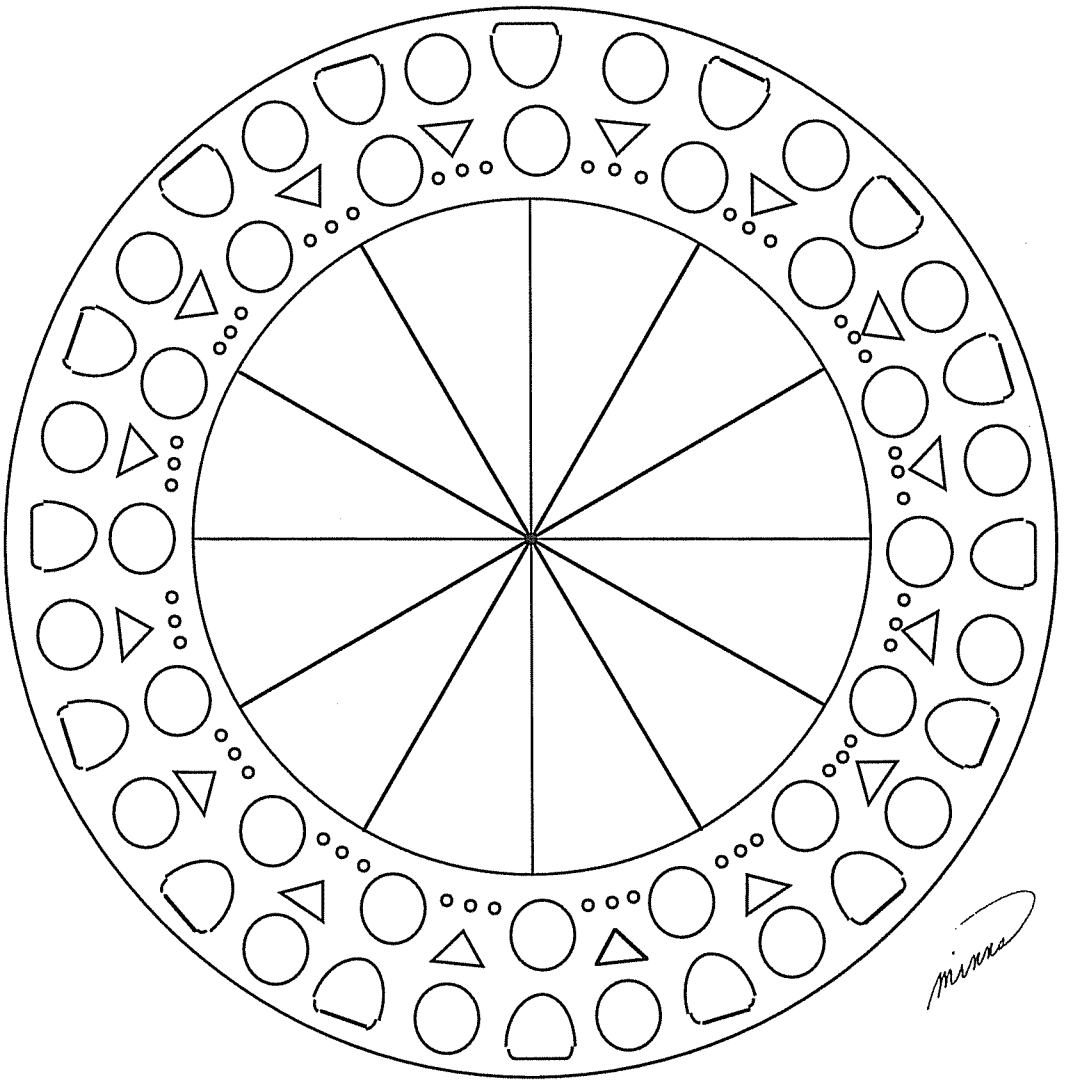
Patterns in Arithmetic

Fractions - Booklet 6 PDF

Multiplication

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 6 PDF - Multiplication

Contents

This booklet is dedicated to all the wonderful parents who had their children educated at the Farm School. Thank you for trusting us with your precious sons and daughters.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the last booklets and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Pre-Assessment: Part 2 is the same as the Post-Assessment.

Prerequisites

Patterns in Arithmetic: Fractions - Booklets 1 - 5

Materials

Fractions: Booklet 6 - Pre-Assessment - Parts 1 and 2 - Worksheets 1 - 3, pages 1 - 3
Score sheets, pages 5 - 8, in this booklet

Note

Do not provide manipulatives of any kind.

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.
Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. Addition of Like Fractions (Fractions: Booklet 2) and Simplification (Fractions: Booklet 5)
   A. add the numerators and not the denominators in two of the three problems?
   B. simplify two of the three sums?

2. Subtraction of Like Fractions (Fractions: Booklet 2)
   A. subtract the numerators and not the denominators in problem a?
   B. visualize the whole as three-thirds and subtract two-thirds in problem b?
   C. Bonus Point: Extend what he knows to get the correct answer to problem c, an unfamiliar but accessible problem? This item tests to see if he can visualize the $2\frac{1}{2}$ subtract one whole and then one-fourth, which the student knows is half of the half, leaving one and one-fourth. A correct answer here indicates strong growth in conceptual understanding, which is formally taught in Fractions: Booklet 7.

3. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)
   A. identify $\frac{1}{2}$ as being the least?
   B. identify $\frac{5}{6}$ as being the greatest?
   C. order correctly $\frac{6}{8}$, $\frac{2}{3}$, and $\frac{6}{10}$?
   D. Bonus Point: simplify $\frac{6}{8}$ or $\frac{6}{10}$?

This item tests conceptual understanding of fractional values. It identifies students who can use reason or drawings to solve a problem without the use of a manipulative.

4. Fractions as Parts of Wholes (Fractions: Booklet 2)
   A. divide the eight items into four equal groups of two?
   B. give the correct answer to all three problems?

5. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
   A. give the correct answer to two of three problems?
   B. calculate rather than draw a graphic to get the answers?

6. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
   A. give the correct answer on two of three problems?
   B. simplify the fraction in problem b?
   C. calculate rather than draw a graphic to get the answers?

7. Equivalence (Fractions: Booklet 1), The Mighty One (Fractions: Booklet 4)
   A. fill in the correct numbers to show the values of one?

8. Equivalence: Calculating (Fractions: Booklet 4)
   A. fill in the correct numerator in two of three problems?
   B. calculate rather than draw a graphic to get the answers?
   C. use the short notation on both problems?

9. Simplification: Calculating (Fractions: Booklet 5)
   A. write in the multiplier in items a and c?
   B. give the correct answer in items b and d?
   C. give the correct answer in item e as a mixed number or an improper fraction?

D. identify the multiplier in item a as 1 on item f?

10. Associative and Commutative Properties (Multiplication: Booklet 2)
    A. fill in the numbers without calculation except for the last number?
    B. identify item a as the Commutative Property?
    C. identify item b as the Associative Property?

11. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)
    A. add the numerators correctly and not the denominators?
    B. identify the multiplication problem as $\frac{2}{5} \times 5 = \frac{10}{5}$?

If he scores a Yes on B, it shows ability to generalize to a new situation. This topic was briefly introduced in Fractions: Booklet 2.

12. Equivalence: Calculating (Fractions: Booklet 4)
    A. pick out two of the three equal fractions?

13. Fractions as Parts of Wholes (Fractions: Booklet 2)
    A. cut the group into three sections and shade two of them to get the answer in item a?
    B. correctly fill in the number sentence in item b?

14. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)
    A. show the multiplication problem as an addition problem?

Booklet Selection Guide based on results of Pre-Assessment: Part 1

If the student scores less than 20 points, excluding items 13 and 14, on Pre-Assessment: Part 1, remediate weak areas and retest. Fractions are complicated and difficult for many students. Be sure you are not moving too quickly. Give more practice. Do not give Pre-Assessment: Part 2 until this work is completed.

Readiness concepts: Items 1A, 5A, 6A, 6B, 9A - D, 10B, and 10C. Any of these items with a No marked on the score sheet need immediate reteaching. Refer back to the appropriate booklets. These skills are needed to complete this booklet. Reteach and retest before giving Pre-Assessment: Part 2.

Important concepts to review: 2A, 3A, 3B, 8A, and 12A. Any of these items marked No should be retaught concurrently with this booklet. These skills will be needed for Fractions: Booklet 7.

All Parts of Wholes topics are reviewed in the beginning of this booklet. If Items 4A, 4B, 11A, 11B, 13A, 13B and 14A are marked with No on the score sheet, complete the review section at the beginning of Fractions: Booklet 6, then retest these items to make sure they are mastered. Not having mastered these concepts will not stop you at this point. Give Pre-Assessment: Part 2 and begin the booklet if these are the only weak areas.

If Items 4A, 4B, 11B, 13A, 13B, and 14A are marked with Yes on the score sheet, you might consider skimming over some of the review parts at the beginning of this booklet.

Assessment Criteria for Pre-Assessment: Part 2

All criteria in Pre-Assessment: Part 2 are taught in Fractions: Booklet 6.

Can the student:

1. Multiplication
   A. identify the multiplication of fractions problem shown in the drawing?

2. Multiplication - ones in the numerator
   A. give the correct answer?
   B. prove the answer with a drawing?

It is common to find that a student can answer A but not B. This is an indication of a memorized procedure without understanding. This is also true of Item 3.

3. Multiplication - numbers other than one in the numerator
   A. give the correct answer?
   B. prove the answer with a drawing?

4. Cancelling
   A. get the correct answer on both problems?
   B. show the cancelling work?
   C. use the standard short notation for this procedure?

The student may still be using the rewrite and reorder notation. This is acceptable but indicates the student needs more work to master the shorter, faster notation.

5. Commutative Property Meets Multiplication of Fractions
   A. explain why cancelling works?
   0 point - Did not answer or the answer made no sense.
   1 point - It works because you simplify the fractions before you multiply.
   2 points - It works because you change the order of the numerators to create fractions that can be simplified before you multiply.
   B. give at least one of the properties, Associative or Commutative?

6. Multiplication
   A. give the correct answer in one of two problems? May do it the long way and simplify at the end.
   B. use cancelling? May use the rewrite, reorder strategy.
   C. use the short notation for cancelling?

7. Multiplying Mixed Numbers
   A. change both mixed numbers to the correct improper fraction?
   B. use cancelling?
   C. get the correct answer?

8. Multiplying Mixed Numbers
   A. get the correct answer on problem a?
   B. get the correct answer on problem b?

These last two problems don't have any shortcuts and have lots of arithmetic. We are testing to see if the student can navigate all the steps and get the correct answer in the end.

Booklet Selection Guide based on results of Pre-Assessment: Part 2

The student needs to score 15 points or more to pass. Each Yes counts for one point. If the student scores 15 points or more, move on to Fractions: Booklet 7 - Addition and Subtraction of Uncommon Denominators. If the student scores 13 points or less, Fractions: Booklet 6 is the correct booklet.

Pre-Assessment: Part 1 Score Sheet Name _____ Date _____

Can the student:

1. Addition of Like Fractions (Fractions: Booklet 2)
   Simplification (Fractions: Booklet 5)
   - Yes No A. add correctly on two of the three problems?
   - Yes No B. simplify two of the three sums?

2. Subtraction of Like Fractions (Fractions: Booklet 2)
   - Yes No A. subtract correctly in problem a?
   - Yes No B. subtract correctly in problem b?
   - Yes No C. Bonus Point: give correct answer to problem c?

3. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)
   - Yes No A. identify $\frac{1}{2}$ as being the least?
   - Yes No B. identify $\frac{5}{6}$ as being the greatest?
   - Yes No C. correctly order $\frac{6}{8}$, $\frac{2}{3}$, and $\frac{6}{10}$?
   - Yes No D. Bonus Point: simplify $\frac{6}{8}$ or $\frac{6}{10}$?

4. Fractions as Parts of Wholes (Fractions: Booklet 2)
   - Yes No A. divide the eight items into four equal groups of two?
   - Yes No B. give the correct answer to all three problems?

5. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
   - Yes No A. give the correct answer to two of three problems?
   - Yes No B. calculate rather than draw a graphic to get the answers?

6. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
   - Yes No A. give the correct answer on two of three problems?
   - Yes No B. simplify the fraction in problem b?
   - Yes No C. calculate rather than draw a graphic to get the answers?

7. Equivalence (Fractions: Booklet 1), The Mighty One (Fractions: Booklet 4)
   - Yes No A. fill in the correct numbers to show the values of one?

8. Equivalence: Calculating (Fractions: Booklet 4)
   - Yes No A. fill in the correct numerator in two of three problems?
   - Yes No B. calculate rather than draw a graphic to get the answers?
   - Yes No C. use the short notation on both problems?

9. Simplification: Calculating (Fractions: Booklet 5)
   - Yes No A. write in the multiplier in problems a and c?
   - Yes No B. give the correct answer in problems b and d?
   - Yes No C. give the correct answer in problem e as a mixed number or an improper fraction?
   - Yes No D. identify the multiplier in problem a as 1 on item f?

10. Associative and Commutative Properties (Multiplication: Booklet 2)
    - Yes No A. fill in the correct numbers?
    - Yes No B. identify item a as the Commutative Property?
    - Yes No C. identify item b as the Associative Property?

11. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)
    - Yes No A. add correctly in problem a?
    - Yes No B. identify the multiplication problem as $\frac{2}{5} \times 5 = \frac{10}{5}$?

12. Equivalence: Calculating (Fractions: Booklet 4)
    - Yes No A. pick out two of the three equal fractions?

13. Fractions as Parts of Wholes (Fractions: Booklet 2)
    - Yes No A. shade in six of the nine circles?
    - Yes No B. write $(9 \div 3) \times 2$ in problem b?

14. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)
    - Yes No A. show the multiplication problem as an addition problem?

Items Correct = _____
Items Possible = 33

Notes:

Pre-Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Multiplication

   Yes No A. identify the multiplication of fractions problem as $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$?

2. Multiplication - ones in the numerator

   Yes No A. give the correct answer of $\frac{1}{6}$?

   Yes No B. prove the answer with a drawing?

3. Multiplication - numbers other than one in the numerator

   Yes No A. give the correct answer of $\frac{8}{15}$?

   Yes No B. prove the answer with a drawing?

4. Cancelling

   Yes No A. get the correct answer on both problems?

   Yes No B. show the cancelling work?

   Yes No C. use the standard short notation for this procedure?

5. Commutative Property Meets Multiplication of Fractions

   A. explain why cancelling works?

   0 point - Did not answer or the answer made no sense.

   1 point - It works because you simplify the fractions before you multiply.

   2 points - It works because you change the order of the numerators to create fractions that can be simplified before you multiply.

   Yes No B. give at least one of the properties, Associative or Commutative?

6. Multiplication

   Yes No A. give the correct answer? May do it the long way.

   Yes No B. use cancelling? May use the rewrite, reorder strategy.

   Yes No C. use the short notation for cancelling?

7. Multiplying Mixed Numbers

   Yes No A. change both mixed numbers to the correct improper fraction?

   Yes No B. use cancelling?

   Yes No C. get the correct answer?

8. Multiplying Mixed Numbers

   Yes No A. get the correct answer on problem a?

   Yes No B. get the correct answer on problem b?

Items Correct = _____

Items Possible = 19

78% needed to begin Fractions: Booklet 7
This is 15 or more Yes items.
11 - 13 Yes: review weak areas, retest, and move to Fractions: Booklet 7 when skills are improved.

Post-Assessment Score Sheet Name _____ Date _____

Can the student:

1. Multiplication

   Yes No A. identify the multiplication of fractions problem as $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$?

2. Multiplication - ones in the numerator

   Yes No A. give the correct answer of $\frac{1}{6}$?

   Yes No B. prove the answer with a drawing?

3. Multiplication - numbers other than one in the numerator

   Yes No A. give the correct answer of $\frac{8}{15}$?

   Yes No B. prove the answer with a drawing?

4. Cancelling

   Yes No A. get the correct answer on both problems?

   Yes No B. show the cancelling work?

   Yes No C. use the standard short notation for this procedure?

5. Commutative Property Meets Multiplication of Fractions

   A. explain why cancelling works?

   0 point - Did not answer or the answer made no sense.

   1 point - It works because you simplify the fractions before you multiply.

   2 points - It works because you change the order of the numerators to create fractions that can be simplified before you multiply.

   Yes No B. give at least one of the properties, Associative or Commutative?

6. Multiplication

   Yes No A. give the correct answer? May do it the long way.

   Yes No B. use cancelling? May use the rewrite, reorder strategy.

   Yes No C. use the short notation for cancelling?

7. Multiplying Mixed Numbers

   Yes No A. change both mixed numbers to the correct improper fraction?

   Yes No B. use cancelling?

   Yes No C. get the correct answer?

8. Multiplying Mixed Numbers

   Yes No A. get the correct answer on problem a?

   Yes No B. get the correct answer on problem b?

Items Correct = _____

Items Possible = 19

78% needed to begin Fractions: Booklet 7
This is 15 or more Yes items.
11 - 13 Yes: review weak areas, retest, and move to Fractions: Booklet 7 when skills are improved.

Pre-Assessment: Part 1 - Worksheet 1 Date_____

Put a question mark next to anything you do not know how to do..

1. Solve. Simplify all answers.

a. $\frac{2}{3}+\frac{2}{3}=$ b. $1\frac{1}{4}+\frac{1}{4}=$ c. $1\frac{5}{8}+\frac{1}{8}=$

2. Solve. Challenge.

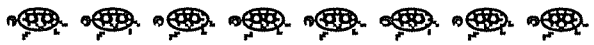
a. $\frac{5}{6}-\frac{2}{6}=$ b. $1-\frac{2}{3}=$ c. $2\frac{1}{2}-1\frac{1}{4}=$

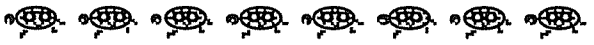
3. Put these fractions in order from the least to the greatest.

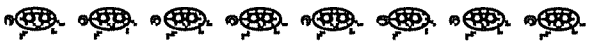
$\frac{2}{3}$ $\frac{6}{8}$ $\frac{6}{10}$ $\frac{5}{6}$ $\frac{1}{2}$

_____ _____ _____ _____ _____

4. Solve and show:

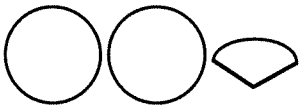
 $\frac{1}{4}$ of 8 = _____

 $\frac{2}{4}$ of 8 = _____

 $\frac{3}{4}$ of 8 = _____

5. Change to improper fractions.

a. $2\frac{1}{3}=\frac{\quad}{3}$ b. $3\frac{1}{2}=$ c. $4\frac{3}{5}=$



6. Change to mixed numbers.

a. $\frac{8}{3}=$ b. $\frac{22}{10}=$ c. $\frac{47}{6}=$

7. Fill in the blanks. $1=\frac{\quad}{5}=\frac{\quad}{10}=\frac{\quad}{100}=\frac{200}{\quad}$

8. Solve.

a. $\frac{2}{3}=\frac{\quad}{12}$ b. $\frac{\quad}{4}=\frac{9}{12}$ c. $\frac{4}{7}=\frac{\quad}{28}$

Pre-Assessment:: Part 1 - Worksheet 2 Date_____

9. Simplify. a. b. c. d. e.

$\frac{14}{21} \div \_\_ = \_\_$ $\frac{10}{100} \div \_\_ = \_\_$ $1\frac{3}{15} =$

f. What is the value of this number? _____

10. Solve. a. $8 \times 5 = 5 \times \square$ b. $2 \times 4 \times 3 = 3 \times \square \times 2 =$ _____

Fill in the blanks. What property is shown by problem a? _____

What property is shown by problem b?_____

Properties: Associative Commutative Distributive Mighty One

11. Solve. a. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$ ___

b. Write it as a multiplication problem. _____ x _____ = _____

12. Circle all the fractions below that are equal to $\frac{3}{5}$.

$\frac{6}{10}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{3}{10}$ $\frac{9}{25}$ $\frac{15}{25}$

13. $\frac{2}{3}$ of 9 a. Use the picture to show $\frac{2}{3}$ of ○○○○○○○○○

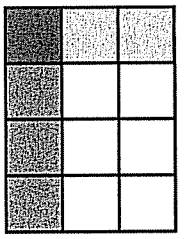
b. Write the number sentence. (___ ÷ ___) x ____ = ____

14. $\frac{2}{3} \times 9 =$ ____ Find the answer with iterative addition.

_____

Pre-Assessment: Part 2 Date_____

1. What multiplication problem does this show? _____



2. a. What is $\frac{1}{3}$ of $\frac{1}{2}$? ____ b. Draw a picture to prove the answer is correct.

3. Solve. a. $\frac{4}{5} \times \frac{2}{3} =$ b. Draw a picture to prove the answer is correct.

4. Use cancelling to solve. a. $\frac{4}{7} \times \frac{7}{8} =$ b. $\frac{15}{4} \times \frac{8}{9} =$

5. a. Explain why cancelling works. _____

_____

_____

b. What property of mathematics is used?_____

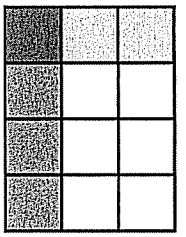
6. Solve. a. $\frac{2}{3} \times \frac{15}{22} =$ b. $\frac{8}{3} \times \frac{9}{4} =$

7. Solve. $2\frac{2}{3} \times 2\frac{1}{4} =$

8. Solve. a. $3\frac{2}{3} \times \frac{8}{9} =$ b. $2\frac{1}{5} \times 3\frac{1}{5} =$

Post-Assessment Date_____

1. What multiplication problem does this show? _____



2. a. What is $\frac{1}{3}$ of $\frac{1}{2}$? ____ b. Draw a picture to prove the answer is correct.

3. Solve. a. $\frac{4}{5} \times \frac{2}{3} =$ ____ b. Draw a picture to prove the answer is correct.

4. Use cancelling to solve. a. $\frac{4}{7} \times \frac{7}{8} =$ b. $\frac{15}{4} \times \frac{8}{9} =$

5. a. Explain why cancelling works. _____

_____

_____

b. What property of mathematics is used?_____

6. Solve. a. $\frac{2}{3} \times \frac{15}{22} =$ b. $\frac{8}{3} \times \frac{9}{4} =$

7. Solve. $2\frac{2}{3} \times 2\frac{1}{4} =$

8. Solve. a. $3\frac{2}{3} \times \frac{8}{9} =$ b. $2\frac{1}{5} \times 3\frac{1}{5} =$

Parts of Wholes: Review and Fractions as Parts of Wholes

Purpose

This lesson's purpose is to review a topic originally taught in Fractions: Booklet 2 of this series. For many students the gap between that book and this may be a year or more. For a student beginning the series at this point, it is important to use a manipulative to establish the physical concept behind the procedure. This lesson assumes the student discovered the following procedure in previous lessons. The lesson reviews the idea that to take a fraction of a set, you must count the number of members of the set, divide it by the denominator of the fractional unit you are using and then multiply the number in that set by the number of sets the numerator indicates you need. For example, $\frac{2}{5}$ of $10 = (10 \div 5) \times 2$. If this is a new concept, have the student go back and do Parts of Wholes in Fractions: Booklet 2. The following dialogue is from a student who is just reviewing what was previously figured out.

Prerequisites

Fractions: Booklet 2 - Parts of Wholes

Materials

Parts of Wholes: Review - Worksheets 1 and 2, pages 4 and 5
Fractions as Parts of Wholes: Worksheets 1 - 3, pages 6 - 8
Blocks and fraction pieces

Warm Up

Give Parts of Wholes: Review - Worksheet 1. Watch while the student does the work. Notice if she counts the total number of pictures in the set and then divides it.

"How do you know how many fish to color in?" "There are ten in the set and I need to divide it into half, so I divide by two."

"How do you know how many squares to color in?" "There are four in the set and I need to divide it into half, so I divide by two."

"How do you know how many stars to color in?" "There are fifteen in the set and I need to divide it into thirds, so I divide by three and get five. Then, since it asks for two-thirds, I need to take two groups of five to make ten. So I color in ten stars."

If by the end of the Warm Up she can't remember or is not able to construct the procedure for these problems, reteach this concept. Use blocks or another manipulative to pose the same problems. Let her divide them in half, prove it is half, and verify her answer. It also appears in the last part of Fractions: Booklet 2: Parts of Wholes of this series, or make up more problems like these. For example, draw a group of nine objects and have her circle one-third of the objects.

Lesson Part 1

"Do the first two problems on Parts of Wholes: Review - Worksheet 2. How can you use a pattern to figure out the answer?" "If I know what one-third of fifteen is, which is five, then all I have to do is go up by fives for each new part." Have blocks available for her to work with if needed.

Lesson Part 2

"Does this pattern also work for the next four problems on Parts of Wholes: Review - Worksheet 2?" "If I know that one-fifth of fifteen is three, then all I have

to do is go up by threes for each new part. Yes, the pattern works for these too."

Worksheet

Fractions as Parts of Wholes - Worksheet 1, page 6 Watch her solve the first problem on the page. Be sure she can clearly articulate how to do the problem. Then let her finish the other problems on the page alone. Check when finished.

Lesson Part 3

Fractions as Parts of Wholes - Worksheet 2, page 7 This page helps her translate the procedure she wrote out the page before to the number sentence form. Take out nine blocks.

"What is the first step of finding two-thirds of nine?" "You divide the nine by three." Point out how that is recorded on the top problem of the worksheet.

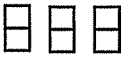
"What is the second step of finding two-thirds of nine?" "You multiply the quotient of the division you just did by two. This gives you six."
Point out how this is recorded on the worksheet. $(9 \div 3) \times 2 = 6$

Have her do the next problem while you watch. If she is correct, have her finish the worksheet alone.

If she has difficulty, take out some blocks and record the first step and then the second step of getting the answer. Then have her record it on the worksheet.

Lesson Part 4

Fractions as Parts of Wholes - Worksheet 3, page 8 This page introduces iterative addition. When multiplication was first learned in this math series, multiplication problems were built and the addition problem was recorded first. For example, the problem 2 x 3 means 2 + 2 + 2, or two added three times.

To review this concept, have the student build 2 x 3  and record the iterative addition sentence 2 + 2 + 2 = 6, then record the multiplication sentence 2 x 3. Build the example problem on the worksheet with fraction pieces.

$\frac{1}{2} \times 2$ $= \frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

Have her complete the page independently if there are no problems. If necessary do a few more problems together.

Parts of Wholes Meet Iterative Addition

Purpose

The purpose of this lesson is to review the previous strategy to find a fractional part of a set, e.g., $\frac{2}{5}$ of 10, by partitioning a set into equal groups as indicated by the denominator and then multiplying by the numerator. Writing the mathematical sentence for this process is also reviewed. We also introduce the idea that the problem can be seen as iterative addition of a fraction. Both procedures produce the same answer but have different physical applications. It hones the mind to see both; how they are alike and how they are different.

Prerequisites

Fractions as Parts of Wholes, Reduction of Improper Fractions and the multiplication tables

Materials

Parts of Wholes Meet Iterative Addition - Worksheets 1 - 3, pages 9 - 11

Warm Up

Review the number sentence that you did on Fractions as Parts of Whole - Worksheet 2, page 7.
Give this word problem: **"A recipe you are making for a big cake calls for nine cups of flour and six eggs. But oops you only have four eggs. You will have to cut your recipe down. So if you only use four eggs, how much flour should you use?"**

Lesson

Parts of Wholes Meet Iterative Addition - Worksheet 1, page 9
"What does iterative mean? Look at problem 2 and see if you can figure it out." "Iterative means the same thing done over and over." **"Make up another iterative problem."** "2 + 2 + 2 + 2."

"How is the first addition problem, 2 + 3 + 4, different from 3 + 3 + 3 + 3?" "The first ones add different numbers. The second one adds the same number over and over."

"How is iterative addition connected to multiplication?" "They are the same thing. Multiplication is a shortcut for adding the same number over and over again."

"Do you think taking two-thirds of twelve will have the same answer as two-thirds added twelve times? Try it." Many students will be surprised that the answer is the same. He will add two twelve times to get twenty-four thirds. He must then simplify the twenty-four thirds using division, which is twenty-four divided by three. Twenty-four thirds is equal to eight.

Note

Most students will need help to translate adding the two twelve times and then dividing by three into the number sentence. Do not help any more than is needed.

"How do you write what you just did as a number sentence?" "First I multiply the numerator, the two, times the whole number, twelve. I would put that inside the parentheses. That gives me twenty-four. Then I would divide the twenty-four by

the denominator, three, to get eight $(2 \times 12) \div 3 = 24 \div 3 = 8$."

Worksheets

Parts of Wholes Meet Iterative Addition - Worksheet 2, page 10 Make sure he writes the addition problem out. He might want to take a shortcut and use his trusty parts of whole formula. Ask him to resist and work on understanding this new way.

Parts of Wholes Meet Iterative Addition - Worksheet 3, page 11 This page works on the idea that the two procedures can be seen differently. It is hard to see them both at the same time.

Test for Understanding

Compare the two methods. How are they different and how are they alike? Which way of thinking fits each of the two word problems?

Give these word problems.

1. Take a whole group of bricks that are each a fraction of a foot long. How long will they be altogether? For example, place twelve bricks, each eight inches long, in a line. How many feet long is the whole line? Write the number sentence and solve it.

Answer.

$\frac{8}{12} = \frac{2}{3}$

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{24}{3} = 8$

$\frac{2}{3} \times 12 = (2 \times 12) \div 3 = 8$

Or $\frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} + \frac{8}{12} = \frac{24}{3} = 8$

$\frac{8}{12} \times 12 = (8 \times 12) \div 12 = 8$

2. Take part of a whole section of rope. For example, twelve feet of rope divided into three sections so as to give away two of three sections. Write the number sentence.

Answer.

12 feet $\div$ 3 = 4 feet 4 feet x 2 sections = 8 feet

$\frac{2}{3} \times 12 = (2 \times 12) \div 3 = 8$

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{24}{3} = 8$

The numerical answer is the same, but the physical process is different.
Either method will work. But it hones the mind to see both: how they are alike and how they are different.

Think up other examples like these.

Multiplication

Purpose

The purpose of this lesson is to use a manipulative to construct the concept of what multiplying a fraction means. In the previous lessons the student worked with the idea that the 'of' in a parts of wholes problem, two-thirds of twelve, for example, is the same thing as two-thirds added twelve times or two-thirds times twelve. In this lesson, this idea is extended to a fraction of a fraction, which is what multiplication of fractions is. Again, the inverse quality of fractions is encountered. When you multiply a fraction, it feels like division because you are taking a part of a part. The product of a multiplication of fractions problem always comes out less than either of the numbers you started with. Those who memorize this simple procedure will not notice or question this.

Prerequisites

Parts of Wholes

Preparation

Read through the lesson and try the problems yourself. Most teachers are not familiar with this way of seeing multiplication of fractions. Check the Answer Key to be sure you know how to record the problems. Read Multiplication - Worksheets 1 and 5 carefully.

Note

Allow the student to play with the clear acetate pieces before you begin the lesson. She will want to hold them up to the light to see the pretty colors.

Materials

Multiplication - Worksheets 1 - 11, pages 12 - 22
Scratch paper and scissors
Colored pencils
Clear and colored acetate Prism Fractions, or Translucent Overhead fraction pieces if you are working with a class

Warm Up

Review basic multiplication like 5 x 4 = 20 and Parts of Wholes with problems like $\frac{1}{4}$ x 20.

"Why does the answer come out with less than you started with? When you multiply, don't you usually get an answer that is larger than either of the numbers you start with?" "It comes out less because you are taking a part of something away, or, since you are only adding $\frac{1}{4}$, your total will be four times less than twenty." She may say something such as "It's like division" but not be able to clearly articulate why.

"How is $\frac{1}{4}$ x 20 like division?" "You are dividing the twenty up into four groups, and that is division. So multiplying a fraction is like going backwards of whole numbers."

Lesson Part 1

"If $\frac{1}{4}$ x 20 means $\frac{1}{4}$ of 20, then what do you think $\frac{1}{4}$ of a $\frac{1}{2}$ might be?" Wait for her to think about it. She may draw it, use the pieces, or, more likely, picture it in her head. "If I cut a half into four parts, then each part would be an eighth."

"Prove it with a piece of paper. Can you prove that one of these small pieces is

one-eighth?" "I would have to cut up the other one too to show that there would be eight little pieces in all, so one would be one-eighth."
"What would three-fourths of one-half be?" "It would be three-eighths because I would just pick up three pieces instead of one."
Try these problems with paper and scissors. Give them orally. Remind her that the X sign means 'of.'
$\frac{2}{3} \times \frac{1}{2} =$ ____ $\frac{1}{3} \times \frac{1}{4} =$ ____ $\frac{2}{3} \times \frac{1}{4} =$ ____

Worksheets

Do Multiplication - Worksheets 1 and 2, pages 12 and 13. These two worksheets work with the denominator of the products. Use the clear Acetate Prism Fractions. Cut the squares out along the dashed lines. Lay a square with thirds over a square with halves and rotate the piece with thirds 90 degrees. This creates six little rectangles. The small rectangles are sixths of the square's whole. Have her use the clear squares and draw the resulting little rectangles in the box.

Now, switch to the colored acetate pieces. The bottom half of Multiplication - Worksheet 2 has her record the results of crossing two fractions and getting a new denominator.

Follow the instructions on Multiplication - Worksheets 3 and 4. The colors show the numerators. The overlap color between the two fraction pieces is the numerator of the product (the answer). The number of squares in the whole square is the denominator, as you saw on Multiplication - Worksheets 1 and 2. This is a good stopping point.

Lesson Part 2

The next session, do Multiplication - Worksheet 5 and part of Worksheet 6, pages 16 and 17 together. If you are working with a class, use an overhead.

"Draw a picture of three-fourths times one-half as if we were using the acetate fraction pieces. Describe how you did it." "I drew a square. Then I cut the square in half with an up and down (vertical) line and colored $\frac{1}{2}$ of it with the red pencil. Then I drew three lines across the other way (horizontal) to cut the one-half into fourths. I colored in one-fourth of it with yellow."

"Show the answer to three-fourths times one-half from your drawing." "One-eighth." She points to the orange double shaded intersection of the $\frac{1}{4}$ and $\frac{1}{2}$.

"How do you know what the denominator is?" "Eight." She points to the eight small rectangles.

Note

Some students are confused by shading the entire fourth all the way across. Tell her to shade only one-fourth of the shaded half. It is just that this does not match the manipulative. But it will clear up confusion.

Many students will quickly see the pattern here and resist drawing. Have her draw anyway on another piece of paper. Draw only two on the worksheet.
After you do two or three problems together on Multiplication - Worksheet 6,

look at the top two problems on Multiplication - Worksheet 7. It asks her to predict what will happen when the numerator is not one. It asks her to predict what will happen if one of the numerators doubles.
"What would happen if you did 3 x 4 and then doubled one of the numbers? What would happen to the product?" "It would double."
"Do you think that will happen with fractions too? Why?" "Yes, because you are doubling the amount of shaded space you start with."
Have her write her prediction on the lines. Then have her finish Multiplication - Worksheets 6 and 7 on her own.

Lesson Part 3

Multiplication - Worksheets 8 and 9, pages 19 and 20 Insist that she does a nice careful drawing for three of the problems. The drawings should be in color.
At the bottom of Multiplication - Worksheet 8, she is asked to explain the pattern she sees to 'do' these kinds of problems. After she writes it in common words, which will probably be something such as "You multiply the top two numbers and then multiply the bottom two numbers," have her write it using precise mathematical language: which is "Put the product of the numerators over the product of the denominators and then simplify." After she completes Multiplication - Worksheet 9, pick a problem at random and ask her to draw it to prove her answer is correct.

Note

Do not teach how to cancel at this point. Some students will have the brilliant idea of simplifying the fraction before they multiply. Give her a high five if she does, but do not tell her to do it.

Warm Up

Draw a picture of a problem on Multiplication - Worksheet 9, page 20.

Lesson Part 4

This is a bit of a trick on the students. Ask her to do the problem at the top of Multiplication - Worksheet 10, page 21, using the pattern she wrote out on Multiplication - Worksheet 9. Most students will get $\frac{20}{180}$ and then simplify to $\frac{1}{9}$.

"What would happen if the fraction was simplified before being multiplied? Do you think we would get the same answer?" Most students will be unsure. A brilliant answer would be, "Yes, it would have to be the same because a simplified fraction is equal to the original one."

Try it. Simplify $\frac{5}{15}$ to $\frac{1}{3}$ and $\frac{4}{12}$ to $\frac{1}{3}$ and multiply. Wow, we get the answer without having to do such hard work! It is easier to multiply smaller fractions than larger ones. Most students will think this is a neat trick.
Have her complete the page on her own.

Practice Worksheet

Multiplication - Worksheet 11, page 22

Test for Understanding

1. Solve $\frac{3}{4} \times \frac{2}{3} =$ Prove your answer. $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$
2. Explain in words why simplifying before multiplying produces the same product as multiplying first and then simplifying.
   Answer: Each simplified fraction is equal to the original one, so the products would be equal.

Commutative Property Meets Multiplication of Fractions

Purpose

The purpose of this lesson is to teach what is commonly called cross cancelling. Cancelling is a tool used to make multiplication of fractions easier. The process is based on the Commutative Property of Multiplication. What is basically happening when cross cancelling is the position of the numerators are switched to create a reducible fraction. The student learns the basic process of mathematics that underlies the trick-like shortcut taught in most books.

Prerequisites

Commutative Property of Multiplication and simplification of fractions

Materials

Commutative Property Meets Multiplication of Fractions - Worksheets 1 - 5, pages 23 - 27

Warm Up

2 x 3 = 6

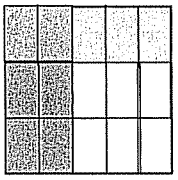
3 x 2 = 6

Review the Commutative Property of Multiplication with whole numbers.
2 x 3 = 3 x 2 Have the student build or draw pictures of each problem.
2 x 3 is 2 + 2 + 2 and 3 x 2 is 3 + 3. They are not the same problem, but the products are equal. Have him practice spelling *commutative*. Do three or four simplification of fractions problems, e.g., $\frac{3}{9} = \text{-}$, $\frac{6}{8} = \text{-}$, $\frac{14}{21} = \text{-}$. Answers: $\frac{3}{9} = \frac{1}{3}$, $\frac{6}{8} = \frac{3}{4}$, $\frac{14}{21} = \frac{2}{3}$.

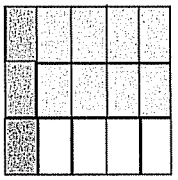
Lesson Part 1

Problem 1

$\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$



$\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$



Have him turn back to Multiplication - Worksheet 10, page 21, and reexamine the last problem. **"Why was this particular problem so difficult?"** ($\frac{15}{12} \times \frac{4}{15}$) "Neither problem could be simplified, so you had to do it the long way with lots of arithmetic."

Commutative Property Meets Multiplication of Fractions - Worksheet 1, page 23. Do it with him or if you have a class, have students work in small groups. Insist on answers to all the questions.
"What happens when I switch the positions of the numerators?" "The problem looks different, but the answer is the same."

Have him draw both fraction problems in the first box on another piece of paper. Notice that the drawings do not look the same but give the same answer.
"What happens if you switch the denominators instead of the numerators?" "You get the same problems as if you switched the numerators; they're just in different places."

"How could we use this idea to help with that really hard problem on Multiplication - Worksheet 10?" Let him think about it. Many students will not be able to answer this question. Commutative Property Meets Multiplication of Fractions - Worksheet 2 answers this question.

Worksheet

$\frac{5}{12} \times \frac{4}{15} =$
$\frac{4}{12} \times \frac{5}{15} =$
$\_\_ \times \_\_ = \_\_$

Commutative Property Meets Multiplication of Fractions - Worksheet 2, page 24
A possible discussion for the last problem on the page:
"If the positions of the four and the five are switched and then multiplied, do you get the same answer?" "Yes, because 5 x 4 and 4 x 5 both equal twenty." Have him write that answer on the worksheet.

"If I commute and then simplify the new fractions, will I get the correct answer? How do you know?" "You know because you already know that the answer is one-ninth, so you can tell it works because the answer stays the same."

Note

When the positions of the numerators are switched, it creates two new fractions that can be simplified. *As usual, do not show the student the kitty-corner shortcut.* This will be worked on in the next lesson.

Worksheets

Commutative Property Meets Multiplication of Fractions - Worksheets 3 and 4, pages 25 and 26

Lesson Part 2

Review the Associative Property of Multiplication. Work with 2 x 3 x 4 and change the arrangements. Use proper mathematical grammar to show all the ways to do this problem. Only two numbers can be multiplied at one time. So the pair to be done first must be grouped with a parentheses. (2 x 3) x 4, (3 x 4) x 2, (2 x 4) x 3. On Commutative Property Meets Multiplication of Fractions - Worksheet 5, page 27, the Associative Property is shown with algebraic symbols. Help him connect what is written with numbers to what is written with variables. The algebra shows the general pattern. It does not matter what the numbers are.

Now look at the problems that have three fractions. Discuss what would be the best way to rearrange the numerators to produce simple fractions. Do one or two together. Do not help unless it is needed. See if he notices that in some problems, you are able to create one. This is always a good way to make the arithmetic easier.

Worksheet

Commutative Property Meets Multiplication of Fractions - Worksheet 5, page 27

Test for Understanding

Watch him do the 'in your head' problems at the bottom of Commutative Property Meets Multiplication of Fractions - Worksheet 5. Note if he sees how he can make ones and then simplify those easy fractions. Ask him to explain in writing how the Commutative Property of Multiplication helps make multiplication of fractions easier. If you have a group, have them write a few sentences on how these can be done easily in their heads.

$\frac{7}{3} \times \frac{13}{7} \times \frac{3}{13} = \frac{3}{3} \times \frac{7}{7} \times \frac{13}{13} = 1 \times 1 \times 1$

Answer: When numbers are multiplied together, the order can change and the answer will remain the same. In fractions the position of the numerators changes, or commutes, to make fractions that can be simplified before multiplying.

Cancelling

Purpose

The purpose is to formally teach the procedure called cancelling. Cancelling is used to simplify problems. In the last lesson, students were taught to reorder the numerators of fractions to create reducible fractions. In this lesson, they will be taught the shortcut that allows cancelling without rewriting the problem. This is done carefully, step-by-step, to assure that the student understands each transition.

Prerequisites

Previous lessons

Materials

Cancelling - Worksheets 1 - 13, pages 28 - 40

Lesson Part 1

Cancelling - Worksheet 1 Do and then discuss the two problems at the top of Cancelling - Worksheet 1. This worksheet focuses the student's attention on the numbers kitty-corner from each other in the original problem. She will notice how they can so easily be turned into ones. Watch her complete the worksheet.

Cancelling - Worksheet 2, page 29
Do the problem at the top and discuss the question of shortcuts. Most students will be noticing that the cross numbers can be put together to make one.
"In Step 2, you rewrite the problem to reorder it. Do you think you could do that step in your head?" "Yes, I could just write one over one in Step 3."
Notice that in problems 7 and 8, Step 2 is still visible but lightened. Encourage her to do Step 2 in her head and write the simplified answer in Step 3. A few students will not be confident enough to do this yet and will want to use Step 2 anyway. Let them do this, but have them write so lightly you can barely see it.
Since the simplification in these problems is so easy, most students can also now skip Step 3. Have her do problems 9 - 11 on her own and then discuss the work and check it.
"How come you did not simplify that big fraction in problem 11?" "I can't, they are both prime numbers."

Worksheet

Cancelling - Worksheet 3, page 30 The student commutes and simplifies both fractions before writing them in Step 3. Check the Answer Key to be sure she is doing these correctly.

Note

The answers to the questions with the asterisk after them are on page 30.

Practice Worksheets

Cancelling - Worksheets 4 - 7, pages 31 - 34 Check each page as she finishes. Do not allow her to practice incorrectly. Some students who have been taught this shortcut before will resist doing all the steps. Encourage them to go through the steps, as it will cement understanding.

Test for Understanding

Closely examine her work on Cancelling - Worksheet 7, page 34. Note how much figuring there is on the side and if the answers are correct. If she is using shortcuts, ask her to explain them.

Note

Watch out for errors when the simplified fraction is a whole number. In problem 3, the fourteen is in the numerator, the seven in the denominator. The simplified fraction is two over one. Many students will write ½ here. **"Look again at your answer."**

Warm Up

Reexamine her completed problems numbers 6 and 7 on Cancelling - Worksheet 6. If she skipped that part, have her do it now.
"What do you have to think about when you construct a problem like this?" "You have to think about the multiplication tables."

Lesson Part 2

Cancelling - Worksheet 8, page 35 Study the material in the box. It asks her to circle what equals ½ in each line. This refers to the steps written out on the left. In Step 2 you see the five-tenths. She should circle this, because that is where ½ shows up. Read the worksheet with her. Be sure she understands each step.

"Why is there a picture of the Mighty One here? What does it have to do with this?" "It is there because you are dividing both the 5 and the 10 by 5. This means you are really dividing by 1 in the form of 5 over 5." If she can not clearly explain this, have her write out the steps of simplifying. $\frac{5}{10} \div \frac{5}{5} = \frac{1}{2}$

On the bottom of the worksheet it asks, "Why does this work?" "It works because you are really changing the order of the numerators and putting the five over the ten and simplifying it to make ½. I am just doing it all in my head."

Do Cancelling - Worksheet 9, page 36, together. Check the Answer Key to be sure all the answers are correct. In the boxes, we want her to do both the long way by writing all the steps and the short way by using cross cancelling. The lower box asks her to do it the old way by multiplying and then simplifying, then to reorder using all the steps and then to use cancelling. She should get the same answer every time.

Practice Worksheets

Cancelling - Worksheet 10, page 37 Have her put in the loops to show which numbers she is simplifying. A few students will be reluctant to trust the loops and will want to show all the steps. Have them do the steps on another piece of paper and then do the loops with them immediately.
Cancelling - Worksheet 11, page 38 has word problems.
Cancelling - Worksheet 12, page 39, has more practice. Allow her to stop using the loops and just do the cross cancelling.

Test for Understanding

Can she fill out the questions in the box at the bottom of Cancelling - Worksheet 12? Use Cancelling - Worksheet 13, page 40, as a test. Tell her it is a test. Score it immediately. She should not be using the loops. It is not wrong, but it tends to clutter up the page. Note if she is reordering under each problem. If she is, she is not yet secure in the cross cancelling shortcut. Work with her more to develop confidence.

Cancelling Meets Parts of Wholes

Purpose The purpose of this lesson is to allow the student to connect the thought process he used for Parts of Wholes with the standard procedure for multiplication of fractions and cancelling. This is a short lesson to connect all the pieces.

Prerequisites Previous lessons, and Fractions: Booklet 2 - Fractions as Parts of Wholes

Materials Cancelling Meets Parts of Wholes - Worksheets 1 and 2, pages 41 and 42

Lesson Give Cancelling Meets Parts of Wholes - Worksheet 1, which should be self-explanatory. Check with the Answer Key to be sure.

Worksheet Cancelling Meets Parts of Wholes - Worksheet 2, page 42

Test for Understanding None.

Multiplying Mixed Numbers

Purpose This lesson's purpose is to guide the student through all the things he knows about multiplication of whole numbers, multiplication of fractions, changing mixed numbers to improper fractions, and cancelling. The integration of all of this allows the student to construct the algorithm for the multiplication of mixed numbers.

Prerequisites Previous lessons

Materials Multiplying Mixed Numbers - Worksheets 1 - 6, pages 43 - 48
Colored pencils
Fraction pieces and pattern blocks
Graph paper

Warm Up Multiplying Mixed Numbers - Worksheet 1 begins with a warm up in drawing whole number multiplication problems onto graph paper to create an array. Have the student color in the array. **"What does the word 'product' mean?"** "The answer in a multiplication problem."

Lesson Part 1 Do the bottom half of Multiplying Mixed Numbers - Worksheet 1. **"What is one equal to?"** "Four grid squares." The problem shows him how to draw $3\frac{1}{2}$ two times. Shading in the array will give seven as the answer. He then does it on his own with the numbers commuted.
"Why are the answers to the two problems the same?" "Because you just switched the positions of the whole numbers."

Worksheets Multiplying Mixed Numbers - Worksheet 2, page 44, has him do the problem three ways: once with iterative addition, the second time with pattern blocks, and the third time as an array drawing.

With the pattern blocks, have him build $3\frac{1}{2}$ with three yellows and a red, build it four times, and add up the result. Pattern blocks provide a concrete model of the problem. Then, draw it on the grid, which is representation of the problem. Shade in the product. The product is fourteen.

On the second problem, have him build the problem with fraction pieces. He will build with three wholes and a half. He then needs to take half of that and will need to do some trades to do this. As usual, have him figure it out. On the array, have him draw $3\frac{1}{2}$ and then cut it in half with a colored pencil and shade in the product. The solution is shown on the next page in his booklet.

Multiplying Mixed Numbers - Worksheet 3, page 45
He may not be able to come up with a solution to the question in the middle of the page. Have him build $3\frac{1}{2}$ with the fraction pieces. Then have him trade so all the pieces are the same color. He will trade each of the wholes for two halves. Have him count up the halves and write it as a fraction.
"What is this kind of fraction called?" "An improper fraction."

"Can you change $3\frac{1}{2}$ to an improper fraction and then multiply it with your old procedure?" "Yes!"

Finish the worksheet and check the results. Remind him to use cancelling. The purpose of this exercise is to develop confidence in the rightness of the standard procedure used for multiplication of mixed numbers. He needs to see that it does produce correct answers, answers that he can verify with a drawing if he wants to.

Lesson Part 2

Use graph paper to draw this series of problems: 3×2, $3\frac{1}{2} \times 2$, $3\frac{1}{2} \times \frac{1}{2}$, and $3\frac{1}{2} \times 2\frac{1}{2}$. To get the answer to $3\frac{1}{2} \times 2\frac{1}{2}$, you could add the results of $(3\frac{1}{2} \times 2) + (3\frac{1}{2} \times \frac{1}{2})$ using the Distributive Property of Multiplication. Try to draw $3\frac{1}{2} \times 2\frac{1}{2}$. It can be done. The product is eight and three-fourths. You can also build it with fraction pieces. It would look like this:

$3\frac{1}{2}$ once twice a half a time Add it all up and it is eight and three-fourths.

Now try changing them both to improper fractions and using the standard procedure for multiplication. You get the same answer of course. Which is easier? No contest!

Worksheet

Multiplying Mixed Numbers - Worksheet 4, page 46
These problems are too hard to build and difficult to draw. Do a few problems together and then let him finish on his own. Check the work immediately. There are lots of arithmetic errors possible here.

Practice Worksheets

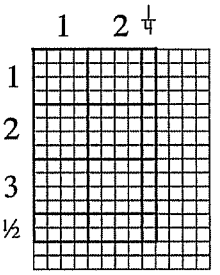
Multiplying Mixed Numbers - Worksheets 5 and 6, pages 47 and 48

Test for Understanding

Find the answer to $2\frac{1}{4} \times 3\frac{1}{2}$ three different ways.

Build it.

Draw it.



Calculate it.

$(2\frac{1}{4} \times 3) + (2\frac{1}{4} \times \frac{1}{2})$

$\frac{9}{4} \times \frac{7}{2} = \frac{63}{8} = 7\frac{7}{8}$

$$\begin{array}{r} 2\frac{1}{4} \\ 2\frac{1}{4} \\ 2\frac{1}{4} \\ +1\frac{1}{8} \\ \hline 7\frac{7}{8} \end{array}$$

Do the Cumulative Review, pages 49 and 50, before taking the Post-Assessment, page 51, in the Student Workbook.

Patterns in Arithmetic

Fractions: Booklet 6

Multiplication

Answer Key for the Student Workbook

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

By Suki Glenn, Susan Carpenter, and Alysia Krafel

Patterns in Arithmetic: Fractions - Booklet 6
Answer Key for the Student Workbook

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Fractions - Booklet 6

Pre-Assessment: Part 1 - Worksheet 1

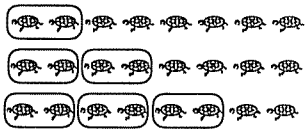
1. Solve. Simplify all answers.
   a. $\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}$ b. $1\frac{1}{4}+\frac{1}{4}=1\frac{2}{4}=1\frac{1}{2}$ c. $1\frac{5}{8}+\frac{1}{8}=1\frac{6}{8}=1\frac{3}{4}$

2. Challenge.
   a. $\frac{5}{6}-\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$ b. $1-\frac{2}{3}=\frac{1}{3}$ c. $2\frac{1}{2}-1\frac{1}{4}=2\frac{2}{4}-1\frac{1}{4}=1\frac{1}{4}$

3. Put these fractions in order from the least to the greatest

$\frac{2}{3}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{5}{6}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{6}{8}$	$\frac{5}{6}$

4. Solve and show:



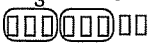
$\frac{1}{4}$ of 8 = 2

$\frac{2}{4}$ of 8 = 4

$\frac{3}{4}$ of 8 = 6

5. Change to improper fractions.
   a. $2\frac{1}{3}=\frac{7}{3}$ b. $3\frac{1}{2}=\frac{7}{2}$ c. $4\frac{3}{5}=\frac{23}{5}$

6. Change to mixed numbers.
   a. $\frac{8}{3}=2\frac{2}{3}$ b. $\frac{22}{10}=2\frac{2}{10}=2\frac{1}{5}$ c. $\frac{47}{6}=7\frac{5}{6}$



7. Fill in the blanks. $1=\frac{5}{5}=\frac{10}{10}=\frac{100}{100}=\frac{200}{200}$

8. Solve.
   a. $\frac{2}{3}=\frac{8}{12}$ b. $\frac{3}{4}=\frac{9}{12}$ c. $\frac{4}{7}=\frac{16}{28}$

Pre-Assessment: Part 1 - Worksheet 2

9. Simplify. a. $\frac{14}{21}\div\frac{7}{7}=\frac{2}{3}$ b. $\frac{10}{100}\div\frac{10}{10}=\frac{1}{10}$ c. d. e. $1\frac{3}{15}=1\frac{1}{5}$
   f. What is the value of this number? 1

10. Solve. a. $8 \times 5 = 5 \times \boxed{8}$ b. $2 \times 4 \times 3 = 3 \times \boxed{4} \times 2 = 24$
    Fill in the blanks. What property is shown by problem a? Commutative
    What property is shown by problem b? Associative
    Properties: Associative Commutative Distributive Mighty One

11. Solve. a. $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{10}{5}=2$
    b. Write it as a multiplication problem. $\frac{2}{5} \times 5 = \frac{10}{5} = 2$

12. Circle all the fractions below which are equal to $\frac{3}{5}$.
    $\frac{6}{10}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{3}{10}$ $\frac{9}{25}$ $\frac{15}{25}$

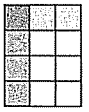
13. $\frac{2}{3}$ of 9 a. Use the picture to show $\frac{2}{3}$ of 9

    b. Write the number sentence. (9 ÷ 3) x 2 = 6

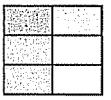
14. $\frac{2}{3} \times 9 = 6$ Find the answer with iterative addition. $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{18}{3}=6$

Pre-Assessment: Part 2

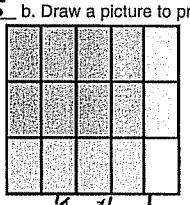
1. What multiplication problem does this show? $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



2. a. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{6}$ b. Draw a picture to prove the answer is correct.



3. Solve. a. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ b. Draw a picture to prove the answer is correct.



4. Use cancelling to solve. a. $\frac{\cancel{4}^{1}}{\cancel{7}_{1}} \times \frac{1}{\cancel{8}_{2}} = \frac{1}{2}$ b. $\frac{5}{\cancel{15}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = \frac{10}{3} = 3\frac{1}{3}$

5. a. Explain why cancelling works.
   You can change the order of the numerators to create fractions that can be simplified before multiplying.
   b. What property of mathematics is used? Commutative or Associative

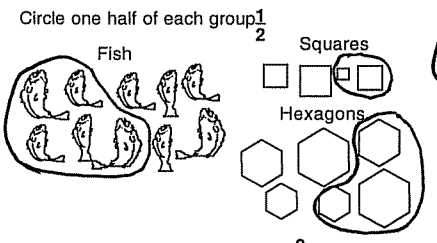
6. Solve. a. $\frac{\cancel{4}^{1}}{\cancel{8}_{1}} \times \frac{15}{\cancel{22}_{11}} = \frac{5}{11}$ b. $\frac{\cancel{2}^{1}}{\cancel{8}_{1}} \times \frac{3}{\cancel{9}_{1}} = 6$

7. Solve. $2\frac{2}{3} \times 2\frac{1}{4} = \frac{8}{3} \times \frac{9}{4} = \frac{72}{12} = 6$

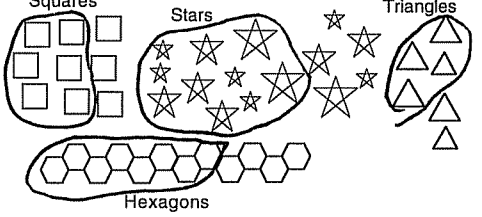
8. Solve. a. $3\frac{2}{3} \times \frac{8}{9} = 3\frac{7}{27}$ $\frac{11}{3} \times \frac{8}{9} = \frac{88}{27}$ b. $2\frac{1}{5} \times 3\frac{1}{5} = 7\frac{1}{25}$ $\frac{11}{5} \times \frac{16}{5} = \frac{176}{25}$

Parts of Wholes: Review - Worksheet 1

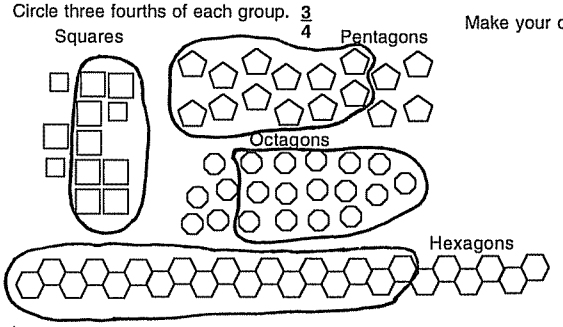
Circle one half of each group $\frac{1}{2}$



Circle two thirds of each group. $\frac{2}{3}$



Circle three fourths of each group. $\frac{3}{4}$



Parts of Wholes: Review - Worksheet 2

Look for a pattern to solve these problems.

1/5 of 15 = 3
2/5 of 15 = 6
3/5 of 15 = 9
4/5 of 15 = 12
1/7 of 14 = 2
2/7 of 14 = 4
3/7 of 14 = 6
4/7 of 14 = 8
5/7 of 14 = 10
6/7 of 14 = 12

1/2 of 20 = 10
1/4 of 20 = 5
2/4 of 20 = 10
3/4 of 20 = 15
1/5 of 20 = 4
2/5 of 20 = 8
3/5 of 20 = 12
4/5 of 20 = 16

What patterns do you see?

If $\frac{1}{5}$ of 15 = 3 then $\frac{2}{5}$ = 2x3 or 6, $\frac{3}{5}$ = 3x3 or 9
Once I find the size of the fractional part I multiply it by how many fractional parts I have, the numerator.

Fractions as Parts of Wholes - Worksheet 1

Solve. Look for a pattern.

1/2 of 48 = 24
1/3 of 54 = 18
2/3 of 54 = 36
1/6 of 42 = 7
5/6 of 42 = 35
1/9 of 72 = 8
4/9 of 72 = 32
1/18 of 36 = 2

1/2 of 64 = 32
1/4 of 64 = 16
3/4 of 64 = 48
1/5 of 40 = 8
3/5 of 40 = 24
1/20 of 80 = 4
3/20 of 80 = 12

Write out a procedure for calculating trhe answer to $\frac{4}{5}$ of 20.

Divide the denominator of the fraction into the whole number, then multiply the numerator by that quotient. Or multiply the numerator by the whole number, then divide by the denominator.

Fractions as Parts of Wholes - Worksheet 2

In fractions the "of " is written with a X sign.

$\frac{1}{3}$ of 9 = $\frac{1}{3}$ x 9

Write the procedure with a number sentence.

$\frac{2}{3}$ of 9 = (9 ÷ 3) x 2 = 6

$\frac{1}{4}$ of 8 = (8 ÷ 4) x 1 = 2

$\frac{3}{4}$ of 8 = (8 ÷ 4) x 3 = 6

$\frac{1}{5}$ of 25 = (25 ÷ 5) x 1 = 5

$\frac{3}{5}$ of 25 = (25 ÷ 5) x 3 = 15

$\frac{5}{6}$ of 24 = (24 ÷ 6) x 5 = 20

Fractions as Parts of Wholes - Worksheet 3

$\frac{1}{2}$ x 2 can also mean $\frac{1}{2}$ added 2 times. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

This is called iterative addition.

Write these problems as iterative addition problems and solve.

$\frac{1}{4}$ x 2 = $\frac{1}{2}$ $\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{4}$ x 4 = 1 $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$\frac{1}{6}$ x 5 = $\frac{5}{6}$ $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}$

$\frac{2}{3}$ x 4 = $2\frac{2}{3}$ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2\frac{2}{3}$

$\frac{3}{4}$ x 6 = $4\frac{1}{2}$ $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{14}{18} = 4\frac{2}{4}$ or $4\frac{1}{2}$

$\frac{5}{6}$ x 3 = $2\frac{1}{2}$ $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$ or $2\frac{3}{6}$ or $2\frac{1}{2}$

Parts of Wholes Meet Iterative Addition- Worsheet 1

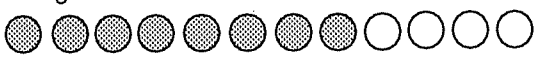
1. What does iterative mean? To repeat.

2. This is addition: $2+3+4=$ 9

This is iterative addition, $3+3+3+3=$ 12

also known as multiplication. $3 \times 4 =$ 12

$\frac{2}{3}$ of 12 can be done as a part of a whole.



Shade in $\frac{2}{3}$ of the 12 circles.

$\frac{2}{3} \times 12 =$ 8 You did these in Booklet 2.

$\frac{2}{3} \times 12$ can mean two thirds added twelve times.

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{24}{3} = 8$

Do you think iterative addition (adding $\frac{2}{3}$ twelve times) will give you the same or a different answer as taking two thirds of 12? Same

Try it. How many thirds do you get if you add $\frac{2}{3}$ twelve times? $\frac{24}{3}$

$\frac{2}{3} \times 12 = (12 \times 2) \div 3 = 24 \div 3 = 8$

Now, simplify the answer. 8

Parts of Wholes Meet Iterative Addition- Worsheet 2

$\frac{2}{3}$ of $\times 9$ can also mean $\frac{2}{3}$ added 9 times. This is called iterative addition.

$\frac{2}{3} \times 9 = \frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = 6$

$\frac{2}{3} \times 9 = (9 \times 2) \div 3 = 18 \div 3 = 6$

Write as an iterative addition problem.

Write the procedure with a number sentence.

$\frac{1}{4}$ of $\times 8 = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = \frac{8}{4} = 2$

$\frac{1}{4}$ of $\times 8 = (8 \times 1) \div 4 = 8 \div 4 = 2$

$\frac{3}{4}$ of $\times 8 = \frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4} = \frac{24}{4} = 6$

$\frac{3}{4}$ of $\times 8 = (8 \times 3) \div 4 = 24 \div 4 = 6$

$\frac{1}{5}$ of $\times 15 = \frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5} = \frac{15}{3} = 5$

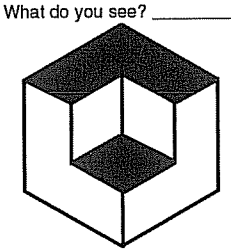
$\frac{1}{5}$ of $\times 15 = (15 \times 1) \div 5 = 15 \div 5 = 3$

$\frac{3}{5}$ of $\times 15 = \frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5} = \frac{45}{5} = 9$

$\frac{3}{5}$ of $\times 15 = (15 \times 3) \div 5 = 45 \div 5 = 9$

Parts of Wholes Meet Iterative Addition- Worsheet 3

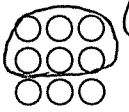
Look at this figure. What do you see? _____



Some people see a big big cube in back and a small cube floating in front.
Other people see a big cube with the corner taken out.
Can you see both? yes Can you see both at the same time? no

Notice, your mind has to "pop" between the two images.

$\frac{2}{3} \times 9 =$ 6 Parts of wholes



$(9 \div 3) \times 2 = 6$

and

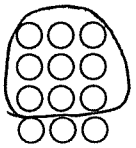
$\frac{2}{3} \times 9 =$ 6 Iterative

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{18}{3} = 6$

$2 \times 9 \div 3 = 6$

These two ways of solving the same problem are just like the cube above.
They fade and feel different but produce the same answer.
Try another one.

Parts of wholes



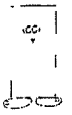
$\frac{3}{4} \times 12 =$ 9

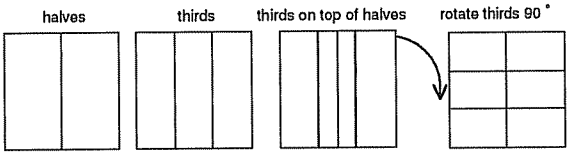
and

Iterative $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4} = \frac{36}{4} = 9$

Multiplication - Worksheet 1

Use **clear** Prism FractionsAcetate squares to find the solution to these problems.
Start with the square divided in two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°.

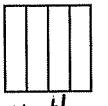




This shows halves divided into thirds.
How many little rectangles are there after rotating the squares? 6
What fraction of the whole is the little rectangle? $\frac{1}{6}$

Now do several more rotations and record the number of rectangles.

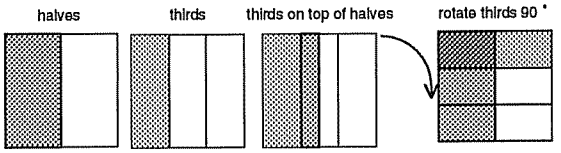
Record here.



strips 4 x strips 3 = 12 little rectangle

Multiplication - Worksheet 2

Use **colored** Prism Fractions Acetate squares to find the solution to these problems.
Start with the square divided in two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°

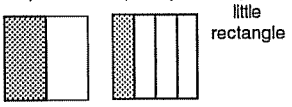


This shows halves divided into thirds.
How many little rectangles are there after rotating the squares? 6
What fraction of the whole is the little rectangle? 1/6
What happens to the color at the intersection of the two squares?
changes color

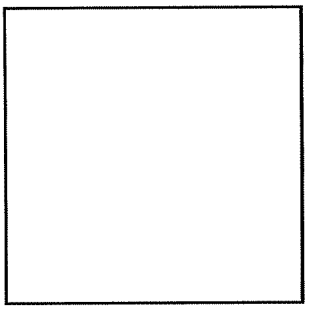
The intersection of the colors show 1/3 of 1/2. What fraction of the whole is in the interection? 1/6

Now do several more rotations and record the number of rectangles.

Record here. Color with colored pencils.



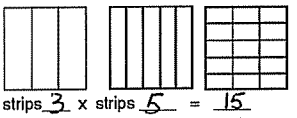
strips 2 x strips 4 = 8 little rectangle



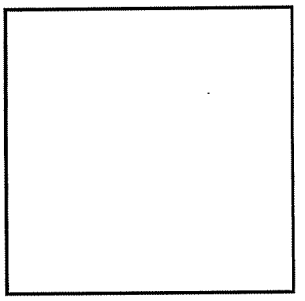
Multiplication - Worksheet 3

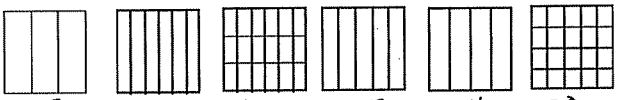
Use clear Prism Factions Acetate squares to find the solution to these problems.
Do several more rotations and record the number of little rectangles.

Record here.



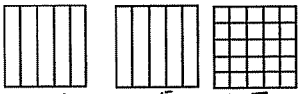
strips 3 x strips 5 = 15 little rectangles





strips 3 x strips 6 = 18 little rectangles

strips 5 x strips 4 = 20 little rectangles



strips 5 x strips 5 = 25 little rectangles

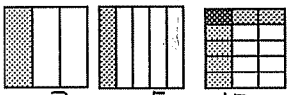
Make your own.

strips ___ x strips ___ = ___ little rectangles

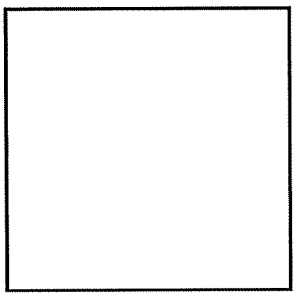
Multiplication - Worksheet 4

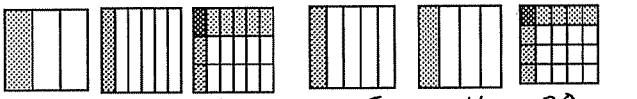
Use colored Prism Factions Acetate squares to find the solution to these problems.
Do several more rotations and record the number of little rectangles.

Record here.



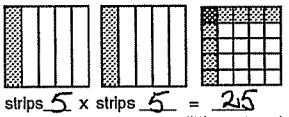
strips 3 x strips 5 = 15 little rectangles





strips 3 x strips 6 = 18 little rectangles

strips 5 x strips 4 = 20 little rectangles



strips 5 x strips 5 = 25 little rectangles

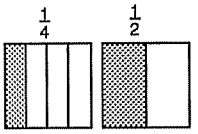
Make your own.

strips ___ x strips ___ = ___ little rectangles

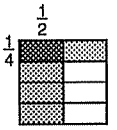
Multiplication - Worksheet 5

Prism Fractions Acetate colored squares must be used to see the crosses.

Learn to draw these. $\frac{1}{4}$ of $\frac{1}{2}$ =

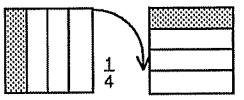
1. 

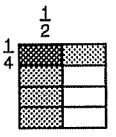
Start with the second number, the $\frac{1}{2}$.
A chunk is going to be taken out of this $\frac{1}{2}$.

2. 

Shade $\frac{1}{4}$ of $\frac{1}{2}$

Rotate the $\frac{1}{4}$ 90°. Put the $\frac{1}{4}$ piece on top of the $\frac{1}{2}$ piece.



3. 

Look at the overlap where the colors from $\frac{1}{2}$ and the $\frac{1}{4}$ are both in the little rectangle.
This is called "cross shading".

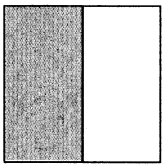
$\frac{1}{8}$ of the whole is cross shaded.

$\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$ $\quad \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Multiplication - Worksheet 6

Use Prism Fractions Acetate colored squares to find the solution to these problems.
Record how many rectangles there are after rotating the squares.
What happens to the color at the intersection of the two squares? ____

	Number of rectangles	New colored part of the whole
$\frac{1}{2}$ of $\frac{1}{3}$	6	$\frac{1}{6}$
$\frac{1}{2}$ of $\frac{1}{4}$	8	$\frac{1}{8}$
$\frac{1}{2}$ of $\frac{1}{2}$	4	$\frac{1}{4}$
$\frac{1}{2}$ of $\frac{1}{8}$	16	$\frac{1}{16}$
$\frac{1}{2}$ of $\frac{1}{6}$	12	$\frac{1}{12}$
$\frac{1}{2}$ of $\frac{1}{5}$	10	$\frac{1}{10}$

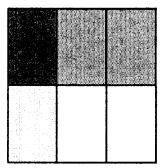


Draw two of your solutions.

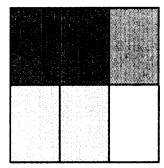
What patterns do you notice?
the number of little rectangles is the new denominator in the answer. Multiply the denominators to get the number of little rectangles.

Multiplication - Worksheet 7

Use Prism Fractions Acetate colored squares to find the solution to these problems.
Draw a picture of $\frac{1}{2} \times \frac{1}{3}$ and shade with colored pencils.



Draw a picture of $\frac{1}{2} \times \frac{2}{3}$.



Predict what will happen to the answer if one numerator doubles. AWV
The answer will double.

Solve.

$\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$ $\quad$ $\frac{1}{4}$ of $\frac{2}{3} = \frac{2}{12}$

$\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$ $\quad$ $\frac{2}{5}$ of $\frac{1}{2} = \frac{2}{10}$

$\frac{1}{6}$ of $\frac{1}{3} = \frac{1}{18}$ $\quad$ $\frac{2}{6}$ of $\frac{1}{3} = \frac{2}{18}$

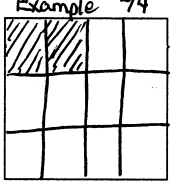
$\frac{1}{4}$ of $\frac{1}{8} = \frac{1}{32}$ $\quad$ $\frac{1}{4}$ of $\frac{2}{8} = \frac{2}{32}$

$\frac{1}{5}$ of $\frac{1}{3} = \frac{1}{15}$ $\quad$ $\frac{1}{5}$ of $\frac{2}{3} = \frac{2}{15}$

Multiplication - Worksheet 8

Use Prism Fractions Acetate colored squares to find the solution to these problems.
Choose three problems and draw pictures with colored pencils or shaded areas.

Example 2/4



$\frac{2}{6} \times \frac{1}{3} = \frac{2}{18}$

$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$

$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

$\frac{1}{6} \times \frac{2}{3} = \frac{2}{18}$

$\frac{3}{4} \times \frac{1}{8} = \frac{3}{32}$

$\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$

$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

$\frac{3}{5} \times \frac{2}{6} = \frac{6}{30}$

$\frac{3}{4} \times \frac{2}{6} = \frac{6}{24}$

$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$

Make your own.

— x — =

— x — =

Use the patterns you see to make up a rule.
What is the rule for multiplying fractions?
Multiply the numerator times the numerator. Multiply the denominator times the denominator.

Multiplication - Worksheet 9

Simplify all answers. Be ready to use the Prism Fractions Acetate colored squares to prove the answer is correct.

$\frac{3}{6} \times \frac{3}{5} = \frac{9}{30} = \frac{3}{10}$ $\quad$ $\frac{2}{4} \times \frac{3}{8} = \frac{6}{32} = \frac{3}{16}$

$\frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$ $\quad$ $\frac{3}{4} \times \frac{6}{8} = \frac{18}{32} = \frac{9}{16}$

$\frac{4}{5} \times \frac{4}{10} = \frac{16}{50} = \frac{8}{25}$ $\quad$ $\frac{4}{10} \times \frac{5}{10} = \frac{20}{100} = \frac{1}{5}$

$\frac{5}{8} \times \frac{4}{6} = \frac{20}{48} = \frac{5}{12}$ $\quad$ $\frac{3}{10} \times \frac{2}{8} = \frac{6}{80} = \frac{3}{40}$

$\frac{6}{8} \times \frac{8}{12} = \frac{48}{96} = \frac{1}{2}$

Challenge.

$\frac{3}{9} \times \frac{3}{7} = \frac{9}{63} = \frac{1}{7}$

On these problems you multiplied first, then simplified the answer

Multiplication - Worksheet 10

Use your multiplication of fractions procedure on this problem.

$\frac{5}{15} \times \frac{4}{12} = \frac{20}{180}$

Work Space

Now simplify the product.

$\frac{20}{180} = \frac{10}{90} = \frac{5}{45} = \frac{1}{9}$

$\frac{5}{15} \times \frac{4}{12} = \frac{20}{180}$

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\frac{4}{8} \times \frac{9}{12} = \frac{36}{96}$

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{8}{20} \times \frac{6}{24} = \frac{48}{480}$

$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

$\frac{35}{70} \times \frac{4}{6} = \frac{140}{420}$

$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

$\frac{50}{100} \times \frac{30}{90} = \frac{1,500}{9,000}$

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\frac{5}{12} \times \frac{4}{15} = \frac{20}{180}$

$\frac{5}{12} \times \frac{4}{15} = \frac{20}{180} = \frac{10}{90} = \frac{5}{45} = \frac{1}{9}$

Why was this one harder?

$\frac{5}{12}$ and $\frac{4}{15}$ can't be simplified.

Did it occur to your that you might make the problem easier if your simplified the fraction before you multiplied? Would you still get the same final product if you did this? yes Try it. Go back and redo each problem above. Simplify first, then multiply.

Multiplication - Worksheet 11

Solve. Simplify all answers.

$\frac{3}{6} \times \frac{2}{4} =$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{2}{4} \times \frac{2}{8} =$

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\frac{4}{6} \times \frac{12}{15} =$

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

$\frac{8}{12} \times \frac{9}{12} =$

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$\frac{10}{12} \times \frac{15}{20} =$

$\frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}$

$\frac{21}{27} \times \frac{15}{25} =$

$\frac{7}{9} \times \frac{3}{5} = \frac{21}{45} = \frac{7}{15}$

$\frac{7}{7} \times \frac{14}{16} =$

$\frac{1}{1} \times \frac{7}{8} = \frac{7}{8}$

$\frac{10}{15} \times \frac{12}{16} =$

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

Commutative Property Meets Multiplication of Fractions - Worksheet 1

Solve these problems.

$\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$ $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$

$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$ $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$

Make your own. AWV

$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$ $\frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$

—x— = —x— =

What do you notice about the answers to each pair of problems? same

What do you notice about the numerators in each pair of problems? switched

What is the difference between the problems? The numbers are switched.

Predict what will happen to the answer if you switch or commute the denominators instead of the numerators in a problem. _____

$\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$ $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$ $\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$

Make your own.

$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$ $\frac{3}{3} \times \frac{2}{5} = \frac{6}{15}$

—x— = —x— =

Was your prediction correct? _____

What property in mathematics allows you to do this? Commutative

Commutative Property Meets Multiplication of Fractions - Worksheet 2

$\frac{5}{12} \times \frac{4}{15} =$

— x — =

This problem was hard because neither of the fractions could be reduced. You had to do a lot of arithmetic.

There is a way to make these kinds of problems easier.

In the normal procedure you multiply 5 x 4 to get 20 for the numerator.

Would it be OK to switch 5 x 4 to 4 x 5? yes

Try it a few times.

$\frac{2}{6} \times \frac{1}{3} = \frac{2}{18}$ $\frac{1}{6} \times \frac{2}{3} = \frac{2}{18}$

$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$ $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$

$\frac{1}{6} \times \frac{2}{4} = \frac{2}{24}$ $\frac{2}{6} \times \frac{1}{4} = \frac{2}{24}$

The Commutative Property says "yes," you can do this because in multiplication, the order of the numbers multiplied does not change the product. So, let's use that to make this hard problem easier.

Here goes. Watch what happens when you switch the positions of the two numerators, the 5 and the 4.

$\frac{5}{12} \times \frac{4}{15} =$

$\frac{4}{12} \times \frac{5}{15} =$

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Commutative Property Meets Multiplication of Fractions - Worksheet 3

Simplify and solve. If necessary use the Commutative Property to rewrite.

$\frac{5}{12} \times \frac{4}{15} =$

Commute. $\frac{4}{12} \times \frac{5}{15} =$

Simplify. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Time to practice.

Remember this problem. The Commutative Property can be used to switch the positions of the numerators.
What happens when the numerators are switched? The fractions can be simplified.

$\frac{9}{12} \times \frac{8}{9} =$	$\frac{3}{8} \times \frac{2}{9} =$	
Commute. $\frac{8}{12} \times \frac{9}{9} =$	$\frac{2}{8} \times \frac{3}{9} =$	
Simplify. $\frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$	$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$	
$\frac{2}{21} \times \frac{7}{10} =$	$\frac{5}{6} \times \frac{3}{25} =$	
$\frac{7}{21} \times \frac{2}{10} =$	$\frac{5}{25} \times \frac{3}{6} =$	
$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$	$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$	
$\frac{4}{21} \times \frac{7}{10} =$	$\frac{5}{6} \times \frac{12}{15} =$	
$\frac{4}{10} \times \frac{7}{21} =$	$\frac{5}{15} \times \frac{12}{6} =$	
$\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$	$\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$	25

Commutative Property Meets Multiplication of Fractions - Worksheet 4

Simplify and solve. If necessary use the Commutative Property to rewrite.

1.	2.	3.
$\frac{5}{15} \times \frac{4}{12} =$	$\frac{2}{3} \times \frac{6}{4} =$	$\frac{3}{8} \times \frac{2}{9} =$
Commute. $\frac{4}{12} \times \frac{5}{15} =$	$\frac{2}{4} \times \frac{6}{3} =$	$\frac{3}{9} \times \frac{2}{8} =$
Simplify. $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$	$\frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$	$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

4.	5.	6.
$\frac{9}{12} \times \frac{8}{9} =$	$\frac{2}{21} \times \frac{7}{10} =$	$\frac{5}{6} \times \frac{3}{25} =$
$\frac{9}{9} \times \frac{8}{12} =$	$\frac{2}{10} \times \frac{7}{21} =$	$\frac{5}{25} \times \frac{3}{6} =$
$\frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$	$\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$	$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

Make your own.
AWV

7.	8.	9.
$\frac{2}{21} \times \frac{3}{10} =$	$\frac{5}{6} \times \frac{12}{25} =$	$\_ \times \_ =$
$\frac{2}{10} \times \frac{3}{21} =$	$\frac{5}{25} \times \frac{12}{6} =$	$\_ \times \_ =$
$\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$	$\frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$	$\_ \times \_ =$

26

Commutative Property Meets Multiplication of Fractions - Worksheet 5

Simplify and solve. If necessary use the Commutative Property to rewrite.

More practice.

$\frac{3}{14} \times \frac{7}{9} =$	$\frac{5}{14} \times \frac{4}{5} =$
$\frac{3}{9} \times \frac{7}{14} =$	$\frac{5}{5} \times \frac{4}{14} =$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{1} \times \frac{2}{7} = \frac{2}{7}$

Try rearranging with three fractions. Use the Associative Property.
(A x B) x C = A x C x B = B x C x A and the Commutative Property.

$\frac{1}{4} \times \frac{2}{3} \times \frac{9}{10} =$	$\frac{3}{5} \times \frac{10}{11} \times \frac{2}{9} =$
$\frac{2}{4} \times \frac{3}{1} \times \frac{1}{10} =$	$\frac{3}{9} \times \frac{10}{5} \times \frac{2}{11} =$
$\frac{1}{2} \times \frac{3}{1} \times \frac{1}{10} = \frac{3}{20}$	$\frac{1}{3} \times \frac{2}{1} \times \frac{2}{11} = \frac{4}{33}$
$\frac{7}{9} \times \frac{3}{14} \times \frac{9}{15} =$	$\frac{2}{11} \times \frac{5}{7} \times \frac{11}{12} =$
$\frac{9}{9} \times \frac{7}{14} \times \frac{3}{15} =$	$\frac{2}{12} \times \frac{11}{11} \times \frac{5}{7} =$
$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$	$\frac{1}{6} \times \frac{1}{1} \times \frac{5}{7} = \frac{5}{42}$

Solve these in your head.

$\frac{7}{3} \times \frac{13}{7} \times \frac{3}{13} = 1$

$\frac{2}{19} \times \frac{19}{23} \times \frac{23}{4} = \frac{1}{2}$

27

Cancelling - Worksheet 1

1. $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

2. $\frac{3}{3} \times \frac{2}{5} = \frac{2}{5}$

Which problem is easier? #2

Why is it easier? $\frac{3}{3} = 1$ and $1 \times \frac{2}{5} = \frac{2}{5}$ In problem #1 $\frac{6}{15}$ had to be simplified which added another step.

3. Study this:

Step 1 Look at problem.	Step 2 Commute numbers.	Step 3 Simplify.	Step 4 Get product.
$\frac{2}{3} \times \frac{3}{7} =$	$\frac{3}{3} \times \frac{2}{7} =$	$\frac{1}{1} \times \frac{2}{7} = \frac{2}{7}$	

Circle the two 3s in Step 1.

4. Now you do it.

Step 1	Step 2	Step 3	Step 4
$\frac{5}{7} \times \frac{7}{12} =$	$\frac{7}{7} \times \frac{5}{12} =$	$\frac{1}{1} \times \frac{5}{12} = \frac{5}{12}$	

Circle the two 7s in Step 1.

5. $\frac{5}{6} \times \frac{6}{7} =$ $\quad \frac{6}{6} \times \frac{5}{7} =$ $\quad \frac{1}{1} \times \frac{5}{7} = \frac{5}{7}$

Circle the two 6s in Step 1.

28

Cancelling - Worksheet 2

6. One more time. Do you see any short cuts?

Step 1 Step 2 Step 3 Step 4

$\frac{3}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{\cancel{4}}{\cancel{4}} \times \frac{3}{5} = \frac{3}{5}$ $\quad \frac{1}{1} \times \frac{3}{5} = \frac{3}{5}$

Circle the two 4s in Step 1.

Can you skip Step 2 and go directly toStep 3? (Can you Commute and Simplify in your head?)

7. Step 1 Step 2 Step 3 Step 4

$\frac{3}{5} \times \frac{5}{8} = \frac{5}{5} \times \frac{3}{8} = \frac{1}{1} \times \frac{3}{8} = \frac{3}{8}$

8. Step 1 Step 2 Step 3 Step 4

$\frac{2}{9} \times \frac{9}{11} = \frac{9}{9} \times \frac{2}{11} = \frac{1}{1} \times \frac{2}{11} = \frac{2}{11}$

Can you skip Step 3 too!? What do you have to see in problem 8 to skip Step 3? That $\frac{9}{9}$ is equal to 1.
Try it.

9. Step 1 Step 4

$\frac{9}{11} \times \frac{11}{13} = \frac{9}{13}$

10. Step 1 Step 4

$\frac{2}{7} \times \frac{7}{9} = \frac{2}{9}$

One last time. Looks hard doesn't it? Look closely. It's not so hard afterall.

11. Step 1 Step 4

$\frac{31}{89} \times \frac{89}{97} = \frac{31}{97}$

Cancelling - Worksheet 3

Can you skip Step 2 this time?

1. Step 1 Step 2 Commute Step 3 Simplify Step 4

$\frac{2}{3} \times \frac{3}{4} = \_\_ \times \_\_ = \frac{1}{1} \times \frac{2}{4} = \frac{1}{2}$

What new thing do you have to do to skip Step 2 this time? The $\frac{2}{4}$ has to be reduced or simplified.

Try skipping Step 2 (do it in your head) and writing only the already simplified fractions in Step 3.

2. Step 1 Step 2 Step 3 Step 4

$\frac{5}{7} \times \frac{7}{10} = \_\_ \times \_\_ = \frac{1}{1} \times \frac{5}{10} = \frac{1}{2}$

If you can't, fill in Step 2.

3. Step 1 Step 2 Step 3 Step 4

$\frac{7}{11} \times \frac{11}{14} = \cdots \times \cdots = \frac{1}{1} \times \frac{7}{14} = \frac{1}{2}$

4. $\frac{6}{7} \times \frac{7}{12} = \_\_ \times \_\_ = \frac{1}{1} \times \frac{6}{12} = \frac{1}{2}$

5. Make your own. AWV

$\_\_ \times \_\_ = \cdots \times \cdots = \_\_ \times \_\_ = \_\_$

Challenge. Try to skip Step 2. If you get stuck write in Step 2.

6. Step 1 Step 2 Step 3 Step 4

$\frac{5}{7} \times \frac{14}{15} = \cdots \times \cdots = \frac{2}{1} \times \frac{5}{15} = \frac{10}{15} = \frac{2}{3}$

What changed here?** No more ones.

** No more ones! This one has $\frac{14}{7}$ or $\frac{2}{1}$ instead of a one.

* The $\frac{2}{4}$ has to be reduced in your head and written $\frac{1}{2}$ in Step 3.

Cancelling - Worksheet 4

Imagine a way to do this type of problem without rewriting it.

Go directly from Step 1 to Step 4. Check by doing all the Steps.

Example:

Step 1 Step 4 Step 2 Step 3 Step 4

1. $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$ $\quad \frac{3}{3} \times \frac{2}{5} = \frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$
2. $\frac{3}{5} \times \frac{5}{8} = \frac{3}{8}$ $\quad \frac{5}{5} \times \frac{3}{8} = \frac{1}{1} \times \frac{3}{8} = \frac{3}{8}$
3. $\frac{4}{7} \times \frac{7}{9} = \frac{4}{9}$ $\quad \frac{7}{7} \times \frac{4}{9} = \frac{1}{1} \times \frac{4}{9} = \frac{4}{9}$
4. $\frac{2}{3} \times \frac{3}{6} = \frac{1}{3}$ $\quad \frac{3}{3} \times \frac{2}{6} = \frac{1}{1} \times \frac{1}{3} = \frac{1}{3}$
5. $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5} =$ $\quad \frac{4}{4} \times \frac{3}{5} = \frac{1}{1} \times \frac{3}{5} = \frac{3}{5}$
6. $\frac{5}{6} \times \frac{6}{8} = \frac{5}{8}$ $\quad \frac{6}{6} \times \frac{5}{8} = \frac{1}{1} \times \frac{5}{8} = \frac{5}{8}$
7. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ $\quad \frac{3}{3} \times \frac{2}{4} = \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$

Cancelling - Worksheet 5

Rearrange and Simplify Skip Step 2. On the easier problems skip Step 3.

1. $\frac{7}{15} \times \frac{3}{21} = \frac{3}{15} \times \frac{7}{21} = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$
2. $\frac{16}{32} \times \frac{14}{16} = \frac{16}{16} \times \frac{14}{32} = \frac{1}{1} \times \frac{7}{16} = \frac{7}{16}$
3. $\frac{17}{16} \times \frac{4}{17} = \frac{17}{17} \times \frac{4}{16} = \frac{1}{1} \times \frac{1}{4} = \frac{1}{4}$
4. $\frac{6}{8} \times \frac{5}{12} = \frac{6}{12} \times \frac{5}{8} = \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$
5. $\frac{19}{40} \times \frac{30}{19} = \frac{19}{19} \times \frac{30}{40} = \frac{1}{1} \times \frac{3}{4} = \frac{3}{4}$

Cancelling - Worksheet 6

Rearrange in your head. Record the Simplified answer and multiply.

Example: Skip

1. $\frac{3}{8} \times \frac{8}{24} = \frac{8}{8} \times \frac{3}{24} = \frac{1}{1} \times \frac{1}{8} = \frac{1}{8}$

2. $\frac{2}{3} \times \frac{3}{18} \times \frac{3}{18} = \frac{1}{1} \times \frac{1}{9} \times \frac{1}{6} = \frac{1}{54}$

3. $\frac{3}{4} \times \frac{4}{9} \times \frac{9}{16} = \frac{1}{1} \times \frac{1}{1} \times \frac{3}{16} = \frac{3}{16}$

4. $\frac{100}{200} \times \frac{4}{100} \times \frac{20}{16} = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{40}$

5. $\frac{5}{6} \times \frac{18}{20} = \frac{5}{20} \times \frac{18}{6} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

Cancelling - Worksheet 7

Solve these with as few steps as possible.

1. $\frac{5}{10} \times \frac{3}{9} = \frac{1}{2} \times \frac{1}{3} = \text{—} \times \text{—} = \frac{1}{6}$

Before rearranging, check to see if the fractions can be simplified as they are.

2. $\frac{9}{24} \times \frac{15}{27} \times \frac{9}{15} = \frac{3}{8} \times \frac{1}{1} \times \frac{1}{3} = \frac{3}{24} = \frac{1}{8}$

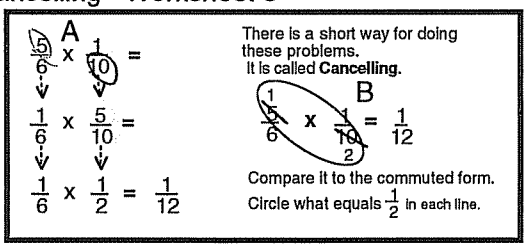
3. $\frac{4}{5} \times \frac{5}{7} \times \frac{14}{8} = \frac{2}{1} \times \frac{1}{1} \times \frac{1}{2} = \frac{2}{2} = 1$

4. $\frac{19}{38} \times \frac{6}{13} \times \frac{13}{12} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{4}$

5. $\frac{25}{50} \times \frac{6}{18} = \frac{1}{2} \times \frac{1}{3} = \text{—} \times \text{—} = \frac{1}{6}$

6. $\frac{15}{26} \times \frac{7}{30} \times \frac{13}{14} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Cancelling - Worksheet 8



In problem A, the 5 and the 10 are put together into $\frac{5}{10}$ then simplified to $\frac{1}{2}$

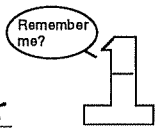
Circle the 5 and the 10 in the top problem $\frac{5}{6} \times \frac{1}{10}$

In problem B, notice the 5 and the 10 in the loop. Where are they in relationship to each other? opposite diagonally

In problem B:

the $\frac{5}{10}$ is diagonal.

visualize $\frac{5}{10}$



What number divides into the 5 and the 10 evenly? 5

Divide both the numbers by that number in your head.

Record the simplified numbers by crossing out the 5 and putting a 1 and by crossing out the 10 and putting a 2.

How did you get the 1? 5÷5 How did you get the 2? 10÷2

Why does this work? You can divide both numbers by 5 which equals 1.

The simplified $\frac{1}{2}$ shows up inside the loop. This is a real time and recording saver.

Cancelling - Worksheet 9

Example:

$\frac{2}{3} \times \frac{1}{10} = \frac{2}{30}$ $\quad \frac{1}{3} \times \frac{1}{10_5} = \frac{1}{15}$

What operation is used to simplify the numbers in a fraction problem? Division

How is the 2 in $\frac{2}{3}$ changed to 1 ? 2÷2=1

How is the 10 in $\frac{1}{10}$ changed to 5? 10÷2=5

$\frac{9}{4} \times \frac{1}{18} =$

$\frac{1}{4} \times \frac{9}{18} =$

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Use cancelling.

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$\frac{5}{6} \times \frac{3}{10} =$

$\frac{3}{6} \times \frac{5}{10} =$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Do this problem without reducing before you multiply. Reduce at the end.

$\frac{5}{6} \times \frac{3}{10} = \frac{15}{60} = \frac{1}{4}$

Use cancelling on the problem below.

$\frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$

Did you get the same answer for all three problems in this box? yes

Cancelling - Worksheet 10

Use the new method of cancelling to solve.

$\frac{4}{8}{}^{3} \times \frac{1}{9} = \frac{4}{27}$ $\quad \frac{1}{5}{}^{3} \times \frac{1}{8} = \frac{1}{24}$ $\quad \frac{1}{7}{}^{1} \times \frac{1}{8} = \frac{1}{8}$

$\frac{2}{14}{}^{3} \times \frac{1}{15} = \frac{2}{45}$ $\quad \frac{1}{21}{}^{3} \times \frac{1}{12} = \frac{1}{36}$ $\quad \frac{3}{10}{}^{5} \times \frac{13}{15} = \frac{39}{50}$

$\frac{5}{8}{}^{3} \times \frac{10}{11} = \frac{25}{33}$ $\quad \frac{1}{8}{}^{2} \times \frac{3}{10} = \frac{3}{20}$ $\quad \frac{1}{2}{}^{7} \times \frac{3}{42} = \frac{3}{14}$

$\frac{1}{10}{}^{2} \times \frac{1}{7} = \frac{1}{14}$ $\quad \frac{1}{8}{}^{2} \times \frac{3}{10} = \frac{3}{16}$ $\quad \frac{1}{2}{}^{5} \times \frac{1}{2} = \frac{1}{5}$

$\frac{1}{4}{}^{1} \times \frac{1}{4} \times \frac{2}{10} = \frac{2}{40} = \frac{1}{20}$ $\quad \frac{1}{6} \times \frac{1}{4}{}^{1} \times \frac{4}{6}{}^{2} = \frac{1}{12}$

Cancelling - Worksheet 11

Solve. Write the number sentence and the answer in the simplest form.

1. Amy milked her cow in the morning and got $\frac{2}{3}$ of a gallon of milk. Kris drank $\frac{1}{4}$ of the milk at lunch. How much milk did Kris drink? $\frac{1}{6}$
   Number sentence $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$
2. The lawn is $\frac{1}{2}$ mowed. Gina mowed $\frac{6}{8}$ of it. How much of the lawn did Gina mow? $\frac{3}{8}$ Number sentence $\frac{1}{2} \times \frac{6}{8}{}^{3} = \frac{3}{8}$
   How much is left to mow? $\frac{1}{2}$ Number sentence $1 - \frac{1}{2} = \frac{1}{2}$
3. Laurel bought $\frac{6}{8}$ of a pound of her favorite cheddar cheese. She ate $\frac{1}{3}$ of it. How much did she eat? $\frac{1}{4}$
   Number sentence $\frac{6}{8} \times \frac{1}{3} = \frac{6}{24} = \frac{1}{4}$

4. Five sixths of the room got painted before lunch. Judi did $\frac{2}{3}$ of the painting. How much of the room did she paint? $\frac{5}{9}$
   Number sentence $\frac{5}{6} \times \frac{2}{3} = \frac{10}{18} = \frac{5}{9}$
5. Peter swam $\frac{3}{4}$ of a mile. He did the back stroke for $\frac{4}{5}$ it. How far did he swim the back stroke? $\frac{3}{5}$
   Number sentence $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

Cancelling - Worksheet 12

Simplify and solve.

$\frac{8}{8}{}^{1} \times \frac{2}{9}{}^{1} = \frac{1}{12}$ $\quad \frac{8}{8}{}^{1} \times \frac{4}{15} = \frac{1}{10}$ $\quad \frac{7}{8} \times \frac{2}{21} = \frac{1}{12}$

$\frac{7}{15} \times \frac{3}{21} = \frac{1}{15}$ $\quad \frac{4}{21} \times \frac{7}{12} = \frac{1}{9}$ $\quad \frac{3}{10} \times \frac{5}{21} = \frac{1}{14}$

$\frac{5}{12} \times \frac{3}{10} = \frac{1}{8}$ $\quad \frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$ $\quad \frac{8}{9} \times \frac{3}{16} = \frac{1}{6}$

$\frac{4}{21} \times \frac{7}{8} = \frac{1}{6}$ $\quad \frac{3}{8} \times \frac{2}{12} = \frac{1}{16}$ $\quad \frac{3}{5}{}^{1} \times \frac{10}{15}{}^{2} = \frac{2}{5}$

$\frac{1}{4} \times \frac{2}{3} \times \frac{9}{10} = \frac{3}{20}$ $\quad \frac{3}{5} \times \frac{2}{9} \times \frac{10}{11} = \frac{4}{33}$

Review. Solve both problems. $\frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$ $\quad \frac{2}{10} \times \frac{9}{3} = \frac{18}{30} = \frac{3}{5}$

What do you notice about the answers to both of these problems? Both are the same.

What was switched in the problems? denominators

What property allows the numbers to be switched? Commutative

Reduce or simplify: $\frac{2}{10} = \frac{1}{5}$ What did you divide by to get $\frac{1}{5}$? $\frac{2}{2}$

You've been recording simplifying like this: $\frac{2}{3}{}^{1} \times \frac{9}{10}{}_{5} = \frac{9}{15}$

Can you do this for the $\frac{9}{3}$? Try it. What did you divide by to get $\frac{3}{1}$? $\frac{3}{3}$

Record the simplified problem here and solve. $\frac{1}{1} \times \frac{3}{5} = \frac{3}{5}$

Cancelling - Worksheet 13

Solve and simplify.

$\frac{5}{10}{}_{2} \times \frac{5}{4}{}^{2} = \frac{25}{6} = 4\frac{1}{6}$ $\quad \frac{12}{5} \times \frac{7}{3} = \frac{28}{5} = 5\frac{3}{5}$

$\frac{1}{8}{}_{2} \times \frac{6}{12}{}^{1} = \frac{6}{1} = 6$ $\quad \frac{2}{8}{}^{1} \times \frac{4}{12}{}^{1} = \frac{8}{1} = 8$

$\frac{2}{12}{}^{1} \times \frac{19}{8}{}_{1} = \frac{38}{7} = 5\frac{3}{7}$ $\quad \frac{15}{7}{}^{1} \times \frac{3}{21}{}^{3} = \frac{15}{3} = 5$

$\frac{4}{16}{}_{2} \times \frac{4}{12}{}^{3} = \frac{16}{3} = 5\frac{1}{3}$ $\quad \frac{7}{21}{}_{4} \times \frac{3}{12}{}^{6} = \frac{21}{6} = 3\frac{1}{2}$

$\frac{5}{15}{}^{1} \times \frac{1}{8}{}_{4} = \frac{5}{4} = 1\frac{1}{4}$ $\quad \frac{19}{6} \times \frac{6}{19} = \frac{1}{1} = 1$

$\frac{3}{15}{}_{1} \times \frac{3}{18}{}_{1} = \frac{9}{1} = 9$ $\quad \frac{24}{5} \times \frac{20}{6} = \frac{16}{1} = 16$

Cancelling Meets Parts of Wholes - Worksheet 1

Remember this?

$\frac{1}{3}$ of 9 

1. What is the fastest way to get the answer? Color in three of the circles. Divide 9 by 3.

2. Here is a new way which uses a fraction multiplication procedure.

$\frac{1}{3} \times \frac{9}{1}$ New Write the 9 as $\frac{9}{1}$ $\frac{1}{\cancel{3}_1} \times \frac{\cancel{9}^{3}}{1} = \frac{3}{1} = 3$

Use cancelling to write the answer

Is this the same way you solved problem 1 or a different way? _____

A different way.

Defend your answer

AWV

3. Parts of wholes.

$\frac{2}{3}$ of 9 = (9 ÷ 3) x 2 = 6

4. Cancelling.

$\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{1} = \frac{6}{1} = 6$

Is the order of your process for parts of wholes the same or different from cancelling? Different.

You will find that writing the whole number 9, as a fraction, $\frac{9}{1}$, makes the next lesson easier

Cancelling Meets Parts of Wholes - Worksheet 2

$\frac{3}{4} \times 8 = \frac{3}{4} \times \frac{8}{1} = \frac{3}{\cancel{4}_1} \times \frac{\cancel{8}^{2}}{1} = 6$

$\frac{5}{7} \times 7 = \frac{5}{\cancel{7}} \times \frac{\cancel{7}^{1}}{1} = \frac{5}{1} = 5$

$\frac{7}{9} \times 6 = \frac{7}{\cancel{9}_3} \times \frac{\cancel{6}^{2}}{1} = \frac{14}{3} = 4\frac{2}{3}$

$\frac{3}{\cancel{20}_5} \times \frac{\cancel{4}^{1}}{1} = \frac{3}{5}$

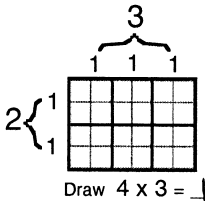
$\frac{3}{\cancel{20}_2} \times \frac{\cancel{10}^{1}}{1} = \frac{3}{2} = 1\frac{1}{2}$

$\frac{\cancel{15}}{\cancel{100}_5} \times \frac{\cancel{20}^{1}}{1} = \frac{15}{5} = 3$

$\frac{2}{\cancel{3}_1} \times \frac{\cancel{90}^{30}}{1} = \frac{60}{1} = 60$

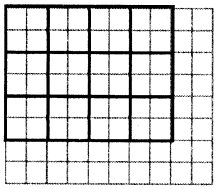
Multiplying Mixed Numbers - Worksheet 1

If ▢ = 1 square unit ——— = 1 unit of length

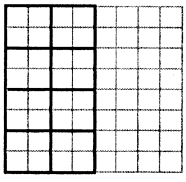


This drawing shows: 3 x 2 = 6

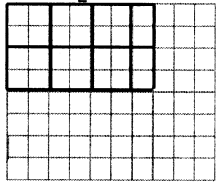
Draw 4 x 3 = 12.



Draw 2 x 4 = 8.

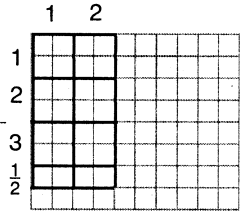


Trace. $3\frac{1}{2} \times 2$



Color in the product.
The product = 7

Draw $2 \times 3\frac{1}{2}$



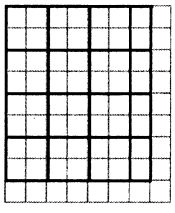
Color in the product.
The product = 7

Multiplying Mixed Numbers - Worksheet 2

Think about what you know about the multiplication of whole numbers and the multiplication of fractions.

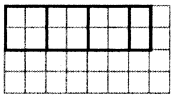
How is this problem solved? $3\frac{1}{2} \times 4 =$ 14

Build this problem with fraction pieces. Draw a picture below of the solution. Record the answer

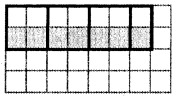


Now try a mixed number by a fraction. Build this problem with fraction pieces. Draw a picture below of the solution. Record the answer

$3\frac{1}{2}$



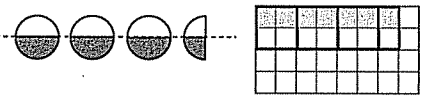
$3\frac{1}{2} \times \frac{1}{2} =$ $1\frac{3}{4}$



Multiplying Mixed Numbers - Worksheet 3

One possible solution. Use fraction pieces or a drawing on graph paper to help.

"What is $\frac{1}{2}$ of $3\frac{1}{2}$?



$\frac{1}{2} \times 3\frac{1}{2} = 1\frac{3}{4}$

A different solution.

The standard procedure for multiplication of fractions is:

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ The product of numerators of 2 x 4 equals 8. $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

The product of denominators 3 x 5 equals 15.

How can the form of $3\frac{1}{2}$ be changed in order to use the standard procedure?

Change the mixed number into an improper fraction.

Solve these problems and compare the answers to the ones on orksheet 1.

$3\frac{1}{2} \times 4 = 14$

$\frac{7}{2} \times \frac{4}{1} = \frac{28}{2} = 14$

$3\frac{1}{2} \times \frac{1}{2} = 1\frac{3}{4}$

$\frac{7}{2} \times \frac{1}{2} = \frac{7}{4} = 1\frac{3}{4}$

Try a few here.

$2\frac{2}{3} \times \frac{1}{2} = 1\frac{2}{3}$

$\frac{10}{3} \times \frac{1}{2} = \frac{10}{6} = 1\frac{4}{6} = 1\frac{2}{3}$

$1\frac{3}{4} \times \frac{1}{7} = \frac{1}{4}$

$\frac{7}{4} \times \frac{1}{7} = \frac{7}{28} = \frac{1}{4}$

Multiplying Mixed Numbers - Worksheet 4

Change the mixed number to an improper fraction. Watch for chances to cancel. Simplify all answers. Do not leave answers as improper fractions.

Example: $2\frac{1}{3} \times 4\frac{1}{2} = \frac{7}{\cancel{8}_{1}} \times \frac{\cancel{9}^{3}}{2} = \frac{21}{2} = 10\frac{1}{2}$

$1\frac{1}{4} \times 3\frac{1}{3} = \frac{5}{\cancel{4}_{1}} \times \frac{\cancel{10}^{5}}{3} = \frac{25}{6} = 4\frac{1}{6}$

$3\frac{3}{5} \times 2 = \frac{18}{5} \times \frac{2}{1} = \frac{36}{5} = 7\frac{1}{5}$

$1\frac{7}{8} \times 1\frac{1}{3} = \frac{\cancel{15}^{5}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}} = \frac{5}{2} = 2\frac{1}{2}$

$3\frac{3}{5} \times 6\frac{2}{3} = \frac{\cancel{18}^{6}}{\cancel{5}_{1}} \times \frac{\cancel{20}^{4}}{\cancel{3}_{1}} = \frac{24}{1} = 24$

$1\frac{1}{3} \times 4\frac{1}{4} = \frac{\cancel{4}^{1}}{3} \times \frac{17}{\cancel{4}_{1}} = \frac{17}{3} = 5\frac{2}{3}$

$2\frac{1}{3} \times 3\frac{1}{3} = \frac{7}{3} \times \frac{10}{3} = \frac{70}{9} = 7\frac{7}{9}$

$3 \times 2\frac{1}{2} = \frac{3}{1} \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}$

$5\frac{1}{4} \times 5\frac{1}{3} = \frac{\cancel{21}^{7}}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{\cancel{3}_{1}} = \frac{28}{1} = 28$

$7\frac{1}{2} \times 4\frac{4}{5} = \frac{\cancel{15}^{3}}{\cancel{2}_{1}} \times \frac{\cancel{24}^{12}}{\cancel{5}_{1}} = \frac{36}{1} = 36$

Multiplying Mixed Numbers - Worksheet 5

1. Daphne has a big doll house for her Barbie. One room has a floor that is $4\frac{1}{2}$ feet long and $2\frac{3}{4}$ feet wide. Find the area of this room. $12\frac{3}{8}$ sq. ft.

   $\frac{9}{2} \times \frac{11}{4} = \frac{99}{8} = 12\frac{3}{8}$

2. The doll house's bedroom is $2\frac{1}{4}$ feet long and $1\frac{1}{3}$ feet wide. How much rug would she need for this room? 3 sq. ft.

   $\frac{\cancel{9}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}} = 3$

3. There are some tiny pictures for the room. One picture is $2\frac{1}{4}$ inches long and $1\frac{1}{6}$ inches wide. Find the area of the picture. $2\frac{5}{8}$ sq. in.

   $\frac{\cancel{9}^{3}}{4} \times \frac{7}{\cancel{6}_{2}} = \frac{21}{8} = 2\frac{5}{8}$

4. Pam is getting new carpet for a room that is $10\frac{2}{3}$ feet long and $8\frac{1}{4}$ feet wide. How big would the new carpet need to be? 88 sq. ft.

   $\frac{\cancel{32}^{8}}{\cancel{3}_{1}} \times \frac{\cancel{33}^{11}}{\cancel{4}_{1}} \quad \frac{88}{1} = 88$

5. Richard's garden is $12\frac{5}{6}$ feet long and $11\frac{1}{3}$ feet wide. Find the area of the garden.

   $\frac{77}{3} \times \frac{34}{3} = \frac{2,618}{9} \quad 290\frac{8}{9}$ sq. ft.

6. Write and solve a multiplication of fractions problem with mixed numbers. Write one that can be simplified by cancelling. AWV

6. Write and solve a multiplication of fractions problem with mixed numbers. Write one that can't be simplified by cancelling. AWV

Multiplying Mixed Numbers - Worksheet 6

Solve. Simplify all answers.

$\frac{3}{6} \times \frac{3}{5} = \frac{3}{10}$

$\frac{2}{4} \times \frac{3}{8} = \frac{3}{16}$

$\frac{4}{7} \times \frac{5}{6} = \frac{10}{21}$

$\frac{7}{8} \times \frac{5}{6} = \frac{35}{48}$

$\frac{4}{5} \times \frac{4}{10} = \frac{8}{25}$

$\frac{8}{12} \times \frac{3}{4} = \frac{1}{2}$

$\frac{7}{10} \times \frac{2}{14} = \frac{1}{10}$

$1\frac{2}{5} \times \frac{7}{8} = 1\frac{9}{40}$

$8 \times 2\frac{1}{12} = 16\frac{2}{3}$

$3 \times \frac{2}{5} = 1\frac{1}{5}$

$\frac{2}{5} \times 6\frac{1}{4} = 2\frac{1}{2}$

$3\frac{1}{8} \times 3\frac{1}{5} = 10$

Cumulative Review - Worksheet 1

1. $\frac{2}{3}$ of 9 Use this picture to show $\frac{2}{3}$ of 9 ⊗⊗⊗⊗⊗○○○

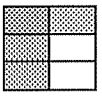
   Write the number sentence. $(9 \div 3) \times 2 = 6$

2. $\frac{2}{3}$ of 9 Find the answer using iterative addition.

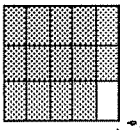
   Write the number sentence. $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{18}{3} = 6$

3. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{6}$

   Draw a picture to prove the answer is correct.



4. Solve. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ Draw a picture to prove the answer is correct.



5. Use cancelling to solve. $\frac{\cancel{4}^{1}}{\cancel{7}_{1}} \times \frac{1}{\cancel{8}_{2}} = \frac{1}{2}$

6. Explain why cancelling works. The numerator's position is being changed to be over a different denominator to create a fraction which can be simplified by using division.

   What property of mathematics is used? commutative or associative

7. Solve. $\frac{2}{3} \times \frac{15}{22} = \frac{5}{11}$

8. Solve. $\frac{8}{3} \times \frac{9}{4} = 6$

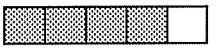
9. Solve. $2\frac{2}{3} \times 2\frac{1}{4} = 6$

10. The scenic route took $2\frac{1}{2}$ hours. The freeway route took $\frac{1}{2}$ of the time with $\frac{1}{4}$ the beautiful views. How long was the freeway route? $1\frac{1}{4}$ hours

    Write the number sentence. $2\frac{1}{2} \times \frac{1}{2} = 1\frac{1}{4}$

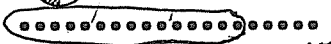
Cumulative Review - Worksheet 2

1. Shade in $\frac{4}{5}$ of this rectangle.



2. Shade in $\frac{2}{3}$ of this circle.

3. Circle $\frac{3}{4}$ of these dots.



4. Change to an improper fraction. $3\frac{3}{4} = \frac{15}{4}$ $7\frac{4}{9} = \frac{67}{9}$

5. Change to an mixed number. $\frac{29}{4} = 7\frac{1}{4}$ $\frac{58}{8} = 7\frac{1}{4}$

5. Fill in the missing numbers. $\frac{4}{5} = \frac{12}{15}$ $\frac{2}{3} = \frac{18}{27}$

7. Simplify these fractions. $\frac{32}{36} = \frac{8}{9}$ $\frac{17}{34} = \frac{1}{2}$ $\frac{19}{23} = \frac{19}{23}$ $\frac{42}{54} = \frac{7}{9}$

8. Solve. $\frac{\cancel{8}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{\cancel{21}_{7}} = \frac{2}{7}$

9. Write a word problem to go with: $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

   AWV

10. Solve. $7\frac{1}{2} \times 3\frac{2}{3} = \frac{\cancel{15}^{5}}{2} \times \frac{11}{\cancel{3}_{1}} = \frac{55}{2} = 27\frac{1}{2}$

Post-Assessment is the same as the Pre-Assessment-Part 2

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Multiplication

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