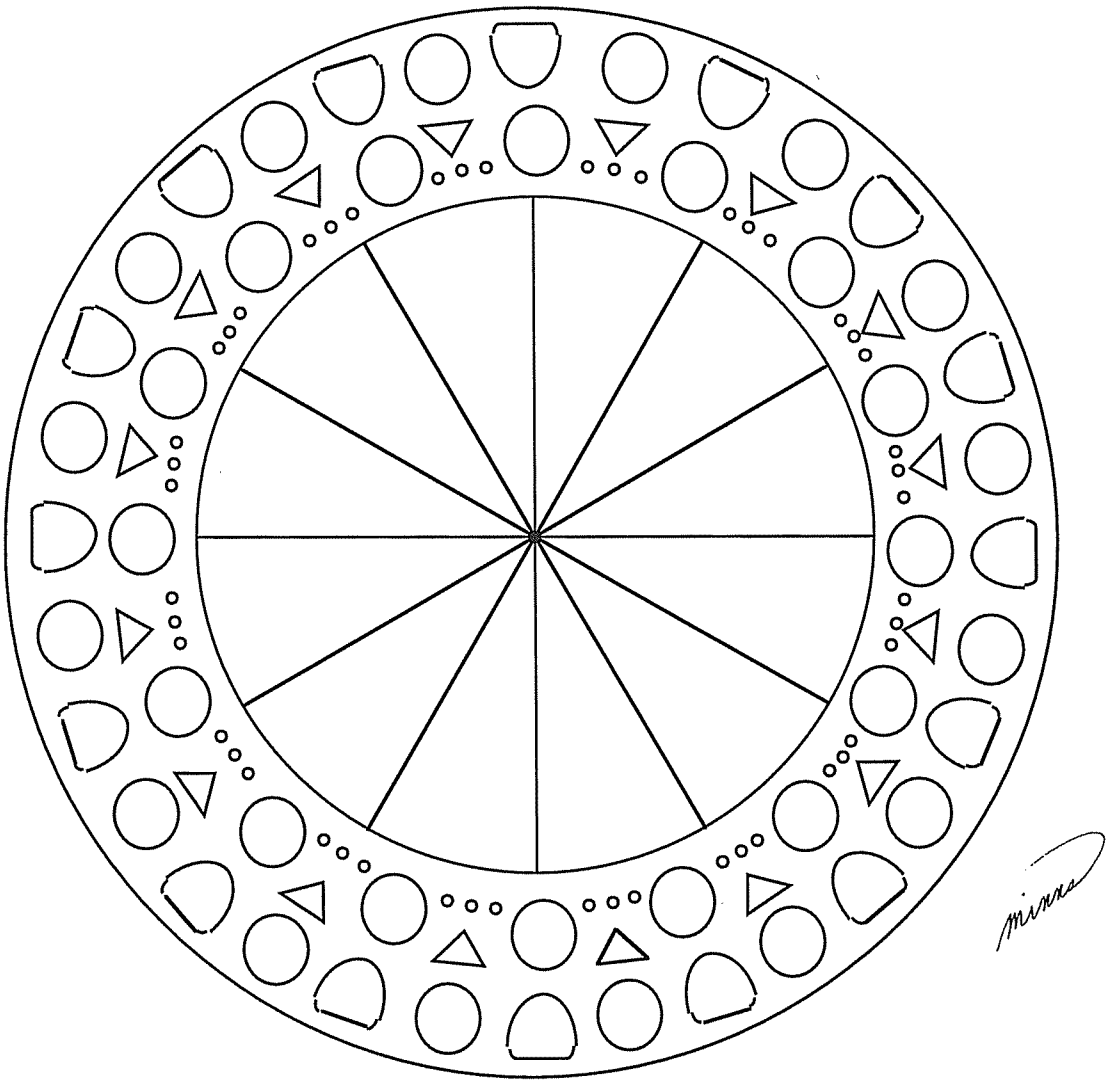
Patterns in Arithmetic

Fractions - Booklet 8 PDF

Understanding Division

Parent/Teacher Guide



By Alysia Krafel

Edited by Susan Carpenter and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 8 PDF - Understanding Division

Contents

This booklet is dedicated to Kelly Delap and Alysia Krafel for their mathematical exploration and discoveries. They paved the way for this method of understanding the division of fractions.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Kaŕan Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 8 PDF
Parent/Teacher Guide

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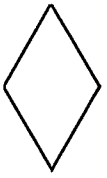
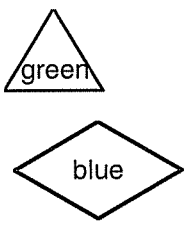
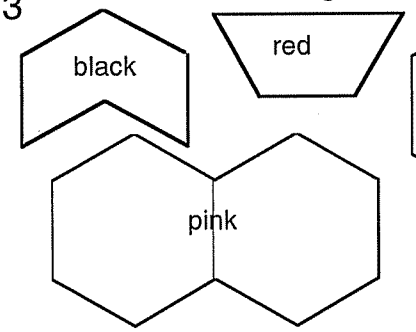
Pre-Assessment - Part 1

Date_____

Put a question mark next to any problem you do not know how to do.

1. If the  is equal to $\frac{2}{3}$, which of these figures is equal to one whole?





_____

2. Change to an improper fraction. a. $3\frac{3}{4} =$ ___ b. $6\frac{7}{8} =$ ___ c. $7\frac{4}{9} =$ ___

3. Change to a mixed number. a. $\frac{17}{5} =$ b. $\frac{29}{4} =$ c. $\frac{58}{8} =$

4. Fill in the missing numbers. a. $\frac{4}{5} = \frac{\quad}{15}$ b. $\frac{\quad}{4} = \frac{15}{20}$ c. $\frac{2}{3} = \frac{18}{\quad}$

5. Simplify. a. $\frac{32}{36} =$ ___ b. $\frac{17}{34} =$ ___ c. $\frac{19}{23} =$ ___ d. $\frac{42}{54} =$ ___

6. Multiply. a. $\frac{1}{3} \times \frac{1}{2} =$ ___ b. $6 \times \frac{3}{4} =$ ___ c. $\frac{4}{5} \times \frac{2}{3} =$ ___

7. Use cancelling to solve. a. $\frac{4}{7} \times \frac{7}{8} =$ ___ b. $\frac{2}{3} \times \frac{15}{22} =$ ___ c. $\frac{8}{3} \times \frac{9}{4} =$ ___

8. Multiply. a. $2\frac{2}{3} \times 2\frac{1}{4} =$ _____ b. $7\frac{1}{2} \times 3\frac{2}{3} =$ _____

9. If 25¢ is one-seventh the cost of a piece of ribbon, how much does the ribbon cost? ____

10. Solve. a. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{2}{3} \\ \hline \end{array}$ b. $\begin{array}{r} \frac{4}{5} \\ +\ \frac{1}{3} \\ \hline \end{array}$ c. $\begin{array}{r} \frac{4}{9} \\ +\ \frac{3}{5} \\ \hline \end{array}$

Pre-Assessment - Part 2 - Worksheet 1 Date_____

Put a question mark next to any problem you do not know how to do.

1. What question does each problem ask?
   a. $12 \div 3$ _____
   b. $6 \div \frac{2}{3}$ _____
   c. $3 \div 4$ _____
   d. Circle the problem whose answer will be less than one.

2. Solve each problem. Make a drawing that clearly proves the answer.
   a. $\frac{1}{2} \div \frac{1}{4} =$ _____
   b. $6 \div \frac{2}{3} =$ _____
   c. $\frac{3}{4} \div \frac{1}{3} =$ _____
   d. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$. Explain where the $\frac{1}{8}$ comes from. Why is it $\frac{1}{8}$?

_____

_____

3. Solve each problem. Make a drawing that clearly proves the answer.
   a. Will the answer to $\frac{1}{4} \div \frac{1}{2}$ be greater than or less than one? _____
   b. How do you know? _____
   c. $\frac{1}{3} \div \frac{1}{2} =$ _____

d. $\frac{1}{2} \div \frac{3}{4} =$ _____

e. $\frac{1}{4} \div \frac{3}{8} =$ _____

4. Write a fraction and its reciprocal. a. _____ b. _____

5. Explain why $12 \div \frac{1}{2} = 12 \times 2$. _____

_____

6. Does the pattern shown in problem 5 work for this problem $\frac{3}{4} \div \frac{1}{2}$? _____

   a. Prove it with a drawing.

   b. Explain or draw why the 'invert and multiply' formula works in this division of fractions problem. _____

_____

_____

7. Solve these problems.

   a. $\frac{3}{4} \div \frac{2}{3} =$ _____ b. $\frac{5}{6} \div \frac{4}{5} =$ _____ c. $\frac{3}{7} \div \frac{7}{12} =$ _____

8. Solve these problems.

   a. $5 \div 2\frac{1}{2} =$ _____ b. $2\frac{2}{3} \div 1\frac{1}{4} =$ _____ c. $6\frac{4}{5} \div 9\frac{1}{2} =$ _____

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the past fraction booklets and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Patterns in Arithmetic: Fractions - Booklets 3 - 7, or proficiency with proper and improper fractions, equivalence and simplification of fractions, common denominators, and multiplication of fractions

Materials

Fractions Booklet 8: Pre-Assessment - Worksheets 1 - 3, pages 1 - 3
Score Sheets
Colored pencils
Patterns Blocks
Fraction manipulative

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. Changing Wholes (Fractions: Booklet 2)
   A. identify that the red block will be equal to the whole if the blue block is designated as $\frac{2}{3}$?

2. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
   A. give the correct answer two of three times? Answers do not have to be simplified to be correct. We want to see if he can perform this operation.

3. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
   A. give the correct answer two of three times? Answers do not have to be simplified to be correct. We want to see if he can perform this operation.

4. Equivalent Fractions (Fractions: Booklet 4)
   A. supply the correct missing number in two of the three items?

5. Simplification of Fractions (Fractions: Booklet 5)
   A. give the correct answer to three of the four problems?

6. Multiplication of Fractions (Fractions: Booklet 6)
   A. give the correct unsimplified answer for two of the three items?
   B. give the correct simplified answer for two of the three items?

7. Cancelling (Fractions: Booklet 6)
   A. supply the correct simplified answer in two of the three problems?
   B. use cancelling to simplify the numbers before he multiplies?

8. Multiplying Mixed Numbers (Fractions: Booklet 6)
   A. supply the correct simplified answer in one of the two problems?
   B. use cancelling to simplify the numbers before he multiplies?

9. Word Problem (Fractions: Booklet 6)
   A. supply the correct answer?

10. Addition of Fractions with Unlike Denominators (Fractions: Booklet 7)
    A. give the correct unsimplified answer two of three times?
    B. give the correct simplified answer two of three times?
    C. use forty-five as the common denominator on item c? If he used ninety, have him practice finding the least common factor of two numbers. This will be important when drawing division of fractions problems.

Booklet Selection Guide based on results of Pre-Assessment: Part 1

All of the skills in this assessment are needed in Division of Fractions. There must be a Yes on all items before proceeding. Any No items must be remediated before beginning this booklet. If the score

is 75% or greater, you can probably begin the Division of Fractions Booklet and remediate needed items concurrently. This is not the case if he receives a No on multiplication of fractions, Items 6, 7, and 8. These items must be remediated before the booklet can be attempted. Begin with Fractions: Booklet 6.

Assessment Criteria for Pre-Assessment: Part 2

All items in this section are taught in Fractions: Booklet 8

Can the student:

1. Understand the question asked by a division problem?
   A. give the answer, 'How many groups of 3 are in 12'?
   B. give the answer, 'How many groups of $\frac{2}{3}$ are in 6'?
   C. give the answer, 'What part of 4 fits into 3'?
   D. circle $3 \div 4$?

2. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the dividend is larger than the divisor?
   A. give the correct answer on two of three problems using any method?
   B. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   C. show some kind of marking in the drawing that clearly indicates the counting of the number of divisor units in the dividend?
   D. use a common denominator to clearly show each fraction?
   E. explain that the remainder of $\frac{1}{12}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$ with a clear drawing to support the answer?

3. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the divisor is larger than the dividend?
   A. state that the quotient to $\frac{1}{4} \div \frac{1}{2}$ will be less than one?
   B. give the answer that it will be less than one because $\frac{1}{2}$ is larger than $\frac{1}{4}$ and thus only part of the $\frac{1}{2}$ will fit into the $\frac{1}{4}$?
   C. give the correct answer to two of three problems in items c, d, and e?
   D. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   E. clearly show the answer in the drawing?

4. Introducing Reciprocals
   A. write a fraction and its reciprocal?

5. Using Multiplication to Solve Division Problems
   A. explain why these two problems give equal answers? 2 Points: These two give equal answers because when you solve $12 \div \frac{1}{2}$, you are asking how many halves fit in twelve. Since there are two halves in each whole, it makes sense that the total number of halves would be gotten by multiplying 2 x 12. 1 Point: Because they give the same answer.

6. Using Multiplication to Solve Division Problems
   A. give the answer 'yes'?
   B. explain or draw a clear picture to show that $\frac{3}{4} \div \frac{1}{2}$ is $1\frac{1}{2}$, which is equal to $\frac{3}{4} \times 2$?

7. Standard Procedure
    A. give the correct answer in two of the three problems?
8. Division with Mixed Numbers
    A. give the correct answer in two of the three problems?

Booklet Selection Guide based on results of Pre-Assessment: Part 2

If he can give correct answers when he calculates Items 2A, 3C, 4A, and 8A but is not able to explain or prove understanding with a clear drawing, then you have some choices:

Move on because he can do division of fractions and leave it at that.
Know that he does not understand the nature of division with fractions.
Or you can have him complete this booklet to investigate why the procedure works the way it does and develop understanding of the physical process that is occurring when fractions are divided. One teacher gave a student a choice of understanding division of fractions and working through this booklet or just being given the recipe for the Standard Procedure. This enlightened student chose to do this booklet. Completing this booklet may take three to four weeks for most students.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Pre-Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Changing Wholes (Fractions: Booklet 2)
   Yes No A. identify that the red block will be equal to the whole?

2. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
   Yes No A. give the correct answer two of three times?

3. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
   Yes No A. give the correct answer two of three times?

4. Equivalent Fractions (Fractions: Booklet 4)
   Yes No A. supply the correct missing number in two of the three items?

5. Simplification of Fractions (Fractions: Booklet 5)
   Yes No A. give the correct answer to three of the four problems?

6. Multiplication of Fractions (Fractions: Booklet 6)
   Yes No A. give the correct unsimplified answer for two of the three items?
   Yes No B. give the correct simplified answer for two of the three items?

7. Cancelling (Fractions: Booklet 6)
   Yes No A. supply the correct simplified answer in two of the three problems?
   Yes No B. use cancelling to simplify the numbers before he multiplies?

8. Multiplying Mixed Numbers (Fractions: Booklet 6)
   Yes No A. give the correct simplified answer in one of the two problems?
   Yes No B. use cancelling to simplify the numbers before he multiplies?

9. Word Problem (Fractions: Booklet 6)
   Yes No A. supply the correct answer?

10. Addition of Fractions with Unlike Denominators (Fractions: Booklet 7)
    Yes No A. give the correct unsimplified answer two of three times?
    Yes No B. give the correct simplified answer two of three times?
    Yes No C. use forty-five as the common denominator on item c?

Items Correct = _____ = _____%
Items Possible = 15

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Understand the question asked by a division problem?
   - Yes No A. give the answer, 'How many groups of 3 are in 12'?
   - Yes No B. give the answer, 'How many groups of $\frac{2}{3}$ are in 6'?
   - Yes No C. give the answer, 'What part of 4 fits into 3'?
   - Yes No D. circle $3 \div 4$?

2. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the dividend is larger than the divisor?
   - Yes No A. give the correct answer on two of three problems using any method?
   - Yes No B. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   - Yes No C. show some kind of marking in the drawing that clearly indicates the counting of the number of divisor units in the dividend?
   - Yes No D. use a common denominator to clearly show each fraction?
   - Yes No E. explain that the remainder of $\frac{1}{12}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$ with a clear drawing to support the answer?

3. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the divisor is larger than the dividend?
   - Yes No A. state that the quotient to $\frac{1}{4} \div \frac{1}{2}$ will be less than one?
   - Yes No B. explain why?
   - Yes No C. give the correct answer to two of three problems in items c, d, and e?
   - Yes No D. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   - Yes No E. clearly show the answer in the drawing?

4. Introducing Reciprocals
   - Yes No A. write a fraction and its reciprocal?

5. Using Multiplication to Solve Division Problems (for points see Assessment Criteria on page 6)
   - 2 1 0 pt. A. explain why these two problems give equal answers?

6. Using Multiplication to Solve Division Problems
   - Yes No A. give the answer yes?
   - Yes No B. explain or draw a clear picture to prove his answer?

7. Standard Procedure
   - Yes No A. give the correct answer in two of the three problems?

8. Division with Mixed Numbers
   - Yes No A. give the correct answer in two of the three problems?

Was he able to score Yes on items 2A, 3C, 4A, and 8A but No on many other items?

Items Correct = _____ = _____%

Items Possible = 21

Post-Assessment Score Sheet

Name _____ Date _____

Can the student:

1. Understand the question asked by a division problem?
   - Yes No A. give the answer, 'How many groups of 3 are in 12'?
   - Yes No B. give the answer, 'How many groups of $\frac{2}{3}$ are in 6'?
   - Yes No C. give the answer, 'What part of 4 fits into 3'?
   - Yes No D. circle $3 \div 4$?

2. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the dividend is larger than the divisor?
   - Yes No A. give the correct answer on two of three problems using any method?
   - Yes No B. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   - Yes No C. show some kind of marking in the drawing that clearly indicates the counting of the number of divisor units in the dividend?
   - Yes No D. use a common denominator to clearly show each fraction?
   - Yes No E. explain that the remainder of $\frac{1}{12}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$ with a clear drawing to support the answer?

3. Solving Problems with Pictures
   Solve a division of fractions problem with a picture where the divisor is larger than the dividend?
   - Yes No A. state that the quotient to $\frac{1}{4} \div \frac{1}{2}$ will be less than one?
   - Yes No B. explain why?
   - Yes No C. give the correct answer to two of three problems in items c, d, and e?
   - Yes No D. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
   - Yes No E. clearly show the answer in the drawing?

4. Introducing Reciprocals
   - Yes No A. write a fraction and its reciprocal?

5. Using Multiplication to Solve Division Problems (for points see Assessment Criteria on page 6)
   - 2 1 0 pt. A. explain why these two problems give equal answers?

6. Using Multiplication to Solve Division Problems
   - Yes No A. give the answer yes?
   - Yes No B. explain or draw a clear picture to prove his answer?

7. Standard Procedure
   - Yes No A. give the correct answer in two of the three problems?

8. Division with Mixed Numbers
   - Yes No A. give the correct answer in two of the three problems?

Was he able to score Yes on items 2A, 3C, 4A, and 8A but No on many other items? See choices on page 7 in Booklet Selection Guide based on results of Assessment: Part 2

Items Correct = _____ = _____%

Items Possible = 21

Introduction to Division of Fractions

A teacher can teach any ten-year-old how to divide fractions by telling her the procedure and then having her practice it for about a, hour. The procedure itself, however, makes no sense to most people, including most adults. The sad thing is not only are they missing out on understanding why 'invert the divisor and multiply' works, but more important, they are missing the chance to develop mathematical power and the delight that comes with it.

And even more sadly, they are learning that their understanding is not important. Just memorize it and confirm your answer with only the authority of an Answer Key.

To understand division of fractions, one must explore the physical process of it. Using fraction manipulatives, the student can explore what it means to solve a problem such as this:

$$\frac{3}{4} \div \frac{2}{3} =$$

What question does this problem ask? What do I need to understand to process this question? First, to approach understanding, a student must understand the basic nature of division itself. In the operations of addition, subtraction, and multiplication, the answer is a number of objects that can be counted. In division this is not the case. In division, the quotient (answer) is always an expression of the relationship between the dividend and the divisor. It is not a quantity, but a relationship. This makes division uniquely challenging to understand. I will repeat that packed sentence.

In division, the quotient (answer) is always an expression of the relationship between the dividend and the divisor. I failed eighth grade math because I could not understand what that sentence meant. So let's look at this a minute.

Dividend ÷ Divisor = Quotient For example consider these three problems:

$8 \div 2 = 4$ $12 \div 3 = 4$ $20 \div 5 = 4$

In all three cases the quotient is four. But the number four is not a quantity. It is four groups of whatever the divisor is. In $8 \div 2 = 4$, the four means four groups of two. In $12 \div 3 = 4$, the four means four groups of three, and in $20 \div 5 = 4$, four groups of five. The relationship between the dividend and the divisor is called a ratio. In the problems above, it is the ratios that are equal, not the quantities of objects in the groups. All three dividends and divisors have a ratio of 4 to 1. In other words, there are four groups of the divisor in each dividend. This is hard to get your head around with whole numbers; it is even more so when working with fractions.

In looking at $8 \div 2 = ?$ or $\frac{3}{4} \div \frac{2}{3} = ?$, the question is the same. That question is how many groups of the divisor, in our first case two, are in the dividend, eight?

The answer is that there are four groups of two in eight. To answer this question, one must think of the divisor, two, as a whole group. We must think of the two as the whole. Mathematicians can define a whole to be whatever they want it to be. This concept is taught as the Changing Wholes. In the second problem, $12 \div 3$, the divisor defines the new whole as a group of three. In $25 \div 5$ the divisor defines one whole as a group of five. The size of the group being made, as defined by the divisor, will always be the new whole. This is easier to understand if you look at the division problems as improper fractions: $\frac{8}{2}, \frac{12}{3}, \frac{20}{5}$. When one looks at the problems as fractions, it is easy to see that the whole is two, three, and

five respectively. It is the denominator of a fraction that tells the size of the whole. The definition of the whole changes depending on what the denominator is.

Now consider this concept with respect to a division of fractions problem. $\frac{3}{4} \div \frac{2}{3} =$

Looking at this division problem as a fraction will make most people's heads spin.

$$\frac{\frac{3}{4}}{\frac{2}{3}}$$

For most, this approach is not immediately understandable. In this fraction, the whole is now defined as $\frac{2}{3}$. It is quite difficult to get your head around the idea that a fraction, $\frac{2}{3}$, can be defined as the whole! Try explaining that to a student!

An easier way to understand it is to ask the same question we ask for any division problem. How many groups of $\frac{2}{3}$ are in $\frac{3}{4}$? $\frac{3}{4} \div \frac{2}{3} =$  =

In order to answer that question, one must think of the $\frac{2}{3}$ as a whole group. The student therefore must have a good grasp of the fact that a whole can be defined as anything, including a fraction. She must also see that since the $\frac{3}{4}$ is larger than the $\frac{2}{3}$, the $\frac{2}{3}$ will fit into the $\frac{3}{4}$ one time with a little left over. Now, even if the procedure of invert the divisor and multiply has been memorized, she is not likely to be able to explain why the answer is more than one, in fact is one and one-eighth! Where in the world did the one-eighth come from?

To build a model of the process of division of fractions and then construct a pattern that will allow someone to calculate an answer and prove it requires patient construction with physical objects and a search for number patterns.

The pursuit of understanding of this concept is fortunately fun for most people. Given the chance to construct understanding of this whole process, most people feel invigorated with lots of 'ahas!' echoing around the classroom.

Teachers, do not be afraid to teach this unit with your students if you do not understand division of fractions. You will both learn it as you go. Trust me on this. That is how I, Alysia Krafel, learned it. These problems feel like puzzles. Have fun!

Changing Wholes: Review

Purpose

The purpose of this lesson is to review the concept of Changing Wholes using three different manipulatives. This concept is critical to the understanding of the division of fractions. The student must understand that a whole can be defined as any quantity since a fractional part is determined by the size of the whole, and its value will change when the whole changes. For example, if the whole is defined as twelve, three equals one-fourth of it. But if the whole is defined as six, three is now equal to one-half of the whole. The amount of stuff in a fraction is determined by the size of the whole.

Prerequisites

Previous lessons on fractions

Materials

Changing Wholes: Review - Worksheets 1 - 5, pages 4 - 8
Cuisenaire Rods
Pattern Blocks
Prism Fractions

Warm Up

If it has been longer than a month since the student worked with the blocks, have him freely explore with them before beginning the lesson.

Lesson

Changing Wholes: Review - Worksheet 1
"How many red blocks fit into the hexagon?" "Two."

"So one red block is what fraction of this whole?" "One-half." Record that. Repeat this question with each of the blocks.

Embedded Assessment

The second figure is twice the area of the first. Pay attention to see if he recognizes that and uses it to find answers for the fractions. If he realizes that the new whole is twice as large as the first, then he may be able to reason that the red trapezoid would have half the value with this whole than it did with the first. In the first whole, the red block had a value of one-half. In the second whole, it will be only one-fourth.

Test for Understanding

Math Journal Question
Explain why the value of the red block is cut in half, from one-half to one-fourth, when the size of the whole is doubled.

Answer: The value of the red block is cut in half when the size of the whole is doubled because the size of the red block did not change. It will take twice as many of them to fill the area of the new whole. The denominator of the fraction is determined by how many blocks it takes to fill the whole. Since the whole is twice as large, there will need to be twice as many red blocks to cover it. Doubling the number of the red blocks in the new whole will cut each red block's fractional value in half.

Have him finish the page on his own. Check his answers with the Answer Key. He

may be able to complete this worksheet correctly without using the blocks. This is okay as long as he is getting the correct answers.

Worksheets

Changing Wholes: Review - Worksheet 2, page 5, using Cuisenaire Rods
Changing Wholes: Review - Worksheets 3 and 4, pages 6 and 7, using Prism Fractions
Changing Wholes: Review - Worksheet 5, page 8 - Division - Relationships

Test for Understanding

1. Watch carefully to see what he does when he confronts the problems on the Changing Wholes: Review - Worksheets 3 and 4, pages 6 and 7. In the second problem, the red rod is defined as the whole, or one, which causes the white piece to be worth two. Hopefully, there is no hesitation at this change from fractional values to whole numbers. This issue becomes pronounced on Changing Wholes: Review - Worksheet 4, page 7.

"This page works with fractions. Where are all these whole numbers coming from?"
He should be able to explain that if a small piece is defined as one, then larger pieces must have values greater than one.

2. Looking at Changing Wholes: Review - Worksheet 5, page 8:

"Which number in a division problem is the whole?" "The divisor."

"Which number changes in a division problem when the value of the whole is changed?" "The quotient."

Division of Fractions into Whole and Mixed Numbers

Purpose

The purpose of this lesson is to begin the development of the ability to use a divisor that is a fraction. Using blocks to build models, the student will investigate division with a fractional divisor as a physical process. How many of those blocks can I fit into this space? All the quotients are whole numbers, and the patterns in the answers are obvious. Many students will see immediately that the multiplication tables are evident in the sequence of answers.

Prerequisites

Patterns in Arithmetic: Division - Booklet 2 or mastery of Single Digit Division from another curriculum, for example, 4,578 ÷ 8. Mastery of division with double digit divisors is not necessary.
Key to Decimals: Booklet 3 is helpful as it demonstrates how the fractional divisors can be converted into different units using the Identity Property (the Mighty One) for ease of calculation. This may be taught concurrently.

Materials

Division of Fractions into Whole and Mixed Numbers - Worksheets 1 - 6, pages 9 - 14
Cuisenaire Rods
Two sets of fraction pieces of any kind for each pair of students

Warm Up

Division of Fractions into Whole and Mixed Numbers - Worksheet 1, page 9
Let the student work down this page and complete the first two sets independently if possible. Require her to write out what patterns she sees.

Math Journal Topics

1. Edit answer to what patterns she sees.
2. Explain why the three times table appears in the answers to the problems in Set 2, on Division of Fractions into Whole and Mixed Numbers - Worksheet 1, page 9. Require her to use the proper vocabulary.

Answer: The definition of one-third is that the whole is divided into three equal pieces. Each time another whole is added, the number of thirds increases by a factor of three (times three) each time. Or, each time another whole is added, three more pieces are added. The pattern shows that to find the total number of pieces in a given number of wholes, you multiply the number of wholes by three.

Enrichment Question: Let x be the number of wholes, and y be the number of thirds. Write the function for this relationship. Answer: $y = 3x$

Lesson Part 1

The lesson begins with Set 3 on Division of Fractions into Whole Numbers - Worksheet 1, Page 9. Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

"When writing a division problem such as 8 ÷ 2 = ___, what question is being asked?" "How many groups of two are in eight."

"The worksheet asks, 'How many fourths are in one?' What division problem is that?" "One divided by one-fourth."

Have her study the next two problems and then continue the pattern to complete Set 3. Move on to the next page.

Note

One dark green rod is defined as the whole, or one. The rectangle on the right that has six long rectangles in it is equal to six wholes. Some students will interpret the large rectangle as a whole as was done in the Changing Wholes exercises. Make sure she understands that the whole, one, is a single dark green rod.

Division of Fractions into Whole and Mixed Numbers - Worksheet 2, page 10
"If the dark green rod is one, then what color rod is equal to one-third?" "The red rod."

"If the dark green rod is one, then what color rod is equal to two-thirds?" "One purple rod or two red rods."

"Record this on the line on the left side of the worksheet."

Critical Concept: She must see the two-thirds in her mind as a single piece or a single group. Many students have difficulty looking at two blocks and thinking of it as one group. They see two groups because they see two pieces. In this exercise, using the purple rod helps students overcome this perceptual problem. She is counting how many purple rods fit into the big rectangle. The attachment of the name of the block, two-thirds, and the size of the rectangle, six wholes, happens after she understands that she is trying to see the divisor in the division problem.

"How many groups of one-third are in six?" It is important that she builds this, even though she can easily use her multiplication pattern. "Eighteen."
"How many groups of two-thirds are in six?" *Wait.* Let her work this out. "Nine."

Test for Understanding/ Math Journal Question

"The first quotient is eighteen; the second one is nine. Explain why it makes sense that the quotient of the second problem is half of the first." "It makes sense because we doubled the size of the divisor, which halves the size of the quotient."

Continue in this way to finish the worksheet. Do not help unless assistance is needed. Check the answers immediately. End the session.

Note

Cuisenaire Rods or fraction pieces can be used for this exercise. If you use the Cuisenaire Rods, use the dark green rod as the whole and the purple rod for two-thirds, or tape two red rods together. If fraction pieces are used, it is important that two

$\frac{1}{3}$ pieces are taped together so she will see the two pieces as a single divisor.

Some students will benefit if the fraction names of the pieces are dropped and just the colors are used. Example: **"How many groups of two red rods fit onto a dark green?"** "One and one-third."
"Build one and one-third. How many groups of one-third are in one and one-third?" "Four."
"So how many groups of two-thirds would there be in that same amount?" "Half as many, or two."

"Show that with the blocks to prove your answer."

Lesson Part 2

Division of Fractions into Whole and Mixed Numbers - Worksheet 3, page 11
The sequence of questions appears on the worksheet. Sit with her while she completes these. Do not help unless assistance is needed. Use the Answer Key to be sure she is doing it correctly.

Some teachers like to work the problems right along with the students. This is particularly important and totally acceptable if the teacher is also constructing understanding of this topic.

Worksheets

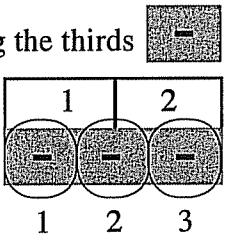
Division of Fractions into Whole and Mixed Numbers - Worksheets 4 and 5, pages 12 and 13

Test for Understanding

Division of Fractions into Whole and Mixed Numbers - Worksheet 6, page 14
Watch her do this page. If you are working with a class, use this in an individual test format.

Note

In the diagram to the right, be sure she sees the dark line grouping the thirds together. $2 \div \frac{2}{3} = 3$
Have her circle each group of $\frac{2}{3}$ with a colored pen to be sure she understands the drawing. Some students will miss this connecting bar and not see the new fraction as a whole.



Before she begins to work, point out the Prediction Lines in the bottom half of the page. Require her to use a predicted answer by calculating it using the patterns she has noticed before. Then require her to confirm her answers with the manipulative. If you are in a one on one situation, watch her do the work and have her explain her predictions to you.

Application

Have her copy the problems into the Math Journal and explain how she is calculating each problem. Then have her use a drawing similar to the one on the worksheet to confirm her answer. If you are working with a group, use this Math Journal problem.

Use the Answer Key to check the work. If you are scoring these, make up a rubric for points on a 1 - 3 scale for the clarity of the explanations and precision of the drawings.

Fractions as Divisors: Whole Number Quotients

Purpose The purpose of this lesson is to model the physical process of division with a fraction as a divisor and to teach students to make representational drawings of that process.

Prerequisites Previous lessons

Materials Fractions as Divisors: Whole Number Quotients - Worksheets 1 - 3, pages 15 - 17
Paper Square Prism Fractions (Do not use fraction circles for this lesson as they are too difficult for students to draw accurately.)
Colored pencils

Warm Up Have the student explore the paper square Prism Fractions for as long as she needs. Give her a chance to figure out how to manipulate and then draw a division with a fractional divisor on her own.

"Pick up the $\frac{1}{2}$ and $\frac{1}{4}$ pieces. How many groups of $\frac{1}{4}$ are in $\frac{1}{2}$?" *Wait.* Give her time to figure out how to model this. "There are two groups of $\frac{1}{4}$ in $\frac{1}{2}$."

"How did you arrive at that answer?" Have her explain her idea to you or to her classmates.

Math Journal **"Pick up the $\frac{1}{2}$ and $\frac{1}{4}$ pieces. How many groups of $\frac{1}{4}$ are in $\frac{1}{2}$? Draw the pieces. Use the colored pencils to help you make a clear drawing. Explain how you decided on your answer."**

Answers will vary. You are looking to see if she counts the number of one-fourth pieces that cover or match up with the one-half piece. Listen to hear if she uses the concept of equivalent fractions to justify her answer, such as one-half equals two-fourths. This would be an indicator of developing understanding.

Examine how she drew the problem. If the drawing does not make sense to you, have her redraw it while she explains her thinking.

Lesson Fractions as Divisors: Whole Number Quotients - Worksheet 1, page 15
Read the top box together. Discuss it to be sure she understands how the problem, $\frac{1}{2} \div \frac{1}{4}$, was drawn this time, and how it compares with the drawing from the Warm Up. Have her color the page to match the color of the pieces she is using.

Continue reading the worksheet together. Use both the pieces and drawings as you work.

Watch her solve problems 1 - 4 at the bottom of the page. Check the answers for problems 5 - 8, page 15 and correct them. Check these answers. Make sure the answers are correct before proceeding.

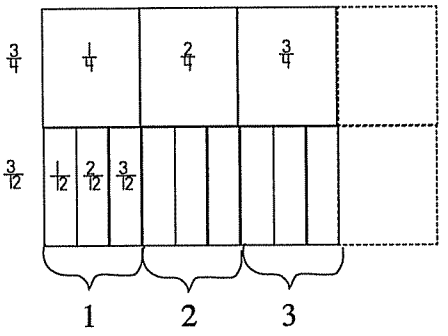
Practice Worksheets

Fractions as Divisors: Whole Number Quotients - Worksheet 2, page 16. Correct it.

Test for Understanding

Use Fractions as Divisors: Whole Number Quotients - Worksheet 3, page 17, as a test. You are looking for correct answers and clear drawings that show the dividend, the divisor, and the quotient. Require her to use the colored pencils to support her in making the drawings clear.

Example: $\frac{3}{4} \div \frac{3}{12} = \underline{3}$



Quotients with Remainders: Whole Number Dividends

Purpose

The purpose of this lesson is to build a model for division problems whose divisor is a fraction and where the quotient has a remainder. We review problems with both the dividend and the divisor as whole numbers, with a quotient that has a fractional remainder. Then we move to a fractional divisor. The student develops the ability to see the remainder as a fraction of the divisor even when that divisor is also a fraction.

Teacher Background

When there is a remainder in a division problem, there are three choices as to how to express it. Example: $22 \div 4 = ?$

The first way is to show the number of groups with a remainder: $22 \div 4 = 5$ r 2.

The second way is to show the quotient as a mixed fraction with the remainder as a fraction of the divisor, e.g., $22 \div 4 = 5\frac{1}{2}$. The remainder of two is half of a group of four. So there are five and a half groups of four in twenty-two. Another way to think of it is: There are twenty-two really good, soft moist cookies and four children. How many cookies does each child get? Each child gets five cookies. No children I know would leave those extra cookies on the table! The children will take the last the two cookies and divide them up into four equal pieces. ⓘⓘ Each child will get a half a cookie more. So each child gets a total of $5\frac{1}{2}$ cookies and a tummy ache. Notice that in the first problem, we thought of the divisor as the size of the group. In the second problem, we thought of the divisor as the number in each group. Either way of thinking about it will produce the same quotient. Most of the time when dividing with fractions, we think of the divisor as the size of the group because it is easier to build it that way.

The third way is to show the quotient as a decimal. The decimal also shows a mixed fraction but in a different form. A good example of this would be twenty-two dollars divided between four people. How much does each person get? Five dollars and fifty cents. Note that the remainder is two dollars divided between four people. Each person gets fifty cents of that remaining two dollars.

The identical thought process is applied when the divisor is a fraction. The question posed about the remainder is the same. "The remainder is what fraction of the divisor?" To answer this question, one must see the divisor as a whole group, even when it is a fraction. This can be quite confusing for children and adults alike. You will get better at it as you go. It feels great when you get it.

Prerequisites

Previous lessons, and being able to show remainders as fractions in whole number division is helpful. *Patterns in Arithmetic*: Division - Booklet 3, first lesson

Materials

Quotients with Remainders: Whole Number Dividends - Worksheets 1 and 2. pages 18 and 19
Soft cookies are very helpful and fun. Paper ones will do as well; so will drawings

of cookies.
Fraction manipulative - most students prefer fraction circles for this activity.

Warm Up

Place twenty-two cookies on the table.
"Show me how four hungry children would share these cookies." "They would give each person five cookies and then cut the last two cookies into halves and give each child another half. So each child will get 5 $\frac{1}{2}$ cookies."

"Do the same problem again with a different way of seeing the divisor. Now the four will be the number in each group, not the number of groups as it was in the first problem."
Do the top problem on Quotients with Remainders: Whole Number Dividends - Worksheet 1, page 18. Do 22 ÷ 4 = ___ .
"How many dots are there?" "Twenty-two."

"What is the size of the group being made?" "Four."

"Circle the groups of four in the picture." The student circles the groups of four.

"How many groups of four are in twenty two?" "Five."

"What is the remainder?" "Two."

"The remainder is what fraction of the group being made? Or what fraction of four is the remainder of two?" "The remainder of two is half of a group of four." Use the graphic to help him see this.

"What is the quotient of the problem written as a mixed fraction?" "5$\frac{1}{2}$."
Have him fill in the answers to these questions on his worksheet.
Repeat a dialogue with 15 ÷ 4.

Do Problems 1 - 4 above the line on Quotients with Remainders: Whole Number Dividends - Worksheet 1, page 18. Show the remainders as fractions.
Answers: 1. 7 $\frac{1}{2}$ 2. 3 $\frac{3}{4}$ 3. 5 $\frac{2}{5}$ 4. 7$\frac{2}{3}$

Lesson

Read the section below the line on page 18 together. The remainder is $\frac{1}{3}$. The quotient can be shown as 4 r $\frac{1}{3}$. To show the quotient as a mixed fraction, the remainder must be seen as a fraction of the divisor. The group we are making is $\frac{2}{3}$. The remaining piece is $\frac{1}{3}$ of the group of $\frac{2}{3}$. This perception is tricky for many. What he sees is $\frac{1}{3}$ sticking out at the end. You as a teacher must keep asking the question, **"What is the group being made, and what fraction of that group is this remainder?"** Sometimes it helps to pull away a group of two-thirds from the problem and place the one-third piece beneath it as we did on page 15. This makes it more obvious that the remainder piece is one-half of the group above.
Answer the questions on the page. The answers are upside-down on the bottom of the page.

Note

Do not give up if this is confusing for you. You will get it. That lovely 'aha' will

happen. Just keep going.

Turn the page and try the next problem. $2 \div \frac{3}{4}$. Look at the remainder of two-fourths. Place them under a group of three-fourths. This makes it much easier to see that the remainder is two-thirds of the group of three-fourths.
This is a mind-bender sure enough, but isn't it interesting to start to understand why the answer to the problem $2 \div \frac{3}{4} = 2\frac{2}{3}$? Before, when you just used your memorized procedure, you might never have stopped to wonder why the answer had a two-thirds in it. How does that make sense? Now you know.

Worksheet

Do Quotients with Remainders: Whole Number Dividends - Worksheet 2, page 19. Check his answers (upside-down at the bottom of the page) as soon as he does each problem to be sure he is thinking correctly.

Note

Great insights! A pattern a very few students will see is that the denominator of the remainder matches the numerator of the divisor. This pattern makes sense because the numerator of the divisor tells you how many pieces will be in each group. If the problem is $2 \div \frac{3}{4} = 2\frac{2}{3}$, the size of the group being made is three pieces. It does not matter that those pieces are fourths. So if the size of the group is three pieces, each piece then becomes a third of the divisor. This is why the denominator of the answer matches the numerator of the divisor.

Some students will realize that the easiest way to calculate these is to turn the whole number into an improper fraction where the denominator matches the denominator of the divisor. Then all you have to do is divide the numerators and treat it like a whole number division problem. This works well. Do not tell him to do this unless he sees it on his own. It will get in the way of what comes next.

Here is a list of more practice problems. Answers are below.*
a. $1 \div \frac{3}{4}$ b. $1 \div \frac{2}{5}$ c. $1 \div \frac{3}{5}$ d. $1 \div \frac{3}{8}$ e. $1 \div \frac{4}{5}$ f. $1 \div \frac{5}{12}$ g. $1 \div \frac{5}{6}$ h. $1 \div \frac{7}{8}$

Test for Understanding

If he can do problem f, $1 \div \frac{5}{12} = 2\frac{2}{5}$, on his own, you know he understands.

Math Journal

Math Journal Question: Copy this sequence of problems.
$1 \div \frac{2}{5} = 2\frac{1}{2}$ $1 \div \frac{3}{5} = 1\frac{2}{3}$ $1 \div \frac{4}{5} = 1\frac{1}{4}$
Explain why the denominator of the fraction in the quotient is the same as the numerator of the divisor.
Answer: This pattern makes sense because the numerator of the divisor tells you how many pieces will be in each group. In the problem $1 \div \frac{3}{5} = 1\frac{2}{3}$, the size of the group being made is three pieces. It does not matter that those pieces are fifths. So if the size of the group is three pieces, each piece then becomes a third of the divisor. This is why the denominator of the answer matches the numerator of the divisor.

*a. $1 \div \frac{3}{4} = 1\frac{1}{3}$, b. $1 \div \frac{2}{5} = 2\frac{1}{2}$, c. $1 \div \frac{3}{5} = 1\frac{2}{3}$, d. $1 \div \frac{3}{8} = 1\frac{2}{3}$, e. $1 \div \frac{4}{5} = 1\frac{1}{4}$,
f. $1 \div \frac{5}{12} = 2\frac{2}{5}$, g. $1 \div \frac{5}{6} = 1\frac{1}{5}$, h. $1 \div \frac{7}{8} = 1\frac{1}{7}$

Quotients with Remainders: Fractional Dividends

Purpose The purpose of this lesson is to continue the work of constructing understanding of remainders when working with divisors that are fractions. This lesson progresses from the last lesson with whole number dividends to fractional dividends. For example: $1 \div \frac{3}{8} = 2\ \frac{2}{3}$ to $\frac{1}{2} \div \frac{3}{8} = 1\ \frac{1}{3}$. The student uses the same models as before.

Prerequisites Previous lessons

Materials Quotients with Remainders: Fractional Dividends - Worksheets 1 - 5, pages 20 - 24
Fraction pieces

Warm Up Read Quotients with Remainders: Fractional Dividends - Worksheet 1, page 20 and the top section of Worksheet 2, page 21.

Lesson Work on the problems on Quotients with Remainders: Fractional Dividends - Worksheet 2, page 21, Set 1. Build each one with the fraction pieces.

Note It is often easier for the student to use the names of the colors of the pieces rather than the number name. If the fourth pieces are yellow and the halves red, for example, you can ask; **"How many red pieces fit into three yellows?"** The answer to that question is the same as if you were using numbers. There are one and one-half reds in three yellows. It is a large leap of understanding to see that it is the physical sizes of the pieces in relationship to each other that reveal the answers.

It is on this page that most students begin to feel some confidence in the process.

Worksheets Quotients with Remainders: Fractional Dividends - Worksheets 3 - 5, pages 22- 24

Test for Understanding If the student is able to do the problems in Set 4 on Quotients with Remainders: Fractional Dividends - Worksheet 5, page 24, independently and get the correct answer 80% of the time, it is a demonstration of the understanding of fractional remainders.

Fractional Quotients

Purpose The purpose of this lesson is to extend the previous lessons to construct understanding of quotients that are less than one; in other words, the entire quotient is a fraction.

Prerequisites Previous lessons, and knowing how to change a fraction into a decimal is a helpful skill to have to address this concept. A brief lesson follows in the Warm Up section below.

Materials Fractional Quotients - Worksheets 1 and 2, pages 25 and 26
Paper square Prism Fractions or other rectangular or square fraction pieces
Do not use fraction circles; they are too hard to draw.

Warm Up To change a fraction into a decimal, divide the denominator into the numerator. Example: To change the fraction $\frac{3}{4}$ to a decimal, divide four into three.

$$\begin{array}{r} 0. \\ 4\overline{)3.00} \end{array} \qquad \begin{array}{r} 0.7 \\ 4\overline{)3.00} \\ \underline{2\,8} \end{array} \qquad \begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 00 \end{array}$$

Give the student several more examples to practice. Change these fractions to decimals: $\frac{1}{2}, \frac{3}{5}, \frac{1}{3}$
Answers:

$$\begin{array}{r} 0. \\ 2\overline{)1.00} \end{array} \qquad \begin{array}{r} 0.5 \\ 2\overline{)1.00} \\ \underline{1\,0} \end{array} \qquad \begin{array}{r} 0. \\ 5\overline{)3.00} \end{array} \qquad \begin{array}{r} 0.6 \\ 5\overline{)3.00} \\ \underline{3\,0} \end{array} \qquad \begin{array}{r} 0. \\ 3\overline{)1.00} \end{array} \qquad \begin{array}{r} 0.33...* \\ 3\overline{)1.00} \\ \underline{9} \\ 1.0 \\ \underline{9} \\ 1.0 \end{array}$$

Lesson Fractional Quotients - Worksheet 1, page 25
Read page 25 together and discuss the answers. Most students will see that these problems, especially when shown with the graphics, are simply fractions.

This is a good time to impress upon her that a fraction *is* a division problem.
"How do you think a fraction is like a division problem?" "One over two means one divided by two, or one of two parts. It also means one divided by two. One divided by two is ½ . They are the same thing."

Have her write this in her Math Journal.

Worksheet Fractional Quotients - Worksheet 2, page 26
Use the Answer Key to be sure you are doing these problems correctly. Check each problem immediately to prevent incorrect models from forming.
Read the top of the page and study the examples. Have her build the examples with her fraction pieces.

*repeating decimal

Begin Set 1 problems. The problems are in pairs to plant the seed that it is helpful to use common denominators to help find answers.

Note

If you have a mathematically talented student, give problems 2 and 4 on a piece of paper. Do not give her the hint that common denominators will really help. Let her discover this.

In problem 1 of Set 1, build the problem as is shown in the picture to the right. The denominators match, so it is easy to see that $\frac{3}{4}$ of the $\frac{4}{12}$ fits under the $\frac{3}{12}$. The answer is three-fourths.

Problem 2 is exactly the same problem as problem 1, except that the fractions are simplified and shown as single pieces. The student must look at the empty space to the right of the $\frac{1}{4}$ and see what fraction piece will fit into the space to match up with the fraction below, in this case the $\frac{1}{3}$. It is not unlike the thinking pattern we used to show remainders. The twelfths piece fits into the space. This clues the student to see that the one-third must be thought about as four-twelfths, just like it was in problem 1. Have her cover both pieces with twelfths to see the relationship.

The same strategy is used in problems 3 and 4. The two problems are exactly the same problem. The first uses a common denominator; the second uses simplified pieces. When she solves problem 4, wait to see if she builds it with the larger pieces and then covers them with twelfths. If she does not, suggest it.

Watch to see what she does on problems 5 and 6. Have her explain her thinking as she works on these problems. After she builds them, have her sketch each problem.

Practice Worksheet

Fractional Quotients - Worksheet 2, page 26, Set 2

Test for Understanding

If the student can solve problems 9 and 10 in Set 2 without assistance, she is demonstrating that she understands how to think about these problems.

$\frac{3}{8} \div \frac{3}{4} = \frac{1}{2}$ $\frac{1}{2} \div \frac{3}{5} = \frac{5}{6}$

Introducing Reciprocals

Purpose The purpose of this lesson is to introduce the concept of reciprocal fractions and discover an important rule that the product of any pair of reciprocal fractions is equal to one. Later in Using Multiplication to Solve Problems, this important rule will be used to develop the algorithm for division of fractions and to solve equations where division terms appear.

Prerequisites Previous lessons

Materials Introducing Reciprocals - Worksheets 1 and 2, pages 27 and 28

Lesson The lesson is presented in the Student Workbook and needs little explanation.
Answers to the questions on Introducing Reciprocals - Worksheet 1, page 27

1. The student studies the example.

2. The fractions in each set have the same numbers in them, but they are upside-down from each other.

3. Have her practice saying and spelling this word. Have her enter it in her Math Journal glossary and write an example.

4. Yes, $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals. 5. No 6. Yes 7. No 8. Yes 9. No

In problems 10 to 12, mixed numbers must be converted to improper fractions.
10. Yes 11. No 12. Yes, but the fraction must be simplified to see the reciprocal.

Check the work now to be sure it is done correctly. Answer any questions.

Continue with Introducing Reciprocals - Worksheet 2, page 28
Follow the worksheet as written. Use the Answer Key to be sure she has done problems 1 - 12 correctly before going on to Item 13.

13. Let her experiment with multiplying both the mixed number and then the improper fractions when multiplying the pairs.

14. The answer to every single product pair is one. Make sure she simplifies each answer and converts it to one. It is acceptable for her to use cancelling to get the answers.

15. The rule is best stated: The product of any pair of reciprocal fractions is equal to one.

Test for Understanding Ask her to write the reciprocal of $3\frac{3}{4}$ and check the pair by multiplying them together. It is $\frac{4}{15}$. $\frac{15}{4} \times \frac{4}{15} = 1$

Solving Problems with Pictures

Purpose The purpose of this lesson is to teach the student to solve problems with graphic representation instead of manipulative pieces. This will move the student from the concrete stage of working on division with fractions to the representational stage, which is more abstract.

Prerequisites Previous lessons

Materials Solving Problems with Pictures - Worksheets 1 - 4, pages 29 - 33
Colored pencils

Warm Up Solving Problems with Pictures - Worksheet 1, page 29

Lesson Solving Problems with Pictures - Worksheets 2 and 3, pages 30 and 31

Math Journal Topic Use problems 3 and 4, Solving Problems with Pictures - Worksheet 3, page 31, to explain the pattern in the quotients in the pair of problems. Answer: They are reciprocals of each other. Every time!

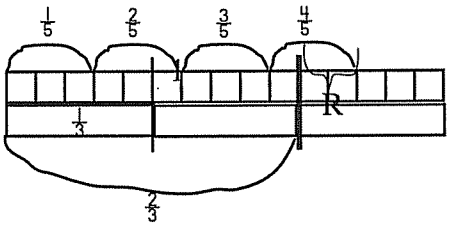
Practice Worksheets Solving Problems with Pictures - Worksheet 4, page 32
Problems 7 and 8 are an embedded assessment.

Solving Problems with Pictures - Worksheet 5, page 33

Test for Understanding 1. Watch the student do problems 7 and 8 on page 32. If he can do these without assistance and find the correct solutions, he understands the procedure for making drawings to solve problems.

2. Using a piece of blank graph paper, ask him to draw this problem and solve it with the drawing. He can not solve this one with the pieces. Require him to use colored pencils to show the different fractions. Ask him to explain his answer in words.

$\frac{4}{5} \div \frac{2}{3} =$ Answer: $\frac{4}{5} \div \frac{2}{3} = 1\frac{2}{10} = 1\frac{1}{5}$



$1\frac{2}{10} = 1\frac{1}{5}$

Explanation: The common denominator of three and five is fifteen. So I used fifteen boxes to show both thirds and fifths. In the divisor, the $\frac{2}{3}$ used ten boxes to show the whole group of $\frac{2}{3}$. The $\frac{4}{5}$ is larger than the $\frac{2}{3}$. There are two little boxes left over. Since the $\frac{2}{3}$ uses ten boxes, those two remainder boxes from the $\frac{4}{5}$ are two tenths of the $\frac{2}{3}$, which is simplified to $1\frac{1}{5}$.

Finding Patterns

Purpose

The purpose of this lesson is for the student to find a number pattern that will work for solving all division of fractions problems. The student is guided through several groups of problems and solves them first with fraction pieces and/or drawings. A number pattern will emerge in which a general rule is developed using a calculation. The same solution may not work for every group, which allows the student to refine the rule over time and practice.

Prerequisites

Previous lessons. The teacher needs to read and do all the worksheet pages before giving them to the student.

Materials

Finding Patterns - Worksheets 1 - 8, pages 34 - 41
Paper Prism Fractions pieces
Graph paper
Colored pencils

Warm Up

Vocabulary words to practice, spell and discuss: Dividend ÷ Divisor = Quotient, Denominator, Numerator, and Product. This will make it easier for him to explain his ideas.

Lesson Part 1

In each successively harder group of problems the student solves the problems with fraction pieces or drawings. The student looks for a pattern that can be applied to solve all of the problems on that page. Then the rule is tested on a set of problems. If the rule works for a group of problems, see if the same rule works for the previous groups.

Note

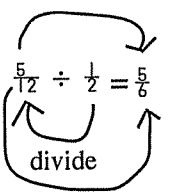
Each of these patterns can be directly linked to the standard 'invert and multiply' algorithm. Although some students will have discovered standard procedure from their explorations and will practice that pattern with these problems, please do not push other students to see that pattern. They need to explore their patterns first. By the end, all students will be able to derive the standard algorithm and understand why it works.

Here is an explanation of the reasoning behind the Finding Patterns worksheets.

Finding Patterns - Worksheet 1, page 34
Group 1 $\frac{1}{6} \div \frac{1}{3} = \frac{1}{2}$
A pattern that most students will see is to divide the second denominator of the divisor into the first one to make the denominator of the answer. Since both numbers are one, bring the one over for the numerator of the answer.

Finding Patterns - Worksheet 2, page 35
Group 2 $\frac{5}{12} \div \frac{1}{2} = \frac{5}{6}$
The first pattern will work for denominators. Divide the second denominator (of the divisor) into the first one (of the dividend) to make the denominator of the

answer. There is a slight change for numerators. Bring the larger numerator into the numerator of the answer or multiply numerator times numerator.
Calculated answers will come out as unreduced fractions.
Manipulated answers will come out as simplified, lowest term fractions.



Pattern for Groups 1 and 2

dividend ÷ divisor = quotient

Finding Patterns - Worksheet 3, page 36
Group 3 $\frac{1}{2} \div \frac{5}{6} = \frac{6}{10} = \frac{3}{5}$

These problems are different because now the larger numbers in the numerator and denominators are in the divisor rather than the dividend.

Group 2 $\frac{5}{12} \div \frac{1}{2}$ Group 3 $\frac{1}{2} \div \frac{5}{6}$

Larger numbers
Old pattern will not work

quotient
divisor | dividend

Discuss 'why' does it make sense, in problems 2, 3, 4, and 5, that the denominator of the answer is fifths, for example, $\frac{1}{2} \div \frac{5}{8} = \frac{4}{5}$? The divisor defines the new whole, or size of groups being made. Locate it on each problem.

Worksheets

A pattern to see for calculation is to take the numerator of the divisor and put it into the denominator of the answer. Divide the denominator of the dividend into the denominator of the divisor to determine the numerator of the quotient.

Finding Patterns - Worksheet 4, page 37

Group 3 continued
Some students will notice that if you change the fractions to have a common denominator (which is what the drawings do), you can divide the numerators to get the answer, e.g., $6 \div 7$ or $\frac{6}{7}$.

$\frac{1}{2} \div \frac{7}{12} = \frac{6}{12} \div \frac{7}{12} = \frac{6}{7}$

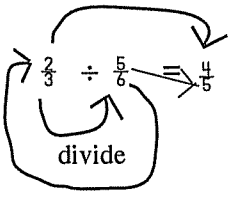
"Check to see if the pattern works with other groups of problems."

Finding Patterns - Worksheet 5, page 38
Group 4
The pattern for Group 4 involves using the numerator of the *divisor* with the two denominators.

$\frac{5}{8} \div \frac{3}{4} = \frac{5}{6}$ Divide 4 into 8 = 2. Take that 2 and multiply by 3 (numerator of the divisor) and put the product as the denominator of the quotient.
Put 5 in the numerator of the quotient.
Above is the reason for this specific problem.
This statement applies to any problem: Divide the denominator of the

divisor into the denominator of the dividend, multiplying by the numerator of the divisor to be the denominator of the quotient. Place the numerator of the dividend as the numerator of the quotient. Figure out why this works. Analyze the drawn figure too.

Finding Patterns - Worksheet 6, page 39
Group 5
The pattern for Group 5 involves using the numerator of the *dividend* with the two denominators.

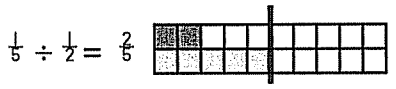


Divide 3 into 6 = 2.
Multiply by 2 in numerator 2 x 2 = 4.
Move 5 to denominator of answer.

This statement applies to any of this type of problem: Divide the denominator of the dividend into the denominator of the divisor, multiply that quotient by the numerator of the dividend to equal the numerator of the quotient. Move the denominator of the divisor to be the denominator of the quotient.

What other groups of problems does the pattern work on successfully? Group 3.

Finding Patterns - Worksheet 7, page 40
Group 6
The pattern students find will vary. What is important is that she tests it on a problem and verifies it works. One pattern is shown by the drawing. The squares colored in are the answer or quotient. For example:



Test for Understanding

Finding Patterns - Worksheet 8, page 41. This is an embedded assessment.
Group 7
The patterns a student finds will vary. Do not help her unless she asks for help.

Remember to ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Patterns with Reciprocals

Purpose The purpose of this lesson is to use the concept of an inverse operation and reciprocal numbers to create equal number sentences. This pattern forms the base of the standard algorithm for division of fractions–our old friend 'invert and multiply.'

Prerequisites Previous lessons

Materials Patterns with Reciprocals - Worksheets 1 and 2, pages 42 and 43
Blocks

Warm Up Have the student do the top half of Patterns with Reciprocals - Worksheet 1. The point of this exercise is to demonstrate that multiplication and division 'go together' or 'reverse each other.' The math words for this are *inverse operations*.
Add *inverse operations* to the Math Journal Glossary.

Lesson The puzzle does not use the Families of Facts. It does use a change from division to multiplication–inverse operations. He may be stumped by a problem that asks him to come up with a number sentence that uses multiplication to produce the same answer as a division problem. Have him think about it a while.

Put eight blocks on the table. Ask him to pick up half of them.
"Did you think half of eight is four? Or did you think eight divided by two is four?" The answers will vary.

"Write the words, 'half of eight is four' as a number sentence. Write the answer also." "*Of* means to multiply, so $\frac{1}{2} \times 8$ says half of eight, $\frac{1}{2} \times 8 = 4$."

"Write the words, 'eight divided by two' as a number sentence. Write the answer also." "$8 \div 2 = 4$."

"So both of these number sentences have the same answer."

Here is similar problem said with different words.
"What number can you multiply by eight and get two?" *Wait*. If he doesn't know, a possible dialogue follows.

> **"Two is only a part of eight. What number word means 'part of'?"** "Fraction of."
>
> **"Can you multiply with a fraction?"** "Yes."
>
> **"What fraction could you multiply by eight that would give the answer two? Or, two is what fraction of eight?"** "One-fourth."
>
> **"Test it."** "$8 \times \frac{1}{4} = \frac{8}{4} = 2$."

Wait again and see if he can finish all four problems.

"What patterns do you see?" He may or may not see the reciprocal pattern. The operation changes from division to multiplication.

Note

The pattern he is looking for is that dividing by four produces the same answer as multiplying by $\frac{1}{4}$. Give him time to notice that two things happen. First, the second number 'flips over.' In the division problem, the four is in the numerator; in the multiplication problem, the four is in the denominator with a one in the numerator. Second, the operators always switch from ÷ to x (from division to multiplication). After he does all four problems, ask again if he can see the pattern.

The worksheet prompts, 'Each puzzle pair can be written like this: $8 \div \frac{4}{1} = 8 \times \frac{1}{4}$.' In other words, the whole number operator, x 4, can be written $\frac{4}{1}$. Have him write the other three pairs of problems in this same format.

It is easy to see the inverse operation and the use of the reciprocal.
"Is this pattern always true?" "Yes."

Patterns with Reciprocals - Worksheet 2 explores this pattern further. The problems are easy so that he can confirm the truth of the relationships with arithmetic he already knows how to do.
The first set is partially completed. If he needs help on $6 \div \_\_\_\_ = \frac{1}{2}$, ask:
"What number fits into six only a half a time? Or, six is half of what number?"
"Twelve."

"If you put twelve in the blank, does it make a true number sentence?" "Yes, $6 \div 12 = \frac{1}{2}$."

Have him work on the rest of the page as independently as possible.

Problem 5 is a Test for Understanding. In the past, this kind of problem has required him to use drawings and deep thinking about division to find the answer. The problems in these sets ask him to use the reciprocal and this inverse operation of division (which is multiplication) to obtain matching answers. These problems are also introducing a fractional dividend.

Look at Problem 5–Set 3. The first problem in the set is $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$. The second problem in the set uses the inverse operation of multiplication and the reciprocal of $\frac{1}{2}$ to make $\frac{3}{4} \times \frac{2}{1} = 1\frac{1}{2}$. The second problem in each set is much easier to solve than the first.

End the session at this point. This new pattern and how to use it is continued in the next lesson Using Multiplication to Solve Problems.

Practice Worksheets

None

Test for Understanding

After the next lesson there is a Test for Understanding.

Using Multiplication to Solve Problems

Purpose The purpose of this lesson is to further develop the pattern of using the inverse operation and the reciprocal of the divisor to solve division of fractions problems.

Prerequisites Previous lessons

Materials Using Multiplication to Solve Problems - Worksheets 1 - 3, pages 44 - 46
Colored pencils

Warm Up Using Multiplication to Solve Problems - Worksheet 1 is a review of the last part of the previous lesson. Use it as a warm up.

Lesson Follow the instructions and problems on Using Multiplication to Solve Problems - Worksheets 2 and 3. Using Multiplication to Solve Problems - Worksheet 3 requires the student to use both the pattern and the drawing. Insist on clear, clean drawings.

If you can multiply with a reciprocal and get the same answer to those problems you had to draw, why not do it the easy way? You get the same answer!
This is the core of the standard procedure commonly used to calculate the answers to division of fractions problems.
Now you have been shown why you 'invert and multiply.'

Practice Worksheets Go back to Finding Patterns, Group 7, page 41, and try this new pattern on the problems on that page to see if it works.

Test for Understanding Explain the pattern of using multiplication by the reciprocal of the divisor to solve division of fractions problems. How could you use this pattern to make division of fractions problems easier?

Answer: I have found that when I get a fraction division problem, such as $\frac{2}{3} \div \frac{1}{4}$, instead of puzzling it out or drawing, I can flip the $\frac{1}{4}$ over, or take its reciprocal, $\frac{4}{1}$, and multiply it by the dividend and get the same answer. $\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1}$. The second problem is so much easier to do than the first one. This makes division of fractions much easier because some problems, such as $\frac{11}{32} \div \frac{13}{24}$, would be impossible to draw. Multiplying is so much easier, and I can cancel too.

Math Journal Explain the pattern of using multiplication and reciprocals to solve division of fractions problems. Use the answer above in Test for Understanding if she needs help clarifying her thoughts.

Standard Procedure

Purpose

The purpose of this lesson is to rebuild the division problems with Cuisenaire Rods to integrate the physical process of dividing by a fraction with the computational 'invert and multiply' pattern. Then the student begins the practice process to attain fluency and mastery.

Prerequisites

Previous lessons

Materials

Standard Procedure - Worksheets 1 - 6, pages 47 - 52
Cuisenaire Rods

Lesson

Review the procedure with a physical model of easy division problems. Have the student build the physical solution to each of the division problems and then calculate it with the multiplication of the reciprocal.

In the first problem, the whole is defined as the yellow rod. Have him fill out the chart on the left to show the color of each rod that matches the fraction asked for.

One-fifth = white two-fifths = red three-fifths = light green four-fifths = purple

To solve $4 \div \frac{2}{5}$, have him cover the rectangle with red blocks. Two have to be turned sideways to fit. The answer will be ten.
To model the other fractional divisors, some trading for whites may be needed to make them fit and show the remainder.

Build $4 \div \frac{3}{5}$. **"What is the size of the new group?"** "Three-fifths."

"How many groups fit in?" "Six."

"What is the remainder?" "Two white blocks or two- thirds of the group."

Practice Worksheets

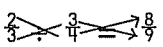
Have him finish Standard Procedure - Worksheet 1, page 47

Standard Procedure - Worksheet 2, page 48, continues using Cuisenaire Rods. The student builds and then calculates the problem to connect the standard procedure with a physical model.

Standard Procedure - Worksheet 3, page 49, has a Test for Understanding that asks him to draw three of the problems to prove that his answers are correct. Have him do this before he checks his answers with the Answer Key.

Standard Procedure - Worksheet 4, page 50, continues the discussion of why the Standard Procedure of 'invert and multiply' works. The example given assumes the solution of this pattern was discovered by the student on Finding Patterns, Worksheet 8.
Give the problem $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$. If the student has figured out the solution of 'invert

and multiply' from Patterns with Reciprocals and Using Multiplication to Solve Problems, then this pattern will be recognized.



The next example given, $\frac{5}{6} \div \frac{3}{4}$, explores how to deal with the problem of an answer in the form of an improper fraction. The student uses a drawing on graph paper and then changes the improper fraction to a mixed number.

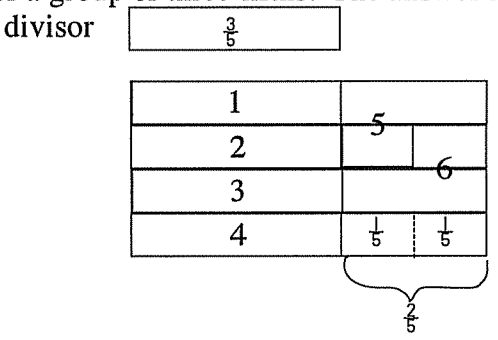
Standard Procedure - Worksheet 5, page 51, is a formal explanation with drawings of why–'invert and multiply' procedure works. It gives an example of a formal proof.

Standard Procedure - Worksheet 6, page 52, is a practice page using the standard procedure.

Math Journal Topic

Explain why the answer to $4 \div \frac{3}{5}$ is $6\frac{2}{3}$. Use Cuisenaire Rods to prove your answer.

Answer: The divisor is the green rod, a group of three-fifths. Six green rods fit. A white rod is one-third of the light green rod. So the two remaining white rods are two-thirds of a group of three-fifths. The answer is *not* because $4 \times 5 \div 3$ is $6\frac{2}{3}$.



Have the student record her definition of and process for solving the standard procedure for the division of fractions.

The standard procedure refers to the frequently taught method of attaining the answer. In division of fractions, the standard procedure is: Invert the divisor and multiply it by the dividend.

Division of Mixed Numbers

Purpose The purpose of this lesson is to extend the division of fractions procedure to working with mixed numbers.

Prerequisites Previous lessons

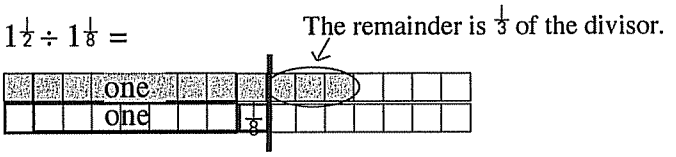
Materials Division of Mixed Numbers - Worksheets 1 and 2, pages 53 and 54

Warm Up Solve the multiplication of mixed numbers problem at the top of Division of Mixed Numbers - Worksheet 1. **"How did you solve the first problem $5\frac{1}{3} \times 3\frac{3}{4}$?"**

"I changed both mixed numbers into improper fractions and then multiplied."
$\frac{16}{3} \times \frac{15}{4} = 20$

Lesson **"Will the same strategy used for mixed numbers in multiplication work for mixed numbers in division too?"** "I don't know. Maybe?"

"Draw Problem 2." $1\frac{1}{2} \div 1\frac{1}{8} =$



"How many times does $1\frac{1}{8}$ fit into $1\frac{1}{2}$?" "One whole time with three boxes left over."

"These three boxes are what fraction of the $1\frac{1}{8}$ (which takes nine boxes)?" "They are one-third of it. So the answer is $1\frac{1}{3}$."

"Now calculate the answer knowing everything you know."
Did he figure out that you can change both mixed numbers into improper fractions? Then he can invert the divisor just like before and then multiply to get the correct answer. This has been confirmed by his drawing. Now that is what mathematicians do. $\frac{3}{2} \div \frac{9}{8} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \times \frac{8}{\underset{1}{\cancel{3}}} = \frac{4}{1} = 1\frac{1}{3}$

Give Problem 4 as a Test for Understanding. $1\frac{2}{3} \div 2\frac{2}{3} = \frac{5}{3} \div \frac{8}{3} = \frac{5}{3} \times \frac{3}{8} = \frac{5}{8}$

"To draw and prove Problem 4, where you have to show thirds and also almost three wholes, what size do you have to draw the whole?" "The whole will be six."

Practice Worksheet Division of Mixed Numbers - Worksheet 2, page 54

Math Journal Topic Write out the procedure for doing $3\frac{3}{5} \div 1\frac{1}{2}$. Have him confirm his answer by multiplying his answer by the divisor. He should get back to $3\frac{3}{5}$. This is a tricky bit of arithmetic and may take more than one try.
Answer: Change both numbers $3\frac{3}{5}$ and $1\frac{1}{2}$ to improper fractions. That gives $\frac{18}{5} \div \frac{3}{2}$. Invert the $\frac{3}{2}$ to $\frac{2}{3}$ and multiply. $\frac{18}{5} \times \frac{2}{3} = \frac{36}{15} = 2\frac{2}{5}$.
He can confirm his answer by multiplying $2\frac{2}{5} \times 1\frac{1}{2}$. The correct answer is $3\frac{3}{5}$.

Post-Assessment - Worksheet 1 Date_____

1. What question does each problem ask?
   a. $12 \div 3$ _____
   b. $6 \div \frac{2}{3}$ _____
   c. $3 \div 4$ _____
   d. Circle the problem whose answer will be less than one.

2. Solve each problem. Make a drawing that clearly proves the answer.
   a. $\frac{1}{2} \div \frac{1}{4} =$ _____
   b. $6 \div \frac{2}{3} =$ _____
   c. $\frac{3}{4} \div \frac{1}{3} =$ _____
   d. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$. Explain where the $\frac{1}{8}$ comes from. Why is it $\frac{1}{8}$?

_____

_____

3. Solve each problem. Make a drawing that clearly proves the answer.
   a. Will the answer to $\frac{1}{4} \div \frac{1}{2}$ be greater than or less than one? _____
   b. How do you know? _____
   c. $\frac{1}{3} \div \frac{1}{2} =$ _____

d. $\frac{1}{2} \div \frac{3}{4} =$ _____

e. $\frac{1}{4} \div \frac{3}{8} =$ _____

4. Write a fraction and its reciprocal. a. _____ b. _____

5. Explain why $12 \div \frac{1}{2} = 12 \times 2$. _____

_____

6. Does the pattern shown in problem 5 work for this problem $\frac{3}{4} \div \frac{1}{2}$? _____

   a. Prove it with a drawing.

   b. Explain or draw why the 'invert and multiply' formula works in this division of fractions problem. _____

_____

_____

7. Solve these problems.

   a. $\frac{3}{4} \div \frac{2}{3} =$ _____ b. $\frac{5}{6} \div \frac{4}{5} =$ _____ c. $\frac{3}{7} \div \frac{7}{12} =$ _____

8. Solve these problems.

   a. $5 \div 2\frac{1}{2} =$ _____ b. $2\frac{2}{3} \div 1\frac{1}{4} =$ _____ c. $6\frac{4}{5} \div 9\frac{1}{2} =$ _____

Patterns in Arithmetic

Fractions: Booklet 8

Understanding Division

Answer Key for the Student Workbook

By Suki Glenn, Susan Carpenter, and Zephyr Alfanash

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

Patterns in Arithmetic: Fractions - Booklet 8
Parent/Teacher Guide
Answer Key for the Student Workbook

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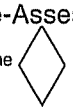
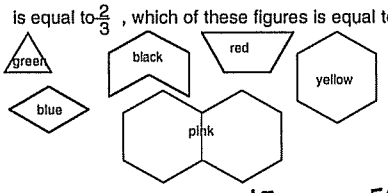
Fractions - Booklet 8 - Division Answer Key

Pre-Assessment - Part 1

1. If the  is equal to $\frac{2}{3}$, which of these figures is equal to one whole? red



2. Change to an improper fraction. a. $3\frac{3}{4} = \frac{15}{4}$ b. $6\frac{7}{8} = \frac{55}{8}$ c. $7\frac{4}{9} = \frac{67}{9}$

3. Change to a mixed number. a. $\frac{17}{5} = 3\frac{2}{5}$ b. $\frac{29}{4} = 7\frac{1}{4}$ c. $\frac{58}{8} = 7\frac{1}{4}$

4. Fill in the missing numbers. a. $\frac{4}{5} = \frac{12}{15}$ b. $\frac{3}{4} = \frac{15}{20}$ c. $\frac{2}{3} = \frac{18}{27}$

5. Simplify. a. $\frac{32}{36} = \frac{8}{9}$ b. $\frac{17}{34} = \frac{1}{2}$ c. $\frac{19}{23} = \frac{19}{23}$ d. $\frac{42}{54} = \frac{7}{9}$

6. Multiply. a. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ b. $6 \times \frac{3}{4} = 4\frac{1}{2}$ c. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

7. Use cancelling to solve. a. $\frac{1}{\cancel{4}_{2}} \times \frac{\cancel{8}^{2}}{\cancel{3}_{1}} = \frac{1}{2}$ b. $\frac{\cancel{6}^{1}}{\cancel{3}_{1}} \times \frac{5}{\cancel{22}_{11}} = \frac{5}{11}$ c. $\frac{\cancel{2}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{1}} = 6$

8. Multiply. a. $2\frac{2}{3} \times 2\frac{1}{4} = 6$ b. $7\frac{1}{2} \times 3\frac{2}{3} = \frac{165}{6} = 27\frac{1}{2}$

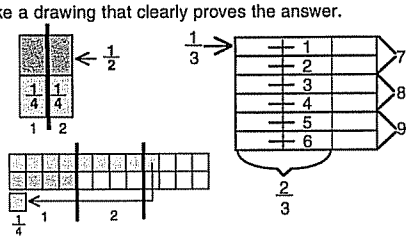
9. If 25¢ is one seventh cost of a piece of ribbon, how much the does the ribbon cost? $1.75

10. Solve. a. $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = 1\frac{5}{12}$ b. $\frac{4}{5} + \frac{1}{3} = \frac{17}{15} = 1\frac{2}{15}$ c. $\frac{4}{9} + \frac{3}{5} = \frac{47}{45} = 1\frac{2}{45}$

Pre-Assessment - Part 2 - Worksheet 1

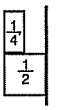
1. What question does each problem ask?
   a. $12 \div 3$ How many groups of 3 are in 12?
   b. $6 \div \frac{2}{3}$ How many groups of $\frac{2}{3}$ are in 6?
   c. $3 \div 4$ What part of 4 fits into 3?
   d. Circle the problem whose answer will be less than one.

2. Solve each problem. Make a drawing that clearly proves the answer.
   a. $\frac{1}{2} \div \frac{1}{4} = 2$
   b. $6 \div \frac{2}{3} = 9$
   c. $\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$

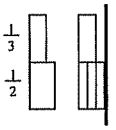


   d. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$. Explain where the $\frac{1}{8}$ comes from. Why is it $\frac{1}{8}$? The remainder of $\frac{1}{12}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$. The new whole = $\frac{8}{12} = \frac{2}{3}$.

3. Solve each problem. Make a drawing that clearly proves the answer
   a. Will the answer $\frac{1}{4} \div \frac{1}{2}$ be greater than or less than one? less



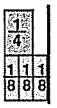
   b. How do you know? Because $\frac{1}{2}$ is greater than $\frac{1}{4}$, so only part of the $\frac{1}{2}$ will fit in the $\frac{1}{4}$.
   c. $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$



Pre-Assessment - Part 2 - Worksheet 2

d. $\frac{1}{2} \div \frac{3}{4} = \frac{4}{6} = \frac{2}{3}$

e. $\frac{1}{4} \div \frac{3}{8} = \frac{8}{12} = \frac{2}{3}$

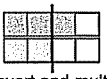


AWV

4. Write a fraction and its reciprocal. a. $\frac{2}{3}$ b. $\frac{3}{2}$

5. Explain why $12 \div \frac{1}{2} = 12 \times 2$. When solving $12 \div \frac{1}{2}$ you are asking how many halves fit in 12. Since there are 2 halves in each whole, the total number of halves is solved by 12×2.

6. Does the pattern shown in problem 5 work for this problem $\frac{3}{4} \div \frac{1}{2}$? yes
   a. Prove it with a drawing.



   b. Explain or draw why the 'invert and multiply' formula works in this division of fractions problem. When you divide $\frac{3}{4}$ by $\frac{1}{2}$, it is equal to $\frac{1}{2}$, which is the same as $\frac{3}{4} \times 2$.

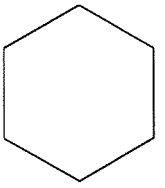
$6 \div \frac{2}{3} = \frac{6 \times 3}{1 \times 2} = \frac{6 \times 3}{1 \times 2} = \frac{6}{1} \times \frac{3}{2} = \frac{6}{1} \times \frac{3}{2} = \frac{6}{1} \div \frac{2}{3}$

7. Solve these problems.
   a. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$ b. $\frac{5}{6} \div \frac{4}{5} = \frac{1}{24}$ c. $\frac{3}{7} \div \frac{7}{12} = \frac{36}{49}$

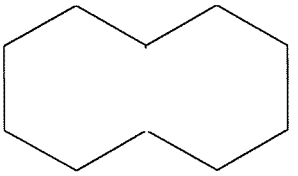
8. Solve these problems.
   a. $5 \div 2\frac{1}{2} = 2$ b. $2\frac{2}{3} \div 1\frac{1}{4} = 2\frac{2}{15}$ c. $6\frac{4}{5} \div 9\frac{1}{2} = \frac{68}{95}$

Changing Wholes: Review - Worksheet 1

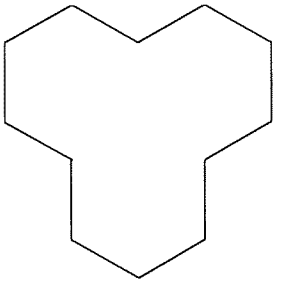
If: 1 yellow = 1
Then: 1 red = $\frac{1}{2}$
1 blue = $\frac{1}{3}$
1 green = $\frac{1}{6}$

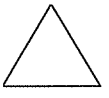


If: 2 yellows = 1
Then: 1 red = $\frac{1}{4}$
1 blue = $\frac{1}{6}$
1 green = $\frac{1}{12}$
1 yellow = $\frac{1}{2}$



If: 3 yellows = 1
Then: 1 red = $\frac{1}{6}$
1 blue = $\frac{1}{9}$
1 green = $\frac{1}{18}$
1 yellow = $\frac{1}{3}$





If: 1 green = 1
Then: 1 red = 3 or $\frac{3}{1}$
1 blue = 2 or $\frac{2}{1}$
1 yellow = 6 or $\frac{6}{1}$

Changing Wholes: Review - Worksheet 2

If this = 1, twelve

then the red rod = $\frac{1}{6}$
the light green rod = $\frac{1}{4}$
the dark green rod = $\frac{1}{2}$
1 purple rod = $\frac{1}{3}$
1 white rod = $\frac{1}{12}$

If this = 1, twenty- four

then the red rod = $\frac{1}{12}$
the light green rod = $\frac{1}{8}$
the dark green rod = $\frac{1}{4}$
1 purple rod = $\frac{1}{6}$
1 white rod = $\frac{1}{24}$

If this = 1, six

then the red rod = $\frac{1}{3}$
the light green rod = $\frac{1}{2}$
the dark green rod = $\frac{1}{1}$
1 white rod = $\frac{1}{6}$

If this = 1, sixteen

then the brown rod = $\frac{1}{2}$
the purple rod = $\frac{1}{4}$
the red rod = $\frac{1}{8}$
1 white rod = $\frac{1}{16}$

If this = 1, thirty-two

then the brown rod = $\frac{1}{4}$
the purple rod = $\frac{1}{8}$
the red rod = $\frac{1}{16}$
1 white rod = $\frac{1}{32}$

5

Changing Wholes: Review - Worksheet 3

1 white is 1 whole
1 red = $\frac{1}{2}$
1 orange = $\frac{1}{3}$
1 yellow = $\frac{1}{4}$
1 green = $\frac{1}{5}$
1 blue = $\frac{1}{6}$
1 indigo = $\frac{1}{8}$
1 violet = $\frac{1}{10}$
1 black = $\frac{1}{12}$
1 magenta = $\frac{1}{16}$

If 1 red = 1 whole, then:
1 white = 2
1 yellow = $\frac{1}{2}$
1 blue = $\frac{1}{3}$
1 indigo = $\frac{1}{4}$

If 1 orange = 1 whole, then:
1 white = 3
1 blue = $\frac{1}{2}$
1 black = $\frac{1}{4}$
1 yellow = $\frac{3}{4}$

If 1 blue = 1 whole, then:
1 white = 6
1 red = 3
1 orange = 2
1 black = $\frac{1}{2}$

6

Changing Wholes: Review - Worksheet 4

If 2 oranges = 1 whole,
2 whites = 3
*1 white = $1\frac{1}{2}$
1 blue = $\frac{1}{4}$
2 blues = $\frac{1}{2}$
*1 red = $\frac{3}{4}$
1 black = $\frac{1}{8}$
**3 yellows = $\frac{9}{8}$ or $1\frac{1}{8}$

If 2 black pieces = 1 whole, then:
one white = $\frac{1}{2}$
one red = $\frac{1}{4}$
one orange = $\frac{1}{6}$
one yellow = $\frac{1}{8}$
one green = $\frac{1}{10}$
one blue = $\frac{1}{12}$
one indigo = $\frac{1}{16}$
one violet = $\frac{1}{20}$
one black = $\frac{1}{24}$
one magenta = $\frac{1}{32}$

If 3 yellows = 1 whole, then:
1 white = $1\frac{1}{3}$
1 green = $\frac{2}{3}$
1 black = $\frac{1}{9}$
Hint: Use the black pieces to help.
**2 oranges = $\frac{8}{9}$
**4 blues = $\frac{8}{9}$
1 indigo = $\frac{1}{6}$

7

Changing Wholes: Review - Worksheet 5

Review *Patterns in Arithmetic: General Math: Booklet 5*, Division - Relationships

Solve these problems

quotient / divisor | dividend

1. $3\overline{)24}$ = 8; $6\overline{)24}$ = 4; $12\overline{)24}$ = 2
2. $1\overline{)12}$ = 12; $2\overline{)12}$ = 6; $4\overline{)12}$ = 3

3. What is changing in each set?
The divisor is doubling and the quotient is halving.

4. Make a set of problems that change in the same way. AWV

5. What will happen to the answer (quotient) of a division problem if the size of the divisor is doubled?
It will be halved.

6. State it as a relationship. Keeping the same dividend,
if you double the divisor, it will halve the quotient.
If you halve the divisor, it will double the quotient.

8

Division of Fractions into Whole and Mixed Numbers - Worksheet 1

Use the fraction pieces to help you find or prove the answers. Look for patterns.

1. How many 1/2s in 1? 2
   How many 1/2s in 2? 4
   How many 1/2s in 3? 6
   How many 1/2s in 4? 8
   How many 1/2s in 5? 10
   How many 1/2s in 6? 12

Two times table.
Counting by twos.

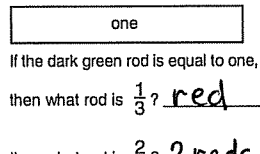
2. How many 1/3s in 1? 3
   How many 1/3s in 2? 6
   How many 1/3s in 3? 9
   How many 1/3s in 4? 12
   How many 1/3s in 5? 15
   How many 1/3s in 6? 18

What patterns do you see?
Three times table.
Counting by threes.
Multiply the denominator by the whole number.

Write as a division problem.

3. How many 1/4s in 1? 4 — $1 \div 1/4 = 4$
   How many 1/4s in 2? 8 — $2 \div 1/4 = 8$
   How many 1/4s in 3? 12 — $3 \div 1/4 = 12$
   How many 1/4s in 4? 16 — $4 \div \frac{1}{4} = 16$
   How many 1/4s in 5? 20 — $5 \div \frac{1}{4} = 20$
   How many 1/4s in 6? 24 — $6 \div \frac{1}{4} = 24$

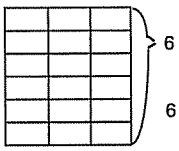
Division of Fractions into Whole and Mixed Numbers - Worksheet 2



If the dark green rod is equal to one,

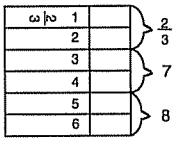
then what rod is $\frac{1}{3}$? red

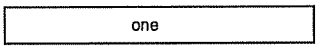
then what rod is $\frac{2}{3}$? 2 reds or purple



$6 \div \frac{1}{3} =$

$6 \div \frac{2}{3} =$





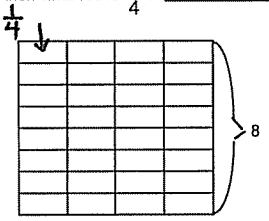
Try trading.

If the brown is equal to one,

then what rod is $\frac{1}{4}$? red or 2 whites

then what rod is $\frac{2}{4}$? 2 reds or 1 purple

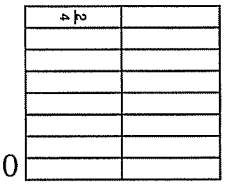
then what rod is $\frac{3}{4}$? 3 reds or 1 dark green

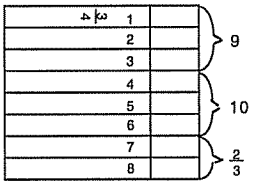


$8 \div \frac{1}{4} = 32$

$8 \div \frac{2}{4} = 16$

$8 \div \frac{3}{4} = 10\frac{2}{3}$





Division of Fractions into Whole and Mixed Numbers - Worksheet 3

Use any fraction set to help you find the answers. Look for patterns.

1. Build 1 1/3 with fraction pieces.
   How many groups of 2/3 are in 1 1/3? 2

2. Build 2 with fraction pieces.
   How many groups of 2/3 are in 2? 3

3. Build 2 1/3 with fraction pieces.
   How many groups of 2/3 are in 2 1/3? $3\frac{1}{2}$

4. Build 3 1/3 with fraction pieces.
   How many groups of 2/3 are in 3 1/3? 5

5. What number has 6 groups of 2/3 in it? 4

6. What number has 9 groups of 2/3 in it? 6

Guess!

7. How many groups of 2/3 do you think will be in 5 1/3? AWV
   Now try it.
   How many groups of 2/3 are in 5 1/3? 8

Guess again!

8. How many groups of 2/3 do you think will be in 4 2/3? AWV
   Now try it.
   How many groups of 2/3 are in 4 2/3? 7

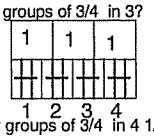
Division of Fractions into Whole and Mixed Numbers - Worksheet 4

Use the 1/4 fraction pieces.

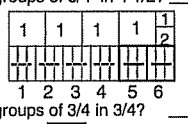
Write as a division problem.
Example:

$3 \div \frac{3}{4}$ or $\frac{3}{4}\overline{)3}$

9. How many groups of 3/4 in 3? 4



10. How many groups of 3/4 in 4 1/2? 6



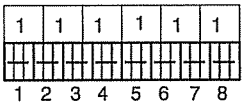
$\frac{3}{4}\overline{)4\frac{1}{2}}$ $4\frac{1}{2} \div \frac{3}{4}$

11. How many groups of 3/4 in 3/4? 1

$\frac{3}{4}\overline{)\frac{3}{4}}$ $\frac{3}{4} \div \frac{3}{4}$

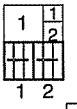
12. How many groups of 3/4 in 6? 8

$6 \div \frac{3}{4}$ $\frac{3}{4}\overline{)6}$



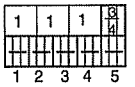
13. How many groups of 3/4 in 1 1/2? 2

$1\frac{1}{2} \div \frac{3}{4}$ $\frac{3}{4}\overline{)1\frac{1}{2}}$



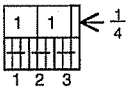
14. How many groups of 3/4 in 3 3/4? 5

$3\frac{3}{4} \div \frac{3}{4}$ $\frac{3}{4}\overline{)3\frac{3}{4}}$



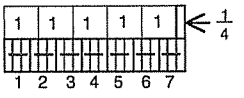
15. How many groups of 3/4 in 2 1/4? 3

$2\frac{1}{4} \div \frac{3}{4}$ $\frac{3}{4}\overline{)2\frac{1}{4}}$



16. How many groups of 3/4 in 5 1/4? 7

$\frac{3}{4}\overline{)5\frac{1}{4}}$ $5\frac{1}{4} \div \frac{3}{4}$



Division of Fractions into Whole and Mixed Numbers - Worksheet 5

On Division of Fractions into Whole and Mixed Numbers - Worksheet 1 you found a pattern that would allow you to calculate the answers to a kind of division problem with fractions. Explain how you can find the answer to this problem without having to use the fraction pieces.

How many groups of $\frac{1}{6}$ are in 3? or $3 \div \frac{1}{6} =$

Now check your answer with the fraction pieces. Does your pattern work? __
Use your pattern on these problems.

1. How many groups of $\frac{1}{5}$ are in 4? $4 \div \frac{1}{5} =$ 20
2. How many groups of $\frac{1}{6}$ are in 4? $4 \div \frac{1}{6} =$ 24
3. How many groups of $\frac{1}{8}$ are in 3? $3 \div \frac{1}{8} =$ 24
4. How many groups of $\frac{1}{10}$ are in 3? $3 \div \frac{1}{10} =$ 30
5. How many groups of $\frac{1}{12}$ are in 2? $2 \div \frac{1}{12} =$ 24
6. How many groups of $\frac{1}{16}$ are in 2? $2 \div \frac{1}{16} =$ 32
7. Why are there more 16ths in 2 than 12ths in 2? 16th are smaller than 12th, so more would fit into a whole.
8. In a division problem, this number is called a divisor.

$3 \div \underset{\downarrow}{\frac{1}{12}} = ?$

What will happen to the answer of a division problem if the size of the divisor is doubled and the dividend remains constant? AWV
Extend the pattern.

$16 \div 1 =$ 16 $16 \div 2 =$ 8 $16 \div 4 =$ 4 $16 \div 8 =$ 2

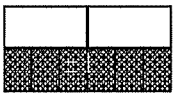
What pattern do you see in the answers? The quotient halves.

Division of Fractions into Whole and Mixed Numbers - Worksheet 6

Here are two division problems. What will happen to the answer when the divisor is doubled?
$18 \div 3 =$ 6
$18 \div 6 =$ 3
Do you think this pattern will be true for fractions also? AWV
Try it.

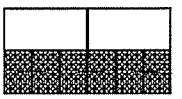
Use fraction pieces to make a group of $\frac{1}{3}$

How many groups of $\frac{1}{3}$ are in 2? 6



Now make a groups of $\frac{2}{3}$.

How many groups of $\frac{2}{3}$ are in 2? 3



What happened to the answer when you doubled the size of the group you were making? It was cut in half. The larger the divisor, the smaller the quotient.

Use your patterns to predict the answers to each set of problems. Then use fraction pieces to check if your pattern worked.

Set 1

	Predict	Check
$2 \div \frac{1}{6} =$	____	12
$2 \div \frac{2}{6} =$	____	6
$2 \div \frac{4}{6} =$	____	3

Challenge Set
Fill in the missing fractions.

$2 \div \frac{1}{5} = 10$ and $2 \div \frac{2}{5} = 5$

Set 2
The size of the divisor is tripled then doubled.
What will happen to the quotients?

	Predict	Check
$3 \div \frac{1}{8} =$	____	24
$3 \div \frac{3}{8} =$	____	8
$3 \div \frac{6}{8} =$	____	4

Was your prediction correct? _____

Fractions as Divisors: Whole Number Quotients - Worksheet 1

Use fraction pieces to help you figure out the answers.
All the answers will be whole numbers.

Example Problem:

This problem $\frac{1}{2} \div \frac{1}{4} =$ ____ asks you, "How many groups of $\frac{1}{4}$ are in $\frac{1}{2}$?"

Put the $\frac{1}{2}$ piece on the table like this:

Now put the $\frac{1}{4}$ piece like this:

You can see that it will take two $\frac{1}{4}$ pieces to fit under the $\frac{1}{2}$ piece.

So the answer to the question, "How many groups of $\frac{1}{4}$ are in $\frac{1}{2}$?" is 2.

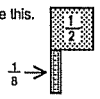
There are 2 groups of $\frac{1}{4}$ in $\frac{1}{2}$

Now you try one.

The problem $\frac{1}{2} \div \frac{1}{8} =$ ____ asks, "How many groups of $\frac{1}{8}$ are in $\frac{1}{2}$?

Take out a $\frac{1}{2}$ piece and several $\frac{1}{8}$ pieces.

Put them on the table like this.



How many groups of $\frac{1}{8}$ fit under the $\frac{1}{2}$?

Therefore, $\frac{1}{2} \div \frac{1}{8} =$ 4.

Solve these problems. Use fraction pieces to find the answers. Record the answers on the line.

1. $\frac{1}{2} \div \frac{1}{6} =$ 3
2. $\frac{1}{2} \div \frac{1}{12} =$ 6
3. $\frac{1}{2} \div \frac{1}{10} =$ 5
4. $\frac{1}{2} \div \frac{1}{16} =$ 8
5. $\frac{3}{4} \div \frac{1}{8} =$ 6
6. $\frac{2}{3} \div \frac{1}{12} =$ 8
7. $\frac{5}{8} \div \frac{1}{16} =$ 10
8. $\frac{3}{5} \div \frac{1}{10} =$ 6

Fractions as Divisors: Whole Number Quotients - Worksheet 2

Use fraction pieces to solve these problems. Record your answers.

$\frac{1}{4} \div \frac{1}{8} =$ 2 $\frac{3}{4} \div \frac{1}{8} =$ 6 $\frac{3}{4} \div \frac{1}{12} =$ 9 $\frac{1}{4} \div \frac{1}{12} =$ 3

5. $\frac{3}{8} \div \frac{1}{16} =$ 6 $\frac{7}{12} \div \frac{1}{12} =$ 7 $\frac{1}{4} \div \frac{1}{16} =$ 4 $\frac{3}{4} \div \frac{1}{16} =$ 12

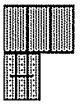
Sometimes the group you are making will have more than one piece in it.

Look at this problem. $\frac{3}{4} \div \frac{3}{8} =$ ____ The question is, "How many groups of $\frac{3}{8}$ are in $\frac{3}{4}$?"

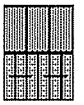
Build $\frac{3}{4}$

Now build a group of $\frac{3}{8}$

Put your pieces like this:



Now build a another group of $\frac{3}{8}$



There are 2 groups of $\frac{3}{8}$ in $\frac{3}{4}$.

Use your fraction pieces to solve these problems. Record your answers.

9. $\frac{3}{8} \div \frac{3}{8} =$ 1
10. $\frac{3}{8} \div \frac{3}{16} =$ 2
11. $3\frac{3}{8} \div \frac{1}{8} =$ 3
12. $\frac{5}{6} \div \frac{2}{12} =$ 5
13. $\frac{3}{4} \div \frac{3}{16} =$ 4
14. $\frac{7}{8} \div \frac{2}{16} =$ 7
15. $3\frac{3}{4} \div \frac{3}{8} =$ 2
16. $\frac{5}{6} \div \frac{1}{12} =$ 10

Fractions as Divisors: Whole Number Quotients - Worksheet 3

Solve these problems. Draw your solutions or check the answers with fraction pieces.

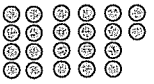
Example: $\frac{1}{2}$

1. $\frac{1}{2} \div \frac{1}{16} =$ 8
2. $\frac{1}{3} \div \frac{1}{12} =$ 4 — $\frac{1}{3}$
3. $\frac{3}{4} \div \frac{9}{12} =$ 1 — $\frac{3}{4}$ → 1; $\frac{9}{12}$ →
4. $\frac{3}{4} \div \frac{3}{12} =$ 3 — $\frac{3}{4}$ → 1 2 3; $\frac{3}{12}$ →
5. $\frac{2}{3} \div \frac{1}{12} =$ 8 — $\frac{2}{3}$ →
6. $\frac{8}{10} \div \frac{2}{5} =$ 2 — $\frac{8}{10}$ → 1 2
7. $\frac{7}{8} \div \frac{7}{16} =$ 2 — $\frac{7}{8}$ → 1 2
8. $\frac{10}{16} \div \frac{1}{8} =$ 5 — $\frac{10}{16}$ →

Quotients with Remainders: Whole Number Dividends Worksheet 1

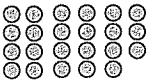
Sometimes the answers to division problems do not come out evenly. They have remainders. How do remainders work in problems when the divisor is a whole number?

In the problem 22 ÷ 4 = ___:



What is the size of the group being made? 4
How many groups of 4 are in 22? 5
What is the remainder? 2
The remainder is what fraction of the group being made? $\frac{2}{4}$ or $\frac{1}{2}$
Write the quotient as a mixed number. $5\frac{1}{2}$

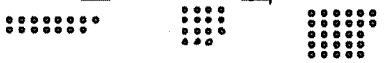
In the problem 23 ÷ 4 = ___:



What is the size of the group being made? 4
How many groups of 4 are in 23? 5
What is the remainder? 3
The remainder is what fraction of the group being made? $\frac{3}{4}$
Write the quotient as a mixed number. $5\frac{3}{4}$

Solve these problems. Write the quotient as a mixed number.

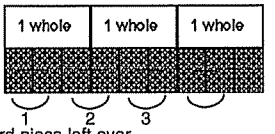
1. 15 ÷ 2 = $7\frac{1}{2}$ 2. 15 ÷ 4 = $3\frac{3}{4}$ 3. 27 ÷ 5 = $5\frac{2}{5}$ 4. 23 ÷ 3 = $7\frac{2}{3}$



When the divisor is a fraction, how do you think remainders will work?

Try this problem. 3 ÷ $\frac{2}{3}$ = ___.

Count the groups of $\frac{2}{3}$.



There are 4 groups of $\frac{2}{3}$ with a one third piece left over.

That $\frac{1}{3}$ remainder is what fraction of the group we are making? $\frac{1}{2}$

We are making groups of $\frac{2}{3}$. $\frac{1}{3}$ is what part of $\frac{2}{3}$? $\frac{1}{2}$

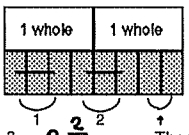
Answer
The answer is four and one half.
One third is one half of two thirds.

3 ÷ $\frac{2}{3}$ = $4\frac{1}{2}$

Quotients with Remainders: Whole Number Dividends Worksheet 2

Another more difficult example:

2 ÷ $\frac{3}{4}$ = ___



There are 2 groups of $\frac{3}{4}$ with two pieces left over.

Therefore, 2 ÷ $\frac{3}{4}$ = $2\frac{2}{3}$

These two pieces are what fraction the group being made? two thirds

Use fraction pieces to help you figure out the answers to these problems. The pictures are to remind you how to arrange the pieces. Watch for patterns.

1. 1 ÷ $\frac{2}{3}$ = $1\frac{1}{2}$ — 1 whole
2. 2 ÷ $\frac{2}{3}$ = 3 — 1 whole, 1 whole
3. 1 ÷ $\frac{3}{8}$ = $2\frac{2}{3}$ — 1 whole
4. 2 ÷ $\frac{3}{8}$ = $5\frac{1}{3}$ — 1 whole, 1 whole
5. 2 ÷ $\frac{5}{6}$ = $1\frac{1}{5}$ — 1 whole
6. 2 ÷ $\frac{5}{6}$ = $2\frac{2}{5}$ — 1 whole, 1 whole

Quotients with Remainders: Fractional Dividends Worksheet 1

We have been working with problems which divided fractions into whole numbers. Now we will look at problems which divide fractions into fractions. This may sound hard to you. Fortunately the basic ways of doing problems with whole numbers also works with fractions.

All you have to do is count the number of times one fraction fits into another fraction.

In the problem $\frac{1}{2} \div \frac{1}{3}$ = ___, the question is, "How many groups of $\frac{1}{3}$ are in $\frac{1}{2}$?"

Take out the $\frac{1}{2}$ and $\frac{1}{3}$ fraction pieces. $\frac{1}{2}$ $\frac{1}{3}$

Place them on the table like this: $\frac{1}{2}$ / $\frac{1}{3}$ Count how many groups of $\frac{1}{3}$ fit in under the $\frac{1}{2}$.

You can see there is one group of $\frac{1}{3}$ and a little more.

Find a fraction piece that will fill the space left under the $\frac{1}{2}$ piece. $\frac{1}{2}$ / $\frac{1}{3}$

The piece that fits in the space is the $\frac{1}{6}$ piece.

Is the answer $\frac{1}{6}$? no Remember, you are making groups of $\frac{1}{3}$.

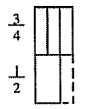
The $\frac{1}{6}$ piece that fills the space left is what fraction of the group you are making? $\frac{1}{2}$

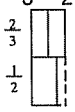
The $\frac{1}{6}$ piece is what fraction of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{3}$

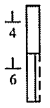
The answer to the problem $\frac{1}{2} \div \frac{1}{3}$ = ___ is 1 $\frac{1}{2}$ not 1 $\frac{1}{6}$.

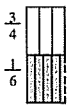
Quotients with Remainders: Fractional Dividends Worksheet 2

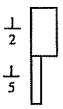
Set 1

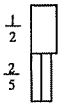
1. $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$ 

2. $\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}$ 

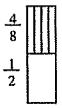
3. $\frac{1}{4} \div \frac{1}{6} = 1\frac{1}{2}$ 

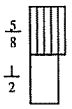
4. $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$ 

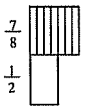
5. $\frac{1}{2} \div \frac{1}{5} = 2\frac{1}{2}$ 

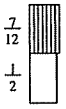
6. $\frac{1}{2} \div \frac{2}{5} = 1\frac{1}{4}$ 

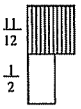
7. $\frac{1}{2} \div \frac{3}{8} = 1\frac{1}{3}$ 

8. $\frac{4}{8} \div \frac{1}{2} = 1$ 

9. $\frac{5}{8} \div \frac{1}{2} = 1\frac{1}{4}$ 

10. $\frac{7}{8} \div \frac{1}{2} = 1\frac{3}{4}$ 

11. $\frac{7}{12} \div \frac{1}{2} = 1\frac{1}{6}$ 

12. $\frac{11}{12} \div \frac{1}{2} = 1\frac{5}{6}$ 

Quotients with Remainders: Fractional Dividends Worksheet 3

Use the fraction pieces to solve these problems. Look for patterns. Find a connection between the numbers in the problem and the numbers in the answer.

Set 2

13. $\frac{3}{8} \div \frac{1}{4} = 1\frac{1}{2} = \frac{3}{2}$

14. $\frac{5}{8} \div \frac{1}{4} = 2\frac{1}{2} = \frac{5}{2}$

15. $\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2} = \frac{7}{2}$

16. $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2} = \frac{5}{2}$

17. $\frac{5}{6} \div \frac{2}{3} = 1\frac{1}{4} = \frac{5}{4}$

18. $\frac{1}{3} \div \frac{1}{4} = 1\frac{1}{3} = \frac{4}{3}$

19. $\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3} = \frac{8}{3}$

20. $\frac{5}{6} \div \frac{1}{4} = 3\frac{1}{3} = \frac{10}{3}$

21. $\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4} = \frac{9}{4}$

22. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8} = \frac{9}{8}$

23. $\frac{7}{12} \div \frac{1}{4} = 2\frac{1}{3} = \frac{7}{3}$

24. $\frac{11}{12} \div \frac{1}{4} = 3\frac{2}{3} = \frac{11}{3}$

Explain any pattern you noticed in your work. Do the patterns work on every problem? Record your discoveries in your Math Journal also.

The answer will vary. There is no 'right answer'. Do not push the pattern of the standard procedure. Just notice how the thoughts are constructed. Here are some patterns that have been found.

If the divisor has a 1 as a numerator and I can divide the denominator of the divisor into the denominator of the dividend evenly, it then equals the quotients denominator. To get the numerator of the quotient, multiply the two numerators together. $\frac{5}{8} \div \frac{1}{4} = \frac{5}{2} = 2\frac{1}{2}$ $8 \div 4 = 2$ $5 \times 1 = 5$

If the dividend is doubled and the divisor is the same, the quotient doubles.

$\frac{1}{3} \div \frac{1}{4} = 1\frac{1}{3}$ $\frac{2}{3} \div \frac{1}{4} = 2\frac{1}{3}$

Quotients with Remainders: Fractional Dividends Worksheet 4

Use the fraction pieces or a drawing to solve these problems. Look for patterns. Record each answer as both an improper fraction and a mixed number.

Set 3

25. $\frac{9}{16} \div \frac{1}{2} = \frac{9}{8} = 1\frac{1}{8}$

26. $\frac{11}{16} \div \frac{1}{2} = \frac{11}{8} = 1\frac{3}{8}$

27. $\frac{13}{16} \div \frac{1}{2} = \frac{13}{8} = 1\frac{5}{8}$

28. $\frac{15}{16} \div \frac{1}{2} = \frac{15}{8} = 1\frac{7}{8}$

29. $\frac{16}{16} \div \frac{1}{2} = \frac{32}{16}$ or $\frac{16}{8} = 2$

30. $\frac{5}{6} \div \frac{1}{2} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

31. $\frac{1}{2} \div \frac{3}{16} = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}$

32. $\frac{1}{2} \div \frac{3}{10} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

33. $\frac{1}{2} \div \frac{5}{12} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$

34. $\frac{3}{5} \div \frac{1}{2} = \frac{6}{5} = 1\frac{1}{5}$

35. $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5} = 1\frac{3}{5}$

36. $\frac{9}{10} \div \frac{1}{2} = \frac{18}{10} = \frac{9}{5} = 1\frac{4}{5}$

Quotients with Remainders: Fractional Dividends Worksheet 5

Use the fraction pieces to solve these problems. Look for patterns.

Set 4

37. $\frac{1}{2} \div \frac{5}{16} = \frac{8}{5} = 1\frac{3}{5}$

38. $\frac{3}{4} \div \frac{5}{8} = \frac{6}{5} = 1\frac{1}{5}$

39. $\frac{3}{4} \div \frac{5}{12} = \frac{9}{5} = 1\frac{4}{5}$

40. $\frac{7}{12} \div \frac{1}{3} = 1\frac{3}{4}$

41. $\frac{11}{12} \div \frac{1}{3} = 2\frac{3}{4}$

42. $\frac{11}{12} \div \frac{2}{3} = 1\frac{3}{8}$

43. $\frac{2}{3} \div \frac{5}{12} = \frac{8}{5} = 1\frac{3}{5}$

44. $\frac{5}{12} \div \frac{1}{3} = \frac{5}{4} = 1\frac{1}{4}$

45. $\frac{2}{3} \div \frac{7}{12} = \frac{8}{7} = 1\frac{1}{7}$

46. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{6} = 1\frac{1}{6}$

47. $\frac{3}{4} \div \frac{7}{12} = 1\frac{2}{7}$

48. $\frac{11}{12} \div \frac{3}{4} = 1\frac{2}{9}$

Find patterns that would help you calculate the answers. The patterns found will vary. Here are some patterns that have been found.

A pattern: $\frac{1}{2} \div \frac{5}{16}$ $16 \div 2 = 8$
$8 \times 1 = 8 \div 5 = \frac{8}{5} = 1\frac{3}{5}$

Take 2 into 16 which = 8. Take that 8 and multiply it by 1 which equals 8.
Take the 8 and divide by 5 which equals $1\frac{3}{5}$.

Or, take the denominator of the first number (dividend) and divide it into the denominator of the second number (divisor). Take that quotient and multiply it by the first numerator. Take that product and divide it by the second numerator. This pattern will work for all problems whose first denominator is smaller than, and a factor of the second denominator.

Or, change into common denominators. $\frac{1}{2} \div \frac{5}{16} = \frac{8}{16} \div \frac{5}{16} = \frac{8}{5} = 1\frac{3}{5}$

quotient = $\frac{\text{numerator of dividend}}{\text{numerator of divisor}}$

Fractional Quotients - Worksheet 1

Sometime a division problem divides a larger number into a smaller number.

Study this problem. $2 \div 4 =$ ___ or $4\overline{)2}$

The question this problems asks is: "How many fours are in two?" You may t there are no fours in two. Four does not go into two. But part of four will go or fit into two. Study this picture.

2 divided 4 

What part of the four fits under the two?*
One half of the four.

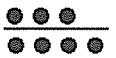
Therefore, $2 \div 4 = \frac{1}{2}$ or $4\overline{)2}^{\frac{1}{2}}$. Four "goes into" two one half of a time.

Solve these problems. Use the pictures to help you.

1. $1 \div 4 = \frac{1}{4}$ $4\overline{)1}$

1 divided 4 

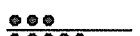
2. $3 \div 4 = \frac{3}{4}$ $4\overline{)3}$

3 divided 4 

3. $2 \div 3 = \frac{2}{3}$ $3\overline{)2}$

2 divided 3 

Show your answers in the lowest term.

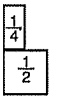
4. $1 \div 8 = \frac{1}{8}$ 
5. $2 \div 8 = \frac{1}{4}$ 
6. $3 \div 5 = \frac{3}{5}$ 
7. $9 \div 12 = \frac{3}{4}$ 
8. $6 \div 9 = \frac{2}{3}$

Fractional Quotients - Worksheet 2

It is possible to divide a larger fraction into a smaller one. What can you predict about the answer? AWV < 1

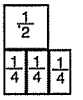
Study these problems.

$\frac{1}{4} \div \frac{1}{2} =$ ___ Place the fraction pieces on the table like this:



What part of the $\frac{1}{2}$ fits under the $\frac{1}{4}$?* $\frac{1}{2}$

$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$



What part of the $\frac{3}{4}$ fits under the $\frac{1}{2}$?** $\frac{2}{3}$

Solve these problems. Use fraction pieces to figure out the answers. Draw pictures of the pieces to show how you did it.

Set 1 Examples:

1. $\frac{3}{12} \div \frac{4}{12} = \frac{3}{4}$
2. $\frac{1}{4} \div \frac{1}{3} = \frac{3}{4}$
3. $\frac{3}{6} \div \frac{4}{6} = \frac{3}{4}$
4. $\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$

Draw the pictures.

5. $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$
6. $\frac{1}{6} \div \frac{1}{4} = \frac{2}{3}$

Set 2 Draw the pictures.

7. $\frac{1}{3} \div \frac{5}{6} = \frac{6}{15} = \frac{2}{5}$
8. $\frac{3}{8} \div \frac{1}{2} = \frac{3}{4}$
9. $\frac{3}{8} \div \frac{3}{4} = \frac{1}{2}$ $\frac{12}{24} = \frac{1}{2}$
10. $\frac{1}{2} \div \frac{3}{5} = \frac{5}{6}$

*Answer: One half **Answer: Two thirds

Introducing Reciprocals - Worksheet 1

In this exercise, write all whole numbers as fractions. Example: $3 = \frac{3}{1}$

Convert all mixed numbers to improper fractions. Example: $3\frac{1}{2} = \frac{7}{2}$

1. Study the pairs of fractions in each set.

Set 1	Set 2	Set 3
$\frac{8}{4}$ $\frac{4}{8}$	$\frac{12}{3}$ $\frac{3}{12}$	$\frac{3}{8}$ $\frac{8}{3}$

2. What pattern do you see in each set? Fractions in each set have the same numbers but they are upside down from each other.

3. Pairs of fractions that have this pattern are called reciprocals.

Write yes or no on the line to tell whether the fractions in each pair are reciprocals.

4. $\frac{2}{3}$ and $\frac{3}{2}$ yes
5. $\frac{5}{6}$ and $\frac{5}{6}$ no
6. $\frac{1}{6}$ and $\frac{6}{1}$ yes
7. $\frac{1}{4}$ and $\frac{4}{3}$ no
8. $\frac{3}{4}$ and $\frac{4}{3}$ yes
9. $\frac{5}{6}$ and $\frac{10}{12}$ no
10. $2\frac{4}{5}$ and $\frac{5}{14}$ yes
11. $\frac{3}{17}$ and $5\frac{1}{3}$ no
12. $\frac{8}{36}$ and $4\frac{1}{2}$ yes

Write three pairs of reciprocals. AWV

13. 14. 15.

Introducing Reciprocals - Worksheet 2

Write the reciprocal of each of the following numbers.

1. $\frac{5}{8}$ $\frac{8}{5}$ $\frac{5}{8} \times \frac{8}{5} = \frac{40}{40} = 1$
2. $\frac{1}{8}$ $\frac{8}{1}$ $\frac{1}{8} \times \frac{8}{1} = \frac{8}{8} = 1$
3. 6 $\frac{1}{6}$ $\frac{6}{1} \times \frac{1}{6} = \frac{6}{6} = 1$
4. $\frac{9}{4}$ $\frac{4}{9}$ $\frac{9}{4} \times \frac{4}{9} = \frac{36}{36} = 1$
5. $\frac{5}{6}$ $\frac{6}{5}$ $\frac{5}{6} \times \frac{6}{5} = 1$
6. $1\frac{1}{3}$ $\frac{3}{4}$ $\frac{4}{3} \times \frac{3}{4} = \frac{12}{12} = 1$
7. $2\frac{2}{3}$ $\frac{3}{8}$ $\frac{8}{3} \times \frac{3}{8} = \frac{24}{24} = 1$
8. $\frac{1}{12}$ $\frac{12}{1}$ $\frac{1}{12} \times \frac{12}{1} = \frac{12}{12} = 1$
9. 18 $\frac{1}{18}$ $\frac{18}{1} \times \frac{1}{18} = 1$
10. 1 $\frac{5}{3}$ $\frac{3}{5} \times \frac{5}{3} = 1$
11. $3\frac{3}{5}$ $\frac{5}{18}$ $\frac{18}{5} \times \frac{5}{18} = \frac{90}{90} = 1$
12. $\frac{15}{5}$ $\frac{5}{15}$ $\frac{15}{5} \times \frac{5}{15} = \frac{75}{75} = 1$

13. Under each pair of reciprocal fractions above, rewrite the pair as a multiplication problem. Find the product. Simplify.

Example: $\frac{5}{8} \times \frac{8}{5} = \frac{40}{40} = 1$

14. What patterns did you notice in the products? All products are 1.

15. Write this pattern as a rule. Use proper mathematical vocabulary with correct spelling, too. Add the rule to your Math Journal.
The product of any pair of reciprocal fractions is equal to one.

This rule will be used later to solve division of fraction problems.

Solving Problems with Pictures - Worksheet 1

Mathematicians can create any and all size wholes to picture fraction problems.

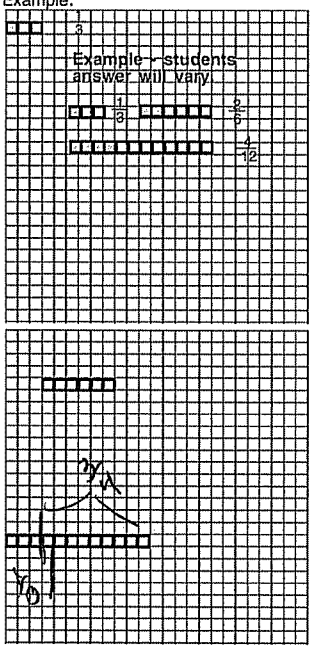
1. Make three different size box wholes and shade $\frac{1}{3}$ of each. Draw a line around the whole, then shade in the fraction. Record the fraction next to the drawing.

2. Draw four dif ferent size wholes and shade in $\frac{1}{2}$ of each one.

3. Create a whole that can show both $\frac{1}{3}$ and $\frac{1}{2}$.

4. Create a whole that can show both $\frac{3}{4}$ and $\frac{1}{3}$.

Example:



When there are two fractions in a problem, how many little boxes or fractional parts should the whole have to show both fractions? In other words, how do you find the common denominator? Multiply the two denominators or see if one denominator divides evenly into the other. If it will the denominator of the smallest sized fraction is the common denominator.

Solving Problems with Pictures - Worksheet 2

dividend ÷ divisor = quotient

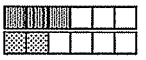
One way to solve division of fraction problems is with a drawing.

The example problem is: $\frac{1}{2} \div \frac{1}{3} =$ ____. You solved this with manipulatives on page 20.

The whole needs to have _____ parts so both fractions can easily be shown. _____ is the Least Common Denominator for one half and one third, so it is best to make the whole have _____ squares in it.

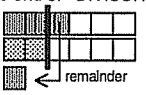
The top row shows the $\frac{1}{2}$.

The bottom row shows the $\frac{1}{3}$.



Draw a heavy line at the end of DIVISOR. The divisor is shown on the bottom whole.

$\frac{1}{2} \div \frac{1}{3} =$ ____.



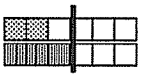
The question this problem asks is, "How many groups of one third are in one half? The drawing shows that there is one group of one third and one square left over. That one square is what part of the divisor?*

The drawing shows the answer to the problem is $1\frac{1}{2}$. There are $1\frac{1}{2}$ groups $\frac{1}{3}$ in $\frac{1}{2}$.

Now reverse the dividend and the divisor $\frac{1}{3} \div \frac{1}{2} =$ ____

The top row shows the $\frac{1}{3}$.

The bottom row shows the $\frac{1}{2}$.



What part of the one half is under the one third? $\frac{2}{3}$**

$\frac{2}{3}$ of the $\frac{1}{2}$ fits into $\frac{1}{3}$. So the answer to the problem is $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

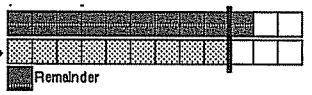
*Answer: One half.

**Answer: $\frac{2}{3}$ of the $\frac{1}{2}$ fits into $\frac{1}{3}$. So the answer to the problem is $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$

Solving Problems with Pictures - Worksheet 3

Here is another example of how to solve a division of fraction problem with a picture.

The example problem is: $\frac{5}{6} \div \frac{3}{4} =$



The Common Denominator for these two fractions is twelve. This means the whole will have twelve squares in it.

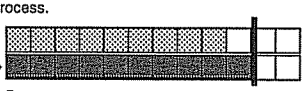
The $\frac{5}{6}$ is shown in the top row.

The $\frac{3}{4}$ is shown in the bottom row.

You can see that the $\frac{3}{4}$ fits under the $\frac{5}{6}$ one time with a small remainder. The remainder square is one ninth of the group of $\frac{3}{4}$. Therefore, $\frac{5}{6} \div \frac{3}{4} = 1\frac{1}{9}$.

Now reverse the process.

$\frac{3}{4} \div \frac{5}{6} =$



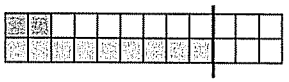
Only part of the $\frac{5}{6}$ fits under the $\frac{3}{4}$. Notice that the $\frac{5}{6}$ uses ten squares. Only nine of the ten squares fit under the $\frac{3}{4}$

This show that $\frac{9}{10}$ of the group of $\frac{5}{6}$ is in $\frac{3}{4}$ Therefore, $\frac{3}{4} \div \frac{5}{6} = \frac{9}{10}$

Use pictures to solve these

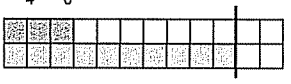
1. $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$

2. $\frac{1}{6} \div \frac{3}{4} = \frac{2}{9}$



3. $\frac{5}{6} \div \frac{1}{4} = 3\frac{1}{3}$

4. $\frac{1}{4} \div \frac{5}{6} = \frac{3}{10}$



Solving Problems with Pictures - Worksheet 4

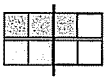
Solve these problems with pictures. Change any quotients that are mixed numbers to improper fractions to see a neat pattern.

1. $\frac{2}{3} \div \frac{1}{3} = 2 = \frac{2}{1}$

2. Reverse problem 1

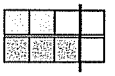
$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$

3. $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2} = \frac{3}{2}$

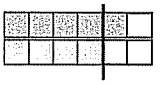


4. Reverse problem 3

$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$

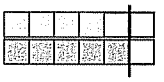


5. $\frac{5}{6} \div \frac{2}{3} = 1\frac{1}{4} = \frac{5}{4}$

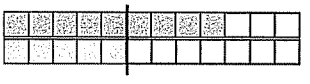


6. Reverse problem 5

$\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$

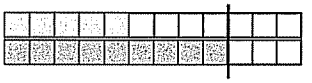


7. $\frac{3}{4} \div \frac{5}{12} = 1\frac{4}{5} = \frac{9}{5}$

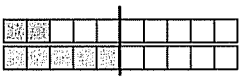
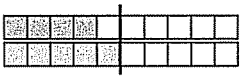
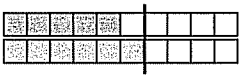
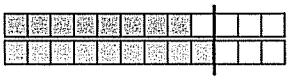


8. Reverse problem 7

$\frac{5}{12} \div \frac{3}{4} = \frac{5}{9}$



Solving Problems with Pictures - Worksheet 5

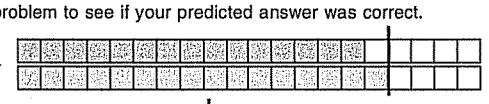
1. $\frac{1}{5} \div \frac{1}{2} = \frac{2}{5}$ 
2. $\frac{2}{5} \div \frac{1}{2} = \frac{4}{5}$ 
3. $\frac{1}{2} \div \frac{3}{5} = \frac{5}{6}$ 
4. $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ 

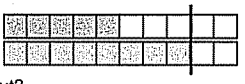
What number pattern do you see in every problem that will allow you to calculate the answer? Multiply the denominator of the dividend by the numerator of the divisor which becomes the quotient's denominator. Multiply the numerator of the dividend by the denominator of the divisor which becomes the numerator of the quotient.

Use only your pattern to predict the answer to the next two problems.

5. $\frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$ Predicted answer _____
6. $\frac{1}{2} \div \frac{4}{5} = \frac{5}{8}$ Predicted answer _____

Now draw each problem to see if your predicted answer was correct.

$\frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$ 

$\frac{1}{2} \div \frac{4}{5} = \frac{5}{8}$ 

What did you find out?

My pattern worked ↘↗↘

Finding Patterns - Worksheet 1

You have found that you can use fraction pieces or pictures to find the answers to division of fractions problems. Doing this helps you understand the sense of your answers, but it is a slow process. It is much faster to calculate the answers. In order to calculate the answers, we must find a number pattern that works for all the problems.

First we will use our fraction pieces or drawings to find the answers. Then we will look at all the numbers in the problem and try to spot a pattern that will allow us to calculate the answers.

Sometimes you will find a pattern that works for one group of problems but not for another group. When this happens, try to define how the two kinds of problems are different. Sometimes you can make a small change in how you work the pattern to make it work for the new group of problems.

Eventually, we want to find one pattern that works for all division of fractions problems. This is not always easy, but the search is interesting and fun. This is the work that mathematicians do.

GROUP 1

Here is your first group of similar problems. Find the answers with your fractions pieces or with drawings. Look for a pattern.
All of the answers to the problems in this group will be fractions.

1. $\frac{1}{6} \div \frac{1}{3} = \frac{1}{2}$
2. $\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$
3. $\frac{1}{8} \div \frac{1}{2} = \frac{1}{4}$
4. $\frac{1}{12} \div \frac{1}{6} = \frac{1}{2}$
5. $\frac{1}{12} \div \frac{1}{3} = \frac{1}{4}$
6. $\frac{1}{12} \div \frac{1}{4} = \frac{1}{3}$

What patterns do you see?

See notes on the next page.

How can you calculate the answers?

Test your pattern on these two problems. Use pattern to get the answers. Then check with the fraction paper pieces.

7. $\frac{1}{12} \div \frac{1}{2} = \frac{1}{6}$ Does your pattern work? _____
8. $\frac{1}{16} \div \frac{1}{8} = \frac{1}{2}$

Finding Patterns - Worksheet 2

In the first group of problems, all of the numerators were 1. You probably noticed that the second denominator could be evenly divided into the first denominator.

What will happen if *one* of the numerators is changed to a number *other than one*? Do you think your pattern will still work? _____ AWV

Try it and find out.

Predict what the answer to this problem will be.

1. $\frac{3}{8} \div \frac{1}{2} = $ ___ _____ prediction $\frac{3}{4}$ check

Now check your answer with the fraction pieces.
Does your old pattern work? _____ AWV

Solve these problems with fraction pieces or drawings and look for a pattern.

GROUP 2

2. $\frac{5}{12} \div \frac{1}{2} = \frac{10}{12} = \frac{5}{6}$
3. $\frac{3}{12} \div \frac{1}{2} = \frac{6}{12} = \frac{1}{2}$
4. $\frac{3}{16} \div \frac{1}{2} = \frac{6}{16} = \frac{3}{8}$
5. $\frac{5}{16} \div \frac{1}{2} = \frac{10}{16} = \frac{5}{8}$
6. $\frac{7}{16} \div \frac{1}{2} = \frac{14}{16} = \frac{7}{8}$

Can you see a pattern that would allow you to calculate the answers to these problems? _____

Explain it. _____

Test the pattern on this problem Calculate what the answer should be. Then check it.

7. $\frac{3}{10} \div \frac{1}{2} = \frac{3}{5}$

Did your pattern work? _____

Challenge. Use the pattern you found above to calculate the answers to these problems. First predict what the answer should be. Then check it out with the fraction pieces.

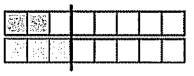
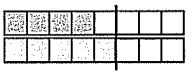
8. $\frac{5}{6} \div \frac{1}{3} = $ ___ _____ prediction $\frac{5}{2} = 2\frac{1}{2}$ check
9. $\frac{5}{12} \div \frac{1}{4} = $ ___ _____ prediction $\frac{20}{12} = 1\frac{8}{12} = 1\frac{2}{3}$ check

Hint: Check to see if the value of your predicted answer equals the answer the pieces gave. The answers may be in a different form.

Finding Patterns - Worksheet 3

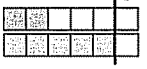
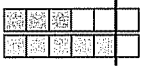
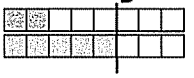
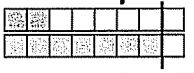
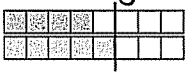
Group 3

Predict what the answer to these two problems will be.

1. $\frac{1}{4} \div \frac{3}{8} = $ ___ _____ prediction $\frac{2}{3}$ check 
2. $\frac{1}{2} \div \frac{5}{8} = $ ___ _____ prediction $\frac{4}{5}$ check 

Does your old pattern work?

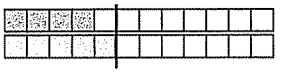
Solve these problems with a drawing. Watch for a pattern.

3. $\frac{1}{3} \div \frac{5}{6} = \frac{2}{5}$ 
4. $\frac{1}{2} \div \frac{5}{6} = \frac{3}{5}$ 
5. $\frac{1}{4} \div \frac{5}{8} = \frac{2}{5}$ 
6. $\frac{1}{4} \div \frac{7}{8} = \frac{2}{7}$ 
7. $\frac{1}{2} \div \frac{5}{8} = \frac{4}{5}$ 

Do you see a pattern that could help calculate these answers? _____
If so, explain it. It not, keep thinking.

See notes on the next page.

If you see a pattern, use it to predict the answer to the next problem. Then check it.

8. $\frac{1}{3} \div \frac{5}{12} = $ ___ _____ prediction $\frac{4}{5}$ check 

Reasoning behind Finding Patterns worksheets

Finding Patterns - Worksheet 1 (page 34)

Group 1

A pattern that most students will see is to divide the second denominator into the first one to make the denominator of the answer. Bring the top one over for the numerator of the answer.

Note! Each of these patterns can be directly linked to the standard invert and multiply algorithm. Although some students will have discovered this from their explorations and will practice that pattern with these problems, please do not push other students to see that pattern. They need to explore their patterns first. By the end, all students will be able to derive the standard algorithm and understand why it works.

Finding Patterns - Worksheet 2 (page 35)

The old pattern will work for denominators. There is a slight change for numerators. Bring larger numerators into the numerator of the answer.

Group 2 $\frac{5}{12} \div \frac{1}{2} = \frac{5}{6}$

Calculated answers will come out as improper fractions. Manipulated answers will come out as mixed, lowest term fraction.

Finding Patterns - Worksheet 3 (page 36)

These problems are different because now the larger numbers in the numerator and denominators are in the divisor rather than the dividend.

Group 2 $\frac{5}{12} \div \frac{1}{2}$ Group 3 $\frac{1}{2} \div \frac{5}{6}$

Larger numbers

Old pattern will not work.

dividend ÷ divisor = quotient

quotient

divisor | dividend

A pattern to see for calculation is to take the numerator of the divisor and put it into the denominator of the answer. Divide the denominator of the dividend into the denominator of the divisor to determine the numerator of the quotient.

Discuss "why" does it make sense, in #2, 3, 4, and 5, that the denominator of the answer is fifths. For example: $\frac{1}{2} \div \frac{5}{8} = \frac{4}{5}$

Finding Patterns - Worksheet 4 (page 37)

Group 3

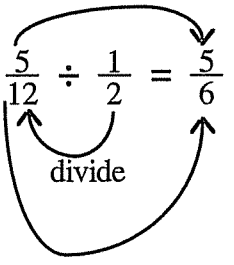
Some students will notice that if you change the fractions to have a common denominator (which is what the drawings do), you can simply divide the numerator to get the answer.

$\frac{1}{2} \div \frac{7}{12} = \frac{6}{12} \div \frac{7}{12}$

$6 \div 7$ or $\frac{6}{7}$

This one is hard.

Pattern for groups 1 & 2 is:



Finding Patterns - Worksheet 5 (page 38)

Pattern for Group 4 involves using the numerator of the divisor with the two denominators.

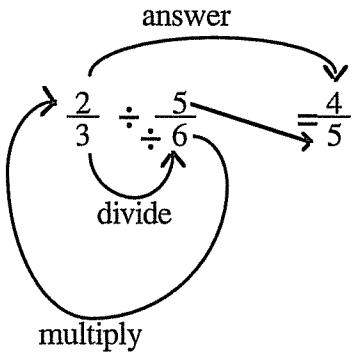
$\frac{5}{8} \div \frac{3}{4} = \frac{5}{6}$

Divide the denominator of the divisor into denominator of the dividend, multiply it by the numerator of the divisor to be the denominator of the quotient. Place the numerator of the dividend as the numerator of the quotient.

Finding Patterns - Worksheet 6 (page 39)
Group 5
The pattern goes like this: $\frac{1}{2} \div \frac{7}{12} = \frac{6}{7}$

divide

For this one the numerator of the dividend works with the two denominators as they did for Group 4.

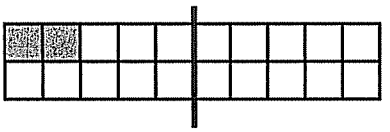


Divide the denominator of the dividend into denominator of the divisor. Multiply that quotient by the numerator of the dividend to equal the numerator of the quotient. Move the denominator of the divisor to the denominator of the quotient.

Finding Patterns - Worksheet 7 (page 40)
Group 6

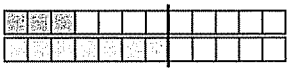
The pattern students find will vary. What is important is that she tests it on problems and verifies it works. One pattern is shown by the drawing. The squares colored in are the answer or quotient. For example:

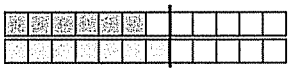
$\frac{1}{5} \div \frac{1}{2} = \frac{2}{5}$

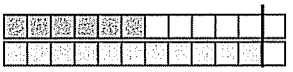


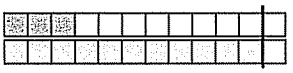
This set also has the familiar  pattern. $\frac{2}{3} \times \frac{3}{4} = \frac{18}{9}$

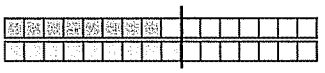
Finding Patterns - Worksheet 4

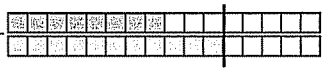
9. $\frac{1}{4} \div \frac{7}{12} =$ ___ $\frac{12}{28}$ by pattern $\frac{3}{7}$ check 

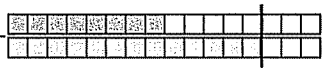
10. $\frac{1}{2} \div \frac{7}{12} =$ ___ $\frac{12}{14}$ by pattern $\frac{6}{7}$ check 

11. $\frac{1}{2} \div \frac{11}{12} =$ ___ $\frac{12}{22}$ by pattern $\frac{6}{11}$ check 

12. $\frac{1}{4} \div \frac{11}{12} =$ ___ $\frac{12}{44}$ $\frac{6}{22}=\frac{3}{11}$ by pattern check 

13. $\frac{1}{2} \div \frac{9}{16} =$ ___ $\frac{16}{18}$ by pattern $\frac{8}{9}$ check 

14. $\frac{1}{2} \div \frac{11}{16} =$ ___ $\frac{16}{22}$ by pattern $\frac{8}{11}$ check 

15. $\frac{1}{2} \div \frac{13}{16} =$ ___ $\frac{16}{26}$ by pattern $\frac{8}{13}$ check 

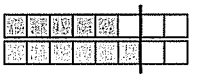
16. $\frac{1}{2} \div \frac{15}{16} =$ ___ $\frac{16}{30}$ by pattern $\frac{8}{15}$ check 

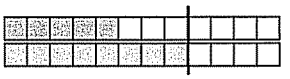
Finding Patterns - Worksheet 5

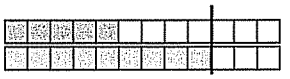
Group 4

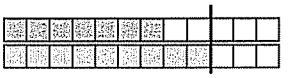
Solve using a drawing. The pattern to look for is related to the one you found for the problems in Groups 1 and 2.
Go back and study the patterns you found for those groups.

Caution! This pattern is not so easy to see. The problems in Groups 1 and 2 had at least one 1 in the numerators. These problems have no ones in the numerators. Can you adapt your pattern to work for these? It is possible.

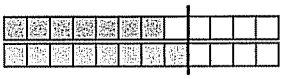
1. $\frac{5}{8} \div \frac{3}{4} =$ $\frac{5}{6}$ 

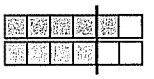
2. $\frac{5}{12} \div \frac{2}{3} =$ $\frac{5}{8}$ 

3. $\frac{5}{12} \div \frac{3}{4} =$ $\frac{5}{9}$ 

4. $\frac{7}{12} \div \frac{3}{4} =$ $\frac{7}{9}$ 

If you think you see the pattern, predict the answer to the next two problems.

5. $\frac{7}{12} \div \frac{2}{3} = \frac{21}{24} = \frac{7}{8}$ ___ predict ___ check 

6. $\frac{5}{6} \div \frac{2}{3} = \frac{5}{4} = 1\frac{1}{4}$ ___ predict ___ check 

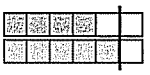
If your predicted answer is different from the one in which you got from the picture, check to see if the prediction is equal in value to the answer you got from the picture.

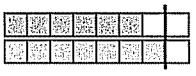
Finding Patterns - Worksheet 6

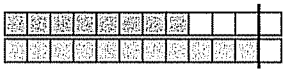
Group 5

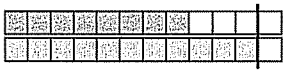
Solve using a drawing. The pattern to look for is related to the one you found for the problems in Group 3.
Go back and study the patterns you found for that group.

Caution! This pattern is not so easy to see. The problems in Group 3 had at least one 1 in the numerators. These problems have no ones in the numerators. Can you adapt your pattern to work for these? It is possible.

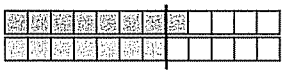
1. $\frac{2}{3} \div \frac{5}{6} =$ $\frac{4}{5}$ 

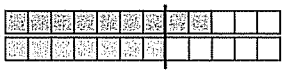
2. $\frac{3}{4} \div \frac{7}{8} =$ $\frac{6}{7}$ 

3. $\frac{2}{3} \div \frac{11}{12} =$ $\frac{8}{11}$ 

4. $\frac{3}{4} \div \frac{11}{12} =$ $\frac{9}{11}$ 

If you think you see the pattern, predict the answer to the next two problems.

5. $\frac{2}{3} \div \frac{7}{12} = \frac{8}{7} = 1\frac{1}{7}$ ___ predict ___ check 

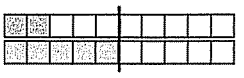
6. $\frac{3}{4} \div \frac{7}{12} = \frac{9}{7} = 1\frac{2}{7}$ ___ predict ___ check 

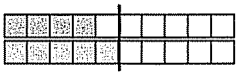
If your predicted answer is different from the one in which you got from the picture, check to see if the prediction is equal in value to the answer you got from the picture.

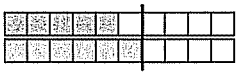
Finding Patterns - Worksheet 7

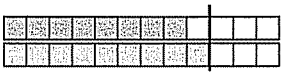
Group 6

Solve using a drawing. This pattern is fairly easy to see.

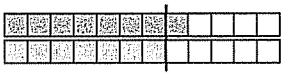
1. $\frac{1}{5} \div \frac{1}{2} =$ $\frac{2}{5}$ 

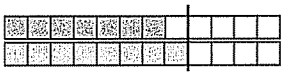
2. $\frac{2}{5} \div \frac{1}{2} =$ $\frac{4}{5}$ 

3. $\frac{1}{2} \div \frac{3}{5} =$ $\frac{5}{6}$ 

4. $\frac{2}{3} \div \frac{3}{4} =$ $\frac{8}{9}$ 

If you think you see the pattern, predict the answer to the next two problems.

5. $\frac{2}{3} \div \frac{7}{12} =$ $\frac{24}{21}$ ___ predict $1\frac{1}{7}$ check 

6. $\frac{7}{12} \div \frac{2}{3} =$ $\frac{21}{24}$ ___ predict $\frac{7}{8}$ check 

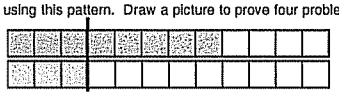
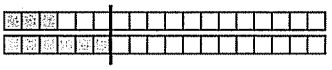
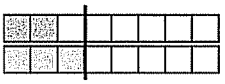
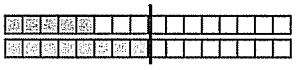
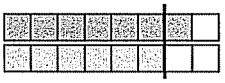
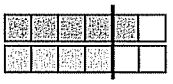
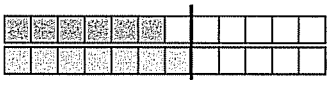
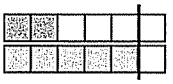
If your predicted answer is different from the one in which you got from the picture, check to see if the prediction is equal in value to the answer you got from the picture.

Finding Patterns - Worksheet 8

Group 7

Record the pattern you found. Show how the pattern works with a diagram.

Solve these problems using this pattern. Draw a picture to prove four problems.

1. $\frac{2}{3} \div \frac{1}{4} =$ 
2. $\frac{1}{6} \div \frac{1}{3} =$ 
3. $\frac{1}{4} \div \frac{3}{8} =$ 
4. $\frac{5}{16} \div \frac{1}{2} =$ 
5. $\frac{7}{8} \div \frac{3}{4} =$ 
6. $\frac{5}{6} \div \frac{2}{3} =$ 
7. $\frac{1}{2} \div \frac{7}{12} =$ 
8. $\frac{1}{3} \div \frac{5}{6} =$ 

Patterns with Reciprocals - Worksheet 1

Review Families of Facts for multiplication and division.

9 4 36	7 56 8
9 x 4 = 36	7 x 8 = 56
4 x 9 = 36	8 x 7 = 56
36 ÷ 9 = 4	56 ÷ 7 = 8
36 ÷ 4 = 9	56 ÷ 8 = 7

Multiplication and division are called inverse operations. What do you think inverse operation means? opposite

Add 'inverse' to the glossary in your Math Journal.

==========

Fraction Puzzle with Inverse Operations

Put the inverse operator, 'x' or '÷', in the second operator triangle. Then find a number to put in the blank that will make a true number sentence.

Examples: 8 ÷ 4 = 2; 8 x $\frac{1}{4}$ = 2; 12 ÷ 3 = 4; 12 x $\frac{1}{3}$ = 4

Solve these puzzles.

6 ÷ 2 = 3; 6 x $\frac{1}{2}$ = 3; 12 x 3 = 36; 12 ÷ $\frac{1}{3}$ = 36

Explain the pattern you found. Dividing by a whole number gives the same answer as multiplying by the upside down fraction, Or 6 one half a time or $\frac{1}{2}$ of 6 is the same as*

Write the whole numbers operators as fractions, and show the puzzle pair relationship like this: $8 \div \frac{4}{1} = 8 \times \frac{1}{4} = 2$

Write the other pairs in this format;

$8 \div \frac{4}{1} = 8 \times \frac{1}{4} = 2$ $\quad 12 \div 3 = 12 \times \frac{1}{3} = 4$

$6 \div \frac{2}{1} = 6 \times \frac{1}{2} = 3$ $\quad 12 \div \frac{1}{3} = 12 \times 3 = 36$

*dividing the 6 into 2 groups.

Patterns with Reciprocals - Worksheet 2

Fill in the blanks.

Write all whole number operators such as x 4 as fractions x $\frac{4}{1}$.

Example:

1. $12 \div \frac{6}{1} = 2$
   $12 \times \frac{1}{6} = 2$
   $6 \div \frac{12}{1} = \frac{1}{2}$
   $6 \times \frac{1}{12} = \frac{1}{2}$
2. $10 \div \frac{5}{1} = 2$
   $10 \times \frac{1}{5} = 2$
   $5 \div \frac{10}{1} = \frac{1}{2}$
   $5 \times \frac{1}{10} = \frac{1}{2}$
3. $6 \div \frac{3}{1} = 2$
   $6 \times \frac{1}{3} = 2$
   $3 \div \frac{6}{1} = \frac{1}{2}$
   $3 \times \frac{1}{6} = \frac{1}{2}$
4. $18 \div \frac{9}{1} = 2$
   $18 \times \frac{1}{9} = 2$
   $9 \div \frac{18}{1} = \frac{1}{2}$
   $9 \times \frac{1}{18} = \frac{1}{2}$

5. Use the pattern to solve harder problems.

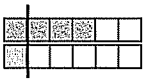
Set 1

$3 \div \frac{1}{4} = 12$

$3 \times \frac{4}{1} = 12$

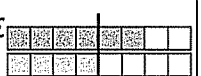
Set 2

$\frac{2}{3} \div \frac{1}{6} = 4$

$\frac{2}{3} \times \frac{6}{1} = 4$ 

Set 3

$\frac{3}{4} \div \frac{1}{2} = \frac{6}{4} = 1\frac{1}{2}$

$\frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{1}{2}$ 

Circle the problem in each set that was the easiest to do. What did you discover?

The second problem is easier because it's easier to do multiplication problems. The answers are the same.

Using Multiplication to Solve Problems - Worksheet 1

Here is a review of the very interesting relationship between inverse operations, multiplication, division, and reciprocal fractions.

Remember to show all whole number operators , e.g., x 4 as fractions, $\frac{4}{1}$.

Put in the correct sign and fraction to make these true number sentences.

$2\overline{)10}$ $\quad 10 \div \frac{2}{1} = 5 \rightarrow 10 \times \frac{1}{2} = 5$

$4\overline{)12}$ $\quad 12 \div \frac{4}{1} = 3 \rightarrow 12 \times \frac{1}{4} = 3$

$6\overline{)48}$ $\quad 48 \div \frac{6}{1} = 8 \rightarrow 48 \times \frac{1}{6} = 8$

Does this pattern work with fractions too? guess yes

Try it.

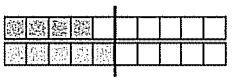
Does $\frac{1}{2} \div \frac{1}{4} = \frac{2}{1}$ also equal $\frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$? yes (yes or no)

Does this pattern also work on a more complicated fraction? yes

Use a pattern to get the answer

$\frac{2}{5} \div \frac{1}{2} = \frac{4}{5}$ Does $\frac{2}{5} \times \frac{2}{1} = \frac{4}{5}$ equal the same answer? yes

Draw a picture to prove the answer.



You have learned that sometimes the pattern and drawings produce answers different forms, but they are equivalent.

Since $\frac{2}{5} \div \frac{1}{2} = (\frac{2}{5} \times \frac{2}{1})$ either one can be used to find the answer

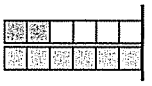
Circle the one that is the easiest to solve.

Using Multiplication to Solve Problems - Worksheet 2

Let's test this relationship one more time with patterns and drawings.

What part of 2 fits into $\frac{2}{3}$? $\frac{2}{3} \div 2 = \frac{2}{6}$ pattern

$\frac{2}{3} \div 2 =$ _____ drawing



Now if the new pattern holds true, we should be able to find the answer to $\frac{2}{3} \div 2 =$ _____ with multiplication. Try it.

$\frac{2}{3} \div \frac{2}{1} = \frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$

$\frac{2}{6} = \frac{1}{3}$

Does it work? yes

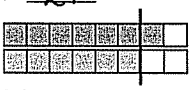
Which problem is easiest to do? $\frac{2}{3} \div \frac{2}{1}$ or $\frac{2}{3} \times \frac{1}{2}$

Now to try it with a more complicated problem to test if it works again.

Solve. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{6}$

Does $\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3}$? $\frac{28}{24} = \frac{7}{6}$

Prove it with a drawing.

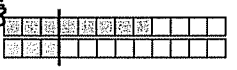


Explain what you found out. The divisor can be flipped over, $\frac{3}{4} \rightarrow \frac{4}{3}$, and then multiply the dividend by the flipped over divisor (or reciprocal) to get the answer.

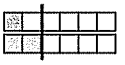
Using Multiplication to Solve Problems - Worksheet 3

Solve these problems. Simplify all answers. You must prove each answer with a drawing.

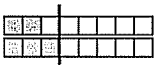
1. $\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$ pattern / drawing



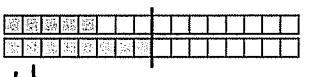
2. $\frac{1}{6} \div \frac{1}{3} = \frac{1}{6} \times \frac{3}{1} = \frac{3}{6} = \frac{1}{2}$ pattern / drawing



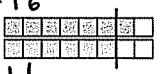
3. $\frac{1}{4} \div \frac{3}{8} = \frac{1}{4} \times \frac{8}{3} = \frac{8}{12} = \frac{2}{3}$ pattern / drawing



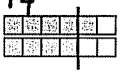
4. $\frac{5}{16} \div \frac{1}{2} = \frac{5}{16} \times \frac{2}{1} = \frac{10}{16} = \frac{5}{8}$ pattern / drawing



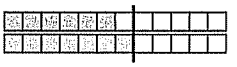
5. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3} = \frac{28}{24} = \frac{7}{6} = 1\frac{1}{6}$ pattern / drawing



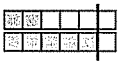
6. $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = \frac{5}{4} = 1\frac{1}{4}$ pattern / drawing



7. $\frac{1}{2} \div \frac{7}{12} = \frac{1}{2} \times \frac{12}{7} = \frac{12}{14} = \frac{6}{7}$ pattern / drawing



8. $\frac{1}{3} \div \frac{5}{6} = \frac{1}{3} \times \frac{6}{5} = \frac{6}{15} = \frac{2}{5}$ pattern / drawing



Standard Procedure - Worksheet 1
Cuisenaire Rods

1

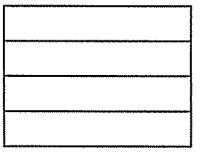
If the yellow rod is equal to one,

then what rod is $\frac{1}{5}$? white

then what rod is $\frac{2}{5}$? red

then what rod is $\frac{3}{5}$? light green

then what rod is $\frac{4}{5}$? purple



$4 \div \frac{1}{5} = 20$

$4 \times \frac{5}{1} = 20$

$4 \div \frac{2}{5} = 10$ $\quad 4 \times \frac{5}{2} = \frac{20}{2} = 10$

$4 \div \frac{3}{5} = 6\frac{2}{3}$ $\quad 4 \times \frac{5}{3} = 6\frac{2}{3}$

$4 \div \frac{4}{5} = 5$ $\quad 4 \times \frac{5}{4} = \frac{20}{4} = 5$

1

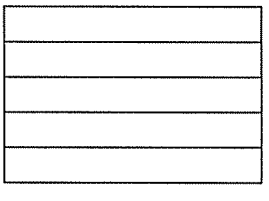
The black rod is equal to one,

then what rod is $\frac{2}{7}$? red

then what rod is $\frac{3}{7}$? light green

then what rod is $\frac{4}{7}$? purple

then what rod is $\frac{5}{7}$? yellow



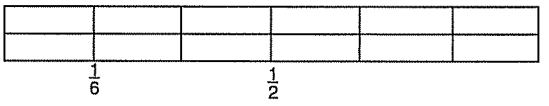
$5 \div \frac{2}{7} = 17\frac{1}{2}$

$5 \times \frac{7}{2} = \frac{35}{2} = 17\frac{1}{2}$

Build it. $5 \div \frac{2}{7} = 17\frac{1}{2}$ $\quad 5 \div \frac{3}{7} = 15$ $\quad 5 \div \frac{4}{7} = 8\frac{3}{4}$ $\quad 5 \div \frac{5}{7} = 7$

Calculate it. $5 \times \frac{7}{2} = \frac{35}{2} = 17\frac{1}{2}$ $\quad 5 \times \frac{7}{3} = \frac{35}{3} = 15$ $\quad 5 \times \frac{7}{4} = \frac{35}{4} = 8\frac{3}{4}$ $\quad 5 \times \frac{7}{5} = \frac{35}{5} = 7$

Standard Procedure - Worksheet 2
Cuisenaire Rods



Build it.	Calculate it.	Build it.	Calculate it.
$\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} = \frac{6}{2} = 3$		$\frac{1}{6} \div \frac{1}{2} = \frac{1}{6} \times \frac{2}{1} = \frac{2}{6} = \frac{1}{3}$	
$\frac{1}{2} \div \frac{2}{6} = \frac{1}{2} \times \frac{6}{2} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$		$\frac{2}{6} \div \frac{1}{2} = \frac{2}{6} \times \frac{2}{1} = \frac{4}{6} = \frac{2}{3}$	
$\frac{1}{2} \div \frac{3}{6} = \frac{1}{2} \times \frac{6}{3} = \frac{6}{6} = 1$		$\frac{3}{6} \div \frac{1}{2} = \frac{3}{6} \times \frac{2}{1} = \frac{6}{6} = 1$	
$\frac{1}{2} \div \frac{4}{6} =$ $\times \frac{6}{4} = \frac{6}{8} = \frac{3}{4}$		$\frac{4}{6} \div \frac{1}{2} = \frac{4}{6} \times \frac{2}{1} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$	
$\frac{1}{2} \div \frac{5}{6} = \frac{1}{2} \times \frac{6}{5} = \frac{6}{10} = \frac{3}{5}$		$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{10}{6} = 1\frac{4}{6} = 1\frac{2}{3}$	

This process of solving a division problem and turning it into a multiplication problem is commonly called 'invert and multiply'. How would you describe it using the word 'reciprocal'?

Flip the divisor over, that makes it the reciprocal of this divisor, and multiply it by the dividend.

Standard Procedure - Worksheet 3

Describe the standard producure for dividing fraction (which you have been doing). _____

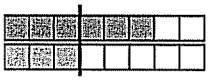
_____

Solve using the standard procedure for dividing fractions.

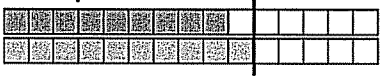
1. $8 \div \frac{3}{4} = 10\frac{2}{3}$
2. $10 \div \frac{3}{5} = 16\frac{2}{3}$
3. $\frac{3}{4} \div \frac{3}{8} = 2$
4. $\frac{5}{16} \div \frac{1}{3} = \frac{15}{16}$
5. $\frac{7}{8} \div \frac{3}{4} = 1\frac{1}{6}$
6. $\frac{3}{5} \div \frac{2}{3} = \frac{9}{10}$
7. $\frac{3}{7} \div \frac{7}{12} = \frac{36}{49}$
8. $\frac{1}{3} \div \frac{5}{12} = \frac{4}{5}$
9. $\frac{2}{5} \div \frac{1}{4} = 1\frac{3}{5}$
10. $\frac{5}{6} \div \frac{4}{5} = 1\frac{1}{24}$
11. $\frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$
12. $\frac{5}{16} \div \frac{2}{3} = \frac{15}{32}$
13. $\frac{3}{8} \div \frac{3}{8} = 1$
14. $\frac{3}{5} \div \frac{6}{8} = \frac{4}{5}$

Test for Understanding

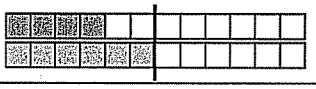
15. Draw problem 3.



16. Draw problem 6.



17. Draw problem 8.

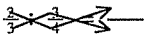


Standard Procedure - Worksheet 4

Solve this problem using the Standard Procedure of invert and multiply.

$\frac{2}{3} \div \frac{3}{4}$

This is the pattern that clever Kelly Delap, a Farm School student, figured out.



Multiply the first numerator by the second denominator. Put the product on top (numerator of the answer). Multiply the first denominator by the second numerator and put that product on the bottom (denominator of the answer).

How is Kelly Delap's pattern like the 'invert and multiply' pattern? _____

_____

How is it different? She cross multiplied instead of flipping over (inverting) the divisor.

Try the pattern on this problem.* $\frac{5}{6} \div \frac{3}{4} = \frac{20}{18}$

Now do the problem on graph paper and see if it matches.

$\frac{5}{6} = \frac{10}{12}$

$\frac{3}{4} = \frac{9}{12}$

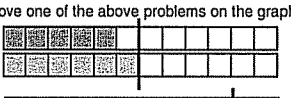
Answer = $1\frac{1}{9}$ Do the answers match? Not at first glance. Look again.

$\frac{20}{18} = 1\frac{2}{18} = 1\frac{1}{9}$

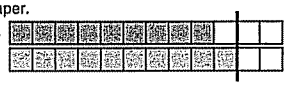
Solve these problems. 1. $\frac{5}{12} \div \frac{1}{2} = \frac{10}{12} = \frac{5}{6}$ 2. $\frac{3}{4} \div \frac{5}{6} = \frac{18}{20} = \frac{9}{10}$ $\frac{1}{4} \div \frac{5}{6} = \frac{6}{20} = \frac{3}{10}$

Prove one of the above problems on the graph paper.

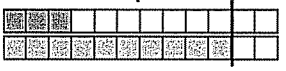
1.



2.



3.

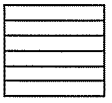


*The answer is $\frac{20}{18}$

Standard Procedure - Worksheet 5
Why It Works

Now we will figure out why the tip over and multiplication pattern works.

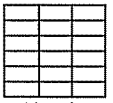
The problem is $6 \div \frac{2}{3} =$ Start with six wholes.



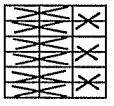
In order to find out how many groups of $\frac{2}{3}$ there are, we have to divide the six into thirds.

1. How many little pieces do we have now? 18

We have increased our pieces from 6 to 18.



2. How many times more pieces do we than before? 3
3. Why do we multiply the 6 x 3? The whole has been divided into 18 pieces.
4. Then what do we do with our 18 pieces? Group them into two thirds.
5. Why? Because we are dividing by two thirds.
6. When you group the 18 pieces into groups of two, what operation are you doing +, -, X, or ÷? Division



You are going to divide by 2.

7. How many groups? 9

This is what it looks like when recorded. $6 \div \frac{2}{3} = \frac{6 \times 3}{1 \times 2} =$

Look what happened in the last step.

8. Have you seen this before? _____ $\frac{6 \times 3}{1 \times 2} = \frac{6}{1} \times \frac{3}{2}$

So now you know how to divide fractions. $\frac{6}{1} \times \frac{3}{2} = \frac{6}{1} \div \frac{2}{3}$

This is a formal proof. Kelly Delap and Alysia discovered this. This is the first formal proof you have been shown..

Standard Procedure - Practice - Worksheet 6

Solve using the standard procedure for dividing fractions.

1. $4 \div \frac{3}{4} = \frac{4}{1} \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$
2. $5 \div \frac{3}{5} = \frac{5}{1} \times \frac{5}{3} = \frac{25}{3} = 8\frac{1}{3}$
3. $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times \frac{8}{1} = \frac{24}{4} = 6$
4. $\frac{5}{16} \div \frac{2}{3} = \frac{5}{16} \times \frac{3}{2} = \frac{15}{32}$
5. $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{14}{8} = 1\frac{6}{8} = 1\frac{3}{4}$
6. $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$
7. $\frac{3}{5} \div \frac{5}{12} = \frac{3}{5} \times \frac{12}{5} = \frac{36}{25} = 1\frac{11}{25}$
8. $\frac{1}{3} \div \frac{5}{12} = \frac{1}{3} \times \frac{12}{5} = \frac{12}{15} = \frac{4}{5}$
9. $\frac{3}{5} \div \frac{1}{4} = \frac{3}{5} \times \frac{4}{1} = \frac{12}{5} = 2\frac{2}{5}$
10. $\frac{5}{6} \div \frac{3}{5} = \frac{5}{6} \times \frac{5}{3} = \frac{25}{18} = 1\frac{7}{18}$
11. $\frac{1}{4} \div \frac{5}{8} = \frac{1}{4} \times \frac{8}{5} = \frac{8}{20} = \frac{2}{5}$
12. $\frac{5}{16} \div \frac{3}{4} = \frac{5}{16} \times \frac{4}{3} = \frac{20}{48} = \frac{5}{12}$
13. $\frac{5}{8} \div \frac{5}{8} = \frac{5}{8} \times \frac{8}{5} = \frac{40}{40} = 1$
14. $\frac{4}{5} \div \frac{6}{8} = \frac{4}{5} \times \frac{8}{6} = \frac{32}{30} = 1\frac{1}{15}$

Make your own.

15.

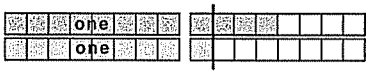
16.

Mixed Fractions - Worksheet 1

1. Solve. $5\frac{1}{3} \times 3\frac{3}{4} =$ 20 $\quad \frac{16}{3} \times \frac{15}{4} = \frac{240}{12} = 20$

Describe how to solve a problem like this. *Change both mixed numbers Into improper fractions, multiply the improper fractions, then simplify.*

2. Solve. $1\frac{1}{2} \div 1\frac{1}{8} =$ $\frac{1}{3}$

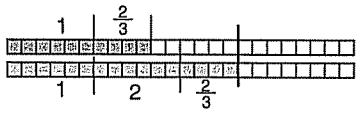


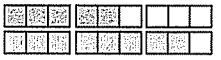
3. Draw.

Challenge

4. Solve. $1\frac{2}{3} \div 2\frac{2}{3} = \frac{5}{8}$ $\quad \frac{5}{3} \div \frac{8}{3} = \frac{5}{3} \times \frac{3}{8} = \frac{5}{\cancel{3}} \times \frac{\cancel{3}}{8} = \frac{5}{8}$

5. Draw.





or $\frac{5}{3} \times \frac{3}{8} = \frac{15}{24} = \frac{5}{8}$

What would be an easier way to solve this problem? *Change both mixed numbers Into improper fractions. Then use cancelling after inverting the divisor.*

Write a rule for dividing this problem. $3\frac{3}{5} \div 1\frac{1}{2} =$ $2\frac{2}{5}$

Change both mixed numbers Into improper fractions. invert the divisor, $\frac{3}{2}$ to $\frac{2}{3}$, Then multiply using the standard procedure.

$3\frac{3}{5} \div 1\frac{1}{2} = \frac{18}{5} \div \frac{3}{2} = \frac{\cancel{18}^{6}}{5} \times \frac{2}{\cancel{3}_{1}} = \frac{12}{5} = 2\frac{2}{5}$

Confirm your answer by multiplying the quotient by the divisor. You should get $3\frac{3}{5}$ as a product.

$2\frac{2}{5} \times 1\frac{1}{2} = \frac{12}{5} \times \frac{3}{2} = \frac{36}{10} = 3\frac{6}{10} = 3\frac{3}{5}$

Mixed Fractions - Worksheet 2

Solve. Write each answer in simplest form. Write the equations.

1. $2\frac{1}{2} \div \frac{1}{4} = 10$

2. $2\frac{1}{2} \div 1\frac{1}{4} = 2$

3. $3\frac{1}{2} \div 1\frac{3}{4} = 2$

4. $6 \div 1\frac{1}{3} = 4\frac{1}{2}$

5. $4 \div 1\frac{1}{3} = 3$

6. $1\frac{2}{5} \div 3 = \frac{7}{15}$

7. $2\frac{1}{2} \div 3 = \frac{5}{6}$

8. Casey practices basketball $1\frac{1}{2}$ hours a day. How many days will it take for him to practice 15 hours?

It will take 10

$15 \div 1\frac{1}{2} = 15 \div \frac{3}{2} = 15 \times \frac{2}{3} = \frac{30}{3} = 10$

9. Rice comes in twenty-five pound If each person wants $1\frac{1}{4}$ pounds, how many people can get rice?

20 people can get rice.

$25 \div 1\frac{1}{4} = 25 \div \frac{5}{4} = 25 \times \frac{4}{5} = \frac{100}{5} = 20$

10. If each person wants $2\frac{1}{2}$ pounds, how many people can get rice?

10 people can get rice.

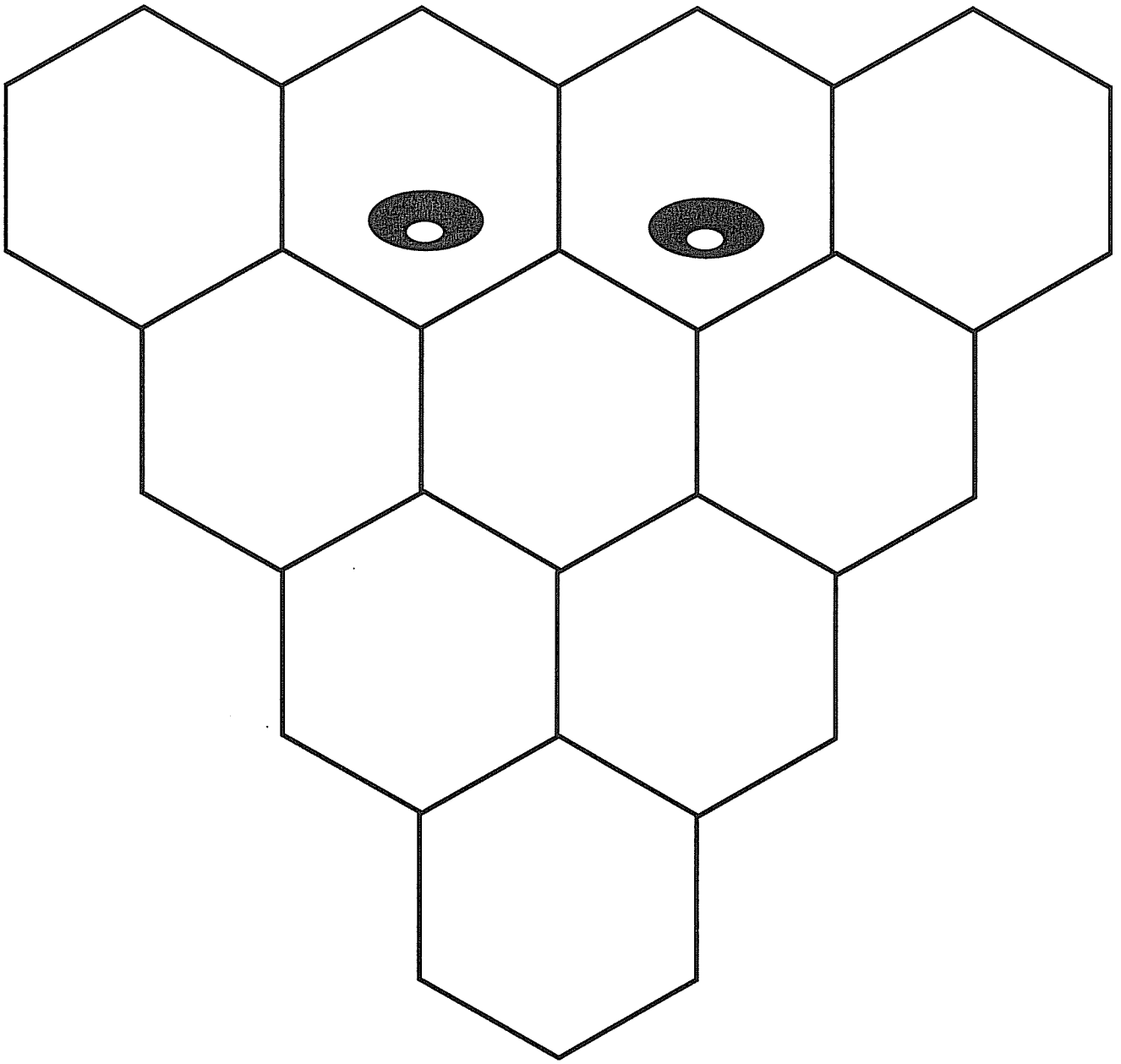
$25 \div 2\frac{1}{2} = 25 \div \frac{5}{2} = 25 \times \frac{2}{5} = \frac{50}{5} = 10$

The Post-Assessment is the same as Pre-Assessment - Part 2

Answers are on the first page of the Answer Key.

Predator Game Date_____

2 - 4 players
Use yellow, red, green, and blue pattern blocks, a Predator Game board for each player, and one die.
Take turns rolling the die and take as many green blocks as rolled. Trade blue blocks (or any other color block) for green blocks whenever possible. If you do not trade immediately, your opponent can take your two greens. Cover the board with yellow blocks. The first player to cover the board is the winner. Remember, if you do not trade ASAP, your opponent can steal them on his or her turn.



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