Fractions: Booklet 6 - Multiplication

Contents

Assessment Guide	1
Pre-Assessment	. 9
Post-Assessment	12
Parts of Wholes: Review and Fractions as Parts of Wholes	13
Parts of Wholes Meet Iterative Addition	15
Multiplication	17
Commutative Property Meets Multiplication of Fractions	20
Cancelling	22
Cancelling Meets Parts of Wholes	24
Multiplying Mixed Numbers	25

This booklet is dedicated to all the wonderful parents who had their children educated at the Farm School. Thank you for trusting us with your precious sons and daughters.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 6

Parent/Teacher Guide ©2014 Pattern Press

Published by Pattern Press P.O. Box 2737 Fallbrook, CA 92088 (760)728-3731

www.patternpress.com E-mail: Patternpress1@gmail.com All rights reserved.

Practice worksheets may be duplicated.

Printed on recycled paper.



ISBN 978-1-935559-39-9

Parts of Wholes: Review and Fractions as Parts of Wholes

Purpose

This lesson's purpose is to review a topic originally taught in Fractions: Booklet 2 of this series. For many students the gap between that book and this may be a year or more. For a student beginning the series at this point, it is important to use a manipulative to establish the physical concept behind the procedure. This lesson assumes the student discovered the following procedure in previous lessons. The lesson reviews the idea that to take a fraction of a set, you must count the number of members of the set, divide it by the denominator of the fractional unit you are using and then multiply the number in that set by the number of sets the numerator indicates you need. For example, $\frac{2}{5}$ of $10 = (10 \div 5) \times 2$. If this is a new concept, have the student go back and do Parts of Wholes in Fractions: Booklet 2. The following dialogue is from a student who is just reviewing what was previously figured out.

Prerequisites Fractions: Booklet 2 - Parts of Wholes

Materials Parts of Wholes: Review - Worksheets 1 and 2, pages 4 and 5

Fractions as Parts of Wholes: Worksheets 1 - 3, pages 6 - 8

Blocks and fraction pieces

Warm Up

Give Parts of Wholes: Review - Worksheet 1. Watch while the student does the work. Notice if she counts the total number of pictures in the set and then divides it.

"How do you know how many fish to color in?" "There are ten in the set and I need to divide it into half, so I divide by two."

"How do you know how many squares to color in?" "There are four in the set and I need to divide it into half, so I divide by two."

"How do you know how many stars to color in?" "There are fifteen in the set and I need to divide it into thirds, so I divide by three and get five. Then, since it asks for two-thirds, I need to take two groups of five to make ten. So I color in ten stars."

If by the end of the Warm Up she can't remember or is not able to construct the procedure for these problems, reteach this concept. Use blocks or another manipulative to pose the same problems. Let her divide them in half, prove it is half, and verify her answer. It also appears in the last part of Fractions: Booklet 2: Parts of Wholes of this series, or make up more problems like these. For example, draw a group of nine objects and have her circle one-third of the objects.

Lesson Part 1

"Do the first two problems on Parts of Wholes: Review - Worksheet 2. How can you use a pattern to figure out the answer?" "If I know what one-third of fifteen is, which is five, then all I have to do is go up by fives for each new part." Have blocks available for her to work with if needed.

Lesson Part 2

"Does this pattern also work for the next four problems on Parts of Wholes: Review - Worksheet 2?" "If I know that one-fifth of fifteen is three, then all I have

Patterns in Arithmetic: Fractions - Booklet 6
Parent/Teacher Guide

Parts of Wholes: Review and Fractions as Parts of Wholes

13

Parts of Wholes: Review - Worksheet 2

How can you figure out what $\frac{4}{5}$ of 15 is if you know what $\frac{1}{5}$ of fifteen is?

$$\frac{1}{3}$$
 of 15 = ____

$$\frac{1}{2}$$
 of 20 = ____

$$\frac{2}{3}$$
 of 15 = ____

$$\frac{1}{4}$$
 of 20 = ____

$$\frac{1}{5}$$
 of 15 = ____

$$\frac{2}{4}$$
 of 20 = ____

$$\frac{2}{5}$$
 of 15 = ____

$$\frac{3}{4}$$
 of 20 = _____

$$\frac{3}{5}$$
 of 15 = ____

$$\frac{1}{5}$$
 of 20 = ____

$$\frac{4}{5}$$
 of 15 = _____

$$\frac{2}{5}$$
 of 20 = ____

$$\frac{1}{7}$$
 of $14 =$ _____

$$\frac{3}{5}$$
 of 20 = ____

$$\frac{2}{7}$$
 of $14 =$ _____

$$\frac{4}{5}$$
 of 15 = ____

$$\frac{3}{7}$$
 of $14 =$ _____

Make up a set.

$$\frac{4}{7}$$
 of $14 =$ _____

$$\frac{5}{7}$$
 of 14 = _____

$$\frac{6}{7}$$
 of $14 =$ _____

Explain in words how you find out what $\frac{3}{5}$ of 20 is.

Step 1

Parts of Wholes Meet Iterative Addition

Purpose

The purpose of this lesson is to review the previous strategy to find a fractional part of a set, e.g., $\frac{2}{5}$ of 10, by partitioning a set into equal groups as indicated by the denominator and then multiplying by the numerator. Writing the mathematical sentence for this process is also reviewed. We also introduce the idea that the problem can be seen as iterative addition of a fraction. Both procedures produce the same answer but have different physical applications. It hones the mind to see both; how they are alike and how they are different.

Prerequisites

Fractions as Parts of Wholes, Reduction of Improper Fractions and the multiplication tables

Materials

Parts of Wholes Meet Iterative Addition - Worksheets 1 - 3, pages 9 - 11

Warm Up

Review the number sentence that you did on Fractions as Parts of Whole - Worksheet 2, page 7.

Give this word problem: "A recipe you are making for a big cake calls for nine cups of flour and six eggs. But oops you only have four eggs. You will have to cut your recipe down. So if you only use four eggs, how much flour should you use?"

Lesson

Parts of Wholes Meet Iterative Addition - Worksheet 1, page 9

"What does iterative mean? Look at problem 2 and see if you can figure it out." "Iterative means the same thing done over and over." "Make up another iterative problem." "2 + 2 + 2 + 2."

"How is the first addition problem, 2 + 3 + 4, different from 3 + 3 + 3 + 3?"

"The first ones add different numbers. The second one adds the same number over and over."

"How is iterative addition connected to multiplication?" "They are the same thing. Multiplication is a shortcut for adding the same number over and over again."

"Do you think taking two-thirds of twelve will have the same answer as two-thirds added twelve times? Try it." Many students will be surprised that the answer is the same. He will add two twelve times to get twenty-four thirds. He must then simplify the twenty-four thirds using division, which is twenty-four divided by three. Twenty-four thirds is equal to eight.

Note

Most students will need help to translate adding the two twelve times and then dividing by three into the number sentence. Do not help any more than is needed.

"How do you write what you just did as a number sentence?" "First I multiply the numerator, the two, times the whole number, twelve. I would put that inside the parentheses. That gives me twenty-four. Then I would divide the twenty-four by

Parts of Wholes Meet Iterative Addition - Worksheet 1

Date

1. What does iterative mean?

This is addition:

This is iterative addition:

$$3+3+3+3=$$

also known as multiplication.

 $\frac{2}{3}$ of 12 can be done as a part of a whole.



Shade in $\frac{2}{3}$ of the 12 circles.

 $\frac{2}{3}$ x 12 = _____ You did these in Fractions: Booklet 2.

 $\frac{2}{3}$ x 12 can mean two-thirds added twelve times.

$$\frac{2}{3} + \frac{2}{3} = \underline{\qquad}$$





Do you think iterative addition (adding $\frac{2}{3}$ 12 times) will give you the same or a different answer as taking two-thirds of 12? ._____

Try it. How many thirds so you get if you add $\frac{2}{3}$ twelve times?

$$\frac{2}{3}$$
 x 12 = (___X ___) ÷ ___ = ___ ÷ ___ = ___

Now, simplify the answer. ____

Multiplication

Purpose

The purpose of this lesson is to use a manipulative to construct the concept of what multiplying a fraction means. In the previous lessons the student worked with the idea that the 'of' in a parts of wholes problem, two-thirds of twelve, for example, is the same thing as two-thirds added twelve times or two-thirds times twelve. In this lesson, this idea is extended to a fraction of a fraction, which is what multiplication of fractions is. Again, the inverse quality of fractions is encountered. When you multiply a fraction, it feels like division because you are taking a part of a part. The product of a multiplication of fractions problem always comes out less than either of the numbers you started with. Those who memorize this simple procedure will not notice or question this.

Prerequisites Parts of Wholes

Preparation

Read through the lesson and try the problems yourself. Most teachers are not familiar with this way of seeing multiplication of fractions. Check the Answer Key to be sure you know how to record the problems. Read Multiplication - Worksheets 1 and 5 carefully.

Note

Allow the student to play with the clear acetate pieces before you begin the lesson. She will want to hold them up to the light to see the pretty colors.

Materials

Multiplication - Worksheets 1 - 11, pages 12 - 22

Scratch paper and scissors

Colored pencils

Clear and colored acetate Prism Fractions, or Translucent Overhead fraction pieces if you are working with a class

Warm Up

Review basic multiplication like 5 x 4 = 20 and Parts of Wholes with problems like $\frac{1}{4}$ x 20.

"Why does the answer come out with less than you started with? When you multiply, don't you usually get an answer that is larger than either of the numbers you start with?" "It comes out less because you are taking a part of something away, or, since you are only adding $\frac{1}{4}$, your total will be four times less than twenty." She may say something such as "It's like division" but not be able to clearly articulate why.

"How is $\frac{1}{4}$ x 20 like division?" "You are dividing the twenty up into four groups, and that is division. So multiplying a fraction is like going backwards of whole numbers."

Lesson Part 1

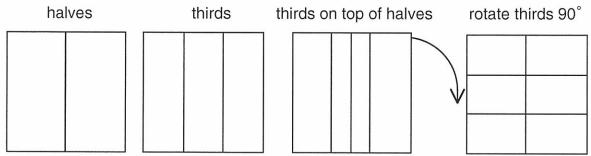
"If $\frac{1}{4}$ x 20 means $\frac{1}{4}$ of 20, then what do you think $\frac{1}{4}$ of a $\frac{1}{2}$ might be?" Wait for her to think about it. She may draw it, use the pieces, or, more likely, picture it in her head. "If I cut a half into four parts, then each part would be an eighth."

"Prove it with a piece of paper. Can you prove that one of these small pieces is

Use **clear** Prism Fractions acetate squares to find the solution to these problems.

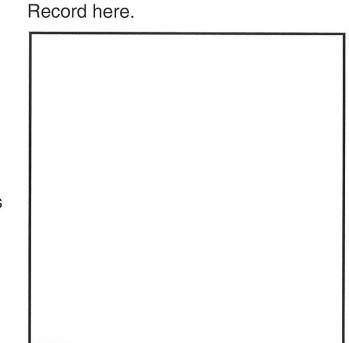


Start with the square divided into two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°.



How many little rectangles are there after rotating the squares? _____ What fraction of the whole is the little rectangle? ____

Now do several more rotations and record the number of rectangles.

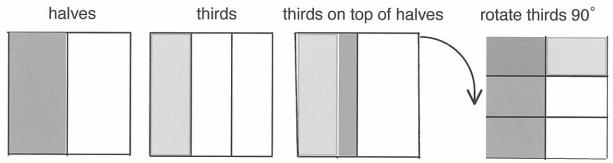


Multiplication - Worksheet 2

Use **colored** Prism Fractions acetate squares to find the solution to these problems.

Start with the square divided into two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°.





This shows halves divided into thirds.

How many little rectangles are there after rotating the squares? _____

What fraction of the whole is the little rectangle? _____

What happens to the color at the intersection of the two squares?

The intersection of the colors shows 1/3 of 1/2. What fraction of the whole is in the intersection? _____

Now do several more rotations and record the number of rectangles.

Record here. Color with colored pencils.

strips___ x strips ___ = ___ little rectangles

Parts of Wholes: Review - Worksheet 2 Look for a pattern to solve these problems.

1/5 of 15 = 3	1/2 of 20 = 10
2/5 of 15 = 6	1/4 of 20 = 5
3/5 of 15 = 9	2/4 of 20 = 10
4/5 of 15 = 12	3/4 of 20 = 15
1/7 of 14 = 2	1/5 of 20 = 4
2/7 of 14 = 4	2/5 of 20 = 8
3/7 of 14 = 6	3/5 of 20 = 1/2
4/7 of 14 = 8	4/5 of 20 = 1/6
5/7 of 14 = IO	
6/7 of 14 = 12	

What patterns do you see? $\frac{2}{5} = 2x3 \text{ or } 6 \cdot \frac{3}{5} = 3x3 \text{ or } 9$ Once I find the size of the fractions nart I multiply it by how man fractional parts I have, the numerator,

Fractions as Parts of Wholes - Worksheet 2 In fractions the "of" is written with a X sign.

$$\frac{1}{3}$$
 of $9 = \frac{1}{3} \times 9$

Write the procedure with a number sentence.

$$\frac{2}{3} \stackrel{\text{of}}{\times} 9 = (9 \div 3) \times 2 = 6$$

$$\frac{1}{4} \stackrel{\text{of}}{\times} 8 = (\underline{8} \div \underline{4}) \times \underline{1} = \underline{2}$$

$$\frac{3}{4} \times 8 = (8 \div 4) \times 3 = 6$$

$$\frac{1}{5} \overset{\text{of}}{X} 25 = (25 \div 5) \times 1 = 5$$

$$\frac{3}{5} \times 25 = (25 \div 5) \times 3 = 15$$

$$\frac{5}{6} \times 24 = (24 \div 6) \times 5 = 20$$

Fractions as Parts of Wholes - Worksheet 1 Solve. Look for a pattern.

$$1/2 \text{ of } 48 = 24$$
 $1/2 \text{ of } 64 = 32$
 $1/3 \text{ of } 54 = 18$
 $1/4 \text{ of } 64 = 16$
 $2/3 \text{ of } 54 = 36$
 $3/4 \text{ of } 64 = 48$
 $1/6 \text{ of } 42 = 7$
 $1/5 \text{ of } 40 = 8$
 $5/6 \text{ of } 42 = 35$
 $3/5 \text{ of } 40 = 24$
 $1/9 \text{ of } 72 = 8$
 $1/20 \text{ of } 80 = 4$
 $1/3 \text{ of } 36 = 2$
 $1/3 \text{ of } 36 = 2$

Write out a procedure for calculating trhe answer to $\frac{4}{5}$ of 20. Divide the denominator of the fraction into the whole number, then multiply the numerator by that quotient. Or multiply the numerator by the whole number, then divide by the denominator.

6

5

Fractions as Parts of Wholes - Worksheet 3

$$\frac{1}{2}$$
 X $\frac{1}{2}$ can also mean $\frac{1}{2}$ added $\frac{1}{2}$ times. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$. This is called iterative addition.

Write these problems as iterative addition problems and solve.

$$\frac{1}{4} \times 2 = \frac{1}{2} \qquad \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{4} \times 4 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{1}{6} \times 5 = \frac{5}{6} \qquad \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}$$

$$\frac{2}{3} \times 4 = \frac{22}{3} \qquad \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2 \qquad \frac{2}{3}$$

$$\frac{3}{4} \times 6 = \frac{11}{2} \qquad \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{14}{18} = 4 \qquad \frac{2}{4} \qquad \text{or} \qquad 4 \qquad \frac{1}{2}$$

$$\frac{5}{6} \times 3 = \frac{2}{2} \qquad \frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6} \text{ or} \qquad 2 \qquad \frac{3}{6} \text{ or} \qquad 2 \qquad \frac{1}{2}$$

Parts of Wholes Meet Iterative Addition- Worsheet 1

1. What does iterative mean? To repeat.

2. This is addition:

This is iterative addition,

3+3+3+3=12also known as multiplication.

$$3 \times 4 = 12$$

 $\frac{2}{3}$ of 12 can be done as a part of a whole.







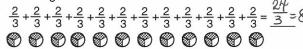




Shade in $\frac{2}{3}$ of the 12 circles.

 $\frac{2}{3}$ x 12 = $\frac{9}{2}$ You did these in Booklet 2.

 $\frac{2}{3}$ x 12 can mean two thirds added twelve times.



Do you think iterative addition (adding $\frac{2}{3}$ twelve times) will give you the same or a different answer as taking two thirds of 12? .__Same_

Try it. How many thirds do you get if you add $\frac{2}{3}$ twelve times? $\frac{3}{3}$

$$\frac{2}{3} \times 12 = (12 \times 2) \div 3 = 24 \div 3 = 8$$

Now, simplify the answer. 8

Parts of Wholes Meet Iterative Addition- Worsheet 2

 ${\textstyle\frac{2}{3}}\stackrel{\text{of}}{\times}9$ can also mean ${\textstyle\frac{2}{3}}$ added 9 times. This is called iterative addition.

$$\frac{2}{3} \times 9 = \frac{2}{3} + \frac{2}{3} = 6$$

$$\frac{2}{3} \times 9 = (9 \times 2) \div 3 = 18 \div 3 = 6$$

Write as an iterative addition problem.

Write the procedure with a number sentence

$$\frac{1}{4} \times 8 = \frac{1}{4} + \frac{1}{4} = \frac{8}{4} = 2$$

$$\frac{1}{4} \stackrel{\text{of}}{\times} 8 = (\cancel{2} \times \cancel{1}) \div \cancel{4} = \cancel{2} \div \cancel{4} = \cancel{2}$$

$$\frac{3}{4} \times 8 = \frac{3}{4} + \frac{3}{4} = \frac{24}{4} = 6$$

$$\frac{3}{4}$$
 $\stackrel{\text{of}}{\times}$ $8 = (8 \times 3) \div 4 = 24 \div 4 = 4$

$$\begin{array}{l} \frac{1}{5} \stackrel{\text{of}}{\times} 15 = \frac{1}{5} + \frac{1}$$

$$\frac{1}{5} \stackrel{\text{of}}{\times} 15 = (15 \times 1) \div 5 = 15 \div 5 = 3$$

$$\begin{array}{l} \frac{3}{5} \stackrel{\text{of}}{\times} 15 = \frac{3}{5} + \frac{3}$$

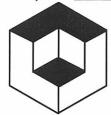
$$\frac{3}{5} \stackrel{\text{of}}{X} 15 = (\cancel{5} \times \cancel{3}) \div 5 = \cancel{45} \div 5 = 9$$

10

9

Parts of Wholes Meet Iterative Addition- Worsheet 3

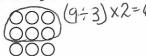
Look at this figure. What do you see?



Some people see a big big cube in back and a small cube floating in front. Other people see a big cube with the corner taken out. Can you see both? __yes_ Can you see both at the same time? __no_

Notice, your mind has to "pop" between the two images

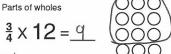
$$\frac{2}{3} \times 9 = 6$$
 Parts of wholes



$$\frac{2}{3} \times 9 = \frac{\ell_{\rho}}{4} \quad \text{Herative} \quad \frac{2}{3} + \frac{2}{3} = \frac{18}{3} = \ell_{\rho}$$

These two ways of solving the same problem are just like the cube above. They fade and feel different but produce the same answer.

Try another one.



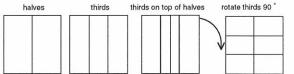
and Iterative

 $\frac{3}{4} + \frac{3}{4} = \frac{\frac{36}{4}}{11} = 9$

Multiplication - Worksheet 1

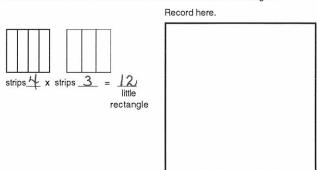
Use clear Prism FractionsAcetate squares to find the solution to these problems.

Start with the square divided in two parts and place the square divided into three parts on top of it. Rotate one of the squares 90°. ec.



This shows halves divided into thirds. How many little rectangles are there after rotating the squares? 🗘 What fraction of the whole is the little rectangle?

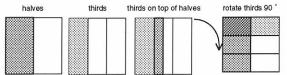
Now do several more rotations and record the number of rectangles.



Multiplication - Worksheet 2

Use **colored** Prism Fractions Acetate squares to find the solution to these problems.

Start with the square divided in two parts and place the square divided into three parts on top of it. Rotate one of the squares 90*



This shows halves divided into thirds.

How many little rectangles are there after rotating the squares? What fraction of the whole is the little rectangle?

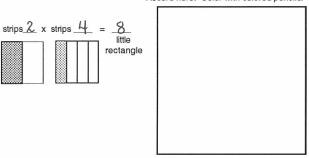
What happens to the color at the intersection of the two squares?

Changes Color

The intersection of the colors show 1/3 of 1/2. What fraction of the whole is in the intersection? $\frac{1}{2}$

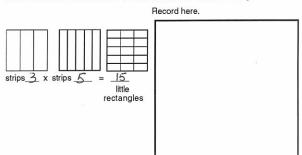
Now do several more rotations and record the number of rectangles.

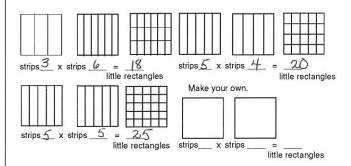
Record here. Color with colored pencils.



Multiplication - Worksheet 3

Use clear Prism Factions Acetate squares to find the solution to these problems. Do several more rotations and record the number of little rectangles.



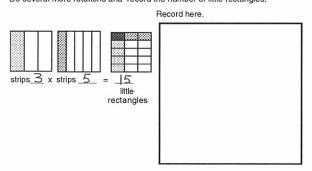


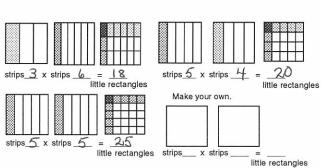
13

14

Multiplication - Worksheet 4

Use colored Prism Factions Acetate squares to find the solution to these problems. Do several more rotations and record the number of little rectangles.

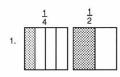




Multiplication - Worksheet 5

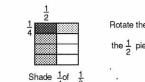
Prism Fractions Acetate colored squares must be used to see the crosses.

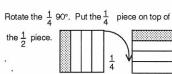
Learn to draw these. $\frac{1}{4} \times \frac{0}{2} =$

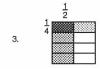


Start with the second number, the $\frac{1}{2}$.

A chunk is going to be taken out of this $\frac{1}{2}$.







Look at the overlap where the colors from

 $\frac{1}{2}$ and the $\frac{1}{4}$ are both in the little rectangle. This is called "cross shading".

 $\frac{1}{8}$ of the whole is cross shaded.

$$\frac{1}{4}$$
 of $\frac{1}{2} = \frac{1}{8}$ $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$