Fractons: Booklet 7 - Addition and Subtraction of Unlike Fractions

Contents

Pre-Assessment: Part 1 and Part 2
Assessment Guide
Addition of Unlike Fractions: Manipulative
Addition of Unlike Fractions: Recording
Subtraction of Unlike Fractions: Recording
Common Denominators with Graph Paper
Least Common Multiples
Fraction Card Game
Working with Mixed Numbers
Half Inch Graph Paper
Answer Key

This booklet is dedicated to all the wonderful students who attended the Farm School. You all taught us so much more than you will ever know.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karan Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 7

Parent/Teacher Guide ©2014 Pattern Press

©2014 Pattern Press

Published by Pattern Press P.O. Box 2737 Fallbrook, CA 92088 (760)728-3731

www.patternpress.com

E-mail: Patternpress1@gmail.com

All rights reserved.

Practice worksheets may be duplicated.

Printed on recycled paper.



ISBN 978-1-935559-11-5

Addition of Unlike Fractions: Manipulative

Purpose

To add fractions with different denominators, manipulatively changing them so the units match. The basic concept behind adding fractions with uncommon denominators is that you cannot combine things that are expressed in different units. You cannot combine three dogs plus two cats to get five unless you rename the dogs to animals and the cats to animals. Then you can combine them into one group of five animals. Essentially you are regrouping to create common units. This basic idea is not difficult for students to grasp. We begin with common things like cats and dogs and then move to pattern blocks of adding greens and blues, then to fraction circles with fractional numbers added in. Most students enjoy this lesson. The full lesson will take four to five sessions.

Prerequisites

Familiarity with pattern blocks, fraction circles, and mastery of the concept of the Changing Whole taught in Fractions: Booklet 2.

Materials

Addition of Unlike Fractions: Manipulative - Worksheets 1 - 6, pages 4 - 9 An assortment of objects such as apples, oranges, little toy cars or trucks, silverware, or bowls and cups, etc.

Pattern blocks

Fraction pieces - both circles and squares

Note

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: "How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."

Warm Up

Play with the pattern blocks for a while. Tell the student that she can make designs and stacks now but not during the lesson.

Lesson

Play a verbal game.

Part 1

"Three cats plus two cats is five ...?" "Cats."

"Three cats plus two kittens is five ...?" You will get various answers. Five cats, or five animals. Cats is the easier solution because only one quantity, kittens, needs to be renamed. If she says 'animals' ask her if kittens are also cats.

"Three puppies plus two dogs is five ...?" "Dogs."

Note

Most students intuitively understand that in order to combine objects, they must be expressed in the same units. Kittens can be renamed as cats, puppies renamed to dogs. Think of other things that are like this where one thing is a subset of another thing. Example: two women plus three girls.

Now set a group of three apples and a group of two apples on the table.

Patterns in Arithmetic: Fractions - Booklet 7 Addition of Unlike Fractions: Manipulative Parent/Teacher Guide

"Three apples plus two apples is ...?" "Five apples."

Leave the group of three apples and replace the group of two apples with two oranges on the table.

"Three apples plus two oranges is five ...?" Many people will say "Apple-oranges." Most do not think initially of changing the units to fruits.

"There is no such thing as apple-oranges! It has to be a real thing that fits.

"What do we have five of here on the table?" "Five pieces of fruit."

"So, three apples plus two oranges is five ...?" "Fruits."

Try another one.

"Three bowls plus two cups are five ...?" "Dishes."

Make up more problems. Have her make up some too.

"Why to we have to say five fruits when we add three apples plus two oranges?" "Because we can not add them unless they both use the same words or units." "Three kittens plus two puppies are five ...?" "Baby animals."

Now look at the top of Addition of Unlike Fractions: Manipulative Worksheet 1. Have her do those problems, writing the answers and the units. (Suns are stars.) Point out that these problems are called kitty cat problems because one of the groups, kittens, can be renamed using the other group, cats. This same thing happens when halves and fourths are added. The half can be traded for fourths.

Now look at the Apple Orange Problems. In these problems, both groups must be renamed in order to combine the numbers. On the first problem, have her write in

3 apples = 3 fruits

+2 oranges = 2 fruits

= 5 fruits

Finish the worksheet. Make sure she reads all the words on the page.

Worksheets

Take out the pattern blocks and do Addition of Unlike Fractions: Manipulative - Worksheet 2. Read the instructions with her and do the first two problems together. Use the Answer Key to check the work immediately.

End the lesson with some play time with the fraction circles or the Prism Fractions you are going to use for the lesson in Part 2.

Lesson Part 2

Play a verbal game with the colored fraction pieces. Use the names of the colors to give the problem. For example, if the whole piece is white and the half piece is red, ask, "What is one white plus two reds?" "Four reds or two whites."

Note

Do not teach how to calculate common denominators at this time. Allow the student to construct the idea that fractional units can be traded and renamed just like the apples and oranges. Manipulatives must be used to do this.

Worksheet

Addition of Unlike Fractions: Manipulative - Worksheet 3. Do the first problem together. Take out the one-half and the one-fourth pieces and place them on the table. "How can you trade one of these pieces so that they stay the same amount but change to all one color?" "I can trade the one-half for two of the fourths." Watch her complete the page to be sure she is doing the work correctly. Have her use the

Addition of Unlike Fractions: Manipulative

Patterns in Arithmetic: Fractions - Booklet 7
Parent/Teacher Guide

Addition of Unlike Fractions: Manipulative - Worksheet 1 Date _____

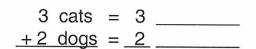
Kitty Cat Problems

Sometimes one thing can change into another. Kittens are also cats. Kittens change into cats.



To add things, units must _____.

Apple Orange Problems



+2 planets = 2



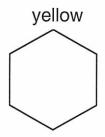


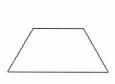
Sometimes the things are so different that they have to be changed into a new group. Apples can't change into oranges, but they are both fruits.

Often units have to be changed to match before they can be worked with. Practice changing units with pattern blocks.

Addition of Unlike Fractions: Manipulative - Worksheet 2 Date _______Use pattern blocks.

In order to add objects of unlike units, you must rename them in some way to make the units the same. On this page the unit is size and color. Trade blocks so the colors match.





red









1 red block + 1 yellow block = 1 red block + 2 red blocks = 3 red blocks

1 blue + 1 green

= ____ greens + ___ green = ___ greens

3 reds + 1 green

= ____ greens + ___ green = ____ greens

4 blues + 1 green

= ___ greens + ___ green = ___ greens

1 yellow+ 1 green

=

5 blues + 2 greens

3 reds + 2 greens

_

1 red + 5 greens

= _____

Make 2 problems of your own.



Addition of Unlike Fractions: Recording

Purpose

The purpose of this lesson is to add unlike fractions with a different manipulative and to record the trades. As usual, the manipulative is changed two to three times to require the student to reconstruct the procedure several times. Many students who have mastered equivalent fractions will begin to use that skill at this point. This lesson takes two to three sessions.

Prerequisites Understanding the concept of Equivalent Fractions is helpful. If it is not mastered, this concept should be worked on in conjunction with this lesson. See Fractions: Booklet 4 - Equivalent Fractions.

Materials

Addition of Unlike Fractions: Recording, Worksheets 1 - 4, pages 10 - 13 pattern blocks and fraction pieces Crayons

Warm Up

Take out the pattern blocks and review the concept of the Changing Whole. In the last lesson, Addition of Unlike Fractions: Manipulative - Worksheet 6, the whole was defined as three yellow pattern blocks. Review what fraction each different colored block then represents. Now repeat the same process, this time defining the whole as two yellow pattern blocks (or a pink if you have those pieces). What is the value of each block now?

"If the whole is two yellows, what is one red plus one yellow equal to?" "Three reds because I can trade one yellow for two reds and add that to the one red I already have."

Lesson Part 1

Take out Addition of Unlike Fractions: Recording - Worksheet 1, page 10. Have him color in the pictures of the blocks at the top of the page to match the colors of the actual blocks. Now study the example problem that is just above the line. The only new thing is the writing of the fractional value of each block just beneath the number and color of the block. On the right side, the trade of the yellow block for two red blocks is recorded both in words and as a fraction. The total is shown in both words and a fractional number.

Do the first problem together.

"What is the fractional value of three reds?" "Three-fourths." Have him write that fraction under the words 'three reds.'

"What is the fractional value of one green?" "One-twelfth." Have him write that fraction under the words 'one green.'

"How are three reds added to one green?" "The three reds are traded for nine greens and then added to the one green we already have." Have him record like this:

3 reds + 1 green = 9 greens + 1 green = 10 greens

$$\frac{3}{4}$$
 + $\frac{1}{12}$ = $\frac{9}{12}$ + $\frac{1}{12}$ = $\frac{10}{2}$

Note

When he writes the fraction under the words, have him write the numerator above

the little line and the denominator below it.

Do not ask him to simplify at this point. If he does it on his own, fine, but do not require it. It adds another level of complexity that can confuse some students at this point.

On the next problem, he must write in the number and colors of the blocks himself. The result will look like this:

4 blues + 1 green = 8 greens + 1 green = 9 greens

$$\frac{4}{6}$$
 + $\frac{1}{12}$ = $\frac{8}{12}$ + $\frac{1}{12}$ = $\frac{9}{12}$

Have him finish the page on his own.

Worksheets

Addition of Unlike Fractions: Recording - Worksheet 2, page 11

Have him color in the pictures of the blocks with the correct colors at the top of the page. Do the first problem with him. He will put one red (\(\frac{1}{4}\)) and one yellow block $(\frac{1}{2})$ on the table. He will trade the one yellow for two reds. Now have him record as shown below.

When completed the first problem will look like this:

"How many red blocks?" "Three."

"What happens to the fractions in the two bottom problems?" "The whole changes."

Have him complete the rest of the page alone.

Practice Worksheets

Addition of Unlike Fractions: Recording - Worksheets 3 and 4, pages 12 and 13. Change the manipulative to fraction pieces to complete these problems. There are two problems with sixteenths in them. Most fraction sets do not have sixteenths. Use these two problems as a Test for Understanding. The Prism Fractions Paper set does have sixteenths.

Test for

See if he can do the problems with the sixteenths without a manipulative. Have Understanding him draw a picture to help. Just watch and see what he does.

$$\frac{\frac{1}{2} + \frac{3}{16} = \frac{8}{16} + \frac{3}{16} = \frac{11}{16}}{2}$$
Example: $\frac{1}{2} = \frac{1}{16} = \frac{1}{1$

If he can not do the problem, just leave it for now. It means he is still constructing the general idea of trading in the concrete stage and needs the physical pieces to do it.

If he can draw a picture, he may realize that halves can be traded for eight-sixteenths and be able to record this. He can represent it and draw it.

If he can do it without a picture or manipulative, you know that he is internalizing and generalizing the abstract, symbolic pattern of trading.

Addition of Unlike Fractions: Recording - Worksheet 3 Date ______ Use fraction pieces to find the sums. Figure out how to trade or regroup pieces so the answers come out as a single color or same denominator.

Review:

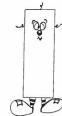
$$\frac{1}{3}$$
 + $\frac{1}{3}$

Example:

Record the trades.

$$\frac{1}{3} = \frac{2}{6}$$

$$+\frac{1}{6}=\frac{1}{6}$$



Record the trades. Show the Common Denominators.

$$\frac{1}{2} = + \frac{1}{4} =$$

$$\frac{1}{6} =$$
 $+ \frac{1}{12} =$

$$\frac{\frac{1}{5}}{=}$$
 + $\frac{1}{10}$ =

$$\frac{1}{4} = + \frac{1}{8} =$$

Make your own.

What is the relationship between the original denominator and the new one?

Fraction Card Game

Purpose

The purpose of this lesson is to give practice in greater than, less than, and common denominators, and to play a more complex variation than the first Fraction Card Game. It is Fraction War without manipulatives. In Fractions: Booklet 2, students did this activity with a manipulative to determine which fraction was the largest. This time they have to use a common denominator to determine this.

Prerequisites Common denominators and multiplication tables

Materials

Fraction Cards, pages 29 - 37 in the Fractions: Booklet 7 - Student Workbook Paper and pencil

Warm Up

Show the cards for five eighths and three fifths. "Which one is larger?" "They are both a little bit more than $\frac{1}{2}$. But it is hard to tell." "How do you check?" Have the student develop his thoughts. Change both fractions into equivalent fractions with common denominators. The common denominator is forty.

$$\frac{5}{8} = \frac{25}{40} \quad \frac{3}{5} = \frac{24}{40}$$

Game Rules

- 1. A player deals out all the cards, an even amount to each player. Leftover cards are placed in the center of the table face down.
- 2. Each player pulls the top card from his or her face-down stack and lays it on the table face up.
- 3. The fraction card with the larger fraction wins. The player who drew that card keeps all the cards in that turn and puts them in a separate pile.
- 4. If both cards have equal fractions, each player deals out three cards face down and one card face up. The larger fraction wins all. If another tie occurs, the process is repeated.
- 5. When a player exhausts his original stack of cards, he picks up the pile of cards he has won and plays with these cards.
- 6. The person who runs out of cards first is the loser.

This time, if the players are unsure of which card is larger, they change both fractions to a common denominator. The larger fraction wins, or if you choose, the smaller wins.

Game Variation

Sum Game

In this game, each player can see all of their cards and can choose them in whatever order they wish. This introduces a strategy element.

Each player chooses two cards and lays them on the table. He then takes the sum of his cards. The opponent does the same. Whichever sum is the greatest wins all four cards.

If it is unclear which sum is greatest, then common denominators must be found for those two numbers. This could get to be a fairly difficult operation. A calculator might be helpful here.

Fractions - Booklet 7

Pre-Assessment - Part 1 - Worksheet 1

- 1. Solve. a. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ b. $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ c. $\frac{3}{11} + \frac{6}{11} = \frac{9}{11}$
- 2. Solve and change any improper fractions to mixed numbers. a. $\frac{4}{5} + \frac{3}{5} = \frac{1}{5} = \frac{12}{5}$ b $\frac{6}{7} + \frac{5}{7} = \frac{11}{7} = \frac{14}{7}$ c. $\frac{4}{9} + \frac{7}{9} = \frac{11}{9} = \frac{12}{9}$
- 3. Solve and change all improper fractions to mixed numbers.

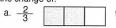
Example: 3 3

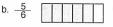
a.
$$2\frac{3}{5}$$

+ $1\frac{4}{5}$
 $3\frac{7}{5}$ = $4\frac{2}{5}$

c.
$$2\frac{3}{7} + 5\frac{6}{7} = 8\frac{2}{7}$$

4. Draw the change of:





- c. What is the multiplier in problem a?
- 5. Fill in the blanks.

a.
$$\frac{1}{3} \times \frac{9}{9} = \frac{9}{2}$$

b.
$$\frac{5}{7} \times \frac{6}{42} = \frac{30}{42}$$

c.
$$\frac{8}{9} \times \frac{4}{4} = \frac{32}{36}$$

6. Fill in the missing numbers. a. $\frac{7}{9} = \frac{49}{63}$ b. $\frac{3}{5} = \frac{18}{30}$

a.
$$\frac{7}{9} = \frac{49}{63}$$

$$\frac{3}{5} = \frac{18}{30}$$

a.
$$\frac{5}{6} = \frac{15}{18}$$

Pre-Assessment - Part 2

Put a question mark next to any problem you do not know how to do.

- 1. If you add 2 oranges + 3 apples, you will have 5 fruits
- 2. Solve.

- 7. Why must you find 'common denominators' to add fractions?
 The unit numbers must match because
 you can't add or subtract unlike units.
- 8. List four common multiples of 3 and 2. 6, 12, 18, 24
- 9. a. When solving this fraction problem. 3/4 + 5/6 list three common denominators that are possible. 12, 24, 36
 b. What is the Least Common Denominator?
 c. Why does it matter? The arithmetic is easier and the answer is in the simplest form.
- 10. Solve. a. 2 4

- Pre-Assessment Part 1 Worksheet 2
- - b. Why does this number have to be equal to one? Because of the $\frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$ The times any number is the same number.
- 8. Simplify these fractions. Show the Mighty One.

 - a. $\frac{15}{18} \div \frac{3}{3} = \frac{5}{6}$ b. $\frac{7}{21} \div \frac{7}{7} = \frac{1}{3}$
- 9. a. Where does this number come from? It has to equal one and $\frac{12}{16} \div \frac{4}{4} = -$ of 12 and 16.
 - b. What is the value of this number? The value of the number is one.
- 10. List the common factors of 24 and 32.
 - a. 1 2 4 (8)
 - b. Circle the common factor you would use to simplify $\frac{24}{22}$.
 - c. What would happen if you used four to simplify $\frac{24}{32}$?

24 ÷ 4 = 6 The answer would not be in the lowest terms or simplified all

d. Why is the Greatest Common Factor used to simplify a fraction instead of any factor? The GCF will simplify the fraction to its lowest term or to the fewest pieces.

Addition of Unlike Fractions: Manipulative Worksheet 1

Kitty Cat Problems

2

1

3 cats 3 cats + 2 cats 5 **CaTS** + 2 kittens 5 felines Sometimes one thing can change into another. Kittens are also cats. Kittens change into cats.

3 dogs 3 stars +2 pups +2 suns 5 canines

To add things, units must be the same. 5 stars (sunsare stars)

Apple Orange Problems

- 3 apples = 3 fruits +2 oranges= 2 fruits 5 fruits
 - 3 cats = 3 <u>animals</u> +2 dogs = 2 <u>animals</u>



Sometimes the objects are so different they have to be changed into a new group. Apples can't change into oranges, but they are both

3 cars = 3 Vehicles +2 trucks = 2 Vehicles 5 Vehicles

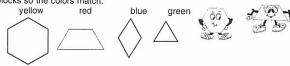
3 stars = 3 heavenly bodies +2 planets = 2 heavenly bodies 5 heavenly bodies

Often units have to be changed to match before they can be worked with. Practice changing units with pattern blocks.

Addition of Unlike Fractions: Manipulative Worksheet 2

Use pattern blocks.

In order to add objects of unlike units, you must rename them in some way to make the units the same. On this page the unit is size and color Trade blocks so the colors match.



1 red block + 1 yellow block = 1 red block + 2 red blocks = 3 red blocks

1 blue + 1 green =
$$2$$
 greens + 1 green = 3 greens

$$3 \text{ reds} + 1 \text{ green} = \underline{q} \text{ greens} + \underline{1} \text{ green} = \underline{10} \text{ greens}$$

4 blues + 1 green =
$$\frac{9}{2}$$
 greens + $\frac{1}{2}$ green = $\frac{9}{2}$ greens

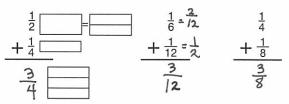
Make 2 problems of your own. AมีV

Addition of Unlike Fractions: Manipulative Worksheet 3

Use fraction pieces to find the sums.

These fractions are in different units. Units must match in order to add. Figure out how to trade pieces so the answers come out as a single color or single fraction. Trading fractions so the units match is called finding a COMMON DENOMINATOR. This means the two fractions have the same denominator.



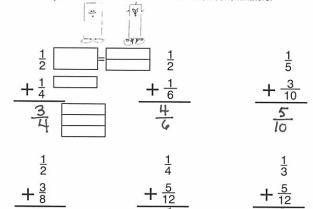


5 | 6

(1 PE)

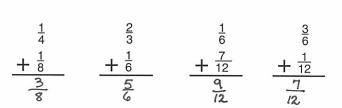
Addition of Unlike Fractions: Manipulative Worksheet 4

Use fraction pieces to find the sums. Find common denominators.



Addition of Unlike Fractions: Manipulative Worksheet 5

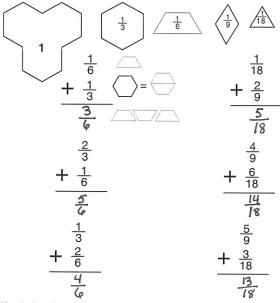
Use fraction pieces to find the sums. Find common denominators.



Addition of Unlike Fractions: Manipulative Worksheet 6



Use pattern blocks to find the sums. Find common denominators.



What is the relationship between the original denominator and the new one?

They are all multiples. The smaller denominator is half the size of the larger denominator.

Addition of Unlike Fractions: Recording Worksheet 1

Use pattern blocks. Find the sums. Trade blocks so the colors match.



Change to all reds.

1 red block + 1 yellow block = 1 red block + 2 red blocks = 3 red blocks
$$\frac{1}{4}$$
 + $\frac{1}{2}$ = $\frac{1}{4}$ + $\frac{2}{4}$ = $\frac{3}{4}$

$$3 \text{ reds} + 1 \text{ green}$$
 = $9 \text{ greens} + 1 \text{ green} = 10 \text{ greens}$

$$\frac{3}{11} + \frac{1}{12} = \frac{9}{12} + \frac{1}{12} = \frac{10}{12}$$

$$4 \text{ blues} + 1 \text{ green} = \frac{8 \text{ greens} + 1 \text{ green}}{12} = \frac{9 \text{ greens}}{12}$$

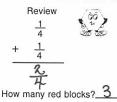
$$\frac{3 \text{ reds} + 2 \text{ greens}}{3} + \frac{2}{12} = \frac{9 \text{ greens} + 2 \text{ greens}}{12} = \frac{11}{12}$$

9

10

Addition of Unlike Fractions: Recording Worksheet 2

Use pattern blocks to find the sums. Figure out how to trade or regroup pieces so the answers come out as a single color or same denominator Record the trades. Show the Common Denominators.









J. 10

How many green blocks? 3

$$\frac{\frac{1}{4} = \frac{1}{4}}{\frac{1}{2} = \frac{2}{4}}$$

 $\frac{\frac{1}{12} = \frac{1}{12}}{+ \frac{1}{6} = \frac{2}{12}}$ $\frac{3}{12}$

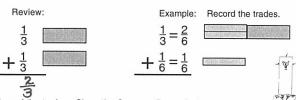
How many red blocks? 3

How many green blocks? 3

$\frac{\frac{1}{3}}{\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}}$ $\frac{\frac{1}{6}}{\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}}$ $\frac{\frac{1}{9}}{\frac{1}{9}} = \frac{\frac{2}{18}}{\frac{18}{18}}$ $\frac{\frac{1}{9}}{\frac{1}{18}} = \frac{\frac{1}{18}}{\frac{1}{18}}$ $\frac{\frac{3}{18}}{\frac{1}{18}}$

Addition of Unlike Fractions: Recording Worksheet 3

Use fraction pieces to find the sums. Figure out how to trade or regroup pieces so the answers come out as a single color or same denominator



Record the trades. Show the Common Denominators.
$$\frac{\frac{1}{2} = \frac{2}{4}}{\frac{1}{4}} \qquad \qquad \frac{\frac{1}{6} = \frac{2}{12}}{\frac{1}{4}} \\ + \frac{1}{4} = \frac{1}{4} \\ \frac{\frac{3}{4}}{\frac{1}{4}} \qquad \qquad \frac{1}{12} = \frac{1}{12} \\ \frac{\frac{1}{12}}{\frac{1}{12}} = \frac{1}{12} \\ \frac{1}{12} = \frac{1}{12} \\$$

What is the relationship between the original denominator and the new one? The Smaller denominators divide evenly into the larger denominator. Change the smaller denominator into the larger denominator.