is J♥-9♦-9♣

[315] The caveat here is that this strategy only applies against players who understand the power of betting and thus think at a higher level than Bob.

- ♠ The pot is $200
- ♠ Alex and Bob both have $200 behind
- ♠ Bob checked to Alex on the flop
- ♠ Alex can only bet $200 (ALL-IN) or check (for the sake of simplicity)
- ♠ Bob will call with half his range if Alex bets
- ♠ Bob will check the hand all the way to the river if Alex does not bet

The correct play for Alex would be:

- ♠ Bet $200 all-in if she holds A♠-J♣
- ♠ Check back if she holds 10♣-10♠
- ♠ Check back if she holds 5♠-4♠

This is because according to our analysis, she should make:

- ♠ $192 is better than $154 in the 1st case
- ♠ $120 is worse than $128 in the 2nd case
- ♠ $21 is worse than $27 in the 3rd case

We conclude that, *assuming* Bob's strategy is exactly as described above, the true value of A♠-J♣ is $192, the true value of 10♣-10♠ is $128, while 5♠-4♠ has a true value of $27. Alex can realize those values by going ALL-IN, CHECK and CHECK respectively.

Of course, knowing Bob's exact strategy is crucial here and it is absolutely necessary for her to compute the true value of her hand. Although it is unlikely that Bob will openly reveal his strategy to Alex, she can approximate it to a reasonable degree. As we have seen, like Bob, the majority of players have predictable patterns which Alex can identify quickly using her experience, meticulous observation and deductive skills (Range Carving).