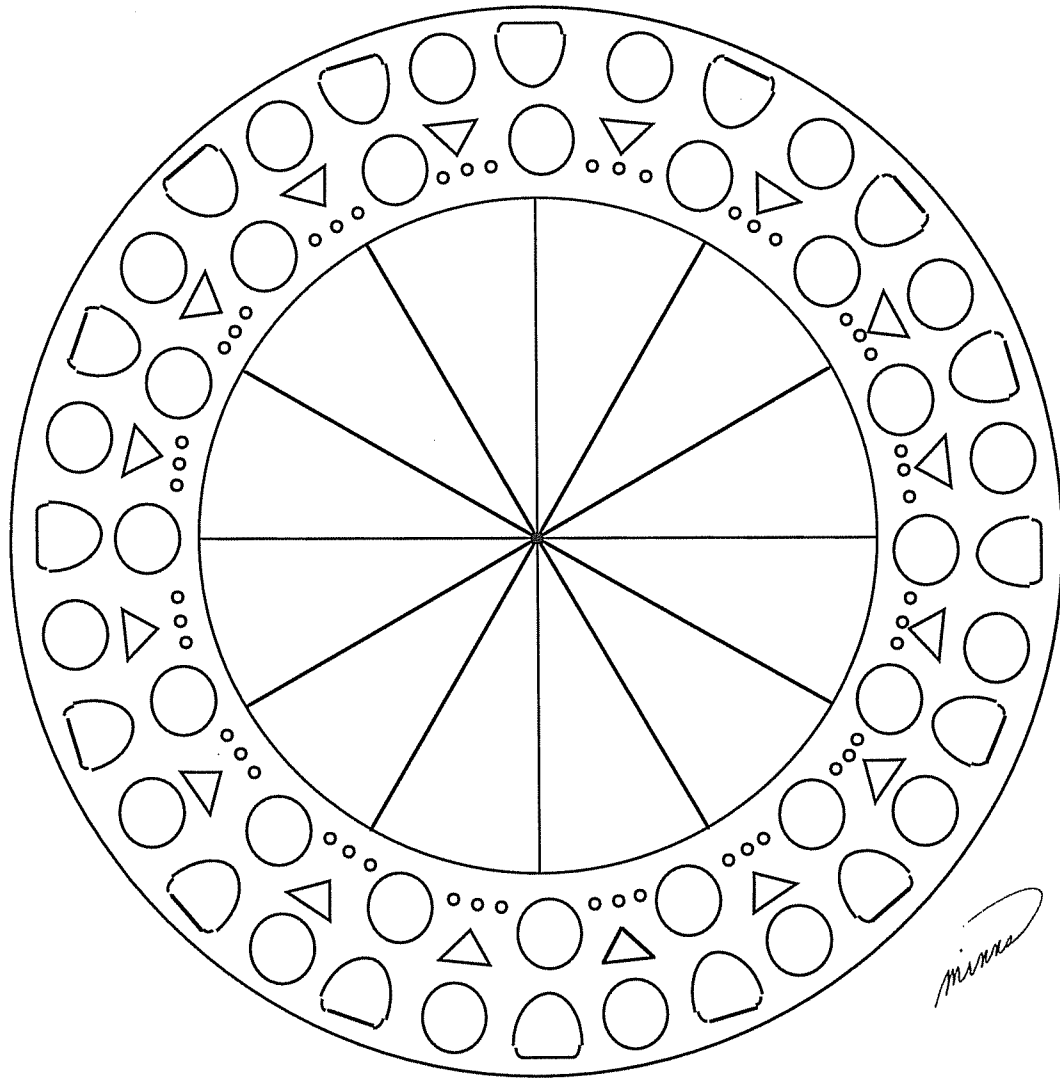


**Patterns in Arithmetic**  
**Fractions - Booklet 7 PDF**  
Addition and Subtraction of Unlike Fractions  
**Parent/Teacher Guide**



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn  
Based on methods developed by Prof. Michael Butler at the  
UCI Farm Elementary School  
University of California, Irvine

# Fractons: Booklet 7 PDF - Addition and Subtraction of Unlike Fractions

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To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

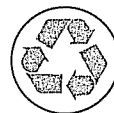
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**Pre-Assessment - Part 1 - Worksheet 1** Name \_\_\_\_\_ Date \_\_\_\_\_

Put a question mark next to any problem you do not know how to do.

1. Solve. a.  $\frac{1}{3} + \frac{1}{3} =$       b.  $\frac{2}{5} + \frac{1}{5} =$       c.  $\frac{3}{11} + \frac{6}{11} =$

2. Solve and change any improper fractions to mixed numbers.

a.  $\frac{4}{5} + \frac{3}{5} =$       b.  $\frac{6}{7} + \frac{5}{7} =$       c.  $\frac{4}{9} + \frac{7}{9} =$

3. Solve and change all improper fractions to mixed numbers.

Example: 
$$\begin{array}{r} 3\frac{3}{4} \\ + 1\frac{3}{4} \\ \hline 4\frac{6}{4} = 5\frac{1}{2} \end{array}$$

a. 
$$\begin{array}{r} 2\frac{3}{5} \\ + 1\frac{4}{5} \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 6\frac{2}{9} \\ + 5\frac{8}{9} \\ \hline \end{array}$$

c. 
$$\begin{array}{r} 2\frac{3}{7} \\ + 5\frac{6}{7} \\ \hline \end{array}$$

4. Draw the change of:

a.  $\frac{2}{3}$   to  $\frac{\quad}{12}$

b.  $\frac{5}{6}$   to  $\frac{\quad}{12}$

c. What is the multiplier in problem a? \_\_\_\_\_

5. Fill in the blanks.

a.  $\frac{1}{3} \times \frac{\quad}{\quad} = \frac{\quad}{27}$       b.  $\frac{5}{7} \times \frac{\quad}{\quad} = \frac{\quad}{42}$       c.  $\frac{8}{9} \times \frac{\quad}{\quad} = \frac{32}{\quad}$

6. Fill in the missing numbers.

a.  $\frac{7}{9} = \frac{\quad}{63}$       b.  $\frac{\quad}{5} = \frac{18}{30}$       a.  $\frac{5}{6} = \frac{15}{\quad}$

7. a. How do you know what this number is? \_\_\_\_\_

$$\frac{2}{3} \times \frac{x}{x} = \frac{8}{12}$$

b. Why does this number have to be equal to one? \_\_\_\_\_

$$\frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$$

8. Simplify these fractions. Show the Mighty One.

a.  $\frac{15}{18} \div \frac{1}{1} = \text{---}$

b.  $\frac{7}{21} \div \frac{1}{1} = \text{---}$

c.  $\frac{35}{56} \div \frac{1}{1} = \text{---}$

9. a. Where does this number come from? \_\_\_\_\_

$$\frac{12}{16} \div \frac{4}{4} = \text{---}$$

b. What is the value of this number? \_\_\_\_\_

10. List the common factors of 24 and 32.

a. \_\_\_\_\_

b. Circle the common factor you would use to simplify  $\frac{24}{32}$ .

c. What would happen if you used four to simplify  $\frac{24}{32}$  ?

$$\frac{24}{32} \div \frac{4}{4} = \text{---}$$

d. Why is the greatest common factor used to simplify a fraction instead of any factor? \_\_\_\_\_

Put a question mark next to any problem you do not know how to do.

1. If you add 2 oranges + 3 apples, you will have \_\_\_\_\_

2. Solve.

$$\begin{array}{r} \frac{2}{5} \\ + \frac{1}{5} \\ \hline \end{array}$$

3. Solve.

$$\begin{array}{r} \frac{1}{4} \\ + \frac{1}{12} \\ \hline \end{array}$$

4. Solve.

$$\text{a. } \begin{array}{r} \frac{3}{4} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\text{b. } \begin{array}{r} \frac{3}{5} \\ + \frac{3}{6} \\ \hline \end{array}$$

5. Solve.

$$\begin{array}{r} \frac{1}{3} \\ - \frac{1}{12} \\ \hline \end{array}$$

6. Solve.

$$\text{a. } \begin{array}{r} \frac{5}{6} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$\text{b. } \begin{array}{r} \frac{5}{8} \\ - \frac{5}{12} \\ \hline \end{array}$$

7. Why must you find 'common denominators' to add fractions?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

8. List four common multiples of 3 and 2. \_\_\_\_\_

9. a. When solving this fraction problem,  $\frac{3}{4} + \frac{5}{6}$ , list three common denominators that are possible. \_\_\_\_\_

b. What is the least common denominator? \_\_\_\_\_

c. Why does it matter? \_\_\_\_\_

\_\_\_\_\_

10. Solve. a.

$$\begin{array}{r} 3 \\ - \frac{3}{4} \\ \hline \end{array}$$

b.

$$\begin{array}{r} 3 \frac{1}{4} \\ - 1 \frac{3}{4} \\ \hline \end{array}$$

c.

$$\begin{array}{r} 7 \frac{1}{3} \\ - 2 \frac{5}{8} \\ \hline \end{array}$$

## Assessment Guide

**Purpose** The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Pre-Assessment: Part 2 is the same as the Post-Assessment.

**Prerequisites** *Patterns in Arithmetic*: Fractions - Booklet 4 (Equivalent Fractions) and Fractions - Booklet 5 (Simplifying Fractions)

**Materials** Fractions: Booklet 7 - Pre-Assessment: Part 1 - Worksheets 1 and 2, pages 1 and 2, and Pre-Assessment: Part 2, page 3. A copy of the Post-Assessment is on page 27 in this booklet. In the Student Workbook the page numbers are: Pre-Assessment: Part 1 - Worksheets 1 and 2, pages 1 and 2, Part 2, page 3, and Post-Assessment: page 45. Score sheets, pages 9 - 11 in this booklet.

**Instructions** Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

**Assessment Guide** This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and Post-Assessment follow.

## Assessment Criteria for Pre-Assessment: Part 1

The main skills needed to add and subtract fractions with unlike denominators are the abilities to find equivalent fractions, convert improper fractions to mixed fractions, and to simplify a fraction. If these skills are weak, they must be remediated before attempting this booklet. Use this Pre-Assessment to uncover areas that need reteaching.

Can the student:

### 1. Addition of Like Fractions (Fractions: Booklet 2)

- A. give the correct answer on two of the three problems?

A student who adds the denominators at this point is demonstrating lack of understanding of the function of a denominator.

### 2. Addition of Like Fractions (Fractions: Booklet 2) (Fractions: Booklet 3 - Improper to Mixed Fractions)

- A. give the correct improper fraction in two of the three problems?
- B. give the correct mixed number in two of the three problems?

### 3. Addition of Mixed Numbers with Like Denominators (Fractions: Booklet 3)

- A. add the whole numbers and the fractions in two of the three problems?

Point out the example to the student. Do not explain. We are testing to see if the student can perform a series of operations to get the correct answer. First, the whole numbers and the fractions need to be added. This creates a whole number and an improper fraction. Then the student must simplify to create a mixed number with a whole number and a proper fraction.

- B. give the correct mixed number sum on two of the three problems?

### 4. Equivalence: Representational (Fractions: Booklet 4)

- A. draw the two-thirds and the five-sixths correctly?

The box should be divided into three or six roughly equal sections. Then two of the three and five of the six should be lightly shaded in.

- B. draw the change from two-thirds to twelfths and the change from five-sixths to twelfths?

In Item B, each third should be broken down into four smaller sections. Each sixth should be broken down into two sections.

- C. identify the multiplier as  $\frac{4}{3}$ ?

### 5. Equivalence: Calculating (Fractions: Booklet 4)

- A. fill in the correct multiplier on two of the three problems?
- B. fill in the correct missing numerators on problems a and b?
- C. fill in the correct missing denominator on problem c?

### 6. Equivalence: Calculating (Fractions: Booklet 4)

- A. fill in the correct missing number on two of the three problems?

### 7. Equivalence: Calculating (Fractions: Booklet 4)

- A. explain how the multiplier is determined?

1 point - You divide by four.

2 points - You find the relationship between the denominators by dividing the three into the twelve. You then use that number as the multiplier. Or, you find the relationship

between the denominators by dividing the three into the twelve and multiplying the four by both the three and the two.

- B. explain why the multiplier must be equal to one?

1 point - Because that is the only way to make the two fractions equal.

2 points - Because one times any number is the same number. To keep the two fractions equal, the only number you can multiply by is one.

8. Simplification: Calculating (Fractions: Booklet 5)

- A. give the correct answer in two of the three problems?

- B. show the Mighty One in two of the three problems?

Some students will do this part mentally. If the numbers in the Mighty One are left blank, ask him to fill them in. If he can not, he does not understand how this process works.

9. Simplification: Calculating (Fractions: Booklet 5)

- A. explain where the number that makes up the Mighty One when simplifying come from?

- B. state that the value of the number being divided into the numerator and denominator is one?

10. Simplification: Calculating (Fractions: Booklet 5)

- A. list the four common factors of 24 and 32 in problem a?

- B. circle the greatest of the common factors, eight, in the list in problem a?

- C. explain that the resulting fraction would not be in lowest terms or simplified all the way?

D. explain that the greatest common factor will simplify the fraction to its lowest terms or to the fewest possible pieces?

**Booklet Selection Guide based on results of Pre-Assessment: Part 1**

The student should score 80% or better on the Pre-Assessment: Part 1 before moving into this booklet without any review. Reteach any items on which the student received a No on the score sheet. The Assessment Guide indicates where each concept is taught.

If the student is new to this program and gets No scores on the explanation sections 7A, 7B, 8B, 9A, and 9B but Yes on the items that require only calculation, you can proceed with this booklet. The student will be able to do the problems. It would be wise to go back and redo Fractions: Booklets 4 and 5 if the student does not understand how the multiplier and the greatest common factors used are equal to one. This is a very, very important concept. The Mighty One is a major tool for many arithmetic and algebraic procedures.

**Assessment Criteria for Pre-Assessment: Part 2**

Items in this part will be taught in Fractions: Booklet 7 unless otherwise indicated.

Can the student:

1. Addition of Unlike Fractions: Manipulative

- A. change the unit to fruits or any other common unit?

The general principle in combining any quantity is that the units must match. You must add inches to inches, feet to feet, and so on. This is the reason fractions must have common denominators in order to be combined. This is a very basic principle.

2. Addition of Like Fractions (Fractions: Booklet 2)

- A. add two fractions with like denominators?



3. Addition of Unlike Denominators - easy one
  - A. find the correct common denominator?
  - B. find the correct equivalent fraction to  $\frac{1}{4}$ ?
  - C. give the correct sum?
  - D. simplify the sum?
  
4. Addition of Unlike Fractions - more difficult
  - A. find a correct common denominator on both problems?
  - B. find the correct equivalent fraction in three of the four fractions?
  - C. convert the improper fraction sum into a mixed number in problem a?
  - D. find the correct answers on both problems?
  - E. give the sum in the lowest terms on problem b?
  - F. Bonus Point - simplify the  $\frac{3}{6}$  to  $\frac{1}{2}$  before adding?
  
5. Subtraction of Unlike Fractions - easy one
  - A. find the least common denominator?
  - B. find the correct equivalent fraction in both numbers?
  - C. find the correct difference?
  - D. simplify the answer?
  
6. Subtraction of Unlike Fractions - more difficult
  - A. find common denominators for both problems?
  - B. find the lowest common denominators for both problems?
  - C. find the correct equivalent fraction for all four fractions?
  - D. give the correct answer to one of the two problems?
  
7. Least Common Multiples
  - A. explain why the denominators must match?  
 Rubric: 1 point You can't add them if they don't match.  
 2 points In order to add or subtract, the unit the numbers are in must match because you can not add unlike units.
  
8. Least Common Multiples
  - A. list four common multiples of 2 and 3?
  
9. Least Common Multiples
  - A. list three common multiples for 4 and 6?
  - B. give the least common denominator?
  - C. explain why it matters?  
 Rubric: 1 point Because it makes it easier  
 2 points It makes the arithmetic easier, and the answer is in a simpler form at the end.
  
10. Working with Mixed Numbers
  - A. give the correct answer for problem a?
  - B. in problems b and c, regroup a whole number to increase the fraction to an improper fraction to make the top fraction larger?
  - C. get the correct answer on problem b?

D. get the correct answer on problem c?

This problem is quite difficult. A common denominator must be found first and then the top number regrouped.

### **Booklet Selection Guide based on results of Pre-Assessment: Part 2**

If your student scores 75% or better on Pre-Assessment: Part 2, use the error analysis chart and reteach any weak areas. You can then move on to Fractions: Booklet 8 (Division of Fractions) or switch your focus to decimals and complete the *Key to Decimals* - Booklets 2, 3, and 4.

If understanding of the Identity Property of Multiplication (Mighty One) is weak, now would be a good time to do Fractions: Booklet 4 and Fractions: Booklet 5.

If the score is 74% or less, this booklet is a good match for your student.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic). This links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements, such as: “**Are you sure?**” or “**Build it.**” or “**What gave you the clue?**” or “**Show me how you got that.**” or “**Prove it.**” even when a student is correct. This is important to do often. Many students will ask an adult, “Am I right?” rather than answering definitively. Confidence in a student’s response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What’s most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: “**How did you get that?**” “**Show me what you mean.**” “**Build a model of that.**” “**Tell me more so I can understand what you are saying.**”

Can the student:

1. Addition of Like Fractions

Yes No A. give the correct answer on two of the three problems?

2. Addition of Like Fractions

Yes No A. give the correct improper fraction in two of the three problems?

Yes No B. give the correct mixed number in two of the three problems?

3. Addition of Mixed Numbers with Like Denominators (Fractions: Booklet 3 )

Yes No A. add the whole numbers and the fractions in two of the three problems?

Yes No B. give the correct mixed number sum in two of the three problems?

4. Equivalence: Representational (Fractions: Booklet 4)

Yes No A. draw the two-thirds and the five-sixths correctly?

Yes No B. draw the change of both fractions to twelfths?

Yes No C. identify the multiplier as  $\frac{4}{3}$ ?

5. Equivalence: Calculating (Fractions: Booklet 4)

Yes No A. fill in the correct multiplier on two of the three problems?

Yes No B. fill in the correct missing numerators on problems a and b?

Yes No C. fill in the correct missing denominator on problem c?

6. Equivalence: Calculating (Fractions: Booklet 4)

Yes No A. fill in the correct missing number on two of the three problems?

7. Equivalence: Calculating (Fractions: Booklet 4)

0 1 2 A. explain how the multiplier is determined?

0 1 2 B. explain why the multiplier must be equal to one?

8. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. give the correct answer in two of the three problems?

Yes No B. show the Mighty One in two of the three problems?

9. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. explain the Mighty One when simplifying is in the form of a common factor?

Yes No B. state that the value of this number is one?

10. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. list the four common factors as 1, 2, 4, and 8?

Yes No B. circle 8, the greatest of the common factors in the list?

Yes No C. explain that the resulting fraction would not be in lowest terms or simplified all the way?

Yes No D. explain that the greatest common factor will simplify the fraction to its lowest terms or to the fewest possible pieces?

Items Correct = \_\_\_\_ = \_\_\_\_\_% 19 points or more to pass

Items Possible = 24

**Pre-Assessment: Part 2 Score Sheet**

Name \_\_\_\_\_ Date \_\_\_\_\_

Can the student:

1. Addition of Unlike Fractions: Manipulative  
Yes No A. change the unit to fruits or any other common unit?
2. Addition of Like Fractions (Fractions: Booklet 2)  
Yes No A. add the numerators and not the denominators?
3. Addition of Unlike Denominators - easy one  
Yes No A. find the correct common denominator of twelfths?  
Yes No B. find the correct equivalent fraction of  $\frac{3}{12}$ ?  
Yes No C. give the correct sum of  $\frac{4}{12}$ ?  
Yes No D. simplify the sum to  $\frac{1}{3}$ ?
4. Addition of Unlike Fractions - more difficult  
Yes No A. find a correct common denominator on both problems?  
Yes No B. find the correct equivalent fraction in three of the four fractions?  
Yes No C. convert the improper fraction sum into a mixed number in problem a?  
Yes No D. find the correct answers on both problems?  
Yes No E. give the sum in the lowest terms on problem b?  
Yes No F. Bonus Point - simplify the  $\frac{3}{8}$  to  $\frac{1}{2}$  before adding?
5. Subtraction of Unlike Fractions - easy one  
Yes No A. find the least common denominator?  
Yes No B. find the correct equivalent fraction in both numbers?  
Yes No C. find the correct difference?  
Yes No D. simplify the answer?
6. Subtraction of Unlike Fractions - more difficult  
Yes No A. find common denominators for both problems?  
Yes No B. find the lowest common denominators for both problems?  
Yes No C. find the correct equivalent fraction for all four fractions?  
Yes No D. give the correct answer to one of the two problems?
7. Least Common Multiples  
0 1 2 A. explain why the denominators must match?
8. Least Common Multiples  
Yes No A. list four common multiples of 2 and 3?
9. Least Common Multiples  
Yes No A. list three common multiples for 4 and 6?  
Yes No B. give the least common denominator?  
0 1 2 C. explain why it matters?
10. Working with Mixed Numbers  
Yes No A. give the correct answer for problem a?  
Yes No B. regroup a whole number correctly in problems b and c?  
Yes No C. get the correct answer on problem b?  
Yes No D. get the correct answer on problem c?

Items Correct = \_\_\_\_\_ = \_\_\_\_\_ %  
Items Possible = 30

**Post-Assessment Score Sheet**

Name \_\_\_\_\_ Date \_\_\_\_\_

See Error Analysis after this score sheet to help you focus on any areas not mastered in the combining of unlike fractions.

Can the student:

1. Addition of Unlike Fractions: Manipulative
  - Yes No A. change the unit to fruits or any other common unit?
2. Addition of Like Fractions (Fractions: Booklet 2)
  - Yes No A. add the numerators and not the denominators?
3. Addition of Unlike Denominators - easy one
  - Yes No A. find the correct common denominator of twelfths?
  - Yes No B. find the correct equivalent fraction of  $\frac{3}{12}$ ?
  - Yes No C. give the correct sum of  $\frac{4}{12}$ ?
  - Yes No D. simplify the sum to  $\frac{1}{3}$ ?
4. Addition of Unlike Fractions - more difficult
  - Yes No A. find a correct common denominator on both problems?
  - Yes No B. find the correct equivalent fraction in three of the four fractions?
  - Yes No C. convert the improper fraction sum into a mixed number in problem a?
  - Yes No D. find the correct answers on both problems?
  - Yes No E. give the sum in the lowest terms on problem b?
  - Yes No F. Bonus Point - simplify the  $\frac{3}{8}$  to  $\frac{1}{2}$  before adding?
5. Subtraction of Unlike Fractions - easy one
  - Yes No A. find the least common denominator?
  - Yes No B. find the correct equivalent fraction in both numbers?
  - Yes No C. find the correct difference?
  - Yes No D. simplify the answer?
6. Subtraction of Unlike Fractions - more difficult
  - Yes No A. find common denominators for both problems?
  - Yes No B. find the lowest common denominators for both problems?
  - Yes No C. find the correct equivalent fraction for all four fractions?
  - Yes No D. give the correct answer to one of the two problems?
7. Least Common Multiples
  - 0 1 2 A. explain why the denominators must match?
8. Least Common Multiples
  - Yes No A. list four common multiples of 2 and 3?
9. Least Common Multiples
  - Yes No A. list three common multiples for 4 and 6?
  - Yes No B. give the least common denominator?
  - 0 1 2 C. explain why it matters?
10. Working with Mixed Numbers
  - Yes No A. give the correct answer for problem a?
  - Yes No B. regroup a whole number correctly in problems b and c?
  - Yes No C. get the correct answer on problem b?
  - Yes No D. get the correct answer on problem c?

Items Correct = \_\_\_\_\_ = \_\_\_\_\_% 80%, or 24 points, needed to pass.

Items Possible = 30

Patterns in Arithmetic: Fractions - Booklet 7

Parent/Teacher Guide

## Error Analysis for Post-Assessment or Pre-Assessment: Part 2\*

Circle items scored No on the score sheet on the chart below. It may show patterns that will tell you where to focus on reteaching if needed.

| Common Denominators | Equivalence | Answer Accuracy | Simplification |
|---------------------|-------------|-----------------|----------------|
| 12A 13A             | 13B         | 13C             | 13D            |
| 14A                 | 14B         | 14D             | 14E            |
| 15A                 | 15B         | 15C             | 15D            |
| 16A                 | 16C         | 16D             |                |
|                     |             | 20A, B, C, D    |                |

\* Pre-Assessment: Part 2 and the Post-Assessment are the same test.

## Addition of Unlike Fractions: Manipulative

- Purpose** To add fractions with different denominators, manipulatively changing them so the units match. The basic concept behind adding fractions with uncommon denominators is that you cannot combine things that are expressed in different units. You cannot combine three dogs plus two cats to get five unless you rename the dogs to animals and the cats to animals. Then you can combine them into one group of five animals. Essentially you are regrouping to create common units. This basic idea is not difficult for students to grasp. We begin with common things like cats and dogs and then move to pattern blocks of adding greens and blues, then to fraction circles with fractional numbers added in. Most students enjoy this lesson. The full lesson will take four to five sessions.
- Prerequisites** Familiarity with pattern blocks, fraction circles, and mastery of the concept of the Changing Whole taught in Fractions: Booklet 2.
- Materials** Addition of Unlike Fractions: Manipulative - Worksheets 1 - 6, pages 4 - 9  
An assortment of objects such as apples, oranges, little toy cars or trucks, silverware, or bowls and cups, etc.  
Pattern blocks  
Fraction pieces - both circles and squares
- Note** Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **“How did you get that?”** **“Show me what you mean.”** **“Build a model of that.”** **“Tell me more so I can understand what you are saying.”**
- Warm Up** Play with the pattern blocks for a while. Tell the student that she can make designs and stacks now but not during the lesson.
- Lesson Part 1** Play a verbal game.  
**“Three cats plus two cats is five ...?”** “Cats.”  
**“Three cats plus two kittens is five ...?”** You will get various answers. Five cats, or five animals. Cats is the easier solution because only one quantity, kittens, needs to be renamed. If she says ‘animals’ ask her if kittens are also cats.  
**“Three puppies plus two dogs is five ...?”** “Dogs.”
- Note** Most students intuitively understand that in order to combine objects, they must be expressed in the same units. Kittens can be renamed as cats, puppies renamed to dogs. Think of other things that are like this where one thing is a subset of another thing. Example: two women plus three girls.
- Now set a group of three apples and a group of two apples on the table.

**“Three apples plus two apples is ...?” “Five apples.”**

Leave the group of three apples and replace the group of two apples with two oranges on the table.

**“Three apples plus two oranges is five ...?”** Many people will say “Apple-oranges.” Most do not think initially of changing the units to fruits.

**“There is no such thing as apple-oranges! It has to be a real thing that fits.**

**“What do we have five of here on the table?”** “Five pieces of fruit.”

**“So, three apples plus two oranges is five ...?”** “Fruits.”

Try another one.

**“Three bowls plus two cups are five ...?”** “Dishes.”

Make up more problems. Have her make up some too.

**“Why to we have to say five fruits when we add three apples plus two oranges?”** “Because we can not add them unless they both use the same words or units.”

**“Three kittens plus two puppies are five ...?”** “Baby animals.”

Now look at the top of Addition of Unlike Fractions: Manipulative Worksheet 1. Have her do those problems, writing the answers and the units. (Suns are stars.) Point out that these problems are called kitty cat problems because one of the groups, kittens, can be renamed using the other group, cats. This same thing happens when halves and fourths are added. The half can be traded for fourths.

Now look at the Apple Orange Problems. In these problems, both groups must be renamed in order to combine the numbers. On the first problem, have her write in

$$\begin{array}{r} 3 \text{ apples} = 3 \text{ fruits} \\ + 2 \text{ oranges} = 2 \text{ fruits} \\ \hline = 5 \text{ fruits} \end{array}$$

Finish the worksheet. Make sure she reads all the words on the page.

**Worksheets** Take out the pattern blocks and do Addition of Unlike Fractions: Manipulative - Worksheet 2. Read the instructions with her and do the first two problems together. Use the Answer Key to check the work immediately. End the lesson with some play time with the fraction circles or the Prism Fractions you are going to use for the lesson in Part 2.

**Lesson Part 2** Play a verbal game with the colored fraction pieces. Use the names of the colors to give the problem. For example, if the whole piece is white and the half piece is red, ask, **“What is one white plus two reds?”** “Four reds or two whites.”

**Note** *Do not teach how to calculate common denominators at this time. Allow the student to construct the idea that fractional units can be traded and renamed just like the apples and oranges. Manipulatives must be used to do this.*

**Worksheet** Addition of Unlike Fractions: Manipulative - Worksheet 3. Do the first problem together. Take out the one-half and the one-fourth pieces and place them on the table. **“How can you trade one of these pieces so that they stay the same amount but change to all one color?”** “I can trade the one-half for two of the fourths.” Watch her complete the page to be sure she is doing the work correctly. Have her use the



fraction pieces for each problem.

## Note

Notice if she begins to do the trades in her head and then puts down the answer. That will indicate that she is moving to the next stage of a more abstract understanding of renaming and combining. Students move at all different speeds here.

Some students will do better using the colors of the pieces rather than the numbers initially. Use a crayon to shade a fractional number on the worksheet the same color as the pieces you are using. A dialogue may sound like this (red might be one-half and yellow might be one-fourth):

**“What color is the half?”** “Red.” She puts the piece on the table and colors the one-half on the worksheet red.

**“What color is the one-fourth?”** “Yellow.” She puts the piece on the table and colors the one-fourth on the worksheet yellow.

**“Which piece is larger, the red or the yellow?”** “The red.”

**“How many yellows trade for a red?”** “Two.”

**“Can you turn one yellow into reds?”** “No, the yellow is smaller than the red.”

**“If two yellows equal one red, then what is one yellow plus one red?”** “Three yellows.”

**“What fraction is three yellows?”** “Three-fourths.” She writes that number below the line on the worksheet. She can shade the answer in yellow also if she wishes.

## Practice Worksheets

Do Addition of Unlike Fractions: Manipulative - Worksheet 4, page 7, together. **“Build  $\frac{1}{2} + \frac{3}{8}$ .”** “I will put one-half piece and three-eighths pieces together.” Watch to see if she makes the jump from the numerator always being one to a numerator larger than one. If she has difficulty, have her build a few more problems together with you.

## Test for Understanding

The next session, have her do the assessment using Addition of Unlike Fractions: Manipulative - Worksheet 5, page 8. Give her the fraction pieces she used on the last worksheets.

Check the work with the Answer Key. If she can not do the page alone with correct answers, do Addition of Unlike Fractions: Manipulative - Worksheet 5, page 8, with her to help develop the concept.

If she can do Addition of Unlike Fractions: Manipulative - Worksheet 5 alone and get it correct, then give her the pattern blocks. Have her do Addition of Unlike Fractions: Manipulative - Worksheet 6, page 9. Watch to see if she can figure out how to make the fractions she needs. If she needs help, give that help and be ready to offer closer supervision in the next session when we begin to record the trades.

Most students will be independent at this point. Some will need assistance, especially when the manipulative and the definition of the whole are changed.

## Addition of Unlike Fractions: Recording

- Purpose** The purpose of this lesson is to add unlike fractions with a different manipulative and to record the trades. As usual, the manipulative is changed two to three times to require the student to reconstruct the procedure several times. Many students who have mastered equivalent fractions will begin to use that skill at this point. This lesson takes two to three sessions.
- Prerequisites** Understanding the concept of Equivalent Fractions is helpful. If it is not mastered, this concept should be worked on in conjunction with this lesson. See Fractions: Booklet 4 - Equivalent Fractions.
- Materials** Addition of Unlike Fractions: Recording, Worksheets 1 - 4, pages 10 - 13  
pattern blocks and fraction pieces  
Crayons
- Warm Up** Take out the pattern blocks and review the concept of the Changing Whole. In the last lesson, Addition of Unlike Fractions: Manipulative - Worksheet 6, the whole was defined as three yellow pattern blocks. Review what fraction each different colored block then represents. Now repeat the same process, this time defining the whole as two yellow pattern blocks (or a pink if you have those pieces). What is the value of each block now?  
**“If the whole is two yellows, what is one red plus one yellow equal to?”** “Three reds because I can trade one yellow for two reds and add that to the one red I already have.”
- Lesson Part 1** Take out Addition of Unlike Fractions: Recording - Worksheet 1, page 10. Have him color in the pictures of the blocks at the top of the page to match the colors of the actual blocks. Now study the example problem that is just above the line. The only new thing is the writing of the fractional value of each block just beneath the number and color of the block. On the right side, the trade of the yellow block for two red blocks is recorded both in words and as a fraction. The total is shown in both words and a fractional number.  
Do the first problem together.  
**“What is the fractional value of three reds?”** “Three-fourths.” Have him write that fraction under the words ‘three reds.’  
  
**“What is the fractional value of one green?”** “One-twelfth.” Have him write that fraction under the words ‘one green.’  
  
**“How are three reds added to one green?”** “The three reds are traded for nine greens and then added to the one green we already have.” Have him record like this:  
$$3 \text{ reds} + 1 \text{ green} = 9 \text{ greens} + 1 \text{ green} = 10 \text{ greens}$$
$$\frac{3}{4} + \frac{1}{12} = \frac{9}{12} + \frac{1}{12} = \frac{10}{12}$$
- Note** When he writes the fraction under the words, have him write the numerator above

the little line and the denominator below it.

Do not ask him to simplify at this point. If he does it on his own, fine, but do not require it. It adds another level of complexity that can confuse some students at this point.

On the next problem, he must write in the number and colors of the blocks himself. The result will look like this:

4 blues + 1 green = 8 greens + 1 green = 9 greens

$$\frac{4}{6} + \frac{1}{2} = \frac{8}{12} + \frac{1}{2} = \frac{9}{12}$$

Have him finish the page on his own.

## Worksheets

Addition of Unlike Fractions: Recording - Worksheet 2, page 11

Have him color in the pictures of the blocks with the correct colors at the top of the page. Do the first problem with him. He will put one red ( $\frac{1}{4}$ ) and one yellow block ( $\frac{1}{2}$ ) on the table. He will trade the one yellow for two reds. Now have him record as shown below.

When completed the first problem will look like this:

$$\begin{array}{r} \frac{1}{4} = \frac{1}{4} \\ + \frac{1}{2} = \frac{2}{4} \\ \hline \frac{3}{4} \end{array}$$

“How many red blocks?” “Three.”

“What happens to the fractions in the two bottom problems?” “The whole changes.”

Have him complete the rest of the page alone.

## Practice Worksheets

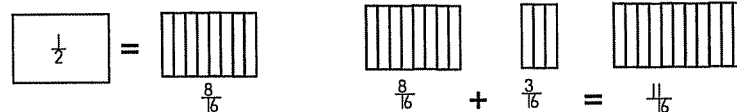
Addition of Unlike Fractions: Recording - Worksheets 3 and 4, pages 12 and 13. Change the manipulative to fraction pieces to complete these problems. There are two problems with sixteenths in them. Most fraction sets do not have sixteenths. Use these two problems as a Test for Understanding. The Prism Fractions Paper set does have sixteenths.

## Test for Understanding

See if he can do the problems with the sixteenths without a manipulative. Have him draw a picture to help. Just watch and see what he does.

$$\frac{1}{2} + \frac{3}{16} = \frac{8}{16} + \frac{3}{16} = \frac{11}{16}$$

Example:



If he can not do the problem, just leave it for now. It means he is still constructing the general idea of trading in the concrete stage and needs the physical pieces to do it.

If he can draw a picture, he may realize that halves can be traded for eight-sixteenths and be able to record this. He can represent it and draw it.

If he can do it without a picture or manipulative, you know that he is internalizing and generalizing the abstract, symbolic pattern of trading.

## Subtraction of Unlike Fractions: Recording

- Purpose** The purpose of this lesson is to subtract unlike fractions using the same format as the previous lesson, Addition of Unlike Fractions: Recording.
- Prerequisites** Addition of Unlike Fractions: Recording
- Materials** Subtraction of Unlike Fractions - Worksheets 1 - 4, pages 14 - 17  
Addition and Subtraction of Unlike Fractions: Assessment, page 18  
Addition of Unlike Fractions: Assessment, page 19  
Pattern blocks  
Prism Fractions pieces
- Lesson** Look through the worksheets together. Have the student look at the problems. Do one or two together. She should be able to do all the sheets with minimal help. Require her to show all the trades, have her write out the equivalent fractions and then get the answers.
- Worksheets** Subtraction of Unlike Fractions - Worksheets 1 - 4, pages 14 - 17  
Addition and Subtraction of Unlike Fractions: Assessment, page 18  
Addition of Unlike Fractions: Assessment, page 19
- Test for Understanding** Addition and Subtraction of Unlike Fractions: Assessment, page 18.  
On the first page, 'Show You Know,' allow her to work concretely with manipulatives.
- On Addition of Unlike Fractions: Assessment, page 19, have her do the problems without the fraction pieces. This will let you know if she has internalized and abstracted what she has learned. Answers are in the Answer Key.

## Common Denominators with Graph Paper

**Purpose** The purpose of this lesson is to find common denominators using graph paper. This is done by going from using concrete fraction pieces to representational drawings. It sets the stage for developing the concept of using the multiplication tables to find common multiples to use for finding common denominators.

**Prerequisites** Previous lessons in this booklet and fluency with multiplication tables

**Materials** Common Denominators with Graph Paper - Worksheets 1 - 3, pages 20 - 22  
A Multiplication Chart may be handy for those who are not fluent in their tables  
Fraction pieces  
Colored pencils or crayons  
Graph paper with  $\frac{1}{2}$  inch squares work great. Copy page 26 in this book.

**Warm Up** “Draw a long 1 x 12 rectangle on the graph paper like the one shown on Worksheet 1. Divide the rectangle into fourths.”



If the student draws groups of four, have her take out the twelfths and fourths from the fraction pieces set. If she draws groups of three, see the note below.

“How many twelfths are in one-fourth?” “Three.”

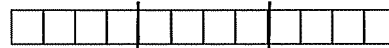
“So how many of the twelve graph paper boxes should be in a fourth?” “Three.”

“If you make a group of four, how many groups will you get?” “Three.”

“What fraction is it when the whole is divided into three groups?” “Thirds.”

“So, making groups of four boxes gives you \_\_\_\_\_ and making groups of three boxes gives you \_\_\_\_\_?” “Making groups of four boxes makes thirds; making groups of three boxes makes fourths.”

**Note** Watch to see if she draws groups of four creating thirds, not fourths. This is a common error for students at this level. Groups of four.



If she draws groups of three, tell her to “draw rectangles of fifteen boxes. Draw fifths and thirds. Groups of three make \_\_\_\_\_, and groups of five make \_\_\_\_\_.”

“Groups of three make fifths, and groups of five make thirds.” Then, repeat the problem with twelve boxes divided into fourths. Also use fraction pieces. If she still has difficulty with this idea, repeat the exercise with eight squares in a box. Groups of two make fourths, groups of four make halves.

**Lesson** Use Common Denominators with Graph Paper - Worksheets 1 and 2, pages 20 and 21. Read the instructions at the top of the page. Have her notice the heavy lines used to break the rectangles into fractional units. Have her use a colored pencil to shade in the fraction (color over the gray). Choose a color that matches the color of the fraction pieces you have been using. A gray shading of the regular pencil works too, but many students benefit from the color association.

Copy the example at the top of the page and do the second problem. Be sure she makes the correct groupings, draws heavy lines between the groups and shades in the boxes with a colored pencil that matches the color of the fraction pieces you use. When she shades in the addition problem, have her use both colors to show the combination.

Repeat for the third section.

At the bottom of Common Denominators with Graph Paper - Worksheet 1 is the question: **“Why are fifteen boxes chosen to show fifths and thirds?”**

Have her write an answer as best she can. Then talk it over and polish it up. We are looking for an answer something like this:

“Because both five and three go into fifteen evenly.” Or, “Because fifteen can be evenly divided into groups of five and groups of three, so you have a common denominator.”

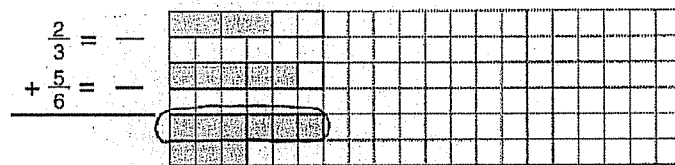
Proceed to Common Denominators with Graph Paper - Worksheet 2. Do the top problem together. Answer this question to the right of the graphic: Why were eighths chosen? Because you can make both fourths and eighths from eight boxes. **“In the next problem, you have thirds and sixths. How many boxes do you need in the whole to make both thirds and sixths?”** Answers will vary. If she does not choose the lowest number, let it be. If she has difficulty, get the fraction pieces out again and do it with fraction pieces first and then transfer it to the graph paper. The fraction pieces will have her use sixths or possibly twelfths. Have her finish the page alone. Check the work.

## Practice Worksheet

Common Denominators with Graph Paper - Worksheet 3, page 22, gives some problems where the common denominator is not in the fraction pieces set. *Watch* to see what she does with the second problem on this page.

Have her simplify all the answers and show this simplification in the drawing of the answer. On the top problem, she will get the answer nine-sixths. Have her use a bright colored pencil to circle six sixths to show the whole.

The simplified answer is  $1\frac{3}{6}$  or  $1\frac{1}{2}$ .



## Test for Understanding

After she has completed Common Denominators with Graph Paper - Worksheet 3, ask her about the second problem. **“Why did you choose the denominator you did?”**

You are listening for words such as ‘multiplication table,’ both three and nine ‘go into’ nine and eighteen.

**“Could you have chosen a different one?”** “Yes, I could have used nine, eighteen, or even twenty-seven.”

**“Why couldn’t you use twelfths?”** “Because you can not make ninths with twelve boxes.”

## Least Common Multiple

- Purpose** The purpose of this lesson is to formally introduce the concept of the least common multiple as a tool for finding common denominators. It also explores why you want the least common denominator.
- Prerequisites** Fluency with multiplication tables, previous lessons in this book
- Materials** Least Common Multiple - Worksheets 1 - 6, pages 23 - 28  
Fraction pieces  
Colored pencils or crayons  
Graph paper with one-half inch squares work great.
- Warm Up** Orally skip count the two, three, and four times tables up to times twelve.
- Lesson** Least Common Multiple - Worksheets 1 and 2. Do the pages together. They are self-explanatory. Check for accuracy before you go on.
- Worksheet** Least Common Multiple - Worksheet 3, page 25 Observe the student's work.
- Practice Worksheets** Least Common Multiple - Worksheets 4 and 5, pages 26 and 27. Correct the work immediately.
- Test for Understanding**
1. Give him this cooking problem: A corn bread recipe calls for a third of a cup of flour, a fourth of a cup of corn meal and an eighth of a cup of honey. Can all that fit into a one cup measuring cup?  
Answer:  $\frac{1}{3} + \frac{1}{4} + \frac{1}{8} = \frac{8}{24} + \frac{6}{24} + \frac{3}{24} = \frac{17}{24}$  Yes, it will fit.
  2. Give the Least Common Multiple - Worksheet 6, page 28. Have him do this page alone without graph paper or fraction pieces. Note any problems he has difficulty with or misses. Answers are in the Answer Key.
- The ones most likely to cause trouble are the sevenths and ninths. Look for common arithmetic errors. The most likely errors are multiplication errors. If he uses a denominator that is not the least common one, have him redo the problem next to the one he did. Check to see that the answers are equal.
- Many students will have discovered that the easiest way to get a common denominator is to multiply the two denominators together. While this always works, it does not always produce the lowest common denominator.

## Fraction Card Game

**Purpose** The purpose of this lesson is to give practice in greater than, less than, and common denominators, and to play a more complex variation than the first Fraction Card Game. It is Fraction War without manipulatives. In Fractions: Booklet 2, students did this activity with a manipulative to determine which fraction was the largest. This time they have to use a common denominator to determine this.

**Prerequisites** Common denominators and multiplication tables

**Materials** Fraction Cards, pages 29 - 37 in the Fractions: Booklet 7 - Student Workbook  
Paper and pencil

**Warm Up** Show the cards for five eighths and three fifths. “Which one is larger?” “They are both a little bit more than  $\frac{1}{2}$ . But it is hard to tell.” “How do you check?” Have the student develop his thoughts. Change both fractions into equivalent fractions with common denominators. The common denominator is forty.  
 $\frac{5}{8} = \frac{25}{40}$      $\frac{3}{5} = \frac{24}{40}$

**Game Rules**

1. A player deals out all the cards, an even amount to each player. Leftover cards are placed in the center of the table face down.
2. Each player pulls the top card from his or her face-down stack and lays it on the table face up.
3. The fraction card with the larger fraction wins. The player who drew that card keeps all the cards in that turn and puts them in a separate pile.
4. If both cards have equal fractions, each player deals out three cards face down and one card face up. The larger fraction wins all. If another tie occurs, the process is repeated.
5. When a player exhausts his original stack of cards, he picks up the pile of cards he has won and plays with these cards.
6. The person who runs out of cards first is the loser.

This time, if the players are unsure of which card is larger, they change both fractions to a common denominator. The larger fraction wins, or if you choose, the smaller wins.

**Game Variation** Sum Game  
In this game, each player can see all of their cards and can choose them in whatever order they wish. This introduces a strategy element.  
Each player chooses two cards and lays them on the table. He then takes the sum of his cards. The opponent does the same. Whichever sum is the greatest wins all four cards.  
If it is unclear which sum is greatest, then common denominators must be found for those two numbers. This could get to be a fairly difficult operation. A calculator might be helpful here.



## Working With Mixed Numbers

- Purpose** The purpose of this lesson is to include whole numbers in the addition and subtraction of fractions with uncommon denominators. It also introduces regrouping from whole numbers to fractions when the top fraction is smaller than the bottom fraction in a mixed number, e. g.,  $4\frac{1}{4} - 2\frac{3}{4}$ .
- Prerequisites** Addition and Subtraction of Fractions with Uncommon Denominators and converting Mixed Numbers to Improper Fractions
- Materials** Working with Mixed Numbers - Worksheets 1 - 7, pages 38 - 44  
Prism Fractions pieces
- Warm Up** Give the problems  $\frac{3}{4} + \frac{2}{3} = \underline{\quad}$  and  $\frac{3}{4} + \frac{2}{5} = \underline{\quad}$ . Remediate as needed.
- Lesson Part 1** Give Working with Mixed Numbers - Worksheet 1. Do the first one together using the work you did in the warm up;  $1\frac{3}{4} + 3\frac{2}{5}$ . **“What is different?”** “A whole number has been added in front of the fractions.” **“Simplify all answers.”** To explain what you mean, have the student do the left hand problem in the fourth row. The answer is  $5\frac{10}{20}$ . The improper fraction must be converted to a mixed number, and the  $1\frac{1}{5}$  is added to the 5 to create  $6\frac{1}{5}$ . The worksheet will take a while. End the session when she is finished. Check the work. Have her fix any errors the next day.
- Warm Up** Give her the whole piece from a fraction pieces set. Ask her to take out one-half from the whole. *Wait*. See what she does. She will probably trade the whole for two half pieces and give you one of the halves. If she does not do that, give her a ten’s block and ask her to give you a one out of it. She will regroup to do this. Then give the fraction problem again.  
Now give her two whole pieces and ask her to give you a half again. *Watch*. Does she trade both wholes or just one? If she trades both, ask her why she traded the second one. Ask her if trading the second one is necessary.
- Lesson Part 2** Do Working with Mixed Numbers - Worksheet 2, page 39, together. The picture problems repeat the problems you did in the warm up. Make sure the fraction pieces are available in case the pictures are confusing. Be sure she answers question number 3. We want her to see that the problems in set three are nearly the same as the ones she just did in set two. Hopefully she will see that the answer is the same except one higher. Have her prove her answers with the pictures at the bottom.  
  
Do Working with Mixed Numbers - Worksheet 3, page 40. This problem presents a new complication. Have her build  $3\frac{1}{4}$  with her pieces.  
  
**“How could you subtract  $\frac{3}{4}$  from this? You only have  $\frac{1}{4}$ .”** “I would have to trade one of my wholes in for fourths to get more.”  
  
**“Then how many fourths would you have in all?”** “Five.”

**“How many whole pieces would you have then?” “Only two.”**

**“Do you still have  $3\frac{1}{4}$ ?” “Yes, but it is broken up differently.”**

**“Draw what you just did on the worksheet.”** She must realize that to subtract the  $\frac{3}{4}$ , she must regroup one of her wholes into  $\frac{4}{4}$  to change  $3\frac{1}{4}$  into  $2\frac{5}{4}$ .

See if she can do the problem in the center without help. Give her a chance to figure out how to work the problem. If she needs help, have her read through the Last Resort Help Box. It will walk her through the problem.

Give Working with Mixed Numbers - Worksheet 4, page 41. Have her study the example problem. Provide paper for her to draw with and fraction pieces. Make sure she does the ‘Make your own’ problems. Help her only if needed. Check the work and end the session.

### Part 3

Now we will add the complication of regrouping and uncommon denominators.

### Warm Up

Have her make up the first problem as a warm up with a problem such as the ones she did on the last page.

### Lesson

Now give her this problem,  $2\frac{1}{2} - \frac{3}{4}$ . *Watch.* See if she can figure out what to do on her own. She needs to change the first number to  $2\frac{2}{4}$  and then regroup it to  $1\frac{6}{4}$  in order to subtract the  $\frac{3}{4}$ . The answer is  $1\frac{3}{4}$ .

If she needs help, try this question sequence:

**“Check the denominators. What do you notice?” “They are different.”**

**“What do you have to do about that?” “Get a common denominator.”**

**“What would be the common denominator in this case?” “Fourths.”**

**“So go ahead and change the first fraction into fourths. Now what?” “I do not have enough fourths to take away three-fourths because I only have two-fourths.”**

**“So how can you get more fourths?” “I can get more from one of the wholes.”**

**“What would you have then in the first number?” “I would have  $1\frac{6}{4}$ , then I could subtract. I would have  $1\frac{3}{4}$  left.”** Now give her this problem:  $5\frac{1}{4} - 2\frac{1}{3}$ .

### Note

*Have her rewrite the common denominators with the whole number attached. Do not change to mixed numbers.*

She can drop all the extra writing when she is solid in what she is doing. These harder problems will look like this:

$$\begin{array}{r} 5\frac{1}{4} = 5\frac{3}{12} = 4\frac{15}{12} \\ - 2\frac{1}{3} = \quad \quad 2\frac{4}{12} \\ \hline 2\frac{11}{12} \end{array}$$

Do problems like these until she can do them alone without help. Use fractions in your problems that she can build with fraction pieces.

**Practice**

Working with Mixed Numbers - Worksheet 5, page 42, the top six problems.

**Worksheets**


Working with Mixed Numbers - Worksheets 6 and 7, pages 43 and 44, which are all word problems

**Test for**

**Understanding**


Watch her do the last two problems on Working with Mixed Numbers - Worksheet 5, page 42.


Start With  


Take Away  


Draw What's Left?

Ask her to solve the first problem using only the drawing, no numbers. Can she regroup one of the top wholes into fourths, not into halves?

If she draws , rejoice and watch her do the next problem.

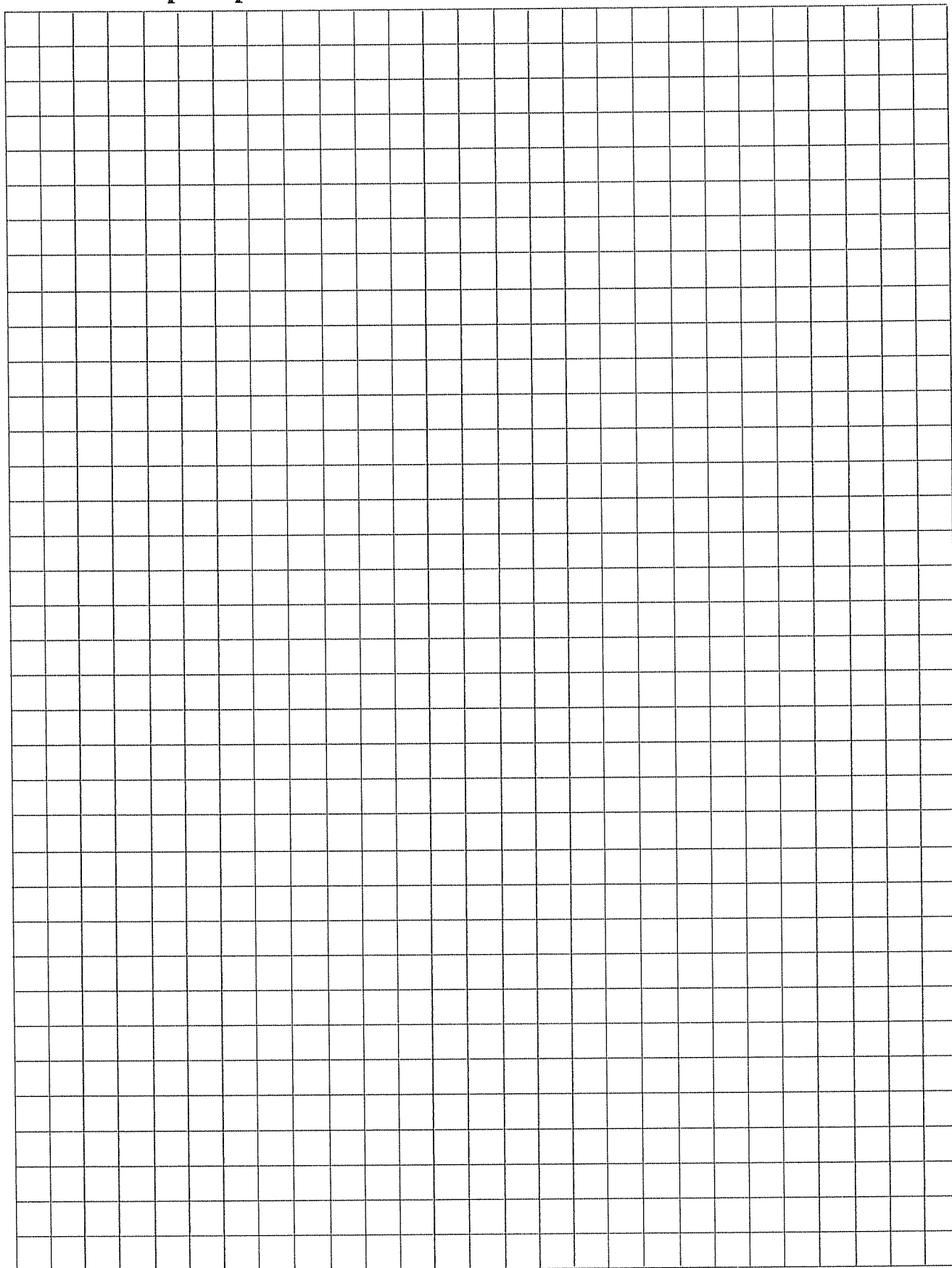
If she draws , ending up with one whole, one-half, and one fourth, ask her to read the answer. She will have to use a common denominator at this point.

Now give her the constraint of changing the drawings so that the common denominator shows up in the picture.

The second problem is about as difficult as problems like these get.\* There is a lot of arithmetic, and one error will give an incorrect answer. If she misses it, check to see if the error was that she got confused or if she made a multiplication or subtraction error.

$$\begin{array}{r} * 6 \frac{2}{7} = 6 \frac{18}{63} = 5 \frac{81}{63} \\ - 2 \frac{5}{7} = 2 \frac{35}{63} = - 2 \frac{35}{63} \\ \hline 3 \frac{46}{63} \end{array}$$

# Half Inch Graph Paper



**Post-Assessment**

Name \_\_\_\_\_ Date \_\_\_\_\_

1. If you add 2 oranges + 3 apples, you will have \_\_\_\_\_

2. Solve.

$$\begin{array}{r} \frac{2}{5} \\ + \frac{1}{5} \\ \hline \end{array}$$

3. Solve.

$$\begin{array}{r} \frac{1}{4} \\ + \frac{1}{12} \\ \hline \end{array}$$

4. Solve.

$$\text{a. } \begin{array}{r} \frac{3}{4} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\text{b. } \begin{array}{r} \frac{3}{5} \\ + \frac{3}{6} \\ \hline \end{array}$$

5. Solve.

$$\begin{array}{r} \frac{1}{3} \\ - \frac{1}{12} \\ \hline \end{array}$$

6. Solve.

$$\text{a. } \begin{array}{r} \frac{5}{6} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$\text{b. } \begin{array}{r} \frac{5}{8} \\ - \frac{5}{12} \\ \hline \end{array}$$

7. Why must you find 'common denominators' to add fractions?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

8. List four common multiples of 3 and 2. \_\_\_\_\_

9. a. When solving this fraction problem,  $\frac{3}{4} + \frac{5}{6}$ , list three common denominators that are possible. \_\_\_\_\_

b. What is the least common denominator? \_\_\_\_\_

c. Why does it matter? \_\_\_\_\_

\_\_\_\_\_

10. Solve. a.  $3$

$$\begin{array}{r} 3 \\ - \frac{3}{4} \\ \hline \end{array}$$

b.  $3 \frac{1}{4}$

$$\begin{array}{r} 3 \frac{1}{4} \\ - 1 \frac{3}{4} \\ \hline \end{array}$$

c.  $7 \frac{1}{3}$

$$\begin{array}{r} 7 \frac{1}{3} \\ - 2 \frac{5}{8} \\ \hline \end{array}$$

# Patterns in Arithmetic

## Fractions: Booklet 7

### Addition and Subtraction of Unlike Fractions

## Answer Key

### for the

## Student Workbook

By Suki Glenn, Susan Carpenter, and Alysia Krafel

#### Answer Key Legend

AWV = answer(s) will vary Cuisenaire Rods

BUWV = break up will vary 1 w = white

OWV = order will vary 2 r = red

#### Pattern Blocks

r = red trapezoid

g = green triangle

y = yellow hexagon

o = orange square

b = blue parallelogram

t = tan rhombus

3 lg = light green

4 p = purple

5 y = yellow

6 dg = dark green

7 bk = black

8 bn = brown

9 bl = blue

10 o = orange

Note: Some items and pages are left out of the answer key.

1) Some pages in which the answers are open-ended or will vary.

2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.

3) Blank practice pages

4) Workboards

5) Games

6) Self correcting pages

7) Instructions only pages

Patterns in Arithmetic: Fractions - Booklet 7

Student Workbook Answer Key

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# Fractions - Booklet 7

## Pre-Assessment - Part 1 - Worksheet 1

- Solve. a.  $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$  b.  $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$  c.  $\frac{3}{11} + \frac{6}{11} = \frac{9}{11}$
- Solve and change any improper fractions to mixed numbers.  
a.  $\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$  b.  $\frac{6}{7} + \frac{5}{7} = \frac{11}{7} = 1\frac{4}{7}$  c.  $\frac{4}{9} + \frac{7}{9} = \frac{11}{9} = 1\frac{2}{9}$
- Solve and change all improper fractions to mixed numbers.  
Example:  $3\frac{3}{4} + 1\frac{3}{4} = 4\frac{6}{4} = 5\frac{1}{2}$   
a.  $2\frac{3}{5} + 1\frac{4}{5} = 3\frac{7}{5} = 4\frac{2}{5}$   
b.  $6\frac{2}{9} + 5\frac{8}{9} = 11\frac{10}{9} = 12\frac{1}{9}$   
c.  $2\frac{3}{7} + 5\frac{6}{7} = 7\frac{9}{7} = 8\frac{2}{7}$
- Draw the change of:  
a.  $\frac{2}{3}$   to  $\frac{8}{12}$    
b.  $\frac{5}{6}$   to  $\frac{10}{12}$    
c. What is the multiplier in problem a?  $\frac{4}{4}$
- Fill in the blanks.  
a.  $\frac{1}{3} \times \frac{9}{9} = \frac{9}{27}$  b.  $\frac{5}{7} \times \frac{6}{6} = \frac{30}{42}$  c.  $\frac{8}{9} \times \frac{4}{4} = \frac{32}{36}$
- Fill in the missing numbers.  
a.  $\frac{7}{9} = \frac{14}{18}$  b.  $\frac{3}{5} = \frac{18}{30}$  a.  $\frac{5}{6} = \frac{15}{18}$

1

## Pre-Assessment - Part 1 - Worksheet 2

- How do you know what this number is?  $12 \div 3 = 4$  so  $4 \times \frac{2}{3} = \frac{8}{3}$   
Or  $3 \times 4 = 12$  and the multiplier has to equal one, also  $2 \times 4 = 8$   
a.  $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$   
b. Why does this number have to be equal to one? Because of the Identity Property of Multiplication. One times any number is the same number.  
 $\frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$
- Simplify these fractions. Show the Mighty One.  
a.  $\frac{15}{18} \div \frac{3}{3} = \frac{5}{6}$  b.  $\frac{7}{21} \div \frac{7}{7} = \frac{1}{3}$  c.  $\frac{35}{56} \div \frac{7}{7} = \frac{5}{8}$
- Where does this number come from? It has to equal one and 4 is a common factor of 12 and 16.  
a.  $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$   
b. What is the value of this number? The value of the number is one.
- List the common factors of 24 and 32.  
a. 1 2 4 8  
b. Circle the common factor you would use to simplify  $\frac{24}{32}$ .  
c. What would happen if you used four to simplify  $\frac{24}{32}$ ?  
 $\frac{24}{32} \div \frac{4}{4} = \frac{6}{8}$  The answer would not be in the lowest terms or simplified all the way.  
d. Why is the Greatest Common Factor used to simplify a fraction instead of any factor? The GCF will simplify the fraction to its lowest term or to the fewest pieces.

2

## Pre-Assessment - Part 2

Put a question mark next to any problem you do not know how to do.

- If you add 2 oranges + 3 apples, you will have 5 fruits
- Solve.  $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$
- Solve.  $\frac{1}{4} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$
- Solve. a.  $\frac{3}{4} = \frac{9}{12}$  b.  $\frac{3}{5} = \frac{18}{30}$   
 $\frac{2}{3} = \frac{8}{12}$   $\frac{3}{6} = \frac{15}{30}$   
 $\frac{11}{12} = \frac{15}{12}$   $\frac{33}{30} = \frac{11}{10}$
- Solve.  $\frac{1}{3} = \frac{4}{12}$  6. Solve. a.  $\frac{5}{6} = \frac{10}{12}$  b.  $\frac{5}{8} = \frac{15}{24}$   
 $\frac{1}{12} = \frac{1}{12}$   $\frac{3}{4} = \frac{9}{12}$   $\frac{3}{12} = \frac{1}{4}$   $\frac{5}{12} = \frac{10}{24}$   
 $\frac{12}{12} = \frac{1}{1}$   $\frac{1}{12}$   $\frac{5}{24}$
- Why must you find 'common denominators' to add fractions?  
The unit numbers must match because you can't add or subtract unlike units.
- List four common multiples of 3 and 2. 6, 12, 18, 24
- a. When solving this fraction problem,  $\frac{3}{4} + \frac{5}{6}$ , list three common denominators that are possible: 12, 24, 36  
b. What is the Least Common Denominator? 12  
c. Why does it matter? The arithmetic is easier and the answer is in the simplest form.
- Solve. a.  $2\frac{4}{8} - \frac{3}{4} = 1\frac{1}{4}$  b.  $2\frac{1}{4} - \frac{3}{4} = 1\frac{1}{2}$  c.  $6\frac{8+24}{24} - 2\frac{5}{8} = \frac{32}{24} - \frac{15}{24} = \frac{17}{24}$

3

4

## Addition of Unlike Fractions: Manipulative Worksheet 1

### Kitty Cat Problems

$$\begin{array}{r} 3 \text{ cats} \\ + 2 \text{ cats} \\ \hline 5 \text{ cats} \end{array}$$

$$\begin{array}{r} 3 \text{ cats} \\ + 2 \text{ kittens} \\ \hline 5 \text{ felines} \end{array}$$

Sometimes one thing can change into another. Kittens are also cats. Kittens change into cats.



$$\begin{array}{r} 3 \text{ dogs} \\ + 2 \text{ pups} \\ \hline 5 \text{ canines} \end{array}$$

$$\begin{array}{r} 3 \text{ stars} \\ + 2 \text{ suns} \\ \hline 5 \text{ stars} \end{array}$$

To add things, units must be the same. (suns are stars)

### Apple Orange Problems

$$\begin{array}{r} 3 \text{ apples} = 3 \text{ fruits} \\ + 2 \text{ oranges} = 2 \text{ fruits} \\ \hline 5 \text{ fruits} \end{array}$$



Sometimes the objects are so different they have to be changed into a new group. Apples can't change into oranges, but they are both fruits

$$\begin{array}{r} 3 \text{ cats} = 3 \text{ animals} \\ + 2 \text{ dogs} = 2 \text{ animals} \\ \hline 5 \text{ animals} \end{array}$$

$$\begin{array}{r} 3 \text{ cars} = 3 \text{ vehicles} \\ + 2 \text{ trucks} = 2 \text{ vehicles} \\ \hline 5 \text{ vehicles} \end{array}$$

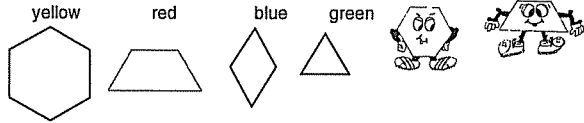
$$\begin{array}{r} 3 \text{ stars} = 3 \text{ heavenly bodies} \\ + 2 \text{ planets} = 2 \text{ heavenly bodies} \\ \hline 5 \text{ heavenly bodies} \end{array}$$

Often units have to be changed to match before they can be worked with. Practice changing units with pattern blocks.

## Addition of Unlike Fractions: Manipulative Worksheet 2

Use pattern blocks.

In order to add objects of unlike units, you must rename them in some way to make the units the same. On this page the unit is size and color. Trade blocks so the colors match.



$$1 \text{ red block} + 1 \text{ yellow block} = 1 \text{ red block} + 2 \text{ red blocks} = 3 \text{ red blocks}$$

$$1 \text{ blue} + 1 \text{ green} = 2 \text{ greens} + 1 \text{ green} = 3 \text{ greens}$$

$$3 \text{ reds} + 1 \text{ green} = 9 \text{ greens} + 1 \text{ green} = 10 \text{ greens}$$

$$4 \text{ blues} + 1 \text{ green} = 8 \text{ greens} + 1 \text{ green} = 9 \text{ greens}$$

$$1 \text{ yellow} + 1 \text{ green} = 6 \text{ greens} + 1 \text{ green} = 7 \text{ greens}$$

$$5 \text{ blues} + 2 \text{ greens} = 10 \text{ greens} + 2 \text{ greens} = 12 \text{ greens}$$

or  $5 \text{ blues} + 1 \text{ blue} = 6 \text{ blues}$

$$3 \text{ reds} + 2 \text{ greens} = 9 \text{ greens} + 2 \text{ greens} = 11 \text{ greens}$$

$$1 \text{ red} + 5 \text{ greens} = 3 \text{ greens} + 5 \text{ greens} = 8 \text{ greens}$$

Make 2 problems of your own. AWV



5

## Addition of Unlike Fractions: Manipulative Worksheet 3

Use fraction pieces to find the sums.

These fractions are in different units. Units must match in order to add. Figure out how to trade pieces so the answers come out as a single color or single fraction. Trading fractions so the units match is called finding a COMMON DENOMINATOR. This means the two fractions have the same denominator.



$$\begin{array}{r} \frac{1}{2} \\ + \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

$$\begin{array}{r} \frac{1}{6} = \frac{2}{12} \\ + \frac{1}{12} = \frac{1}{12} \\ \hline \frac{3}{12} \end{array}$$

$$\begin{array}{r} \frac{1}{4} \\ + \frac{1}{8} \\ \hline \frac{3}{8} \end{array}$$

$$\begin{array}{r} \frac{1}{3} \\ + \frac{1}{6} \\ \hline \frac{3}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{5} \\ + \frac{1}{10} \\ \hline \frac{3}{10} \end{array}$$

$$\begin{array}{r} \frac{1}{8} \\ + \frac{1}{16} \\ \hline \frac{3}{16} \end{array}$$

6

## Addition of Unlike Fractions: Manipulative Worksheet 4

Use fraction pieces to find the sums. Find common denominators.

$$\begin{array}{r} \frac{1}{2} \\ + \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

$$\begin{array}{r} \frac{1}{2} \\ + \frac{1}{6} \\ \hline \frac{4}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{5} \\ + \frac{3}{10} \\ \hline \frac{5}{10} \end{array}$$

$$\begin{array}{r} \frac{1}{2} \\ + \frac{3}{8} \\ \hline \frac{7}{8} \end{array}$$

$$\begin{array}{r} \frac{1}{4} \\ + \frac{5}{12} \\ \hline \frac{8}{12} \end{array}$$

$$\begin{array}{r} \frac{1}{3} \\ + \frac{5}{12} \\ \hline \frac{9}{12} \end{array}$$

Make your own.

$$\begin{array}{r} \frac{3}{8} \\ + \frac{1}{4} \\ \hline \frac{5}{8} \end{array}$$

$$\begin{array}{r} \frac{2}{6} \\ + \frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$

$$\begin{array}{r} + \\ \hline \text{AWV} \end{array}$$

7

## Addition of Unlike Fractions: Manipulative Worksheet 5

Use fraction pieces to find the sums. Find common denominators.

$$\begin{array}{r} \frac{1}{4} \\ + \frac{1}{8} \\ \hline \frac{3}{8} \end{array}$$

$$\begin{array}{r} \frac{2}{3} \\ + \frac{1}{6} \\ \hline \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{6} \\ + \frac{7}{12} \\ \hline \frac{9}{12} \end{array}$$

$$\begin{array}{r} \frac{3}{6} \\ + \frac{1}{12} \\ \hline \frac{7}{12} \end{array}$$

$$\begin{array}{r} \frac{2}{5} \\ + \frac{1}{10} \\ \hline \frac{5}{10} \end{array}$$

$$\begin{array}{r} \frac{1}{2} \\ + \frac{1}{8} \\ \hline \frac{5}{8} \end{array}$$

$$\begin{array}{r} \frac{1}{6} \\ + \frac{5}{12} \\ \hline \frac{7}{12} \end{array}$$

$$\begin{array}{r} \frac{3}{4} \\ + \frac{1}{8} \\ \hline \frac{7}{8} \end{array}$$

$$\begin{array}{r} \frac{3}{5} \\ + \frac{2}{10} \\ \hline \frac{8}{10} \end{array}$$

$$\begin{array}{r} \frac{1}{5} \\ + \frac{3}{10} \\ \hline \frac{5}{10} \end{array}$$

$$\begin{array}{r} \frac{2}{6} \\ + \frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$

$$\begin{array}{r} \frac{1}{4} \\ + \frac{6}{8} \\ \hline \frac{8}{8} \end{array}$$

8



### Addition of Unlike Fractions: Manipulative Worksheet 6



Use pattern blocks to find the sums. Find common denominators.

$$\begin{array}{r} 1 \\ + \frac{1}{6} \\ + \frac{1}{3} \\ \hline \frac{3}{6} \\ + \frac{2}{3} \\ + \frac{1}{6} \\ \hline \frac{5}{6} \\ + \frac{1}{3} \\ + \frac{2}{6} \\ \hline \frac{4}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{18} \\ + \frac{2}{9} \\ \hline \frac{5}{18} \\ + \frac{4}{9} \\ + \frac{6}{18} \\ \hline \frac{14}{18} \\ + \frac{5}{9} \\ + \frac{3}{18} \\ \hline \frac{13}{18} \end{array}$$

What is the relationship between the original denominator and the new one?

They are all multiples. The smaller denominator is half the size of the larger denominator.

### Addition of Unlike Fractions: Recording Worksheet 1

Use pattern blocks. Find the sums. Trade blocks so the colors match.

$$\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$\frac{3}{4} + \frac{1}{12} = \frac{9}{12} + \frac{1}{12} = \frac{10}{12}$$

$$\frac{4}{6} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12}$$

$$\frac{5}{6} + \frac{2}{12} = \frac{10}{12} + \frac{2}{12} = \frac{12}{12}$$

$$\frac{3}{4} + \frac{2}{12} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$$

### Addition of Unlike Fractions: Recording Worksheet 2

Use pattern blocks to find the sums. Figure out how to trade or regroup pieces so the answers come out as a single color or same denominator. Record the trades. Show the Common Denominators.

Review:  $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

How many red blocks? 3

How many green blocks? 3

$\frac{1}{4} = \frac{1}{4}$

$\frac{1}{12} = \frac{1}{12}$

$+\frac{1}{2} = \frac{2}{4}$

$+\frac{1}{6} = \frac{2}{12}$

$\frac{3}{4}$

$\frac{3}{12}$

How many red blocks? 3

How many green blocks? 3

$\frac{1}{6} = \frac{1}{6}$

$\frac{1}{9} = \frac{2}{18}$

$+\frac{1}{3} = \frac{2}{6}$

$+\frac{1}{18} = \frac{1}{18}$

$\frac{3}{6}$

$\frac{3}{18}$

### Addition of Unlike Fractions: Recording Worksheet 3

Use fraction pieces to find the sums. Figure out how to trade or regroup pieces so the answers come out as a single color or same denominator.

Review:  $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Example: Record the trades.  $\frac{1}{3} = \frac{2}{6}$

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{6} = \frac{2}{12}$

$+\frac{1}{4} = \frac{1}{4}$

$+\frac{1}{12} = \frac{1}{12}$

$\frac{3}{4}$

$\frac{3}{12}$

$\frac{1}{5} = \frac{2}{10}$

$\frac{1}{4} = \frac{2}{8}$

$+\frac{1}{10} = \frac{1}{10}$

$+\frac{1}{8} = \frac{1}{8}$

$\frac{3}{10}$

$\frac{3}{8}$

What is the relationship between the original denominator and the new one?  
The smaller denominators divide evenly into the larger denominators. Change the smaller denominator into the larger denominator.

### Addition of Unlike Fractions: Recording Worksheet 4

Addition of Unlike Fractions: Recording Worksheet 4  
Use fraction pieces to find the sums.  
Show the trades to get Common Denominators.



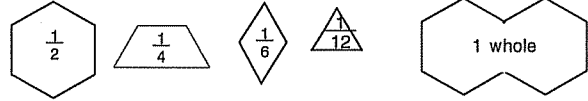
|                                |                                |
|--------------------------------|--------------------------------|
| $\frac{1}{2} = \frac{4}{8}$    | $\frac{3}{12} = \frac{3}{12}$  |
| $+\frac{3}{8} = \frac{3}{8}$   | $+\frac{1}{4} = \frac{3}{12}$  |
| <hr/>                          | <hr/>                          |
| $\frac{7}{8}$                  | $\frac{6}{12}$                 |
| $\frac{2}{3} = \frac{8}{12}$   | $\frac{1}{4} = \frac{3}{12}$   |
| $+\frac{1}{12} = \frac{1}{12}$ | $+\frac{5}{12} = \frac{5}{12}$ |
| <hr/>                          | <hr/>                          |
| $\frac{9}{12}$                 | $\frac{8}{12}$                 |
| $\frac{1}{2} = \frac{3}{6}$    | $\frac{1}{2} = \frac{8}{16}$   |
| $+\frac{2}{6} = \frac{2}{6}$   | $+\frac{3}{16} = \frac{3}{16}$ |
| <hr/>                          | <hr/>                          |
| $\frac{5}{6}$                  | $\frac{11}{16}$                |
| $\frac{1}{3} = \frac{4}{12}$   | $\frac{1}{4} = \frac{4}{16}$   |
| $+\frac{5}{12} = \frac{5}{12}$ | $+\frac{3}{16} = \frac{3}{16}$ |
| <hr/>                          | <hr/>                          |
| $\frac{9}{12}$                 | $\frac{7}{16}$                 |

What patterns do you notice? The smaller denominators divide into the larger denominators evenly.

### Subtraction of Unlike Fractions - Worksheet 1



Use pattern blocks.



1 yellow block - 1 red block = 2 red blocks - 1 red block = 1 red block

$$\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

3 reds - 1 green = 9 greens - 1 green = 8 greens

$$\frac{3}{4} - \frac{1}{4} = \frac{9}{12} - \frac{1}{12} = \frac{8}{12}$$

4 blues - 1 green = 8 greens - 1 green = 7 greens

$$\frac{4}{6} - \frac{1}{12} = \frac{8}{12} - \frac{1}{12} = \frac{7}{12}$$

4 blues - 2 greens = 8 greens - 2 greens = 6 greens

$$\frac{4}{6} - \frac{2}{12} = \frac{8}{12} - \frac{2}{12} = \frac{6}{12}$$

5 blues - 2 greens = 10 greens - 2 greens = 8 greens

$$\frac{5}{6} - \frac{2}{12} = \frac{10}{12} - \frac{2}{12} = \frac{8}{12}$$

3 reds - 2 greens = 9 greens - 2 greens = 7 greens

$$\frac{3}{4} - \frac{2}{12} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

### Subtraction of Unlike Fractions - Worksheet 2

Use fraction pieces. Record the trades. Show the Common Denominators.

|                                |                              |  |
|--------------------------------|------------------------------|--|
| $\frac{1}{2} = \frac{2}{4}$    | $\frac{1}{3} = \frac{2}{6}$  |  |
| $-\frac{1}{4} = \frac{1}{4}$   | $-\frac{1}{6} = \frac{1}{6}$ |  |
| <hr/>                          | <hr/>                        |  |
| $\frac{1}{4}$                  | $\frac{1}{6}$                |  |
| $\frac{1}{6} = \frac{2}{12}$   | $\frac{2}{3} = \frac{4}{6}$  |  |
| $-\frac{1}{12} = \frac{1}{12}$ | $-\frac{1}{6} = \frac{1}{6}$ |  |
| <hr/>                          | <hr/>                        |  |
| $\frac{1}{12}$                 | $\frac{3}{6}$                |  |
| $\frac{5}{6} = \frac{10}{12}$  | $\frac{2}{3} = \frac{4}{6}$  |  |
| $-\frac{1}{12} = \frac{1}{12}$ | $-\frac{3}{6} = \frac{3}{6}$ |  |
| <hr/>                          | <hr/>                        |  |
| $\frac{9}{12}$                 | $\frac{1}{6}$                |  |
| $\frac{5}{6} = \frac{10}{12}$  | Make your own. AWW           |  |
| $-\frac{2}{12} = \frac{2}{12}$ | $- =$                        |  |
| <hr/>                          | $- =$                        |  |
| $\frac{8}{12}$                 |                              |  |

### Subtraction of Unlike Fractions - Worksheet 3

Use fraction pieces. Record the Common Denominators.



|                              |                               |
|------------------------------|-------------------------------|
| $\frac{7}{8} = \frac{7}{8}$  | $\frac{7}{12} = \frac{7}{12}$ |
| $-\frac{1}{2} = \frac{4}{8}$ | $-\frac{1}{4} = \frac{3}{12}$ |
| <hr/>                        | <hr/>                         |
| $\frac{3}{8}$                | $\frac{4}{12}$                |

|                                 |                                |
|---------------------------------|--------------------------------|
| $\frac{15}{16} = \frac{15}{16}$ | $\frac{1}{2} = \frac{6}{12}$   |
| $-\frac{1}{2} = \frac{8}{16}$   | $-\frac{1}{12} = \frac{1}{12}$ |
| <hr/>                           | <hr/>                          |
| $\frac{7}{16}$                  | $\frac{5}{12}$                 |

|                              |                                |
|------------------------------|--------------------------------|
| $\frac{1}{2} = \frac{3}{6}$  | $\frac{1}{2} = \frac{5}{10}$   |
| $-\frac{1}{6} = \frac{1}{6}$ | $-\frac{1}{10} = \frac{1}{10}$ |
| <hr/>                        | <hr/>                          |
| $\frac{2}{6}$                | $\frac{4}{10}$                 |

|                                |                                |
|--------------------------------|--------------------------------|
| $\frac{1}{3} = \frac{4}{12}$   | $\frac{1}{4} = \frac{4}{16}$   |
| $-\frac{1}{12} = \frac{1}{12}$ | $-\frac{1}{16} = \frac{1}{16}$ |
| <hr/>                          | <hr/>                          |
| $\frac{3}{12}$                 | $\frac{3}{16}$                 |

### Subtraction of Unlike Fractions - Worksheet 4

Use fraction pieces. Record the Common Denominators.

|                               |                               |
|-------------------------------|-------------------------------|
| $\frac{1}{2} = \frac{3}{6}$   | $\frac{5}{6} = \frac{5}{6}$   |
| $-\frac{1}{3} = \frac{2}{6}$  | $-\frac{1}{2} = \frac{3}{6}$  |
| $\frac{1}{6}$                 | $\frac{2}{6}$                 |
| $\frac{2}{3} = \frac{8}{12}$  | $\frac{1}{4} = \frac{3}{12}$  |
| $-\frac{1}{4} = \frac{3}{12}$ | $-\frac{1}{6} = \frac{2}{12}$ |
| $\frac{5}{12}$                | $\frac{1}{12}$                |
| $\frac{5}{6} = \frac{5}{6}$   | $\frac{3}{4} = \frac{9}{12}$  |
| $-\frac{2}{3} = \frac{4}{6}$  | $-\frac{2}{3} = \frac{8}{12}$ |
| $\frac{1}{6}$                 | $\frac{1}{12}$                |
| $\frac{2}{3} = \frac{4}{6}$   | $\frac{3}{4} = \frac{9}{12}$  |
| $-\frac{3}{6} = \frac{3}{6}$  | $-\frac{3}{6} = \frac{6}{12}$ |
| $\frac{1}{6}$                 | $\frac{3}{12}$                |

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### Addition and Subtraction of Unlike Fractions

Show You Know

Show the Common Denominators. Simplify the answer.

|                                 |                                |
|---------------------------------|--------------------------------|
| $\frac{1}{3} = \frac{2}{6}$     | $\frac{1}{3} = \frac{2}{6}$    |
| $+\frac{1}{6} = \frac{1}{6}$    | $-\frac{1}{6} = \frac{1}{6}$   |
| $\frac{3}{6}$                   | $\frac{1}{6}$                  |
| $\frac{2}{3} = \frac{8}{12}$    | $\frac{2}{3} = \frac{8}{12}$   |
| $+\frac{1}{4} = \frac{3}{12}$   | $-\frac{1}{4} = \frac{3}{12}$  |
| $\frac{11}{12}$                 | $\frac{5}{12}$                 |
| $\frac{2}{3} = \frac{8}{12}$    | $\frac{2}{3} = \frac{8}{12}$   |
| $+\frac{5}{12} = \frac{5}{12}$  | $-\frac{5}{12} = \frac{5}{12}$ |
| $\frac{13}{12} = 1\frac{1}{12}$ | $\frac{3}{12}$                 |
| $\frac{5}{6} = \frac{10}{12}$   | $\frac{5}{6} = \frac{10}{12}$  |
| $+\frac{1}{4} = \frac{3}{12}$   | $-\frac{1}{4} = \frac{3}{12}$  |
| $\frac{7}{12}$                  | $\frac{7}{12}$                 |

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### Addition Unlike Fractions- Assessment

Show You Know

Trade without using fraction pieces. Show the Common Denominator:

Example:

|                                |                                |
|--------------------------------|--------------------------------|
| $\frac{1}{2} = \frac{4}{8}$    | $\frac{1}{12} = \frac{1}{12}$  |
| $+\frac{1}{8} = \frac{1}{8}$   | $+\frac{1}{4} = \frac{3}{12}$  |
| $\frac{5}{8}$                  | $\frac{4}{12}$                 |
| $\frac{1}{3} = \frac{4}{12}$   | $\frac{1}{2} = \frac{6}{12}$   |
| $+\frac{1}{12} = \frac{1}{12}$ | $+\frac{1}{12} = \frac{1}{12}$ |
| $\frac{5}{12}$                 | $\frac{7}{12}$                 |
| $\frac{1}{2} = \frac{3}{6}$    | $\frac{1}{2} = \frac{5}{10}$   |
| $+\frac{1}{6} = \frac{1}{6}$   | $+\frac{1}{10} = \frac{1}{10}$ |
| $\frac{4}{6}$                  | $\frac{6}{10}$                 |

Challenge.

|                                |                                |
|--------------------------------|--------------------------------|
| $\frac{1}{2} = \frac{8}{16}$   | $\frac{1}{4} = \frac{4}{16}$   |
| $+\frac{1}{16} = \frac{1}{16}$ | $+\frac{1}{16} = \frac{1}{16}$ |
| $\frac{9}{16}$                 | $\frac{5}{16}$                 |

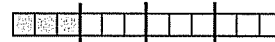
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### Common Denominators with Graph Paper - Worksheet 1

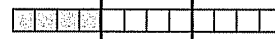
Use graph paper

Example:

This shows one-fourth.

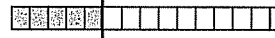


This shows one-third.

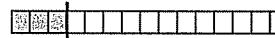


The same number of boxes are in the whole.

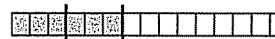
Show one-third.



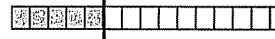
Show one-fifth.



Show two-fifths.



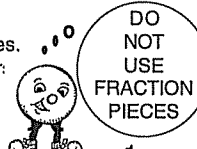
Show one-third.



Why are fifteen boxes chosen to show fifths and thirds?

Both 3 and 5 are factors of 15. Multiplied together, they equal 15.

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### Common Denominators with Graph Paper - Worksheet 2

$\frac{1}{4} = \frac{2}{8}$  One-fourth equals how many eighths? 2

Example:

Draw  $\frac{1}{4}$  with a whole represented by 8 boxes. Draw the equivalent fractions.

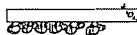
|               |  |   |
|---------------|--|---|
| $\frac{1}{4}$ |  | Why were eighths chosen?<br><u>Best common denominator.</u><br><u><math>4 \times 2 = 8</math></u> |
| +             |  |   |
| $\frac{1}{8}$ |  |   |
| $\frac{3}{8}$ |  |   |

Draw and color the fractions on the graph paper. Find a common denominator for each set. Solve.

|                              |  |
|------------------------------|--|
| $\frac{1}{3} = \frac{6}{18}$ |  |
| +                            |  |
| $\frac{1}{6} = \frac{3}{18}$ |  |
| $\frac{9}{18}$               |  |

|                               |  |
|-------------------------------|--|
| $\frac{1}{6} = \frac{2}{12}$  |  |
| +                             |  |
| $\frac{1}{12} = \frac{1}{12}$ |  |
| $\frac{3}{12}$                |  |

|                               |  |
|-------------------------------|--|
| $\frac{1}{5} = \frac{2}{10}$  |  |
| +                             |  |
| $\frac{1}{10} = \frac{1}{10}$ |  |
| $\frac{3}{10}$                |  |



### Common Denominators with Graph Paper - Worksheet 3

Draw and color the fractions on the graph paper. Find a common denominator for each set. Solve.

|   |  |
|---|--|
| $\frac{2}{3} = \frac{4}{6}$                 |  |
| +   |  |
| $\frac{5}{6} = \frac{5}{6}$                 |  |
| $\frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$ |  |

|                               |  |
|-------------------------------|--|
| $\frac{2}{3} = \frac{6}{9}$   |  |
| +                             |  |
| $\frac{4}{9} = \frac{4}{9}$   |  |
| $\frac{10}{9} = 1\frac{1}{9}$ |  |

|                                 |  |
|---------------------------------|--|
| $\frac{2}{5} = \frac{6}{15}$    |  |
| +                               |  |
| $\frac{2}{3} = \frac{10}{15}$   |  |
| $\frac{16}{15} = 1\frac{1}{15}$ |  |

|                              |  |
|------------------------------|--|
| $\frac{1}{2} = \frac{9}{18}$ |  |
| +                            |  |
| $\frac{1}{9} = \frac{2}{18}$ |  |
| $\frac{10}{18}$              |  |

### Least Common Multiple - Worksheet 1

Count by threes to sixty.

3 6 9 12 15 18 21 24 27 30  
33 36 39 42 45 48 51 54 57 60

Count by fours to sixty.

4 8 12 16 20 24 28 32 36 40  
44 48 52 56 60

Circle the numbers which show up in both lists.

These are called common multiples of 3 and 4.

Write all of the common multiples. 12, 24, 36, 48, 60

Which number is the lowest number in the list? 12

This is called the Least Common Multiple.

Count by sixes to sixty.

6 12 18 24 30 36 42 48 54 60

Count by fives to sixty.

5 10 15 20 25 30 35 40 45 50 55 60

Circle all of the common multiples of 6 and 5.

Write all of the common multiples. 30, 60

What is the Least Common Multiple? 30

### Least Common Multiple - Worksheet 2

List the multiples of:

9 9 18 27 36 45 54 63 72 81 90 99 108  
6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102

Circle all of the common multiples of 9 and 6.

Write all of the common multiples 18, 36, 54, 72, 90

What is the least common multiple? 18

4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68  
6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102

Circle all of the common multiples 4 and 6.

Write all of the common multiples 12, 24, 36, 48, 60

What is the least common multiple? 12

3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54  
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85  
6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102

Circle all of the common multiples of 3, 5, and 6.

Write all of the common multiples 30

What is the least common multiple? 30

The least common multiple is used to decide which denominator to use match fractions so they can be added or subtracted.

Aren't you glad you already know how to calculate an equivalent fraction?

### Least Common Multiple - Worksheet 3

What are the common multiples of 2 and 3? 6, 12, 18, 30, 60

Why do you want to choose the Least Common Multiple to make common denominators? There is less arithmetic and it is easier.

Here are four ways to solve  $\frac{1}{2} + \frac{1}{3}$ .

Simplify the answers.

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ + \frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{2} = \frac{6}{12} \\ + \frac{1}{3} = \frac{4}{12} \\ \hline \frac{10}{12} = \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{2} = \frac{9}{18} \\ + \frac{1}{3} = \frac{6}{18} \\ \hline \frac{15}{18} = \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{2} = \frac{30}{60} \\ + \frac{1}{3} = \frac{20}{60} \\ \hline \frac{50}{60} = \frac{5}{6} \end{array}$$

Are all the answers the same? yes, if simplified.

Circle the problem which was the easiest.

6, 12, 18, and 60 are all common multiples of 2 and 3.

Which is the easiest common multiple to use? 6

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### Least Common Multiple - Worksheet 4

Use the Least Common Multiple to decide the easiest common denominator to use. Use graph paper if needed.



$$\text{LCM} = (6)$$

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ + \frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

$$\text{LCM} = (12)$$

$$\begin{array}{r} \frac{2}{3} = \frac{8}{12} \\ + \frac{1}{4} = \frac{3}{12} \\ \hline \frac{11}{12} \end{array}$$

$$\text{LCM} = (30)$$

$$\begin{array}{r} \frac{1}{6} = \frac{5}{30} \\ + \frac{2}{5} = \frac{12}{30} \\ \hline \frac{17}{30} \end{array}$$

$$\text{LCM} = (24)$$

$$\begin{array}{r} \frac{1}{3} = \frac{8}{24} \\ + \frac{3}{8} = \frac{9}{24} \\ \hline \frac{17}{24} \end{array}$$



$$\text{LCM} = (18)$$

$$\begin{array}{r} \frac{1}{6} = \frac{3}{18} \\ + \frac{1}{9} = \frac{2}{18} \\ \hline \frac{5}{18} \end{array}$$

$$\text{LCM} = (12)$$

$$\begin{array}{r} \frac{1}{4} = \frac{3}{12} \\ + \frac{1}{6} = \frac{2}{12} \\ \hline \frac{5}{12} \end{array}$$

$$\text{LCM} = (20)$$

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{3}{5} = \frac{12}{20} \\ \hline \frac{17}{20} \end{array}$$

$$\text{LCM} = (12)$$

$$\begin{array}{r} \frac{1}{4} = \frac{3}{12} \\ + \frac{3}{6} = \frac{6}{12} \\ \hline \frac{9}{12} \end{array}$$

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### Least Common Multiple - Worksheet 5

Use the Least Common Multiple to decide the easiest common denominator to use. Use graph paper if needed.

$$\begin{array}{r} \frac{1}{3} = \frac{5}{15} \\ + \frac{2}{5} = \frac{6}{15} \\ \hline \frac{11}{15} \end{array} \quad \text{LCM} = (15)$$

$$\begin{array}{r} \frac{5}{12} = \frac{5}{12} \\ + \frac{1}{2} = \frac{6}{12} \\ \hline \frac{11}{12} \end{array} \quad \text{LCM} = (12)$$

$$\begin{array}{r} \frac{1}{3} = \frac{7}{21} \\ + \frac{2}{7} = \frac{6}{21} \\ \hline \frac{13}{21} \end{array} \quad \text{LCM} = (21)$$

$$\begin{array}{r} \frac{5}{9} = \frac{10}{18} \\ + \frac{1}{18} = \frac{1}{18} \\ \hline \frac{11}{18} \end{array} \quad \text{LCM} = (18)$$

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{2}{5} = \frac{8}{20} \\ \hline \frac{13}{20} \end{array} \quad \text{LCM} = (20)$$

$$\begin{array}{r} \frac{1}{3} = \frac{8}{24} \\ + \frac{5}{8} = \frac{15}{24} \\ \hline \frac{23}{24} \end{array} \quad \text{LCM} = (24)$$

$$\begin{array}{r} \frac{1}{6} = \frac{5}{30} \\ + \frac{3}{5} = \frac{18}{30} \\ \hline \frac{23}{30} \end{array} \quad \text{LCM} = (30)$$

$$\begin{array}{r} \frac{2}{9} = \frac{4}{18} \\ + \frac{1}{2} = \frac{9}{18} \\ \hline \frac{13}{18} \end{array} \quad \text{LCM} = (18)$$

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### Least Common Multiple - Worksheet 6

Use the Least Common Multiple to decide the easiest common denominator to use. Use graph paper if needed. Simplify all answers.

$$\begin{array}{r} \frac{2}{3} = \frac{10}{15} \\ + \frac{4}{5} = \frac{12}{15} \\ \hline \frac{22}{15} = 1\frac{7}{15} \end{array} \quad \text{LCM} = (15)$$

$$\begin{array}{r} \frac{5}{12} = \frac{5}{12} \\ + \frac{5}{6} = \frac{10}{12} \\ \hline \frac{15}{12} = 1\frac{3}{12} = \frac{17}{12} \end{array} \quad \text{LCM} = (12)$$

$$\begin{array}{r} \frac{2}{3} = \frac{6}{9} \\ + \frac{5}{9} = \frac{5}{9} \\ \hline \frac{11}{9} = 1\frac{2}{9} \end{array} \quad \text{LCM} = (9)$$

$$\begin{array}{r} \frac{3}{4} = \frac{15}{20} \\ + \frac{4}{5} = \frac{16}{20} \\ \hline \frac{31}{20} = 1\frac{11}{20} \end{array} \quad \text{LCM} = (20)$$

$$\begin{array}{r} \frac{5}{6} = \frac{15}{18} \\ + \frac{4}{9} = \frac{8}{18} \\ \hline \frac{23}{18} = 1\frac{5}{18} \end{array} \quad \text{LCM} = (18)$$

$$\begin{array}{r} \frac{5}{12} = \frac{5}{12} \\ + \frac{2}{3} = \frac{8}{12} \\ \hline \frac{13}{12} = 1\frac{1}{12} \end{array} \quad \text{LCM} = (12)$$

$$\begin{array}{r} \frac{2}{7} = \frac{18}{63} \\ + \frac{4}{9} = \frac{28}{63} \\ \hline \frac{46}{63} \end{array} \quad \text{LCM} = (63)$$

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ + \frac{5}{6} = \frac{10}{12} \\ \hline \frac{19}{12} = 1\frac{7}{12} \end{array} \quad \text{LCM} = (12)$$

Fraction Card Game: pages 29 - 37

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Working with Mixed Numbers - Worksheet 1  
Now solve mixed numbers. Simplify all answers.

$$\begin{array}{r} 1 \frac{3}{4} = 1 \frac{27}{36} \\ + 3 \frac{2}{9} = 3 \frac{8}{36} \\ \hline 4 \frac{35}{36} \end{array}$$

$$\begin{array}{r} 1 \frac{3}{4} = 1 \frac{3}{4} \\ + 4 \frac{2}{7} = 4 \frac{2}{7} \\ \hline 6 \frac{1}{28} \end{array}$$

$$\begin{array}{r} 2 \frac{3}{8} = \frac{19}{8} \\ + 1 \frac{2}{8} = \frac{10}{8} \\ \hline 3 \frac{5}{8} = \frac{29}{8} \end{array}$$

$$\begin{array}{r} 4 \frac{2}{3} = 4 \frac{2}{3} \\ + 5 \frac{3}{7} = 5 \frac{3}{7} \\ \hline 10 \frac{21}{21} \end{array}$$

$$\begin{array}{r} 6 \frac{5}{8} = 6 \frac{5}{8} \\ + 7 \frac{6}{9} = 7 \frac{6}{9} \\ \hline 14 \frac{7}{24} \end{array}$$

$$\begin{array}{r} 1 \frac{5}{12} = 1 \frac{5}{12} \\ + 2 \frac{2}{3} = 2 \frac{2}{3} \\ \hline 4 \frac{1}{12} \end{array}$$

$$\begin{array}{r} 3 \frac{2}{3} = 3 \frac{2}{3} \\ + 2 \frac{4}{9} = 2 \frac{4}{9} \\ \hline 6 \frac{1}{9} \end{array}$$

$$\begin{array}{r} 1 \frac{3}{4} = 1 \frac{3}{4} \\ + 3 \frac{2}{6} = 3 \frac{2}{6} \\ \hline 5 \frac{1}{12} \end{array}$$

Working with Mixed Numbers - Worksheet 3

Regrouping Subtraction with fractions. Here is a complication. Try this. Use fraction pieces and draw it.



You do not have enough fourths in  $3 \frac{1}{4}$  to subtract  $\frac{3}{4}$ . What can you do about this? Where can you get more fourths?  
From one of the wholes.

What do you have to change  $3 \frac{1}{4}$  to in order to be able to subtract  $\frac{3}{4}$ ?  
Change one whole to 4/4 add that to the 1/4 = 2 5/4

$$3 \frac{1}{4} \text{ changes to } 2 \frac{5}{4}$$

$$\begin{array}{r} 3 \frac{1}{4} = 2 \frac{5}{4} \\ - \frac{3}{4} = - \frac{3}{4} \\ \hline 2 \frac{2}{4} = 2 \frac{1}{2} \end{array}$$

Last resort Help Box

Before you take away the  $\frac{3}{4}$ , you must regroup one of the wholes into fourths.

This regrouping written down would look like this:  $3 \frac{1}{4} = 2 \frac{5}{4}$

Circle the  $\frac{5}{4}$  in the picture above. Now can you take away the  $\frac{3}{4}$ ? yes

Cross of what you take away in the picture above. Rewrite the problem to look like this:

$$3 \frac{1}{4} - \frac{3}{4} = 2 \frac{5}{4} - \frac{3}{4}$$

Working with Mixed Numbers - Worksheet 2

Regrouping Subtraction with fractions. Use pictures or fraction pieces. Cross out what is being subtracted or taken away.

Examples:

1. Solve.

$$1 - \frac{1}{2} = \frac{1}{2} \quad 1 - \frac{1}{3} = \frac{2}{3} \quad 1 - \frac{3}{4} = \frac{1}{4} \quad 1 - \frac{5}{6} = \frac{1}{6}$$

To subtract fractions out of a whole, what do you have to first do to the whole?  
The whole has to be changed to a fractional unit.

2. Solve.

$$2 - \frac{1}{2} = 1 \frac{1}{2} \quad 2 - \frac{1}{4} = 1 \frac{3}{4} \quad 2 - \frac{3}{4} = 1 \frac{1}{4} \quad 2 - \frac{5}{6} = 1 \frac{1}{6}$$

Do you have to regroup both wholes to do this subtraction?  
No, only one whole.

3. How can the problems above help you solve these problems?  
Only the whole has changed. The fractional part is the same.

$$3 - \frac{1}{2} = 2 \frac{1}{2} \quad 3 - \frac{1}{4} = 2 \frac{3}{4} \quad 3 - \frac{3}{4} = 2 \frac{1}{4} \quad 3 - \frac{5}{6} = 2 \frac{1}{6}$$

4. Draw it. Cross out what is being subtracted.

$$3 - \frac{1}{2} = 2 \frac{1}{2}$$

How many wholes have to be regrouped into halves to solve this problem?

Solve.

$$3 - \frac{1}{4} = 2 \frac{3}{4} \quad 3 - \frac{3}{4} = 2 \frac{1}{4} \quad 3 - \frac{5}{6} = 2 \frac{1}{6}$$

Working with Mixed Numbers - Worksheet 4

Now solve mixed numbers with regrouping. Record every step.

Example::

$$\begin{array}{r} 4 \frac{1}{3} = 3 \frac{4}{3} \\ - 2 \frac{2}{3} = 2 \frac{2}{3} \\ \hline 1 \frac{2}{3} \end{array}$$

$$\begin{array}{r} 5 \frac{1}{4} = 4 \frac{5}{4} \\ - 2 \frac{3}{4} = 2 \frac{3}{4} \\ \hline 2 \frac{2}{4} = 2 \frac{1}{2} \end{array}$$

$$\begin{array}{r} 7 \frac{1}{3} = 6 \frac{4}{3} \\ - 4 \frac{2}{3} = 4 \frac{2}{3} \\ \hline 2 \frac{2}{3} \end{array}$$

$$\begin{array}{r} 6 \frac{1}{7} = 5 \frac{8}{7} \\ - 1 \frac{5}{7} = 1 \frac{5}{7} \\ \hline 4 \frac{3}{7} \end{array}$$

$$\begin{array}{r} 4 \frac{1}{9} = 3 \frac{10}{9} \\ - 2 \frac{8}{9} = 2 \frac{8}{9} \\ \hline 1 \frac{2}{9} \end{array}$$

$$\begin{array}{r} 5 \frac{1}{8} = 4 \frac{9}{8} \\ - 1 \frac{7}{8} = 1 \frac{7}{8} \\ \hline 3 \frac{2}{8} = 3 \frac{1}{4} \end{array}$$

Working with Mixed Numbers - Worksheet 5  
 Now solve mixed numbers with regrouping. Record every step.  
 Simplify all answers.

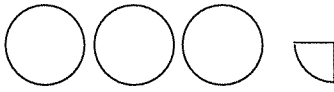
$$\begin{array}{r} 4\frac{1}{5} = 3\frac{6}{5} \\ - 2\frac{2}{5} = 2\frac{2}{5} \\ \hline 1\frac{4}{5} \end{array}$$

$$\begin{array}{r} 7\frac{2}{3} = 7\frac{10}{15} = 6\frac{25}{15} \\ - 4\frac{4}{5} = 4\frac{12}{15} = 4\frac{12}{15} \\ \hline 2\frac{13}{15} \end{array}$$

$$\begin{array}{r} 4\frac{1}{9} = 4\frac{8}{72} = 3\frac{80}{72} \\ - 2\frac{1}{8} = 2\frac{9}{72} = 2\frac{9}{72} \\ \hline 1\frac{71}{72} \end{array}$$

Test for Understanding

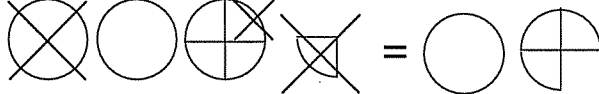
Start With



Take Away



Draw What's Left



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Working with Mixed Numbers - Worksheet 7

5. You need some ribbon too. A remnant at half price has  $\frac{3}{4}$  yard of ribbon.

You need  $\frac{2}{3}$  yard of ribbon. How much will you have left over?  $\frac{1}{12}$

$$\ominus \quad \frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

6. You are repairing an animal pen. There is a board which is  $5\frac{3}{4}$  feet long.

$3\frac{5}{8}$  feet is needed to fix the hole in the fence. How much should be sawed off?  $2\frac{1}{8}$

$$\ominus \quad \begin{array}{r} 5\frac{3}{4} = 5\frac{6}{8} = \frac{46}{8} \\ - 3\frac{5}{8} = \frac{29}{8} \\ \hline 2\frac{1}{8} \end{array} \quad \frac{17}{8} = 2\frac{1}{8}$$

7. You buy a bag of apples which weighs  $2\frac{3}{4}$  pounds. Apples cost 88¢ per pound. How much will the apples cost?  $\$2.42$

$$\otimes \quad 2\frac{3}{4} \times .88 = \$2.42$$

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Working with Mixed Numbers - Worksheet 6

There are addition, subtraction, multiplication and two-step problems in this set.

1. Read the problem
2. Circle the important numbers and underline key words.
3. In the circle under the problem put +, -, or x to identify what kind of problem it is. If it is a two-step problem, show which two steps will be needed.

Before you do the arithmetic, check the answer key to see if you have the correct operation.

Key words - addition: together, in all  
 subtraction: left, how much more, difference  
 multiplication: per, of, each, times



1. A recipe calls for  $2\frac{1}{2}$  cups of flour,  $1\frac{1}{3}$  cups of corn flour,  $\frac{1}{2}$  cup of honey,

and  $\frac{1}{8}$  cup of baking powder. Will a  $4\frac{1}{2}$  cup bowl hold all of this? yes

$$\oplus \quad 2\frac{1}{2} + 1\frac{1}{3} + \frac{1}{2} + \frac{1}{8} = 4\frac{11}{24} \quad - 4\frac{1}{2} = \frac{1}{24}$$

2. How much room will be left over in the bowl?  $\frac{1}{24}$

$$\ominus \quad \begin{array}{r} 4\frac{1}{2} \\ - 4\frac{11}{24} \\ \hline \frac{1}{24} \end{array}$$

3. What if you only wanted to make half of this recipe? How many cups will the bowl need to hold?  $2\frac{1}{4}$

$$\div \quad 4\frac{11}{24} \div 2 = 2\frac{11}{48} \quad 2\frac{1}{4} \text{ is the closest cup measurement}$$

4. A costume you are making calls for  $\frac{1}{4}$  yards of material. The material is \$4.00 per yard. How much will the costume material cost?  $\$1.00$

$$\otimes \quad \$4 \times \frac{1}{4} = \$1.00$$

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Post-Assessment is the same as the Pre-Assessment - Part 2

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