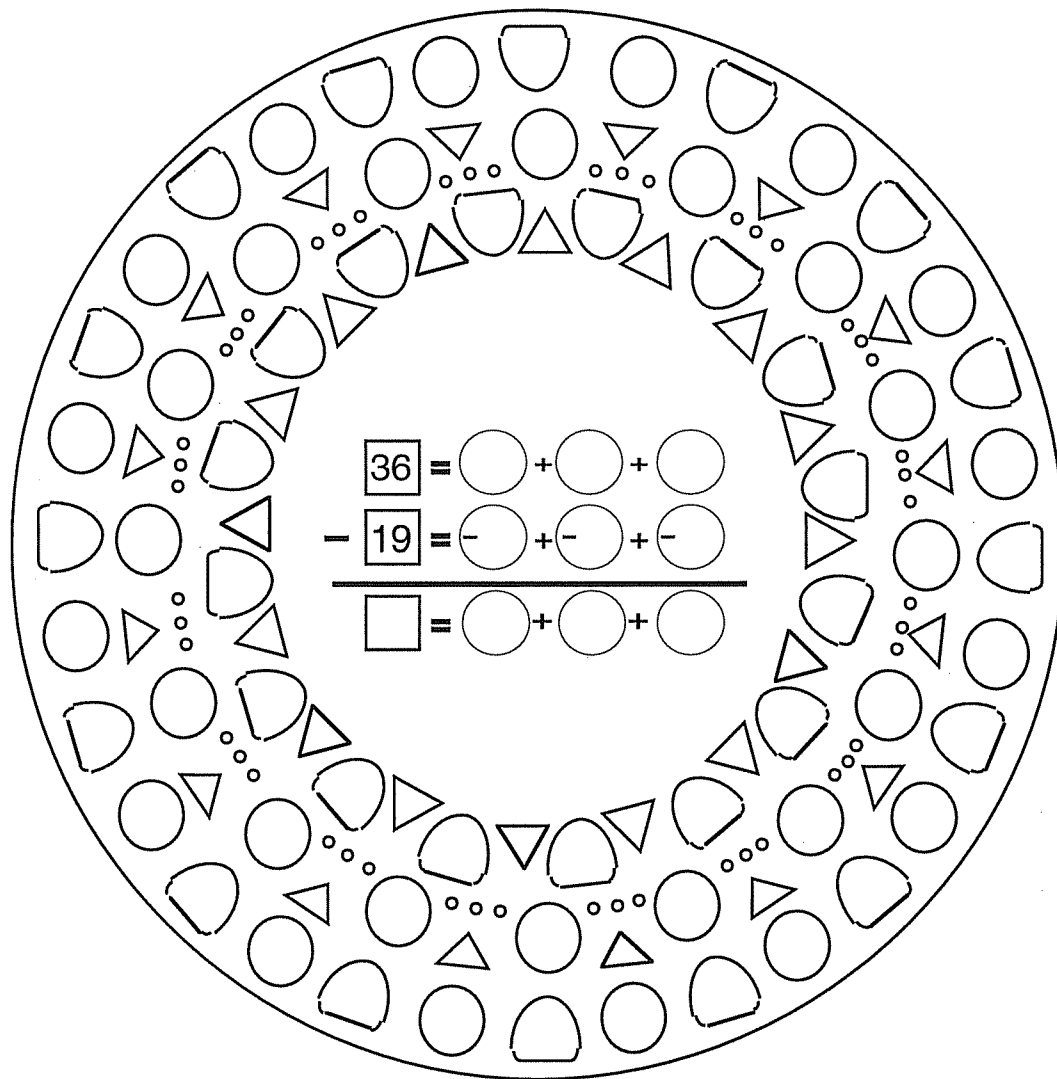


**Patterns in Arithmetic**  
**Subtraction - Booklet 3 PDF**  
General Principles of Regrouping  
**Parent/Teacher Guide**



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the

UCI Farm Elementary School

University of California, Irvine

# Subtraction: Booklet 3 - General Principles of Regrouping

## Contents

Pre-Assessment: Part 1 .....	i	Nonstandard Regrouping and IOUs .....	19
Pre-Assessment: Part 2 .....	ii	Standard Regrouping: Manipulative .....	22
Assessment Guide .....	iv	Standard Regrouping: Circle Math .....	25
Post-Assessment .....	x	Standard Regrouping: Short Way .....	29
Farm School Teaching Techniques .....	xii	Hundreds, Tens, and Ones: Manipulative ....	34
Regrouping: Introduction and Philosophy .....	1	Three Digits Regrouping: Manipulative .....	35
Circle Math: Review .....	2	Three Digits Regrouping: Recording .....	37
Fixing Problems .....	4	Answer Key .....	40
Difference Between .....	6		
Discovering Easier Ways .....	8		
Tens and Ones: Manipulative .....	12		
Nonstandard Regrouping .....	16		

The Circle Subtraction series is dedicated to Michael Butler and Alysia Krafel. His unique approach to subtraction and teaching mathematics is the foundation of the entire Farm School math series. Alysia learned how to use his principles and apply it to a practical way for all of us to be able to teach mathematics in a Farm School way.

## Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

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Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide  
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**Pre-Assessment - Part 1**

Name \_\_\_\_\_ Date \_\_\_\_\_

Put a question mark next to any problem you do not know how to do.

1. Solve. a.

$$\begin{array}{r} \boxed{15} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{12} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

b.

$$\begin{array}{r} \boxed{14} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{7} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2. Solve each problem a different way.

a.

$$\begin{array}{r} \boxed{26} = \bigcirc + \bigcirc \\ - \boxed{17} = -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc \end{array}$$

b.

$$\begin{array}{r} \boxed{26} = \bigcirc + \bigcirc \\ - \boxed{17} = -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc \end{array}$$

c.

$$\begin{array}{r} \boxed{26} = \bigcirc + \bigcirc \\ - \boxed{17} = -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc \end{array}$$

3. Find the mistakes. Circle them with a red pencil.

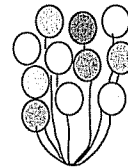
a.

$$\begin{array}{r} \boxed{21} = \bigcirc 16 + \bigcirc 12 + \bigcirc 3 \\ - \boxed{9} = -\bigcirc 2 + -\bigcirc 6 + -\bigcirc 1 \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

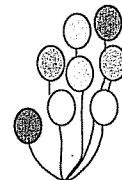
b.

$$\begin{array}{r} \boxed{21} = \bigcirc 20 + \bigcirc 1 \\ - \boxed{9} = -\bigcirc 2 + -\bigcirc 7 \\ \hline \boxed{24} = \bigcirc 18 + \bigcirc 6 \end{array}$$

4. a. Who has more balloons? \_\_\_\_\_  
 How many more? \_\_\_\_\_  
 Number sentence \_\_\_\_\_



Jackie



Joseph

b. How many more is 18 than 7? \_\_\_\_\_  
 Number sentence \_\_\_\_\_

c. How many more is 60 than 40? \_\_\_\_\_  
 Number sentence \_\_\_\_\_

5. Fill in the missing numbers. The difference between each number is \_\_\_\_.

$$\begin{array}{ccccccccc} 0 & , & \underline{\quad} & , & \underline{\quad} & , & 8 & , & \underline{\quad} & , & \underline{\quad} & , & \underline{\quad} \\ & & \vee & & \vee & & \vee & & \vee & & \vee & & \vee \\ & & 4 & & 4 & & 4 & & 4 & & 4 & & 4 \end{array}$$

1. Solve each problem a different way.

a. 
$$\begin{array}{r} \boxed{62} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{37} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

b. 
$$\begin{array}{r} \boxed{62} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{37} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2. Solve each problem a different way.

a. 
$$\begin{array}{r} \boxed{35} = \bigcirc(15) + \bigcirc(15) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

b. 
$$\begin{array}{r} \boxed{35} = \bigcirc(15) + \bigcirc(15) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

c. 
$$\begin{array}{r} \boxed{35} = \bigcirc(10) + \bigcirc(10) + \bigcirc(10) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(6) + -\bigcirc(2) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

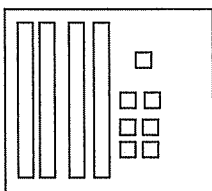
d. 
$$\begin{array}{r} \boxed{35} = \bigcirc(30) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(20) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc \end{array}$$

Take out Base Ten Blocks. You may use them for any problem from here to the end of the test.

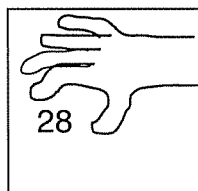
3. a. Take out a 100 flat. Cover it with 43.  
How much is not covered? \_\_\_\_\_
- b. Build 132. Cover it with 67.  
How much is not covered? \_\_\_\_\_

4. Use Base Ten Blocks to solve the problems shown by the picture. Draw the answer. Write the problem

a. Start With

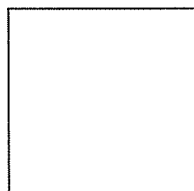


Take Away



=

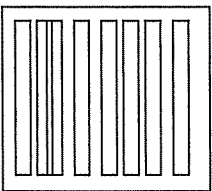
What's Left?



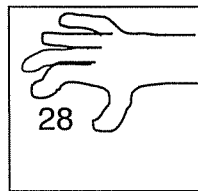
Example:

$$\begin{array}{r} 47 \\ - 28 \\ \hline \end{array}$$

b. Start With

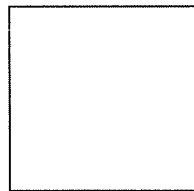


Take Away



=

What's Left?



Now, you write the problem and solve it.

$$\begin{array}{r} \phantom{00} \\ - \phantom{00} \\ \hline \phantom{00} \end{array}$$

5. Solve each problem two ways. First do it the expanded way. Then do it the short way.

<p>a. Expanded way</p> $\begin{array}{r} \boxed{50} = \textcircled{50} + \textcircled{0} \\ - \boxed{32} = \textcircled{-30} + \textcircled{-2} \\ \hline \end{array}$ <p>_____ = _____ + _____                   tens           ones</p>	<p>Short way</p> $\begin{array}{r} 50 \\ - 32 \\ \hline \end{array}$ <p>_____</p>	<p>b. Expanded way</p> $\begin{array}{r} \boxed{86} = \textcircled{80} + \textcircled{6} \\ - \boxed{39} = \textcircled{-30} + \textcircled{-9} \\ \hline \end{array}$ <p>_____ = _____ + _____                   tens           ones</p>	<p>Short way</p> $\begin{array}{r} 86 \\ - 39 \\ \hline \end{array}$ <p>_____</p>
---	---	---	---

6. Use Base Ten Blocks to solve these problems. Draw the answer in the What's Left? box.

<p>a.</p> <table border="1" style="width: 100%; height: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">Start With</td> <td style="width: 33%; text-align: center;">Take Away</td> <td style="width: 33%; text-align: center;">What's Left?</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> </table>	Start With	Take Away	What's Left?				<p>b.</p> <table border="1" style="width: 100%; height: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">Start With</td> <td style="width: 33%; text-align: center;">Take Away</td> <td style="width: 33%; text-align: center;">What's Left?</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> </table>	Start With	Take Away	What's Left?			
Start With	Take Away	What's Left?											
Start With	Take Away	What's Left?											

7. Solve each problem. Record the expanded way and the short way.

<p>a. Expanded way</p> $\begin{array}{r} \boxed{347} = \textcirc{\quad} + \textcirc{\quad} + \textcirc{\quad} \\ - \boxed{156} = \textcircled{-} + \textcircled{-} + \textcircled{-} \\ \hline \end{array}$ <p>_____ = _____ + _____ + _____                   hundreds   tens       ones</p>	<p>Short way</p> $\begin{array}{r} 347 \\ - 156 \\ \hline \end{array}$ <p>_____</p>
<p>b. Expanded way</p> $\begin{array}{r} \boxed{435} = \textcirc{\quad} + \textcirc{\quad} + \textcirc{\quad} \\ - \boxed{279} = \textcircled{-} + \textcircled{-} + \textcircled{-} \\ \hline \end{array}$ <p>_____ = _____ + _____ + _____                   hundreds   tens       ones</p>	<p>Short way</p> $\begin{array}{r} 435 \\ - 279 \\ \hline \end{array}$ <p>_____</p>

## Assessment Guide

**Purpose** The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is a review of problems from previous books. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment determines if the student learned the material that was presented. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

**Prerequisites** The ability to find answers to basic subtraction problems with numbers up to twenty and *Patterns in Arithmetic: Book 2*, the Circle Subtraction Section is preferred but not absolutely necessary.

**Materials** Subtraction: Booklet 3 - Pre-Assessment - Worksheets, pages i - iii  
Post-Assessment - Worksheets, pages 74 and 75. A copy of Pre-Assessment - Worksheets, pages i - iii, precedes this guide.

**Assessment Guide Part 1** This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow.

### Assessment Criteria for Pre-Assessment: Part 1

Can the student:

#### 1. Circle Math Review

A student new to this program will get A correct but probably not B or C. The teacher guide gives instructions for how to teach this concept using the material in Subtraction: Booklet 3 - Circle Math Subtraction: Review. Most students learn this concept easily. It is not necessary for you to obtain *Patterns in Arithmetic: Book 2* to teach your student this concept.

- A. get the correct answer in the box?
- B. break up the numbers into the circles correctly?
- C. perform the little subtraction problems in the circles and sum for the result?

#### 2. Circle Math Review

The student is asked to do a single problem three different ways by breaking up the numbers in different combinations. The answer should always come out the same. Take note if the student uses a standard regrouping procedure. If the student

can do D but not C, there is a good chance he does not understand the place value of regrouping.

- A. get the correct answer?
- B. do the problems three different ways?
- C. use a standard tens and ones breaking up as one of the ways?
- D. use a standard regrouping of tens strategy?

### 3. Fixing Problems

Can the student locate errors? Sometimes errors happen in breaking up the numbers. Sometimes the top number is less than the bottom number. A common error is the student will take the difference between the two numbers, which will result in an incorrect answer. There are several ways to work with a top number less than the bottom one. The standard way to deal with this problem is to move a ten from the first circle in the top row to the second circle in the top row to increase the size of the second number. You can actually regroup with a five or a two also. There is nothing unique about ten except it makes record keeping easier. Many students will take only what is needed to zero out the column. Another way to deal with it is to use a negative number.  $1 - 7$  is  $-6$ . This negative number can then be subtracted when the answers to the little subtractions are summed (See IOUs). A student may also rearrange the numbers to subtract without regrouping at all. All of these strategies work.

- A. locate the errors in the calculations by circling them?
- B. fix the mistakes to produce a correct final result?
- C. deal effectively with a larger number on the bottom than on the top? Make a note of which strategies he uses.

### 4. Difference Between

Difference Between is a common form that shows up in word problems. Questions that ask, "How many more?" are working with this concept.

- A. use a picture to determine how many more balloons?
- B. write the number sentence to go with the balloon picture?
- C. get the correct answer on both the problems with no picture?
- D. put in the correct number sentence for both the problems with no picture?

### 5. Difference Between Patterns

These kinds of problems are used to detect patterns in strings of numbers.

- A. fill in the correct numbers?

## Assessment Criteria for Pre-Assessment: Part 2

### 1. Discovering Easier Ways

One of the early discoveries children make about breaking up numbers is that some ways are easier than others. Using multiples of ten is one of those ways. Having the top and bottom numbers match, which zeros out a column, is another way. We want students to learn that the standard way is *a* way, not *the* way.

- A. break up the two problems differently?
- B. use factors of ten to make the break up easier? E.g.,  $56 = 30 + 10 + 16$
- C. avoid creating situations where regrouping is needed?
- D. get correct matching answers?

### 2. Nonstandard Regrouping

This is related to the discussion in Fixing Problems, Item C Pre-Assessment: Part 1. There are hun-

dreds of ways to do a subtraction problem.

- A. show that the answer to each problem is the same?
- B. get all the answers in the squares correct?
- C. solve each problem differently (instead of just copying the answers)?

There are no scores on the items below; just make a note about strategies.

- D. regroup any problem by moving a ten?
- E. use a negative number (an IOU) in any problem?
- F. zero out a column so that the number in the circle below the line is 0?

### 3. Covering

This is a variation of Difference Between in Pre-Assessment: Part 1 with larger numbers. The student should use Base Ten Blocks. Have him build the given number. Then he should put the covering number on top of the one hundred flat (barge).

- A. understand how to manipulate the blocks to do a covering problem?
- B. get the correct answer to problem a?
- C. get the correct answer to problem b?

### 4. Standard Regrouping Manipulative

Sometimes you will find a student who can do the problems but not be able to show regrouping with the blocks. This is an indication of lack of understanding of the regrouping process. Some students do not know they are regrouping a ten, thinking it is a one.

- A. use the blocks to show how the answer to problem a is obtained?
- B. use the blocks to show how the answer to problem b is obtained?
- C. show the correct answer in the written problems?

### 5. Standard Regrouping: Short Way

These are two digit problems. The student is asked to do the problems both the standard expanded way and the short way. The first problem is  $50 - 32$ . It is shown in both formats. In the standard expanded way, the fifty must be crossed off and a forty written above it, not a four. The student must demonstrate that he knows the number being regrouped is a ten. In the short notation, the five is crossed off and replaced by a four. In both cases a one is placed in front of the zero. It is possible for a student to not know that a ten is moving into the ones place. Many think it is a one. We want to be sure he can do the problems using the standard procedure but at the same time understand the place value of the numbers being regrouped.

- A. get the correct answers to both problems?
- B. use the standard expansion correctly showing the real place value of the number being moved from the tens place to the ones place?

### 6. Three Digits Regrouping: Manipulative

- A. use the blocks to show how the answer to problem a is obtained?
- B. get the correct answer to problem a?
- C. use the blocks to show how the answer to problem b is obtained?
- D. get the correct answer to problem b?

### 7. Three Digits Regrouping: Recording

- A. get the correct answers to both problems?
- B. use the standard expansion correctly showing the real place value of the number being regrouped?



**Pre-Assessment Score Sheet: Part 1**

Name \_\_\_\_\_ Date \_\_\_\_\_

Can the student:

1. Circle Math Review

- Yes No A. get the correct answer in the box?
- Yes No B. break up the numbers into the circles correctly?
- Yes No C. perform the little subtraction problems in the circles and sum for the result?

2. Circle Math Review

- Yes No A. get the correct answer?
- Yes No B. do the problems three different ways?

Note which strategies are present. Tens and Ones? Standard Regrouping?

3. Fixing Problems

- Yes No A. locate the errors in the calculations by circling them?
- Yes No B. fix the mistakes to produce a correct final answer?
- Yes No C. deal effectively with a larger number on the bottom than on the top?  
Make a note of which strategies he uses.

4. Difference Between

- Yes No A. use a picture to determine how many more balloons?
- Yes No B. write the number sentence to go with the balloon picture?
- Yes No C. get the correct answer on both the problems with no picture?
- Yes No D. put in the correct number sentence for both the problems with no picture?

5. Difference Between Patterns

- Yes No A. fill in the correct numbers?

Items Correct = \_\_\_\_\_ = \_\_\_\_\_%  
Items Possible = 13

Note: A student new to this program will need more instruction with lessons Circle Math Subtraction: Review and Fixing Problems and additional practice using this method of subtraction.

Can the student:

## 1. Discovering Easier Ways (of breaking up)

- Yes No A. break up the two problems differently?  
 Yes No B. use factors of ten to make the break up easier? E.g.,  $56 = 30 + 10 + 16$   
 Yes No C. avoid creating situations where regrouping is needed?  
 Yes No D. get correct matching answers?

## 2. Non-standard Regrouping

- es No A. show that the answer to each problem is the same?  
 Yes No B. get all the answers in the squares correct?  
 Yes No C. solve each problem differently (instead of just copying the answers)?  
 D. regroup a problem by moving a ten?  
 E. use a negative number (an IOU) in any problem?  
 F. zero out a column so that the number in the circle below the line is 0?

## 3. Covering as a Strategy

- Yes No A. understand how to manipulate the blocks to do a covering problem?  
 Yes No B. get the correct answer to both problems?

## 4. Standard Regrouping: Manipulative

- Yes No A. use the blocks to show how the answer to problem a is obtained?  
 Yes No B. use the blocks to show how the answer to problem b is obtained?  
 Yes No C. show the correct answer in the written problems?

## 5. Standard Regrouping: Short Way

- Yes No A. get the correct answers to both problems?  
 Yes No B. correctly use the standard expansion showing the real place value of the number being moved from the tens place to the ones place?

## 6. Three Digits Regrouping: Manipulative

- Yes No A. use the blocks to show how the answer to problem a is obtained?  
 Yes No B. get the correct answer to problem a?  
 Yes No C. use the blocks to show how the answer to problem b is obtained?  
 Yes No D. get the correct answer to problem b?

## 7. Three Digits Regrouping: Recording

- Yes No A. get the correct answers to both problems?  
 Yes No B. correctly use the standard expansion showing the real place value of the number being regrouped?

Items Correct = \_\_\_\_\_ = \_\_\_\_\_%

Items Possible = 20

Can the student:

1. Discovering Easier Ways (of breaking up)

- Yes No A. break up the two problems differently?
- Yes No B. use factors of ten to make the break up easier? E.g.,  $56 = 30 + 10 + 16$
- Yes No C. avoid creating situations where regrouping is needed?
- Yes No D. get correct matching answers?

2. Nonstandard Regrouping

- Yes No A. show that the answer to each problem is the same?
- Yes No B. get all the answers in the squares correct?
- Yes No C. solve each problem differently (instead of just copying the answers)?
- D. regroup a problem by moving a ten?
- E. use a negative number (an IOU) in any problem?
- F. zero out a column so that the number in the circle below the line is 0?

3. Covering as a Strategy

- Yes No A. understand how to manipulate the blocks to do a covering problem?
- Yes No B. get the correct answer to both problems?

4. Standard Regrouping: Manipulative

- Yes No A. use the blocks to show how the answer to problem a is obtained?
- Yes No B. use the blocks to show how the answer to problem b is obtained?
- Yes No C. show the correct answer in the written problems?

5. Standard Regrouping: Short Way

- Yes No A. get the correct answers to both problems?
- Yes No B. correctly use the standard expansion showing the real place value of the number being moved from the tens place to the ones place?

6. Three Digits Regrouping: Manipulative

- Yes No A. use the blocks to show how the answer to problem a is obtained?
- Yes No B. get the correct answer to problem a?
- Yes No C. use the blocks to show how the answer to problem b is obtained?
- Yes No D. get the correct answer to problem b?

7. Three Digits Regrouping: Recording

- Yes No A. get the correct answers to both problems?
- Yes No B. correctly use the standard expansion showing the real place value of the number being regrouped?

Items Correct = \_\_\_\_\_ = \_\_\_\_\_% 80%, or 16 points, needed to pass.

Items Possible = 20

1. Solve each problem a different way.

a. 
$$\begin{array}{r} \boxed{62} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{37} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

b. 
$$\begin{array}{r} \boxed{62} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{37} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2. Solve each problem a different way.

a. 
$$\begin{array}{r} \boxed{35} = \bigcirc(15) + \bigcirc(15) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

b. 
$$\begin{array}{r} \boxed{35} = \bigcirc(15) + \bigcirc(15) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

c. 
$$\begin{array}{r} \boxed{35} = \bigcirc(10) + \bigcirc(10) + \bigcirc(10) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(10) + -\bigcirc(10) + -\bigcirc(6) + -\bigcirc(2) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

d. 
$$\begin{array}{r} \boxed{35} = \bigcirc(30) + \bigcirc(5) \\ - \boxed{28} = -\bigcirc(20) + -\bigcirc(8) \\ \hline \boxed{\phantom{00}} = \bigcirc + \bigcirc \end{array}$$

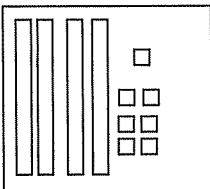
Take out Base Ten Blocks. You may use them for any problem from here to the end of the test.

3. a. Take out a 100 flat. Cover it with 43.  
How much is not covered? \_\_\_\_\_

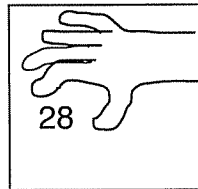
b. Build 132. Cover it with 67.  
How much is not covered? \_\_\_\_\_

4. Use Base Ten Blocks to solve the problems. Draw the answer. Write the problem shown by the picture.

a. Start With

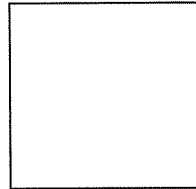


Take Away



=

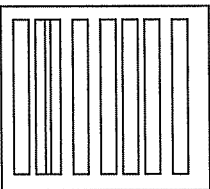
What's Left?



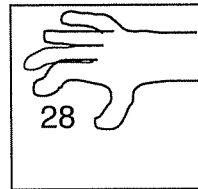
Example:

$$\begin{array}{r} 47 \\ - 28 \\ \hline \end{array}$$

b. Start With

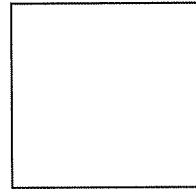


Take Away



=

What's Left?



Now, you write the problem and solve it.

$$\begin{array}{r} - \\ \hline \end{array}$$



## Farm School Teaching Techniques

The following process is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept:

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract, symbolic), which links the concrete to the pictorial to the abstract.
4. Practice to fluency.
5. Practice for speed.

Expect math to make sense. Look for sources of enjoyment.

Treat yourself and student as a mathematician. Mathematicians make up problems for themselves and others. Students who do this increase their control over the learning process, reinforce and show understanding, and strengthen their image of themselves as mathematicians.

Begin each lesson with a warm-up and review. Always end the lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements such as: "**Are you sure?**" or "**Build it.**" or "**What gave you the clue?**" or "**Show me how you got that.**" even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Some things students and teachers should be doing routinely:

1. *Inventing problems*: For practice and self-teaching. For other people, including the teacher. As puzzles, to give and get pleasure. To show understanding of a principle. To link math to the world. To stretch creative power. To develop mathematical taste.
2. *Finding patterns*: In numbers, shapes, etc. Find the rule; 'black box.' Math or formal patterns in worldly stuff--motion, people, plants, cars, stars. 'Getting' a puzzle.
3. *Giving plausible answers and why*: Estimation/prediction/plausible guessing. Error bounds. Recognizing that some estimates are better than others. Iteration to improve estimates. Quickly inventing problems whose answers are 'about such-and-such--e.g., between m and n. Believing that answers ought to make sense.
4. *Learning about and using kinds of relations*: The more the more, the more the much more, etc. Plots; worldly cases. 'Variable' thinking. 'Control' – 'other things being equal.' Refining explanations ('true for boys but not girls') in terms of variables.

## Regrouping: Introduction and Philosophy

Some subtraction problems can be more efficiently handled by a special kind of regrouping also called borrowing. *Patterns in Arithmetic: Subtraction - Booklet 3* uses the Farm School discovery, inductive approach to achieve this goal.

We believe students can and should construct a regrouping procedure for themselves. They will understand it that way. When regrouping is taught by recipe or rote, math becomes an accumulated set of memorized procedures shown to you by someone else. But when students (and adults) construct their own unique ways to regroup, they act as mathematicians, leading to the realization that although the standard regrouping method has particular advantages in the base ten system, it is only one way among many. Students have invented unique, successful ways to regroup.

In addition to supporting a student's understanding, learning regrouping through this inductive approach, continues to lay a foundation for understanding *all* arithmetic algorithms (addition or subtraction or multiplication or division) because each is built on the same model, a process with three parts: decomposing a large problem into many small problems, solving each small problem, and adding the results. When students understand this, each new arithmetic procedure has a familiar feeling, because it is seen to be part of the same family of algorithms. Some people ask, why go to all this trouble? If the student's experience of learning is one of memorized things that don't really make sense, not only does he not learn to think for himself, but he learns that understanding is not important. This hidden lesson is devastating to one's education.

$$\begin{array}{r} 21 \\ \cancel{4} \\ - 15 \\ \hline 19 \end{array}$$

Many students I worked with who were taught the short form process didn't understand that the *little one* they wrote in the ones place stood for a ten. This is a dialogue from one such student, a bright seven-year-old. The problem is 34 - 15.

**"Why did you write that little one there?"** "To make the four bigger."

**"When you put it there, it makes the four into a bigger number?"** "Yes."

**"What number does it make it into?"** "Into a fourteen."

**"That confuses me. I thought that a one and a four makes five. How did you make it into a fourteen?"** (An expected answer is: "Because the one is really a ten.")

"It just is. That's the way you write it."

**"How do you know that it is OK to do that?"**

"I just learned it that way. It's right, isn't it?"

This student does not understand the notation he is using. If he did, he would be able to explain how the one plus one equals fourteen. He does not even possess the tools to reason it out and see that the *one* must be a ten because one plus one equals fourteen. It is important that every piece of new knowledge is securely grounded in experience and derived from what the student already understands. Manipulation, with concrete materials, used to verify answers, representational solutions with drawings, and finally symbolic numbers build a strong foundation. Anything less will result in memorized, nonsensical (to the student) procedures. If the student's experience of learning is memorized things that don't really make sense, not only does he not learn to think for himself, but he learns that understanding is not important. The hidden lesson is devastating to one's education.

The sensory system of some people can best validate what they learn by using a concrete, manipulative, and guided approach. Students who have difficulty generalizing patterns and thinking abstractly will benefit from this method. *Chrysalis Charter School: Base Ten Subtraction* uses such an approach.

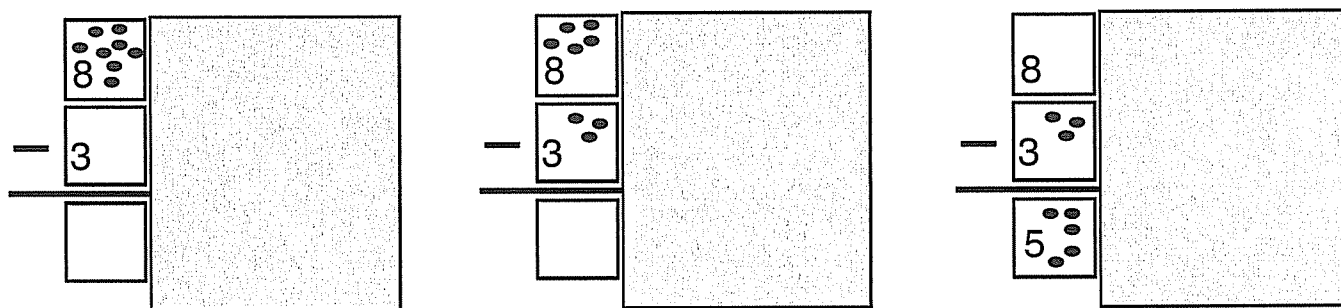
## Circle Math: Review

**Purpose** The purpose is to review the position of the numbers in a subtraction problem and expanding and recording numbers into two circles.

**Prerequisites** Fluency in subtracting numbers

**Materials** Circle Math Subtraction: Two Circles Workboard, page 1\*  
Erasable pen  
Beans or blocks  
Scratch paper and pencil.

**Lesson 1** For the first time, cover up the circles on the erasable board. **"Please build and solve the problem 8 - 3."**



**"What does the eight stand for?"** "What I start with." **"What does the three stand for?"** "What I take away." **"What does the five stand for?"** "What's left." Make sure the student takes away five beans from the eight beans and can identify each number position. Then reveal the circles.

Give a problem such as 9 - 4. Record the problem on the erasable board. **"Solve this problem and prove your answer with beans."** Below is an idealized version an experienced student might use. See *Patterns in Arithmetic: Book 2 - Subtraction* for a complete description of the process.

Begin with:

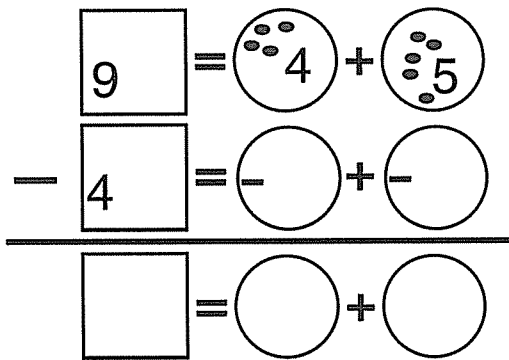
Nine is broken up into 4 + 5 and recorded.

$\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$ $\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$ $\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$	$\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$ $\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$ $\begin{array}{r} \square \\ 9 \\ - 4 \\ \hline \square \end{array} = \bigcirc + \bigcirc$
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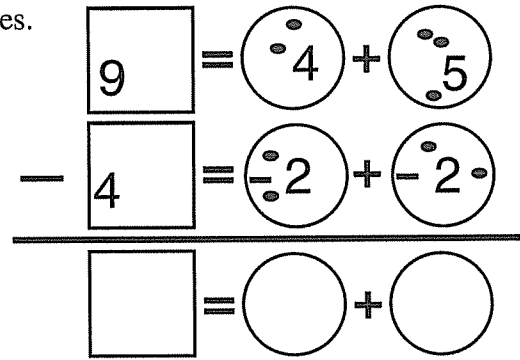
\*Make a copy and cover it with clear contact paper or put the page in a clear sheet protector. Or use poster board, copy the page, and cover it with clear contact paper.



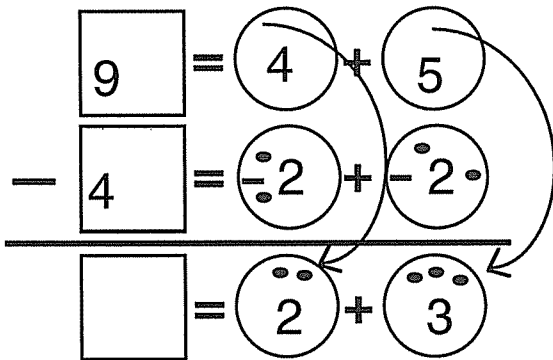
Beans are moved to the respective numbers.



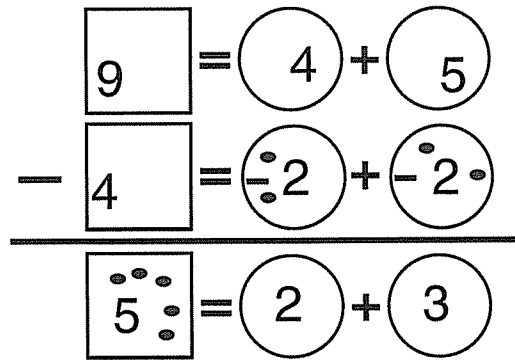
Two beans are taken away from the four and the five, and the numbers are recorded in the middle circles.



The beans left in the top circle are moved to the bottom circle and the numbers are recorded.



Two plus three are added together and the answer is recorded in the difference square.



**Worksheets**

Circle Math: Review: Worksheets 1 - 2, pages 4 and 5

**Lesson 2**

Review a problem with three circles using Circle Math: Three Circles Workboard, page 2.

**Worksheet**

Circle Math: Review: Worksheet 3, page 6

## Fixing Problems

**Purpose** The purpose is to practice fixing subtraction problems that were broken up poorly.

**Prerequisites** Fluency with expanding numbers and Circle Math Subtraction: Review.

**Materials** Erasable Three Circles Workboard, page 2\*  
Any manipulative  
An erasable pen

$$\begin{array}{r} 9 \text{ Minuend} \\ - 4 \text{ Subtrahend} \\ \hline 5 \text{ Difference} \end{array}$$

**Lesson** Give a circle math subtraction problem in which the broken up "subtrahend" is larger than the "minuend." Many students will already have created this situation and solved it on their own. This lesson is to insure that every student can find a solution. It is an important step to inventing nonstandard regrouping.

On an erasable Three Circles Workboard give a problem such as  $8 - 7$ .

Set up the problem this way:

"Here is a problem for you to finish on your own today."

$$\begin{array}{r} 8 = 3 + 2 + 3 \\ - 7 = -1 + -3 + -3 \\ \hline \square = \square + \square + \square \end{array}$$

Allow the student to complete the problem independently. Several possible outcomes:

A. She solves the problem with beans. "I can't take away three from two." "**How can you fix it?**" "Trade some beans." One bean is taken from the first three in the top row and given to the two in the middle circle of the same row. Change each number as the beans are moved around by crossing it out and rewriting the new number. It will help you and the student later to track what was done.

$$\begin{array}{r} 8 = 3 + 2 + 3 \\ - 7 = -1 + -3 + -3 \\ \hline \square = \square + \square + \square \end{array} \rightarrow \begin{array}{r} 8 = 2 + 3 + 3 \\ - 7 = -1 + -3 + -2 \\ \hline \square = \square + \square + \square \end{array}$$

You will be able to follow the thought process, and the student can recall what she did when asked to prove the answer.

B. Another solution would be to change how the numbers are expanded in the middle row: change the one to a two and the three to a two, (see below.) If a student chooses this way, allow her to continue through the problem. It is a way to solve the problem, but it will make later work with regrouping more difficult if she gets into the habit of rearranging the middle row of numbers. After she has done the problem, compliment her on her success and ask her to fix the problem again with the constraint that she has to use the top row of numbers only to fix the problem.

$$\begin{array}{r}
 \boxed{8} = \textcircled{3} + \textcircled{2} + \textcircled{3} \\
 - \boxed{7} = \textcircled{-1} + \textcircled{-3} + \textcircled{-3} \\
 \hline
 \boxed{\phantom{0}} = \textcircled{\phantom{0}} + \textcircled{\phantom{0}} + \textcircled{\phantom{0}}
 \end{array}
 \quad \longrightarrow \quad
 \begin{array}{r}
 \boxed{8} = \textcircled{3} + \textcircled{2} + \textcircled{3} \\
 - \boxed{7} = \textcircled{\cancel{-1}} + \textcircled{\cancel{-2}} + \textcircled{-3} \\
 \hline
 \boxed{\phantom{0}} = \textcircled{\phantom{0}} + \textcircled{\phantom{0}} + \textcircled{\phantom{0}}
 \end{array}$$

C. "I can't take away three from two. I don't know how to do it." **"Think about it for a while."** Let her think about it longer. If no solution is found within this lesson, either give the same problem to work on independently or give more practice with Circle Math Subtraction without this constraint. Try it again next lesson.

D. She completes the problem but does it incorrectly in this manner: Three is taken from two and one is recorded as the answer.

$$\begin{array}{r}
 \boxed{8} = \textcircled{3} + \textcircled{2} + \textcircled{3} \\
 - \boxed{7} = \textcircled{-1} + \textcircled{-3} + \textcircled{-3} \\
 \hline
 \boxed{\phantom{0}} = \textcircled{2} + \textcircled{1} + \textcircled{0}
 \end{array}$$

This happens often if no manipulatives are used. Allow her to complete the problem without interruption. Don't say, "No, that answer is wrong." Instead ask, "Are you sure?" Or "Prove it with beans." Or "Is that true; is eight take away seven equal to three?" Then ask her to prove the answer with beans (or whatever manipulative she used).

Or she may notice that  $8 - 7$  is not 3. The error will become apparent when she tries to take three beans from the two in the middle set of circles. Have her work on the problem until she discovers a possible way of fixing the problem.

E. A student may understand negative numbers and use them to solve this dilemma. It is a clever solution. Teach the lesson IOUs if you want to make this solution available to your student.

$$\begin{array}{r}
 \boxed{8} = \textcircled{3} + \textcircled{2} + \textcircled{3} \\
 - \boxed{7} = \textcircled{-1} + \textcircled{-3} + \textcircled{-3} \\
 \hline
 \boxed{1} = \textcircled{2} + \textcircled{-1} + \textcircled{0}
 \end{array}$$

## Test for Understanding

Fixing Problems - Test for Understanding, page 7

\*Make a copy and cover it with clear contact paper or put the page in a clear sheet protector. Or use poster board, copy the page, and cover it with clear contact paper.

## Difference Between

### Purpose

The purpose is to explore subtraction by the comparison of the sizes of numbers. To link the operation of subtraction and counting. For example, "How much larger is nine than four?"

### Materials

Any manipulative

### Lesson

**"Make a pile of nine pennies. Then make a pile of five pennies."**

### Session 1

**"Which set has the most?"** "The nine pile." **"How do you know?"**

**"How much larger?"** Or, **"How many more pennies in the larger pile? How can you figure this out?"**

There are different ways to solve this. Encourage each student to solve this in her own way. Here are a few solutions students have come up with:

"I have five in this pile and counted pennies until I got to nine. Nine is four more than five." (Started with five and "counted on"—six, seven, eight, nine.)

The counters are placed in two rows, one above the other. The student compares the two lines with one-to-one correspondence and counts the difference.

"I know that five plus five is ten, and nine is one less than ten, so nine is four more than five."

"I subtracted. I have nine and I took away five and I have four left."

Nine pennies are laid out and each of the five pennies are placed on top of a penny that is laid out already. There are four pennies not covered up. "Five is four less than nine."

**"Count out thirteen counters and eight counters. Which number is larger? How much larger? What is the difference between thirteen and eight? How much smaller is eight than thirteen? How did you get the answer?"** Continue with several more examples. Encourage the student to ask the questions.

Finally, have the student draw a picture of the counters and write a number sentence or write a statement. If she counted on, as in the first example above, she may write the problem as an addition problem. Another student may write the problem as a subtraction problem. Compare solutions and encourage different ways to solve it.

### Session 2

Eventually we do want a student to see the problem as a subtraction problem. There are several ways to do this. Ask the student to find different ways of solving the problem. Or if more than one student is working on the problem, have the students compare the different ways in which they solved the problem. Have the students take turns being teacher and student and teach the other person a different way to solve the problem.

Another approach is to have the student build a model. One solution is to line up the counters.



**"What numbers go with this problem?"** "Fifteen, eight, and seven." **"Show what each number represents."** Compare the difference. **"How much larger is fifteen than eight?"**

"Fifteen is seven larger than eight." Then line the numbers up: 15 8 7. **"Can you make a number sentence with these numbers?"**  $15 - 8 = 7$  or  $15 = 8 + 7$

Or, try this way using the Families of Facts lesson. **"The answer is seven. What were the other numbers in this problem?"** Fifteen and eight. **"What can you do with a fifteen and an eight to get seven as an answer?"**  $15 - 8 = 7$ ." Have her prove each equation with counters. The student may need to play around with the manipulatives and the numbers to figure this out. If the student writes  $8 - 15 = 7$ , the use of counters will prove it is not correct. Let her self correct by asking, **"Show me the eight, show me the fifteen, and show me the seven,"** instead of saying, **"That is not right."** Allow the student time to reason this out. The relationship is not always obvious to students.

## Worksheets

Difference Between, page 8, and Difference Between: Patterns, page 9 Make up more of your own with your student.

Once the student has figured out that a difference can be calculated by subtraction, have a discussion about the difference between comparison problems and 'take away' or 'what's left?' problems. Both are solved by the same operation, but they appear differently in real world situations.

How much change will I get when I buy this ball with my dollar? This is a 'take away' or 'what's left?' problem.

How much more expensive is a candy bar than a package of gum? This is a comparison problem, solved by finding the difference.

Make up several examples of both together. Then have the student make up both types. Look for Difference Between problems in the real world. Car ads often boast, "We'll beat their price and pay you the difference."

## Discovering Easier Ways

**Purpose** The purpose is to develop and discuss strategies for decomposing and composing larger numbers easily.

**Prerequisites** Circle Math Subtraction: Review, Fixing Problems, and mastery of basic circle subtraction with numbers up to twenty without a manipulative

**Materials** Discovering Easier Ways - Worksheet 1, page 11  
Erasable Three Circles Workboard  
A manipulative

**Lesson** Give  $19 - 12$  as the first problem and have the student solve without assistance. Check the solution on the Circle Math Workboard with a manipulative. Proceed only if the student can do the problem alone. Have the student solve the same problem another way. **"You are now going to start working with larger numbers. Sometimes you may get confused and need to start over. Starting over is fine and may help you find a new way of solving a problem."** During the rest of the lesson the main role of the teacher is as an observer and troubleshooter. For the student it is an exercise in pathfinding and discovery. It is important to comment on discoveries. Ask the student to explain to you what he is doing and why.

**"Solve the problem  $26 - 17$  on four circles two different ways."** This is a regrouping problem (because the top number in the ones place is smaller than the number below it), but in circle subtraction this is not a problem and regrouping is not necessary.

Here the student did each expansion separately and created a small subtraction problem,  $3 - 4$ , which he can't do with his current knowledge. **"How can you fix this?"** He can fix it by rearranging the expansion or by starting over.

$$\begin{array}{r} \boxed{26} = \textcircled{8} + \textcircled{9} + \textcircled{6} + \textcircled{3} \\ - \boxed{17} = \textcircled{-4} + \textcircled{-8} + \textcircled{-1} + \textcircled{-4} \\ \hline \boxed{\phantom{00}} = \textcircled{4} + \textcircled{1} + \textcircled{5} + \textcircled{?} \end{array}$$

One labor saving method commonly discovered is to use large numbers in the expansion and to put zeros in the extra circles. As long as the subtraction of the larger expanded numbers is done easily and correctly, this is a workable method.

$$\begin{array}{r} \boxed{26} = \textcircled{17} + \textcircled{9} + \textcircled{0} + \textcircled{0} \\ - \boxed{17} = \textcircled{-17} + \textcircled{-0} + \textcircled{-0} + \textcircled{-0} \\ \hline \boxed{9} = \textcircled{0} + \textcircled{9} + \textcircled{0} + \textcircled{0} \end{array}$$

A related strategy:

Another way to make the work easier is to make the expansion easier. Using round numbers such as ten or twenty is a common route. This discovery can be facilitated by having the student practice expanded subtractions such as  $80 - 60 = 20$  or  $600 - 200 = 400$ .

$$\begin{array}{r} \boxed{26} = \textcircled{10} + \textcircled{10} + \textcircled{6} \\ - \boxed{17} = \textcircled{-10} + \textcircled{-7} + \textcircled{-0} \\ \hline \boxed{9} = \textcircled{0} + \textcircled{3} + \textcircled{6} \end{array}$$

If the student uses this expansion, which is the standard expansion, he will be unable to subtract with his current knowledge. **Do not introduce regrouping at this time.** Encourage him to fix the problem in whatever fashion seems best to him.

$$\begin{array}{r} \boxed{26} = \textcircled{20} + \textcircled{6} \\ - \boxed{17} = \textcircled{-10} + \textcircled{-7} \\ \hline \boxed{\phantom{00}} = \textcircled{0} + \textcircled{?} \end{array}$$

### Worksheet

Give Discovering Easier Ways - Worksheets 2 - 5, pages 12 - 15 Give Discovering Easier Ways: Practice in the Black Line Master following this lesson for additional practice.

### Troubleshooter

One problem that commonly occurs when working with larger numbers is shown here:

The student has created a small subtraction problem that is as difficult as the original problem in that it must be broken up to find the answer. Have the student use more circles or find an easier expansion. This difficulty often immediately precedes and facilitates the discovery of using the round number expansions discussed earlier.

$$\begin{array}{r} \boxed{58} = \textcircled{35} + \textcircled{13} + \textcircled{8} \\ - \boxed{27} = \textcircled{-17} + \textcircled{-10} + \textcircled{-0} \\ \hline \boxed{\phantom{00}} = \textcircled{?} + \textcircled{3} + \textcircled{8} \end{array}$$

### Student example

Mark's way. Mark was a bright seven-year-old trying to figure out subtraction. He was using Cuisenaire Rods when he found this method of doing subtraction problems. He was doing this problem:  $43 - 27$ .

First, he built forty-three with four ten rods and one three rod. He recorded his expansion as  $10 + 10 + 10 + 10 + 3$ . Then, when he tried to remove twenty-seven, he found he couldn't because he didn't have a seven rod. After thinking a minute, he traded one ten rod for a seven rod and a three rod. His expansion now read  $10 + 10 + 7 + 3 + 3$ . He then removed two ten rods and the seven rod. In his recording he crossed out the numbers he removed so it looked like this:  
 $10 + 10 + 10 + 7 + 3 + 3$  He then added up the remainder, which is sixteen, and recorded the answer under the problem.

What Mark had developed was a manipulative way to "zero out" his columns to use round numbers in his expansion. Here is this method transferred to circle math format:

$$\begin{array}{r} \boxed{43} = \textcircled{10} + \textcircled{10} + \textcircled{10} + \textcircled{7} + \textcircled{3} + \textcircled{3} \\ - \boxed{27} = \textcircled{-10} + \textcircled{-10} + \textcircled{-0} + \textcircled{-7} + \textcircled{-0} + \textcircled{-0} \\ \hline \boxed{16} = \textcircled{0} + \textcircled{0} + \textcircled{10} + \textcircled{0} + \textcircled{3} + \textcircled{3} \end{array}$$

Using Cuisenaire Rods to do the expansions can assist the process of discovery because it prevents a student from becoming bogged down in the calculation of the expansions. It allows him to focus on the process of the subtraction. Mark dropped the use of the rods as soon as he saw his method written down as shown above. He no longer needed them, and the circle way was faster.

This was one student's way of making it easier. The entire process took Mark three weeks. This is an example. You don't necessarily need to teach it to your student. Your student may come up with a different method. Some students will develop many methods, never settling on just one. They will learn by trial and error, revision of previous methods, and from each other. The point is that a student can learn a great deal working on his own if given the chance to play around without constant teacher intervention or pressure. Don't be afraid to make suggestions when a fresh vision is needed or conceptual errors are made, but be sensitive to the student's discovery process.

## Notes

This lesson takes several weeks to complete. Do one or two problems, two different ways a day. Have students write their own problems. Go on to other things during this exploration stage. Continue to work on addition problems, Difference Between, word problems, the Trading Game played backwards, and general math.

A few students will not progress to easier and easier ways. They will find a method that works, even one that is cumbersome, and will persist with it in spite of repeated suggestions, or will wander aimlessly through methods, almost at random, never building on previous experience. If this occurs, simply use the constraint that the student uses round numbers in an expansion. Require them to make certain changes to get them moving, then proceed to the next lesson.

Students who have had major difficulties with this lesson should use the Base Ten Subtraction Booklet instead of this booklet. The Base Ten Subtraction Booklet uses a rote, algorithm, or recipe approach and is useful for students who learn best with a guided teaching style and concrete models.

## Worksheet

For additional practice in breaking up numbers into tens and ones, give Discovering Easier Ways - Practice on the following page in this booklet (duplicate first).

## Test for Understanding

1. The main test here is fluency and independent working. Watch for the ability to self-correct errors, to spot potential problems with certain expansions, to proceed when a problem arises, and to revise willingly.
2. Ask a student to explain what he is doing and why.
3. **"What does it mean if you do the same problem two ways and the answers come out differently?"** "A mistake was made."



**Discovering Easier Ways**  
Practice - Make multiple copies.

Date \_\_\_\_\_

1.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \bigcirc - + \bigcirc - + \bigcirc - \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2.

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \bigcirc - + \bigcirc - \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

3.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - \square = \bigcirc - + \bigcirc - + \bigcirc - + \bigcirc - \\ \hline \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

4.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - \square = \bigcirc - + \bigcirc - + \bigcirc - + \bigcirc - \\ \hline \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

Do the answers to problems 2 and 3 match?



What does it mean if they don't?

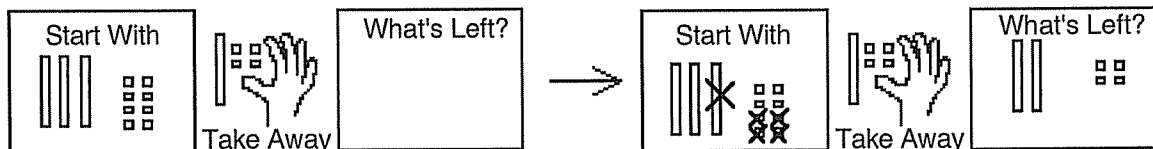
## Tens and Ones: Manipulative

**Purpose** The purpose is to manipulatively break up numbers in a subtraction problem into tens and ones.

**Prerequisites** Nonstandard Regrouping

**Materials** Tens and One: Manipulative - Worksheets 1 and 2, pages 18 and 19  
Base Ten blocks  
Blank paper

**Lesson Session 1** Give Tens and Ones: Manipulative - Worksheet 1. **"Build thirty-eight with Base Ten blocks. Your blocks should match the picture in the Start With box in the first problem."** There are three tens and eight ones. **"The Take Away hand tells you how many blocks to pick up. How many do you take away? How can you show that?"** "Fourteen, or one ten and four ones." **"Draw a picture of the remaining blocks in the What's Left? box. How many are there?"** "Two tens and four ones." **"Read the whole problem."** "Thirty-eight minus fourteen equals twenty-four." Complete the worksheet independently.



Use the completed Tens and Ones - Worksheet 1 and a blank piece of paper.

**"Now change the pictures into numbers. Look at the Start With box."**

**"How many tens are in the box?"** "Three."

**"How much is three tens worth?"** "Thirty."

**"How many ones are in the box?"** "Eight."

**"How much is there altogether?"** "Thirty-eight."

Have him write:  $38 = 30 + 8$

**"What should you write for how many were taken away?"**

$14 = 10 + 4$

Have him write  $14 = 10 + 4$  under the  $38 = 30 + 8$ .

**"How many did you put in your What's Left box?"** "Twenty-four."

**"How can you record that answer?"**  $24 = 20 + 4$  written under the  $14 = 10 + 4$ .

Example:

$$\begin{array}{r}
 1. \quad \boxed{38} = \boxed{30} + \boxed{8} \\
 - \boxed{14} = \boxed{-10} + \boxed{-4} \\
 \hline
 \boxed{24} = \boxed{20} + \boxed{4}
 \end{array}$$

Answers:

$$\begin{array}{r}
 2. \quad \boxed{25} = \boxed{20} + \boxed{5} \\
 - \boxed{13} = \boxed{-10} + \boxed{-3} \\
 \hline
 \boxed{12} = \boxed{10} + \boxed{2}
 \end{array}$$

$$\begin{array}{r}
 3. \quad \boxed{59} = \boxed{50} + \boxed{9} \\
 - \boxed{35} = \boxed{-30} + \boxed{-5} \\
 \hline
 \boxed{24} = \boxed{20} + \boxed{4}
 \end{array}$$

Repeat for problems 2, 3, and 4.

**Worksheet** Tens and Ones: Manipulative - Worksheet 2, page 19

**Session 2** **"In the box that goes with the drawing of the Start With blocks, the number is written. Write the number that goes with the drawing of the Take Away blocks in the box in the numbers column. What number did you write?"** "Twenty-three." **"Subtract and record the answer."** Do the second problem together and have the student do the third problem independently.

## Worksheets

Tens and Ones: Manipulative - Worksheet 3, page 20 Problems 9 - 12 are a test of abstract understanding of this work. If a student is not ready to generalize to the abstract yet, then have him continue to use the blocks and pictures. That is common and is fine. Tens and Ones: Manipulative - Worksheets 4 - 6, pages 21 - 23, are puzzle pages. Worksheet 7, page 24, is for the student to make up problems. After a student is fluent, continue Tens and Ones: Practice - Worksheets 1 and 2, pages 25 and 26.

## Notes

This lesson is easy for most students and goes very fast. Do not be tempted to rush it, drop the use of the blocks too soon, or skip the long practice period. These problems lay the foundation for the more difficult work ahead. A clear mental picture of the standard subtraction process, with place value emphasized, will be needed.

$$\begin{array}{r} 24 \\ -13 \\ \hline 11 \end{array}$$

Short way

## Time Span

Have the student do ten to thirty of these problems each week for a few weeks. Do the problems the short way, without expansions. The student is ready when he can solve the problems quickly without the use of blocks but can easily check with the blocks if asked.

## Test for Understanding

1. Watch for fluency in using the blocks. Does he select the correct size, the correct number? Does he easily perform the subtraction and record the problem?
2. When doing a problem the short way, can the student identify the place value of each digit?
3. **"Do this problem:  $35 - 72 = ?$ "** Watch the reaction. Does he recognize immediately that (with present knowledge) the problem can't be done? Can he explain why? Or does he hesitate or need to check with the blocks? The latter reaction indicates a weakness in understanding the notation or possibly a conceptual uncertainty.

4. Give this problem:

$$\begin{array}{r} \boxed{38} = \textcircled{3} + \textcircled{8} \\ - \boxed{21} = -\textcircled{2} + -\textcircled{1} \\ \hline \boxed{8} = \textcircled{1} + \textcircled{7} \end{array}$$

**"What is wrong with this problem? Can you find the mistakes and fix them?"**

5. The person who did the problem above also did this one:  $85 - 23 = 8$ . **"How can you tell by just looking at the answer that it can't be right?"**
6. **"Jean started with thirty-six and ended up with fourteen. How many did she give away?"**

Here we are checking to see if the student understands the relationships between the numbers in a subtraction problem and what their positions in the problem mean. The student should be able to start with three tens and six ones, and know that they go in the Start With box. He should recognize that the fourteen goes in the What's Left? box. He may have difficulty figuring out the answer. That's fine, we are looking for the ability to set up the problem.

## Answers

- 1 and 2. You must evaluate these. There are no set answers.
3. This problem can't be done because you can't take a larger number (seventy-two) from a smaller number.

4. The mistake here is that the student made a place value error. The three should have been a thirty and the two should have been a twenty. The answer should be  $10 + 7 = 17$  not  $1 + 7 = 8$ .

$$\begin{array}{r} \boxed{38} = \textcircled{30} + \textcircled{8} \\ - \boxed{21} = \textcircled{-20} + \textcircled{-1} \\ \hline \boxed{17} = \textcircled{10} + \textcircled{7} \end{array}$$

5. One can quickly tell that this answer is wrong because it's much too small. The difference between eighty-five and twenty-three is obviously much greater than eight.

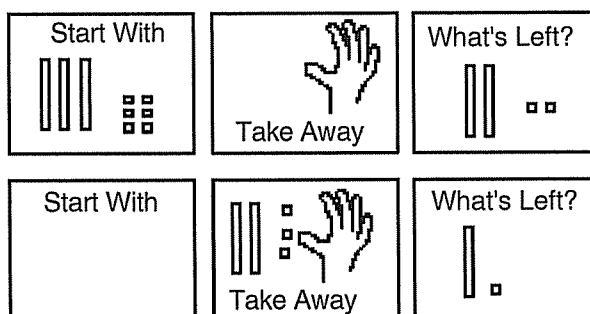
6. Thirty-six in the Start With box and fourteen in the What's Left? box means that two tens and two ones were taken away.

The Challenge Problem on Tens and Ones: Manipulative - Worksheet 3 is an extension of the skills being worked on in this lesson. It involves figuring out missing numbers other than the one after the equal sign. This relates to the Family of Facts and could be done concurrently with that lesson.

## Extensions

1. A fun activity is to put a small sum of money in dimes and pennies on the table (you could also use the blocks, but students like the money better). Have the student count and record how much is there. Then he closes his eyes while you take some of the money away. He must figure out how much you took.

2. Students also enjoy making puzzle pages for each other. Tens and Ones: Manipulative - Practice Black Line Master page follows.  
Examples:



3. *Hands On: Base Ten Blocks - Book 2* Pages 21 - 24 relate specifically; however, the whole series of activities is supportive.


4. *Spectrum Mathematics - Grade 3* Basic drill and word problems

**Tens and Ones: Manipulative**  
Practice - Make multiple copies.


Date \_\_\_\_\_

Use Base Ten Blocks to make up problems for yourself.


1. 

Start With	 Take Away	What's Left?
------------	---	--------------


2. 

Start With	 Take Away	What's Left?
------------	---	--------------

3. 

Start With	 Take Away	What's Left?
------------	--	--------------

4. 

Start With	 Take Away	What's Left?
------------	---	--------------

5. Write a subtraction word problem that goes with the numbers you used in problem 4.

## Nonstandard Regrouping

**Purpose** The purpose is to learn to work with a regrouping situation without having to reorganize the entire expansion or to start over. It sets the stage for formal regrouping.

**Prerequisites** Discovering Easier Ways and Tens and Ones: Manipulative

**Materials** Nonstandard Regrouping - Worksheet 1, page 27  
Any manipulative

**Lesson** Warm up with problems 1 and 2 on Worksheet 1.  
Then look at problem 3 together.

$$\begin{array}{r} \boxed{27} = \textcircled{10} + \textcircled{10} + \textcircled{7} \\ - \boxed{18} = \textcircled{-5} + \textcircled{-5} + \textcircled{-8} \\ \hline \boxed{\phantom{00}} = \textcircled{\phantom{0}} + \textcircled{\phantom{0}} + \textcircled{\phantom{0}} \end{array}$$

"What is wrong with this expansion?"

"You can't take eight from seven."

"How can you fix it?"

Watch to see what she does. If the student puts a one under the 7 - 8, review basic Circle Subtraction before going on. If the student fixes the situation by changing only two numbers in the top expansion, then have her finish the worksheet alone.

Examples of satisfactory solutions. **Note:** The regrouping process does not require the borrowing of a ten. That is *a way*, not *the way*. Allow the student to borrow any amount she wants.

$$\begin{array}{r} \phantom{9} \phantom{8} \\ \boxed{27} = \textcircled{10} + \textcircled{\cancel{10}} + \textcircled{\cancel{7}} \\ - \boxed{18} = \textcircled{-5} + \textcircled{-5} + \textcircled{-8} \\ \hline \boxed{9} = \textcircled{5} + \textcircled{4} + \textcircled{0} \end{array}$$

$$\begin{array}{r} \phantom{7} \phantom{10} \\ \boxed{27} = \textcircled{\cancel{10}} + \textcircled{10} + \textcircled{\cancel{7}} \\ - \boxed{18} = \textcircled{-5} + \textcircled{-5} + \textcircled{-8} \\ \hline \boxed{9} = \textcircled{2} + \textcircled{5} + \textcircled{2} \end{array}$$

There are many variations possible in the exact number changes, but a satisfying solution should involve as few changes as possible. Note that in both of the above examples, only two numbers in the top row were changed. A similar change in the middle row is possible and should be complimented as a possible solution.

Example of changing the numbers in the middle row.

$$\begin{array}{r} \boxed{27} = \textcircled{10} + \textcircled{10} + \textcircled{7} \\ - \boxed{18} = \textcircled{-5} + \textcircled{\cancel{-5}^6} + \textcircled{\cancel{-8}^7} \\ \hline \boxed{9} = \textcircled{5} + \textcircled{4} + \textcircled{0} \end{array}$$

However, ask her to make the changes in the top row only. There is more room for recording, and it will make the work on very large numbers easier. Also, some students will make the changes by erasing rather than crossing out. Discourage this because not only does it take longer but it makes it difficult to follow what the student has done.

If, however, the student starts over or tries to reorganize the entire top or bottom line, let her complete the problem as she wishes and validate her solution. Then ask her to do problem 4, with the constraint of changing only two numbers in the top row. Present it as a puzzle. Give enough time for the student to think and experiment. If she has difficulty with this problem, you might want to write some additional, easier problems before going on. Watch to see how she completes problems 5 and 6.

**Worksheets** Nonstandard Regrouping - Worksheets 2 - 4, pages 28 -30 A Nonstandard Regrouping practice page follows; duplicate and write practice problems appropriate to the ability of each student.

**Note** Students who have had major difficulties with this lesson should use Base Ten Subtraction instead of this booklet. Base Ten Subtraction uses a rote, algorithm, or recipe approach and is useful for students who learn best with a guided teaching style and concrete models.

**Time Span** Continue to practice for the next few days to weeks. Have the student do each problem several ways on various numbers of circles. Include at least one problem with an expansion that requires regrouping on practice pages. She should then make the changes worked on in this lesson. This exercise will generate fluency in recognizing and operating with regrouping problems.

**Nonstandard Regrouping**  
Practice - Make multiple copies.

Date \_\_\_\_\_

1

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \ominus + \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \ominus + \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

3.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \ominus + \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

4.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \ominus + \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

5.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - \square = \ominus + \ominus + \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

6.

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \ominus + \ominus \\ \hline \square = \bigcirc + \bigcirc \end{array}$$



## Nonstandard Regrouping and IOUs

**Purpose** The purpose is to learn how to use negative numbers in a standard subtraction problem offering students yet another way to quickly solve problems.

**Prerequisites** Fluency in circle subtraction and nonstandard regrouping

**Materials** Nonstandard Regrouping and IOUs Worksheets 1, page 31  
Black and red erasable board markers (for a classroom teacher)  
Red colored pencil and a pencil  
Paper money if a manipulative is needed

**Lesson** Tell this familiar scenario: **"You are in the store with your mom or dad. You have three dollars in your pocket. You see a very cool notebook that you would really like to have. The notebook costs five dollars. What would you say to your parent?"** "Mom, can I borrow two dollars to buy this notebook?"

Then, have a discussion about paying back loans.

Continue: **"If your mom says yes, she loans you the two dollars and you use your three dollars to buy the notebook. How much money do you have now?"** "Five dollars."

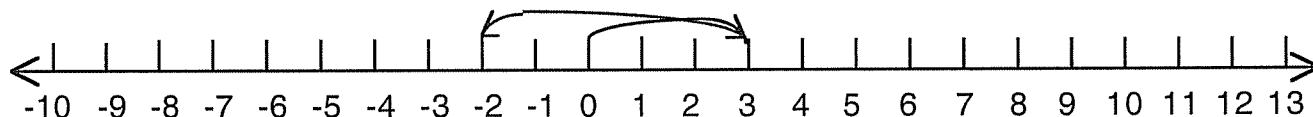
**"After buying the notebook, how much money do you have?"** Most students will say, "Zero." Correct them and say, **"No, you have less than zero. You are in debt and have an IOU of two dollars. You are in the hole by two dollars. You have minus two dollars. You are in the red."**

Language Arts Connection: These are idioms for debt: 'IOU's,' 'in the hole,' and 'in the red'; all mean we owe money. Do not skip this at this point. Discuss all these idioms. Students have heard these and often know what they mean. Also put the word "debt" on their spelling lists.

History: In the old days before computers, people used to keep track of money on long sheets of paper called ledgers. The ledgers were written in black. If there was a debt or if you spent more money than you had, you would write the debt in red ink so you would be sure to see it. Many computer programs still print these debts in red ink so you will see them. So "in the red" now means that you have spent more money than you have and you are in debt and must pay it back.

Continue: **"So you owe your mom how much?"** "Two dollars."  
Write this on the board. Have three dollars Spend five dollars = owe two dollars  
Write the 2 in red. Write this:  $\$3 - \$5 = -\$2$  Again, write the 2 in red.

Use a number line to show negative numbers. Draw the first line going from 0 to the 3 in black. Draw the second line going from the 3 to the -2 in red. Discuss that a debt is shown as a negative number. **"Is a negative number greater than or less than zero?"** "Less than." **"Is being in the red a good thing?"** "No."



Do these problems on the board in a classroom. You can do them orally or have students copy them onto a piece of paper. All negative numbers should be written in red. Have the students use a number line to prove the solutions.

Have \$10 spend \$4 leaves \_\_\_\_\_ Have \$10 spend \$12 leaves \_\_\_\_\_

Have \$12 spend \$4 leaves \_\_\_\_\_ Have \$10 spend \$15 leaves \_\_\_\_\_

Have \$14 spend \$21 leaves \_\_\_\_\_ Have \$20 spend \$14 leaves \_\_\_\_\_

Let's apply this to subtraction. On the Nonstandard Regrouping and IOUs Worksheet 1, you see this problem, Diagram A.

A.

$$\begin{array}{r} \boxed{47} = \overset{29}{\cancel{30}} + \textcircled{10} + \overset{8}{\cancel{7}} \\ - \boxed{28} = \textcircled{-10} + \textcircled{-10} + \textcircled{-8} \\ \hline \boxed{19} = \textcircled{19} + \textcircled{0} + \textcircled{0} \end{array}$$

You could solve the problem of the top number, the seven, being smaller than the bottom number, the eight, several ways. The way students are familiar with is to regroup the top number and move some numbers around to increase the size of the seven. In the example, I took a one from the thirty and gave it to the seven to make it eight.

B.

$$\begin{array}{r} \boxed{47} = \textcircled{30} + \textcircled{10} + \textcircled{7} \\ - \boxed{28} = \textcircled{-10} + \textcircled{-10} + \textcircled{-8} \\ \hline \boxed{19} = \textcircled{20} + \textcircled{0} + \textcircled{-1} \end{array}$$

Another way to solve the problem is to use an IOU or a negative number. It is not true that you can't take eight from seven. You can; you just get a negative number as an answer. In this case you get a minus one in the right hand row. *Write it in red.* In the end when you sum up your answers (bulldozer) you must pay back the debt. So  $20 - 1 = 19$ , which is the correct answer. See Diagram B.

Any subtraction problem can be solved using negative numbers. But you must watch the place value. Some students love this method, while others find it confusing.

Even larger subtraction problems can be done using negative numbers as long as one pays attention to the place value.

Example:

$$\begin{array}{r} 4 \ 3 \ 7 \\ - 1 \ 5 \ 9 \\ \hline 300 - 20 - 2 = 278 \text{ (Which is the correct answer.)} \end{array}$$

This method is often favored by students who like to do math in their heads.

If you get the answer 298 for this problem it would mean the student didn't remember that the minus two in the tens place is a twenty not a two. To prevent this error, I have students write answers as I have above. At first have them write negative numbers in red.

**Worksheets** Nonstandard Regrouping and IOUs Worksheets 2 - 5, pages 32 -35

**Test for Understanding** Nonstandard Regrouping and IOUs: Assessment - Show You Know - Worksheets 1 and 2, pages 36 and 37

Worksheet 1, problem 4. What was done wrong here? You can't subtract nine from two. The number on the top must be larger than the number on the bottom before you can subtract. One way to fix it is by making the two larger and making a thirty smaller by the same amount. Redo the subtractions and compute the correct answer of twenty-three.

$\begin{array}{r} \boxed{62} \\ - \boxed{39} \\ \hline \boxed{51} \end{array}$	$= (\boxed{30}) + (\boxed{30}) + (\boxed{2})$	$= (-\boxed{30}) + (-\boxed{0}) + (-\boxed{9})$	$\begin{array}{r} \boxed{62} \\ - \boxed{39} \\ \hline \boxed{23} \end{array}$	$= (\boxed{30}) + (\cancel{\boxed{30}}) + (\cancel{\boxed{2}})$	$= (-\boxed{30}) + (-\boxed{0}) + (-\boxed{9})$	$= (\boxed{0}) + (\boxed{23}) + (\boxed{0})$
--	---	---	--	---	---	--

Worksheet 1, problem 5. What is weird with this problem?

$$\begin{array}{r} 35 \\ - 49 \\ \hline \end{array}$$

A larger number can't be taken away from a smaller number. The subtrahend is larger than the minuend. Answers:

**Extension**

Give the students pairs of numbers like these:

$9 - 7 =$	and	$7 - 9 =$	$9 - 7 = 2$	and	$7 - 9 = -2$
$10 - 7 =$	and	$7 - 10 =$	$10 - 7 = 3$	and	$7 - 10 = -3$
$12 - 9 =$	and	$9 - 12 =$	$12 - 9 = 3$	and	$9 - 12 = -3$
$15 - 6 =$	and	$6 - 15 =$	$15 - 6 = 9$	and	$6 - 15 = -9$

## Standard Regrouping: Manipulative

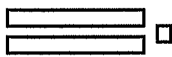
**Purpose** The purpose is to learn standard regrouping manipulatively with tens and ones blocks.

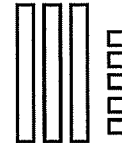
**Prerequisites** Tens and Ones Trading Game, Difference Between, Discovering Easier Ways, Tens and Ones: Manipulative, Nonstandard Regrouping, Nonstandard Regrouping and IOUs, and Covering as a Strategy (in Student Workbook, pages 38-40).

**Materials** Base Ten Blocks or Cuisenaire Rods (orange and white rods). This lesson is oral. Have a set of Base Ten Blocks on the table.

**Lesson Session 1**  
(Oral/  
Manipulative)

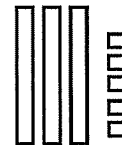
"Build thirty-five with the blocks. Put it on the table."  
"Take away twenty-one."

Student picks up. 



Repeat the process with forty-six take away fourteen. Have the student make up several problems.

If a student does not suggest one like this, give:



"Build thirty-five with the blocks. Put it on the table."  
"Take away nineteen."  
Give the student lots of time to think about this puzzle.

This last problem is a classic regrouping (or borrowing) problem. This last question "Build thirty-five and remove nineteen" is not easy. To do it requires an insight rarely obvious to the student on the first try.

Most students will say, "I can't. I don't have enough ones."

You say, "**It is possible, but the 'way' is a puzzle that you will have to figure out.**" Remain quiet or leave them to puzzle it out. Don't be too quick to help. Most will get it in a day or two if not sooner.

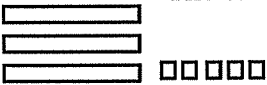
The insight she needs is to reverse the process that was used in the Trading Game. In that game tens were built with ones. Whenever there were enough ones to make a ten, it was traded up. Now, the trading is down, undoing a ten to obtain more ones. This is the main concept behind our system of subtraction. *This solution is not obvious to the beginner.*

You can tell a student to trade tens for one and get it done very quickly, but it will be forgotten many times and then drilled in before it is remembered. If the student figures it out for herself now as a puzzle, not only is it more fun, rewarding, and essential to understanding, but the student will remember the path to the solution, if not the solution itself. The solution becomes hers, not yours. A great deal of personal power comes from that ownership of solutions.

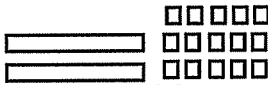
**Helping Out** If a student is having major difficulties figuring out the puzzle: Play the Trading Game, trading up and down. If the student really needs a hint, tell her to think about the game when thinking about the puzzle.

If after a few days the solution still has not been discovered, try this: Build thirty-five with three tens and five ones. **"Is this the only way to build thirty-five with these blocks? Try some other ways."**

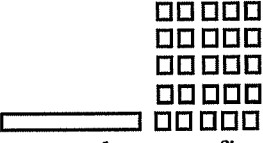
The student builds three models.



Three tens and five ones



Two tens and fifteen ones



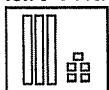
One ten and twenty-five ones

Compare all the ways to make thirty-five with tens and ones. **"Which ones will allow you to pick up nineteen?"** "Two tens and fifteen ones or one ten and twenty-five ones." **"Of these two, which is the easiest to use?"** "Two tens and fifteen ones because it is easier to count out." Then give another problem, such as,  $25 - 17$ . Let her work it out on her own. After she has solved the puzzle, describe to her what she did and do a few more problems together.

**Manipulative/ Standard Regrouping: Recording**  
Recording

**"Build thirty-five with Base Ten Blocks. Take away nineteen."** She trades in one ten for ten ones and removes the nineteen ones. **What is left?** "Sixteen." Give Standard Regrouping: Manipulative - Worksheet 1, page 42. **"Study the example and figure out how to record what you just did."**

Start With



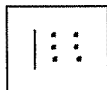
The first picture tells what to build. How many to Start With.

Take Away



The second picture tells how many to Take Away.

What's Left?



The student figures the trades, takes away the given amount, and draws a picture of what is left in the What's Left? box.

$$\begin{array}{r} 35 \\ - 19 \\ \hline 16 \end{array}$$

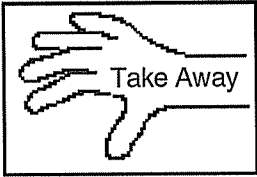
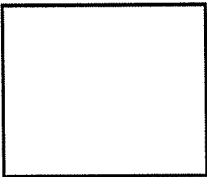
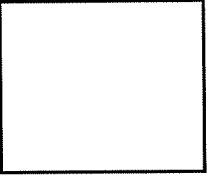
She records the pictures as numbers to the right of each problem. Have the student do another problem while you watch. If the student has difficulty, continue to work together until she is secure in what she is doing. Then have her complete the worksheet independently.

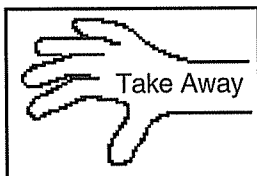
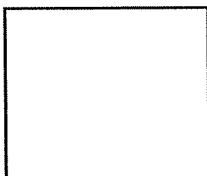
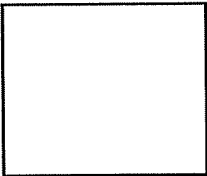
**Worksheets** Give Standard Regrouping: Manipulative - Worksheets 2 - 4, pages 43 - 45, over the next few days. Standard Regrouping: Manipulative - Practice is on the next page in this booklet; duplicate and write practice problems appropriate to the ability of each student and also have the student make up problems.

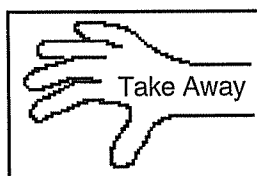


Students who have had major difficulties with this lesson should skip Standard Regrouping: Circle Math, and go on to Standard Regrouping: Short Way, page 49, or use Base Ten Subtraction.

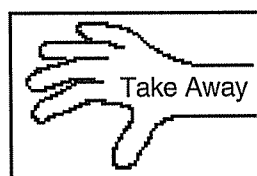


**Standard Regrouping: Manipulative**  
Practice - Make multiple copies.

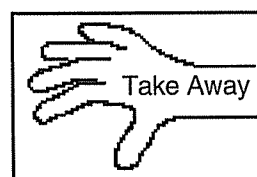
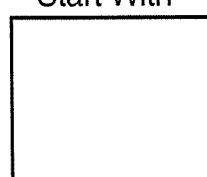

Write subtraction problems that use trading. Draw the blocks. Write in the Take Away number and write the answer under the line.

1. Start With  = What's Left? -  
  \_\_\_\_\_

2. Start With  = What's Left? -  
  \_\_\_\_\_

3. Start With  = What's Left? -  
  \_\_\_\_\_

4. Start With  = What's Left? -  
  \_\_\_\_\_

5. Start With  = What's Left? -  
  \_\_\_\_\_

6. Write a subtraction word problem that uses the numbers in problem 5.

## Standard Regrouping: Circle Math

**Purpose** The purpose is to learn standard regrouping using the constraint that the answer must come out in tens and ones.

**Prerequisites** Tens and Ones Trading Game, Circle Math Subtraction: Review, Difference Between, Discovering Easier Ways, Tens and Ones: Manipulative, Nonstandard Regrouping, Nonstandard Regrouping and IOUs, and Standard Regrouping: Manipulative.

**Materials** Standard Regrouping: Circle Math - Worksheet 1, page 46

**Lesson Session 1** The student breaks up  $47 - 39$  three different ways independently.

**Session 2** Look over the completed worksheet. Note the way the third problem was done. Did the student use nonstandard regrouping or did she generalize how to trade ten in the lesson Standard Regrouping: Manipulative? Many students do not experience the regrouping of tens as easier than nonstandard regrouping at this point. It is not until the student begins to use the short notation that its value as a technique becomes apparent. Using nonstandard regrouping is not a sign of lack of progress or understanding.

A. If the student used nonstandard regrouping, such as:

$$\begin{array}{r}
 \boxed{47} = \overset{38}{\cancel{40}} + \overset{+2}{\cancel{7}} \\
 - \boxed{39} = \overset{-30}{\cancel{30}} + \overset{-9}{\cancel{9}} \\
 \hline
 \boxed{8} = \overset{8}{\circ} + \overset{0}{\circ} \\
 \\
 \boxed{47} = \overset{40}{\circ} + \overset{7}{\circ} \\
 - \boxed{39} = \overset{-30}{\circ} + \overset{-9}{\circ} \\
 \hline
 \boxed{\phantom{47}} = \overset{\circ}{\phantom{40}} + \overset{\circ}{\phantom{7}}
 \end{array}$$

give the student a piece of paper and record the problem  $47 - 39$  as the student expanded it into tens and ones on her worksheet.

Give this constraint to the student: **"Solve this problem so the answer comes out in tens and ones."** The constraint applies only to the answer. She can change whatever she wants with the numbers above the line. Be sure she understands this.

Let her think and ponder as long as she needs. It may take several days or longer for her to figure it out. It may seem obvious since she has made the trades with blocks, but it is not obvious for many students. Occasionally a student who is able to generalize and think abstractly with ease figures it out quickly.

It is not a failure to not know the answer immediately. The opportunity to ponder and discover a solution to a difficult concept is the right of every learner. By pressuring to come up with quick answers constantly inhibits deep thinking and second thoughts.

The insight she needs is to trade or borrow one ten and give it to the ones column.

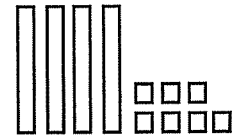
Nonstandard regrouping gives answers that are not in tens and ones. The old technique will not work under this new constraint.

A. If the student needs a little help, suggest using the Base Ten Blocks. Be sure she has had a good chance to reason it out first. Once she has figured it out, move on to recording.

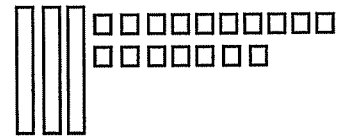
B. If the student is unable to generalize the concept of regrouping and completely "hits a blank" after being given the hint of using blocks, move on to recording.

### Session 3 Recording

Start with the problem  $47 - 39$ . Have the student use the Base Ten Blocks to build the problem. The blocks are four ten rods and seven one rods.



"Take away 39." In order to take away thirty-nine, one of the tens is traded in for ten ones. The ten ones are placed with the seven ones, which will now be a total of seventeen ones. Three tens are left.



*Stop at this point.* "Do you still have forty-seven?" "Yes." Some students will stop and count at this point. "Does the forty-seven look the same as it did before you traded it?" "No."

"How is it different?" Answers will vary, but the general idea is that, before, there were four tens and seven ones and now there are three tens and seventeen ones.

"On the paper you show the forty-seven as  $40 + 7$ . How could you record the change you just made?" Answers will vary: some students will erase the  $40 + 7$ ; some will cross out the  $40 + 7$  and write  $30 + 17$ ; others will rewrite the new expansion to the side of the first problem. All of these are acceptable.

"Now finish the problem." Thirty-nine can be taken away in any order, ones or tens first.

Count the blocks left and record it under the line.  $\square\square\square\square\square\square\square$

Tell the student there is an agreed upon way that people record this kind of problem. It is a conventional way of recording. The important part is that she figured out how it works and knows why the conventional way is done. Go through the problem and record step by step as you move through the steps.

Step 1	Step 2	Step 3	Step 4
$\boxed{47} = \boxed{40} + \boxed{7}$	$\boxed{47} = \overset{30}{\cancel{40}} + \boxed{7}$	$\boxed{47} = \overset{30}{\cancel{40}} + \overset{17}{\boxed{7}}$	$\boxed{47} = \overset{30}{\cancel{40}} + \overset{17}{\boxed{7}}$
$-\boxed{39} = \boxed{-30} + \boxed{-9}$	$-\boxed{39} = \boxed{-30} + \boxed{-9}$	$-\boxed{39} = \boxed{-30} + \boxed{-9}$	$-\boxed{39} = \boxed{-30} + \boxed{-9}$
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
$\square = \bigcirc + \bigcirc$	$\square = \bigcirc + \bigcirc$	$\square = \bigcirc + \bigcirc$	$\boxed{8} = \boxed{0} + \boxed{8}$

Solve another problem or two together and have the student record. Assist only if asked or if an error is made. Have the student write a subtraction problem that requires trading.



## Worksheets

Standard Regrouping: Circle Math - Worksheets 2 - 3, pages 47 and 48 Have Base Ten Blocks available for the students at all times. Eventually the process will become internalized and the student will stop using the blocks.

It is important that continued practice in the nonstandard Circle Math solutions are done along with the new tens and ones answers. At least once a week give Standard Regrouping: Circle Math Practice, page follows in this booklet; duplicate and write practice problems appropriate to the ability of each student. Also have the student make up problems. Students may moan and groan about this, but that is good because they have come to realize why the standard form is standard. It's much faster and easier. Finding faster and easier ways to solve problems is the work of mathematicians. Students do the work of mathematicians when they figure out faster and easier ways to solve problems. However, students often only use the new way and are in danger of forgetting that regrouping by tens is *a way*, not *the way* to do subtraction. Continued practice of the former way will remind them.

A few students may not think the standard way is better and will continue to use the nonstandard regrouping when given a choice. This is fine. They may even discover a faster and better way that works for them. The one-to-one transfer section may show them why the standard method is preferred by most people.

## Textbook

If you have a large class and/or are using a standard second or third grade textbook, you may find it useful to use the same set of problems several times over a period of a few weeks. The first time through, they do the problems the nonstandard way as they are doing in this lesson. The next time through, they do the problems using a tens and ones expansion with nonstandard regrouping. The third time through, they use the standard expanded form. The fourth time, they can do the problems the short, standard way. If you keep the work, the student can use the old work to help check the new work. This is good for the student and a time saver for you.

## Test for Understanding

1. Have the student make up one regrouping problem, such as  $26 - 18$ , and one problem that does not require regrouping, such as  $26 - 22$ . Have her prove both with Base Ten Blocks.

2. Ask her to articulate how the process of finding the answers is slightly different between a regrouping and a non-regrouping problem.

3. Give this problem:

**"How much was borrowed from the forty to increase the five?"** "Five."

**"How can you fix it so the answer comes out in tens and ones?"** "Trade a ten from the forty instead of five. There would be fifteen ones in the ones place and three tens in the tens place.  $30 - 20$  is 10 and  $15 - 7$  is 8, so the answer is  $10 + 8 = 18$ ."

$$\begin{array}{r} 35 \quad 10 \\ \boxed{45} = \cancel{40} + \cancel{5} \\ - \boxed{27} = \textcircled{-20} + \textcircled{-7} \\ \hline \boxed{18} = \textcircled{15} + \textcircled{3} \end{array}$$

$$\begin{array}{r} 30 \quad \textcircled{10} \\ \boxed{45} = \cancel{40} + \textcircled{5} \\ - \boxed{27} = \textcircled{-20} + \textcircled{-7} \\ \hline \boxed{18} = \textcircled{10} + \textcircled{8} \end{array}$$

**Standard Regrouping: Circle Math**  
Practice - Make multiple copies.

Date \_\_\_\_\_

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc \end{array}$$

## Standard Regrouping: Short Way

**Purpose** The purpose is to form a bridge from the standard Circle Math regrouping process to the standard place value notation, the short way notation.

**Prerequisites** Mastery of Standard Regrouping: Circle Math and regrouping in addition (see *Patterns in Arithmetic: Book 2* - Standard Regrouping: Short Way in the Addition section, Parent/Teacher Guide, pages 86 - 88, and Student Workbook, pages 166 -168).

**Materials** Standard Regrouping: Short Way - Worksheet 1, page 50  
Base Ten Blocks or Cuisenaire Rods (orange and white rods)

**Lesson Session 1** Give Standard Regrouping: Short Way - Worksheet 1. Have the student prove each problem with blocks. If he has difficulty with any of the problems, work on it until the difficulty has been remedied.

**"Look at problem 6. How is the expanded  $40 + 5$  shown in the short way notation?"** "45."

**"How are  $40 + 5$  and 45 the same? How are they different?"** "They both have the same value. The  $40 + 5$  has a zero and a '+' sign, which the 45 doesn't."

**"In the forty-five, what does the four mean?"** "Four tens, or forty." If the student doesn't know this, review place value notation before proceeding.

**"How can you tell that the four stands for four tens and not four ones or four hundreds?"** "Because it is in the second row or column (counting from the right). The second row stands for tens."

**"What does the first row stand for?"** "Ones."

**"Look at problem 1. What does the 'little one' over the three mean and why did you write it there?"**

"It stands for one ten. It was put there because you can't have more than nine in the ones place. I took a ten from the twelve and gave it to the tens place. That left two in the ones place."

$$\begin{array}{r} \text{Short way} \\ 1 \\ 35 \\ +27 \\ \hline 62 \end{array}$$

Now have the student look at problems 5. Challenge question: **"This is a challenge question. Can you rewrite this subtraction problem the short way and do the short way notation like you did with addition?"** A few students will be able to figure it out but most will not. It's important to give a student the opportunity first. Give the student time to think about it, but let the student know it's fine if he isn't able to do the short way without instructions first. If he does it correctly, give a few more similar problems.

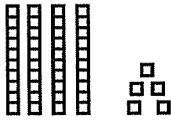
Many students will not be able to solve the challenge problem. Use Standard Regrouping: Short Way, page 49, and the one-to-one transfer process. This process is called one-to-one transfer. It helps the student form a bridge from the expanded form of the subtraction problem to the new, short notation. The student will first make a manipulation with the blocks. That move will be recorded on the expanded problems as has been done before. Then the same single step recording will be done

on the short notation problem.

Begin by having the student rewrite the problem of  $45 - 29$  on a piece of paper as shown below.

**Step 1**      **Build**

Build forty-five with the blocks.

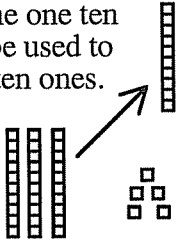


Expanded Notation	Short Notation
Write this on a piece of paper.	
$45 = 40 + 5$	$\begin{array}{r} 45 \\ - 29 \\ \hline \end{array}$
$- 29 = -20 + -9$	
$\square = \bigcirc + \bigcirc$	

When the student removes the ten to be traded, stop and record the change in the tens place.

**Step 2**

Pick up the one ten that will be used to trade for ten ones.

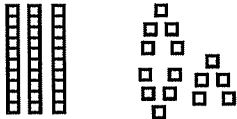


Expanded Notation	Short Notation
Write this on a piece of paper.	
$45 = 30 + 5$	$\begin{array}{r} 30 \\ 3 \\ \sqrt{4} 5 \\ - 29 \\ \hline \end{array}$
$- 29 = -20 + -9$	
$\square = \bigcirc + \bigcirc$	

When the student gets to short notation recording, ask, "**How many tens are left?**" "Three." "**How do you show three tens in the short notation?**" "You show it as a three in the second row (or column)." "**Now record that.**" Or, you cross out the four and write the three above it and ask, "**Why did I do that and what does the three mean?**"

**Step 3**

Trade the one ten for ten ones.

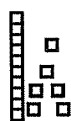


Expanded Notation	Short Notation
Write this on a piece of paper.	
$45 = 30 + 5$	$\begin{array}{r} 30 \\ 3 \\ \sqrt{4} 5 \\ - 29 \\ \hline \end{array}$
$- 29 = -20 + -9$	
$\square = \bigcirc + \bigcirc$	

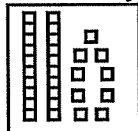
"What does the 'little one' next to the five stand for?" "The ten that was given to the five. It was taken from the forty."

#### Step 4

Take the twenty-nine away. Count up what's left.



Take away.



Expanded Notation

Short Notation

$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$	$\begin{array}{r} 30 \\ + 10 \\ \hline 40 \\ + 5 \\ \hline 45 \end{array}$	$\begin{array}{r} 3 \\ \downarrow \\ 45 \\ - 29 \\ \hline 16 \end{array}$
$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$	$\begin{array}{r} 40 \\ + 5 \\ \hline 45 \\ - 20 \\ \hline 20 \\ - 9 \\ \hline 11 \end{array}$	$\begin{array}{r} 3 \\ \downarrow \\ 45 \\ - 29 \\ \hline 16 \end{array}$
$\begin{array}{r} 16 \\ = 10 + 6 \end{array}$	$\begin{array}{r} 10 \\ + 6 \\ \hline 16 \end{array}$	$\begin{array}{r} 3 \\ \downarrow \\ 45 \\ - 29 \\ \hline 16 \end{array}$

When the student finishes, ask, "How many tens are left? How many ones are left?" Repeat this process several times with different numbers. Have the student explain what he is doing at each step.

### Worksheet

When the student is competent, give Standard Regrouping: Short Way - Worksheet 2, page 51.

### Troubleshooter

These are the most common recording errors when learning the short notation:

A.

$$\begin{array}{r} 30 \\ \downarrow \\ 45 \\ - 29 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 3 \\ \downarrow \\ 4105 \\ - 29 \\ \hline \end{array}$$

If the student does either of these, talk about the difference between the expanded notation and the short notation.

Write a sequence, such as, 11, 21, 31, 41, 51, ..., 91 and, 101.

"Each time you put a new number into the tens place, what does the number stand for?" "One ten, two tens, three tens, etc. When you get to 101 it is ten tens, or one hundred." Or refer to the place value recording work in the General Math booklet and ask similar questions.

Example A: "Look at how you recorded the trading. What does a thirty written in the tens place mean?" "Thirty tens." "Did you intend to say there are thirty tens in the tens place? Do you have thirty tens?" "No." "How can you change the recording?" "Write 3 instead of 30." Example B: "Read this number. "One hundred and five." "Did you intend to say there are one hundred and five ones in the ones place?" "No, there are fifteen ones." "How can you change the recording?" "Write a one instead of a ten."

Other questions to ask: "In the short notation, the value of the number is determined by what?" "Its place, which column it is in." "What in the expanded

**notation determines the value of the number?"** "How many zeros are after it." This can be confusing at first. It takes practice and time to learn to use the new notation. Do lots of practice with the transfer process using the new form and the blocks right alongside. Drilling a memorized process risks instilling a disconnected procedure that won't make sense to the student.

## **Worksheets**

Standard Regrouping: Short Way - Worksheets 3 - 5, pages 52 - 54

The student should continue to do problems using the transfer process until he is fluent. Most students will need to do from twenty to fifty problems to master the short notation. The completed Standard Regrouping: Circle Math - Worksheets 3 and 4 can be used. Have the student record each problem using the short way notation next to the already completed problems.

Continue giving expansion practice using Standard Regrouping: Short Way - Practice on the following page with problem sizes gradually increasing according to the student's ability.

Have the student write his own problems for drill practice and word problems to go along with the problems. Continue word problems such as the ones in Gus the Bus Driver. When the student is ready, the blocks and the expanded form can be dropped. Next work on speed and accuracy with two digit regrouping and non-regrouping problems. There are many commercial products on the market for this type of practice. Use blocks to check any problems done incorrectly. A goal is to have the student attempt to find and correct errors independently.

## **Test for Understanding**

Standard Regrouping: Short Way - Assessment - Test for Understanding, page 55

**Standard Regrouping: Short Way**  
**Practice - Make multiple copies.**

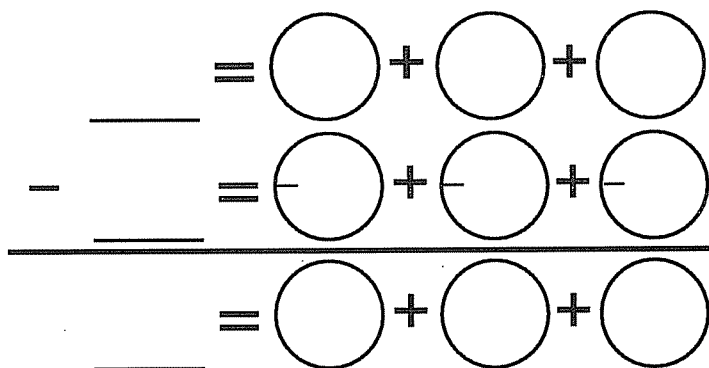
Date \_\_\_\_\_

Use blocks to help you solve this problem.  
 Write a problem that uses trading.

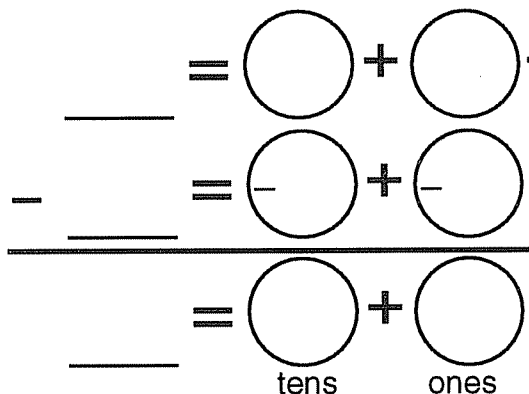
My problem for today is \_\_\_\_\_ - \_\_\_\_\_ = \_\_\_\_\_



Break this one up not using tens and ones.



Break this one up into tens and ones, please.



Now, record it the short way.



Write a subtraction word problem that uses your number sentence.

## Hundreds, Tens, and Ones: Manipulative

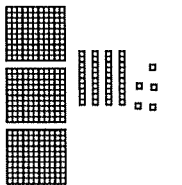
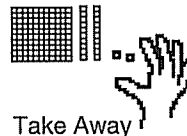
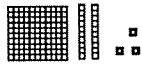
**Purpose** The purpose is to develop the physical model for standard subtraction procedure with three digit numbers. To understand multidigit numbers in terms of place value.

**Prerequisites** Mastery of Standard Regrouping: Short Way

**Materials** Hundreds, Tens, and Ones: Manipulative - Worksheets 1 to 3, pages 58 - 60  
Base Ten Blocks

**Lesson Session 1** Give Hundreds, Tens, and Ones: Manipulative - Worksheet 1, page 58. It reviews two digit subtraction. If he has any problems, review standard regrouping with two digits until the student is fluent and confident.

**Worksheets** Give Hundreds, Tens, and Ones: Manipulative - Worksheets 2 and 3, pages 59 and 60. The student builds three hundred forty-five with Base Ten Blocks, takes away one hundred twenty-two, and counts what's left. The difference is drawn in the What's Left? box. This problem requires no regrouping. On Worksheet 2 record this problem using the expanded form and the short form.

<p>Start With</p> 	<p>Take Away</p> 	<p>What's Left?</p> 	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; padding-right: 20px;">Expanded way</th> <th style="text-align: left;">Short way</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;"><math>345 = 300 + 40 + 5</math></td> <td style="padding: 5px;"><math>345</math></td> </tr> <tr> <td style="padding: 5px;"><math>- 122 = -100 + -20 + -2</math></td> <td style="padding: 5px;"><math>- 122</math></td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 5px;"><math>123 = 100 + 20 + 3</math></td> <td style="padding: 5px;"><math>123</math></td> </tr> </tbody> </table>	Expanded way	Short way	$345 = 300 + 40 + 5$	$345$	$- 122 = -100 + -20 + -2$	$- 122$	$123 = 100 + 20 + 3$	$123$
Expanded way	Short way										
$345 = 300 + 40 + 5$	$345$										
$- 122 = -100 + -20 + -2$	$- 122$										
$123 = 100 + 20 + 3$	$123$										

Have the student complete the page independently.

**Worksheet** Give Hundreds, Tens, and Ones: Manipulative - Worksheet 4, page 61, for independent work. Have the student check the first five problems with Base Ten Blocks.

**Extension** *Hands on Base Ten Blocks: Book 3* has problems using hundreds blocks for additional practice.



## Three Digits Regrouping: Manipulative

<b>Purpose</b>	The purpose is to develop the physical model for standard subtraction procedure with three digit numbers with regrouping in one place.
<b>Prerequisites</b>	Hundreds, Tens, and Ones: Manipulative
<b>Materials</b>	Three Digits Regrouping: Manipulative - Worksheets 1 and 2, pages 62 and 63 Base Ten Blocks
<b>Lesson</b>	<p>Give the problem <math>249 - 163</math>. Have the student build and solve it with Base Ten Blocks. This problem requires the student to trade a one hundred block for ten tens. Allow the student to discover this. If the student doesn't immediately know what to do, give him time to ponder it.</p> <p>If he is completely stumped, ask, "<b>What is causing your confusion?</b>" "I can't take away six tens because I have only four tens." "<b>How did you solve it when this happened in the ones place, for example, in the problem <math>34 - 19</math>?</b>" "I took more ones from the tens place." "<b>Where could you get more tens?</b>"</p> <p>If this doesn't help, stop the lesson and have the student think about it some more. The next day play the trading game starting with two hundreds blocks and three dice. Trade down.</p>
<b>Worksheet</b>	For students who successfully solve the first problem, give Three Digits Regrouping: Manipulative - Worksheets 1 and 2 and the Base Ten Blocks to use. Problems 1 - 4 and problem 6 require trading one hundred in for ten tens. Problem 5 uses the more familiar trading one ten for ten ones. There is a Regrouping Review: Hundreds, Tens and Ones on the following page.
<b>Time Span</b>	The student should do problems like this with the blocks, with no other recording other than drawing the answer in pictures or writing numbers in the answer for a week or two. Some students will need to practice this until the end of the year. Since each person proceeds at his own pace, this is fine. Continue with the recording of the process next year. Use lessons in General Math while the student is practicing these problems.

**Regrouping Review: Hundreds, Tens and Ones**  
 Practice - Make multiple copies.

Date \_\_\_\_\_

My problem today is:  $\square - \square$

Use this space for your scratch work.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} + \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

Break your numbers into hundreds, tens, and ones. Then regroup to solve.

$$\begin{array}{l} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = \overset{-}{\bigcirc} + \overset{-}{\bigcirc} + \overset{-}{\bigcirc} \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

Do all your answers agree?

## Three Digits Regrouping: Recording

**Purpose** The purpose is to extend and to record the physical model for regrouping standard subtraction with three digit numbers within one place.

**Prerequisites** Three Digits Regrouping: Manipulative, Difference Between, Sums and Differences, and Rounding Off in the General Math book

**Materials** Three Digits Regrouping: Recording - Example, page 64, Worksheet 1, page 65  
Base Ten Blocks

**Lesson Session 1** Give the problem  $347 - 156$  (use Worksheet 1). Have the student build and solve it with Base Ten Blocks. **"Write the problem in expanded form (Circle Math) and short way."** If the student is hesitant or asks for help with the recording, stop. Do not proceed with this session. Review the previous session before going on. If no problems arise, ask the student to try to record the problem using short notation. If assistance is needed, have him write a problem such as  $45 - 28$ . Ask him to do it the expanded way and the short way. Then have him apply what he knows from the two digit problem to the three digit problem. For a student who is unable to record the problem the short way on his own, use a one-to-one transfer through the recording process step by step as outlined on Three Digits Regrouping: Recording - Example, page 64, and the next page in this guide. Repeat this process with the rest of the problems on Worksheet 1.

**Session 2** Give the student a problem such as  $459 - 187$  in which the trading is between the hundreds and the tens. **"Predict what the answer will be. Can you get within one hundred of the actual answer? Think about what you think about when you make a guess."**

Write this:  $539 - 285$   
900 550 250 100 50 10

**"Which of these estimated answers to the problem seem the most reasonable?"**  
Have the student explain the reason for choosing the one he chooses.

**"Which answers seem too large to be reasonable guesses? Can you end up with more than you started with in a subtraction problem?"**

**"Which answers seem too small to be reasonable guesses?"** This is where the concept of Difference Between helps. The difference between 539 and 285 is at least one hundred; all guesses below that are unreasonable. Have the student calculate the answer to  $539 - 285$  using the expanded notation, with or without the blocks.

**Worksheets** Three Digits Regrouping: Recording - Worksheets 2 - 4, pages 66 - 68

**Note** Discuss the strategies students use to make their estimates. Downplay getting the "right" answers on the estimating part of these worksheets. This is more of an activity to play around with guessing. Let the student know that it is fine to guess wrong. If you traditionally grade your students' work, note that learning to estimate will be more effective if no grade is given.

Example:

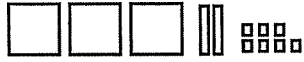
**Build**

**Expanded Notation**

**Short Notation**

**Step 1**

Build 327 with the blocks.



Write this on a piece of paper.

$$\begin{array}{r} (300) + (20) + (7) \\ - (-100) + (-40) + (-5) \\ \hline \phantom{(} + \phantom{(} + \phantom{(} \end{array}$$

$$\begin{array}{r} 327 \\ - 145 \\ \hline \end{array}$$

**Step 2** (Some students will wait and combine this step with Step 5.)

Take away the five ones from the 7 ones.



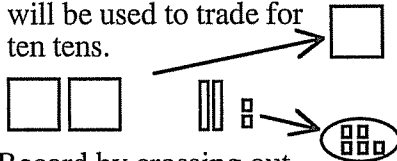
Record what's left of the 7 ones.

$$\begin{array}{r} (300) + (20) + (7) \\ - (-100) + (-40) + (-5) \\ \hline \phantom{(} + \phantom{(} + (2) \end{array}$$

$$\begin{array}{r} 327 \\ - 145 \\ \hline 2 \end{array}$$

**Step 3**

Pick up one hundred that will be used to trade for ten tens.



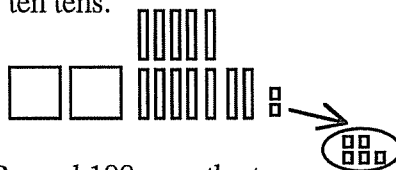
Record by crossing out the 300 and writing 200.

$$\begin{array}{r} 200 \\ \cancel{(300)} + (20) + (7) \\ - (-100) + (-40) + (-5) \\ \hline \phantom{(} + \phantom{(} + (2) \end{array}$$

$$\begin{array}{r} 2 \\ \cancel{3}27 \\ - 145 \\ \hline 2 \end{array}$$

**Step 4**

Trade the one hundred for ten tens.



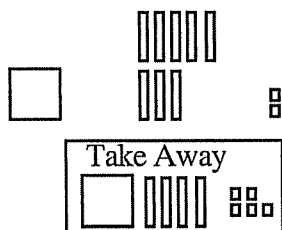
Record 100 over the tens circle.

$$\begin{array}{r} 200 \quad (100) \\ \cancel{(300)} + (20) + (7) \\ - (-100) + (-40) + (-5) \\ \hline \phantom{(} + \phantom{(} + (2) \end{array}$$

$$\begin{array}{r} 2 \\ \cancel{3}127 \\ - 145 \\ \hline 2 \end{array}$$

**Step 5**

Take away the four tens and the one hundred. Count what's left.



$$\begin{array}{r} 200 \quad (100) \\ \cancel{(300)} + (20) + (7) \\ - (-100) + (-40) + (-5) \\ \hline (100) + (80) + (2) \end{array}$$

$$\begin{array}{r} 2 \\ \cancel{3}127 \\ - 145 \\ \hline 182 \end{array}$$

**Test for Understanding**

1. Show the student these two problems:

A.	$\begin{array}{r} 3 \\ 6\cancel{4}5 \\ - 518 \\ \hline 127 \end{array}$
----	---

B.	$\begin{array}{r} 3 \\ \cancel{4}58 \\ - 181 \\ \hline 277 \end{array}$
----	---

**"How is the little one next to the five in problem A different from the little one next to the five in problem B?"** "The little one next to the five in problem A stands for one ten traded for ten ones. In problem B, it stands for one hundred traded in for ten tens. The three above the four in problem A stands for three tens or thirty, while in problem B it stands for three hundreds."

2. **"How much larger is the answer to problem B than the answer to problem A?"** "The answer to problem B is exactly one hundred fifty larger than the answer to problem A."

3. **"Two of these problems have the wrong answer. How can you tell which two are wrong just by looking?"** "B and C."

A.	B.	C.	D.
$\begin{array}{r} 247 \\ - 184 \\ \hline 63 \end{array}$	$\begin{array}{r} 358 \\ - 196 \\ \hline 554 \end{array}$	$\begin{array}{r} 608 \\ - 195 \\ \hline 163 \end{array}$	$\begin{array}{r} 714 \\ - 292 \\ \hline 422 \end{array}$

"Problem B can be identified as wrong because you can't subtract a number from another number and come out with more than you started with. The answer is too large to be reasonable. Problem C can be identified as wrong because the difference between six hundred eight and one hundred ninety-five is much more than one hundred sixty-three. That answer is too small to be reasonable."

**Worksheet**

Three Digits Regrouping: Recording - Assessment - Worksheet 5, Show You Know, page 69.

**Session 3 Game**

Play this game before giving Three Digits Regrouping: Recording - Worksheet 6, page 70. For a group of four to six students: Have one student write on a slip of paper a three digit subtraction problem that uses trading (regrouping). The other students write a three digit number on a small piece of paper and put it in the center of the table. The students look at the problem and all the numbers on the papers and guess which number is closest to the answer to the problem. The teacher should solve the problems for verification, or the student who wrote the problem can do it with blocks or a calculator. In a tutorial setting, the student can write four or five three digit numbers on a paper and then write a problem. The student then follows the same procedure outlined above.

**Worksheet**

Once the student is fluent, give Three Digits Regrouping: Recording - Worksheet 7, page 71. Some students will need to use the blocks. When the student is ready, the blocks will be dropped. Copy and use Regrouping Review: Hundreds, Tens, and Ones from the previous lesson, page 36, in this guide.

**Time Span**

Have the student continue to do the expanded form along with the short form until he is fluent with the short form. Then, he should continue to practice these types of problems, using the short form, to develop speed and accuracy. Anytime the student makes an error, the problem should be done with blocks to find and correct the error.

# Patterns in Arithmetic

## Subtraction: Booklet 3

### General Principles of Regrouping

## Answer Key

### for the

## Student Workbook

By Suki Glenn and Susan Carpenter

#### Answer Key Legend

AWV = answer(s) will vary  
BUWV = break up will vary  
OWV = order will vary

#### Pattern Blocks

r = red trapezoid  
g = green triangle  
y = yellow hexagon  
o = orange square  
b = blue parallelogram  
t = tan rhombus

#### Cuisenaire Rods

1 w = white  
2 r = red  
3 lg = light green  
4 p = purple  
5 y = yellow  
6 dg = dark green  
7 bk = black  
8 bn = brown  
9 bl = blue  
10 o = orange

Note: Some items and pages are left out of the Answer Key.

- 1) Some pages in which the answers are open-ended or will vary.
- 2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
- 3) Blank practice pages
- 4) Workboards
- 5) Games
- 6) Self correcting pages
- 7) Instructions only pages

Patterns in Arithmetic: Subtraction - Booklet 3  
Answer Key for the Student Workbook  
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# Subtraction - Booklet 3

## Speed Test

$$\begin{array}{r} 7 \\ -3 \\ \hline 4 \end{array} \quad \begin{array}{r} 8 \\ -8 \\ \hline 0 \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline 1 \end{array} \quad \begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array} \quad \begin{array}{r} 8 \\ -7 \\ \hline 1 \end{array} \quad \begin{array}{r} 7 \\ -1 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 2 \\ -2 \\ \hline 0 \end{array} \quad \begin{array}{r} 8 \\ -0 \\ \hline 8 \end{array} \quad \begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array} \quad \begin{array}{r} 8 \\ -4 \\ \hline 4 \end{array} \quad \begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array} \quad \begin{array}{r} 8 \\ -8 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 8 \\ -2 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array} \quad \begin{array}{r} 9 \\ -3 \\ \hline 6 \end{array} \quad \begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array} \quad \begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array} \quad \begin{array}{r} 9 \\ -7 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 4 \\ -0 \\ \hline 4 \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array} \quad \begin{array}{r} 7 \\ -0 \\ \hline 7 \end{array} \quad \begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline 1 \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array} \quad \begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array} \quad \begin{array}{r} 6 \\ -5 \\ \hline 1 \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array} \quad \begin{array}{r} 6 \\ -2 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 8 \\ -4 \\ \hline 4 \end{array} \quad \begin{array}{r} 7 \\ -3 \\ \hline 4 \end{array} \quad \begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array} \quad \begin{array}{r} 9 \\ -8 \\ \hline 1 \end{array} \quad \begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array} \quad \begin{array}{r} 7 \\ -4 \\ \hline 3 \end{array}$$

3

## Circle Math Subtraction: Review - Worksheet 3

BUWV

$$\begin{array}{r} 15 = \bigcirc + \bigcirc + \bigcirc \\ -12 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 3 = \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 14 = \bigcirc + \bigcirc + \bigcirc \\ -7 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 7 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 13 = \bigcirc + \bigcirc + \bigcirc \\ -13 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 0 = \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 16 = \bigcirc + \bigcirc + \bigcirc \\ -11 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 5 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 17 = \bigcirc + \bigcirc + \bigcirc \\ -2 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 15 = \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 17 = \bigcirc + \bigcirc + \bigcirc \\ -12 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 5 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 19 = \bigcirc + \bigcirc + \bigcirc \\ -19 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 0 = \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 16 = \bigcirc + \bigcirc + \bigcirc \\ -5 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 11 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

6

Pre-Assessments are on the last page.

BUWV

## Circle Math Subtraction: Review - Worksheet 1

$$\begin{array}{r} 9 = \bigcirc + \bigcirc \\ -3 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 6 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 10 = \bigcirc + \bigcirc \\ -0 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 10 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 17 = \bigcirc + \bigcirc \\ -6 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 11 = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 6 = \bigcirc + \bigcirc \\ 10 = \bigcirc + \bigcirc \\ 11 = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 16 = \bigcirc + \bigcirc \\ -5 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 11 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 19 = \bigcirc + \bigcirc \\ -7 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 12 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 8 = \bigcirc + \bigcirc \\ -5 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 3 = \bigcirc + \bigcirc \end{array}$$

4

## Circle Math Subtraction: Review - Worksheet 2

Break up each problem a different way.

BUWV

$$\begin{array}{r} 18 = \bigcirc + \bigcirc \\ -6 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 12 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 18 = \bigcirc + \bigcirc \\ -6 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 12 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 18 = \bigcirc + \bigcirc \\ -6 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 12 = \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 25 = \bigcirc + \bigcirc \\ -15 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 10 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 25 = \bigcirc + \bigcirc \\ -15 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 10 = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 25 = \bigcirc + \bigcirc \\ -15 = \bigcirc - \bigcirc + \bigcirc - \bigcirc \\ \hline 10 = \bigcirc + \bigcirc \end{array}$$

5

## Fixing Problems - Test for Understanding

$$\begin{array}{r} 19 = \overset{11}{\cancel{15}} + \overset{6}{\cancel{2}} + 2 \\ -9 = \overset{1}{\cancel{2}} + \overset{6}{\cancel{-6}} + \overset{-2}{\cancel{-2}} \\ \hline 10 = 10 + 0 + 0 \end{array}$$

Fix this problem.  
Show your work.  
The problem is broken up incorrectly

$$\begin{array}{r} 19 = 8 + 6 + \overset{6}{\cancel{5}} \\ -9 = \overset{-2}{\cancel{-2}} + \overset{-1}{\cancel{-1}} + \overset{-6}{\cancel{-6}} \\ \hline 10 = \overset{6}{\cancel{5}} + 4 + \overset{0}{\cancel{0}} \end{array}$$

Fix this problem.  
Show your work.

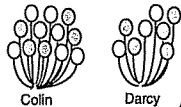
Do the answers to these two problems match? yes

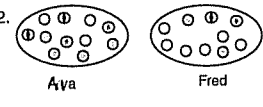
Should they? yes

Why? The problems are the same, only the expansions are different.



7

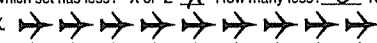
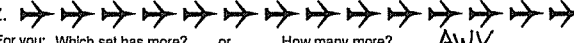
## Difference Between

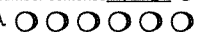
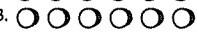
- 

Who has more balloons? Colin  
 How many more? 4  
 Number sentence  $12 - 8 = 4$
- 

Who has more marbles? Ava  
 How many more? 1  
 Number sentence  $11 - 10 = 1$
- Which set has more? S or T S How many more? 2 Number sentence  $13 - 11 = 2$

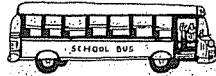
S.   
 T. 
- Which set has less? X or Z X How many less? 5 Number sentence  $14 - 9 = 5$

X.   
 Z. 
- For you: Which set has more?     or     How many more? AWV  
 Number sentence
- Which set has more? A or B neither How many more? 0  
 Number sentence  $6 - 6 = 0$

A.   
 B. 
- How many more is 18 than 6? 12 What is the difference? 12  
 Number sentence  $18 - 6 = 12$
- How many more is 60 than 40? 20 Number sentence  $60 - 40 = 20$   
 17 than 4? 13 Number sentence  $17 - 4 = 13$   
 50 than 27? 23 Number sentence  $50 - 27 = 23$

8

## Gus the Bus Driver



Gus drives a bus down Main Street every day. He drives up the street from First Street to Ninth Street. Then he drives back down again. Gus' bus does not have a back door. First people get off and then people get on. Gus keeps a log at each stop. Help Gus do his math. Use counters if you need to.

At First Street: The bus was empty then 19 people got on. 
$$\begin{array}{r} 19 \\ +27 \\ \hline 46 \end{array}$$

At Second Street: Nobody got off and 27 more people got on. 
$$\begin{array}{r} 46 \\ +15 \\ \hline 61 \end{array}$$

At Third Street: 15 people got off, then 25 got on. 
$$\begin{array}{r} 61 \\ -15 \\ \hline 46 \\ +25 \\ \hline 71 \end{array}$$

How many people are on the bus now? 56 people

Show your work or explain how you got the answer

At Fourth Street: 16 people got off, then 18 got on.

$$56 - 16 = 40 \quad 40 + 18 = 58$$

At Fifth Street: 21 people got off then Gus forgot to count how many people got on.  $58 - 21 = 37$

So, he counted all the people on the bus. He had 50 people on the bus.

How many people got on at Fifth Street? 13 
$$\begin{array}{r} 50 \\ -37 \\ \hline 13 \end{array}$$
  
 Explain how you got your answer

Now you fill in Gus's Log.

When Gus pulled away from the curb at Sixth Street he had 58 people on the bus. Some people got off and some got on. You write in the numbers of what could have happened. AWV

At Sixth Street     people got off and     people got on.

Optional Challenge:

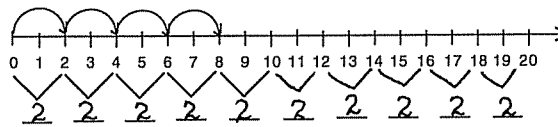
At what point did Gus have the most people on the bus? Fourth and Sixth

10

## Difference Between: Patterns



- Skip count by twos. Continue the pattern.  
 Record the numbers. 0 2 4 6 8 10 12 14 16 18 20



Record the difference between each number and the next one.
- Complete the sequence and solve for the difference between each number.  
 The difference between: 0, 3, 6, 9, 12, 15, 18, 21, 24, 27  
3 3 3 3 3 3 3 3 3 3  
 How did you get it? Subtraction or skip counting
- Complete the sequence and solve for the difference between each number.  
 The difference between: 40, 35, 30, 25, 20, 15, 10, 5, 0  
5 5 5 5 5 5 5 5 5
- Complete the sequence and solve for the difference between each number.  
 The difference between: 0, 2, 6, 12, 20, 30, 42, 56  
2 4 6 8 10 12 14
- Complete the sequence and solve for the difference between each number.  
 The difference between: 0, 4, 8, 12, 16, 20  
4 4 4 4 4 4

9

## Discovering Easier Ways - Worksheet 1

BUWV Example:

Use this space for your scratch

$$35 = 20 + 10 + 5$$

$$-18 = -10 - 3 - 5$$

$$17 = 10 + 7 + 0$$

2. 
$$47 = \bigcirc + \bigcirc + \bigcirc + \bigcirc$$

$$-29 = -\bigcirc - \bigcirc - \bigcirc - \bigcirc$$

$$18 = \bigcirc + \bigcirc + \bigcirc + \bigcirc$$

3. 
$$47 = \bigcirc + \bigcirc + \bigcirc + \bigcirc$$

$$-29 = -\bigcirc - \bigcirc - \bigcirc - \bigcirc$$

$$18 = \bigcirc + \bigcirc + \bigcirc + \bigcirc$$

Do the answers to problems 2 and 3 match? yes

What does it mean if they don't?

The problems were solved incorrectly.

11



### Discovering Easier Ways - Worksheet 2

There are harder ways to break up numbers and easier ways.

$$57 = 15 + 13 + 20 + 9$$

$$57 = 30 + 20 + 7$$

Using the second way is easier to break up.

Some ideas.

1.	2.
$\boxed{67} = \textcircled{30} + \textcircled{30} + \textcircled{7}$	$\boxed{67} = \textcircled{50} + \textcircled{10} + \textcircled{7}$
$-\boxed{25} = \textcircled{-10} + \textcircled{-10} + \textcircled{-5}$	$-\boxed{25} = \textcircled{-20} + \textcircled{-0} + \textcircled{-5}$
<hr/>	<hr/>
$\boxed{42} = \textcircled{20} + \textcircled{20} + \textcircled{2}$	$\boxed{42} = \textcircled{30} + \textcircled{10} + \textcircled{2}$

3.	4.
$\boxed{45} = \textcircled{15} + \textcircled{10} + \textcircled{10} + \textcircled{10}$	$\boxed{45} = \overset{30}{\textcircled{40}} + \overset{+10}{\textcircled{5}}$
$-\boxed{27} = \textcircled{-0} + \textcircled{-10} + \textcircled{-10} + \textcircled{-7}$	$-\boxed{27} = \textcircled{-20} + \textcircled{-7}$
<hr/>	<hr/>
$\boxed{18} = \textcircled{15} + \textcircled{0} + \textcircled{0} + \textcircled{3}$	$\boxed{18} = \textcircled{10} + \textcircled{8}$

12

### Discovering Easier Ways - Worksheet 4

Fill all circles. No single zeros.

1.	2.
$\boxed{80} = \textcircled{20} + \textcircled{50} + \textcircled{10}$	$\boxed{70} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{20} = \textcircled{-10} + \textcircled{-5} + \textcircled{-5}$	$-\boxed{30} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{60} = \textcircled{10} + \textcircled{45} + \textcircled{5}$	$\boxed{40} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
3.	4.
$\boxed{90} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	<small>Solve it differently than 3.</small> $\boxed{90} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{50} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{50} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{40} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{40} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
5.	6.
$\boxed{60} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{60} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{40} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{40} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{20} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{20} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
7.	8.
$\boxed{800} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$	<small>Solve it differently than 7.</small> $\boxed{800} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$
$-\boxed{400} = \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}}$	$-\boxed{400} = \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}}$
<hr/>	<hr/>
$\boxed{400} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$	$\boxed{400} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$

14

### Discovering Easier Ways - Worksheet 3

**BUWV**

Use this space for your scratch work.

1.	
$\boxed{58} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	
$-\boxed{26} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	
<hr/>	
$\boxed{32} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	
2.	3. Solve it differently than 2.
$\boxed{67} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{67} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{32} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{32} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{35} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{35} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
4.	5. Solve it differently than 4.
$\boxed{56} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{56} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{43} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{43} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{13} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{13} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$

13

### Discovering Easier Ways - Worksheet 5

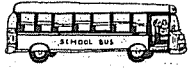
1.	
$\boxed{67} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{67} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{24} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{24} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{43} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{43} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
2.	
$\boxed{56} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{56} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$
$-\boxed{23} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$	$-\boxed{23} = \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}} + \textcircled{-\phantom{00}}$
<hr/>	<hr/>
$\boxed{33} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$	$\boxed{33} = \textcircled{\phantom{00}} + \textcircled{\phantom{00}} + \textcircled{\phantom{00}}$

Challenge!

3.
$\boxed{118} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$
$-\boxed{72} = \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}} + \textcircled{-\phantom{000}}$
<hr/>
$\boxed{46} = \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}} + \textcircled{\phantom{000}}$

15

## Gus the Bus Driver: Bus Stop Problems Worksheet 1



Gus drives a bus down Main Street every day. He drives up the street from First Street to Ninth Street. Then he drives back down again. Gus's bus does not have a back door. First people get on and then people get off.

Gus keeps a log at each stop. Help Gus do his math. Use counters if you need to.

At First Street: The bus was empty, then 31 people got on.

At Second Street: Nobody got off and 29 more people got on. Write the number sentence that shows how many people are on the bus now.

$$31 + 29 = 60 \text{ people}$$

At Third Street: 18 people got off, then 38 got on.

How many people are on the bus now? 80 people  
Show your work below or explain how you got the answer

$$\begin{array}{r} 60 \\ -18 \\ \hline 42 \\ +38 \\ \hline 80 \end{array}$$

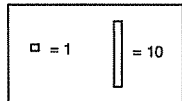
At Fourth Street: 14 people got off, then 18 got on.

How many people are on the bus now? 84  
Show your work below or explain how you got the answer

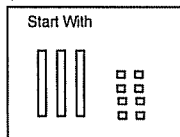
$$\begin{array}{r} 80 \\ -14 \\ \hline 66 \\ +18 \\ \hline 84 \end{array}$$

16

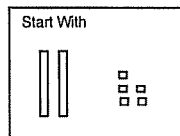
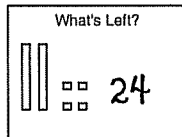
## Tens and Ones: Manipulative - Worksheet 1



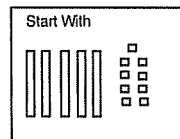
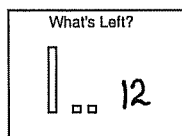
Build with Base Ten Blocks.



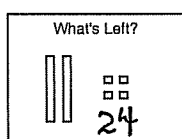
Take Away



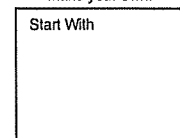
Take Away



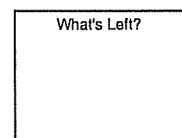
Take Away



Make your own.



Take Away



18

## Gus the Bus Driver: Bus Stop Problems Worksheet 2



At Fifth Street: 21 people got off, then Gus forgot to count how many people got on.

So, he counted all the people on the bus. He had 75 people on the bus.

How many people got on at Fifth Street? 12 people  
Show your work below or explain how you got the answer

$$\begin{array}{r} 84 \\ -21 \\ \hline 63 \\ +12 \\ \hline 75 \end{array}$$

Now you fill in Gus's Log.

When Gus pulled away from the curb at Sixth Street he had 81 people on the bus. Some people got off and some got on. You write in the numbers of what could have happened.

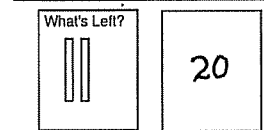
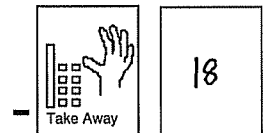
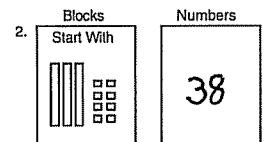
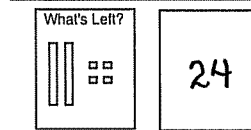
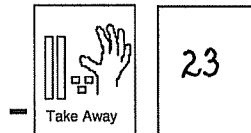
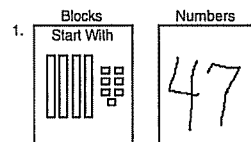
AWV

At Sixth Street \_\_\_\_\_ people got off and \_\_\_\_\_ people got on.

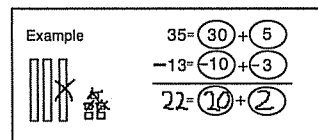
At what point did Gus have the most people on the bus? Fourth Street

17

## Tens and Ones: Manipulative - Worksheet 2

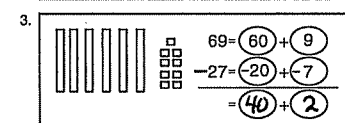


This time cross out what is taken away. Then write in the numbers.



You make up one now.

AWV




19


### Tens and Ones: Manipulative - Worksheet 3

Cross out what is taken away.  
Write in all the numbers.


Solve these problems.

4.   $85 = 80 + 5$   
 $-51 = 50 + 1$   
 $34 = 30 + 4$

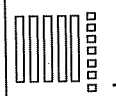
9.  $62 = 60 + 2$   
 $-42 = 40 + 2$   
 $= 20 + 0$

5.   $32 = 30 + 2$   
 $-22 = 20 + 2$   
 $10 = 10 + 0$


10.  $78 = 70 + 8$   
 $-53 = 50 + 3$   
 $25 = 20 + 5$

6.   $45 = 40 + 5$   
 $-34 = 30 + 4$   
 $11 = 10 + 1$

11.  $99 = 90 + 9$   
 $-66 = 60 + 6$   
 $33 = 30 + 3$

7.   $57 = 50 + 7$   
 $-15 = 10 + 5$   
 $42 = 40 + 2$

12.  $68 = 60 + 8$   
 $-68 = 60 + 8$   
 $0 = 0 + 0$

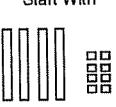
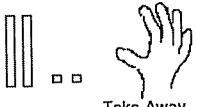
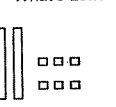
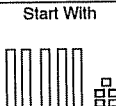

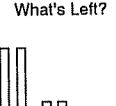
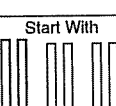

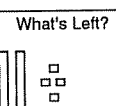
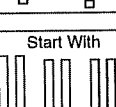
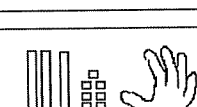
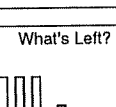
8.   $47 = 40 + 7$   
 $-11 = 10 + 1$   
 $36 = 30 + 6$

Challenge Problem!  
Fill in the missing numbers.  
 $68 = 60 + 8$   
 $-45 = 40 + 5$   
 $23 = 20 + 3$

20

### Tens and Ones: Manipulative - Worksheet 5 Puzzle Worksheet

Draw pictures of the blocks that go in the empty squares.

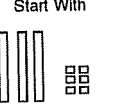
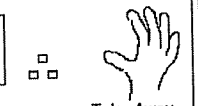
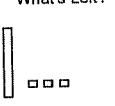
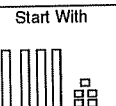
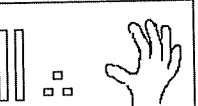
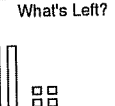
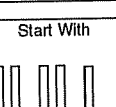
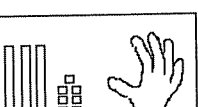
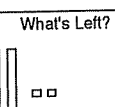
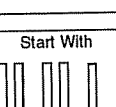
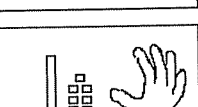
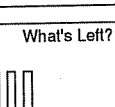
Fill in the missing numbers. Use blocks to help you.

$57 - 22 = 35$     $47 - 24 = 23$     $39 - 27 = 12$   
 $36 - 5 = 31$     $38 - 23 = 15$     $68 - 22 = 46$

22

### Tens and Ones: Manipulative - Worksheet 4 Puzzle Worksheet

Draw pictures of the blocks that go in the empty squares.

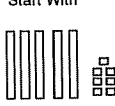
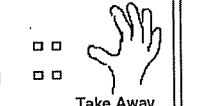
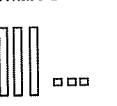
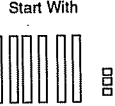

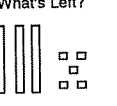
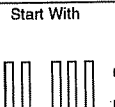
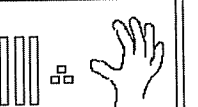
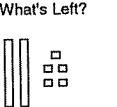
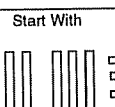
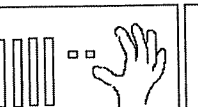
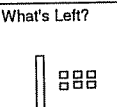
Fill in the missing numbers. Use blocks to help you.

$43 - 22 = 21$     $67 - 32 = 35$     $28 - 23 = 5$   
 $26 - 7 = 19$     $24 - 12 = 12$     $43 - 30 = 13$

21

### Tens and Ones: Manipulative - Worksheet 6

Draw pictures of the blocks that go in the empty squares.

Fill in the missing numbers. Use blocks to help you.

$47 - 34 = 13$     $56 - 12 = 44$     $74 - 43 = 31$   
 $65 - 2 = 63$     $67 - 52 = 15$     $89 - 48 = 46$

23

Tens and Ones: Practice - Worksheet 1

$$\begin{array}{r} 57 = (50) + (7) \\ -33 = (-30) + (-3) \\ \hline 24 = (20) + (4) \end{array} \quad \begin{array}{r} 98 = (90) + (8) \\ -47 = (-40) + (-7) \\ \hline 51 = (50) + (1) \end{array} \quad \begin{array}{r} 54 = (50) + (4) \\ -32 = (-30) + (-2) \\ \hline 22 = (20) + (2) \end{array}$$
  

$$\begin{array}{r} 44 = (40) + (4) \\ -24 = (-20) + (-4) \\ \hline 20 = (20) + (0) \end{array} \quad \begin{array}{r} 56 = (50) + (6) \\ -41 = (-40) + (-1) \\ \hline 15 = (10) + (5) \end{array} \quad \begin{array}{r} 99 = (90) + (9) \\ -68 = (-60) + (-8) \\ \hline 31 = (30) + (1) \end{array}$$
  

$$\begin{array}{r} 59 = (50) + (9) \\ -37 = (-30) + (-7) \\ \hline 22 = (20) + (2) \end{array} \quad \begin{array}{r} 69 = (60) + (9) \\ -47 = (-40) + (-7) \\ \hline 22 = (20) + (2) \end{array} \quad \begin{array}{r} 79 = (70) + (9) \\ -52 = (-50) + (-2) \\ \hline 27 = (20) + (7) \end{array}$$

Make your own.

$$\begin{array}{r} = \bigcirc + \bigcirc \\ - = \bigcirc + \bigcirc \\ \hline = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} = \bigcirc + \bigcirc \\ - = \bigcirc + \bigcirc \\ \hline = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} = + \\ - = + \\ \hline = + \end{array}$$

$$\begin{array}{r} 59 \\ -43 \\ \hline 16 \end{array} \quad \begin{array}{r} 67 \\ -32 \\ \hline 35 \end{array} \quad \begin{array}{r} 73 \\ -13 \\ \hline 60 \end{array} \quad \begin{array}{r} 65 \\ -42 \\ \hline 23 \end{array} \quad 25$$

Nonstandard Regrouping - Worksheet 1

Write yourself a subtraction problem. The numbers must be between 15 and 40. Do the problem two different ways. Circle the easiest way.

$$\begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc \quad \begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc$$
  

$$\begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc \quad \begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc$$

Build these problems. Trading (fixing) will vary. Example:  $9 \quad 8$

$$\begin{array}{r} 27 = (10) + (10) + (7) \\ -18 = (-5) + (-5) + (-8) \\ \hline 9 = (5) + (4) + (0) \end{array} \quad \begin{array}{r} 27 = (10) + (10) + (7) \\ -18 = (-5) + (-5) + (-8) \\ \hline 9 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 35 = (15) + (10) + (10) + (0) \\ -27 = (-5) + (-7) + (-8) + (-7) \\ \hline 8 = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 35 = (30) + (5) \\ -27 = (-20) + (-7) \\ \hline 8 = \bigcirc + \bigcirc \end{array}$$

Tens and Ones: Practice - Worksheet 2

$$\begin{array}{r} 57 = (50) + (7) \\ -36 = (-30) + (-6) \\ \hline 21 = (20) + (1) \end{array} \quad \begin{array}{r} 26 = (20) + (6) \\ -13 = (-10) + (-3) \\ \hline 13 = (10) + (3) \end{array} \quad \begin{array}{r} 14 = (10) + (4) \\ -11 = (-10) + (-1) \\ \hline 3 = (0) + (3) \end{array}$$
  

$$\begin{array}{r} 99 = (90) + (9) \\ -28 = (-20) + (-8) \\ \hline 71 = (70) + (1) \end{array} \quad \begin{array}{r} 46 = (40) + (6) \\ -24 = (-20) + (-4) \\ \hline 22 = (20) + (2) \end{array} \quad \begin{array}{r} 48 = (40) + (8) \\ -38 = (-30) + (-8) \\ \hline 10 = (10) + (0) \end{array}$$
  

$$\begin{array}{r} 87 = (80) + (7) \\ -54 = (-50) + (-4) \\ \hline 33 = (30) + (3) \end{array} \quad \begin{array}{r} 85 = (80) + (5) \\ -45 = (-40) + (-5) \\ \hline 40 = (40) + (0) \end{array} \quad \begin{array}{r} 64 = (60) + (4) \\ -32 = (-30) + (-2) \\ \hline 32 = (30) + (2) \end{array}$$

Make your own.

$$\begin{array}{r} = \bigcirc + \bigcirc \\ - = \bigcirc + \bigcirc \\ \hline = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} = \bigcirc + \bigcirc \\ - = \bigcirc + \bigcirc \\ \hline = \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} = + \\ - = + \\ \hline = + \end{array}$$

$$\begin{array}{r} 62 \\ -31 \\ \hline 31 \\ 26 \end{array} \quad \begin{array}{r} 56 \\ -24 \\ \hline 32 \end{array} \quad \begin{array}{r} 55 \\ -41 \\ \hline 14 \end{array} \quad \begin{array}{r} 79 \\ -52 \\ \hline 27 \end{array}$$

Nonstandard Regrouping - Worksheet 2

Write yourself a subtraction problem. The numbers must be between 15 and 40. Do the problem two different ways. Circle the easiest way.

$$\begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc \quad \begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = + +$$
  

$$\begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = \bigcirc + \bigcirc + \bigcirc \quad \begin{array}{r} \square \\ - \square \\ \hline \square \end{array} = + +$$

Build these problems. Trading (fixing) will vary.

$$\begin{array}{r} 32 = (10) + (20) + (2) \\ -16 = (-5) + (-5) + (-6) \\ \hline 16 = \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 32 = (10) + (20) + (2) \\ -16 = (-5) + (-5) + (-6) \\ \hline 16 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 32 = (10) + (10) + (10) + (2) \\ -16 = (-5) + (-4) + (-2) + (-5) \\ \hline 16 = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array} \quad \begin{array}{r} 32 = (30) + (2) \\ -16 = (-10) + (-6) \\ \hline 16 = \bigcirc + \bigcirc \end{array}$$

### Nonstandard Regrouping - Worksheet 3

Write yourself a subtraction problem. The numbers must be between 15 and 40. Do the problem two different ways. Circle the easiest way.

1. 
$$\begin{array}{r} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2. Make your own

$$\begin{array}{r} \square = \quad + \quad + \quad \\ - \square = \quad + \quad + \quad \\ \hline \square = \quad + \quad + \quad \end{array}$$

3. Fixing will vary.

$$\begin{array}{r} \boxed{31} = \boxed{20} + \boxed{10} + \boxed{1} \\ - \boxed{14} = \boxed{-5} + \boxed{-5} + \boxed{-4} \\ \hline \boxed{17} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

4. Fix it differently from 3.

$$\begin{array}{r} \boxed{31} = \boxed{20} + \boxed{10} + \boxed{1} \\ - \boxed{14} = \boxed{-5} + \boxed{-5} + \boxed{-4} \\ \hline \boxed{17} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

5.

$$\begin{array}{r} \boxed{23} = \boxed{5} + \boxed{10} + \boxed{5} + \boxed{3} \\ - \boxed{18} = \boxed{-5} + \boxed{-5} + \boxed{-3} + \boxed{-5} \\ \hline \boxed{5} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

6.

$$\begin{array}{r} \boxed{23} = \boxed{20} + \boxed{3} \\ - \boxed{18} = \boxed{-10} + \boxed{-8} \\ \hline \boxed{5} = \bigcirc + \bigcirc \end{array}$$

29

### Nonstandard Regrouping and IOUS - Worksheet 1

When a number is broken up like this:

$$\boxed{42} = \boxed{30} + \boxed{10} + \boxed{2} \leftarrow \text{You have a problem here. The 2 is less than the 7.}$$

$$\begin{array}{r} \boxed{42} = \boxed{30} + \boxed{10} + \boxed{2} \\ - \boxed{27} = \boxed{-10} + \boxed{-10} + \boxed{-7} \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

You have solved the problem before different ways. One way was to regroup the top number by taking some amount from 30 and making the 2 larger. This is called nonstandard regrouping. There are many ways to regroup.

$$\begin{array}{r} \boxed{42} = \overset{24}{\cancel{30}} + \boxed{10} + \cancel{2} + 6 = 8 \\ - \boxed{27} = \boxed{-10} + \boxed{-10} + \boxed{-7} \\ \hline \boxed{15} = \boxed{14} + \boxed{0} + \boxed{1} \end{array}$$

Or, it can be solved by using an IOU.

$$\begin{array}{r} \boxed{42} = \boxed{30} + \boxed{10} + \boxed{2} \\ - \boxed{27} = \boxed{-10} + \boxed{-10} + \boxed{-7} \\ \hline \boxed{15} = \boxed{20} + \boxed{0} + \boxed{-5} \\ 15 = 20 - 5 \end{array}$$

31

### Nonstandard Regrouping - Worksheet 4

Write yourself a subtraction problem. The numbers must be between 15 and 40. Do the problem two different ways. Circle the easiest way.

1. 
$$\begin{array}{r} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

2.

$$\begin{array}{r} \square = \bigcirc + \bigcirc + \bigcirc \\ - \square = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \square = \bigcirc + \bigcirc + \bigcirc \end{array}$$

Build these problems.

3.

$$\begin{array}{r} \boxed{35} = \boxed{15} + \boxed{15} + \boxed{5} \\ - \boxed{28} = \boxed{-10} + \boxed{-8} + \boxed{-10} \\ \hline \boxed{7} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

4. Fix it differently from 3.

$$\begin{array}{r} \boxed{35} = \boxed{15} + \boxed{15} + \boxed{5} \\ - \boxed{28} = \boxed{-10} + \boxed{-8} + \boxed{-10} \\ \hline \boxed{7} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

5.

$$\begin{array}{r} \boxed{35} = \boxed{10} + \boxed{10} + \boxed{10} + \boxed{5} \\ - \boxed{28} = \boxed{-10} + \boxed{-10} + \boxed{-6} + \boxed{-2} \\ \hline \boxed{7} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

6.

$$\begin{array}{r} \boxed{35} = \boxed{30} + \boxed{5} \\ - \boxed{28} = \boxed{-20} + \boxed{-8} \\ \hline \boxed{7} = \bigcirc + \bigcirc \end{array}$$

30

### Nonstandard Regrouping and IOUS - Worksheet 2

In each set there is an A and a B problem. Use regrouping to fix all the A problems. Use IOUs to fix all the B problems.

Set 1

Example:

A. 
$$\begin{array}{r} \overset{29}{\boxed{47}} = \overset{8}{\cancel{30}} + \boxed{10} + \boxed{7} \\ - \boxed{28} = \boxed{-10} + \boxed{-10} + \boxed{-8} \\ \hline \boxed{19} = \boxed{19} + \boxed{0} + \boxed{0} \end{array}$$

B. 
$$\begin{array}{r} \boxed{47} = \boxed{30} + \boxed{10} + \boxed{7} \\ - \boxed{28} = \boxed{-10} + \boxed{-10} + \boxed{-8} \\ \hline \boxed{19} = \boxed{20} + \boxed{0} + \boxed{-1} \\ 20 - 1 = 19 \end{array}$$

Set 2

A. 
$$\begin{array}{r} \boxed{72} = \boxed{40} + \boxed{30} + \boxed{2} \\ - \boxed{57} = \boxed{-20} + \boxed{-30} + \boxed{-7} \\ \hline \boxed{15} = \bigcirc + \bigcirc + \bigcirc \end{array}$$

B. 
$$\begin{array}{r} \boxed{72} = \boxed{40} + \boxed{30} + \boxed{2} \\ - \boxed{57} = \boxed{-20} + \boxed{-30} + \boxed{-7} \\ \hline \boxed{15} = \boxed{20} + \boxed{0} + \boxed{-5} \\ 20 - 5 = 15 \end{array}$$

Set 3

A. 
$$\begin{array}{r} \boxed{85} = \boxed{15} + \boxed{10} + \boxed{60} + \boxed{0} \\ - \boxed{27} = \boxed{-5} + \boxed{-5} + \boxed{-10} + \boxed{-7} \\ \hline \boxed{58} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

B. 
$$\begin{array}{r} \boxed{85} = \boxed{80} + \boxed{5} \\ - \boxed{27} = \boxed{-20} + \boxed{-7} \\ \hline \boxed{58} = \boxed{60} + \boxed{-2} \\ 60 - 2 = 58 \end{array}$$

32

### Nonstandard Regrouping and IOUS - Worksheet 3

In each set there is an A and a B problem. Use regrouping to fix all the A problems. Use IOUs to fix all the B problems.

Set 1 *Fixing problems will vary.*

A. 
$$\begin{array}{r} 51 \\ - 37 \\ \hline 14 \end{array} = \begin{array}{r} (20) + (30) + (1) \\ - (-10) + (-20) + (-7) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 51 \\ - 37 \\ \hline 14 \end{array} = \begin{array}{r} (20) + (30) + (1) \\ - (-10) + (-20) + (-7) \\ \hline \end{array}$$
  
 $20 - 6 = 14$

Set 2

A. 
$$\begin{array}{r} 67 \\ - 29 \\ \hline 38 \end{array} = \begin{array}{r} (50) + (10) + (7) \\ - (-10) + (-10) + (-9) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 67 \\ - 29 \\ \hline 38 \end{array} = \begin{array}{r} (50) + (10) + (7) \\ - (-10) + (-10) + (-9) \\ \hline \end{array}$$
  
 $40 - 2 = 38$

Set 3

A. 
$$\begin{array}{r} 83 \\ - 54 \\ \hline 29 \end{array} = \begin{array}{r} (80) + (3) \\ - (-50) + (-4) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 83 \\ - 54 \\ \hline 29 \end{array} = \begin{array}{r} (80) + (3) \\ - (-50) + (-4) \\ \hline \end{array}$$
  
 $30 - 1 = 29$

### Nonstandard Regrouping and IOUS - Worksheet 4

In each set there is an A and a B problem. Use regrouping to fix all the A problems. Use IOUs to fix all the B problems.

Set 1

A. 
$$\begin{array}{r} 90 \\ - 52 \\ \hline 38 \end{array} = \begin{array}{r} (90) + (0) \\ - (-50) + (-2) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 90 \\ - 52 \\ \hline 38 \end{array} = \begin{array}{r} (90) + (0) \\ - (-50) + (-2) \\ \hline \end{array}$$
  
 $38 = (40) + (-2)$

Set 2

A. 
$$\begin{array}{r} 167 \\ - 68 \\ \hline 99 \end{array} = \begin{array}{r} (100) + (60) + (7) \\ - (-50) + (-10) + (-8) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 167 \\ - 68 \\ \hline 99 \end{array} = \begin{array}{r} (100) + (60) + (7) \\ - (-50) + (-10) + (-8) \\ \hline \end{array}$$
  
 $99 = (50) + (50) + (-1)$

Set 3

A. 
$$\begin{array}{r} 235 \\ - 129 \\ \hline 106 \end{array} = \begin{array}{r} (200) + (30) + (5) \\ - (-100) + (-20) + (-9) \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 235 \\ - 129 \\ \hline 106 \end{array} = \begin{array}{r} (200) + (30) + (5) \\ - (-100) + (-20) + (-9) \\ \hline \end{array}$$
  
 $106 = (100) + (10) + (-4)$

33

34

### Nonstandard Regrouping and IOUS - Worksheet 5

This time make your own circles in Set 3. Solve the problems in any way that makes sense to you. Solve Problem A and Problem B differently. You can add circles if you need to. *BUWV*

Set 1

A. 
$$\begin{array}{r} 42 \\ - 17 \\ \hline 25 \end{array} = \begin{array}{r} \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 42 \\ - 17 \\ \hline 25 \end{array} = \begin{array}{r} \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc \\ \hline \end{array}$$

Set 2

A. 
$$\begin{array}{r} 135 \\ - 86 \\ \hline 49 \end{array} = \begin{array}{r} \bigcirc + \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc + \bigcirc \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 135 \\ - 86 \\ \hline 49 \end{array} = \begin{array}{r} \bigcirc + \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc + \bigcirc \\ \hline \end{array}$$

Set 3

A. Put in your own circles.

$$\begin{array}{r} 346 \\ - 128 \\ \hline 218 \end{array}$$

B. Solve the same problem again a different way

$$\begin{array}{r} 346 \\ - 128 \\ \hline 218 \end{array}$$

### Nonstandard Subtraction: Assessment Worksheet 1

Show You Know

1. Add circles if you need to.

$$\begin{array}{r} 42 \\ - 28 \\ \hline 14 \end{array} = \begin{array}{r} \bigcirc + \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc + \bigcirc \\ \hline \end{array}$$

2. Add circles if you need to.

$$\begin{array}{r} 62 \\ - 39 \\ \hline 23 \end{array} = \begin{array}{r} \bigcirc + \bigcirc + \bigcirc \\ - \bigcirc + \bigcirc + \bigcirc \\ \hline \end{array}$$

3. Make your own circles.

$$\begin{array}{r} 56 \\ - 38 \\ \hline 18 \end{array} =$$

4. What was done wrong here? Please correct the error.

$$\begin{array}{r} 62 \\ - 39 \\ \hline 23 \end{array} = \begin{array}{r} (30) + (30) + (2) \\ - (-30) + (-0) + (-9) \\ \hline \end{array}$$
  
 $23 \begin{array}{r} 37 \\ - 14 \\ \hline 23 \end{array} = \begin{array}{r} (0) + (30) + (-7) \\ \hline \end{array}$   
*forgot - sign*

5.

What is weird with this problem?

$$\begin{array}{r} 35 \\ - 49 \\ \hline \end{array}$$

*The subtrahend is greater than the minuend. The larger number is being subtracted from the smaller number.*

Note to teacher: Reteach pages of these concepts can be found in the Black Line Masters.

35

36

## Nonstandard Subtraction: Assessment Worksheet 2

### Show You Know: Challenge

Solve the problem three different ways.

$$\begin{array}{r} \boxed{234} = \bigcirc + \bigcirc + \bigcirc \\ - \boxed{146} = -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{88} = \bigcirc + \bigcirc + \bigcirc \end{array}$$
  

$$\begin{array}{r} \boxed{234} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - \boxed{146} = -\bigcirc + -\bigcirc + -\bigcirc + -\bigcirc \\ \hline \boxed{88} = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

Break your numbers into hundreds, tens and ones.

$$\begin{array}{r} \boxed{234} = \overset{185}{\boxed{200}} + \overset{+10}{\boxed{30}} + \overset{+4}{\boxed{19}} \\ - \boxed{146} = -\overset{100}{\boxed{100}} + -\overset{40}{\boxed{40}} + -\overset{6}{\boxed{6}} \\ \hline \boxed{88} = \overset{85}{\boxed{85}} + \overset{0}{\boxed{0}} + \overset{3}{\boxed{3}} \end{array}$$

Example: Fixing the problem will vary.

Do all your answers agree? yes

37

## Covering as a Strategy - Worksheet 2

Use Base Ten Blocks.

Take out a 100 flat barge. Cover it with 43.

How much is not covered? 57 squares

Cover the 100 flat barge with 62.

How much is not covered? 38 squares

Cover the 100 flat barge with 59.

How much is not covered? 41 squares

Cover the 100 flat barge with 37.

How much is not covered? 63 squares  
What is the difference between 100 and 37? 63

Cover the 100 flat barge with 74.

How much is not covered? 26 squares  
Write this problem in a number sentence.

$$100 - 74 = 26$$

What is the difference between 100 and 88? 12

$$100 - 88 = 12$$

Make your own.

Cover the 100 flat barge with \_\_\_\_\_.

How much is not covered? \_\_\_\_\_

39

## Covering as a Strategy - Worksheet 1

Use Cuisenaire Rods or Base Ten Blocks.

How much longer is a 15 centimeter (cm) train than a 9 cm train? 6 cm

How did you get your answer? Awv

How else could you get an answer? count on, subtract, compare, cover up

Compare your method to other's strategies.

A strategy that Alysia uses to solve this type of problem is covering.

Build a 17 cm train.

Take out a 8 cm train and use it to cover the 17 cm train.

How much longer is the 17 cm train? 9 cm Prove it.

How much shorter is an 8 cm train than a 17 cm train? 9 cm

Build a 23 cm train.

Take out a 10 cm train. Compare them.

How much shorter is the 10 cm train? 13 cm

Build a 25 cm train.

Take out a 19 cm train. Compare them.

How much longer is the 25 cm train? 6 cm

A 29 cm train is 13 centimeters longer than a 16 cm train.

An 18 cm train is 8 centimeters shorter than a 26 cm train.

A 24 cm train is 7 centimeters longer than a 17 cm train.

A 21 cm train is 7 centimeters longer than a 14 cm train.

38

## Covering as a Strategy - Worksheet 3

Use Base Ten Blocks.

Build a 137 flat barge. Cover it with 94.

How much is not covered? 43 squares

Cover the 137 flat barge with 102.

How much is not covered? 35 squares

Build a 200 flat barge. Cover it with 165.

How much is not covered? 35 squares

Build a 250 flat barge. Cover it with 187.

How much is not covered? 63 squares  
What is the difference between 250 and 187? 63

Build a 300 flat barge. Cover it with 184.

How much is not covered? 116 squares  
What is the difference between 300 and 184? 116  
Write this problem in a number sentence.

$$300 - 184 = 116$$

Make your own.

Cover the \_\_\_\_\_ flat barge with \_\_\_\_\_.

How much is not covered? \_\_\_\_\_

40

### Gus the Bus Driver: Differences



Gus, the bus driver, is working on his bus. He noticed the front two tires were very worn and he needs new tires.

Cost of tires:  
Goodyear tires \$80  
Michelin X Radial cost \$111

What is the difference in their cost? \$31  
Hint: If you get stuck, take out the tens blocks and use the covering strategy.

If he bought two Goodyear tires, it would cost? \$160  
If he bought two Michelin X Radial tires, it would cost? \$222

What would the difference between the cost of the two sets? 62 dollars

When Gus drove the town's southern route it took him 60 minutes to deliver all the students. His bus driver friend, Frank, could deliver the students to the north side of town in 45 minutes.

What is the difference between their route times? 15 minutes

Please write a "difference between" story for Gus.

### Standard Regrouping: Manipulative - Worksheet 1

Solve the problems shown by the pictures. Draw the answer. Write the problem.

Example:  
Start With: Take Away: = What's Left?   
$$\begin{array}{r} 35 \\ - 19 \\ \hline 16 \end{array}$$

1. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 47 \\ - 29 \\ \hline 18 \end{array}$$

2. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 80 \\ - 35 \\ \hline 45 \end{array}$$

3. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 66 \\ - 49 \\ \hline 17 \end{array}$$

4. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 50 \\ - 31 \\ \hline 19 \end{array}$$

Challenge!  
5. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 50 \\ - 17 \\ \hline 33 \end{array}$$

### Standard Regrouping: Manipulative - Worksheet 2

Solve the problems shown by the pictures. Draw the answer. Write the problem.

6. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 48 \\ - 18 \\ \hline 28 \end{array}$$

7. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 30 \\ - 17 \\ \hline 13 \end{array}$$

8. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 53 \\ - 44 \\ \hline 9 \end{array}$$

9. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 55 \\ - 28 \\ \hline 27 \end{array}$$

10. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 61 \\ - 42 \\ \hline 19 \end{array}$$

Challenge!  
11. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 33 \\ - 17 \\ \hline 16 \end{array}$$

### Standard Regrouping: Manipulative - Worksheet 3

Challenge! May be hard.

Solve the problems shown by the pictures.

Write the problem.

12. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 46 \\ - 25 \\ \hline 21 \end{array}$$

13. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 30 \\ - 17 \\ \hline 13 \end{array}$$

14. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 53 \\ - 30 \\ \hline 23 \end{array}$$

15. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 55 \\ - 23 \\ \hline 32 \end{array}$$

16. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 45 \\ - 26 \\ \hline 19 \end{array}$$

17. Start With: Take Away: = What's Left?   
$$\begin{array}{r} 35 \\ - 19 \\ \hline 16 \end{array}$$



### Standard Regrouping: Manipulative - Worksheet 4

This time the problem is written for you. You draw the blocks and write the answer under the line.

	Start With 	Take Away 	=	What's Left? 	$\begin{array}{r} 47 \\ - 19 \\ \hline 28 \end{array}$
18.	Start With 	Take Away 	=	What's Left? 	$\begin{array}{r} 28 \\ - 19 \\ \hline 9 \end{array}$
19.	Start With 	Take Away 	=	What's Left? 	$\begin{array}{r} 22 \\ - 15 \\ \hline 7 \end{array}$
20.	Start With 	Take Away 	=	What's Left? 	$\begin{array}{r} 43 \\ - 27 \\ \hline 16 \end{array}$
21.	Start With 	Take Away 	=	What's Left? 	$\begin{array}{r} 52 \\ - 18 \\ \hline 34 \end{array}$
			=		$\begin{array}{r} \phantom{00} \\ - \phantom{00} \\ \hline \phantom{00} \end{array}$

45

### Standard Regrouping: Circle Math - Worksheet 1

My problem today is:  $47 - 39$

Use this space for your scratch work.

BUWV

$$\begin{array}{r} 47 = \bigcirc + \bigcirc + \bigcirc \\ - 39 = \bigcirc + \bigcirc + \bigcirc \\ \hline 8 = \bigcirc + \bigcirc + \bigcirc \end{array}$$

$$\begin{array}{r} 47 = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ - 39 = \bigcirc + \bigcirc + \bigcirc + \bigcirc \\ \hline 8 = \bigcirc + \bigcirc + \bigcirc + \bigcirc \end{array}$$

Break your numbers into tens and ones. Then regroup to solve.

$$\begin{array}{r} 47 = \overset{30}{\cancel{40}} + \overset{17}{7} \\ - 39 = \overset{30}{\cancel{30}} + \overset{17}{-9} \\ \hline 8 = \overset{30}{\cancel{0}} + \overset{17}{8} \end{array}$$

Do all your answers agree?

yes

46

### Standard Regrouping: Circle Math - Worksheet 2

Now, use the answer circles for place value and have the answer come out in tens and ones.

1.	$\begin{array}{r} 55 \\ 62 = \cancel{60} + \cancel{2} \\ - 37 = \cancel{-30} + \cancel{-7} \\ \hline 25 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$	2.	$\begin{array}{r} 30 + 10 = 11 \\ 62 = \cancel{40} + \cancel{1} \\ - 37 = \cancel{-10} + \cancel{-9} \\ \hline 22 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$
----	--	----	--

**ILLEGAL:** The answer is correct except that there are ones in the tens place. This is now not allowed.

3.	$\begin{array}{r} 20 \quad 16 \\ 36 = \cancel{30} + \cancel{6} \\ - 28 = \cancel{-20} + \cancel{-8} \\ \hline 8 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$	4.	$\begin{array}{r} 40 \quad 13 \\ 53 = \cancel{50} + \cancel{3} \\ - 48 = \cancel{-40} + \cancel{-8} \\ \hline 5 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$
5.	$\begin{array}{r} 30 \quad 15 \\ 45 = \cancel{40} + \cancel{5} \\ - 26 = \cancel{-20} + \cancel{-6} \\ \hline 19 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$	6.	$\begin{array}{r} 50 \quad 14 \\ 64 = \cancel{60} + \cancel{4} \\ - 35 = \cancel{-30} + \cancel{-5} \\ \hline 29 = \bigcirc + \bigcirc \\ \text{tens ones} \end{array}$

47

### Standard Regrouping: Circle Math - Worksheet 3

Solve these problems. Draw in the circles. The answer must come out in tens and ones.

Use only two circles. If you get stuck, take out the Base Ten Blocks.

$$\begin{array}{r} 84 = \overset{30}{\cancel{80}} + \overset{14}{4} \\ - 58 = \overset{30}{\cancel{50}} + \overset{14}{-8} \\ \hline 26 = \overset{30}{\cancel{20}} + \overset{14}{6} \end{array}$$

$$\begin{array}{r} 52 = \overset{40}{\cancel{50}} + \overset{12}{2} \\ - 35 = \overset{40}{\cancel{30}} + \overset{12}{-5} \\ \hline 17 = \overset{40}{\cancel{10}} + \overset{12}{7} \end{array}$$

$$\begin{array}{r} 74 = \overset{60}{\cancel{70}} + \overset{14}{4} \\ - 39 = \overset{60}{\cancel{30}} + \overset{14}{-9} \\ \hline 35 = \overset{60}{\cancel{30}} + \overset{14}{5} \end{array}$$

$$\begin{array}{r} 84 = \overset{70}{\cancel{80}} + \overset{4}{4} \\ - 37 = \overset{70}{\cancel{30}} + \overset{4}{-7} \\ \hline 47 = \overset{70}{\cancel{40}} + \overset{4}{7} \end{array}$$

$$\begin{array}{r} 52 = \overset{40}{\cancel{50}} + \overset{12}{2} \\ - 26 = \overset{40}{\cancel{20}} + \overset{12}{-6} \\ \hline 26 = \overset{40}{\cancel{20}} + \overset{12}{6} \end{array}$$

$$\begin{array}{r} 403 = \overset{300}{\cancel{400}} + \overset{13}{3} \\ - 97 = \overset{300}{\cancel{400}} + \overset{13}{-7} \\ \hline 306 = \overset{300}{\cancel{300}} + \overset{13}{6} \end{array}$$

48

### Standard Regrouping: Short Way - Worksheet 1

1. Expanded  $35 = 30 + 5$  Short way  $35$   
 $+ 27 = 20 + 7$   $+ 27$   
 $62 = 60 + 2$   $62$   
 tens ones BUWV

3.  $45 = \square + \square + \square$   
 $- 29 = \square + \square + \square$   
 $16 = \square + \square + \square$

4.  $45 = \square + \square + \square + \square$   
 $- 29 = \square + \square + \square + \square$   
 $16 = \square + \square + \square + \square$

5. The answer must come out in tens and ones.  
 $45 = 40 + 5$   
 $- 29 = 20 + 9$   
 $16 = 10 + 6$

6. Expanded  $45 = 40 + 5$  Short way  $45$   
 $- 36 = 30 + 6$   $- 36$   
 $9 = 0 + 9$   $9$

### Standard Regrouping: Short Way - Worksheet 3

Use Base Ten Blocks to help you do these problems. Build each problem and record what you do in the Expanded way and in the new Short way.

7.  $45 = 30 + 15$   $31$   
 $- 26 = 20 + 6$   $- 26$   
 $19 = 10 + 9$   $19$   
 tens ones

9. You write in the expansions this time.  
 $60 = 50 + 10$   $51$   
 $- 35 = 30 + 5$   $- 35$   
 $25 = 20 + 5$   $25$   
 tens ones

11. Challenge! Puzzle Problem  
 Fill in the missing numbers.  
 $47 = 40 + 7$   $31$   
 $- 29 = 20 + 9$   $- 29$   
 $18 = 10 + 8$   $18$   
 tens ones  
 Scratch space.

8. You write in the expansions this time.  
 $52 = 40 + 12$   $44$   
 $- 33 = 30 + 3$   $- 33$   
 $19 = 10 + 9$   $19$   
 tens ones

10. Make your own.  
 $= \square + \square$   
 $- = \square + \square$   $-$   
 $= \square + \square$   
 tens ones

12. Challenge! Puzzle Problem  
 Fill in the missing numbers.  
 $41 = 30 + 11$   $41$   
 $- 25 = 20 + 5$   $- 25$   
 $16 = 10 + 6$   
 tens ones  
 Scratch space.

### Standard Regrouping: Short Way - Worksheet 2

Use Base Ten Blocks to help you do these problems. Build each problem and record what you do in the Expanded way and in the new Short way. Before you do each problem, estimate what you think the answer will be. Draw an arrow in the target. The closer the estimate, the closer to the bull's-eye.

1. Can you guess within 10 of the right answer?  
 $25 = 20 + 5$   $25$   
 $- 17 = 10 + 7$   $- 17$   
 $8 = 0 + 8$   $8$   
 tens ones

3.  $47 = 30 + 17$   $37$   
 $- 28 = 20 + 8$   $- 28$   
 $19 = 10 + 9$   $19$   
 tens ones

5.  $50 = 40 + 10$   $40$   
 $- 32 = 30 + 2$   $- 32$   
 $18 = 10 + 8$   $18$   
 tens ones

2.  $37 = 20 + 17$   $27$   
 $- 19 = 10 + 9$   $- 19$   
 $18 = 10 + 8$   $18$   
 tens ones

4.  $32 = 20 + 12$   $22$   
 $- 13 = 10 + 3$   $- 13$   
 $19 = 10 + 9$   $19$   
 tens ones

6.  $86 = 70 + 16$   $76$   
 $- 39 = 30 + 9$   $- 39$   
 $47 = 40 + 7$   $47$   
 tens ones

### Standard Regrouping: Short Way - Worksheet 5

Solve these problems the Short way.

$$\begin{array}{r} 4 \\ 56 \\ - 37 \\ \hline 19 \end{array}$$

$$\begin{array}{r} 4 \\ 52 \\ - 33 \\ \hline 19 \end{array}$$

$$\begin{array}{r} 5 \\ 61 \\ - 36 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 4 \\ 57 \\ - 29 \\ \hline 28 \end{array}$$

$$\begin{array}{r} 3 \\ 45 \\ - 27 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 7 \\ 80 \\ - 43 \\ \hline 37 \end{array}$$

$$\begin{array}{r} 69 \\ - 47 \\ \hline 22 \end{array}$$

$$\begin{array}{r} 5 \\ 67 \\ - 49 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 46 \\ - 24 \\ \hline 22 \end{array}$$

$$\begin{array}{r} 5 \\ 263 \\ - 48 \\ \hline 215 \end{array}$$

$$\begin{array}{r} 4 \\ 150 \\ - 35 \\ \hline 115 \end{array}$$

$$\begin{array}{r} 5 \\ 524 \\ - 106 \\ \hline 418 \end{array}$$

## Standard Regrouping: Short Way - Assessment

Test for Understanding

1. Solve this problem:

$$\begin{array}{r} 31 \\ -15 \\ \hline 27 \end{array}$$

2. Explain what is wrong with this notation:

$$\begin{array}{r} 30 \\ 47 \\ -18 \\ \hline 299 \end{array} \quad \begin{array}{r} 31 \\ 47 \\ -18 \\ \hline 29 \end{array}$$

The "30" should be written with the short notation "3". There are not 30 tens in the tens column.

3. Explain what is wrong with this notation:

$$\begin{array}{r} 4105 \\ -27 \\ \hline 248 \end{array} \quad \begin{array}{r} 45 \\ -27 \\ \hline 28 \end{array}$$

The "10" should be written with the short notation "1". The student may change the value to 105 instead of 15.

4. Solve these problems:

$$\begin{array}{r} 40 \\ -29 \\ \hline 21 \end{array} \quad \begin{array}{r} 37 \\ -18 \\ \hline 29 \end{array} \quad \begin{array}{r} 45 \\ -27 \\ \hline 28 \end{array}$$

5. Make up one regrouping problem and solve it.

6. Write a subtraction word problem. Write the number sentence that goes with the problem.

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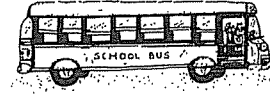


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55

## Gus the Bus Driver

Goes on a field trip



Gus, the bus driver, gets to take Mrs. Zarves' class on a field trip to the Nature Preserve. It is 73 miles away and should take about 1 1/2 hours or 90 minutes to get there.

Write a number sentence for each problem to show how you figured out answer.

Gus planned to drive the bus which holds 44 students. Mrs. Zarves has 28 students. The other seats can be used for parents. How many adults can go to fill the bus?  $44 - 28 = 16$  adults

Of course, Mrs. Zarves is going with the class. She loves to go places with her students. How many parents can go?  $16 - 1 = 15$  parents

If each seat holds two people, how many seats are on the bus?  $44 \div 2 = 22$  seats

After traveling 47 miles, according to the trip odometer, Gus heard the first question of the drive: "How much farther?" Figure out the answer.

$74 - 47 = 26$  miles

After 55 minutes a student asked, "How much longer are we going to be on the bus?"  $90 - 55 = 35$  minutes.

56

## Families of Facts

Complete these Families of Facts.

$24 + 28 = 52$	$12 + 38 = 26$	$52 - 18 = 34$
$28 + 24 = 52$	$26 + 12 = 38$	$34 + 18 = 52$
$52 - 24 = 28$	$38 - 26 = 12$	$52 - 18 = 34$
$52 - 28 = 24$	$38 - 12 = 26$	$52 - 34 = 18$

Use the Family of Facts to check these subtraction problems. Correct each wrong one.

$\begin{array}{r} 25 \\ -16 \\ \hline 9 \end{array}$  If  $25 - 16 = 9$   
Then  $9 + 16 = \underline{\quad}$

$\begin{array}{r} 46 \\ -17 \\ \hline 29 \end{array}$  If  $46 - 17 = 29$   
Then  $29 + 17 = \underline{46}$

$\begin{array}{r} 47 \\ -15 \\ \hline 32 \\ + \\ \hline 47 \end{array}$  Some folks cover up the top number and add.

$\begin{array}{r} 43 \\ -27 \\ \hline 16 \\ + \\ \hline 43 \end{array}$

$\begin{array}{r} 42 \\ -18 \\ \hline 24 \\ + \\ \hline 42 \end{array}$

$\begin{array}{r} 35 \\ -19 \\ \hline 16 \\ + \\ \hline 35 \end{array}$	$\begin{array}{r} 71 \\ -28 \\ \hline 43 \\ + \\ \hline 71 \end{array}$	$\begin{array}{r} 71 \\ -28 \\ \hline 43 \\ + \\ \hline 71 \end{array}$	$\begin{array}{r} 60 \\ -46 \\ \hline 24 \\ + \\ \hline 60 \end{array}$	$\begin{array}{r} 60 \\ -46 \\ \hline 24 \\ + \\ \hline 60 \end{array}$	$\begin{array}{r} 53 \\ -46 \\ \hline 17 \\ + \\ \hline 53 \end{array}$	$\begin{array}{r} 53 \\ -46 \\ \hline 17 \\ + \\ \hline 53 \end{array}$
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57

## Hundreds, Tens, and Ones: Manipulative - Worksheet 1

### Two Digits Review

	Expanded way	Short way	check
1.	$64 = 60 + 4$	$64$	
	$- 43 = 40 + 3$	$- 43$	$21$
	$\hline 21 = 20 + 1$	$\hline 21$	$+ 43$
			$\hline 64$
2.	$75 = 70 + 5$	$75$	
	$- 27 = 20 + 7$	$- 27$	$48$
	$\hline 48 = 40 + 8$	$\hline 48$	$+ 27$
			$\hline 75$
3.	$68 = 60 + 8$	$68$	
	$- 28 = 20 + 8$	$- 28$	$40$
	$\hline 40 = 40 + 0$	$\hline 40$	$+ 28$
			$\hline 68$
4.	$77 = 70 + 7$	$77$	
	$- 19 = 10 + 9$	$- 19$	$58$
	$\hline 58 = 50 + 8$	$\hline 58$	$+ 19$
			$\hline 77$

58



### Three Digits Regrouping: Recording - Worksheet 1

1. Expanded way

$$\begin{array}{r} 347 = 300 + 40 + 7 \\ - 156 = -100 + -50 + -6 \\ \hline 191 = 100 + 90 + 1 \end{array}$$

hundreds tens ones

Short way

$$\begin{array}{r} 347 \\ - 156 \\ \hline 191 \end{array}$$

2. Expanded way

$$\begin{array}{r} 483 = 400 + 80 + 3 \\ - 248 = -200 + -40 + -8 \\ \hline 235 = 200 + 30 + 5 \end{array}$$

hundreds tens ones

Short way

$$\begin{array}{r} 483 \\ - 248 \\ \hline 235 \end{array}$$

3. Expanded way

$$\begin{array}{r} 632 = 600 + 30 + 2 \\ - 269 = -200 + -60 + -9 \\ \hline 363 = 300 + 60 + 3 \end{array}$$

hundreds tens ones

Short way

$$\begin{array}{r} 632 \\ - 269 \\ \hline 363 \end{array}$$

4. Expanded way

$$\begin{array}{r} 806 = 800 + 0 + 6 \\ - 278 = -200 + -70 + -8 \\ \hline 528 = 500 + 20 + 8 \end{array}$$

hundreds tens ones

Short way

$$\begin{array}{r} 806 \\ - 278 \\ \hline 528 \end{array}$$

65

### Three Digits Regrouping: Recording - Worksheet 2

Look at the problem in the box. Round the numbers in the subtraction problem to the nearest hundred. Look at the list of possible answers that are written next to the problem in the box. Circle the estimate that seems the most reasonable. Then do the problem with blocks. Record your work on the expanded problem.

A. 345  
-101

Round it.  
345  $\rightarrow$  300  
- 101  $\rightarrow$  100  
 $\hline$   
200

Now solve it.

$$\begin{array}{r} 300 + 40 + 5 \\ - 100 + -90 + -1 \\ \hline 200 + 40 + 4 \end{array}$$

Which estimate was closest to the answer? c or d

a. 900 b. 500 c. 250 d. 150 e. 75 f. 25

B. 252  
-180

Round it.  
252  $\rightarrow$  300  
- 180  $\rightarrow$  200  
 $\hline$   
100

Now solve it.

$$\begin{array}{r} 200 + 50 + 2 \\ - 100 + -60 + -0 \\ \hline 100 + 90 + 2 \end{array}$$

Which estimate was closest to the answer? e

a. 900 b. 500 c. 250 d. 150 e. 75 f. 25

C. 624  
-291

Round it.  
624  $\rightarrow$  600  
- 291  $\rightarrow$  300  
 $\hline$   
300

Now solve it.

$$\begin{array}{r} 600 + 20 + 4 \\ - 200 + -90 + -1 \\ \hline 300 + 30 + 3 \end{array}$$

Which estimate was closest to the answer? c

a. 900 b. 500 c. 250 d. 150 e. 75 f. 25

D. 976  
-895

Round it.  
976  $\rightarrow$  1,000  
- 895  $\rightarrow$  900  
 $\hline$   
100

Now solve it.

$$\begin{array}{r} 900 + 70 + 6 \\ - 800 + -90 + -5 \\ \hline 100 + 80 + 1 \end{array}$$

Which estimate was closest to the answer? e

a. 900 b. 500 c. 250 d. 150 e. 75 f. 25

66

### Three Digits Regrouping: Recording - Worksheet 3

In column A, there are five subtraction problems. In column B, there are five answers to the problems. Draw a line from each problem in column A to the most reasonable answers in column B. Then do each problem in column C. Draw a line from the problem in column C to its answer in column B. See if you guessed right.

A	B	C
$\begin{array}{r} 952 \\ - 171 \\ \hline 781 \end{array}$	156	$\begin{array}{r} 900 + 50 + 2 \\ - 100 + -70 + -1 \\ \hline 700 + 80 + 1 \end{array}$
$\begin{array}{r} 275 \\ - 183 \\ \hline 92 \end{array}$	92	$\begin{array}{r} 200 + 70 + 5 \\ - 100 + -80 + -3 \\ \hline 100 + 90 + 2 \end{array}$
$\begin{array}{r} 285 \\ - 129 \\ \hline 156 \end{array}$	781	$\begin{array}{r} 200 + 80 + 5 \\ - 100 + -20 + -9 \\ \hline 100 + 50 + 6 \end{array}$
$\begin{array}{r} 539 \\ - 286 \\ \hline 253 \end{array}$	253	$\begin{array}{r} 500 + 30 + 9 \\ - 200 + -80 + -6 \\ \hline 300 + 50 + 3 \end{array}$
$\begin{array}{r} 842 \\ - 339 \\ \hline 503 \end{array}$	503	$\begin{array}{r} 800 + 40 + 2 \\ - 300 + -30 + -9 \\ \hline 500 + 0 + 3 \end{array}$

Write three different subtraction problems whose answers are 156. AWV

67

### Three Digits Regrouping: Recording - Worksheet 4

Each box has five problems in it. The answer for two of the problems appears in the square. Circle the two problems you think have that answer.

Set 1

A	B	C	D	E
<span style="border: 1px solid black; padding: 2px;">100</span>	$\begin{array}{r} 679 \\ - 579 \end{array}$	$\begin{array}{r} 693 \\ - 445 \end{array}$	$\begin{array}{r} 289 \\ - 159 \end{array}$	$\begin{array}{r} 789 \\ - 689 \end{array}$
	$\begin{array}{r} 101 \\ - 59 \end{array}$			

Set 2

A	B	C	D	E
<span style="border: 1px solid black; padding: 2px;">151</span>	$\begin{array}{r} 786 \\ - 485 \end{array}$	$\begin{array}{r} 321 \\ - 170 \end{array}$	$\begin{array}{r} 389 \\ - 298 \end{array}$	$\begin{array}{r} 564 \\ - 413 \end{array}$
	$\begin{array}{r} 956 \\ - 285 \end{array}$			

Set 3

A	B	C	D	E
<span style="border: 1px solid black; padding: 2px;">309</span>	$\begin{array}{r} 301 \\ - 159 \end{array}$	$\begin{array}{r} 782 \\ - 291 \end{array}$	$\begin{array}{r} 976 \\ - 309 \end{array}$	$\begin{array}{r} 963 \\ - 654 \end{array}$
	$\begin{array}{r} 594 \\ - 285 \end{array}$			

Set 4

A	B	C	D	E
<span style="border: 1px solid black; padding: 2px;">495</span>	$\begin{array}{r} 594 \\ - 99 \end{array}$	$\begin{array}{r} 832 \\ - 337 \end{array}$	$\begin{array}{r} 369 \\ - 174 \end{array}$	$\begin{array}{r} 842 \\ - 707 \end{array}$
	$\begin{array}{r} 986 \\ - 171 \end{array}$			

Set 5

A	B	C	D	E
<span style="border: 1px solid black; padding: 2px;">612</span>	$\begin{array}{r} 489 \\ - 177 \end{array}$	$\begin{array}{r} 909 \\ - 296 \end{array}$	$\begin{array}{r} 870 \\ - 668 \end{array}$	$\begin{array}{r} 634 \\ - 429 \end{array}$
	$\begin{array}{r} 803 \\ - 191 \end{array}$			

68

### Three Digits Regrouping: Recording - Assessment Worksheet 5 - Show You Know

Show two different ways to solve this problem. Draw your own circles.

*BUVV*

$$\begin{array}{r} 472 \\ - 198 \\ \hline 274 \end{array}$$

$$\begin{array}{r} 472 \\ - 198 \\ \hline 274 \end{array}$$

$$534 = 400 + 20 + 10 + 4$$

$$-189 = -100 + -80 + -9$$

$$345 = 300 + 40 + 5$$

Now solve the same problem with the Short way without expanding.

$$\begin{array}{r} 534 \\ - 189 \\ \hline 345 \end{array}$$

What was done wrong when solving this problem?

$$\begin{array}{r} 2^{100} \\ 304 \\ - 123 \\ \hline 1981 \end{array}$$

*Incorrect short notation in the tens column which lead to incorrect subtraction of 100-2=98*  
 $304 - 123 = 181$

Solve these two problems any way you choose.

$$\begin{array}{r} 324 \\ - 129 \\ \hline 195 \end{array}$$

$$\begin{array}{r} 405 \\ - 162 \\ \hline 243 \end{array}$$

69

### Practice

$$\begin{array}{r} 590 \\ - 479 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 760 \\ - 120 \\ \hline 640 \end{array}$$

$$\begin{array}{r} 336 \\ - 176 \\ \hline 160 \end{array}$$

$$\begin{array}{r} 775 \\ - 156 \\ \hline 619 \end{array}$$

$$\begin{array}{r} 724 \\ - 488 \\ \hline 236 \end{array}$$

$$\begin{array}{r} 434 \\ - 342 \\ \hline 92 \end{array}$$

$$\begin{array}{r} 782 \\ - 637 \\ \hline 145 \end{array}$$

$$\begin{array}{r} 535 \\ - 344 \\ \hline 191 \end{array}$$

$$\begin{array}{r} 744 \\ - 655 \\ \hline 89 \end{array}$$

$$\begin{array}{r} 835 \\ - 535 \\ \hline 300 \end{array}$$

$$\begin{array}{r} 520 \\ - 289 \\ \hline 231 \end{array}$$

$$\begin{array}{r} 473 \\ - 328 \\ \hline 145 \end{array}$$

$$\begin{array}{r} 994 \\ - 438 \\ \hline 556 \end{array}$$

$$\begin{array}{r} 824 \\ - 735 \\ \hline 89 \end{array}$$

$$\begin{array}{r} 536 \\ - 183 \\ \hline 353 \end{array}$$

$$\begin{array}{r} 747 \\ - 557 \\ \hline 190 \end{array}$$

$$\begin{array}{r} 500 \\ - 251 \\ \hline 249 \end{array}$$

$$\begin{array}{r} 254 \\ - 107 \\ \hline 147 \end{array}$$

$$\begin{array}{r} 576 \\ - 268 \\ \hline 308 \end{array}$$

$$\begin{array}{r} 843 \\ - 689 \\ \hline 154 \end{array}$$

72

Answer Key: Subtraction - Booklet 3

### Three Digits Regrouping: Recording - Worksheet 6

Solve each problem the Expanded way and the Short way. Before you do each problem, round off both numbers in your head and record the rounded answer in the estimate circle. Build with Base Ten Blocks if you need to.

Estimate How did you do?

Estimate: 100

$$300 + 40 + 5 = 345$$

$$-100 + -60 + -4 = -164$$

$$100 + 80 + 1 = 181$$

Estimate: 500

$$500 + 70 + 3 = 573$$

$$-200 + -80 + -2 = -282$$

$$300 + 90 + 1 = 291$$

Estimate: 400

$$400 + 10 + 6 = 416$$

$$-100 + -60 + -4 = -160$$

$$300 + 50 + 6 = 256$$

Estimate: 400

$$400 + 20 + 7 = 427$$

$$-200 + -40 + -4 = -284$$

$$100 + 40 + 3 = 143$$

Estimate: 700

$$700 + 0 + 5 = 705$$

$$-300 + -20 + -3 = -323$$

$$300 + 80 + 2 = 382$$

Estimate: 200

$$200 + 70 + 2 = 272$$

$$-100 + -40 + -6 = -146$$

$$200 + 20 + 6 = 226$$

### Challenge!

Try to solve this problem. If you get stuck, use the blocks and write the answer.

Question: What makes this problem harder than the others? *Zero in the ones place. Regrouping in two places.*

Estimate: 300

$$300 + 20 + 0 = 320$$

$$-100 + -40 + -1 = -141$$

$$100 + 70 + 9 = 179$$

70

### Riddle

What did the math book tell the pencil? Solve for the value of the letter in each equation. Write the letter above it's value to solve the riddle.

$$141 - O = 87$$

$$O = 54$$

$$19 + 37 - B = 17$$

$$B = 39$$

$$10 + 47 + M = 93$$

$$M = 36$$

$$246 - 158 + S = 97$$

$$S = 9$$

$$18 + 56 - A = 50$$

$$A = 24$$

$$E + 27 + 35 = 108$$

$$E = 46$$

$$128 + O - 79 = 89$$

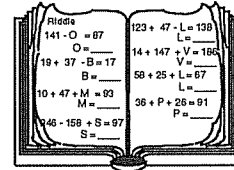
$$O = 40$$

$$118 + 52 - T = 144$$

$$T = 26$$

$$24 + 25 + E = 93$$

$$E = 44$$



$$123 + 47 - L = 138$$

$$L = 32$$

$$14 + 147 + V = 186$$

$$V = 25$$

$$58 + 25 + L = 67$$

$$L = 16$$

$$36 + P + 26 = 91$$

$$P = 29$$

$$130 + 125 + A = 272$$

$$A = 17$$

$$15 + 23 + H = 66$$

$$H = 28$$

$$F + 129 - 56 = 84$$

$$F = 11$$

$$29 + 44 - O = 36$$

$$O = 37$$

$$28 + 27 + I = 88$$

$$I = 33$$

$$100 + 138 - R = 219$$

$$R = 19$$

I H A V E A L O T O F  
 33 28-24-25-44 17 16-40-26 37-11  
 P R O B L E M S  
 29-19-54 39-32-46-36-9

Joke by Chrissy B., New Iberia, LA

73

Pre-Assessment - Part 1 - Worksheet 1

1. Solve. a. BUWV Example:  $\begin{array}{r} 15 \\ -12 \\ \hline 3 \end{array} = \begin{array}{r} 6 \\ -4 \\ +1 \\ \hline 3 \end{array}$  b. BUWV  $\begin{array}{r} 14 \\ -7 \\ \hline 7 \end{array} = \begin{array}{r} 0 \\ -0 \\ +7 \\ \hline 7 \end{array}$
2. Solve each problem a different way. BUWV Examples:  $\begin{array}{r} 26 \\ -17 \\ \hline 9 \end{array} = \begin{array}{r} 14 \\ -14 \\ +9 \\ \hline 9 \end{array}$   $\begin{array}{r} 26 \\ -17 \\ \hline 9 \end{array} = \begin{array}{r} 9 \\ -8 \\ +9 \\ \hline 9 \end{array}$   $\begin{array}{r} 26 \\ -17 \\ \hline 9 \end{array} = \begin{array}{r} 20 \\ -10 \\ +9 \\ \hline 9 \end{array}$
3. Find the mistakes. Circle them with a red pencil.
- a.  $\begin{array}{r} 21 \\ -9 \\ \hline 12 \end{array} = \begin{array}{r} 16 \\ -2 \\ +6 \\ \hline 12 \end{array}$  b.  $\begin{array}{r} 21 \\ -9 \\ \hline 24 \end{array} = \begin{array}{r} 20 \\ -2 \\ +6 \\ \hline 24 \end{array}$
4. a. Who has more balloons? Jackie  
How many more? 3  
Number sentence  $11 - 8 = 3$
- b. How many more is 18 than 7? 11  
Number sentence  $18 - 7 = 11$
- c. How many more is 60 than 40? 20  
Number sentence  $60 - 40 = 20$
5. Fill in the missing numbers. The difference between each number is 4.
- 0, 4, 8, 12, 16, 20

Pre-Assessment - Part 2 - Worksheet 1 BUWV

1. Solve each problem a different way. Examples:  $\begin{array}{r} 62 \\ -37 \\ \hline 25 \end{array} = \begin{array}{r} 25 \\ -25 \\ +0 \\ \hline 25 \end{array}$   $\begin{array}{r} 62 \\ -37 \\ \hline 25 \end{array} = \begin{array}{r} 30 \\ -20 \\ +5 \\ \hline 25 \end{array}$
2. Solve each problem a different way.  $\begin{array}{r} 35 \\ -28 \\ \hline 7 \end{array} = \begin{array}{r} 15 \\ -10 \\ +9 \\ \hline 7 \end{array}$   $\begin{array}{r} 35 \\ -28 \\ \hline 7 \end{array} = \begin{array}{r} 15 \\ -10 \\ +8 \\ \hline 7 \end{array}$   $\begin{array}{r} 35 \\ -28 \\ \hline 7 \end{array} = \begin{array}{r} 5 \\ -5 \\ +9 \\ \hline 7 \end{array}$
- Take out Base Ten Blocks. You may use them for any problem from here to the end of the test.
3. a. Take out a 100 flat. Cover it with 43.  
How much is not covered? 57
- b. Build 132. Cover it with 67.  
How much is not covered? 65
4. Use Base Ten Blocks to solve the problems. Draw the answer. Write the problem shown by the picture.
- a. Start With: Take Away: What's Left?: Example:  $\begin{array}{r} 47 \\ -28 \\ \hline 19 \end{array}$
- b. Start With: Take Away: What's Left?: Now, you write the problem and solve it.  $\begin{array}{r} 80 \\ -28 \\ \hline 52 \end{array}$

Pre-Assessment - Part 2 - Worksheet 2

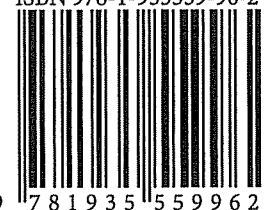
5. Solve each problem two ways. First do it the expanded way. Then do it the short way.
- a. Expanded way  $\begin{array}{r} 50 \\ -32 \\ \hline 18 \end{array} = \begin{array}{r} 40 \\ -30 \\ +8 \\ \hline 18 \end{array}$  Short way  $\begin{array}{r} 50 \\ -32 \\ \hline 18 \end{array}$
- b. Expanded way  $\begin{array}{r} 86 \\ -39 \\ \hline 47 \end{array} = \begin{array}{r} 70 \\ -30 \\ +7 \\ \hline 47 \end{array}$  Short way  $\begin{array}{r} 86 \\ -39 \\ \hline 47 \end{array}$
6. Use Base Ten Blocks to solve these problems. Draw the answer in the What's Left? box.
- a.  $463 - 172 = 291$
- b.  $287 - 168 = 119$
7. Solve each problem. Record the expanded way and the short way.
- a. Expanded way  $\begin{array}{r} 347 \\ -156 \\ \hline 191 \end{array} = \begin{array}{r} 300 \\ -100 \\ +40 \\ -50 \\ +7 \\ -6 \\ \hline 191 \end{array}$  Short way  $\begin{array}{r} 347 \\ -156 \\ \hline 191 \end{array}$
- b. Expanded way  $\begin{array}{r} 435 \\ -279 \\ \hline 156 \end{array} = \begin{array}{r} 300 \\ -200 \\ +30 \\ -70 \\ +5 \\ -9 \\ \hline 156 \end{array}$  Short way  $\begin{array}{r} 435 \\ -279 \\ \hline 156 \end{array}$

Post-Assessment is the same as Pre-Assessment - Part 2

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