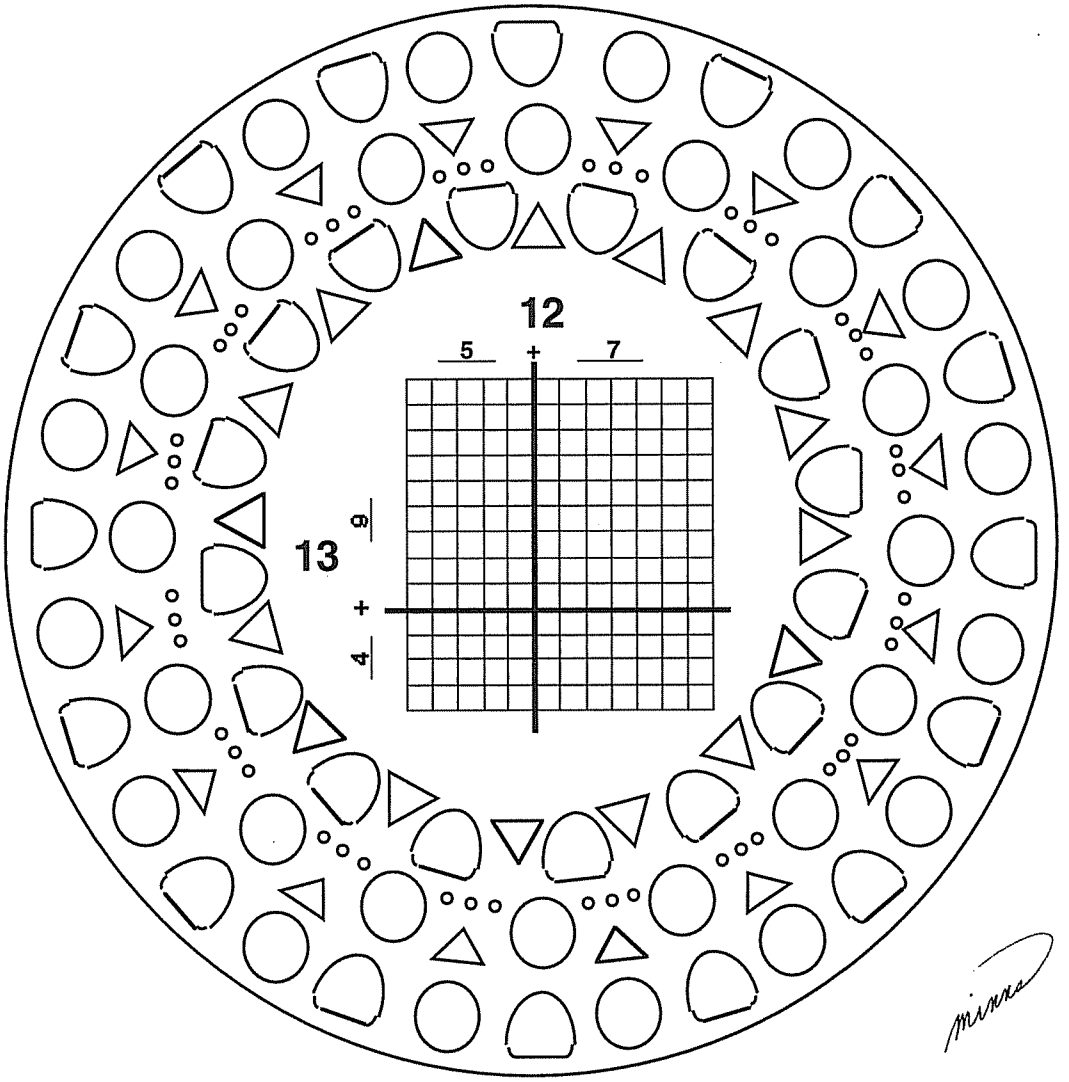
Patterns in Arithmetic

Multiplication - Booklet 4 PDF

Working with Large Numbers and Decimals

Parent/Teacher Guide



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Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Multiplication: Booklet 4 - PDF - Working with Large Numbers and Decimals

Contents

Purchase *Key to Decimals:* Books 1 and 2 to use concurrently

Acknowledgment

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication. This booklet is dedicated to all the undergraduate students who took the class "Creative Learning in Children" at UC Irvine. Your talents enriched the learning of students and teachers alike at the Farm School. Together we learned how to be better teachers.

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Assessment Guide

Purpose

The purpose is to determine if this booklet is the correct starting place. Pre-Assessment: Part 1 assesses the fundamental knowledge necessary for success in this booklet.

Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Patterns in Arithmetic: Multiplication - Booklet 3, or previous instruction in the basic concepts of the Distributive Property of Multiplication as it applies to single digit multiplication, for example 345 x 6. Also, the student must have had work on the use of the Expanded Tables, for example 3 x 4 =12, 30 x 4 = 120, and 300 x 4 =1,200. And Multiplication - Booklet 3 on Factoring, or other instruction in prime numbers and prime factoring.

Note

Knowing the standard procedure for multiplication we all learned is not the same thing as understanding the Distributive Property of Multiplication. Understanding this property is key to understanding both multiplication and division. See Introduction to the Distributive Property of Multiplication in this teacher guide.

Materials

Multiplication: Booklet 4 - Pre-Assessment - Worksheets 1 - 3, pages 53 - 55 in this booklet and pages ii - iv in the Student Workbook
Post-Assessment, pages 56 and 57 in this booklet and pages 72 and 73 in the Student Workbook
Cuisenaire Rods
Colored pencils or crayons

Instructions

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If the student is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment

Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.
Score sheets that match the Assessment Guide for Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow.

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Assessment Criteria for Pre-Assessment: Part 1

Can the student

1. Short Notation (Multiplication: Booklet 2)

A. use the standard short notation for multiplication and get two of three answers correct?

You are looking to see if the student knows the standard short procedure for multiplication. If he uses addition, you can assume he does not know how to use the multiplication procedure. Students who have completed Multiplication: Booklet 2 may do the problems the long way. Point out the instruction to use the short way. He can use the long way on item 2.

2. Expanded Multiplication (Multiplication: Booklet 2)

A. write the partial products on each line in two of the three problems? Partial products for problem a would be twenty-four and one hundred eighty. The student may write twenty-four and eighteen and still get a Yes on this item. It means he knows what to do but made a place value error, which is common at this point. You are looking to see if he knows what to do, even if there is an error in calculation.

B. write the correct partial products with the correct place value in two of the three problems? Partial products for problem a would be twenty-four and one hundred eighty. The order he writes them in is not important.

C. sum the partial products to get a final answer in two of the three problems? The answer may be incorrect if the student used eighteen instead of one hundred eighty. If he summed his partial products he gets a Yes on this item. It shows he knew to add to complete the process.

D. sum the partial products and get the correct answer in two of the three problems?

3. Expanded Tables and Breaking Up Times (Multiplication: Booklet 2)

A. break up the fifty-seven into 50 + 7 on the top line and fill in x 3 correctly?

B. show the correct little multiplication problems on the second line?

C. show the partial products on the third line? Give a Yes even if he made a multiplication error. It shows he knows what to do.

D. add to get the final product? Give a Yes if the addition was done even if there is an arithmetic error. He may need to review his tables.

E. get the correct final product?

4. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain and/or show with an example what a factor is?

2 Points: A factor is a number that results in a product (answer) when multiplied by another factor. Example: 2 x 3 = 6. Two and three are factors of six.

1 Point: A factor is a number used to make an answer such as 2 x 3 = 6.

5. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain what a prime number is?

2 Points: A number that has only itself and one as factors.

1 Point: A number that can only be made one way (referring to the rectangular arrays drawn in the lesson).

6. Prime Factoring (Multiplication: Booklet 3)
   A. correctly prime factor 120?
   B. place the prime factors in order from largest to smallest in the box?

7. Factoring by Tens (Multiplication: Booklet 3)
   A. correctly factor 120 by factors of ten? Answer can be 12 x 10 x 10 or 12 x 100

8. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   A. give the correct product at the bottom as 6,300?
   B. show that the 700 x 9 is factored into (7 x 100) x 9 in the second line?
   C. show the reordering of the factors to combine the (7 x 9) x 100 in the third line?
   D. show 63 x 100 in the fourth line?

Booklet Selection Guide based on results of Pre-Assessment: Part 1

To begin Multiplication: Booklet 4, the student must receive a Yes on either 1A or 2A – 2D. If both of these are a No, the student is not ready for Multiplication: Booklet 4, and should begin in Multiplication: Booklet 2. Do not give Pre-Assessment: Part 2.

If the student received 15 or more Yes marks on Pre-Assessment: Part 1, give Pre-Assessment: Part 2 and begin Multiplication: Booklet 4.

For the students who did *Patterns in Arithmetic*: Multiplication - Booklets 2 and 3 previously:

a. If he receives between 12 and 14 Yes marks, give Pre-Assessment: Part 2 and remediate the places that are weak using the pages in the early sections of Multiplication: Booklet 4. You may also want to reteach items in the earlier booklets. Begin Multiplication: Booklet 4 carefully.

b. If the score is 11 or less, go back to Booklet 2 and reteach the Expanded Multiplication using Base Ten Blocks and tens and ones notation only and move to the short notation and practice. Allow him to use the zero pattern to find answers and reteach factoring by tens. Reteach prime numbers and prime factoring. In Multiplication: Booklet 4, teach only the tens and ones way of breaking up the numbers. Skip the nonstandard ways of breaking up numbers. Consider skipping the factoring and let him use the zero patterns to get partial products. Do not go to the short notation until he can do it the long way with two digit numbers times two digit numbers. Then you can teach the short way for longer problems.

For students new to *Patterns in Arithmetic*:

a. If the student receives a Yes on 1A and 3E and a No on criteria in section 2A - 2D and 3A - D, strongly consider beginning with Multiplication: Booklet 2 in the section on Breaking Up Times. The short delay here will pay off in the long run. There is a review in Multiplication: Booklet 4, but it is not a good place to teach the concept of distribution. But if time is a factor and your student is in fifth grade or higher, you can begin the teaching of distribution in Multiplication: Booklet 4, but the results will not be as strong.

b. If the student receives a Yes on 2A - 2D, 3A - D, and No on 5 - 8, begin with Multiplication: Booklet 3 on prime numbers, prime factoring, and factoring in general. These are important tools

students need in Multiplication: Booklet 4 and in fractions.

Assessment Criteria for Assessment: Part 2

All criteria in this section come from Multiplication: Booklet 4.

Can the student:

1. Multiplying Factors of Ten

A. get the correct product?

B. give a clear and correct explanation of how he got his answer?

3 points: I factored out the four from the one hundred and the three from the ten. Then I rearranged the factors and combined the 4 x 3 to get twelve, and the 100 x 10 to get one thousand. Then I multiplied 12 x 1,000 to get twelve thousand.

2 points: I took the four off of the one hundred and the three off the ten. Then I multiplied the 4 x 3 and added the three zeros from the one hundred and the ten.

1 point: I multiplied 4 x 3 and added three zeros.

2. Factoring by Tens

A. use a factor tree to show how the final product is obtained? Give a Yes even if the student did not use factors of ten but other sets of factors including prime factors.

B. use numbers in the factor tree that are factors of ten? The student should not use any numbers other than four, seven, ten, and one hundred in the factoring.

C. obtain the correct final product? The correct answer may appear without the factor tree being filled in.

3. Factoring by Tens

A. explain how the adding the zeros trick works?

3 points: It works because of factoring. The Associative Property of Multiplication allows the rearrangement of the factors in any combination. So you multiply 9 x 5 and then the 100 x 100. The four zeros come from multiplying 100 x 100 to get 10,000, which has the four zeros.

2 points: It works because of factoring. So you multiply 9 x 5. The four zeros come from multiplying 100 x 100.

1 point: The four zeros come from multiplying the hundreds.

If he gets a Yes on criterion 4a and only 1 point on 4b, followed by No on criteria 5A, 5B, and 6A, it means he memorized the trick with no understanding.

4. Breaking Up Times

Most students will be unable to do this item on Pre-Assessment: Part 2. On the Post-Assessment we are looking to see if he can look at the graphic and see that the numbers are being broken up into tens and ones, the standard method. Then we want to see if he can correctly fill in the distribution number sentence to match the diagram.

A. fill in the numbers 10 + 2 and 10 + 3 on the diagram?

B. fill in the top line of the distribution number sentence correctly?

C. show all the little multiplication problems correctly in the second line?

D. obtain the correct partial products in the third line?

E. write in the correct answer on the dark line? This answer may be correctly done using a memorized short procedure. Give credit for a correct answer with a Yes on this criterion.

5. Using Arrow Patterns to Multiply Tens and Ones

Most students will be unable to do this item on the Pre-Assessment. We are looking to see if the

student understands the arrow pattern (distribution) tested by the use of nonstandard numbers. The nonstandard numbers will confuse a student who has only memorized the short procedure.

A. break up the thirteen using nonstandard method of nine and four instead of ten and three?

B. draw in the arrow pattern correctly?

C. write all the small multiplication problems and partial products correctly?

D. obtain the correct final answer using this nonstandard procedure only?

6. Breaking Up Times

A. explain why it makes sense that the problems would have the same answers?

2 points: Even though the problems were broken up differently, the answers will still be the same because it is the same problem both times.

1 point: Yes, because it is the same problem both times.

7. Breaking Up Times: Long Way

A. write down all the correct little multiplication problems on the lines in the left hand column? Order is not important.

B. show the correct place value of each partial product in the right hand column?

C. obtain the correct answer?

8. Standard Way

This problem is solved using the procedure we all learned in school. If the student can get the correct answer for this item and not for criteria 4 and 7, then he can do the problem but does not understand the procedure.

A. use the standard procedure? Even if the final answer is incorrect, does it appear that he is using the correct procedure?

B. place a zero or placeholder in the second row of the partial products below the line?

C. obtain the correct answer?

9. Short Notation

A. give a clear explanation as to why there is a zero in the second row of the partial product?

3 points: The zero is there because the four is in the tens place, so you are really multiplying forty times five, which is two hundred, which has a zero in the ones place.

2 points: The zero is there because you are multiplying by a number in the tens place.

1 point: Because it's tens.

0 points: It is a placeholder. Or, you have to do it to get the right answer.

Booklet Selection Guide based on results of Pre-Assessment: Part 2

If he scores 24 or more points, he does not need to do this book.

If he scores 21 - 23 points, remediate the weak answers and move on.

If he scores 20 or less, this is the correct booklet for the student.

Pre-Assessment: Part 1 **Score Sheet** Name _____ Date _____

Can the student:

1. Short Notation (Multiplication: Booklet 2)
   - Yes No A. use the standard short notation and get two of three answers correct?

2. Expanded Multiplication (Multiplication: Booklet 2)
   - Yes No A. write the partial products on each line in two of the three problems?
   - Yes No B. write the correct partial products with the correct place value in two of the three problems?
   - Yes No C. sum the partial products to get a final answer in two of the three problems?
   - Yes No D. sum the partial products and get the correct answer in two of the three problems?

3. Expanded Tables and Breaking Up Times (Multiplication: Booklet 2)
   - Yes No A. fill in the top line correctly?
   - Yes No B. show the correct little multiplication problems on the second line?
   - Yes No C. show the partial products on the third line?
   - Yes No D. add to get the final product?
   - Yes No E. get the correct final product?

4. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 2, 1, 0 points A. explain and/or show with an example what a factor is?

5. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 2, 1, 0 points A. explain what a prime number is?

6. Prime Factoring (Multiplication: Booklet 3)
   - Yes No A. prime factor 120 correctly?
   - Yes No B. place the prime factors in order in the box?

7. Factoring by Tens (Multiplication: Booklet 3)
   - Yes No A. factor 120 by factors of ten correctly?

8. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct product at the bottom as 6,300?
   - Yes No B. write (7 x 100) x 9 in the second line?
   - Yes No C. show the reordering of the factors in the third line?
   - Yes No D. show 63 x 100 in the fourth line?

Items Correct = ____ = _____%
Items Possible = 21

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Multiplying Factors of Ten
   - Yes No A. get the correct product?
   - 3, 2, 1, 0 points B. give a clear and correct explanation of how he got his answer?

2. Factoring by Tens
   - Yes No A. use a factor tree to show how the final product is obtained?
   - Yes No B. use numbers in the factor tree that are factors of ten?
   - Yes No C. obtain the correct final product?

3. Factoring by Tens
   - 3, 2, 1, 0 points A. explain how the adding the zeros trick works?

4. Breaking Up Times
   - Yes No A. fill in the numbers 10 + 2 and 10 + 3 on the diagram?
   - Yes No B. fill in the top line of the distribution number sentence correctly?
   - Yes No C. show all the little multiplication problems correctly?
   - Yes No D. obtain the correct partial products in the third line?
   - Yes No E. write in the correct answer on the dark line?

5. Using Arrow Patterns to Multiply Tens and Ones
   - Yes No A. break up the thirteen using the nonstandard method?
   - Yes No B. draw in the arrow pattern correctly?
   - Yes No C. write all the multiplication problems and partial products correctly?
   - Yes No D. obtain the correct answer using the nonstandard procedure only?

6. Breaking Up Times
   - 2, 1, 0 points A. explain why the problems would have the same answers?

7. Breaking Up Times: Long Way
   - Yes No A. write correctly little multiplication problems in the left hand column?
   - Yes No B. show the correct place value of each partial product in the right hand column?
   - Yes No C. obtain the correct answer?

8. Standard Way
   - Yes No A. use the standard procedure?
   - Yes No B. place a zero or placeholder in the second row below the line?
   - Yes No C. obtain the correct answer?

9. Short Notation
   - 3, 2, 1, 0 points A. give a clear explanation as to why there is a zero in the second row?

Items Correct = _____ = _____%

Items Possible = 30

Post-Assessment Score Sheet

Name _____ Date _____

Can the student:

1. Multiplying Factors of Ten
   - Yes No A. get the correct product?
   - 3, 2, 1, 0 points B. give a clear and correct explanation of how he got his answer?

2. Factoring by Tens
   - Yes No A. use a factor tree to show how the final product is obtained?
   - Yes No B. use numbers in the factor tree that are factors of ten?
   - Yes No C. obtain the correct final product?

3. Factoring by Tens
   - 3, 2, 1, 0 points A. explain how the adding the zeros trick works?

4. Breaking Up Times
   - Yes No A. fill in the numbers 10 + 2 and 10 + 3 on the diagram?
   - Yes No B. fill in the top line of the distribution number sentence correctly?
   - Yes No C. show all the little multiplication problems correctly?
   - Yes No D. obtain the correct partial products in the third line?
   - Yes No E. write in the correct answer on the dark line?

5. Using Arrow Patterns to Multiply Tens and Ones
   - Yes No A. break up the thirteen using the nonstandard method?
   - Yes No B. draw in the arrow pattern correctly?
   - Yes No C. write all the multiplication problems and partial products correctly?
   - Yes No D. obtain the correct answer using the nonstandard procedure only?

6. Breaking Up Times
   - 2, 1, 0 points A. explain why the problems would have the same answers?

7. Breaking Up Times: Long Way
   - Yes No A. write correctly little multiplication problems in the left hand column?
   - Yes No B. show the correct place value of each partial product in the right hand column?
   - Yes No C. obtain the correct answer?

8. Standard Way
   - Yes No A. use the standard procedure?
   - Yes No B. place a zero or placeholder in the second row below the line?
   - Yes No C. obtain the correct answer?

9. Short Notation
   - 3, 2, 1, 0 points A. give a clear explanation as to why there is a zero in the second row?

Items Correct = _____ = _____%

Items Possible = 30

Introduction to the Distributive Property of Multiplication

Why does this booklet take so long to teach such an easy operation as long multiplication? Why all the square grids and crayons? Why do I see pages with the same problem over and over? Why not do it the easy way and get it done in less than a week?

$$\begin{array}{r} 46 \\ \times\ 23 \\ \hline \end{array}$$

Remember this? You probably learned this back in fourth or fifth grade. It was pretty easy for most of us if we knew our times tables. The talk went something like this: Multiply the three by the six to get eighteen. Put down the eight and carry the one. Then multiply the three by the four to get twelve and add one to get thirteen. Put the thirteen to the left of the eight to get one hundred thirty-eight. Then begin a second row. Put a zero in the ones place. Then multiply the two by the six to get twelve. Put the two to the left of the zero and carry the one. Then multiply the two by the four to get eight, add the one to get nine. Put the nine to the left of the two to get nine hundred twenty. Add this to the one hundred thirty-eight in the top row to get one thousand fifty-eight. Easy. Most likely it took less than a week for you to master this. It was easy because you were taught the shortcut–the procedure. No conceptual understanding is needed until you got to algebra.

This shortcut, taught as a memorized recipe, completely obscures one of the most important patterns in arithmetic, the Distributive Property of Multiplication. The shortcut also obscures the place value of the numbers being multiplied and their products. When I ask most incoming students why you put a zero in the ones place of the second row, most say, "That's what my teacher told me to do." Or "It's a placeholder." When asked what the placeholder is for, most students say, "The teacher told me that's what you do." Most have no idea the zero is there because they are really multiplying twenty times six, not two times six. So what? What difference does it make? They get the right answer, don't they?

For most students, the lack of understanding of this fundamental pattern results in general weakness in mathematics overall and vastly increases the chances they will not do well in algebra, higher mathematics, or advanced science courses. Little is gained by teaching the shortcut only, yet we stand to lose much in the long run, including careers in the scientific and mathematical fields.

$$(a + b)(c + d) = ac + ad + bc + bd$$

Remember this? If you took algebra in ninth grade, you were taught this. If you are fluent in algebra you recognize this as a multiplication of a polynomial. Did you learn how to do it by memorizing FOIL–first, outer, inner, last? If you did, then you were not taught how to understand the principles of mathematics but taught only another recipe to be memorized.

If you are not fluent in algebra, maybe you sort of remember seeing this in ninth or tenth grade. If you did not do very well in algebra, there is a good chance that not understanding how 46 x 23 works is a major cause of your troubles (that and not understanding fractions).

The great sadness of this is that mathematics is really a very interesting subject that many of us learn to hate or fear because it makes no sense and we fail tests. It makes no sense because we were not given the time or the tools to construct understanding when we were children. Mathematics is a mental

banquet of ideas and tools. We bring students to the banquet and then only let them lick the used plates in the kitchen. All the fun, the power, and the discovery are taken from them in the name of getting it done quickly in time for the tests or simply because the teacher does not know any better.

This booklet will help you correct this loss in yourself and prevent it in your student.

Here is a preview of the connection.

First, in Multiplication: Booklet 2 of this series, we looked at problems like this: 46 x 3 = 138

46 x 3 is equal to 3 x 46. The forty-six can be broken up into 3 x (40 + 6). Each part inside the parentheses is multiplied by the three to get 120 + 18. The sum of 120 and 18 is the answer. The forty-six does not even have to be broken up into tens and ones. It can be broken up any way you want to, but the rule of multiplying each of the parts by the multiplier (the three in this case) is always the same no matter what the number is. That is why we can show the general pattern with letters: a(b + c) = ab + ac. The a is the multiplier, and the b and c are a larger number broken up into parts. The Distributive Property states you will get the right answer if the larger number is broken up into parts, then each part is multiplied by the three and then results are summed (added). The three is distributed to both the forty and the six. If it is put in a vertical format you will recognize it.

46	40 + 6	B + C
x 3	x 3	x A
138 =	120 + 18	AB + AC

Now, if another digit is added to the bottom number,

$$\begin{array}{r} 46 \\ \times\ 23 \end{array}$$

there is a new twist to the Distributive Property. The bottom number must be broken up also since I do not know the twenty-three times table.

Step 1 is the same as before:

46	40 + 6	B + C
x 3	x 3	x A
138 =	120 + 18	AB + AC

Step 2 requires that the 40 + 6 be multiplied by twenty. The twenty-three had to be broken up into twenty plus three. I already multiplied by the three. Now the twenty must be distributed to both the forty and the six. More attention must be paid to place values. The reason for the zero in the second row is easy to see in the center arithmetic problem: 20 x 6 = 120. There is the zero the teacher told you to put in as a 'placeholder' in bold print below. It is needed because you are multiplying by a number in the tens place. If you understand that you do not need to worry about the placeholder, you will put the zero in the ones place automatically.

46	40 + 6	B + C
x 23	x 20 + 3	D + A
138	120 + 18	AB + AC
+920	800 + 120	DB + DC
1,058	800 + 120 + 120 + 18	AB + AC + DC + DB

It is sad that many math teachers do not realize that the basic arithmetic problems on the left and the algebraic one on the right are the same procedure. Algebra is just the general pattern written with letters to stand for any number you want to use. If the arithmetic is understood in fourth grade, the algebra will be understood in eighth or ninth grade. It isn't necessary to memorize it. It will be simply and delightfully clear.

Most of us simply memorized the shortcut and then, when the same kind of problem was encountered in algebra, we did not recognize it and had to memorize it again as if it were something new. Those of us who memorized, or tried to memorize, our way though algebra did not do very well when we had to do word problems or use the pattern in an unfamiliar form or use it to divide. Higher mathematics and science classes became huge hurdles. This is very unfortunate because, while understanding takes a little longer to develop, it is not that hard. Most students not only get it, they enjoy it.

One of the big, big ideas in arithmetic is that when adding large numbers, the big addition problem is broken up into a bunch of little addition problems and then added up at the end.

In a problem such as

645

+ 324

the ones are added first, then the tens, and so on. In other words, the problem is broken up into ones, tens, and one hundreds, and each part is added. Then all the parts are pushed together to get the answer. Carrying as you go is a shortcut of this big idea. This is done in subtraction, multiplication, and division. All arithmetic procedures work this way. There are rules for how the numbers can be broken up and recombined, but the big idea is always the same. A student who understands this big pattern has power to do problems in multiple ways and thus has more mathematical strength overall than a student who simply memorizes recipes. She also tends to like math better because it makes sense.

In multiplication and division, the guiding rule for breaking up large numbers to multiply or divide them is the Distributive Property.

Understanding the Distributive Property of Multiplication is fundamental to algebra. Not understanding it is one of the major reasons students fail in algebra. Why would you set students up to fail when learning to understand this critical concept is not that hard? It will take only a few extra weeks.

That is why this book has all those drawings, blocks, and crayons. In the end, your student will not

only be able to solve multiplication problems, she will understand how the general idea works and will then be able to apply it to the general algebraic form later.

This book teaches the general pattern of distribution using small numbers like 12 x 13 shown in a square grid array where students can count the number of squares to confirm the answer. The student then already knows the answer is one hundred fifty-six. She will break up the twelve and the thirteen in multiple ways and use both simple areas of small rectangles and the Distributive Property to get the answer. She will know the pattern works because she will always get the same answer, one hundred fifty-six, which she can confirm for herself. She stays connected to the concrete every step of the way. It is only when the pattern is understood concretely that it is extended to the abstract. Then the shortcuts, called algorithms, are looked for.

In addition to learning to understand a critical concept in mathematics, she will also have fun. Most students really enjoy this book. It makes sense all the way and uses lots of fun colors. Enjoy.

Multiplying Factors

Purpose The purpose is to remind the students of the Associative Property of Multiplication that allows us to factor numbers and recombine those factors in any way we wish. This property underpins many arithmetic procedures to come, including multiplication of large numbers and decimals, simplification of fractions, and scientific notation. For many students, this booklet is the opening booklet in the fall of fifth grade. The reviews of factoring and distribution will brush up skills.

Prerequisites *Patterns in Arithmetic:* Multiplication - Booklet 2, and Basics of Distribution in Multiplication: Booklet 3 - Factoring

Note Many students who already know the procedures for multiplication of large numbers do not understand or even know about distribution. The Distributive Property of Multiplication looks like this in algebra a(b+c), (a+b)(c+d) for multiplication and like this in division: $\frac{a+b}{a}$

A student who gives the answer b because the a's cancel out doesn't understand division. This lack of understanding causes students to, unnecessarily, fail to do well in algebra. If your student does not understand distribution, now is the easiest time to address it. It is time well spent.

Materials Multiplying Factors - Worksheets 1 and 2, pages 1 and 2

Warm Up **"What is 4 x 8?"** "Thirty-two."

"What are the prime factors of four?" "Two and two."

"What about eight?" "Two times two times two."

"What will happen if all of those twos are multiplied together? Will you still get thirty-two?" Answers will vary.

"Try it." "(2 x 2) x (2 x 2 x 2) = 32. It is the same as 4 x 8."

"What would happen if we did not use the prime factors of eight? What if we just used 4 x 2 and multiplied that by 2 x 2 from the four? Would it change the answer?" Answers will vary.

An answer that demonstrates understanding is "No, it will not change the answer because when you factor numbers you can multiply all the factors back together and get the same answer." Most students will have to try it to be sure.

"What if we changed the order of the factors and put the four first and then all the twos? Would we still get the same answer?" "Yes."

Repeat this process with 6 x 8. Try factoring them with (2 x 3) x (2 x 4) and change the order. Try 4 x 3 x 2 x 2 and (2 x 2) (4 x 3). The product does not change. **"What property of multiplication are we seeing operating here?"** "Associative."

Lesson

Record the first two examples on Multiplying Factors - Worksheet 1, page 1. In the 6 x 8 example, show the factors of six right under the six and those of the eight right under the eight. Then on the third line, reorder the numbers. Change the order like you did in the warm-up. Many students do not see the point of this, but it drives the idea in that the order does not matter and that factoring numbers and recombining them produces correct products that they know to be correct from experience. It also teaches them how to get the answers to multiplication facts they can not remember by factoring them down.

Have the student complete Multiplying Factors - Worksheet 1. Check the answers. If you are working with a group, have students compare the different solutions and combinations.

Note

This sureness is needed so when very large numbers are factored, such as 8,000 x 600, students will be confident that they are getting the correct product.

Practice Worksheet

Multiplying Factors - Worksheet 2, page 2

Test for Understanding

"Imagine that you can not remember the answer to nine times ten. Show more than one way to figure it out."
Possible solutions: (10 x 3) x 3, (2 x 3 x 3) x 5, which could be solved by counting by fives.

Breaking Up Times: Review

Purpose

The purpose of this lesson is to review the Distributive Property of Multiplication with small numbers and a manipulative. Each problem is done three different ways so the student can both self-confirm the answer and demonstrate the general process of distribution. The student pages repeat some of the pages he encountered in Multiplication: Booklet 2.

Note

Teacher preparation is needed to teach this lesson. This is especially true if the teacher is unfamiliar with Multiplication: Booklet 2.

Prerequisites

Multiplication: Booklet 2, Lessons on Arrays: Manipulative, Arrays: Recording, Arrays to Boxes, and Distributive Property. If your student did not do Multiplication: Booklet 2, it is highly recommended that he does it before he does this booklet. It is possible he can pick up the concepts from the short review sections in this booklet. If he does not pick them up easily, consider doing the relevant lessons in Multiplication: Booklet 2 before going on to Breaking Up: Two Ways.

Materials

Breaking Up Times: Review - Worksheets 1- 4, pages 3 - 6
Cuisenaire Rods
Crayons
Index card

Note

If the student already knows how to do single digit multiplication, tell him you are going to do it a new way, which he will do when he goes to algebra in a few years. Ask him to tell you when he sees the connection between this new way and his old way. Most students have memorized the short form and do not see or understand the distribution being used to run the procedure.

Warm Up

Allow play time with the Cuisenaire Rods the day before the lesson. Make a chart of the length of each rod by color if he is not highly familiar with the rods.
To begin, have him build three different trains of rods (rods placed end-to-end) equal to fourteen. Ask him to make a two-rod train, a three-rod train and a four-rod train.
"Do they all equal fourteen?" "Yes."

"Write an addition number sentence for each train you built." For example: a three-rod train made of dark green, yellow, and light green would be written as $6 + 5 + 3 = 14$.

Before he opens the workbook, have him build the problem shown at the top of Breaking Up Times: Review - Worksheet 1, page 3.
"Build a three-rod train that equals fourteen with a dark green, a yellow and a light green in that order. What is the addition number sentence for this train?" "$6 + 5 + 3 = 14$."

Lesson

"Now build three more fourteen trains exactly like the one you already have. What multiplication problem do you have here?" "Fourteen times four."

"What small multiplication problem is shown by the four dark greens?" "Six times four."

"What small multiplication problem is shown by the four yellows?" "Five times four."

"What small multiplication problem is shown by the four light greens?" "Three times four."

"Add up (six times four) + (five times four) + (three times four). What do you get?" "Fifty-six."

"What is the answer to a multiplication problem called?" "The product."

"Take out Breaking Up Times: Review - Worksheet 1. Using the index card, cover all but the topmost line of squares."

Have him separate the top fourteen train from the others. The top line shows how the fourteen is broken up. Have him color in the top line the same colors as the rods. Then remove the index card. Color the rectangle labeled 6 x 4 dark green. Color the rectangle labeled 5 x 4 yellow. Color the rectangle labeled 3 x 4 light green. Point out that in the example, the small multiplication problems are written in ( ) under the colored rectangles and the products of each written on the line below. The final product is written on the left. It is fifty-six.

Repeat the procedure for the next problem. This time he is asked to build a fourteen train with two rods, the first of which is a dark blue nine rod. Have him build a fourteen train with a blue rod and a yellow rod. Again, cover all but the topmost line of squares with the index card. Have him fill in the number sentence (9 + 5) x 4. It is written x 4 because we are going to multiply this train four times. Have him build it with four blue rods and four yellow rods. Each color must be in a rectangle as shown in the picture. Have him color in the rectangles.

"What multiplication problem is shown by the four blue blocks?" "9 x 4." He writes that under the blue rectangle.
"What is the product of 9 x 4?" "Thirty-six." He writes that underneath the (9 x 4). Repeat with the yellow block. Use the Answer Key to be sure he is filling in the numbers correctly.

The last problem asks him to fill in the numbers in the number sentence. Cover all but the top line of boxes so he can see the numbers. They are 5 + 2 + 6 + 1 = 14. Have him color in each rectangle with the color that matches the rod color. Then he labels each small multiplication problem and sums them to get the answer.

Be sure he answers the questions at the bottom of the page. End the lesson here if you have been going for more than thirty minutes. Continue the next day.

Practice Worksheet

Breaking Up Times: Review - Worksheets 2 and 3, pages 4 and 5

Note

The little smiley face is for him to estimate the product before he does the problem.

Have him do Breaking Up Times: Review - Worksheet 4, page 6 by himself. Require that he write in all the parts and color the rectangles. He must draw in the lines between the rectangles in the picture. The answers are in the Answer Key.

Test for Understanding

The Challenge is the true test of understanding. If he can not do the problem, have him build a three-rod thirteen train different from the one at the top of the page. Have him see if he can move from the rods to the top number sentence and then fill in the multiplication problems. If he can, you can go on to the next lesson.

If he can not, please do the relevant lessons in Multiplication: Booklet 2. This topic is so critical for success in higher mathematics it should not be skipped.

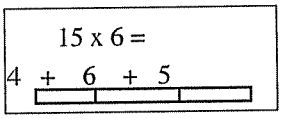
Arrays: Review

Purpose The purpose of this lesson is to review the Distributive Property of Multiplication with small numbers using a graphic array instead of a manipulative to move from the concrete to the representational level. The general process is the same as the previous lesson.

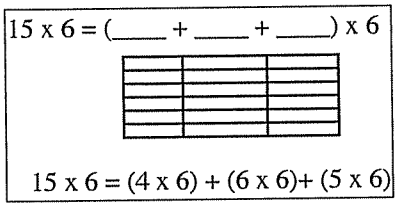
Note Teacher preparation is needed to teach this lesson. This is especially true if the teacher is unfamiliar with Multiplication: Booklet 2.

Prerequisites Breaking Up Times: Review

Materials Arrays: Review - Worksheets 1 and 2, pages 7 and 8
Cuisenaire Rods
A piece of unlined paper
Colored pencils
Index card



Warm Up Write the problem 15 x 6 = in fairly large print at the top of the blank paper. Ask the student to build the 15 with a three-rod train with no repeating colors. Have her place the rods to the right and slightly below the = sign.
Have her write the addition number sentence for the rod train above the train as was done in the last lesson.



Then have her build the other five trains. Example: She writes in the (__x___) under each rectangle with the partial products written under each one. Then sum the total. The more she can do the problem unassisted the better. Have her study the problem she did on Breaking Up Times: Review - Worksheet 2. The problem shown is 17 x 3.

Have a review discussion on what distribution means. Talk about newspaper distribution or distribution of papers in a class.

"What number in the problem 17 x 3 is being distributed? What number is showing up in each little multiplication problem under the arrays?" "The three."

"Which number was broken up into parts?" "The seventeen."

"Which number was multiplied by all those little parts?" "The three."

Note This is how all multiplication and division problems work. We take the large number and break it into parts and multiply or divide all the parts by the small number and add up the totals to get the final answer.
Remind her what an array is. (An array is an ordered set of objects, in this case

squares arranged in rows and columns.) Multiplication problems are often shown graphically as arrays.

Lesson

Review the word, 'iterative': repetition of a series of an operation, e.g., 2 + 2 + 2 + 2. The problem shown on the right of the worksheet is the iterative addition solution of 36 x 7.
Above the array the equal number sentence shown is:

$$36 \times 7 = (6 + \_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_) \times 7$$

Cover all but the top row of the array with the index card. Shade in those first six squares dark green. (Colored pencils work better on these small squares.) She must then break up the rest of the row of thirty-six squares into four other numbers. Each number must be ten or less. Have her color in each section to match the color of the Cuisenaire Rod for the number she chose. Then remove the index card, extend the dividing lines between the colors down through the array, and color each rectangle with the appropriate color. Then fill in the multiplication problem shown by each colored array and its product. The first one has been done. It shows the partial product of forty-two for that part of the array. Check the Answer Key to be sure which numbers could go in the blanks.

The second problem allows her to choose how she wants to break up the thirty-six. She may use 30 + 6. She may not choose any number for which she can not multiply it by seven in her head. Have her fill in the parentheses below and the partial products and sum them to get the answer. All three answers should match. Students who have completed Multiplication: Booklet 2 will have no difficulty with this at all and may not need any assistance to complete these worksheets.

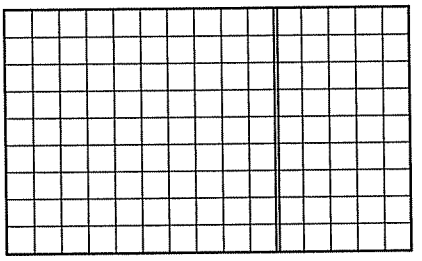
Practice Worksheet Test for Understanding

Arrays: Review - Worksheet 2, page 8

Give her a piece of graph paper. Ask her to draw an array for 15 x 9. Require her to write out the number sentence that goes above the array, the small multiplications below and the partial products.
If she can't, please do the relevant lessons in Multiplication: Booklet 2. This topic is so critical for success in higher mathematics it should not be skipped.

Example:

$15 \times 9 = (10 + 5) \times 9$



$$15 \times 9 = (10 \times 9) + (5 \times 9)$$
$$= 90 + 45$$
$$= 135$$

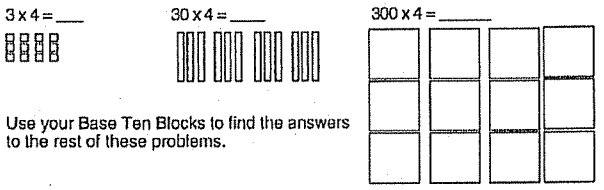
Expanded Multiplication: Review

Purpose The purpose of this lesson is to review the use of the expanded multiplication table in breaking up and multiplying large numbers. The expanded table is 3 x 4 = 12, 30 x 4 = 120, 300 x 4 = 1,200, etc.

Prerequisites Multiplication: Booklet 2 and Expanded Tables Flip Book. If your student did not do Multiplication: Booklet 2, give Expanded Multiplication: Review - Worksheets 4 - 6, pages 22 - 24, which follow this lesson. These pages are not in the Student Workbook. The answers are at the end of the Answer Key.

Materials Expanded Multiplication: Review - Worksheets 1 - 3, pages 9 - 11 (in the Student Workbook)
Expanded Tables Flip Book
Base Ten Blocks

Warm Up Build 3 x 4 = 12, 30 x 4 = 120, 300 x 4 = 1,200 with Base Ten Blocks.



"What changes from problem to problem here?" "The size of the blocks and the place value of the answers."

"What stays the same?" "The number twelve; there are always twelve blocks." Have the student write out the expanded table for the four times table.

1 x 4 = 4	10 x 4 = 40	100 x 4 = 400
2 x 4 = 8	20 x 4 = 80	200 x 4 = 800
3 x 4 = 12	30 x 4 = 120	300 x 4 = 1,200
4 x 4 = 16	40 x 4 = 160	400 x 4 = 1,600
5 x 4 = 20	50 x 4 = 200	500 x 4 = 2,000 etc.

Record all the problems up to 9 x 4.

Note Study the Expanded Multiplication: Review - Worksheet 1 for an example of what this lesson does. Notice that all the little squares in the array are not shown. In its place is just a blank box. The student no longer needs all the little squares. Notice that in the top number sentence: 237 x 4 = (200 + 30 + 7) x 4, the numbers 200 + 30 + 7 are spread out, and the line that would have divided the little squares into groups is placed under the + sign. The little multiplication problems that used to appear in the (__ x __) are done mentally, 200 x 4 is 800. That is written in the box where the squares used to be. This saves time yet allows the student to stay connected to what has gone before as she begins to use the more abstract written forms shown in the middle and bottom of the worksheet.

Lesson

At the top of the Expanded Multiplication: Review - Worksheet 1 is the box or array multiplication.

In the middle the distribution of the four to each piece is written out.

At the bottom, you will see the long way shown. In the long way, the student shows each partial product in reverse order from the way it is shown in the box. The order does not matter in the long way, but it does in the short way. She multiplies the 7 x 4 and writes 28 below the line. Then she multiplies not 3 x 4, but 30 x 4 to get the 120. This is written below the 28. Finally, she multiplies not 2 x 4, but 200 x 4 to get the 800 written below the 120. Doing the problem the long way keeps her in touch with the Distributive process she has learned.

The short way is the way we all memorized in school. Here the distribution is present but is obscured by the carrying. In the short way, we do not think at all about the place value of the numbers we are multiplying. It is easy and not as prone to errors as the long way. That is why we use it. But, now she understands how the process works, the place value of the numbers she is using, in addition to getting the correct answer.

Worksheets

Do Expanded Multiplication: Review - Worksheet 2 together. Check the answer Key to be sure it is done correctly.

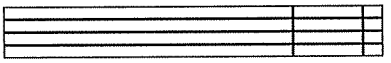
Expanded Multiplication: Review - Worksheet 3, page 11

Practice Worksheets

Practice: Worksheets 1 and 2, pages 12 and 13

Test for Understanding

Give Practice: Worksheet 1, page 12 and say, **"Choose a problem and draw the box and show the distribution, then solve it."**
Example: (Taken from Expanded Multiplication: Review - Worksheet 1)



$$
\begin{aligned}
237 \times 4 &= (200 + 30 + 7) \times 4 \\
&= (200 \times 4) + (30 \times 4) + (7 \times 4) \\
&= \underline{800} + \underline{120} + \underline{28} \\
237 \times 4 &= 948
\end{aligned}
$$

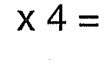
Before going on to the next section, she must understand how to write out the distribution without help.

Expanded Multiplication: Review - Worksheet 4 Date_____

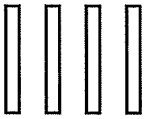
Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the "four times" table.
Use your Base Ten Blocks.

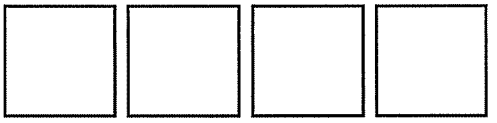
1 x 4 = 4



10 x 4 = 40

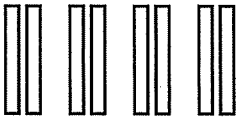


100 x 4 = 400

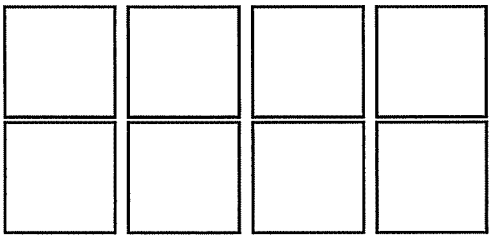


2 x 4 = ___

20 x 4 = ___

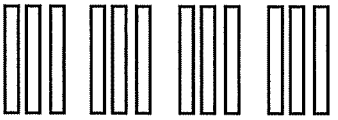


200 x 4 = ____

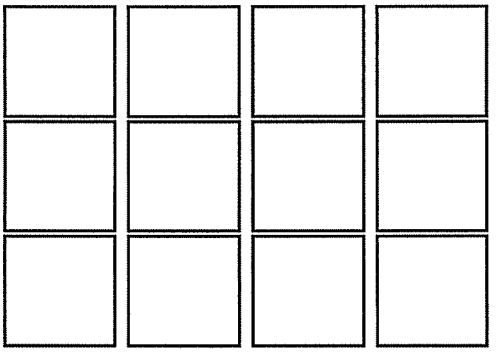


3 x 4 = ___

30 x 4 = ____



300 x 4 = _____



Use Base Ten Blocks to find the answers to the rest of these problems.

4 x 4 = 16

40 x 4 = 160

400 x 4 = 1,600

5 x 4 = ___

50 x 4 = ____

500 x 4 = _____

6 x 4 = ___

60 x 4 = ____

600 x 4 = _____

7 x 4 = ___

70 x 4 = ____

700 x 4 = _____

8 x 4 = ___

80 x 4 = ____

800 x 4 = _____

9 x 4 = ___

90 x 4 = ____

900 x 4 = _____

Expanded Multiplication: Review - Worksheet 5 Date_____

Use Base Ten Blocks and expanded tables from the previous page.

29 x 4 = (20 + 9) x 4

80 + **36**

29 x 4 = _____

38 x 4 = (_____ + _____) x 4

_____ + _____

38 x 4 = _____

57 x 4 = (_____ + _____) x 4

_____ + _____

57 x 4 = _____

79 x 4 = (_____ + _____) x 4

_____ + _____

79 x 4 = _____

Try 56 x 4 = (___ + ___) x 4 You draw the box.

56 x 4 = _____

Expanded Multiplication: Review - Worksheet 6 Date_____

Use Base Ten Blocks and expanded tables to solve these box problems.

Example:

87 x 4 = (80 + 7) x 4

320 +	28

87 x 4 = 348:

$$\begin{array}{r} 87 \\ \times\ 4 \\ \hline 28 \\ 320 \\ \hline 348 \end{array}$$

79 x 4 = (_____ + _____) x 4

_____ +	_____

79 x 4 = _____

$$\begin{array}{r} 79 \\ \times\ 4 \\ \hline \end{array}$$

47 x 4 = (_____ + _____) x 4

_____ +	_____

47 x 4 = _____

$$\begin{array}{r} 47 \\ \times\ 4 \\ \hline \end{array}$$

Breaking Up Times: Two Ways

Purpose The purpose of this lesson is to introduce the procedure for breaking up and distributing both numbers in a multiplication problem. This procedure is needed for working with double digit numbers and for working in algebra with polynomials (multiple numbers and terms). The student works the same problem using different combinations of numbers but always the same pattern that is seen when the Distributive Property is being applied. The lesson uses graphic arrays rather than the blocks as it seems to make the concept easier for students to grasp than do the confusing two-by-two block arrays used with Base Ten blocks.

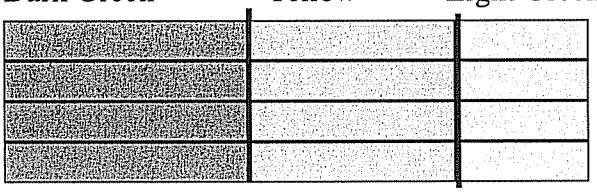
Note Teacher preparation required before this lesson. Study Breaking Up Times: Two ways - Worksheet 1 and follow the lesson instructions yourself so you can understand what your student will do. If working with a group, an overhead will be needed for Breaking Up Times: Two Ways - Worksheets 1 and 2.

Prerequisites Previous lessons

Materials
Breaking Up Times: Review - Worksheet 1, page 3
Breaking Up Times: Two Ways - Worksheets 1 - 7, pages 14 - 20
Colored pencils
Index card
Cuisenaire Rods

Warm Up Rebuild with 14 x 4 with Cuisenaire Rods as shown on Breaking Up Times: Review - Worksheet 1. The blocks will look like this:

Dark Green Yellow Light Green



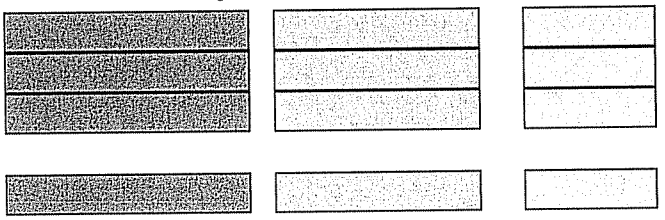
"How is the fourteen broken up?" "Six, five, and three."
When the number being multiplied was only four, it was not necessary to break it up. But we could have.

"In the blocks right now, there are three multiplication problems. List them." "6 x 4, 5 x 4, and 3 x 4."

"When you add all the multiplication products (answers) together, what do you get?" "Fifty-six."

"If I break up the four by doing this (move the blocks so they look like the graphic below), will it change the answer?" "No."

"How do you know?" "Because all you did was move the blocks. It is still the same amount."
"Now how many multiplication problems do you have?" "Six."



"Can you list all six?" "6 x 3, 5 x 3, 3 x 3 and 6 x 1, 5 x 1, 3 x 1."

"If you add up all the products, what will you get?" "Fifty-six."

Give a problem from a previous practice page, 349 x 5, for example. Ask the student what does he think would need to be done if the problem was 349 x 15? Multiple answers are possible. He could add, for one. Many students think of this. An insightful answer would be to find out what 349 x 5 was and multiply that answer by three. He may give the standard procedure answer. If the latter occurs, ask him if he thinks he can do it the long way like he did on the practice sheet. Have him do 349 x 5 the long way; then add on the one in front of the five and ask, **"Now what?"** He may or may not know. If he does know what to do, ask how he broke up the fifteen. Does it have to be broken up into 10 + 5? Is that the only way? (It isn't.) If you are working with a group, you may get all of these answers. Keep the discussion open and admit all answers.

Note

Students who have been taught the standard algorithm may show it to you at this point. Asking him to do it the long way may bring out a defensive response. If he can do it the long way (which is very rare), give a high five and ask if he can write it out using the distributive form used in the middle of the page on Expanded Multiplication: Review - Worksheet 1. If he can, see if he can do it with algebraic terms like (a + b)(c + d). Would the pattern change if the numbers change? If he can do this, the rest of this book can be done with only one page per lesson and skate through very fast just to be sure the bases are covered. The latter situation would be very rare.

Lesson

Begin with Breaking Up Times: Two Ways - Worksheet 2, page 15.
"What multiplication problem is shown in the array?" "12 x 13."

"Do you have any ideas on how to break this up so we could use numbers in the multiplication table to solve it?" Wait and see what happens.

As usual, give him a chance to figure this out on his own from his experience. Some will figure out how to break it up, though only a rare student will be able to fill out the distribution pattern in the lower part of the page.
Decide when to move in and show him what to do.
Turn back a page to Breaking Up Times: Two Ways - Worksheet 1, page 14.
Cover all but the top line of the array with the index card.

"How is the twelve broken up?" "Into five plus seven."

Color the top row of the array with the colors that match the Cuisenaire Rods. Yellow for the five and black (not too heavy) for the seven.
Turn the index card sideways and cover all but the left-most column.

"How is the thirteen broken up?" "Into nine and four."
Color in the upper part of the first column blue for the nine. Color the lower one magenta for the four. Then have some fun. Have him pick up both the yellow and the blue and hold them in one hand. Have him color in the 5 x 9 array with both colors. You could also blend them, which is interesting.

"How many multiplication problems are there?" "Four."

"Can you list them?" "5 x 9, 7 x 9, 4 x 5 and 4 x 7."

"Look at the distribution number sentences at the bottom of the page. Look at the second line where the little problems are listed. Is this the same list you just gave me?" "Yes."

Color in each of the arrays using both colors. Some students will draw diagonal stripes; others will blend; some will alternate columns or rows. Give him time to do this. It puts a visual image in his head.

"Look at the top line of the distribution sentence where it says
12 x 13 = (5 + 7) x (9 + 4). What is 5 + 7 ?" "Twelve."
"What is 9 + 4 ?" "Thirteen."

Over the (5 + 7), have him write 12. Over the (9 + 4), have him write 13.
"How do you find out what the answer to 12 x 13 is?" "I add up all the answers to the little multiplication problems."
This is a good breaking point if the lesson has gone on longer than forty minutes.

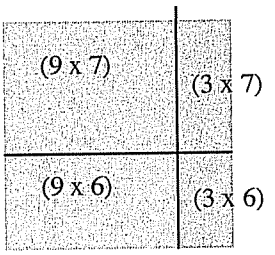
Now take out Breaking Up Times: Two Ways - Worksheet 2, page 15.
Start with the top distribution number sentence at the bottom of the page. Write 12 x 13 on the left hand side of the equal sign, in the same position they were on the example page.

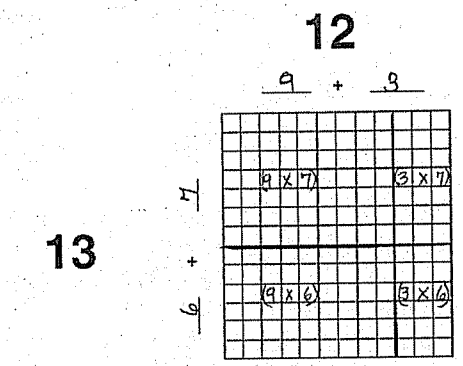
Note

When learning the pattern for breaking up two ways, make sure you avoid using the same number twice. Double numbers, such as 6 + 6, confuse students.

"Break up the 12 a new way. Do not use 6 + 6 or 5 + 7." If you are working with a group, have everyone use the same numbers.
For example: Use 9 + 3 for the twelve.
Have him write 9 + 3 just under the very large twelve at the top of the array.
Cover all but the top row of the array with the trusty index card.
Color the first nine squares blue.

Color the last three squares light green.
Draw a heavy line between the nine and the three.
Remove the index card and extend that heavy line all the way to the bottom of the array. In the top distribution number sentence, just to the right of the equal sign, write into the first set of parentheses 12 x 13 = (9 + 3) x (__ + __).
Now do the thirteen using 6 + 7. Turn the paper sideways so the big thirteen is at the top.
Have him write 6 + 7 just under the very large thirteen at the top of the array.
Cover all but the left hand column of the array with the index card.
Color the top seven squares soft black (or use a pencil).
Color the lower six squares dark green.
Draw a heavy line between the seven and the six.
Remove the index card and extend that heavy line all the way to the other side of the array. Turn the paper right side up.
In the top distribution number sentence, just to the right of the 9 + 3, write into the second set of parentheses 6 + 7.
12 x 13 = (9 + 3) x (6 + 7)





12 x 13 = (9 + 3) x (6 + 7)
List all little problems = (9 x 7) + (3 x 7) + (9 x 6) + (3 x 6)
156 = 63 + 21 + 54 + 18
Final Product

Using the graphic to help, list the four little multiplication problems in the second line of the distribution number sentence.
Color in each multiplication problem.
Write the answers to each small multiplication problem on the third line and sum it all up.
You should get one hundred fifty-six, the same answer you got on Breaking Up Times: Two Ways - Worksheet 1, page 14.
Repeat this procedure again, with different numbers on Breaking Up Times: Two Ways - Worksheet 3, page 16.

Practice Worksheets

Breaking Up Times: Two Ways - Worksheets 4 - 6, pages 17 - 19 Help him as needed.

Test for Understanding

Breaking Up Times: Two Ways - Worksheet 7, page 20 Use this page as an assessment. As usual with an assessment do not help him at all. This will reveal what he understands and what he does not. Check answers in the Answer Key.

Breaking Up Times: Arrow Pattern

Purpose This lesson's purpose is to help the student see the pattern created by the Distributive Property when the problem is viewed vertically as it is when the standard procedure is being used. The student uses the same graphic and distributive number sentence as was presented in the last lesson with the addition of drawing arrows between the little multiplication problems when the problem is shown in the standard vertical form.

Prerequisites Previous lessons

Materials Breaking Up Times: Arrow Pattern - Worksheets 1 - 4, pages 21 - 24

Warm Up Begin with the problem shown on Breaking Up Times: Arrow Pattern - Worksheet 1. It is the same problem we began with in the previous lesson. Have the student fill in the numbers and colors on the graphic. Then see if she can remember how to fill in the distribution number sentences under the graphic. If she needs help, have her refer to Breaking Up Times: Two Ways - Worksheet 1. Make sure she does not just copy it. Make sure she knows where the numbers are coming from. If she does not know, reteach it at this point.

Lesson On the worksheet Breaking Up Times: Arrow Pattern - Worksheet 1, look to the right of the graphic. You will see 12 x 13 written vertically. Notice that the 12 is broken up into 5 + 7.

"How is the thirteen broken up?" "Into nine plus four." Write 9 + 4 in the parentheses to the right of the equal sign.

Point to the little multiplication problem on the array graphic of 5 x 9.

"What little multiplication problem is this?" "Five times nine."

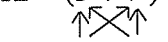
"Point to where this little multiplication problem is written in your distributive number sentence (second row)." Watch to see if she points to the correct location.

"On the vertical problem to the right of the array graphic, draw an arrow between the five and the nine."

Repeat this process for every little multiplication problem shown on the array graphic. You should get the arrow pattern shown at the bottom of the page. Check the Answer Key to see that the numbers and arrows are correctly inserted.

Have her write the answers to the little multiplication problems under the line of the vertical problem as shown below. Notice any patterns?

$$12 = (5 + 7)$$



$$\times\ 13 = (9 + 4)$$

$$5 \times 9 = 45$$
$$7 \times 9 = 63$$
$$5 \times 4 = 20$$
$$7 \times 4 = 28$$
$$156$$

Practice Worksheets

Breaking Up Times: Arrow Pattern - Worksheets 2 - 4, pages 22 - 24

Test for Understanding

"How is the distribution number sentences (the three number sentences under the array graphic) the same as the arrow pattern?"

They are the same in that they both show that the numbers being multiplied can be broken up in several different ways. They both show the little multiplication problems being done. They both allow the writing of the partial products (the answers to the little multiplication problems) to be seen and summed up to get the answer.

"How is the distribution number sentences different from the arrow pattern?"

One is written sideways and the other up and down. The up and down format uses less writing and is faster.

Using Arrow Patterns to Multiply in Tens and Ones

Purpose The purpose of this lesson is to connect the arrow pattern learned in the previous lesson to breaking up numbers in tens and ones. This leads the student to the standard algorithm for long multiplication.

Prerequisites Previous lesson

Materials Using Arrow Patterns to Multiply in Tens and Ones: Worksheets 1 - 4, pages 25 - 28 and the assessment, page 29
Colored pencils
Index card
Graph paper

Warm Up Using the graph paper, draw an array that is 14 x 15 squares. Have her break up both numbers and write the distribution number sentences and the arrow pattern as shown on Breaking Up Times: Arrow Pattern - Worksheet 1, page 21.

Lesson Using Arrow Patterns to Multiply in Tens and Ones - Worksheet 1, page 25, begin with the top problem. This time both numbers are going to be broken up into tens and ones.

Have her put 10 + 6 under the large sixteen, count over from the left ten squares and draw the line all the way down. Use the index card only if needed. Color in the top line to match the rods. Turn the paper sideways and repeat this with the seventeen. It does not matter which number–the ten or the seven–is at the top of the graphic, so the positions of the rectangles for the little multiplication problems may vary.

On the graphic to the right, have her break up the sixteen and the seventeen into tens and ones. She will then use the rectangles on the graphic and the arrow pattern to list the little multiplication problems and write their products. Have her list the little multiplication problems to the left of the lines provided on the worksheet. See the following example for the completed problem.

$$\begin{array}{rl} 16 = & 10 + 6 \\ \times\ 17 = & 10 + 7 \\ \hline 10 \times 10 & = 100 \\ 10 \times 6 & = 60 \\ 10 \times 7 & = 70 \\ 7 \times 6 & = 42 \\ \hline & 272 \end{array}$$

The order of the list can vary.

Do the rest of Arrow Patterns - Worksheet 1 together. Check the Answer Key to be sure she is doing the work correctly. Most students do not have difficulty at this point.

Worksheets Using Arrow Patterns to Multiply in Tens and Ones: Worksheets 2 - 4 and Word Problems That Use Parentheses, page 30

Test for Understanding

Use the Using Arrow Patterns to Multiply in Tens and Ones - Assessment, page 29. The answers are in the Answer Key.

The top problem assesses the lesson just taught. She should be able to do this part correctly without assistance.

The middle problem assesses her control of the distribution number sentences without a grid and asks her to use tens and ones to break up both the fourteen and the sixteen. She should be able to use the distribution number sentences without any assistance. She may use graph paper to help herself if needed. Note if she uses graph paper or not.

The next problem tests if she can use the arrow pattern without the array.

The Challenge problem tests to see if she can use the tens and ones arrow patterns without the arrow pattern graphic, without an array, and with the numbers in the tens place being changed from one to two. She will have to realize that she must multiply twenty times twenty and find the product of four hundred. This problem requires her to generalize the expanded table to a new situation.

Breaking Up Times: Three Ways

Purpose

The purpose of this lesson is to illustrate the arrow pattern when dealing with three and four digit numbers. How does the pattern change? It is also to convince the student that using the standard breaking up (decomposition) of numbers into hundreds, tens, and ones is not the only way to multiply large numbers, but it is certainly the easiest. The standard procedure is not the way to do any problem, but it is the preferred way. There is a reason for this.

Prerequisites

Fluency with the Arrow Pattern

Concurrent Topic

Multiplying Factors of Ten, and Factoring, the next two lessons

Materials

Breaking Up Times: Three Ways - Worksheets 1 - 3, pages 31 - 33

Lesson

Begin with Breaking Up Times: Three Ways - Worksheet 1, page 31.

"In this problem the thirteen is broken up into three parts and the twenty-two into four parts. How do you think this will change the arrow pattern you have already learned?"

Answers will vary. Listen to what the student's ideas are. Your role as a teacher is to figure out what her idea of the problem is. If you don't understand what she says, work on it until you do. The student tells you her best understanding of the solution. It may or may not work in this case. Have her test the model.
1) It works. Test again. Will it always work? Where won't it work?

2) It doesn't work. Test again. Where will it work? How is it different?
Build a model, plan, test, check again. This is what learning is all about.

The best answer is, "There will be more arrows, but every number on the top will have an arrow to every number on the bottom."

"What order are you going to do the problems in?" "I will work across the rows." Or, " I will work down the columns."

"You must put in the arrows as you work. This is important to help you feel how the pattern works."

Notice if she is neat and keeps her columns straight for the long addition problem. If she doesn't keep it neat all along she will need to do it at the end.

Watch as she does the work. Do not help unless needed. It is not required for her to use the colored pencils or the index card unless she feels the need to do so.

Inform the student that she will be practicing breaking up arrays and using the arrow pattern. She will also be drilled on her multiplication and addition accuracy.

Note

Do only one of these problems each day. They are tedious and most students make several errors. Work on the next lesson in your instruction time.

The worksheets should be done independently. If the correct answer is not obtained, have her do the problem again. Have her first check her list, then her partial products, and finally the addition. If she still gets it wrong, use the Answer Key to ferret out where the error is. It is most likely in the addition. *Have her stay with it until she gets it right.*

Practice Worksheets

Breaking Up Times: Three Ways - Worksheets 2 and 3, pages 32 and 33

Note

If you are working with a group, assign Breaking Up Times: Three Ways - Worksheet 3 as homework so collaboration does not happen. How she approaches this problem is part of the Test for Understanding.

Test for Understanding

Look at how she approached the problem on Breaking Up Times: Three Ways - Worksheet 3. Did she use tens and ones when given the opportunity? Did she break the twenty into two tens, or did she simply multiply by twenty? If she used tens and ones, it shows you she understands that it is much easier to use these. Most students break the twenty-nine into $10 + 10 + 9$. Some students will not realize they can leave spaces blank and not use them. How she responds to this problem will let you know how flexible her thinking is and if she can generalize the pattern to make it work for her.

"What did you notice about the arrow pattern? How was it different from when the big numbers were broken into only two parts? How is it the same?"

"There were more arrows, but every number on the top was still connected by an arrow to every number on the bottom."

"What made these problems difficult?" "There were too many parts! It was hard to add and get the correct answer."

Multiplying Factors of Ten

Purpose The purpose of this lesson is to teach the student how to get partial products with large numbers and to understand the trick of multiplying the first two numbers and then adding zeros.

Prerequisites Prime Factoring, Factoring Numbers by Ten in Multiplication: Booklet 2

Materials Multiplying Factors of Ten, Worksheets 1 - 5, pages 34 - 38 Start with Multiplying Factors of Ten - Worksheet 2, page 35.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Warm Up Explore the pattern of multiplying a number by ten and then by one hundred.

3 x 10 =	3 x 100 =	6 x 10 =	6 x 100 =
15 x 10 =	15 x 100 =	12 x 10 =	12 x 100 =

Confirm your answers. Allow the student to figure out how to confirm it. One way is with addition.

"What happens when you multiply a number by ten?" "It gets bigger." **"How much bigger?"** "Ten times bigger."

"What else? How do you know?" "It adds a zero to the end."

"What happens when you multiply a number by one hundred?" "It adds two zeros to the end."

Refer to the first lesson she did in this booklet. It was Multiplying Factors.

"What happens if you factor two numbers and then multiply all the factors together?" "You will get the product of the two original numbers."

Give her the problem 20 x 30. *Wait*. See what she does. A few students will be stumped. A few more will break the twenty into tens and reason that ten times thirty is three hundred, so twice that would be six hundred. A common response is that you multiply the two by the three and add two zeros. If the last response occurs, ask her how she knows that gives the right answer. Can she confirm her answer another way? Find out if she knows why that trick works.

Very, very few students who give this last answer will be able to explain it. Someone showed it to them. The answer you would be looking for would be, "The trick

works because you can break the two and the three off and multiply the tens last. This works because you can multiply the factors in any order you choose."

Do not tell her the pattern. Simply proceed with the lesson.

Give her the chance to figure out this pattern before you show her the example on Multiplying Factors of Ten - Worksheet 1.

Lesson

After looking at 40 x 7 on Multiplying Factors - Worksheet 2, ask her what would be the easiest way to factor out the problem if she were going to solve 40 x 70. See if she thinks of using at least one ten in the final multiplication. A very few students will use the pattern shown on Multiplying Factors of Ten - Worksheet 1, which is the most direct way to get to the answer.

If she says, you just multiply 4 x 7 and add two zeros, ask, **"Where do the two zeros come from?"** If she is unable to give a plausible explanation, show her the formal factoring process shown on Multiplying Factors of Ten - Worksheet 1.

It is important for students to become familiar with the formal grammar of factoring. Point out the little words 'Number Value' and 'Place Value' under the lines. What we do when we want to know the product of 40 x 70 is to factor the Number Value from the Place Value. Then using the Associative Property of Multiplication, the numbers are reordered. The Number Values and the Place Values are placed together. This is the basis of the trick.

Then continue doing Multiplying Factors of Ten - Worksheet 2 and the top problem of Multiplying Factors of Ten - Worksheet 3 together.

Practice Worksheets

Multiplying Factors of Ten - Worksheets 3 - 5, pages 36 - 38

Test for Understanding

Watch to see what she does on Problems 7 and 8 on Multiplying Factors of Ten - Worksheet 5, page 38. Watch to see if she factors the two hundred into 2 x 100 rather than 20 x 10.

This lets you know if she understands that the Number Value, the two,.must be separated from the Place Value, the one hundred.

Factoring

Purpose This purpose is of this lesson to review and contrast prime factoring with factoring by tens.

Prerequisites Previous lessons, Prime Factoring in *Patterns in Arithmetic:* Multiplication - Booklet 3

Materials Factoring - Worksheets 1 - 6, pages 39 - 44

Lesson Session 1 Factoring - Worksheet 1, page 39, will serve as a warm-up and review of prime factoring. Have the student prime factor both fifty and twenty. Remember, a prime number has only itself and one as factors. It can not be evenly divided by any other number.

Note Prime factoring is used as a tool in fractions to both simplify and to find least common multiples. The prime factors of fifty are 2 x 5 x 5. The prime factors of twenty are 2 x 2 x 5.

"We are asked to take all of these factors and multiply them together. What do you think would be the easiest order to do them in?" Let him think about it and do it in whatever way makes sense to him.

The easiest way is to put all the fives first, 5 x 5 x 5, which is one hundred twenty-five, and then the twos, which then double the number three times. It looks like: 125 x 2 = 250, 250 x 2 = 500, and 500 x 2 = 1,000.

Now repeat the process using factoring by tens. Do not factor the ten into its primes. He should use the same procedure he used in the previous lesson. This one basically ends up with 2 x 5 and adds on two zeros from the one hundred. Check the Answer Key to see how this factoring by ten spaces should be filled in.

"Did you get the same answer both times?" "Yes, because it was the same problem both times."

Most students will agree that factoring by tens is less work than prime factoring and multiplying.

Have him complete the bottom of the page on his own. Notice if he does it in his head. That is fine at this point.

Do Factoring - Worksheet 2, page 40. This page reviews more on factoring in general. He should be able to do this page independently. If you are working with a group, have small groups work on this together. Check the work immediately.

Factoring - Worksheet 3, page 41, do the first problem together. Assign the rest for independent work or homework. Require him to write out all the steps. Most students will groan at this. This is good. It means he is understanding the faster way, but have him write out each step anyway, just to prove he can do it.

Worksheets Assign the practice page Factoring - Worksheet 4, page 42

Lesson Session 2

In the next session do Factoring - Worksheets 5 and 6, pages 43 and 44. The arrows will confuse some students. Have him notice that on the previous page, there were four lines in a factoring sequence. Have him note that on this page, there are only three. The arrows indicate which numbers are put together, the four and the two, which are number values, but this time instead of writing out that step, he is asked to multiply those two numbers together in his head and simply record the product. This is also true for the 10 x 10. The part that is done in his head is shown in small parentheses off to the side.

Have him finish the page with you helping. Use the Answer Key to be sure he is filling out the spaces correctly.

On Factoring - Worksheet 6, he finally gets to do the whole shebang in his head.

Make sure he does these neatly as it is very easy to drop a zero.

Test for Understanding

Do one of the problems in the bottom row, Factoring - Worksheet 6, the long way.

8,000 x 900
(8 x 9) x (1,000 x 100)
72 x 100,000 = 7,200,000

Some students will not know that seventy-two, one hundred thousands is seven million, two hundred thousand. Check on your student's understanding of regrouping in place values. He may be able to add on the zeros without really understanding that ten, one hundred thousands is a million. Review the place value pattern that each time you multiply a number by ten, the place value is bumped up to the next group.

"How much is ten, one hundreds?" "A thousand."

"How much is ten, one thousands?" "Ten thousand."

"How much is ten, ten thousands?" "One hundred thousand."

"How much is ten, one hundred thousands?" "One million."

"So here you have seventy, one hundred thousands. How many millions will that be?" "Seven million."

"What happens to the two that was attached to the seventy-two?" "It stays next to the seven but it ends up in the one hundred thousands place, and a comma goes between them."

This is confusing for many, many students. Place value here gets beyond imagination. If he has difficulty with this last problem, ask, **"What is seven hundred twenty, ten times?"** The seven hundred bumps up to the one thousands place and the two ends up in the one hundreds place. Many students have difficulty if the number value gets spread over two 'families' such as hundreds and thousands or thousands and millions.

Breaking Up Times: Tens and Ones

Purpose The purpose of this lesson is to integrate the concept of distribution, the distribution number sentence, the arrow pattern, and factoring for the purpose of multiplying large numbers.

Prerequisites Previous lessons

Materials Breaking Up Times: Tens and Ones, Worksheets 1 - 6, pages 45 - 50

Warm Up Stroll though Memory Lane with a look at the lessons the student has done so far.

Examine Breaking Up Times: Two Ways - Worksheet 1, page 14. **"How were you doing these problems back then?"** "Breaking the numbers any old way and using the distribution number sentences to find the answer."

Now go to Breaking Up Times: Arrow Pattern - Worksheet 1, page 21. **"How were you doing these problems back then?"** "Breaking the numbers any old way and using the distribution number sentences to find the answer and added the arrow pattern to that."

Travel on to Using Arrow Patterns to Multiply in Tens and Ones - Worksheet 1, page 25. **"How did your work change on this page?"** "I was using only the arrow pattern and breaking the numbers into tens and ones."

Remember Breaking Up Times: Three Ways, page 31? **"What made these problems difficult?"** "There were too many numbers, and the addition was hard because it was easy to make a mistake."

Now we are going to put all that together with factoring to do these problems in just about the easiest way possible.

Lesson Look at Breaking Up Times: Tens and Ones - Worksheet 1, page 45.

"How are the numbers broken up?" "Into tens and ones." Notice the dotted line in the middle of the twenty. The purpose of that dotted line is to show you the two sections of one hundred.

Do the page together. Use the arrow pattern first. List all the little multiplication problems and their products. Sum to find the final product.

Then see if she can fill out the distribution number sentences without help. Most students find this page very easy. If she can do it alone, skip Breaking Up Times: Tens and Ones - Worksheet 2, page 46, and assign it for independent work.

Move on to Breaking Up Times: Tens and Ones - Worksheet 3, page 47. This page gives the same problem twice, once with the array grid and once with a blank box. Both problems should be broken up exactly the same way. On the box, in the lower half of the page, have her draw in the lines in the same location they appear on the array above.

Notice if she uses tens and ones or if she drops back to nonstandard breaking up. This will tell you how secure she is with the tens and ones procedure. Most students use the tens and ones without prompting.

"Does anything change if the little squares in the array grid disappear?" "No, it is exactly the same."

Breaking Up Times: Tens and Ones - Worksheet 4, page 48 is the same as Breaking Up Times: Tens and Ones - Worksheet 3 except it requires the use of tens and ones in the breakup. She is asked to write the distribution number sentence only once down at the bottom of the page.

Practice Worksheets

Breaking Up Times: Tens and Ones - Worksheets 2, 4, 5, and 6, pages 46, 48, 49, and 50

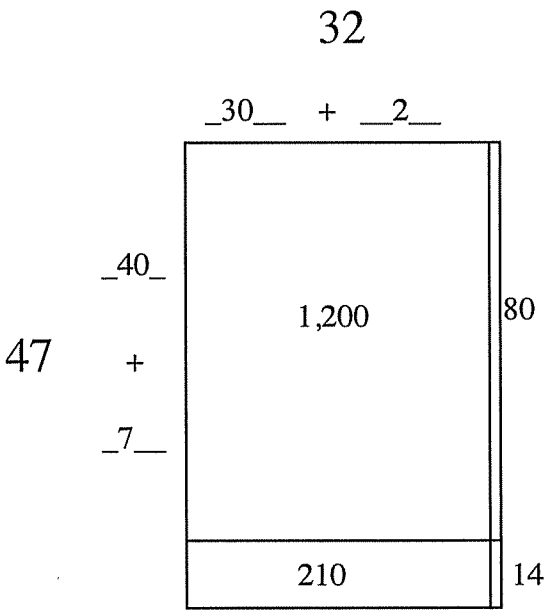
Test for Understanding

Give her the problem 32 x 47. Draw a large box and ask her to draw in the lines, and use the arrow pattern to the side. This is what you want to see:

32 = 30 + 2
x 47 = 40 + 7

32 = 30 + 2
x 47 = 40 + 7
40 x 30 = 1,200
40 x 2 = 80
30 x 7 = 210
7 x 2 = 14
1,504

The order may vary on the small problems list.



Breaking Up Times: Standard Procedure

Purpose The purpose of this lesson is to practice what will become the standard procedure for long multiplication. The lesson uses boxes for reference, and the arrow pattern and adds a new requirement for doing the small multiplication problems in a specific order to facilitate moving to the short procedure where carrying is used. That procedure requires starting in the ones place.

Prerequisites Previous lessons

Materials Breaking Up Times: Standard Procedure - Worksheets 1 - 4, pages 51 - 54

Warm Up Give a problem such as 23 x 29. Ask her to draw the box and put in the arrow pattern. She should be able to do this alone.

Then give her the addition problem 23 + 29. When she is finished ask:

"Why did you start in the ones place?" Some students will answer, "Because that is how you do it." A better answer is, "Because it is easier."

"Could you start in the tens place?" "Yes, but then you would get forty and then when you add on the twelve you have to change the forty to fifty. It is easier to do the ones and carry that ten and add all the tens together."

"So we begin with the ones because of the carrying?" "Yes."

"The same thing is going to be true later when we use the shortcut for doing long multiplication."

Lesson Begin with Breaking Up Times: Standard Procedure - Worksheet 1, page 51. Study the example at the top.

"What is new?" "The little numbers in the circles."

Study the example. Have her put a one in the array box where the problem 5 x 5 is. Have her put a two in the box where 30 x 5 is, a three where the 20 x 5 is and a four in the box where the 20 x 30 is.

"What pattern do you notice?" "It is backwards from the way I have always done it." Or, "It goes from the bottom to the top."

"Will changing the order you usually do it change the answer?" "No."

Have her do the problem at the bottom of the page and do it in the standard order as shown in the example above. Some students like to put the numbers one to four in the boxes to remind themselves of what order to go in. Check the Answer Key to be sure the work is done correctly.

Give Breaking Up Times: Standard Procedure - Worksheet 2, page 52. She must put in the lines in the box and write the tens and ones numbers in the parentheses above the box. Watch her do the first two to be sure they are done correctly.

Practice Worksheet

Breaking Up Times: Standard Procedure - Worksheets 3 and 4, pages 53 and 54

Test for Understanding

See if she can generalize from her experience of Breaking Up Times: Three Ways. Have her look back at those difficult problems she did back then. Draw her attention to the arrow pattern for numbers broken into three or four parts.

Then give her this problem: 234 x 85. Ask her to use the arrow pattern to do the problem and see what she does. If she understands the pattern, she will realize that the two hundred must be multiplied by both the eighty and the five. If she does not know what to do, show her Breaking Up Times: Three Ways and try again copying the arrow pattern on those problems. This would mean that she does not yet get the big picture of distribution. Solution: Order of the little problems will vary.

234 =	200 + 30 + 4
x 85 =	80 + 5
5 x 4 =	20
5 x 30 =	150
5 x 200 =	1,000
80 x 4 =	320
80 x 30 =	2,400
80 x 200 =	16,000
	19, 890

Breaking Up Times: Long Way

Purpose The purpose of this lesson is to move the student from the representational array graphics to a purely abstract method of solving long multiplication problems. The student still uses the distributive process but records only the partial products. These are done the 'long way,' requiring the student to use mental factoring and write the answers with the correct place values attached.

Prerequisites Previous lessons

Materials Breaking Up Times: Long Way - Worksheets 1 - 4, pages 55 - 58

Warm Up Review factoring by tens with large numbers such as 600 x 70, and 600 x 800. Have the student write out the factoring notation. He will forget it if it is not reinforced regularly.

Give the problem 35 x 73 and have him write out the distributive number sentences. He will forget this also if he does not practice it.

$35 \times 73 = (30 + 5) \times (70 + 3)$

$= (30 \times 70) + (30 \times 3) + (5 \times 70) + (5 \times 3)$

$= 2{,}100 + 90 + 350 + 15$

$= 2{,}555$

Challenge Try using algebra to show the distributive number sentences.

"Does the arrow pattern work for all numbers when they are being multiplied?" "Yes."

When a pattern works for all numbers, algebra is used. Just below the number sentence he wrote for 35 x 73, write (a + b) x (c + d). The (a + b) will be like the 30 + 5. The (c + d) will be like 70 + 3.

"How could you show the pattern in algebra?" Some students will get this in a snap and some won't. Don't push it. Answer:

$35 \times 73 = (30 + 5) \times (70 + 3)$ $= (a+b) \times (c+d)$

$= (30 \times 70) + (30 \times 3) + (5 \times 70) + (5 \times 3)$ $= (ac) + (ad) + (bc) + (bd)$

$= 2{,}100 + 90 + 350 + 15$ The above is the answer because there are no number values.

$= 2{,}555$

Lesson Use the problem you did in Warm Up to demonstrate the 'long way' of doing problems without writing all that stuff you had to write when doing the arrow pattern. The way to write this is shown on Breaking Up Times: Long Way -

Worksheet 1. It will go in better if you write the numbers with him rather than have him just look at it on the worksheet.

Write the problem 35 x 73 vertically as shown on the worksheet.

"Turn back to Breaking Up Times: Standard Procedure - Worksheet 1. What order is the standard way used to write the answers to these problems?" "It goes backwards using the circled numbers."

"So in this problem you just did in the Warm Up, which little multiplication problem would you do first?" "The three times five."

"Now you can use a shortcut. You do not have to write the 3 x 5 like you did before. Just write the 15 below the line. Keep the place value lines straight so you can add easily."

"What's next?" "Three times thirty (or thirty, three times)."

Note

If he says three times three, correct him to say "three times thirty" or "thirty, three times."

Proceed like this until you get to the end of the problem. Have him check his work by looking at Breaking Up Times: Long Way - Worksheet 1. Then have him check his work with a calculator.

Do the next three problems together. Make sure he writes the partial products in the correct order. If he has difficulty, draw the box.

Practice Worksheet

Breaking Up Times: Long Way - Worksheets 2 - 4, pages 56 - 58

Spread these worksheets out over a few days. Each day before he begins, review factoring. Insist the work is done carefully. Those dropped zeros and sloppy columns will cause errors.

Test for Understanding

Breaking Up Times: Long Way - Worksheet 4, page 58, has a built in Test for Understanding. Do not warn him or prepare him for the longer problems. Just watch and see if he can correctly apply the distributive principle to the new, three digit number. Correct his order if need be. If he has difficulty, go back to the previous lesson where this was dealt with. You can also go back to those difficult three way problems to see the general pattern. See Breaking Up Times: Three Ways.

Long Way to Standard Way

Purpose The purpose of this lesson is to move from writing out all the small multiplication problems and their products and use carrying to make it even faster. This is the standard procedure everyone memorized in school. The student will move step-by-step from the long way to the short standard procedure that uses carrying.

Prerequisites Previous lessons

Materials Long Way to Standard Way - Worksheets 1 and 2, pages 59 and 60

Lesson These two worksheets are examples of what is called a one-to-one transfer. The student performs one step of the familiar long way notation where each partial product is written out, then repeats that same step using the new short notation.

Have her write the problem 42 x 23 vertically, two times as shown in the example. Together follow the example. Have her do the long way first, then the short way.

Step 1 **"What do you do first?"** "Multiply the two by the three."

"Where do you record the product?" "In the ones place under the line." Have her do this on both the long way and the short way.

"What do you do next?" "Multiply the forty by the three to get one hundred twenty."

"Where do you record the one hundred twenty in the long way?" "Under the six." Have her do this on the long way.

Stop. Point out that in the short way, she will write the one hundred twenty but the six is in the ones place where the zero is. So the zero is not written. The one is placed in the hundreds place and the two in the tens place. Explain that this allows her to not worry about the place value. She can just think 3 x 4 and write the twelve right next to the six. This makes the problem easier and faster.

Step 2 **"What do you do next?"** "Multiply the two by the twenty to get forty."

"Where do you record the forty in the long way?" "Under the one hundred twenty." Have her do this on the long way.

Stop. Point out that in the short way, she will write the forty under the one hundred twenty-six. Point out that most people do not think 20 x 2 = 40 when they work the short way. They think 2 x 2 is 4. But they must remember to put the zero in this time because the four must be written in the tens place.

Step 3 **"What do you do next?"** "Multiply the forty by the twenty to get eight hundred."

"Where do you record the eight hundred in the long way?" "Under the forty." Have her do this on the long way.

Stop. Point out that in the short way, she will think 2 x 4 = 8 and simply put the

eight next to the four. The place value is correct automatically.

Step 4 **"Why is the short way here easier than the long way?"** "You only have to add two numbers instead of four."

Worksheet Long Way to Standard Way - Worksheet 2, page 60

Have her write the problem 37 x 92 vertically, two times as shown in the example. Together follow the example. Have her do each step the long way first, then each step the short way.

Step 1 **"What do you do first?"** "Multiply the seven by the two to get fourteen."

"Where do you record the product?" "In the ones place under the line." Have her do this on the long way.

Stop. Point out that in the short way, the four is written in the ones place and the one is carried into the tens place.

Step 2 **"What do you do next in the long way?"** "Multiply the thirty by the two to get sixty."

"Where do you record the sixty in the long way?" "Under the fourteen." Have her do this on the long way.

Stop. Point out that in the short way, she will multiply the three tens by the two and get six tens. Then she will add on the one ten from the fourteen to get seven tens.

Point out the addition of the six and the one in the tens row on the long way.

It is even easier to not even think about the fact that these are tens. All you have to do is multiply three times two and add the one to get seven. Since the ones place is already occupied by the four, you just put the seven to the left of it and it is automatically in the right place value. Very cool.

Step 3 **"What do you do next?"** "Multiply the seven by the ninety to get six hundred thirty."

"Where do you record the six hundred thirty in the long way?" "Under the sixty." Have her do this on the long way.

Stop and ask, **"How do you think you record the six hundred thirty in the short way?"** Wait, some students can figure this out.

Have her do Step 4 the long way.

"Find the six hundred thirty in your long way list. What number will the six in the hundreds place be added to when you do the final sum?" "The seven in the hundreds place."

"Where did that seven come from?" "From the seven in the two thousand seven hundred I got when I multiplied ninety by thirty."

"Now go back to the short way in Step 3. So the six hundred needs to be held onto until you finish step four. Look in the example on the short way side in Step 3. Where is the six hundred recorded?" "Above the three."

"Be sure you scratch out the one from Step 2. Now what do you do?" "Multiply the nine times three to get the twenty-seven hundreds and add on the six hundreds."

"What do you get when you do that?" "Three thousand three hundred thirty."

Note

Here is a mathematical strangeness hardly anyone ever notices. In the short way, Step 3, we carry the six into the tens place. But we know that six is actually a six hundred. Why do we carry a six hundred into the tens place?

An answer could be that when we are multiplying by a number in the tens place all the place values are moved up one. Most students will not get this. You could just say the six is merely held above the three because that is the number you are multiplying by.

Step 4

"Now finish the problem."

Many students will say the long way is easier than the short way. They say this because the carrying in Step 3 does not really make sense to them. They have told me the long way makes sense where the short way does not. You will need to require them to do problems the long way and then the short way to overcome this. Some students never drop the long way until the problems get into the hundreds and thousands and the column adds become tedious. Here is a true story about one such student.

An Amusing Story (reported by a parent whose son had an honest teacher)

I had a student leave our school just after he had finished learning the long way to multiply with hundreds, tens, and ones. When he did this in his new school, the teacher looked at his paper a long time. She would have said, "You are doing these problems incorrectly," except that he had all the correct answers. She could not figure out what he was doing. All she knew was the standard memorized procedure. She went and asked her mentor to help her figure out what her student was doing. The mentor told the teacher that the student understood multiplication better than she did. The teacher went back to my student and asked him to teach her what he was doing. He taught her and she was amazed that no one had ever taught her this concept.

Practice Worksheets

Breaking Up Times: Long Way - Worksheets 2 and 3, pages 56 and 57

These worksheets have already been done the long way. Have her redo each problem the short way. She will know if she got the correct answer by comparison. You can also point out using the long way, where the carried numbers in the short way are being added to the next product.

Test for Understanding

Watch her do a problem the short way using problems from Breaking Up Times: Long Way - Worksheet 4, page 58. See if she generalizes the pattern to include working with numbers in the hundreds. The answers are in the Answer Key.

Challenge Problem

Give her the first problem from Standard Way: Practice - Worksheet 4, page 64

$$\begin{array}{r} 837 \\ \times\ 152 \\ \hline 1{,}674 \\ 41{,}850 \\ 83{,}700 \\ \hline 127{,}224 \end{array}$$

See if she can generalize the pattern to a distribution from the hundreds place. She has not yet been asked to use a three digit multiplier. Watch to see if she either places the 7 into the hundreds place or writes 700, thus placing zeros in the tens and ones place.

If she can do this at this point, she understands.

Standard Way: Practice

Purpose

The purpose of this lesson is to practice the standard procedure for long multiplication and to extend what the student already knows to longer problems.

Prerequisites

Area of a rectangle and previous lessons

Materials

Standard Way: Practice - Worksheets 1 - 4, pages 61 - 64

Warm Up

Solve 42 x 23 with a box, with distribution number sentences and the long way as was done on Breaking Up Times: Tens and Ones - Worksheet 3, page 47, on the lower half of the page.

Lesson Part 1

Solve 42 x 23 the short way using the standard short notation learned in the last lesson. Review the carrying and connect it to the long way so the student will be reminded of where the numbers he is carrying are coming from.

Have him do the first three rows of Standard Way: Practice - Worksheet 1. Check the work before proceeding.

Give the problem 234 x 23 (first problem of row 4) and have him do it the long way. The solution to the long way will look like this:

$$\begin{array}{r} 234 \\ \underline{\times\ 23} \\ 12 \\ 90 \\ 600 \\ 80 \\ 600 \\ +\ \underline{4{,}000} \\ 5{,}382 \end{array}$$

Now have him do the problem the short way on Standard Way: Practice - Worksheet 1. Have him finish the page.

Practice Worksheets

Standard Way: Practice - Worksheets 2 and 3, pages 62 and 63 as independent work. End the session.

Part 2

Give him practice sessions over the next two days to complete the next two worksheets. Insist on neat work and aligned columns. Make sure he is using a sharp pencil.

Please, do not just tell him to add two zeros when doing a three digit multiplication. Let him figure this out. Doing this will strengthen his deductive logic.

Begin with Standard Way: Practice - Worksheet 4, page 64.
Draw a large box and solve 837 x 152 the long way. Sum the results.

837 x 152 =

800	+ 30	+ 7
800 x 100 = 80,000	30 x 100 = 3,000	7 x 100 = 700
800 x 50 = 40,000	30 x 50 = 1,500	7 x 50 = 350
800 x 2 = 1,600	30 x 2 = 60	7 x 2 = 14

Do 837 x 152 the short way.

```
  837
x 152
1,674
41,850
83,700
127,224
```

Discuss the zeros in the partial product.

"How should we record the multiplication of 100 x 7 in the standard way? What we really want to think about is 1 x 7 to make it easier. Where should we put the seven?" "You have to put it in the one hundreds place because the answer is 700."
"So what should go in the ones and tens places?" "Zeros."

"Why?" "Because when you multiply by a one hundred, you are automatically going to have zeros in the tens and ones places."

"So if all I want to think about is 1 x 7, what should I do to make sure the seven gets into the hundreds place?" "Put zeros in the ones and tens places automatically."

Practice Worksheets

Part 1 - Standard Way: Practice - Worksheets 1 - 3, pages 61 - 63

Part 2 - Standard Way: Practice - Worksheet 4, page 64 and check the answers. Have him correct any incorrect answers.

Test for Understanding

Standard Multiplication: Assessment, page 65. The answers are in the Answer Key.

Give problem 925 x 357 and ask him to solve it four ways: Write out the boxes, the arrow pattern in the long way, the distributive number sentences, and the standard notation. All four answers (330,225) must match.

Frequently review how to do the distributive sentences. Memory of this skill is very important for algebra.

Multiplying Decimal Factors

Purpose The purpose of this lesson is to extend factoring to find the place value of a product from whole numbers to decimal place values.

Prerequisites Multiplication of Fractions: Booklet 6, Decimal Place Value, and *Key to Decimals: Book 2* - See the Preparation below.

Preparation Purchase *Key to Decimals: Book 2*. This is not a *Patterns in Arithmetic* publication. *Key to Decimals* can be purchased from www.keycurriculum.com.

Materials Multiplying Decimal Factors - Worksheets 1 - 3, pages 66 - 68
Times Tables Quiz - Worksheets 1 and 2, pages 70 and 71

Warm Up Review factoring by tens. Redo a few problems from Multiplying Factors of Ten done earlier in this booklet, page 35, e.g., 60 x 80, 300 x 20, 4,000 x 300.

Practice several examples. Give problems from Multiplying Factors of Ten and check the results with the Answer Key.

Review multiplication of fractions with easy examples, e.g., $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9} = \frac{2}{3}$. All we want to do here is re-establish the procedure of multiplying the numerators and then the denominators and simplifying.

Note Multiplication with decimals uses the exact same principles that multiplication with whole numbers does. A difference is that the place values of the numbers are less than one. Use the multiplication of fractions procedure to find the answer to seven tenths times nine tenths, $\frac{7}{10} \times \frac{9}{10} = \frac{63}{100}$.

Lesson **"Use what you know about factoring to find the answer to .7 x .9."**
Wait. Some students will be able to figure this out and some will not. Give her the chance and time to do it on her own.

Multiplying Decimal Factors - Worksheet 1, page 66 Now, try the procedure on problems you know the answer to such as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Change the $\frac{1}{2}$ to 0.5 and do the factoring procedure. You already know the answer should be 0.25. The work will look like this:

$.5 \times .5 = (5 \times \frac{1}{10}) \times (5 \times \frac{1}{10})$
$= (5 \times 5) \times (\frac{1}{10} \times \frac{1}{10})$
$= 25 \times \frac{1}{100}$
$0.25 = \frac{25}{100}$

Try problems, such as 8 x .5 = _____.
"What is 8 x $\frac{1}{2}$?" "Four."
"What would the answer to 8 x 0.5 be?" "It would have to be four also."

"Now use your factoring process to work with the decimal. What will the place value of the eight be when you factor?" "The eight will be shown as 8 x 1 because the place value of the eight is in the ones place."

$$\begin{aligned} 8 \times 0.5 &= (8 \times 1) \times (5 \times \tfrac{1}{10}) \\ &= (8 \times 5) \times (1 \times \tfrac{1}{10}) \\ &= 40 \times \tfrac{1}{10} \\ &= 4.0 \end{aligned}$$

Practice Worksheets

Multiplying Decimal Factors - Worksheets 2 and 3, pages 67 and 68
Times Tables Quiz - Worksheets 1 and 2, pages 70 and 71 Practice on basic times tables facts. Instructions are on the worksheets.

Test for Understanding

Give the problem 0.25 x 0.25. See if she can generalize the factoring pattern to double digit multiplication.

$$.25 \times .25 = (.25 \times \tfrac{1}{100}) \times (.25 \times \tfrac{1}{100})$$
$$(.25 \times .25) \times (\tfrac{1}{100} \times \tfrac{1}{100}) = 625 \times \tfrac{1}{10,000} = \tfrac{625}{10,000}$$

To confirm the answer ask her to change the decimals to fractions and get the answer. She already knows the answer is one sixteenth.
"What is one sixteenth as a decimal?" ".0625"

What do you have to divide a fourth by to get a sixteenth?" "Divide 0.25 by 4 to find out what the decimal answer should be."

$$4\overline{)\,.25}\qquad .0625$$

Assignments for *Key to Decimals - Book 2* are below and on page 69 in the Student Workbook.
Do these pages in this order:
A. 25 and 26
B. 19
C. 27
D. Discuss why the trick of moving decimal points works. Connect it to what is done with factoring on the previous pages. Change every problem on page 27 into a factoring problem.

$$\begin{array}{r} 1.23 \\ \times\ .9 \end{array} = \frac{123}{100} \times \frac{9}{10} = \frac{1,107}{1,000} = 1.107$$

Multiply the numbers from the numerators, then set the place value with the denominators.

E. 28 - 33
F. 35 Connections to two-way multiplication
G. 34 - 42 Practice pages
H. 23 and 24 Getting ready for division
I. 44 and 45 Take the Practice Test.

Pre-Assessment: Part 1 Date _____

Put a question mark next to any problem you don't know how to do.

1. Solve the Short Way.

a. 34 x 6 b. 258 x 3 c. 1,735 x 4

2. Solve the Long Way.

a. 34 x 6 b. 258 x 3 c. 1,735 x 4

3. Break up the 57 using tens and ones. Fill in all the blanks.

57 x 3 = (_____ + _____) x ____

= (____ x ____) + (____ x ____)

_____ = _____ + _____

4. What is a factor? You may use a numerical example to help you explain.

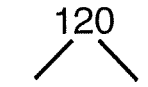
_____

_____

5. What is a prime number? _____.

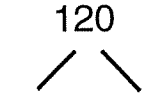
_____

6. a. Prime factor 120.
   b. Record the prime factors in this box.



____ x ____

7. Factor 120 into factors of ten.



____ x ____

8. Use factoring by tens to show how to get the product of 700 x 9.

700 x 9

____ x ____ x ____

(____ x ____) x ____

____ x ____

_____

Pre-Assessment: Part 2 - Worksheet 1 Date _____

1. a. Solve. 400 x 30 = _____

   b. Explain how you got your answer. _____

   _____

   _____

2. Use factoring by tens to show how to get the answer to 40 x 700.

   40 x 700

   (___x___) x (___ x ___)

   (___x___) x (___ x ___)

   _____ x _____ = _____

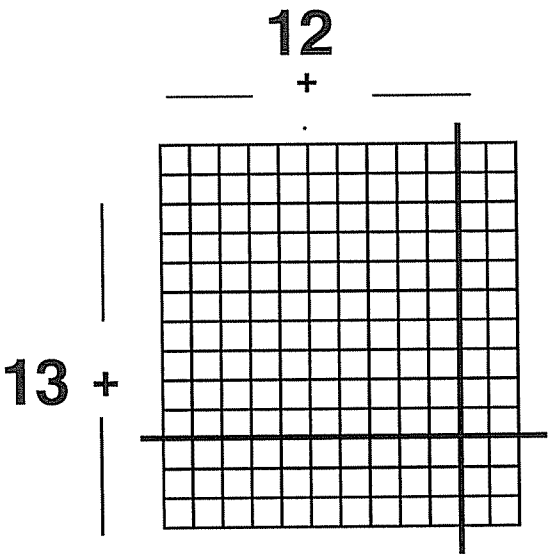
3. There is a trick to solving a problem such as 900 x 500. Multiply 9 x 5 and add four zeros. Explain why this trick works.

   _____

   _____

   _____

4. Use this diagram to help you fill in the distribution number sentences.



___ x ___ = (___ +___) x (___ + ___)

= (___x___) + (___x___) + (___x___) + (___x___)

_____ = ____ + ____ + ____ + ____

Pre-Assessment: Part 2 - Worksheet 2 Date _____

5. Show the distribution pattern with arrows between the numbers.
   Use this diagram to help.

$$12 = 5 + 7$$

$$\times 13 = 9 + 4$$

Write the little problems here.

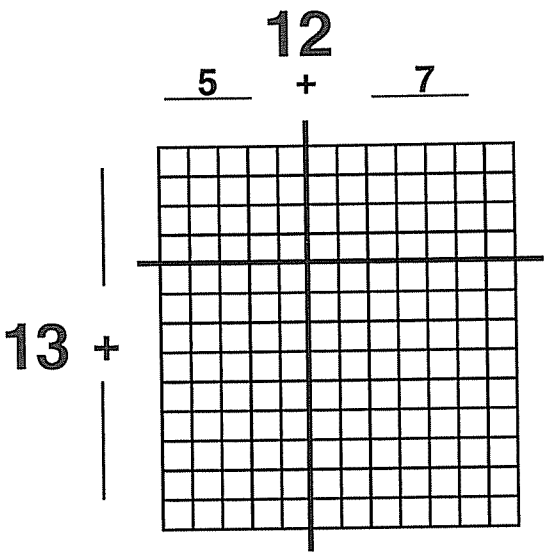
4 x 7 = _____

_____ = _____

_____ = _____

_____ = _____

Final answer. = _____



6. You should have gotten the same answer for problems 4 and 5. Explain why.

_____

_____

7. Solve this problem the long way using the arrow pattern.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

little multiplication problems		partial products
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____

= _____

8. Solve the same problem the short way.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

9. Why do you have to put a zero in the ones place in the second row of the partial product in problem 8?

_____

_____

(Note: The answers are on the last page of the Answer Key.)

Post-Assessment - Worksheet 1 Date _____

1. a. Solve. 400 x 30 = _____

   b. Explain how you got your answer. _____

   _____

   _____

2. Use factoring by tens to show how to get the answer to 40 x 700.

   40 x 700

   (___x___) x (___ x ___)

   (___x___) x (___ x ___)

   _____ x _____ = _____

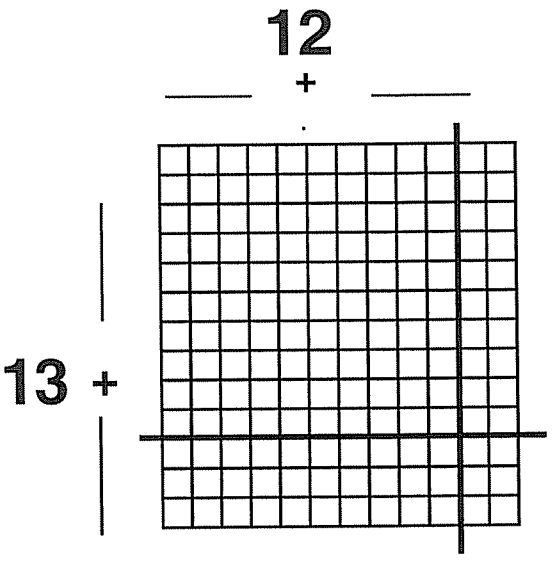
3. There is a trick to solving a problem such as 900 x 500. Multiply 9 x 5 and add four zeros. Explain why this trick works.

   _____

   _____

   _____

4. Use this diagram to help you fill in the distribution number sentences.



___ x ___ = (___ + ___) x (___ + ___)

= (___x___) + (___x___) + (___x___) + (___x___)

_____ = ____ + ____ + ____ + ____

Post-Assessment - Worksheet 2 Date _____

5. Show the distribution pattern with arrows between the numbers.
   Use this diagram to help.

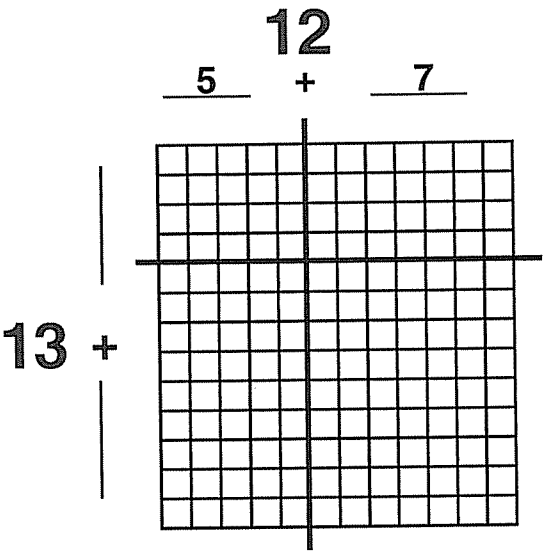
$$12 = 5 + 7$$

$$\underline{\quad} \times 13 = 9 + 4$$

Write the little problems here.

$$\underline{4 \times 7} = \underline{\quad}$$
$$\underline{\quad} = \underline{\quad}$$
$$\underline{\quad} = \underline{\quad}$$
$$\underline{\quad} = \underline{\quad}$$

Final answer. = _____



6. You should have gotten the same answer for problems 4 and 5. Explain why.

_____

_____

7. Solve this problem the long way using the arrow pattern.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

little multiplication problems		partial products
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
	=	_____

8. Solve the same problem the short way.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

9. Why do you have to put a zero in the ones place in the second row of the partial product in problem 8?

_____

_____

(Note: The answers are on the next to the last page of the Answer Key.)

Patterns in Arithmetic

Multiplication: Booklet 4

Working with Large Numbers and Decimals

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Practice pages.
4) Workboards.
5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: Multiplication - Booklet 4
Answer Key for the Student Workbook

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Multiplication - Book 4

Assessments are on the second to the last page.

Multiplying Factors - Worksheet 1

4 x 8 = 2 x 2 x 2 x 4 = 2x 2 x 2 x 4 = 32

6 x 8 = 1 x 6 x 2 x 4 = 1x 6 x 2 x 4 = 48

What happens when you multiply the factors of a number together?
You get the same number you started with.

7 x 6 = 1 x 7 x 2 x 3 = 1x 7 x 2 x 3 = 42

6 x 4 = 3 x 2 x 2 x 2 = 3x 2 x 2x 2 = 24

20 x 8 = 2 x 10 x 2 x 4 = 2x 10 x 2 x 4 = 160

30 x 6 = 5 x 6 x 2 x 3 = 5x 6 x 2 x 3 = 180

Change the order of the numbers.
2x 2 x 4 x 10 = 160 (OWV)

6x 5 x 3 x 2 = 180

1

Multiplying Factors - Worksheet 2

9 x 8 = 3 x 3 x 2 x 4 = 3x 3 x 2 x 4 = 72

4 x 8 = 2 x 2 x 2 x 4 = 2x 2 x 2 x 4 = 32

What happens when you multiply the factors of a number together?

10 x 6 = 2 x 5 x 3 x 2 = 2x 5 x 3 x 2 = 60

20 x 4 = 4 x 5 x 2 x 2 = 4x 5 x 2 x 2 = 80

40 x 7 = 4 x 10 x 1 x 7 = 10 x 4 x 7 x 1 = 280

2

Breaking Up Times: Review - Worksheet 1

Do these problems with your teacher.

14 x 4 = (6 + 5 + 3) x 4

Guess

= (6 x 4) + (5 x 4) + (3 x 4)

56 = 24 + 20 + 12

Again, please.

14 x 4 = (9 + 5) x 4

= (9 x 4) + (5 x 4)

56 = 36 + 20

One more time.

14 x 4 = (5 + 2 + 6 + 1) x 4

= (5 x 4) + (2 x 4) + (6 x 4) + (1 x 4)

56 = 20 + 8 + 24 + 4

Do all of your products match? yes
Should they? yes

3

Breaking Up Times: Review - Worksheet 2

Solve this problem three different ways. Estimate the answer first.

17 x 3 = (7 + 6 + 4) x 3

Guess

= (7 x 3) + (6 x 3) + (4 x 3)

51 = 21 + 18 + 12

Again, please. You draw the lines in the picture this time.

17 x 3 = (8 + 6 + 3) x 3

= (8 x 3) + (6 x 3) + (3 x 3)

51 = 24 + 18 + 9

One more time. Break the 17 into four sections this time. BUWV

17 x 3 = (___ + ___ + ___ + ___) x 3

= (__ x __) + (__ x __) + (__ x __) + (__ x __)

51 = ___ + ___ + ___ + ___

Do all of your final products match? _____

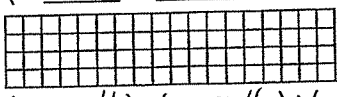
4

Breaking Up Times: Review - Worksheet 3

Solve this problem three different ways. Estimate the answer first.

18 x 4 = (____ + ____ + ____) x 4

Guess

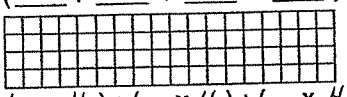


= (__ x 4) + (__ x 4) + (__ x 4)

72 = ____ + ____ + ____

Break the 18 into four sections.

18 x 4 = (____ + ____ + ____ + ____) x 4

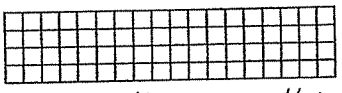


= (__ x 4) + (__ x 4) + (__ x 4) + (__ x 4)

72 = ____ + ____ + ____ + ____

Challenge! Break the 18 into only two sections.

18 x 4 = (____ + ____) x ___



= (__ x 4) + (__ x 4)

72 = ____ + ____

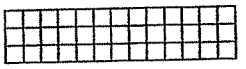
Do all of your final products match? yes

Breaking Up Times: Review - Worksheet 4

Solve this problem three different ways. Estimate the answer first.

13 x 3 = (____ + ____ + ____) x 3

Guess

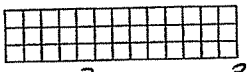


= (__ x 3) + (__ x 3) + (__ x 3)

39 = ____ + ____ + ____

Again, please. Break the 13 into only two sections.

13 x 3 = (____ + ____) x 3



= (__ x 3) + (__ x 3)

39 = ____ + ____

Change 13 x 3 = ___ into an addition problem. Then find the answer. Do the work here.

13
13
+13
39

Challenge! Can you break up the 13 and get the answer without a picture?

13 x 3 = (____ + ____ + ____) x 3

= (__ x 3) + (__ x 3) + (__ x 3)

39 = ____ + ____ + ____

Use different numbers the ones in problem at top of this page.

Arrays: Review - Worksheet 1

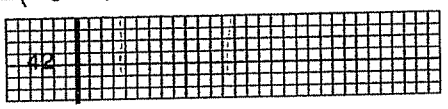
Do this problem three ways. BUWV

36 x 7 = 252 One way is by addition.

36
36
36
36
36
36
+ 36
252

Another way is by breaking the 36 into smaller numbers and multiplying each part by 7.
Do this two times. Break up the 36 two different ways.
Draw the lines to show each small or partial product.

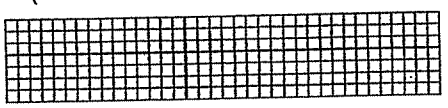
36 x 7 = (6 + + + +)x 7



= (6 x 7) + (__ x 7) + (__ x 7) + (__ x 7) + (__ x 7)

252 = 42 + ____ + ____ + ____ + ____

36 x 7 = ( )x 7



= (__ x 7) +

252 = ____ +

Arrays: Review - Worksheet 2

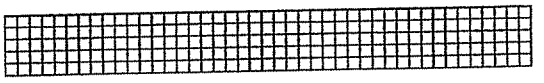
Solve this problem three ways. BUWV

43 x 5 = 215 One way is by addition.

43
43
43
43
+ 43
215

Another way is by breaking the 43 into smaller numbers and multiplying each part by 5.
Break up the 43 two different ways.
You will need to supply lines and + signs for your partial products.

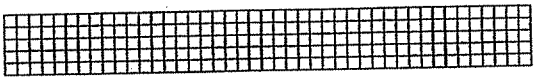
43 x 5 = ( )x 5



= (__ x __) +

215 = ____ +

43 x 5 = ( )x 5



= (__ x __) +

215 = ____ +

Expanded Multiplication: Review Worksheet 1

Study this example:

237 x 4 = (200 +30 +7)x4

800	+	120	+	28

237 x 4 = 948

Here is the same problem again without the box. It is important that you remember this way of writing the problem because this is how multiplication is done in algebra. The only thing that is different is that you must write out each little multiplication that you did in the box. This is how you did it when you were first learning.

237 x 4 = (200 + 30 + 7) x4

= (200 x 4) + (30 x 4) + (7 x 4)
= 800 + 120 + 28

237 x 4 = 948

Your new way has you do this same problem this way:

Long way	Short way
237	12 237
x 4	x 4
28	948
120	
800	
948	

Expanded Multiplication: Review Worksheet 2

Solve this problem all ways shown on the last page.

357 x 5 = (300 + 50 + 7)x5

1500	+	250	35

357 x 5 = 1,785

357 x 5 = (300 + 50 + 7)x5

= (300 x 5) + (50 x 5) + (7 x 5)

= 1,500 + 250 + 35

357 x 5 = 1,785

Now solve it the long way.

357
x 5
35
250
1,500
1,785

The short way.

2 3
357
x 5
1,785

Expanded Multiplication: Review Worksheet 3

Solve this problem all ways.

1238 x 4 = (1,000 + 200 + 30 + 8)x4

4,000	+	800	+	120	+	32

1238 x 4 = 4,952

1238 x 4 = (1,000 + 200 + 30 + 8)x4

= (1,000 x 4) + (200 x 4) + (30 x 4) + (8 x 4)

= 4,000 + 800 + 120 + 32

1238 x 4 = 4,952

Now solve it again the long way.

1238
x 4
32
120
800
4,000
4,952

The short way.

1 3
1238
x 4
4,952

Practice - Worksheet 1

Long way	Short way	Long way	Short way
434	1 1 434	349	2 4 349
X3	X3	X5	X5
12	1,302	45	1,745
90		200	
1,200		1,500	
1,302		1,745	

Long way	Short way	Long way	Short way
506	506	865	1 1 865
X7	X7	X2	X2
42	3,542	10	1,730
3,500		120	
3,542		1,600	
		1,730	

Long way	Short way	Long way	Short way
197	2 2 197	426	1 4 426
X3	X3	X7	X7
21	591	42	2,982
270		140	
300		2,800	
12 591		2,982	

Practice - Worksheet 2

Long way	Short way
2,456 X3	2,456 X3
18	7368
150	
1,200	
6,000	
7,368	

Long way	Short way
3,247 X7	3,247 X7
49	22729
280	
1,400	
21,000	
22,729	

Long way	Short way
2,832 X8	2,832 X8
16	22,656
240	
6,400	
16,000	
22,656	

Long way	Short way
8,765 X4	8,765 X4
20	35,060
240	
2,800	
32,000	
35,060	

Long way	Short way
2,394 X6	2,394 X6
24	14,264
540	
1,800	
12,000	
14,264	

Long way	Short way
1,657 X9	1,657 X9
63	14,913
450	
5,400	
9,000	
14,913	

13

Breaking Up Times: Two Ways - Worksheet 2
BUWV - Example:

Do the same problem again but break up the 12 and 13 a different way.
Use the same color pattern you used on the previous page.

12

13 +

13 x 12 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

156 = ___ + ___ + ___ + ___

Final Product

15

Breaking Up Times: Two Ways - Worksheet 3

BUWV

Answer to both problems 156

16

Breaking Up Times: Two Ways - Worksheet 4
BUWV

Solve these problems two different ways.

12

17 +

12 x 17 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

204 = ___ + ___ + ___ + ___

Final Product

12

17 +

12 x 17 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

204 = ___ + ___ + ___ + ___

Final Product

17

Breaking Up Times: Two Ways - Worksheet 5
BUWV

Solve these problems two different ways.

21

17 +

21 x 17 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

357 = ___ + ___ + ___ + ___

Final Product

21

17 +

21 x 17 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

357 = ___ + ___ + ___ + ___

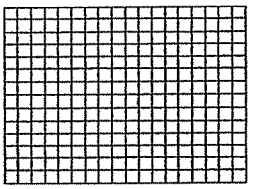
Final Product

18

Breaking Up Times: Two Ways - Worksheet 6
BUWV

Solve these problems two different ways.

18 = ___ + ___

14 + 

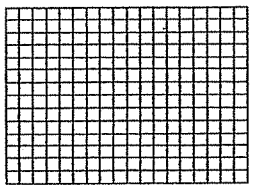
18 x 14 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

252 = ___ + ___ + ___ + ___

Final Product

18 = ___ + ___

14 + 

18 x 14 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

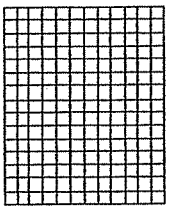
252 = ___ + ___ + ___ + ___

Final Product

Breaking Up Times: Two Ways - Worksheet 7
BUWV

Break up both numbers in these problems by drawing the lines and recording the little problems.

12 = ___ + ___

15 + 

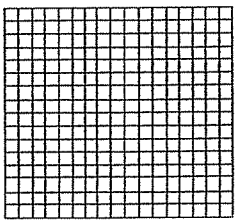
12 x 15 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

180 = ___ + ___ + ___ + ___

Final Product

17 = ___ + ___

16 + 

12 x 15 = (___ + ___) x (___ + ___)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

180 = ___ + ___ + ___ + ___

Final Product

Breaking Up Times: Arrow Pattern - Worksheet 1
BUWV

12 = 5 + 7

13 | 5 | 7

+ 4 | 5x4 | 7x4

5x9 | 7x9

Fill in the blanks.
Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

12 = (5 + 7)

x13 = (9 + 4)

12 x 13 = (5 + 7) x (9 + 4)

= (5 x 9) + (5 x 4) + (7 x 9) + (7 x 4)

= 45 + 20 + 63 + 28

Here is one way to draw the arrows.

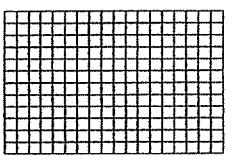
12 = (5 + 7)

x13 = (9 + 4)

Breaking Up Times: Arrow Pattern - Worksheet 2
BUWV

Break up both numbers in these problems by drawing the lines and recording the little problems.

18 = ___ + ___

12+ 

Fill in the blanks.
Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

18 = ___ + ___

x12 = ___ + ___

18 x 12 = (+) x (+)

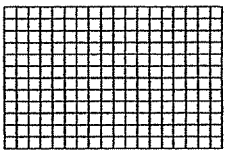
List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

216 = ___ + ___ + ___ + ___

Final Product

Break up the problem a different way. Find the easiest way to break up the problem.

18 = ___ + ___

12+ 

18 = ___ + ___

x12 = ___ + ___

18 x 12 = (+) x (+)

List all little problems = (___ x ___) + (___ x ___) + (___ x ___) + (___ x ___)

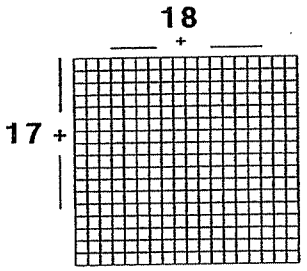
216 = ___ + ___ + ___ + ___

Final Product

Breaking Up Times: Arrow Pattern - Worksheet 3

BUWV

Break up both numbers by drawing the lines and recording the little problems.



Fill in the blanks.
Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

18 = ___ + ___

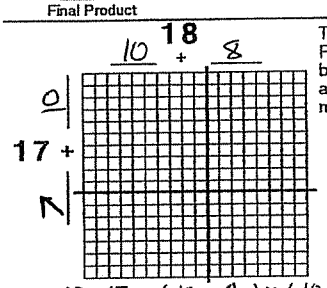
x17 = ___ + ___

18 x 17 = (+)x (+)

List all little problems = (___x___) + (___x___) + (___x___) + (___x___)

306 = ___ + ___ + ___ + ___

Final Product



This problem is broken up for you. Fill in the blanks. Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

18 = 10 + 8

x17 = 10 + 7

18 x 17 = (10 + 8)x (10 + 7)

List all little problems = (10x10) + (8x10) + (10x7) + (8x7)

306 = 100 + 80 + 70 + 56

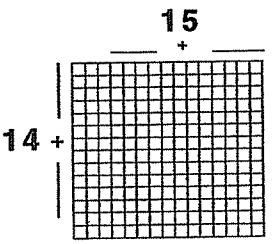
Final Product

23

Breaking Up Times: Arrow Pattern - Worksheet 4

BUWV

Break up both numbers by drawing the lines and recording the little problems.



Fill in the blanks.
Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

15 = ___ + ___

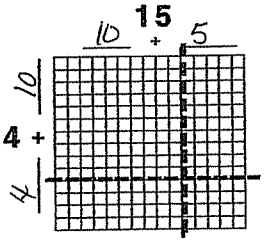
x14 = ___ + ___

15 x 14 = (+)x (+)

List all little problems = (___x___) + (___x___) + (___x___) + (___x___)

210 = ___ + ___ + ___ + ___

Final Product



This problem is broken up for you. Fill in the blanks. Draw arrows between the pairs of numbers that are multiplied in the little multiplication problems.

15 = 10 + 5

x14 = 10 + 4

15 x 14 = (10 + 5)x (10 + 4)

List all little problems = (10x10) + (10x4) + (5x10) + (5x4)

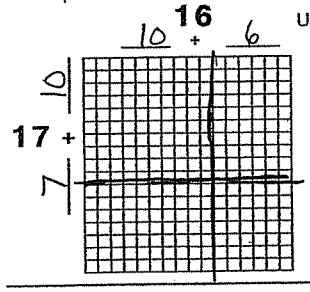
210 = 100 + 40 + 50 + 20

Final Product

24

Using Arrow Patterns to Multiply in Tens and Ones Worksheet 1

Break up both numbers into tens and ones and draw the lines.



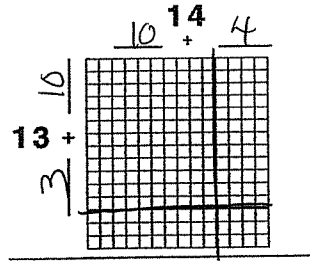
Use the arrow pattern to fill in the blanks.

16 = 10 + 6

x 17 = 10 + 7

42
70
60
100

Final Product 272

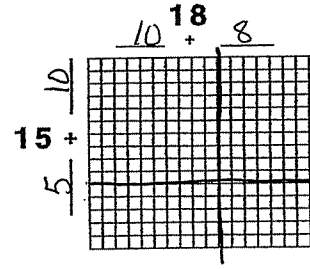


14 = 10 + 4

x 13 = 10 + 3

12
30
40
100

Final Product 182



18 = 10 + 8

x 15 = 10 + 5

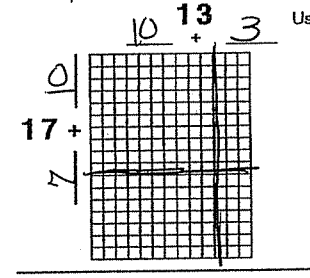
40
50
80
100

Final Product 270

25

Using Arrow Patterns to Multiply in Tens and Ones Worksheet 2

Break up both numbers into tens and ones and draw the lines.



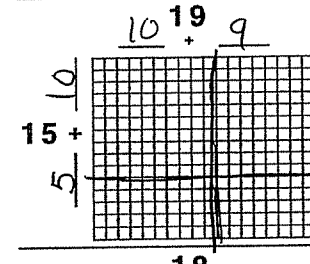
Use the arrow pattern to fill in the blanks.

13 = 10 + 3

x 17 = 10 + 7

21
70
30
100

Final Product 221

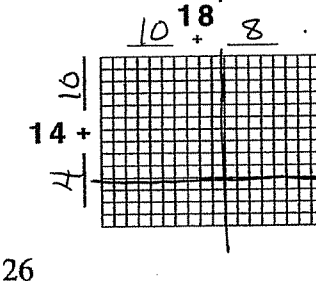


19 = 10 + 9

x 15 = 10 + 5

45
50
90
100

Final Product 285



18 = 10 + 8

x 14 = 10 + 4

32
40
80
100

Final Product 252

26

Using Arrow Patterns to Multiply in Tens and Ones Worksheet 3

Break up these problems into tens and ones and use the arrow pattern to solve.

16 = 10 + 6
x 12 = 10 + 2
12
20
60
100
Final Product 192

19 = 10 + 9
x 18 = 10 + 8
72
80
90
100
Final Product 342

14 = 10 + 4
x 16 = 10 + 6
24
40
60
100
Final Product 224

13 = 10 + 3
x 15 = 10 + 5
15
50
30
100
Final Product 195

15 = 10 + 5
x 18 = 10 + 8
40
80
50
100
Final Product 270

17 = 10 + 7
x 11 = 10 + 1
7
10
70
100
Final Product 187

Using Arrow Patterns to Multiply in Tens and Ones Worksheet 4

Break up these problems into tens and ones and use the arrow pattern to solve.

13 = 10 + 3
x 16 = 10 + 6
18
60
30
100
Final Product 208

17 = 10 + 7
x 12 = 10 + 2
14
20
70
100
Final Product 204

11 = 10 + 1
x 19 = 10 + 9
9
90
10
100
Final Product 209

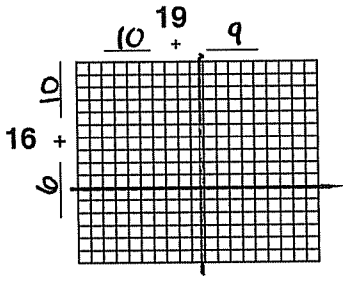
16 = 10 + 6
x 18 = 10 + 8
48
80
60
100
Final Product 288

15 = 10 + 5
x 16 = 10 + 6
30
60
50
100
Final Product 240

19 = 10 + 9
x 14 = 10 + 4
36
40
90
100
Final Product 266

Using Arrow Patterns to Multiply Tens and Ones - Assessment

Show You Know



Show the arrows.

19 = 10 + 9
x 16 = 10 + 6
54
60
90
100
Final Product 304

Fill in the blanks. Break the 14 and 16 into tens and ones.

14 x 16 = (10 + 4) x (10 + 6)

224 = (10 x 10) + (10 x 6) + (4 x 10) + (4 x 6)
224 = 100 + 60 + 40 + 24

Solve it without an array. Break the 12 and 13 into tens and ones

12 = 10 + 2
X 13 = 10 + 3
6
30
20
+100
156

Challenge!
Extend what you know to a new problem. Use the arrow pattern. Write all four little problems below the line and show their products.

25
X 23
15
60
100
400
575

Word Problems That Use Parentheses

(3 x 4) + 2

Price Sheet for Honey Apiaries (An apiary is a place that raises bees.)

Box of 5 queen cells	$8.00
1 single queen cell	$2.00
1 jar of honey	$5.00
1 hive of bees + queen	$50.00

How much would it cost to buy:

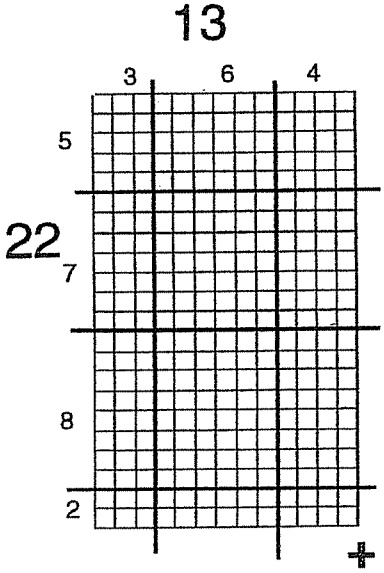
Three boxes of five queen cells and a jar of honey?

Number sentence = (3 x $8.00) + $5.00 = $29.00

Write two more problems like this using the menu above.

AWV

Breaking Up Times: Three Ways -Worksheet 1
Transition



13 = 3 + 6 + 4

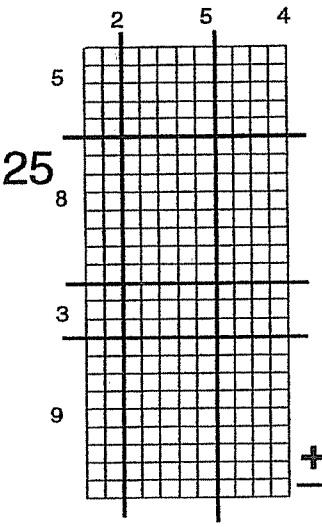
x 22 = 5 + 7 + 8 + 2

List all little problems

3 x 5 = 15
6 x 5 = 30
4 x 5 = 20
3 x 7 = 21
6 x 7 = 42
4 x 7 = 28
3 x 8 = 24
6 x 8 = 48
4 x 8 = 32
3 x 2 = 6
6 x 2 = 12
+ 4 x 2 = 8

286

Breaking Up Times: Three Ways - Worksheet 2
Transition



11 = 2 + 5 + 4

x 25 = 5 + 8 + 3 + 9

List all little problems

2 x 5 = 10
5 x 5 = 25
4 x 5 = 20
2 x 8 = 16
5 x 8 = 40
4 x 8 = 32
2 x 3 = 6
5 x 3 = 15
4 x 3 = 12
2 x 9 = 18
5 x 9 = 45
+ 4 x 9 = 36

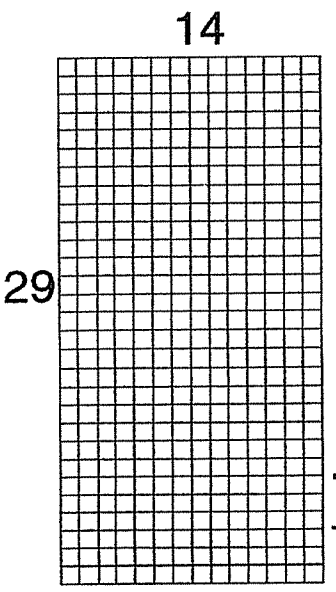
275

Breaking Up Times: Three Ways - Worksheet 3
Transition

Break up the 14 and 29 any way you want.

BUWV

Use only as many lines as you need.



14 = ___ + ___ + ___

x 29 = ___ + ___ + ___ + ___

List all little problems

___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
+ ___ x ___ = ___

406

Multiplying Factors of Ten - Worksheet 2

Examples of multiplying factors of ten. Do not use Prime Factors.

Step 1
Factor into multiples of ten.

70 x 90 =
(7 x 10) x (9 x 10)
Number Value, Place Value, Number Value, Place Value

Step 2
Rearrange the numbers*.

70 x 90 =
(7 x 9) x (10 x 10)
Number Value, Number Value, Place Value, Place Value
Sets Number, Sets Place Value
63 x 100 = 6,300

Now, you try it.

60 x 80 =
(6 x 8) x (10 x 10)
Number Value, Number Value, Place Value, Place Value
Sets Number, Sets Place Value
48 x 100 = 4,800

*When all numbers in an equation are multiplied, does the order matter? No.
What property of multiplication is this called? Associative

Multiplying Factors of Ten - Worksheet 3

Factor each problem by multiplying factors of ten. # = number value
P.V. = place value

20 x 30 =

(2 x 10) x (3 x 10)
(Number Value / Place Value / Number Value / Place Value)

(2 x 3) x (10 x 10) =
(Number Value, Number Value / Place Value, Place Value)

6 x 100 = 600
(Number Value / Place Value)

30 x 40 =

(3 x 10) x (4 x 10)
(# / P.V. / # / P.V.)

(3 x 4) x (10 x 10) =
(# / # / P.V., P.V.)

12 x 100 = 1,200
(# / P.V.)

40 x 50 =

(4 x 10) x (5 x 10)
(# / P.V. / # / P.V.)

(4 x 5) x (10 x 10) =
(# / # / P.V., P.V.)

20 x 100 = 2,000
(# / P.V.)

60 x 30 =

(6 x 10) x (3 x 10)
(# / P.V. / # / P.V.)

(6 x 3) x (10 x 10) =
(# / # / P.V., P.V.)

18 x 100 = 1,800
(# / P.V.)

80 x 40 =

(8 x 10) x (4 x 10)
(# / P.V. / # / P.V.)

(8 x 4) x (10 x 10) =
(# / # / P.V., P.V.)

32 x 100 = 3,200
(# / P.V.)

Multiplying Factors of Ten - Worksheet 4

Factor into multiplies of ten. Rearrange the numbers so the number values and the place value numbers are together

1.

80 x 70 =

(8 x 10) x (7 x 10) =

(8 x 7) x (10 x 10) =

56 x 100 = 5,600

2.

60 x 50 =

(6 x 10) x (5 x 10) =

(6 x 5) x (10 x 10) =

30 x 100 = 3,000

3.

50 x 20 =

(5 x 10) x (2 x 10) =

(5 x 2) x (10 x 10) =

10 x 100 = 1,000

Multiplying Factors of Ten - Worksheet 5

Factor into multiplies of ten. Rearrange the numbers so the number values and the place value numbers are together

4.

80 x 80 =

(8 x 10) x (8 x 10) =

(8 x 8) x (10 x 10) =

64 x 100 = 6,400

5.

30 x 90 =

(3 x 10) x (9 x 10) =

(3 x 9) x (10 x 10) =

27 x 100 = 2,700

6.

60 x 70 =

(6 x 10) x (7 x 10) =

(6 x 7) x (10 x 10) =

42 x 100 = 4,200

7.

200 x 70 =

(2 x 100) x (7 x 10) =

(2 x 7) x (100 x 10) =

14 x 1,000 = 14,000

8.

300 x 200 =

(3 x 100) x (2 x 100) =

(3 x 2) x (100 x 100) =

6 x 10,000 = 60,000

9. Make your own. AWV

____ x ____ =

(__ x __) x (__ x __) =

(__ x __) x (__ x __) =

__ x __ = __

Factoring - Worksheet 1

Prime factor:	List the factors:	Prime factor:	List the factors:
50	5, 5, 2	20	5, 2, 2

50 → 2 x 25 → 5 x 5

20 → 2 x 10 → 2 x 5

Now take the prime factors of 50 and 20 and multiply them together

5 x 5 x 2 x 5 x 2 x 2 = 1,000

Now factor by tens, rearrange the numbers, and multiply the factors.

50 → 5 x 10 → (5 x 2) → 10

20 → 2 x 10 → (10 x 10) → 100

10 x 100 = 1,000

Which way is easier for you? Factoring by tens.

Solve these by factoring by tens:

60 x 40 = (6 x 4) x (10 x 10)
24 x 100 = 2,400

300 x 30 = (3 x 3) x (100 x 10)
9 x 1,000 = 9,000

80 x 9 = (8 x 9) x 10
72 x 10 = 720

20 x 400 = (2 x 4) x (10 x 100)
8 x 1,000 = 8,000

Factoring - Worksheet 2

Take Them Apart

What is a factor? Numbers that are multiplied together.

What is a prime factor? A factor which is a prime number.

Are all factors prime? no

What are the prime factors of 36? 3, 3, 2, 2

What are the prime factors of 54? 3, 3, 3, 2

Factors and multiples of ten. Use only tens!

10 = 10 x 1
100 = 10 x 10
1,000 = 10 x 10 x 10
10,000 = 10 x 10 x 10 x 10
100,000 = 10 x 10 x 10 x 10 x 10
1,000,000 = 10 x 10 x 10 x 10 x 10 x 10

Write Yes if the statements are equal and No if they are not equal.

yes 10 x 10 x 10 = 100 x 10

no 10 x 10 x 10 x 10 = 100 x 100,000

yes 10 x 10 x 10 = 100 x 10

yes 1,000 x 10 x 10 = 100,000

yes 10 x 10 x 100 = 10 x 10 x 10 x 10

no 100 x 1,000 = 10 x 10 x 10 x 10 x 10 x 10

Factoring - Worksheet 3

Use factoring to solve. Change the order to make it easier.

40 x 40 = (4 x 10) x (4 x 10) = (4 x 4) x (10 x 10) = 16 x 100 = 1,600

60 x 70 = 4,200

300 x 20 = 6,000

600 x 30 = 18,000

90 x 80 = 7,200

50 x 700 = 35,000

Watch out!

800 x 300 = 240,000

You make up one:

___ x ___ =

Factoring - Worksheet 4

Show two ways to factor 100.
1. 10 X 10
2. 1 X 100

Show two ways to factor 1,000.
1. 10 X 10 X 10
2. 10 X 100 2 X 500

Show two ways to factor 4,000.
1. 4 X 1,000
2. 40 X 100

Use factoring to solve. Change the order to make it easier.

4,000 x 100 = (4 x 1,000) x (10 x 10) = (4 x 10) x (1,000 x 10) = 40 x 10,000 = 400,000
or 4 x (10 x 10 x 1,000)

4,000 x 1,000 = 4,000,000

6,000 x 2,000 = 12,000,000

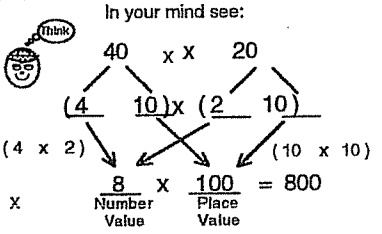
200 x 40 = 8,000

600 x 300 = 180,000

Factoring - Worksheet 5

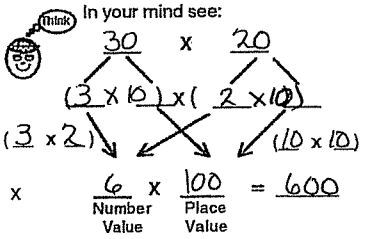
In your mind see:

See: 40 x 20

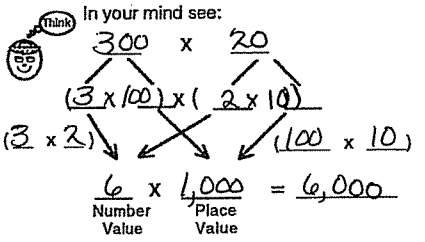


Now you do some.

See: 30 x 20



See: 300 x 20



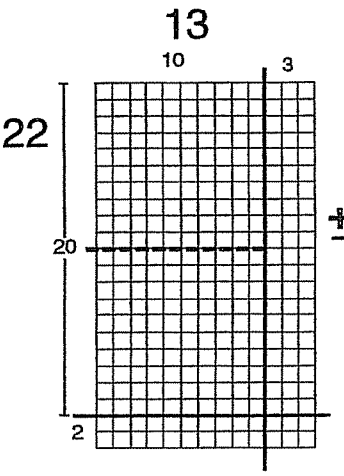
Do the Factoring - Worksheet 6: Just Look and Know on the next page as fast as you can.

Factoring - Worksheet 6

Just Look and Know

40 x 2 = 80	70 x 3 = 210	80 x 6 = 480	90 x 5 = 450
200 x 6 = 1,200	500 x 7 = 3,500	300 x 8 = 2,400	400 x 3 = 1,200
6,000 x 3 = 18,000	9,000 x 7 = 63,000	8,000 x 4 = 32,000	5,000 x 2 = 10,000
90 x 60 = 5,400	70 x 50 = 3,500	80 x 60 = 4,800	40 x 20 = 800
500 x 40 = 20,000	700 x 60 = 42,000	200 x 50 = 10,000	300 x 90 = 27,000
2,000 x 80 = 160,000	3,000 x 30 = 90,000	8,000 x 70 = 560,000	4,000 x 90 = 360,000
2,000 x 800 = 1,600,000	6,000 x 500 = 3,000,000	9,000 x 800 = 7,200,000	8,000 x 900 = 7,200,000

Breaking Up Times: Tens and Ones Worksheet 1



13 = 10 + 3

x 22 = 20 + 2

3 x 2 = 6
10 x 3 = 20
3 x 20 = 60
+ 10 x 20 = 200
286

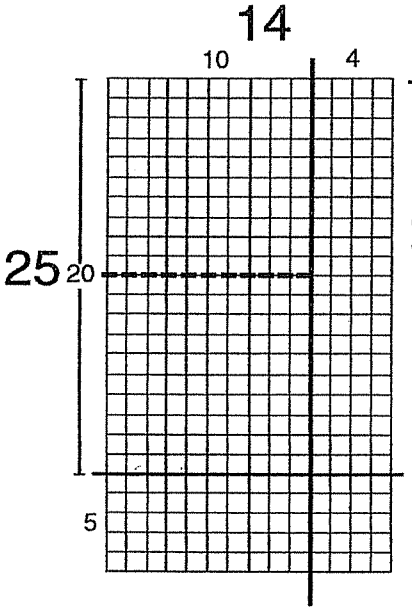
13 x 22 = (10 + 3) x (20 + 2)

List all little problems = (10 x 20) + (10 x 2) + (3 x 20) + (3 x 2)

286 = 200 + 20 + 60 + 6

Final Product

Breaking Up Times: Tens and Ones Worksheet 2



14 = 10 + 4

x 25 = 20 + 5

4 x 5 = 20
10 x 5 = 50
4 x 20 = 80
+ 10 x 20 = 200
350

14 x 25 = (10 + 4) x (20 + 5)

List all little problems = (10 x 20) + (10 x 5) + (4 x 20) + (4 x 5)

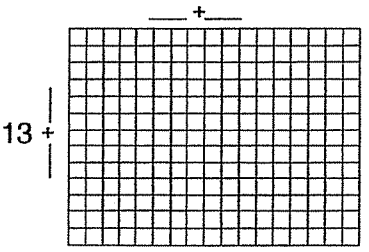
350 = 200 + 50 + 80 + 20

Final Product

Breaking Up Times: Tens and Ones Worksheet 3

BUWV

Now, solve the problem with and without the grid. You decide how to break up the 17 and the 13. Draw lines if you need to.



17 = ___ + ___

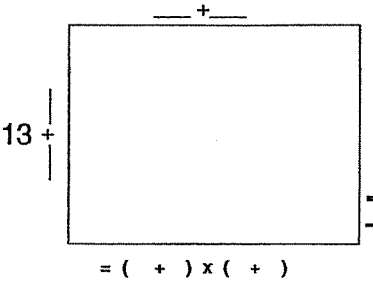
x 13 = ___ + ___

___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
+ ___ x ___ = ___
221

= (+) x (+)

= (x) + (x) + (x) + (x)

221 = ___ + ___ + ___ + ___



17 = ___ + ___

x 13 = ___ + ___

___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
+ ___ x ___ = ___
221

= (+) x (+)

= (x) + (x) + (x) + (x)

221 = ___ + ___ + ___ + ___

Breaking Up Times: Tens and Ones
Worksheet 4

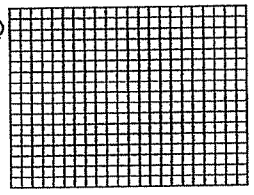
Fill in the missing numbers.

22 = 20 + 2

22

20 + 2

10
17 +
7



x 17 = 10 + 7

2 x 7 = 14
20 x 7 = 140
2 x 10 = 20
+ 20 x 10 = 200
374

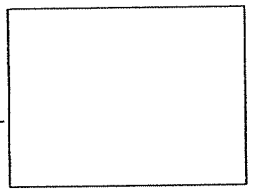
Fill in the missing numbers.

22 = 20 + ___

22

20 + ___

___
17 +
___



x 17 = ___ + ___

___ x ___ = ___
___ x ___ = ___
___ x ___ = ___
+ ___ x ___ = ___
374

22 x 17 = (20 + 2) x (10 + 7)
374 = (20x10) + (20x7) + (2x10) + (2x7)
374 = 200 + 140 + 20 + 14
Final Product

Breaking Up Times: Tens and Ones
Worksheet 5

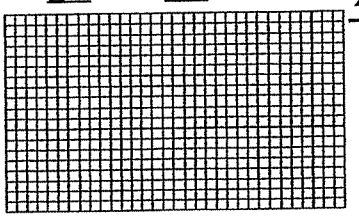
Break up the numbers into tens and ones.

32 = 30 + 2

32

30 + 2

16
19 +
9



x 19 = 10 + 9

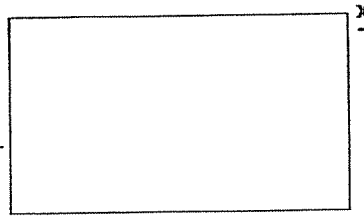
2 x 9 = 18
30 x 9 = 270
2 x 10 = 20
+ 30 x 10 = 300
608

32 = 30 + 2

32

30 + 2

10
19 +
9



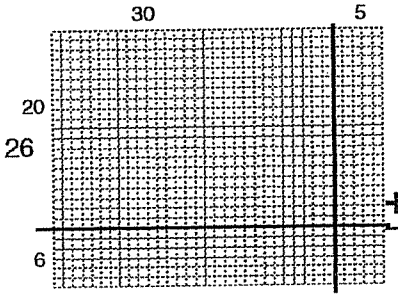
x 19 = 10 + 9

2 x 9 = 18
30 x 9 = 270
2 x 10 = 20
+ 30 x 10 = 300
608

Breaking Up Times: Tens and Ones
Worksheet 6

35

30 5



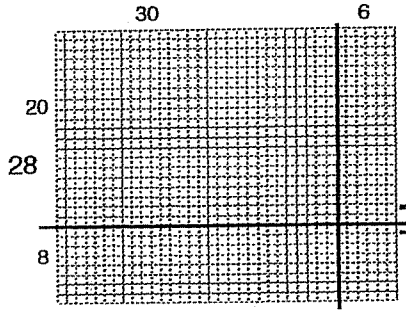
20
26
6

35 = 30 + 5
x 26 = 20 + 6

5 x 6 = 30
30 x 6 = 180
5 x 20 = 100
+ 30 x 20 = 600
910

36

30 6



20
28
8

36 = 30 + 6
x 28 = 20 + 8

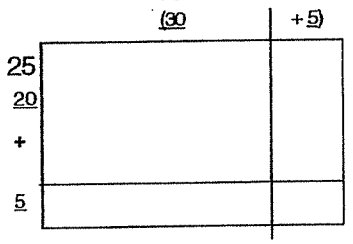
6 x 8 = 48
30 x 8 = 240
6 x 20 = 120
+ 30 x 20 = 600
1008

Breaking Up Times: Standard Procedure
Worksheet 1

Use tens and ones to break up the numbers.

35

(30) +5)



25
20
+
5

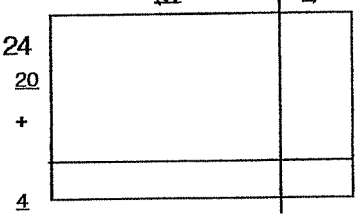
35 = 30 + 5
x 26 = 20 + 5

① 5 x 5 = 25
② 5 x 30 = 150
③ 20 x 5 = 100
+ ④ 20 x 30 = 600
875

You put in the arrows this time.

37

(30) +7



24
20
+
4

37 = 30 + 7
x 24 = 20 + 4

7 x 4 = 28
30 x 4 = 120
20 x 7 = 140
+ 30 x 20 = 600
888

Breaking Up Times: Standard Procedure Worksheet 2

Use tens and ones to break up the numbers.

43
(40 + 3)

28
20 +
8

You put in the arrows this time.

43 = 40 + 3

x 28 = 20 + 8

8 x 3 = 18
8 x 40 = 320
20 x 3 = 60
+ 20 x 40 = 800
1,204

27
(20 + 7)

46
40 +
6

You put in the arrows this time.

27 = 20 + 7

x 46 = 40 + 6

6 x 7 = 42
6 x 20 = 120
40 x 7 = 280
+ 40 x 20 = 800
1,242

Breaking Up Times: Standard Procedure Worksheet 3

Use tens and ones to break up the numbers.

36
(30 + 6)

42
40 +
2

36 = 30 + 6

x 42 = 40 + 2

2 x 6 = 12
2 x 30 = 60
40 x 6 = 240
+ 40 x 30 = 1,200
1,512

31
(30 + 1)

53
50 +
3

31 = 30 + 1

x 53 = 50 + 3

3 x 1 = 3
3 x 30 = 90
50 x 1 = 50
+ 50 x 30 = 1,500
1,643

Breaking Up Times: Standard Procedure Worksheet 4

27 = 20 + 7

x 19 = 10 + 9

① 9 x 7 = 63
② 9 x 20 = 180
③ 10 x 7 = 70
+ ④ 10 x 20 = 200
513

Work space for factoring.

48 = 40 + 8

x 29 = 20 + 9

① 8 x 9 = 72
② 9 x 40 = 360
③ 20 x 8 = 160
+ ④ 20 x 40 = 800
1,392

Work space for factoring.

Make your own.

___ = 0 + ___

x ___ = 0 + ___

① ___ + ___ = _____
② ___ + ___ = _____
+ ③ ___ + ___ = _____
④ ___ + ___ = _____

___ = 0 + ___

x ___ = 0 + ___

① ___ + ___ = _____
② ___ + ___ = _____
+ ③ ___ + ___ = _____
④ ___ + ___ = _____

Breaking Up Times: Long Way - Worksheet 1

Here is a sample problem. What is 35 x 73? _____

35
x 73
15
90
350
+ 2,100
2,555

Check your answer with a calculator.
If your answer is incorrect you need to check the _____.*

Now solve these problems:

46	29	56
x 83	x 37	x 35
18	63	30
120	140	250
480	270	180
3,200	600	1,500
+	+	+
3,818	1,073	1,960

*addition

Breaking Up Times: Long Way - Worksheet 2

Show all work.

22 x 13	59 x 15	41 x 16
6	45	6
60	250	240
20	90	10
200	500	400
286	885	656

56 x 27	70 x 28	65 x 22
42	0	10
350	560	120
120	0	100
1,000	1,400	1,200
1,512	1,960	1,430

Breaking Up Times: Long Way - Worksheet 3

Show all work.

94 x 39	97 x 32	27 x 37
36	14	49
810	180	140
120	210	210
2,700	2,700	600
3,666	3,104	999

56 x 41	87 x 42	89 x 46
6	14	54
50	160	480
240	280	360
2,000	3,200	3,200
2,214	3,654	4,094

Breaking Up Times: Long Way - Worksheet 4

Show all work.

494 x 35	197 x 46	427 x 57
20	42	49
450	540	140
2,000	600	2,800
120	280	350
2,700	3,600	1,000
12,000	4,000	20,000
17,290	9,062	24,339

356 x 41	387 x 48	789 x 52
6	56	18
50	640	160
300	2,400	1,400
240	280	450
2,000	640	4,000
12,000	12,000	35,000
14,596	18,576	41,028

Long Way to Standard Way Transfer Page

Step 3.

42 x 23	42 x 23
6	126
120	+ 840
40	966
+ 800	
966	

Long Way to Standard Way - Worksheet 2 Transfer Page

Step 4.

37 x 92	6 37 x 92	
14	64	Multiply the 3 x 9, add the 6 and record the 33. Add.
60	+ 3330	
630	3394	
+ 2700		
3394		

Go back to Breaking Up Times: Long Way - Worksheets 2 and 3 and solve all the problems the new standard way right next to the old long way. Check your answers to be sure they match.

Standard Way: Practice - Worksheet 1

42 x 23 126 84 966	71 x 15 355 710 1,065	81 x 36 486 2,430 2,916
34 x 29 306 680 986	73 x 55 365 3,650 4,015	53 x 62 106 3,180 3,286
44 x 28 352 880 1,232	48 x 36 288 1,440 1,728	57 x 34 228 1,710 1,938
234 x 23 702 4,680 5,382	148 x 45 740 5,920 6,660	231 x 52 462 11,550 12,012
327 x 43 981 13,080 14,061	408 x 67 2,856 24,480 27,336	278 x 54 1,112 13,900 15,012

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Standard Way: Practice - Worksheet 2

1 46 x 23 138 92 1,058	73 x 13 219 73 949	87 x 39 783 2,610 3,393
92 x 39 828 2,760 3,588	39 x 56 234 1,950 2,184	41 x 76 246 2,870 3,116
238 x 68 1,904 14,280 16,184	142 x 87 994 11,360 12,356	376 x 62 752 22,560 23,312
323 x 38 2,584 9,690 12,274	254 x 75 1,270 17,780 19,050	172 x 93 516 15,480 15,996
3,347 x 79 264,413	3,318 x 38 126,084	6,906 x 56 386,736

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Standard Way: Practice - Worksheet 3

What ten things you can always count on? _____

Figure out the area of these figures. Place the letter in the box below its area to solve this riddle.

2,795	1,190	4,536	2,805	1,998	2,773	3,200	2,408	2,000	2,304	2,464
Y	O	U	R	F	I	N	G	E	R	S

63

Standard Way: Practice - Worksheet 4

837 x 152 127,224	902 x 260 234,520	748 x 134 100,232
507 x 293 148,551	375 x 327 122,625	507 x 170 86,190
2,324 x 27 23,312	3,638 x 38 138,244	6,236 x 74 461,464
145 x 236 34,220	248 x 127 31,496	367 x 150 55,050
421 x 308 129,668	523 x 873 456,579	925 x 357 330,225

64

Standard Multiplication: Assessment

Show You Know

1. 300 x 70 = 21,000

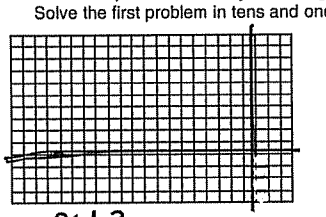
Prove your answer with factoring.

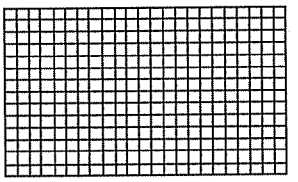
300×70
$= (3 \times 100) \times (7 \times 10)$
$= (3 \times 7) \times (100 \times 10)$
$21{,}000 = 21 \times 1{,}000$

2. What is 10 x 10 x 10 x 10 = 10,000 ?

3. Solve this problem two ways on the grid: 23 x 14 = 322
Solve the first problem in tens and ones.

BUWV





20+3
x 10+4
4x3=12
4x20=80
10x3=30
10x20=200
322

4. Solve this problem the long way

47
x 39
63
360
210
+1,200
1,833

5. Solve this problem the standard way

53
x 64
212
318
3,392

65

Multiplying Decimal Factors - Worksheet 1

Examples of multiplying factors of 10. Do not use prime factors.

Step 1
Factor into multiples of $\frac{1}{10}$.

0.7 x 0.9

(7 x $\frac{1}{10}$) x (9 x $\frac{1}{10}$)

Number Value | Place Value | Number Value | Place Value

Step 2
Rearrange the numbers.*

0.7 x 0.9

(7 x 9) x ($\frac{1}{10}$ x $\frac{1}{10}$)

Number Value | Number Value | Place Value | Place Value

Sets Number: 63 Sets Place Value: $\frac{1}{100}$ product: $\frac{63}{100}$ = .63

Now, you try it.

Step 1

0.6 x 0.8

(6 x $\frac{1}{10}$) x (8 x $\frac{1}{10}$)

Number Value | Place Value | Number Value | Place Value

Step 2

0.6 x 0.8

(6 x 8) x ($\frac{1}{10}$ x $\frac{1}{10}$)

Number Value | Number Value | Place Value | Place Value

Sets Number: 48 x Sets Place Value: $\frac{1}{100}$ = $\frac{48}{100}$ = .48

*When all numbers in an equation are multiplied, does the order matter? No
What property of multiplication is this called? Associative

Multiplying Decimal Factors - Worksheet 2

Factor into multiples of ten. Rearrange the numbers so the number values and the place value numbers are together.

1. 0.8 x 0.7 =

(8 x $\frac{1}{10}$) x (7 x $\frac{1}{10}$) =

(8 x 7) x ($\frac{1}{10}$ x $\frac{1}{10}$) =

56 x $\frac{1}{100}$ = .56

2. 0.6 x 0.5 =

(6 x $\frac{1}{10}$) x (5 x $\frac{1}{10}$) =

(6 x 5) x ($\frac{1}{10}$ x $\frac{1}{10}$) =

30 x $\frac{1}{100}$ = .3

3. 0.5 x 0.2 =

(5 x $\frac{1}{10}$) x (2 x $\frac{1}{10}$) =

(5 x 2) x ($\frac{1}{10}$ x $\frac{1}{10}$) =

10 x $\frac{1}{100}$ = .1

Multiplying Decimal Factors - Worksheet 3

Factor into multiples of ten. Rearrange the numbers so the number values and the place value numbers are together

4. .08 x .8 =

(8 x $\frac{1}{100}$) x (8 x $\frac{1}{10}$) =

(8 x 8) x ($\frac{1}{100}$ x $\frac{1}{10}$) =

64 x $\frac{1}{1,000}$ = .064

5. 0.3 x .09 =

(3 x $\frac{1}{10}$) x (9 x $\frac{1}{100}$) =

(3 x 9) x ($\frac{1}{10}$ x $\frac{1}{100}$) =

27 x $\frac{1}{1,000}$ = .027

6. .6 x .07 =

(6 x $\frac{1}{10}$) x (7 x $\frac{1}{100}$) =

(6 x 7) x ($\frac{1}{10}$ x $\frac{1}{100}$) =

42 x $\frac{1}{1,000}$ = .042

7. .002 x 0.7 =

(2 x $\frac{1}{1,000}$) x (7 x $\frac{1}{10}$) =

(2 x 7) x ($\frac{1}{1,000}$ x $\frac{1}{10}$) =

14 x $\frac{1}{10,000}$ = .0014

8. .03 x .02 =

(3 x $\frac{1}{100}$) x (2 x $\frac{1}{100}$) =

(3 x 2) x ($\frac{1}{100}$ x $\frac{1}{100}$) =

6 x $\frac{1}{10,000}$ = .0006

9. Make your own. AWV

___ x ___ =

(___ x ___) x (___ x ___) =

(___ x ___) x (___ x ___) =

___ x ___ = ___

Times Tables Quiz - Worksheet 1

How many problems can you do in one minute? _____
How many problems can you do in 30 seconds? _____

Three Times Table

12 x 3 = 36	9 x 3 = 27	10 x 3 = 30	0 x 3 = 0
5 x 3 = 15	3 x 3 = 9	2 x 3 = 6	4 x 3 = 12
6 x 3 = 18	7 x 3 = 21	1 x 3 = 3	8 x 3 = 24

Four Times Table

0 x 4 = 0	5 x 4 = 20	3 x 4 = 12	4 x 4 = 16
10 x 4 = 40	9 x 4 = 36	12 x 4 = 48	8 x 4 = 32
6 x 4 = 24	7 x 4 = 28	1 x 4 = 4	2 x 4 = 8

Five Times Table

6 x 5 = 30	7 x 5 = 35	4 x 5 = 20	3 x 5 = 15
12 x 5 = 60	9 x 5 = 45	10 x 5 = 50	2 x 5 = 10
1 x 5 = 5	5 x 5 = 25	0 x 5 = 0	8 x 5 = 40

Six Times Table

8 x 6 = 48	9 x 6 = 54	4 x 6 = 24	1 x 6 = 6
10 x 6 = 60	5 x 6 = 30	12 x 6 = 72	7 x 6 = 42
3 x 6 = 18	6 x 6 = 36	0 x 6 = 0	2 x 6 = 12

Times Tables Quiz - Worksheet 2

How many problems can you do in one minute? _____
How many problems can you do in 30 seconds? _____

Seven Times Table

12 x 7 = 14	9 x 7 = 63	10 x 7 = 70	0 x 7 = 0
5 x 7 = 35	3 x 7 = 21	2 x 7 = 14	4 x 7 = 28
6 x 7 = 42	7 x 7 = 49	1 x 7 = 7	8 x 7 = 56

Eight Times Table

0 x 8 = 0	5 x 8 = 40	3 x 8 = 24	4 x 8 = 32
10 x 8 = 80	9 x 8 = 72	12 x 8 = 96	8 x 8 = 64
6 x 8 = 48	7 x 8 = 56	1 x 8 = 8	2 x 8 = 16

Nine Times Table

6 x 9 = 54	7 x 9 = 63	4 x 9 = 36	3 x 9 = 27
12 x 9 = 108	9 x 9 = 81	10 x 9 = 90	2 x 9 = 18
1 x 9 = 9	5 x 9 = 45	0 x 9 = 0	8 x 9 = 72

Two Times Table

8 x 2 = 16	9 x 2 = 18	4 x 2 = 8	1 x 2 = 2
10 x 2 = 20	5 x 2 = 10	12 x 2 = 24	7 x 2 = 14
3 x 2 = 6	6 x 2 = 12	0 x 2 = 0	2 x 2 = 4

Pre-Assessment: Part 1 Worksheet 1

1. Solve the Short Way.

a. 34 x 6 = 204 b. 258 x 3 = 774 c. 1,735 x 4 = 6,940

2 Solve the Long Way.

a. 34 x 6: 24 + 180 = 204
b. 258 x 3: 24 + 150 + 600 = 774
c. 1,735 x 4: 20 + 120 + 2800 + 4,000 = 6,940

3. Break up the 57 using tens and ones. Fill in all the blanks.

57 x 3 = (50 + 7) x 3
= (50 x 3) + (7 x 3)
171 = 150 + 21

4. What is a factor? You may use a numerical example to help you explain.
2 x 3 are factors of 6. A factor is a number that results in a product when multiplied by another factor.

5. What is a Prime Number? A number that has only one and itself as factors.

6. a. Prime Factor 120.
b. Record the Prime Factors in this box.

120 → 2 x 60 → 6 x 10 → 3 x 2 x 2 x 5

5, 3, 2, 2, 2

7. Factor 120 into factors of ten.

120 → 100 x 12 or 12 x 10 x 10

8. Use factoring by tens to show how to get the product of 700 x 9.

700 x 9
7 x 100 x 9
(7 x 9) x 100
63 x 100
6,300

Pre-Assessment: Part 2 - Worksheet 1

1. a. Solve. 400 x 30 = 12,000

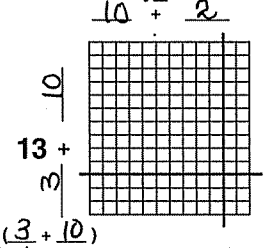
b. Explain how you got your answer Factor 4 from the 100 and 3 from the 10, rearrange the factors and combine the 4 x 3 = 12 and the 100 x 10 to get 1,000. Multiply 12 x 1,000 = 12,000

2. Use factoring by tens to show how to get the answer to 40 x 700.

40 x 700
(4 x 10) x (7 x 100)
(4 x 7) x (10 x 100)
28 x 1,000 = 28,000

3. There is a trick to solve a problem such as 900 x 500. Multiply 9 x 5 and add four zeros. Explain why this trick works.
Associative Property allows the rearrangement of factors. Multiply 9 x 5 and 100 x 100. The four zeros come from multiplying 100 x 100 = 10,000 which has four zeros.

4. Use this diagram to help you fill in the distribution number sentences.



12 x 13 = (10 + 2) x (3 + 10)
= (3 x 2) + (3 x 10) + (10 x 2) + (10 x 10)
= 6 + 30 + 20 + 100
156

Pre-Assessment: Part 2 - Worksheet 2

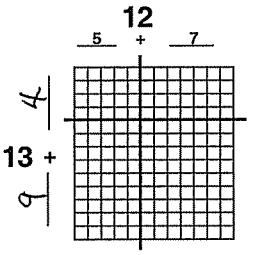
5. Show the distribution pattern with arrows between the numbers. Use this diagram to help.

12 = 5 + 7
x 13 = 9 + 4

Write the little problems here.

4 x 7 = 28
4 x 5 = 20
9 x 7 = 63
9 x 5 = 45

Final answer = 156



6. You should have gotten the same answer for problems 4 and 5. Explain why.
Even though the problems are broken up differently, the answers will be the same because it's the same problem.

7. Solve this problem the long way using the arrow pattern.

235
x 46

little multiplication problems		partial products
5 x 6	=	30
30 x 6	=	180
200 x 6	=	1,200
5 x 40	=	200
30 x 40	=	1,200
200 x 40	=	8,000
	=	10,810

8. Solve the same problem the short way.

235
x 46
1410
940
10,810

9. Why do you have to put a zero in the ones place in the second row of the product in problem 8?
Because the four is in the tens place, so the multiplication problem is 40 x 5 = 200 which has a zero in the ones place.

Post-Assessment is the same as Pre-Assessment: Part 2

Expanded Multiplication: Review - Worksheet 4

Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the "four times" table.
Use your Base Ten Blocks.

1 x 4 = 4	10 x 4 = 40	100 x 4 = 400
2 x 4 = 8	20 x 4 = 80	200 x 4 = 800
3 x 4 = 12	30 x 4 = 120	300 x 4 = 1,200

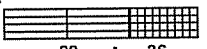
Use your Base Ten Blocks to find the answers to the rest of these problems.

4 x 4 = 16	40 x 4 = 160	400 x 4 = 1,600
5 x 4 = 20	50 x 4 = 200	500 x 4 = 2,000
6 x 4 = 24	60 x 4 = 240	600 x 4 = 2,400
7 x 4 = 28	70 x 4 = 280	700 x 4 = 2,800
8 x 4 = 32	80 x 4 = 320	800 x 4 = 3,200
9 x 4 = 36	90 x 4 = 360	900 x 4 = 3,600

Expanded Multiplication: Review - Worksheet 5

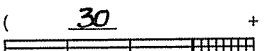
Use Base Ten Blocks and expanded tables from the previous page.

29 x 4 = (20 + 9) x 4



29 x 4 = 116 (80 + 36)

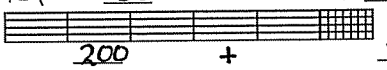
38 x 4 = (30 + 8) x 4



120 + 32

38 x 4 = 152

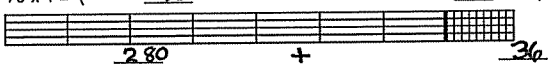
57 x 4 = (50 + 7) x 4



200 + 28

57 x 4 = 228

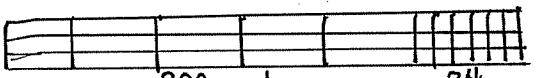
79 x 4 = (70 + 9) x 4



280 + 36

79 x 4 = 316

Try 56 x 4 = (50 + 6) x 4 You draw the box.



56 x 4 = 224 (200 + 24)

Expanded Multiplication: Review - Worksheet 6

Use Base Ten Blocks and expanded tables to solve these box problems.

Example:

87 x 4 = (80 + 7) x 4

320	+	28

87 x 4 = 348:

87
x 4
28
320
348

79 x 4 = (70 + 9) x 4

280	+	36

79 x 4 = 316

79
x 4
316

47 x 4 = (40 + 7) x 4

160	+	28

47 x 4 = 188

47
x 4
188

Pages 22, 23, and 24 are from the Parent/Teacher Guide.

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Parent/Teacher Guide

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