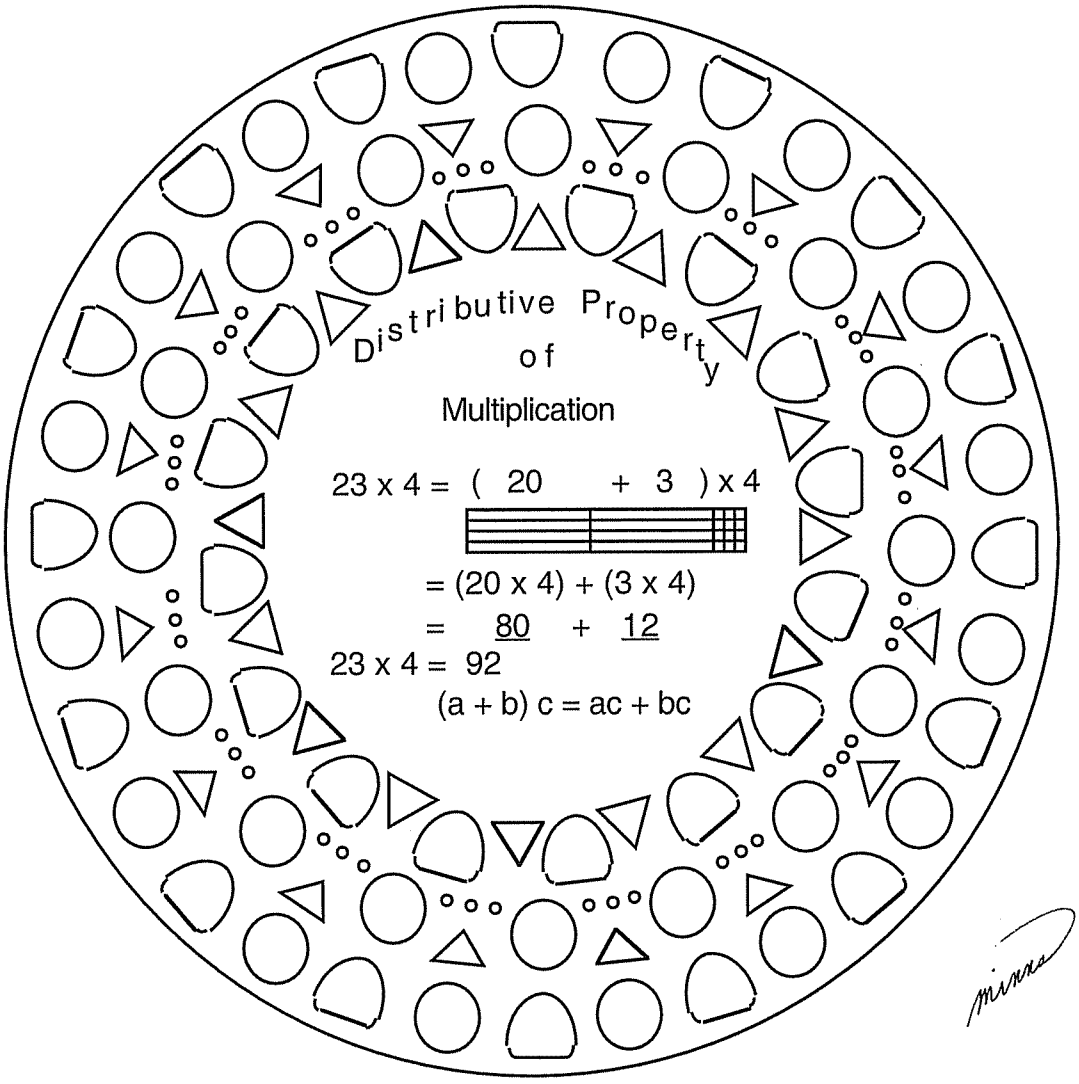
Patterns in Arithmetic

Multiplication - Booklet 2 PDF

Beginning Long Multiplication
and Basics of Distribution

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Multiplication: Booklet 2 - PDF - Beginning Long Multiplication and Basics of Distribution

Contents

This booklet is dedicated to my mother, Beryl Lueck, a constructivist teacher for many years. She would be delighted to know I followed in her footsteps. Thanks also to Whittier College for the teaching program where she learned this teaching method. Suki Glenn

Acknowledgment

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Multiplication - Booklet 2 PDF
Parent/Teacher Guide

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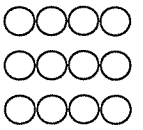
Pre-Assessment - Part 1 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. What multiplication problem is shown in this drawing?



_____ x _____ = _____

2. Draw a picture like the one above for 7 x 3 =_____.

3. Write this multiplication problem as an addition problem and solve: 23 x 4.

4. Skip count by fours from 0 to 40. _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

5. Solve these problems.

a. 6 x 4

b. 5 x 5

c. 9 x 3

d. 7 x 6

e. 8 x 8

f. 9 x 6

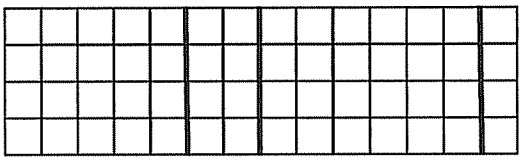
6. Write all multiplication problems that equal 24.
You may use Cuisenaire Rods and a meter stick if you like.
Hint: There are more than two.

Pre-Assessment - Part 2 Date _____

1. Fill in the missing numbers.

   a. 14
   x 4

   b. 14 x 4 = (___ + ___ + ___ + ___) x 4



   c. (__ x __) + (__ x __) + (__ x __) (__ x __)

   e. _____ = d. _____ + _____ + _____ + _____

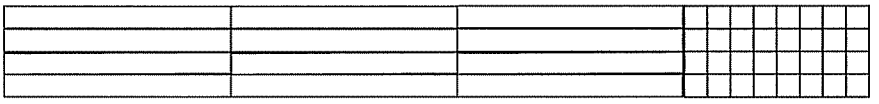
2. Show how you would break up the 63. Label the partial products in the box. Show the final product below the box on the line.

   63 x 8 = ( ) x 8

   63 x 8 = _____

3. Break up the 38 into tens and ones; fill in the missing numbers.

   a. 38 x 4 = (_____ + _____) x 4



   = b. (__ x __) + (__ x __)

   d. _____ = c. _____ + _____

4. In problem 3, what number is being distributed? _____

5. Solve these problems the Long way and the Short way.

   a. Long way
   1,439
   x 5

   b. Short way
   1,439
   x 5

6. Solve. a. 138
   x 6

   b. 2,375
   x 4

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from *Patterns in Arithmetic:* Multiplication: Booklet 1. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Multiplication: Booklet 1

Materials

Multiplication: Booklet 2 - Pre-Assessment: Part 1 and Part 2, pages 1 and 2
Pre-Assessment: Part 1 and Part 2 Score sheets, pages 7 and 8 in this booklet
Cuisenaire Rods
Meter stick
Post-Assessment (Student Workbook, page 77, and page 41 in this booklet)

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Part 1

Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)
   A. identify a multiplication problem from a drawing?
   B. record the correct product?

2. Multiplication Concept (Multiplication: Booklet 1)
   A. draw a picture of a given multiplication number sentence?
   B. record the correct product?

3. Multiplication by Addition (Multiplication: Booklet 1)
   A. show 23 x 4 as 23 + 23 + 23 + 23?

If he shows the problem as 23 x 4 and solves it with the standard procedure, mark this item a No.

   B. give the correct answer?

4. Building Times Tables (Multiplication: Booklet 1)
   A. skip count by fours to forty? There may be a few errors.
   B. insert the correct numbers?

The items in criterion 5 are broken into lower times tables 2 - 5 for problems a, b, and c and into upper times tables 6 - 9 for problems d, e, and f.

5. Building Times Tables (Multiplication: Booklet 1)
   A. give the correct answers for two of three lower times table problems a, b, and c?
   B. give the correct answers for two of three upper times table problems d, e, and f?

6. Factoring: Recording (Multiplication: Booklet 1)
   A. list three of the four pairs of factors for twenty-four?
   B. use the Cuisenaire Rods to do it ? No score on this item. Just a note.

Booklet Selection Guide

Use the score sheet to record the results.

Readiness for Multiplication: Booklet 2 is based on results of Pre-Assessment: Part 1. If the student's score is 7 or less, don't give Pre-Assessment: Part 2 and begin with Multiplication: Booklet 1. If his score is 8 or more give, Pre-Assessment: Part 2.

Assessment Criteria for Pre-Assessment: Part 2

Can the student:

1. Breaking Up Times
   A. write the correct answer for problem a?
   B. fill in line b correctly?
   C. fill in line c correctly?
   D. fill in line d correctly?
   E. fill in the correct product at e?

Note the procedure used by the student in problem a. Did he use repeated addition or the standard multiplication procedure?

This format for a multiplication problem may be unfamiliar to you. If so, simply use the Answer Key to determine if your student wrote in the correct numbers or not. After you teach this section in the booklet, it will make sense to you.

2. Box Multiplication
    A. break up the sixty-three in the parentheses?
    Example: 63 x 8 = (30 + 20 + 10 + 3) x 8

The way the sixty-three is broken up will vary. Check to see if the addition is correct.

B. draw the same number of sections in the box as the number of addition terms he wrote in the parentheses? For example: If he wrote 63 x 8 = (30 + 20 + 10 + 3) x 8–in the parentheses he used four numbers to break up the sixty-three. The box should be divided into four sections also.

C. write the correct partial product in each section of the box? For example: If he wrote 63 x 8 = (30 + 20 + 10 + 3) x 8–in the parentheses on the top row, he would multiply 30 x 8 in his head and write 240 in the first section of the box. The second section would contain 20 x 8, or 160; the third section 10 x 8, or 80, and the last section 3 x 8, or 24. The numbers will vary, but the pattern should be the same.

D. write the correct product of 63 x 8 on the last line?

3. Expanded Tables
    A. break the thirty-eight into 30 + 8 in the parentheses on the top line, line a?
    B. write in the correct little multiplication problems on line b?
    C. write in the correct partial products on line c?
    D. write in the correct final product on line d?

4. Distributive Property
    A. identify the number four as the number being distributed in problems 3?

5. Expanded Multiplication and Short Notation
    A. use the long, expanded notation correctly in problem a?
    B. give the correct answer to problem a? Note any place value errors.
    C. use the short notation correctly on problem b?
    D. give the correct answer on problem b?

6. Short Notation
    A. give the correct answer to problem a?
    B. give the correct answer to problem b?

Booklet Selection Guide (based on results of Pre-Assessment: Part 2)

If the student scores 80% or better on Pre-Assessment: Part 2–this is 16 or more Yes items–move on to Multiplication: Booklet 3.

If the student gets a Yes on the following items: 1E, 2D, 3D, 5C, 6A, 6B, and a No on items 3A, 3B, 3C, 4A, and 5A, you have a student who knows how to multiply but does not understand it. You have two choices.

First, you could take the time to do Multiplication: Booklet 2, which is what you should do if you have an eight- or nine-year-old student. Follow Multiplication: Booklet 2 with Multiplication: Booklet 3, which is on factoring and prime numbers.

Second, you could begin Multiplication: Booklet 4, which teaches the same concept with problems like 35 x 27. This may be a good choice for a strong fifth-grader.

Multiplication: Booklet 3 can be taught concurrently with Multiplication: Booklet 4.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm-up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements such as, **"Are you sure?"** or **"Build it,"** or **"What gave you the clue?"** or **"Show me how you got that,"** or **"Prove it,"** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answer definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as, **"How did you get that? "Show me what you mean," "Build a model of that," "Tell me more."**

Pre-Assessment: Part 1 Score Sheet Name _____ Date _____

Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. identify a multiplication problem from a drawing?
   - Yes No B. record the correct product?

2. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. draw a picture of a given multiplication number sentence?
   - Yes No B. record the correct product?

3. Multiplication by Addition (Multiplication: Booklet 1)
   - Yes No A. show 23 x 4 as 23 + 23 + 23 + 23?
   - Yes No B. give the correct answer?

4. Building Times Tables (Multiplication: Booklet 1)
   - Yes No A. skip count by fours to forty?
   - Yes No B. insert the correct numbers? There may be a few errors.

5. Building Times Tables (Multiplication: Booklet 1)
   - Yes No A. give the correct answers for two of three on a, b, and c?
   - Yes No B. give the correct answers for two of three on d, e, and f?

6. Factoring: Recording (Multiplication: Booklet 1)
   - Yes No A. list three of the four pairs of factors for twenty-four?
   - B. use the Cuisenaire Rods to do it? No score on this item.

Items Correct = _____ = _____% 72% needed to begin Booklet 2
Items Possible = 11 This is 8 or more Yes items

Pre-Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Breaking Up Times
   - Yes No A. write the correct answer for problem a?
   - Yes No B. fill in line b correctly?
   - Yes No C. fill in line c correctly?
   - Yes No D. fill in line d correctly?
   - Yes No E. fill in the correct product at e?

2. Box Multiplication
   - Yes No A. break up the sixty-three in the parentheses?
   - Yes No B. draw the correct number of sections in the box?
   - Yes No C. write the correct partial product in each section of the box?
   - Yes No D. write the correct product of 63 x 8 on the last line?

3. Expanded Tables
   - Yes No A. break the 38 into 30 + 8 in the parentheses on line a?
   - Yes No B. write in the correct little multiplication problems on line b?
   - Yes No C. write in the correct partial products on line c?
   - Yes No D. write in the correct final product on line d?

4. Distributive Property
   - Yes No A. identify the number four as the number being distributed?

5. Expanded Multiplication and Short Notation
   - Yes No A. use the correct long, expanded notation in problem a?
   - Yes No B. give the correct answer to problem a?
   - Yes No C. use the correct short notation on problem b?
   - Yes No D. give the correct answer on problem b?

6. Short Notation
   - Yes No A. give the correct answer to problem a?
   - Yes No B. give the correct answer to problem b?

Items Correct = _____ = _____% 80% needed to move on to Multiplication: Booklet 3
Items Possible = 20 This is 16 or more Yes items.

Post-Assessment Score Sheet Name _____ Date _____

Can the student:

1. Breaking Up Times
   - Yes No A. write the correct answer for problem a?
   - Yes No B. fill in line b correctly?
   - Yes No C. fill in line c correctly?
   - Yes No D. fill in line d correctly?
   - Yes No E. fill in the correct product at e?

2. Box Multiplication
   - Yes No A. break up the sixty-three in the parentheses?
   - Yes No B. draw the correct number of sections in the box?
   - Yes No C. write the correct partial product in each section of the box?
   - Yes No D. write the correct product of 63 x 8 on the last line?

3. Expanded Tables
   - Yes No A. break the 38 into 30 + 8 in the parentheses on line a?
   - Yes No B. write in the correct little multiplication problems on line b?
   - Yes No C. write in the correct partial products on line c?
   - Yes No D. write in the correct final product on line d?

4. Distributive Property
   - Yes No A. identify the number four as the number being distributed?

5. Expanded Multiplication and Short Notation
   - Yes No A. use the correct long, expanded notation in problem a?
   - Yes No B. give the correct answer to problem a?
   - Yes No C. use the correct short notation on problem b?
   - Yes No D. give the correct answer on problem b?

6. Short Notation
   - Yes No A. give the correct answer to problem a?
   - Yes No B. give the correct answer to problem b?

Items Correct = _____ = _____%
Items Possible = 20

80% needed to begin Multiplication: Booklet 3
This is 16 or more Yes items.
Less than 80%–do additional practice in the areas missed and retest.

Introduction to the Distributive Property of Multiplication

Why does this booklet take so long to teach such an easy operation as long multiplication? Why all the square grids and crayons? Why do I see pages with the same problem over and over? Why not do it the easy way and get it done in less than a week?

645
x 3

Remember this? You probably learned this back in third or fourth grade. It was pretty easy for most of us. The talk went something like this: Multiply the three by the five to get fifteen. Put down the five and carry the one. Then multiply the three by the four to get twelve and add one to get thirteen. Put down the three and carry the one. Finally, multiply the three by the six to get eighteen and add one to get nineteen. Put down the nineteen. The answer is 1,935. Easy. Well, this little pattern you learned so easily is a shortcut. This shortcut completely obscures one of the most important patterns in arithmetic, the Distributive Property of Multiplication. For most students, the lack of understanding of this fundamental pattern results in general weakness in mathematics overall. You gain little by teaching the shortcut only, and stand to lose much in the long run.

$a (b + c + d) = ab + ac + ad$

Remember this? If you are fluent in algebra you recognize this as a multiplication of a polynomial. If you are not fluent in algebra, maybe you sort of remember seeing it in ninth or tenth grade. If you did not do very well in algebra, there is a good chance that not understanding how 645 x 3 works is a major cause.

Do you understand that $4 \times 3 = 3 \times 4$? Or that $645 \times 3 = 3 \times 645$? You probably do.

Do you understand that 3 x 645 is the exact same thing as $a (b + c + d)$? For most of you the answer is no. If you can understand the first, then you understand the second. If you cannot make the connection, it is because you were not taught to understand the Distributive Property of Multiplication back in grade school. This booklet will help you correct this loss in yourself and prevent it in your student.

Here is a preview of the connection.

3 x 645 can be broken up into 3 x (600 + 40 + 5). Each part inside the parentheses is multiplied by the three to get 1,800 + 120 + 15. The sum of these is the answer. The 645 does not even have to be broken up into hundreds, tens, and ones. You can break it up any way you want to, but the rule of multiplying each of the parts by the multiplier (the three in this case) is always the same no matter what the number is. That is why we can show the general pattern with letters: $a (b + c + d) = ab + ac + ad$. The a is the multiplier, and the b, c, and d are the large number broken up into parts. If put in a vertical format you will recognize it now.

645	600 + 40 + 5	B + C + D
x 3	x 3	x A
1,935 =	1,800 + 120 + 15	AB + AC + AD

Algebra is just the general pattern written with letters to stand for numbers. If you understand the numbers in third or fourth grade, you will understand the algebra in eighth or ninth grade.

Most of us simply memorized the shortcut, and then when we encountered the same kind of problem in algebra, we did not recognize it and had to memorize again. Those of us who memorized, or tried to memorize, our way though algebra did not do very well when we had to do word problems or use the pattern in an unfamiliar form or use it to divide. Higher mathematics and science classes became huge hurdles for us. This is very unfortunate because understanding takes a little longer to develop, but it is not that hard.

One of the big, big ideas in arithmetic is that to add with large numbers, the big addition problem is broken into a bunch of little addition problems, and then they are all added up at the end.

In a problem such as 645
$\underline{+\,324}$,

the ones are added first, then the tens; in other words, the problem is broken into ones, tens, and one hundreds, and each part is added. Then all the parts are pushed together to get the answer. This is done in subtraction, multiplication, and division. All arithmetic procedures work this way. There are rules for how the numbers can be broken up and recombined, but the big idea is always the same. A student who understands this big pattern has power to do problems in multiple ways and thus has more mathematical strength overall.

In multiplication and division, the guiding rule for breaking up large numbers to multiply or divide them is the Distributive Property.

Understanding the Distributive Property of Multiplication is fundamental to algebra. Not understanding it is one of the major reasons students fail in algebra. Why would one set students up to fail when learning to understand this critical concept is not that hard? It takes only a few extra weeks.

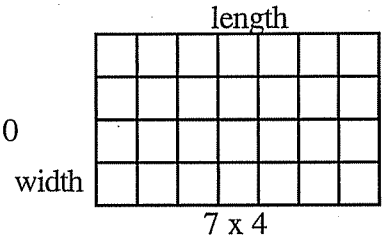
That is why this book has all those drawings, blocks, and crayons. In the end, your student will not only be able to solve multiplication problems, she will understand how the general idea works and will then be able to apply it to the general algebraic form later.

This book teaches the general pattern of distribution using very small numbers such as 7 x 4. The student already knows the answer is twenty-eight. She will break up the seven in multiple ways and use the Distributive Property to get the answer. She will know the pattern works because she will always get the same answer, twenty-eight, which she knows and can confirm for herself. We stay connected to the concrete every step of the way. It is only when the pattern is understood concretely that it is extended to the abstract, and we then look for the shortcuts.

In addition to learning to understand a critical concept in mathematics, she will also have fun. Most students really enjoy this book. It makes sense all the way and uses pretty blocks and fun colors. Enjoy.

Arrays: Manipulative

Purpose The purpose is to introduce the concept of an array and to begin breaking up multiplication problems and to identify the little problems created by breaking up the larger problem.



Prerequisites Mastery of basic multiplication

Materials Arrays: Manipulative - Worksheets 1 - 6, pages 5 - 10
Cuisenaire Rods or tiles in several colors
Crayons or markers

Lesson Session 1 Manipulative

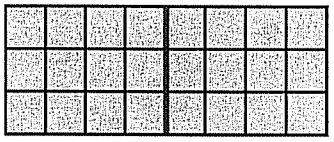
The student builds models of an array with Cuisenaire Rods in the first two worksheets. In this book, the longer side of an array is called the length and the shorter side is called the width.

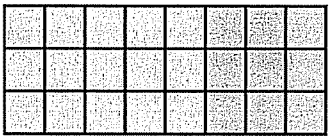
Worksheets Arrays: Manipulative - Worksheets 1 and 2, pages 5 and 6

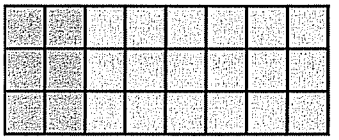
Session 2 Recording

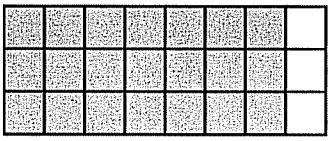
In this session, the student first builds an array with one color of Cuisenaire Rods. The same array is built with two colors of rods two different ways and then with three different rods into little multiplication problems. She figures out that a problem can be broken up many ways and this does not change the final answer. After studying the example problem on Worksheet 6, page 10, the student records the multiplication number sentence for the arrays on Worksheets 3 and 4, pages 7 and 8.

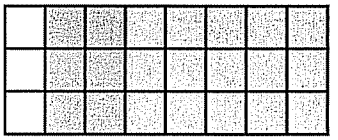
Worksheets Arrays: Manipulative - Worksheets 3 and 4, pages 7 and 8 *Keep these worksheets. They will be used again in the next session.*

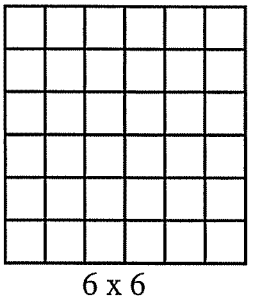

4 x 3 4 x 3

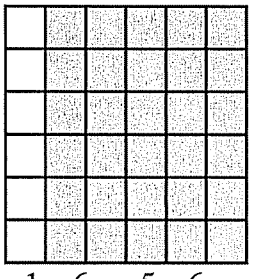

5 x 3 3 x 3

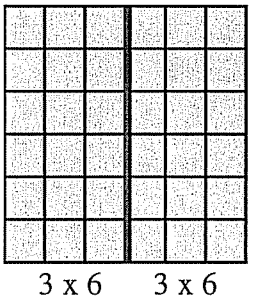

2 x 3 6 x 3

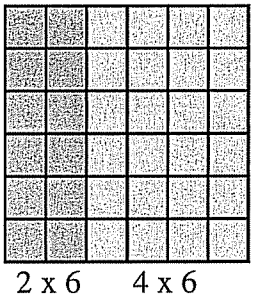

7 x 3 1 x 3


1 x 3 2 x 3 5 x 3


6 x 6


1 x 6 5 x 6


3 x 6 3 x 6

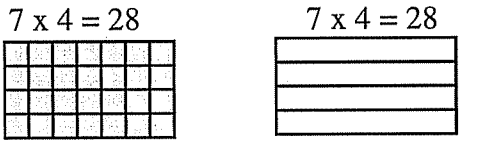

2 x 6 4 x 6

Session 3

Begin by reviewing the physical models for various basic multiplication problems. **"Build 3 x 5 = ? with blocks. What is the answer?"** Give two or three of these.

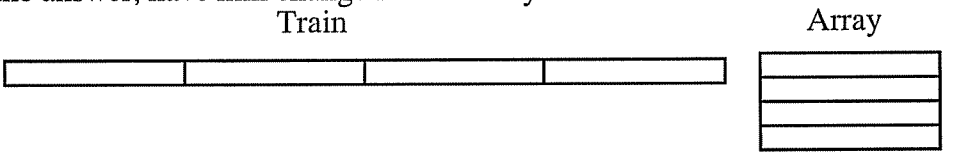
Warm Up

Give 7 x 4 = ? as your last problem. The model for this problem is shown below.
Example: 7 x 4 = 28 *Keep the 7 x 4 = 28 model on the table.* It will be used again.



Tiles Cuisenaire Rods

If the student shows the problem as a 'train,' after he finishes measuring the length to get the answer, have him change it to an array.



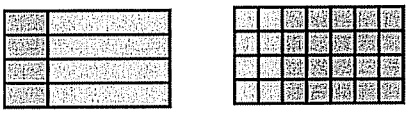
Lesson

Give the student Arrays: Manipulative - Worksheet 5. **"Build a seven with Cuisenaire Rods (or tiles) using two different colors."** See below:.

Cuisenaire Rods Tiles

"Do you still have seven blocks?" "Yes."
"How have you broken up the seven?" "2 + 5 (for the example above)."
"How many blocks of each color?" "One red and one yellow rod."

"Build three more rows of seven *exactly* as you made the first row." There will be a total of four rows of seven, shown in two colors.
Example:



Cuisenaire Rods Tiles

"Place this new, two colored model of 7 x 4 = 28 on *top of the array*." From this he can *see* that the two are equal, that breaking up the 7 x 4 into two smaller problems has not changed the total amount.

"Look at the model of the problem in two colors. Do you still have 7 x 4 = 28?" "Yes."
"How can you tell?" Most students say something such as, "You can tell that you still have 7 x 4 because it fits over the other blocks (the other blocks being the single-colored, original, non-broken-up 7 x 4 = 28). The other blocks made 7 x 4, so these do too because they are the same amount."

"What *big* multiplication problem do you have?" "7 x 4."
"Separate the colors a little bit." See the illustration below.

Cuisenaire Rods Tiles

"When you broke up the seven into red and yellow rods, how many little multiplication problems did you make?" "Two."
"What *little* multiplication problem is shown by your first color?" (Point to the red rods.) "2 x 4."

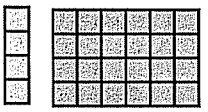
"What *little* multiplication problem is shown by your second color?" (Point to the yellow rods.) "5 x 4."

Have him repeat the procedure twice, breaking up the seven differently each time and identifying the little multiplication problems. See the illustrations below for the other ways he can break up the 7 x 4 with two colors.

The little multiplications are:

(1 x 4) (6 x 4)

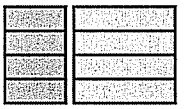
Cuisenaire Rods



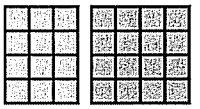
Tiles

One block of one color and six of another

(3 x 4) (4 x 4)



Cuisenaire Rods

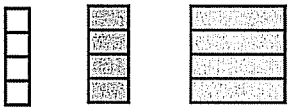


Tiles

Three blocks of one color and four of another

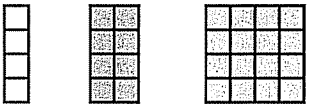
"Break the seven into three different colors."

(1 x 4) (2 x 4) (4 x 4)



Cuisenaire Rods

(1 x 4) (2 x 4) (4 x 4)



Tiles

Repeat this session with several different multiplication problems that can be made with centimeter graph paper. Students can also make their own problems on graph paper.

Worksheet Arrays: Manipulative - Worksheet 5, page 9

Practice Times Tables with Merry Mix Up Drill, pages 3 and 4

Arrays: Recording

Purpose

The purpose is to identify and to label the little multiplication problems created by breaking up the larger problem in an array.

Prerequisites

Mastery of basic multiplication, familiarity with the basic facts in the multiplication times tables two through six, Arrays: Manipulative - Worksheets 1 - 5 (completed in the Arrays: Manipulative lesson)

Materials

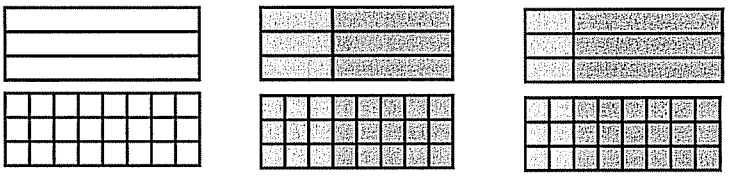
Completed Arrays: Manipulative - Worksheet 3, page 7
Cuisenaire Rods or tiles in several colors
Crayons or markers
Meter stick

Lesson Warm Up

"Build 8 x 3 = ? with the blocks using only one color and find the answer to the problem." Keep this model on the table. See the first illustration below.

"Take out Arrays: Manipulative - Worksheet 3 and build the models just like the ones you did on that page."

Single color model Two and three color models



There will be four different models of the same problem. Keep all of the models on the times table.

"What is the BIG problem?" "8 x 3 = 24."

Test for Understanding

"Will the two and three colored (broken up models) fit exactly on top of the single colored (non-broken-up one)?"
Watch for sureness of his response. Does he know that they will fit on top of each other exactly without having to test it? If he is unsure, ask, **"How could you find out if they were the same?"** "Try it."

Then do more problems orally until he is sure that all the models of the same problem will be of equal size. He should be sure enough that the models are exactly the same size to "bet his lunch" or some other prized possession on it before you go on.

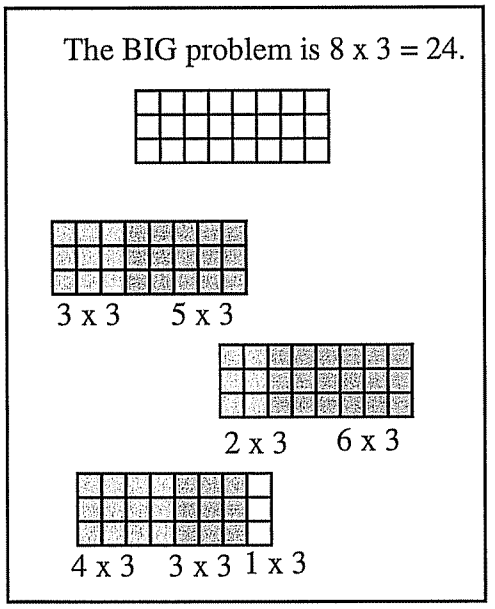
If he is sure, ask, **"How sure are you on a scale of 1 to 10? How do you know you are right?"** If he says, "I know because the models are all eight, three times." Or, "Because it's the same problem every time." Or, "Because it is always eight times three, and when you break it up it's still the same." The last answer is an even stronger answer. You know that he is beginning to build an understanding of the Distributive Principle of Multiplication. That is, when you break up a big multiplication problem into little multiplication problems it does not change the answer or the sum total of blocks you have. **"Prove it, or show that they are the same."** Be sure to ask the question, "How sure are you?" even when the student is *correct,* thus eliminating the student's dependency on *your reactions* to check his work.

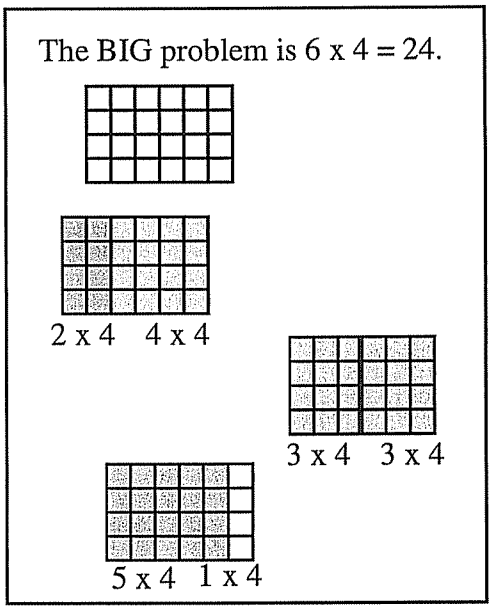
A reply such as "Because they always fit before" indicates a physical rather than an abstract understanding. The physical understanding precedes the abstract understanding. Experience and time will move the student from the physical level to the abstract level.

Session 1

Look at Arrays: Manipulative - Worksheet 3, page 7. Have the student identify and label each little multiplication problem in the model and on the picture. As he identifies each little multiplication problem, he writes it under the picture.

The completed worksheets will look something like this if the student broke up the problems like these examples. Actual answers will vary.





If after he completes both sheets, you feel he needs more practice on this work before going on, make up problems and have him draw them on centimeter grid paper.
Keep these worksheets; they will be used in the next session.

Session 2

In this session, the student will find the answers to the partial products (the little multiplication problems) and sum them to find the final product (the BIG multiplication problem).

Materials

Completed Arrays: Manipulative - Worksheet 3, page 7
Manipulative

"Look at the first multicolored drawing on Worksheet 3. What is the BIG problem? What is the answer?" "8 x 3 = 24."

Under each picture, he has labeled each little multiplication problem. **"Point to the little multiplication problems."**

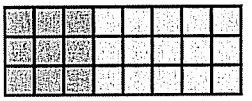
Put in ( ).

"In order to keep the numbers from getting mixed up, since often they are very close to each other, we will put 'fences' or parentheses ( ) around each of the little multiplication problems."

(3 x 3) + (5 x 3)

Write in answers.

"Find the answer to the first little multiplication problem and write it on the worksheet *under* the little multiplication problem it goes with."



(3 x 3) (5 x 3)

"Find the answer to the other little multiplication problem, record the answer in the same way."

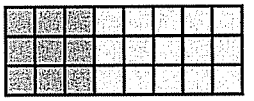
9 **15**

"How could you use the answers to these little multiplication problems to find the answer to the BIG multiplication problem?" "Add them." For most students, this is not a difficult question to answer.

"Write plus + signs between the little problems and between the answers." Then he adds them and writes = 24.

The = 24 can be written on either the right or left end of the sentence.

Add and place + signs.



(3 x 3) + (5 x 3)
9 + 15 = 24

After he has added the two answers to the little multiplication problems, ask, **"Does your answer agree with the answer you got for the BIG multiplication problem?"** "Yes." **"What if the answers aren't the same?"** "There is an error somewhere."

Repeat the same procedure with the other two problems on the page.

"What patterns do you see?" "The answers to all four problems are the same." **"Why does it make sense that they would be the same?"** "Because all the problems are eight times three. They are just broken up differently. Breaking up the eight in different ways doesn't change the answer."

Many students may not know *why* it makes sense at this point. If they don't know, tell them to think about it when they do these kinds of problems again. *Do not explain to them the sense of the answers all being the same.* It is important for this recognition and understanding to grow within the student. Resist explanations on your part.

Another pattern he may notice is that the 'x 3' appears in all the little multiplication problems. He might also observe that the first numbers in the little multiplication problems (**2** x 3 and **6** x 3) or (**5** x 3 and **3** x 3) on each problem add up to eight.

Encourage him to find more than one pattern. Don't press if he can't and suggest that he keep his eyes open for patterns as he does the other worksheets.

Worksheets

Arrays: Manipulative - Worksheets 6 - 8, pages 10 - 12 The first problem on Worksheet 6 is an example. Do the second problem together to make sure he understands the procedure. Give only what help he needs.

Note

Many students will want to stop building the models at this point. *It is important to build the models.* Tell them that you know that they can do the problems without the blocks but that they will need a clear memory of the models to do the harder problems later. It is important that their hands and minds work together to build a strong mental picture.

Use centimeter graph paper to make additional problems if more practice is needed. You might want to put in all the ( ) , + , ____ s and = signs for him. Some students can do this for themselves.

Breaking Up Times

Purpose

The purpose is to build a physical model for the Distributive Property of Multiplication and to record pictures of the models in the form of an array and to identify and label the little multiplication problems contained in the larger problem.

Prerequisites

Mastery of basic multiplication, familiarity with the basic facts in the multiplication tables of two through six, Arrays: Manipulative

Materials

Arrays: Manipulative - Worksheet 6 - 8, pages 10 -12
Completed Arrays: Manipulative - Worksheet 3, page 7
Cuisenaire Rods or tiles in several colors
Crayons or markers
A large index card or blank piece of paper

Note
Symbolic/
Recording

In this session, we will add the final piece of this process called Distribution. The student has already learned to physically break up the first number of a BIG multiplication problem, to record and solve the little multiplication problems, and to add the parts to get the final product. To complete the process, he must learn to symbolically record the breaking up of the first number.
In the problem 9 x 3 = 27, for example, the nine is broken up into 4 + 5. It is recorded as follows: 9 x 3 = **(4 + 5)** x 3. Both the four and the five are multiplied by three and the results are added. The entire process is recorded like this:

9 x 3	=	(4 + 5) x 3	=	**(4 x 3) + (5 x 3)**	=	**12 + 15 = 27**
The BIG problem		Breaking up the 9		Multiplying the parts of the 9 by 3		Adding the parts

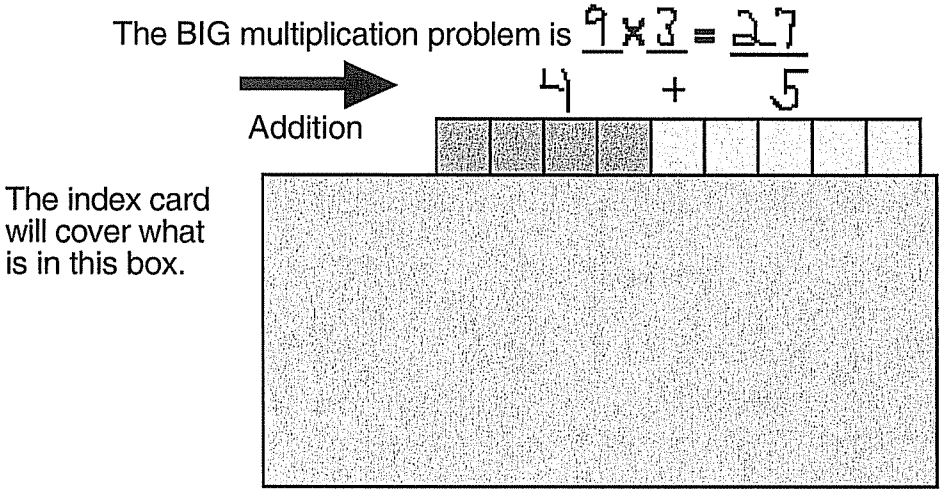
Notice that equality was maintained in every step. Try it again yourself. Break up the nine in a different way. In algebra, we learned this same process expressed like this:

(a + b) c	=	(ac) + (bc)
Breaking up the problem,		multiplying each part (a and b) by c, and adding the results

When you break up the numbers in a multiplication problem, multiply the parts together, and sum the results to obtain a final answer, you are using the *Distributive Property of Multiplication.* It makes long multiplication and algebraic multiplication possible.

Lesson
Session 1

"Look at the first problem you did on Arrays: Manipulative - Worksheet 6. What is the Big problem?" "9 x 3 = 27."
"How is the nine broken up?" "Into two colors, green and yellow."



Have him take the index card or blank paper and cover all but the top row of the first picture. It is important that he use the index card to cover all but the top row of the drawings. This focuses him on the first number, the nine, of the multiplication problem and covers the repeated rows of rods. It helps avoid distraction and confusion.

"How is the nine broken up?" "4 + 5."
"Above each color in the drawing, write the numbers that show how the nine is broken up."

Worksheets

Have him repeat this same process with all of the problems on all of the Arrays: Manipulative - Worksheets 6 - 8, pages 10 - 12.

Session 2

Equalities
An investigation of equalities at this point would be useful. Start with a simple equation like 5 + 4 = 9. Then play around with it. Here are some changes you might make: (2 + 3) + 4 = 9; 5 + 4 = 7 + 2; 5 + 4 = 15 - 6; (6 - 1) + (2 x 2) = 3 x 3, and so on. The variations are endless. Ask each person to find ten or twenty ways to write 5 + 4 = 9. Repeat it with equations like 7 x 4 = 28.

You can also give equations with missing numbers. Try this one: (6 + 1) - _?_ = 2 x 2. Students enjoy writing these kinds of puzzle problems for each other. This activity will strongly increase the understanding of mathematical notation. They like it and it is a good math lab.

Session 3

Worksheets

Breaking Up Times - Worksheets 1 - 8, pages 13 - 20

Give the student Breaking Up Times - Worksheet 1, page 13. **"Build seven with two colors. Color the picture to look like his blocks and write the number sentence."**

Example: 7 = 2 + 5

"What BIG multiplication problem have you built?" "7 x 4."
Have him record that on his worksheet where that question is asked.
"Color the picture on the worksheet to match the blocks." The top row of numbers is already labeled as an example.
This labelling of the top row is new material.

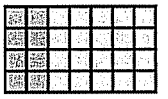
"Remember, the ( ) are like fences that keep numbers together: in this case, the seven. Notice that the two is above the first color and that the five is above the second color."

"In this equation 7 x 4 = (2 + 5) x 4, how are the two sides the same and how are they different? Where is the seven on the right side of the equation?" "In the (2 + 5)."

"What little multiplication problem is made by each color? "(2 x 4) and (5 x 4)."

See Estimation Note on the next page.

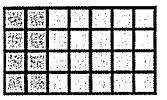
7 x 4 = (2 + 5) x 4



= (_ x _) + (_ x _)

28 = ___ + ___

7 x 4 = (2 + 5) x 4



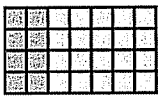
= (2 x 4) + (5 x 4)

28 = ___ + ___

"How much does each little multiplication problem make?" "(2 x 4) = 8 and (5 x 4) = 20"

"How do I find out how much there is in both sections?" "Add the eight and the twenty." **"What is the final product?"** "Twenty-eight."

$7 \times \underline{4} = (\underline{2} + \underline{5}) \times 4$



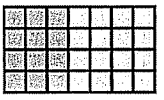
$= (2 \times 4) + (5 \times 4)$

$\underline{28} = 8 + 20$

"Here is Breaking Up Times - Worksheet 2. We'll do the first problem together with a different expansion of the seven." Here are two completed examples.

If at this point the student is tiring of coloring in the squares he can simply outline the squares in the appropriate color to show the break ups.

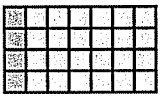
$7 \times 4 = (\underline{3} + \underline{4}) \times 4$



$= (3 \times 4) + (4 \times 4)$

$\underline{28} = 12 + 16$

$7 \times 4 = (\underline{1} + \underline{6}) \times 4$



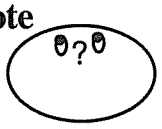
$= (1 \times 4) + (6 \times 4)$

$\underline{28} = 4 + 24$

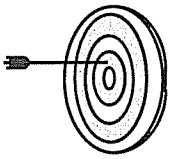
Worksheets

Give Breaking Up Times - Worksheets 3 - 7, pages 15 - 19, to be completed independently. Most students will be able to complete the rest with little assistance.

Estimation Note



On Breaking Up Times - Worksheet 1, the estimation face appears. **"Guess what the final answer will be and record your guess in the face. After completing the problem, compare your guess with the results. Evaluate this guess by drawing an arrow in the target. The closer to the correct answer, the closer the arrow is drawn to the bull's-eye."** A good guess should be within fifteen of the right answer.



Practice Worksheet

Breaking Up Times - Worksheet 8, page 20 The student writes a problem. The first number should be no larger than fifteen and the second no larger than nine.

He builds the problem with the blocks, breaking the first number into three sections. He then records the problem on the worksheet using the same process as he did on the last three pages of Breaking Up Times. He does not have to draw the picture if he builds a model but may draw a picture or record on centimeter graph paper.

He then builds the problem a second time, breaking the first number into four sections, and a third time into two sections. A challenge problem is given. In this, he must break the problem up in his own way, write in all the lines, parentheses, signs, and numbers.

Sit quietly beside him the first time or two through the worksheet. After he has mastered the process, he should do one of these practice sheets per week until he reaches Arrays to Boxes. After a few weeks he may want to drop the use of the blocks and draw grid pictures on centimeter grid paper instead.

Watch for the use of tens and ones expansions (breakups), especially when the student is breaking the problem into two sections. Using tens is a sign that the student is finding easier ways to do the problems. *This is an important discovery. Do not tell the student to use tens and ones at this time.* If he does not discover this on his own,

it will be suggested to use them at a later time.

Session 4 Oral/ Mental Math

In this session, the first number is broken up in our heads, the partial products are found, and the final product is added.

Start off with easy problems, such as 5 x 4 = 20. **"Break up the first number."** A possible response is 2 + 3. Write the 2 + 3 on a paper or the board.
"What are the little problems?" "2 x 4 and 3 x 4."
"What are the answers and what is the sum of both of them?" " Eight and twelve equals twenty."

The goal is for the student to be able to do a problem like 8 x 7 by thinking 4 x 7 is 28 and there are two fours in eight, so I double the twenty-eight to get fifty-six. Breaking the even number in half and doubling the partial product (the answer to the little multiplication problem) to obtain the final answer is a common mental math strategy for these types of problems. Formally written, this problem looks like this: 8 x 7 = (4 + 4) x 7 = (4 x 7) + (4 x 7) = 28 + 28 = 56.

Another strategy is to break a number into 5 + something. Five is always easy to multiply by. Take this problem for example, 9 x 8 = ? The student might think 5 x 8 is 40 and 4 x 8 is 32, 40 + 32 is 72.

Some people like to break up the second number when doing mental math. That's OK, too. When seeing or hearing the problem 7 x 6 = ?, they think 7 x 3 and 7 x 3 is 21 + 21, or 42.

Have the student describe his strategy when he has finished the problem. Help him learn to recognize his own strategies for finding answers.

If you are working with a large class, have several students describe their way of finding the answers. Write their answers out on the board (or have them do it) using a format like the one shown for 8 x 7 above.

Have the student practice doing the problems mentally every day. One or two a day as a warm-up is good mental exercise. Gradually increase the difficulty of the problems. Try ones like 12 x 4 and so on.

Notes

Large groups: For Sessions 1 and 2: Instead of having everyone do the problem several ways you might simply have them compare each other's work. Have them look at the different small multiplication problems that were created by the different colors. Have each student identify, aloud, how the little multiplication problems were made. In Session 2, have the students each make 8 x 3 a different way. Let them use each other's models to do the worksheets. This works well if the supply of blocks is limited.

Checking these worksheets can be very time-consuming if you have a large group. It is suggested that you teach the students to check each other's work. A way to check in four steps:
First, check the addition in the top row. Do the 'breakup' numbers add up to the first number in the problem?
Second, check the little multiplications in the second row to see if they are the correct problems.
Third, check the partial products to the little multiplications.
Fourth, check the final addition.

The checkers should lightly circle each row after they have checked it. Have the

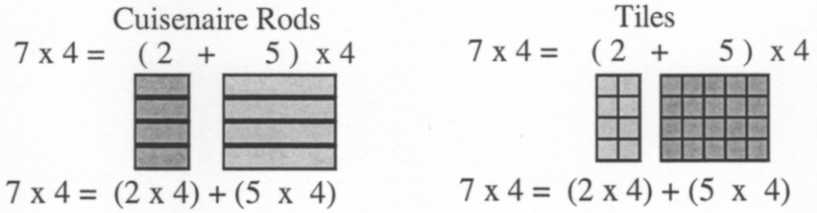
person who checked the page sign his or her name to the paper. Praise accurate checking. The use of a calculator for checking is fun.

One thing you will have to watch for is glossed-over small multiplication and addition errors. As the student comes to realize that the answer to a problem being done several ways is always the same, there is a tendency to ignore or override arithmetic errors. Simply looking at his paper to see if all the final answers match is not a routinely safe practice. But if the students are checking each other, a little practice will allow you to scan quickly to see how each student is progressing.

It is critical for the understanding of long multiplication, long division, and algebra that the comprehension of this basic idea is deep. This takes many exposures over a long period of time. In everyday practice this is not difficult, as the work is easily done independently by most students.

Test for Understanding

1. In the problem below, the top row says, 7 x 4 = (2 + 5) **x 4.** The (2 + 5) tells how the 7 is broken up, but why is the **x 4** written again?



In terms of the given problem, **x 4** is written again because we've multiplied both parts of the seven by four. Or in terms of the idea of an equation, so that the two sides of the equation will be equal. 7 x 4 does not equal (2 + 5). The (2 + 5) is another way to write seven. The seven is being multiplied by four, so (2 + 5) must be multiplied by four also.

2. **"If I do the above problem again, but I break the seven up into 3 + 4 , what will happen to the final answer?"** "It will not change the final answer."

3. **"Explain your answer to number 2."** "It does not change because in both problems you are multiplying 7 x 4. The only thing that changes is the way that the seven is broken up."

4. Challenge Question: **"If you did a problem two different ways and the two answers came out differently, what happened?"** "I made a mistake." **"What things can you check to find the error?"**

Check the expansions (breakups) on both problems. Are they broken up right? If so, then check the multiplications. If they are correct, check the addition. If that doesn't work, get someone else to look at it.

Concurrent work on basic times tables should be going on. Use Merry Mix Up Drill, pages 3 and 4.

Extension

Spectrum Mathematics - Grade 3, pages 86 through 95, drills facts and word problems. *Grade 4*, pages xi and xii, drills facts in all times tables.

Arrays to Boxes

Purpose The purpose is to extend the model of distribution developed in Breaking Up Times to larger problems, to move from a physical model to a picture, and then to a box with a grid.

Prerequisites Breaking Up Times, Times Tables - Mastery of the 1 - 5 times tables is helpful.

Materials Arrays to Boxes - Worksheets 1 - 5, pages 21 - 25
One-fourth inch grid paper

Warm Up Before beginning this lesson, review the student's ability to break up a problem such as 9 x 4 = __ with blocks and recording.

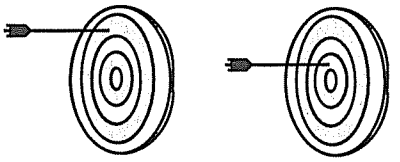
Lesson Session 1 Give Arrays to Boxes - Worksheet 1. The first problem is done as an example.
"What is the BIG multiplication problem?" "14 x 4."
"Where is the fourteen shown in the picture?" "Across the top."
"Where is the four shown in the picture?" "Down the side (or up and down)."
"How is the fourteen broken up?" "Into 6 + 5 + 3."

If a student has difficulty seeing how the fourteen is broken up, cover all but the top line of the picture with another piece of paper or your hand. This will leave only one row of fourteen squares showing. This makes it easy to see just the fourteen. The student can then see how the fourteen is broken up.
This technique also works on problems where the student has to break up the number on her own. Sometimes the student gets confused on which way to draw the lines. Have her cover up all but the top line of the picture and break up the row into sections like she used to do with the rods. Then she simply extends the lines downward through the picture.

"What are the little multiplication problems?"
"(6 x 4), (5 x 4), and (3 x 4)."
"What do you think the answer will be? Write the estimate in the face."

"What are the little multiplication problems below the pictures?" "The answers." **"What do you do now?"** "Add up the answers." The completed problem will look like this:

$$14 \times 4 = (6 + 5 + 3) \times 4$$
$$= (6 \times 4) + (5 \times 4) + (3 \times 4)$$
$$56 = 24 + 20 + 12$$



Have her evaluate the estimate by drawing an arrow in the target. The closer to the correct answer, the closer the arrow is drawn to the bull's-eye. Solve the second problem together. The problem is already broken up, but one of the numbers, the 5, is not written in. The third problem has the fourteen broken up into four sections. The numbers are not written in.

Session 2 Do Arrays to Boxes - Worksheet 2, page 22, together. The first two problems have been broken up for the student. In the first problem, the student supplies the numbers used in the expansion. In the second problem, the numbers are given and the student must draw in the lines on the picture without a grid. The third problem has no grid, and the student supplies all the numbers for the breakup and the lines to show how

the problem was broken up.
If the student has difficulty with the first two pages, continue working together. Then try to have her work the rest of the pages alone. If a student is having extreme difficulty with this material, resume practice on the Breaking Up Times type problems with blocks, grids, and recording.

Worksheets

Give Arrays to Boxes - Worksheets 3 - 4, pages 23 - 24 if the student has been functioning easily. The Challenge problems are optional. If the student did not do a Challenge problem, ask the student if she read and understood the problem. Some students will skip the Challenge problems without even reading them. The Challenge problem on the bottom of Arrays to Boxes - Worksheet 4 is often skipped by many students (do not push on this one).

Arrays to Boxes: Show You Know - Worksheet 5, page 25

Note for Large Groups

If you have a large group, you may want to have one student make up the problem for the whole group. Even better, have the students break up into groups of four. Have one person in the group give the problem for that group. The people in the group check each other's work. For some groups, you will need to require that no two people use the same expansion.

Test for Understanding

Oral Test for Understanding

1. **"You are given this problem, 19 x 6 = __. You decide to break up the problem like this: 19 x 6 = (12 + 7) x 6. When you start to find the answers to the little multiplication problems, you discover that you don't know the answer to 12 x 6. What can you do? Think of two ways to get yourself out of this difficulty (other than looking up 12 x 6 on a table or asking someone else for the answer)."**

If you wanted to break it into only two sections, you could try a different expansion like 10 + 9 or 11 + 8. You could break the nineteen into more sections so the numbers would be smaller, 5 + 6 + 4 + 4 for example. You could also break up 12 x 6 into (6 x 6) + (6 x 6) or some variation of that. You could also find the answer to 12 x 6 by addition, although this is not a preferred way because it's slow.

2. Challenge Problem: **"Find the answer to this problem: 59 x 3 = ___ ?"**

The student would need to breakup the fifty-nine into many sections, do the little multiplication problems, and sum the results. It is not as important that they get the right answer as it is for them to know how to attack the problem. The answer is likely to be wrong because of the massive number of possible arithmetic errors. (It is this problem that leads the student to try to find more efficient expansions like 50 + 9. A student who uses a series of tens in their expansion is already discovering this. *Do not tell the student to do this.* Most will figure this out as a result of the upcoming activities and more experience.) If the answer does come out wrong, tell the student what kind of error or errors she made. Then have her try to find the errors.

Distributive Property

Purpose

The purpose is to develop the formal vocabulary to fit all the manipulations and pictures the student has been doing with arrays and boxes. The mathematical procedure of breaking up the number to be multiplied, doing the small multiplications, and then adding the results is called the Distributive Property of Multiplication. This lesson connects the old process to a new word and then connects the new word to the concept of distribution as it is seen in the retail world and on the freeways.

Prerequisites

Previous lessons in this book

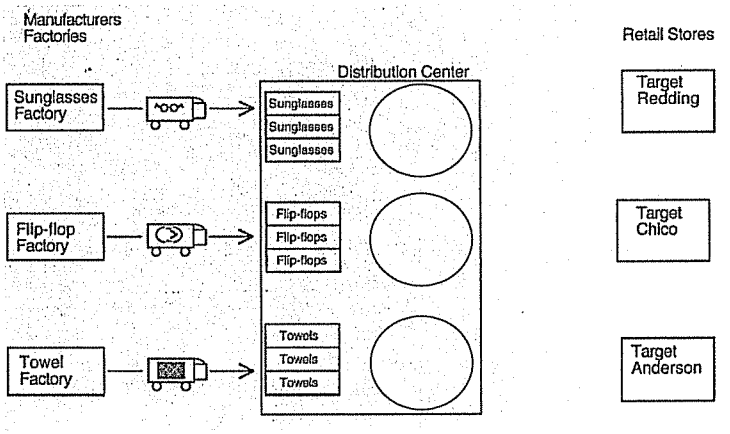
Materials

Distributive Property, page 26
A large piece of white paper (12 x 18 works best)
Crayons

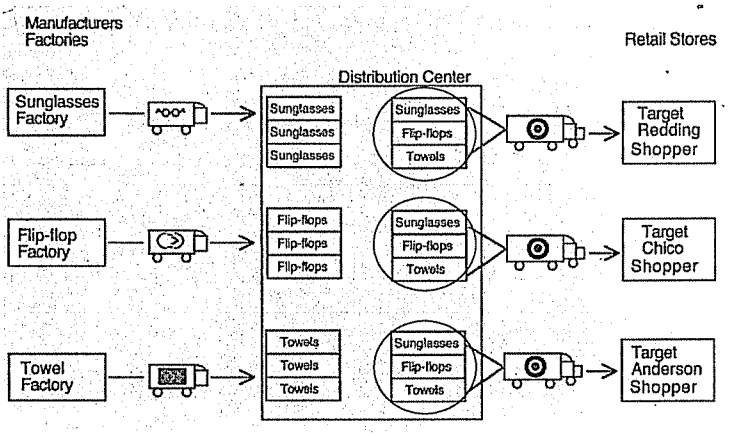
Social Studies Connection: Have her write the word 'Distribution' at the top of the large white paper. Have a conversation such as:
"We go to the store (you can put in a specific store like Safeway or Target) to buy summer supplies. How did those summer supplies get to the store?" "They get there in a truck."
"Where did the truck pick up the summer supplies?" "From the place where they are made."
"Do you think that one truck went first to the sunglasses factory, and then over to the flip-flop factory and then to the towel factory?" "Maybe." (Many students will not know.)
On the left hand side of her large paper, have her write the word 'Manufacturers/ Factories.'
Under the words draw four 2 x 3 inch rectangles down the left hand side of the paper. Label each box as a factory for sunglasses, flip-flops, and towels.
On the right hand side of her paper have her write the words 'Retail Stores.' Under the words draw three 2 x 3 inch rectangles down the right hand side of the paper. Label each box as a local store in three nearby towns like Target in Redding, Target in Anderson, and Target in Chico.
In the center of the paper, draw a large 6 x 6 box. Label it the 'Distribution Center.' She may have seen one of these huge buildings with rows and rows of trucks backed into them.

Now have her draw a road from each factory to the Distribution Center. Have all of the roads come into the left hand side of the distribution box. Then have her draw a truck on each road. On the side of the truck, give the name of the factory it came from. Just inside the distribution box, where the truck docks, have her draw and label the product that was unloaded. Draw three boxes of sunglasses. In the next truck bay, draw three boxes of flip-flops, in the next truck bay draw three boxes of towels.

The drawing should now look like this:



Draw a box in the top dock on the right side of the distribution box that has a box from each of the three products. Now we will load the trucks that will go to each store. Draw three trucks with the store label on them (I drew Target), on the right hand side of the Distribution Center. Draw and label one box of sunglasses, one box of flip-flops, and one box of towels in each circle. Then draw lines from each circle in the Distribution Center to the Target trucks. These are loaded into the trucks going to specific stores.



This is what distribution is in the retail world. The pieces coming from the factories are broken up and then distributed to the trucks going to individual stores.
When you are traveling around, look at the trucks. Many are not marked by brand but by the hauler. These are usually trucks carrying loads to the distribution centers from the ships and trains that brought the goods to your area. The trucks going out of the distribution center to the stores are frequently carrying the logo of that store on them.

If you travel to larger towns, you may see on the outskirts of the towns the huge, windowless distribution centers.
This same process takes place with newspapers. How do the papers get to all the homes and to the stores where people buy newspapers? Do you have them distributed to your home? The entire mail system works this way also.

Lesson

After you have developed the general idea of what distribution is, have her do the Distributive Property, page 26. She will look at all the steps of the process she uses

to multiply numbers and write it out in words. The number being distributed is the five. It is delivered to each of the parts of the broken up number.

The Distributive Property is the fundamental property of multiplication that underpins all of our algorithms for multiplication and division.

Practice Worksheets

None

Test for Understanding

Write these words on a piece of paper:

Commutative Associative Distributive

Then write these three problems. See if she can identify the properties being used in each problem.

1. John earned thirty-seven dollars each week for four weeks. How much did he earn? (Distributive) One hundred forty-eight dollars

2. There are two posters. One is five feet long and four feet wide. The other is four feet long and five feet wide. Which one has more area? (Commutative) Both have equal areas.

3. There are three sizes of boxes at the post office. One is five inches high, ten inches deep, and eight inches wide. The second is eight inches tall, five inches wide, and ten inches deep. The third is ten inches high, five inches wide, and eight inches deep.

   Which one has the greatest volume? (Associative)
   They have different shapes, but equal volumes.
   1) 5 x 10 x 8 2) 8 x 5 x 10 3) 10 x 5 x 8

Arrays: Easier Ways

Purpose

The purpose is to extend the model of distribution developed in Arrays to Boxes to discover easier ways to break up numbers, i.e., into tens and ones, base ten numeration.

Prerequisites

Arrays to Boxes

Materials

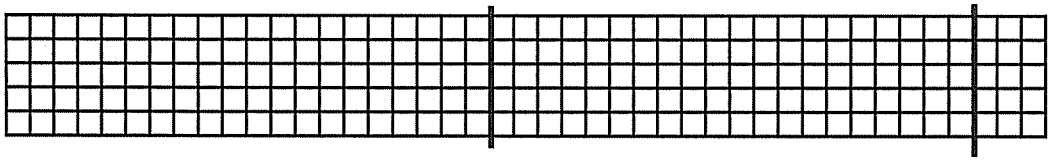
Arrays: Easier Ways - Worksheet 1, page 27
One-fourth inch grid paper

Lesson

Give Arrays: Easier Ways - Worksheet 1, page 27. On Worksheet 1 the student solves 36 x 7 three different ways. First by addition and then by breaking up into grids two different ways.

The same process is repeated on Arrays: Easier Ways - Worksheet 2, page 28, with different numbers. Be sure the student is drawing in the lines for the break-ups and writing the partial products.

$43 \times 5 = (\quad 20 \quad + \quad 20 \quad + \quad 3 \quad) \times 5$



$= (20 \times 5) + (20 \times 5) + (3 \times 5)$

$215 = 100 + 100 + 15$

Check her progress on Arrays: Easier Ways - Worksheet 3 to see what type of breaking up she is doing. Give suggestions on easier ways only if needed. Allow her the time to figure out breaking up by tens and ones for herself. An in-between step may be to break up in multiples of ten, such as breaking up fifty-four into 10 + 40 + 4 or 30 + 20 + 4. By the time the student completes Arrays: Easier Ways - Worksheet 4, she will have realized easier ways to break up larger numbers.

Practice Worksheets

Arrays: Easier Ways - Worksheets 3 and 4, pages 29 and 30

Arrays to Boxes: Breaking Up into Tens

Purpose

The purpose is to lead the student toward the standard method of breaking up numbers, which is to use tens and ones. Some students will have discovered this shortcut on their own in their explorations in past lessons.

Prerequisites

Previous lessons

Materials

Arrays to Boxes: Breaking Up into Tens - Worksheets 1 - 3, pages 31 - 33
Colored pencils
Cuisenaire Rods
Meter ruler

Warm Up

Arrays to Boxes: Breaking Up into Tens - Worksheet 1, page 31 Remind him that he can not use numbers in his breakup greater than twelve. These are the numbers in the six times table. I would recommend he doesn't use numbers larger than ten. Remind him to write in all the numbers, plus signs, parentheses, etc. Check the work before you go on.

Lesson

Study Arrays to Boxes: Breaking Up into Tens - Worksheet 2, page 32. Help him examine the examples.

"How is the 43 broken up?" "Into ten, plus ten, plus ten, plus ten, plus three."

"What is 10 x 8?" "Eighty."

"Why is this way of breaking up 43 easier than some other ways you could do it?" "Because breaking up by ten and multiplying by ten are really easy."
If he notices at this point that the four eighties can be quickly added by doing 80 x 4, give him a high five!

Draw his attention to the box in the middle of the page. It is the same exact problem with the grid removed. This is a shortcut. It allows us to simply write the partial products in the boxes and skip all the parentheses.
Do the problem at the bottom of the page with him. Watch to see if he knows what to do. Help only if needed and check the answers.

Practice Worksheets

Arrays to Boxes: Breaking Up into Tens - Worksheet 3, page 33

Test for Understanding

The test is to check the work on Arrays to Boxes: Breaking Up into Tens - Worksheet 3 and see that he put in all the pieces. Make sure he put in all the plus signs, parentheses, etc. The process is more important, at this time, than the correct answer.
"Which number is being distributed in the problem on Worksheet 3?" "The four."

Box Multiplication

Purpose The purpose is to move the student into using tens to decompose the numbers and to use the shorter version of the Distributive Property offered by the boxes.

Prerequisites Previous lessons and readiness to drop the graph paper boxes shown in the arrays in previous lessons

Materials Box Multiplication - Worksheets 1 - 3, pages 34 - 36
Word Problems: Review, page 37

Warm Up Guide her to study the example at the top of the page of Box Multiplication - Worksheet 1, page 34. It is a problem she has already done in the previous lesson. Watch her work the first problem 39 x 8 and watch for any difficulty. If there are no difficulties, go to the next page and save the last two problems on this page for independent practice.

Lesson Box Multiplication - Worksheet 2, page 35 This is left blank on purpose. She must put in the 10 + 10 + 10 + 4 in the parentheses above the box. Then she draws the lines down through the box under each plus sign. Have her write 10 x 6 in each box for now. Later she will drop this and just write the sixty. Have her write the sixties in and then the 4 x 6 = 24 in the last box. She will then add 60 + 60 + 60 + 24 on the side and add it up. Watch to see if she uses 3 x 6 when she is adding the tens column.

Note *Do not* tell her to get the total by multiplying 6 x 3 and adding a zero. She will discover this on her own.
If she has no difficulty, have her finish the previous page and the rest of this page on her own. Check the work immediately and correct any errors. Do not let her write the breakups in a slap dash way on the top line.

Practice Worksheets Box Multiplication - Worksheet 3, page 36

Test for Understanding Word Problems: Review Watch her process on problem 2. She must do the problem 215 x 4. which is a much larger problem than she has done before. Watch to see how she approaches it. First, can she write the problem that needs to be done? Can she write it as an addition problem? How does she solve the problem? Does she extend what she has learned with box multiplication to include hundreds. or does she add? Either solution is fine. This problem opens the door for her to generalize if she is ready to do so.

Expanded Tables

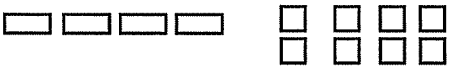
Purpose The purpose is to extend the times tables into the tens and hundreds places, an important part of the multiplication of large numbers.

Prerequisites Times Tables, Breaking Up Times, and Arrays to Boxes

Materials Expanded Tables Worksheets, pages 38 - 45
Base Ten Blocks, or beans, or Cuisenaire Rods (whites and oranges only) and paper 10 cm. x 10 cm. squares to stand for hundreds

Lesson Using the ones blocks, have the student build models of small multiplication problems such as 2 x 4 or 4 x 3, etc. Remember that 2 x 4 = says, "How much is two added four times?" A model of 2 x 4 looks like this:

Cuisenaire Rods Base Ten Blocks



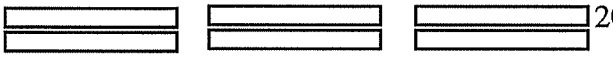
Now you build a model of a problem and have the student identify what problem it is. If you have a group, have students make models for each other to identify. See the example below.

3 x 4 Cuisenaire Rods 3 x 4 Base Ten Blocks

Give the student a blank piece of paper.
Just to the left of the paper, have the student build a model of 2 x 3 = ___ (using the ones blocks). Next to the model have the student record the problem and its answer.

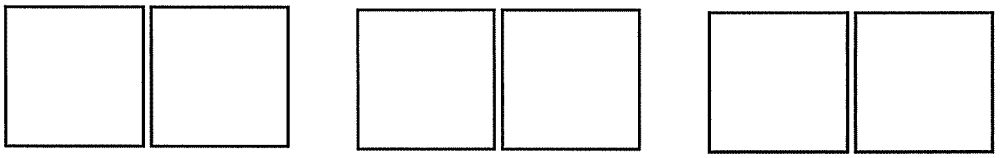
2 x 3 = 6

Just to the left of the paper, have the student build a model of 20 x 3 = ___ (using the tens rods). Next to the model have the student record the problem and its answer.



20 x 3 = 60

Repeat the procedure for 200 x 3 = ___ (using the hundreds flats).



200 x 3 = 600

"What patterns do you see in the answers?"
"The first number is always the same but the number of zeros following it are different." "The six is always there but its place value changes."

Repeat this process for 4 x 2, 40 x 2 and 400 x 2. **"What patterns do you see in the answers?"**

Then give 3 x 4, 30 x 4, and 300 x 4. **"Predict the answers and then find them with the blocks."** The answers to these problems read as follows: 3 x 4 = 12, or twelve ones; 30 x 4 = 120, or twelve tens; 300 x 4 = 1,200, or twelve one hundreds. See Extensions for further play with this important place value concept.

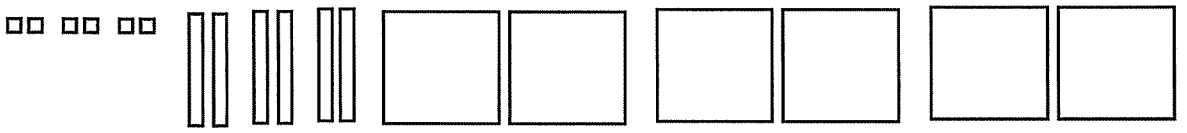
Worksheets

Expanded Tables - Worksheets 1 - 8, pages 38 - 45 are to be completed independently. Complete the *Expanded Tables Flip Book* and then have the student do Expanded Tables - Worksheets 9 - 11, pages 46 - 48. Then give Box Multiplication - Worksheet 1, page 50. Box Multiplication - Worksheet 2, page 51, is a blank practice page to copy and give additional problems if needed.

Test for Understanding

1. Give a series problem such as 3 x 4 = 12, ___ x 4 = _____, 300 x 4 = 1,200. **"Fill in the missing numbers in the series."** "3 x 4 = 12, 30 x 4 = 120, 300 x 4 = 1,200. **"How do you know which numbers to put into the blanks?"** "I just added a zero to the first and the last numbers."

2. **"Build it with the blocks to show what you mean."** This will test if the student understands what it means to add a zero to a number in an expanded fact such as 2 x 3, 20 x 3, and 200 x 3.



3. **"How many groups of the six ones blocks would it take to make the same number as the six tens blocks?"** Another way to ask this: Point to the group of six ones blocks and ask, **"How many groups of these would it take to cover this?"** Point to the group of six tens rods as you say, **"...cover this?"**
The answer should be a prompt and sure 'tens'. If he is unsure or needs to build to find the answer, his understanding of what is happening in an expanded table is not complete. What is happening in the expanded tables is that 20 x 3 has the same answer as (2 x 3) x 10. Each time a zero is added to a number it is multiplied by ten. If the student needs to use blocks (and he probably will the first few times you ask the question), have him answer the question using the blocks. Make this into an activity, repeating it for different facts. Keep asking the question as the weeks go by until you get a sure, firm ten as an answer.

4. **"How many groups of six tens blocks will it take to cover the group of six one hundreds blocks?"** "Ten."

Extensions

1. Names of Numbers. Play around with the names of numbers to increase understanding of place value. It is common (especially on car commercials) to call the number 1,500, fifteen hundred. It could also be called 150 tens or one thousand-five hundred (ones). Sometimes students have difficulty seeing that a number such

as 540 is the same as five hundred forty ones and fifty-four tens. This idea becomes important when learning long division.
To illustrate this idea, have the student build five hundred forty with the Base Ten Blocks. Usually, he will do so with five one hundreds blocks and four tens blocks. **"How many tens blocks would it take to build this number if you had no one hundreds blocks? Record three ways to build five hundred forty with these blocks."** 1. Five hundreds blocks and four tens blocks. 2. Fifty-four tens blocks. 3. Five hundred forty ones blocks.

2. It is important for the student to learn to recognize constant patterns. 2, 20, 200 is one of those patterns. To get from two to twenty, you multiply by ten. To get from twenty to two hundred, you multiply by ten. In the series 3 x 10 = 30, 4 x 10 = 40, 5 x 10 = 50, each number is being multiplied by ten.
When you are looking for a pattern that uncovers a hidden operation, you look at what you start with and what you end with and try to figure out what happened to the number in between. In the expanded tables patterns the answers look like this: 2 , 20, 200 or 4, 40, 400, etc. An interesting problem is figuring out how to get from two to twenty, or from four to forty. What's the pattern?

Have your student write out this series of numbers:

1 10
2 20
3 30
4 .....40 and so on to 10 ..... 100.

"What do you do to the first number to get the second number?"
There are two solutions to the problem.
There are several things a student will try. One of the most common responses is, "I could add 9 to the one to get ten." **"What would you have to add to two to get twenty?"** Pushing this approach further reveals a very nice pattern.

It is a very rare eight-year-old who will think to subtract two from twenty to get eighteen. This seemingly obvious reversal of operations is a major conceptual leap for any student under nine or so (unless he has had extensive work in Families of Facts and has internalized him). Don't tell him to do this. He may not understand, and it will pull him off the task at hand. If the student gets bogged down in the arithmetic, allow him to use a calculator to figure this out. The computations will get in the way of enjoying the pattern. If you don't have a calculator, use money or Cuisenaire Rods. To use the rods, the student puts the red rod against the meter stick and adds rods until he gets to twenty. He then measures the length of the entire string of rods that was added to the first two rods. It measures 18 cm. The answer is eighteen.

Continuing the pattern, **"What do you add to the three to get thirty?"** "Twenty-seven."

You now have this series:

1 + 9 = 10
2 + 18 = 20
3 + 27 = 30 What is the lovely pattern that is emerging?

Here's a puzzle for you to try to explain. Why would this series produce the nine times table? The answer is at the end of the lesson.

Another response the student could give is to multiply the one by ten to get ten and the two by ten to get twenty and so on. This is the obvious response for an adult to give but a rare one for a student. Students are more familiar with addition, so they will tend to think of it first. After the student has explored and been validated for the addition pattern, ask him to find another solution. If he needs a hint, suggest that he try another operation. If that doesn't help, tell him to try multiplication. If he needs more help, ask him how many twos it takes to make twenty, threes to make thirty (use the Rods to find answers if the student does not know). Give as few hints as you can. Give the student ample time to think between hints.

Answer to the teacher's puzzle.
The reason the nine times table appears in the addition series above is because when you move from two to twenty you have done this problem: $2 \times 10 = 20$. It takes ten twos to make twenty. When working with the addition pattern you have this: $2 + \_\_\_ = 20$. You have already added one two here: $2 + \_\_\_ = 20$. So you must add nine more twos in the second number, the eighteen, to make the ten twos, or twenty. You could rewrite the problem as $2 + 18 = 20$ $(2 \times 1) + (2 \times 9) = (2 \times 10)$. The general pattern for all numbers in the series is $X + 9X = 10X$.

Expanded Tables Flip Book

This is a separate book and is not included within this booklet.
Begin filling out the multiplication part of the *Expanded Tables Flip Book.*
Directions: The first page is best printed on heavy card stock that when cut becomes the front and back covers.
The inside pages are printed double sided. Cut the pages in half. The top half of page 17, Expanded Division - 9s has these directions. Remove this page from the stack and place page 9, Expanded Division - 5s, and the rest of the cut stack under the first half of cut pages. The first page is the title page and is followed by Expanded Multiplication - 2s, page 2. The pages should follow in numerical order. Place the front and back covers on the outside of the stack of papers. Staple the left side of the booklet together. Students fill out the booklet a page or so at a time.

$1 \times 2 = 2$	$10 \times 2 = 20$	$100 \times 2 = 200$
$2 \times 2 = 4$	$20 \times 2 = 40$	$200 \times 2 = 400$
$2 \div 2 = 1$	$20 \div 2 = 10$	$200 \div 2 = 100$
$4 \div 2 = 2$	$40 \div 2 = 20$	$400 \div 2 = 200$

Keep this booklet - it is used for several years.

Expanded Numbers: Vertical Way

Purpose

The purpose is to link the model of distribution developed in Expanded Tables to a vertical format.

Prerequisites

Arrays to Boxes, Arrays: Easier Ways, and Expanded Tables

Materials

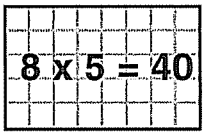
Expanded Numbers: Vertical Way - Worksheet 1, page 52, and a copy of Expanded Numbers: Vertical Way - Practice, the last page of this lesson

Lesson Session 1

Give Expanded Numbers: Vertical Way. On Worksheet 1 the student breaks up 28 x 5 in the expanded format using an array on a grid. The student is asked to study the example and explain the reasoning behind the new vertical format.

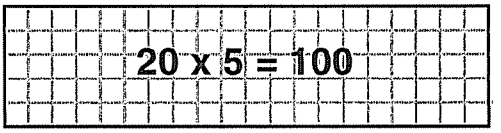
In this format the factors in the ones column are multiplied first and the product is written under the line. This relates to this model in the grid:

$$\begin{array}{r} 28 \\ \times\ 5 \\ \hline 40 \end{array}$$



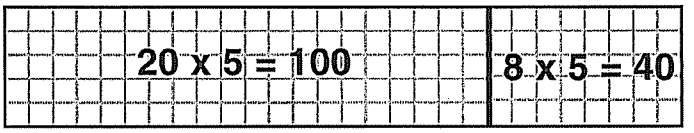
Secondly, the factors in the tens column are multiplied and the product is written below the first product. This relates to this model in the grid:

$$\begin{array}{r} 28 \\ \times\ 5 \\ \hline 40 \\ 100 \end{array}$$



The products are added together and the final product is recorded.

28 x 5 = (20 + 8) x 5



28 x 5 = 140

New way:

$$\begin{array}{r} 28 \\ \times\ 5 \\ \hline 40 \\ 100 \\ \hline 140 \end{array}$$

Explain how this new way works.

Practice Worksheets

Expanded Numbers: Vertical Way, Worksheets 2 and 3, pages 53 and 54. For additional practice use Expanded Numbers: Vertical Way: Practice - Worksheet 5 , page 56. On this page the student reviews expanding numbers with boxes and expanded numbers. An additional copy is on page 37, at the end of this lesson.

Session 2

On Expanded Numbers: Vertical Way - Worksheet 4, the student breaks up 23 x 4 in the expanded format using an array on a box as in the previous worksheets. The student is asked to study the example.

$$23 \times 4 = (\ 20 + 3\ ) \times 4$$

80 +	12

$$23 \times 4 = 92$$

In the second example the first factor (23) is broken up into 20 + 3 (tens and ones). Each number is then multiplied by the second factor (4).

$$23 \times 4 = (\ 20 + 3\ ) \times 4$$
$$= (20 \times 4) + (3 \times 4)$$
$$= 80 + 12$$
$$23 \times 4 = 92$$

$$\begin{array}{r} 23 \\ \times\ \ 4 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$$

The factors in the ones column are multiplied first and the product is written under the line. This relates to the model in the box and the little multiplication problem:

12

(3×4)

Secondly, the factors in the tens column are multiplied and the product is written below the first product. This relates to the model in the box and the little multiplication problem:

80

(20×4)

The products from the little multiplication problems are added together and the final product is recorded.

$$= 80 + 12$$
$$23 \times 4 = 92$$

The standard breaking-up into tens and ones is easier and requires less multiplication and addition. Standard procedures have developed over time because they are usually the quickest way to solve problems. It is important for students to know there are different ways to solve problems, and they may discover a way that they understand or works better for them other than the standard procedure.

Test for Understanding

Expanded Numbers: Vertical Way - Show You Know - Worksheets 1 and 2, pages 57 and 58

Expanded Numbers: Vertical Way Date_____

Practice - Make multiple copies.

Do this problem five ways.
First do it with addition.
Then write a multiplication word problem to go with this problem.
Then do it with a box.
Then do it with a box again, but this time break up the large number differently.
Then use your new vertical way.

_____ X____ = _____ Do it with addition here.

Write your word problem here.

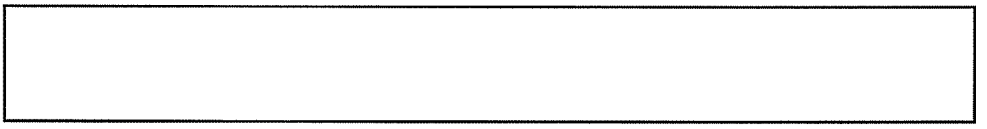
Solve it with a box.

_____ X ____ = ( ) X ___

_____ X ____ = _____

Solve it with a box but with the numbers broken up a different way.

_____ X ____ = ( ) X ___



_____ X ____ = _____

Try it the vertical way: _____ _____ _____

X_____ X_____ X_____

Expanded Multiplication

Purpose

The purpose is to integrate all the forms of distribution to multiply that the students have used to date. The box multiplication is a simple graphic that represents the problem they are doing, the distribution number sentence is the abstract statement of the basic rules of multiplication as the student will see it in algebra, and finally the vertical way is the method students will use when they actually calculate products. In a later lesson, this vertical way will be shortened to include the carrying, which is the shortcut algorithm we all learned in school.

Prerequisites

Previous lessons

Materials

Expanded Multiplication - Worksheets 1 - 5, pages 59 - 63
Practice: Two Digit - Worksheets 1 - 3, pages 64 - 66
Expanded Multiplication: Practice - Worksheets 1 - 3, pages 67 - 69
Practice: Three and Four Digit - Worksheets 1 and 2, pages 70 and 71

Warm Up

Review the worksheets the student has already completed to remind the student of the parts he is now controlling. First, study Box Multiplication - Worksheet 2, page 35. Remember when the numbers were broken up into tens? Review what the lines in the box refer to. Now go to Expanded Tables - Worksheet 8, page 45.
"How were you doing the problems on this page?" "I was breaking the large numbers into tens and ones."

"What did you do on the next page?" (Expanded Tables - Worksheet 9, page 46)
"I dropped the pictures of the tens blocks and just used lines to show the divisions of tens and ones."
"What was added under the boxes?" "The vertical way of writing the problem."

Worksheets

Give Practice: Two Digit - Worksheets 1 - 3, pages 64 - 66

Lesson

Study Expanded Multiplication: Practice - Worksheet 1, page 67. Make sure he answers the question at the bottom of the page. Do Expanded Multiplication - Practice - Worksheet 2, page 68, together to be sure he knows how to do all the problems. Check the work immediately with the Answer Key. The 'new way' is the vertical way showing each partial product.

Practice Worksheets

Expanded Multiplication: Practice - Worksheet 3, page 69
Do several problems together on Practice: Three and Four Digit - Worksheet 1, page 70. Have him complete this page on his own. Then give Practice: Three and Four Digit - Worksheet 2, page 71.

Test for Understanding

Draw a box like the one shown on Expanded Multiplication: Practice - Worksheet 3. Give the problem 89 x 4 and ask him to break the eighty-nine into four parts to fill in the four sections shown on the worksheet. Then ask him to write out the distribution number sentence for that breakup. You are checking to see that he has not forgotten that the number can be broken up into numbers other than hundreds, tens, and ones.

Short Notation

Purpose The purpose is to teach the student the short notation commonly used for multiplication. The 'long way' shows each partial product under the line and then sums them at the end. The short notation combines the multiplication and the addition. It obscures the Distributive Property somewhat, which is why it hasn't been used up until now, but the short notation is fast because there is less writing and you do not have to worry about the place value of your partial products.

Prerequisites Previous lessons

Materials Short Notation, page 72 and Practice, page 73
Show You Know, page 74
Real World Math, pages 75 and 76
Post-Assessment, page 77

Lesson Do a few problems as they were done in the last lesson. Have him use a box, show the distributive number sentences, and do the problem the vertical way using the long format shown on the practice pages.

As you do this lesson, use a one-to-one transfer process. This means you make one calculation using the old long way, then stop. You then make that same calculation using the short notation.
The talk on the second problem would sound something like this:
Looking at 245 x 3:
"Start with the long way. What do you do first?" "Multiply the three times the five, which is fifteen."

"Where do you record that?" "Write the fifteen under the line."

"Now switch to the short way. When you multiply the three times the five, you get fifteen. How do you record this in the short way?" "Put the five in the ones place and carry the ten into the tens place."

"What is the second step in the long way?" "Multiply the three times the forty and get one hundred twenty. Record that by putting the one hundred twenty under the fifteen."

"What do you do in the short way? Do you add the one ten to the four in the tens place before or after you multiply? Study the long way to figure this out." "To get the one hundred twenty, I multiply the forty and the three. Then I would add the ten on from the fifteen. So in the short way, I multiply the three times the four to get twelve and then add the one to get thirteen."

"How do you record this?" "Put the three in the tens place and carry the one into the hundreds place."

"How can you tell that the one you carried into the hundreds place is in fact a one hundred?" "I can see that it is a one hundred by looking at the long way. The one in the one hundred twenty is obviously a one hundred."

"What is the last step in the long way?" "Multiply the two hundred by the three to get six hundred. This is written under the one twenty, and now all the numbers are added up to get seven hundred thirty-five."

"How is this done in the short way?" "The two is multiplied by the three to get six and then the one is added to get seven. This seven is placed in the one hundreds place."

"What makes the short way easier?" "You do not have to write so many numbers, and you do not have to think about the place value at all. You just fill in the slots as you go across."

"Where is the Distributive Property showing in the short way?" "The three is still distributed, multiplied by each number in the top row. The partial products are added as you go."

Worksheet

Look at the first problem 34 x 7 on Practice, page 73. Have him do it the long way and the short way step by step. There is space to do it the long way in between the problems. Once you are sure he understands the connection between the long way and the short way, let him practice for speed and accuracy.

Test for Understanding

Give Show You Know, page 74. Do not help him. Press for clear explanations of the questions. If he is uncertain, reteach that aspect.

Practice Worksheets

Real World Math, pages 75 and 76

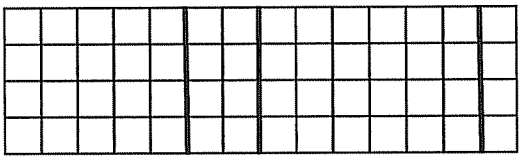
Then have him complete the Post-Assessment, page 77.

Post-Assessment Date _____

1. Fill in the missing numbers.

   a. 14
   x 4

   b. 14 x 4 = (___ + ___ + ___ + ___) x 4



   c. (__ x __) + (__ x __) + (__ x __) + (__ x __)

   e. _____ = d. _____ + _____ + _____ + _____

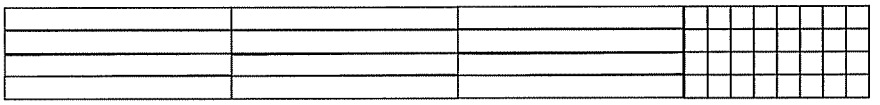
2. Show how you would break up the 63. Label the partial products in the box. Show the final product below the box on the line.

   63 x 8 = ( ) x 8

   63 x 8 = _____

3. Break up the 38 into tens and ones; fill in the missing numbers.

   a. 38 x 4 = (_____ + _____) x 4



   = b. (__ x __) + (__ x __)

   d. _____ = c. _____ + _____

4. In problem 3, what number is being distributed? _____

5. Solve this problem the Long way and the Short way.

   a. Long way
   1,439
   x 5

   b. Short way
   1,439
   x 5

6. Solve.

   a.
   138
   x 6

   b.
   2,375
   x 4

Patterns in Arithmetic

Multiplication: Booklet 2

Beginning Long Multiplication

and Basics of Distribution

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

Patterns in Arithmetic: Multiplication - Booklet 2
Student Workbook Answer Key

Published by Pattern Press
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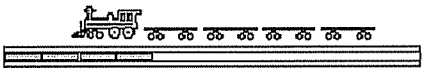
Fallbrook, CA 92088

Printed on recycled paper.

ISBN 978-1-935559-41-2

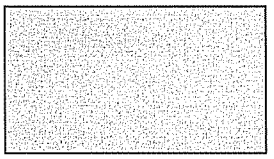
Multiplication - Booklet 2 Pre-Assessment is on the last page of the answer key.

Arrays: Manipulative - Worksheet 1
Train Wreck

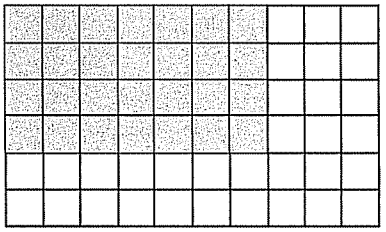


Use Cuisenaire Rods to make a twenty eight train with four black cars. If you run the train of the track, you would get a train wreck. Go ahead and do this with your Cuisenaire Rods.

Now take the train to the train wreck fix-it shop and rebuild the cars to fit into the following shape or array.



On the centimeter graph paper below use four black rods and build the shape. Color the area covered by the four rods.



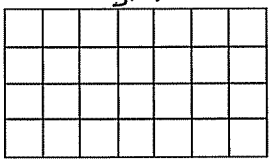
Examples: Coloring will vary.

Arrays: Manipulative - Worksheet 2
Number Sentence

This is called an array in mathematics.
The longer side is called the length.
The shorter side is called the width.
Write the name of each side on the lines below

7 length

4
width



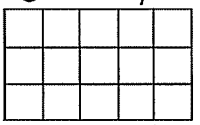
Use Cuisenaire Rods and place black rods to cover the array

The length is seven. The width is 4.
The number sentence for this array is 7 x 4 = 28.

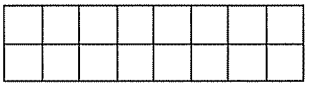
In arrays the length is always written first and the width is always written second. This number sentence is called the BIG multiplication problem.

Cover each array with one color rods.
Write the number sentence for each

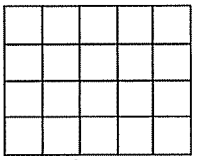
5x3=15 yellow



8x2=16 brown



5x4=20 yellow

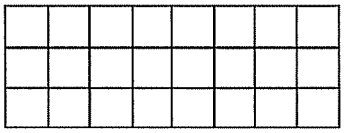


4x1=4 purple

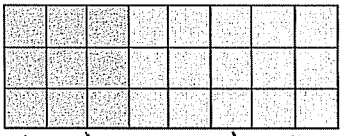
Arrays: Manipulative - Worksheet 3

The BIG multiplication problem is 8 x 3 = 24.

Build this array with Cuisenaire Rods of only one color. Color the picture to look like the rods.

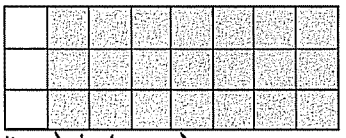


Build this array using two colors. Color the picture the same colo as the rods.



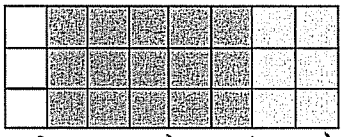
=(3x3) + (5x3) little multiplication problem

Build this array using two colors different than the array above. Color the picture the same color as the rods.



=(1x3) + (7x3)

Build this array using three colors. Color the picture the same color as the rods.



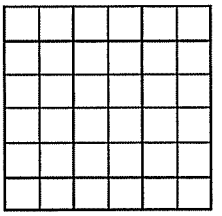
=(1x3) + (5x3) + (2x3)

Arrays: Manipulative - Worksheet 4

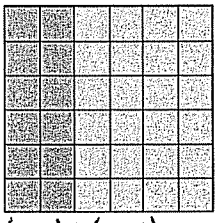
Use Cuisenaire Rods.

The BIG multiplication problem is 6 x 6 = 36.

Build this array with Cuisenaire Rods of only one color. Color the picture to look like the rods.

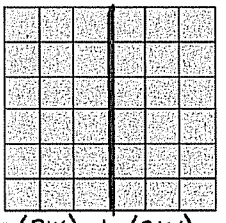


Build three more arrays using two colors, each broken up a different way. Color the picture to look like the rods.

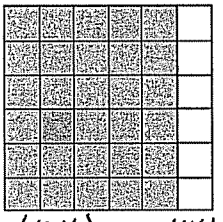


=(2x6)+(4x6)

Examples



=(3x6) + (3x6)



=(5x6) + (1x6)

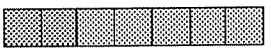
Examples: Coloring will vary.

Arrays: Manipulative - Worksheet 5

Multiplication Problems - Seven with two colors

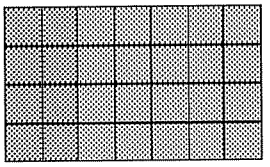
Build seven with two colors of Cuisenaire Rods.

2 + 5



Color the picture to look like your rods.

Now make three more sevens with the blocks. Make them just like the first one and arrange them in an array.

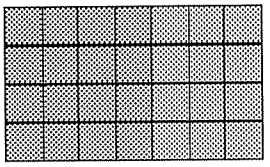


What BIG multiplication problem do you now have?

7 x 4

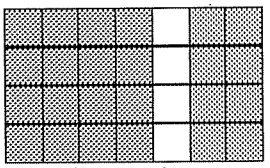
Build two more arrays using two different colors.

Color the picture to look like your rods.



4 + 3

Build one array using three different colors.



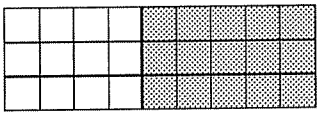
4 + 1 + 2

Examples: Coloring will vary. BuWV

Arrays: Manipulative - Worksheet 6

Use Cuisenaire Rods. Color the picture the same color as the rods.

The BIG multiplication problem is 9 x 3 = 27.

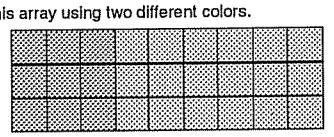


Study this example. Go back to Worksheets 3 and 4 and label all the little multiplication problems before doing this page.

= (4 x 3) + (5 x 3)
little multiplication problems

27 = 12 + 15
Final Product — Answers to the little multiplication problems

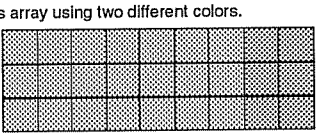
Build this array using two different colors.



= (3 x 3) + (6 x 3)
little multiplication problems

27 = 9 + 18
Final Product — Answers to the little multiplication problems.

Build this array using two different colors.



= (7 x 3) + (2 x 3)
little multiplication problems

27 = 21 + 6
Final Product — Answers to the little multiplication problems.

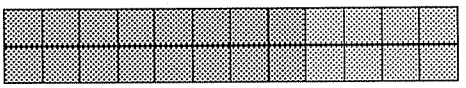
Are the answers the same? yes

Arrays: Manipulative - Worksheet 7

The BIG multiplication problem is 12 x 2 = 24.

Use Cuisenaire Rods. Color the picture the same color as the rods.

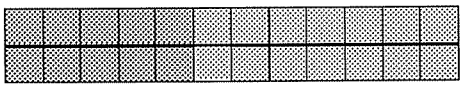
Build this array using two colors.



= (8 x 2) + (4 x 2)
little multiplication problems

24 = 16 + 8
Final Product — Answers to the little multiplication problems

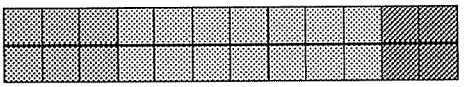
Build this array using two different colors.



= (5 x 2) + (7 x 2)
little multiplication problems

24 = 10 + 14
Final Product — Answers to the little multiplication problems.

Build this array using three different colors.



= (3 x 2) + (7 x 2) + (2 x 2)
little multiplication problems

24 = 6 + 14 + 4
Final Product — Answers to the little multiplication problems.

Are the answers the same? yes

Explain? The same problem is broken up into smaller parts in different ways. When the smaller parts are added up, the answer stays the same.

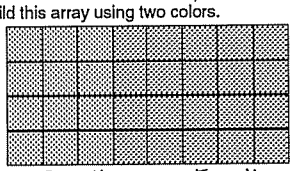
Examples: Coloring will vary. BuWV

Arrays: Manipulative - Worksheet 8

The BIG multiplication problem is 8 x 4 = 32

Use Cuisenaire Rods. Color the picture the same color as the rods.

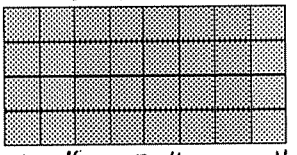
Build this array using two colors.



= (3 x 4) + (5 x 4)
little multiplication problems

32 = 12 + 20
Final Product — Answers to the little multiplication problems

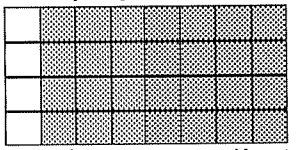
Build this array using three different colors.



= (2 x 4) + (5 x 4) + (1 x 4)
little multiplication problems

32 = 8 + 20 + 4
Final Product — Answers to the little multiplication problems

Build this array using three different colors.



= (1 x 4) + (3 x 4) + (4 x 4)
little multiplication problems

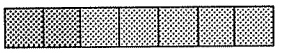
32 = 4 + 12 + 16
Final Product — Answers to the little multiplication problems

Are the answers the same? yes

Breaking Up Times - Worksheet 1

Build 7 with a red and a yellow Cuisenaire Rod.
Write in the numbers to show how you broke up the 7.

7 = 2 + 5



Color the picture to look like your rods.

Now make three more 7s with the rods.
Make them just like the first one.

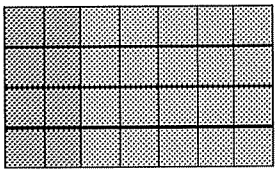
What BIG multiplication problem do you now have?

7 x 4

Color the picture below to look like your rods.

7 x 4 = (2 + 5) x 4

BIG multiplication problem



Guess

= (2 x 4) + (5 x 4)
little multiplication problems

28 = 8 + 20
FINAL PRODUCT

Check - draw an arrow, the closer the answer, the closer to the

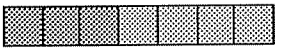
Product is the name for the answer to a multiplication problem.

13

Breaking Up Times - Worksheet 2

Do the same problem again. Break up the 7 differently.

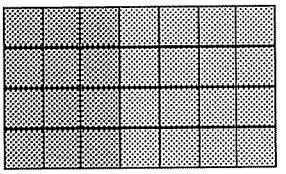
7 = 3 + 4



Color the picture to look like your rods.

What BIG multiplication problem are you working on? ___ x ___

7 x 4 = (3 + 4) x 4

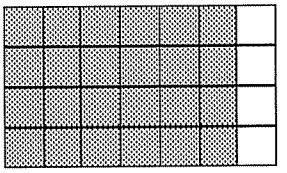


= (3 x 4) + (4 x 4)

28 = 12 + 16

Do the problem one more time. Break up the 7 a different way.

7 x 4 = (6 + 1) x 4



= (6 x 4) + (1 x 4)

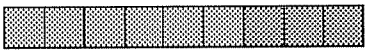
28 = 24 + 4

14

Breaking Up Times - Worksheet 3

Build 9 with two colors. Write in the numbers to show how you broke up the 9.

9 = 6 + 3



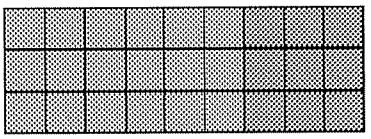
Color the picture to look like your rods.

What BIG multiplication problem are you working on? 9 x 3

Color the picture below to look like your rods. Fill in the blanks.

9 x 3 = (6 + 3) x 3

BIG multiplication problem



Guess

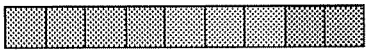
= (6 x 3) + (3 x 3)
little multiplication problems

27 = 18 + 9
FINAL PRODUCT

Chec

Now build 9 with three different colors. Fill in the blanks.

9 = 4 + 3 + 2



Color the picture to look like your rods.

Now solve the problems on the next page.

15

Breaking Up Times - Worksheet 4

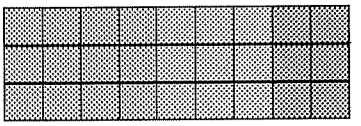
Build your three colored 9 twomore times.

What BIG multiplication problem do you have? 9 x 3

Color the picture below to look like your rods. Fill in the blanks.

9 x 3 = (4 + 3 + 2) x 3

BIG multiplication problem



= (4 x 3) + (3 x 3) + (2 x 3)
little multiplication problems

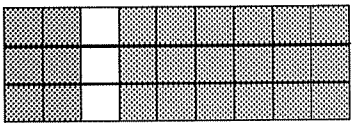
27 = 12 + 9 + 6
FINAL PRODUCT

9 = 2 + 1 + 6

Solve the problems below. Color the pictures

9 x 3 = (2 + 1 + 6) x 3

BIG multiplication problem



= (2 x 3) + (1 x 3) + (6 x 3)
little multiplication problems

27 = 6 + 3 + 18
FINAL PRODUCT

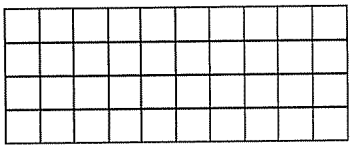
16

Breaking Up Times - Worksheet 5
Break Up Will Vary (BUWV)

Break up 10 into three rods.

10 x 4 = (_____ + _____ + _____) x 4

BIG Multiplication Problem Guess



= (___ x 4) + (___ x 4) + (___ x 4)
little multiplication problems

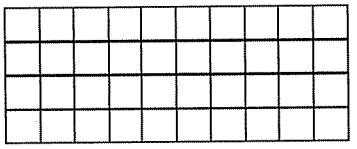
40 = _____ + _____ + _____
Final product — partial products

Check

Break up 10 a different way.

10 x 4 = (_____ + _____ + _____) x 4

BIG Multiplication Problem Guess



= (___ x ___) + (___ x ___) + (___ x ___)
little multiplication problems

40 = _____ + _____ + _____
Final product — partial products

Check

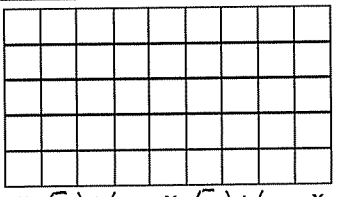
Do your answers match? yes

Breaking Up Times - Worksheet 6 (BUWV)

Break up 9 into three rods or numbers.

9 x 5 = (_____ + _____ + _____) x 5

BIG Multiplication Problem Guess



= (___ x 5) + (___ x 5) + (___ x 5)
little multiplication problems

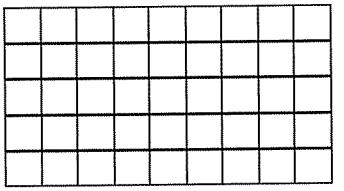
45 = _____ + _____ + _____
Final product — partial products

Check

Break up 9 a different way.

9 x 5 = (_____ + _____ + _____) x ___

BIG Multiplication Problem Guess



= (___ x 5) + (___ x 5) + (___ x 5)
little multiplication problems

45 = _____ + _____ + _____
Final product — partial products

Check

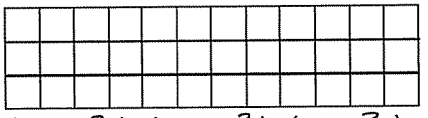
Do your answers match? _____

Breaking Up Times - Worksheet 7 (BUWV)

Break up 12 into three rods or numbers.

12 x 3 = (_____ + _____ + _____) x ___

BIG Multiplication Problem Guess



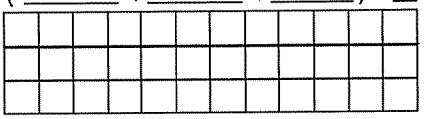
= (___ x 3) + (___ x 3) + (___ x 3)

36 = _____ + _____ + _____
Final product

Check

Break up 12 a different way.

12 x 3 = (_____ + _____ + _____) x ___

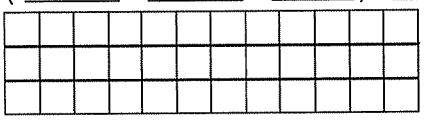


= (___ x 3) + (___ x 3) + (___ x 3)

36 = _____ + _____ + _____
Final product

Solve the problem again a different way with rods. You do not have to draw the rods this time. Record your work below.

12 x 3 = (_____ + _____ + _____) x ___



= (___ x 3) + (___ x 3) + (___ x 3)

36 = _____ + _____ + _____

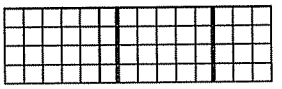
Do your answers match? _____

Arrays to Boxes - Worksheet 1

Do these problems with your teacher.

14 x 4 = (6 + 5 + 3) x 4

Guess



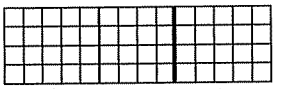
= (6 x 4) + (5 x 4) + (3 x 4)

56 = 24 + 20 + 12

Check

Again, please.

14 x 4 = (9 + 5) x 4

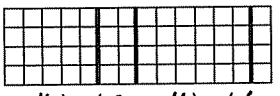


= (9 x 4) + (5 x 4)

56 = 36 + 20

One more time.

14 x 4 = (5 + 2 + 6 + 1) x 4



= (5 x 4) + (2 x 4) + (6 x 4) + (1 x 4)

56 = 20 + 8 + 24 + 4

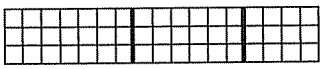
Do all of your final products match? yes
Should they? yes

Arrays to Boxes - Worksheet 2

Solve this problem three different ways. Estimate the answer

17 x 3 = (7 + 6 + 4) x 3

Guess



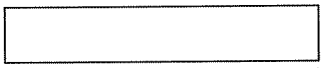
= (7 x 3) + (6 x 3) + (4 x 3)

51 = 21 + 18 + 12

Check

Again, please. You draw the lines in the picture this time.

17 x 3 = (8 + 6 + 3) x 3

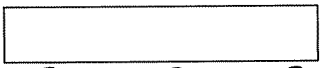


= (8 x 3) + (6 x 3) + (3 x 3)

51 = ____ + ____ + ____

One more time. Break the 17 into four sections this time. BUWV

17 x 3 = (____ + ____ + ____ + ____) x 3



= (__ x 3) + (__ x 3) + (__ x 3) + (__ x 3)

51 = ____ + ____ + ____ + ____

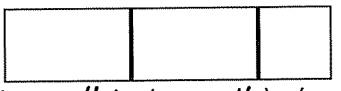
Do all of your final products match? yes

Arrays to Boxes - Worksheet 3

Solve this problem three different ways. Estimate the answer first. BUWV

18 x 4 = (____ + ____ + ____) x 4

Guess



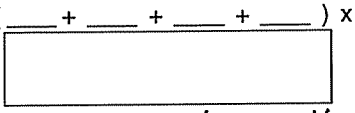
= (__ x 4) + (__ x 4) + (__ x 4)

72 = ____ + ____ + ____

Check

Break the 18 into four sections.

18 x 4 = (____ + ____ + ____ + ____) x 4



= (__ x 4) + (__ x 4) + (__ x 4) + (__ x 4)

72 = ____ + ____ + ____ + ____

Challenge! Break the 18 into only two sections.

18 x 4 = (____ + ____) x ____

= (__ x 4) + (__ x 4)

72 = ____ + ____

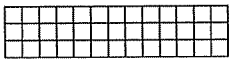
Do all of your final products match? yes

Arrays to Boxes - Worksheet 4

Solve this problem three different ways. Estimate the answer first.

13 x 3 = (____ + ____ + ____) x 3

Guess



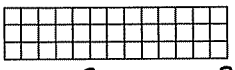
= (__ x 3) + (__ x 3) + (__ x 3)

39 = ____ + ____ + ____

Check

Again, please. Break the 13 into only two sections.

13 x 3 = (____ + ____) x 3



= (__ x 3) + (__ x 3)

39 = ____ + ____

Change 13 x 3 = ___ into an addition problem. Then find the answer.
Do the work here.

13
13
+13
39

BUWV

Challenge! Can you break up the 13 and get the answer without a picture?

13 x 3 = (____ + ____ + ____) x 3

= (__ x 3) + (__ x 3) + (__ x 3)

39 = ____ + ____ + ____

Use different numbers than the ones in the problem at the top of this page.

Arrays to Boxes: Assessment - Worksheet 5
Show You Know

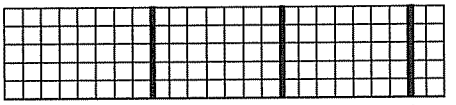
Estimate the answer. Do you think the answer will be more than 100? _____

Record the way this problem is broken up and solve.

Guess

24 x 5 = (8 + 7 + 7 + 2) x 5

My Way.



Check

= (8 x 5) + (7 x 5) + (7 x 5) + (2 x 5)

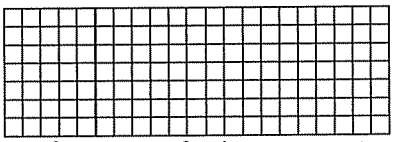
120 = 40 + 35 + 35 + 10

Estimate the answer. Draw the lines and solve.

Guess

21 x 7 = (9 + 8 + 4) x 7

My Way.



Check

= (9 x 7) + (8 x 7) + (4 x 7)

147 = 63 + 56 + 28

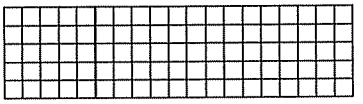
BUWV

Break it up your own way into three sections. You put in the lines.

Guess

19 x 5 = (____ + ____ + ____) x 5

Your Own Way.



Check

= (__ x 5) + (__ x 5) (__ x 5)

95 = ____ + ____

Distributive Property

You have just used another law of multiplication, probably without realizing you were
It is called the Distributive Property. It is very important. It is the law that allows us to break big numbers into little numbers, multiply the little numbers and add it all up at the end.

Get a dictionary. Look up the word distribution.
What does it mean?

1. To divide among several or many.

2. To spread out; scatter.

What does it mean when we say the newspaper is distributed every day to all the customers?
The newspapers are spread out to each house where a customer lives.

Solve this problem. Break up the 13 into two numbers. Example:

13 x 5 = (10 + 3) x 5
(10 x 5) + (3 x 5)
65 = 50 + 15

What did you do first?
Broke up the 13 into smaller parts.

Then what did you do?
Multiplied each part by 5. Then added the products together.

What number got "distributed" to all the little parts? 3

What would happen if you distributed that number to only some of the parts but not to all them?
The answer would be wrong.

Arrays: Easier Ways - Worksheet 1

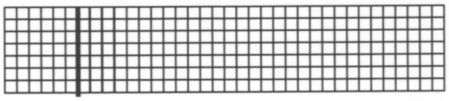
Do this problem three ways.

36 x 7 = 252 One way is by addition.

36
36
36
36
36
36
+ 36
252

Another way is by breaking the 36 into smaller numbers and multiplying each part by 7.
Do this two times. Break up the 36 two different ways.
Draw the lines to show each small or partial product.

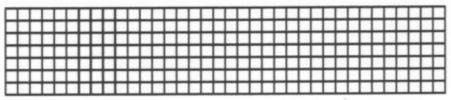
36 x 7 = (6 + + + +) x 7



= (6 x 7) + (__ x 7) + (__ x 7) + (__ x 7) + (__ x 7)

252 = 42 + ____ + ____ + ____ + ____

36 x 7 = ( ) x 7



= (__ x 7) +

252 = ____ +

Arrays: Easier Ways - Worksheet 2

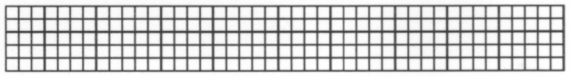
Solve this problem three ways.

43 x 5 = 215 One way is by addition.

43
43
43
43
+43
215

Another way is by breaking the 43 into smaller numbers and multiplying each part by 5.
Break up the 43 two different ways.
You will need to supply lines and + signs for your partial products.

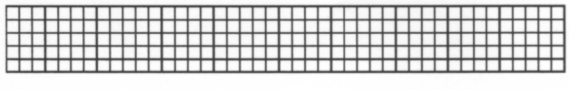
43 x 5 = ( ) x 5



= (__ x 5) +

215 = ____ +

43 x 5 = ( ) x 5



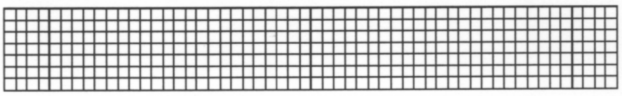
= (__ x 5) +

215 = ____ +

Arrays: Easier Ways - Worksheet 3

Break up this problem three ways. You will need to supply the lines and + signs for the partial products.
Find the easiest way to break up the problem. 54 x 7 = 378

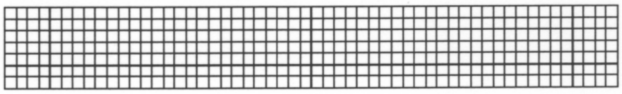
54 x 7 = ( ) x 7



= (__ x 7) +

378 = ____ +

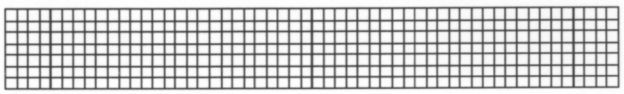
54 x 7 = ( ) x 7



= (__ x 7) +

378 = ____ +

54 x 7 = ( ) x 7



= (__ x 7) +

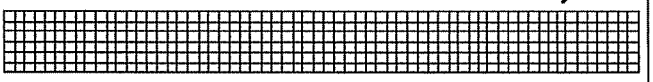
378 = ____ +

Arrays: Easier Ways - Worksheet 4

Break up this problem three ways. You will need to supply the lines and + signs for the partial products.
Find the easiest way to break up the problem. BUWV

62 x 6 = 372

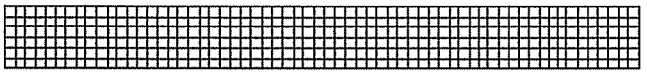
62 x 6 = ( )x6



= (__ x 6) +

372 = _____ +

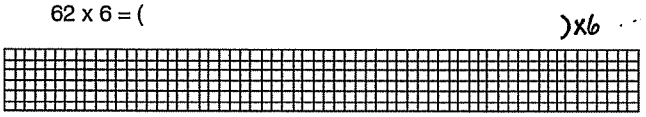
62 x 6 = ( )x6



= (__ x 6) +

372 = _____ +

62 x 6 = ( )x6



= (__ x 6) +

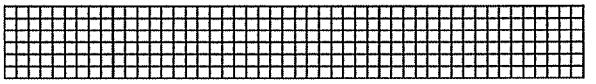
372 = _____ +

Arrays to Boxes: Breaking Up into Tens Worksheet 1

Break up and solve these problems the easist way you know. You will need to supply the lines and + signs for the partial products.

48 x 6 = 288

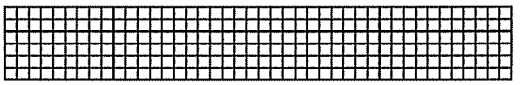
48 x 6 = ( )x6



= (__ x 6) +

288 = _____ +

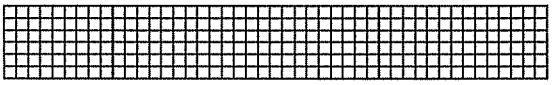
42 x 6 = ( )x6



= (__ x 6) +

252 = _____ +

45 x 6 = ( )x6



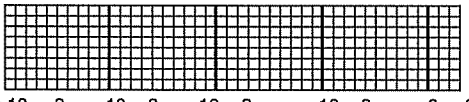
= (__ x 6) +

270 = _____ +

Arrays to Boxes: Breaking Up into Tens Worksheet 2

Study the example below.

43 x 8 = (10 + 10 + 10 + 10 + 3) x 8



= (10 x 8) + (10 x 8) + (10 x 8) + (10 x 8) + (3 x 8)

344 = 80 + 80 + 80 + 80 + 24

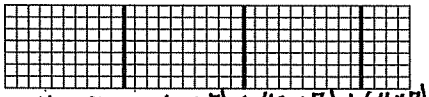
Now we will do the exact same problem again. But this time we will drop all the little squares. We will just use lines to show about how we would break the 43 up. We can also do the multiplication right in the box and save some space.

43 x 8 = (10 + 10 + 10 + 10 + 3) x 8

10 x 8 → 80	80	80	80	24

Try this problem using the array and then the box. Do the problem the same way both times.

34 x 7 = (10 + 10 + 10 + 4) x7



= (10 x 7) + (10 x 7) + (10 x 7) + (4 x 7)

238 = 70 + 70 + 70 + 28

34 x 7 = (10 + 10 + 10 + 4) x 7

70	70	70	28

(10 x 7) + (10 x 7) + (10 x 7) + (4 x 7)

238 = 70 + 70 + 70 + 28

Arrays to Boxes: Breaking Up into Tens Worksheet 3

43 x 8 = (10 + 10 + 10 + 10 + 3) x

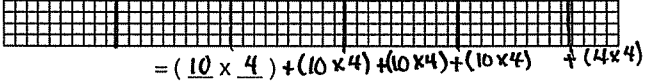
Remember You can do this part in your head.

10 x 8 → 80	80	80	80	24

43 x 8 = 344

We will use the array (grid) and the box together one more time. Then we will drop the arrays and use only the boxes. If you have difficulty doing the problems with just the box, draw the array you need on graph paper. Do this problem with the array.

54 x 4 = (10 + 10 + 10 + 10 + 10 + 4) x 4



= (10 x 4) + (10 x 4) + (10 x 4) + (10 x 4) + (4 x 4)

216 = _____

Now do it with the box.

54 x 4 = (10 + 10 + 10 + 10 + 10 + 4) x 4

10 x 4 → 40	40	40	40	40	16

(10 x 4) + (10 x 4) + (10 x 4) + (10 x 4) + (10 x 4) + (4 x 4)

54 x 4 = 40 + 40 + 40 + 40 + 40 + 16

216 =

Box Multiplication - Worksheet 1

43 x 8 = (10 + 10 + 10 + 10 + 3) x 8

Remember, You can do this part in your head.

10 x 8				
80	80	80	80	24

43 x 8 = 344

We will use the same sized box for all the problems, even though the numbers will change. Solve these problems.

39 x 8 = (10 + 10 + 10 + 9) x 8

80	80	80	72

39 x 8 = 312

46 x 8 = (10 + 10 + 10 + 10 + 6) x 8

80	80	80	80	48

46 x 8 = 368

53 x 6 = (10 + 10 + 10 + 10 + 10 + 3) x 6

60	60	60	60	60	18

53 x 6 = 318

Box Multiplication - Worksheet 2

34 x 6 = ( ) x 6

34 x 6 = 204

27 x 5 = ( ) x 5

27 x 5 = 135

56 x 7 = ( ) x 7

56 x 7 = 392

43 x 9 = ( ) x 9

43 x 9 = 387

63 x 8 = ( ) x 8

63 x 8 = 504

Box Multiplication - Worksheet 3

68 x 3 = ( ) x 3

68 x 3 = 204

37 x 4 = ( ) x 4

37 x 4 = 148

34 x 5 = ( ) x 5

34 x 5 = 170

42 x 4 = ( ) x 4

42 x 4 = 168

53 x 8 = ( ) x 8

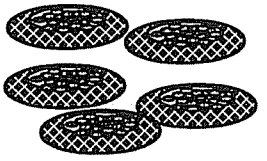
53 x 8 = 424

Word Problems: Review

43 x 5 = 215

43 + 43 + 43 + 43 + 43 ← Write the addition problem.

43 x 5 = 215



1. Mary had five nests for her large flock of chickens. Each nest had forty-three eggs. How many eggs did she collect? 215

2. She sold the eggs for 4¢ each. How much money were all of the eggs worth? 8.60

Write the multiplication problem two ways for the second problem. Also write it as an addition problem.

215 x 4 = 860

215 x 4 = 8.60

215 + 215 + 215 + 215 = 860

Make up your word problem and then write three number sentences that go along with the word problem.

Expanded Tables - Worksheet 1

Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the "two times" table.
Use your Base Ten Blocks

1 x 2 = 2	10 x 2 = 20	100 x 2 = 200
2 x 2 = 4	20 x 2 = 40	200 x 2 = 400
3 x 2 = 6	30 x 2 = 60	300 x 2 = 600

Use your Base Ten Blocks to find the answers to the rest of these problems.

4 x 2 = 8	40 x 2 = 80	400 x 2 = 800
5 x 2 = 10	50 x 2 = 100	500 x 2 = 1,000
6 x 2 = 12	60 x 2 = 120	600 x 2 = 1,200
7 x 2 = 14	70 x 2 = 140	700 x 2 = 1,400
8 x 2 = 16	80 x 2 = 160	800 x 2 = 1,600
9 x 2 = 18	90 x 2 = 180	900 x 2 = 1,800

Expanded Tables - Worksheet 2

Use Base Ten Blocks and expanded tables on the previous page to solve these box problems.

56 x 2 = (50 + 6) x 2

100 + 12

56 x 2 = 112

68 x 2 = (60 + 8) x 2

120 + 16

68 x 2 = 136

73 x 2 = (70 + 3) x 2

140 + 6

73 x 2 = 146

Try 89 x 2 = (80 + 9) x 2 You draw the box.

89 x 2 = 178

89
x2
178

Expanded Tables - Worksheet 3

Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the "three times" table.
Use your Base Ten Blocks.

1 x 3 = 3	10 x 3 = 30	100 x 3 = 300
2 x 3 = 6	20 x 3 = 60	200 x 3 = 600
3 x 3 = 9	30 x 3 = 90	300 x 3 = 900

Use your Base Ten Blocks to find the answers to the rest of these problems.

4 x 3 = 12	40 x 3 = 120	400 x 3 = 1,200
5 x 3 = 15	50 x 3 = 150	500 x 3 = 1,500
6 x 3 = 18	60 x 3 = 180	600 x 3 = 1,800
7 x 3 = 21	70 x 3 = 210	700 x 3 = 2,100
8 x 3 = 24	80 x 3 = 240	800 x 3 = 2,400
9 x 3 = 27	90 x 3 = 270	900 x 3 = 2,700

Expanded Tables - Worksheet 4

Use Base Ten Blocks and expanded tables on the previous page to solve these box problems.

54 x 3 = (50 + 4) x 3

150 + 12

54 x 3 = 162

47 x 3 = (40 + 3) x 3

120 + 21

47 x 3 = 141

62 x 3 = (60 + 2) x 3

180 + 6

62 x 3 = 186

Try 73 x 3 = (70 + 3) x 3 You draw the box.

73 x 3 = 219

Expanded Tables - Worksheet 5

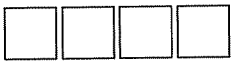
Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the "four times" table.
Use your Base Ten Blocks.

1 x 4 = 4 □□□□

10 x 4 = 40

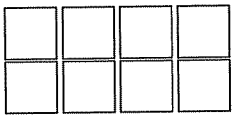
100 x 4 = 400



2 x 4 = 8

20 x 4 = 80

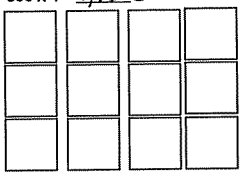
200 x 4 = 800



3 x 4 = 12

30 x 4 = 120

300 x 4 = 1,200



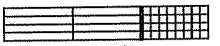
Use your Base Ten Blocks to find the answers to the rest of these problems.

4 x 4 = 16	40 x 4 = 160	400 x 4 = 1600
5 x 4 = 20	50 x 4 = 200	500 x 4 = 2,000
6 x 4 = 24	60 x 4 = 240	600 x 4 = 2,400
7 x 4 = 28	70 x 4 = 280	700 x 4 = 2,800
8 x 4 = 32	80 x 4 = 320	800 x 4 = 3,200
9 x 4 = 36	90 x 4 = 360	900 x 4 = 3,600

Expanded Tables - Worksheet 6

Use Base Ten Blocks and expanded tables from the previous page.

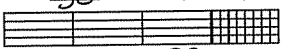
29 x 4 = (20 + 9) x 4



80 + 36

29 x 4 = 116

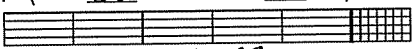
38 x 4 = (30 + 8) x 4



120 + 32

38 x 4 = 152

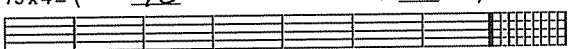
57 x 4 = (50 + 7) x 4



200 + 28

57 x 4 = 228

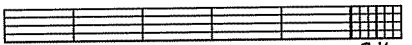
79 x 4 = (70 + 9) x 4



280 + 36

79 x 4 = 316

Try 56 x 4 = (50 + 6) x 4 You draw the box.



200 + 24

56 x 4 = 224

Expanded Tables - Worksheet 7

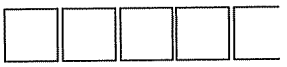
Knowing expanded tables makes it easier to do large multiplication and division problems.

Here is the expanded table for the five times table.
Use your Base Ten Blocks

1 x 5 = 5 □□□□□

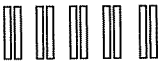
10 x 5 = 50

100 x 5 = 500

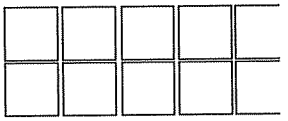


2 x 5 = 10

20 x 4 = 100



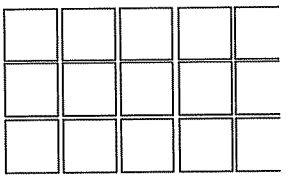
200 x 5 = 1,000



3 x 5 = 15

30 x 5 = 150

300 x 5 = 1,500



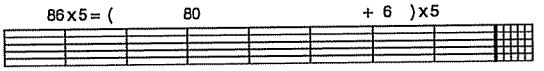
Use your Base Ten Blocks to find the answers to the rest of these problems.

4 x 5 = 20	40 x 5 = 200	400 x 5 = 2,000
5 x 5 = 25	50 x 5 = 250	500 x 5 = 2,500
6 x 5 = 30	60 x 5 = 300	600 x 5 = 3,000
7 x 5 = 35	70 x 5 = 350	700 x 5 = 3,500
8 x 5 = 40	80 x 5 = 400	800 x 5 = 4,000
9 x 5 = 45	90 x 5 = 450	900 x 5 = 4,500

Expanded Tables - Worksheet 8

Use your Base Ten Blocks and expanded tables on the previous page.

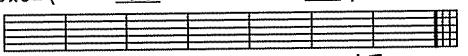
86 x 5 = (80 + 6) x 5



400 + 30

86 x 5 = 430

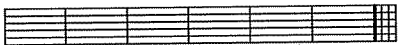
73 x 5 = (70 + 3) x 5



350 + 15

73 x 5 = 365

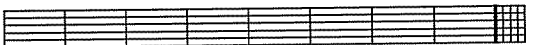
63 x 5 = (60 + 3) x 5



300 + 15

63 x 5 = 315

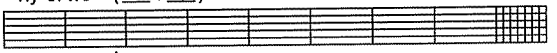
84 x 5 = (80 + 4) x 5



400 + 20

84 x 5 = 420

Try 87 x 5 = (80 + 7) x 5 You draw the box.



87 x 5 = 435

Expanded Tables - Worksheet 9

Use Base Ten Blocks and expanded tables booklet to solve these box problems.

87 x 4 = (80 + 7)x4

80	+	28

87 x 4 = 348

87 x4 348

79 x 4 = (70 + 9)x4

280	+	36

79 x 4 = 316

79 x4 316

47 x 4 = (40 + 7)x4

160	+	28

47 x 4 = 188

47 x4 188

Expanded Tables - Worksheet 10

Use Base Ten Blocks and expanded tables booklet to solve these box problems.

78 x 5 = (70 + 8)x5

350	+	40

78 x 5 = 390

78 x5 390

61 x 5 = (60 + 1)x5

300	+	5

61 x 5 = 305

61 x5

132 x 5 = (100 + 30 + 2)x5

100	+	150 +	10

132 x 5 = 660

132 x 5 660

Factor these to prime numbers. List the prime factors in the box. Remember to put them in order from largest to smallest. Put commas between the prime numbers.

7,3,2

42

21 x 2

3x7

3,3,2,2

36

6 x 6

2x3 2x3

5,5,3

75

5 x 15

3x5

Expanded Tables - Worksheet 11

Use Base Ten Blocks and expanded tables booklet to solve these box problems.

54 x 6 = (50 + 4)x6

300	+	24

54 x 6 = 324

54 x6

82 x 6 = (80 + 2)x6

480	+	12

82 x 6 = 492

82 x6

137 x 6 = (100 + 30 + 7)x6

600	+	180 +	42

137 x 6 = 822

137 x 6 822

How many one inch tiles could you put in a space that was 6 inches wide and 125 inches long? 750 Show your work.

6

125

What would be the perimeter of this shape? 262
What would be the area? 750

Word Problems: Two Step

Use money to help you solve these.

Menu			
soup	$ 1.50	salad small	$2.50
cracker	$ 0.60	salad large	$5.75
sandwich	$ 2.50	cookie	$0.95
milk	$ 1.00	juice	$0.95

1. Sam bought soup, a sandwich and a glass of milk. Mrs. K paid with a $5 bill. How much does Sam owe Mrs. K? $5.00

$1.50
2.50
1.00
$5.00

2. How much change did Mrs. K get? 0

3. Andrew, Michael, Karly and Cossette each bought a cookie for a snack. Mrs. K gave them a $5 bill also. How much change should they bring back? $1.20

95 x 4 = $3.80

5.00 - 3.80 = $1.20

4. Trudy bought a large salad, a sandwich, a cookie and some milk. Kaela had the same thing Sam bought.
How much more did Trudy spend than Kaela? $5.20

$5.75
2.50
.95
1.00
$10.20

10.20 - 5.00 = $5.20

Box Multiplication - Worksheet 1

96 x 6 = ( ) x 6

96 x 6 = 576

49 x 5 = ( ) x 5

49 x 5 = 245

158 x 8 = ( ) x 8

158 x 8 = 1,264

158 x 4 = ( ) x 4

158 x 4 = 632

209 x 8 = ( ) x 8

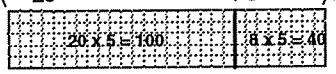
209 x 8 = 1,672

50

Expanded Numbers: Vertical Way - Worksheet 1

Now let's learn a new way to write down what we are doing in an expanded multiplication problem. Shown below is the box way you have been using. Under the box there is a new way to write the problem down.
Figure out the pattern.

28 x 5 = (20 + 8) x 5



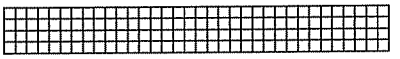
28 x 5 = 140

New way:
28
x 5
40
100
140

Explain how this new way works.

Try another one. Solve it the box way and then the new vertical way.

34 x 4 = (30 + 4) x 4



34 x 4 = 136

34
x 4
16
+120
136

Solve this problem. Show your work.
If you sold 35 puppies for $4 each and sold 27 kittens for $3 each, how much money would you take in?

35
x 4
20
+120
140

27
x 3
21
+60
81

140
+81
221

52

Expanded Numbers: Vertical Way - Worksheet 2

Do this problem five ways.
First do it with addition.
Then write a multiplication word problem to go with this problem.
Then do it with a box.
Then do it with a box again, but this time break the large number up differently.
Then use your new vertical way.

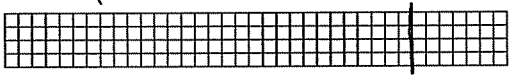
37 x 4 = 148

Write your word problem here.

Do it with addition here.
37
37
37
+37
148

Solve it with a box.

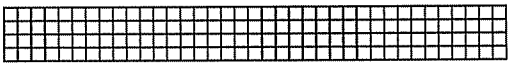
37 x 4 = (30 + 7) x 4



37 x 4 = 148

Solve it with a box but with the numbers broken up a different way.

37 x 4 = ( ) x 4



37 x 4 = 148

Try it the vertical way:
37
x 4
28
+120
148

53

Expanded Numbers: Vertical Way - Worksheet 3

Do this problem five ways.
First do it with addition.
Then write a multiplication word problem to go with this problem.
Then do it with a box.
Then do it with a box again, but this time break the large number up differently.
Then use your new vertical way.

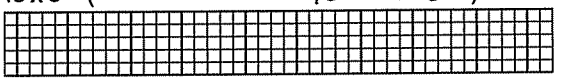
43 x 5 = 215

Write your word problem here.

Do it with addition here.
43
43
43
43
+43
215

Solve it with a box, use standard tens and ones expansion.

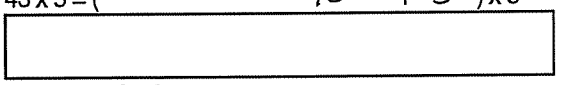
43 x 5 = (40 + 3) x 5



43 x 5 = 215

Solve it with a box.

43 x 5 = (40 + 3) x 5



43 x 5 = 215

Try it the vertical way:
43
x 5
15
+200
215

Try these the vertical way without the boxes.

38
x 4
32
+120
152

29
x 3
27
+60
87

54

Expanded Numbers: Vertical Way - Worksheet 4

Before you do this page, do an Arrays to Boxes Practice Page with boxes and expanded numbers.

Study this example.

$23 \times 4 = (\ 20 \ + 3\ ) \times 4$

80 +	12

$23 \times 4 = 92$

Here is the same problem again without the box. It is important that you remember this way of writing the problem because this is how multiplication is done in algebra. You will use this a lot in coming years. The only thing that is different is that you must write out each little multiplication that you did in the box. This is how you did it when you were first learning.

$23 \times 4 = (\ 20 \ + \ 3\ ) \times 4$

$= (20 \times 4) + (3 \times 4)$

$= 80 + 12$

$23 \times 4 = 92$

Your new way has you do this same problem this way:

$$\begin{array}{r} 23 \\ \times\ 4 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$$

There are many other ways you could break up the 23 also.
You used to break up numbers this way:
10 + 10 + 3 or 5 + 5 + 8 or ...

Why don't you use these ways much anymore? There are more little problems which makes it harder and there are more chances for mistakes.

Expanded Numbers: Vertical Way Show You Know - Worksheet 1

Solve this problem with addition.

$32 \times 7 = 224$

$$\begin{array}{r} 32 \\ 32 \\ 32 \\ 32 \\ 32 \\ 32 \\ +\ 32 \\ \hline 224 \end{array}$$

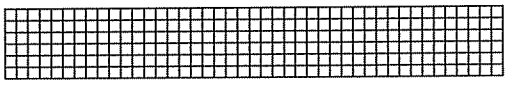
Use the Vertical Way to solve this problem.

$$\begin{array}{r} 57 \\ \times\ 3 \\ \hline 21 \\ +150 \\ \hline 171 \end{array}$$

Solve this one with a grid.

BUWV

$43 \times 6 = (\ \_\_\_\_ + \_\_\_\_ + \_\_\_\_\ ) \times 6$



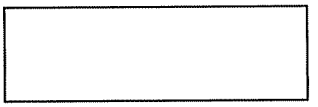
$= (\_\_ \times 6) + (\_\_ \times 6) + (\_\_ \times 6)$

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

258 =

Solve this one without the grid.

$25 \times 7 = (\ \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_\ ) \times 7$



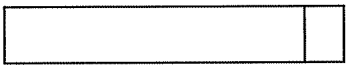
$= (\_\_ \times 7) + (\_\_ \times 7) + (\_\_ \times 7) + (\_\_ \times 7)$

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

175 =

Expanded Numbers: Vertical Way Show You Know - Worksheet 2

$26 \times 4 = (\ 20 \ + \ 6\ ) \times 4$



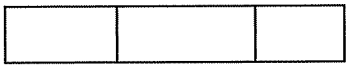
$26 \times 4 = (20 \times 4) + (6 \times 4)$

$= 80 + 24$

104 =

Remember, tens and ones expansions are not the only way.
Break it up with a nonstandard expansion.

$26 \times 4 = (\ \_\_\_\_ + \_\_\_\_ + \_\_\_\_\ ) \times \_\_\_\_$



$26 \times 4 = (\_\_ \times 4) + (\_\_ \times 4) + (\_\_ \times 4)$

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

104 =

Solve using the Vertical Way.

$$\begin{array}{r} 26 \\ \times\ 4 \\ \hline 24 \\ +80 \\ \hline 104 \end{array}$$

Expanded Multiplication - Worksheet 1

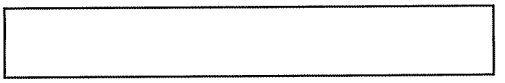
Fill in the blanks. You may use graph paper.
Be sure to attach it to this page if you use it.

1. $\begin{array}{r} 21 \\ \times\ 4 \\ \hline 84 \end{array}$ $21 \times 4 = (20 + 1) \times 4$

$= (20 \times 4) + (1 \times 4)$

$= 80 + 4$

$= 84$

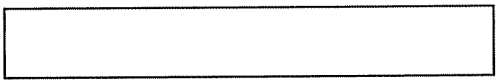


2. $\begin{array}{r} 21 \\ \times\ 4 \\ \hline 84 \end{array}$ $21 \times 4 = (\_\_ + \_\_ + \_\_) \times 4$

$= (\_\_ \times 4) + (\_\_ \times 4) + (\_\_ \times 4)$

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

$= 84$



Write a word problem to go with the numbers in problem 2.

Expanded Multiplication - Worksheet 2

Fill in the blanks. You may use graph paper. Be sure to attach it to this page if you use it.

BUWV

Example:

3. 25 x 3 = 75

25 x 3 = (20 + 5) x 3

= (20 x 3) + (5 x 3)

= 60 + 15

= 75

4. 25 x 3 = 75

25 x 3 = (___ + ___ + ___) x 3

= (___ x 3) + (___ x 3) + (___ x 3)

= ___ + ___ + ___

= 75

Write a word problem to go with the numbers in problem 3.

Expanded Multiplication - Worksheet 3

Fill in the blanks. You may use graph paper. Be sure to attach it to this page if you use it. Then solve the problem in the vertical way.

1. 24 x 6 = 144

24 x 6 = (20 + 4) x 6

= (20 x 6) + (4 x 6)

= 120 + 24

= 144

2. 23 x 5 = 115

23 x 5 = (20 + 3) x 5

= (20 x 5) + (3 x 5)

= 100 + 15

= 115

3. 26 x 3 = 78

26 x 3 = (20 + 6) x 3

= (20 x 3) + (6 x 3)

= 60 + 18

= 78

Expanded Multiplication - Worksheet 4

Fill in the blanks. You may use graph paper. Be sure to attach it to this page if you use it. Then solve the problem in the vertical way.

1. 27 x 5 = 135

27 x 5 = (20 + 7) x 5

= (20 x 5) + (7 x 5)

= 100 + 35

= 135

2. 25 x 4 = 100

25 x 4 = (20 + 5) x 4

= (20 x 4) + (5 x 4)

= 80 + 20

= 100

3. 28 x 3 = 84

28 x 3 = (20 + 8) x 3

= (20 x 3) + (8 x 3)

= 60 + 24

= 84

Expanded Multiplication - Worksheet 5

Fill in the blanks. You may use graph paper. Be sure to attach it to this page if you use it. Then solve the problem in the vertical way.

1. 21 x 7 = 147

21 x 7 = (20 + 1) x 7

= (20 x 7) + (1 x 7)

= 140 + 7

= 147

2. 29 x 3 = 87

29 x 3 = (20 + 9) x 3

= (20 x 3) + (9 x 3)

= 60 + 27

= 87

3. 27 x 5 = 135

27 x 5 = (20 + 7) x 5

= (20 x 5) + (7 x 5)

= 100 + 35

= 135

Practice: Two Digit - Worksheet 1

23 x 4 = 92	22 x 7 = 154	31 x 5 = 155	27 x 3 = 81	16 x 7 = 112
32 x 5 = 160	19 x 6 = 114	25 x 3 = 75	24 x 8 = 192	29 x 2 = 58
27 x 5 = 135	28 x 3 = 84	13 x 9 = 117	17 x 4 = 68	24 x 6 = 144
18 x 6 = 108	34 x 3 = 102	32 x 5 = 160	16 x 8 = 128	34 x 4 = 136
21 x 9 = 189	26 x 6 = 156	23 x 8 = 184	24 x 7 = 168	29 x 5 = 145

64

Practice: Two Digit - Worksheet 2

15 x 3 = 45	22 x 6 = 132	32 x 4 = 128	24 x 6 = 144	15 x 4 = 60
24 x 5 = 120	38 x 7 = 266	24 x 8 = 192	36 x 4 = 144	38 x 3 = 114
35 x 4 = 140	27 x 3 = 81	12 x 8 = 96	18 x 5 = 90	25 x 6 = 150
17 x 5 = 85	33 x 2 = 66	31 x 4 = 124	14 x 6 = 84	36 x 7 = 252
20 x 8 = 160	25 x 7 = 175	34 x 6 = 204	25 x 4 = 100	28 x 3 = 84

65

Show Your Know

Solve these problems without any help.

7 x 3 = 21	9 x 2 = 18	4 x 4 = 16	6 x 5 = 30	8 x 2 = 16
5 x 2 = 10	8 x 5 = 40	3 x 3 = 9	4 x 5 = 20	5 x 3 = 15
5 x 7 = 35	6 x 3 = 18	9 x 3 = 27	3 x 4 = 12	9 x 5 = 45
4 x 2 = 8	7 x 4 = 28	3 x 8 = 24	8 x 4 = 32	7 x 5 = 35
3 x 5 = 15	7 x 2 = 14	6 x 4 = 24	5 x 5 = 25	4 x 9 = 36

How long to finish? _____ Working rate _____

35 x 4 = 20 + 120 = 140	12 x 5 = 10 + 50 = 60	26 x 3 = 18 + 60 = 78
37 x 6 = 42 + 180 = 222	29 x 4 = 36 + 80 = 116	132 x 4 = 8 + 120 + 400 = 528

66

Expanded Multiplication: Practice - Worksheet 1

Before you do this page, do an Expanded Numbers: Vertical Way - Practice page with boxes and expanded numbers.

Study this example.

1. 237 x 4 = (200 + 30 + 7) x 4

800	+	120	+	28

237 x 4 = 948

Here is the same problem again without the box. It is important that you remember this way of writing the problem because this is how multiplication is done in algebra. You will use this often in coming years. The only thing that is different is that you must write out each little multiplication that you did in the box. This is how you did it when you were first learning.

2. 237 x 4 = (200 + 30 + 7) x 4

= (200 x 4) + (30 x 4) + (7 x 4)
= 800 + 120 + 28

237 x 4 = 948

3. Your new way vertical or standard way looks like this:

237
x 4
28
120
800
948

There are also many other ways you could break up the 237.
You used to break up numbers this way:
100 + 100 + 10 + 10 + 10 + 7

Why don't you use this way much anymore? AWV
There is less arithmetic to do. It's easier.

67

Expanded Multiplication: Practice - Worksheet 2

Solve this problem all three ways as shown on the last page.

378 x 5 = (300 + 70 + 8) x 5

1,500	350	40

378 x 5 = 1890

378 x 5 = (300 + 70 + 8) x 5

= (300 x 5) + (70 x 5) + (8 x 5)

= 1,500 + 350 + 40

378 x 5 = 1,890

Now solve it again the new way.

378
x 5
40
350
1,500
1,890

Expanded Multiplication: Practice - Worksheet 3

Solve this problem all three ways as shown on Expanded Multiplication: Practice - Worksheet 1

1238 x 4 = (1000 + 200 + 30 + 8) x 4

4,000	800	120	32

1238 x 4 = 4,952

1238 x 4 = (1,000 + 200 + 30 + 8) x 4

= (1,000 x 4) + (200 x 4) + (30 x 4) + (8 x 4)

= 4,000 + 800 + 120 + 32

1238 x 4 = 4,952

Now solve it again the vertical way.

1238
x 4
32
120
800
4,000
4,952

Practice: Three and Four Digit - Worksheet 1

See how fast you can solve these problems.

Show all the partial products.

Example:

422
X 3
6
60
1200

349
X5
45
200
1,500
1,745

641
X6
6
240
3600
3,846

506
X7
3,542

470
X8
3,760

865
X2
1,730

494
X9
4,446

197
X3
591

427
X7
2,989

Practice: Three and Four Digit - Worksheet 2

2,452
X 3
6
150
1,200
6,000
7,356

3,247
X 5
35
200
1,000
15,000
16,235

2,468
X 6
14,808

2,832
X 7
19,824

3,675
X 8
29,400

8,765
X 2
17,530

2,394
X 9
26,546

1,697
X 4
6,788

1,357
X 7
9,499

Short Notation

In the long way, you multiply then add. This is also true in the short way
Now you must multiply the 3 x 4, then add that 1.
3 x 4 = 12 plus 1 = 13
You do not need to be concerned about the place value.
But just to check your understanding, in the short way you multiplied (3 x 4) + 1 = 13. This 13 is actually what number? 13 tens

13 ones
13 tens
13 hundreds

```
 1 1
 2 4 5
   X 3
 7 3 5
```

Finish this problem. Why is there a "1" above the 2? Regrouping 100 from 120 (40 x 3)

How much is that "1" worth? 100

Show You Know

1. Explain how you would find the answer to this problem: AWV
   400 x 7 = 2,800
   Add 400 seven times
   or Multiply 4x7 = 28 and add two zeros
   400x7 = (4x100)x7 = (7x4)x100 = 28x100

2. Explain how to do a long multiplication problem such as this one 325 x 4.
   What are the procedures that you use over and over again? Add and multiply.
   Break up the 325 into parts and multiply each part by 4. Then add it all up.
   Example: 325 = (300+20+5)x5 = (300x5)+(20x5)+(5x5)
   1,625 = 1500 + 100 + 25

3. Solve this problem the long way and the short way.

```
 1439
 x   5
   45
  150
 2000
 5000   7,195
```

```
 1439
 x   5
 7,195
```

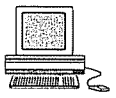
4. What does it mean when we say that multiplication is "distributive"?
   Large numbers are broken up into smaller numbers and the smaller numbers are multiplied by one number. This number is distributed, or spread out, to all the smaller problems.

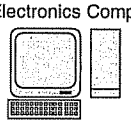
Real World Math - Worksheet 1

You are in business selling older (therefore inexpensive) computers and printers. What will you call your business?

You buy your merchandise (your stuff) from the 21st Century Electronics Company. You then sell the computers to families. You sell the equipment for more than you paid 21st Century Electronics for it. The difference between what you bought it for and what you sell it for is called your profit. Your profit is what buys everything your family needs to buy.

Here is the price list for 21st Century Electronics Company


iMac $700


PC $500


Printer 1 $400


Printer 2 $150

Here is the order you sent into to 21st Century Electronics.

21st. Century Electronics Company Order Form.
Fill in the blanks.

Quantity	Item	Unit Price	Total Price
3	iMac	$ 700	$ 2,100
6	PC	$ 500	$ 3,000
4	Printer 1	$ 400	$ 1,600
2	Printer 2	$ 150	$ 300
		Total Order Cost	$ 7,000

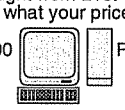
Practice

34 x 7 = 238	39 x 5 = 195	79 x 3 = 237	55 x 8 = 440
64 x 9 = 576	29 x 8 = 232	88 x 3 = 264	39 x 7 = 273
85 x 7 = 595	32 x 6 = 192	79 x 4 = 316	45 x 9 = 405
93 x 2 = 186	138 x 8 = 1,104	335 x 7 = 2,345	186 x 9 = 1,674
275 x 4 = 1,110	358 x 8 = 2,864	128 x 7 = 896	538 x 9 = 4,842
167 x 8 = 1,336	234 x 3 = 702	845 x 7 = 5,915	236 x 9 = 2,124
2,375 x 4 = 9,500	3,583 x 8 = 28,664	2,284 x 7 = 15,988	1,385 x 9 = 12,465

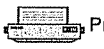
Real World Math - Worksheet 2

You took all the stuff you bought from 21st Century Electronics down to the local flea market. This is what your price list looked like.


iMac $900


PC $600


Printer 1 $550


Printer 2 $200

You sold all of the stuff you brought.

How much money did you take in that day? Show all your work.
You may use a calculator.

3 x 900 = 2,700
6 x 600 = 3,600
4 x 550 = 2,200
2 x 400 = 800
$9,300

How much is your profit? Show your work. To get profit, subtract what you paid for the computers from what you received when you sold the them.

$ 9,300
− 7,000
$ 2,300

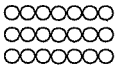
This is the amount of money you made that day. Was this a good day? If you worked every Saturday of the year and averaged this much profit, how much would you make? $119,600 (You may use a calculator.)
$2,300 x 52 = $119,600

Make your own problem.

Pre-Assessment - Part 1

1. What multiplication problem is shown in this drawing?
   4 x 3 = 12

2. Draw a picture like the one above for 7 x 3 = 21



3. Write this multiplication problem as an addition problem: 23 x 4.

```
 23
 23
 23
+23
 92
```

4. Skip count by fours from 0 to 40. 0 4 8 12 16 20
   24 28 32 36 40

5. Solve these problems.

a.	b.	c.
6 x 4 = 24	5 x 5 = 25	9 x 3 = 27
d.	**e.**	**f.**
7 x 6 = 42	8 x 8 = 64	9 x 6 = 54

6. Write all multiplication problems that equal 24.
   You may use Cuisenaire Rods and a meter stick if you like.
   Hint: There are more than two.

1 x 24 = 24

2 x 12 = 24

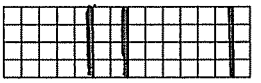
3 x 8 = 24

4 x 6 = 24

1

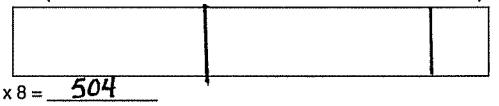
Pre-Assessment - Part 2

1. Fill in the missing numbers.
   a. 14 x 4 = 56
   b. 14 x 4 = (5 + 2 + 6 + 1) x 4



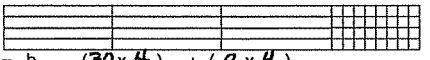
   c. (5 x 4) + (2 x 4) + (6 x 4) (1 x 4)
   e. 56 = d. 20 + 8 + 24 + 4

2. Show how you would break up the 63. Label the partial products in the box. Show the final product below the box on the line. **Example:**
   63 x 8 = (24 + 32 + 7) x 8



   63 x 8 = 504

3. Break up the 38 into tens and ones; fill in the missing numbers.
   a. 38 x 4 = (30 + 8) x 4



   = b. (30 x 4) + (8 x 4)
   d. 152 = c. 120 + 32

4. In problem 3, what number is being distributed? 4

5. Solve these problems the Long way and the Short way.

a. Long way	b. Short way
1,439 x 5 = 7,195	1,439 x 5 = 7,195

   1,439 x 5 = 45 + 150 + 2,000 + 5,000 = 7,195

6. Solve. a. 138 x 6 = 828 b. 2,375 x 4 = 9,500

2

Post-Assessment is the same as Pre-Assessment - Part 2

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Parent/Teacher Guide

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