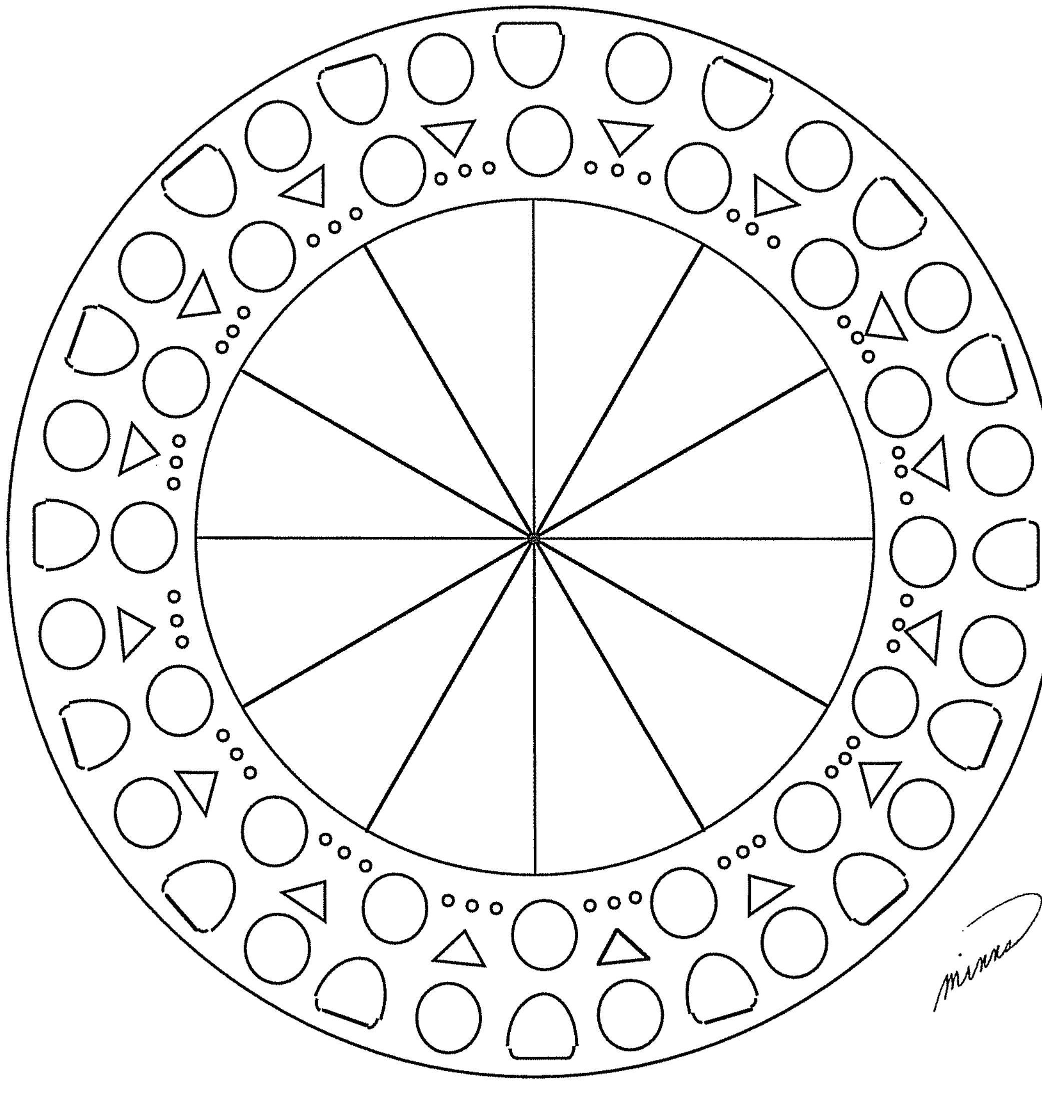
Patterns in Arithmetic

Fractions - Booklet 4 PDF

Equivalent Fractions

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 4 PDF - Equivalent Fractions

Contents

This booklet is dedicated to Jo Service, Michael's beloved wife. Her contribution to the Farm School is beyond measure in her support and dedication. Thank you for all of your valuable advice for many years.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 4 PDF
Parent/Teacher Guide

Published by Pattern Press
P.O. Box 2737
Fallbrook, CA 92088
(760)728-3731

www.patternpress.com
E-mail: Patternpress1@gmail.com

Printed on recycled paper.

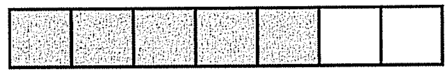
ISBN 978-1-935559-63-4

Pre-Assessment - Part 1 - Worksheet 1 Date ________

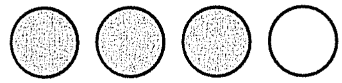
Put a question mark next to any problem you do not know how to do.
The student may use a manipulative.

1. Shade in $\frac{2}{3}$ of this rectangle.

2. What fraction of this rectangle is shaded? ______



3. ____ out of ____ of the circles are shaded.



4. Write the name of this fraction. $\frac{2}{5}$ ______________________________

Use fraction pieces or fraction circles. Fill in the missing numbers.

5. a. $\frac{1}{2} = \frac{\quad}{4}$ b. $\frac{1}{3} = \frac{\quad}{12}$ c. $\frac{3}{4} = \frac{\quad}{8}$

6. a. $\frac{4}{6} = \text{—}$ b. $\frac{9}{12} = \text{—}$ c. $\frac{8}{10} = \text{—}$

7. Put these fractions in order from the least to the greatest.

$\frac{7}{8}$ $\frac{1}{2}$ $\frac{2}{6}$ $\frac{3}{5}$ $\frac{3}{4}$

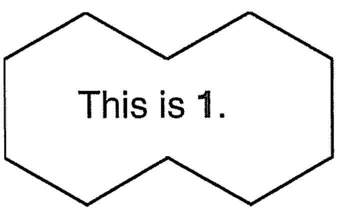
_____ _____ _____ _____ _____

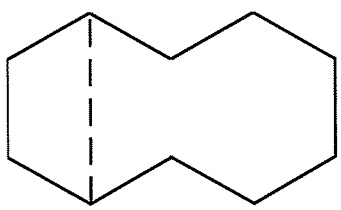
8. Fill in this sequence. $\frac{1}{3} = \frac{\quad}{6} = \frac{3}{\quad} = \text{—} = \text{—}$

Use pattern blocks for the ones you can't do with fraction pieces.

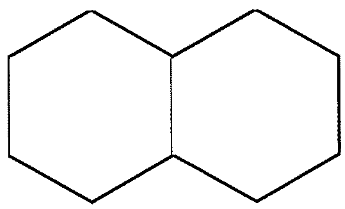
---

9. Use pattern blocks.



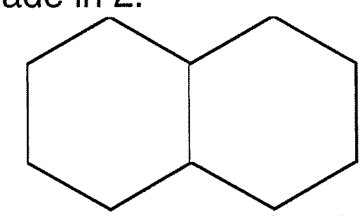


Draw in fourths, shade in 2.



Draw in sixths.

Use your drawings to find the equivalent fraction.



$\frac{2}{4} = \frac{\quad}{6}$

Draw the equivalent fraction here.

10. If the red block is equal to 1, what is the green block equal to? _____
What is the blue block equal to? _____
What is the yellow block equal to? _____

Pre-Assessment - Part 1 - Worksheet 2 Date ___________

Do not use fraction pieces or pattern blocks for this set of problems.

11. Fill in this sequence. $\frac{2}{3} = \frac{\quad}{6} = \frac{6}{\quad} = \frac{\quad}{12}$

12. If 25¢ is a third of what you need to buy a ribbon, how much does the ribbon cost? ____________

13. Solve.

a. $\frac{1}{2} + \frac{1}{2} =$ b. $\frac{3}{5} + \frac{1}{5} =$ c. $\frac{6}{10} + \frac{2}{10} =$

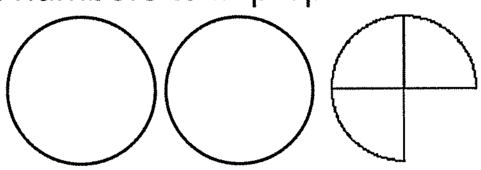
d. $\frac{7}{8} - \frac{1}{8} =$ e. $\frac{2}{3} - \frac{1}{3} =$ f. $\frac{5}{5} - \frac{1}{5} =$

Do not use fraction pieces or pattern blocks for this set of problems.

14. Write an improper fraction. ________________

15. Change these mixed numbers to improper fractions.

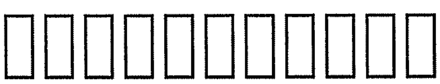
a. $2\frac{3}{4} = \text{—}$



b. $4\frac{1}{6} = \text{—}$

16. Change these improper fractions to mixed numbers.

a. $\frac{11}{3} =$ b. $\frac{8}{5} =$ c. $\frac{10}{4} =$



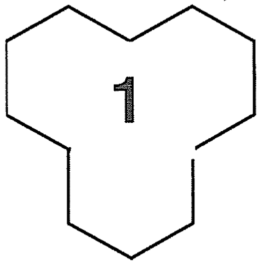
17. If you couldn't do problems 15 and 16 without blocks or pictures, can you do them with the blocks? _____ If yes, do them; if no, put a question mark.

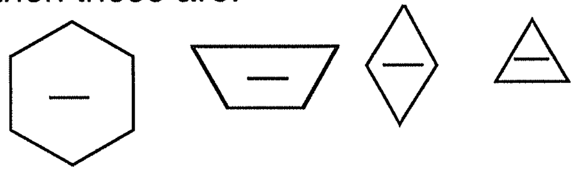
18. Put $1\frac{3}{4}$ on this number line.

19. Put $\frac{5}{3}$ on this number line.

Pre-Assessment - Part 2 Date ____________

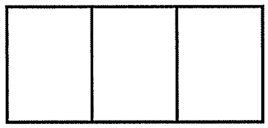
1. Identify each block. If this is 1, then these are:





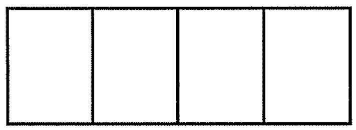
Draw it.

2. Change $\frac{2}{3}$ into twelfths. a.



b. $\frac{2}{3} = \frac{\_\_}{12}$

Change $\frac{3}{4}$ into sixteenths. c.



d. $\frac{3}{4} = \frac{\_\_}{16}$

3. Fill in the missing numbers.

a. b. $\frac{2}{3} \frac{\times}{\times} \_\_ = \frac{\_\_}{9}$

c. d. $\frac{4}{5} \frac{\times}{\times} \_\_ = \frac{8}{\_\_}$

e. f. $\frac{5}{8} \frac{\times}{\times} \_\_ = \frac{15}{\_\_}$

g. How do you know what this number is? ______________________________

____

4. Fill in the missing numbers.

a. b. $\frac{3 \times}{4 \times} \frac{\_\_}{\_\_} = \frac{\_\_}{12}$

c. d. $\frac{3 \times}{5 \times} \frac{\_\_}{\_\_} = \frac{\_\_}{20}$

5. a. Explain what the "Mighty One" is. ______________________________

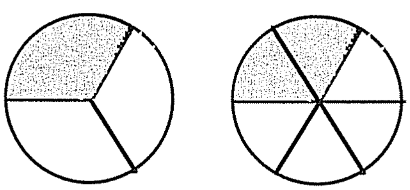
____

b. Explain how the "Mighty One" is used to calculate equivalent fractions.

____

____

6.



$\frac{1}{3} = \frac{2}{6}$

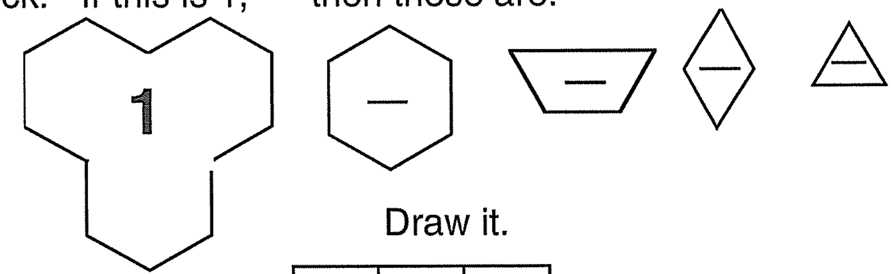
Explain why multiplying $\frac{1}{3}$ by $\frac{2}{2}$ changes it to two sixths. ______________________________

____

____

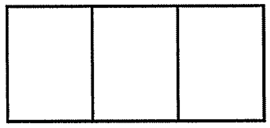
Post-Assessment Date ____________

1. Identify each block. If this is 1, then these are:



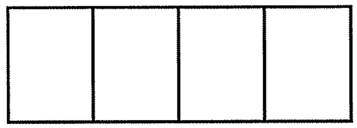
Draw it.

2. Change $\frac{2}{3}$ into twelfths. a.



b. $\frac{2}{3} = \frac{}{12}$

Change $\frac{3}{4}$ into sixteenths. c.



d. $\frac{3}{4} = \frac{}{16}$

3. Fill in the missing numbers.

a. b. $\frac{2}{3}\frac{\times}{\times}$ ___ $=$ $\frac{}{9}$

c. d. $\frac{4}{5}\frac{\times}{\times}$ ___ $=$ $\frac{8}{}$

e. f. $\frac{5}{8}\frac{\times}{\times}$ ___ $=$ $\frac{15}{}$

g. How do you know what this number is? ______________________________

____

4. Fill in the missing numbers.

a. b. $\frac{3 \times}{4 \times}$ ___ $= \frac{}{= 12}$

c. d. $\frac{3 \times}{5 \times}$ ___ $= \frac{}{= 20}$

5. a. Explain what the "Mighty One" is. ______________________________

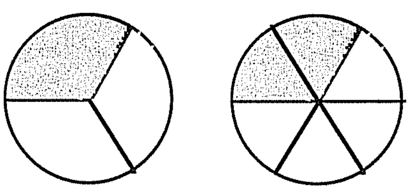
____

b. Explain how the "Mighty One" is used to calculate equivalent fractions.

____

____

6.



$\frac{1}{3} = \frac{2}{6}$

Explain why multiplying $\frac{1}{3}$ by $\frac{2}{2}$ changes it to two sixths. ______________________________

____

____

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered at the end to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Fractions: Booklet 2 - Beginning Operations With Fractions, Fractions: Booklet 3 - Mixed Numbers and Improper Fractions

Materials

Fractions: Booklet 4 - Pre-Assessment: Part 1 - Worksheets 1 and 2, pages 1 and 2, Pre-Assessment: Part 2, page 3
Fractions: Booklet 4 - Post-Assessment, page 39 and page 4 in this booklet
Score Sheets: Pre-Assessment: Part 1, pages 11 and 12; Pre-Assessment: Part 2, page 13; and Post-Assessment, page 14 in this booklet
Prism Fractions Circles
Pattern blocks

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If the student is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness Items, then give Pre-Assessment: Part 2.

Note

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

This particular assessment assumes the student is beginning fractions instruction in Grade 5 after many months of having not worked on fractions. Pre-Assessment: Part 1 serves as a warm-up as well as a placement assessment. If you have just completed Fractions: Booklet 3, you can skip Pre-Assessment: Part 1 and move directly to Pre-Assessment: Part 2.

Important

If he is new to this program and has never used fraction circles or pattern blocks, give him time to explore these manipulatives before trying to use them in the assessment. After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. Numerators Greater Than One (Fractions: Booklet 1)

A. divide the rectangle into three roughly equal sections and shade in two sections to indicate that he knows what $\frac{2}{3}$ is?

2. Numerators Greater Than One (Fractions: Booklet 1)

A. write the correct denominator of the fraction to show that he knows to count the total number of squares to obtain this number?

B. write the correct numerator of the fraction to show that he knows to count the shaded squares to obtain this number?

3. Numerators Greater Than One (Fractions: Booklet 1)

A. recognize the language of fractions '3 out of 4 parts' are shaded?

4. My Fractions Book (Fractions: Booklet 1)

A. write the correct name of a given fraction with no picture given?

5. My Fractions: Equivalence (Fractions: Booklets 1 and 2)

A. select the correct fraction pieces for each problem?

B. fill in two of the three missing numbers correctly?

We want to see if he can select the correct fractional units, halves and fourths, and use the correct number of fourths to cover the half.

If he correctly fills in the missing numbers without using the fraction pieces, check to see if he can prove his answers are correct by using the pieces. If he can not, he may have memorized a procedure he does not understand.

6. My Fractions Book: Equivalence (Fractions: Booklet 1)

A. select the correct fraction pieces to use in the problems?

B. find and record a second fraction that covers the same area as the given fraction?

Does he know that two equal fractions cover the same area, or the same amount of space? Students who are taught the rote procedure for 'reducing fractions' often do not know this.

7. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)

A. use the pieces to put the fractions in order from least to greatest?

B. write the fractions in the correct sequence?

8. Equivalence: Recording (Fractions: Booklet 1)

A. use the manipulative to find equal fractions and fill in the missing numerators for sixths and twelfths?

B. use the number pattern to fill in the ones he does not have pieces for?

9. Equivalence: Manipulative (Fractions: Booklet 1)

A. draw in fourths after having been given the definition of the whole?

B. draw in sixths after having been given the definition of the whole?

C. find the numerator of the equivalent fraction?

Getting a Yes on Item C and not on Items A or B indicates a possible memorized procedure.

10. Changing Wholes (Fractions: Booklet 2)

A. figure out the fractional value of two smaller blocks when given a different definition of the whole than was given on the previous problem?

B. label the yellow block as having a value of two if the red block is one?

In Items 9 and 10, the definition of the whole has been changed three times using the same manipulative, the pattern blocks. Does he understand that the fractional value of a block is relative to the definition of what the whole is? In fractions, the whole can be defined as anything you like: a yellow block or a hamburger.

11. Equivalence: Recording (without a manipulative) (Fractions: Booklet 1)

A. use the multiplication number pattern to fill in the missing numbers?

12. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)

A. find the correct answer to a basic fraction word problem?

13. Addition and Subtraction of Like Fractions (Fractions: Booklet 2)

A. give the correct answer to two of three addition of fractions problems?

B. give the correct answer to two of three subtraction of fractions problems?

14. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)

A. write an improper fraction?

15. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)

A. convert a mixed number to an improper fraction in problem a, which gives a picture assist?

B. convert a mixed number to an improper fraction with no manipulative or picture?

16. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)

A. convert an improper fraction to a mixed number in problem a, which gives a picture assist?

B. convert an improper fraction to a mixed number in both problems b and c?

17. Manipulative

A. complete Items 15 and 16 with a manipulative if he was unable to do it without a manipulative?

18. Number Lines (Fractions: Booklet 3)

A. locate a mixed number on a number line?

19. Number Lines (Fractions: Booklet 3)
    A. locate an improper fraction on a number line?

Examine the student work while reading the criteria listed above. Use the score sheet to answer all the questions Yes or No. Evaluate those results using the next section.

Booklet Selection is based on results of scoring on Pre-Assessment: Part 1
Students six - nine years old
If Items 1 - 6 are marked No on the score sheet in 6 or more criteria, begin with Fractions: Booklet 1. Do not give Part 2 of this assessment.
If Items 1 - 6 are Yes, but Items 7 - 13 have 8 or more criteria marked No, begin with Fractions: Booklet 2. Do not give Part 2 of this assessment.
If Items 1 - 13 are Yes, but Items 14, 15B, and 16B have a No, begin with Fractions: Booklet 3.

Students ten and up
If Items 1 - 8 are marked No on the score sheet on more than 8 of the criteria, begin with Fractions: Booklet 1 with the 'My Fraction Booklet' in the middle of the book. Some ten-year-olds will consider it 'baby stuff' and resist. Try it and see what happens. If the student complains too much or if he is in a class, after having him make a chart of all the fraction pieces and their values and giving free exploration time with the manipulatives, start with Fractions: Booklet 2.
If Items 1 - 13 are Yes, but Items 14, 15B, 16B have a No, begin with Fractions: Booklet 3. Fractions: Booklet 3 can be taught concurrently with Fractions: Booklet 4.
If, on all of Part 1, he has scored 20 or more, he is ready for Fractions: Booklet 4 - Equivalent Fractions.

All areas of weakness should be remediated as instruction on this book moves forward. Either redo sections of previous books or use other resources in the Fractions Tool Chest. Students new to this program may be able to calculate answers but may not understand what they are doing. This will be revealed by the inability to draw or prove answers, or with an answer of "I don't know, that's how I was taught."

It is not expected that students know this material on Pre-Assessment: Part 2. He may use pattern blocks for Items 1, 2a, and 2b.

Assessment Criteria for Pre-Assessment: Part 2
Can the student:
1. Changing Wholes Meets Manipulative Equivalence
    A. correctly identify three out of four of the fractional units of the smaller blocks once the whole has been identified as three yellow blocks?

2. Representational Equivalence: Cutting Up Fractions
    A. draw how thirds change to twelfths?
    B. give the correct numerator to form an equivalent fraction?
    C. draw how fourths change to sixteenths?
    D. give the correct numerator to form an equivalent fraction?

The drawing shows a graphic representation of the multiplier. To change thirds to twelfths, each third must be cut up into four smaller sections. The multiplier then is four.

3. Equivalence: Calculating
   A. show the correct multiplier in two of three problems? See problems a, c, and e.
   B. supply the correct missing number to form equivalent fractions in two of three problems? See problems b, d, and f.
   C. explain in words how he knows what the multiplier is?
   One Point - Divide the three into the nine.
   Two points - Find the relationship between the two denominators and use that same number in the numerator.

4. Equivalence: Calculating
   A. place the correct multiplier in the dotted 1 in problems a and c?
   B. supply the correct numerators to create equivalent fractions b and d?

5. Equivalence: Calculating
   A. explain what the Mighty One is?
      One point: You have to do the same on the top and bottom.
      Two points: It is the multiplier over the multiplier.
   B. explain how the Mighty One is used to calculate equivalent fractions?
      One point: You have to multiply by one to get an equivalent number.
      Two points: Multiplying by one changes the form of the fraction, or changes the denominator but keeps the value of the two fractions the same.

6. Equivalence: Calculating
   A. explain where the multiplier comes from physically?
      One point: Because three times two is six.
      Two points: There are two-sixths for every one-third. When multiplying by 2 over 2, the area is kept the same, but it is changed from one large piece to two smaller pieces.

Booklet Selection after Pre-Assessment: Part 2

If a student scores 13 or more on this part of the assessment, this whole booklet is not needed. Remediate weak areas and move on. If the weak areas are in 2A or C, 3C, 5, and 6, do the section on Representational Equivalence and Calculating Equivalence to strengthen understanding. Then move on to Fractions: Booklet 5 on Simplification.

Post-Assessment/Pre-Assessment Comparisons

Hopefully, you and the student will see a significant increase in the score on the Post-Assessment over that of the Pre-Assessment. Be sure to share this result with your student.
A score of 16 or more on the Post-Assessment is excellent.
A score of 14 (82%) or 15 (88%) on the Post-Assessment is very good.
A score of 12 points is 70%, a passing score, with some remediation and retesting needed. If the weak areas are in 2C, 5, and 6, work on the Math Journal to improve his ability to explain mathematical concepts.

If the Post-Assessment score is less than 10, remediate weak areas and retest. Some students are unable to clearly explain their thinking process at this time. If he scores Yes on all criteria for Items 1, 2A, 2B, 3, and 4, you can move on and work on the Math Journal to improve his ability to explain mathematical concepts in words.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/repre sentation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm-up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Pre-Assessment: Part 1 Score Sheet Name ________________ Date ______________

Make copies of this sheet before you score.

Can the student:

1. Numerators Greater Than One (Fractions: Booklet 1)
   Yes No A. divide the rectangle into three roughly equal sections and shade in two sections?

2. Numerators Greater Than One (Fractions: Booklet 1)
   Yes No A. write the correct numerator of the fraction?
   Yes No B. write the correct denominator of the fraction?

3. Numerators Greater Than One (Fractions: Booklet 1)
   Yes No A. recognize the language of fractions '3 out of 4 parts'?

4. My Fractions Book (Fractions: Booklet 1)
   Yes No A. write the correct name of a given fraction with no picture?

5. My Fractions: Equivalence (Fractions: Booklets 1 and 2)
   Yes No A. select the correct fraction pieces for each problem?
   Yes No B. fill in two of the three missing numbers correctly?

6. My Fractions Book: Equivalence (Fractions: Booklet 1)
   Yes No A. select the correct fraction pieces?
   Yes No B. find and record a second fraction equal to the first?

7. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)
   Yes No A. use pieces to put the fractions in order from least to greatest?
   Yes No B. write the fractions in the correct sequence?

8. Equivalence: Recording (Fractions: Booklet 1)
   Yes No A. use the manipulative to find equal fractions and fill in the missing numerators for sixths and twelfths?
   Yes No B. use the number pattern to fill in the fractions he does not have pieces for?

9. Equivalence: Manipulative (Fractions: Booklet 1)
   Yes No A. draw in fourths after being given the definition of the whole?
   Yes No B. draw in sixths after being given the definition of the whole?
   Yes No C. find the missing numerator of the equivalent fraction?

10. Changing Wholes (Fractions: Booklet 2)
    Yes No A. figure out the fractional value of two smaller blocks when given a different definition of the whole than was given on the previous problem?
    Yes No B. label the yellow block as having a value of 2?

11. Equivalence: Recording (without a manipulative) (Fractions: Booklet 1)
    Yes No A. use the multiplication number pattern to fill in the missing numbers?

12. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)
Yes No A. find the correct answer to a basic fraction word problem?

13. Addition and Subtraction of Like Fractions (Fractions: Booklet 2)
Yes No A. give the correct answer to two of three addition of fractions problems?
Yes No B. give the correct answer to two of three subtraction of fractions problems?

14. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
Yes No A. write an improper fraction?

15. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
Yes No A. convert a mixed number to an improper fraction in problem a, which gives a picture assist?
Yes No B. convert a mixed number to an improper fraction with no manipulative or picture?

16. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
Yes No A. convert an improper fraction to a mixed number in problem a, which gives a picture assist?
Yes No B. convert an improper fraction to a mixed number in both problems b and c?

17. Manipulative
NA* Yes No A. complete Items 15 and 16 with a manipulative if he was unable to do it without a manipulative? *Score NA if Items 15 and 16 were a Yes.

18. Number Lines (Fractions: Booklet 3)
Yes No A. locate a mixed number on a number line?

19. Number Lines (Fractions: Booklet 3)
Yes No A. locate an improper fraction on a number line?

End of Pre-Assessment: Part 1
29 points possible Each Yes counts as 1 point

Items Correct = _______ = ______%
Items Possible = 29

Notes:
Place in Booklet _____. Continue to Pre-Assessment: Part 2 Yes No

Pre-Assessment: Part 2 Score Sheet Name _________ Date _________

Student may use pattern blocks for Items 1, 2a, and 2b.

Can the student:

1. Changing Wholes Meets Manipulative Equivalence

   Yes No A. identify correctly three out of four of the fractional units of the smaller blocks?

2. Representational Equivalence: Cutting Up Fractions

   Yes No A. draw how thirds change to twelfths?

   Yes No B. give the correct numerator to form an equivalent fraction?

   Yes No C. draw how fourths change to sixteenths?

   Yes No D. give the correct numerator to form an equivalent fraction?

3. Equivalence: Calculating

   Yes No A. show the correct multiplier in two of three problems?

   Yes No B. supply the correct missing number in two of three problems?

   0 1 2 C. explain in words how he knows what the multiplier is?

4. Equivalence: Calculating

   Yes No A. place the correct multiplier in the dotted one in both problems?

   Yes No B. supply the correct numerators to create equivalent fractions?

5. Equivalence: Calculating

   0 1 2 A. explain what the Mighty One is?

   0 1 2 B. explain how the Mighty One is used to calculate equivalent fractions?

6. Equivalence: Calculating

   0 1 2 A. explain where the multiplier comes from physically?

End of Part 2 Each Yes counts as 1 point; short answer scores are 0, 1, or 2 points

17 points possible Score $\frac{\_\_\_\_\_\_}{17}$ = _______ %

Notes:

Post-Assessment Score Sheet Name _________ Date ________

Student may use pattern blocks for Items 1, 2a, and 2b.

Can the student:

1. Changing Wholes Meets Manipulative Equivalence

Yes No A. identify correctly three out of four of the fractional units of the smaller blocks?

2. Representational Equivalence: Cutting Up Fractions

Yes No A. draw how thirds change to twelfths?

Yes No B. give the correct numerator to form an equivalent fraction?

Yes No C. draw how fourths change to sixteenths?

Yes No D. give the correct numerator to form an equivalent fraction?

3. Equivalence: Calculating

Yes No A. show the correct multiplier in two of three problems?

Yes No B. supply the correct missing number in two of three problems?

0 1 2 C. explain in words how he knows what the multiplier is?

4. Equivalence: Calculating

Yes No A. place the correct multiplier in the dotted one in both problems?

Yes No B. supply the correct numerators to create equivalent fractions?

5. Equivalence: Calculating

0 1 2 A. explain what the Mighty One is?

0 1 2 B. explain how the Mighty One is used to calculate equivalent fractions?

6. Equivalence: Calculating

0 1 2 A. explain where the multiplier comes from physically?

End of Part 2 Each Yes counts as 1 point; short answer scores are 0, 1, or 2 points

17 points possible Score ______ = _______ %

17

Notes:

Introduction to the Concept of Equivalent Fractions

This booklet has students playing with blocks, cutting paper, and drawing pictures and will take two to three weeks to complete. Why bother with all this cutting paper and drawing pictures and working with blocks to teach equivalent fractions? It takes so much time. The pattern is so easy to teach by rote application of the multiplication tables. Why not do it in a day or two and get it over with? Because.

.Understanding the concept of equivalent fractions contributes a major piece of understanding that underlies a large number of mathematical procedures. There are four major concepts embedded in the calculation of equivalent fractions: the **multiplier** used to create sets of equivalent fractions, the **Identity Property of multiplication (also called the Mighty One)** behind the selection of the number used as the multiplier, the use of **proportions** to solve word problems, and the **inverse relationship** of the parts of the equivalent fractions. The memorized procedure we all learned makes only the first one in the list, the multiplier, obvious.

Seems So Easy

The 'way' to get an equivalent fraction, we were taught in school, is to multiply the numerator (the top number) and denominator (the bottom number) by the same number and you will get an equivalent fraction.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

Solve this problem: $\frac{1}{2} = \frac{}{6}$

You are asked to find out how many sixths are equal to one-half. You are told to divide the first denominator (the two) into the second (the six). $6 \div 2 = 3$ Then, take the three and multiply it by the numerator in the first fraction (the 1). $3 \times 1 = 3$. The numerator of the new fraction is three. Seems so easy.

$$\frac{1}{2} = \frac{3}{6}$$

Most of us memorized this procedure with few problems. Why this works and the mathematical concepts behind it are not at all obvious to students. The loss of the why of this simple little procedure is one of the reasons students do not do well in algebra!

If you yourself do not understand all four of these major concepts, you will learn them by teaching this material to your student using manipulatives and patterns. You will also increase your confidence in your math and have fun too. What a bargain!

The four concepts listed in the opening paragraph of this article are fundamental to all operations with fractions, ratios, proportions, and percents and are key strategies for working with equations in algebra and formulas in science classes.

Because the operation of creating an equivalent fraction is so fundamental, yet comprehensive mastery so elusive for most students, many hands-on experiences are needed to construct understanding. Do not skip the construction of understanding at this early stage to save time. It will cost you much later on. In our experience, most students who do not do well in algebra can trace much of their difficulty to not understanding equivalent fractions and the tools used to create and operate with them.

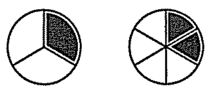
A stitch in time saves nine, and understanding these four concepts may be the difference between passing algebra and not passing.

You can read on to see why, or you can take my word for it and begin the work in the first lesson. Whichever you do, resist the temptation to show your student the easy pattern to produce an answer. Look for the patterns yourself while you are at it.

What is hidden in the simple calculation of an equivalent fraction is:

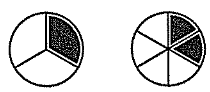
1. The Multiplier

You use this concept when you change inches to feet. In this case the multiplier is twelve because it takes twelve inches to make one foot. How many inches would you need for two feet? You would multiply 2 times 12 inches to get 24 inches. All measurement units use this concept of the multiplier to make conversions from one unit to another. You use it every day when you make change with your money or when you see an ad that has percent in it. We use this idea to find relationships between numbers. Those relationships are called ratios. Try this: $\frac{1}{3} = \frac{\quad}{6}$ Now look at the graphic.



You divide the six by three to get 2. You multiply the 1 by that 2 to get two sixths. But where is the two that you multiplied by in the picture? Where is the multiplied 2? Where does it come from physically?

Where is the multiplier two in the picture?



The two is in the relationship between thirds and sixths. It takes two sixths to make one-third. The ratio between sixths and thirds is 2 to 1. This is because the sixths are only half the size of the third, so it takes two sixths to cover the same area as one-third.

A child cutting up paper fractions will see this ratio right away. If you have one-third, you need to cut it in half to get two sixths. Here is a strange complexity you will not notice just working with the abstract numbers. To change paper thirds into sixths you cut each third in half. Cutting something in half is division. Right? So why do you end up **multiplying** the three **by two** when calculating the equivalent fraction, but dividing by two when you create the paper sixths from the paper thirds with your scissors? Why this paradox? What do you need to know about fractions to get your head around this multiplication division weirdness?

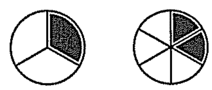
Notice that while the third was divided into two pieces with scissors, the number of pieces increased by two, or was multiplied. Students who simply memorize the recipe never even notice this. They see only abstract numbers or the multiplication tables, but not the relative sizes of the pieces or the relationships between them. If you ask them about the paradox above, they are likely to not even understand the question. A student with paper fractions and a pair of scissors can work out that you multiply the three by two because it takes twice as many of those smaller pieces to equal the area of the larger piece, and the student can prove it to you. You are multiplying the number of pieces by a factor of two. This knowing is the fundamental root of ratio and proportion, a major item in mathematics.

(I, Alysia, failed eighth grade math, and hence algebra, and did not get to go to medical school because I could not understand what ratio and proportion are, let alone how to use them as a tool. A manipulative and some time to construct understanding would have changed the path of my life. One of my missions as a math teacher is to make sure others do not needlessly suffer the same fate.)

The multiplier also controls what set of fractions are equal to each other. Because you are cutting all the larger pieces in the whole the same way, you will get denominators that are multiples of the original denominator. Fourths can be cut into eighths, twelfths, or sixteenths, but not into tenths. Students who are cutting paper discover this pattern on their own. The relationship between the multiplier and the denominator of the fraction controls many operations with fractions. Since fractions are the way we divide in algebra, it is a major controller of algebraic operations as well.

2. The Identity Property of Multiplication (The Mighty One)

Using the same example of changing thirds into sixths, we can see the action of another major tool in algebra. You can see the multiplier is two.



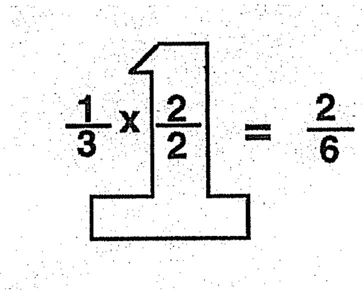
$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

Please note that the multiplier looks and feels like two. Most of us were taught to multiply both top number and bottom number by two. Most students think they are multiplying by two. You are *not* multiplying by two, but in fact you are multiplying by two over two, $\frac{2}{2}$, which is equal to one. You are actually multiplying by one, not by two. When anything is multiplied by one, the value of the product, the answer, is the same as the number you started with. This is called the identity property of multiplication.

($4 \times 1 = 4 \quad 4 = 4$.) Try multiplying with any other number and you will not get an equality between the answer and the number you started with. ($4 \times 2 = 8 \quad 4 \neq 8$.)

$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

In his books, John Saxon showed it like this

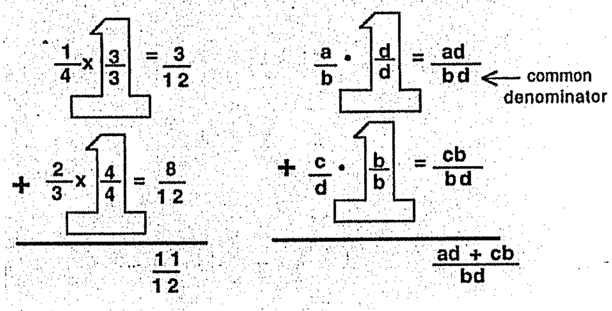


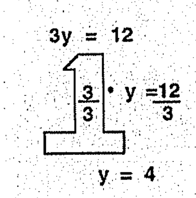
This technique of multiplying by one to change the form of a fraction, but not its value, is an *important* tool in algebra. Calculating equivalent fractions is the first time students overtly use the 'Mighty One' as a mathematical tool. It is very easy to teach this important concept at this time.

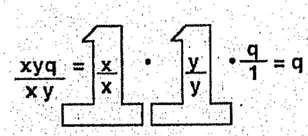
The Mighty One also helps us solve equations when there is a number attached to the variable, like 3y. Let's say we have the equation $3y = 12$. The way to solve this algebraically is to divide both sides of the equation by three. This is done, because on the left hand side of the equation, three divided by three is one. One times y is y. We want to have only one y on the left side of the equation and using the Mighty One is the way we peel off extra numbers from the variable, the y. On the right side of the equation, the twelve divided by three is four. So one y is equal to four.
This one is easy.

This one $\frac{2}{5}x = 395$ is not so easy. But it is easier if you know how to use the Mighty One.

Here is the Mighty One being used to add unlike fractions.







The Mighty One is also used to simplify algebraic fractions.

These are both simple procedures to do if you understand equivalent fractions, the multiplier and the Mighty One. These procedures, when shown with letters as is done in algebra, are very difficult and confusing if all the student knows is the memorized use of the multiplier.

3. Identity Property of Multiplication (Mighty One) and Proportions

A proportion is made of two equal ratios. Ratios look and act like fractions. We use ratios and proportions to solve all kinds of problems in science and engineering.

Multiplication by one is used to change the forms of numbers as you saw in the section on the Mighty One. The forms of numbers are changed to make problems easier or to get them to match the units we want to use. Ratios and proportions are used to do this. Here is an example: Let's say you ask twenty people what their favorite food is and nine say pizza. This data is usually reported as a percent. But the question was not asked to one hundred people. So how do we get a percent? How is that conversion done? You got it. With two equal ratios constructed with a multiplier and the Mighty One acting together. First, we show the data as a fraction or a ratio: nine out of twenty liked pizza. Then we use a multiplier to change the denominator to one hundred to get a percent. (Percent means per one hundred.) The multiplier is five because there are five one hundredths in each twentieth. But in order to have the percent number have the exact same value as the nine-twentieths, we must multiply by one. So we show the multiplier as five-fifths, which equals one.

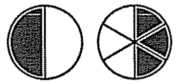
$\frac{9}{20} \times \frac{5}{5} = \frac{45}{100}$ or 45%.

So we can say that 45% of the people in our sample liked pizza best. (Many students and adults have difficulty with percent because they do not understand how to construct a proportion.)

4. Inverse Relationships

Fractions such as $\frac{1}{2}$ and $\frac{3}{6}$ are called equivalent fractions because even though they look different, they are equal in value. Equal, in terms of fractions, means that the two fractions cover exactly the same amount of area even though they have different numbers of pieces.

For example: one-half of a large pizza is the same amount of pizza as three-sixths of that same pizza.

It just looks different. 

Cutting the parts of the pizza into smaller pieces does not change the amount of area they cover. Try to convince a young child of this in the pizza parlor! She will pitch a fit when she gets only one large piece and her brother gets three small pieces. She thinks he has more because three is greater than one. Older children will make the same error with equivalent fractions. We have seen middle school students, with As in math, tell us that $\frac{4}{12}$ is larger than $\frac{1}{3}$ because both four and twelve are larger than one and three.

Many students do not understand that at the same time the whole is cut into smaller pieces, it is also cut into more pieces. You would think this would be obvious, but for most students it is not unless they are working with a manipulative. This kind of relationship is called an **inverse relationship**. Professor Michael Butler of the University of California, Irvine, Farm Elementary School taught us to see these relationships as "the less, the more" relationships.

The smaller the pieces, the more pieces needed to cover the same area. The less, the more.

The larger the pieces, the fewer pieces needed to cover the same area. The more, the less.

The smaller the denominator, the larger the pieces. The less, the more.

The larger the denominator, the smaller the pieces. The more, the less.

In division, if the size of the divisor goes up, the quotient goes down proportionally.

Example: $12 \div 3 = 4$ $12 \div 6 = 2$ If the size of the divisor is double, the size of the quotient is halved. This is known as an inverse proportion. If I spend more money, I have less in the bank. Savings is inversely proportional to spending. Inverse relationships are very tricky to understand and will, if not solidly mastered, play havoc with anyone's ability to grasp some very basic concepts in both physical science and algebra. Inverse relationships are what cause negative slope in linear equations. A famous example of an inverse proportion is Isaac Newton's Law of Gravitation. The attractive gravitational force between two objects is proportional to the masses of two objects and **inversely** proportional to the square of the distance between them. What that means is that this formula has a fraction in it. The inverse part means that as the distance between the objects goes up, the strength of the gravitational force between them goes down. The first and best place to tackle this concept is with equivalent fractions.

Many people will not understand this inverse geometric relationship that equivalent fractions present, or even think of it, unless they have a physical model of some kind. Most mathematics books gloss over the physical experience of the inverse quality of fractions, thinking that the memorized multiplication procedure we all learned will make the physical relationships obvious. It does not for most people. In the larger sense, this is true of all arithmetic operations taught in our math series *Patterns in Arithmetic*. The student builds mathematical strength by exploring a physical model coupled with an explicit search for patterns. The student then uses those patterns to create procedures for working with numbers. Those procedures are constructed and reconstructed by the student. This is what creates understanding and mastery of mathematics. It is also a lot more interesting and empowering than simply memorizing and drilling and letting the mathematicians have all the fun.

Relative Sizes: A Cutting Activity

Purpose

The purpose of this lesson is to use paper cutting to demonstrate that larger numbers of smaller pieces cover the same area as a smaller number of larger pieces. It also introduces the physical concept of multiplier in the costume of the scissors. This activity has students cut paper to directly experience the multiplier effect. It also uses measurement of the area covered by each fractional unit to graph the relationship between the size of the denominator and the area covered by that piece. This reveals the inverse nature of fractions.

Prerequisites

Basic concept of a fraction, what the numerator and the denominator are, fraction notation, calculation of the area of a rectangle, and coordinate graphing

Materials

Relative Sizes: A Cutting Activity, Worksheets 1 - 3, pages 4 - 6
Scissors and pencil
Several sheets of white, blank paper
Inch ruler
Graph paper

Preparation

Pre cut several 3 x 8 inch strips of paper. *Do not use 8 ½ inch wide strips; cut off the last half inch.* We are going to be measuring the area of each fractional unit, so the strips need to be carefully cut.

Copy these four sentences onto a piece of paper or an overhead:

1. The *more* pieces in the whole, the *greater* the size of each piece.
2. The *more* pieces in the whole, the *less* the size of each piece.
3. The *less* pieces in the whole, the *greater* the size of each piece.
4. The *less* pieces in the whole, the *less* the size of each piece.

Lesson Part 1 Halves of Halves

"Each strip is defined here as the whole, or one. Label one of your strips as the whole. Do not cut this one up."

"Take another strip, fold it in half using a hamburger fold,* and cut it in half." "Label each piece as one-half, $\frac{1}{2}$."

"Put both of the one-half pieces on top of the whole. What do you notice?" "They match. I still have the same amount."

"Take another strip, use a hamburger fold and cut it in half. Now use a hamburger fold to cut each of your halves in half. How many pieces will you have then?" "Four."

"How should you label these pieces? How do you know?" "I will label each one as $\frac{1}{4}$ because there are four parts in the whole."

"Put all of your $\frac{1}{4}$ pieces on top of your whole. What do you notice?" "They cover the whole thing. I still have the same amount."

---

* A hamburger fold is a fold that creates two squarish sections. It is the shortest line fold. A hot dog fold is a long fold that looks like a hot dog bun.

"Put two of your fourths on top of one of your halves. What do you notice?" "They match. I still have the same amount."

"Take another strip. Cut it into halves and then fourths like you did before. Now cut each one of your fourths in half using a hamburger fold. How should you label these pieces? How do you know?" "I will label each one as $\frac{1}{8}$ because there are eight parts in the whole."

"How many eighths do you have?" "Eight."

"Which one covers more area, your whole piece or all the little one-eighth pieces together?" "They both cover the same area." Most students at this level will know this with certainty.

Challenge that certainty.

"How can that be? Here you have only one piece of paper (pointing to the one whole) **and here you have eight pieces** (pointing to the one-eighth pieces)**?"** "There are more pieces in the eighths but they are really little so it takes more of them to cover the whole. With the big piece you need only one."

"So which one of these statements would be correct?" Write these down on a piece of paper or the board. Then hold up a whole piece of paper.

Four Sentences

1. The *more* pieces in the whole, the *greater* the size of each piece.

2. The *more* pieces in the whole, the *less* the size of each piece.

3. The *less* pieces in the whole, the *greater* the size of each piece.

4. The *less* pieces in the whole, the *less* the size of each piece.

Wait and let them think about it. There are two correct answers (both 2 and 3 are correct). Notice that in the correct answers, both *more* and *less* are in the same statement. That is an indicator of an inverse relationship.

"To record the folds and cuts, draw a picture of the four strips. What pattern do you see in the numbers of pieces?" "One, two, four, eight–the numbers double."

"What created that pattern?" "The pattern is doubled because each time the pieces are cut in half. Cutting the pieces in half doubles the number of pieces."

Lesson Part 2 — Graphing Relationships

Materials

Relative Sizes: A Cutting Activity - Worksheet 1, page 4
The cutup fractions from the last activity and an inch ruler

You are going to make a graph of two of the relationships you saw in the last activity.

Fill in the T chart on the worksheet.

"Pick up the whole strip. Write on it 1/1. One over one is equal to our one whole piece. On the T chart, write a 1 under the X where it says 'Denominator.'" Wait for him to do this.

How many pieces does it take to make a whole with a whole?" "One."

Have him place that number 1 under the Y where it says Number of pieces in the whole. The two ones should be even with each other.

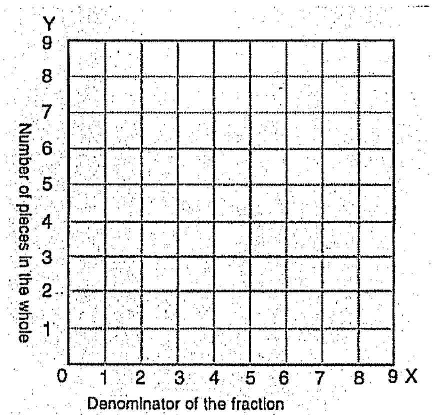
"Now look at your $\frac{1}{2}$ pieces. What is the denominator?" "Two."

"How many halves are needed to cover the whole?" "Two."

Have him write these numbers on the T chart also. Continue this with the fourths and eighths pieces he cut out. So under the X, write the denominators of the fractions made with the strips. They would be 1 (for the whole 1/1), 2, 4 and 8. Under the Y, write the number of pieces it takes to make the whole for a fraction with that denominator. The completed T chart will look like this:

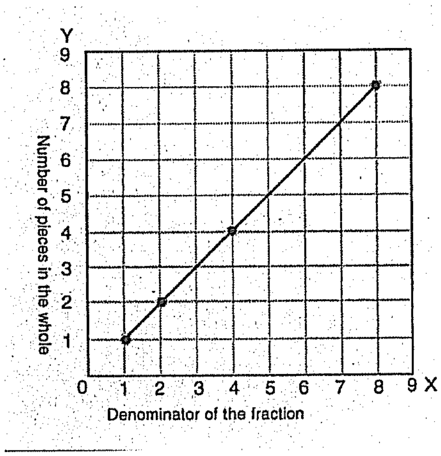
Denominator X	Number of pieces in the whole Y
1	1
2	2
4	4
8	8

The graph in the workbook has no numbers on the X or Y axis and the titles are also not labeled. Guide him to prepare the graph so it looks like this by recording the numbers on the X and Y axes and filling in the titles:



"You are now going to graph the denominator and the number of pieces in the whole from your T chart onto the graph. For the first pair, the (1, 1), trace the pencil up the vertical line labeled 1 and stop at the horizontal line labeled 1. Put a dot there."

"Trace the pencil up the vertical line labeled 2 and stop at the horizontal line labeled 2. Put a dot there. This shows that $\frac{1}{2}$ has a denominator of two and it takes two pieces to make the whole."



Repeat this process with the fourths and then the eighths. Next, use a ruler to connect the dots into a line.

"What relationship pattern do you see in this line?" "The more, the more. The more the denominator of a fraction, the more pieces are needed to cover the whole."

"What can be told about thirds from this line, even though we did not work with thirds this time?" "The same relationship applies. It takes three pieces to cover a whole with thirds."

Have him trace from the three on the X axis up until it hits the line he just made. Trace from that intersection back to the Y axis and it will tell you that if a fraction has a denominator of three, it will have three pieces in the whole.

"This is called a positive linear graph. The line is straight and slopes up left to right. When a 'the more, the more' relationship is put on a graph it will look like this picture."

Now begin work on the second T chart on Relative Sizes: A Cutting Activity - Worksheet 2, page 5.

"What is the area in square inches of your paper strip we labeled as the whole?" "Twenty four square inches. The length is eight inches, the width is three inches for a total of twenty-four square inches."

"Write 24 square inches on the whole strip you have."

"We are going to label the denominator of the whole as one. Put the number 1 under the X where the word 'Denominator' is on the T chart. Under the Y, where it says 'Area of the fraction in square inches,' write the 24. Make sure the 1 and the 24 are in line with each other."

Have him figure out the area of each fractional part, label it, and record the information on the T chart.

"What is the area of the one-half strip?" "Twelve square inches."

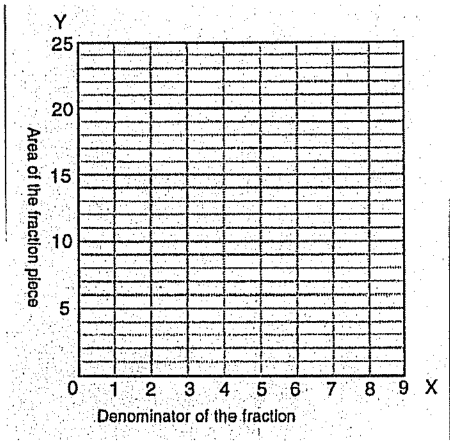
"What is the area of the one-fourth strip?" "Six square inches."

"What is the area of the one-eighth strip?" "Three square inches."

Your T chart will look like this:

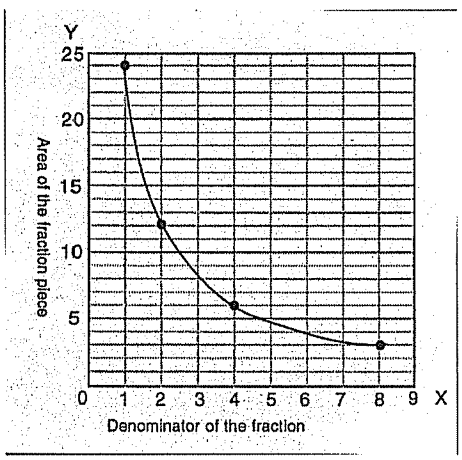
Denominator X	Area of the fraction in square inches Y
1	24
2	12
4	6
8	3

Have him prepare the second graph like the one shown below.



"What do you think the line on the graph will look like this time?" Many students will not know.

Try it. This time the whole with a denominator of one has an area of twenty-four square inches. The coordinate of this point will be (1, 24). Go over 1 and up 24. Put a dot on that intersection. Do the same for the half, the fourth, and the eighth. The finished graph will look like this:



WOW! You get a curve! (That is because we were using a doubling pattern.) And it slopes the opposite direction as the other graph. The slope direction here is negative. In other words, the line starts high and drops as it goes away from the origin. This is what an inverse relationship looks like on a graph.

Write this at the bottom of the graph:

The _______ the denominator the _______ the area covered by each piece.

Answer: The more the denominator, the less the area covered by each piece.

A 'the more, the less' relationship is called *inverse*.

Note

This same kind of relationship comes up if you graph the relationship between the divisor and the quotient of a division problem. This is because fractions are division problems.

Practice Worksheets

Relative Sizes: A Cutting Activity - Worksheet 3, page 6 Repeat the paper cutting process with the activities on this worksheet. See the Answer Key to be sure it is being done correctly.

After he does the cutting activity with thirds ask, **"What pattern do you see in the denominators of pieces this time?"** "Three, six, twelve. It is also a doubling pattern."

"What created that pattern?" "The pattern is doubling again because each time the pieces were cut in half."

Test for Understanding

Give the student more strips and ask him to cut the strips into thirds to begin with, then ask him to cut the thirds into three.

"How many pieces will you get if you cut the thirds into three?" "Ninths."

"Predict what the answer would be if the ninths were then cut into three equal parts." "Twenty-sevenths."

"What would the pattern be if you keep cutting each new piece into three parts?"

"It would be a tripling pattern: 3, 9, 27, 81, 243, 729."

If the student can't predict the pattern, keep giving the cutting activity once or twice each week as you move forward. Also work on skip counts in the times tables to help him see the patterns.

Equivalence: Manipulative Review

Purpose

The purpose of this lesson is to use a fraction manipulative, such as Fraction Circles, to physically build equivalent fractions. The main concept being worked on is that fewer large pieces can cover the same area as several smaller pieces. The concept of the multiplier is not noted in this lesson.

Prerequisites

Fractions: Booklets 1 and 2

Materials

Equivalence: Manipulative Review - Worksheets 1 and 2, pages 7 and 8
Fraction manipulative such as Fraction Circles, squares, or Prism Fractions

Warm Up

Give the student time to make designs and stacks with the pieces before beginning the lesson. Have the student refresh her memory on what fraction each colored piece represents.

Lesson

Orally, have the student find all the ways of covering the one-half piece. The pieces must match exactly. Review how to write the fractions. Do not try to teach the multiplier pattern at this time.

Begin Equivalence: Manipulative Review - Worksheet 1 by having the student place the one-half pieces over the one whole piece. Review the concept of numerator and denominator. Show her how to record the missing number in the numerator of the first problem.

When you get to the second row, one-half will appear. Note if she writes the correct number from memory. Have her build each one to check her work.

If you notice that she is able to write the missing numerator in without using the fraction pieces, ask her how she knows. See the conversation in Test for Understanding.

If she uses a manipulative for every problem, just note that.

Worksheets

Equivalence: Manipulative Review - Worksheets 1 and 2, pages 7 and 8

Test for Understanding

A conversation about $\frac{1}{3} = \frac{4}{12}$ could go something like this:

"How did you know to put the four in the numerator?" "I know that there are four twelfths in one-third because you have to put the twelfths into three groups."

"Why three groups?" "Because to make thirds you have to have three equal parts."

"So where did the four in the numerator come from?" "I am working with twelfths, so I have to put the twelve pieces into three equal groups. One of the groups would have four-twelfths in it. So one-third is equal to four-twelfths."

"What if you had two-thirds?" "Then I would need two groups. One of the groups would have four-twelfths in it. So two would have eight. Two-thirds is equal to eight-twelfths."

"Are you sure? Check it with your pieces." "I am sure. See."

Changing Wholes Meet Manipulative Equivalence

Purpose

The purpose of this lesson is to repeat the concept used in the last lesson, of building equal fractions, and adds the concept of the changing whole. The block that is defined as one, changes on each page. This builds the concept that all fractional units are determined by the definition of the whole. The student has to rethink the fractional units and find the blocks that build equal fractions. The abstract multiplier is not yet discussed formally.

Prerequisites

Previous lessons in this book. Previous experience or exploration with Cuisenaire Rods and pattern blocks

Materials

Changing Wholes Meet Manipulative Equivalence, Worksheets 1 - 7, pages 9 - 15
Cuisenaire Rods and pattern blocks

Note

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world he is presenting. From your understanding of what he says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Lesson

Have him take out the brown rod. Ask him to find all the one color trains that cover the brown rod. A train is a series of rods of the same color.

He will find that the brown rod can be evenly covered by two purple rods, four red rods, or eight tan rods.

"One purple rod is what fraction of a brown rod?" "One-half."

"How do you know it is one-half?" "Because it takes two to cover the brown rod. That divides the brown rod into two parts."

"One red rod is what fraction of a brown rod?" "One-fourth."

"One tan rod is what fraction of a brown rod?" "One-eighth."

"Take out your worksheet and color in the brown whole, the purple half, the red fourth, and the tan eighth."

Have him complete Changing Wholes Meet Manipulative Equivalence - Worksheet 1 independently. This may only take a few minutes or so.

Worksheet

"Now go to Changing Wholes Meet Manipulative Equivalence - Worksheet 2. What color will the whole be this time?" "Dark green."

"What fractions of this whole can you make with the rods?" "Halves (light green), thirds (red), and sixths (tan)."

Ask him to color in one-half, one-third, and one-sixth with the correct colors.

"What did you say the red rod was?" "One-third."

"But on Changing Wholes Meet Manipulative Equivalence - Worksheet 1, you said the red was one-fourth. How can it be one-fourth on one page and one-third on the next?" "It changes because the size of the whole rod changed. This whole is smaller than the last one. It takes only three reds to cover instead of four. This makes one red rod one-third instead of one-fourth. It changes."

Have him complete Changing Wholes Meet Manipulative Equivalence - Worksheet 2. An embedded assessment is at the bottom. The denominator is given and he must supply the numerator. Many students will have a bit of difficulty with this. This will show you how secure his understanding of numerators and denominators is.

If he has difficulty, have the student go to the problems with a 1 = first, then back-track.

Have him build with rods if he can not see the pattern.

Worksheets

Changing Wholes Meet Manipulative Equivalence - Worksheets 3 and 4, pages 11 and 12

Test for Understanding

Changing Wholes Meet Manipulative Equivalence - Worksheet 5, page 13, is an assessment. Do not help the student. He can make wholes, halves and fifths. Watch to see that he correctly labels each fractional unit. The answers are in the Answer Key. In the definitions, we want him to explain what that number does, not simply that it is the top or bottom number. The numerator tells you how many rods or pieces you have. The denominator tells you how many pieces are needed to make the whole.

Practice Worksheets

The manipulative is changed again to require him to reconstruct his idea of the whole and all possible equal fractions. Changing Wholes Meet Manipulative Equivalence - Worksheets 6 and 7, pages 14 and 15

"What pattern do you see in the challenge number series at the bottom of Changing Wholes Meet Manipulative Equivalence - Worksheet 7?"

"I see the three times table in the denominators and the two times table on the top, the numerators."

Graph Paper Fractions

Purpose

The purpose of this lesson is to use manipulatives to introduce the multiplier relationship between the denominators of equal fractions. The big conceptual issue here is understanding that when you take fifteenths and group them into groups of five, you get thirds. The multiplier effect between thirds and fifteenths is a times five relationship. Each third has five-fifteenths in it. This is a geometric relationship. This concept is the foundation for the understanding of proportions.

Prerequisites

The multiplication tables

Materials

Graph Paper Fractions - Worksheets 1 and 2, pages 16 and 17
Colored pencils
Prism Fractions pieces for the Warm Up

Warm Up

Ask the student to find all the ways to make one-half using the Prism Fractions Pieces.

"How are one-half and three-sixths the same?" "They cover the same area."

"How are one-half and three-sixths different?" "They use different sized pieces."

Lesson

Drawings are going to be used to help her figure out equivalent fractions. On Graph Paper Fractions - Worksheet 1, have her take a pencil and darken the perimeter, the outside line, of the rectangle with the sixteen squares. This rectangle stands for the whole.

"What fraction of the whole is one little box?" "One-sixteenth."

"How does the example show fourths in this whole?" "It divided the sixteen squares into four equal groups."

"So how many sixteenths are in one-fourth?" "Four."

Have her use a colored pencil to shade over the gray in the example. Point out the equivalent fractions to the right of the drawing. Have her again darken the perimeter of the next rectangle.

"How are you going to show one-half?" "I will divide the rectangle into two equal parts."

Note

At this point some students will make groups of two instead of two large groups. If this happens, count the number of sections made (in this case eight) and ask what fraction of the whole each one of those parts is and how do you know.

"How many little boxes are in each half?" "Eight."

"What is the size of each little box?" "One-sixteenth."

Have her color in one-half with her colored pencils and fill in the missing number in the equivalent fraction on the right. The missing number will be eight, indicating $\frac{8}{16}$.

Repeat this procedure with problems on the rest of the page. Check the Answer Key to be sure she is doing them correctly.

Worksheet

Graph Paper Fractions - Worksheet 2, page 17

Draw her attention to the rectangle with the fifteen boxes in the third row on the grid shown at the top of the worksheet. Have her darken the perimeter of the rectangle. Point out to her that the rectangle has the same number of squares in the whole as the largest number in the denominator of the fractions you are working with, in this case fifteenths.

Have her finish the page on her own. Draw her attention to the fact that there are problems on both sides of the grid paper in the lower part of the worksheet.

Check the work to be sure it is done correctly.

Test for Understanding

The bottom question is a Test for Understanding. "What pattern is there between equivalent fractions?" If she notices the multiplication relationships between the numbers or can explain them, you can be confident she is understanding the pattern. If she can not answer the question, leave it alone and continue, knowing the concrete and representational work will move understanding toward articulating the abstract pattern. The numerator and denominator are both multiplied by the same number, e.g., $\frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$, to get an equivalent fraction.

Equivalence: Recording

Purpose The purpose of this lesson is to use manipulatives and charts to begin the formal search for the multiplication patterns underlying the calculation of equivalent fractions.

Prerequisites Fluency in the multiplication tables

Materials Equivalence: Recording - Worksheets 1 - 8, pages 18 - 25
Prism Fractions - Circles or Squares
Graph paper and colored pencils

Warm Up Ask the student to build three fractions equal to two-thirds using a manipulative.

Lesson Equivalence: Recording - Worksheets 1 and 2 This is called the 'Make With' Chart. The horizontal row along the top shows which number is being constructed. He will pick up the fraction piece with this value. The first square shows 1. He will pick up the piece that is the whole. He will then proceed to make wholes out of each number listed in the 'With' Column.

"Can you cover or make a whole with halves?" "Yes, it takes two halves." Record that: $1 = \frac{2}{2}$.

"Can you cover or make a whole with thirds?" "Yes, it takes three thirds."

"What pattern do you see?" "To make a whole the top number and the bottom number are always the same."

Finish this column.

The second column shows $\frac{1}{2}$. He will pick up the piece that is the half. I will walk you though doing the $\frac{1}{2}$. We will move down the $\frac{1}{2}$ column.

First, have him pick up the $\frac{1}{2}$ piece. The first box says $\frac{1}{2} = \frac{1}{2}$. This means we can build another half on top of our first half and they will exactly match in size. The second box down the column shows a half and a third. Put a 1 in the numerator above the 2. Now pick up your one-half piece and place it on the table. Now pick up the $\frac{1}{3}$ piece.

"Can you exactly cover a one-half piece with a $\frac{1}{3}$ piece?" "No, it is too small."

"Can you exactly cover a one-half piece with two one-third pieces?" "No, it is too large."

"So, can you make a half with thirds?" "No, they do not match up."

"Put a big X in this box to show you can not make halves with thirds."

Now move to the next box in the column. It is fourths.

"Can you exactly cover a one-half piece with a one-fourth piece?" "No, it is too small."

"Can you exactly cover a one-half piece with two one-fourth pieces?" "Yes."

"Do they match exactly? Are they equal to each other in size?" "Yes."

"So, can you make a half with two-fourths?" "Yes."

Record that in the box. Put a 2 in the numerator above the 4 to show that $\frac{1}{2} = \frac{2}{4}$. Repeat this process with fifths, sixths, etc., all the way down to the bottom. You probably do not have a manipulative for sevenths, ninths, or elevenths, so use cubes.

"Can you divide seven cubes into two equal halves?" "No, because seven is not an even number."

"What about ninths and elevenths?" "No, because they are odd numbers also."

"What pattern do you see in the one-half column?" "They go every other one. Halves can only be made with even numbers of parts."

Repeat this process with all the fractions in the Make With column. You will see a pattern emerge in the Xs. In the half column, there is a match in every other square. In the thirds column, there is a match in every third square.

Note

As you get to the smaller pieces, you can not make $\frac{1}{4}$ with $\frac{1}{2}$. The half is larger than the fourth. Put an X in this box. Check the Answer Key to be sure you are doing the work correctly. This page can be tedious for students. Work on it for only fifteen to twenty minutes at a time.

Continuing the lesson: When he has completed the chart, begin to fill in the information on the far right of Equivalence: Recording - Worksheet 2. Where it says to list all the fractions equal to one-half, have him go down his chart and list the fractions that were equal to one-half in order. Then have him describe the pattern he sees in words. Most students will say the top goes up by ones and the bottom goes up by twos. Check the Answer Key. Be sure to have him describe the patterns clearly.

Worksheets

Equivalence: Recording - Worksheet 3, page 20, begins with a recap of the patterns noticed on Equivalence: Recording - Worksheet 2. Then the patterns are extended to include numerators greater than 1. It's important to have him build all the possible fractions with the manipulative. He will be able to build sixths and twelfths. Then look for the number pattern to fill in the rest. Use graph paper if he needs a visual aid.

"What pattern do you see in the numerator of the $\frac{2}{3}$ fraction sequence?" "The

two times table."
"What pattern do you see in the denominator of the two-third fraction sequence?" "I see the three times table."

It's important to use the chart, manipulatives, and graph paper to fill out the sequences on Equivalence: Recording - Worksheet 4. Do not let him jump to conclusions about what the pattern is before he builds or draws it. Stop the lesson at this point.

Practice Worksheets

Equivalence: Recording - Worksheet 5, page 22
Build the first three rows with manipulatives. Notice we are building whole numbers with fractional units. This is a slightly different take, and some students will jump to conclusions and fill in the numbers incorrectly if they are not observant.

Test for Understanding

Equivalence: Recording - Worksheet 6, page 23 Make available the manipulative and graph paper. Watch him to see if he has memorized some of the values, or if he sketches or builds. You are mostly concerned here with correct answers. The challenge at the bottom will tell you if he sees, and can apply, the multiplication pattern yet.

Equivalence: Recording - Worksheet 7, page 24 Do not use any manipulatives or graph paper here. Watch to see if he uses the patterns. If he can't, reteach the lessons using this sheet and graph paper.

Equivalence: Recording - Worksheet 8, page 25 Describing his method is the most important step to show understanding and to allow him to own the knowledge. He may or may not be able to do this page. If he can't, drop it for now and come back later. Some students will figure out the pattern and apply it correctly.

Equivalence: Representational

Purpose

The purpose of this lesson is to have students manipulatively change fractions from larger units to equivalent smaller ones, which makes the multiplier obvious. This lesson helps students see the pattern that underlies the procedure for calculating equivalent fractions. It also explicitly brings out the inverse nature of fractions. That is, the lower/smaller the number in the denominator, the larger the piece, for example, $\frac{1}{2} > \frac{1}{4}$. And, when equivalent fractions are made, the smaller the size of the fractional part, the more pieces are needed to cover the area, for example $\frac{4}{8} = \frac{1}{2}$.

Teacher Preparation Required

This lesson requires that you read carefully ahead of meeting with your student. The lesson will go easily if you are prepared. Before beginning this lesson with your student, study Equivalence: Representational - Worksheet 2. The paper cutting activity shown there is what you will be exploring. You can have her cut the strips of paper or you can precut them.

Prerequisites

Fluency in the multiplication tables or a multiplication chart

Materials

Equivalence: Representational - Worksheets 1 - 8, pages 26 - 33
Strips of paper 8 inches long and 3 inches wide (It is important that each strip is the same size.)
Scissors
12 x 18 construction paper
Glue stick
Thin line markers and colored pencils

Lesson Part 1

Exploring the Denominators

Do Equivalence: Representational - Worksheet 1 together to set the context. Then begin the paper strip cutting as an oral activity. The student needs the strips and scissors to begin.

"This strip of paper is the whole." Hold up a strip. **"Fold it into halves and cut the halves apart."** Allow time to do this.

"Have you changed the size of the whole by cutting it into two halves?" "No."

"Are you sure? You used to have only one piece and now you have two pieces." "That is true, but the two pieces are smaller and they are still the same size together."

Test for Understanding

"Can you prove that?" "I can put the two halves over another whole that is the same size." Have her do this.

"Have you changed the size of the whole by cutting it into two halves?" "No, it is still the same."

"What is the same?" "The space it covers."

Note

Most students will make the cuts the way they are shown in the picture on Equivalence: Representational - Worksheet 2. Long horizontal cuts tend to get difficult to handle later, so encourage her to make vertical cuts as shown in the picture for the first fraction.

"Take out another strip of paper and cut it in half also using a vertical cut." Give time for her to do this.

"Can you change these halves into fourths?" Watch to see what she does. She may fold each half into two parts. Or she may cut each half into two equal parts to produce four smaller pieces. She may cut vertically or horizontally. Accept either.

Note

If she cuts each half into four pieces, it means she is confused about the denominator. Have her finish and then put all her pieces back over one of the uncut wholes and count all the pieces. She will see she has eighths and not fourths.

"How can you prove these are fourths?" "Because it takes four of them to cover the whole." Have her cover an uncut whole to show that.

"How can you change your fourths into twelfths?" Watch to see what she does. Let her experiment. Have her put all her cut pieces over an uncut whole to check her creations. Most students will cut each fourth into three smaller parts without guidance. Let her know the smaller parts need to be roughly the same size but do not have to be measured or exactly precise.

"How can you prove that your pieces are twelfths?" "By putting them back over the whole and seeing that there are twelve equal little pieces in all."

"How did you know to cut each fourth into three parts?" "Because I know that four times three is twelve. I had four pieces already, so I needed to cut each of them into three smaller pieces."

The whole strip is cut with vertical cuts to make the first fraction, like the fourths. To make twelfths out of those fourths, the cuts can again be made vertically or they can be made horizontally. At this point, direct her to make all the cuts for the second fraction, the twelfths in this case, horizontally as shown in the picture. The reason for this is that the multiplication array is easier to see and that is the way we are going to draw them in the next part of the lesson.

"Make thirds." Give her time to do this.

"Now make horizontal cuts to change the thirds into twelfths." Watch her.

"Last time you made twelfths you cut each piece into three parts. This time you cut them into four parts. Why did you change how you cut out your twelfths?" "I cut each part into four parts this time because I had thirds to begin with. Since 3 x 4 is 12, I need to cut each third into four small parts to make twelfths."

"What would have happened if you had cut each section into three parts like you did last time?" "I would have gotten ninths not twelfths." If she does not know, have her try it.

"How would you cut thirds if you wanted to end up with fifteenths?" "I would cut each third into five little parts because three times five is fifteen."

"See if you can draw a picture of thirds being turned into fifteenths." Give her time to puzzle this out. Most students will produce a drawing something like the drawing on Worksheet 3. Have her explain her drawing.

"Get a new whole strip. Cut it into fourths." Wait for her to do this.

"Now change it into tenths." This can not be easily done. Wait to see what happens.

She will probably say, "I can't do it."

"**Why**?" "I can not cut four pieces into ten equal pieces." She may even say, "I can't because four does not go into ten, or even better, ten and four do not share any factors."

Note

It is not impossible to change fourths to tenths but you have to cut each fourth into two and one-half pieces and recombine the halves to make the two extra pieces. If she does this, give her a high five for a brilliant piece of work. Then tell her, while this conversion is not illegal, it creates a compound fraction, so we avoid these if possible. It is very, very rare for a student to come up with this solution.

This is a good stopping place. Continue the next day.

Investigation and a Poster: Give her more whole paper strips and a 12 x 18 piece of construction paper. It is better if the strips are different colors. Have her make a poster that shows all the fractions that can be made if you start with fourths. Begin with a labeled, uncut, whole strip at the top. Just below the whole, paste a strip cut up to show fourths. Have her use fine tip markers to label the fractions on her poster. It is important that all the cut up fractions are tightly spaced so they appear to be the exact same size as the whole. She should darken the cut lines so they can be seen. Have her show the vertical cuts in one color and the horizontal cuts in a second color. Some students will not want to cut the strips at all but simply show with a dark line where the cuts would be. It is preferred for them to do the cutting to reinforce the concept. (If necessary, stipulate that you can't make fractional cuts as discussed in the note above. Do not bring this up unless she came up with that solution in the last problem.) If you have more than one student, have each student make a poster with a different base fraction.

You or another student should make one with thirds for comparison. Halves are a good one for a younger child or a less advanced student. Thirds and fifths are harder to fold and can be given to more dexterous students. Challenge a hot shot to make sevenths using a centimeter ruler to measure even pieces. Let her go down to as small a piece as she has patience for, but at least four levels down.

When she is finished, ask her to read the list, in order, of the equivalent wholes she was able to make. Write down the list in front of her as she reads: fourths, eighths, twelfths, sixteenths, etc.

"What pattern do you notice?" "It is the four times table."

"Yes, the denominators are multiples of four. Mathematicians call it *multiples*." The only equivalent fractions that can be made with the cuts are fractions whose denominators are multiples of the starting denominator, in this case, fourths.

"What pattern do you think will happen if you look at the chart with the thirds?"

"The three times table." Then confirm that by listing the multiples as shown on the chart.

Discussion

"What relationship can you describe between fractions of different sizes? Make a 'the more, the more' statement. Make a 'the more, the less' statement. Then, prove it or disprove it."

Write the sentences below on a piece of paper and have her copy them onto the bottom of her poster. See if she can put the correct words in the blanks without help.

1. The smaller the size of the pieces of a fraction, the ______ pieces needed to cover the same area.

2. The larger the pieces of a fraction, the ______ pieces needed to cover the same area.

3. The smaller the denominator, the _______ the size of the pieces covering the whole.

4. The larger the denominator, the _______ the size of the pieces covering the whole.

5. The smaller the denominator, the _______ the number of pieces needed to make the whole.

6. The larger the denominator, the _______ the number of pieces needed to make the whole.

Have her label each sentence in a short form as 'the less, the more'; or 'the more', 'the less'; or 'the more, the more'; or the 'less, the less.'

1. the less, the more 2. the more, the less

3. the less, the more 4. the more, the less

5. the less, the less 6. the more, the more

Part 2

Drawing the changing of the Denominator

Warm Up

Revisit the posters. **"Draw a rectangle to show the change from thirds to eighteenths."** Watch what she does. If she has difficulty, have her do it with paper cutting. If she can do this, then proceed with the lesson. If she has difficulty, repeat Part 1 with different problems.

Have her redraw the thirds to eighteenths and guide her to draw it so that the colors show the multiplication array clearly. Draw the whole rectangle with a regular pencil. Then switch to a purple or blue pencil to cut the rectangle into thirds vertically. Then change it to eighteenths using a warm color such as orange or red to draw the horizontal lines that convert it to eighteenths.

Lesson

Do Equivalence: Representational - Worksheets 2 and 3, pages 27 and 28, with your student following the instructions on those pages. These pages formally introduce the concept of the multiplier she discovered in the paper cutting activity.

Equivalence: Representational - Worksheet 3 introduces the recording of the multiplier. In the lower half of Worksheet 3, notice the words to the right of the boxes for changing fourths to eighths. They say, **"Cut each larger piece** (this refers to the first fraction drawn on the left box with vertical lines) **into _____ smaller parts because 4 x 2 = 8."** The number two that goes in the blank is the multiplier. It is the number of sections that she had to cut the original larger fraction into (the fourths in this case) to create the smaller eighths. In this case the multiplier is two. You have to cut each fourth into two smaller equal parts to create eighths.

Introduce the word *Multiplier* to her. **"The number of smaller parts you choose to cut the larger piece into is called the Multiplier."**

Do the second problem changing the fourths to sixteenths. Leave the top box alone. Show the horizontal lines to create the sixteenths in the lower box. Then have her fill in the words to the side. The correct drawing is shown for her at the bottom of the page.

Use the Answer Key to guide you. Insist that she use the horizontal lines to change fourths to twelfths when drawing, as shown on Equivalence: Representational - Worksheet 2 in the center of the page and on the top of Equivalence: Representational - Worksheet 3. Using dotted lines horizontally helps students see the multiplication array. It also sets them up to do multiplication of fractions later. Correct the work before going on.

Practice Worksheets

Equivalence: Representational - Worksheet 4, page 29

Do the first one together. **"We are going to add a new twist with colored pencils."** This worksheet asks her to draw in both the original fraction and then the equivalent fraction. The first problem asks her to change thirds to sixths. First, use a dark colored (not black) pencil to draw in the thirds in the first box. Then draw the thirds again in the second box *using the same color.* Now switch colors *to a lighter color* and draw the horizontal lines that change the thirds to sixths *only in the second box.*

Insist on neat work. Use colored pencils. Have her make the first vertical lines with one color and the horizontal lines in another color. Check to see each space for filling in numbers is done correctly. Use the Answer Key to check her work.

Lesson Part 3 — **Drawing the Numerator**

Equivalence: Representational - Worksheet 5, page 30

Note

Before you begin, have her fold the bottom half of the page under, or cover it with another piece of paper so she can not see the solution to the problem at the top of the page.

"In the last worksheet we have only been looking at the denominator. How do we show the three in three-fourths?" "By shading in three of the four boxes."

"You already know how to change fourths into twelfths. How can you figure out how many twelfths are in three-fourths?" Give her time to think about it. Watch what she does. See if she can extend the pattern she already knows for the changing of the denominator and use it to change the numerator also.

Each fourth has three twelfths in it. So if she shades in three-fourths and changes the entire box to twelfths, all she has to do is count up the number of shaded twelfths. It would be nine. Give her time to figure this out. Many students will do this easily. If she can not find the solution, ask her, **"How many twelfths are in the shaded part?"**

The last two problems on Equivalence: Representational - Worksheet 5 bring in the familiar words on the right hand side of the page. It also adds a place to put in the new numerator of the converted fraction.

"How can you prove that twelve-sixteenths is equal to three-fourths?" "The area of the shaded part of each box is exactly the same. We did not change the amount of shaded box; we only cut it into smaller pieces."

Equivalence: Representational - Worksheets 6 and 7, pages 31 and 32

On these pages, the student must again draw both the first and the second fractions. Have her study the example. Have her again use a colored pencil to draw the thirds line in the first and second boxes. She will then lightly shade in the two-thirds. Then, using a second color, draw in the horizontal lines for the fifteenths. Important: Instruct her to write the word Multiplier or put an M next to the space where it says: "_____ smaller parts." The number that goes in the blank is the multiplier.

Worksheet

Equivalence: Representational - Worksheet 8, page 33

Test for Understanding

Watch to see if the student uses the drawings first and then fills in the missing number or if she skips the drawing and fills in the numbers first. Either way is acceptable, but a student who fills in the numbers first is demonstrating that she has strong control of the concept. What you are looking for is fluency with or without the drawing. It is okay if the student does lovely, carefully drawn pictures and then fills in the numbers. The assessment asks her to identify the multiplier without the prompt of "Cut each larger piece into __ smaller parts." If your student needs any help on this, repeat the lessons from Equivalence: Representational - Worksheets 6 - 8 with similar problems you make up on your own.

Equivalence: Calculating

Purpose The purpose of this lesson is to use the multiplier and the identity property of multiplication to calculate equivalent fractions. This is the standard procedure we all learned in school with the illumination of the Mighty One.

Prerequisites Two out of three problems done correctly on Equivalence: Representational - Assessment,Worksheet 8, page 33.

Materials Equivalence: Calculating - Worksheets 1 - 5, pages 34 -38

Teacher Preparation Read Equivalence: Calculating - Worksheets 1 - 3 yourself and read the Answer Key before beginning this lesson. Look up the Identity Property of Multiplication online to review: a x 1 = a, any number times one equals itself.

Warm Up Make up some problems like the ones on Equivalence: Representational - Assessment - Worksheet 8. Review the vocabulary of the multiplier.
"Draw a rectangle and change thirds to eighteenths. What multiplier will you choose?" "Six, because 3 x 6 = 18."

"Now shade in two-thirds. How many eighteenths are in those two-thirds?" "Twelve."

"Does the multiplier change the shaded area too?" "Yes."

Lesson Do Equivalence: Calculating - Worksheet 1, page 34, together. Fill in the blanks as you go. When you get to the question "What happens when you multiply by one?", do several examples.
"What is 4 x 1 equal to?" "Four."

"7 x 1 =?" "Seven."

"$\frac{1}{2}$ x 1 = $\frac{1}{2}$?" "One-half."

"What pattern to you see?" "The number you multiply by one does not change."

"But when we multiplied three-fourths by three over three the answer changed. It did not stay three-fourths. How can you explain that?" "It changed the number of pieces, but it is still the same number."

"Can you use the drawing above to prove that the two fractions are the same?" "They are not the same in that they have different numbers of pieces, but you can see that they both cover the same area. So they are equal."

Direct her attention again to the drawing at the top of the page. **"Draw a rectangle and divide it into thirds. Shade in two-thirds. What would happen if you used**

a different multiplier like five?" Give her time to do it.

"Do you still have a fraction that equals two-thirds?" "Yes."
"Can you write the number sentence for what you just did? Use the example where it says 'Check this out.'" She will write $\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$.

"Can you make a drawing that proves that twelve-eighteenths is equal to ten-fifteenths?" "Yes."

Turn to Equivalence: Calculating - Worksheet 2, page 35.
Read the top of the page together and follow the instructions.
A Test for Understanding is coming up.

Watch her solve the first problem on the lower half of the page. Does she know what the multiplier is without drawing a box? If she writes in 3 as the multiplier, ask, **"How do you know to use three?"** If she gets stuck, have her draw a box, cut it into fifths, shade in two-fifths and then use horizontal lines to change fifths to fifteenths. If she needs the box, let her use it until she drops it on her own.
Make sure she draws a Mighty One around the multiplier every time.

Finish the rest of the page. Have her do it on her own if she can. Do not help unless it is needed. Do not skip the 'Make your own' problem. Doing these demonstrates to her that she is in control of the math, not the other way around.

Practice Worksheet

Equivalence: Calculating - Worksheet 3, page 36 *Make sure she draws a Mighty One around the multiplier every time*. Check the work before going on.

Test for Understanding

Equivalence: Calculating - Worksheet 4, page 37, is an assessment. A new variation is given. The numerator of the changed fraction is given instead of the denominator. How will she approach this?

A student who understands the big idea will understand that the multiplier affects both the numerator and the denominator. She will use the change in the numerator to find the multiplier and then apply it to the denominator.

A student who is still constructing understanding will falter at this new problem type. Prompt her to draw a box to help. Wait to see if she can work out how to do that.

Some students will not know how to show the fifteen as a numerator. Help them with this sequence.

"Draw a box, cut it into fourths, and shade in three of those fourths. Where do you see the numerator three in your drawing?" "In the shaded part."

"So focus only on the shaded part of your drawing. How can you make fifteen smaller parts so there are the same number in each of the three shaded boxes?"

Give her time to think and draw. "I could divide each of the shaded boxes into five sections, because three times five equals fifteen."
"What is the multiplier for the numerator?" "Five."

"What is the multiplier for the denominator?" "It has to be five also."

If she does not know, have her extend the lines she drew in the shaded area into the unshaded area and count the total number of smaller sections, which will be twenty. Have the student finish the worksheet on her own, giving help only if needed.

Practice Worksheet

Equivalence: Calculating - Worksheet 5, page 38 Check the work before going on.

Test for Understanding

Watch to see how fluent she is on Equivalence: Calculating - Worksheet 5. If she is not fluent, give her more practice until she is. Make up your own problems or find worksheets on the Internet or in other books.

If she cruises through and uses the multiplier on both the numerator and denominator, she has mastered this topic. Have her do the Post-Assessment.

Remediate any weak points and move on to Fractions: Booklet 5 - Simplifying Fractions.

Patterns in Arithmetic

Fractions: Booklet 4

Equivalent Fractions

Answer Key for the Student Workbook

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods

1	w	= white
2	r	= red
3	lg	= light green
4	p	= purple
5	y	= yellow
6	dg	= dark green
7	bk	= black
8	bn	= brown
9	bl	= blue
10	o	= orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

By Suki Glenn, Susan Carpenter, and Alysia Krafel

Patterns in Arithmetic: Fractions - Booklet 4
Answer Key for the Student Workbook

Published by Pattern Press
P.O. Box 2737
Fallbrook, CA 92088

Printed on recycled paper.

ISBN 978-1-935559-10-8

Fractions - Boolet 4 Answer Key

Pre-Assessment - Part 1 - Worksheet 1

The student may use a manipulative.

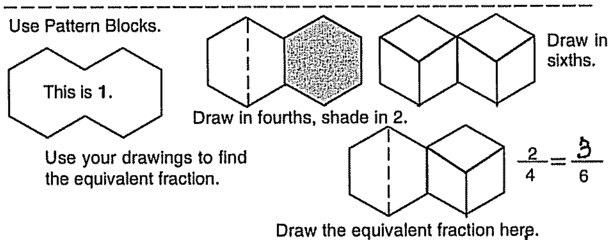
1. Shade in $\frac{2}{3}$ of this rectangle.
2. What fraction of this rectangle is shaded? $\frac{5}{7}$
3. 3 out of 4 of the circles are shaded.
4. Write the name of this fraction. $\frac{2}{5}$ two fifths

Use Fraction pieces or Fraction Circles. Fill in the missing numbers.

5. a. $\frac{1}{2} = \frac{2}{4}$ b. $\frac{1}{3} = \frac{4}{12}$ c. $\frac{3}{4} = \frac{6}{8}$
6. a. $\frac{4}{6} = \frac{2}{3}$ or $\frac{8}{12}$ b. $\frac{9}{12} = \frac{3}{4}$ c. $\frac{8}{10} = \frac{4}{5}$
7. Put these fractions in order from the least to the greatest.
   $\frac{7}{8}$ $\frac{1}{2}$ $\frac{2}{6}$ $\frac{3}{5}$ $\frac{3}{4}$
   $\frac{1}{2}$ $\frac{2}{6}$ $\frac{3}{5}$ $\frac{3}{4}$ $\frac{7}{8}$
8. Fill in this sequence. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$

Use Pattern Blocks for the ones you can't do with Fraction pieces.

9. Use Pattern Blocks.



Use your drawings to find the equivalent fraction. $\frac{2}{4} = \frac{3}{6}$

Draw the equivalent fraction here.

10. If the red block is equal to 1, what is the green block equal to? $\frac{1}{3}$
    What is the blue block equal to? $1\frac{1}{3}$ or $1\frac{2}{6}$
    What is the yellow block equal to? 2

1

Pre-Assessment - Part 1 - Worksheet 2

11. Fill in this sequence. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$
12. If 25¢ is a third of what you need to buy a ribbon, how much the does the ribbon cost? 75¢
13. Solve.
    a. $\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ b. $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$ c. $\frac{6}{10}+\frac{2}{10}=\frac{8}{10}$
    d. $\frac{7}{8}-\frac{1}{8}=\frac{6}{8}$ e. $\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$ f. $\frac{5}{5}-\frac{1}{5}=\frac{4}{5}$

Do not use Fraction pieces or Pattern Blocks for this set of problems.

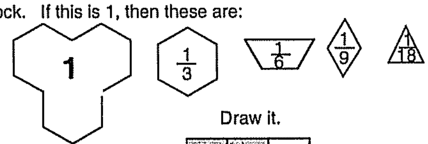
14 Write an improper fraction. $\frac{9}{4}$, $\frac{18}{5}$, AWV

15. Change these mixed numbers to improper fractions.
    a. $2\frac{3}{4} = \frac{11}{4}$ b. $4\frac{1}{6} = \frac{25}{6}$
16. Change these improper fractions to mixed numbers.
    a. $\frac{11}{3} = 3\frac{2}{3}$ b. $\frac{8}{5} = 1\frac{3}{5}$ c. $\frac{10}{4} = 2\frac{2}{4}$
17. If you couldn't do problems 15 and 16 without blocks or pictures, can you do them with the blocks? AWV If yes, do them, if no, put a question mark.
18. Put $1\frac{3}{4}$ on this number line. 0 1 $1\frac{3}{4}$ 2 3
19. Put $\frac{5}{3}$ on this number line. 0 1 $\frac{5}{3}$ 2 3

2

Pre-Assessment - Part 2

1. Identify each block. If this is 1, then these are:



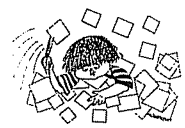
Draw it.

2. Change $\frac{2}{3}$ into twelfths. a. b. $\frac{2}{3} = \frac{8}{12}$
   Change $\frac{3}{4}$ into sixteenths. c. d. $\frac{3}{4} = \frac{12}{16}$
3. Fill in the missing numbers.
   a. b. c. d. e. f.
   $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$ $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$ $\frac{5}{8} \times \frac{3}{3} = \frac{15}{24}$
   g. How do you know what this number is? Divide the 3 into the 9 to find the multiplier. Then top and bottom numbers are the same.
4. Fill in the missing numbers.
   a. b. c. d.
   $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$
5. a. Explain what the "Mighty One" is. It is the fractional equivalent to one that changes a number into an easier, useful form.
   b. Explain how the "Mighty One" is used to calculate equivalent fractions. Any number can be multiplied by 1 in fractional form to create an equivanlent fraction.
6. $\frac{1}{3} = \frac{2}{6}$

Explain why multiplying $\frac{1}{3}$ by $\frac{2}{2}$ changes it to two sixths. $\frac{2}{2}$ is equal to 1, so the value doesn't change. Since it takes two-sixths to make a third, multiplying by 2 doubles the number of pieces. Multiplying by $\frac{2}{2}$ doubles the number of pieces but halves the size.

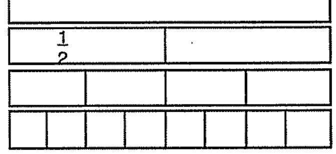
3

Relative Sizes: A Cutting Activity - Worksheet 1



Cut strips of paper 3 inches by 8 inches.
If I cut one strip into halves, I have 2 pieces.
Draw a picture of each strip beginning with one whole, or $\frac{1}{1}$
Take another strip. Cut it in halves, then cut the halves in two. I have 4 pieces, or $\frac{4}{4}$

Take the last strip, cut it just like the one before, then cut each piece in two more equal pieces. Altogether I have 8 pieces, or $\frac{8}{8}$.



Record the denominators. Count the number of pieces in each whole.

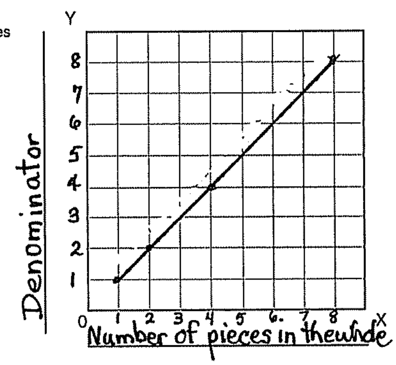
Record the titles and the numbers on this graph. Graph the relationship between the denominator and the number of pieces in each whole.

Fill in this T chart.

Denominator X	Number of pieces in the whole Y
1	1
2	2
4	4
8	8

Predict more in this pattern.

16	16
72	72



4

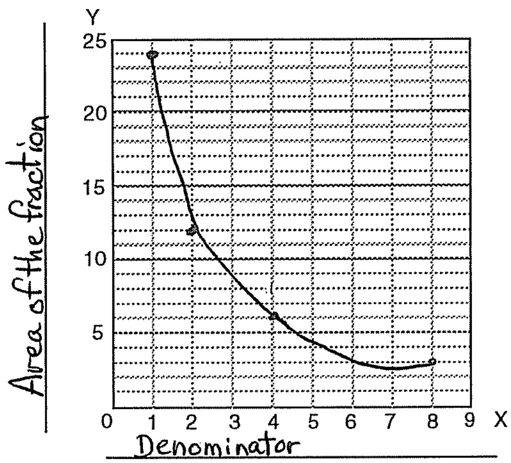
Relative Sizes: A Cutting Activity - Worksheet 2

Measure the area of each 3 inch by 8 inch piece.

Area of the whole = 24 square inches.
Area of the one-half = 12 square inches.
Area of the one-fourth = 6 square inches.
Area of the one-eighth = 3 square inches.

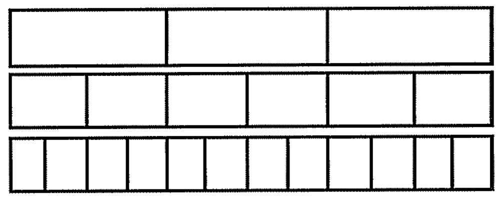
1. Record the titles and the numbers on this graph.
2. Graph the areas.

Denominator X	Area of the fraction in square inches Y
1	24
2	12
4	6
8	3

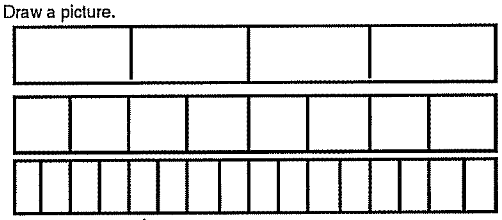


Relative Sizes: A Cutting Activity - Worksheet 3

If I cut my strip into thirds, I have 3 pieces.
Then I cut each third into two equal pieces.
All together I have 6 pieces.
Then I cut each new piece into two equal pieces.
All together I have 12 pieces.
Draw a picture.



If I cut my strip into fourths, I have 4 pieces.
Then I cut each fourth into two equal pieces. All together I have 8 pieces.
Then I cut each new piece into two equal pieces. All together I have 16 pieces.
Draw a picture.



Describe the relationship.
The fewer the pieces, the larger the size.
The smaller the pieces, the more the number of pieces.
Make your own relationship.

____

Equivalence: Manipulative - Review Worksheet 1

$1 = \frac{2}{2}$ $1 = \frac{3}{3}$ $1 = \frac{4}{4}$ $1 = \frac{5}{5}$ $1 = \frac{6}{6}$

$1 = \frac{8}{8}$ $1 = \frac{12}{12}$ $\frac{1}{2} = \frac{2}{4}$ $\frac{2}{2} = \frac{4}{4}$ $\frac{1}{2} = \frac{4}{8}$

$\frac{2}{2} = \frac{8}{8}$ $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{2} = \frac{6}{6}$ $\frac{1}{2} = \frac{6}{12}$ $\frac{2}{2} = \frac{12}{12}$

$\frac{1}{3} = \frac{2}{6}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{2}{3} = \frac{4}{6}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{3}{3} = \frac{12}{12}$

$\frac{1}{4} = \frac{2}{8}$ $\frac{2}{4} = \frac{4}{8}$ $\frac{3}{4} = \frac{6}{8}$ $\frac{4}{4} = \frac{8}{8}$ $\frac{5}{4} = \frac{10}{8}$

$\frac{1}{4} = \frac{3}{12}$ $\frac{2}{4} = \frac{6}{12}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{4}{4} = \frac{12}{12}$ $\frac{5}{4} = \frac{15}{12}$

Equivalence: Manipulative - Review Worksheet 2

$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6}$

$2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \frac{12}{6}$

$3 = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = \frac{15}{5} = \frac{18}{6}$

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12}$

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$

$\frac{2}{4} = \frac{4}{8} = \frac{6}{12}$ $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$ $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$

$\frac{4}{4} = \frac{8}{8} = \frac{12}{12}$ $\frac{3}{3} = \frac{6}{6} = \frac{12}{12}$

Changing Wholes Meet Manipulative Equivalence Worksheet 1

This is equal to one.

Example:

Write the names of fractions.

Place Cuisenaire Rods on top. Build all of the even fractions. Build one-color trains.

$\frac{\text{numerator}}{\text{denominator}}$

Fraction name	Train
halves	purple
fourths	red
eighths	tan

ESTIMATE

Build and record the following equations:

$1 = \frac{2}{2}$

$1 = \frac{4}{4}$ $\quad 1 = \frac{8}{8}$

$\frac{1}{2} = \frac{2}{4}$ $\quad \frac{1}{4} = \frac{2}{8}$

$\frac{2}{2} = \frac{4}{4}$ $\quad \frac{2}{4} = \frac{4}{8}$

$\frac{1}{2} = \frac{4}{8}$ $\quad \frac{3}{4} = \frac{6}{8}$

$\frac{2}{2} = \frac{8}{8}$ $\quad \frac{4}{4} = \frac{8}{8}$

9

Changing Wholes Meet Manipulative Equivalence Worksheet 2

This is equal to one.

Write the names of the fractions.

Place Cuisenaire Rods on top. Build all of the even fractions. Build one-color trains.

halves

thirds

sixths

ESTIMATE

Build and record the following equations:

$1 = \frac{2}{2}$ $\quad \frac{1}{2} = \frac{3}{6}$ $\quad \frac{1}{3} = \frac{2}{6}$

$1 = \frac{3}{3}$ $\quad \frac{2}{2} = \frac{6}{6}$ $\quad \frac{2}{3} = \frac{4}{6}$

$1 = \frac{6}{6}$ $\quad \frac{3}{3} = \frac{6}{6}$

$\frac{1}{2} = \frac{3}{\boxed{6}}$ $\quad \frac{2}{2} = \frac{6}{\boxed{6}}$ $\quad 1 = \frac{2}{\boxed{2}}$ $\quad 1 = \frac{3}{\boxed{3}}$

$\frac{0}{2} = \frac{\boxed{0}}{6}$ $\quad 1 = \frac{\boxed{6}}{6}$ $\quad 2 = \frac{\boxed{6}}{3}$ $\quad 2 = \frac{\boxed{12}}{6}$

10

Changing Wholes Meet Manipulative Equivalence Worksheet 3

This is equal to one.

Write the names of the fractions.

Place Cuisenaire Rods on top. Build all of the even fractions. Build one-color trains.

thirds

ninths

$\frac{\text{numerator}}{\text{denominator}} = \frac{1}{3}$

ESTIMATE

Build and record the following equations:

$1 = \frac{3}{3}$ $\quad \frac{1}{3} = \frac{3}{9}$ $\quad \frac{3}{9} = \frac{1}{3}$

$1 = \frac{9}{9}$ $\quad \frac{2}{3} = \frac{6}{9}$ $\quad \frac{6}{9} = \frac{2}{3}$

$2 = \frac{6}{3}$ $\quad \frac{3}{3} = \frac{9}{9}$ $\quad \frac{0}{9} = \frac{0}{3}$

$1 = \frac{3}{\boxed{3}}$ $\quad 1 = \frac{9}{\boxed{9}}$

11

Changing Wholes Meets Manipulative Equivalence Worksheet 4

This is equal to one.

Write the names of the fractions.

Place Cuisenaire Rods on top. Build all of the even fractions. Build one-color trains.

halves

sixths

thirds

fourths

twelfths

Build and record the following equations:

ESTIMATE

$1 = \frac{2}{2}$ $\quad \frac{1}{2} = \frac{1}{2}$ $\quad \frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

$1 = \frac{3}{3}$ $\quad \frac{2}{3} = \frac{4}{6} = \frac{8}{12}$

$1 = \frac{4}{4}$ $\quad \frac{1}{2} = \frac{2}{4}$ $\quad \frac{3}{3} = \frac{6}{6} = \frac{12}{12}$

$1 = \frac{6}{6}$ $\quad \frac{1}{2} = \frac{3}{6}$

$1 = \frac{12}{12}$ $\quad \frac{1}{2} = \frac{6}{12}$ $\quad \frac{1}{4} = \frac{3}{12}$

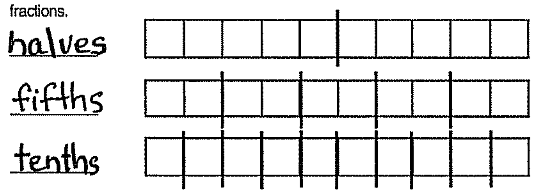
12

Changing Wholes Meet Manipulative Equivalence Assessment - Worksheet 5

This is equal to one.

Write the names of the fractions.

Place Culsenaire Rods on top. Build all of the even fractions. Build one-color trains.



Build and record all the equivalent equations.

$1=\frac{5}{5}$	$\frac{1}{2}=\frac{5}{10}$	$\frac{1}{5}=\frac{2}{10}$
$1=\frac{2}{2}$	$\frac{5}{5}=\frac{10}{10}$	$\frac{2}{5}=\frac{4}{10}$
$1=\frac{10}{10}$	$\frac{4}{5}=\frac{8}{10}$	$\frac{3}{5}=\frac{6}{10}$

The top number of a fraction is the numerator.

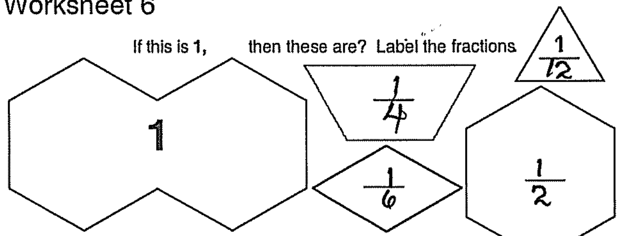
The bottom number of a fraction is the denominator.

Define numerator. The term of a fraction written above the line to indicate the number of parts of the whole.

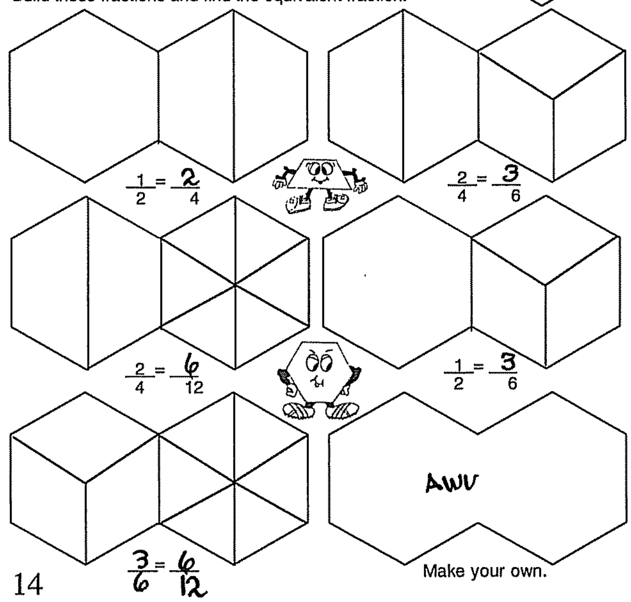
Define denominator. Written below the line. It indicates the number of parts into which the whole is divided. The bottom number of a fraction.

Changing Wholes Meet Manipulative Equivalence Worksheet 6

If this is 1, then these are? Label the fractions

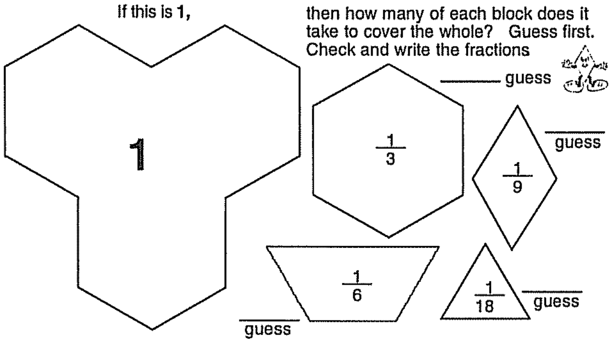


Build these fractions and find the equivalent fraction.

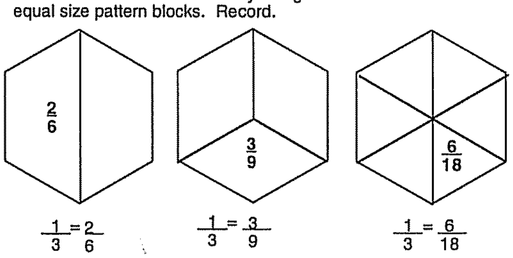


Changing Wholes Meet Manipulative Equivalence Worksheet 7

If this is 1, then how many of each block does it take to cover the whole? Guess first. Check and write the fractions



Cover each third a different way using equal size pattern blocks. Record.



$\frac{1}{3}=\frac{2}{6}$ $\frac{1}{3}=\frac{3}{9}$ $\frac{1}{3}=\frac{6}{18}$

Challenge! Another way to record.

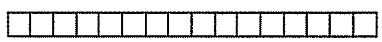
$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{6}{18}$

Graph Paper Fractions - Worksheet 1

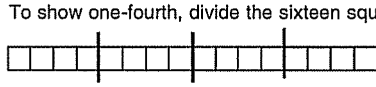
Use graph paper.

Sixteenths $\frac{1}{16}$

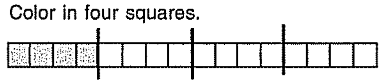
First draw a box around sixteen squares.



To show one-fourth, divide the sixteen squares into four equal parts.



Color in four squares.



$\frac{1}{4}=\frac{4}{16}$

Now show one-half.

$\frac{1}{2}=\frac{8}{16}$

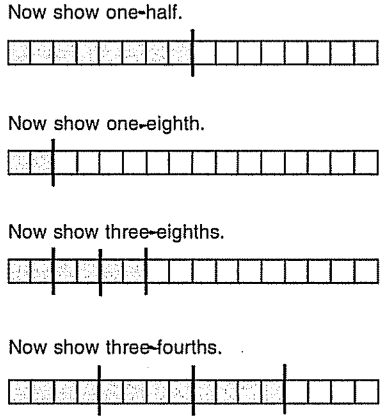
Now show one-eighth.

$\frac{1}{8}=\frac{2}{16}$

Now show three-eighths.

$\frac{3}{8}=\frac{6}{16}$

Now show three-fourths.



$\frac{3}{4}=\frac{12}{16}$

Circle the equivalent fraction with fewest number of pieces.

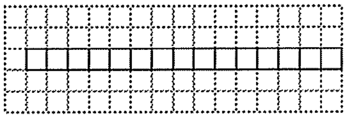
Graph Paper Fractions - Worksheet 2

Graph Paper Equivalence

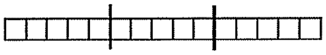
Use graph paper.

This is the problem: $\frac{1}{3} = \frac{}{15}$

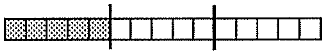
Create a whole that contains thirds and fifteenths.



Show thirds.

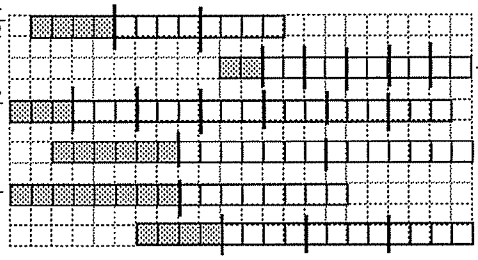


Color in one third of fifteen.
How many fifteenths are colored? 5



$\frac{1}{3} = \frac{5}{15}$

Use graph paper to solve these problems.



$\frac{1}{3} = \frac{4}{12}$ $\frac{1}{6} = \frac{2}{12}$

$\frac{1}{7} = \frac{3}{21}$ $\frac{1}{3} = \frac{7}{21}$

$\frac{1}{2} = \frac{8}{16}$ $\frac{1}{4} = \frac{4}{16}$

What pattern is there between equal fractions? ____________________

Equivalence: Recording - Worksheet 1

Cross out the impossible ones and look for patterns. Use fraction pieces or cut up of paper.

MAKE / WITH	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
halves	$1 = \frac{2}{2}$	$\frac{1}{2} = \frac{1}{2}$	~~$\frac{1}{3} =$~~	~~$\frac{1}{4} =$~~
thirds	$1 = \frac{3}{3}$	~~2 = 3~~	$\frac{1}{3} = \frac{1}{3}$	~~$\frac{1}{4} =$~~
fourths	$1 = \frac{4}{4}$	$\frac{1}{2} = \frac{2}{4}$	~~$\frac{1}{3} =$~~	$\frac{1}{4} = \frac{1}{4}$
fifths	$1 = \frac{5}{5}$	~~1 =~~	~~1 =~~	~~1 =~~
sixths	$1 = \frac{6}{6}$	$\frac{1}{2} = \frac{3}{6}$	$\frac{1}{3} = \frac{2}{6}$	~~=~~
sevenths	$1 = \frac{7}{7}$	~~1 =~~	~~1 =~~	~~1 =~~
eighths	$1 = \frac{8}{8}$	$\frac{1}{2} = \frac{4}{8}$	~~1 =~~	$\frac{1}{4} = \frac{2}{8}$
ninths	$1 = \frac{9}{9}$	~~1 =~~	$\frac{1}{3} = \frac{3}{9}$	~~1 =~~
tenths	$1 = \frac{10}{10}$	$\frac{1}{2} = \frac{5}{10}$	~~1 =~~	~~1 =~~
elevenths	$1 = \frac{11}{11}$	~~1 =~~	~~1 =~~	~~1 =~~
twelfths	$1 = \frac{12}{12}$	$\frac{1}{2} = \frac{6}{12}$	$\frac{1}{3} = \frac{4}{12}$	$\frac{1}{4} = \frac{3}{12}$

Equivalence: Recording - Worksheet 2

$\frac{1}{5}$	$\frac{1}{6}$
~~$\frac{1}{5} =$~~	~~$\frac{1}{6} =$~~
~~$\frac{1}{5} = \frac{1}{3}$~~	~~$\frac{1}{6} =$~~
~~$\frac{1}{5} =$~~	~~$\frac{1}{6} =$~~
$\frac{1}{5} = \frac{1}{5}$	~~=~~
~~=~~	$\frac{1}{6} = \frac{1}{6}$
~~1 =~~	~~1 =~~
~~1 =~~	~~1 =~~
~~1 =~~	~~1 =~~
$\frac{1}{5} = \frac{2}{10}$	~~1 =~~
~~1 =~~	~~1 =~~
~~1 =~~	$\frac{1}{6} = \frac{2}{12}$

You can make $\frac{1}{2}$ with eighths but you can't make $\frac{1}{8}$ with a half.

Why not?
1/2 is larger than 1/8

List all the fractions equal to $\frac{1}{2}$ in order going down the page.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

What patterns do you notice?
Top number goes up by ones.
bottom number goes up by twos

List all the fractions equal to $\frac{1}{3}$ in order going down the page.

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$

What patterns do you notice?
Bottom number goes up by threes

List all the fractions equal to $\frac{1}{4}$ in order going down the page.

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24}$

What patterns do you notice?
Bottom number goes up by fours.

How are all three of these patterns alike?
Same pattern in the top numbers.

How are they different?
Different pattern in bottom numbers. different multiples of the denominators

Equivalence: Recording - Worksheet 3

Use the information on the chart to find new patterns.

This collection of fractions is from the $\frac{1}{3}$ column. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

They are all equivalent fractions.

What patterns do you notice?
In the numerator, I see go up by one number.
In the denominator, I see go up by multiples of three.

Use the fraction pieces and patterns to create a list of fractions equivalent to $\frac{2}{3}$. Use patterns to fill in blank spaces.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$

Collect all the fractions equal to $\frac{1}{4}$

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} =$

Use the fraction pieces and patterns to create a list of fractions equivalent to $\frac{3}{4}$. Use patterns to fill in blank spaces.

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}$

Use the chart, patterns, and Fraction pieces to fill in blank spaces.

$\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30}$

$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$

$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30}$

$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{15}{20} = \frac{20}{25} = \frac{24}{30}$

Equivalence: Recording - Worksheet 4

What do you notice in the set of fifths? ____________________

____

Use what you have learned to fill in the blanks.

$\frac{3}{5} = \frac{9}{15}$ $\frac{4}{5} = \frac{16}{20}$ $\frac{2}{5} = \frac{12}{30}$

Use the chart, patterns, and Fraction pieces to fill in blank spaces.

$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36}$

$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36}$

Use only patterns to fill in blank spaces.

$\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48}$

How does the pattern above connect to the multiplication tables?

Each denominator is a multiple of eight.

Solve this series.

$\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32} = \frac{25}{40} = \frac{30}{48}$

Equivalence: Recording - Worksheet 5

An interesting chart can be made with each fraction for all whole numbers up to ten. Fill in the blanks.

	halves	thirds	fourths	fifths	sixths	eighths	tenths	twefths
1 =	2/2 =	3/3 =	4/4 =	5/5 =	6/6 =	8/8 =	10/10 =	12/12
2 =	4/2 =	6/3 =	8/4 =	10/5 =	12/6 =	16/8 =	20/10 =	24/12
3 =	6/2 =	9/3 =	12/4 =	15/5 =	18/6 =	24/8 =	30/10 =	36/12
4 =	8/2 =	12/3 =	16/4 =	20/5 =	24/6 =	32/8 =	40/10 =	48/12
5 =	10/2 =	15/3 =	20/4 =	25/5 =	30/6 =	40/8 =	50/10 =	60/12
6 =	12/2 =	18/3 =	24/4 =	30/5 =	36/6 =	48/8 =	60/10 =	72/12
8 =	16/2 =	24/3 =	32/4 =	40/5 =	48/6 =	64/8 =	80/10 =	96/12
9 =	18/2 =	27/3 =	36/4 =	45/5 =	54/6 =	72/8 =	90/10 =	108/12
10 =	20/2 =	30/3 =	40/4 =	50/5 =	60/6 =	80/8 =	100/10 =	120/12

Equivalence: Recording - Assessment Worksheet 6

Try to solve these from memory or draw a quick sketch. Use fraction pieces only if you really need them.

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{4} = \frac{2}{8}$

$\frac{2}{4} = \frac{4}{8}$

$\frac{1}{8} = \frac{2}{16}$

$\frac{3}{8} = \frac{6}{16}$

$\frac{1}{4} = \frac{4}{16}$

$\frac{1}{3} = \frac{2}{6}$

$\frac{2}{3} = \frac{4}{6}$

$\frac{1}{6} = \frac{2}{12}$

$\frac{5}{6} = \frac{10}{12}$

Challenge! Can you use a pattern to solve these?

$\frac{1}{12} = \frac{2}{24}$

$\frac{3}{12} = \frac{6}{24}$

$\frac{1}{4} = \frac{6}{24}$

Equivalence: Recording - Assessment Worksheet 7

Record all the equivalent fractions.
Use patterns to help you.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14}$

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{8}{24}$

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21}$

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28}$

$\frac{2}{4} = \frac{4}{8} = \frac{6}{12} = \frac{8}{16} = \frac{10}{20} = \frac{12}{24} = \frac{14}{28}$

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28}$

$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36} = \frac{7}{42}$

$\frac{2}{6} = \frac{4}{12} = \frac{6}{18} = \frac{8}{24} = \frac{10}{30} = \frac{12}{36} = \frac{14}{42}$

$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \frac{35}{42}$

$\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48} = \frac{7}{56}$

$\frac{2}{8} = \frac{4}{16} = \frac{6}{24} = \frac{8}{32} = \frac{10}{40} = \frac{12}{48} = \frac{14}{56}$

Equivalence: Recording - Worksheet 8

Think of a rule to help you find an easier way. What operation can you do to both the denominator and the numerator to solve these without a manipulative? Describe your method. Record all the equivalent fractions using this method.

Multiply both numerator and the denominator by the same number, e.g. $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

$\frac{1}{2} = \frac{2}{4}$ $\quad \frac{1}{2} = \frac{3}{6}$ $\quad \frac{1}{4} = \frac{2}{8}$ $\quad \frac{1}{4} = \frac{4}{16}$

$\frac{1}{3} = \frac{2}{6}$ $\quad \frac{2}{3} = \frac{4}{6}$ $\quad \frac{1}{4} = \frac{3}{12}$ $\quad \frac{1}{4} = \frac{2}{8}$

$\frac{3}{4} = \frac{9}{12}$ $\quad \frac{1}{4} = \frac{2}{8}$ $\quad \frac{1}{2} = \frac{3}{6}$ $\quad \frac{3}{12} = \frac{6}{24}$

Make your own. AWV

$\frac{2}{4} = \frac{4}{8}$ $\quad$ ___ = ___ $\quad$ ___ = ___

Equivalence: Representational - Worksheet 1

Equivalent fractions are also used when making change with money.

For example, you want to buy a drink for 75¢ from a vending machine. All you have is a one dollar bill. What do you do? You get change of course.

What is the easiest coin to trade for? quarters

How many quarters do you get for one dollar? 4

$1.00 = 25¢ + 25¢ + 25¢ + 25¢

In fractions, this trade would look like this: $1 = \frac{4}{4}$

How many nickels are there in $1.00? 20

In fractions, that trade would look like this: $1 = \frac{20}{20}$

We can also have fractions which are equal to each other. How many nickels are equal to one quarter? 5 nickels

In fractions that trade would look like this: $\frac{1}{4} = \frac{5}{20}$
quarter equals nickles

What would the equivalent fraction look like if two quarters were changed into nickels? $\frac{2}{4} = \frac{10}{20}$ How many nickels are equal to 2 quarters? 10

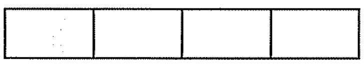
How would the equivalent fraction look if you wanted to change one quarter into pennies $\frac{1}{4} = \frac{25}{100}$

In many arithmetic problems making "change" with fractions is often needed. It's time to make sure you can do this without pieces or pictures. You need to understand the patterns that make this

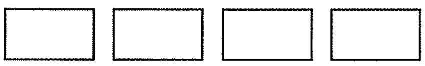
Equivalence: Representational - Worksheet 2

How to change fourths into eighths.

Take a strip of paper and fold it into fourths.

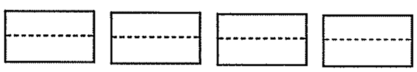


Now cut the fourths apart.

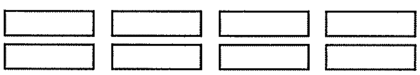


In order to change these four equal pieces into eight equal pieces, each of the pieces need to be cut into two smaller equal sections.

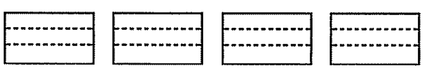
Use scissors to cut each fourth into eighths. Cut horizontally as shown.



The fourths have been changed to eighths.



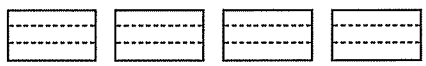
Repeat again. This time change fourths into twelfths. Cut the fourths horizontally as was done above. Draw the results.



Equivalence: Representational - Worksheet 3

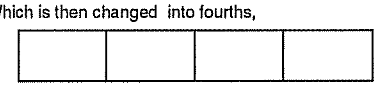
Equivalence: - Worksheet

When the fourths were changed to twelfths, each fourth was cut into three sections. It came out looking like this:

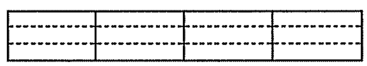


This is the whole rectangle.

Which is then changed into fourths,

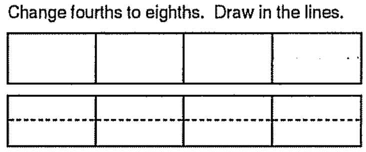


and then changed into twelfths by cutting each fourth into three smaller



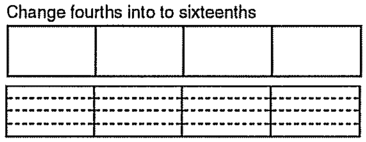
Each fourth is cut into 3 smaller parts because 4 x 3 = 12.

Change fourths to eighths. Draw in the lines.



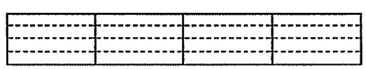
Cut each larger piece into 2 smaller parts becaus 4 x 2 = 8.

Change fourths into to sixteenths



Cut each larger piece into 4 smaller parts because 4 x 4 = 16.

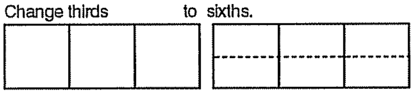
You should have cut each fourth into four smaller equal parts because 4 x 4 = 16.



Equivalence: Representational - Worksheet 4

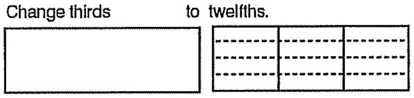
Now you draw all the lines.

Change thirds to sixths.



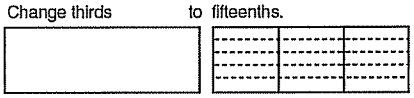
Cut each larger piece into 2 smaller parts because 3 x 2 = 6.

Change thirds to twelfths.



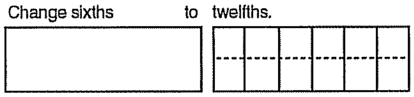
Cut each larger piece into 4 smaller parts because 3 x 4 = 12

Change thirds to fifteenths.



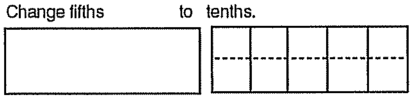
Cut each larger piece into 5 smaller parts because 3 x 5 = 15.

Change sixths to twelfths.



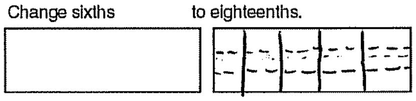
Cut each larger piece into 2 smaller parts because 6 x 2 = 12.

Change fifths to tenths.



Cut each larger piece into 2 smaller parts because 5 x 2 = 10

Change sixths to eighteenths.



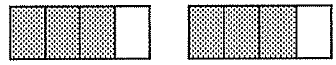
Cut each larger piece into 3 smaller parts because 6 x 3 = 18.

Equivalence: Representational - Worksheet 5

If $\frac{3}{4}$ are changed into twelfths, then 12 would be the new numerator.

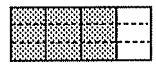
$$\frac{3}{4} = \frac{?}{12}$$

Be a mathematician. Use the drawings or a pattern to figure out the missing numbers.



$$\frac{3}{4} = \frac{9}{12}$$

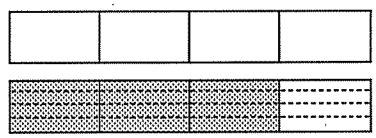
Draw the dotted lines to change fourths to twelfths.



Count how many of your twelfths are shaded in. How many? 9

Therefore $\frac{3}{4} = \frac{9}{12}$

Change three fourths into __ /16.

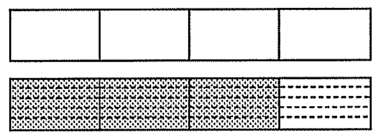


This number is called the multiplie.

Cut each larger piece into 4 smaller parts because 4 x 4 = 16

$$\frac{3}{4} = \frac{12}{16}$$

Change three fourths into __ /20.



Cut each larger piece into 5 smaller parts because 4 x 5 = 20

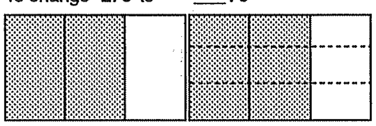
$$\frac{3}{4} = \frac{15}{20}$$

Equivalence: Representational - Worksheet 6

Example:

To change 2/3 to 6 /9

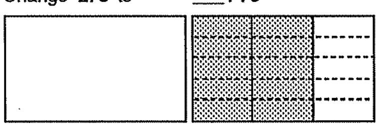
Circle the multiplier.



Cut each larger piece into (3) smaller parts because 3 x (3) = 9. 9

$$\frac{2}{3} = \frac{6}{9}$$

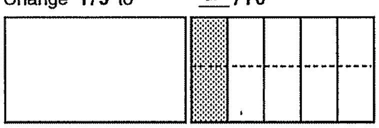
Change 2/3 to 10/15



Cut each larger piece into (5) smaller parts because 3 x (5) = 15.

$$\frac{2}{3} = \frac{10}{15}$$

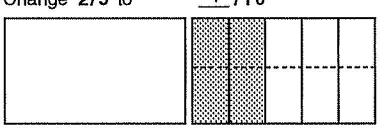
Change 1/5 to 2 /10



Cut each larger piece into (2) smaller parts because 5 x (2) = 10.

$$\frac{1}{5} = \frac{2}{10}$$

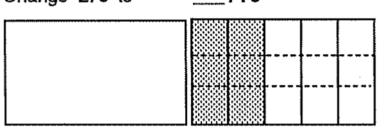
Change 2/5 to 4 /10



Cut each larger piece into (2) smaller parts because 5 x (2) = 10.

$$\frac{2}{5} = \frac{4}{10}$$

Change 2/5 to 6 /15

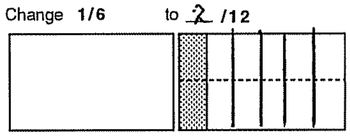


Cut each larger piece into (3) smaller parts because 5 x (3) = 15.

$$\frac{2}{5} = \frac{6}{15}$$

Equivalence: Representational - Worksheet 7

Change 1/6 to 2 /12

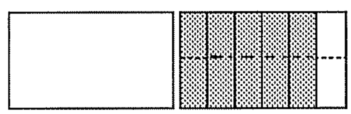


Cut each larger piece into 2 smaller parts because 6 x 2 = 12.

What is the multiplier?

$$\frac{1}{6} = \frac{2}{12}$$

Change 5/6 to 10 /12

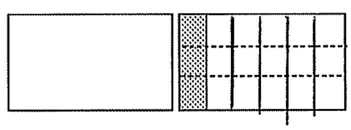


Cut each larger piece into 2 smaller parts because 6 x 2 = 12.

What is the multiplier?

$$\frac{5}{6} = \frac{10}{12}$$

Change 1/6 to 3 /18

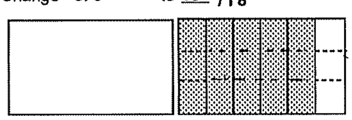


Cut each larger piece into 3 smaller parts because 6 x 3 = 18.

What is the multiplier?

$$\frac{1}{6} = \frac{3}{18}$$

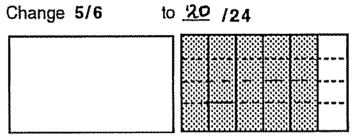
Change 5/6 to 15 /18



What is the multiplier? 3

$$\frac{5}{6} = \frac{15}{18}$$

Change 5/6 to 20 /24



What is the multiplier? 4

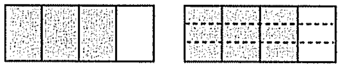
$$\frac{5}{6} = \frac{20}{24}$$

Equivalence: Representational - Worksheet 8

Use the drawing to figure out these equivalent fractions.

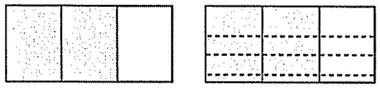
Example:

Change $\frac{3}{4}$ into $\frac{?}{12}$ — What is the multiplier? 3



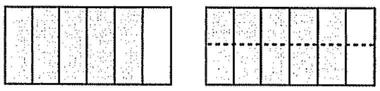
$\frac{3}{4} = \frac{9}{12}$

$\frac{2}{3}$ into $\frac{?}{12}$ — What is the multiplier? 4



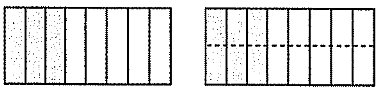
$\frac{2}{3} = \frac{8}{12}$

Change $\frac{5}{6}$ into $\frac{?}{12}$ — What is the multiplier? 2



$\frac{5}{6} = \frac{10}{12}$

Change $\frac{3}{8}$ into $\frac{?}{16}$ — What is the multiplier? 2

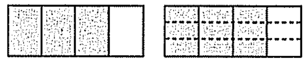


$\frac{3}{8} = \frac{6}{16}$

Equivalence: Calculating - Worksheet 1
The Power of 1

Now you can use what you know about cutting up fractions, drawing boxes and shading them, and your multiplication tables to calculate what ever equivalent fraction you want.

In the fraction shown below, you changed $\frac{3}{4}$ to $\frac{9}{12}$. You did this by choosing to cut into3. You chose 3 to be the multiplier because you know that 4 x 3 will give you 12.



Multiplier because

3   4 x 3 =12 $\frac{3}{4} = \frac{9}{12}$

The shaded sections showed you the numerators.

What did the 3 in the $\frac{3}{4}$ get multiplied by to make it the 9 in $\frac{9}{12}$? 3

Notice it is the same number as the multiplier

Check this out:. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Draw a box around the $\frac{3}{3}$. What does $\frac{3}{3}$ equal ? 1

What happens when you multiply by one? Example 5 x 1= 5, 243 x 1 = 243 or $\frac{1}{2}$ x 1 = $\frac{1}{2}$ The answer doesn't change

Remember the identity factor? Just silly math names? Not quite. Multiplying a fraction by one allows us to change the numerator and denominator into any equivalent fraction, just by choosing the way we want the fraction to look. This will allow you to add and subtract unlike fractions without using fraction pieces or drawings. Cool, don't you think?

The identy factor is one of the most important tools you will ever learn about in mathematics. It is used in algebra constantly.

Equivalence: Calculating - Worksheet 2
The Power of 1

John Saxon, the author of the Saxon math books, shows it this way.

$\frac{3 \times}{4 \times} \boxed{\frac{3}{3}} = \frac{9}{12}$ This is called the MIGHTY ONE!

This reminds you that even though it looks and feels like you are multiplying by three, you are in fact multiplying by one.

Now you do it. Put the multiplier in and draw in the Mighty One.

$\frac{2 \times}{3 \times} \frac{\_\_}{\_\_} = \frac{\_\_}{12}$

Solve these problems and draw the Mighty One on each one.

$\frac{2 \times}{5 \times} \frac{3}{3} = \frac{6}{15}$ $\frac{3 \times}{5 \times} \frac{3}{3} = \frac{9}{15}$

$\frac{1 \times}{5 \times} \frac{5}{5} = \frac{5}{25}$ $\frac{4 \times}{5 \times} \frac{5}{5} = \frac{20}{25}$

$\frac{1 \times}{8 \times} \frac{3}{3} = \frac{3}{24}$ $\frac{3 \times}{8 \times} \frac{3}{3} = \frac{9}{24}$

Equivalence: Calculating - Worksheet 3
The Power of 1

Solve these problems and draw the Mighty One on each one.

$\frac{1 \times}{3 \times} \frac{5}{5} = \frac{5}{15}$ $\frac{2 \times}{3 \times} \frac{5}{5} = \frac{\_\_}{15}$

$\frac{1 \times}{3 \times} \frac{6}{6} = \frac{6}{18}$ $\frac{2 \times}{3 \times} \frac{6}{6} = \frac{12}{18}$

$\frac{1 \times}{3 \times} \frac{7}{7} = \frac{7}{21}$ $\frac{2 \times}{3 \times} \frac{7}{7} = \frac{14}{21}$

$\frac{1 \times}{3 \times} \frac{10}{10} = \frac{10}{30}$ $\frac{2 \times}{3 \times} \frac{10}{10} = \frac{20}{30}$

Make your own. AWV

$\frac{\_\_ \times}{\_\_ \times} \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$ $\frac{\_\_ \times}{\_\_ \times} \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

$\frac{1 \times}{4 \times} \frac{2}{2} = \frac{2}{8}$ $\frac{1 \times}{4 \times} \frac{3}{3} = \frac{3}{12}$ $\frac{1 \times}{4 \times} \frac{4}{4} = \frac{4}{16}$

$\frac{1 \times}{4 \times} \frac{5}{5} = \frac{5}{20}$ $\frac{1 \times}{4 \times} \frac{6}{6} = \frac{6}{24}$

Make your own. AWV

$\frac{\_\_ \times}{\_\_ \times} \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

Equivalence: Calculating - Worksheet 4

Solve by making equivalent fractions. This time the numerators are given and denominators are missing.

$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$	$\frac{1 \times 10}{2 \times 10} = \frac{10}{\ }$	$\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$
$\frac{1 \times 3}{7 \times 3} = \frac{3}{21}$	$\frac{2 \times 4}{5 \times 4} = \frac{8}{\ }$	$\frac{6 \times 6}{6 \times 6} = \frac{36}{36}$
$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$	$\frac{3 \times 3}{10 \times 3} = \frac{9}{30}$	$\frac{5 \times 10}{5 \times 10} = \frac{50}{50}$
$\frac{1 \times 15}{2 \times 15} = \frac{15}{30}$	$\frac{1 \times 6}{6 \times 6} = \frac{6}{36}$	$\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$
$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$	$\frac{2 \times 5}{9 \times 5} = \frac{10}{45}$	$\frac{3 \times 4}{8 \times 4} = \frac{12}{32}$
$\frac{1 \times 5}{3 \times 5} = \frac{5}{15}$	$\frac{5 \times 5}{7 \times 5} = \frac{25}{35}$	$\frac{9 \times 4}{9 \times 4} = \frac{36}{36}$
$\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$	$\frac{8 \times 2}{9 \times 2} = \frac{16}{18}$	$\frac{1 \times 25}{2 \times 25} = \frac{25}{50}$

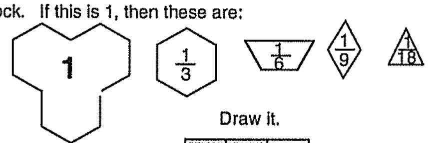
Equivalence: Calculating - Worksheet 5

Find the missing numerators and denominators.

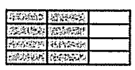
$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$	$\frac{1 \times 15}{2 \times 15} = \frac{15}{30}$	$\frac{3 \times 8}{4 \times 8} = \frac{24}{32}$
$\frac{1 \times 3}{7 \times 3} = \frac{3}{21}$	$\frac{2 \times 7}{5 \times 7} = \frac{14}{35}$	$\frac{3 \times 7}{6 \times 7} = \frac{21}{42}$
$\frac{5 \times 6}{8 \times 6} = \frac{30}{48}$	$\frac{3 \times 4}{10 \times 4} = \frac{12}{40}$	$\frac{5 \times 7}{5 \times 7} = \frac{35}{35}$
$\frac{1 \times 12}{2 \times 12} = \frac{12}{24}$	$\frac{1 \times 9}{6 \times 9} = \frac{9}{54}$	$\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$
$\frac{3 \times 6}{5 \times 6} = \frac{18}{30}$	$\frac{2 \times 12}{9 \times 12} = \frac{24}{108}$	$\frac{3 \times 7}{8 \times 7} = \frac{21}{56}$
$\frac{1 \times 9}{3 \times 9} = \frac{9}{27}$	$\frac{5 \times 6}{7 \times 6} = \frac{30}{42}$	$\frac{9 \times 8}{9 \times 8} = \frac{72}{72}$
$\frac{3 \times 11}{5 \times 11} = \frac{33}{55}$	$\frac{8 \times 4}{9 \times 4} = \frac{32}{36}$	$\frac{1 \times 24}{2 \times 24} = \frac{24}{48}$
$\frac{5 \times 8}{8 \times 8} = \frac{40}{64}$	$\frac{5 \times 6}{8 \times 6} = \frac{30}{48}$	$\frac{3 \times 8}{4 \times 8} = \frac{24}{32}$

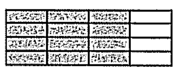
Post-Assessment

1. Identify each block. If this is 1, then these are:



Draw it.

2. Change $\frac{2}{3}$ into twelfths. a.  b. $\frac{2}{3} = \frac{8}{12}$

Change $\frac{3}{4}$ into sixteenths. c.  d. $\frac{3}{4} = \frac{12}{16}$

3. Fill in the missing numbers.

a. b. c. d. e. f.

$\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$ $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$ $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$

g. How do you know what this number is? Divide the 3 into the 9 to find the multiplier. Then top and bottom numbers are the same.

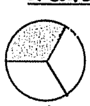
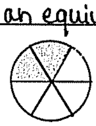
4. Fill in the missing numbers.

a. b. c. d.

$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

5. a. Explain what the "Mighty One" is. It is the fractional equivalent to one that changes a number into an easier, more useful form.

b. Explain how the "Mighty One" is used to calculate equivalent fractions. Any number can be multiplied by 1 in fractional form to create an equivalent fraction.

6.  

$\frac{1}{3} = \frac{2}{6}$

Explain why multiplying $\frac{1}{3}$ by $\frac{2}{2}$ changes it to two sixths. $\frac{2}{2}$ is equal to 1, so the value doesn't change. Since it takes two-sixths to make a third, multiplying by 2 doubles the number of pieces! Multiplying by $\frac{2}{2}$ doubles the number of pieces but halves the size.

Patterns in Arithmetic: Fractions - Booklet 4 PDF
Parent/Teacher Guide
Equivalent Fractions

ISBN 978-1-935559-63-4

ISBN 978-1-935559-63-4

9 781935 559634