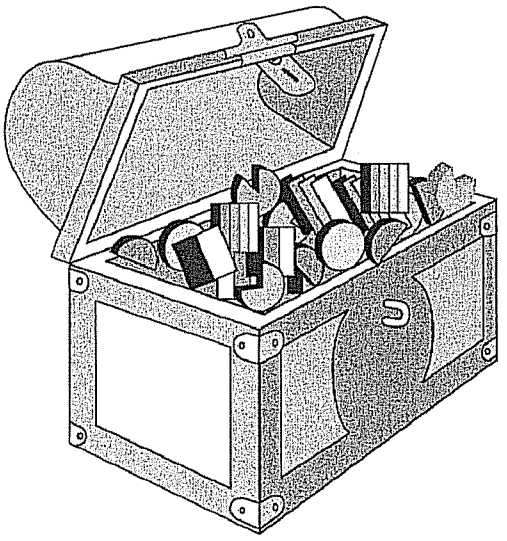
Patterns in Arithmetic

Fractions Tool Chest PDF

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions Tool Chest PDF

Contents

Thank you, Todd Martin, for keeping the ancient Mac computers functioning for the many years it took to complete this math series.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karan Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copyediting was done by Jacqueline Logue.

Many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker. Cover illustration by Vic Mackenzie.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions Tool Chest PDF
Parent/Teacher Guide

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Foreword

To the reader,
I am delighted to write a foreword to this lovely book on mathematics. The work described here reflects a more general Farm School approach, but thinking about how people come to like and be good at mathematics played an important part in developing that approach.

Years ago I was a young professor at UC Irvine, and although I had long been fascinated by the act of learning, this was my first teaching job. Among other things I taught mathematics. The experience was immensely rewarding but unsettling. I thought of math as beautiful, richly ordered, and fun. Most of my students in those required courses appeared to think of it, at least at first, as arbitrary, impenetrable, incoherent, and dull; some of them found it scary.

A few students did not, cheerfully pushing and pulling at a formula, for instance, and asking: What would happen if this part of the denominator were in the numerator? What would happen if I reversed this and that part? What would happen if I made this piece very large or very small? What would be a simpler form or a more general form of the expression? They engaged in this systematic play for the fun of it, but their reinventing or recasting of the material of mathematics also helped them see why something was the way it was; it helped them understand. In fact, the students I started listening to each seemed to carry with them a kind of 'understanding kit.' They had an expectation that math would make sense; they knew when a particular expression or idea did not yet make sense to them, and when it did; and they had developed skills and stamina for getting from the first state to the second, and the habit of doing so. The math they came to know in this way, they owned.

These happy few were regarded by the others (and by most of my colleagues) as having a peculiar knack. There was no shame in not having it; that was just the luck of the genetic draw. Or did the attitude of the rest of the class toward math have to do with the way they had been educated? Their reports of their pre-college math study matched what I found when I started visiting schools, especially elementary schools, and reading texts of that era: my students had been spending most of their time memorizing calculation recipes and learning to run them more or, often, less well.

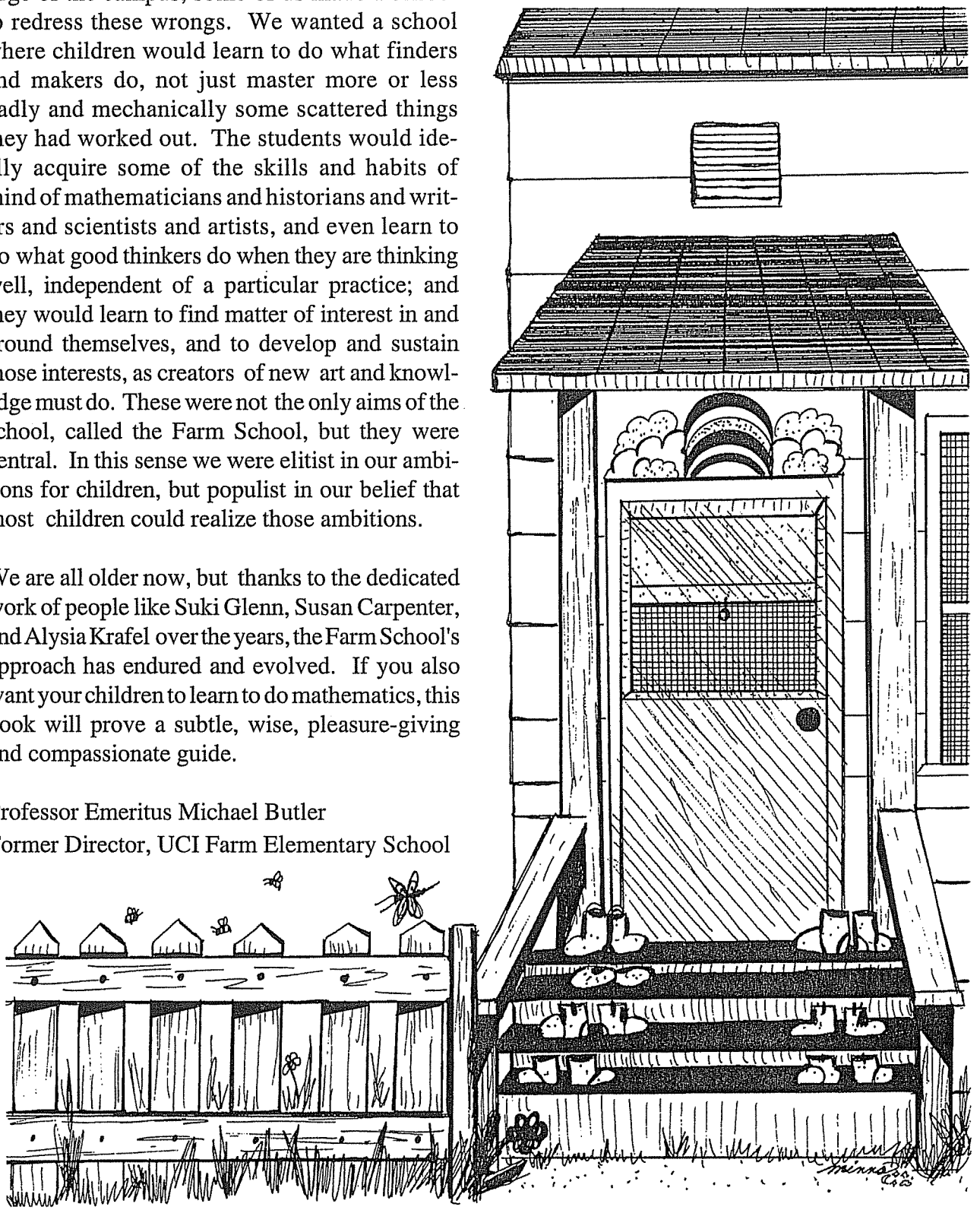
But that wasn't at all what the kind of people who had discovered the math did. Mathematicians look for and find patterns in formal objects, extend them, seek counter examples, figure out why the patterns work, and then, finally, publish an account of one way that they work. The last is the public part, but the rest is what they do. Almost none of my undergraduate students seemed to have had much experience with that. There was an odd disjunction between what practitioners did and what schools asked students to do, a disjunction that was deeper and odder the more you looked at it. It was as though we had plucked the fruit "mathematics" for use in schools, peeled it, and fed students the rind instead of the flesh.

Much the same thing seemed to be true in other areas. What working historians did, for example, or scientists was rarely much like what school children did, so it wasn't surprising that undergraduates found it hard to think that way when asked. Again, it was as though teachers had discovered what it was that delighted practitioners, that drew them to their discipline—and in fact kept it a discipline, a thing that people were willing to spend their lives in, over generations—and having found these sources of delight in practice, schools threw them away and taught the residue.
So we and the times and UCI being young, and

there being some farmhouses available on the edge of the campus, some of us made a school to redress these wrongs. We wanted a school where children would learn to do what finders and makers do, not just master more or less badly and mechanically some scattered things they had worked out. The students would ideally acquire some of the skills and habits of mind of mathematicians and historians and writers and scientists and artists, and even learn to do what good thinkers do when they are thinking well, independent of a particular practice; and they would learn to find matter of interest in and around themselves, and to develop and sustain those interests, as creators of new art and knowledge must do. These were not the only aims of the school, called the Farm School, but they were central. In this sense we were elitist in our ambitions for children, but populist in our belief that most children could realize those ambitions.

We are all older now, but thanks to the dedicated work of people like Suki Glenn, Susan Carpenter, and Alysia Krafel over the years, the Farm School's approach has endured and evolved. If you also want your children to learn to do mathematics, this book will prove a subtle, wise, pleasure-giving and compassionate guide.

Professor Emeritus Michael Butler
Former Director, UCI Farm Elementary School



Introduction

University of California, Irvine, Professor Emeritus Michael Butler: "We wanted a school where children would learn to do what finders and makers do, not just master more or less badly and mechanically some scattered things they had worked out. The students would ideally acquire some of the skills and habits of mind of mathematicians and historians and writers and scientists and artists, and even learn to do what good thinkers do when they are thinking well, independent of a particular practice; and they would learn to find matters of interest in and around themselves, and to develop and sustain those interests, as creators of new art and knowledge must do. These were not the only aims of the school, called the Farm School, but they were central. In this sense we were elitist in our ambitions for children, but populist in our belief that most children could realize those ambitions."

Patterns in Arithmetic is the math program developed at the UCI Farm Elementary School for elementary aged students and now also published with adult learners in mind. At the heart of mathematics is the idea of patterns. Mathematicians are people who find and use patterns in numbers, shapes, and relationships, exploring through these patterns the workings of the world and universe. Mathematicians also explore patterns simply for their beauty and for the great pleasure discovering patterns can give.

This mathematics program aims to teach students and their teachers to think like mathematicians, not just memorize some of the things that mathematicians have found out. Students "do" mathematics from the beginning, so there is an emphasis on inquiry and discovery, on invention, and on learning what genuine understanding feels like and how to achieve it.

Students create their own methods for solving arithmetic problems. They use concrete materials to build and prove their answer. Then they are asked to solve it a different way and to verify that answer. The goal is understanding: knowing what it feels like when one understands (which often is accomplished through "Aha" flashes of insight); knowing what it feels like when one does not understand and what to do then; learning perseverance and how to push through feelings of uncertainty and frustration.

Learners create, routinely look for, and discover patterns in numbers, shapes, the sky, the world. They invent problems for themselves and others, estimate, predict, and look for answers that make sense; they develop heuristics (practices that help them solve problems, e.g., draw a picture), and look for relationships (the more the more).

Ideally they develop the meta-skills of a good mathematician: *the 'understanding' loop*: They know when they understand and when they don't, and they keep at it until they do; *the 'conjecture and test' loop:* They sense when something can be generalized and press to the limit of their ability to do so.

The Parent/Teacher Guide scripts the lesson to create this learning interaction. Ultimately, the learner will ask herself these thinking questions.

The following process is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept:

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract, symbolic), which links the concrete to the pictorial to the abstract.
4. Practice to fluency.
5. Practice for speed.

Free Exploration, or How Not to Be a Nag

by Alysia Krafel

I recall an incident years ago when my six-year-old nephew was learning to set the table. Dinner was almost ready and there he was popping his fists down onto fork tines to flip them into the air. The stage was set for an unpleasant confrontation. I see now that the problem is one of time frames. We needed him to get the job done; he needed to explore. When we press a child to use a tool seriously and with skill before we have provided fiddle time, we set him up to be seen as a behavior problem and ourselves to be seen as nags. If a child can do his necessary playing and exploring with materials when he is not under pressure or expectations, before he needs to use them as we wish him to use them, not only will he learn the new skill faster and with more understanding, he will learn it with more delight (or at least with less complaining). The first association will be of joy and competence rather than frustration and forced anxiety.

Learning to use math manipulatives is like learning to use other tools. An adult will look at a screwdriver and use it to drive screws. A child will roll it on the floor, listening intently as the grooves in the plastic handle thump. She will roll it over her tongue, poke it into cracks, bang it on the table, spin it, or balance it on her finger. When finally satisfied with the investigation, she will watch you screw in the screws and then try it herself. And so it is with all of our doodads.

Play, unhindered exploration, is very important to children. At the Farm School, we came to realize that this self-directed learning style was not an impediment but a boon to our aims. We found that children who had enough time to fiddle with the plexiglass tiles came to their first fractions lessons knowing that the smaller the pieces there were, the more of them they had; that you could cover the black ($1/2$) tile[1] with the yellow ($1/4$) tile or orange ($1/6$) pieces, but you couldn't with the clear ones ($1/3$). When they began to work with the formal fractions lessons, they brought into action all that their senses already knew about these tiles. As soon as the labels "halves" and "fourths" were attached to the colored tiles, they knew that $1/2 = 2/4$. They found the patterns very quickly and mastered the materials faster than the previous children who had not been allowed fiddle time.

The problem the teachers at the Farm School had encountered was that we wanted to teach x amount of material in a certain time frame. When the children needed three weeks of fiddle time with the fraction tiles, we experienced that as a delay, a delay that made us nervous. We just had to teach and have the children master fractions by the end of the school term. The solution was to introduce the materials long, long before (in many cases years before) we intended to use them as teaching tools. After ample free exploration time, not only did the children not resist the use of the materials in the specific way that the formal lessons demanded, they eagerly attended to the new way to interact with the now familiar materials. The new work was for them a continuation of the old play. The experience did not feel all that different to them. Since the struggle between teacher and child had been eased, the adversarial relationship became a co-worker relationship. The instruction then proceeded easily. The children and the teachers learned together to truly understand the world of numbers and enjoyed doing it.

My life suddenly changed when I had a baby. When giving her a bath one day, I had the idea to give her as a toy one thing she would later use as a tool, a cup. In the tub, she could pour, drink (or sputter), spill, bang, and splash to her heart's content without undoing whatever housework managed to get done. She would talk into the cup and put it on her head. Doing these things at the table with a cup filled with milk would have caused an uproar just as fiddling with colored tiles during a fractions lesson did. She had had the cup as a toy many months before we ever put milk into it for

[1] As used in *Fraction Tiles* by Lee Jenkins and Peggy McLean

her to drink at the table. Interestingly enough, when given milk in the cup at the table, she reacted in the same way as the children in the fractions lesson did. Since pouring liquid from the cup was old hat and drinking milk from it like Mommy and Daddy was new, she tried very hard to do it right because that's what she was ready to do, that was the new fun. She wouldn't pour milk out at the table and say "oh no" when she spilled it. Later, when she showed an interest, spoons and bowls were added to the toy shelf and to the bath set. While in the tub recently, she filled her bowl with water, bent down and began to blow bubbles into it. She dumped out the water and put the bowl on her head and said, "Hat." I thought to myself, "I'm glad that was warm water on her naked body instead of hot tomato soup on her white sweater!"

An unintended side effect of all of this was that she became competent with a cup at 14 months and with a spoon and bowl by 16.

Having done all of that, I was surprised recently to find myself remaking that same mistake of not allowing exploration as a parent as I had as a beginning teacher. When my daughter put eggs and Cheerios in her cup of milk I said, "No put food in cup! No put eggs in milk!" I was expecting her to be like me, to use tools as tools, food as food, instead of tools and food as toys. Resisting my constant scolding, she persisted in this behavior. Obviously, she needed to explore how things, like scrambled eggs and Cherrios, would interact with milk in her cup. My choice was either to press her to "act properly at the table," triggering tears and tantrums, or to allow her to explore with the knowledge (or at least great hope) that when she was satisfied, she would stop. This is always a difficult decision for me to make. One doesn't want to be so permissive as to develop a totally undisciplined child. On the other hand, one doesn't want to be a repressive nag either. Since food couldn't be a toy anywhere else but at the table, I gave in on unconventional mixing, squashing food through fingers and food painting but drew the line on throwing. So with a certain amount of plaintive sighing, I watched her put my lovingly prepared, warm eggs into her milk, got out the dish towels, and added them to her toy box. To my delight, she ate the cold, milky eggs out of her cup with the spoon without as much mess as one would expect! My husband realized that the current fascination was food and liquid together. Now we put the liquid in the bowl with the food to begin with, and everyone is happy. She certainly is not like me.

I wonder how many things children must learn, like setting the table, feeding the cats, making the beds and so on, would move more easily if we parents allowed the child to play with the new materials long before we or society demanded that they use them properly and with skill.

Farm School Math Notes by Michael Butler

A. Some things students and teacher should be doing routinely:

1. *Inventing problems*: For practice and self-teaching. For other people, including the teacher. As puzzles; to give and get pleasure. To show understanding of a principle. To link math to the world. To stretch creative power. To develop mathematical taste.

2. *Finding patterns:* In numbers, shapes, etc. Find the rule; 'black box.' Math or formal patterns in worldly stuff--motion, people, plants, cars, stars. 'Getting' a puzzle.

3. *Giving plausible answers and why:* Estimation/prediction/plausible guessing. Error bounds. Recognizing that some estimates are better than others. Iteration to improve estimates. Quickly inventing problems whose answers are 'about' such-and-such--e.g., between m and n. Believing that answers ought to make sense.

4. *Learning about and using heuristics:* Make a picture. Bookkeeping and, in general, 'cases.' Reduction to absurdity. If then. Plausible reasoning. Find a solution for simpler numbers, for a simpler case. Get <u>some</u> result; change it. What's a problem like this? What have I got so far?

5. *Learning about and using kinds of relations:* The more the more, the more the much more, etc. Plots; worldly cases. 'Variable' thinking. 'Control' -- 'other things being equal.' Refining explanations ['true for boys but not girls'] in terms of variables.

6. *Exploring the big ideas:* Scale. Symmetry. Congruence. Ratio. Equivalence. Measurement. Teacher alertness to these in anything.

B. Meta-skills for good mathematicians and students of mathematics:

--the 'understanding' loop: They know when they understand, and when they don't, and they keep at it until they do.

--the 'conjecture and test' loop: They sense when something can be generalized, and press to the limit of their ability to do so.

That children can do these things is one big test of the teacher's success.

C. The other is:

Children <u>like</u> working with numbers, 'thinking stuff' --as shown by their spontaneously choosing to do math and thinking stuff, visibly enjoying it, being absorbed in it, and not saying, "I don't want to, this is icky," etc.

They <u>can</u> do it.

They <u>do</u> do it.

Farm School Math Notes: Routine Questions, Challenges, and Responses

1. *Inventing problems:* Make your own.
   Invent a problem like this for me, for your friend, and for a younger child.
   Find a story in your life that is like this problem.

2. *Finding patterns:* What patterns do you see?
   What will happen next?
   What do you notice about_____? (plants, days, people, cars, playing, the sky)
   What is the same?
   What is different?

3. *Estimating:* Guess the answer.
   What do you predict will happen?
   The answer will be about how big?
   What is an impossible answer to this?
   In what ball park will the answer be?

4. *Heuristics:* (Methods of discovering answers.) How can you show that?
   Draw a picture of it.
   How did you get that answer?
   What part of it can you do?
   What problems like this are easier (or harder)?
   Solve it another way.
   Build it.
   What gave you the clue?

5. *Relationships:* Is there a relationship here? For example: The more the words, the more the time it takes to read the writing.
   The smaller the number, the fewer the combinations.
   Is the relationship true for everything?
   What part of this is in this?

6. *Big Ideas:* How can we measure it? Is that design symmetrical?
   Can this pattern be repeated bigger?

   --the 'understanding' loop: Build that answer.
   Prove that answer.
   How do you know it is right?
   What gives you the clue?
   Challenge both correct and incorrect answers.
   Create more problems using your methods.

   --the 'conjecture and test' loop: What other problems work like this?
   When won't it work?

Teaching Techniques

Expect math to make sense. Look for sources of enjoyment.

Treat yourself and student as a mathematician. Mathematicians make up problems for themselves and others. Students who do this increase their control over the learning process, reinforce and show understanding, and strengthen their image of themselves as mathematicians.

Use two or more manipulatives for the concrete stages of concept development. It will increase the student's flexibility in thinking. Make many copies of the blank practice pages so you can gradually increase the difficulty of the problems and still have extra for periodical review. Cover pages with contact paper or put them in clear sheet protectors to make them reusable.

Begin each lesson with a warm-up and review. Always end the lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions such as, **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** when a student is correct. This is important to do often. A student needs to rely on the self for knowledge ("Am I right?"). Confidence in knowing if a student is right or wrong or at least close, must come from within the student, not from an external source. By asking the student if he or she is right even when right will make the student be confident and learn how to self-check.

Be sure the student is the one to solve the problem. Avoid giving too much help or direction. Time constraints during a lesson or impatience can lead a teacher to push too hard and show the student how to solve a problem the student could have solved without interference.

Explore other ways of solving a problem. Ask, who has another idea over and over again. Mathematicians test each idea several ways to see if it will work or not. Encourage second and third thoughts.

Listen to each student explain how he or she solved a problem. It reinforces auditory learners and lets all students solidify concepts. Persist, talk, and think about what the student has done until you understand it.

Give some unsolvable problems (e.g., break up 4 six different ways). Allow a student to realize no solution is a possibility, a limit, and important information. It stretches the student's mind and teaches what failure is not.

Challenge a student who appears bored. It could be working with higher numbers, recording answers for the group, or making up problems for all to solve.

Math Journal

It is interesting to keep a math journal of work and discoveries. Have students get in the habit of recording their thoughts, equations, and methods for solving problems.

Math Journal

Materials A blank lined-paper journal–this can be a hardbound composition book, a colored report folder with lined paper in it, or a loose-leaf folder in which additional pages can be added as needed. Use wide-lined paper for this age group.

Post-it® Notes for Tabs–these are optional but very helpful to mark journal sections such as thoughts, discoveries, vocabulary glossary, algorithms, and formulas for easy reference.

Computer/Internet (optional, but wonderful)–amathsdictionaryforkids.com. This site has a complete, illustrated, interactive glossary of mathematical terms.

Set Up Set up the Math Journal. Number the first fifty pages. Set aside the first three pages for a table of contents. Label the opening page as the Table of Contents. The first entry will be Vocabulary Glossary pages 4 - 30.
On page 4 label the top of the page Vocabulary Glossary A for all of the vocabulary words starting with the letter A. Label subsequent pages Glossary B on page 5, Glossary C on page 6, and so on to Z.

Place a labeled tab on the first page of the Glossary.

Students record thoughts, methods of solving problems, definitions, vocabulary, and formulas they discover. Many lessons ask the student to record a short answer or an essay question in the journal. It is useful for the student to write a first draft of an answer in the workbook. Then discuss it, refine the explanation using the glossary to substitute mathematical vocabulary for vague wording. Then have her copy the edited version of her answer into the Math Journal.

Although pictures will be useful to illustrate many concepts, discourage doodling in the Math Journal.

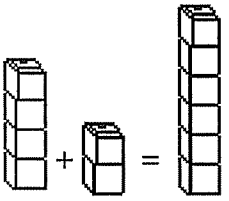
An example of a journal entry:
Area: The space inside the boundaries of a flat figure.
Area of a rectangle: The formula is length x width (l x w).

Note Warning Note on Online Resources: There are thousands of sites online that provide worksheets, lessons, and games for kids to learn math. Take care when you use 'free' sites. Many of them are free because they are installing cookies in your computer to collect data on your student's computer use for marketing purposes. This data is used for pop-up ads and can be sold to third parties. If you do not want this collection to occur, avoid free game sites especially. If your child is using free applications on smartphones and the like, the GPS data on the physical location of the user is being collected by third parties. The sites that require subscription are much less likely to monetize your data in this way. Use online resources with care. There is a mountain of good stuff online, but you must be picky and read the privacy notices. If there are no privacy notices, that should act as a red flag.

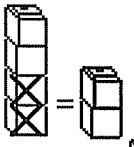
Arithmetic Conventions

It is essential that learners build models of problems, draw pictures to prove their own answers, show understanding, and communicate their ideas to others. This program adopts the following conventions. The first number of an equation tells the amount or number of blocks to start with. The second number is seen as part of the operator, with the sign, and tells what to do next. The order of the words and the action of the hands correspond. Each operation–addition, subtraction, multiplication, and division–uses the same convention throughout the entire *Patterns in Arithmetic* series, Books 1 and 2 and Booklets.

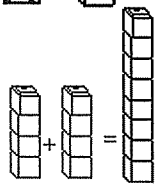
4 + 2 = ? means start with four and add two more.



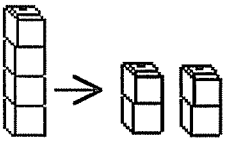
4 - 2 = ? means start with four and take away two.



4 x 2 = ? means start with four and add it two times (4 + 4 = 8).



4 ÷ 2 = ? means start with four and divide it into groups of two (or make two groups).



Make Your Own

Make your own problems should be done routinely. The student acting as a mathematician invents problems, which strengthens her control over the learning process. This reveals to the teacher her understanding of the concepts, how her mind works, and her current level of competency and confidence. It is a creative act for an empowered learner, and through its use powerful habits of mind will grow.

Time Span

Time span is a general guide for how long a lesson may take or the length of a practice phase. Since each individual will vary in her pace based upon age and experience, do not worry about externally suggested time frames or compare one student to another. The time spans given are for an average student at the third-grade level (about eight years old). Adults who are teaching themselves will pace themselves by their understanding, ease, and fluency with each concept.

At the Farm School students invented unique, successful ways to solve problems and discovered that although the standard regrouping algorithm has particular advantages in a base ten system, it is only one way among many. This invention laid a foundation for understanding *all* arithmetic algorithms (addition or subtraction or multiplication or division) because they are all built on the same model, a process with three parts: decomposing a large problem into many small problems, solving each small problem, and adding the results. When students understand this, each new arithmetic procedure has a familiar feeling, because it is seen to be part of the same family of algorithms.

The idea of breaking up difficult problems into smaller ones and then solving the little problems first can be applied to all of life. For example, a difficult piece of music is broken up into smaller parts to be practiced separately, then played all together later. And specifically to *Patterns in Arithmetic* a large third-, fourth-, and fifth-grade set of arithmetic lessons is broken up in a different way: by operations and by fractions and then recombined in booklet form for learners of all ages to do in their own chosen order.

The philosophy of the Farm School is based on the insights, ideas, and teachings of Professor Michael Butler. Over the years the work of many teachers has resulted in a mathematics program that aims to teach students to think like mathematicians.

"Anything children learn from adults can be learned more readily from another child." Welcome to this learning adventure.

Take Delight

The following are some words of wisdom from Alysia Krafel, who helped develop many of the math materials contained in the lessons:

> Take genuine delight in the learning of your student. This is your most powerful tool. There is much that is delightful in good thinking, a new solution, or an insightful comment. There is much danger for us adults when we come to think that we have seen it all, particularly when working on seemingly simple, "childish" problem. Often we are so intent on getting the student to see things our way that we miss the new perspective uttered by the student seeing the world for the first time. Take delight in those new perspectives...they are a gift to you. Often these perspectives will be a simple and obvious yet profound in their truth. Be careful that you don't cast them aside with a "but of course."
>
> Be aware enough to separate a good thought path from the correctness of an answer. A student [in the process of] explaining will often find a good path but follow it poorly. The original idea may be a good one even if it does not produce the correct solution. Instead of saying "No, that is not right," honor the attempt. "That was a good idea; I understand what you think even though it doesn't work here. It was a good try." Or ask questions that will test the hypothesis in other ways. "Are there other cases where this will work? Can you think of problems where it won't work?"
>
> Be aware also that a student in a comfortable learning situation rarely says nonsensical things. Even an answer which may seem absurd to you makes sense to the student. Try to understand the model the student is using, the model that will make sense of what the student says. When you listen carefully to what the student is struggling to articulate, you show that you value his thoughts by your behavior. Your act of valuing and delighting in your student's thoughts will go many miles further than any reprimand you could give. You will, by your praise, cultivate a desire to search and test, and this will strengthen him against the frustration of error. The student will learn that thinking is fun.
>
> If delight is to be your greatest asset, then impatience will be your greatest liability. Impatient teaching is humiliating to the learner. One aspect of impatience is expecting students to give snap answers to questions. The better the question the more time a student will need to mull it over . You must allow the student time to think. Silent times in my teaching have been as long as three to five minutes. Give students time to think.

A question often asked by teachers is, "How do I know what to do next, and in what order do things need to be done?" Use the evaluation section and the sequence guides to determine the next step in the learning process. The guides are for sequencing. They are not a time schedule. The Tests for Understanding help teachers determine when students are ready for the next step. And ultimately let self-motivated students of all ages know what they understand and their next moves.

Patterns in Arithmetic: Fractions Series Overview

Books

Contains:
Addition
Subtraction
Multiplication
Division
General Math
Place Value
Beginning Fractions

Grade 1	Grade 2
Parent/Teacher Guide Book 1	Parent/Teacher Guide Book 2
Student Workbook Book 1	Student Workbook Book 2

Booklets

Fractions Tool Chest - Understanding Fractions
Includes: Philosophy, Farm School Math Notes, Teaching Techniques, Arithmetic Conventions, Fraction Series Overview, Sequence Guide, Prism Fractions Instructions, Materials and Resources, Series Overview for Patterns in Arithmetic

Student Workbook and Parent Teacher/Guide come as a set

Booklet 1 - Basic Concepts

Booklet 2 - Developing Concepts and Beginning Operations

Booklet 3 - Mixed Numbers and Improper Fractions

Booklet 4 - Equivalent Fractions

Booklet 5 - Simplifying Fractions

Booklet 6 - Multiplication

Booklet 7 - Addition and Subtraction of Unlike Fractions

Booklet 8 - Understanding Division

Set of Fraction Booklets 1 - 8

Placement Assessment: in every booklet and available for free on the website: www.patternpress.com
An Answer Key to the Student Workbook is in each Parent/Teacher Guide

Prism Fractions® manipulatives
Prism Fractions Paper Set - squares and circles
Colored acetate set for Booklet 6 - Multiplication
Plastic Prism Fractions Circles Set - 51 pieces

Sequence Guide

The following sequence presents a model from which a teacher can get ideas for designing a plan. Use the Assessment booklets by subject to precisely plan a student's program. Plan a week at a time and revise as needed.

This is a proposed sequence, not a schedule. Each student will move at his or her own pace, taking a year, more or less for some booklets. Booklets 3, 4, and 5 are exceptions. Booklet 3 teaches Mixed Numbers and Improper Fractions, Booklet 4 teaches Equivalent Fractions, and Booklet 5 teaches Simplifying Fractions. These three subjects can be taught in a different order according to a teracher's personal preference. These three booklets take less time than a year.

In addition, adults who want to understand arithmetic can try a strand and proceed independently and quickly.

"Use your professional judgment and trust your intuition. If you feel you need to move more quickly, do so. If you feel you need to move more slowly, do that. *Always remember*: our focus is the children. Our goal is to meet those children's needs. The focus is *never* on "getting through" these lessons or keeping to the sequence in spite of children's needs."
Mary Baratta-Lorton, *Mathematics Their Way*

Lessons progress developmentally from top to bottom according to the order in which they may be taught. Most lessons depend upon concepts and skills learned in earlier lessons.

Year-round schools, homeschoolers, and adults will expand or contract the schedule as needed.

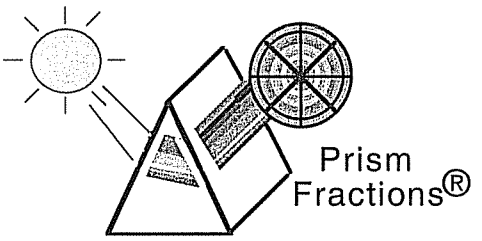
Begin year

Months ↓	Grades →			
	Title and location of lesson and/or worksheet			

	Fractions: Booklet 1	Fractions: Booklet 2	Fractions: Booklets 3 - 7	Fractions: Booklet 8	
Month	Grade 3	Grade 4	Grade 5	Grade 6	Month
1	*Pre-Assessment* *Free Exploration* *In the Kitchen* *Fraction Circles - make designs*	*Pre-Assessment - Part 1* *Free Exploration:* *Pattern Blocks and Fraction Circles - make designs* *Flower Power*	*Free Exploration:* *Prism Fractions Circles* *Pattern Blocks* *Cuisenaire Rods*	*Free Exploration of fraction manipulatives*	1
2	*My Fractions Book (MFB)* *What Fraction Is This?*	*Pattern Block Fractions: Review* *Predator Game* *Recording: Prism Fractions*	Fractions: Booklet 3 *Addition of Mixed Numbers with Like Denominators* *Mixed Numbers Meet Subtraction*	*Review:* *Equivalent Fractions* *Simplifying Fractions* *Multiplication*	2
3	*Fractions Guessing Game* *Numerators Greater Than One, Shrinking Circles, Whole Numbers as Fractions, Fraction Card Game*	*Fraction Card Game* *Greater Than, Less Than, or Equal To*	*Subtraction of Mixed Numbers with Regrouping* *Post-Assessment*	*Review:* *Addition and Subtraction of Unlike Fractions*	3
4	*Greater Than and Less Than* *Recording Fractions: Prism Set* *Sorting Fractions* *Spelling* *Pattern Block Games*	*Changing Wholes*	Pattern Block art and design - students create on their own	Fractions: Booklet 8 *Pre-Assessment - Part 1* *Understanding Division* at the student's pace for the rest of the year	4
5	*MFB: Pattern Blocks* *Pattern Block Fractions* *Changing Wholes: Beginning*	*Addition and Subtraction of Like Fractions*	Teachers may vary the teaching order of Booklets 3, 4, and 5.		5
6	*Fractional Parts of Sets* *Pattern Block Designs* *Rulers as Number Lines* *MFB: Number Lines*	*Fractions as Ratios* *Parts of Wholes* *Parts of Wholes as Multiplication of Fractions* *Post-Assessment*	Fractions: Booklet 4 - Equivalent Fractions		6
7	*Whole Numbers as Fractions, Numerators Greater Than 1: Number Lines* *MFB: Equivalence*	Teachers may vary the teaching order of Booklets 3, 4, and 5.	Fractions: Booklet 5 - Simplifying Fractions		7
8	*MFB: Equivalence - Number Lines* *Equivalence: Manipulative* *Equivalence: Recording*	Fractions: Booklet 3 - Mixed Numbers and Improper Fractions, *Pre-Assessment - Part 1*, *Mixed Numbers to Improper Fractions*	Fractions: Booklet 6 - Multiplication		8
9	*Word Problems* *Post-Assessment*	*Improper Fractions to Mixed Numbers*	Fractions: Booklet 7 - Addition and Subtraction of Unlike Fractions	*Post-Assessment*	9

Italic = lesson title, Booklet Title, MFB = My Fractions Book

Prism Fractions Instructions

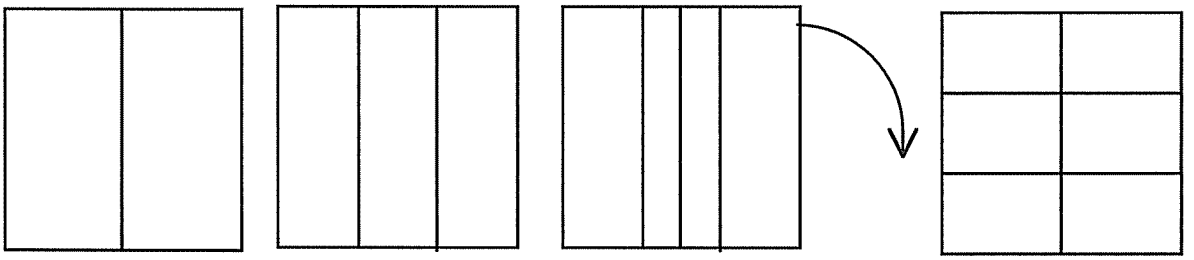


Prism Fractions® Acetate Set
Cut the fraction squares along the thin lines.

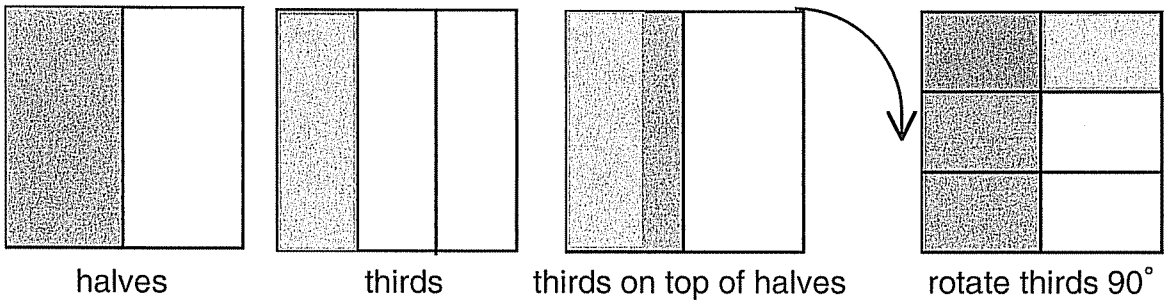
Prism Fractions Paper Set
Carefully cut the fraction squares out along the heavy lines but do not cut the squares into individual units. Fold the paper along the heavy inside lines to make smaller fraction units such as 2/3. (It's easier to compare the fractional units with the squares cut exactly the same size.) Use with the clear acetate set for Equivalent Fractions and Understanding Division booklets.

Multiplication of Fractions–use two acetate sets.

Start with the square divided into two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°.



There are six little rectangles after rotating the squares.
Each little rectangle is now one-sixth fraction of the whole.

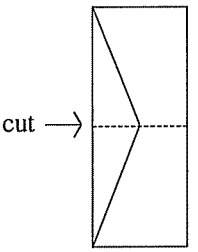


The color at the intersection of the two squares changes color.
The intersection of the colors shows 1/2 of 1/3.

Prism Fractions Storage Folder

Materials

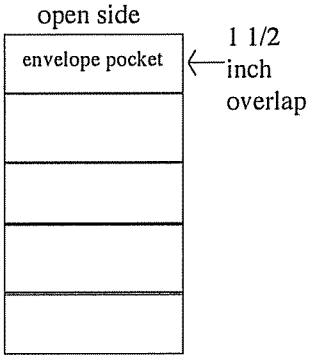
2 file folders
11 plain business envelopes (no. 10 envelopes)
Tape and paper glue
Stapler
Prism Fractions Paper set–squares and circles



Directions for one folder

One folder is needed for squares and one for circles.

Seal envelopes shut and cut five envelopes in half across the short side.
Line up five halves one above the other with about an inch and a half overlap.
Staple the top envelope pocket to the second pocket.
Continue stapling this stack of five pockets.
Repeat these steps with the other five envelope halves.

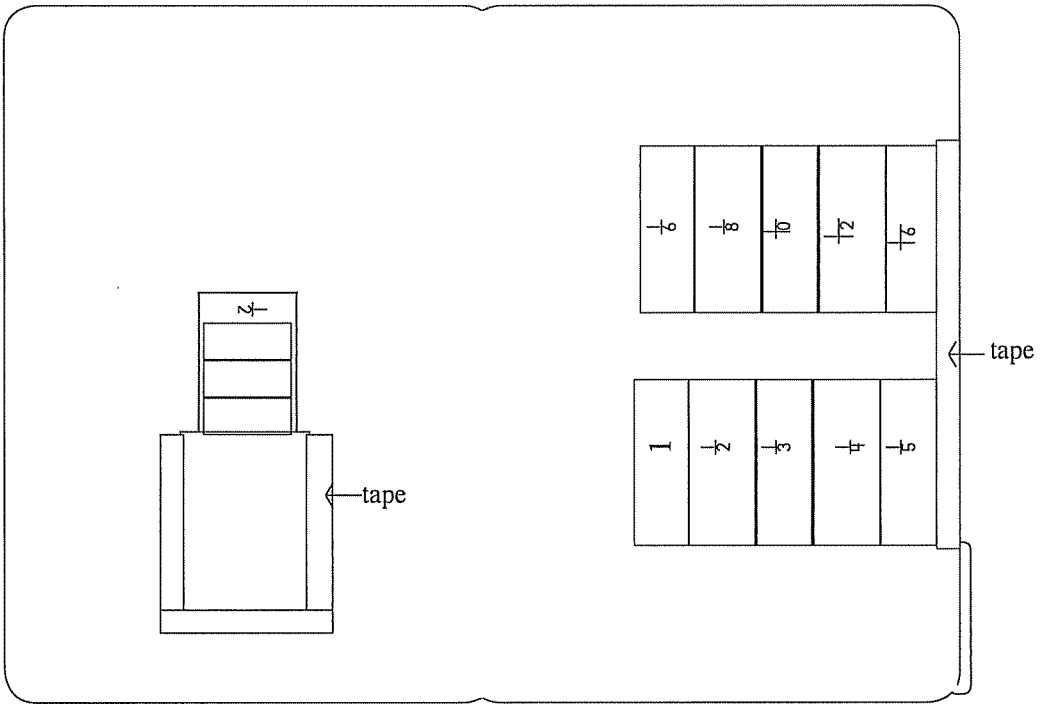


Open the file folder. Place the two sets of stapled envelopes side by side (see below).
Glue or tape in place. Tape across the bottom. Be sure to leave enough space for the paper fractions to slip inside.

Cut the last envelope in half. (Not needed for paper circles.)
Tape it toward the bottom of the empty half of the file folder.

Cut out the Prism Fractions Paper set along the fine lines. A paper cutter helps keep the cuts straight.

Slip all the whole fraction pieces in a top pocket, the halves in next pocket down. Continue placing all the fraction pieces in the pockets in an orderly manner. All the long fraction pieces go in the pocket on left side of the file folder.
Decorate folder.



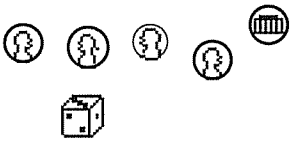
Materials and Resources

Manipulatives

Pattern blocks
Cuisenaire Rods
Prism Fractions® (paper set included)
Colored acetate set for Booklet 6: Multiplication–available from Pattern Press.
Prism Fractions Circles (a plastic set)–available from Pattern Press

To make your own plastic circle set:
Rainbow Circle Fraction® Deluxe can be used with Prism Fractions and follow the same color scheme with the exceptions of the whole and the half pieces. To match Prism Fractions, change the whole to white and the one-half pieces to red. White labels or spray paint can be used for the whole, and red permanent marker or spray paint can be used for the half pieces. Available from ETA-Cuisenaire and Nasco Math.*

Variety of counters
- plastic counters
- pennies, nickels, dimes, and quarters
- die



Recording Materials

Prismacolor colored pencils
soft lead pencil
crayons and/or markers
graph paper
scratch paper

Resources

Books

Fraction Factory (Wright Group)
Patterns in Arithmetic Series (Pattern Press)
Key to Decimals (McGraw-Hill)

Manipulatives Sources

Activity Resources (510) 782-1300 www.activityresources.com
ETA-Cuisenaire (800) 445-5985 www.etacuisenaire.com
Cuisenaire Rods, Pattern blocks, and Unifix Cubes
McGraw-Hill www.mheonline.com
Nasco Math (800) 558-9595 www.enasco.com
Pattern Press (760) 728-3731 www.patternpress.com
School Specialty Publications (800) 845-8149 www.schoolspecialtypublishing.com
Wright Group (800) 523-2371 www.wrightgroup.com

*Rainbow Fractions Deluxe set purchased from ETA-Cuisenaire and Nasco Math need the modifications described above to be used with this program.

Patterns in Arithmetic: Series Overview

Books
(Contain Addition, Subtraction, Multiplication, Division, Place Value, General Math, and Fractions)

Grade 1	Grade 2
Parent/Teacher Guide Book 1	Parent/Teacher Guide Book 2
Student Workbook Book 1	Student Workbook Book 2

Booklets by Subject

Parent/Teacher Guide
Student Workbook

General Organization
Includes:
Teaching Techniques
Arithmetic Convention
Sequence Guides
Lesson Planning
Series Overview
Materials
Resources
Philosophy
Math Journal

General Math
Includes:
Patterns
Pre-algebra
Geometry
Measurement
Place Value
Graphing
Functions
and more

	Grade 3	Grade 4	Grade 5	Grade 6
General Organization	Grade 3	Grades 4 and 5	Grades 4 and 5	
Addition	Booklet 3			
Subtraction	Booklet 3 Base Ten	Booklet 4		
Multiplication	Booklet 1	Booklet 2 Expanded Tables Flip Bk *Booklet 3	Booklet 4	
Division	Booklet 1	Booklet 2 Base Ten	Booklet 3	
General Math	Booklet 3	Booklet 4	Booklet 5 Booklet 6	
Place Value		*Booklet 2	Booklet 3	
Fractions Fractions Tool Chest	Booklet 1	Booklet 2 *Booklet 3	Booklet 4 Booklet 5 Booklet 6 Booklet 7	Booklet 8
Decimals		*Key to Decimals* Book 1	*Key to Decimals* Book 2	*Key to Decimals* Book 3

*These booklets are used in grades 4 and 5.

Patterns in Arithmetic Series Overview

Books - Contains all subjects listed below
Book 1 - Grade 1
Book 2 - Grade 2

General Organization and Philosophy
General Organization - Grade 3
General Organization - Grades 4 and 5
Fractions Tool Chest - Understanding Fractions

Booklets by Subject

Addition

Booklet 3 - Regrouping (carrying) into Four Digits - Grade 3

Subtraction

Booklet 3 - General Principles of Regrouping (borrowing) - Grade 3
Base Ten Subtraction - Chrysalis Charter School - Grade 3
Booklet 4 - Large Numbers and Practice - Grade 4

General Math

Booklet 3 - Place Value, Measurement, and Geometry - Grade 3
Booklet 4 - Patterns and Algebraic Thinking - Grade 4
Booklet 5 - Geometric Formulas, Linear Functions, and Division Relationships - Grade 5
Booklet 6 - Geometry of Circles, Algebraic Properties, and More Functions - Grades 5 and 6

Multiplication

Booklet 1 - Basic Concepts - Grade 3
Booklet 2 - Beginning Long Multiplication and Basics of Distribution - Grade 4
Expanded Tables Flip Book - Grades 3, 4, and 5
Booklet 3 - Properties and Factoring - Grades 4 and 5
Booklet 4 - Working with Large Numbers and Decimals - Grade 5

Division

Booklet 1- Basic Concepts - Grades 3 and 4
Booklet 2 - Breaking Up Division - Grade 4
Base Ten Division - Chrysalis Charter School - Grade 4
Booklet 3 - Working with Double Digit Division - Grade 5

Place Value

Booklet 2 - Rounding Off and Estimating - Grades 3 and 4
Booklet 3 - Into the Millions, Exponential Notation, and Operations Review - Grades 4 and 5

Fractions

Booklet 1 - Basic Concepts - Grade 3
Booklet 2 - Developing Concepts and Beginning Operations - Grade 4
Booklet 3 - Mixed Numbers and Improper Fractions - Grades 4 and 5
Booklet 4 - Equivalent Fractions - Grades 4 and 5
Booklet 5 - Simplifying Fractions - Grades 4 and 5
Booklet 6 - Multiplication - Grades 4 and 5
Booklet 7 - Addition and Subtraction of Unlike Fractions - Grade 5
Booklet 8 - Understanding Division - Grades 5 and 6

Patterns in Arithmetic: Fractions Tool Chest PDF
Parent/Teacher Guide

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